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Heterogeneity in Price Stickiness and the New Keynesian Phillips Curve

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Abstract

Standard sticky price models do not account for heterogeneity in firms' price setting behavior. However, there is ample evidence that firms differ substantially in the frequency of price adjustments (e.g. Bils and Klenow, 2004 and Dhyne et al., 2005). In this paper I derive a generalized new Keynesian Phillips curve that accounts for heterogeneity in price stickiness across sectors. It differs from the standard one in two important ways: first, the coefficient on the output gap now depends on the distribution of frequencies of price adjustment, and second, it features a new, endogenous shift term that is proportional to a weighted average of the sectoral relative prices. Despite inflation being more sensitive to the output gap in heterogeneous economies than in comparable identical firms economies, the process of adjustment to nominal shocks tends to be more sluggish in the former, owing to the endogenous shift term associated with the distribution of sectoral relative prices. In the presence of strategic complementarities in price setting, the decisions of firms with higher adjustment frequencies are influenced by the existence of firms with lower frequencies, which end up having a disproportionate effect on the aggregate price level. I calibrate the model using an empirical distribution of frequencies of price changes in the U.S. economy derived from Bils and Klenow (2004), and find that reproducing the dynamics of a truly heterogeneous economy with a model based on identical firms usually requires larger degrees of nominal rigidity than the average found in the data.

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1 Introduction

Standard models of sticky prices usually do not account for heterogeneity in firms' price setting behavior. In particular, models based on the seminal work of Calvo (1983), which gives rise to the new Keynesian Phillips curve (NKPC) and its variants, and Taylor (1979, 1980) typically assume that all firms change prices with the same frequency. However, there is ample evidence that firms differ substantially in the frequency of price adjustments (see Blinder et al., 1998 and Bils and Klenow, 2004 for the U.S. economy; Dhyne et al., 2005, and references cited therein for the Euro area). So, apart from analytical convenience, the only reason not to take heterogeneity explicitly into account would be if it did not matter qualitatively in aggregate terms, or at least not quantitatively.

In this paper I argue that this is not the case by introducing heterogeneity into otherwise standard sticky price models, and comparing them with identical firms counterparts under different calibrations. I assume that firms are divided into sectors which differ in the frequency of price adjustment, and study both economies with Calvo and Taylor pricing. I present qualitative and quantitative results that help understand the effects of heterogeneity in price stickiness for the dynamic properties of these models, and which show that heterogeneity leads monetary shocks to have larger and more persistent real effects. I focus my exposition on the model with Calvo pricing, and present all the derivations for the model with Taylor pricing in the Appendix. Throughout the paper I indicate whenever the results for the two models differ.

I start by deriving a generalized NKPC in continuous time that accounts for heterogeneity in price stickiness, in section (2). It differs from the standard NKPC in two important ways: first, the coefficient on the output gap now depends on the first two moments of the distribution of frequencies of price adjustment and second, it features a new, endogenous shift term that is proportional to a weighted average of the sectoral relative prices, where the weights are adjustment-frequency-based transformations of the sectoral weights.

In general, the dynamics of a heterogeneous economy depend on the whole distribution of adjustment frequencies. In section (3), for arbitrary distributions, I characterize the steady state and the equations which determine the impulse responses of endogenous variables to AR(1) shocks to the level and growth rate of nominal aggregate demand.

I study the implications of heterogeneity for the real effects of monetary shocks in section (4). I compare the dynamic response of heterogeneous economies to nominal shocks to those of identical firms economies under alternative calibrations. The first result, based on the generalized NKPC, is that heterogeneous economies display a steeper Phillips curve relative to two benchmark identical firms economies: one with the same average frequency of price changes and the other with the same average duration of price rigidity. However, owing to the endogenous shift term in the generalized NKPC associated with the distribution of sectoral relative prices, the coefficient on the output gap no longer summarizes the degree of stickiness in the economy. In fact, it turns out to be a misleading indicator of the degree of nominal rigidity. Despite inflation being more sensitive to the output gap in heterogeneous economies, i.e. the Phillips curve being steeper, in general the adjustment process in the latter need not be faster (nor slower) than in comparable identical firms economies. The reason is that, right after a shock hits, heterogeneous economies tend to display faster adjustment owing to a relatively higher measure of firms with higher adjustment frequencies, which get to change prices earlier. However, as time passes, the distribution of the frequency of price changes among firms which have not yet adjusted becomes progressively more dominated by firms with relatively lower adjustment rates, and the process becomes more sluggish.

For the case of very persistent level and growth rate shocks, I am able to derive analytically the implications of heterogeneity for the cumulative real effects of monetary shocks for arbitrary distributions. Albeit particular, these results turn out to illustrate qualitatively the role of heterogeneity in the more general cases. For the latter, where the whole distribution of adjustment frequencies matters, I use the statistics reported recently by Bils and Klenow (2004) (henceforth BK) for the U.S. economy to calibrate the model. More specifically, I identify each sector in the model with one of the goods and services categories analyzed in their sample. Accordingly, I set the sectoral weights equal to the weights for these categories, which are also reported in their paper. This results in 350 sectors, with an "average frequency based" duration of price spells of 3.3 months, an average duration of price rigidity of 6.6 months, and a standard deviation of durations of price rigidity of 7.1 months.

The first general finding is that calibrating identical firms models based on the average frequency of price adjustments understates the real effects of monetary shocks relative to the underlying heterogeneous economy. The reason is that such effects are more directly related to the average *duration of price rigidity* rather than to the average frequency of price changes, and, because of Jensen's inequality, the former is greater than the duration of price spells implied by the latter. With the distribution I derive from BK, the difference is approximately 100%.

Accounting for this difference, however, does not suffice in general. For the more realistic case of growth rate shocks, heterogeneity has a direct impact on the real effects of monetary shocks, on top of the bias engendered by Jensen's inequality. I show that in the case of persistent shocks, cumulative real effects are approximately equal to the second moment of the distribution of durations of price rigidity. Moreover, a simple calibration with the distribution derived from BK shows that the effect of heterogeneity can be as large as that of the average duration of price rigidity itself.

In the presence of strategic complementarities in price setting (or real rigidities as in Ball and Romer, 1990), the interaction of firms with higher and lower frequencies of price changes tends to amplify the role of heterogeneity in generating persistence. The intuition is the same as for Taylor's "contract multiplier." The decisions of firms with higher adjustment frequencies are influenced by the existence of firms with lower frequencies, since the former do not want to set prices that will deviate "too much" from the aggregate price in the future. Therefore, firms from sectors in which prices are more sticky end up having a disproportionate effect on the aggregate price level.[1]

How large are these effects likely to be in quantitative terms? What fraction of the persistence not accounted for by models with identical firms can be explained with realistic degrees of heterogeneity? Addressing these questions with the continuous time version of the model requires solving it in the presence of strategic complementarities, which is an extremely demanding task in computational terms. However, for discrete time linear rational expectations models the standard techniques do not impose any computational constraint. Therefore, to render the results more applicable in empirical terms, I derive the underlying discrete time model and present the corresponding version of the generalized NKPC.[2]

I start by analyzing the impulse response functions of the output gap and inflation to monetary shocks in the presence of strategic complementarities, and comparing them to case of no real rigidities. The results show that heterogeneity and real rigidities interact in a quantitatively important way. In particular, because of strategic complementarities, even firms in the sector with the highest frequency of price changes may respond to shocks more slowly than the typical firm in an identical firms economy with the same average duration of price rigidity as the heterogeneous economy.

Finally, in section (5) I analyze the problem of fitting the impulse response functions of a heterogeneous economy with an identical firms model. I perform different exercises. In the first one, I constrain the identical firms economy to have the same degree of real rigidities as the heterogeneous economy, and find the frequency of price changes that minimizes the sum of squared deviations of its impulse response function for the output gap from the heterogeneous economy's. I also perform a slightly different exercise in which I constrain the identical firms economy to have no strategic complementarities. This is an interesting exercise, given that some papers build on models that rule out strategic complementarities, or adopt calibrations that imply strategic substitutability rather than complementarity in price setting (for example, Chari et al., 2000). Of course, in this second exercise the "best fitting adjustment frequency" would be distorted even if the target impulse response function were that of an identical firms economy *with* strategic complementarities. Therefore, to properly assess the distortion that arises from assuming no heterogeneity, I also fit an identical firms economy without real rigidities to one with strategic complementarities. In addition, for the model based on Taylor pricing I perform a third exercise in which, in order to find the best fitting identical firms economy, I optimize over the level of real rigidities as well as the "contract length."[3]

The results show that reproducing the dynamics of a truly heterogeneous economy with a model based on identical firms usually requires larger degrees of nominal rigidity than the average found in the data. However, this is not always the case. In the model

[1] Dixon and Kara (2005) study this mechanism in a model with Taylor staggered wage setting and heterogeneous contract lengths.

[2] Underlying in the sense that the continuous time model obtains as the limit when the period length goes to zero. The discrete time version of the model with Taylor pricing is also presented in the Appendix.

[3] The model based on Calvo pricing is not suitable for this exercise, since real rigidities and the probability of price changes are not separately identified.

with Taylor pricing and heterogeneity in contract lengths, I find that reproducing the dynamics of a heterogeneous economy with low degrees of real rigidity using an identical firms model requires a shorter contract length (relative to the actual average contract length) coupled with a higher degree of real rigidities.

Some recent papers which involve heterogeneity in price setting behavior are Ohanian et al. (1995), Bils and Klenow (2004), Bils et al. (2003) and Barsky et al. (2003). Taylor (1993), in particular, extended his original model (1979, 1980) to account for contract lengths of different durations.[4] However, none of these papers focuses on isolating the role of heterogeneity. This requires comparing models with heterogeneous firms with otherwise equivalent models in which all firms are identical. Exceptions are Aoki (2001) and Benigno (2004), who explore this comparison to show that ex-ante heterogeneity in the context of Calvo pricing does affect optimal monetary policy, and Dixon and Kara (2005), who study heterogeneity in the context of Taylor staggered wage setting. Aoki (2001) presents a NKPC with some features in common with the generalized NKPC derived here, but does not analyze the role of heterogeneity in price stickiness.

On the empirical front, Jadresic (1999) presents econometric evidence that heterogeneity improves the performance of sticky price models when applied to U.S. data, and Coenen and Levin (2004) document promising performance of a dynamic stochastic general equilibrium (DSGE) model with heterogeneity in price rigidity, using data for Germany.

2 A Generalized New Keynesian Phillips Curve[5]

In the economy there is a continuum of imperfectly competitive firms divided into sectors that differ in the frequency of price adjustments. Firms are indexed by their sector, $k \in [0,1]$, and by $i \in [0,1]$. The distribution of firms across sectors is summarized by a density function f on $[0,1]$.

All firms set prices as in Calvo (1983). For each firm in sector k, the opportunity to change prices arrives according to a Poisson process with rate given by $\alpha_k > 0$. These "price lotteries" are independent across all firms in the economy.

In the absence of frictions to price adjustment, the optimal level of an individual firm's relative price, which is the same for all firms, is given by:[6]

$$p^*(t) - p(t) = \theta y(t), \tag{1}$$

where p^* is the individual frictionless optimal price, p is the aggregate price level and y is the output gap. All variables should be interpreted as log-deviations from a deterministic,

[4] There are a few papers that introduce heterogeneity in a different framework in which firms follow state- rather than time-dependent pricing rules (for example, Caballero and Engel 1991, 1993).

[5] Although the model with Taylor pricing yields a qualitatively similar aggregate supply, the derivation of an explicit generalized version of the NKPC is particular to the model with Calvo pricing.

[6] Given that the basic setup is quite standard, I start with loglinear equations, which are straighforward to derive from first principles (see, for example, Woodford 2003).

zero inflation steady state.[7]

The aggregate price level, $p(t)$, is given by:

$$p(t)=\int_0^1 f(k)\int_0^1 p_{k,i}(t)didk, \tag{2}$$

where $p_{k,i}(t)$ is the price charged by firm i from sector k at time t.

Whenever a firm from sector k has a chance to change its price, it sets $x_k(t)$ according to:

$$\begin{aligned} x_k(t) &= \arg\min_x \int_0^\infty e^{-(\delta+\alpha_k)s}E_t\left[x-p^*(t+s)\right]^2 ds \\ &= (\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)s}E_t p^*(t+s)\,ds, \end{aligned} \tag{3}$$

where $\delta \geq 0$ is a discount rate.

Given this price setting behavior, the aggregate price level can be written as:

$$p(t)=\int_0^1 f(k)\,p_k(t)\,dk, \tag{4}$$

where the sectoral price indices, $p_k(t)$, are given by:

$$p_k(t)\equiv\int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)}x_k(s)\,ds. \tag{5}$$

Solving for the aggregate supply relation implied by (1), (3), (4), and (5) yields a *generalized new Keynesian Phillips curve* in continuous time, which accounts for heterogeneity in price stickiness:[8]

$$\dot{\pi}(t)=\delta\pi(t)-\varphi\theta y(t)-\varphi g(t), \tag{6}$$

where

$$\begin{aligned} \varphi &\equiv \delta\overline{\alpha}+\overline{\alpha}^2+\sigma_\alpha^2; \\ g(t) &\equiv \int_0^1 \widetilde{f(k)}\,(p(t)-p_k(t))\,dk; \\ \overline{\alpha} &\equiv \int_0^1 f(k)\,\alpha_k dk; \end{aligned}$$

[7] Although this is not without loss of generality, I assume a zero inflation steady state to isolate the impact of heterogeneity on the new Keynesian Phillips curve. For the effects of steady state inflation see Ascari (2004), and Cogley and Sbordone (2005).

[8] The details of all derivations are in the Appendix. As usual, $\dot{z}(t)\equiv\frac{\partial z(t)}{\partial t}$.

$$\begin{aligned}\sigma_\alpha^2 &\equiv \int_0^1 f(k)(\alpha_k - \overline{\alpha})^2\, dk; \\ \widetilde{f(k)} &\equiv \frac{\delta\alpha_k + \alpha_k^2}{\int_0^1 f(k)\left(\delta\alpha_k + \alpha_k^2\right) dk} f(k).\end{aligned}$$

The standard continuous time version of the NKPC first derived by Calvo (1983) obtains as a particular case when there is no discounting ($\delta = 0$) and all sectors have the same adjustment frequency, $\overline{\alpha}$ say, in which case (6) simplifies to:

$$\dot{\pi}(t) = -\overline{\alpha}^2 \theta y(t).$$

Heterogeneity in price stickiness changes the standard NKPC in two important ways. First, the coefficient on the output gap now depends on the first two moments of the distribution of frequencies of price adjustment. In addition, heterogeneity produces an endogenous shift term that is proportional to a weighted average of the sectoral relative prices, where the weights $\widetilde{f(k)}$ are adjustment-frequency-based transformations of the actual sectoral weights $f(k)$. This is akin to the relative-price shift term obtained by Aoki (2001).

The precise nature of the effects of heterogeneity on the NKPC is hard to gauge from direct inspection of (6), since it is clear that the dynamic behavior of the economy will depend on the whole distribution of adjustment frequencies.[9] Nevertheless, it is easy to prove general results on how heterogeneity changes the coefficient on the output gap. Owing to the presence of the endogenous shift term, however, these results only provide a partial view of the effects of heterogeneity on the dynamic behavior of the economy. Therefore, I postpone the analysis of the impact of heterogeneity to sections (4) and (5), where the roles of the new shift term and the different coefficient on the output gap will become clear.

3 Steady State and Monetary Shocks

To study the implications of heterogeneity in price stickiness for the real effects of monetary shocks I use a simple specification for the demand side of the model. In particular, I assume that nominal aggregate demand, $m(t) = y(t) + p(t)$, follows an exogenous process.[10] Then, departing from a deterministic inflationary steady state, I study the real effects of one time AR(1) type shocks to the level and growth rate of nominal aggregate demand. In this section I derive the steady state and the equations which describe the dynamic response of the economy to nominal shocks.

[9] This can actually be valuable for empirical work, since, in contrast to the standard Calvo (1983) model, the level of nominal and real rigidities become separately identified.

[10] This simplification is also used by Mankiw and Reis (2002), among others. See also Woodford (2003).

3.1 Steady State

In an inflationary steady state nominal aggregate demand grows at a constant rate $\mu \geq 0$. This implies that, after a normalization, $m(t) = \mu t$. Firms in sector k set prices as:

$$x_k(t) = (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)s} \left(\theta\left(\mu t + \mu s\right) + (1-\theta)\, p(t+s)\right) ds.$$

The aggregate price level is given implicitly by:

$$\begin{aligned} p(t) &= \theta \int_0^1 f(k) \int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)} (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)r} (\mu s + \mu r)\, drdsdk \\ &\quad + (1-\theta) \int_0^1 f(k) \int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)} (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)r} p(s+r)\, drdsdk. \end{aligned}$$

Using the method of undetermined coefficients, it is straightforward to show that the aggregate price level also grows at rate μ, and is given by:

$$p(t) = \mu t - \int_0^1 f(k) \frac{\delta\mu}{\theta\left(\delta\alpha_k + \alpha_k^2\right)} dk.$$

Individual prices are set according to:

$$x_k(t) = \mu t + \mu \left(\frac{1}{\alpha_k} - \frac{\delta}{\theta\left(\delta\alpha_k + \alpha_k^2\right)}\right),$$

and output is constant at the natural rate $y(t) = \int_0^1 f(k) \frac{\delta\mu}{\theta\left(\delta\alpha_k+\alpha_k^2\right)} dk$. Incidentally, notice that the usual result of non-superneutrality in the Calvo model extends to the setting with heterogeneous firms. The distribution of adjustment frequencies interacts with the degree of strategic complementarities in price setting in determining the steady state output gap.

3.2 Monetary Shocks

3.2.1 Level shocks

Assume that $\mu = 0$, and that at $t = 0$ nominal aggregate demand is hit by a shock of size $\overline{m}$, which then decays exponentially at rate $\rho \geq 0$. For $t \geq 0$, the path for nominal aggregate demand is therefore given by $m(t) = \overline{m}e^{-\rho t}$.

After learning of the shock, whenever a firm from group k gets a chance to adjust its price it sets:

$$\begin{aligned} x_k(t) &= (\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)s}E_t p^*(t+s)\,ds \\ &= (\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)s}\left(\theta\overline{m}e^{-\rho(t+s)}+(1-\theta)\,p(t+s)\right)ds. \end{aligned}$$

The corresponding path for the aggregate price level is defined implicitly by:

$$\begin{aligned} p(t) &= \int_0^1 f(k)\int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)}x_k(s)\,dsdk \qquad (7) \\ &= \theta\int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}(\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)r}\overline{m}e^{-\rho(s+r)}drdsdk \\ &\quad +(1-\theta)\int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}(\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)r}p(s+r)\,drdsdk, \end{aligned}$$

where the second integrals in each term of the last expression range from 0 (and not from $-\infty$) to t because $\mu=0$ implies $p(0)=0$.

3.2.2 Growth rate shocks

In the case of a growth rate shock, $\dot{m}(t)$ jumps at $t=0$ from μ to $\mu+\Delta\mu$, where $\Delta\mu$ is the size of the shock. Thereafter the shock decays exponentially at rate $\gamma\geq 0$, so that $\dot{m}(t)=\mu+\Delta\mu e^{-\gamma t}$, and $m(t)=\int_0^t \dot{m}(s)\,ds=\mu t+\Delta\mu\frac{1-e^{-\gamma t}}{\gamma}$.

After learning of the shock, whenever a firm from group k gets a chance to change its price it sets:

$$\begin{aligned} x_k(t) &= (\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)s}E_t p^*(t+s)\,ds \\ &= (\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)s}\left(\theta\left(\mu t+\mu s+\Delta\mu\frac{1-e^{-\gamma(t+s)}}{\gamma}\right)+(1-\theta)\,p(t+s)\right)ds. \end{aligned}$$

The corresponding path for the aggregate price level is defined implicitly by:

$$\begin{aligned} p(t) &= \int_0^1 f(k)\int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)}x_k(s)\,dsdk \qquad (8) \\ &= \int_0^1 f(k)\int_{-\infty}^0 \alpha_k e^{-\alpha_k(t-s)}\left(\mu s+\mu\left(\frac{1}{\alpha_k}-\frac{\delta}{\theta\left(\delta\alpha_k+\alpha_k^2\right)}\right)\right)dsdk \\ &\quad +\theta\int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}(\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)r}\mu(s+r)\,drdsdk \end{aligned}$$

$$+\theta\int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}(\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)r}\frac{1-e^{-\gamma(s+r)}}{\gamma}\Delta\mu drdsdk$$
$$+(1-\theta)\int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}(\delta+\alpha_k)\int_0^\infty e^{-(\delta+\alpha_k)r}p(s+r)\,drdsdk.$$

4 The Real Effects of Monetary Shocks

In order to isolate the effects of heterogeneity in price stickiness on the dynamic response of the economy to nominal shocks, I need to construct a benchmark economy with identical firms, retaining the same degree of nominal rigidity, in some sense. But what does that mean exactly? Does that mean matching the average duration of price rigidity, or the average frequency of price adjustments? While with identical firms the degree of nominal price rigidity can be equivalently summarized by either of these measures, with heterogeneity this is no longer the case.

As a first step in the analysis, I study how the coefficient on the output gap of the generalized NKPC compares to that of the standard NKPC in two different identical firms economies: one with the same average frequency of price adjustments and the other with the same average duration of price rigidity as the heterogeneous economy. In the context of the standard NKPC the slope is the most important determinant of the dynamic response of the economy to monetary shocks.[11] However, notice that in the heterogeneous economy this may no longer be the case, because of the shift term arising from the distribution of sectoral relative prices. With that in mind, the results are summarized in the following:[12]

Proposition 1 *Let $\overline{\alpha}$ and $\overline{d}$ denote, respectively, the average frequency of price adjustment and the average duration of price rigidity in an arbitrary heterogeneous economy. Let ψ denote the (absolute value of the) coefficient on the output gap in the generalized NKPC associated with such economy. Likewise, let $\psi^{\overline{\alpha}}$ and $\psi^{\overline{d}}$ denote the (absolute values of the) coefficients on the output gap in the standard NKPCs associated, respectively, with an identical firms economy with average frequency of price adjustment equal to $\overline{\alpha}$, and with an identical firms economy with average duration of price rigidity equal to $\overline{d}$. Then, $\psi \geq \psi^{\overline{\alpha}} \geq \psi^{\overline{d}}$, with equality if and only if there is no heterogeneity in price stickiness.*

The generalized NKPC is steeper than the NKPC in an identical firms economy with the same average frequency of price adjustment, which in turn is steeper than the NKPC in an identical firms economy with the same average duration of price rigidity (as that of

[11] It is the only factor in the case of no discounting ($\delta = 0$).

[12] Unlike the other results presented in this section, **Proposition 1** does not have a counterpart in the case of the model with Taylor pricing.

the heterogeneous economy). This result might mislead one to conclude that, following a monetary shock, the adjustment process in the heterogeneous economy is faster than in those two identical firms economies. However, this need not be the case. The reason is that the dynamic effects introduced by the shift term associated with the distribution of sectoral relative prices eventually tend to make the adjustment process more sluggish in the heterogeneous economy.[13] As a result, despite a steeper Phillips curve, heterogeneity may actually lead monetary shocks to have larger and more persistent real effects.

To illustrate why this may arise, I simulate the dynamic response of three economies to a permanent level shock to nominal aggregate demand:[14] one heterogeneous economy and two identical firms economies like the ones used in the comparison in **Proposition 1**. To better isolate the role of heterogeneity in this example I rule out strategic complementarities, and set $\theta = 1$. In addition, to avoid the effects of having different steady state output gaps, which arise in the case of permanent shocks to the growth rate of nominal income, from now on I abstract from discounting and set $\delta = 0$ to obtain superneutrality.[15]

As noted earlier, the dynamics of the heterogeneous economy depend on the whole distribution of adjustment frequencies. To handle this issue I choose to use the statistics on price setting behavior in the U.S. economy reported by BK to obtain the distribution used in the simulations. More specifically, I identify each sector in the model with one of the goods and services categories listed in their appendix. Accordingly, I set $\alpha_k = -\ln(1-\lambda_k)$, where λ_k is the monthly frequency of price changes of the category identified with sector k, so that the unit of time is one month. Then, I set the sectoral weights equal to the weights for these categories, which are also reported in the same table.[16] This results in 350 sectors, with an "average frequency based" duration of price rigidity of 3.3 months,[17] an average duration of price rigidity $\left(\sum_{k=1}^{350} f(k)\,\alpha_k^{-1}\right)$ of 6.6 months, and a standard deviation of durations of price rigidity of 7.1 months.

Figure 1 presents the results of the simulation. Contrary to what the naive analysis of the slopes of the NKPCs suggested, the adjustment process in the identical firms economy with the same average frequency of price changes is clearly too fast relative to the heterogeneous economy.[18] The identical firms economy with the average duration of price rigidity found in the BK data seems to provide a better representation of the heterogeneous economy.[19] A qualitative difference between the latter two economies is

[13] The direct comparison between the two identical firms economies based on the slope of the NKPC is, of course, valid.

[14] The qualitative features illustrated with this example are common to the other types of shock.

[15] An additional advantage is more tractability.

[16] I normalize the reported weights so that they add up to one.

[17] By "average frequency based" duration of price rigidity I mean $-\left(\ln\left(1-\overline{\lambda}\right)\right)^{-1}$, where $\overline{\lambda} = \sum_{k=1}^{350} f(k)\,\lambda_k$. An alternative measure would be the inverse of the average rate of price change arrivals: $\left(\sum_{k=1}^{350} f(k)\,\alpha_k\right)^{-1}$. It equals 2.9 months in the BK data.

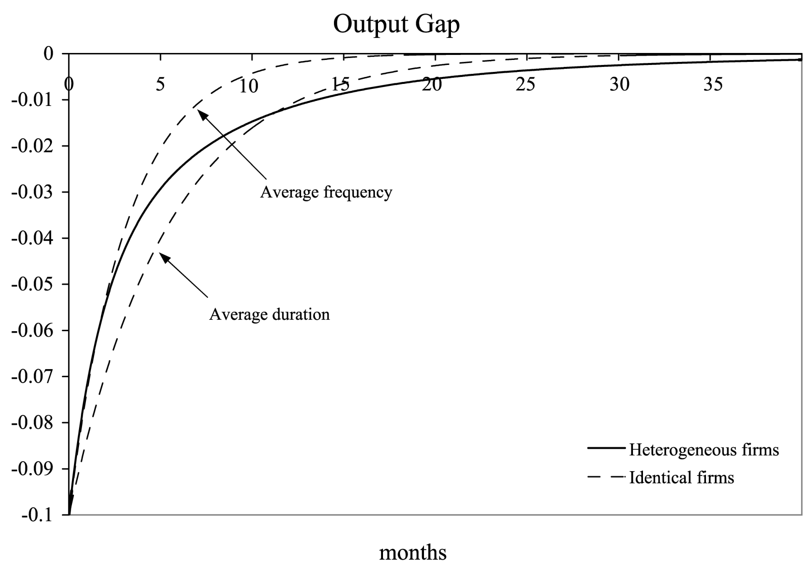
[18] Another possibility is to use the median duration of price rigidity, as advocated by BK. Although not shown here, it also performs poorly in tracking the behavior of the heterogeneous economy.

[19] Baharad and Eden (2004) argue in favor of the average duration of price rigidity instead of the

that, initially, adjustment is faster in the heterogeneous economy, because a relatively larger measure of firms with higher frequency of adjustment gets to change prices earlier. As time passes, the distribution of the frequency of price changes among firms which have not yet adjusted becomes progressively more dominated by firms with relatively lower adjustment rates. So, the speed of adjustment slows down through time, and eventually the process becomes more sluggish in the heterogeneous economy. This illustrates how the dynamics of the economy are affected by the shift term in the generalized NKPC.

Figure 1: Permanent Level Shock

$\overline{m} = -0.1,\ \rho = 0,\ \theta = 1$



A natural question to ask is whether there is a general rule to determine which parameterization for an identical firms economy will best mimic the dynamics of a given heterogeneous economy, in terms of its impulse response functions, say. Given the dependence of the latter on the underlying distribution of price stickiness, the answer is unfortunately, but not surprisingly, negative. It is possible, however, to get useful guidance from the results presented in the next subsections and in section (5).

4.1 Real Effects without Strategic Complementarities

In the absence of strategic complementarities in price setting it is possible to derive analytical results which shed additional light on the relationship between a model with

average frequency of price adjustments when there is heterogeneity, in the context of a Taylor type model.

arbitrary heterogeneity, and identical firms models with calibrations based on the average duration of price rigidity and the average frequency of price changes, as previously illustrated in Figure 1. To circumvent the dependence of the impulse response functions (IRFs) of the heterogeneous economy on the specific distribution of price stickiness, I focus on a sensible measure of the overall effects of monetary disturbances, which takes into account both the intensity and the persistence of the real effects of monetary shocks: the (normalized) cumulative effect on the output gap.[20] In the interesting case of persistent shocks, it turns out to depend only on the first two moments of the distribution of adjustment frequencies.

4.1.1 Level Shocks

The first result shows why, without strategic complementarities, the IRFs for identical firms economies with the same average duration of price rigidity will, in some sense, track their heterogeneous firms counterparts: for permanent level shocks the cumulative real effects that they imply are the same.

Proposition 2 *For an arbitrary heterogeneous economy, in the context of level shocks to nominal aggregate demand and no strategic complementarities ($\theta = 1$), the (normalized)*[21] *cumulative real effect as measured by $\frac{1}{\overline{m}} \int_0^\infty y(t)\, dt$ is equal to:*

$$\int_0^1 f(k) \frac{1}{\rho + \alpha_k} dk.$$

In the limiting case of permanent shocks ($\rho = 0$), it reduces to the average duration of price rigidity in the economy:

$$\overline{d} = \int_0^1 f(k) \frac{1}{\alpha_k} dk.$$

An identical firms economy with the same average frequency of price adjustment, on the other hand, will systematically understate the real effects of monetary shocks in the heterogeneous economy in the context of permanent level shocks, as shown below:

Proposition 3 *For an arbitrary heterogeneous economy, in the absence of strategic complementarities ($\theta = 1$) the (normalized) cumulative real effect of permanent level shocks to nominal aggregate demand ($\rho = 0$) (as measured by $\frac{1}{\overline{m}} \int_0^\infty y(t)\, dt$) will always be greater than in an identical firms economy with the same average frequency of price adjustments.*

[20] This measure is also discussed, for example, in Christiano et al. (2005).

[21] Normalized by the size of the shock, $\overline{m}$.

This is a direct result of Jensen's inequality. The intuition as to why the average frequency of price adjustments is misleading as an indicator of the overall degree of nominal rigidity can be developed from the following limiting case: imagine a heterogeneous economy with a non-negligible fraction of firms which adjust prices continuously. Then, irrespective of how low the frequencies of price adjustment of the remaining firms are, the average frequency in the economy will be infinite. Nevertheless, monetary shocks may still have large real effects due to the firms with finite adjustment frequencies. The intuition of this extreme example carries through to more realistic distributions: in heterogeneous economies, a high average frequency of price adjustment need not imply a low degree of nominal rigidity (note that the implication does hold in identical firms economies).[22] One might conjecture that getting rid of the extremes in the distribution of adjustment frequencies by using the median rather than the average frequency would solve this problem, but this is not what is to be taken from the above results. **Proposition 2** clearly states that the average duration of price rigidity, not the median adjustment frequency nor the median duration of price rigidity, is directly related to the real effects of monetary shocks.

To give a first idea of how large these effects are likely to be in quantitative terms, take the case of permanent shocks. By this measure, using the BK data the increase in total real effects when heterogeneity is accounted for is of approximately 100% in the case of the U.S. economy: the "average frequency based" duration of price rigidity is 3.3 months, while the average duration of price rigidity is 6.6 months.[23]

4.1.2 Growth Rate Shocks

In the case of growth rate shocks, taking Jensen's inequality into account and using the average duration instead of the average frequency to calibrate the identical firms economy does not suffice. The reason is that heterogeneity has an additional, direct impact on cumulative real effects, as shown below:

Proposition 4 *For an arbitrary heterogeneous economy, in the context of temporary shocks to the growth rate of nominal aggregate demand* ($\gamma > 0$) *and no strategic comple-*

[22] More formally, an identical firms economy with "Calvo parameter" equal to the weighted average of the sectoral Calvo parameters in the heterogeneous economy will have the same frequency of price changes as the latter, simply because a linear combination of independent Poisson processes is a Poisson process with rate given by the linear combination of the individual rates. But the two models are not observationally equivalent, because the arrival rate is not all that matters: the behavior of the economies also depends on the prices that firms set. In aggregating the individual prices to get the overall price level, the sectoral price adjustment rates enter in a non-linear fashion, and this is why the aggregate price levels in the two economies differ.

[23] The results for the Euro area are similar. Based on the statistics reported by Dhyne et al. (2005), the "average frequency based" duration of price rigidity is 6.1 months months, while the average duration of price rigidity ranges from 13 to 15.1 months, depending on how the individual country data are aggregated.

mentarities ($\theta = 1$), the (normalized)[24] *cumulative real effect as measured by* $\frac{1}{\Delta\mu}\int_0^\infty y(t)\,dt$ *is equal to:*

$$\int_0^1 f(k)\,\frac{1}{\gamma\alpha_k + \alpha_k^2}dk.$$

In the case of very persistent shocks ($\gamma \approx 0$), it is approximately equal to the second moment of the distribution of (expected) durations of price rigidity in the economy:[25]

$$\overline{d}^2 + \sigma_d^2 = \int_0^1 f(k)\,\frac{1}{\alpha_k^2}dk,$$

where $\sigma_d^2 \equiv \int_0^1 f(k)\left(\frac{1}{\alpha_k} - \overline{d}\right)^2 dk$ *is the variance of the the distribution of (expected) durations of price rigidity in the economy.*

The intuition for why heterogeneity has a direct effect on cumulative real effects in the case of growth rate shocks, but not in the case of level shocks, can be developed from the identical firms case. With level shocks, a change in the frequency of price adjustments affects the speed of the adjustment process, but not the magnitude of real effects on impact. In the case of growth rate shocks, however, a lower frequency of price changes both increases the magnitude of real effects, and reduces the speed at which they fade away. Jointly, these two features lead total real effects to depend on the square of the frequency of price changes. This difference is illustrated in Figures 2 (a-b) using two identical firms economies with different frequencies of price adjustment (average durations of 1 and 2 years). With heterogeneity, the mechanism at work is qualitatively the same, and the overall effect is the weighted average of the effect for each sector, thus depending as well on the second moment of the distribution of adjustment frequencies.

To give a first idea of how large this effect is likely to be in quantitative terms, I compute the ratio of the approximate normalized cumulative real effect of a permanent growth rate shock in a heterogeneous economy to the same measure in an identical firms economy with the same average frequency of price changes. I use once more the empirical distribution for the U.S. economy obtained from BK. In that case, the standard deviation of the distribution of (expected) durations of price rigidity in the economy is $\sigma_d = 7.1$ months. Recall that the average duration of price rigidity is 6.6 months, while the "average frequency based" duration of price rigidity is 3.3 months. Therefore, the ratio referred to above is $\frac{(6.6)^2+(7.1)^2}{(3.3)^2} = 8.6$. Even correcting for Jensen's inequality and using the average duration of price rigidity to calibrate the identical firms economy produces cumulative effects which are less than half of the those in the heterogeneous economy, since in that case the ratio is $\frac{(6.6)^2+(7.1)^2}{(6.6)^2} = 2.2$.

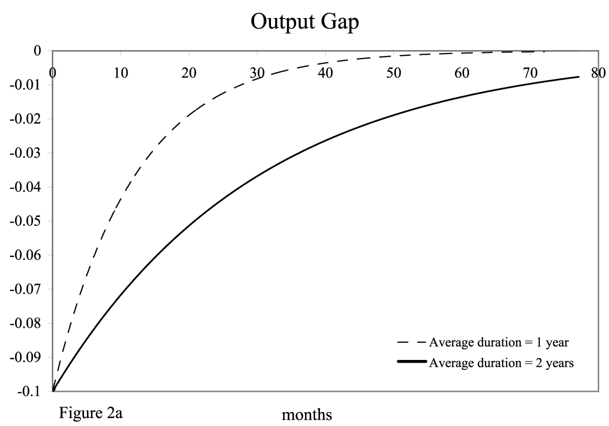
[24] Normalized by the size of the shock, $\Delta\mu$ in this case.

[25] The approximation error is of order $\mathcal{O}(\gamma)$.

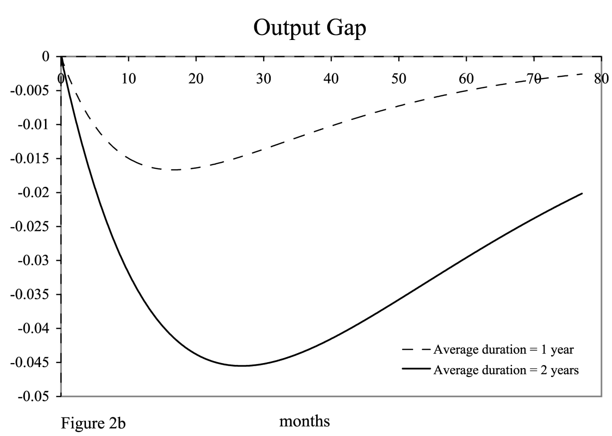
Why the Approximation? The reason why the second result in **Proposition 4** only holds approximately is that the expression derived for the normalized cumulative real effects is not valid for the case of permanent growth rate shocks ($\gamma = 0$). In that case, the output gap is identically zero in the model with heterogeneity, as well as with identical firms. This is a feature of Calvo pricing which has not received much attention in the literature, although it has been documented elsewhere. Mankiw and Reis (2002), for example, find the same result with the NKPC in their "sudden disinflation" experiment. When they consider temporary shocks to the growth rate of the money supply, however, they find that they do have real effects in the Calvo economy. The result is not due to the lack of real rigidities, because Mankiw and Reis's (2002) experiments do account for them.

Figures 2(a-b): Identical Firms

Level Shock: $\overline{m} = -0.1$, $\rho = 0$, $\theta = 1$



Growth Rate Shock: $\mu = 0.1$, $\Delta\mu = -0.1$, $\gamma = 0.5$, $\theta = 1$



So, treating the normalized cumulative effect as a function of the degree of persistence of the shock, the fact that they differ for a permanent and an arbitrarily persistent shock implies a discontinuity at $\gamma = 0$.[26] Economically, what underlies this discontinuity is the fact that in the case of extremely persistent (but temporary) shocks, most of its real effects are spread out into the distant future. When the shock hits, firms know that nominal income growth will remain at the new level for a long time, but that it will, eventually, return to the previous level. Because of nominal rigidity, this generates small, but nevertheless non-zero real effects, which also last for a long time. When the shock is permanent, however, the real effects are exactly zero.

This discontinuity disappears, however, when there is discounting ($\delta > 0$).[27] In that case, as shown in the Appendix, starting from the zero inflation steady state ($\mu = 0$), normalized and discounted cumulative real effects are instead given by:

$$\int_0^1 f(k) \frac{1}{(\alpha_k + \delta)(\alpha_k + \delta + \gamma)} dk.$$

As $\gamma \to 0$, this clearly converges to:

$$\int_0^1 f(k) \frac{1}{(\alpha_k + \delta)^2} dk,$$

which is exactly the normalized (and discounted) cumulative real effect in the case of a permanent shock (see Appendix).[28]

This result need *not*, however, undermine the usefulness of the approximation used in **Proposition 4**. Since the discontinuity disappears when there is discounting, the approximation derived for permanent shocks when $\delta = 0$ may still be useful for realistic values of the discount rate. This is indeed the case, as the results in Table 1 show. As a matter of fact, the exact result for temporary shocks can also be viewed as an approximation to the more realistic case of $\delta > 0$. For varying degrees of persistence of the shock,[29] Table 1 presents the normalized cumulative real effects as derived in **Proposition 4**, as well as the exact normalized and discounted cumulative effect, for discount rates ranging from 0.01 to 0.05. It is clear that the approximation is quite accurate, and that for very persistent shocks the real effects are indeed approximately proportional to the second moment of the distribution of expected durations of price rigidity.

[26] Technically, it arises because, although $y(t) \to 0$ pointwise as $\gamma \to 0$, it is not uniformly integrable on $[0, \infty)$, and the exchange of limit and integration yields a different result.

[27] Mankiw and Reis (2002) abstract from discounting.

[28] Technically, the reason why the discontinuity disappears is that, with discounting, $e^{-\delta t} y(t)$ becomes uniformly integrable on $[0, \infty)$.

[29] I also present the half lives. The time unit is one year.

Table 1: Normalized Cumulative Real Effects of Growth Rate Shock

			Discount Rate (δ)				
γ	Half Life (years)	Proposition 4	0.01	0.02	0.03	0.04	0.05
0.00	∞	0.65	0.62	0.59	0.57	0.55	0.53
0.07	10	0.56	0.54	0.52	0.50	0.49	0.47
0.14	5	0.50	0.49	0.47	0.46	0.44	0.43
0.23	3	0.45	0.43	0.42	0.41	0.40	0.39
0.35	2	0.39	0.38	0.37	0.36	0.36	0.35
0.46	1.5	0.36	0.35	0.34	0.33	0.32	0.32
0.69	1	0.30	0.29	0.29	0.28	0.27	0.27
1.39	0.5	0.21	0.20	0.20	0.20	0.19	0.19

4.2 Real Effects with Strategic Complementarities

Introducing strategic complementarities amplifies the role of heterogeneity in generating more persistent real effects of monetary shocks. As it is well known, real rigidities do make the process of adjustment to monetary shocks more sluggish, even in the identical firms case. With heterogeneity, however, this is even more so. The reason can be understood in the context of the framework developed by Haltiwanger and Waldman (1991). With strategic complementarities the decisions of firms with higher adjustment frequencies are influenced by the existence of firms with lower frequencies, since the former do not want to set prices that will deviate "too much" from the aggregate price in the future. Therefore, firms from sectors in which prices are more sticky end up having a disproportionate effect on the price level.

Solving the model for the dynamic response to monetary shocks in the presence of strategic complementarities amounts to finding the fixed points in (7) and (8) when $\theta < 1$, and requires numerical methods. For the continuous time version of the model this is an extremely demanding task in computational terms. However, for discrete time linear rational expectations models there are standard tools which do not impose any computational constraint. Therefore, to render the results more applicable in empirical terms, I present them in the context of the discrete time model that underlies the continuous time model analyzed previously.

I keep the details of the derivation to the Appendix, including the proof that the continuous time model obtains as the limit of the discrete time model when the period length goes to zero. Here I only present the *discrete time version of the generalized NKPC*:

$$\pi_t = \beta E_t \pi_{t+h} + \widehat{\varphi}\theta y_t + \widehat{\varphi} g_t,$$

where h is the period length, $\pi_t = p_t - p_{t-h}$ is the inflation rate in period t, $\beta = e^{-\delta h}$ is the per period discount factor,

$$
\begin{aligned}
\widehat{\varphi} &\equiv \int_0^1 f(k) \frac{\lambda_k^2}{1-\lambda_k} dk + (1-\beta)\overline{\lambda}; \\
g_t &\equiv \int_0^1 \widehat{f(k)} \left(p_t - p_t^k\right) dk; \\
\overline{\lambda} &\equiv \int_0^1 f(k) \lambda_k dk; \\
\widehat{f(k)} &\equiv \frac{\frac{\lambda_k}{1-\lambda_k} - \beta\lambda_k}{\int_0^1 f(k)\left(\frac{\lambda_k}{1-\lambda_k} - \beta\lambda_k\right) dk} f(k),
\end{aligned}
$$

and where $\lambda_k = 1 - e^{-\alpha_k h}$ is the probability that a firm from sector k can change its price in any given period.

This discrete time version generalizes the standard discrete time NKPC in a similar way as the continuous time version presented before does with the original version derived by Calvo (1983). In particular, the coefficient on the output gap depends on the distribution of frequencies of price adjustment and second, it features an endogenous shift term that is proportional to a weighted average of the sectoral relative prices, where the weights are adjustment-probability-based transformations of the sectoral weights. Also, the same qualitative and quantitative results derived before have their counterparts in the discrete time case.

Once more, I use the BK data to calibrate the distribution of adjustment probabilities. I proceed as before, identifying each category with a sector and setting the weights accordingly. The difference is that now I set the λ_k's equal to the actual monthly frequencies of price changes reported by BK. Therefore, in the results presented below each period represents one month.[30]

To study the interaction of heterogeneity in price stickiness and real rigidities, I postulate that (log) nominal income follows an AR(2) process:

$$m_t = \phi_1 (1+\phi_2) m_{t-1} - \phi_2 m_{t-2} + \varepsilon_t,$$

where ε_t is a zero mean i.i.d. process with variance given by σ_ε^2, and I set:

$$
\begin{aligned}
0 &< \phi_1 \leq 1;\ \phi_2 = 0 \text{ to obtain an AR(1) in levels; or} \\
\phi_1 &= 1;\ 0 < \phi_2 \leq 1 \text{ to obtain an AR(1) in growth rates.}
\end{aligned}
$$

I rewrite the model in state space form and solve it using Sims' (2001) "gensys" method to obtain the impulse response functions to the ε_t innovations.[31]

[30] In all exercises I set $\beta = 0.9967$, so that the discount rate is 4% p.a.

[31] The variance of the shock only affects the scale of the impulse response functions. For concreteness, in all simulations I set $\sigma_\varepsilon = 0.015$ analyze one standard deviation negative shocks.

Figures 3 (a-d) display the results for growth rate shocks with different degrees of persistence. All cases include IRFs both with and without strategic complementarities. Following Mankiw and Reis (2002), I set $\theta = 0.1$.[32] The identical firms economy is calibrated to have the same average duration of price rigidity. The effects of heterogeneity are even larger when compared with an identical firms economy with the same average frequency of price changes (not shown).

Figures 3(a-b): Growth Rate Shock

$\phi1 = 1$; $\phi2 = 0.89$ (half life = 0.5 years)
$\beta = 0.9967$ (4% p.a.)

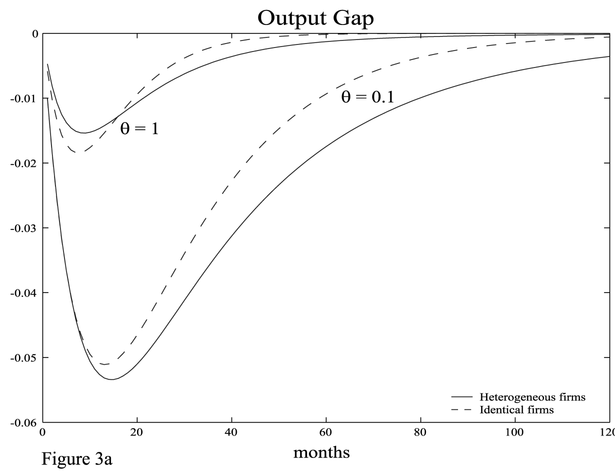


Figure 3a

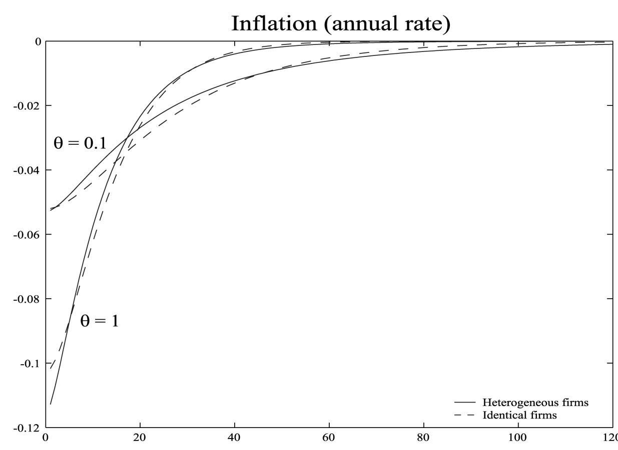


Figure 3b

Note: To avoid cluttering, in both figures the labels indicating the degree of strategic complementarity ($\theta = 1$, $\theta = 0.1$) apply to the two nearest lines.

[32] Woodford (2003, ch. 3) argues that values in the 0.10-0.15 range can be obtained with plausible assumptions for various sources of real rigidities in the context of identical firms models. Results with larger values of θ (less real rigidities), which imply smaller differences between heterogeneous and identical firms economies, are presented in section (5).

These results illustrate how strategic complementarities interact with heterogeneity to generate more persistent real effects of monetary shocks, according to several metrics: the recession troughs are delayed, the output gap is now lower than in the identical firms economy essentially during the whole process, and takes longer to return to the steady state. Inflation is, accordingly, also less responsive: it falls less on impact, and then takes longer to return to zero.

Figures 3(c-d): Growth Rate Shock

$\phi 1 = 1$; $\phi 2 = 0.96$ (half life = 1.5 years)
$\beta = 0.9967$ (4% p.a.)

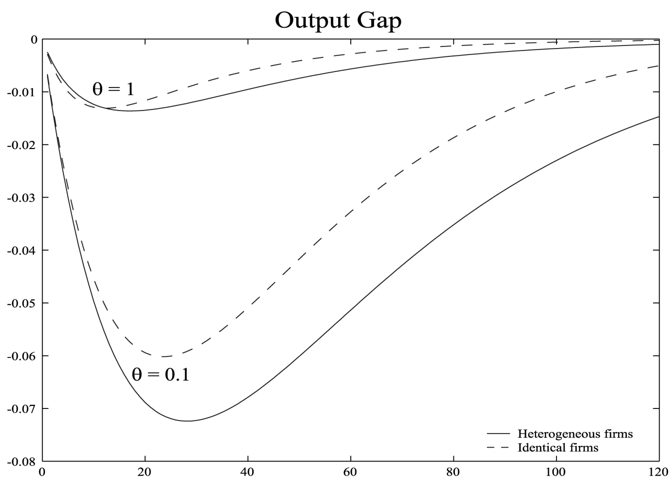


Figure 3c

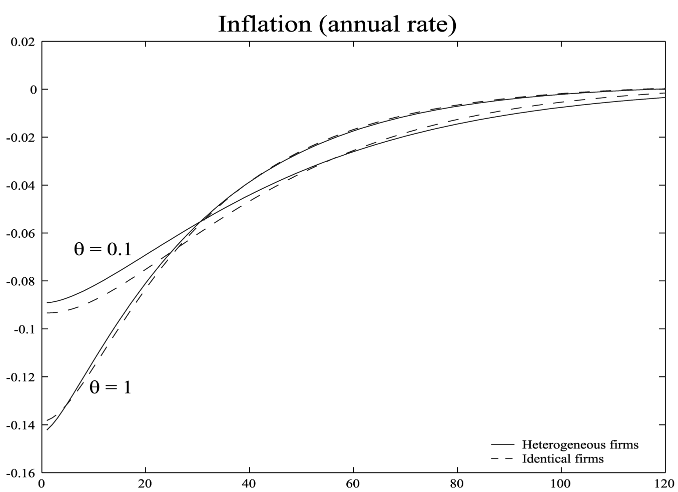


Figure 3d

Note: To avoid cluttering, in both figures the labels indicating the degree of strategic complementarity (θ = 1, θ = 0.1) apply to the two nearest lines.

The interaction becomes even more evident with the analysis of the IRF of prices chosen by firms at each point in time. These are depicted in Figures 4 (a-b), for shocks with different degrees of persistence. For the heterogeneous economy I plot the price set in period t by firms in the sector with the highest adjustment frequency (x_t^1), while for the identical firms economy I plot the price that is set by any firm that gets to adjust at t (x_t). Without strategic complementarities the price set by firms in the least sticky sector respond faster than the price set by a typical firm in the identical firms economy. With strategic complementarities prices respond more slowly in both economies, but even more so in the heterogeneous economy: eventually, even firms in the sector with the highest adjustment frequency respond less than the typical firm in the identical firms economy, because of the influence of sectors with lower adjustment probabilities.

5 Fitting IRFs with an Identical Firms Model

After developing a better understanding of how heterogeneity in price stickiness introduces persistence in monetary economies, in this section I finally revisit a question posed earlier in the paper: which parameterization for an identical firms economy will best mimic the dynamics of a given heterogeneous economy in terms of its impulse response functions?

To address this question I perform a few exercises. In the first one, given the empirical distribution of adjustment probabilities obtained from BK, and a degree of real rigidities in the heterogeneous economy, I find the adjustment probability λ_{id} in an identical firms economy that minimizes the sum of squared deviations of its IRFs from the heterogeneous economy's IRFs. In this case the identical firms economy is constrained to have the same degree of real rigidities as the heterogeneous economy that actually generated the "target IRF." Additionally, I also perform a slightly different exercise in which I impose no strategic complementarities in the identical firms economy. This is an interesting exercise, given that some papers build on models that rule out strategic complementarities, or adopt calibrations that imply strategic substitutability rather than complementarity in price setting (for example, Chari et al., 2000). Of course, in this second exercise the "best fitting adjustment probability" would be distorted even if the target IRF had been generated by an identical firms economy with strategic complementarities. Therefore, I also calculate the adjustment probability in an identical firms economy without real rigidities that best fits the IRF of an identical firms economy with the average duration of price rigidity found in the data, and same degree of strategic complementarities used for the heterogeneous economy.[33]

[33] The fit is perfect in this case, since the degree of real rigidities and the adjustment probability are not separately identified in the Calvo model. Actually, in this case the best fitting adjustment probability λ_{id} can be found analytically. It solves:

$$\frac{\lambda_{id}^2}{1-\lambda_{id}} + (1-\beta)\,\lambda_{id} = \theta\left(\frac{\lambda^2}{1-\lambda} + (1-\beta)\,\lambda\right),$$

Figures 4(a-b): Calvo Pricing - Growth Rate Shocks

$\phi 1 = 1$; $\phi 2 = 0.89$ (half life = 0.5 years)
$\beta = 0.9967$ (4% p.a.)

Chosen Prices: x_t^1 and x_t

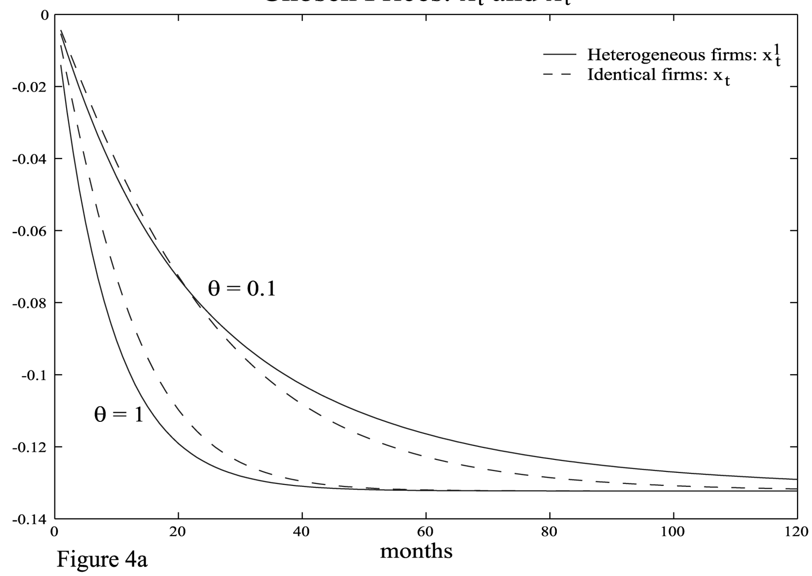


Figure 4a

$\phi 1 = 1$; $\phi 2 = 0.96$ (half life = 1.5 years)
$\beta = 0.9967$ (4% p.a.)

Chosen Prices: x_t^1 and x_t

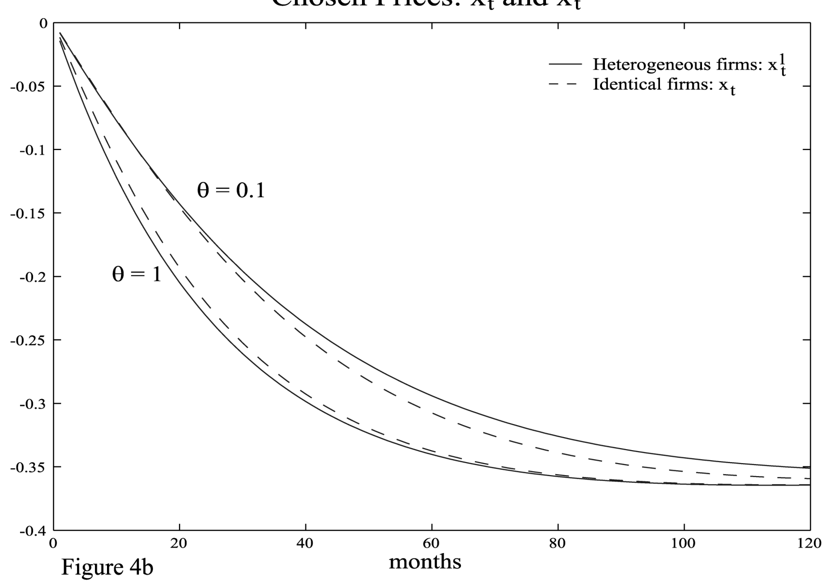


Figure 4b

Note: To avoid cluttering, in both figures the labels indicating the degree of strategic complementarity ($\theta = 1$, $\theta = 0.1$) apply to the two nearest lines.

Using the IRF for the output gap, I do these calculation for several degrees of strategic complementarities, and for shocks with varying levels of persistence.[34] For the same reasons outlined in subsection (4.2), I conduct these exercises with the discrete time version of the models.

I also present separate results for the model based on Taylor pricing, since it allows for an additional exercise in which I optimize over the level of real rigidities in the identical firms economy, in addition to the "contract length." As noted earlier, the model based on Calvo pricing is not suitable for this exercise, since real rigidities and the probability of price changes are not separately identified in identical firms models.

5.1 Model with Calvo Pricing

The results are presented in Tables 2 (a-d). The first two tables report the results of the first exercise, in which the degree of strategic complementarity in the identical firms economy $(1-\theta_{id})$ and in the heterogeneous economy $(1-\theta)$ are equal. To better approximate the dynamic behavior of the heterogeneous economy, the average duration of price rigidity in the identical firms economy has to exceed the actual average duration (6.6 months). This occurs even without strategic complementarities, but the more so the more persistent the shock and the higher the degree of real rigidities.

The results of the second exercise, in which I impose $\theta_{id} = 1$, are reported in the next two tables. For each level of strategic complementarities, the first column displays the best fitting average duration when the "data generating process" (DGP) is the heterogeneous economy, while the second column presents the result when the DGP is an identical firms economy with the same average duration of price rigidity and the given level of strategic complementarities. In both cases the average duration of price rigidity in the identical firms economy without strategic complementarities has to exceed the actual average duration, but the distortion needs to be greater if the data is generated by the heterogeneous economy. As with the previous exercise, this is the more so the more persistent the shock and the higher the degree of real rigidities.

Tables 2(a-b): Calvo Pricing (Growth Rate Shocks)

Setting $\theta_{id} = \theta$

Half life = 12 months

θ	$-1/\ln(1-\lambda_{id})$
0.10	8.83
0.15	8.63
0.25	8.37
0.50	8.01
1.00	7.67

Table 2a

Half life = 36 months

θ	$-1/\ln(1-\lambda_{id})$
0.10	9.21
0.15	9.11
0.25	8.98
0.50	8.80
1.00	8.62

Table 2b

Obs: all durations are reported in months.

where λ is the adjustment probability in the identical firms economy that generated the target IRF.

[34] I focus on growth rate shocks because they are empirically more relevant.

Tables 2(c-d): Calvo Pricing (Growth Rate Shocks)

Setting $\theta_{id} = 1$

Half life = 12 months

θ	$-1/\ln(1-\lambda_{id})$ Heterog. economy	Ident. Firms economy
0.10	27.57	20.05
0.15	21.89	16.37
0.25	16.41	12.73
0.50	11.16	9.10
1.00	7.67	6.57

Table 2c

Half life = 36 months

θ	$-1/\ln(1-\lambda_{id})$ Heterog. economy	Ident. Firms economy
0.10	28.85	20.05
0.15	23.19	16.37
0.25	17.67	12.73
0.50	12.29	9.10
1.00	8.62	6.57

Table 2d

Obs: all durations are reported in months.

5.2 Model with Taylor Pricing[35]

The results are presented in Tables 3 (a-f).[36] The first two tables report the results of the first exercise, in which the variable used to fit the IRF of the heterogeneous economy is the contract length in the identical firms economy (d_{id}), while the degrees of strategic complementarity in both economies are equal ($\theta_{id} = \theta$). The second row of tables presents the results of the second exercise, in which I impose $\theta_{id} = 1$. For each level of strategic complementarities, the first column displays the best fitting contract length when the "data generating process" (DGP) is the heterogeneous economy, while the second column presents the result when the DGP is an identical firms economy with the average contract length and the given level of strategic complementarities.[37]

Both sets of results are similar to the ones found in the model with Calvo pricing. To better approximate the dynamic behavior of the heterogeneous economy, the average contract length in the identical firms economy has to exceed the actual average contract length. This is the more so the more persistent the shock and the higher the degree of real rigidities. When there are no strategic complementarities in the best fitting identical

[35] Bils and Klenow (2004) report that in their NBER working paper version they perform the kind of exercise described in this subsection, and find, in contrast to the results presented here, that the best fit to their heterogeneous economy is obtained with an identical firms model with roughly the median duration of price rigidity found in their data (4 months). The reasons for the differences in results are twofold. First, they adopt a calibration that implies strategic substitutability in price setting. More importantly, they consider the less realistic case of permanent level shocks to the money supply, in which the role of heterogeneity is reduced. With permanent level shocks and strategic substitutability ($\theta = 2$) I find the same results as they do.

[36] The procedure to calibrate the distribution of contract lengths in the model with Taylor pricing is described in the Appendix. It gives rise to an average contract length of 6.7 months and a standard deviation of contract lengths of 5.6 months.

[37] Due to the constraint that contract lengths be integers, I round up the average contract length to 7 months.

firms economy, its contract length needs to be distorted to match the IRF of economies that feature real rigidities, but relatively more so if the target IRF is generated by the heterogeneous economy.

The last two tables present the results of the additional exercise performed with the model based on Taylor pricing. For each level of strategic complementarity in the heterogeneous economy, it presents the contract length (d_{id}) and θ_{id} in an identical firms economy that best track the IRF of the heterogeneous economy. It is clear that the parameters in the identical firms economy need to be distorted to provide a better fit. However, it is no longer the case that the contract length in the identical firms economy always exceeds the actual average contract length. This still occurs when the heterogeneous economy features relatively high degrees of real rigidity (relatively low θ's). For lower levels of strategic complementarity in the heterogeneous economy, however, a better fit is obtained with contract lengths that fall short of the actual average contract length coupled with a higher degree of real rigidities.

Tables 3(a-f): Taylor Pricing (Growth Rate Shocks)

Setting $\theta_{id} = \theta$

Half life = 12 months

θ	d_{id}
0.10	9.00
0.15	9.00
0.25	9.00
0.50	9.00
1.00	7.00

Table 3a

Half life = 36 months

θ	d_{id}
0.10	10.00
0.15	10.00
0.25	9.00
0.50	9.00
1.00	7.00

Table 3b

Setting $\theta_{id} = 1$

Half life = 12 months

	d_{id}	
θ	Heterog. economy	Ident. Firms economy
0.10	21.00	15.00
0.15	18.00	13.00
0.25	14.00	10.00
0.50	10.00	8.00
1.00	7.00	7.00

Table 3c

Half life = 36 months

	d_{id}	
θ	Heterog. economy	Ident. Firms economy
0.10	21.00	14.00
0.15	18.00	12.00
0.25	14.00	10.00
0.50	10.00	8.00
1.00	7.00	7.00

Table 3d

Optimizing over both d_{id} and θ_{id}

Half life = 12 months

θ	d_{id}	θ_{id}
0.10	9.00	0.09
0.15	8.00	0.10
0.25	6.00	0.09
0.50	4.00	0.06
1.00	3.00	0.06

Table 3e

Half life = 36 months

θ	d_{id}	θ_{id}
0.10	11.00	0.15
0.15	10.00	0.17
0.25	8.00	0.17
0.50	6.00	0.16
1.00	4.00	0.12

Table 3f

Obs: all durations are reported in months.

6 Conclusion

Standard sticky price models usually build on the assumption that firms change prices with the same frequency. This would be a good approximation either if empirically the degree of heterogeneity in price stickiness were small or if, despite significant, heterogeneity turned out not to matter.

In this paper I argued that this is not the case, and that heterogeneity in price stickiness should have a larger role in models used to analyze the real effects of monetary shocks. Recent papers document a substantial degree of heterogeneity in the frequency of price changes across firms in different sectors, both in the U.S. economy and in the Euro area. This paper presented results which show that heterogeneity affects the dynamic response of economies to monetary shocks.

I derived a generalized NKPC which accounts for heterogeneity in price stickiness. It differs from the standard NKPC in that the inflation-output dynamics are now affected by the distribution of frequencies of price changes in the economy. Relative to comparable identical firms economies, heterogeneous economies initially display faster adjustment after a shock owing to a relatively higher measure of firms with higher frequencies of price changes, which get to adjust earlier. As time passes, the distribution of adjustment frequencies among firms which have not yet reacted to the shock becomes progressively more dominated by firms with relatively lower adjustment rates, slowing down the adjustment process.

In the presence of strategic complementarities in price setting or real rigidities, the decision to adjust by firms in sectors with higher adjustment frequencies is influenced by the existence of sectors with lower adjustment rates, which end up having a disproportionate effect on the aggregate price level.

I calibrated the model based on a distribution of the frequency of price adjustments in the U.S. economy derived from Bils and Klenow (2004), and obtained important quantitative differences between models with identical firms and models with heterogeneity. In particular, I showed that reproducing the dynamics of a truly heterogeneous economy with a model based on identical firms usually requires larger degrees of nominal rigidity than the average found in the data. However, this is not always the case. In a model with Taylor pricing and heterogeneity in contract lengths, I found that reproducing the dynamics of a heterogeneous economy with low degrees of real rigidity using an identical firms model requires a shorter contract length (relative to the actual average contract length) coupled with a higher degree of real rigidities.[38]

These results might help shed some additional light on the so called *persistence problem* (Chari et al., 2000). Some recent papers which carry out quantitative evaluations of sticky price DSGE models based on identical firms find that in order to obtain good

[38] Smets and Wouters (2003) hint at the possibility that heterogeneity could bias their results (n. 3). Christiano et al. (2005) claim that "inference about nominal rigidities is sensitive to getting the real side of the model right," but ignore the possibility that the same applies to heterogeneity in the frequency of price changes. In addition, inference about *real* rigidities can also be sensitive to getting the *nominal* side of the model right.

empirical performance one needs unrealistically low frequencies of price adjustment, in light of the microeconomic evidence on price setting. A natural step is to fully assess the empirical relevance of the results obtained in this paper for this question, by introducing heterogeneity in a standard DSGE model, and taking the model to the data. Promising results in this direction appear in Coenen and Levin (2004).

Albeit derived here in the context of sticky price models, the results on the effects of heterogeneity extend to sticky information models as well (Carvalho, 2006). This suggests that heterogeneous behavior in price setting may have an important role to play in models of monetary economies, irrespective of the nature of frictions to price adjustment. In my opinion, two extensions appear particularly worth exploring, because they might prove to be important for subsequent quantitative work on the effects of monetary shocks. The first one is to combine heterogeneity in both price and wage setting behavior, which may leverage the interaction between sticky prices and sticky wages already documented in models with identical agents. The other is to allow for sectoral heterogeneity in the fraction of firms that set prices according to some "rule-of-thumb" (such as indexation) in each period, and/or in the degree of indexation.

Finally, the results show that we must be cautious when interpreting estimates of parameters of price stickiness and real rigidities based on identical firms models, or calibrating them in light of the microeconomic evidence. The issue is not whether identical firms models are able to provide a reasonable description of a more complex, heterogeneous reality, but rather that this is likely to require parameter values which will seem unrealistic if interpreted literally. Given the empirical evidence documenting a high degree of heterogeneity in price stickiness and the fact that it does matter, the parameters of (misspecified) identical firms models should not be treated as structural.

Appendix

I start by presenting the derivation of the continuous time version of the generalized NKPC, and the proofs of Propositions 1-4, followed by the derivation of the underlying discrete time model. Next, I present all the derivations for the model with Taylor pricing, including the counterparts of Propositions 2-4, and the derivation of its discrete time version.

1) The generalized NKPC in continuous time

Substitute (1) into (3) and differentiate with respect to time:[39]

$$\begin{aligned}
\dot{x}_k(t) &= -(\delta+\alpha_k)(p(t)+\theta y(t)) + (\delta+\alpha_k)\int_t^\infty e^{-(\delta+\alpha_k)(s-t)}E_t(p(s)+\theta y(s))\,ds \\
&= (\delta+\alpha_k)(x_k(t)-p(t)-\theta y(t));
\end{aligned}$$

Next, differentiate (4) and (5) with respect to time:

$$\begin{aligned}
\pi(t) &= \dot{p}(t) = \int_0^1 f(k)\left(\alpha_k x_k(t) - \alpha_k\int_{-\infty}^t \alpha_k e^{-\alpha_k(t-s)}x_k(s)\,ds\right)dk \\
&= \int_0^1 f(k)\,\alpha_k(x_k(t)-p_k(t))\,dk; \\
\pi_k(t) &= \dot{p}_k(t) = \alpha_k\left[x_k(t)-p_k(t)\right].
\end{aligned}$$

So, differentiating $\pi(t)$ w.r.t. time yields:

$$\begin{aligned}
\dot{\pi}(t) &= \int_0^1 f(k)\,\alpha_k\left(\dot{x}_k(t)-\dot{p}_k(t)\right)dk \\
&= \int_0^1 f(k)\,\alpha_k\left((\delta+\alpha_k)(x_k(t)-p(t)-\theta y(t)) - \alpha_k(x_k(t)-p_k(t))\right)dk \\
&= -\left(\theta\int_0^1 f(k)\left(\delta\alpha_k+\alpha_k^2\right)dk\right)y(t) + \int_0^1 f(k)\,\alpha_k^2(p_k(t)-p(t))\,dk \\
&\quad + \int_0^1 f(k)\,\alpha_k\delta(x_k(t)-p_k(t)+p_k(t)-p(t))\,dk \\
&= -\left(\theta\int_0^1 f(k)\left(\delta\alpha_k+\alpha_k^2\right)dk\right)y(t) + \int_0^1 f(k)\,\alpha_k^2(p_k(t)-p(t))\,dk \\
&\quad + \delta\int_0^1 f(k)\,\alpha_k(x_k(t)-p_k(t))\,dk + \int_0^1 f(k)\,\delta\alpha_k(p_k(t)-p(t))\,dk \\
&= \delta\pi(t) - \left(\theta\int_0^1 f(k)\left(\delta\alpha_k+\alpha_k^2\right)dk\right)y(t) + \int_0^1 f(k)\left(\delta\alpha_k+\alpha_k^2\right)(p_k(t)-p(t))\,dk.
\end{aligned}$$

[39] In the context of the shocks analyzed in this paper, this and all subsequent time derivatives exist for all $t \neq 0$. At $t=0$ they should be interpreted as right derivatives.

So,

$$\dot{\pi}(t) = \delta\pi(t) - \varphi\theta y(t) - \varphi g(t),$$

where

$$\begin{aligned}
\varphi &\equiv \delta\overline{\alpha} + \overline{\alpha}^2 + \sigma_\alpha^2; \\
g(t) &\equiv \int_0^1 \widetilde{f(k)}\left(p(t) - p_k(t)\right) dk; \\
\overline{\alpha} &\equiv \int_0^1 f(k)\,\alpha_k dk; \\
\sigma_\alpha^2 &\equiv \int_0^1 f(k)\left(\alpha_k - \overline{\alpha}\right)^2 dk; \\
\widetilde{f(k)} &\equiv \frac{\delta\alpha_k + \alpha_k^2}{\int_0^1 f(k)\left(\delta\alpha_k + \alpha_k^2\right) dk} f(k).
\end{aligned}$$

2) Proof of Proposition 1

Define:

$$\overline{d} \equiv \int_0^1 f(k)\frac{1}{\alpha_k} dk.$$

From the generalized NKPC, the absolute values of the slope coefficients are:

$$\begin{aligned}
\psi &\equiv \theta\left(\delta\overline{\alpha} + \overline{\alpha}^2 + \sigma_\alpha^2\right); \\
\psi^{\overline{\alpha}} &\equiv \theta\left(\delta\overline{\alpha} + \overline{\alpha}^2\right); \\
\psi^{\overline{d}} &\equiv \theta\left(\delta\overline{d}^{-1} + \overline{d}^{-2}\right).
\end{aligned}$$

$\psi \geq \psi^{\overline{\alpha}}$ follows trivially, since $\sigma_\alpha^2 \geq 0$. Now, because of Jensen's inequality, $\overline{\alpha} \geq \overline{d}^{-1}$. Therefore, $\psi^{\overline{\alpha}} \geq \psi^{\overline{d}}$. The inequalities hold strictly if and only if $\sigma_\alpha^2 \equiv \int_0^1 f(k)\left(\alpha_k - \overline{\alpha}\right)^2 dk > 0$.

3) Proof of Proposition 2

Recall that I assume $\delta = 0$. Set $\theta = 1$ in (7). Then:

$$\begin{aligned}
p(t) &= \int_0^1 f(k)\int_0^t \alpha_k e^{-\alpha_k(t-s)}\alpha_k \int_0^\infty e^{-\alpha_k r}\overline{m}e^{-\rho(s+r)} dr ds dk \\
&= \overline{m}\int_0^1 f(k)\left[-\frac{e^{-\alpha_k t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)} + \frac{e^{-\rho t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}\right] dk.
\end{aligned}$$

The normalized cumulative real effects, $\frac{1}{\overline{m}}\int_0^\infty m(t)-p(t)\,dt$, are given by:

$$\begin{aligned}
&\frac{1}{\overline{m}}\int_0^\infty\left[\overline{m}e^{-\rho t}-\overline{m}\int_0^1 f(k)\left(-\frac{e^{-\alpha_k t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}+\frac{e^{-\rho t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}\right)dk\right]dt\\
= &\int_0^\infty\int_0^1 f(k)\left[e^{-\rho t}+\frac{e^{-\alpha_k t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}-\frac{e^{-\rho t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}\right]dkdt\\
= &\int_0^1 f(k)\int_0^\infty\left[e^{-\rho t}+\frac{e^{-\alpha_k t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}-\frac{e^{-\rho t}\alpha_k^2}{(\alpha_k-\rho)(\alpha_k+\rho)}\right]dtdk\\
= &\int_0^1 f(k)\frac{1}{\rho+\alpha_k}dk.
\end{aligned}$$

4) Proof of Proposition 3

Again, from Jensen's inequality $\overline{d}\geq\overline{\alpha}^{-1}$. Then, just apply the result in **Proposition 2** for the limiting case $\rho=0$.

5) Proof of Proposition 4

Recall that I assume $\delta=0$. Set $\theta=1$ in (8). Then:

$$\begin{aligned}
p(t) &= \int_0^1 f(k)\left[\int_{-\infty}^0\alpha_k e^{-\alpha_k(t-s)}\left(\mu s+\frac{\mu}{\alpha_k}\right)ds\right]dk\\
&\quad+\int_0^1 f(k)\int_0^t\alpha_k e^{-\alpha_k(t-s)}\left[\alpha_k\int_0^\infty e^{-\alpha_k r}\left(\mu s+\mu r+\Delta\mu\frac{1-e^{-\gamma(s+r)}}{\gamma}\right)dr\right]dsdk\\
&= \mu t+\Delta\mu\int_0^1 f(k)\frac{\alpha_k^2\left(e^{-\gamma t}-1\right)+\gamma^2(1-e^{-\alpha_k t})}{\gamma\left(\gamma^2-\alpha_k^2\right)}dk.
\end{aligned}$$

The normalized cumulative real effects, $\frac{1}{\Delta\mu}\int_0^\infty m(t)-p(t)\,dt$, are given by:

$$\begin{aligned}
&\frac{1}{\Delta\mu}\int_0^\infty\mu t+\Delta\mu\frac{1-e^{-\gamma t}}{\gamma}-\mu t-\Delta\mu\int_0^1 f(k)\frac{\alpha_k^2\left(e^{-\gamma t}-1\right)+\gamma^2(1-e^{-\alpha_k t})}{\gamma\left(\gamma^2-\alpha_k^2\right)}dkdt\\
= &\int_0^\infty\int_0^1 f(k)\left[\frac{1-e^{-\gamma t}}{\gamma}-\frac{\alpha_k^2\left(e^{-\gamma t}-1\right)+\gamma^2(1-e^{-\alpha_k t})}{\gamma\left(\gamma^2-\alpha_k^2\right)}\right]dkdt\\
= &\int_0^1 f(k)\frac{1}{\alpha_k^2+\gamma\alpha_k}dk.
\end{aligned}$$

Denote the normalized cumulative real effects as a function of γ by:

$$h(\gamma)\equiv\int_0^\infty\frac{m(t)-p(t)}{\Delta\mu}dt=\int_0^1 f(k)\frac{1}{\alpha_k^2+\gamma\alpha_k}dk.$$

A first order approximation around $\gamma = 0$ yields:

$$h(\gamma) = h(0) - \left(\int_0^1 f(k) \frac{1}{\alpha_k^3} dk \right) \gamma + O(\gamma^2).$$

Therefore, for $\gamma \approx 0$, normalized cumulative real effects are approximately equal to:

$$h(0) = \int_0^1 f(k) \frac{1}{\alpha_k^2} dk = \overline{d}^2 + \sigma_d^2,$$

where $\sigma_d^2 \equiv \int_0^1 f(k) \left(\frac{1}{\alpha_k} - \overline{d} \right)^2 dk$.

6) Effects of growth rate shocks with discounting

For a temporary shock, the relevant equations for $t \geq 0$ are:

$$\begin{aligned} x_k(t) &= (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)s} E_t p^*(t+s) \, ds \\ &= (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)s} \left(\mu t + \mu s + \Delta\mu \frac{1 - e^{-\gamma(t+s)}}{\gamma} \right) ds; \end{aligned}$$

$$\begin{aligned} p(t) &= \int_0^1 f(k) \int_{-\infty}^0 \alpha_k e^{-\alpha_k(t-s)} \left[\mu s + \mu \left(\frac{1}{\alpha_k} - \frac{\delta}{\delta\alpha_k + \alpha_k^2} \right) \right] ds dk \\ &\quad + \int_0^1 f(k) \int_0^t \alpha_k e^{-\alpha_k(t-s)} (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)r} (\mu s + \mu r) \, dr ds dk \\ &\quad + \int_0^1 f(k) \int_0^t \alpha_k e^{-\alpha_k(t-s)} (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)r} \frac{1 - e^{-\gamma(s+r)}}{\gamma} \Delta\mu \, dr ds dk. \end{aligned}$$

Computing the normalized and discounted cumulative real effects yields:

$$\frac{1}{\Delta\mu} \int_0^\infty e^{-\delta t} (m(t) - p(t)) \, dt = \int_0^1 f(k) \frac{1}{\Delta\mu} \frac{(\delta + \gamma)\mu + \alpha_k(\Delta\mu + \mu)}{\alpha_k(\alpha_k + \delta)(\alpha_k + \delta + \gamma)} dk.$$

Setting $\mu = 0$ yields the expression presented in the text:

$$\int_0^1 f(k) \frac{1}{(\alpha_k + \delta)(\alpha_k + \delta + \gamma)} dk.$$

Now, in the case of a permanent shock ($\gamma = 0$), the relevant equations are:

$$\begin{aligned} x_k(t) &= (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)s} E_t p^*(t+s) \, ds \\ &= (\delta + \alpha_k) \int_0^\infty e^{-(\delta+\alpha_k)s} (\mu t + \mu s + \Delta\mu t + \Delta\mu s) \, ds; \end{aligned}$$

$$\begin{aligned} p(t) &= \int_0^1 f(k) \int_{-\infty}^0 \alpha_k e^{-\alpha_k (t-s)} \left[\mu s + \mu \left(\frac{1}{\alpha_k} - \frac{\delta}{\delta \alpha_k + \alpha_k^2} \right) \right] ds dk \\ &+ \int_0^1 f(k) \int_0^t \alpha_k e^{-\alpha_k (t-s)} (\delta + \alpha_k) \int_0^\infty e^{-(\delta + \alpha_k) r} (\mu s + \mu r + \Delta \mu s + \Delta \mu r) \, dr ds dk. \end{aligned}$$

Computing the normalized and discounted cumulative real effects yields:

$$\frac{1}{\Delta \mu} \int_0^\infty e^{-\delta t} (m(t) - p(t)) \, dt = \int_0^1 f(k) \frac{1}{\Delta \mu} \frac{\delta \mu + \alpha_k (\Delta \mu + \mu)}{\alpha_k (\alpha_k + \delta)(\alpha_k + \delta)} dk.$$

Setting $\mu = 0$ yields the expression presented in the text:

$$\int_0^1 f(k) \frac{1}{(\alpha_k + \delta)^2} dk,$$

which equals $\lim_{\gamma \to 0} \int_0^1 f(k) \frac{1}{(\alpha_k + \delta)(\alpha_k + \delta + \gamma)} dk$, as stated in the text.

7) The generalized NKPC in the underlying discrete time model

Firms from sector k set prices as:

$$\begin{aligned} x_t^k &= \arg\min_x \sum_{j=0}^\infty h \beta^j (1 - \lambda_k)^j E_t \left(x - p_{t+jh}^* \right)^2 \qquad (9) \\ &= (1 - (1 - \lambda_k) \beta) \sum_{j=0}^\infty ((1 - \lambda_k) \beta)^j E_t p_{t+jh}^* \\ &= (1 - (1 - \lambda_k) \beta) p_t^* + (1 - \lambda_k) \beta E_t x_{t+h}^k, \end{aligned}$$

where h is the period length, $\lambda_k = 1 - e^{-\alpha_k h}$ is the probability that a firm from sector k can change its price in any given period, and $\beta = e^{-\delta h}$ is the discount factor.

Sectoral price indices are given by:

$$\begin{aligned} p_t^k &= \lambda_k \sum_{j=0}^\infty (1 - \lambda_k)^j x_{t-jh}^k \qquad (10) \\ &= \lambda_k x_t^k + (1 - \lambda_k) p_{t-h}^k. \end{aligned}$$

Multiplying by $f(k)$ and integrating across sectors yields the aggregate price index:

$$\begin{aligned} p_t &= \int_0^1 f(k) p_t^k dk \qquad (11) \\ &= \int_0^1 f(k) \left(\lambda_k x_t^k + (1 - \lambda_k) p_{t-h}^k \right) dk. \end{aligned}$$

Leading the above equation and taking expectations as of time t yields:

$$E_t p_{t+h} = \int_0^1 f(k)\,\lambda_k E_t x_{t+h}^k dk + \int_0^1 f(k)\,(1-\lambda_k)\,p_t^k dk. \tag{12}$$

From (9), solve for $E_t x_{t+h}^k$ to get:

$$E_t x_{t+h}^k = \frac{x_t^k}{(1-\lambda_k)\,\beta} - \frac{1-(1-\lambda_k)\,\beta}{(1-\lambda_k)\,\beta} p_t^*.$$

Multiplying by $f(k)\,\lambda_k$ and integrating across sectors yields:

$$\begin{aligned}
\int_0^1 f(k)\,\lambda_k E_t x_{t+h}^k dk &= \int_0^1 f(k)\left[\frac{\lambda_k x_t^k}{(1-\lambda_k)\,\beta} - \frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta} p_t^*\right] dk \qquad (13)\\
&= \int_0^1 f(k)\left[\frac{\lambda_k x_t^k}{(1-\lambda_k)\,\beta} - \frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\,(p_t+\theta y_t)\right] dk.
\end{aligned}$$

Subtracting (11) from (12), and using (13) delivers:

$$\begin{aligned}
E_t p_{t+h} - p_t &= \int_0^1 f(k)\,\lambda_k E_t x_{t+h}^k dk - \int_0^1 f(k)\,\lambda_k x_t^k dk + \int_0^1 f(k)\,(1-\lambda_k)\left(p_t^k - p_{t-h}^k\right) dk\\
&= \int_0^1 f(k)\,\frac{\lambda_k}{(1-\lambda_k)\,\beta} x_t^k dk - \int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\,(p_t+\theta y_t)\,dk\\
&\quad - \int_0^1 f(k)\,\lambda_k x_t^k dk + \int_0^1 f(k)\,(1-\lambda_k)\,p_t^k dk - \int_0^1 f(k)\,(1-\lambda_k)\,p_{t-h}^k dk\\
&= \int_0^1 f(k)\left(\frac{\lambda_k}{(1-\lambda_k)\,\beta} - \lambda_k\right) x_t^k dk - \int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\,(p_t+\theta y_t)\,dk\\
&\quad + \underbrace{\int_0^1 f(k)\,p_t^k\,dk}_{=p_t} - \int_0^1 f(k)\,\lambda_k p_t^k dk - \underbrace{\int_0^1 f(k)\,p_{t-h}^k\,dk}_{=p_{t-h}} + \int_0^1 f(k)\,\lambda_k p_{t-h}^k dk,
\end{aligned}$$

so that

$$\begin{aligned}
E_t p_{t+h} - p_t &= p_t - p_{t-h} + \int_0^1 f(k)\,\frac{1-(1-\lambda_k)\,\beta}{(1-\lambda_k)\,\beta}\lambda_k\left(x_t^k - p_t\right) dk \qquad (14)\\
&\quad - \int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\theta y_t dk - \int_0^1 f(k)\,\lambda_k\left(p_t^k - p_{t-h}^k\right) dk.
\end{aligned}$$

Now, from (10),

$$\begin{aligned}
\lambda_k x_t^k &= p_t^k - (1-\lambda_k)\,p_{t-h}^k \iff\\
\frac{1-(1-\lambda_k)\,\beta}{(1-\lambda_k)\,\beta}\lambda_k x_t^k &= \frac{1-(1-\lambda_k)\,\beta}{(1-\lambda_k)\,\beta}p_t^k - \frac{1-(1-\lambda_k)\,\beta}{(1-\lambda_k)\,\beta}\,(1-\lambda_k)\,p_{t-h}^k
\end{aligned}$$

$$
\begin{aligned}
&= \left(\frac{1}{(1-\lambda_k)\,\beta}-1\right)p_t^k-\left(\frac{1}{(1-\lambda_k)\,\beta}-1\right)(1-\lambda_k)\,p_{t-h}^k \\
&= \frac{1}{(1-\lambda_k)\,\beta}p_t^k-p_t^k-\lambda_k p_t^k-\left(\frac{1}{\beta}-1\right)p_{t-h}^k+\lambda_k\left(p_t^k-p_{t-h}^k\right) \\
&= \left(\frac{1}{(1-\lambda_k)\,\beta}-1-\lambda_k-\frac{1-\beta}{\beta}\right)p_t^k+\left(\frac{1-\beta}{\beta}+\lambda_k\right)\left(p_t^k-p_{t-h}^k\right) \\
&= \frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}p_t^k+\frac{1-\beta}{\beta}\left(p_t^k-p_{t-h}^k\right)+\lambda_k\left(p_t^k-p_{t-h}^k\right). \qquad (15)
\end{aligned}
$$

Replacing (15) into (14) yields:

$$
\begin{aligned}
E_t p_{t+h}-p_t &= p_t-p_{t-h}+\int_0^1 f(k)\left[\begin{array}{c}\frac{\lambda_k(1-(1-\lambda_k)\beta)}{(1-\lambda_k)\beta}p_t^k+\frac{1-\beta}{\beta}\left(p_t^k-p_{t-h}^k\right) \\ +\lambda_k\left(p_t^k-p_{t-h}^k\right)\end{array}\right]dk \\
&\quad -\int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\,(p_t+\theta y_t)\,dk-\int_0^1 f(k)\,\lambda_k\left(p_t^k-p_{t-h}^k\right)dk \\
&= p_t-p_{t-h}+\int_0^1 f(k)\left[\begin{array}{c}\frac{\lambda_k(1-(1-\lambda_k)\beta)}{(1-\lambda_k)\beta}\left(p_t^k-(p_t+\theta y_t)\right) \\ +\frac{1-\beta}{\beta}\left(p_t^k-p_{t-h}^k\right)\end{array}\right]dk \\
&= \frac{1}{\beta}\,(p_t-p_{t-h})+\int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\left(p_t^k-p_t\right)dk \\
&\quad -\int_0^1 f(k)\,\frac{\lambda_k\,(1-(1-\lambda_k)\,\beta)}{(1-\lambda_k)\,\beta}\theta y_t dk.
\end{aligned}
$$

Multiplying by β and rearranging yields:

$$
\begin{aligned}
p_t-p_{t-h} &= \beta E_t\,(E_t p_{t+h}-p_t)+\left[\int_0^1 f(k)\left(\frac{\lambda_k}{1-\lambda_k}-\beta\lambda_k\right)dk\right]\theta y_t \qquad (16) \\
&\quad +\int_0^1 f(k)\left(\frac{\lambda_k}{1-\lambda_k}-\beta\lambda_k\right)\left(p_t-p_t^k\right)dk.
\end{aligned}
$$

Finally, let $\pi_t\equiv p_t-p_{t-h}$ denote the inflation rate in period t. So,

$$
\pi_t=\beta E_t\pi_{t+h}+\widehat{\varphi}\theta y_t+\widehat{\varphi}g_t,
$$

where

$$
\begin{aligned}
\widehat{\varphi} &\equiv \int_0^1 f(k)\,\frac{\lambda_k^2}{1-\lambda_k}dk+(1-\beta)\,\overline{\lambda}; \\
g_t &\equiv \int_0^1 \widehat{f(k)}\left(p_t-p_t^k\right)dk;
\end{aligned}
$$

$$
\begin{aligned}
\overline{\lambda} &\equiv \int_0^1 f(k)\,\lambda_k dk; \\
\widehat{f(k)} &\equiv \frac{\left(\frac{\lambda_k}{1-\lambda_k}-\beta\lambda_k\right)}{\int_0^1 f(k)\left(\frac{\lambda_k}{1-\lambda_k}-\beta\lambda_k\right)dk} f(k)\,.
\end{aligned}
$$

To obtain the continuous time version of the generalized NKPC, start from (16). Subtract $\beta\left(p_t - p_{t-h}\right)$ from both sides, divide by β and rearrange to get:

$$
\begin{aligned}
\left(E_t p_{t+h} - p_t\right) - \left(p_t - p_{t-h}\right) &= \frac{1-\beta}{\beta}\left(p_t - p_{t-h}\right) - \left[\int_0^1 f(k)\left(\frac{\lambda_k}{(1-\lambda_k)\,\beta} - \lambda_k\right)dk\right]\theta y_t \\
&\quad - \int_0^1 f(k)\left(\frac{\lambda_k}{(1-\lambda_k)\,\beta} - \lambda_k\right)\left(p_t - p_t^k\right)dk.
\end{aligned}
$$

Dividing by h^2, and taking the limit as $h \to 0$ yields the desired result.

8) The model with Taylor pricing

In the economy there is a continuum of imperfectly competitive firms divided into sectors that differ in the duration of price rigidity, or "contract lengths." Firms are indexed by their sector, $k \in [0, k^*]$, and by $i \in [0, 1]$. The distribution of firms across sectors is summarized by a density function $f(\cdot)$ on $[0, k^*]$.

All firms set prices for fixed periods of time. Each firm in sector k sets prices for a period of length k.[40] Adjustments are uniformly staggered through time both in terms of firms and sectors.

The frictionless optimal level of an individual firm's relative price is given by (1).[41]

Firms from sector k sets prices according to:

$$
\begin{aligned}
x_k(t) &= \arg\min_x \int_0^k e^{-\delta s} E_t\left[x - p^*(t+s)\right]^2 ds \\
&= \frac{\delta}{1-e^{-\delta k}} \int_0^k e^{-\delta s} E_t p^*(t+s)\,ds.
\end{aligned}
$$

where $\delta \geq 0$ is a discount rate.

Given this price setting behavior, the aggregate price level can be written as:

$$
p(t) = \int_0^{k^*} f(k)\,p_k(t)\,dk,
$$

[40] This is just for notational convenience. I could specify contracts of length n_k for firms of sector k, and everything would go through with a change of variables.

[41] Again, all variables should be interpreted as log-deviations from a deterministic, zero inflation steady state.

where the sectoral price indices, $p_k(t)$, are given by:

$$p_k(t) \equiv \frac{1}{k} \int_0^k x_k(t-s)ds.$$

8a) Steady State

In the inflationary steady state nominal aggregate demand grows at a constant rate $\mu \geq 0$. This implies that, after a normalization, $m(t) = \mu t$. Firms in sector k set prices as:

$$x_k(t) = \frac{\delta}{1-e^{-\delta k}} \int_0^k e^{-\delta s} \left[\theta(\mu t + \mu s) + (1-\theta) p(t+s)\right] ds.$$

The aggregate price level is given by:

$$p(t) = \int_0^{k^*} f(k) \frac{1}{k} \int_0^k \frac{\delta}{1-e^{-\delta k}} \int_0^k e^{-\delta s} \left[\theta\mu(t-s+r) + (1-\theta) p(t-s+r)\right] drdsdk.$$

Using the method of undetermined coefficients, it is straightforward to show that the aggregate price level also grows at rate μ, and is given by:

$$p(t) = \mu t - \mu \int_0^{k^*} f(k) \frac{2-2e^{\delta k} + \delta k + e^{\delta k}\delta k}{2\left(e^{\delta k}-1\right)\delta\theta} dk.$$

Individual prices are set according to:

$$x_k(t) = \mu t + \mu \frac{2\left(e^{\delta k}-1\right) + \delta k \left(e^{\delta k}-1\right)\theta - \left(1+e^{\delta k}\right)}{2\left(e^{\delta k}-1\right)\delta\theta},$$

and output is constant at the natural rate $y(t) = \mu \int_0^{k^*} f(k) \frac{\left(2-2e^{\delta k}+\delta k+e^{\delta k}\delta k\right)}{2\left(e^{\delta k}-1\right)\delta\theta} dk$. Notice, again, the usual non-superneutrality result.

8b) Monetary Shocks

Level shocks

After the shock, firms from group k set:

$$\begin{aligned} x_k(t) &= \frac{\delta}{1-e^{-\delta k}} \int_0^k e^{-\delta s} E_t p^*(t+s)\, ds \\ &= \frac{\delta}{1-e^{-\delta k}} \int_0^k e^{-\delta s} \left(\theta \overline{m} e^{-\rho(t+s)} + (1-\theta) p(t+s)\right) ds. \end{aligned}$$

For $0 \leq t \leq k^*$ there are firms with prices set before the shock. So:

$$\begin{aligned} p(t) &= \int_0^t f(k)\frac{1}{k}\int_0^k x_k(t-s)dsdk + \int_t^{k^*} f(k)\frac{1}{k}\int_0^t x_k(t-s)dsdk \qquad (17) \\ &= \int_0^t \frac{f(k)}{k}\int_0^k \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left(\theta\overline{m}e^{-\rho(t-s+r)} + (1-\theta)\,p(t-s+r)\right)drdsdk \\ &+ \int_t^1 \frac{f(k)}{k}\int_0^t \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left(\theta\overline{m}e^{-\rho(t-s+r)} + (1-\theta)\,p(t-s+r)\right)drdsdk. \end{aligned}$$

For $t > k^*$ all firms set prices with knowledge of the shock, and $p(t)$ is given by:

$$p(t) = \int_0^{k^*} \frac{f(k)}{k}\int_0^k \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left(\theta\overline{m}e^{-\rho(t-s+r)} + (1-\theta)\,p(t-s+r)\right)drdsdk. \qquad (18)$$

Growth rate shocks

After the shock, firms from group k set:

$$\begin{aligned} x_k(t) &= \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta s}E_t p^*(t+s)\,ds \\ &= \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta s}\left[\theta\left(\mu(t+s) + \Delta\mu\frac{1-e^{-\gamma(t+s)}}{\gamma}\right) + (1-\theta)\,p(t+s)\right]ds. \end{aligned}$$

For $0 \leq t \leq k^*$ there are firms with prices set before the shock, and so:

$$\begin{aligned} p(t) &= \int_0^t \frac{f(k)}{k}\int_0^k x_k(t-s)dsdk + \int_t^{k^*} \frac{f(k)}{k}\int_0^k x_k(t-s)dsdk \qquad (19) \\ &= \int_0^t \frac{f(k)}{k}\int_0^k \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left[\begin{array}{c}\theta\left(\mu(t-s+r) + \Delta\mu\frac{1-e^{-\gamma(t-s+r)}}{\gamma}\right) \\ +(1-\theta)\,p(t-s+r)\end{array}\right]drdsdk \\ &+ \int_t^{k^*} \frac{f(k)}{k}\int_0^t \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left[\begin{array}{c}\theta\left(\mu(t-s+r) + \Delta\mu\frac{1-e^{-\gamma(t-s+r)}}{\gamma}\right) \\ +(1-\theta)\,p(t-s+r)\end{array}\right]drdsdk. \end{aligned}$$

For $t > k^*$ all firms set prices with knowledge of the shock, and $p(t)$ is given by:

$$p(t) = \int_0^{k^*} \frac{f(k)}{k}\int_0^k \frac{\delta}{1-e^{-\delta k}}\int_0^k e^{-\delta r}\left[\begin{array}{c}\theta\left(\mu(t-s+r) + \Delta\mu\frac{1-e^{-\gamma(t-s+r)}}{\gamma}\right) \\ +(1-\theta)\,p(t-s+r)\end{array}\right]drdsdk. \qquad (20)$$

8c) Counterpart to Proposition 2

For an arbitrary heterogeneous economy with Taylor pricing, in the context of level shocks to nominal aggregate demand and no strategic complementarities ($\theta = 1$), the (normalized) cumulative real effect as measured by $\frac{1}{\overline{m}}\int_0^\infty y(t)\,dt$ is equal to:

$$\int_0^{k^*} f(k)\,\frac{\rho k - 1 + e^{-\rho k}}{\rho^2 k}dk.$$

In the limiting case of permanent shocks ($\rho = 0$), it reduces to the average duration of price rigidity in the economy:[42]

$$\frac{\overline{k}}{2} = \int_0^{k^*} f(k)\,\frac{k}{2}dk.$$

Proof

Set $\theta = 1$ in (17) and (18). Then, $\frac{1}{\overline{m}}\int_0^\infty m(t) - p(t)\,dt$ equals:[43]

$$\begin{aligned}
&\frac{1}{\overline{m}}\left(\int_0^\infty m(t)\,dt - \int_0^{\mathbf{k}^*} p(t)\,dt - \int_{\mathbf{k}^*}^\infty p(t)\,dt\right)\\
=\;& \int_0^\infty e^{-\rho t}dt - \int_0^{\mathbf{k}^*} e^{-\rho t}\int_0^t \frac{f(k)}{k^2\rho^2}\left(e^{k\rho} - 2 + e^{-k\rho}\right)dkdt\\
&+\int_0^{\mathbf{k}^*} e^{-\rho t}\int_t^{k^*} \frac{f(k)}{k^2\rho^2}\left(e^{\rho t} + e^{-\rho k} - e^{-\rho(k-t)} - 1\right)dkdt\\
&-\int_{\mathbf{k}^*}^\infty e^{-\rho t}\int_0^{k^*} \frac{f(k)}{k^2\rho^2}\left(e^{k\rho} - 2 + e^{-k\rho}\right)dkdt\\
=\;& \frac{1}{\rho} - \int_0^{\mathbf{k}^*}\int_0^t \frac{f(k)}{k^2\rho^2}e^{-\rho t}\left(e^{k\rho} - 2 + e^{-k\rho}\right)dkdt - \int_0^{\mathbf{k}^*}\int_t^{k^*} \frac{f(k)}{k^2\rho^2}e^{-\rho t}\left(e^{-k\rho} - 1\right)dkdt\\
&-\int_0^{\mathbf{k}^*}\int_t^{k^*} \frac{f(k)}{k^2\rho^2}\left(1 - e^{-k\rho}\right)dkdt - \int_{\mathbf{k}^*}^\infty\int_0^{k^*} \frac{f(k)}{k^2\rho^2}e^{-k\rho}e^{-\rho t}\left(e^{2k\rho} - 2e^{k\rho} + 1\right)dkdt\\
=\;& \frac{1}{\rho} + \left[\int_0^t \frac{f(k)}{k^2\rho^2}\left(e^{k\rho} - 2 + e^{-k\rho}\right)dk\frac{e^{-\rho t}}{\rho}\right]_{t=0}^{k^*} - \int_0^{\mathbf{k}^*} \frac{f(t)}{t^2\rho^2}\frac{e^{-\rho t}}{\rho}\left(e^{\rho t} - 2 + e^{-\rho t}\right)dt
\end{aligned}$$

[42] Note that the relevant average duration is not the average contract length, but rather the average time until the next price adjustment of a randomly selected firm. While for a firm from sector k it is true that upon setting a new price at t it will remain fixed until $t+k$, at any point in time the duration of price rigidity for a randomly selected firm from the same sector will be less than k. In fact, given the assumption of uniform staggering of adjustment dates across time, it will be equal to $\frac{k}{2}$. For the heterogeneous economy as a whole this yields $\frac{\overline{k}}{2}$ as the relevant average duration. On this issue, see also Dupor and Tsuruga (2005), and Dixon and Kara (2006).

[43] The only non-trivial step is to compute expressions such as $\int_0^{\mathbf{k}^*}\int_0^t \frac{f(k)}{k^2\rho^2}e^{-\rho t}\left(e^{k\rho} - 2 + e^{-k\rho}\right)dkdt$. The trick is to rearrange them as $\int_0^{\mathbf{k}^*}\left(\int_0^t h_1(k,\rho)\,dk\right)h_2(t,\rho)\,dt$, and integrate by parts by differentiating $\left(\int_0^t h_1(k,\rho)\,dk\right)$ and integrating $h_2(t,\rho)\,dt$.

$$
\begin{aligned}
&+\left[\int_t^{k^*}\frac{f(k)}{k^2\rho^2}\left(e^{-k\rho}-1\right)dk\frac{e^{-\rho t}}{\rho}\right]_{t=0}^{k^*}+\int_0^{\mathbf{k}^*}\frac{e^{-\rho t}}{\rho}\frac{f(t)}{t^2\rho^2}\left(e^{-\rho t}-1\right)dt\\
&-\left[\int_t^{k^*}\frac{f(k)}{k^2\rho^2}\left(1-e^{-k\rho}\right)dkt\right]_{t=0}^{k^*}+\int_0^{\mathbf{k}^*}\frac{f(t)}{t\rho^2}\left(e^{-\rho t}-1\right)dt\\
&-\frac{e^{-\rho\mathbf{k}^*}}{\rho}\int_0^{k^*}\frac{f(k)}{k^2\rho^2}e^{-k\rho}\left(e^{2k\rho}-2e^{k\rho}+1\right)dk\\
=&\ \frac{1}{\rho}\left(1-\int_0^{\mathbf{k}^*}f(k)\frac{1-e^{-\rho k}}{\rho k}dk\right)\\
=&\ \int_0^{k^*}f(k)\frac{\rho k-1+e^{-\rho k}}{\rho^2 k}dk.
\end{aligned}
$$

For the case of a permanent shock just take the limit as $\rho\to 0$.

8d) Counterpart to Proposition 3

The statement and the proof are the same as for the model with Calvo pricing, replacing α_k by $\frac{1}{k}$.

8e) Counterpart to Proposition 4

For an arbitrary heterogeneous economy, in the context of temporary shocks to the growth rate of nominal aggregate demand ($\gamma>0$) and no strategic complementarities ($\theta=1$), the (normalized) cumulative real effect as measured by $\frac{1}{\Delta\mu}\int_0^\infty y(t)\,dt$ is equal to:

$$\int_0^{k^*}f(k)\frac{(\gamma k-1)^2-3+2e^{-\gamma k}}{2\gamma^3 k}dk.$$

In the case of very persistent shocks ($\gamma\approx 0$), it is approximately proportional to the second moment of the distribution of contract lengths in the economy:[44]

$$\frac{1}{\Delta\mu}\int_0^\infty y(t)\,dt\propto\left(\overline{k}^2+\sigma_k^2\right),$$

where $\sigma_k^2\equiv\int_0^{k^*}f(k)\left(k-\overline{k}\right)^2dk$ is the variance of the the distribution of contract lengths in the economy.

[44] The exact same issues regarding the discontinuity discussed in the context of the model with Calvo pricing arise.

Proof

Set $\theta = 1$ in (19) and (20). Then, $\frac{1}{\Delta\mu}\int_0^\infty m(t) - p(t)\,dt$ equals:

$$\int_0^{k^*} \frac{\mu t}{\Delta\mu} + \frac{\left(1-e^{-\gamma t}\right)}{\gamma} - \frac{\mu t}{\Delta\mu} - \int_0^t \frac{f(k)}{k^2\gamma^2}\left[k^2\gamma + \left(e^{-\gamma(t+k)} - e^{-\gamma t}\right)\frac{e^{\gamma k}-1}{\gamma}\right] dkdt$$
$$-\int_0^{k^*}\int_t^{k^*} \frac{f(k)}{k^2\gamma^2}\left[k\gamma t + \left(e^{-\gamma(t+k)} - e^{-\gamma t}\right)\frac{e^{\gamma t}-1}{\gamma}\right] dkdt + \int_{k^*}^\infty \frac{\mu t}{\Delta\mu} + \frac{1-e^{-\gamma t}}{\gamma} dt$$
$$-\int_{k^*}^\infty \frac{\mu t}{\Delta\mu} + \int_0^{k^*} \frac{f(k)}{k^2\gamma^2}\left[k^2\gamma + \left(e^{-\gamma(t+k)} - e^{-\gamma t}\right)\frac{e^{\gamma k}-1}{\gamma}\right] dkdt$$

$$\begin{aligned}
= \;& -\frac{1}{\gamma^2} + \frac{e^{-\gamma k^*} + \gamma k^*}{\gamma^2} + -\frac{1}{\gamma}\left(k^* - \overline{k}\right) - \frac{e^{-\gamma k^*}}{\gamma^2}\int_0^{k^*} f(k)\,\frac{1-e^{-\gamma k}}{\gamma k}\frac{e^{\gamma k}-1}{\gamma k} dk \\
& + \int_0^{k^*} \frac{f(k)}{\gamma^2}\left(\frac{1-e^{-\gamma k}}{\gamma k}\right)^2 dk - \frac{1}{\gamma}\int_0^{k^*} \frac{k}{2} f(k)\,dk + \frac{1}{\gamma^2}\int_0^{k^*} f(k)\left(\frac{1-e^{-\gamma k}}{\gamma k}\right) dk \\
& + \frac{1}{\gamma^2}\int_0^{k^*} \frac{f(k)}{\gamma k}\frac{e^{-\gamma k}-1}{\gamma k} dk + \int_0^{k^*} \frac{f(k)}{\gamma k}\frac{e^{-\gamma k}}{\gamma^2}\frac{1-e^{-\gamma k}}{\gamma k} dk - \frac{e^{-\gamma k^*}}{\gamma^2} \\
& - \frac{e^{-\gamma k^*}}{\gamma^2}\int_0^{k^*} f(k)\left(\frac{e^{-\gamma k}-1}{\gamma k}\frac{e^{\gamma k}-1}{\gamma k}\right) dk \\
= \;& \frac{1}{2}\frac{\overline{k}}{\gamma} + \frac{1}{\gamma^2}\left[\int_0^{k^*} f(k)\left(\frac{1-e^{-\gamma k}}{\gamma k}\right) dk - 1\right] \\
= \;& \frac{1}{\gamma}\left[\int_0^{k^*} f(k)\,\frac{k}{2} dk - \frac{1}{\gamma}\left(1 - \int_0^{k^*} f(k)\,\frac{1-e^{-\gamma k}}{\gamma k} dk\right)\right] \\
= \;& \int_0^{k^*} f(k)\left(\frac{k}{2\gamma} - \frac{\gamma k - 1 + e^{-\gamma k}}{\gamma^3 k}\right) dk \\
= \;& \int_0^{k^*} f(k)\,\frac{(\gamma k - 1)^2 - 3 + 2e^{-\gamma k}}{2\gamma^3 k} dk.
\end{aligned}$$

For the case of very persistent shocks ($\gamma \approx 0$), by the same arguments presented for the model with Calvo pricing, the result is approximately equal to the limit of the above expression when $\gamma \to 0$, which equals $\frac{1}{6}\left(\overline{k}^2 + \sigma_k^2\right)$. The discontinuity that arises in this case is that the effect of a permanent shock equals $\frac{1}{12}\left(\overline{k}^2 + \sigma_k^2\right)$. As in the case presented in the text, the approximate result without discounting provides a good approximation to realistic discount rates.

8f) Discrete time

Firms from sector k set prices for $\frac{k}{h}$ periods of length h as:[45]

$$\begin{aligned} x_t^k &= \arg\min_x \sum_{j=0,h,\ldots}^{k-h} h\beta^j E_t \left(x - p_{t+h}^*\right)^2 \\ &= \frac{1-\beta}{1-\beta^{\frac{k}{h}}} E_t \sum_{j=0,h,\ldots}^{k-h} \beta^j E_t p_{t+h}^*. \end{aligned}$$

The aggregate price level is given by:

$$p_t = \sum_{k=1}^{k^*} f(k) \frac{1}{k} \sum_{j=0,h,\ldots}^{k-h} h x_{t-h}^k,$$

where now the number of sectors must be finite, since they are indexed directly by their contract length.[46]

8g) Obtaining the distribution from BK

Solving the model with Taylor pricing in discrete time requires relatively more computational resources than with the Calvo model, because for a given number of sectors the dimension of the state space is larger in the former. As a result, solving the model with as many sectors becomes quite difficult. To circumvent this problem I construct the distribution of contract lengths from the BK data in a slightly different way.

I consider contract lengths which are multiples of one month, and aggregate the goods and services categories so that the ones which have a mean duration between price changes (as reported by BK) between zero and one month (inclusive) are assigned to the one month contract length sector; the ones with mean duration between price changes between one (exclusive) and two months (inclusive) are assigned to the two month contract length sector, and so on. The sectoral weights are aggregated accordingly. I proceed in this fashion until the sector with contract lengths of 24 months. Finally, I aggregate all the remaining categories, which have mean durations of price rigidity between 24 and 80 months, into a sector with 25-month contracts.[47] This gives rise to 25 sectors with an average contract length of 6.7 months, and a standard deviation of contracts of 5.6 months.

[45] Note that k is always a multiple of h.

[46] To be precise, in the limit this dicrete time model gives rise to a continuous time model with a finite number of sectors. The results presented with a continuum of sectors, however, extend trivially to the case of a discrete distribution.

[47] The total weight of these categories is approximately 2%.

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