The Microfoundations of Hot and Cold Markets*

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Abstract

This paper considers why markets characterized by costly search, such as housing markets, labor markets, or marriage markets, exhibit excess volatility. Conditions in these markets, including the ratio of buyers to sellers, transaction prices, and expected times-to-transaction, often appear highly sensitive to fundamental shocks. These high sensitivities result from feedback: market participants optimally respond to shocks in a manner that amplifies a shock's initial impact, which in turn elicits further reinforcing response. For example, a positive demand shock brings more buyers into a market. This improves the bargaining position of sellers, who then sell more quickly, decreasing the stock of sellers in the market. This further increasing the relative number of buyers to sellers, amplifying the initial shock.

Keywords: Costly Search, Real Options, Marriage Market, Housing Market, Unemployment, Venture Capital.

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1 Introduction

Why do houses sell so quickly in hot markets, when sellers presumably could, with a little patience, get better prices? And why do houses sell so slowly in cold markets, when sellers presumably could, by lowering their asking prices, sell their properties in a "reasonable" amount of time? The high sensitivity of average time to sale to market conditions, *i.e.*, the observed elasticity of expected time to sale with respect to demand, is quite surprising. Yet this is only one of the puzzling aspects of the housing market. Small shocks to income, or the rate at which people move to or from a city, can also drastically alter several other characteristics of the local housing market, including prices and the relative number of buyers and sellers. What makes the housing market, or other markets characterized by costly search such as labor markets or marriage markets, so sensitive to changes in demand? In short, why are search markets so volatile?

In this paper we demonstrate that markets characterized by costly search can be more volatile than fundamentals (*i.e.*, exogenous shocks that affect market conditions) because market participants' optimal responses to changes in market conditions magnify the impact of fundamental shocks, creating a kind of self-reinforcing feedback loop. In the real estate case, for example, a shock that improves a seller's position in the market induces sellers in general to transact more quickly, reducing the housing stock on the market, making sellers more scarce, further improving a seller's position in the market. This channel, whereby agents' endogenous behavior reinforces the impact of exogenous shocks, is missing from the canonical search literature, as a result of assumptions implicitly made in this literature when modeling markets.

The feedback channel described in this paper has not been considered previously because the search literature implicitly assumes infinitely elastic entry, which shuts down this channel completely. This strong assumption is, however, counter factual in most markets, which are generally characterized by at least partially inelastic entry. Our primary deviation from the canonical search literature spawned by Diamond (1982), Mortensen (1982) and Pissarides (1985) will be, therefore, to assume that entry, while responsive to changes in the expected value of entry, is not perfectly elastic with respect to these changes. That is, we will assume a strictly finite entry-elasticity with respect to the expected value of market participation. This flexible specification should more realistically reflect the dynamics of behavior in the various search markets.

High volatilities relative to fundamentals, similar to those seen in housing markets, are observed in other markets characterized by costly search, including labor markets, marriage markets, and the market for financing new venture-backed firms. Workers' reservation wages, and unemployment duration, are sensitive to labor market tightness. Small reductions in the unemployment rate can produce significant upward wage pressure, while reducing the average time job seekers require to find employment. Similarly, the division of the marriage surplus appears highly sensitive to the relative number of men and women. Unbalanced sex ratios existed in Russia after World War II (due to high male mortality rates), prevail in the U.S. African-American community currently (due to high male incarceration rates), and present an impending crisis for China's next-generation (due to low female birth rates). These unbalanced sex ratios shift the gains from marriage away from the abundant sex toward the scarce sex. The venture capital market is highly cyclical, with large variations in average pre-money valuations and fund-flows over time. In all of these markets, in which participants must trade off the benefits of finding the most appropriate counterparty against the costs required to achieve such a match, participants react to changing market conditions in a manner that reinforces these changes.

The fact that agents' behavior in these markets reinforces exogenous shocks is quite surprising, in light of the fact that agents respond to any given shock in a manner that blunts the shock's first order impact. For example, the first order impact of a positive shock to the number of buyers in the market on average time-to-sale is mitigated by sellers raising prices. More buyers increases the rate at which sellers encounter potential counter-parties, reducing the expected time-to-sale, but this reduction is mitigated by sellers' price response to the shock (*i.e.*, higher prices), which reduces the probability that any given encounter yields a transaction. Such an analysis, however, only considers the *partial equilibrium* effect of agents' response to the shock, and ignores general equilibrium considerations. It fails to consider, for example, how agents respond to changing market conditions induced by the manner in which agents respond to the initial shock. We will show that even though the partial equilibrium effect of agents' behavior tends to mitigate the shock's impact, in general equilibrium the effect of agents' behavior tends to amplify the shock's impact. That is, search market dynamics can be more volatile than underlying demand even though the partial equilibrium response of market participants to changes in demand, which mitigates the changes' direct impacts, seem as if they should *reduce* market volatility.

To illustrate the economic intuition behind how market participants' optimal response to shocks amplifies the shocks' impacts, we will consider a simple example. For the sake of concreteness, let us return to the real estate case and run a simple thought experiment, asking "what happens if the rate at which buyers enter the market increases slightly?"

The initial impact of a slight increase in the rate at which buyers enter the market, in response to an income shock, an interest rate shock, or a change in demographic flows, is of course a slight increase in the number of buyers in the market. This increases the ratio of buyers to sellers, or market "tightness."[1] This increase in buyers relative to sellers decreases the expected time a seller must wait before seeing the next potential buyer. That is, the initial shock leads to an increase in the frequency at which sellers see buyers. Sellers consequently have more opportunities to transact, and this reduces a seller's expected time to sale. Sellers therefore exit the market faster, on average, reducing the number of sellers active in the market at any one time, which further increases the ratio of buyers to sellers, amplifying the initial shock. This intuition for how market participants' optimal behavior feeds back into the initial shock, amplifying its impact, is presented in Figure 1 below.

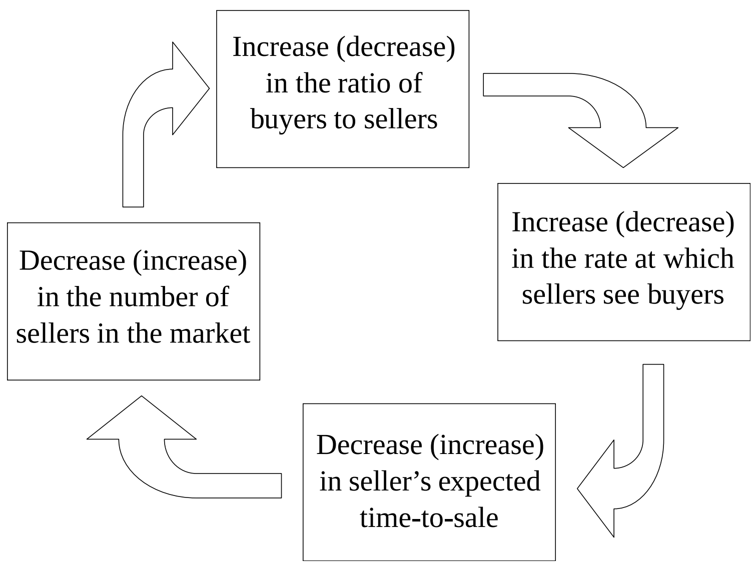

Figure 1: Feedback in Search Markets

The intuition behind how market participants' behavior amplifies demand shocks. A positive demand shock (upward shift in the supply of new buyers) initially results in an increase in the number of buyers. This increases the ratio of buyers to sellers, so sellers meet potential buyers more frequently. Sellers consequently transact more quickly, reducing the number of sellers in the market, which increases the number of buyers relative to sellers. This reinforces the initial shock, leading market participants to further reinforcing behavior. The net effect can dramatically magnify the impact of the initial shock.

[1]The term "tightness" derives from the unemployment literature, where models of costly search were first analyzed in detail (Diamond (1982), Mortensen (1982), Pissarides (1985)). The term is most natural in that context, because if job seekers are looking to "sell" their labor to potential employers, the "buyers," then the ratio of buyers to sellers, *i.e.*, job vacancies to unemployed workers, parameterizes labor market tightness. We employ the term in the more general setting in an effort to use terminology and notation standard in the search literature whenever possible.

The analysis presented above is somewhat naïve, and intended only to convey the simple intuition behind the feedback mechanism. In an effort to convey the intuition as simply as possible we have focused entirely on active market participants, taking their participation for granted, and ignored *potential* participants. A more sophisticated analysis must consider the price response of both current and potential market participants. As market conditions change, due both to fundamental shocks and to the manner in which agents respond to these shocks, the way counterparties in any transaction share the gains from trade will also change. On one side of the market conditions have deteriorated, and agents on this side will capture less of the transaction surplus, because their outside option or "threat point," the expected value of searching for another potential counterparty, is reduced. Some agents on this side, in response to the lessened value of expected participation, will exit the market. On the other side of the market new agents will enter, as improving market conditions increase their expected value of participation. These natural responses to "prices," *i.e.*, changes in the expected value of market participation, will tend to mitigate the magnitude of the feedback, and need to be considered carefully in a proper analysis.

The extent to which agents' behavior amplifies fundamental shocks depends critically on how responsive agents, both current and potential market participants, are to market conditions. In general, the effect will be large in markets where the supply response is relatively inelastic, and smaller in markets where the response is relatively elastic. We would expect, for example, to see a more volatile real estate market, both in terms of transaction prices and expected time-to-sale, in San Francisco, where the supply response is relatively inelastic, due to geography on the extensive margin and regulation on the intensive margin, than in Phoenix, which is situated in a relatively flat, featureless plain and where developers are comparatively unencumbered by regulatory concerns. In the extremes, if agents are completely non-responsive to prices then fundamental shocks are enormously magnified, while as agents become highly responsive the feedback effect disappears.

Ultimately, agents' responsiveness (*i.e.,* price-elasticity of entry), which determines the extent to which agents behavior reinforces fundamental shocks, needs to be determined empirically. However, we have no reason to believe, *a priori*, that very large elasticities are any more likely than very small elasticities.

To more clearly illustrate the tension between the feedback mechanism, whereby agents' behavior amplifies fundamental shocks, and the supply response, which tends to dampen the feedback channel, we will now reconsider the initial thought experiment in a more sophisticated manner. Again, we will consider the effect of a slight increase in the rate at which buyers enter the market, but this time we will pay particular attention to the role played by potential market entrants.

The initial impact of a slight increase in the rate at which buyers enter the market is, again, a slight increase in the number of buyers in the market, increasing the ratio of buyers to sellers, decreasing the expected time a seller must wait before seeing the next potential buyer. The first order effect of the initial shock on market tightness in this more sophisticated analysis has, however, two competing effects on the ratio of buyers to sellers.

On one hand, current market participants' response to the tighter market tends to amplify the impact of the initial shock, by the mechanism previously described. The fact that a seller sees buyers at a higher rate reduces the seller's expected time to transaction, reducing the number of sellers in the market, further increasing the ratio of buyers to sellers, and amplifying the initial shock.

On the other hand, potential market participants response to the tighter market tends to mitigate the impact of the initial shock. The fact that a seller sees buyers at a higher frequency improves the seller's bargaining position. Because sellers see buyers at a higher frequency, the cost of walking away from any potential deal is decreased. This allows the seller to demand more of the transaction surplus, increasing the expected value of market participation. This attracts new sellers to the market, discourages new buyers from entering, and encourages some current buyers to exit, all of which tends to mitigate the impact of the initial shock on the relative number of buyers to sellers.

The relative magnitude of these two competing effects, one which tends to amplify the initial impact of the fundamental shock, the other which tends to mitigate it, depends on how responsive market participation is to changes in the expected value of participation. If net entry into the market is highly elastic with respect to the expected value of participation then a shock that brings new buyers into the market still leads sellers to transact more quickly, reducing the stock on the market, further increasing the ratio of buyers to sellers, but this improves the sellers' position in the market, eliciting a large increase in the rate sellers enter (and discourages new buyers from entering), which brings the ratio of buyers to sellers back toward the pre-shock level. That is, when agents' response is elastic the impact of the increased transaction rate is largely offset by new entry, and the feedback effect is small.

If new entry is less elastic, however, then the feedback effect can dramatically amplify the impact of fundamental shocks, as market participants' response to a shock reinforces the shock, eliciting a further response, which itself reinforces the shock, and each echo elicits still further reinforcing responses.

2 Model

The basic model consists of a relatively standard Diamond-Mortensen-Pissarides two-sided search model, or "mating game," in which agents of two differing types search for potential counterparties of the opposite type in order to enter into a mutually beneficial arrangement. [2] The search literature is most developed in the labor market, but the modeling is equally applicable to other markets characterized by costly search, including the marriage market, real estate markets, and the market for venture capitalists and entrepreneurs. We will assign the titles "buyers" and "sellers" to the two sides of the market, though in some markets we will consider, such as the marriage market, this assignation will be somewhat arbitrary, as there may not actually be anyone "buying" or "selling" *per se*.

While the basic model is quite standard, we will make one unorthodox assumption. We will assume that while agents' entry and exit decisions are sensitive to the expected value of market participation, that the elasticity of net entry/exit is strictly finite with respect to the expected value of participation. This results from embedding the market inside an economy in which agents have heterogeneous market-alternative outside options, and thus downward sloping demand for market participation. This deviates from the canonical search literature, which implicitly assumes that market entry/exit for both buyers and sellers is infinitely price-elastic, *i.e.*, that the supply curves of agents on both sides of the market are horizontal.

Finally, the model presented here includes stochastic matchings. That is, we will assume that the potential transaction surplus realized when a buyer and seller meet is specific to the pair. Stochastic matchings have been studied extensively, but play a special role here.[3] When matching is stochastic, then not every encounter between buyer and seller yields a transaction. The probability that any given encounter yields a transaction is sensitive to market tightness, and consequently to the division of the transaction surplus, which influences entry rates on both sides of the market. Through this channel, agents' "price response" mitigates the feedback channel with which this paper is primarily concerned, and therefore must not be ignored. A more detailed discussion of the role of stochastic matching plays in the context of this paper is left for the appendix.

[2]Mortensen (1982) characterizes a mating game as "a decentralized market in which agents of two types search for one another in order to exploit some joint production or exchange opportunity."

[3]Stochastic matching was first considered by Jovanovic (1979). For more on stochastic matching see, for example, Pissarides (2000).

2.1 The Formal Economy

The measure of buyers and sellers participating in the market, *i.e.*, actively engaged in search, are denoted m_b and m_s, respectively. We will assume, following Pissarides (2000), that the aggregate encounter rate between the two types of agents is homogenous degree-one in numbers of the two types. Under this assumption buyers face a Poisson arrival process of potential sellers at a rate $q(\theta)$, while sellers meet potential buyers at a rate $\theta q(\theta)$, where $\theta \equiv m_b/m_s$ denotes the ratio of buyers to sellers, or market tightness.[4] We will assume, both for simplicity and concreteness, that the matching rate is Cobb-Douglas degree-one in the number of buyers and sellers, and increasing in both the number of buyers and sellers, so $q(\theta) = \theta^{\eta}\lambda$ where $\eta \in (-1, 0)$ and $\lambda \equiv q(1)$ is the Poisson arrival rate at which buyers meet sellers (or sellers meet buyers) in a "balanced" market that has an equal number of buyers and sellers.

When agents meet they realize a potential "transaction utility" specific to the particular buyer-seller pair. That is, when buyer i encounters seller j they observe ϵ_{ij}, the total value a transaction between the two agents would yield. We will assume that while all buyer-seller pairs are *ex ante* identical, the realized value of the match is drawn independently from the random variable ϵ with cumulative distribution function $\Phi_\epsilon(x) = \mathbf{P}(\epsilon \leq x)$. We will assume $\mathbf{E}[\epsilon \mathbb{1}_{\epsilon>0}] < \infty$, and denote the probability density function for ϵ by $\phi_\epsilon(x)$.

A "transaction" occurs whenever the utility of a potential exchange opportunity exceeds the utility of continued search. That is, if the potential transaction utility realized when agents meet matches or surpasses the sum of the agents' threat-points, then agents pursue the joint production or exchange opportunity. Should such a transaction occur, the buyer and seller are assumed to split any surplus in excess of the sum of their threat-points according to their Nash bargaining powers, $\beta_b = \beta$ and $\beta_s = 1 - \beta$, respectively.[5]

Agents would like to split the largest possible surplus, so prefer, *ceteris paribus*, to achieve the best possible match. Consequently, a meeting between a buyer and a seller does not always result in a transaction. The value of any match consists of two components, a common value component $\mu = \mathbf{E}[\epsilon]$, and a mean-zero idiosyncratic component $\epsilon - \mu$.

[4]The Poisson arrival rates $q(\theta)$ and $\theta q(\theta)$ also have elasticities on the intervals $(-1, 0)$ and $(0, 1)$, respectively, and satisfy $\lim_{\theta\to\infty} q(\theta) = \lim_{\theta\to 0} \theta q(\theta) = 0$ and $\lim_{\theta\to 0} q(\theta) = \lim_{\theta\to\infty} \theta q(\theta) = \infty$.

[5]Formally, the asymmetric Nash bargaining solution requires that the transaction price maximize a weighted Nash product of the agents' surpluses. Specifically, the agents will chose to transact at the "price" $V_s = \arg\max_p (p - V_s^*)^{\beta_s}((\epsilon - p) - V_b^*)^{1-\beta_s}$, where the threat-points V_b^* and V_s^* are the values of not transacting (*i.e.*, continued search) to the buyer and seller, respectively. This yields the surplus division given above.

In some markets, such as the real estate market, the common value component is large relative to the variability of the idiosyncratic component (*i.e.*, $\mu \gg \sqrt{\text{var}(\epsilon)}$), while in other markets, such as the marriage market, the common value component is small, or may even be negative. In general, agents will choose not to transact if the realized value of the idiosyncratic component is not sufficiently high. Not transacting, *i.e.*, walking away from the potential "deal," keeps open the possibility of matching with a better suited counterparty, with whom the value of matching would be higher.

While there is a benefit to continued search arising from the potential of a more beneficial future match, search is also assumed to entail costs. These costs consist of both direct search costs, which accrue at a rate c_b for buyers and c_s for sellers, and indirect costs, because agents discount the future at the rate r. Due to these costs agents prefer, *ceteris paribus*, to transact sooner rather than later.

Finally, we would like to capture the fact that agents will enter the market faster when the expected value of entering is high. We will assume, therefore, that the rate of entry is increasing in the expected value of participation, governed by demand curves that are upward sloping with respect to the expected value of entry. This represents our biggest departure from the previous literature, which assumes implicitly that net entry is infinitely elastic with respect to the expected value of participation.

We will assume that the entry supply curves are iso-elastic, and that agents of type $i \in \{b = \text{buyer}, s = \text{seller}\}$ enter the market at a rate $F_i = X_i(V_i + \frac{c_i}{r})^\gamma$, where V_i is the expected value of entry. The entry rate F_i is increasing in V_i, which captures the idea that agents are more likely to enter when the expected value of entering is higher. The c_i/r guarantees that some agents will choose to participate, thereby assuring the existence of the market. This simplifies the analysis, but is not important for the underlying economics. The parameter γ is the participation-value elasticity of entry. If γ is large then small changes in the expected value of participating yield large changes in the rate at which agents enter the market, while if γ is small the entry rates are less sensitive to changes in the value of participation. These entry supply curves effectively assume that market participants are heterogeneous with respect to the value of their market-alternative outside-options: that potential market participants with search-alternative outside-option U_i are "born" at a rate $\gamma X_i(U_i + \frac{c_i}{r})^{\gamma-1}$ for each $U_i > -c_i/r$.

3 Analysis

In the stationary competitive equilibrium agents have rational expectations, transaction utilities V_b and V_s maximize the Nash product $(V_b - V_b^*)^\beta (V_s - V_s^*)^{1-\beta}$ where V_b^* and V_s^* are the buyer's and seller's respective threat points, and the inflows of agents of both types, $F_b(V_b^*)$ and $F_b(V_b^*)$, equal the outflows, which depend both on encounter rates and transaction probabilities.

Before considering agents' general equilibrium behavior, it is convienient to first consider the partial equilibrium solution. Consequently, we begin our analysis in section 3.1 taking market conditions as given, and determine which encounters yield transactions and, should a transaction occur, at what "prices" (*i.e.*, the transaction surplus division). That is, we will determine the optimal microeconomic behavior of agents facing the costly search problem under exogenously specified market conditions. Only then, in section 3.2, will we consider the full general equilibrium solution, embedding the search model in an economy in which market entry is determined endogenously.

3.1 Conditions Yielding a Transaction

An agent will not agree to a transaction that benefits her less than the continuation value of her transaction-alternative, or "threat point" in the parlance of bilateral bargaining problems. Upon meeting a potential counter-party an agent will only transact if the value of doing so exceeds the value of continuing to look for the next potential counter-party. We will denote the agent's threat point V_i^*. If a buyer and seller realize a successful match they will transact and receive their outside options V_b^* and V_s^*, respectively, plus share in any realized surplus $\epsilon - V_b^* - V_s^*$ in proportion to their bargaining powers. A successful match will only occur, therefore, if the realized idiosyncratic valuation ϵ is at least as great as the transaction threshold $\epsilon^* \equiv V_b^* + V_s^*$.

An agent's threat-point, *i.e.*, the continuation value of walking away from a potential match, is the discounted value of waiting for the next potential counter-party,

$$V_i^* = \mathbf{E}[e^{-r\tau_i}] \left(\Phi_\epsilon(\epsilon^*) V_i^* + (1 - \Phi_\epsilon(\epsilon^*))\, \mathbf{E}[V_i | \epsilon > \epsilon^*] \right) - \int_0^{\tau_i} c_i e^{-rt} dt \qquad (1)$$

where τ_i is the time until the agent meets the next potential counter-party. In each successful transaction the transaction values V_b and $V_s = \epsilon - V_b$ maximize the Nash product $(V_b - V_b^*)^\beta (V_s - V_s^*)^{1-\beta}$, so

$$\mathbf{E}[V_i | \epsilon > \epsilon^*] \;=\; V_i^* + \beta_i \mathbf{E}[\epsilon - \epsilon^* | \epsilon > \epsilon^*]. \qquad (2)$$

Letting λ_i denote the encounter rate for type i agents (*i.e.*, $\lambda_b = \theta^\eta \lambda$ and $\lambda_s = \theta \lambda_b$), we have

$$\mathbf{E}[e^{-r\tau_i}] = \int_0^\infty e^{-rt} \lambda_i e^{-\lambda_i t} dt = \frac{\lambda_i}{r + \lambda_i}, \tag{3}$$

$$\int_0^{\tau_i} c_i e^{-rt} dt = c_i \int_0^\infty \left(\int_0^t e^{-rs} ds \right) \lambda_i e^{-\lambda_i t} dt = \frac{c_i}{r + \lambda_i}. \tag{4}$$

Together equations (1) - (4) imply

$$r V_i^* + c_i = \lambda_i \beta_i \nu_\epsilon(\epsilon^*), \tag{5}$$

where $\nu_\epsilon(\epsilon^*) \equiv \mathbf{E}[(\epsilon - \epsilon^*) \mathbb{1}_{\epsilon > \epsilon^*}] = \int_{\epsilon^*}^\infty (z - \epsilon^*) \phi_\epsilon(z) dz$ is the expected transaction surplus generated when a buyer and seller meet. The left hand side of equation (5) is the instantaneous cost of continued search, while the right hand side is the instantaneous expected gain, and reflects the fact than in equilibrium these must balance.

Equation (5), taken simultaneously for buyers and sellers, says

$$\frac{m_b (r V_b^* + c_b)}{m_s (r V_s^* + c_s)} = \frac{\beta_b}{\beta_s}. \tag{6}$$

This equation implies that in equilibrium buyers and sellers lose value, in aggregate, at rates proportional to their respective bargaining powers.

3.1.1 The transaction threshold

Equations (5), in conjunction with the fact that agents will only transact if the value of transacting exceeds the sum of their threat points, $\epsilon^* = V_b^* + V_s^*$, implies that

$$r\epsilon^* + c_b + c_s = (\beta_b \lambda_b + \beta_s \lambda_s)\, \nu_\epsilon(\epsilon^*). \tag{7}$$

The transaction threshold ϵ^* is therefore implicitly defined as the solution in x to

$$\Lambda(\theta) \nu_\epsilon(x) - rx - c_b - c_s = 0, \tag{8}$$

where $\Lambda(\theta) \equiv \beta_b \lambda_b + \beta_s \lambda_s = \left(\beta \theta^\eta + (1 - \beta) \theta^{1+\eta} \right) \lambda$ is the bargaining power weighted average encounter rate. Given r, β, θ, c_b and c_s there exists a unique transaction threshold ϵ^*, because the left hand side is continuous and strictly decreasing in x, positive for x sufficiently small, and negative for x sufficiently large.

The transaction threshold ϵ^* is, as expected, decreasing in r, c_b and c_s, and increasing in λ. The real economic cost of search is increasing in both the discount rate and the direct search costs, and decreasing in the average encounter rate. Agents are more likely to transact, and therefore search less, when search is more costly. It is interesting to note that the transaction threshold does *not* depend independently on c_b and c_s, but only on the sum $c_b + c_s$, though of course conditional on $c_b + c_s$ the relative value of market participation for buyers and sellers does depend on the search costs individually (see Equation (6)).

Throughout the rest of the paper we will assume that ϵ is normally distributed.[6] If $\epsilon \sim N(\mu, \sigma^2)$ then

$$\nu_\epsilon(x) \;=\; \int_x^\infty \tfrac{(y-x)}{\sqrt{2\pi}\sigma}\, e^{\frac{-(y-\mu)^2}{2\sigma^2}}\, dy \;=\; (\mu - x) N\big(\tfrac{\mu-x}{\sigma}\big) + \sigma n\big(\tfrac{\mu-x}{\sigma}\big)\,. \tag{9}$$

This, in conjunction with equation (8), says that $\epsilon^* = \mu - \sigma y$ where y satisfies

$$yN(y) + n(y) + \frac{ry}{\Lambda(\theta)} \;=\; \frac{r\mu + c_b + c_s}{\sigma\Lambda(\theta)}. \tag{10}$$

3.1.2 An Example

The value of the buyers' and sellers' threat points, as well as the minimum realization of the joint transaction surplus that yields a transaction, are shown on the following page, in Figure 2, as a function of market tightness (the ratio of buyers to sellers).

As expected, the value of the seller's outside option is increasing in market tightness (*i.e.*, in demand side competition), while the value of the buyer's outside option is decreasing in market tightness. The figure also shows that the probability that any given encounter yields a transaction is greatest when the market is "balanced" (*i.e.*, when there are equal numbers of buyers and sellers, if buyers and sellers have the same Nash bargaining powers). When markets are balanced, then the threat of walking away from any given deal is not disproportionately strong on either side of the market, and neither side of the market can "hold up" the other.

[6]Other choices would be more convienient, analytically. For example, if ϵ was distributed uniformly on the interval $(\mu - \delta, \mu + \delta)$, then

$$\nu_\epsilon(x) \;=\; \begin{cases} \mu - x & \text{if } x < \mu - \delta, \\ \frac{(\mu+\delta-x)^2}{4\delta} & \text{if } x \in (\mu - \delta, \mu + \delta), \\ 0 & \text{if } x > \mu + \delta. \end{cases}$$

Substituting for $\nu_\epsilon(x)$ in equation (8) and solving then yields a simple analytic characterization of the transaction threshold.

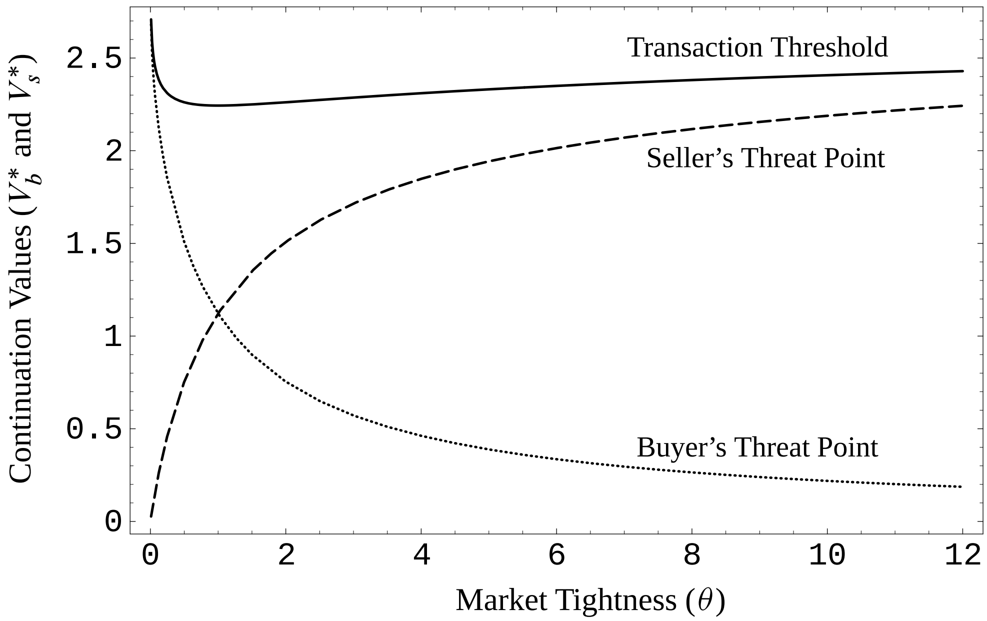

Figure 2: Threat Points and Minimum Transaction Surplus
The value of the agents' threat points (value of waiting for the next deal) for both buyers (dotted line) and sellers (dashed line), and the minimum realized idiosyncratic value that results in a transaction (solid line), are plotted as a function of the ratio of buyers to sellers. Parameters are $\eta = \beta = .5$, $r = .1$, $\lambda = 52$, and $c_b = c_s = 0$.

3.2 Stationary Equilibrium

At this point we will turn our attention to the larger economy, and consider how the entry decision of potential market participants and the exit decision of current participants interacts with the behavior of those agents actively searching in the market. That is, we will embed the search market into the macroeconomy, and determine the equilibrium behavior of market participants.

Stationarity requires, in the steady state equilibrium, that buyers and sellers both enter the market at the same rate they exit. Buyers and sellers exit when they transact, and so exit the market at the same rate. Consequently, buyers and sellers enter the market at the same rates, $F_b(V_b^*) = F_s(V_s^*)$,

$$X_b \left(V_b^* + \tfrac{c_b}{r}\right)^\gamma \;=\; X_s \left(V_s^* + \tfrac{c_s}{r}\right)^\gamma . \tag{11}$$

In conjunction with equation (6), which specifies the relative total rates at which aggregate buyers and aggregate sellers incur costs searching, this implies

$$\theta \;=\; \rho^{-1}\zeta^{1/\gamma}, \tag{12}$$

where $\rho \equiv \beta_b/\beta_s = \beta/(1-\beta)$ is the ratio of buyers' and sellers' bargaining powers, and $\zeta = X_b/X_s$ is the relative level of the buyer and seller entry supply curves. In order to simplify the exposition we will refer to $zeta$ hereafter as "fundamental demand/supply." Equation (12) says a one percent shift in the level of the supply curve for new entrants of buyers or sellers, *i.e.,* a one percent shift in "fundamental demand/supply," results in a $1/\gamma$ percent change in the ratio of buyers to sellers searching at any given time, and a correspondingly large change in the manner in which the realized idiosyncratic value is shared. That is, the elasticity of market tightness with respect to shifts in fundamental demand/supply is the inverse of the entry elasticity with respect to entry value. If agents' entry responds strongly to prices (*i.e.,* large γ), then shifts in the supply curves of new buyers or new sellers have a relatively small impact on market tightness (*i.e.,* small $1/\gamma$), and consequently on market conditions more generally. If agents' entry responds weakly to prices, however, then even small shifts in fundamental demand/supply can have a large impact on market tightness, and consequently dramatically impact general market conditions, the division of the transaction surplus, and expected time-to-sale.

4 The Equilibrium Impact of Supply and Demand Shocks

In the preceeding section we considered how agents' microeconomic decision making interacts with macroeconomic market conditions. In particular, we showed how individual agents' optimal behavior aggregate to reinforce fundamental shocks, and derived explicit implications for market tightness, the ratio of buyers to sellers. In this section we consider how fundamental shocks translates into other market variables, including "prices" (*i.e.,* the division of the transaction surplus) and expected times-to-transaction on both sides of the market.

4.1 Division of the Transaction Surplus

In section 3.1.1 we calculated the transaction threshold and agents' continuation values as a function of an exogenously specified market tightness (Figure 2). Using equation (12), which specifies market tightness as endogenously determined by agents' optimal response to market fundamentals, we can instead express the transaction threshold and agents' continuation values as a function of fundamental demand/supply, *i.e.,* as a function of the relative levels of the buyers' and sellers' entry supply curves. This is shown on the following page, in Figure 3. In the figure we can see that when agents' participation decision is relatively insensitive to the expected value of participation the division of the transaction surplus is highly sensitive to fundamental demand/supply.

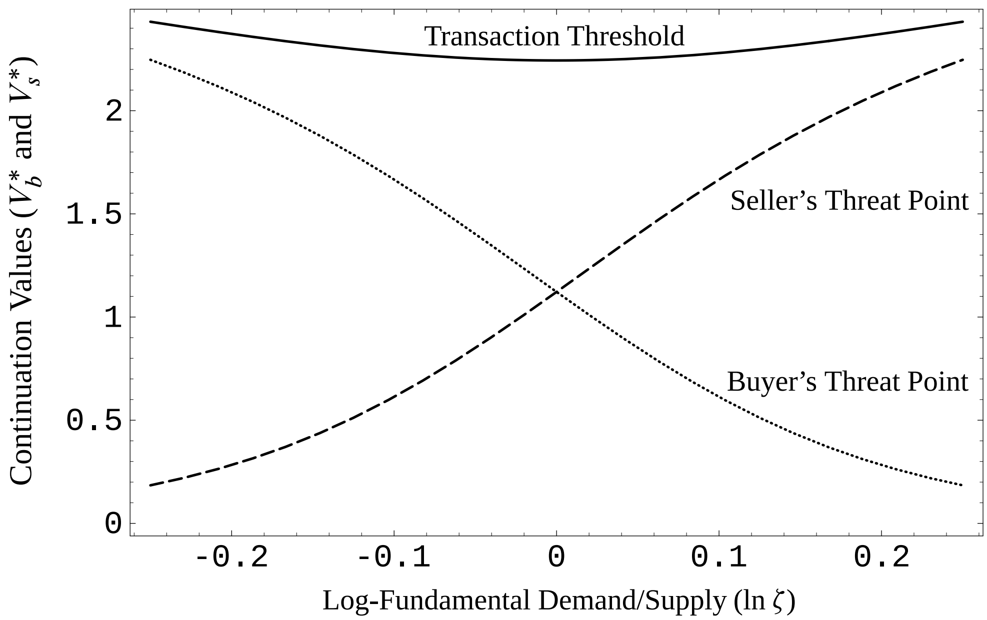

Figure 3: Threat Points in General Equilibrium
The value of the agents' threat points (value of waiting for the next deal) for both buyers (dotted line) and sellers (dashed line), and the minimum realized idiosyncratic value that results in a transaction (solid line), are plotted as a function of the log of fundamental demand/supply. Parameters are $\eta = \beta = .5$, $r = .1$, $\lambda = 52$, $c_b = c_s = 0$, and $\gamma = 0.1$.

4.2 Transaction Rates

We can also express the expected time agents search prior to successfully transacting as a function of our fundamental market condition variable $\zeta = X_b/X_s$, the fundamental demand/supply.

Let $\mathbf{E}[T_i]$ denote the expected time an agent of type i is "in the market" before successfully transacting. Because of the Markovian nature of the market, the expected time-to-transaction may be expressed as

$$\mathbf{E}[T_i] \quad = \quad \mathbf{E}[\tau_i] + \Phi_\epsilon(\epsilon^*)\mathbf{E}[T_i]. \tag{13}$$

In the previous equation the first term is the expected time until the next encounter, and the second term is the probability that the encounter will be unsuccessful times the expected time the agent will spend in the market in the event that she continues searching. Rearranging the previous equation yields

$$\mathbf{E}[T_i] \;=\; \frac{\mathbf{E}[\tau_i]}{1-\Phi_\epsilon(\epsilon^*)}, \tag{14}$$

which, using $\mathbf{E}[\tau_i] = \lambda_i^{-1}$ and equation (12), implies

$$\mathbf{E}[T_b] \;=\; \frac{1}{(1-\Phi_\epsilon(\epsilon^*))\lambda}\left(\frac{\zeta^{1/\gamma}}{\rho}\right)^{-\eta} \tag{15}$$

and $\mathbf{E}[T_s] = \rho\mathbf{E}[T_b]/\zeta^{1/\gamma}$. These also imply $\frac{d}{d\ln\zeta}\ln\frac{\mathbf{E}[T_b]}{\mathbf{E}[T_s]} = \gamma^{-1}$, consistent with the direct implications of market clearing in conjunction with equation (12), which specifies market tightness as a function of fundamental demand/supply. The expected time-to-transaction on both sides of the market are depicted in Figure 4, as a function of $\zeta = X_b/X_s$, fundamental demand/supply.

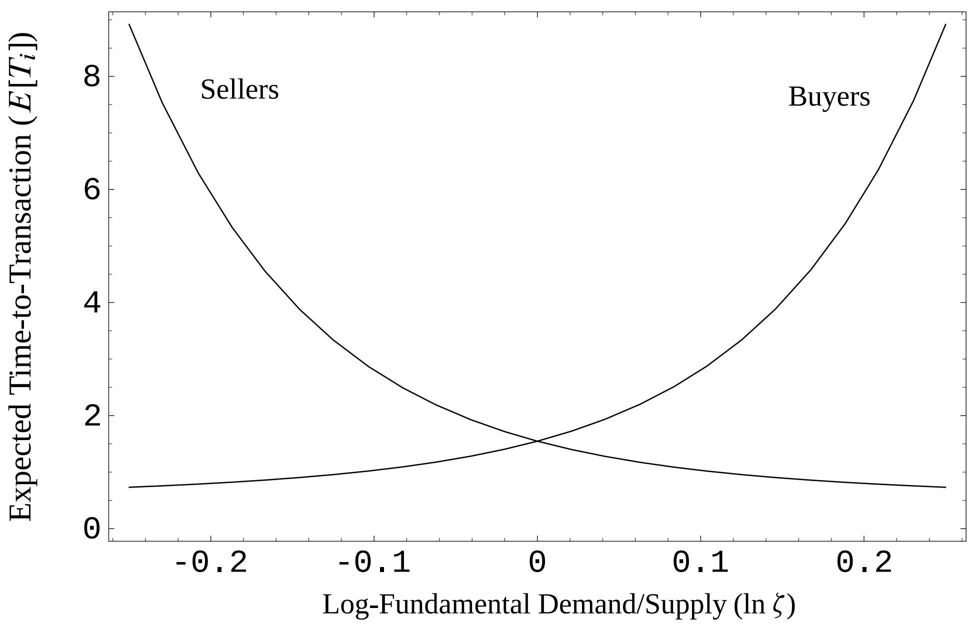

Figure 4: Expected Time in the Market

The buyers' (upward sloping curve) and sellers' (downward sloping curve) expected times-to-transaction are plotted as a function of the log of the market condition variable $\zeta = X_b/X_s$, the fundamental demand/supply. Parameters are $\eta = \beta = .5$, $r = .1$, $\lambda = 52$, $c_b = c_s = 0$, and $\gamma = 0.1$.

Inspection of Figure 4 reveals that the magnitude of the elasticity of expected time-to-transaction with respect to fundamental demand/supply is roughly five, on either side of the market. This elasticity is lower than $1/\gamma = 10$, the elasticity of market tightness with respect to fundamental demand/supply, because the division of the transaction surplus, and consequently the transaction probability, responds to market tightness, somewhat mitigating the feedback channel described in this paper. However, five still greatly exceeds one, reflecting the high sensitivity of expected time-to-transaction to fundamental shifts in supply or demand, and the continued presence of the feedback channel through which agents' behavior amplifies fundamental shocks.

5 Conclusion

Markets characterized by costly search often appear more volatile than fundamentals, with market conditions, including market tightness, prices and average time-to-transaction, exhibiting high sensitivities to fundamental shocks. In this paper we shows that these high sensitivities result from interactions between macro-level market conditions and the microeconomic decision making process of the participants that comprise the market, whose optimal response to fundamental shocks reinforce the shocks, amplifying their impact. For example, a positive demand shock that increases the rate at which buyers enter the market improves the bargaining position of the sellers. Sellers then transact more quickly, decreasing the stock of sellers in the market, further increasing the relative number of buyers and sellers, thereby amplifying the initial shock. As a result, even small changes in the demand can lead to large changes in market tightness, the manner in which buyers and sellers split any transaction surplus, and the time agents must expect to spend in the market prior to transacting.

A Appendix: The Importance of Stochastic Matchings

In some markets, such as the real estate market, the common value component of the potential transaction surplus is an order of magnitude larger than the standard deviation in the potential transaction surplus (*i.e.,* $\mu/\sigma \gg 1$). In other markets, such as the marriage market, the common component may be small relative to idiosyncratic component, or even negative. The relative magnitude of the two components of the potential transaction surplus plays an important role in agents' search behavior. Agents will search longer in markets where the common value component is low (*i.e.,* when the value of a match is primarily idiosyncratic), and less in markets where the value of the match is primarily common value. A large common value component leads agents to transact more quickly, because the discounting of the high common component makes search costly, while the low probability of realizing a large idiosyncratic transaction surplus limits the potential benefits of continued search. A large idiosyncratic component leads to longer search times, as the gains from a "good match" are more important when the variance potential transaction surpluses is high.

To see the impact of the common value component on agents' transaction decision, let $\varepsilon = \epsilon - \mu$. Then rewriting equation (8) in terms of ε we have that $\epsilon^* = \mu + \varepsilon^*$ where ε^* is implicitly defined by

$$\Lambda(\theta)\nu_\varepsilon(\varepsilon^*) - r\varepsilon^* \quad = \quad r\mu + c_b + c_s. \tag{16}$$

Conditional on the distribution of ε, the previous equation yields, through implicit differentiation,

$$\frac{d\varepsilon^*}{d\mu} \quad = \quad \frac{-1}{1 - r^{-1}\Lambda(\theta)\nu'_\varepsilon(\varepsilon^*)}. \tag{17}$$

Now $r^{-1}\Lambda(\theta)\nu'_\varepsilon(\varepsilon^*) < 0$, so $\frac{d\varepsilon^*}{d\mu} \in (-1, 0)$, therefore $\frac{d}{d\mu}\mathbf{P}(\epsilon > \epsilon^*) > 0$. That is, the probability of transacting is increasing in μ, and consequently the expected time in the market is decreasing, *ceteris paribus.*

The dependence of the transaction probability and the expected time-to-transaction on the magnitude of the common value component are shown in Figures 5 and 6. Seemingly insignificant variation in the potential transaction surplus can dramatically alter agents' behavior. For example, even if the standard deviation in the potential transaction surplus is only 1 percent of the mean potential transaction surplus then agents will still choose not to transact in almost 75 percent of encounters. This suggest stochastic matching should not be ignored.

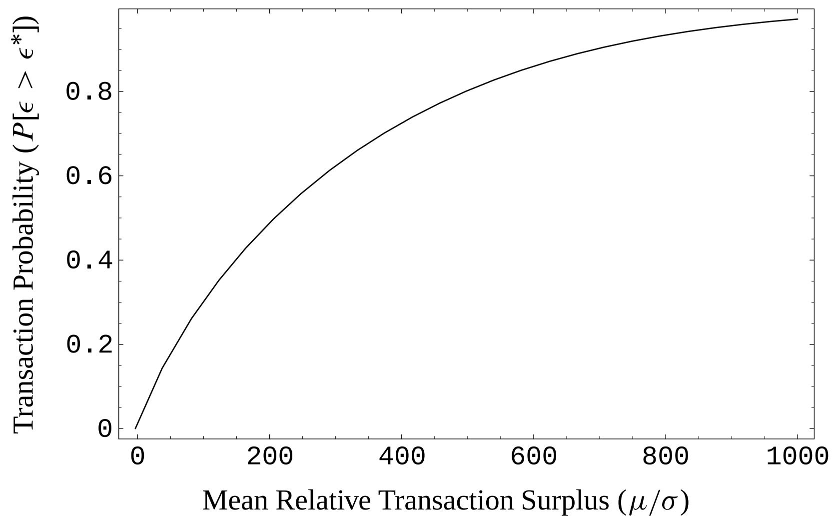

Figure 5: Transaction Probability and the Common Value Component
The probability of an encounter resulting in a transaction in a "balanced market" (*e.g.*, $\beta = .5$ and $\zeta = 1$) is plotted as a function of the magnitude of the common value component (mean transaction surplus scaled by the standard deviation). Parameters are $\eta = .5$, $r = .1$, $\lambda = 52$, $c_b = c_s = 0$, and $\gamma = 0.1$.

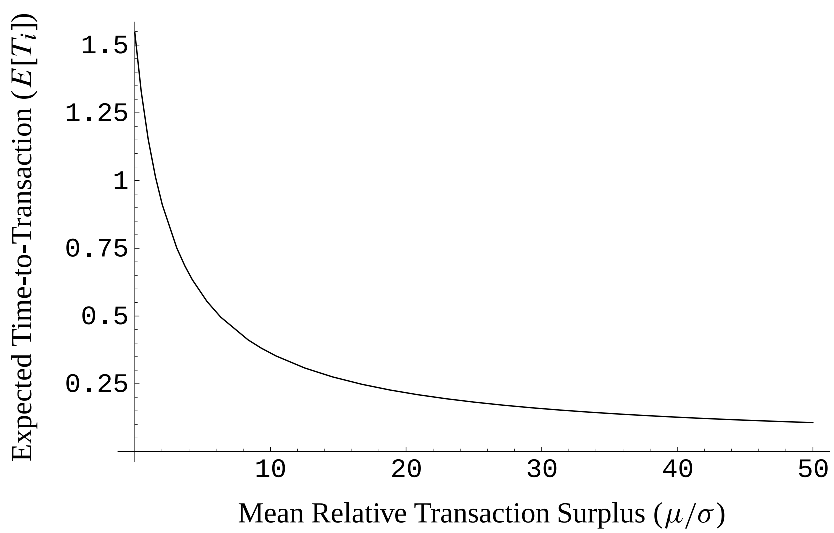

Figure 6: Time-to-Transaction and the Common Value Component
The expected time-to-transaction in a "balanced market" (*e.g.*, $\beta = .5$ and $\zeta = 1$) is plotted as a function of the magnitude of the common value component (mean transaction surplus scaled by the standard deviation). Parameters are $\eta = .5$, $r = .1$, $\lambda = 52$, $c_b = c_s = 0$, and $\gamma = 0.1$.

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