Bailouts and the Incentive to Manage Risk

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In this paper I investigate the incentive effects of bailouts. I present a model in which a firm's liabilities are implicitly guaranteed by the government. Contrary to common intuition, I show that the ability of the government to withdraw its implicit guarantee constrains the firm's ability to exploit the freely provided insurance and take risks. The optimal dynamic policy of the firm is to reduce the riskiness of its portfolio when its net worth declines below an endogenously determined threshold. I show that this is the most effective way to induce the government to continue extending its protection. I argue more generally that the model provides a potential rationale for the optimality of existing risk management practices (e.g. Value at Risk) that require institutions to abandon risky investments when their net assets deteriorate.

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Abstract

In this paper I investigate the incentive effects of bailouts. I present a model in which a firm's liabilities are implicitly guaranteed by the government. Contrary to common intuition, I show that the ability of the government to withdraw its implicit guarantee constrains the firm's ability to exploit the freely provided insurance and take risks. The optimal dynamic policy of the firm is to reduce the riskiness of its portfolio when its net worth declines below an endogenously determined threshold. I show that this is the most effective way to induce the government to continue extending its protection. I argue more generally that the model provides a potential rationale for the optimality of existing risk management practices (e.g. Value at Risk) that require institutions to abandon risky investments when their net assets deteriorate.

1 Introduction

There are few phenomena that capture the public interest more than bailouts of financial institutions and private firms. Certain firms or financial institutions become sufficiently large and important that their potential liquidation poses a threat to the financial system, or presents society with adverse redistributive choices. It is not uncommon for governments or other third parties (such as the IMF) to intervene in such situations and try to bailout such firms. Economic history provides an astoundingly large list of such events, and a significant amount of research has attempted to understand the consequences of such guarantees[1].

Economists are typically opposed to such bailouts, because of the moral hazard issues that they introduce. The most common argument against a bailout is that its anticipation will distort a firm's choice of risk. If the government or another institution (e.g. the IMF) is willing and able to guarantee the downside of an investment, then the firm undertaking the investment should have a stronger incentive to take risk. This insight is widely accepted, well understood, and is presented in many modern day textbooks.[2] Moreover, this fundamental insight has been used by many authors to understand financial crises, to evaluate policy recommendations to the IMF and to derive the cost of such implicit guarantees.

In this paper I take a closer look at this fundamental intuition. My main departure point from pre-existing literature is that I model the private sector's incentives to take risks and the government's incentive to continue extending its implicit guarantee *jointly*. In such a setting it is no longer true that a firm will choose the largest possible level of risk in order to exploit the implicit protection granted by the government. The intuition is that the government

[1]For specific examples, see e.g. Caballero, Hoshi, and Kashyap (2006) who discuss the "Zombie Lending" phnomenon in Japan, Lucas and McDonald (2005) who discuss implicit government guarantees to Government Sponsored Enterprises, Jeanne and Zettelmeyer (2001) who discuss incentive problems created by IMF lending, Schneider and Tornell (2004) who use risk shifting ideas to explain financial crises, Stern and Feldman (2004) who analyze applications to banking and Constantinides, Donaldson, and Mehra (2002) who value the implicit guarantee that would be created by privatizing Social Security.

[2]See e.g. Milgrom and Roberts (1992)

could credibly choose to withdraw its implicit guarantee and leave the firm to its fate. By restricting the riskiness of its portfolio, a firm can reduce the cost (to the government) of the free protection. By doing so, the firm incentivizes the government to continue extending the guarantee.

I illustrate this basic intuition in a simple model where an infinitely lived firm is occasionally dependent on the government's support in order to avoid being liquidated by its creditors. The government derives a benefit from bailing out the firm for exogenous reasons (say because politicians want to avoid the redistributive costs of company liquidation or because the continued operation of the firm has a positive externality). However, this benefit is bounded. Hence, the government has to decide whether it is optimal or not to continue extending its implicit guarantee as a matter of weighing the cost of continuation against the benefit. In calculating the cost, the government rationally anticipates the riskiness of the projects that the firm is going to choose in the future. Anticipation of riskier investments raise the cost of the guarantee, and may lead the government to abandon the firm to its fate. Therefore the firm has an incentive to appropriately restrain itself in its choice of risk.

I derive the firm's optimal policy explicitly and illustrate that it has a particularly simple form: choose projects with high risk levels when net worth (defined as assets minus liabilities) is sufficiently high and switch to projects with low risk levels when net worth falls below a threshold. The intuition for this result is the following: The government's decision to continue extending the guarantee or not is made when the firm's net worth crosses zero. Hence, it is the cost of the implicit guarantee in the proximity of zero net worth that affects the government's decision most strongly. An implication of this fact is that firm volatility choices close to zero net worth will have a *particularly strong influence* on the government's decision to continue extending the implicit guarantee. This is why the firm has an incentive to switch to a less risky portfolio in the proximity of zero net worth.

The analysis suggests two main conclusions:

First, the link between bailouts and the associated increased incentive to take risks depends crucially on the horizon of the government and whether the guarantee is implicit or explicit. If the government has a sufficiently long horizon and its guarantee is implicit, then it will remove its protection whenever there is excessive risk taking. Hence, the model suggests that the standard intuition provided in textbooks relies not only on the convexity of the freely provided option, but also on the finite horizon of the players (firm and government).

Second, the model provides a potential rationale for the risk averse investment choices of financial firms during times of crisis (i.e. when their net worth comes close to zero). Furthermore, the paper proposes potential micro-foundations for existing risk management practices. The model suggests that financial institutions have an incentive to scale back their risk substantially during a crisis, in order to incentivize the government to extend its protection to them.

1.1 Relation to the literature

This paper belongs to the same strand of literature that was initiated by the seminal Merton (1974),Merton (1978) papers and was adopted by many other papers ever since.[3] This literature typically uses the risk neutral pricing approach of Cox and Ross (1976) to price implicit guarantees by familiar option valuation techniques. I differ with that framework in three important ways: First, I do not only consider a one-shot option but rather the endogenously determined appropriate sequence of transfers that are required to keep the firm alive. This allows me to operate in an infinite horizon setting and study the government's continuation value. Second, I explicitly model the benefits and the costs of guarantee extension. Third, I allow the firm to choose volatility endogenously.

The model can form a potential rationale for portfolio restrictions and guidelines that are observed in reality. Several authors have (exogenously) incorporated such restrictions in general equilibrium models.[4] A common theme of this literature is that effective risk aversion increases as net worth declines. The present paper naturally complements this literature by

[3] E.g. Ronn and Verma (1986), Leland (1994), Leland and Toft (1996), Pennacchi (1999), Pennacchi and Lewis (1994), Lucas and McDonald (2005). See also Duffie (2001) for a textbook treatment.

[4] See e.g. Grossman and Zhou (1996), Basak (1995) Pavlova and Rigobon (2005), Basak and Shapiro (2001)

providing a micro-economic reasoning behind such behavior.

The paper shares some remote similarities with models of endogenous default. Models with endogenous default typically place endogenous limits on the borrowing activity[5] so as to ensure repayment. In a similar fashion, the present paper restricts volatility choices so as to ensure that the government has no incentive to renege on its implicitly provided guarantee. There are however distinct differences, since endogenous default models study consumption smoothing problems, while the present paper studies value maximization of options.

The structure of the paper is as follows. Section 2 presents the setup of the basic model. Section 3 presents the solution and the main results. Section 4 presents an intuitive discussion of the results. Section 5 highlights the importance of the horizon of the government. Section 6 concludes. All proofs are relegated to the appendix.

2 Model

There are four types of agents in the model: a continuum of competitive lenders, a single firm, a single government and a risk manager.

2.1 Lenders and the government

The lenders hold a fixed liability of the firm in the amount L. This liability remains constant throughout time for simplicity. The firm also owns assets in the amount W_t, so that the firm's net worth at time t is $W_t - L$. The assets of the firm satisfy $W_0 > L$ at time 0. Throughout the paper, I shall assume that the firm's volatility policy is determined by its risk manager, who acts in the best interest of shareholders, i.e. so as to maximize shareholder value.

The firm can invest its assets in projects involving high or low risk. Under the risk neutral measure both yield an expected return equal to the interest rate r per unit of time dt. However, projects involving high risk have instantaneous volatility σ_2, while less risky

[5] See for example Eaton and Gersovitz (1981),Alvarez and Jermann (2000) among others.

projects have a lower volatility $\sigma_1 < \sigma_2$. The firm can costlessly adjust the fraction that it invests in high and low risk projects. As a result its assets follow a geometric Brownian Motion under the risk neutral measure:

$$\frac{dW_t}{W_t} = rdt + \sigma_t dZ_t \tag{1}$$

where $r > 0$ is the prevailing (real) interest rate in the economy, dZ_t is a standard Brownian motion, and σ_t presents the volatility of total assets. By constantly adjusting the fraction it invests in high risk and low risk assets a firm can attain any level of $\sigma_t \in [\sigma_1, \sigma_2]$ for all $t \geq 0$. I shall assume that a firm can never fully eliminate risk[6], i.e. that $\sigma_1 > 0$.

To keep the analysis simple, I shall assume that the firm can pay no intermediate dividends to its shareholders until a random time τ. In particular, I shall assume that the firm only pays a liquidating dividend to its shareholders in the magnitude of $W_\tau - L$ at time τ. By imposing such a simple exogenous dividend policy, all the risk shifting incentives will be reflected in the firm's volatility choice[7]. The firm also pays a flow of rL to its lenders, up to the time of its liquidation.

The arrival of time τ is assumed to be exogenous with constant hazard $\lambda > 0$. This will facilitate the use of infinite horizon optimization techniques by making all solutions independent of time. In addition to this exogenous arrival of termination, lenders can terminate the firm prior to τ : By covenant,[8] they can enforce liquidation if the assets of the firm fall below its liabilities, i.e. if $W_t < L$. As a result lenders hold riskless debt: they liquidate the

[6]This assumption captures the idea that the firm is a productive entity that cannot fully eliminate the risks associated with its operation. This is a common assumption on the evolution of the capital stock in production economies (see e.g. Cox, Ingersoll, and Ross (1985)). One could relax this assumption without loss in generality by assuming that the firm cannot invest more than a fraction $\phi < 1$ in riskless securities. In that case the lower bound on the volatility would be $(1 - \phi)\,\sigma_1$.

[7]By contrast, if one allowed the firm to distribute dividends, part of the risk shifting incentives would take the form of excessive dividend payouts. Hence, imposing an exogenous dividend policy guarantees that all risk shifting motives will be reflected in excessive risk taking, which is the focus of the analysis.

[8]An alternative assumption that would be equivalent is that all debt is short term and hence needs to be renewed constantly.

firm once W_t crosses the threshold L, and seize the assets W_t. Hence they are guaranteed that they will receive a flow of rL until liquidation and repayment of the principal L upon liquidation.

When the firm gets terminated, the government incurs a (monetary) cost B^* irrespective of the reason for liquidation (endogenous or exogenous). The source of this cost is unmodelled and does not form the focus of the analysis. It could have political origins (e.g. the political cost associated with the firm going bankrupt). Alternatively, it could be assumed that the operation of the firm produces some positive externality, that will be lost if the firm is terminated. Whatever the reason, the government has the option of making transfers to the firm in order to keep its assets above L, and hence prevent liquidation. In mathematical terms

$$dW_t = rW_t dt + \sigma_t W_t dZ_t + dG_t \tag{2}$$

where $dG_t \geq 0$ represents incremental transfers that can be used once $W_t = L$ in order to enforce $W_t \geq L$ for all t. By the Skorohod equation (Karatzas and Shreve (1991), p.210) the unique process for G_t that will only increase only when $W_t = L$ and will safeguard that $W_t \geq L$ for all t is given by:

$$\int_0^t dG_s = \max\left[1, \frac{L}{W_0} \max_{0\leq s\leq t}\left(e^{-\left(rs-\int_0^s \frac{\sigma_u^2}{2}du+\int_0^s \sigma_u dZ_u\right)}\right)\right] - 1.$$

Modelling bailouts as direct transfers is without loss of generality. Even if the government were to extend loans to the firm at rates that are more attractive than the prevailing market yields, this is economically identical to a direct transfer of the capitalized gain from such a loan.[9]

[9] A loan that is fairly priced would clearly not help salvage a firm with $W_t < L$. To see this, suppose that the government gave a loan of D and required repayment of this loan with payments that would be equal to D in net present value for all possible future histories. Then the firm's assets would increase by D but its liabilities would also increase by D in order to reflect the increase in the net present value of future payouts. Lenders would understand that such a loan would not affect the net worth of the firm, and would still terminate the firm. Hence, for a government loan to be effective in postponing termination, it must be

A key assumption of the model is that the government's protection to the firm is implicit. The government has the option of making the incremental transfers $dG_t \geq 0$, but not the obligation. In particular, once the assets of the firm become equal to its liabilities, the government can decide whether to make the transfers dG_t or to just let the lenders seize the assets and terminate the firm.

I will assume that raising taxes in order to finance bailouts causes a distortion that is equal to the taxes raised. As a result, the government does not view bailouts as pure transfers between shareholders and taxpayers, but perceives them as having a real cost to the economy. A necessary and sufficient condition for the government to always prefer to bailout the firm is that the net present value of the costs associated with keeping the firm alive is less than the benefit of doing so:

$$E_t \left(\int_t^\tau e^{-r(s-t)} dG_s | W_t = L \right) + E_t e^{-r(\tau - t)} B^* \leq B^* \tag{3}$$

where τ is the time of exogenous termination. The left hand side is composed of the the net present value of transfers that need to be made by the government in order to prevent the lenders from terminating the firm every time that $W_t = L$ plus the net present value of the cost to be paid upon exogenous termination. The right hand side is the benefit of keeping the firm alive, namely the avoidance of the cost B^*. Inequality (3) safeguards that the government will protect the firm only if the cost of extending the guarantee is lower than the cost of letting the lenders liquidate the firm. Since $E_t e^{-r(\tau-t)} B^*$ does not depend on W_t, it is a constant:[10]

$$E_t e^{-r(\tau - t)} B^* = \frac{\lambda}{r + \lambda} B^*$$

the case that it is unfairly priced: The net present value of future interest and loan repayments has to be less than the face value of the loan. Clearly, an unfairly priced loan is equivalent to an outright transfer.

[10] The easiest way to see this is to define: $F = E_t e^{-r(\tau - t)} B^*$ and then note that F solves the Bellman equation:

$$-rF + \lambda (B^* - F) = 0.$$

and hence it will be convenient to define

$$B \equiv \frac{r}{r+\lambda} B^{*}$$

and rewrite (3) as:

$$E_t \left(\int_t^{\tau} e^{-r(s-t)} dG_s | W_t = L \right) \leq B. \tag{4}$$

Since the firm controls the volatility process σ_t it also influences the net present value of the governmental transfers on the left hand side of equation (4).

2.2 Shareholders

Shareholder value is given by:

$$V(W_t) \equiv -E_t \left(\int_t^{\tau^l} e^{-r(s-t)} r L ds \right) + E_t \left(e^{-r\left(\tau^l - t\right)} \left(W_{\tau^l} - L \right) \right) \tag{5}$$

The first term in the definition of V captures the net present value of interest payments to the lenders of the firm. The second term captures the net present value of the liquidating dividend paid to shareholders. All expectations in the paper are taken under the risk neutral measure.

The next Lemma states that shareholder value is composed of two components: a) net worth $(W_t - L)$ and b) the value of the implicit option that the government extends to the firm, which is defined as $P(W_t)$.

Lemma 1 *Fix any volatility process $\overline{\sigma}_t$ and any termination time τ^l, and define:*

$$P(W_t) = E_t \left(\int_t^{\tau^l} e^{-r(s-t)} dG_s \right) \tag{6}$$

Then, shareholder value V is given by:

$$V(W_t) = W_t - L + P(W_t) \tag{7}$$

Two things become clear from the above Lemma. First, volatility choices by the managers will affect firm value through their effect on the value of the guarantee $P(W_t)$. Second, shareholders have an incentive to induce the government to always keep extending its guarantee: $P(W_t)$ is a positive number only as long as the government finds it optimal to keep extending its guarantee. However, if the government withdrew its guarantee, then $dG_s = 0$, and hence $P(W_t) = 0$.

To check intuition, it is also useful at this stage to confirm that the firm has an incentive to set high levels of volatility in order to exploit the free guarantee provided by the government. To be more specific, ignore for a minute the constraint (4) and assume that the government credibly committed to extend its protection unconditionally and perpetually, so that $\tau^l = \tau$ in equation (6). Then, the following result is true:

Lemma 2 *Assume that*

$$\tau^l = \tau$$

in expression (6). Assume furthermore that volatility is constant at the level $\overline{\sigma}$ *for all* $t \geq 0$ *and define* α *as:*

$$\alpha(\overline{\sigma}) = \frac{-(r - \frac{1}{2}\overline{\sigma}^2) - \sqrt{\left(r - \frac{\overline{\sigma}^2}{2}\right)^2 + 2\overline{\sigma}^2(r + \lambda)}}{\overline{\sigma}^2} < 0 \tag{8}$$

Then, the value of the government guarantee is given by:

$$P(W_t; \overline{\sigma}) = -\frac{L}{\alpha}\left(\frac{W}{L}\right)^{\alpha} \tag{9}$$

It is also straightforward to show the following result:

Lemma 3 *Assume that*

$$\tau^l = \tau$$

in expression (6). Then the volatility choice that maximizes $P(W_t)$ *is given by:*

$$\sigma_t = \sigma_2 \text{ for all } t > 0$$

In light of the above result, if the government extended an unconditional and perpetual guarantee to the firm, then the shareholder value maximizing choice of volatility would be to set σ_t equal to its upper bound σ_2 for all $t > 0$. This captures the standard moral hazard intuition of free guarantees.

The above two Lemmas only apply if the government guarantee is perpetual and unconditional. The focus of this paper, however, is on guarantees that are implicit, i.e. guarantees that will only be extended if (4) is satisfied. In order to make the problem interesting, I shall place the following restriction on the parameters of the model:

Assumption 1

$$P(L;\sigma_1) = -\frac{L}{\alpha(\sigma_1)} < B < -\frac{L}{\alpha(\sigma_2)} = P(L;\sigma_2) \tag{10}$$

In light of Lemma 2 and (4), assumption 1 has two implications: a) first to ensure that the firm has at least one feasible choice of volatility that will make it possible to satisfy the constraint (4) (namely by setting $\sigma_t = \sigma_1$) and b) that setting volatility equal to the upper bound σ_2 for all $t > 0$ will violate the constraint (4).

2.3 Managers

The previous analysis illustrated the tension between incentives to raise volatility and the restraint that needs to be applied in order to satisfy the government's incentive compatibility constraint (4). I turn next to the analysis of this tension.

The firm's volatility choices are determined by a manager who gets hired by shareholders out of a continuum of identical managers. This manager acts as a commitment device for shareholders: Once hired, she has to announce a policy that will be followed by the company during her tenure. If she ever deviates from her announced plan of action, she will face an infinite personal cost (moral and professional disgrace, poor subsequent career prospects etc.). Hence, a manager will always adhere to the plan that she announces.

However, managers can be fired at any time by shareholders costlessely. If shareholders fire a manager, then the new manager has no obligation to honor the commitments made by her predecessor: Each manager only has to adhere to the plan that she announces, once hired. Therefore, managers present the firm with a very limited ability to commit: If the current manager has committed in the past to a policy that seems suboptimal given the current situation, then a new manager can be hired who will not have to honor the commitments of her predecessor.

A manager's objective is to maximize shareholder value subject to the constraint that she doesn't get fired by shareholders. Accordingly, she can only commit to policies that are maximizing shareholder value not only at the time that she gets hired, but also at every possible point in time thereafter. I will refer to this notion as *time invariant commitment* and formulate it mathematically as follows:

Problem 1 *(The manager's problem under time invariant commitment) Let* $\overline{\sigma}_s$, $s \geq t$ *denote any adapted stochastic process with values in the interval* $[\sigma_1, \sigma_2]$. *Then choose* $\overline{\sigma}_s$ *so as to maximize:*

$$\max_{\overline{\sigma}_s \in [\sigma_1, \sigma_2]} P\left(W_t; \sigma_{s \geq t}\right) \text{ for all } t \tag{11}$$

and for any possible value of W_t in $[L, \infty)$, *subject to the constraint:*

$$P(L; \sigma_{s \geq t}) \leq B \text{ for all } t \tag{12}$$

There are two remarks about this formulation of the problem.

First note that shareholder value maximization is equivalent to maximizing the value of the guarantee provided by the government. This is a direct application of the decomposition of shareholder value provided in Lemma 1, which shows that volatility choices affect shareholder value only through their effect on $P(W_t)$.

Second, note that the manager's commitment needs to maximize $P(W_t)$ not only at time 0 when $W_t = W_0$, but also at any future time t and for any possible value of W_t after that.

In light of this second remark, it is interesting to relate the notion of time invariant commitment to dynamic programming. The two concepts are fundamentally identical, with the exception that time invariant commitment allows us to impose the constraint (12). Under time invariant commitment, each manager has to precommit her policy once hired. More importantly, she knows that every future manager has to do the same. If any manager made the mistake of committing to a policy that violated the constraint (12), then shareholders would have an incentive to fire her as soon as $W_t = L$, else the government wouldn't extend its guarantee at that point. Moreover, the current manager knows that every other manager will think the same way and will impose the constraint (12) on her policies, when she makes her commitment.

However, time invariant commitment is a quite weak notion of commitment. The commitment powers of managers stop at imposing the constraint (12) on her actions. The requirement (11) rules out any commitment that goes beyond (12). Given the forward looking nature of both (11) and the constraint (12), Bellman's principle asserts that optimal policies must be Markovian, namely: $\sigma_s = \sigma(W_s)$. To simplify notation, I shall use $\sigma(W)$ instead of $\sigma(W_s)$ from this point on.

3 Solution

3.1 The set of feasible payoffs

The first step towards solving problem 1 is to characterize the set of payoff functions $P(W)$ that can be attained by Markovian policies of the form $\sigma(W)$, while also satisfying (12). This is the purpose of the next Lemma:

Lemma 4 *Let the payoff function P be defined as in (6), and assume that it satisfies constraint (12). Then the following results hold for any $\sigma(W) \in [\sigma_1, \sigma_2]$:*

1. *In the domain (L, ∞), P satisfies the ordinary differential equation:*

$$\frac{\sigma^2(W)}{2} W^2 P_{WW} + r P_W W - (r + \lambda) P = 0 \tag{13}$$

2. *P satisfies the bounds*

$$0 \leq P \leq B \text{ for all } L \leq W < \infty \tag{14}$$

At $+\infty$ *the function* P *satisfies:*

$$\lim_{W \to \infty} P(W) = 0 \tag{15}$$

3. *The derivatives of* P *satisfy:*

$$P_W(L) = -1, \;\; P_W < 0, P_{WW} > 0 \tag{16}$$

Lemma 4 states formally several properties of any feasible payoff function, that we would expect to be true intuitively. The first property is a familiar Black-Scholes type differential equation. It states that $e^{-(r+\lambda)t} P(W_t)$ will behave as a (local) martingale in (L, ∞), which is to be expected, since P pays no dividends in that domain. It is just a conditional expectation of discounted future payoffs. The second statement in Lemma 4 places upper and lower bounds on the set of feasible payoffs. To see why P will always be between those two bounds, let:

$$\tau^L = \inf\{t : W_t = L\}$$

and rewrite P as:

$$P(W_t) = E_t \left(e^{-r[(\tau^L \wedge \tau^l) - t]} E_{\tau^L} \int_{\tau^L \wedge \tau^l}^{\tau^l} e^{-(r+\lambda)(s-\tau)} dG_s \right) \leq E_t \left(e^{-r[(\tau^L \wedge \tau^l) - t]} B \right) \leq B \tag{17}$$

The first equality follows from the law of iterated expectations. The first inequality follows by constraint (12) and the second inequality follows since $e^{-r[(\tau^L \wedge \tau^l) - t]} \leq 1$. The non-negativity of $P(W_t)$ is obvious since $dG_s \geq 0$.

Property 3 has a somewhat more intricate proof, which is given in the appendix. It is however straightforward to give a heuristic intuition for $P_W(L) = -1$, which turns out to be key in the proofs that follow: Consider a situation where the assets of the firm fall below L

by a small amount ε. Then, the government will intervene in order to restore the assets back to L by making a transfer of ε. Therefore, it is as if the "claim" P pays a "dividend" ε and the state variable W is reset to L:

$$P(L-\varepsilon) = \varepsilon + P(L)$$

Expanding the left hand side in a Taylor fashion around L gives:

$$P(L) - \varepsilon P_W(L) = \varepsilon + P(L)$$

Cancelling $P(L)$ from both sides and dividing by ε gives $P_W(L) = -1$.

Finally, property 2 states that the value of the government guarantee should approach 0 as the financial assets of the firm approach infinity: If financial assets are very high, it becomes unlikely that there will be any payoffs associated with the government guarantee for a long time and hence the discounted value of these payouts approaches 0.

3.2 The optimization problem as an optimal control problem

Lemma 4 asserts that any payoff function satisfying (12) will be convex. This will make it impossible to apply simple dynamic programming techniques since these will not capture the commitment of managers to volatility policies that will satisfy the constraint (12). To avoid this problem, I shall use a direct approach motivated by optimal control. I first solve for the policy $\sigma(W_{s\geq t})$ that maximizes $P(W_t)$ imposing the constraint (12). Then I show that the resulting policy does not depend on the level of wealth W_t at the time of its initiation, and hence it would still be optimal if it were "re-initiated" at any future time $t_2 > t$ when the level of assets is W_{t_2}. From that I conclude that it presents the optimal policy under time invariant commitment.

The body of the text contains a heuristic derivation of the optimal control problem. Exact proofs are given in the appendix.

As a first step, note that $P(W_t)$ in the maximization problem 1 can be rewritten as:

$$P(W_t) = P(L) + \int_L^{\infty} P'(x) 1\{x < W_t\} dx$$

$1\{x < W_t\}$ is an indicator function taking the value 1 if $x < W_t$ and 0 otherwise. One can restrict without loss of generality attention to policies / payoffs that will make the constraint (12) hold as an equality[11]:

$$P(L) = B$$

Furthermore, using the characterization of all attainable payoffs from Lemma 4, one can rewrite the optimization problem 1 as a standard optimal control problem using (P, P') as state variables:

$$\max_{\sigma(x)} \int_L^\infty P'(x) 1\{x < W_t\} dx$$

$$\begin{bmatrix} P' \\ P'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2(r+\lambda)}{\sigma^2}\frac{1}{x^2} & -\frac{2r}{\sigma^2}\frac{1}{x} \end{bmatrix} \begin{bmatrix} P \\ P' \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} P(L) \\ P'(L) \end{bmatrix} = \begin{bmatrix} B \\ -1 \end{bmatrix}, \lim_{x\to\infty} P(x) = 0 \tag{19}$$

Equation (18) is simply a transformation of the second order equation (13) to a system of two first order ordinary differential equations, while equation (19) gives the boundary conditions of the state variables (P, P') at L and ∞.

Letting π_1, π_2 denote the co-state variables for the two state variables (P, P'), the Hamiltonian for this optimal control problem is :

$$H = 1\{x < W_t\}P'(x) + \pi_1 P'(x) + \pi_2 \frac{2}{\sigma^2}\left((r+\lambda)\, P(x)\frac{1}{x^2} - rP'(x)\frac{1}{x}\right) \tag{20}$$

Since $P'' > 0$ (by [16]) maximizing H w.r.t. σ gives the optimal policy

$$\sigma^*(x) = \begin{cases} \sigma_1 \text{ if } \pi_2 > 0 \\ \sigma_2 \text{ if } \pi_2 < 0 \end{cases} \tag{21}$$

[11] If any policy $\sigma(W)$ has the property $P(L) < B$ then increasing volatility by a sufficiently small ε for all $\sigma(W) < \sigma_2 - \varepsilon$, will raise $P(W)$ without violating the constraint. This is due to the fact that $P_{WW} > 0$ and can be shown directly by methods similar to the ones that are used in section 4.

By standard optimal control theory, the co-state variables must satisfy:

$$\dot{\pi}_1 = -\frac{2(r+\lambda)}{[\sigma^*(x)]^2}\pi_2\frac{1}{x^2} \tag{22}$$

$$\dot{\pi}_2 = -(\pi_1 + 1\{x < W_t\}) + r\frac{2}{[\sigma^*(x)]^2}\pi_2\frac{1}{x} \tag{23}$$

3.3 Solving the optimal control problem

In order to solve the optimal control problem, it is easiest to conjecture the form of the optimal policy, construct a solution to the equation for the co-state variables, and verify that the obtained solution along with the conjectured policy satisfy (21). Given the form of (21) it is reasonable to conjecture that the optimal policy will have a "bang-bang" form, with a switch at the point W^* where π_2 changes sign.

In particular, assume that the optimal policy is of the form:

$$\sigma^*(x) = \begin{cases} \sigma_1 \text{ if } x < W^* \\ \sigma_2 \text{ if } x \geq W^* \end{cases} \tag{24}$$

for an appropriately chosen constant W^* that depends on the parameters of the problem. The next Lemma uses policy (24) to determine the value of $P(W_t; \sigma^*(x))$ for an arbitrary $W^* \geq L$ and then determines W^* in such a way as to satisfy the boundary condition $P(L; \sigma^*) = B$.

Lemma 5 *Take an arbitrary $W^* > L$ and suppose that policy (24) is used. Then $P(W_t; \sigma^*)$ is given by*

$$\frac{P(W_t;\sigma^*)}{L} = \begin{cases} \left(\frac{W_t}{L}\right)^{\alpha_1^+} \dfrac{\frac{\alpha_2^- - \alpha_1^-}{\alpha_2^- - \alpha_1^+}\left(\frac{L}{W^*}\right)^{\alpha_1^+ - \alpha_1^-}}{\left[\alpha_1^- - \alpha_1^+ \frac{\alpha_2^- - \alpha_1^-}{\alpha_2^- - \alpha_1^+}\left(\frac{L}{W^*}\right)^{\alpha_1^+ - \alpha_1^-}\right]} - \left(\frac{W_t}{L}\right)^{\alpha_1^-} \dfrac{1}{\left[\alpha_1^- - \alpha_1^+ \frac{\alpha_2^- - \alpha_1^-}{\alpha_2^- - \alpha_1^+}\left(\frac{L}{W^*}\right)^{\alpha_1^+ - \alpha_1^-}\right]} & \text{if } L \leq W_t \leq W^* \\ \dfrac{\left[\frac{\alpha_2^- - \alpha_1^-}{\alpha_2^- - \alpha_1^+} - 1\right]\left(\frac{L}{W^*}\right)^{\alpha_2^- - \alpha_1^-}}{\left[\alpha_1^- - \alpha_1^+ \frac{\alpha_2^- - \alpha_1^-}{\alpha_2^- - \alpha_1^+}\left(\frac{L}{W^*}\right)^{\alpha_1^+ - \alpha_1^-}\right]}\left(\frac{W_t}{L}\right)^{\alpha_2^-} & \text{if } W_t > W^* \end{cases} \tag{25}$$

where

$$\alpha_1^{\pm} = \frac{-\left(r - \frac{\sigma_1^2}{2}\right) \pm \sqrt{\left(r - \frac{\sigma_1^2}{2}\right)^2 + 2\sigma_1^2 (r + \lambda)}}{\sigma_1^2} \tag{26}$$

$$\alpha_2^{\pm} = \frac{-\left(r - \frac{\sigma_2^2}{2}\right) \pm \sqrt{\left(r - \frac{\sigma_2^2}{2}\right)^2 + 2\sigma_2^2 (r + \lambda)}}{\sigma_2^2} \tag{27}$$

Accordingly, $P(L;\sigma^*) = B$ *if and only if* W^* *is chosen as:*

$$W^* = L \left[\frac{\alpha_2^- - \alpha_1^+}{\alpha_2^- - \alpha_1^-} \frac{\left(1 + \frac{B}{L}\alpha_1^-\right)}{\left(1 + \frac{B}{L}\alpha_1^+\right)} \right]^{\frac{1}{\alpha_1^- - \alpha_1^+}} \tag{28}$$

Lemma 5 determines the appropriate value of W^*, that makes (24) a policy that satisfies (12). Given this value of W^*, the next step is to examine whether there exist co-state variables that will "support" such a policy. This can be checked by using the conjectured optimal policy and examining whether there exists a solution to the system of equations (22) and (23), satisfying:

$$\pi_2(W^*) = 0 \tag{29}$$

and

$$\pi_2(x) \begin{cases} \geq 0 \text{ if } x < W^* \\ \leq 0 \text{ if } x > W^* \end{cases} \tag{30}$$

with at least one of the two inequalities being strict for some values x. Furthermore, to provide sufficient conditions for the optimality of policy (24), I shall also require

$$\lim_{x \to \infty} |\pi_1(x)| < \infty \tag{31}$$

$$\lim_{x \to \infty} |\pi_2(x)| < \infty \tag{32}$$

The appendix constructs an explicit continuous solution to π_1, π_2 that satisfies (22), (23) and (29), (30), (31), (32).

Lemma 6 *Let* W^* *be given by (28). Then, there exist continuous functions* π_1 *and* π_2 *(constructed explicitly in the proof) that solve the pair of differential equations (22), (23) and satisfy (29), (30), (31), (32).*

Given this solution to π_1, π_2, it is now possible to establish the key proposition of the paper.

Proposition 1 *Let σ_t^* be defined as in (24) with W^* given by (28). Then*

$$P\left(W_t; \sigma_t^*\right) \geq P\left(W_t; \sigma(W_{s\geq t})\right)$$

for any feasible volatility policy $\sigma(W_{s\geq t})$ and for any $W_t \geq L$. *The inequality becomes an equality only if $\sigma(W_{s\geq t}) = \sigma_t^*$.*

This proposition implies that the firm will always follow a simple policy: keep volatility at the lower bound σ_1 until a threshold level W^*, and then switch to maximal volatility if current assets W_t exceed W^*. The critical wealth level W^* is determined in such a way as to make the key constraint (12) hold as an equality. Most importantly, it does not depend on the level of wealth W_t at the time at which the policy is initiated. Therefore it continues to be optimal if the level of assets becomes W_{t_2} at some later time t_2. Therefore it satisfies all the requirements of an optimal time invariant commitment.

4 The intuition behind the optimal policy

In this section I provide some intuition on two properties of the solution that may seem surprising at first. The first is a substantive issue: Why is it optimal for the firm to lower, instead of raise volatility as its net worth declines ? The second issue is technical, but turns out help in understanding the same substantive properties of the solution: In setting up the optimal control problem, the Hamiltonian (20), depends on the level of assets at which the commitment is entered (W_t). How is it then, that the optimal solution doesn't?

The answer to both questions follows from inspection of the constraint (12). The government's incentive compatibility constraint is checked *only at the lowest* possible level of assets (L), since it is only then that the government has a decision to make. Therefore, the government's decision is affected strongly by the volatility choices of the firm in a neighborhood of $W_t = L$. By contrast, the commitment of the manager needs to be optimal for any level of assets W_t.

The firm cannot set volatility equal to the upper bound σ_2 throughout[12]. Therefore, it has to "promise" the government that volatility will be lower than σ_2 in certain states of the world. As already mentioned, the government only cares about volatility choices close to L. Therefore, the "cheapest" way to satisfy the government's incentive compatibility constraint is to set volatility as low as possible when net assets are in a vicinity of the lower bound L.

Indeed, under the optimal policy, the manager of the firm promises low volatility for just enough states of the world with low levels of assets, that will satisfy the government's incentive compatibility constraint. The constant W^* in (28) is determined so as ensure that. Moreover, since the constant W^* is chosen so as to make the value of $P(L)$ exactly equal to B, it cannot possibly depend on the level of assets at which the commitment is entered (W_t), as $P(L)$ and B do not depend on W_t.

This safeguards that the optimal policy will not depend on the time at which it is entered.

To conclude, the model suggests a potential rationalization for existing approaches to risk management that are widely adopted by firms (value at risk). Analyses such as Basak and Shapiro (2001) (for value at risk) and Grossman and Zhou (1996), Basak (1995) (for portfolio insurance) suggest that popular risk management approaches limit a firm's ability to take risks as its assets decline. Furthermore, the model helps explain phenomena such as flight to quality. In response to a series of negative shocks that will erode its assets, a firm should optimally lower the riskiness of its investments.

The key intuition behind these results is that the government makes its decisions when the level of assets is at its lower bound L. It appears that this intuition is robust and would survive in any variation of the model as long as the party deciding whether to let the firm survive would have to be induced to do so when the level of assets is low.

[12] By assumption 1

5 Discussion

The standard textbook intuition predicts that free options should lead to the maximal possible levels of volatility choices. In this section I argue that a key difference between the standard textbook treatment and the results obtained in this paper is the long horizon of the government.

To see this, suppose that the government were to apply a higher discount rate to the future than the private sector: Current politicians may not care much about the distortions that will be caused by a firm's future volatility choices -say because with a constant hazard rate $\beta > 0$ they could lose power. Then constraint (3) would become:

$$E_t \left(\int_t^\tau e^{-(r+\beta)(s-t)} dG_s | W_t = L \right) + E_t e^{-(r+\beta)(\tau - t)} B^* \leq B^* \tag{33}$$

The following result then illustrates that as β increases, eventually the standard textbook intuition applies:

Lemma 7 *If constraint (3) is replaced with constraint (33) then the optimal solution is to set*

$$\sigma(W_t) = \sigma_2 \text{ for all } W_t \geq L$$

when β becomes sufficiently high.

In the $\beta \to \infty$ limit, the government becomes very short-termist, and the firm will have a maximal incentive to increase risk. Hence, the key difference between the standard textbook intuition and the current model is the government's horizon.

More generally, the present model reinforces a conclusion reached by Panageas and Westerfield (2005) in a model of professional portfolio choice: The standard textbook conclusion that free options lead to maximal volatility choices relies not only on the convexity of the payoff, but also on the horizon of the players.

6 Conclusion

This paper presented a model, whereby a long-horizon government finds it optimal to undertake bailouts so as to prevent liquidation of a firm. The optimal actions for the government, the firm and the lenders are derived endogenously.

The results of the paper allow two broad conclusions:

First, the link between bailouts and the incentive to take maximal risk is not necessarily present if the guarantee is implicit and the government has a long horizon.

Second, the optimal policy of an infinitely lived firm to "appease" the government is to increase volatility when its net worth is high and reduce it when its net worth declines. This policy seems to be in line with most existing risk management practices that force firms to reduce the volatility of their investments in response to a declining net worth. Therefore the model provides a potential justification for existing approaches to risk management, and is consistent with empirical phenomena such as flight to quality.

The key driver of the paper's results is that the party extending the guarantee (the government) must be incentivized to bailout the firm when the firm assets are particularly low. Therefore, it is in these states of the world where the firm has an incentive to set low levels of volatility, in order to induce the government to undertake the bailout.

It is likely that this intuition can be extended to more general models of reorganization, as long as the reorganization decision is undertaken when assets are low and both termination and high volatility choices present the party deciding on continuation with a negative externality.

A Appendix

Proof of Lemma 1. By (5), it follows that:

$$\begin{aligned}
V(W_t) &= -E_t\left(\int_t^{\tau^l} e^{-r(s-t)} rLds\right) - E_t e^{-r\left(\tau^l-t\right)} L + E_t\left(e^{-r\left(\tau^l-t\right)} W_{\tau^l}\right) = \\
&= -E_t\left(\int_t^{\tau^l} e^{-r(s-t)} rLds\right) - E_t e^{-r\left(\tau^l-t\right)} r\frac{L}{r} + E_t\left(e^{-r\left(\tau^l-t\right)} W_{\tau^l}\right) = \\
&= -L + E_t\left(e^{-r\left(\tau^l-t\right)} W_{\tau^l}\right)
\end{aligned}$$

for any stochastic time τ^l, since L and r are constants. Hence it suffices to show that:

$$W_t + P\left(W_t;\overline{\sigma}_t\right) = E_t\left(e^{-r\left(\tau^l-t\right)} W_{\tau^l}\right)$$

To show this, apply Ito's Lemma to $e^{-rs}W_s$ to get:

$$\begin{aligned}
d(e^{-rs}W_s) &= -re^{-rs}W_s ds + e^{-rs} rW_s ds \\
&\quad + e^{-rs}\overline{\sigma}_s W_s dB_s + e^{-rs} dG_s \\
&= e^{-rs}\overline{\sigma}_s W_s dB_s + e^{-rs} dG_s
\end{aligned}$$

Rewriting this in integral form yields:

$$e^{-r\tau^l} W_{\tau^l} = e^{-rt} W_t + \int_t^{\tau^l} e^{-rs}\overline{\sigma}_s W_s dB_s + \int_t^{\tau^l} e^{-rs} dG_s$$

Rearranging, taking expectations, using the definition (6) and the boundedness of $\overline{\sigma}_s$ gives:

$$E\left(e^{-r(\tau^l-t)} W_{\tau^l}\right) = W_t + P\left(W_t;\overline{\sigma}_t\right)$$

■

Proof of Lemma 2. Let $\tau^{\overline{W}}$ denote the first passage time to some $\overline{W}$:

$$\tau^{\overline{W}} = \inf\{t : W_t \geq \overline{W}\}$$

and consider the price of a guarantee that is terminated at either the exogenous liquidation time τ^l or $\tau^{\overline{W}}$, whichever comes first:

$$P^{(\overline{W})}\left(W_t;\overline{\sigma}\right) = E_t\left(\int_t^{\tau^{\overline{W}}\wedge\tau^l} e^{-r(s-t)} dG_s\right) \tag{34}$$

It is easiest to price this claim first and then take the limit as $\overline{W} \to \infty$ in order to arrive at (9). One can use standard results to express $P^{(\overline{W})}$ as:

$$P^{(\overline{W})}\left(W_t; \overline{\sigma}\right) = E_t\left(\int_t^{\tau^{\overline{W}}} e^{-(r+\lambda)(s-t)} dG_s\right) \tag{35}$$

In order to construct $P^{(\overline{W})}$ it is easiest to determine certain properties that function $P^{(\overline{W})}$ should satisfy. Using Ito's Lemma, and taking expectations, it is straightforward to establish that:

$$\begin{aligned} 0 \;=\;& P^{(\overline{W})}(W_t) \\ & -E_t\left[e^{-(r+\lambda)(\tau^{\overline{W}}-t)} P^{(\overline{W})}(\overline{W})\right] + \\ & +E_t\left[\int_t^{\tau^{\overline{W}}} e^{-(r+\lambda)(s-t)}\left(\frac{\overline{\sigma}^2 W_s^2}{2} P_{WW}^{(\overline{W})} + r P_W^{(\overline{W})} W_s - (r+\lambda) P^{(\overline{W})}\right) ds\right] \\ & +E_t\left[\int_t^{\tau^{\overline{W}}} e^{-(r+\lambda)(s-t)} \overline{\sigma} P_W^{(\overline{W})} W_s dB_s\right] \\ & +E_t\left[\int_t^{\tau^{\overline{W}}} e^{-(r+\lambda)(s-t)} P_W^{(\overline{W})}(L) dG_s\right] \end{aligned} \tag{36}$$

Suppose now that one can find a function $P_W^{(\overline{W})}$ such that:

$$\frac{\overline{\sigma}^2 W_s^2}{2} P_{WW}^{(\overline{W})} + r P_W^{(\overline{W})} W_s - (r+\lambda) P^{(\overline{W})} = 0 \tag{37}$$

$$P_W^{(\overline{W})}(L) = -1 \tag{38}$$

$$P^{(\overline{W})}(\overline{W}) = 0 \tag{39}$$

Suppose moreover that $P_W^{(\overline{W})}$ is bounded in the domain $\left[L, \overline{W}\right]$ .Then, it is clear that (36) reduces to:

$$P^{(\overline{W})}(W_t) = -E_t\left[\int_t^{\tau^{\overline{W}}} e^{-(r+\lambda)(s-t)} P_W^{(\overline{W})}(L) dG_s\right] \tag{40}$$

since the second and third line in (36) drop out. Moreover, the fourth line is 0 since $\overline{\sigma} P_W^{(\overline{W})} W_s$ is bounded and hence the local martingale inside the square brackets is indeed a martingale.

Combining (38) and (40) leads to (35).

Finding a $P_W^{(\overline{W})}$ that satisfies (37), along with the boundary conditions (38) and (39) is straightforward. The general solution to (37) is:

$$P^{(\overline{W})}(W_t) = C_1 W_t^{\alpha} + C_2 W_t^{b}$$

where C_1, C_2 are arbitrary constants and α is given by (8) and b is given by

$$b = \frac{-(r - \frac{1}{2}\overline{\sigma}^2) + \sqrt{\left(r - \frac{\overline{\sigma}^2}{2}\right)^2 + 2\overline{\sigma}^2 (r + \lambda)}}{\overline{\sigma}^2} > 0$$

To satisfy (38) and (39), one needs to determine C_1, C_2 such that:

$$\begin{aligned} P_W^{(\overline{W})}(L) &= \alpha C_1 L^{\alpha-1} + b C_2 L^{b-1} = -1 \\ P^{(\overline{W})}(\overline{W}) &= C_1 \overline{W}^{\alpha} + C_2 \overline{W}^{b} = 0 \end{aligned}$$

Solving for C_1, C_2 yields the solution:

$$P^{(\overline{W})}(W_t) = \frac{\frac{L}{\alpha}\left(\frac{\overline{W}}{L}\right)^{\alpha} W_t^{b} - \frac{L}{\alpha}\left(\frac{\overline{W}}{L}\right)^{\alpha} \overline{W}^{b-\alpha} W_t^{\alpha}}{\overline{W}^{b} - \frac{b}{\alpha} L^{b} \left(\frac{\overline{W}}{L}\right)^{\alpha}}$$

To conclude the proof, let $\overline{W} \to \infty$, to obtain:

$$\lim_{\overline{W}\to\infty} P^{(\overline{W})}(W_t) = P\left(W_t\right) = -\frac{L}{\alpha}\left(\frac{W_t}{L}\right)^{\alpha}$$

The statement of the theorem is then a consequence of the monotone convergence theorem.

■

Proof of Lemma 3. Consider the Hamilton Jacobi Bellman equation for $P(W_t)$:

$$\max_{\sigma\in[\sigma_1,\sigma_2]} \left\{\frac{\sigma^2}{2} W^2 P_{WW}\right\} + rWP_W - (r+\lambda)\, P = 0 \tag{41}$$

The boundary conditions at L and at $+\infty$ are the same as in Lemma 2. Now, guess that the value function is convex ($P_{WW} < 0$), so that

$$\max_{\sigma\in[\sigma_1,\sigma_2]} \left\{\frac{\sigma^2}{2} W^2 P_{WW}\right\} = \frac{\sigma_2^2}{2} W^2 P_{WW}.$$

Substituting this into (41) one obtains by Lemma 2 that:

$$P(W_t) = -\frac{L}{\alpha(\sigma_2)} \left(\frac{W}{L}\right)^{\alpha(\sigma_2)}$$

where $\alpha(\sigma_2)$ is given by (8). It is now straightforward to verify that $P_{WW} < 0$, and then invoke a classical verification theorem along the lines of Fleming and Soner (1993) to obtain the result. ■

Proof of Lemma 4. To show result 1 let Q be any domain of the form: (L, W_2) for arbitrarily large W_2 such that $W_t < W_2 < \infty$. Consider now any stopping time τ^Q before W_t exits the domain Q. Then, by the definition of P and for any volatility process $\overline{\sigma}_t$:

$$e^{-(r+\lambda)t} P(W_t) = E_t \left[e^{-(r+\lambda)\tau^Q} P(W_{\tau^Q}) \right]$$

This local martingale property of $e^{-(r+\lambda)t} P(W_t)$ in the domain Q implies that (13) holds (for details see Øksendal (2003), Chapter 9). The first part of the proof of result 2 is contained in the text (see equation [17]). To see why $\lim_{W \to \infty} P(W) = 0$, define

$$\tau^L = \inf\{t : W_t = L\}$$

and note that for arbitrary $x > t$:

$$\begin{aligned} P(W_t) &= E\left(e^{-(r+\lambda)(\tau^L - t)} E\left(\int_{\tau^L \wedge \tau}^{\tau} e^{-r(s-\tau^L)} dG_s | W_{\tau^L} = L\right)\right) \leq E\left(e^{-(r+\lambda)(\tau^L - t)} B\right) = \\ &= \Pr\left(\tau^L < x\right) E\left(e^{-(r+\lambda)\left(\tau^L - t\right)} B | \tau^L < x\right) + \Pr\left(\tau^L \geq x\right) e^{-(r+\lambda)(x-t)} E\left(e^{-(r+\lambda)(\tau^L - x)} B | \tau^L \geq x\right) \\ &\leq B\left[\Pr\left(\tau^L < x\right) + \Pr\left(\tau^L \geq x\right) e^{-(r+\lambda)(x-t)}\right] \end{aligned} \tag{42}$$

Now, fix an arbitrary $\varepsilon > 0$ and choose large x such that $e^{-(r+\lambda)(x-t)} = \frac{\varepsilon}{2B}$. Then there always exists W_t large enough such that $\Pr\left(\tau^L < x\right) < \frac{\varepsilon}{2B}$. In light of (42), this then implies that:

$$P(W_t) < \varepsilon$$

Since ε can be chosen arbitrarily small, the result follows.

Assertion 3 contains three specific statements. The first statement is that $P_W(L) = -1$. To see why this is so, take any $\overline{W}$ and define:

$$\overline{\tau} = \inf\{t : W_t \geq \overline{W}\}$$

Then apply Ito's Lemma to P to obtain:

$$\begin{aligned}
e^{-(r+\lambda)(T\wedge\overline{\tau}-t)}P\left(W_{T\wedge\overline{\tau}}\right) &= P(W_t) + \int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}\left(\frac{\sigma^2(W_s)}{2}W_s^2 P_{WW} + rP_W W_s - (r+\lambda)P\right)ds \\
&+ \int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}P_W\sigma(W_s)W_s dB_s \\
&+ \int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}P_W(L)dG_s
\end{aligned}$$

Taking expectations on both sides and equation (13) leads to:

$$\begin{aligned}
P(W_t) &= -E_t\left(\int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}P_W(L)dG_s\right) \\
&\quad -E_t\left(\int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}P_W\sigma(W_s)W_s dB_s\right) \\
&\quad +E_t\left[e^{-(r+\lambda)(T\wedge\overline{\tau}-t)}P\left(W_{T\wedge\overline{\tau}}\right)\right]
\end{aligned} \tag{43}$$

Since $P(W_t)$ represents the payoff of strategy $\sigma(W)$ it follows that:

$$P(W_t) = E_t\left(\int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}dG_s\right) + E_t\left[e^{-(r+\lambda)(T\wedge\overline{\tau}-t)}P\left(W_{T\wedge\overline{\tau}}\right)\right] \tag{44}$$

for any $\overline{\tau}$. Combining (44) and (43), it follows that:

$$E_t\left(\int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}[1+P_W(L)]dG_s\right) = -E_t\left(\int_t^{T\wedge\overline{\tau}} e^{-(r+\lambda)(s-t)}P_W\sigma(W_s)W_s dB_s\right) \tag{45}$$

As the differential equation (13) has a classical solution[13], P_W is a continuous and hence bounded function in the closed interval $\left[L, \overline{W}\right]$ .Therefore, $P_W\sigma(W_s)W_s$ is bounded in $\left[L, \overline{W}\right]$. Hence the integrand on the right hand side of equation 45 is a martingale. Therefore, the right hand side of equation (45) is clearly 0, and hence so must be the left side. This can only be the case if $P_W(L) = -1$.

[13] See Øksendal (2003), Chapter 9

The proof that $P_W < 0$ proceeds by contradiction. Assume otherwise. In particular assume that there exist a $W^{**} > L$ such that $P_W(W^{**}) > 0$. Since $P_W(L) = -1$ and the differential equation (13) has a continuous first derivative, there must be a point $\widehat{W} > L$ such that $P_W(\widehat{W}) = 0$. Since equation (13) holds at $\widehat{W}$:

$$\frac{\sigma^2(\widehat{W})}{2}\widehat{W}^2 P_{WW}\left(\widehat{W}\right) = (r+\lambda)P\left(\widehat{W}\right) > 0$$

since $P > 0$ and hence $P_{WW}\left(\widehat{W}\right) > 0$. Therefore, at $\widehat{W}$ the function P must have a local minimum. But since $P > 0$ for all $W > L$ and $\lim_{W\to\infty} P(W) = 0$, the function P must also have a local maximum at some point $\widetilde{W} > \widehat{W}$, so that $P_W\left(\widetilde{W}\right) = 0$, and $P_{WW}\left(\widetilde{W}\right) < 0$. But this is impossible, by equation (13), since at $\widetilde{W}$ it would have to be the case that

$$\frac{\sigma^2(\widetilde{W})}{2}\widetilde{W}^2 P_{WW}\left(\widetilde{W}\right) = (r+\lambda)P\left(\widetilde{W}\right) > 0$$

which is a contradiction to $P_{WW}\left(\widetilde{W}\right) < 0$. Hence it must be the case that $P_W(W) \leq 0$ for all W. Given that $P_W \leq 0$ it is now straightforward to use (13) to establish that:

$$P_{WW} = \frac{2}{\sigma^2(W)W^2}\left[-rP_W W + (r+\lambda)P\right] > 0$$

Now $P_{WW} > 0$ implies that P_W is increasing throughout. Moreover it can never cross 0. Hence it must be bounded between $P_W(L) = -1$ and 0 as was asserted above. ■

Proof of Lemma 5. A detailed proof of this theorem would replicate the same steps as Lemma 2. To save space, I only give a sketch of the basic steps. Applying the same logic as in Lemma 2, P should satisfy:

$$0 = \begin{cases} \frac{\sigma_2^2 W^2}{2} P_{WW} + rP_W W - (r+\lambda)P \text{ if } W > W^* \geq L \\ \frac{\sigma_1^2 W^2}{2} P_{WW} + rP_W W - (r+\lambda)P \text{ if } L \leq W \leq W^* \end{cases}$$

The general solution to this equation is

$$P(W) = \begin{cases} C_{21}W^{\alpha_2^+} + C_{22}W^{\alpha_2^-} \text{ if } W > W^* \geq L \\ C_{11}W^{\alpha_1^+} + C_{12}W^{\alpha_1^-} \text{ if } L \leq W \leq W^* \end{cases}$$

In order to be able to replicate the same steps as in Lemma 2, $P(W)$ must be continuous and continuously differentiable[14] at W^*. This implies:

$$C_{21}(W^*)^{\alpha_2^+} + C_{22}(W^*)^{\alpha_2^-} = C_{11}(W^*)^{\alpha_1^+} + C_{12}(W^*)^{\alpha_1^-} \tag{46}$$

$$\alpha_2^+ C_{21}(W^*)^{\alpha_2^+ -1} + \alpha_2^- C_{22}(W^*)^{\alpha_2^- -1} = \alpha_1^+ C_{11}(W^*)^{\alpha_1^+ -1} + \alpha_1^- C_{12}(W^*)^{\alpha_1^- -1} \tag{47}$$

To enforce $\lim_{W\to\infty} P(W) = 0$, it is also necessary to impose

$$C_{21} = 0 \tag{48}$$

Finally, the condition $P_W(L) = -1$ implies:

$$\alpha_1^+ C_{11}(L)^{\alpha_1^+ -1} + \alpha_1^- C_{12}(L)^{\alpha_1^- -1} = -1 \tag{49}$$

Solving for $C_{11}, C_{12}, C_{21}, C_{22}$ from equations (46), (47), (48), (49) leads to (25). Equation (28) follows immediately by setting:

$$P(L) = B$$

and solving for W^*. ■

Proof of Lemma 6. The proof proceeds by explicitly constructing two functions that satisfy all the stated properties. Assume first that $W > W^*$. By the form of the conjectured optimal policy, one needs to distinguish 3 sub-regions for x :

(a) $L \leq x < W^*$

(b) $W^* \leq x \leq W$

(c) $x > W$

Define the four constants $\beta_1^+, \beta_1^-, \beta_2^+, \beta_2^-$ as

$$\beta_1^{\pm} = \frac{-\left(\frac{\sigma_1^2}{2} - r\right) \pm \sqrt{\left(\frac{\sigma_1^2}{2} - r\right)^2 + 2\sigma_1^2 (r+\lambda)}}{\sigma_1^2}$$

$$\beta_2^{\pm} = \frac{-\left(\frac{\sigma_2^2}{2} - r\right) \pm \sqrt{\left(\frac{\sigma_2^2}{2} - r\right)^2 + 2\sigma_2^2 (r+\lambda)}}{\sigma_2^2}$$

[14] In particular, these conditions will make it possible to apply the regular form of Ito's Lemma that is given in Lemma 2.

In light of the conjectured optimal policy, in region (a) the differential equation (22), (23) has the general solution:

$$
\begin{aligned}
\pi_1(x) &= D_{11}x^{\beta_1^+} + D_{21}x^{\beta_1^-} - 1 \\
\pi_2(x) &= -\frac{\sigma_1^2\beta_1^+}{2(r+\lambda)}D_{11}x^{\beta_1^+ +1} - \frac{\sigma_1^2\beta_1^-}{2(r+\lambda)}D_{21}x^{\beta_1^- +1}
\end{aligned}
$$

for appropriate constants D_{11}, D_{21}. Similarly, in region (b) the general solution is:

$$
\begin{aligned}
\pi_1(x) &= D_{12}x^{\beta_2^+} + D_{22}x^{\beta_2^-} - 1 \\
\pi_2(x) &= -\frac{\sigma_2^2\beta_2^+}{2(r+\lambda)}D_{12}x^{\beta_2^+ +1} - \frac{\sigma_2^2\beta_2^-}{2(r+\lambda)}D_{22}x^{\beta_2^- +1}
\end{aligned}
$$

and in region (c):

$$
\begin{aligned}
\pi_1(x) &= D_{13}x^{\beta_2^+} + D_{23}x^{\beta_2^-} \\
\pi_2(x) &= -\frac{\sigma_2^2\beta_2^+}{2(r+\lambda)}D_{13}x^{\beta_2^+ +1} - \frac{\sigma_2^2\beta_2^-}{2(r+\lambda)}D_{23}x^{\beta_2^- +1}
\end{aligned}
$$

It remains to determine the 6 constants in the above equations in order to obtain the solution to π_1, π_2. Starting with region (c), it is clear that (31), (32) can only hold if $D_{13} = 0$, since $\beta_2^+ > 0$. To ensure continuity of $\pi_1(x), \pi_2(x)$ at point W, the constants D_{23}, D_{12}, D_{22} need to satisfy (after some straightforward cancellations):

$$
D_{12}W^{\beta_2^+} + (D_{22} - D_{23})W^{\beta_2^-} = 1 \tag{50}
$$

$$
-\beta_2^+ D_{12}W^{\beta_2^+ +1} - \beta_2^-(D_{22} - D_{23})W^{\beta_2^- +1} = 0 \tag{51}
$$

Similarly, continuity of $\pi_1(x), \pi_2(x)$ at W^* implies that

$$
D_{11}(W^*)^{\beta_1^+} + D_{21}(W^*)^{\beta_1^-} = D_{12}(W^*)^{\beta_2^+} + D_{22}(W^*)^{\beta_2^-} \tag{52}
$$

$$
-\beta_1^+ D_{11}(W^*)^{\beta_1^+ +1} - \beta_1^- D_{21}(W^*)^{\beta_1^- +1} = \tag{53}
$$

$$
= -\left(\frac{\sigma_2}{\sigma_1}\right)^2 \left[\beta_2^+ D_{12}(W^*)^{\beta_2^+ +1} + \beta_2^- D_{22}(W^*)^{\beta_2^- +1}\right]
$$

Finally, to ensure (29) it must also be the case that:

$$-\beta_1^+ D_{11} \left(W^*\right)^{\beta_1^+ + 1} - \beta_1^- D_{21} \left(W^*\right)^{\beta_1^- + 1} = 0 \tag{54}$$

Solving this system of equations leads to the following solution for π_1, π_2 :

(a) $L \leq x < W^*$

$$\begin{aligned}
\pi_1(x) &= \left(\frac{W^*}{W}\right)^{\beta_2^+} \frac{1}{\left(\beta_1^+ - \beta_1^-\right)} \left(\frac{x}{W^*}\right)^{\beta_1^-} \left[\beta_1^+ - \beta_1^- \left(\frac{x}{W^*}\right)^{\beta_1^+ - \beta_1^-}\right] - 1 \\
\pi_2(x) &= -\frac{\sigma_1^2}{2\left(r+\lambda\right)} \left(\frac{W^*}{W}\right)^{\beta_2^+} \frac{\beta_1^- \beta_1^+}{\left(\beta_1^+ - \beta_1^-\right)} \left(\frac{x}{W^*}\right)^{\beta_1^-} \left[1 - \left(\frac{x}{W^*}\right)^{\beta_1^+ - \beta_1^-}\right] x
\end{aligned}$$

(b) $W^* \leq x \leq W$

$$\begin{aligned}
\pi_1(x) &= \frac{1}{\left(\beta_2^+ - \beta_2^-\right)} \left(\frac{x}{W}\right)^{\beta_2^-} \left[\beta_2^+ \left(\frac{W^*}{W}\right)^{\beta_2^+ - \beta_2^-} - \beta_2^- \left(\frac{x}{W}\right)^{\beta_2^+ - \beta_2^-}\right] - 1 \\
\pi_2(x) &= -\frac{\sigma_2^2}{2\left(r+\lambda\right)} \frac{\beta_2^+ \beta_2^-}{\left(\beta_2^+ - \beta_2^-\right)} \left(\frac{x}{W}\right)^{\beta_2^-} \left[\left(\frac{W^*}{W}\right)^{\beta_2^+ - \beta_2^-} - \left(\frac{x}{W}\right)^{\beta_2^+ - \beta_2^-}\right] x
\end{aligned}$$

(c) $x > W$

$$\begin{aligned}
\pi_1(x) &= \frac{\beta_2^+}{\left(\beta_2^+ - \beta_2^-\right)} \left(\frac{W^*}{W}\right)^{\beta_2^-} \left[\left(\frac{W^*}{W}\right)^{\beta_2^+ - \beta_2^-} - 1\right] \left(\frac{x}{W^*}\right)^{\beta_2^-} \\
\pi_2(x) &= -\frac{\sigma_2^2}{2\left(r+\lambda\right)} \frac{\beta_2^+ \beta_2^-}{\left(\beta_2^+ - \beta_2^-\right)} \left(\frac{W^*}{W}\right)^{\beta_2^-} \left[\left(\frac{W^*}{W}\right)^{\beta_2^+ - \beta_2^-} - 1\right] \left(\frac{x}{W^*}\right)^{\beta_2^-} x
\end{aligned}$$

By construction, $\pi_1(x), \pi_2(x)$ are continuous and satisfy (29), (31), (32). It remains to verify that this solution also satisfies (30). This is straightforward upon observing that $\beta_2^+ > 0, \beta_2^- < 0$ and also $\beta_1^+ > 0, \beta_1^- < 0$. The proof for $W < W^*$ follows similar steps and is available upon request. ■

Proof of Proposition 1. To simplify notation let $P^*(W_t)$ denote the value of the implicit guarantee assuming that the policy σ^* is followed. Similarly, let $P(W_t)$ be the value of the implicit guarantee, assuming that any alternative feasible policy $\sigma(x)$ is followed. Also, let H, H^* be defined as:

$$\begin{aligned} H^*(x) &= 1\{x<W\}P^{*\prime}(x)+\pi_1 P^{*\prime}(x)+\pi_2 P^{*''} = \\ &= 1\{x<W\}P^{*\prime}(x)+\pi_1 P^{*\prime}(x)+\pi_2 \frac{2}{\sigma^*(x)^2}\left((r+\lambda)\,P^*(x)\frac{1}{x^2}-rP^{*\prime}(x)\frac{1}{x}\right) \end{aligned}$$

and similarly for $H(x)$:

$$\begin{aligned} H(x) &= 1\{x<W\}P'(x)+\pi_1 P'(x)+\pi_2 P''(x) = \\ &= 1\{x<W\}P'(x)+\pi_1 P'(x)+\pi_2 \frac{2}{\sigma(x)^2}\left((r+\lambda)\,P(x)\frac{1}{x^2}-rP'(x)\frac{1}{x}\right) \end{aligned}$$

Since $P^*(L)=P(L)=B$ for any feasible policy, it follows that:

$$\begin{aligned} P^*(W)-P(W) &= \int_L^\infty 1\{x<W\}\left[P^{*\prime}(x)-P'(x)\right]dx = \\ &= \int_L^\infty \left[H^*(x)-\pi_1 P^{*\prime}(x)-\pi_2 P^{*\prime\prime}(x)\right]dx \\ &\quad -\int_L^\infty \left[H(x)-\pi_1 P'(x)-\pi_2 P''(x)\right]dx \end{aligned} \tag{55}$$

where π_1 and π_2 were determined in Lemma 6. Integrating by parts gives:

$$\int_L^\infty \left[H^*(x)-\pi_1 P^{*\prime}(x)-\pi_2 P^{*\prime\prime}(x)\right]dx-\int_L^\infty \left[H(x)-\pi_1 P'(x)-\pi_2 P''(x)\right] = \tag{56}$$

$$\begin{aligned} = & \int_L^\infty \left\{[H^*(x)-H(x)]+\pi_1'[P^*(x)-P(x)]+\pi_2'[P^{*\prime}(x)-P'(x)]\right\}dx \\ & +\pi_1\left[P^*(L)-P(L)\right]+\pi_2\left[P^{*\prime}(L)-P'(L)\right] \\ & -\lim_{x\to\infty}\left\{\pi_1(x)\left[P^*(x)-P(x)\right]+\pi_2(x)\left[P^{*\prime}(x)-P'(x)\right]\right\} \end{aligned}$$

In light of Lemma 6 the last two lines are 0. The next step is to note that

$$H^*(x)+\pi_1' P^*(x)+\pi_2' P^{*\prime}(x)=0$$

by construction. Moreover, using (22), (23):

$$-\left[H(x)+\pi_1' P(x)+\pi_2' P'(x)\right] =$$

$$
\begin{aligned}
&= -\left\{1\{x<W\}P'(x)+\pi_1 P'(x)+\pi_2\frac{2}{\sigma(x)^2}\left((r+\lambda)\,P(x)\frac{1}{x^2}-rP'(x)\frac{1}{x}\right)\right\} \\
&\quad -\left[-\left(\pi_1+1\{x<W\}\right)+r\frac{2}{\left[\sigma^*(x)\right]^2}\pi_2\frac{1}{x}\right]P'(x) \\
&\quad -\left[-\frac{2\,(r+\lambda)}{\left[\sigma^*(x)\right]^2}\pi_2\frac{1}{x^2}\right]P(x) \\
&= -\pi_2\left(\frac{2}{\sigma(x)^2}-\frac{2}{\left[\sigma^*(x)\right]^2}\right)\left[(r+\lambda)P(x)\frac{1}{x^2}-rP'(x)\frac{1}{x}\right] \qquad (57)
\end{aligned}
$$

Given result 3 in Lemma 4:

$$
\left[(r+\lambda)P(x)\frac{1}{x^2}-rP'(x)\frac{1}{x}\right]>0
$$

Moreover, by the way the optimal policy σ^* is constructed (equation [21]) it follows that:

$$
-\pi_2\left(\frac{2}{\sigma(x)^2}-\frac{2}{\sigma^{*2}(x)}\right)\geq 0
$$

with equality holding *for all* x if and only if $\sigma(x)=\sigma^*(x)$. Combining (55), (56), (57) and (21) establishes that:

$$
P\left(W_t;\sigma_t^*\right)\geq P\left(W_t;\sigma(x)\right)
$$

for any $W_t\geq L$. This concludes the proof. ■

Proof of Lemma 7. If a firm were to set $\sigma\left(W_t\right)=\sigma_2$ for all W_t, then Lemma 2 implies that

$$
E_t\left(\int_t^\tau e^{-\beta(s-t)}dG_s|W_t=L\right)=-\frac{L}{\widetilde{\alpha}\left(\sigma_2\right)}
$$

where:

$$
\widetilde{\alpha}\left(\sigma_2\right)=\frac{-\left(r-\frac{\sigma_2^2}{2}\right)-\sqrt{\left(r-\frac{\sigma_2^2}{2}\right)^2+2\sigma_2^2\left(r+\lambda+\beta\right)}}{\sigma_2^2}
$$

As $\beta\to\infty$, $\widetilde{\alpha}\left(\sigma_2\right)\to-\infty$, and hence:

$$
E_t\left(\int_t^\tau e^{-\beta(s-t)}dG_s|W_t=L\right)=-\frac{L}{\widetilde{\alpha}\left(\sigma_2\right)}\to 0.
$$

Moreover:

$$E_t e^{-(r+\beta)(\tau-t)} B^* \to 0.$$

so that:

$$E_t \left(\int_t^\tau e^{-\beta(s-t)} dG_s | W_t = L \right) + E_t e^{-(r+\beta)(\tau-t)} B^* \to 0 < B^*$$

Therefore setting $\sigma(W_t) = \sigma_2$ for all W_t, clearly maximizes shareholder value (11) (by Lemma 3) while satisfying the constraint (33). ■

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