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A Habit-Based Explanation of the Exchange Rate Risk Premium

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ABSTRACT

This paper presents a risk premium explanation of the uncovered interest rate parity puzzle. In my two-country model, agents are characterized by slow-moving external habit preferences and they incur proportional and quadratic international trade costs. The precautionary savings effect is assumed to be greater than the inter-temporal consumption-smoothing motive. Thus, times of high risk-aversion correspond to low interest rates. The domestic investor receives a positive exchange rate risk premium when he is effectively more risk-averse than his foreign counterpart. As a result, the domestic investor receives a positive risk premium when interest rates are lower at home than abroad.

Keywords: Exchange rate, Time-varying risk premium, Habits.

JEL: F31, G12, G15.

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According to the standard uncovered interest rate parity (U.I.P) condition, the expected change in exchange rate should be equal to the interest rate differential between foreign and domestic securities. Assuming rational expectations, this means that a simple regression of exchange rate variations on interest rate differentials should lead to a regression coefficient of 1. Instead, empirical work following Hansen & Hodrick (1980) and Fama (1984) consistently reveals a regression coefficient smaller than 1 and very often negative.[1] Froot & Thaler (1990) report that, in a survey of 75 published estimates, the slope coefficient of the regression of the nominal exchange rate appreciation on nominal interest rates is always below unity (positive in a very few cases, and -0.88 on average). The international economics literature refers to negative U.I.P slope coefficients as U.I.P puzzles or forward premium anomalies.

A U.I.P slope coefficient below 1 implies nonzero predictable excess returns for an investor borrowing funds at home at a risk-free rate, changing his currency for a foreign equivalent, lending on the corresponding foreign market for a fixed period and finally reconverting his earnings to the original currency.[2] There are two possible explanations for predictable excess returns: time-varying risk premia and/or expectational errors. In this paper, I assume that expectations are rational and I present a risk premium explanation of the uncovered interest rate parity puzzle.

Backus, Foresi, & Telmer (2001) describe the necessary features of a model that might account for the forward premium anomaly under log-normality of pricing kernels: a negative correlation between the difference in conditional means on the one hand and the half difference in conditional variances of the two log pricing kernels (which is the currency risk premium) on the other hand, and great volatility in the risk premium. The two-country model I present in this paper fulfills Backus et al. (2001)'s conditions.

[1]The U.I.P condition appears to be a reasonable assumption only in four cases. Bansal & Dahlquist (2000) show that U.I.P is not rejected at high inflation levels, and likewise Huisman, Koedijka, Kool, & Nissen (1998) find that U.I.P holds for very large forward premia. Chaboud & Wright (2005) show that U.I.P is valid at very short horizons but is rejected for horizons above a few hours. Meredith & Chinn (2005) find that U.I.P cannot be rejected at horizons above 5 years. Lothian & Wu (2005) find positive U.I.P slope coefficients for France/UK and US/UK on annual data over 1800-1999, because of the 1914-1949 subsample. See Engel (1996) and Chinn (2006) for surveys.

[2]Predictability regressions are plagued with small sample bias and persistence in the right hand side variables, but Liu & Maynard (2005) and Maynard (2006) show that these biases can only explain part of the puzzle.

I assume that endowment shocks are *i.i.d.* Agents are characterized by slow-moving external habit preferences similar to Campbell & Cochrane (1999), but different in a key way: the precautionary savings effect is assumed to be greater than the inter-temporal consumption-smoothing motive.[3] Thus, real risk-free rates are low in bad times and high in good times. In goods markets, agents can trade across countries, but they incur proportional and quadratic trade costs.

With this model, I obtain two novel theoretical results. First, the model gives a rationale for the existence of a currency risk premium and for its symmetry.[4] A domestic investor expects to receive a positive foreign currency excess return in times when he is more risk-averse than his foreign counterpart. Times of high risk-aversion correspond to low interest rates at home because of the predominant role of precautionary savings. Thus domestic investors enjoy positive currency excess returns when domestic interest rates are low and foreign interest rates are high. In this model, domestic currency excess returns increase sharply with (foreign minus domestic) interest rate differentials and this leads to a negative U.I.P coefficient.

Second, the introduction of international trade costs resolves the real exchange rate volatility quandary described by Brandt, Cochrane, & Santa-Clara (2006). In complete markets, the real exchange rate is theoretically equal to the ratio of foreign and domestic stochastic discount factors. We know since Mehra & Prescott (1985) and Hansen & Jagannathan (1991) that stochastic discount factors must have a large variance in order to price stock excess returns. Taking into account the low correlation among consumption shocks across countries, and thus the low correlation of stochastic discount factors under power utility, Brandt et al. (2006) show that the actual exchange rate is much smoother than the theoretical one. In my model, endowment shocks are uncorrelated across countries, but countries share some risks because trade costs are finite. As a result, the variance of the theoretical exchange rate remains low.

To assess these theoretical results, two experiments are conducted. I calibrate and

[3]The habit literature has reproduced a wide variety of dynamic asset pricing phenomena. Major examples are Sundaresan (1989), Constantinides (1990), Campbell & Cochrane (1999) and Chan & Kogan (2002). In multi-country models, Shore & White (2003) and Gomez, Priestley, & Zapatero (2004) study international portfolio holdings under external habit preferences.

[4]If a domestic investor gets a positive currency excess return by borrowing at home and lending abroad, his foreign counterpart's return is negative.

simulate this two-country model with habit preferences and proportional and quadratic trade costs. With infinite trade costs, I derive closed-form expressions for the U.I.P slope coefficient, the Sharpe ratio and the mean and variance of real interest rates. The simulation successfully targets all these moments plus the mean and standard deviation of consumption growth. The simulated exchange rate, however, varies three times more than in the data; it is also too highly correlated with consumption growth. When the cost of international trading is finite, the standard deviation of the real exchange rate decreases to its empirical counterpart. The model reproduces the first and second moments of interest rates and exchange rates and their correlation. But it cannot fully account for Backus & Smith (1993)'s puzzle: although the correlation between differences in consumption growth and changes in the real exchange rate is no longer equal to one as with CRRA preferences, it remains higher than in the data.

In addition, I estimate the model by minimizing pricing errors from Euler conditions. As there is only one source of shocks in each country, pricing kernels can be theoretically recovered using either consumption data or interest rates. I use two sets of currency excess returns as test assets. I first consider the investment opportunities of an American investor in 8 other OECD countries. I then focus on the 8 portfolios of currency excess returns built in Lustig & Verdelhan (2005). These portfolios create a large cross-section of excess returns by taking into account many investment opportunities in currencies, without imposing the estimation of a large variance-covariance matrix. Following Hansen, Heaton, & Yaron (1996), a continuously-updating general method of moments (G.M.M) estimator is used. Estimates based either on consumption data or on interest rate data lead to reasonable parameters when pricing the currency excess returns of an American investor. Furthermore, the hypothesis that the pricing errors are zero cannot be rejected at conventional confidence levels.

This paper adds to a large body of empirical and theoretical literature. Empirically, most papers test the U.I.P condition on nominal variables. Two recent studies, however, relate the puzzle to real variables. Hollifield & Yaron (2003) decompose the currency risk premium into conditional inflation risk, real risk, and the interaction between inflation and real risk. They find evidence that real factors, not nominal ones, drive most of the predictable variation in currency risk premia.[5] Lustig & Verdelhan (2005) show that the

[5] Hollifield & Yaron (2003) conclude that:

risk premia produced by asset pricing factors based on real consumption growth risk line up with predictable excess returns in currency markets. One can thus conclude that the forward premium anomaly is primarily a real puzzle.

Theoretically, numerous studies have attempted to explain the U.I.P puzzle under rational expectations, but few models reproduce the negative U.I.P slope coefficient. Appendix (A) presents a literature review and a synthetic view of the assumptions and results of these attempts. I will here present the three most successful studies. Frachot (1996) shows that a financial two-country Cox, Ingersoll, & Ross (1985) framework can account for the U.I.P puzzle but he does not provide an economic interpretation of the currency risk premium. Alvarez, Atkeson, & Kehoe (2005) use endogenously segmented markets. In their model, higher money growth leads to higher inflation, thus inducing more agents to enter the asset market because the cost of non-participation is higher. This leads to a decrease in risk premium. If the segmentation is sufficiently large and sensitive to money growth, this time-varying risk qualitatively generates the forward premium anomaly. To quantitatively reproducing the U.I.P puzzle, the model implies the presence of very large flows in and out of asset markets. Bacchetta & van Wincoop (2005) develop a model where investors face costs of collecting and processing information. Because of these costs, many investors optimally choose to assess available information and revise their portfolios infrequently. Thus, rational inattention produces a negative U.I.P coefficient along the lines suggested by Froot & Thaler (1990) and Lyons (2001): if investors are slow to respond to news of higher domestic interest rates, there will be a continued reallocation of portfolios towards domestic bonds and a appreciation of the currency subsequent to the shock. Bacchetta & van Wincoop (2005) obtain negative U.I.P slope coefficient for information and trading costs higher than 2 percent of total financial wealth.

The rest of this paper is organized as follows. Section II outlines the two-country one-good model. Section III details the mechanism that leads to a negative U.I.P slope coefficient. Section IV summarizes the simulation results with and without proportional or quadratic trade costs. Section V presents the estimation exercises using either consump-

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Virtually none of the predictable variation in returns from currency speculation can be explained empirically by predictable variation in conditional inflation risk and in the interaction between conditional inflation and real risks. Models of a rational currency risk premium should focus on real risk.

tion data or interest rates to compute stochastic discount factors. Section VI concludes.

I. Model

This paper builds on the international economics and finance literature. I first focus on the trade aspect of the model. I then turn to the definition of the real exchange rate before describing in details a representative agent's preferences and their asset pricing implications.

A. International trade

There are two countries -with same initial wealth- and one good. International trade is possible but costly. I abstract from the production side of each country and consider two endowment economies. In each country, the representative agent is characterized by preferences similar to Campbell & Cochrane (1999) but with a time-varying risk-free interest rate. I describe first the international trade mechanism and then preferences.

The shipping costs have two components. The first one is the usual iceberg-like trade cost. When a unit of the good is shipped, only a fraction $1-\tau$ arrives to the foreign shore. The second component is a quadratic cost, which captures the capacity constraints of international trade and ensures that the total cost of trade increases with the volume of international trade. Thus, this quadratic cost is assumed to be proportional (with coefficient δ) to the ratio of exports to endowments as in Backus, Kehoe, & Kydland (1992).

Let X_t denote the amount of the good exported from a domestic to a foreign country at time t. A superscript * refers to the same variable for the foreign country. The amount of exports $X_t \geq 0$ and $X_t^\star \geq 0$ solve the planning problem:

$$\text{Max } E\sum_{t=0}^{\infty}\beta^t \frac{(C_t - H_t)^{1-\gamma}-1}{1-\gamma} + E\sum_{t=0}^{\infty}\beta^t \frac{(C_t^\star - H_t^\star)^{1-\gamma}-1}{1-\gamma}, \tag{1}$$

subject to:

$$C_t = Y_t - X_t + X_t^\star(1 - \tau - \frac{\delta}{2}\frac{X_t^\star}{Y_t}) \text{ and } C_t^* = Y_t^\star - X_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}), \qquad (2)$$

where γ is the risk-aversion coefficient, Y_t and $Y_t^\star$ denote the endowment, H_t and $H_t^\star$ the external habit level and C_t and $C_t^\star$ the amount of consumption in, respectively, the domestic and foreign country. The law of motion of the external habit level in each country does not depend on contemporaneous consumption and the planning problem reduces to a sequence of static problems.[6]

If one country exports, the other does not as there is only one good in the model. Let us assume first that the domestic country exports ($X_t \geq 0$, $X_t^\star = 0$). The first order condition with respect to X_t is then:

$$-[Y_t - X_t - H_t]^{-\gamma} + [1 - \tau - \delta\frac{X_t}{Y_t^\star}][Y_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma} = 0. \qquad (3)$$

The optimal amount of exports is the solution to equation (3) provided that it is positive and satisfies the following conditions: exports are below endowments, consumptions are above habit levels in both countries; a positive fraction of the export makes it to the shore. A closed form solution can be found for log utility ($\gamma = 1$) or when there is no quadratic cost.[7] When it exists, the optimal amount of exports is equal to:

$$X_t = \frac{Y_t - H_t - (1-\tau)^{-\frac{1}{\gamma}}(Y_t^\star - H_t^\star)}{1 + (1-\tau)^{1-\frac{1}{\gamma}}}$$

The case of foreign country exports is obviously symmetric. If the foreign country

[6]As a result, the external habit level can be interpreted as a subsistence level, as a social externality or as a preference shock.

[7]Appendix (B) studies the general case of proportional and quadratic costs. When there is no quadratic cost, the domestic country exports when $(Y_t^\star - H_t^\star)(1-\tau)^{-\frac{1}{\gamma}} < (Y_t - H_t)$. The foreign country exports when $(Y_t^\star - H_t^\star) > (1-\tau)^{-\frac{1}{\gamma}}(Y_t - H_t)$. As a result, there is no trade when $(1-\tau)^{\frac{1}{\gamma}} \leq (Y_t^\star - H_t^\star)/(Y_t - H_t) \leq (1-\tau)^{-\frac{1}{\gamma}}$.

exports ($X_t = 0$, $X_t^\star \geq 0$), the first order condition with respect to $X_t^\star$ is then:

$$-[Y_t^\star - X_t^\star - H_t^\star]^{-\gamma} + [1 - \tau - \delta\frac{X_t^\star}{Y_t}][Y_t + X_t^\star(1 - \tau - \frac{\delta}{2}\frac{X_t^\star}{Y_t}) - H_t]^{-\gamma} = 0. \tag{4}$$

When it exists, the optimal amount of exports is equal to:

$$X_t^* = \frac{Y_t^\star - H_t^\star - (1-\tau)^{-\frac{1}{\gamma}}(Y_t - H_t)}{1 + (1-\tau)^{1-\frac{1}{\gamma}}}.$$

If there are no positive solutions to both export problems, then countries consume their endowments. There is a no-trade zone in which the marginal utility gain of shipping a good is more than offset by the trade cost. Figure (1) summarizes the different cases. Without quadratic costs, the real exchange rate moves behind two constant boundaries when there is no trade and remains on a boundary when one country exports as shown by Dumas (1992). With quadratic costs, real exchange rates are never constant even when countries export.

The setting presented in this paper relates to a large literature in international economics. Proportional (iceberg-like) shipping costs were first proposed by Samuelson (1954), and then used by Dumas (1992), Sercu, Uppal, & Hulle (1995), Sercu & Uppal (2003) and Obstfeld & Rogoff (2000) to study real exchange rates. Yet, none of these papers tackle the forward premium puzzle, and Hollifield & Uppal (1997) show that proportional trade costs are not enough to reproduce the forward premium puzzle when agents are characterized by constant relative risk-aversion (CRRA). They find that the implied U.I.P slope coefficient is never negative, not even for extreme levels of constant risk-aversion or trade costs. In this paper, I show that endogenous time-varying risk-aversion is key to reproduce the forward premium puzzle no matter the assumption on trade costs.

B. Real exchange rate in complete financial markets

I now turn to the assumptions on financial markets and their implications for the definition of the real exchange rate.

Complete financial markets I assume that there are no arbitrage and that financial markets are complete.[8] In each country, at each date, a representative investor has access to a domestic bond that pays off one unit of domestic consumption next period in all states of the world and to a foreign bond that pays off one unit of domestic consumption next period in all states of the world. The change in the real exchange rate is defined as the ratio of the two stochastic discount factors at home and abroad:

$$\frac{Q_{t+1}}{Q_t} = \frac{M^\star_{t+1}}{M_{t+1}}, \tag{5}$$

where Q is expressed in domestic goods per foreign good.[9] Given Q_0, the exchange rate at date 0, equation (5) gives the entire path of Q.

I follow Alvarez et al. (2005) to define Q_0. They assume that at date 0, each representative investor is endowed with claims on domestic and foreign consumptions. Let A and $A^\star$ be the initial claims of the domestic investor on respectively domestic and foreign consumption. Then, the exchange rate at date 0 is equal to $Q_0 = (\overline{A} - A)/A^\star$, where $\overline{A}$ is the equilibrium asset holding. The numerator corresponds to the number of claims on domestic consumption that the domestic investor exchanged for claims on the foreign consumption (in the denominator). Exchange rate at date t is then defined recursively using equation (5).

Exchange rate When there is trade, one first-order condition (3) or (4) of the social planner's problem is satisfied, and the countries share risk. When there is no trade, the real exchange rate is determined on the asset market as the ratio of the two marginal utilities of consumption. To summarize, the real exchange rate Q_t can take the following values:

- If the domestic country exports, $\frac{Q_{t+1}}{Q_t} = \frac{1}{1-\tau-\delta X_t/Y^\star_t}$;

[8] Assuming the "law of one price on the asset markets" implies the existence of a stochastic discount factor M_{t+1}. Assuming the "absence of arbitrage" is stronger: it implies the existence of a *positive* M_{t+1}, see Cochrane (2001). I use the latter assumption because it also implies the uniqueness of M_{t+1} in complete markets. Note that the form of the utility function in this paper guarantees that $M_{t+1} > 0$.

[9] The Euler equation for a foreign investor buying a foreign bond is: $E_t(M^\star_{t+1}R^\star_{t+1}) = 1$. The Euler equation for a domestic investor buying a foreign bond is: $E_t(M_{t+1}R^\star_{t+1}\frac{Q_{t+1}}{Q_t}) = 1$. Because the stochastic discount factor is unique in complete markets, equation (5) follows.

- If the foreign country exports, $\frac{Q_{t+1}}{Q_t} = 1 - \tau - \delta X_t^\star / Y_t$;
- If there is no trade, $\frac{Q_{t+1}}{Q_t} = (\frac{Y_t^\star - X_t^\star}{Y_t - X_t})^{-\gamma}$.

Note that an increase in the trade cost τ or a decrease in the risk aversion coefficient γ enlarges the no-trade zone and thus increases the real exchange rate volatility as in Sercu & Uppal (2003).[10] The amount of trade and the real exchange rate depend on habit levels in each country, which I now turn to.

C. Habit-based preferences

I assume external habit preferences similar to Campbell & Cochrane (1999) but with time-varying risk-free rates.[11] I show in this paper that a model reproducing the equity premium puzzle can also rationalize the forward premium puzzle.[12]

In each country, the habit level is related to consumption through the following AR(1) process of the surplus consumption ratio $S_t \equiv (C_t - H_t)/C_t$:

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g). \tag{6}$$

Lowercase letters correspond to logs, $\lambda(s_t)$ is the sensitivity function, and g is the average growth rate of the log-normal consumption process.

[10] Sercu & Uppal (2003) study the impact of trade costs on exchange rate volatility and international trade using a power-utility framework for a two-country, two-good world. Assuming log-normal outputs, they show that a drop in shipping costs implies a decrease in the variance of the real exchange rate.

[11] Campbell & Cochrane (1999) preferences generate pro-cyclical variations of stock prices, long horizon predictability, counter-cyclical variation of stock market volatility, counter-cyclicality of the Sharpe ratio and the short- and long-run equity premium but Lettau & Ludvigson (2003) note that the variance of the Sharpe ratio implied by this model is smaller than its empirical counterpart.

[12] Abandoning power utility and looking among other asset pricing frameworks that reproduce the equity premium puzzle, several paths seem a-priori possible. These possibilities are based on one of the following assumptions: the introduction of heterogeneity or the use of state-nonseparability with Epstein-Zin preferences (as in Bansal & Yaron (2004)). Sarkissian (2003) notes that heterogeneity alone can not produce a complete explanation of the U.I.P puzzle. Colacito & Croce (2005) study real exchange rates in the Epstein-Zin framework but do not reproduce the U.I.P puzzle.

External habits The habit is assumed here to depend only on aggregate, not on individual, consumption. Thus, the inter-temporal marginal rate of substitution is here:

$$M_{t+1} = \beta \frac{U_c(C_{t+1}, X_{t+1})}{U_c(C_t, X_t)} = \beta (\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t})^{-\gamma} = \beta e^{-\gamma \left[g + (\phi - 1)(s_t - \bar{s}) + (1 + \lambda(s_t))(\Delta c_{t+1} - g) \right]}.$$

Campbell & Cochrane (1999) suggest the following sensitivity function:

$$\lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1, \text{ when } s \leq s_{\max}, \text{ 0 elsewhere},$$

where $\bar{S}$ and $s_{\max}$ are respectively the steady-state and upper bound of the surplus-consumption ratio.

D. Exchange rate risk premium

The exchange rate risk premium is the excess return of a domestic investor who borrows funds at home, changes his currency to a foreign equivalent, lends on the foreign market for a defined period and finally reconverts his earnings to the original currency. Thus, in logs, the foreign currency excess return r^e_{t+1} is equal to:

$$r^e_{t+1} \simeq \Delta q_{t+1} + r^\star_t - r_t, \tag{7}$$

where r_t and $r^\star_t$ are respectively the domestic and foreign risk-free real interest rates. The domestic investor gains $r^\star_t$, but he has to pay r_t, and he loses if the dollar appreciates in real terms - q decreases - when his assets are abroad. Backus et al. (2001) show that the expected foreign currency excess return is equal to the half difference in conditional variances of the two pricing kernels:[13]

$$E_t(r^e_{t+1}) = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m^\star_{t+1}). \quad (8)$$

II. Mechanism

In this section, I abstract from trade and consider post-trade consumption.[14] In this case, I derive a closed-form expression for the currency excess return that highlights the rationale and mechanism of the model. I assume that in both countries idiosyncratic shocks are $i.i.d$ log-normally distributed:

$$\Delta c_{t+1} = g + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0, \sigma^2).$$

Moreover, to keep the model simple and tractable, I assume that the two endowment shocks u_{t+1} and $u^\star_{t+1}$ are independent and that the domestic and foreign investors are characterized by the same underlying structural parameters: $\gamma = \gamma^*$, $S = S^\star$, $\phi = \phi^\star$, $\sigma = \sigma^\star$.[15]

[13]I reproduce here Backus et al. (2001)'s proof in the case of complete markets. Assuming log-normal stochastic discount factors leads to: $r_t = -\log E_t M_{t+1} = -\ E_t \log M_{t+1} - \frac{1}{2}Var_t(\log M_{t+1})$, and $r^\star_t = -\log E_t M^\star_{t+1} = -\ E_t \log M^\star_{t+1} - \frac{1}{2}Var_t(\log M^\star_{t+1})$. The expected change in the exchange rate is then:

$$E_t(\log \frac{Q_{t+1}}{Q_t}) = E_t(\log M^\star_{t+1}) - E_t(\log M_{t+1}) = -r^\star_t + r_t - \frac{1}{2}Var_t(\log M^\star_{t+1}) + \frac{1}{2}Var_t(\log M_{t+1}).$$

Equation (8) follows. Another way to obtain the same result is to start from the definitions of the log currency risk premia in Lustig & Verdelhan (2005) for the domestic and foreign investors, specialized to the case where pricing kernels are uncorrelated.

[14]In this case, the real exchange rate is the rate at which the two countries do not want to trade further.

[15]These assumptions can be relaxed in the simulation. Baxter & Crucini (1995) find that productivity shocks in the US and Europe exhibit a low positive correlation of 0.22. Taking this correlation into account decreases the volatility of real exchange rates (see equation (13) in the third section), but it does not modify substantially the results obtained on the forward premium.

A. Pro-cyclical risk-free rates

Assuming that $\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$ and $s_{\max} = \overline{s} + (1 - \overline{S}^2)/2$ leads to a linear time-varying risk-free rate:

$$r_t = \overline{r} - B(s_t - \bar{s}), \tag{9}$$

where $\overline{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2\sigma^2}{2\bar{S}^2}$ and $B = \gamma(1-\phi) - \frac{\gamma^2\sigma^2}{\bar{S}^2}$.[16] The assumption of a nonzero B has also been used by Buraschi (2004) and Wachter (2006) to model the US yield curve, and by Menzli, Santos, & Veronesi (2004) to study cross-sections of US assets.

In this paper, I emphasize results obtained with a negative B. What is the economic rationale behind the sign of B? Consumption smoothing and precautionary savings affect the real interest rate, and the parameter B here summarizes these two different effects.

- In good times, after a series of positive consumption shocks that result in a high surplus consumption ratio s, the agent wants to save more in order to smooth consumption. This leads to a decrease in the interest rate through an inter-temporal substitution effect.

- But, in good times, the representative agent is less risk-averse (the local curvature of his utility function is γ/s_t). He is less interested in saving, leading to an increase in the real interest rate through a precautionary saving effect. Conversely, in bad times, when the surplus consumption ratio is low, the agent is very risk averse and saves more.

The case of $B < 0$ is thus the one in which the precautionary effect overcomes the substitution effect. As a result, interest rates are low in bad times and high in good times. This framework reproduces the U.I.P puzzle.

B. An interpretation of the U.I.P puzzle

With these preferences, the variance of the log stochastic discount factor is equal to:

$$Var_t(m_{t+1}) = \frac{\gamma^2\sigma^2}{\bar{S}^2}[1 - 2(s_t - \bar{s})]. \tag{10}$$

[16] Note that when the interest rate is allowed to fluctuate in Campbell & Cochrane (1999)'s model, it closely resembles the framework proposed by Cox et al. (1985), which Frachot (1996) has shown reproduces the forward premium.

Then, equation (8) leads to the following expected currency excess return:

$$E_t(r^e_{t+1}) = \frac{\gamma^2\sigma^2}{\bar{S}^2}(s^\star_t - s_t). \tag{11}$$

This formulation of the exchange rate risk premium presents three interesting features.

First, it gives a rationale for the existence of the currency premium and for its symmetry. In this framework, the local curvature of the utility function is equal to $\frac{\gamma}{S_t}$, thus lower surplus consumption ratios entail more risk-averse agents. *The domestic investor gets a positive excess return at date t if he is more risk averse than his foreign counterpart.* The interpretation of the risk premium is perfectly symmetric, thus taking into account that a positive excess return for the domestic investor means a negative one for his foreign counterpart. The currency risk premium is time-varying because risk-aversion is time-varying too.[17]

Second, this proposed formulation of the exchange rate risk premium also offers a possible explanation for the U.I.P puzzle. The expected change in exchange rate is equal to:

$$E_t(\Delta q_{t+1}) = [1 + \frac{1}{B}\frac{\gamma^2\sigma^2}{\bar{S}^2}]\,[r_t - r^\star_t] = \gamma(\frac{1-\phi}{B})\,[r_t - r^\star_t]\,. \tag{12}$$

In this framework, the U.I.P slope coefficient no longer needs to be equal to unity even if consumption shocks are simply *i.i.d.* *Since the risk premium depends on the interest rate gap, the coefficient α in a U.I.P regression can be below* 1 *and, when $B < 0$, even negative.* Since this model can reproduce a negative U.I.P coefficient, it can naturally satisfy the two Fama (1984) conditions presented in Appendix (A). These conditions were derived assuming $\alpha < 0$ for the first one and $\alpha < 1/2$ for the second one. What is the intuition for this result? When the surplus consumption ratio s_t is low, the domestic agent is very risk-averse. As the precautionary savings effect dominates the inter-temporal smoothing one (for a negative B), domestic interest rates are low. A domestic investor expects to receive a positive foreign currency excess return in times when he is more risk-

[17]Lustig & Verdelhan (2005) show that currency excess returns are related to the conditional variances of the log stochastic discount factors and their conditional correlation. Note that here consumption growth shocks are uncorrelated across countries, leading to uncorrelated pricing kernels. Thus, only time-variation in conditional variances of the pricing kernels impacts the currency risk premium.

averse than his foreign counterpart. Thus the domestic investor enjoys positive foreign currency excess returns when domestic interest rates are low and foreign interest rates are high. This translates to a U.I.P coefficient less than 1. It can even be negative because in times of high risk-aversion, a small consumption shock has a large impact on the change in marginal utility, and the stochastic discount factor has a considerable conditional variance $Var_t(\log M_{t+1})$. As a consequence, when interest rates are low, the conditional variance of the stochastic discount factor is high and the excess return is high.

We can refine this interpretation using Backus et al. (2001) conditions to reproduce the U.I.P puzzle: a negative correlation between the difference in conditional means and the half difference in conditional variances of the two pricing kernels, and a greater volatility of the latter.[18] The difference in conditional means of the pricing kernels is equal to $\gamma(1-\phi)(s_t-s_t^\star)$. The currency risk premium, which is the half difference in conditional variances of the two pricing kernels, is given in equation (11). The two are clearly negatively correlated. The risk premium has a larger variance than the difference in conditional means if $\gamma^2\sigma^2/\bar{S}^2$ is above $\gamma(1-\phi)$, which is the case for pro-cyclical interest rates ($B < 0$). As a consequence, the U.I.P coefficient is negative.

Third, *in the very long run, the risk premium disappears if the two countries have the same intrinsic characteristics.* If the two countries are similar (same average consumption growth rate g, risk-aversion γ, persistence ϕ and average surplus consumption ratio $\bar{S}$), then the average real risk free rate is the same in both countries. Taking unconditional expectations of equation (12) shows that the change in the real exchange rate and the risk premium are on average equal to zero. In the long run, two similar countries satisfy P.P.P convergence tests.[19]

C. Exchange rates and consumption

The model presented in this paper offers a simple general equilibrium explanation for the U.I.P puzzle, in which consumption growth shocks drive surplus-consumption ratios, time-

[18] Backus et al. (2001) show that in this framework, reproducing the U.I.P puzzle entails potentially negative interest rates. This is the case here. With a negative parameter B, real interest rates can be negative for very low values of the surplus consumption ratio.

[19] If the two countries have different structural parameters however, the change in the real exchange rate does not have to be zero in the long run: $E(\Delta q) = \overline{r} - \overline{r}^\star + \frac{1}{2}\frac{\gamma^2\sigma^2}{\bar{S}^2} - \frac{1}{2}\frac{\gamma^{\star 2}\sigma^{\star 2}}{\bar{S}^{\star 2}}$.

varying risk-aversions, interest rates and exchange rates. As a result, the model implies a strong and positive correlation between changes in exchange rate and consumption growth.

Yet, Backus & Smith (1993) find that the actual correlation between exchange rates and consumption is low and often negative. Chari, Kehoe, & McGrattan (2002), Corsetti, Dedola, & Leduc (2004) and Benigno & Thoenissen (2006) confirm their findings.[20] In the model presented here, the presence of habits leads to a lower correlation than with power utility, but it still implies too high a correlation between exchange rates and consumption.[21] This shortcoming calls for future work in at least two directions. First, Lustig & Verdelhan (2005) show that the correlation between consumption growth and exchange rates depends on interest rates differentials. Because the correlation switches sign when the interest rate differential fluctuates, a simple unconditional measure might not show an existing link between exchange rates and consumption growth. Second, Chari et al. (2002) show that relaxing the complete markets assumption is not enough to solve the puzzle, but Benigno & Thoenissen (2006) claim that a model with incomplete markets and non-traded intermediate goods goes a long way towards its solution.

III. Simulation

To better assess the performance of the model, I have performed three sets of simulations, with or without proportional and quadratic trade costs. Simple closed-form expressions in the post-trade case can be obtained for a few interesting moments, thus making the calibration straightforward. All simulations use the same set of parameters, but vary in their levels of proportional and/or quadratic trade costs. Below, I describe the calibration parameters and the simulation results. Finally, as a reality check, I compute the time-series of the stochastic discount factor, the surplus consumption ratio and the local curvature using actual US consumption data.

[20] Chari et al. (2002) find for example a low correlation of $corr(\Delta \log q_{t+1}, \Delta c_{t+1} - \Delta c^{\star}_{t+1}) = -0.15$ for US and Germany using HP-filtered data for the 1973:1-1994:4 period.

[21] Backus & Smith (1993) note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient ($\log \Delta q_{t+1} = -\gamma[\Delta c^{\star}_{t+1} - \Delta c_{t+1}]$), thus implying a perfect correlation between the consumption growth and real exchange rate variations.

A. Calibration

I assume that two countries, for example the United States and Germany, can be characterized by the same set of parameters (g, σ, β, γ, ϕ and S) and that endowment shocks are not correlated across countries.

To determine the six independent parameters of the model, I target six simple statistics: the mean g and standard deviation σ of consumption growth rate, the mean $\overline{r}$ and standard deviation σ_r of the real interest rate, the U.I.P coefficient α, and the steady-state Sharpe ratio $\overline{SR}$. The first three moments are clearly linked to structural parameters of the model. Starting from post-trade consumption characteristics, one can obtain closed-form expressions for the last three moments.[22] These six statistics are measured over the 1947:2-2004:4 period for the US economy. Per capita consumption data of non durables and services are from the BEA. US interest rates, inflation and stock market excess returns are from CRSP (WRDS). Expected inflation is computed using a one-lag two-dimension VAR (inflation and interest rates). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. The Sharpe ratio is obtained as the ratio of the unconditional mean of monthly stock excess returns on their unconditional standard deviation. The U.I.P coefficient is computed using the US-Germany exchange rate. German interest rates and inflation rates are from Global Financial Data. Table (I) summarizes the parameters used in this paper. They appear close to the ones proposed by Campbell & Cochrane (1999) and Wachter (2006). The calibration choices outlined above lead to a reasonable risk-aversion coefficient of 2.2. Consumption is on average 8 percent above the habit level, with a maximum spread of 12 percent.

To model trade, I need to calibrate additionally the proportional and quadratic trade costs. Anderson & van Wincoop (2004) provide an extensive survey of the trade cost literature. They conclude that total international trade costs, which include transportation costs and border-related trade barriers, represent an ad-valorem tax of about 74%.[23]

[22] An exact closed-form expression for the standard deviation of the interest rate is difficult to obtain, but the choice of parameters can be based on a simple approximation: supposing that $\lambda(s_t)$ remains equal to its steady-state value ($\lambda(\overline{s}) = (1-\bar{S})/\bar{S}$), the variance of the interest rate is close to $(\sigma/\overline{S})^2 B^2/(1-\phi^2)$, where $\bar{S}$ is defined in terms of σ, γ, ϕ and B. Adding the closed-form expression of the U.I.P coefficient ($\alpha = (1-\phi)\gamma/B$) and the Sharpe ratio at steady-state $\overline{SR} = \gamma\sigma/\bar{S}$ produces three conditions.

[23] Border-related trade barriers represent a 44% cost. This estimate is a combination of direct observation and inferred costs. Transportation costs represent 21%.

Obstfeld & Rogoff (2000) assume a conservative trade cost of 25%. I simulate the model with a proportional trade cost τ equal to 0 (no trade cost), 25%, 50%, 75% and 100% (infinite trade costs). In the baseline scenario, I use $\delta = 0.2$ as in Backus et al. (1992). This ensures that trade costs increase with trade, but reasonably so: when a country imports the equivalent of 20% of its endowment, the trade cost increases by 2 percentage points. When I abstract from proportional trade costs, I simulate the model with δ equal to 0.1, 0.5, 1, 10 and 20.

From $100,000$ endowment shocks and the parameters above, I build surplus consumption ratios, stochastic discount factors, interest rates in both countries and their exchange rate. Appendix (B) details the procedure. I then regress the quarterly variation of the real exchange rate on the real interest rate differential to find the slope coefficient α from a U.I.P test. Table (II) reports results obtained on consumption growth, consumption volatility, mean and volatility of interest rates, volatility and autocorrelation of exchange rates, Sharpe ratios, correlation between exchange rate and relative consumption, mean openness ratio and the U.I.P slope coefficient.[24] I first review results obtained under autarky. I then turn to cases of international trade.

B. Results under autarky

I first review the moments outlined in the calibration process and then turn to the properties of the implied real exchange rate under autarky. Results are presented in the sixth column of panel A, Table (II).

U.I.P coefficient, variance of the interest rate and Sharpe ratio As expected, the U.I.P slope coefficient α is negative and in line with its empirical value. This is also the case for the interest rate standard deviation and the average Sharpe ratio targeted by the calibration.[25] Thus Campbell & Cochrane (1999)'s preferences can, in a two-country model, reproduce the negative U.I.P slope coefficient without either endangering

[24]The openness ratio is computed as the sum of exports and imports divided by the sum of the endowments.

[25]Note however that a high persistence coefficient ϕ imposes a high autocorrelation of real interest rates. Nominal interest rates are highly correlated at both annual and quarterly frequencies, real interest rates are highly correlated at annual frequencies, but quarterly real interest rates are not because quarterly inflation is volatile.

Table I
Calibration parameters

The table presents the parameters of the model and the corresponding actual moments. Data are quarterly. The reference period is here 1947:2-2004:4 (1947-1995 in Campbell & Cochrane (1999), 1952:2-2004:3 in Wachter (2006)). Per capita consumption of non durables and services is from the BEA web site. Interest rates and inflation data are from CRSP(WRDS). Expected inflation is computed using a one-lag two-dimension VAR (inflation and interest rates). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. The Sharpe ratio is obtained as the ratio of the unconditional mean of monthly stocks excess returns on their unconditional standard deviation. The U.I.P coefficient is computed using the US-German exchange rate. German interest rates, inflation rates and exchange rates are from Global Financial Data.

	My parameters	**Campbell & Cochrane (1999)**	**Wachter (2006)**
		Calibrated parameters	
$g(\%)$	0.53	0.47	0.55
$\sigma(\%)$	0.51	0.75	0.43
$\overline{r}(\%)$	0.34	0.23	0.66
γ	2.19	2.00	2.00
ϕ	0.99	0.97	0.97
B	-0.01	–	0.01
τ	0/0.25/0.5/0.75/1	–	–
δ	0/1	–	–
		Implied parameters	
β	1.00	0.97	0.98
$\overline{S}$	0.08	0.06	0.04
$S_{\max}$	0.12	0.09	0.06

Table II
Simulation Results

The table presents the mean (g) and standard deviation (σ) of real per capita consumption growth, the mean ($\overline{r}$) and standard deviation (σ_r) of the real interest rate and the standard deviation ($\sigma_{\Delta q}$) and autocorrelation ($\rho_{q_t,q_{t-1}}$) of the real exchange rate. $\overline{SR}$ denotes the mean Sharpe ratio. $\rho_{\Delta q_t,\Delta c_t^\star-\Delta c_t}$ denotes the correlation between the consumption growth differential and changes in exchange rate. $\overline{T}$ denotes the mean openness ratio. α denotes the U.I.P slope coefficient and $s.e$ the associated standard error. The parameter τ determines the size of the proportional cost while δ determines the importance of the quadratic cost. The last column corresponds to actual data for the US and the US-Germany exchange rate. Data are quarterly. The simulation method is described in the appendix.

	Simulation Results					Data
Panel A: No quadratic cost $\delta = 0$						
	$\tau = 0$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 1$	
g (%)	0.53	0.53	0.53	0.53	0.53	0.53
σ (%)	0.40	0.40	0.40	0.43	0.51	0.51
$\overline{r}$	0.32	0.32	0.36	0.32	0.36	0.34
σ_r (%)	0.44	0.44	0.39	0.43	0.51	0.57
$\sigma_{\Delta q}$ (%)	0.00	1.40	4.29	7.79	22.27	7.53
$\rho_{q_t,q_{t-1}}$	0.00	1.00	1.00	1.00	0.94	0.92
$\overline{SR}$	0.14	0.14	0.14	0.13	0.13	0.15
$\rho_{\Delta q_t,\Delta c_t-\Delta c_t^\star}$	0.00	0.76	0.78	0.76	0.83	-0.01
$\overline{T}$	22.82	23.12	21.04	18.04	0.00	8.40
α	0.00	-1.70	-2.22	-2.95	-1.94	-1.41
$s.e$	[0.00]	[0.35]	[0.20]	[0.19]	[0.19]	[1.29]
Panel B: Proportional and quadratic cost $\delta = 0.2$						
	$\tau = 0$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$		
g (%)	0.53	0.53	0.53	0.53		0.53
σ (%)	0.38	0.37	0.41	0.50		0.51
$\overline{r}$	0.35	0.33	0.31	0.36		0.34
σ_r (%)	0.39	0.38	0.46	0.41		0.57
$\sigma_{\Delta q}$ (%)	0.55	2.63	9.79	10.45		7.53
$\rho_{q_t,q_{t-1}}$	1.00	0.99	0.99	0.99		0.92
$\overline{SR}$	0.14	0.14	0.13	0.13		0.15
$\rho_{\Delta q_t,\Delta c_t-\Delta c_t^\star}$	0.39	0.76	0.80	0.72		-0.01
$\overline{T}$	21.25	13.07	10.05	4.41		8.40
α	-0.00	-3.24	-3.00	-1.80		-1.41
$s.e$	[0.14]	[0.35]	[0.23]	[0.10]		[1.29]
Panel C: No proportional cost $\tau = 0$						
	$\delta = 0.1$	$\delta = 0.5$	$\delta = 1$	$\delta = 5$	$\delta = 20$	
g (%)	0.53	0.52	0.53	0.53	0.53	0.53
σ (%)	0.38	0.37	0.42	0.44	0.51	0.51
$\overline{r}$	0.33	0.36	0.29	0.28	0.38	0.34
σ_r (%)	0.40	0.38	0.45	0.53	0.45	0.57
$\sigma_{\Delta q}$ (%)	0.16	1.58	12.34	15.31	22.09	7.53
$\rho_{q_t,q_{t-1}}$	0.99	0.99	0.95	0.90	0.62	0.92
$\overline{SR}$	0.14	0.14	0.14	0.14	0.13	0.15
$\rho_{\Delta q_t,\Delta c_t-\Delta c_t^\star}$	0.35	0.53	0.79	0.67	0.84	-0.01
$\overline{T}$	18.48	11.70	10.41	3.33	0.00	8.40
α	-0.03	-2.16	-0.55	-2.76	-1.34	-1.41
$s.e$	[0.04]	[0.69]	[0.20]	[0.34]	[0.18]	[1.29]

the stock market implications of the model or overshooting the mean and variance of real interest rates. The Sharpe ratio is sizable even with a reasonable risk-aversion coefficient. This does not mean that risk-aversion is always moderate. As in Campbell & Cochrane (1999), the local curvature coefficient $\eta_t = \gamma/S_t$ sometimes attains very high values, but this happens rarely.

Properties of the real exchange rate The model delivers an autocorrelation coefficient of the exchange rate close to its empirical counterpart. The growth rate of the exchange rate however displays the main drawback of the autarkic model: simulated real exchange rate appreciation has a variance which is three times higher than the actual one. This result can be related to the very definition of the exchange rate in complete markets in equation (5), which implies that its variance is equal to:

$$\sigma^2(\Delta q) = \sigma^2(m) + \sigma^2(m^\star) - 2\rho(m, m^\star)\sigma(m)\sigma(m^\star). \tag{13}$$

To fit the equity premium, we know since Mehra & Prescott (1985) and Hansen & Jagannathan (1991) that the variance of the stochastic discount factor has to be high. Taking into account the low correlation among consumption shocks across countries, and thus the low correlation of stochastic discount factors, Brandt et al. (2006) show that the actual exchange rate is much smoother than the theoretical one implied by asset pricing models.[26] The same tension is present here, because, when countries do not trade, the standard deviation of the change in exchange rate is proportional to the Sharpe ratio.[27] Thus, one cannot obtain a high Sharpe ratio and a low exchange rate volatility at the same time. Leaving autarky for a more realistic world in which trade is possible drastically changes this result.

[26]Consumption shocks are not assumed correlated across countries in this paper. But the variance of the real exchange rate remains high even when the actual small correlation between domestic and foreign consumption processes is taken into account.

[27]The variance of real exchange rate appreciation is here at the steady-state:$\langle Var_t(\Delta q_{t+1})\rangle_{Steady-state} = 2(\gamma\sigma/\bar{S})^2 = 2\overline{SR}^2$.

C. Results with trade

Opening the model up to trade has an impact on both the real side of the economy and on asset prices. When countries can trade, they share risk and the standard deviation of their consumption growth decreases. This in turn decreases the standard deviation of real interest rates and real exchange rates. The U.I.P coefficient remains negative and in the 95% confidence interval of its empirical counterpart. Thus this model reproduces the forward premium puzzle for reasonable levels of international trade costs. I detail below the impact of proportional and quadratic trade costs on trade and the exchange rate distribution.

Proportional trade costs Let us first consider the case of proportional trade costs. Figure (2) reports the time-series of the real exchange rate, the surplus-consumption ratios and the exports/endowments ratios for both countries during the first 10,000 periods of the simulation. As presented in section (I), countries trade when their endowments imply differences in marginal utility of consumption that are not offset by trade costs. When countries trade, the real exchange rate is constant, equal to $1/(1-\tau)$ or $1-\tau$ depending on whether the domestic or foreign country exports. When there is no trade, the real exchange rate fluctuates between these bounds. Thus, with a low trade cost, the exchange rate mostly bounces back and forth between two boundaries and spend most of its time on the boundaries. This has implications for both trade openness and exchange rate volatility.

First, the model implies high openness ratios, computed as the average of imports and exports divided by the endowment. For trade costs equal to 25% and 75%, these openness ratios are on average respectively equal to 23% and 18% with standard deviations of 13% and 18%. These statistics look reasonable for a small open economy but are higher than their US counterpart. The actual global openness ratio for the US is equal to 8.4% on average over the $1957:2-2004:4$ period (with a standard deviation of 2.8%). Note that these figures take into account all international trade with the US and not only bilateral US-Germany trade.[28] One would expect the openness ratio to be smaller and more volatile for one particular bilateral trade than for the sum of all exports and imports.

[28] I used the IFS series 11190C.CZF... and 11198C.CZF... to measure imports and exports in US dollars and 11199B.CZF... for the gross domestic product in US dollars.

Second, the standard deviation of the change in simulated real exchange rates is lower when compared with results obtained under autarky; it is for example divided by 3 at trade costs of 75%. As Sercu & Uppal (2003) noted, the lower the trade cost, the lower the exchange rate variance. At the limit, when there is no trade cost, countries share risk perfectly and the real exchange rate is constant (see table (II) column 2, panel A). The volatility of the simulated exchange rate appears below the post-war value for the US/German rate for a trade cost of 25% and is in line with its empirical counterpart for a trade cost of 75%. As stated in the previous section, the unconditional correlation between simulated exchange rates and consumption growth remains higher than in the data.

Quadratic trade costs Introducing proportional trade costs lowers the real exchange rate volatility, but it implies that the real exchange rate is often constant, which is counterfactual. Adding quadratic trade costs leads to more reasonable patterns as shown in figure (3) and in panel B of Table (II). Even when there is trade, the real exchange rate is no longer constant and it can exceed the previous two fixed boundaries. The increasing marginal trade cost works against large import volumes, even when endowments imply large differences in marginal utility of consumption. Thus trade openness is reduced to 10% for trade costs equal to $\tau = 0.50\%$ and $\delta = 0.2$. For the same parameters, the volatility of the simulated exchange rate roughly matches its empirical counterpart.[29] Panel C in Table (II) highlights the role of quadratic costs by assuming the absence of proportional cost. In this case, countries always trade and the exchange rate's volatility increases and openness decreases with the marginal trade cost parameter.

D. Reality check

Figure (4) shows the time-series of the surplus consumption ratio, stochastic discount factor and local risk curvature for an American investor.[30] The figure is based on the

[29] The model could not reproduce with the same set of parameters both the pre- and post-Bretton Woods exchange rate volatilities because we know since Baxter & Stockman (1989) that real consumption growth shocks have similar volatilities in both sub-periods. Explaining differences in exchange rate regimes is beyond the scope of this paper.

[30] By construction, an infinitesimal rise in consumption always increases habit levels in Campbell & Cochrane (1999)'s model. Ljungqvist & Uhlig (2003) argue that, in some cases, habit levels may decrease

same set of parameters used in the simulation and presented in the first column of Table (I), but uses only actual US consumption data for the $1947:2-2004:4$ period. As actual consumption data correspond to post-trade equilibrium values, I assume that trade costs are infinite when computing pricing kernels. The stochastic discount factor is volatile till the mid-50s and then fluctuates around unity. The surplus consumption ratio varies between 4% and 12%. The local curvature is much higher than the risk-aversion coefficient.

IV. Estimation

The calibration exercise has shown that for parameter values close to the ones used in this literature, the model can reproduce the first two moments of consumption, interest rates and exchange rates and the U.I.P coefficient. In this section, I present results of a direct estimation of the model on foreign currency excess returns. I look for the structural parameters of stochastic discount factors (risk-aversion γ, persistence ϕ, average surplus consumption ratio $\overline{S}$) that minimize the pricing errors of the Euler equation.

A. Method

The model can be estimated without linear approximation by computing the sample equivalent of the Euler equation:

$$E_t[M_{t+1}R^{e,i}_{t+1}] = 0,$$

where $R^e_{t+1} = (1+r^i_t)Q^i_{t+1}/Q^i_t - (1+r_t)$ represents the currency excess return of investing in country i and Q^i and r^i are respectively the real exchange rate and the real interest rate of country i. Theoretically, the model has only one kind of shock that drives both consumption and interest rate processes. Thus I estimate the stochastic discount factor M_{t+1} using either Treasury Bills or consumption data. In each case, I conduct two different experiments:

- first, the model is estimated using moments implied by the pricing behavior of an American investing in 8 other O.E.C.D countries (Australia, Canada, France, Germany,

following a sharp increase in consumption. This does not appear in simulations based on actual data.

Italy, Japan, Sweden, United Kingdom).[31] The model predicts that average currency excess returns should be zero between similar countries. Thus, the estimation is run on conditional moments, using a constant and the domestic lag interest rate as instruments. As a result, this setup gives 16 moments that allow for the estimation of the three parameters (γ, ϕ and $\overline{S}$).

- second, the model is estimated using the 8 portfolios of currency excess returns proposed in Lustig & Verdelhan (2005). These portfolios are built by ranking currencies each period according to interest rates at the end of the previous period. These portfolios offer three advantages: by conditioning on the interest rate, they create a large average spread in excess returns between low and high interest rate portfolios, which is an order of magnitude larger than the average spread for any two given countries; they keep the number of covariances that must be estimated low; they allow to continuously expand the number of countries studied as financial markets open up to international investors, thus including data from the largest possible set of countries.[32]

The estimation relies on the continuously-updating estimator studied by Hansen et al. (1996). The estimator is implemented over a grid of potential parameters. Possible ranges are deduced from the empirical literature on foreign exchange risk premia (the persistence coefficient ϕ should be above 0.8 and below unity), and on habit-based models (the steady-state surplus-consumption ratio $\overline{S} \in [0, 0.10]$). The risk-aversion coefficient γ varies between 0 and 10. For each value of the triplet, the sample equivalent of the Euler pricing errors for investments in country $i = 1, ..., N$ is $\overline{f}^i = \frac{1}{T}\sum_{t=1}^{T} M_{t+1} R_{t+1}^{e,i}$ and the criterion J is equal to $J = T \times \overline{f} \times inv(\Omega) \times \overline{f}$, where $\overline{f} = [\overline{f}^1, ..., \overline{f}^N]$ and Ω is the variance-covariance matrix.[33] The estimation procedure looks for the minimum value of the J criterion over a 100x100x100 grid. Standard errors are computed using GMM

[31] The estimation is run over the pots-Bretton Woods $1971:1-2004:4$ period, for which interest rates and exchange rates are available for all countries considered.

[32] Details on construction and characteristics can be found in Lustig & Verdelhan (2005). The currency portfolios and their composition are available online on the authors' websites. The estimation is run over the post-Bretton Woods $1971:1-2002:4$ period, for which these portfolios are available.

[33] Note here that Ω is computed for each set of parameters, and that Ω is the variance-covariance matrix, not the spectral density matrix. This procedure avoids the production of a non-positive definite matrix and takes into account the limited number of time periods in the estimation. Ω is sometimes singular to working precision. J is computed only for cases when the condition index ($RCOND$) is above $1e-7$ and the rank of Ω is equal to the number of excess returns.

asymptotic theory following Hansen (1982) for the three parameters of the model and by delta-method for the implied coefficients (see Appendix (C) for details).

B. Results

Table (III) presents the estimated values of the model's three structural parameters, the minimized criterion J and the corresponding p-value $p = 1 - \chi^2(J, N-3)$ testing the null hypothesis that pricing errors are zeros. The table also reports the implied interest rate coefficient $B_{implied}$ and the U.I.P slope coefficient $\alpha_{implied} = \gamma(1-\phi)/B$ that the structural parameters would deliver in a two-country symmetric model with post-trade consumption data. Panel A reports results obtained using only consumption growth to compute stochastic discount factors. Panel B reports results obtained using only interest rates to compute stochastic discount factors.

The three structural parameters are estimated within their proposed ranges, and no corner solution is reached, except for one case.[34] The p-values range from 49% to 72%. Risk-aversion coefficients vary between 2 and 9 depending on the set of excess returns and pricing kernels considered. The persistence parameter ϕ is estimated between 0.97 and 0.99 with relatively high standard errors. The average surplus consumption ratio takes low values (between 2% and 3%) with consumption data, but higher values (up to 7%) with interest rates. These values translate in habits ranging from 93% to 98% of consumption. In simulations assuming post-trade consumption shocks, these parameters would deliver negative U.I.P coefficients α.

The estimated values of the model's three structural parameters seem reasonable and line with the literature on domestic excess returns. Chen & Ludvigson (2004) estimate habit-based models without imposing the functional form of habit preferences. They conclude that in order to match moment conditions corresponding to Fama-French portfolios, habits should be equal to a large fraction of current consumption (97% on average). Using a simulation-based method, Tallarini & Zhang (2005) estimate Campbell & Cochrane (1999)'s model on US domestic assets (assuming a constant real risk-free interest rate). They find that the persistence coefficient ϕ is above 0.9 and the risk-aversion coefficient

[34] ϕ reaches its boundary value of 0.999 for the case of an American investor investing in 8 different OECD countries, when the stochastic discount factor is computed using interest rates. In this case, the standard errors derived from asymptotic theory are not valid.

equal to 6.3.

All estimations imply negative values for B, i.e pro-cyclical interest rates, which is consistent with recent results found in the real interest rate literature. Challenging previous findings from Stock & Watson (1999), Dostey, Lantz, & Scholl (2003) conclude that the ex-ante real rate is contemporaneously positively correlated with GDP and with lagged cyclical output. In addition, pro-cyclical risk-free rates lead to downward sloping real yield curves.[35] Evans (1998) documents that term premia for inflation-indexed bonds in the United Kingdom are significantly negative, while term premia for nominal bonds are positive. Thus, this model could be extended to reproduce both risk and term premium, but it would need a sizable inflation risk component.

V. Conclusion

I have shown here that a two-country one-good model in which agents are characterized by slow-moving external habit preferences similar to Campbell & Cochrane (1999) rationalizes the U.I.P puzzle. The model has two main features: a time-varying risk aversion and trade costs.

The failure of the U.I.P condition implies the existence of non-zero currency excess returns when borrowing funds at low interest rates and lending abroad at higher interest rates. But if a domestic investor receives a positive currency excess return, his foreign counterpart receives a negative one. The model rationalizes this stylized fact. In this model, the domestic investor gets positive excess returns in times when he is more risk-averse than his foreign counterpart. The same reasoning applies naturally to the foreign investor. Times of high risk-aversion correspond to low interest rates. Thus, the domestic investor receives a positive risk premium when interest rates are lower at home than abroad.

Model simulations lead to the usual negative covariance between exchange rate variations and interest rate differentials, while simultaneously delivering a sizable Sharpe ratio. Proportional and quadratic trade costs deliver real exchange rates that are neither stale nor too volatile, even as consumption processes among countries are uncorrelated. The

[35] Wachter (2006) shows that a positive parameter B is needed to obtain an upward sloping real yield curve with Campbell & Cochrane (1999) preferences.

Table III
Estimation Results

The table presents the estimated values of the model's three structural parameters (risk-aversion γ, persistence ϕ, average surplus consumption ratio S in percentage) and the implied interest rate coefficient $B_{implied}$ and U.I.P slope coefficient $\alpha_{implied} = \gamma(1-\phi)/B$. It also presents the number of excess returns N, the minimized criterion J and the corresponding p-value $p = 1 - \chi^2(J, N-3)$ testing the null hypothesis that the pricing errors are zeros. Panel A reports results obtained using only consumption growth to compute stochastic discount factors. Panel B reports results obtained using only interest rates to compute stochastic discount factors. In columns 2 and 4, the estimation uses the currency excess returns of an American investor in 8 other O.E.C.D countries (Australia, Canada, France, Germany, Italy, Japan, Sweden, United Kingdom). Using a constant and the US interest rate as instruments, the estimation is run on 16 moment conditions. In columns 3 and 5, the estimation uses the 8 portfolios of currency excess returns proposed in Lustig and Verdelhan (2005). These portfolios are built by sorting currencies on interest rates. Data are quarterly. The sample is 1971:3-2004:4 for individual currencies and 1971:1-2002:4 for currency portfolios. Standard errors are reported between brackets.

	Panel A: Using consumption		**Panel B: Using interest rates**	
	8 Countries	**8 Portfolios**	**8 Countries**	**8 Portfolios**
N	16	8	16	8
J	9.86	3.17	9.65	4.41
p	0.71	0.67	0.72	0.49
γ	5.60	8.20	2.20	8.90
	[1.64]	[1.15]	[0.27]	[1.50]
ϕ	0.97	0.99	1.00	0.99
	[0.14]	[0.14]	[0.17]	[0.25]
$\overline{S}$	2.90	2.50	3.10	7.30
	[0.66]	[0.06]	[0.00]	[0.06]
$B_{implied}$	-1.51	-5.00	-0.22	-0.63
	[0.09]	[0.14]	[0.03]	[0.20]
$\alpha_{implied}$	-0.10	-0.02	-0.00	-0.11
	[0.04]	[0.02]	[0.14]	[0.35]

model's estimation gives reasonable parameters, thus rationalizing the exchange rate risk premium.

These results have been obtained for endowment economies. Could the same set of preferences be transposed into a production framework and thereby reconcile business cycle and asset pricing results? Future attempts in this direction will have to deal with two difficulties. First, using habits in the representative agent's preferences and a "time-to-plan"assumption on investment and labor, Boldrin, Christiano, & Fisher (2001) find that their model generates highly variable risk-free rates. By substituting Campbell & Cochrane (1999) for the Constantinides (1990)' form of habit preferences used by Boldrin et al. (2001), one can hope to overcome this difficulty. This habit form allows the parametrization of the interest rate's sensitivity to the economic stance, which impacts the variance of the risk-free rate. This comes at the price of decreasing the mean interest rate, which can be compensated for by a reasonable increase in risk-aversion. Second, Lettau & Uhlig (2000) show that Campbell & Cochrane (1999) preferences deliver overly smooth consumption in a real business cycle framework. Agents are very risk-averse locally, meaning that the inter-temporal elasticity of substitution is very low. This leads to a desire to use labor to radically smooth consumption. This difficulty might be overcome by introducing pre-determined labor, a time-to-plan assumption and/or adjustment costs and two separate sectors. Considering the many interesting results obtained in endowment economies, the transposition of this class of model onto a general equilibrium framework deserves some future work.

References

Alvarez, Fernando, Andy Atkeson, & Patrick Kehoe, 2005, Time-varying risk, interest rates and exchange rates in general equilibrium, working paper No 627 Federal Reserve Bank of Minneapolis Research Department.

Anderson, James, & Eric van Wincoop, 2004, Trade costs, *Journal of Economic Literature* 42, 691–751.

Bacchetta, Philippe, & Eric van Wincoop, 2005, Rational inattention: A solution to the forward discount puzzle, mimeo.

Backus, David, Silverio Foresi, & Chris Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* LVI, 279–304.

Backus, David, & Gregor Smith, 1993, Consumption and real exchange rates in dynamic economies with non-traded goods, *Journal of International Economics* 35, 297–316.

Backus, David K., Patrick J. Kehoe, & Finn E. Kydland, 1992, International real business cycles, *Journal of Political Economy* 100, 745–775.

Bansal, Ravi, & Magnus Dahlquist, 2000, The forward premium puzzle: Different tales from developed and emerging economies, *Journal of International Economics* 51, 115–144.

Bansal, Ravi, & Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481 – 1509.

Baxter, Marianne, & Mario J. Crucini, 1995, Business cycles and the asset structure of foreign trade, *International Economic Review* 36, 821–854.

Baxter, Marianne, & Alan Stockman, 1989, Business cycles and the exchange-rate regime: Some international evidence, *Journal of Monetary Economics* 23, 377–400.

Benigno, Gianluca, & Christoph Thoenissen, 2006, Consumption and real exchange rates with incomplete markets and non-traded goods, working paper CEPR No. DP5580.

Boldrin, Michele, Lawrence J. Christiano, & Jonas D.M. Fisher, 2001, Habit persistence, asset returns and the business cycle, *American Economic Review* 91, 149–166.

Brandt, Michael W., John Cochrane, & Pedro Santa-Clara, 2006, International risk-sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics, forthcoming* .

Buraschi, Andrea, 2004, Term structure of interest rates implications of habit persistence, working paper Columbia Business School.

Campbell, John Y., & John H. Cochrane, 1995, By force of habit: A consumption-based explanation of aggregate stock market behavior, working paper NBER No 4995.

Campbell, John Y., & John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Chaboud, Alain P., & Jonathan H. Wright, 2005, Uncovered interest parity: it works, but not for long, *Journal of International Economics* 66, 349–362.

Chan, Yeung Lewis, & Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Chari, V. V., Patrick J. Kehoe, & Ellen R. McGrattan, 2002, Can sticky price models generate volatile and persistent real exchange rates?, *The Review of Economic Studies* 69, 533–564.

Chen, Xiaohong, & Sydney C. Ludvigson, 2004, Land of addicts? an empirical investigation of habit-based asset pricing models, working paper New York University.

Chinn, Menzie D., 2006, The (partial) rehabilitation of interest rate parity in the floating rate era: Longer horizons, alternative expectations, and emerging markets, *Journal of International Money and Finance* 25, 7–21.

Cochrane, John H., 2001, *Asset Pricing* (Princeton University Press, Princeton, N.J.).

Colacito, Riccardo, & Mariano Croce, 2005, Risks for the long run and the real exchange rate, mimeo.

Constantinides, George M., 1990, Habit formation: A resolution of the equity premium puzzle, *The Journal of Political Economy* 98, 519–543.

Constantinides, George M., & Darrell Duffie, 1996, Asset pricing with heterogeneous consumers, *The Journal of Political Economy* 104, 219–240.

Corsetti, Giancarlo, Luca Dedola, & Sylvain Leduc, 2004, International risk sharing and the transmission of productivity shocks, working paper series ECB, No. 308.

Cox, John C., Jonathan E. Ingersoll, & Stephen A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–408.

Dostey, Michael, Carl Lantz, & Brian Scholl, 2003, The behavior of the real rate of interest, *Journal of Money, Credit and Banking* 35, 91–110.

Dumas, Bernard, 1992, Dynamic equilibrium and the real exchange rate in a spatially separated world, *Review of Financial Studies* 5, 153–180.

Engel, Charles, 1996, The forward discount anomaly and the risk premium: A survey of recent evidence, *Journal of Empirical Finance* 3, 123–192.

Evans, Martin, 1998, Real rates, expected inflation and inflation risk premia, *Journal of Finance* 53, 187–218.

Fama, Eugene, 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–338.

Frachot, Antoine, 1996, A reexamination of the uncovered interest rate parity hypothesis, *Journal of International Money and Finance* 15, 419–437.

Froot, Kenneth, & Richard Thaler, 1990, Anomalies: Foreign exchange, *The Journal of Economic Perspectives* 4, 179–192.

Gomez, Juan-Pedro, Richard Priestley, & Fernando Zapatero, 2004, An international capm with consumption externalities and non-financial wealth, mimeo.

Gourinchas, Pierre-Olivier, & Aaron Tornell, 2004, Exchange rate puzzle and distorted beliefs, *Journal of International Economics* 64, 303–333.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

Hansen, Lars Peter, John Heaton, & Amir Yaron, 1996, Finite-sample properties of some alternative gmm estimators, *Journal of Business and Economic Statistics* 96, 262–280.

Hansen, Lars Peter, & Robert J. Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, *Journal of Political Economy* 88, 829–853.

Hansen, Lars Peter, & R. Jagannathan, 1991, Implications of security markets data for models of dynamic economies, *Journal of Political Economy* 99, 252–262.

Hollifield, Burton, & Raman Uppal, 1997, An examination of uncovered interest rate parity in segmented international commodity markets, *Journal of Finance* 52, 2145–2170.

Hollifield, Burton, & Amir Yaron, 2003, The foreign exchange risk premium: Real and nominal factors, working paper The Wharton School.

Huisman, Ronald, Kees Koedijka, Clemens Kool, & Francois Nissen, 1998, Extreme support for uncovered interest parity, *Journal of International Money and Finance* 17, 211–228.

Lettau, Martin, & Sydney Ludvigson, 2003, Measuring and modeling variation in the risk-return tradeoff, in Yacine Ait-Sahalia & Lars Peter Hansen, (eds.) *Handbook of Financial Econometrics, forthcoming* (Elsevier).

Lettau, Martin, & Harald Uhlig, 2000, Can habit formation be reconciled with business cycle facts?, *Review of Economic Dynamics* 3, 79–99.

Liu, Wei, & Alex Maynard, 2005, Testing forward rate unbiasedness allowing for persistent regressors, *Journal of Empirical Finance* 12, 613–628.

Ljungqvist, Lars, & Harald Uhlig, 2003, Optimal endowment destruction under campbell-cochrane habit formation, mimeo.

Lothian, James R., & Liuren Wu, 2005, Uncovered interest-rate parity over the past two centuries, mimeo.

Lucas, Robert E. Jr., 1982, Interest rates and currency prices in a two country world, *Journal of Monetary Economics* 10, 335–360.

Lustig, Hanno, & Adrien Verdelhan, 2005, The cross-section of foreign currency risk premia and consumption growth risk, working paper NBER No 11104.

Lyons, Richard K., 2001, *The Microstructure Approach to Exchange Rates* (M.I.T Press).

Maynard, Alex, 2006, The forward premium anomaly: statistical artifact or economic puzzle? new evidence from robust tests, *Canadian Journal of Economics, forthcoming* .

Mehra, Rajnish, & Edward Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Menzli, Lior, Tano Santos, & Pietro Veronesi, 2004, Understanding predictability, *Journal of Political Economy* 112, 1–48.

Meredith, Guy, & Menzie D. Chinn, 2005, Testing uncovered interest rate parity at short and long horizons during the post-bretton woods era, working paper NBER 11077.

Moore, Michael J., & Maurice J. Roche, 2002, Less of a puzzle: a new look at the forward forex market, *Journal of International Economics* 58, 387–411.

Obstfeld, Maurice, & Kenneth Rogoff, 1995, Exchange rate dynamics redux, *Journal of Political Economy* 25, 338–390.

Obstfeld, Maurice, & Kenneth Rogoff, 2000, The six major puzzles in international economics: Is there a common cause?, *NBER Macroeconomics Annual* 5, 338–390.

Samuelson, Paul, 1954, The transfer problem and the transport costs, ii: Analysis of effects of trade impediments, *Economic Journal* 64, 264–289.

Sarkissian, Sergei, 2003, Incomplete consumption risk sharing and currency risk premiums, *Review of Financial Studies* 16, 983–1005.

Sercu, Piet, & Raman Uppal, 2003, Exchange rate volatility and international trade: A general-equilibrium analysis, *European Economic Review* 47, 429–441.

Sercu, Piet, Raman Uppal, & Cynthia Van Hulle, 1995, The exchange rate in the presence of transactions costs: Implications for tests of purchasing power parity, *Journal of Finance* 50, 1309–1319.

Shore, Stephen H., & Joshua S. White, 2003, External habit formation and the home bias puzzle, mimeo.

Stock, James H., & Mark W. Watson, 1999, Business cycle fluctuations in us macroeconomic time series, in John B. Taylor & Michael Woodford, (eds.) *Handbook of Macroeconomics*, 4–64 (Elsevier Science, Amsterdam).

Sundaresan, Suresh, 1989, Intertemporal dependent preferences and the volatility of consumption and wealth, *The Review of Financial Studies* 2, 73–88.

Tallarini, Thomas D., & Harold H. Zhang, 2005, External habit and the cyclicality of expected stock returns, *Journal of Business* 78, 1023–48.

Wachter, Jessica, 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365–399.

A. Literature Review

The existence of the exchange rate risk premium stems from the empirical failure of uncovered interest rate parity (U.I.P).[36] Taking into account this empirical finding, expected exchange rate variations $\Delta e^e_{t,t+1}$ are assumed to be equal to their corresponding interest differentials up to a time-varying risk premium $p_{t,t+1}$:

$$\Delta e^e_{t,t+1} = i_t - i^\star_t - p_{t,t+1},$$

Fama (1984) highlights two characteristics of this risk premium.[37] Assuming rational expectations, he introduces a forecast error η_{t+1}, which is orthogonal to all information dated time t or earlier, defined as: $\Delta e_{t+1} = \Delta e^e_{t,t+1} + \eta_{t+1}$ where $\Delta e_{t,t+1}$ denotes the realized change in exchange rate. Then the U.I.P coefficient α is equal to:

$$\alpha = \frac{cov(\Delta e_{t+1}, i_t - i^\star_t)}{var(i_t - i^\star_t)} = \frac{var(\Delta e^e_{t,t+1}) + cov(p_{t,t+1}, \Delta e^e_{t,t+1})}{var(p_{t,t+1}) + var(\Delta e^e_{t,t+1}) + 2cov(p_{t,t+1}, \Delta e^e_{t,t+1})}.$$

Fama (1984) notes that this simple decomposition has two consequences: a negative U.I.P coefficient α entails a negative covariance between the risk premium and the expected variation in the exchange rate ($cov(p_{t,t+1}, \Delta e^e_{t,t+1}) < 0$); a U.I.P coefficient α less than 1/2 entails a risk premium more volatile than the expected depreciation of the exchange rate,($var(p_{t,t+1}) > var(\Delta e^e_{t,t+1})$.

Keynesian models *a la* Mundell-Fleming or Dornbusch, postulate U.I.P, as do target zone models *a la* Krugman. Flexible price monetary models usually start with the even stronger assumption of continuous purchasing power parity (P.P.P), leading to a constant real exchange rate. For all these models, the stylized fact on U.I.P constitutes a puzzle.

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[36] The U.I.P puzzle has also been called "forward premium bias". The U.I.P condition leads to: $\Delta e^e_{t+1} = i_t - i^\star_t$. Using the covered interest rate parity condition, $f_t - e_t = i_t - i^\star_t$, one obtains a forward rate f_t that should be equal to market expectations of the future spot rate, $f_t = e^e_{t+1}$. Given rational expectations, the expected change in the exchange rate should differ from the realized one only by an expectational error, and the forward rate should hence be a good predictor of the future spot rate. Empirically, however, the forward rate is a very bad predictor of the spot rate; it cannot even correctly forecast the direction of the change in exchange rate.

[37] Note that the risk premium p defined by Fama (1984) is the opposite of the excess return used in this paper.

This paper builds on Backus et al. (2001), who describe the necessary features of a theory that accounts for the forward premium anomaly. When pricing kernels are log-normal, the risk premium equals the difference in their conditional variances. Thus, in order to satisfy Fama (1984)'s first condition and generate a negative U.I.P coefficient, one needs a negative correlation between the difference in conditional means and the difference in conditional variance of the two pricing kernels. To satisfy Fama (1984)'s second condition, one needs a great deal of fluctuation in conditional variances. These necessary features can be directly built in a financial model. For example, in Cox et al. (1985)'s model, the state variable is identified with the spot rate. It is assumed to follow a square-root process, in which the conditional expectation and variance of the short-term interest rates are assumed to be linear in the interest rate itself. Frachot (1996) shows that a two-country version of such a model produces, for certain parameter values, a negative U.I.P slope coefficient. This framework, however, offers no obvious economic explanation for the foreign currency risk premia.[38]

Recently, the development of dynamic stochastic equilibrium models has offered new opportunities for understanding the exchange rate behavior. In these newer models, the exchange rate risk premium is linked to the covariance of excess returns and stochastic discount factors. The proposed theoretical frameworks to date are the following (see table IV for a summary):

- By assuming sticky prices and following Obstfeld & Rogoff (1995)'s pioneering work, Chari et al. (2002) produce volatile and persistent exchange rate fluctuations from the interaction of sticky prices and monetary shocks. They introduce "price-discriminating monopolists in order to get fluctuations in real exchange rate from fluctuations in the relative price of traded goods" and "staggered price-setting in order to get persistent real exchange rates". With prices fixed for one year, and a risk-aversion coefficient of 6, they obtain the real exchange volatility found in the data. Their model cannot, however, produce the right price volatility and the right persistence of the real exchange rate at the same time. And increasing the price-stickiness even to four years leads to an autocorrelation that is too low.[39]

[38]The U.I.P slope coefficient is equal to $(1-e^{-\lambda})/(1-\frac{\partial A^d(1)}{1+\frac{\alpha}{2}A^s A^d(1)})$ where λ, α and A^s are diffusion parameters, and A^d satisfies a unidimensional Riccati differential equation.

[39]Chari et al. (2002) do not report results on the interest rate. Around the steady-state, a log-

- Alvarez et al. (2005) propose an interesting alternative to standard C.I.A models by introducing endogenously segmented markets: higher money growth leads to higher inflation, thus inducing more agents to enter the asset market because the cost of non-participation is higher. This leads to a decrease in risk premium. If segmentation is sufficiently large and sensitive to money growth, this time-varying risk generates the forward premium anomaly. But if inflation is high, for example above a cut-off value $\overline{\pi}$, all agents participate in the market and therefore consumption and risk premia remain constant. Thus, this model can qualitatively reproduce the U.I.P puzzle, while producing U.I.P for high inflation countries, a pattern found empirically by Bansal & Dahlquist (2000). Yet, to reproduce quantitatively the U.I.P puzzle the model implies very large flows in and out of the asset markets or large entry costs in the asset market.

- Moore & Roche (2002) introduce habit-persistence into the classical Lucas (1982) two-country monetary model. They are able to reproduce the relative volatilities of the exchange rate and the risk premium, but not the forward premium bias. Their sample estimates of the U.I.P coefficient α are all negative (the usual forward premium bias), but the results of their calibration experiments are all positive.

- Sarkissian (2003) addresses the issue within the framework of Constantinides & Duffie (1996), assuming heterogenous agents (here countries) that cannot perfectly insure themselves against consumption growth shocks. Two factors, world consumption growth and dispersion, produce a time-varying stochastic discount factor and lead to a negative covariance between the risk premium and depreciation rates. But none of these factors is significant in a beta-pricing framework, and the model can not reproduce the second Fama (1984) condition, $var(p_{t,t+1}) > var(\Delta e^e_{t,t+1})$. This means that the implied U.I.P coefficient is above 1/2.

- Lyons (2001) suggests that investors do not take advantage of arbitrage opportuni-

linearization leads to: $\widehat{q} = 5.94(\widehat{c}^* - \widehat{c}) + 0.06(\widehat{m}^* - \widehat{m})$ where a caret denotes the deviation from the steady-state of the log of each variable (resp. real exchange rate, consumption and real balances). On the one hand, if interest rates are pro-cyclical as in the data, the first term above leads to a positive U.I.P coefficient α. On the other hand, real balances decrease with interest rate (elasticity is equal to 0.39 in their model) and this effect pushes α down. The overall effect is therefore not clear.

ties when the Sharpe ratio remains below unity. This behavior produces an "inaction zone"that can be matched in terms of the U.I.P coefficient. Hence this coefficient should vary between -1 and $+3$. To understand the negative value obtained on short horizons, one needs to introduce another friction that explains why investors do not fully adapt to changes in the interest rate gap. A limited adaptation hypothesis would predict that the U.I.P coefficient is first negative and then switches sign, tending towards unity as the horizon increases.

- Bacchetta & van Wincoop (2005) develop a model where investors face costs of collecting and processing information. Because of these costs, many investors optimally choose to only infrequently assess available information and revise their portfolios. Rational inattention produces a negative U.I.P coefficient along the lines suggested by Froot & Thaler (1990) and Lyons (2001): if investors are slow to respond to news of higher domestic interest rates, there will be a continued reallocation of portfolios towards domestic bonds and an appreciation of the currency subsequent to the shock. Bacchetta & van Wincoop (2005) obtain negative U.I.P slope coefficient for information and trading costs higher than 2 percents of total financial wealth.

- Departing from full rationality, Gourinchas & Tornell (2004) explains the forward premium by assuming that agents misperceive the persistence of interest rate shocks and learning effects. Using survey data, they argue that interest rate forecasts systematically under-react to interest rate innovations. They are able to reproduce both the sign and the magnitude of the U.I.P coefficient.

Table IV: Summary of the literature

The table presents a survey of the results obtained on the U.I.P puzzle (empirically, the U.I.P slope coefficient α is often negative) and the volatility puzzle ($\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$ in the data, where $\sigma^2_{\Delta e}$ is the variance of the change in exchange rates, σ^2_p is the variance of the currency risk premium and $\sigma^2_{i-i^\star}$ is the variance of the interest rate differential).

Papers	*Features*	*UIP puzzle*	*Volatility puzzle*
Lucas (1982)	Two-country, cash-in-advance	$\alpha \simeq 1$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star} > \sigma^2_p$
Bekaert (1996)	Lucas (1982) + Habit persistence	$\alpha < 1/2$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star} > \sigma^2_p$
Moore and Roche (2002)	Lucas (1982) + Habit persistence + Limited participation	$0 < \alpha < 1$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Alvarez et al. (2005)	Lucas (1982) + Endogeneously segmented markets	$\alpha < 0$ for $\pi < \overline{\pi}$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star}$
Sarkissian (2003)	Heterogeneity	$0 < \alpha < 1$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Lyons (2001)	Arbitrage zone + Limited adaptation	$-1 < \alpha < 3$ $-1 < \alpha < 0$	n.a
Bacchetta and van Wincoop (2005)	Information costs	$\alpha < 0$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Gourinchas and Tornell (2004)	Limited rationality	$\alpha < 0$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Obstfled and Rogoff (1995)	Monopolists + Sticky prices + UIP	$\alpha = 1$	$\sigma^2_{\Delta e} = \sigma^2_{i-i^\star}, \sigma^2_p = 0$
Chari et al. (2002)	Monopolists + Sticky prices	n.a	n.a

B. Simulation Method

I first draw $110,000$ $i.i.d$ endowment shocks and delete the first $10,000$. From the $100,000$ endowment shocks and the parameters of the model, I build the endowment process. Then, for each date, I compute the optimal amount of exports, imports and consumption. Real risk-free rates and real exchange rates are also computed from consumption data. I then regress the quarterly variation of the real exchange rate on the real interest rate differential to find the coefficient α from a U.I.P test. Solving the social planner program presents two difficulties that I briefly describe below.

A. Habit and Consumption

Trade at date $t+1$ in equations (3) and (4) depend on the habit level at date $t+1$. The habit level cannot be computed using the exact law of motion described in equation (6) because it requires the value of consumption at date $t+1$, which in turn depends on trade at date $t+1$. But Campbell & Cochrane (1999) chose the sensitivity function $\lambda(s_t)$ so that the habit level at date $t+1$ does not actually depend on consumption level on the same date. This can be shown using a first order Taylor approximation of the law of motion of the habit level x_{t+1} when s_t is close to its steady-state value $\bar{s}$ and the consumption growth Δc_{t+1} is close to its average g. I use the same steps as outlined in footnote 1 page 6 of Campbell & Cochrane (1995).

The log surplus consumption ratio is equal to:

$$s_t = \ln(\frac{e^{c_t} - e^{x_t}}{e^{c_t}}).$$

Let h be the steady-state value of $x_t - c_t$. Then a first-order Taylor approximation of s_t around $\bar{s}$ leads to:

$$s_t - \bar{s} \simeq (1 - \frac{1}{\bar{S}})(x_t - c_t - h).$$

Likewise,

$$\lambda(s_t)(c_{t+1} - c_t - g) \simeq \lambda(\bar{s})(c_{t+1} - c_t - g).$$

Equation (6) leads to:

$$(1-\frac{1}{\bar{S}})(x_{t+1}-c_{t+1}-h)=\phi(1-\frac{1}{\bar{S}})(x_t-c_t-h)+\lambda(\bar{s})(c_{t+1}-c_t-g).$$

Campbell & Cochrane (1999) chose the sensitivity function $\lambda(s_t)$ so that the habit level x_{t+1} does not depend on c_{t+1} ($\lambda(\bar{s})=-(1-\frac{1}{\bar{S}})$).

Thus,

$$x_{t+1}-h=\phi(x_t-c_t-h)+c_t+g,$$

leads to:

$$x_{t+1}=\phi x_t+[(1-\phi)h+g]+(1-\phi)c_t. \tag{14}$$

Equation (14) gives a first guess for the habit level at date $t+1$, thus allowing the computation of trade and consumption at date $t+1$. This new estimate of consumption is used to compute the habit level using the exact law of motion and the process is iterated until convergence.

B. Optimal Trade

In the presence of quadratic costs, there is no closed form solution for the optimal amount of exports (except for log-utility).

To find the optimal amount of exports, let us define and minimize the following function f derived from the first-order condition (3):

$$f(X_t)=-[Y_t-X_t-H_t]^{-\gamma}+[1-\tau-\delta\frac{X_t}{Y_t^\star}][Y_t^\star+X_t(1-\tau-\frac{\delta}{2}\frac{X_t}{Y_t^\star})-H_t^\star]^{-\gamma}.$$

The solution X_t to $f(X_t)=0$ has to satisfy three conditions. First, a country cannot export more than its endowment; thus X_t is in the interval $0\le X_t\le Y_t$. Second, habit preferences prevent consumption from falling below the habit level in both countries; thus $X_t\le Y_t-H_t$ and $Y_t^\star+X_t(1-\tau-\frac{\delta}{2}\frac{X_t}{Y_t^\star})-H_t^\star\ge 0$. The latter condition imposes that $X_t\in[x_{1,t},x_{2,t}]$ where $x_{1,t}=Y_t^\star(1-\tau-\sqrt{\Delta_t})/\delta$ and $x_{2,t}=Y_t^\star(1-\tau+\sqrt{\Delta_t})/\delta$ when $\Delta_t=(1-\tau)^2+2\delta(Y_t^\star-H_t^\star)/Y_t^\star>0$. Third, the foreign country imports X_t only if a positive fraction of the good makes it to its shore, thus $0\le X_t\le 2Y_t^\star(1-\tau)/\delta$. To satisfy the three conditions X_t has to be in the interval $[0, min(Y_t - H_t, 2Y_t^\star(1-\tau)/\delta)] \bigcap [x_{1,t}, x_{2,t}]$.

Note that when the endowment level is above the habit ($Y_t^\star - H_t^\star > 0$), then $\Delta_t > 0$, $x_{1,t} < 0$ and $x_{2,t} > 2Y_t^\star(1-\tau)/\delta$. Thus, the solution of the maximization problem is in the interval $[0, min(Y_t - H_t, 2Y_t^\star(1-\tau)/\delta)]$. In this case, over this simple interval, a solution exists if and only if:

$$\frac{Y_t^\star - X_t^\star}{Y_t - X_t} < (1-\tau)^{\frac{1}{\gamma}}. \tag{15}$$

Note that f is decreasing:

$$\begin{aligned} f'(X_t) &= -\gamma[Y_t - X_t - H_t]^{-\gamma-1} - \frac{\delta}{Y_t^\star}[Y_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma} \\ &- \gamma[1 - \tau - \delta\frac{X_t}{Y_t^\star}]^2[Y_t^\star + X_t(1 - \tau - \frac{\delta}{2}\frac{X_t}{Y_t^\star}) - H_t^\star]^{-\gamma-1}. \end{aligned}$$

Thus, there exists an optimal amount of exports if $f(0) > 0$ and $f(min[Y_t - H_t, 2Y_t^\star(1-\tau)/\delta]) < 0$. The first boundary condition $f(0) > 0$ is equivalent to condition (15). This boundary condition also defines cases when the domestic country exports under no quadratic costs.

Let us check that the second boundary condition $f(min[Y_t - H_t, 2Y_t^\star(1-\tau)/\delta]) < 0$ is always satisfied. When $Y_t - H_t \geq 2Y_t^\star(1-\tau)/\delta$, the boundary condition $f(2Y_t^\star(1-\tau)/\delta) < 0$ is always satisfied:

$$f(2Y_t^\star(1-\tau)/\delta) = -[Y_t - 2Y_t^\star(1-\tau)/\delta) - H_t]^{-\gamma} - [1-\tau][Y_t^\star - H_t^\star]^{-\gamma} < 0.$$

When $Y_t - H_t \leq 2Y_t^\star(1-\tau)/\delta$, there also exists a solution to $f(X_t) = 0$ because $f_{X_t \to Y_t - H_t}(X_t) \to -\infty$.

C. Estimation Method

Let $b = [\gamma\ \phi\ \overline{S}]$ be the vector of parameters to estimate. The criterion is:

$$J(b) = T\ \times\ \overline{f_T(b)}\ \times\ inv(\Omega)\ \times\ \overline{f_T(b)},$$

where $\overline{f_T(b)}$ is the sample mean of the pricing errors estimated with parameters b and Ω is the variance-covariance matrix of $f_t(b)$. Hansen (1982) gives the asymptotic distribution of the GMM estimate:

$$\sqrt{T}(\widehat{b} - b) \rightarrow N[0, (ad)^{-1} a\Sigma a'(ad)^{-1\prime}],$$

where $a = \frac{\partial \overline{f}_T^{\prime}(b)}{\partial b}\Omega^{-1}$ and $d = \frac{\partial \overline{f}_T(b)}{\partial b'}$. Σ is the spectral density matrix $\Sigma = \sum_{j=-\infty}^{\infty} E[f_t(b) f_{t-j}(b)]$ and the precision around b is given by $var(\widehat{b}) = \frac{1}{T}(ad)^{-1} a\Sigma a'(ad)^{-1\prime}$. Due to the sample's size, I use the variance-covariance matrix instead of the spectral density matrix in the estimation. As exchange rate changes are close to random walks and the pricing kernels are here very persistent, this approximation is reasonable.

The stochastic discount factor can be expressed in terms of the parameters b to compute d:

$$M_{t+1} = \beta(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t})^{-\gamma} = \beta G^{-\gamma} e^{-\gamma\left[(\phi-1)(s_t-\bar{s})+(1+\lambda(s_t))v_{t+1}\right]}.$$

For each excess return R^i_{t+1}:

$$\begin{aligned}
\frac{\partial f^i_t(b)}{\partial \gamma} &= -[(s_{t+1} - s_t) + (c_{t+1} - c_t)] M_{t+1} R^i_{t+1}, \\
\frac{\partial f^i_t(b)}{\partial \phi} &= \gamma(s_t - \bar{s}) M_{t+1} R^i_{t+1}, \\
\frac{\partial f^i_t(b)}{\partial S} &= \left[\frac{\gamma(\phi-1)}{\overline{S}} - \gamma v_{t+1}\frac{\partial \lambda(s_t)}{\partial S}\right] M_{t+1} R^i_{t+1},
\end{aligned}$$

where $\lambda(s_t) = \frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1$ and $\frac{\partial \lambda(s_t)}{\partial S} = (-\frac{1}{S^2})\sqrt{1 - 2(s_t - \bar{s})} + \frac{1}{S^2}[1 - 2(s_t - \bar{s})]^{-\frac{1}{2}} = [-\frac{1}{S}\ \lambda(s_t) + \frac{1}{S^3}\frac{1}{\lambda(s_t)}]$.

D. Figures

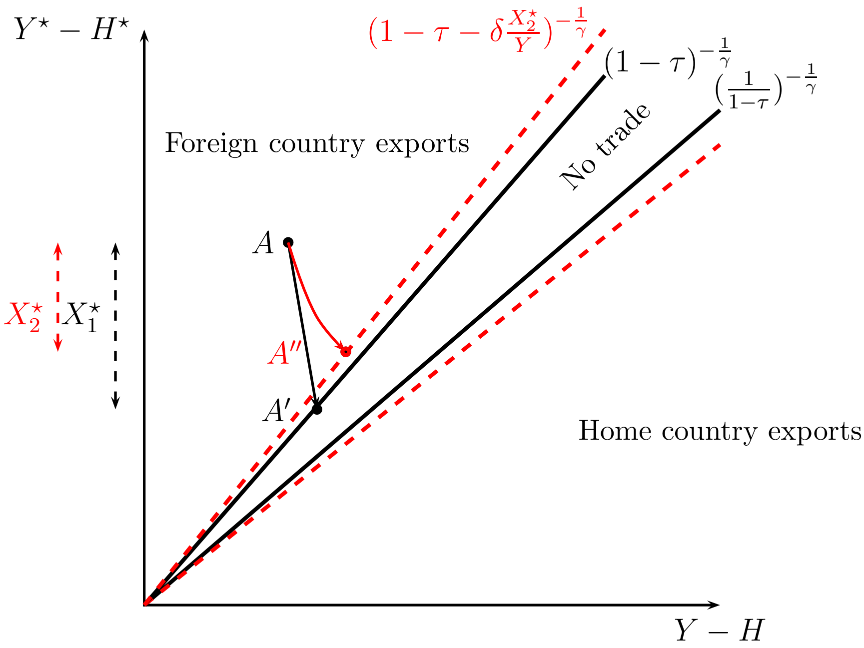

Figure 1. The figure presents the central planner's problem with proportional and quadratic trade costs. Assume that the two countries are characterized by point A where endowments (net of habit levels) are given. If there are only proportional costs, the foreign country exports $X_1^\star$ units. For each unit that the foreign country exports, the domestic country receives $(1-\tau)$. Thus, the slope between A and A' is $-1/(1-\tau)$. At point A', the real exchange rate is equal to $(1-\tau)$. If there are proportional and quadratic costs, the foreign country exports $X_2^\star$ units. The quadratic trade cost incurred is equal to $\delta\frac{X_2^\star}{Y}$. At point A'', the real exchange rate is equal to $(1-\tau-\delta\frac{X_2^\star}{Y})$.

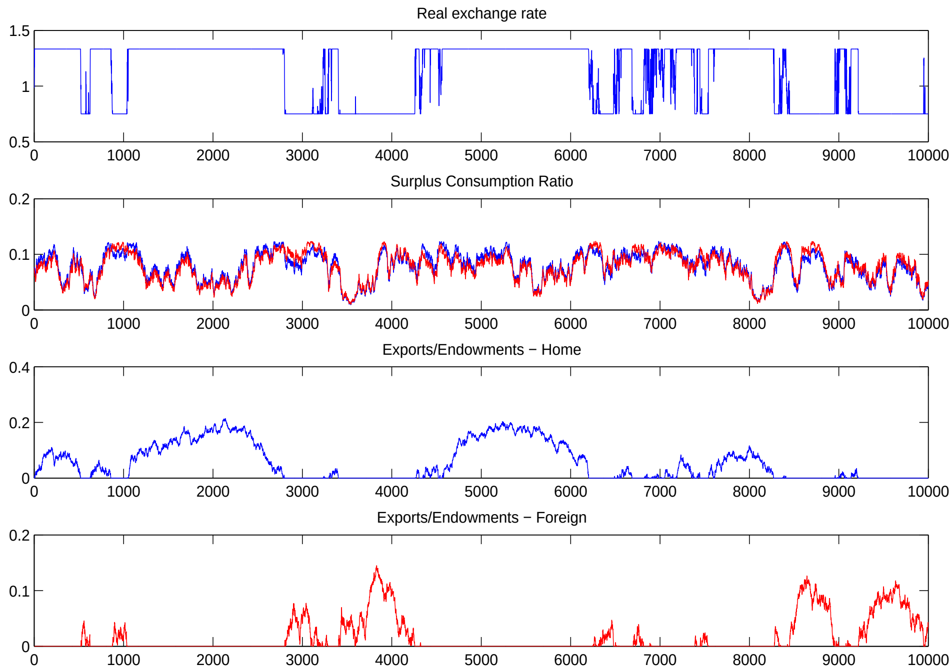

Figure 2. Snapshot of a simulation with proportional trade costs (first 10,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries. The last two panels present the exports/endowments ratios (X/Y and $X^{\star}/Y^{\star}$) at home and abroad. The simulation parameters are reported in the first column of Table (I). The trade cost are $\tau = 25\%$ and $\delta = 0$.

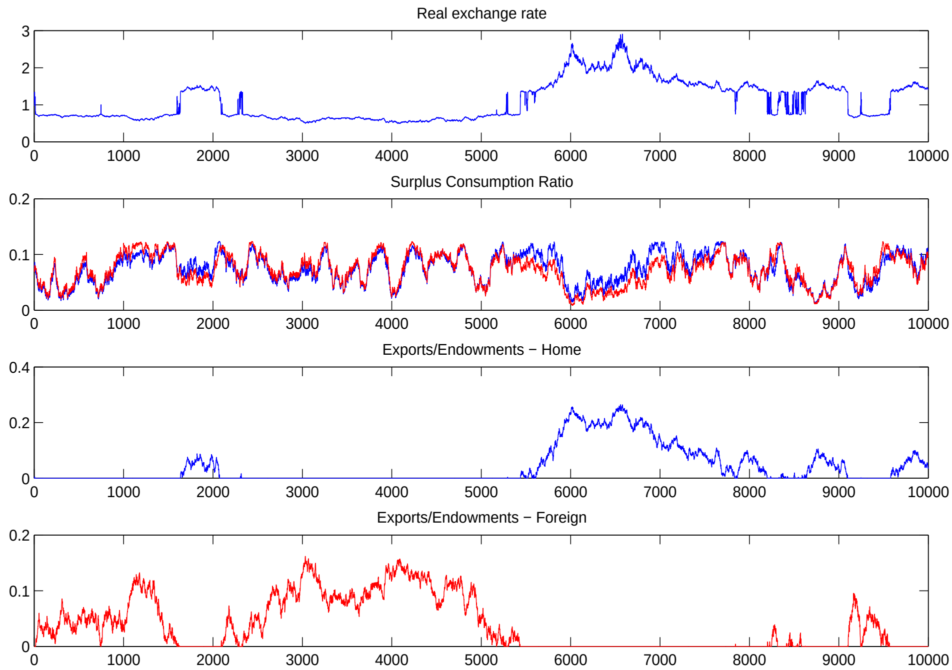

Figure 3. Snapshot of a simulation with proportional and quadratic trade costs (first 10,000 periods). The first panel presents the real exchange rate. The second panel presents the surplus consumption ratios in the two countries. The last two panels present the exports/endowments ratios (X/Y and $X^{\star}/Y^{\star}$) at home and abroad. The simulation parameters are reported in the first column of Table (I). The trade cost are $\tau = 25\%$ and $\delta = 1$.

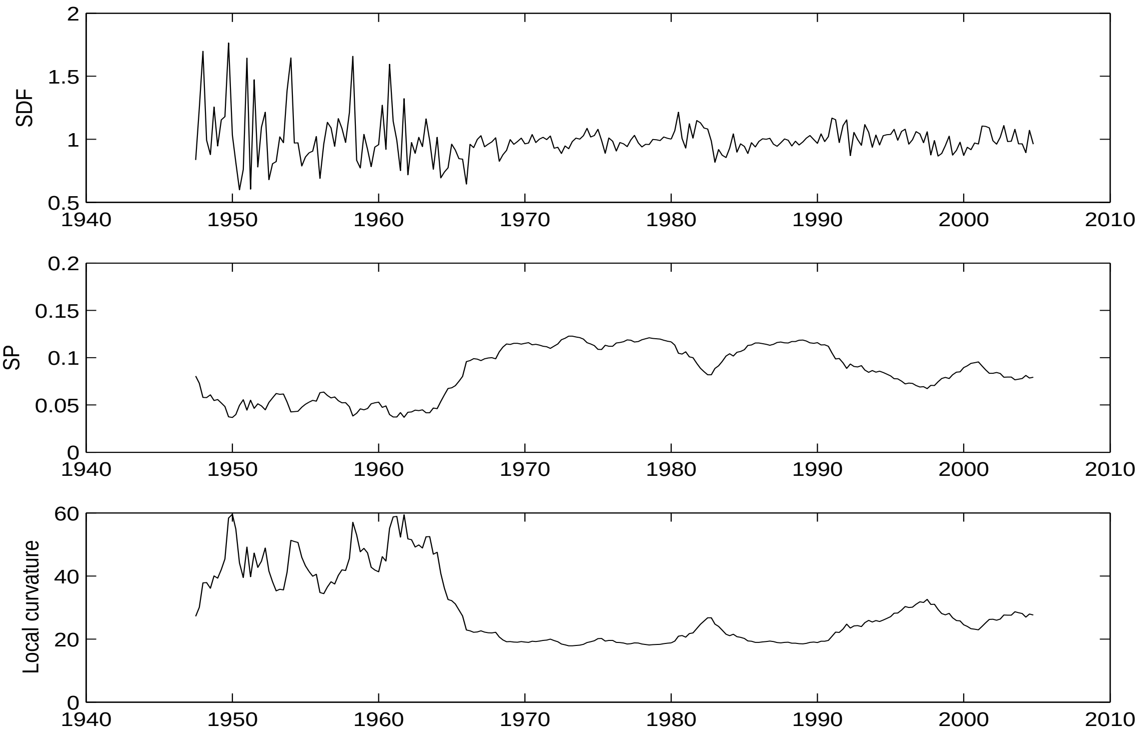

Figure 4. Reality check. Stochastic discount factor (SDF), surplus consumption ratio (SP) and local curvature for an American investor computed with actual US consumption data only over the 1947 : 2 − 2004 : 4 period using the parameters presented in the first column of Table (I) with $\tau = 0$ and $\delta = 0$.