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A Theory of Board Control and Size

by

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ABSTRACT

This paper presents a model of optimal control of corporate boards of directors. In particular, we determine when one would expect inside directors or outside directors to control the board, when the controlling party will delegate decision-making to the other party, the extent of communication with the deciding party, and the number of outside directors. Our model incorporates the traditional view of corporate boards as monitors along with a role for board members as suppliers of expertise or information relevant for the decision. Because of the agency problem between corporate insiders and owners (who are assumed to be represented by the outside directors), neither party will communicate his or her information fully to the other. Outsiders, in our model, control agency problems by making some decisions themselves. When they do, the refusal of insiders to communicate their information fully becomes costly. Therefore, shareholders can sometimes be better off by having boards controlled by insiders, contrary to conventional wisdom. We characterize optimal board control and delegation decisions, the optimal number of outsiders, and resulting profits as functions of the importance of insiders' and outsiders' information, the extent of agency problems, and some other factors. This leads to an endogenous relationship between profits and the number of outside directors that furthers our understanding of some documented empirical regularities.

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A Theory of Board Control and Size

1 Introduction

Corporate governance, and in particular the issue of control of corporate boards by independent directors, has received considerable attention recently, in the wake of corporate scandals afflicting the likes of Enron, Tyco, Adelphi and others.² In 2002, Congress passed the Sarbanes-Oxley act mandating that the audit committees of the boards of directors of firms listed on national exchanges have a majority of independent members. And, in 2003, both the New York Stock Exchange and NASDAQ amended their rules to require the boards of listed firms to have a majority of independent members.³ Meanwhile, the Securities and Exchange Commission has proposed rules that would require mutual funds to have an independent chairman and that at least 75% of directors be independent (they are already required to have a majority of independent directors). Despite the attention given this issue in the popular press, by Congress, and by the major stock exchanges, there has been little theoretical work on this topic.⁴ This paper attempts to contribute toward filling that gap.

The developments mentioned in the previous paragraph suggest that the prevailing view among regulators is that it is in the interest of shareholders for corporate boards to be controlled by independent directors. This view seems to be driven by agency considerations, i.e., that only independent directors (*outsiders*) can effectively curtail agency problems. Although agency problems are clearly important, other considerations may affect the conclusion that boards should be outsider-controlled. In particular, the board's decisions are based on the information available to its members, information provided both by insiders and by outsiders.⁵ Since board control affects the strategic interaction between insiders and outsiders, it also affects the board's decisions and hence shareholder value. For example, when outsiders control the board, insiders may not provide full or completely accurate information. Obviously, this can have an adverse effect on the board's decisions, reducing shareholder value.

In this paper, we take account both of the direct effects of agency problems on corporate decision-making and the indirect effects of agency problems on communication between insiders and outsiders. We model a board of directors that consists of insiders and (possibly) outsiders that must make a decision that affects firm profits. Insiders have private information relevant to the decision but also have private benefits that lead them to choose a decision that does not maximize profits. Outside directors seek to maximize firm profits (they perfectly represent the firm's owners) and may, at a cost, acquire private, decision-relevant expertise.⁶ The party in control of the board, based on its information, may make the decision themselves or delegate decision-making authority to the other party. Whoever makes the decision will base the decision on their own information and any information communicated to

² See, for example, Burns (2004), Luchetti and Lublin (2004), and Solomon (2004).

³ See New York Stock Exchange Rule 303A and NASDAQ Rule 4350(c).

⁴ One important exception that is related to the current paper is Hermalin and Weisbach (1998) which will be discussed below. Hermalin and Weisbach (2003) provide an excellent survey of economic research on boards of directors. As they point out, most of this work is empirical.

⁵ This point has been noted by Adams and Feirrer (2003), discussed below. Grinstein and Tolkowsky (2004) document empirically the informational role of outsiders.

⁶ We assume that, while insiders obtain their information "for free" in the process of performing their duties, outside directors must expend effort to apply their expertise to the specific issues faced by this firm. For example, a board member who is also a professor of finance may understand the theory of corporate investment decisions in general much better than firm insiders but will typically need to invest considerable effort to apply that understanding to the issues that arise on the board.

them by the other party. Because of the agency problem between insiders and outsiders, neither party will choose to communicate his or her information fully to the other.

We determine (i) optimal board control, (ii) when the controlling party will delegate decision-making authority to the other party, (iii) the extent of communication, (iv) the number of outside directors, and (v) firm profits. Despite the fact that the controlling party may delegate to the other, it still matters who controls the board for two reasons. First, who controls will affect the incentives of outsiders to acquire the necessary expertise in the firm's operations. Second, the delegation decision will depend on the controlling party's information and will, therefore, convey information. Whether outsiders or insiders optimally control the board is determined by comparing the two types of board with respect to several factors that affect the final decision. These factors are: the incentives of outsiders to acquire expertise, the likelihoods of the two parties to have decision-making authority, and the extent of communication as a function of who has this authority. We show, contrary to conventional wisdom, that despite the agency problem, it is sometimes optimal to have insiders in control of the board. This is partly because insider-control better exploits insiders' information. This will be especially valuable when insiders' information is important relative to the direct agency costs. In addition, when insiders are in control, they will not delegate to outsiders if outsiders are uninformed. When outsiders control the board, on the other hand, they can make decisions even when uninformed. Consequently, insider-control of the board may provide greater incentives for outsiders to become informed.

The assumption that outside directors must exert effort to apply their expertise also allows us to determine the number of outsiders, along with board control, endogenously. The idea is simple. As the number of outsiders increases, their value to the company in providing expertise increases, provided each outsider continues to expend the same effort. But increasing their number aggravates a free-rider problem for outsiders. That is, when there are more outsiders, each outsider views the importance of his or her contribution as being reduced and, therefore, expends less effort in specializing his or her expertise. Increasing the number of outsiders results in more "heads" but also less informed heads. The optimal number of outsiders appropriately balances the two effects.⁷

Since our model determines both the number of outside directors and profits endogenously, it has implications for the relationship between them. It can thus help us understand some of the empirical findings in the literature on corporate governance. First, Yermack (1996) and Eisenberg, *et al.* (1998) document a negative correlation between profits and the number of board members.⁸ These authors interpret the evidence as supporting the hypothesis of Lipton and Lorsch (1992) and Jensen (1993) that, due to agency problems, large boards are less effective. Hermalin and Weisbach (2003) ask why does board size appear to affect performance and why do we observe large boards if, in fact, they are less effective than small boards. In our model, board size does not affect performance but both are driven by other, exogenous factors such as the importance of the various parties' information, profit potential, and the opportunity cost of outside directors. We show how certain movements in these factors can induce a negative correlation between profits and the number of outside board members. This can explain the observed negative correlation between profits and board size⁹ and why we continue to observe large boards despite the negative correlation. Since our model can explain the negative correlation without any

⁷ Persico (2004) also considers this public-good aspect of information gathering, although in a quite different context.

⁸ Similarly, Graham and Narasimhan (2004) show that "companies with large boards, and boards dominated by insiders are less likely to survive the Depression." (abstract).

⁹ We assume that the optimal number of insiders, given optimal board control, is determined by factors outside our model. With this assumption, given board control, the number of insiders is independent of the factors in our model that determine the number of outsiders. Consequently, board size and the fraction of outside directors will vary one-to-one with the number of outside directors.

implication that large boards are less effective, our approach casts doubt on the interpretation of this evidence as supporting the Lipton-Lorsch-Jensen hypothesis.

Second, many studies fail to find a correlation between performance and the fraction of independent directors (see below for citations). In our model, movements in the exogenous factors can induce a positive correlation or no correlation between profits and the number of outsiders on the board. Thus our model is consistent with this empirical finding.

Third, Rosenstein and Wyatt (1990) find “a positive stock price effect when an outsider is appointed to a company’s board, even where outside directors already constitute a board majority” (Romano (1996)). This finding is puzzling given the two facts just discussed. That is, adding another outsider increases both the fraction of outsiders (which should have no effect on performance) and the size of the board (which should have a negative effect on performance). In our model, a firm will add an outside director only after a shock that increases the optimal number of outsiders. The impact on firm value, in our model, depends on the nature of the shock. These three examples highlight the problem with interpreting empirical correlations as if the number of outside board members were exogenous.

Hermalin and Weisbach (1998) provide a model similar in some respects to ours. They model a board’s decision about whether to retain or replace the CEO. This decision is based on firm performance and a costly additional signal. More independent boards are more inclined to obtain the additional signal. The degree to which the board is independent depends on the firm’s past performance; good prior performance enables the CEO to reduce the board’s independence. A number of interesting results are obtained regarding CEO compensation and turnover and the degree of independence of the board, but the model does not address the issue of the relation between the number of outside board members and contemporaneous profits or optimal board control.

In Raheja (2005), the board consists of insiders with preferences that are not aligned with those of the shareholders and private information and outsiders with the same preferences as shareholders who may acquire information at a cost. Without some insider’s information, the information cost for outsiders is prohibitive. Insiders reveal their information to increase their chances of succeeding the current CEO. More insiders on the board increases competition to become the next CEO, and hence increases the incentives for insiders to reveal their information, but also increases their influence over project choice. This can be countered by increasing the number of outsiders, but this is assumed to increase their information acquisition costs due to increased coordination problems. This can result in firms with more severe agency problems having larger boards with lower performance than similar firms with less severe agency problems. Thus, while Raheja (2005) does not emphasize delegation and communication as we do, the model provides an alternative explanation to ours of the negative correlation between board size and performance that does not involve a causal relationship.

Another related paper is Adams and Feirrer (2003) which also considers directors’ role as advisors to management and management’s incentive to provide information to the board. To obtain better information from managers, the board may want to commit not to use this information in evaluating managerial performance.¹⁰ This results in a theory of the extent to which boards are “manager friendly.” Adams and Feirrer also use the model to comment on the optimality of dual boards (one to play the monitoring role and another the advising role) such as are commonly found in Europe.

Gillette, *et al.* (2003) models board voting among insiders and outsiders. As in the other papers mentioned here, insiders have private information and preferences that diverge from those of shareholders. In particular, informed insiders prefer to accept all projects, even those with negative NPV. Outsiders are uninformed, have preferences that are aligned with those of shareholders, and have veto power. Assuming that the average project (from the point of view of uninformed outsiders) has negative

¹⁰ Spatt (2004) makes a similar point.

NPV, they show that there are essentially two equilibria, one in which all projects are vetoed by outsiders and the efficient equilibrium outcome in which only positive NPV projects are accepted. In experiments, the authors find that, “in the vast majority of times,” the efficient equilibrium prevails (p. 1999). They conclude that having even uninformed outsiders in control can prevent inefficient outcomes. The assumption that the average project has negative NPV corresponds to the case in our model in which the agency problem is especially severe. For this case, our results agree with those of Gillette, *et al.* (2003), i.e., outsider-control is optimal. Gillette, *et al.* (2003) do not consider, either theoretically or experimentally, the case in which the average project has positive NPV. This would correspond to the special case of our model in which agency problems are not severe and outsiders have no useful private information. In that case, our result is that control does not matter. We suspect that, in experiments carried out under the alternative assumption that the average project has positive NPV, the most frequently observed outcome would be acceptance of all projects. This would imply that outside control of the board has no value in that case, essentially in agreement with our results.¹¹

The remainder of this paper is organized as follows. The model is described in section 2. In section 3, we analyze the delegation decision, the extent of communication, the incentives for outsiders to become informed, and the optimal number of outsiders for boards controlled by outsiders. In section 4, we do the same for boards controlled by insiders. We then compare outsider-controlled and insider-controlled boards, taking account of the factors just mentioned, to determine optimal board control in section 5. Section 6 presents some comparative statics results and their empirical predictions. Section 7 applies our model to the Sarbanes-Oxley act, and section 8 concludes.

2 The Model

We consider a firm whose profits depend on a strategic decision denoted s to be determined by the board of directors. For concreteness, we refer to this decision as the scale of the firm, but it could be interpreted as any strategic decision. Board members may be firm employees, whom we call insiders, or independent directors, whom we call outsiders. Either insiders or outsiders may control the board. Control of the board allows the controlling group to choose scale themselves or to delegate this decision to the other group. In the terminology of Aghion and Tirole (1997), the controlling group has “formal” authority but may delegate this authority to the other group. As in Harris and Raviv (2005), the party with formal authority also makes the decision (has “real” authority) with input from the other party.

The firm’s optimal scale depends on private information of insiders (“agents”), \tilde{a} , and private information of outsiders (“principles”), \tilde{p} . In particular, profit, $\tilde{\pi}$, is given by

$$\tilde{\pi} = \pi_0 - (s - (\tilde{a} + \tilde{p}))^2. \quad (1)$$

Equation (1) implies that **potential profit**, π_0 , is reduced to the extent that the scale is chosen to be other than the “first-best” scale, $\tilde{a} + \tilde{p}$. Chosen scale may differ from first-best for two reasons. First, the decision-maker will not have full information about either \tilde{a} or \tilde{p} or both. Second, an agency problem, to be introduced shortly, implies that insiders would not choose the first-best scale, even if they had full information. Consequently, we refer to the quadratic term in (1), or its expectation, as the **information-plus-agency cost**. We assume that potential profit is sufficiently large that expected profit is always positive. This requires that $\pi_0 > \sigma_p^2 + b^2$, where σ_p^2 is the variance of \tilde{p} , and b^2 is a parameter, to be introduced shortly, that measures the extent of the agency problem. Moreover, we assume that the

¹¹ Two other, less closely related, theoretical papers on corporate boards are Hirshleifer and Thakor (1994) and Warther (1998).

compensation paid to outsiders is negligible compared to expected profits and is ignored when choosing the number of outsiders. Clearly, this assumption is appropriate only for large corporations.

We assume that insiders learn their information, \tilde{a} , in the course of their normal duties. Outsiders are presumed to have expertise about the other component of the optimal scale, which may include factors external to the firm. This expertise is costly to specialize to the firm's situation, however. For example, a finance professor who sits on a corporate board may have a deeper understanding of how to evaluate investment projects generally than do insiders but applying that understanding to the company's investment opportunities will require costly study of the particulars of those opportunities. Formally, we model this by assuming that, if any outsider invests non-contractible effort e at personal cost $c(e)$, he or she learns \tilde{p} with probability e (we refer to this as "becoming informed"). Thus the cost function should be interpreted as a measure of the uniqueness of the firm's activities, the degree of difficulty of adapting general business lessons to this particular firm. Outsiders are assumed to have identical interests and, therefore will share any information they produce among themselves (but cannot enter into contracts with each other, for example, to compensate each other for investing effort in becoming informed). As a result, if at least one outsider becomes informed, this information can be used to choose scale.¹² The outsiders' information cannot be obtained by insiders, but may be communicated to them by outsiders. Similarly, outsiders cannot obtain the insiders' information directly, but insiders may communicate it to them. The extent to which one party will communicate its information to the other is limited by an agency problem which we now describe.

All board members are risk-neutral. Insiders have an equity stake and obtain a private benefit from larger scale. In particular, we assume the combined effect results in a payoff to insiders given by¹³

$$\tilde{\pi}_I = \pi_{I0} - (s - (\tilde{a} + \tilde{p} + b))^2. \quad (2)$$

Note that the optimal scale from the point of view of insiders, given \tilde{a} and \tilde{p} , is $\tilde{a} + \tilde{p} + b$. The parameter $b > 0$ measures the extent to which insiders prefer larger scale at the expense of profits, i.e., the extent to which they are biased toward larger scale. Because of the quadratic cost functions in (1) and (2), the difference between the profits that result when the insiders choose scale and maximal profit, for any given information, is b^2 . We therefore refer to b^2 (and sometimes b) as the *agency cost*.

We assume outsiders' compensation consists entirely of an equity stake that is independent of outsiders' effort, since effort is assumed to be non-contractible.¹⁴ Consequently, outsiders prefer the profit-maximizing scale. We denote by $\alpha \in (0,1)$ the share of equity given to each outside board member. Outsiders' equity stake is determined by a participation constraint,

$$\alpha\pi - c(e) = \bar{U}, \quad (3)$$

¹² The purpose of these assumptions is to create a tradeoff in which more outsiders leads to greater information production, holding their effort constant, but also leads them to reduce effort, other things equal. The structure we have assumed leads to a fairly simple determination of the optimal number of outsiders, but we believe the results would not be affected if we assumed any structure that embodies this tradeoff.

¹³ The constant π_{I0} in equation (2) can be different from potential profit to allow insiders' preferences to diverge from profits in addition to their preference for larger scale. This constant plays no role in the analysis.

¹⁴ We could also include a salary without affecting the results. In fact, since outsiders' incentives to exert effort to become informed are greater, the larger their equity stake, for a given total expected compensation, shareholders would prefer that the compensation be entirely through equity. Since board members are assumed to be risk-neutral, this would be optimal. The assumption is also in line with empirical evidence on the compensation of outside directors. Yermack (2004) estimates that outside directors receive, on average, 11 cents for each \$1,000 increase in firm value.

where π is expected profit, and $\bar{U} > 0$ represents the value of an outside board member's opportunity cost of serving on the board.

We make the following assumptions regarding the distributions of \tilde{a} and \tilde{p} :

Assumption 1. \tilde{a} and \tilde{p} are independent. \tilde{a} is uniformly distributed on $[0, A]$; \tilde{p} is uniformly distributed on $[0, P]$.

In some cases, it will be more convenient to work with the standard deviations of the random variables \tilde{a} and \tilde{p} , as well as those of other uniformly distributed random variables, instead of the parameters A and P . Consequently, for any $x \geq 0$, denote by $\sigma(x)$ the standard deviation of a random variable uniformly distributed on an interval of width x , i.e.,

$$\sigma(x) = x/\sqrt{12}.$$

We will use σ_a to denote $\sigma(A)$ and σ_p to denote $\sigma(P)$.

Because of the quadratic cost function in (1), it turns out that, if an unbiased decision-maker chooses scale, the difference in profits between knowing \tilde{p} (respectively, \tilde{a}) and having no information about \tilde{p} (respectively, \tilde{a}) is exactly σ_p^2 (respectively, σ_a^2). We will therefore refer to σ_p^2 (σ_a^2) as the **full value of the outsiders' (insiders') information**. Many of our results depend on a comparison between agency costs and the value of one (or both) party's information. Accordingly, a firm for which $b \geq \sigma_p$ ($b \geq \sigma_a$) is referred to as one for which **agency costs are more important than outsiders' (insiders') information**. We refer to a firm for which agency costs are more important than either party's information ($b \geq \max\{\sigma_a, \sigma_p\}$) as one for which **agency costs are critical**. We label a firm as one for which **agency costs are important** if agency costs are more important than at least one party's information. Finally, we call a firm for which both party's information is more important than agency costs ($b \leq \min\{\sigma_a, \sigma_p\}$) one in which **information is more important than agency costs**. This terminology is illustrated in Figure 1.

Insert Figure 1 Here

3 Outsider-Controlled Boards

We begin by analyzing the scale and delegation decisions of boards that are controlled by the outside directors. We refer to such a board as an **outsider-controlled board or OCB**. In this paper, we assume board control can be determined independently of the number of outsiders or insiders on the board. For example, even if there are more insiders than outsiders, outsiders may control key committees, or they may be explicitly given control over the kinds of decisions we model here, or overruling outsiders may require a supermajority of directors.¹⁵

The results on outsider-controlled boards will be used later when we consider whether boards controlled by insiders may be preferred to those controlled by outsiders. These results are interesting in their own right, however, in situations in which government regulation or other requirements (e.g.,

¹⁵ Romano (1996, note 13) states, "The literature conventionally uses the terms outsider-dominated and insider-dominated to describe a board with a majority of outside directors and inside directors, respectively, and that convention will be adopted [in her paper] . . . , even though it is not, in my view, an apt expression because 'domination' of a board's decisions is likely to be a complex function of individual personalities and expertise rather than a function of the number of directors of a specific type."

exchange listing requirements) mandate that corporate boards be controlled by independent directors. Indeed the New York Stock Exchange and NASDAQ have recently amended their rules to require the boards of listed firms to have a majority of independent directors.

Recall that control of the board empowers the outside directors to choose scale or to delegate the choice of scale to insiders. If outsiders do not delegate, they will choose scale based on their own information and any information communicated to them by insiders.¹⁶ Denote by $r(a)$ the report of insiders to outsiders if insiders observe $\tilde{a} = a$. If outsiders delegate, insiders will choose scale based on their own information and any information communicated to them by outsiders. Denote by $t(p)$ the report of outsiders to insiders if outsiders observe $\tilde{p} = p$. Of course, outsiders will have nothing to report if they do not become informed. Moreover, because of the agency problem, the reports will not fully communicate the reporting party's information, as will be seen below.

The sequence of events in this case is assumed to be the following. First, outsiders simultaneously choose how much effort to invest in becoming informed. Next, either they become informed or they do not, and then decide whether to delegate the scale decision to insiders. It will be shown below, that if outsiders learn that $\tilde{p} = p$, they will delegate if and only if $p \geq p^*$, for some cutoff p^* . If outsiders fail to become informed, they will delegate the scale decision to insiders, unless agency cost is sufficiently large. How large will be determined below.¹⁷ Finally, depending on whether outsiders delegate or not, either insiders or outsiders choose scale and profits are realized. We start by analyzing the scale choice, given a delegation decision, then return to the delegation decision.

3.1 Outsiders Do Not Delegate

Suppose outsiders choose scale themselves. Since they have identical preferences and information, outsiders behave as a single agent. Given their compensation, outsiders will maximize expected profits. It is easy to check that, if outsiders are informed, they will choose

$$s(p, r) = \bar{a}(r) + p, \quad (4)$$

where $\bar{a}(r) = E(\tilde{a}|r)$, and the expectation is with respect to outsiders' posterior belief about \tilde{a} , given insiders' report, r . Thus, $\bar{a}(r)$ is a value of \tilde{a} estimated by outsiders based on what is communicated by insiders. The game in which outsiders choose s is analyzed formally in Harris and Raviv (2005). There it is shown that, because of the agency problem, insiders will not fully reveal their information. They will instead inform outsiders only that \tilde{a} lies in some range.

More precisely, in the Pareto-best Bayes equilibrium of the game in which outsiders choose s , insiders will partition the support of \tilde{a} , $[0, A]$, into cells $[a_i, a_{i+1}]$ and report a value that is uniformly distributed on the cell in which the true realization of \tilde{a} lies. Thus outsiders learn only the cell in which the true value of insiders' information lies, and their posterior belief is that \tilde{a} is uniformly distributed on

¹⁶ We do not consider the possibility of constrained delegation, i.e., that one party allows the other party to choose s subject to a constraint. One can show, for example, that if the outsiders have no private information, it is optimal for them, when they delegate to insiders, to put an upper bound on insiders' choice that is sometimes binding. When outsiders have private information, the problem is more complicated, since their choice of a constraint may convey information. Since this model is sufficiently complicated already, we leave this possibility for future work.

¹⁷ This is also shown in Dessein (2002).

that cell. It follows that, if the report r is in $[a_i, a_{i+1}]$, $\bar{a}(r) = \frac{a_i + a_{i+1}}{2}$. The number of cells is given by $N(b, A)$, where, for any $x > 0$,

$$N(b, x) = \left\langle \frac{1}{2} \left(\sqrt{1 + 2x/b} - 1 \right) \right\rangle, \quad (5)$$

and, for any real number x , $\langle x \rangle$ is the smallest integer greater than or equal to x . Note that the number of cells is a measure of the extent to which insiders communicate their information to outsiders. For example, if there is only one cell, $N(b, A) = 1$, insiders communicate nothing outsiders don't already know. On the other hand as $N(b, A)$ gets very large (as will be the case if agency cost, b , approaches zero), the information communicated approaches perfect information about \tilde{a} . The endpoints of the cells satisfy

$$a_i = \frac{iA}{N(b, A)} - 2i(N(b, A) - i)b, \text{ for } i = 0, \dots, N(b, A). \quad (6)$$

If outsiders are not informed, they will choose

$$s(p, r) = \bar{a}(r) + \bar{p}, \quad (7)$$

where \bar{p} is the unconditional mean of \tilde{p} . The equilibrium report of insiders is the same as before.

3.2 Outsiders Delegate

Now suppose insiders are allowed to choose scale. They will choose

$$s(a, t) = a + \bar{p}(t) + b, \quad (8)$$

where $\bar{p}(t) = E(\tilde{p}|t)$, and the expectation is with respect to insiders' posterior belief about \tilde{p} given outsiders' report, t , and the fact that the decision has been delegated. Since outsiders delegate the scale choice only when $\tilde{p} \geq p^*$, insiders, if given the decision, can infer that $\tilde{p} \geq p^*$. It is shown in Harris and Raviv (2005) that, in the Pareto-best Bayes equilibrium, outsiders will partition the interval $[p^*, P]$ into $N(b, P - p^*)$ cells and report a value that is uniformly distributed on the cell in which the true realization of \tilde{p} lies. Thus insiders' posterior belief about \tilde{p} is that it is uniformly distributed on the cell that contains the true value. It follows that, if the report t is in $[p_i, p_{i+1}]$, $\bar{p}(t) = \frac{p_i + p_{i+1}}{2}$.

The endpoints of the cells satisfy

$$p_i = \frac{iP + (N - i)p^*}{N} + 2i(N - i)b, \text{ for } i = 0, \dots, N, \quad (9)$$

where $N = N(b, P - p^*)$.

3.3 Outsiders' Delegation Decision

Here we analyze the outsiders' decision of whether to delegate the scale choice to insiders. We show that, when outsiders are informed, they will delegate if and only if $\tilde{p} \geq p^*$, and when outsiders are not informed, they will delegate if and only if $b \leq \sigma_a$. Intuitively, when outsiders learn that \tilde{p} is large,

the optimal scale is likely to be large. There is less to lose from delegating to insiders biased in favor of larger scale. When outsiders are uninformed, they will delegate to informed insiders unless agency costs exceed the full value of insiders' information. We also characterize the cutoff value, p^* .

First suppose outsiders are informed. To determine when informed outsiders will delegate the scale decision to insiders, the following notation will be useful. Let $L(b, X)$ be the information cost, i.e., expected loss in profits, of having only information that is transmitted in equilibrium about a random variable that is uniformly distributed on an interval of width X . That is,

$$L(b, X) = E[\bar{x}(r(\tilde{x})) - \tilde{x}]^2, \quad (10)$$

where \tilde{x} is uniform on an interval of width X , r is the equilibrium report of the party that observes \tilde{x} , and $\bar{x}(r)$ is the decision-maker's equilibrium posterior mean of \tilde{x} .

It is shown in Crawford and Sobel (1982) that

$$L(b, X) = \begin{cases} \left(\frac{\sigma(X)}{N(b, X)} \right)^2 + \frac{b^2 (N(b, X)^2 - 1)}{3}, & \forall X > 0, \\ 0, & \text{for } X = 0. \end{cases}$$

Thus $L(b, X)$ depends on how much information is transmitted in the report, as measured by $N(b, X)$. In particular, if $N(b, X) = 1$, i.e., no information is transmitted to the decision-maker, then $L(b, X)$ is the entire variance, $\sigma^2(X)$, of the unobserved variable. If some information is transmitted ($N(b, X) > 1$), the expected cost is smaller than the variance.

Next, suppose outsiders are in control of the board and x is such that outsiders will delegate the scale choice to insiders whenever outsiders' private information is at or above $P - x$. Furthermore, suppose insiders know that the choice will be delegated to them only when outsiders' private information is at or above $P - x$. Define $f(b, x)$ as the expected information-plus-agency cost when scale choice is delegated, given that the outsiders' private information is exactly x , and delegation involves communication as in the above equilibrium.¹⁸ It is shown in Harris and Raviv (2005) that

$$f(b, x) = \begin{cases} \left[\frac{x}{2N(b, x)} + N(b, x)b \right]^2, & \forall x > 0, \\ b^2, & \text{for } x = 0. \end{cases} \quad (11)$$

Some useful properties of f and L are shown in Lemma 1 (in the appendix).

If the scale choice is not delegated to insiders, the actual information cost is given by the square of the deviation of outsiders' choice of s , $\bar{a}(r(\tilde{a})) + \tilde{p}$, from the true realization of $\tilde{a} + \tilde{p}$, i.e.,

$[\bar{a}(r(\tilde{a})) - \tilde{a}]^2$. Thus the expectation of this cost over \tilde{a} , using the above equilibrium value for r , is $L(b, A)$. Essentially, the cutoff, p^* , is determined as the value such that the expected cost of delegating

¹⁸ One can also interpret f symmetrically for the case in which insiders control the board. In that case, one assumes insiders will delegate only when their private information is at or below x . Then f is the expected loss to insiders when they delegate the scale choice to outsiders, given that the insiders' private information is exactly x .

if, in fact, $\tilde{p} = p^*$, $f(b, P - p^*)$, is the same as the cost of not delegating, $L(b, A)$. This is stated formally in the following lemma.

Lemma 2: Delegation Decision of Informed Outsiders. If outsiders are informed that $\tilde{p} = p$, they will delegate the choice of scale to insiders if and only if $p \geq p^*$, where p^* is as follows.

- a. If $b \geq \sigma_a$, $p^* = P$, i.e., outsiders do not delegate, regardless of their information.
- b. If $f(b, P) \leq L(b, A)$, $p^* = 0$, i.e., outsiders delegate the decision to insiders, regardless of their information.
- c. Otherwise, $p^* \in (0, P)$ and is defined by

$$f(b, P - p^*) = L(b, A). \quad (12)$$

In this case, $P - p^*$ is independent of P .

Proof. This is shown in Harris and Raviv (2005).

We refer to case (b) of Lemma 2 as the case in which *insiders' information is critical*. This terminology is justified by the fact (implied by Lemma 1) that if $f(b, P) \leq L(b, A)$, then $\max\{b, 6\sigma_p\} < \sigma_a$, i.e., insiders' information is more than six times as important as outsiders' information, and more important than agency costs. It is also justified by its implication that outsiders always delegate. We refer to the symmetric case for insider-controlled boards, $f(b, A) \leq L(b, P)$, as one in which *outsiders' information is critical*. These terms are illustrated in Figure 2.

Insert Figure 2 Here

Now suppose outsiders do not become informed. If they delegate the scale decision to insiders, insiders choose $s = \bar{p} + \tilde{a} + b$, where \bar{p} is the unconditional mean of \tilde{p} , and the expected information-plus-agency cost is

$$E\left(\left(\bar{p} - \tilde{p} + b\right)^2\right) = \sigma_p^2 + b^2.$$

If outsiders do not delegate, the expected information-plus-agency cost is

$$E\left(\left(\bar{a}(r(\tilde{a})) - \tilde{a} + \bar{p} - \tilde{p}\right)^2\right) = L(b, A) + \sigma_p^2.$$

Thus, when outsiders are not informed, they will choose to delegate if and only if $b^2 < L(b, A)$. It follows from Lemma 1 that this condition is equivalent to $b < \sigma_a$, which implies that $N(b, A) = 1$ and hence that $L(b, A) = \sigma_a^2$. Consequently, when uninformed, outsiders delegate to insiders if and only if $b < \sigma_a$. The information-plus-agency cost in this case, denoted l_U (the subscript U is for "uninformed"), is

$$l_U = \sigma_p^2 + \min\{b^2, \sigma_a^2\}. \quad (13)$$

We summarize the above result in the following lemma.

Lemma 3: Delegation Decision of Uninformed Outsiders. When uninformed, outsiders delegate to insiders if and only if insiders information is more important than agency costs ($b < \sigma_a$). The information-plus-agency cost in this case, l_U , is given in equation (13).

3.4 Outsiders' Equilibrium Effort

Each outsider will choose his or her effort to maximize his or her share of expected profits net of effort costs. Consider the effort investment decision of an individual outsider, given that all $n - 1$ other outsiders invest e .¹⁹ If the given outsider chooses effort x , the probability that at least one outsider will become informed is given by $1 - (1 - e)^{n-1}(1 - x)$. Denote by l_I the information-plus-agency cost given that outsiders are informed (hence the subscript I). An explicit expression for l_I will be developed presently; for now it suffices to note that l_I does not depend on outsider effort. Recall that l_U is the corresponding cost when outsiders are uninformed, defined in (13). Then expected profits are given by

$$\pi_0 - \left[\left(1 - (1 - e)^{n-1}(1 - x) \right) l_I + (1 - e)^{n-1}(1 - x) l_U \right] = \pi_0 - l_I - (1 - e)^{n-1}(1 - x) V_{OCB},$$

where $V_{OCB} = l_U - l_I$ is the *marginal value of outsiders' information* with an outsider-controlled board. The given outsider therefore chooses effort x to maximize his or her share α of the above expected profits minus his or her effort cost, i.e., to maximize

$$\alpha \left[\pi_0 - l_I - (1 - e)^{n-1}(1 - x) V_{OCB} \right] - c(x).$$

The first-order condition for the optimal effort is

$$c'(x) = (1 - e)^{n-1} \alpha V_{OCB}, \quad (14)$$

provided there is an interior solution. To interpret equation (14), note that αV_{OCB} is the individual outsider's share of the marginal value of outsiders' information. By increasing his or her effort, the given outsider increases by that amount the probability of receiving this benefit, but only if no other outsiders become informed. The probability that none of the other $n - 1$ outsiders becomes informed, given that each exerts effort e , is $(1 - e)^{n-1}$. Thus the right hand side of (14) is the marginal expected benefit of extra effort for any given outsider. Obviously, the left hand side is the marginal cost of additional effort. Thus the condition for optimal effort is just the usual marginal-cost-equals-marginal-benefit condition.

All outsiders choosing e is a Nash equilibrium if and only if, given that all other outsiders choose effort e , it is optimal for our given outsider to choose e , i.e., if and only if $x = e$ satisfies (14):

$$c'(e) = (1 - e)^{n-1} \alpha V_{OCB},$$

or

$$c'(e)(1 - e) = (1 - e)^n \alpha V_{OCB}, \quad (15)$$

again provided there is an interior solution of (15).

To make sure the first-order condition (14) is necessary and sufficient for a maximum, and to obtain an interior solution for the equilibrium effort for at least some parameter values, we require the following assumption regarding the cost function c :

¹⁹ We consider only symmetric Nash equilibria of the outsiders' effort choice game.

Assumption 2. The cost c is assumed to be strictly increasing and strictly convex in e . Also, assume $c(0) = 0$, and the function $c'(e)(1-e)$ is convex and has a unique, interior minimum over $[0,1]$ at $e^* \in (0,1)$.²⁰

It is easy to check that, under Assumption 2, for $n > 0$, equation (15) has a unique solution, $e_n(\alpha V_{OCB}) \in (0,1)$, that is increasing in αV_{OCB} and decreasing in n , provided that the marginal cost of effort for an outsider at zero effort is less than the outsider's share of the marginal value of becoming informed, i.e., provided that $\alpha V_{OCB} > c'(0)$ [see Figure 3]. If this condition is satisfied, we say that **effort cost is not prohibitive**. If effort is prohibitively costly, the only Nash equilibrium is $e = 0$, i.e., outsiders will not invest any effort to become informed (and, therefore, will not become informed for sure). Otherwise, outsiders will invest more effort if the marginal value of becoming informed, V_{OCB} , is larger, outsiders' share of profits, α , is larger, and if the number of outside board members, n , is smaller, other things equal. The last implication is the result of free riding by outside board members. This free riding induces a tradeoff in choosing the optimal number of outside board members. For given effort, more outsiders increases the chances that at least one outsider will become informed. Because of free riding, however, having more outsiders on the board reduces the effort of all outsiders. We will exploit this tradeoff in choosing the optimal number of outsiders.

Insert Figure 3 Here

We now develop an explicit expression for the marginal value of outsiders' information, V_{OCB} . If outsiders delegate, the information-plus-agency cost is given by the square of the difference between the insiders' choice of s , $\tilde{a} + \bar{p}(t(\tilde{p})) + b$, and the true realization of $\tilde{a} + \tilde{p}$, i.e., $[\bar{p}(t(\tilde{p})) - \tilde{p} + b]^2$. Since outsiders delegate if and only if $\tilde{p} \in [p^*, P]$, the expected cost when outsiders delegate is

$$E\left([\bar{p}(t(\tilde{p})) - \tilde{p} + b]^2 \mid \tilde{p} \in [p^*, P]\right) = L(b, P - p^*) + b^2.$$

Given that outsiders become informed, the probability that they do not delegate to insiders is $\frac{p^*}{P}$. Recall that if outsiders do not delegate when informed, the expected information-plus-agency cost is $L(b, A)$. Therefore, conditional on outsiders becoming informed, but before knowing the value of \tilde{p} , the expected cost is

$$l_I = \frac{p^*}{P} L(b, A) + \left(1 - \frac{p^*}{P}\right) [L(b, P - p^*) + b^2]. \quad (16)$$

²⁰ If $c'(e)(1-e)$ were monotone decreasing, then the optimal number of outsiders would be either one or zero, depending on whether the marginal value of the outsiders' information does or does not exceed the marginal cost of effort at zero effort. If $c'(e)(1-e)$ were monotone increasing, the optimal number of outsiders would either not be defined (for any number of outsiders, more outsiders would be better) or be zero, again depending on whether the marginal value of the outsiders' information does or does not exceed the marginal cost of effort at zero effort. An example of a cost function that satisfies Assumption 2 is

$$c(e) = -\gamma \ln(1-e) - \varepsilon \left[\frac{1}{2} e^2 + (1-2\delta)(e + \ln(1-e)) \right]$$

for $\varepsilon > 0$, $0 < \delta < 1$ and $\gamma > 2\varepsilon\delta$. In this case $c'(e)(1-e) = \gamma - \varepsilon e(2\delta - e)$, $e^* = \delta$, and $c'(0) = \gamma$.

From (13) and (16), it follows that the marginal value of outsiders' information with an outsider-controlled board is given by

$$V_{OCB} = l_U - l_I = \sigma_p^2 + \min\{b^2, \sigma_a^2\} - \left[\frac{p^*}{P} L(b, A) + \left(1 - \frac{p^*}{P}\right) [L(b, P - p^*) + b^2] \right]. \quad (17)$$

We summarize the results of this subsection and the previous subsection in the following proposition.

Proposition 1: Outsiders' Equilibrium Effort and Delegation Decision When They Are In Control. Assume the board is outsider-controlled.

- If effort cost is not prohibitive ($\alpha V_{OCB} > c'(0)$), then equilibrium effort of outsiders when there are n outsiders, $e_n(\alpha V_{OCB}) \in (0, 1)$, and is given by the unique solution of (15); otherwise the equilibrium effort of outsiders is zero.
- If agency costs are more important than insiders' information ($b \geq \sigma_a$), then outsiders never delegate, whether they become informed ($p^* = P$) or not, and insiders do not communicate any information about \tilde{a} ($N(b, A) = 1$). Consequently, the information-plus-agency cost if outsiders fail to become informed is the full value of both outsiders' and insiders' information, i.e., $l_U = \sigma_p^2 + \sigma_a^2$. For the same reasons, the information-plus-agency cost if outsiders become informed is the full value of insiders' information, σ_a^2 . Thus, the marginal value of outsiders' information is the full value of outsiders' information ($V_{OCB} = \sigma_p^2$).
- If insiders' information is critical ($f(b, P) \leq L(b, A)$), then outsiders always delegate, whether they become informed ($p^* = 0$) or not, so the information-plus-agency cost if outsiders do not become informed is $l_U = \sigma_p^2 + b^2$. Since outsiders always delegate, the information-plus-agency cost if outsiders become informed is the cost of imperfectly communicating this information to insiders, $L(b, P)$ (since $p^* = 0$, insiders learn nothing from the fact that outsiders delegate), plus the agency cost, b^2 . Thus the marginal value of outsiders' information is $V_{OCB} = \sigma_p^2 - L(b, P) \geq 0$.
- If agency costs are less important than insiders' information ($b < \sigma_a$), and insiders' information is not critical ($f(b, P) > L(b, A)$), then outsiders sometimes delegate when they are informed ($p^* \in (0, P)$) and is given by the solution to (12)) and always delegate when they are uninformed. Consequently, the information-plus-agency cost if outsiders do not become informed is $l_U = \sigma_p^2 + b^2$. In this case, $P - p^*$ is independent of P .

Note that p^* is not the optimal cutoff. The reason is that the choice of p^* affects insiders' inference about \tilde{p} . In fact p^* is larger, relative to P , than is optimal, i.e., outsiders, when in control, fail to delegate sufficiently often relative to the importance of their information. This can lead to the counter-intuitive result that the marginal value of outsiders' information may be negative. In particular, when $p^* \in (0, P)$, *not* becoming informed may improve expected profits, because it leads outsiders to delegate to insiders for sure, whereas if outsiders become informed, they will delegate a suboptimal fraction of the time. Obviously, for this to be the case, it must be true that agency costs are relatively small and insiders' information relatively important. Indeed, we see from Proposition 1, that when agency costs are large, so

that outsiders fail to delegate even if they do not become informed, the marginal value of outsiders' information is positive. And, when insiders' information is critical, so that outsiders always delegate, the marginal value is non-negative.

3.5 Optimal Number of Outsiders for Outsider-Controlled Boards

We assume shareholders choose the number of outside board members to maximize the expectation of their share of profits, i.e., profits net of payments to outsiders. Recall that the compensation paid to outsiders is assumed to be negligible compared to expected profits and, hence, is ignored when choosing the number of outsiders. Shareholders will therefore choose the number of outsiders, n_{OCB} , to solve the following problem:

$$\max_{n \in \mathbf{Z}} \pi_0 - l_U + \left[1 - (1 - e_n(\alpha V_{OCB}))^n \right] V_{OCB}, \quad (18)$$

where \mathbf{Z} is the set of non-negative integers. One may interpret the objective function in (18) as potential profits, π_0 , minus the cost of not observing \tilde{p} , l_U , plus the amount of that cost that is saved if \tilde{p} is observed times the probability of observing \tilde{p} . This problem is equivalent to

$$\min_{n \in \mathbf{Z}} (1 - e_n(\alpha V_{OCB}))^n. \quad (19)$$

Thus the object is to minimize the probability that outsiders fail to become informed, given that this probability is determined by the number of outsiders and their share of the marginal value of becoming informed.

Assuming that effort cost is not prohibitive, we can use (15) to write the objective function in (19) as

$$(1 - e_n)^n = \frac{c'(e_n)(1 - e_n)}{\alpha V_{OCB}}. \quad (20)$$

Thus, in this case, n_{OCB} can be chosen to minimize $c'(e_n)(1 - e_n)$. Using Assumption 2, if n were a continuous variable, the optimal n , denoted n^* , would be such that $e_{n^*} = e^*$ [see Figure 3], i.e.,

$$n^*(\alpha V_{OCB}) = \frac{\ln \frac{c'(e^*)(1 - e^*)}{\alpha V_{OCB}}}{\ln(1 - e^*)}. \quad (21)$$

It follows that the optimal number of outsiders with an outsider-controlled board, n_{OCB} , is either $\langle n^*(\alpha V_{OCB}) \rangle$ or $\langle n^*(\alpha V_{OCB}) \rangle - 1$. Since n_{OCB} is approximately equal to $n^*(\alpha V_{OCB})$, and incorporating the integer constraint makes subsequent calculations intractable, hereafter, we ignore the integer constraint on the number of outsiders.

If effort is prohibitively costly, the equilibrium effort of outsiders is zero, and the optimal number of them is one.²¹ Therefore,

²¹ Since we ignore the cost of outsiders in choosing the optimal number of them, strictly speaking, any positive number of outsiders would be equally good (at least one outsider is needed for outsider control). If we use the (negligible) cost to break ties, however, the optimal number of outsiders would be one.

$$n_{OCB} = \begin{cases} n^*(\alpha V_{OCB}), & \text{if } c'(0) < \alpha V_{OCB}, \\ 1, & \text{otherwise.} \end{cases} \quad (22)$$

It is easy to check that n^* is increasing. Intuitively, in our model, regardless of α and V_{OCB} , as long as effort cost is not prohibitive, the equilibrium effort will be e^* . That is, the number of outsiders is determined to make equilibrium effort of outsiders e^* , which depends only on the effort cost function. When outsiders' share of the marginal value of becoming informed, αV_{OCB} , increases, outsiders will increase their effort. To prevent this, the number of outsiders must increase so that the free-rider problem causes them to continue to choose e^* . This result follows from three key assumptions of the model: (i) the function $c'(e)(1-e)$ has a unique minimum on $[0,1]$, (ii) to take advantage of the private information available to outsiders requires only that at least one of them learn it, and (iii) in computing profits, the cost of outside board members is negligible.

It also follows from this observation and (18) that the maximum expected profit with an outsider-controlled board, obtained by choosing the optimal number of outsiders, is given by

$$\pi^*(OCB) = \pi_0 - l_U + \begin{cases} \left[1 - (1 - e^*)^{n_{OCB}}\right] V_{OCB}, & \text{if } c'(0) < \alpha V_{OCB}, \\ 0, & \text{if } c'(0) \geq \alpha V_{OCB}. \end{cases} \quad (23)$$

Note that if effort cost is not prohibitive, $c'(0) < \alpha V_{OCB}$, then information-plus-agency costs consist of the cost when outsiders are uninformed, l_U , minus the probability that outsiders become informed, $1 - (1 - e^*)^{n_{OCB}}$, times the marginal value of their information, V_{OCB} . If effort costs are prohibitive, information-plus-agency costs are simply l_U , since outsiders never become informed.

We can rewrite the upper branch of (23) as

$$\pi^*(OCB) = \pi_0 - l_I - (1 - e^*)^{n_{OCB}} V_{OCB}, \text{ if } c'(0) < \alpha V_{OCB}. \quad (24)$$

This version provides an alternate interpretation, i.e., if effort cost is not prohibitive, then information-plus-agency costs consist of the cost when outsiders are informed, l_I , plus the probability that outsiders don't become informed, $(1 - e^*)^{n_{OCB}}$, times the marginal value of their information, V_{OCB} .

4 Insider-Controlled Boards

Our goal is to determine board control endogenously by comparing the profitability of outsider- and insider-controlled boards. Consequently, having analyzed outsider-controlled boards in the previous section, we now analyze insider-controlled boards (ICBs).

4.1 Outsiders' Equilibrium Effort and Insiders' Delegation Decision

If insiders control the board, they may choose scale or delegate the choice of scale to outsiders. The sequence of events in this case is assumed to be the following. First, outsiders choose effort and become informed or not. Next, insiders observe \tilde{a} and whether outsiders are informed and decide whether to delegate the choice of scale to outsiders. They will not delegate if outsiders fail to become informed. Finally, either outsiders or insiders choose scale, depending on the outcome of the insiders' delegation decision.

It can be shown, using the same argument as in Theorem 2 of Harris and Raviv (2005), that insiders will prefer to delegate if and only if $a \leq a^*$, for some cutoff a^* . The analyses of outsiders' choice of scale and insiders' choice of scale proceeds as in the case in which outsiders are in control of the board

as discussed above, with obvious modification.²² It follows that outsiders' equilibrium effort choice with an insider-controlled board is determined by

$$c'(e) = (1-e)^{n-1} \alpha V_{ICB}, \quad (25)$$

where V_{ICB} is the marginal value of outsiders' becoming informed with an insider-controlled board, i.e., $V_{ICB} = l'_U - l'_I$, and provided (25) has an interior solution, i.e., provided $\alpha V_{ICB} > c'(0)$. The quantities l'_U and l'_I are the counterparts, for insider-controlled boards, of l_U and l_I , i.e., the information costs given outsiders do not (do, respectively) become informed. Formally,

$$l'_U = \sigma_p^2 + b^2, \quad (26)$$

and

$$l'_I = \frac{a^*}{A} L(b, a^*) + \left(1 - \frac{a^*}{A}\right) (L(b, P) + b^2). \quad (27)$$

The basic results for insider-controlled boards are given in the following proposition (the proof is omitted, since it is quite similar to that of Proposition 1).

Proposition 2: Outsiders' Equilibrium Effort and Insiders' Delegation Decision When Insiders' Are in Control. Assume the board is insider-controlled.

- If effort cost is not prohibitive ($\alpha V_{ICB} > c'(0)$), then equilibrium effort of outsiders when there are n outsiders, $e_n(\alpha V_{ICB}) \in (0, 1)$, and is given by the unique solution of (25); otherwise the equilibrium effort of outsiders is zero.
- If outsiders' information is less important than agency costs ($\sigma_p \leq b$) or if outsiders are uninformed, then insiders never delegate ($a^* = 0$), and outsiders do not communicate any information, even if they are informed ($N(b, P) = 1$). In this case, there is nothing to be gained by outsiders' becoming informed ($V_{ICB} = 0$).
- If outsiders' information is critical ($f(b, A) \leq L(b, P)$), then insiders always delegate when outsiders are informed ($a^* = A$). In this case, the marginal value of outsiders' information is

$$V_{ICB} = \sigma_p^2 + b^2 - L(b, A) \geq \sigma_p^2 + b^2 - \sigma_a^2 > 0.$$

- If outsiders' information is more important than agency costs ($\sigma_p > b$), but not critical ($f(b, A) > L(b, P)$), then insiders sometimes delegate if outsiders are informed, i.e., $a^* \in (0, A)$ and is given by the solution of

$$f(b, a^*) = L(b, P), \quad (28)$$

²² The analysis makes use of the fact that, even though outsiders could delegate the choice of s back to insiders when outsiders become informed, they will not choose to do so. This is because, in any situation in which insiders prefer outsiders to choose scale despite losing the benefits of choosing a larger-than-optimal scale, outsiders will prefer to choose scale themselves even more strongly.

which implies $\max\{b, 6\sigma(a^*)\} \leq \sigma_p$. In this case, a^* is independent of A , and the marginal value of outsiders' information is positive ($V_{ICB} > 0$).

Note that, as is the case with p^* , a^* is an equilibrium cutoff, not the optimal cutoff. In fact a^* is smaller than is optimal, i.e., insiders, when in control, fail to delegate sufficiently often relative to the importance of outsiders' information. Nevertheless, when insiders are in control, the marginal value of outsiders' information is always non-negative (and is zero only when insiders never delegate). This is because outsiders have no choice of whether to delegate to insiders when insiders control the board.

4.2 Optimal Number of Outsiders for Insider-Controlled Boards

As in the case of outsider-controlled boards, we assume the number of outsiders is chosen to maximize expected total profits.²³ Therefore, with an insider-controlled board, the optimal number of outsiders, n_{ICB} , solves

$$\min_{n \in \mathbf{Z}} (1 - e_n(\alpha V_{ICB}))^n. \quad (29)$$

Thus, ignoring the integer constraint as mentioned above, the optimal number of outsiders with an insider-controlled board, is

$$n_{ICB} = \begin{cases} n^*(\alpha V_{ICB}), & \text{if } c'(0) < \alpha V_{ICB}, \\ 0, & \text{otherwise,} \end{cases} \quad (30)$$

and the equilibrium effort of outsiders is e^* if $c'(0) < \alpha V_{ICB}$ and zero otherwise.

The maximum profit obtainable with an insider-controlled board is

$$\begin{aligned} \pi^*(ICB) &= \pi_0 - l'_U + \begin{cases} [1 - (1 - e^*)^{n_{ICB}}] V_{ICB}, & \text{if } c'(0) < \alpha V_{ICB}, \\ 0, & \text{if } c'(0) \geq \alpha V_{ICB}, \end{cases} \\ &= \pi_0 - l'_I - \begin{cases} (1 - e^*)^{n_{ICB}} V_{ICB}, & \text{if } c'(0) < \alpha V_{ICB}, \\ V_{ICB}, & \text{if } c'(0) \geq \alpha V_{ICB}. \end{cases} \end{aligned} \quad (31)$$

The two versions of maximum profit shown in the upper branches of equation (31) offer the same two interpretations of information-plus-agency cost as is the case for outsider-controlled boards.

5 Outsider- vs. Insider-Controlled Boards

In this section we derive the optimal board control from the point of view of shareholders.²⁴ In deriving optimal board control, it is important to remember that the party who controls the board need not make the scale decision. Board control bestows an option either to choose scale or to delegate this choice to the other party. Consequently, our results include not only who controls the board but also under what circumstances the controlling party delegates the scale choice.

²³ Even if insiders capture the board selection process, they would choose the same number of outsiders as would shareholders, provided that, as we assume, the choice of board control can be separated from the choice of the number of outsiders.

²⁴ In interpreting our results on board control empirically, one must assume that the board selection process is not captured by insiders. If it is, one expects the board to be insider-controlled, regardless of the implication for profits. See Shivdasani and Yermack (1999) and the references cited therein for some evidence regarding insider control of the board selection process.

5.1 Dependence of Board Control on Importance of Information

Our results in this subsection relate the profit-maximizing board control to the importance of the insiders' and outsiders' information, σ_a and σ_p . The formal results are stated below and are shown graphically in Figure 4, which shows optimal control for various combinations of the importance of the two parties' information for a numerical example. The figure shows that outsider-control is preferred when outsiders' information is sufficiently more important than that of insiders' and conversely for insider-control. In particular the boundary that divides the region where outside control is optimal from that where inside control is optimal is upward sloping and below the forty-five degree line.

Insert Figure 4 Here

One can understand these results by considering the following basic trade-offs. If insiders control the board, they decide, based on their information, who chooses scale. This decision is determined by who will choose s closest to insiders' ideal point, taking account of the extent of communication. If outsiders control the board, they decide, based on their information (if any), who chooses scale. This decision is determined by who will choose s closest to the profit maximizing scale, again taking account of the extent of communication. Moreover, control may also affect outsiders' incentives to become informed. Ignoring the issue of outsiders' incentives to become informed and the expected amount of communication, the other considerations suggest that it is optimal for insiders to control the board only when their information is sufficiently more important than that of outsiders to offset the fact that insiders are biased toward larger scale. This partial intuition is reflected in Figure 4. That is, as suggested by the basic trade-offs, it is optimal for insiders to control the board only when their information is sufficiently more important than that of outsiders.

This partial intuition does not, however, explain the areas in the figure for which shareholders are indifferent with respect to board control. One reason for this indifference is the effect of board control on the incentives of outsiders to become informed. For example, consider the point labeled A in Figure 4. The essential characteristics of point A are that insiders' information is important relative to outsiders' information and agency costs and outsiders' information is less important than agency costs. The first characteristic implies that, if outsiders control the board, they will be very likely to delegate to insiders (notice that point A is relatively close to the boundary at which insiders' information becomes critical and outsiders' always delegate). The second characteristic implies that, when they do delegate to insiders, outsiders will communicate none of their information to insiders. Together, these two facts imply that outsiders, when in control, have little incentive to become informed. In fact, for this example, their incentive to become informed when in control is below their marginal cost of effort at zero effort. Consequently, if they are in control, outsiders never become informed and hence always delegate. If insiders control the board, they never delegate and so always choose scale with no information from outsiders. Thus, in this case, regardless of who controls the board, insiders always choose scale with no information from outsiders.

The previous example highlights the complicated interactions among the delegation decision, the extent of communication, and the incentives for outsiders to invest effort in becoming informed that together determine optimal board control. In the rest of this section, we analyze board control more carefully, taking account of these interactions. To compare the two board types analytically, we must simplify the model by assuming that the share of profits going to outsiders, α , is exogenous.²⁵ We also assume that $c'(0) < \alpha b^2$. This eliminates some uninteresting cases in which effort cost is prohibitive.

²⁵ Numerical simulations with endogenous α indicate that the results on board control are not affected by this assumption.

It is convenient to divide the population of firms into types based on whether each party's information is critical and whether, if it is not critical, it is more or less important than agency costs.²⁶ This results in four types of firms as shown in Figure 5. For both type-I and type-IV firms, neither party's information is critical. For type-I firms, agency costs are more important than at least one party's information. For type-IV firms, agency costs are less important than either party's information. Insiders' information is critical for firms of type II, and outsiders' information is critical for firms of type III. For each type of firm, we derive the optimal board control, taking account of when the controlling party will delegate to the other party (as shown in sections 3 and 4).

Insert Figure 5 Here

Proposition 3: Optimal Board Control When Neither Party's Information is Critical, But Agency Costs Are Important (Type I firms).

- Shareholders prefer an outsider-controlled board. This preference is strict unless effort cost is prohibitive if outsiders are in control.²⁷
- Outsiders delegate to insiders if insiders' information is more important than agency cost and outsiders' are either not informed or their information indicates that the optimal scale is likely to be large ($\tilde{p} \geq p^*$).

Proof. See the appendix.

Proposition 4: Optimal Board Control When Insiders' Information is Critical (Type II firms).

- Shareholders prefer an insider-controlled board. This preference is strict unless agency cost is more important than outsiders' information, or effort cost is prohibitive for an insider-controlled board.
- Insiders delegate if outsiders' information is more important than agency cost, outsiders become informed, and insiders' information indicates that the optimal scale is likely to be small ($\tilde{a} \leq a^*$).

Proof. See the appendix.

Proposition 5: Optimal Board Control When Outsiders' Information is Critical (Type III firms).

- Shareholders prefer an outsider-controlled board. This preference is strict if agency costs are less important than insiders' information, and effort cost is not prohibitive for an outsider-controlled board.
- Outsiders delegate to insiders if insiders' information is more important than agency cost and outsiders' are either not informed or their information indicates that the optimal scale is likely to be large ($\tilde{p} \geq p^*$).

Proof. See the appendix.

Finally, we consider Type IV firms. This is the most complicated case, and, consequently, little can be said in general about the relation between board preference of shareholders and the exogenous

²⁶ Recall that if insiders' information is critical, outsiders always delegate to insiders, and if outsiders' information is critical, insiders always delegate to outsiders if the latter are informed.

²⁷ Even if effort cost is prohibitive for an outsider-controlled board, the preference is still strict if $b > \sigma_a$ and $b \geq \sigma_p$.

parameters in this case. We can, however, relate shareholders' preferences over board control to the marginal value of outsiders' information.

Proposition 6: Optimal Board Control When Neither Party's Information is Critical, but Information is More Important Than Agency Costs (Type IV firms). If effort cost is not prohibitive for at least one type of board, then shareholders prefer the board type that results in the largest marginal value of outsiders' information. If effort cost is prohibitive for both board types, shareholders are indifferent. Outsiders delegate if they are uninformed or their information indicates that the optimal scale is likely to be large. Insiders delegate if outsiders are informed and insiders' information indicates that the optimal scale is likely to be small.

Proof. See the appendix.

In one special case of type-IV firms of particular interest, however, we can say which board type is preferred. This is the case in which outsiders and insiders have more or less equally important information. As argued at the beginning of this section, one might expect that shareholders would prefer an outsider-controlled board. This is indeed the case as is shown in the next result.

Corollary 1. Optimal Board Control When Neither Party's Information is Critical, but Information is More Important Than Agency Costs, and the Parties Have Similar Information. If effort cost is not prohibitive for an outsider-controlled board, and if insiders and outsiders have information of similar importance, i.e., σ_p and σ_a are sufficiently close, then shareholders prefer an outsider-controlled board.

Proof. See the appendix.

Although we cannot characterize theoretically the exact location of the boundary, shown in Figure 4, between outsider-control and insider-control for type-IV firms, the intuition presented at the beginning of this section suggests that this boundary should approach the forty-five degree line as agency costs approach zero. That is, for very small agency costs, it is intuitive that shareholders prefer the board to be controlled by the party with the more important information. In fact, this is shown theoretically in the next corollary.

Corollary 2: Optimal Board Control When Neither Party's Information is Critical, and Agency Costs Are Small. For sufficiently small agency costs, if effort costs are not prohibitive for either board type, shareholders prefer that the board be controlled by the party with the more important information.

Proof. See the appendix.

5.2 Dependence of Board Control on Agency Costs

The above results on how optimal board control varies with the importance of the insiders' and outsiders' information, for given agency costs, are summarized in Figure 4. How board control depends on agency costs is complicated by the way in which these costs enter into the delegation and communication decisions of the two parties. Although we cannot derive theoretically the dependence of control on agency costs, some numerical examples are instructive in highlighting the incompleteness of the partial intuition given earlier in this section. This intuition suggests that increases in agency costs make outsider-controlled boards more desirable. This is often the case in our simulations.²⁸ That is, generally, if agency costs are large, insiders' information must be much more important than outsiders' information for insider-control to be preferred. As agency costs decline, the extent to which insiders'

²⁸ Chester Spatt has suggested that this result of the model, along with the perception that agency costs have risen, can explain the recent trend toward increased representation of outsiders on boards.

information must be more important than that of outsiders decreases. There are cases, however, in which the result is just the opposite. In particular, for some parameter values, increases in b result in a shift from outsider-control to insider-control. In these cases, the increase in b results in an *increase* in delegation when outsiders are in control. This counterintuitive result is due to the fact that the increase in b reduces communication more when outsiders choose scale (insiders communicate) than when insiders choose scale (outsiders communicate). Meanwhile, the increase in b decreases delegation when insiders are in control, as expected. Essentially, increases in delegation imply that the delegating party's information is less useful. Consequently, in these cases, the increase in agency costs renders the outsiders' information relatively less useful and the insiders' information relatively more useful. This leads shareholders to prefer shifting control from outsiders to insiders.

6 Other Comparative Statics and Empirical Implications

In this section, we examine the effects of changes in the various exogenous parameters of the model on the endogenous variables other than board control. That is, we assume, in this section, either that the parameter changes do not change the optimal board type or that the board type is constrained by regulation or some other factor that is outside our model. We also consider only changes in the parameters that do not affect whether effort cost is prohibitive. Consequently, none of the changes in parameters will affect equilibrium effort of outsiders. Following an examination of the model's comparative statics, we take up their empirical implications.

6.1 Comparative Statics

We begin our comparative statics analysis by investigating the effect of changes in the importance of outsiders' information on the number of outsiders, profits, and outsiders' equity share.

Proposition 7 (OCB): The effect of changes in the importance of outsiders' information for outsider-controlled boards. An increase in the importance of outsiders' information, σ_p , results in

- A decrease in the probability that outsiders delegate the decision to insiders;
- An increase in the number of outsiders;²⁹
- A decrease in profits;
- An increase in the share of outsiders;

Proof. See the appendix.

It is not surprising that, when outsiders' information becomes more important, it is optimal to have more of them. Since their effort doesn't change (recall that the number of outsiders is determined to keep effort constant), this increases the probability that they become informed and, by itself, would increase profits. There are two opposing effects, however. If either outsiders do not become informed or become informed but delegate to insiders, the loss due to their failure to become informed or their refusal to communicate the information fully increases. It turns out that the profit-reducing effects dominate. Since profits decrease, outsiders must be given a larger share of profits to assure their participation.

²⁹ If effort cost is prohibitive, the number of outsiders is not affected. Also, if effort cost is not prohibitive, and insiders' information is not critical but is more important than agency costs, we assume that $\sigma_p > b\sqrt{12}$. If this inequality does not hold, V_{OCB} can be decreasing in σ_p , as the following example shows. If $b=0.05$ and $\sigma_a = 0.425$, then $V_{OCB} = 0.000441655789903148$ at $\sigma_p = 0.0626275316444043$ and $V_{OCB} = 0.000077441977273082$ at $\sigma_p = 0.0651275316444044$. In this case, it is unclear whether αV_{OCB} increases or decreases, and hence whether the number of outsiders increases or decreases.

The corresponding result for insider-controlled boards is given in the next proposition.

Proposition 7 (ICB): The effect of changes in the importance of outsiders' information for insider-controlled boards. An increase in the importance of outsiders' information, σ_p , increases the probability that insiders delegate the decision to outsiders. If effort cost is prohibitive, an increase in the importance of outsiders' information, σ_p , decreases profits. Since there are no outsiders in this case, their share is irrelevant, and there is no effect on the number of outsiders. If effort cost is not prohibitive, and outsiders' information is critical, the number of outsiders increases, but profits and the share of outsiders do not change. There is no change in the equilibrium effort of outsiders.

Proof. See the appendix.

If effort cost is prohibitive, outsiders never become informed, and insiders never delegate to outsiders. Since insiders make the decision with no information about \tilde{p} , profits are reduced by the full value of outsiders' information. Any increase in the importance of outsiders' information raises this value and lowers profits.

If effort cost is not prohibitive, and outsiders' information is critical, insiders always delegate if outsiders are informed. Consequently, the increase in the importance of outsiders' information has no effect on profits if outsiders become informed. If they do not become informed, however, insiders choose the scale with no information about \tilde{p} , so profits decline with an increase in the importance of outsiders' information, but the marginal value of outsiders' information increases. This leads to an increase in the number of outsiders and an increase in the probability that outsiders become informed that is just sufficient to keep profits the same on average. Thus no change in the share of outsiders is required.

Note that the proposition says nothing about the case in which effort cost is not prohibitive, and outsiders' information is not critical. It turns out that, in this case, the number of outsiders, profits and the share of outsiders are not monotone in the importance of outsiders' information.

Proposition 8 (OCB): The effect of changes in the importance of insiders' information for outsider-controlled boards. An increase in the importance of insiders' information, σ_a , increases the probability that outsiders delegate the decision to insiders. If insiders' information is less important than agency costs, an increase in the importance of insiders' information, σ_a , results in an increase in the number of outsiders, a decrease in profits, and increase in the equity share of outsiders.³⁰ If insiders' information is critical or if it is more important than agency costs and effort cost is prohibitive, an increase in the importance of insiders' information has no effect on the number of insiders, profits, or the equity share of outsiders. If insiders' information is more important than agency costs but not critical (and effort cost is not prohibitive), the number of outsiders and profits move in the same direction as each other, while the equity share of outsiders moves in the opposite direction.

Proof. See the appendix.

The intuition for Proposition 8 (OCB) is as follows. If insiders' information is less important than agency costs, then outsiders never delegate, even if they're uninformed. Consequently, the marginal value of becoming informed is the full value of outsiders' information, which is not affected by the importance of insiders' information. Profits, however, decrease as a result of the increase in the importance of insiders' information, since outsiders will have less information about \tilde{a} when they choose scale. This requires an increase in outsiders' equity stake to keep their compensation the same. The increased equity stake increases outsiders' share of the marginal value from becoming informed, so to keep outsiders' effort constant at e^* , one must increase the number of outsiders. If either insiders'

³⁰ If effort cost is prohibitive, there is no effect on the number of outsiders.

information is critical or it is more important than agency costs and effort cost is prohibitive, outsiders always delegate, so insiders' information is always fully utilized. There is, therefore, no affect on profits, hence no effect on the equity share of outsiders, no effect on the marginal value of outsiders' becoming informed, and no effect on the optimal number of outsiders.

Unfortunately, one cannot pin down the sign of the effect of a change in the importance of insiders' information when insiders' information is more important than agency costs but not critical, since there are conflicting effects on profits (numerical examples show profits can go either up or down). If, say, profits increase when the importance of insiders' information changes, then so too will the marginal value of outsiders' information. Indeed, the latter will increase by a greater relative amount than the former. Meanwhile, outsiders' share of profits must decrease by the same percentage that profits increase. Consequently, outsiders' share of the marginal benefit of becoming informed increases. This results in an increase in the optimal number of outsiders. Thus the number of outsiders and profits move in the same direction as each other.

The corresponding result for insider-controlled boards is given in the next proposition.

Proposition 8 (ICB): The effect of changes in the importance of insiders' information for insider-controlled boards. An increase in the importance of insiders' information, σ_a , decreases the probability that insiders delegate the decisions to outsiders. If effort cost is prohibitive, changes in the importance of insiders' information has no affect on profits, the share of outsiders, or the number of outsiders. If effort cost is not prohibitive, an increase in the importance of insiders' information results in

- A decrease in the number of outsiders;
- A decrease in profits;
- An increase in the share of outsiders;

Proof. See the appendix.

As in the previous result, if effort cost is prohibitive, outsiders never become informed, and insiders never delegate to outsiders. Since insiders make the decision with full information about \bar{a} , profits are unaffected by the change in the importance of insiders' information. As before, the share of outsiders is irrelevant, and the number of outsiders' is zero, independently of the importance of insiders' information.

Next we consider the effects of changes in effort cost, potential profits, and the opportunity cost of outsiders. To consider comparative statics on effort cost, we introduce a scale factor $g > 0$ for the cost function, i.e., we write $c(e) = gk(e)$, where the function k satisfies Assumption 2.³¹ We also consider the effects of changes in potential profit and the opportunity cost of outside directors. These result are valid for both outsider- and insider-controlled boards.

Proposition 9:

- **The effect of a change in effort cost.** If effort cost is prohibitive, changes in effort cost, g , have no effect. Otherwise, an increase in effort cost results in a decrease in the number of outsiders, a decrease in profits, and a less-than-proportionate increase in the equity share of outsiders.

³¹ Changes in effort cost of the type we consider do not change equilibrium effort when effort cost is not prohibitive, e^* .

- **The effect of a change in potential profit.** An increase in potential profits, π_0 , results in a decrease in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and a decrease in the equity share of outsiders.
- **The effect of a change in the opportunity cost of outsiders.** An increase in the opportunity cost of outsiders, \bar{U} , results in an increase in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and an increase in the equity share of outsiders.

Proof. See the appendix.

The intuition for the results in Proposition 9 is straightforward. An increase in effort costs forces an increase in outsiders' equity share (unless effort cost is prohibitive). The increase in effort costs also dictates a reduction in the number of outsiders to keep them from reducing their effort. The increase in outsiders' equity share partially reverses this effect, but not fully, since the increase in share is less than proportionate to the increase in cost. Since the number of outsiders falls, but their effort stays constant, they are less likely to become informed, and profits fall. An increase in the opportunity cost of outsiders also requires an increase in outsiders' equity share, leading to an increase in the number of outsiders. This increases the probability of outsiders becoming informed (more outsiders, same effort) and hence increases profits. An increase in potential profits dictates a reduction in outsiders' equity share which, in turn, requires a decrease in the number of outsiders so as to keep effort constant. There are opposing effects on profits. The increase in potential profits, *cet. par.*, increases expected profits, and the decrease in the number of outsiders, *cet. par.*, decreases expected profits. The net effect is, however, positive, as shown in the proof in the appendix.

Comparative statics with respect to agency costs, b , are impossible to derive analytically, due to the complicated way in which these costs enter the controlling party's delegation decision and the amount of information conveyed. Consequently, we present only numerical comparative statics for b . For insider-controlled boards, increases in agency costs reduce delegation, the number of outsiders, and profits, but increase outsiders' share of profits. These results are intuitive. For outsider-controlled boards, an increase in agency costs reduces profits and increases outsiders' share of profits. Such an increase decreases delegation when insiders' information is less important, but increases it when insiders' information is more important. The effect on the number of outsiders depends on the importance of the two parties' information in a more complicated way.

We summarize our comparative statics results in Table 1.

Insert Table 1 Here

6.2 Empirical Implications

As can be seen from Table 1, the relationships between the exogenous parameters and the endogenous variables of the model are often quite complicated and, therefore, difficult to test directly. There are some fairly straightforward predictions, however. For example, the model predicts that increases in the importance of the controlling party's information reduce the likelihood that the controlling party delegates the decision to the other party. Just the opposite is true for increases in the importance of the non-controlling party's information. Other examples involve the effect on the number of outsiders, profits, and outsiders' share of profits of changes in the importance of outsiders' information on outsider-controlled boards, the effect of changes in agency costs on insider-controlled boards, and the effects of changes in outsiders' effort costs, potential profits, and outsiders' opportunity costs.

The results of the previous subsection also predict relationships between profitability and the number of outside directors caused by variation in one or more of the underlying parameters of the model. For example, the number of outsiders and profits will be negatively correlated if the driving exogenous variable is the importance of outsiders' information or potential profits. None of these results, of course,

implies that more or fewer outside directors improves profitability *per se*. Consequently, our approach casts doubt on the interpretations of empirical findings that are prevalent in the empirical literature.

For example, Yermack (1996) and Eisenberg, *et al.* (1998) provide evidence that performance is negatively related to board size. These authors have interpreted this empirical regularity as evidence for the view espoused by Jensen (1993) and Lipton and Lorsch (1992) that when boards become too big, agency problems render them less effective. But, as noted by Hermalin and Weisbach (2003), interpreting the empirical results in this way raises the question why do we observe large boards. In our model, profits, board control, and board size are endogenous.³² In particular, we see that, depending on which exogenous variables are driving the variation in profits, board size, and board control, the correlation could be either positive or negative. Thus our model can explain the negative relationship between performance and board size, without any implication that large boards are somehow less effective.

Many researchers have looked for, but failed to find, a relationship between performance, variously measured, and the fraction of outside board members.³³ This seems puzzling given the predominant view that outsider-control of boards is beneficial. Moreover, Rosenstein and Wyatt (1990) finds that appointments of an outsider to a company's board, even where outside directors already constitute a board majority, result in a positive stock price effect. This finding has been taken as evidence in favor of the view that more is better when speaking of outsiders on boards.³⁴ It is also puzzling, however, given the two facts just discussed. That is, adding another outsider increases both the fraction of outsiders (which should have no effect on performance) and the size of the board (which should have a negative effect on performance). In our model, the relationship between profits and board size, as well as the relationship between profits and the number of outsiders is driven by many factors, some of which cause positive co-variation and others that cause negative co-variation. If more than one exogenous variable is changing in the cross section, the effects could cancel out. This could account for the lack of a relationship between performance and the fraction of outside board members. On the other hand, in our model, a firm will add an outside director only after a shock that increases the optimal number of outsiders. Such a shock, e.g., a decrease in effort costs of outsiders, could also increase firm value. Thus, our model can account for the finding of Rosenstein and Wyatt (1990) if the firms in their sample are affected largely by shocks that increase both the optimal number of outsiders and profitability.

These examples highlight the problem with interpreting empirical correlations as if the number of outside board members were exogenous. In particular, our model can account for the empirical regularities without any implication that more outsiders are better or that larger boards are worse.

Another implication of our approach is that in firms for which insiders' information is very important, boards should have few outsiders and be insider-controlled (Propositions 4, 6, and 8 (ICB)). One might expect startups, especially those in the high-tech industries, to be in that category and thus to have insider-controlled boards with few outsiders.

³² The number of insiders on the board is determined by factors outside of our model, so any change in the number of outsiders is also a change in board size by the same amount.

³³ See MacAvoy, *et al.* (1983), Hermalin and Weisbach (1991), Mehran (1995), Klein (1998), and Bhagat and Black (2000).

³⁴ Rosenstein and Wyatt (1990) states, "These results imply that the expected benefits of outside guidance gained from these appointments outweigh the expected costs of potential managerial entrenchment and inefficient decision making [when insiders choose outside directors]." (p. 176). The authors suggest an alternative interpretation along the lines of the present paper, however, namely that "that the addition of an outside director signals a change in firm strategy rather than the benefits of outside guidance." (p. 176).

One can use Proposition 9 to interpret the results of Fich and Shivdasani (2004). They show that firm performance is lower when a majority of outside directors sit on multiple boards. If one assumes that the greater the number of boards on which a director sits the greater is his or her effort costs for any given board, our model predicts exactly the result of Fich and Shivdasani. Our model also predicts that firms with outside directors who sit on many boards will have fewer such directors than similar firms with outside directors who sit on few boards.

7 Application to Sarbanes-Oxley

Recall that the Sarbanes-Oxley act mandates, among other requirements, that audit committees of boards of corporations listed on major exchanges have a majority of independent directors. Presumably, the thinking behind this requirement is that outside directors will improve the accuracy of audited financial statements.

A slight modification of our model, applied to audit committees instead of boards, can be used to evaluate the usefulness of the Sarbanes-Oxley mandate for outsider-controlled audit committees. Take s to be the earnings report and assume that true earnings is the sum of two, independent components, $a + p$. Assume insiders privately observe true earnings, but outsiders observe only p (for now assume they can observe p at no cost).³⁵ Thus outsiders have no information about earnings that insiders do not also possess. Shareholders and outside directors prefer s to be as accurate as possible ($s = a + p$), but insiders want to inflate earnings to some extent. Insiders suffer, however, if the overstatement is too egregious (fines, loss of reputation and credibility if caught), i.e., insiders prefer $s = a + p + b$, with $b > 0$ but finite.

With these assumptions, the model works as before, except that outsiders, if in control, will always delegate to insiders unless insiders' information is less important than agency costs.³⁶ And insiders will never delegate to outsiders (since outsiders have no information that insiders do not possess). Assuming that insiders' information is more important than agency costs, our result is that control of the audit committee does not affect the accuracy of the earnings report. If outsiders are in control, they will always delegate to insiders. If insiders are in control, they will never delegate to outsiders. Consequently, regardless of who controls the audit committee, insiders will always choose the earnings report (and, of course, learn nothing from outsiders). It is clear that, if outsiders must exert effort to acquire their information, the result is the same. That is, the earnings report will be independent of control of the audit committee. Intuitively, as long as agency costs are not too large, outsiders would rather let insiders choose the earnings report than lose even part of insiders' information.

This result implies that the mandate for outsider-control of audit committees is unlikely to increase the accuracy of earnings reports. Indeed, Romano (2005) reaches the same conclusion based on a number of studies that demonstrate the lack of an association between percent (or majority) independence of the audit committee and accounting misstatements.³⁷ Our theoretical result explains the empirical evidence if one assumes that control of the audit committee is assigned in a manner that is

³⁵ It would not matter if insiders observed true earnings plus noise, provided neither party could observe the noise.

³⁶ It is easy to show that outsiders prefer not to delegate if and only if $L(b, A) < b^2$. By Lemma 1, this is equivalent to $\sigma_a < b$.

³⁷ See Agrawal and Chadha (2003), Anderson, *et al.* (2003), Beasley (1996), Chtourou, *et al.* (2001), Felo, *et al.* (2003), Uzun, *et al.* (2004), and Xie, *et al.* (2003). Klein (2002) finds no association between 100% independence of the audit committee and abnormal accruals but a negative association with majority independent or percent independent.

independent of the model's parameters, a reasonable assumption given our conclusion that shareholders are indifferent to this assignment.³⁸

8 Conclusions

We have presented a model to determine the optimal control of corporate boards of directors, the number of outside directors, and resulting profits as functions of the importance of insiders' and outsiders' information, the extent of agency problems, and some other factors. In particular, we find that, in many cases, shareholders prefer an *insider*-controlled board. This result is contrary to conventional wisdom which has it that outside control of boards, or at least of key committees such as the audit committee, is always preferred. This thinking underlies recent legislation, in particular the Sarbanes-Oxley Act, and changes in New York Stock Exchange and NASDAQ listing rules. Our results show, however, that outside board control may, in fact, be value-reducing. In particular, if insiders have important information relative to that of outsiders, giving control to outsiders may result in a loss of information that is more costly than the agency cost associated with inside control.

Our model also leads to an endogenous relationship between profits and the number of outside directors that furthers our understanding of some documented empirical regularities. In particular, our model can account for the observed absence of a relationship between the fraction of outsiders on the board and both profits and accounting misstatements, as well as the observed negative relationship between board size and profits.³⁹ Since these relationships are driven by changes in exogenous parameters in our model, there is no causal relationship. The model thus demonstrates, for example, the pitfall of inferring from the negative relationship between board size and profits that large boards are less effective and suggests that empirical work on corporate board size and composition take the endogeneity issue seriously.

³⁸ Suppose we measure the *inaccuracy* of the earnings report as the expected squared deviation from true earnings. Then, still assuming that insiders' information is more important than agency costs, so that insiders always choose s , inaccuracy of the audit report will be given by $E[s - (\tilde{a} + \tilde{p})]^2 = E[\tilde{a} + \tilde{p} + b - (\tilde{a} + \tilde{p})]^2 = b^2$. Now suppose that in a sample of such firms, some firms have insider-controlled audit committees and some have outsider-controlled audit committees for reasons outside our model, e.g., the outsider-controlled committees are legacies from a time when insiders' information was less important. If the distribution of agency costs across the two groups of firms is similar, then the average accuracy across the two groups of earnings reports will be similar.

³⁹ Another possible application of the model is to the relationship between control of the compensation committee of the board and CEO compensation.

Appendix

Lemma 1. L is continuous and increasing in x . For all $x \geq 0$, $f(b, x) = 3L(b, x) + b^2 + xb$, f is continuous and increasing in x , $2bx \leq f(b, x) \leq 2bx + b^2$, and $\max\left\{0, \frac{b}{3}(x - b)\right\} \leq L(b, x) \leq bx/3$. Also, for any $x > 0$, $b^2 \geq L(b, x)$ if and only if $b \geq \sigma(x)$. Finally, $L(b, x)$ and $f(b, x)$ are strictly increasing in b , for any x .

Proof. That L is continuous and increasing in x is shown in Lemma 1 of Harris and Raviv (2005). That $f(b, x) = 3L(b, x) + b^2 + bx$ follows easily from (10) and (11).

For $n \geq 1$, define $x_n(b)$ as in Harris and Raviv (2005) to be the point at which $N(b, x)$ jumps from $n - 1$ to n , i.e., $N(b, x_n) = n - 1$ and for $x > x_n(b)$ but sufficiently close, $N(b, x) = n$. As shown in the proof of Lemma 1 in Harris and Raviv (2005),

$$x_n(b) = 2bn(n - 1),$$

$L(b, x)$ is continuous in x ,⁴⁰ and

$$L(b, x_n) = bx_n/3$$

(here and below we drop the argument b from the function x_n , since it remains fixed for this argument). It is easy to check that, for all n and $x \in (x_n, x_{n+1})$, $L(b, x) < bx/3$. This proves that $L(b, x) \leq bx/3$, for all $x \geq 0$.

Since $f(b, x) = 3L(b, x) + b^2 + xb$ and L is continuous and increasing in x , so is f . It is also easy to check that

$$f(b, x_n) = 2bx_n + b^2, \forall n$$

and

$$f(b, x) < 2bx + b^2, \forall x \in (x_n, x_{n+1}).$$

Consider the following problem:

$$\max_{x \in (x_n, x_{n+1})} 2bx + b^2 - f(b, x).$$

The solution of this problem is $x = 2bn^2$, and the maximized value of the objective function is

$$4b^2n^2 + b^2 - (2bn)^2 = b^2,$$

independently of n . Consequently, we have that, for any x ,

$$2bx + b^2 - f(b, x) \leq b^2 \text{ or } f(b, x) \geq 2bx,$$

⁴⁰ Continuity at $x = 0$ follows from the fact that the upper branch on the right hand side of (10) approaches zero as x approaches zero.

as claimed. A similar argument shows that $L(b, x) \geq \frac{b}{3}(x - b)$. Since $L \geq 0$, it follows that

$$L(b, x) \geq \max \left\{ 0, \frac{b}{3}(x - b) \right\}.$$

Now, the above argument implies that $L(b, x_2) = 4b^2/3 > b^2$. Therefore, if x is such that $b^2 \geq L(b, x)$, then $x < x_2$. On the other hand, if $b \geq \sigma(x)$, then $x \leq (2\sqrt{3})b < 4b = x_2$. Thus, in either case, $N(b, x) = 1$, so $L(b, x) = \sigma^2(x)$.

That $L(b, x)$ is strictly increasing in b is obvious if the change in b does not change $N(b, x)$, so consider two values b_0 and $b_k(x)$, for some $k \geq 2$, where $b_k(x) = \frac{x}{2bn(n-1)}$, and $b_0 \in (b_{k+1}(x), b_k(x))$. Then $N(b_0, x) = k$ and $N(b_k(x), x) = k - 1$. Therefore, from (10),

$$\begin{aligned} L(b_k(x), x) - L(b_0, x) &= \frac{\sigma^2(x)}{k^2(k-1)^2}(2k-1) + \frac{1}{3} \left[b_k^2(x)k(k-2) - b_0^2(k^2-1) \right], \\ &> \frac{\sigma^2(x)}{k^2(k-1)^2}(2k-1) + \frac{b_k^2(x)}{3}(1-2k), \\ &= \left[\frac{\sigma^2(x)}{k^2(k-1)^2} - \frac{x^2}{12k^2(k-1)^2} \right] (2k-1), \\ &= 0. \end{aligned}$$

Since L is increasing in b and $f(b, x) = 3L(b, x) + b^2 + xb$, f is also increasing in b .

Q.E.D.

Before proceeding, some additional notation will be useful. Whether shareholders prefer an outsider- or an insider-controlled board depends on the sign of the **profit differential**, $\Delta\pi$, defined as

$$\Delta\pi = \pi^*(OCB) - \pi^*(ICB).$$

If $\Delta\pi > 0$, an outsider-controlled board is optimal, and if $\Delta\pi < 0$, insider control is preferred. Of course, if $\Delta\pi = 0$, shareholders are indifferent. Also, let

$$M = \frac{c'(e^*)(1-e^*)}{\alpha}. \quad (32)$$

From (21) and (22), when effort cost is not prohibitive for an OCB, $M = (1-e^*)^{n_{OCB}} V_{OCB}$. Similarly, from (21) and (30), when effort cost is not prohibitive for an ICB, $M = (1-e^*)^{n_{ICB}} V_{ICB}$. Therefore, if effort cost is not prohibitive for both board types, $(1-e^*)^{n_{OCB}} V_{OCB} = (1-e^*)^{n_{ICB}} V_{ICB}$, i.e., the expected cost of not observing outsiders' information is the same for both board types. In this case, it is clear from (24) and (31) that a comparison of maximum profits across the two board types reduces to a comparison of the information-plus-agency costs given that outsiders become informed, i.e., a comparison of l_i with l'_i . This fact is used extensively in the proofs of the results of this section.

Proposition 3: Optimal Board Control When $f(b, A) > L(b, P)$, $f(b, P) > L(b, A)$, and $b \geq \min\{\sigma_p, \sigma_a\}$.

- Shareholders prefer an outsider-controlled board. This preference is strict unless effort cost is prohibitive if outsiders are in control.⁴¹
- Outsiders delegate to insiders if insiders' information is more important than agency cost and outsiders' are either not informed or their information indicates that the optimal scale is likely to be large ($\tilde{p} \geq p^*$).

Proof. We prove only the first bullet point; the second is shown in section 3.

First, suppose $b \geq \max\{\sigma_p, \sigma_a\}$. From Propositions 1 and 2, we have $p^* = P$, outsiders will not delegate even if uninformed, $a^* = 0$, $N(b, A) = N(b, P) = 1$, $V_{OCB} = \sigma_p^2$, and $V_{ICB} = 0$. It follows from the above, using (23) and (31), that

$$\Delta\pi = b^2 - \sigma_a^2 + \begin{cases} \sigma_p^2 - M, & \text{if } c'(0) < \alpha\sigma_p^2, \\ 0, & \text{otherwise.} \end{cases}$$

But $b^2 - \sigma_a^2 \geq 0$ by assumption, and, as noted above, since $V_{OCB} = \sigma_p^2$, $\sigma_p^2 > M$, if $c'(0) < \alpha\sigma_p^2$. Consequently, in this case, $\Delta\pi \geq 0$, with equality if and only if $b = \sigma_a$ and $c'(0) \geq \alpha\sigma_p^2$.

Next suppose $\sigma_p > b \geq \sigma_a$. From Propositions 1 and 2, $p^* = P$, $V_{OCB} = \sigma_p^2$, $a^* \in (0, A)$, and $N(b, A) = 1$. Since $a^* < A$, $N(b, a^*) = 1$. Consequently,

$$V_{ICB} = \sigma_p^2 + b^2 - \lambda(a^*),$$

where

$$\lambda(a^*) = \frac{a^*}{A} \sigma^2(a^*) + \left(1 - \frac{a^*}{A}\right) \left[\left(\frac{a^*}{2} + b\right)^2 + b^2 \right].$$

It is easy to check, using the fact that $N(b, A) = 1$ implies that $A \leq 4b$, that $\lambda(a^*)$ is concave in a^* .

Since $c'(0) < \alpha b^2 < \alpha\sigma_p^2$, it follows from (23) and (31) that

$$\Delta\pi \geq \lambda(a^*) - \sigma_a^2. \quad (33)$$

The right hand side of (33) is positive at $a^* = 0$ and zero at $a^* = A$. Consequently, if $\Delta\pi$ were non-positive anywhere on $(0, A)$, it would have an interior minimum, contradicting the fact that $\lambda(a^*)$ is concave. Therefore, we have shown that $\Delta\pi > 0$.

Finally, suppose $\sigma_a > b \geq \sigma_p$. From Proposition 2, $a^* = 0$, $N(b, P) = 1$, and $V_{ICB} = 0$. Consequently,

⁴¹ Even if effort cost is prohibitive for an outsider-controlled board, the preference is still strict if $b > \sigma_a$ and $b \geq \sigma_p$.

$$\pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2).$$

If $c'(0) < \alpha V_{OCB}$,

$$\pi^*(OCB) = \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M = \pi^*(ICB) + [1 - (1 - e^*)^{n_{OCB}}] V_{OCB},$$

i.e., $\Delta\pi = \pi^*(OCB) - \pi^*(ICB) = [1 - (1 - e^*)^{n_{OCB}}] V_{OCB} > 0$. If $c'(0) \geq \alpha V_{OCB}$, clearly $\Delta\pi = 0$.

Q.E.D.

Proposition 4: Optimal Board Control When $f(b, P) \leq L(b, A)$.

- Shareholders prefer an insider-controlled board. This preference is strict unless agency cost is more important than outsiders' information, or effort cost is prohibitive for an insider-controlled board.
- Insiders delegate if outsiders' information is more important than agency cost, outsiders become informed, and insiders' information indicates that the optimal scale is likely to be small ($\tilde{a} \leq a^*$).

Proof. We prove only the first bullet point; the second is shown in section 4. First suppose $b \geq \sigma_p$. From Propositions 1 and 2, we have $p^* = 0$, outsiders delegate if uninformed, $a^* = 0$, $N(b, P) = 1$, and $V_{ICB} = 0$. Also, $V_{OCB} = 0$ follows immediately from Proposition 1 and the fact that $N(b, P) = 1$. Finally, $\Delta\pi = \sigma_p^2 + b^2 - l_U = 0$, since $b^2 < \sigma_a^2$ and $\alpha V_{OCB} = \alpha V_{ICB} = 0 < c'(0)$.

Now suppose $b < \sigma_p$. From, Propositions 1 and 2, we have $p^* = 0$, outsiders delegate if uninformed, $a^* \in (0, A)$, and $V_{OCB} = \sigma_p^2 - L(b, P)$. Therefore, from (27),

$$V_{OCB} - V_{ICB} = \frac{a^*}{A} [L(b, a^*) - L(b, P) - b^2].$$

From Lemmas 1 and 2,

$$L(b, P) = f(b, a^*) > L(b, a^*) + b^2.$$

Thus, $V_{OCB} - V_{ICB} < -\frac{a^*}{A} 2b^2 < 0$.

Now, from (23) and (31), if $c'(0) < \alpha V_{ICB}$,

$$\pi^*(ICB) = \pi_0 - \sigma_p^2 - b^2 + V_{ICB} - M$$

and

$$\pi^*(OCB) \leq \pi_0 - \sigma_p^2 - b^2 + V_{OCB} - M.$$

Consequently, $\Delta\pi < 0$.

If $c'(0) \geq \alpha V_{ICB} > \alpha V_{OCB}$, then, it is clear from (23) and (31) that $\Delta\pi = 0$.

Q.E.D.

Proposition 5: Optimal Board Control When $f(b, A) \leq L(b, P)$.

- Shareholders prefer an outsider-controlled board. This preference is strict if agency costs are less important than insiders' information, and effort cost is not prohibitive for an outsider-controlled board.
- Outsiders delegate to insiders if insiders' information is more important than agency cost and outsiders' are either not informed or their information indicates that the optimal scale is likely to be large ($\tilde{p} \geq p^*$).

Proof. We prove only the first bullet point; the second is shown in section 3. First suppose $b \geq \sigma_a$. From Propositions 1 and 2, $p^* = P$, outsiders do not delegate even if uninformed, $a^* = A$, $N(b, A) = 1$, $V_{OCB} = \sigma_p^2$, and $V_{ICB} = \sigma_p^2 + b^2 - \sigma_a^2$. Since $b^2 > \sigma_a^2$ for this type of firm, $V_{ICB} > V_{OCB}$.

Since outsiders' information is critical, $\sigma_p \geq b$, so $c'(0) < \alpha b^2 < \alpha \sigma_p^2$. Therefore,

$$\pi^*(OCB) = \pi_0 - (\sigma_p^2 + \sigma_a^2) + \sigma_p^2 - M = \pi_0 - \sigma_a^2 - M,$$

and

$$\pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2) + \sigma_p^2 + b^2 - \sigma_a^2 - M = \pi_0 - \sigma_a^2 - M.$$

Thus $\Delta\pi = 0$.

Now suppose $b < \sigma_a$. From Propositions 1 and 2, p^* is given by (12) and $a^* = A$. Since $b < \sigma_a$,

$$V_{OCB} - V_{ICB} = \left(1 - \frac{P^*}{P}\right) [L(b, A) - L(b, P - p^*) - b^2] = \left(1 - \frac{P^*}{P}\right) [f(b, P - p^*) - L(b, P - p^*) - b^2].$$

From Lemma 1 and the fact that $p^* \in (0, P)$, $f(b, P - p^*) > L(b, P - p^*) + b^2$. Hence $V_{OCB} > V_{ICB}$. If $c'(0) < \alpha V_{OCB}$, then

$$\pi^*(OCB) = \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M,$$

and

$$\pi^*(ICB) \leq \pi_0 - (\sigma_p^2 + b^2) + V_{ICB} - M.$$

Therefore, $\Delta\pi > 0$. If $c'(0) \geq \alpha V_{OCB}$, then $\pi^*(OCB) = \pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2)$.

Q.E.D.

Proposition 6: Optimal Board Control When $f(b, A) > L(b, P)$, $f(b, P) > L(b, A)$, and $\min\{\sigma_a, \sigma_p\} > b$. If effort cost is not prohibitive for at least one type of board, then shareholders prefer the board type that results in the largest marginal value of outsiders' information. If effort cost is prohibitive for both board types, shareholders are indifferent. Outsiders delegate if they are uninformed or their information indicates that the optimal scale is likely to be large. Insiders delegate if outsiders are informed and insiders' information indicates that the optimal scale is likely to be small.

Proof. We prove only the first part; the rest is proved in sections 3 and 4. Suppose $c'(0) < \alpha \max\{V_{OCB}, V_{ICB}\}$. If $V_{OCB} > V_{ICB}$, then

$$\pi^*(OCB) = \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M,$$

and

$$\pi^*(ICB) \leq \pi_0 - (\sigma_p^2 + b^2) + V_{ICB} - M.$$

Thus $\Delta\pi > 0$. If $V_{OCB} < V_{ICB}$, then

$$\pi^*(OCB) \leq \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M,$$

and

$$\pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2) + V_{ICB} - M,$$

so $\Delta\pi < 0$.

$$\text{If } c'(0) \geq \alpha \max\{V_{OCB}, V_{ICB}\}, \text{ then } \pi^*(OCB) = \pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2).$$

Q.E.D.

Corollary 1. If effort cost is not prohibitive for an outsider-controlled board, and if insiders and outsiders have information of similar importance, i.e., σ_p and σ_a are sufficiently close, then shareholders prefer an outsider-controlled board.

Proof. First suppose $\sigma_a = \sigma_p$. If $b \geq \sigma_a = \sigma_p$, Proposition 3 implies the result. If $b < \sigma_a$, then a^* is determined by (28) and p^* is determined by (12). Substituting $P = A$ into (12) and (28) shows that $a^* = A - p^*$. Using this result in (27) and (16) shows that,

$$V_{OCB} - V_{ICB} = \left(1 - 2\frac{a^*}{A}\right)b^2.$$

We claim that $a^*/A < 1/2$. Since f is increasing in x , however, it suffices to show that $f(b, A/2) > L(b, A)$. By Lemma 1, $f(b, A/2) \geq 2b\left(\frac{A}{2}\right) = Ab$, and $L(b, A) \leq \frac{bA}{3}$, so the claim follows from the fact that $b > 0$. Therefore $V_{OCB} > V_{ICB}$, so the result follows from Proposition 6. For σ_p and σ_a sufficiently close, the result follows from continuity.

Q.E.D.

Corollary 2: Optimal Board Control When Neither Party's Information is Critical and Agency Costs Are Small. For sufficiently small agency costs, if effort costs are not prohibitive for either board type, shareholders prefer that the board be controlled by the party with the more important information.

Proof. Suppose b is sufficiently small that we can neglect any terms that are less than or equal to b^2 and that

$$N(b, x) \cong \sqrt{\frac{x}{2b}}. \tag{34}$$

Substituting for N from (34) into the formulas for L and f results in

$$L(b, x) = bx/3 \text{ and } f(b, x) = 2bx. \tag{35}$$

It follows that, for type-IV firms,

$$p^* = P - A/6, \quad a^* = P/6, \quad (36)$$

and

$$6 > A/P > 1/6. \quad (37)$$

Now, since effort costs are not prohibitive for either type board, $\Delta\pi = l'_I - l_I$, and using (35) and (36) in (16) and (27), we have that

$$\Delta\pi > 0 \text{ if and only if } \left(P^2 - \frac{31}{5}PA + A^2 \right) (P - A) < 0. \quad (38)$$

It is easy to check that $P^2 - \frac{31}{5}PA + A^2 < 0$ for all (A, P) that satisfies (37). Consequently, OCB is preferred whenever $P > A$, ICB is preferred whenever $P < A$, and shareholders are indifferent whenever $P = A$.

Q.E.D.

The following lemma will be used in the proofs of several subsequent propositions.

Lemma 3. For either board type, if effort cost is not prohibitive, then π and l_I or l'_I change in opposite directions as a result of a change in σ_a or σ_p . The share of outsiders, α changes in the opposite direction from π if (3) is required.

Proof. We prove this for the case of an outsider-controlled board; the proof for the other board type is symmetric.

Since effort cost is not prohibitive, $\pi = \pi_0 - l_I - M$. If α is exogenous, i.e., we do not require (3), the result is obvious. If α is endogenous, suppose, for example, l_I falls, and π also falls. Then, to satisfy (3), $\alpha\pi$ cannot change, since e^* does not change. Therefore, α must increase. Also since e^* does not change, M falls. Since both l_I and M decline, π increases, contradicting our supposition that π falls. Thus π must increase when l_I falls (a symmetric argument applies if l_I rises). If (3) is required, then, α must clearly change in the opposite direction from π .

Q.E.D.

Proposition 7 (OCB): The effect of changes in the importance of outsiders' information for outsider-controlled boards. An increase in the importance of outsiders' information, σ_p , results in

- An decrease in the probability that outsiders delegate the decision to insiders;
- An increase in the number of outsiders (if insiders' information is not critical but is more important than agency costs, we assume that $\sigma_p > b\sqrt{12}$);
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. If outsiders are informed, by Lemmas 1 and 2, an increase in σ_p either has no effect on p^* or increases P and p^* by the same amount. In either case, the probability of delegation, $\frac{P - p^*}{P}$,

either falls or remains equal to zero. If outsiders are uninformed, by Lemma 3, the change in σ_p has no effect on the probability of delegation.

First suppose effort cost is not prohibitive, so equilibrium effort is e^* , independent of σ_p .

Now suppose $b \geq \sigma_a$, so that $p^* = P$. In this case, an increase in σ_p has no effect on $l_I = L(b, A)$ but increases $l_U = \sigma_p^2 + \sigma_a^2$ and hence increases V_{OCB} . Since the increase in σ_p has no effect on l_I it does not directly affect profits, so that α is not affected. The increase in V_{OCB} , however, requires an increase in n_{OCB} .

Next suppose insiders' information is critical, so that $p^* = 0$. In this case, an increase in σ_p increases $l_I = L(b, P) + b^2$ and, therefore, decreases profits and increases in α by Lemma 3. Now $V_{OCB} = \sigma_p^2 - L(b, P)$. It is clear that this quantity is (weakly) increasing in σ_p as long as the change in σ_p does not change $N(b, P)$ and strictly increasing if $N(b, P) > 1$. If a small increase in σ_p increases $N(b, P)$, then, from the proof of Lemma 1, $P = 2bk(k-1)$ for some $k > 1$, $N(b, P) = k-1$, and $L(b, P) = \frac{2b^2k(k-1)}{3}$. Consider a small increase in P to P' . Then $N(b, P') = k$, and

$$L(b, P') - L(b, P) = \frac{\sigma^2(P')}{k^2} - \frac{b^2(k-1)^2}{3} = \frac{1}{k^2} [\sigma^2(P') - \sigma^2(P)] < \sigma^2(P') - \sigma^2(P).$$

Thus, in this case V_{OCB} is increasing in σ_p . Since both α and V_{OCB} increase, n_{OCB} also increases.

Finally, suppose $p^* \in (0, P)$. Let $p_0 = P - p^* > 0$. Then, using Lemmas 1 and 2 and (17), we can write

$$l_I = L(b, p_0) \left(3 - 2 \frac{p_0}{P} \right) + p_0 b \left(1 - \frac{p_0}{P} \right) + b^2,$$

and

$$V_{OCB} = \sigma_p^2 - \left[L(b, p_0) \left(3 - 2 \frac{p_0}{P} \right) + p_0 b \left(1 - \frac{p_0}{P} \right) \right].$$

Since p_0 is independent of P , clearly l_I increases with σ_p , so profits decrease and α increases as before. Moreover,

$$\frac{\partial V_{OCB}}{\partial \sigma_p} = 2\sigma_p - \frac{\sigma(p_0)}{\sigma_p^2} [2L(b, p_0) + p_0 b].$$

Thus, to show that V_{OCB} increases with σ_p , it suffices to show that

$$\sigma_p^2 > \frac{\sigma(p_0)}{\sigma_p} \left[L(b, p_0) + \frac{1}{2} p_0 b \right], \quad (39)$$

for $\sigma_p \geq \sigma(p_0)$. Since the left side of (39) is increasing in σ_p , and the right side is decreasing in σ_p , it suffices to show that (39) is satisfied at $\sigma_p = \sigma(p_0)$, i.e., that

$$\sigma_p^2 > L(b, P) + \frac{1}{2} Pb. \quad (40)$$

Suppose $N(b, P) = k$. It is easy to check that (40) is satisfied if and only if

$$\sigma_p^2 > \frac{b^2 k^2}{3} + \frac{bPk^2}{2(k^2 - 1)}.$$

From the proof of Lemma 1, we have $P > 2bk(k-1)$. Consequently,

$$\frac{b^2 k^2}{3} + \frac{bPk^2}{2(k^2 - 1)} < \sigma_p^2 \frac{4k+1}{(k-1)^2(k+1)}.$$

Therefore, it suffices to show that

$$\frac{4k+1}{(k-1)^2(k+1)} < 1,$$

or $k(k-1) > 5$, which is true for all $k \geq 3$. Since we assume that $\sigma_p > b\sqrt{12}$, it follows that $k \geq 3$.

Since both α and V_{OCB} increase, n_{OCB} also increases.

If effort cost is prohibitive, then $\pi = \pi_0 - \sigma_p^2 - \min\{\sigma_a^2, b^2\}$. Therefore, an increase in σ_p reduces profits and, hence, increases α . Since we assume that effort cost remains prohibitive, the increase in σ_p does not affect the number of outsiders.

Q.E.D.

Proposition 7 (ICB): The effect of changes in the importance of outsiders' information for insider-controlled boards. An increase in the importance of outsiders' information, σ_p , increases the probability that insiders delegate the decision to outsiders. If effort cost is prohibitive, an increase in the importance of outsiders' information, σ_p , results in a decrease in profits. Since there are no outsiders in this case, their share is irrelevant, and there is no effect on the number of outsiders. If effort cost is not prohibitive, and outsiders' information is critical, the number of outsiders increases, but profits and the share of outsiders do not change. There is no change in the equilibrium effort of outsiders.

Proof. From Proposition 2, an increase in σ_p either increases a^* or does not affect a^* . In the former case, the probability of delegation, a^*/A , increases, while in the latter it does not change.

If effort cost is prohibitive, outsiders never become informed, and insiders never delegate. Therefore, $n_{ICB} = 0$. The share of outsiders is irrelevant and the result on profits follows from the fact that $\pi = \pi_0 - \sigma_p^2 - b^2$.

If effort cost is not prohibitive, and outsiders' information is critical, $a^* = A$, $\pi = \pi_0 - L(b, A) - M$, and $V_{ICB} = \sigma_p^2 + b^2 - L(b, A)$. The results follow trivially.

Equilibrium effort is e^* (or zero if effort cost is prohibitive), which is not affected by changes in σ_p .

Q.E.D.

Proposition 8 (OCB): The effect of changes in the importance of insiders' information for outsider-controlled boards. An increase in the importance of insiders' information, σ_a , increases the probability that outsiders delegate the decision to insiders. If insiders' information is less important than agency costs, an increase in the importance of insiders' information, σ_a , results in an increase in the number of outsiders, a decrease in profits, and increase in the equity share of outsiders (if effort cost is prohibitive, there is no effect on the number of outsiders). If insiders' information is critical or if it is more important than agency costs and effort cost is prohibitive, an increase in the importance of insiders' information has no effect on the number of insiders, profits, or the equity share of outsiders. If insiders' information is more important than agency costs but not critical (and effort cost is not prohibitive), the number of outsiders and profits move in the same direction as each other, while the equity share of outsiders moves in the opposite direction. In any case, there is no effect on equilibrium effort of outsiders.

Proof. If outsiders are informed, from Proposition 1, an increase in σ_a either increases $P - p^*$ or does not affect $P - p^*$. In the former case, the probability of delegation, $\frac{P - p^*}{P}$, increases, while in the latter it does not change. If outsiders are uninformed, from Lemma 3, an increase in σ_a either has no effect on the delegation decision (if the increase in σ_a does not change the relationship between σ_a and b) or increases the probability of delegation from zero to one (if the larger value of σ_a exceeds b , but the original value does not).

As in the previous result, if effort cost is prohibitive, then $\pi = \pi_0 - \sigma_p^2 - \min\{\sigma_a^2, b^2\}$. If insiders' information is less important than agency costs, $\pi = \pi_0 - \sigma_p^2 - \sigma_a^2$, so an increase in σ_a reduces profits and, hence, increases α . If insiders' information is more important than agency costs, $\pi = \pi_0 - \sigma_p^2 - b^2$, so an increase in σ_a has no effect on profits or α . In either case, effort remains zero.

When effort cost is not prohibitive, equilibrium effort is e^* , independent of σ_a , since the number of outsiders is chosen to make effort equal to e^* . Consequently, any change in σ_a must leave $\alpha\pi$ unchanged in order to satisfy (3). Profits, in this case, can be written as $\pi = \pi_0 - l_I - M$.

If $\sigma_a \leq b$, $p^* = P$, $l_I = \sigma_a^2$, and $V_{OCB} = \sigma_p^2$. Thus an increase in σ_a reduces profits but does not change V_{OCB} . The results for this case follow immediately.

If insiders' information is critical, $p^* = 0$, so the change in σ_a has no effect on l_I , V_{OCB} , π , α , or n_{OCB} .

Finally, suppose $\sigma_a > b$, but insiders' information is not critical. Since $\alpha\pi$ cannot change, the change in αV_{OCB} is given by $\frac{\alpha}{\pi}(\pi_0 - l_U)$ times the change in π . Since we assume $\pi_0 > \sigma_p^2 + b^2 \geq l_U$, αV_{OCB} and π must change in the same direction. Consequently, n_{OCB} and π must change in the same direction.

Q.E.D.

Proposition 8 (ICB): The effect of changes in the importance of insiders' information for insider-controlled boards. An increase in the importance of insiders' information, σ_a , decreases the probability that insiders delegate the decisions to outsiders. If effort cost is prohibitive, changes in the

importance of insiders' information has no effect on profits, the share of outsiders, or the number of outsiders. If effort cost is not prohibitive, an increase in the importance of insiders' information results in

- A decrease in the number of outsiders;
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. From Proposition 2, if $\sigma_p \leq b$, an increase in σ_a has no effect on the probability of delegation (which remains equal to zero). If $\sigma_p > b$, an increase in σ_a either has no effect on a^* or increases a^* by the same amount as A . In the former case, the probability of delegation, a^*/A , decreases, while in the latter case, this probability remains equal to one.

The argument for the case in which effort cost is prohibitive is similar to that in the previous proposition, but uses the fact that, in this case, profits do not depend on σ_a .

Suppose effort cost is not prohibitive. Then, we must have $b < \sigma_p$, since otherwise $V_{ICB} = 0$. If outsiders' information is critical, $a^* = A$, and $l'_1 = L(b, A)$. Therefore, an increase in σ_a increases l'_1 , so, from Lemma 3, reduces π and increases α . It is easy to show, using the fact that $\alpha\pi$ doesn't change, that the change in αV_{ICB} is given by $\sigma_p^2 + b^2 - \pi_0$ times the change in α . Since we assume $\pi_0 > \sigma_p^2 + b^2$, and the change in α is positive, αV_{ICB} decreases. Therefore, n_{ICB} decreases.

If outsiders' information is not critical, but $b < \sigma_p$, then an increase in σ_a increases A , but does not affect a^* (Proposition 2). Since $L(b, P) + b^2 > L(b, a^*)$, the increase in σ_a increases l'_1 [equation (27)], thus reducing π and increasing α (Lemma 3). As in the previous case, αV_{ICB} decreases so, n_{ICB} decreases.

As in the previous proposition, equilibrium effort of outsiders is unaffected.

Q.E.D.

Proposition 9:

- **The effect of a change in effort cost.** If effort cost is prohibitive, changes in effort cost, g , have no effect. Otherwise, an increase in effort cost results in a decrease in the number of outsiders, a decrease in profits, and a less-than-proportionate increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.
- **The effect of a change in potential profits.** An increase in potential profits, π_0 , results in a decrease in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and a decrease in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.
- **The effect of a change in the opportunity cost of outsiders.** An increase in the opportunity cost of outsiders, \bar{U} , results in an increase in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and an increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.

Proof. If effort cost is prohibitive, equilibrium effort is zero, so the change in g has no effect on outsiders' effort cost (remember $c(0) = 0$), hence no effect on profits or outsiders' share.

Now suppose effort cost is not prohibitive. Since g is a scale factor, the minimum point of $c'(e)(1-e)$ is not affected, and, hence, the equilibrium effort of outsiders does not change. From (3), α must increase, but, since $\bar{U} > 0$, α increases less than in proportion to g . It follows that M increases, so π decreases. Since α increases less than in proportion to g , and V_{OCB} and V_{ICB} do not change, n_{OCB} and n_{ICB} decrease.

The second and third bullets follow from (3) and the fact that V_{OCB} and V_{ICB} do not change.

Q.E.D.

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Table 1: Comparative Statics Results

Board Type	Parameter	Effect of Parameter Increase On:			
		Delegation Probability	Number of Outsiders	Profits	Outsiders' Share
O C B	σ_p	-	+	-	+
	σ_a	+	+ ^d 0 ^e Same as Profits ^f	- ^d 0 ^e Same as # Outsiders ^f	+ ^d 0 ^e Opposite ^f
	b^g	- for σ_a below cutoff, then +	depends on σ_a and σ_p	-	+
I C B	σ_p	+	+ ^{a,b} 0 ^c	0 ^{a,b} - ^c	0 ^{a,b} 0 ^c
	σ_a	-	- ^a 0 ^c	- ^a 0 ^c	+ ^a 0 ^c
	b^g	-	-	-	+
Both ^a	g	0	-	-	+
	π_0	0	-	+	-
	\bar{U}	0	+	+	+

The interpretation of the parameters is as follows: σ_a and σ_p are, respectively, the importance of insiders' and outsiders' information; b is the agency cost; g is the scale parameter for outsiders' effort cost function; π_0 is potential profit; \bar{U} is the opportunity cost of outsiders for serving on the board.

None of the parameters affects equilibrium effort unless the change in the parameter causes effort cost to switch from being prohibitive to not being prohibitive, or the reverse. We do not consider such cases in this table.

^a Assumes effort cost is not prohibitive.

^b Assumes outsiders' information is critical.

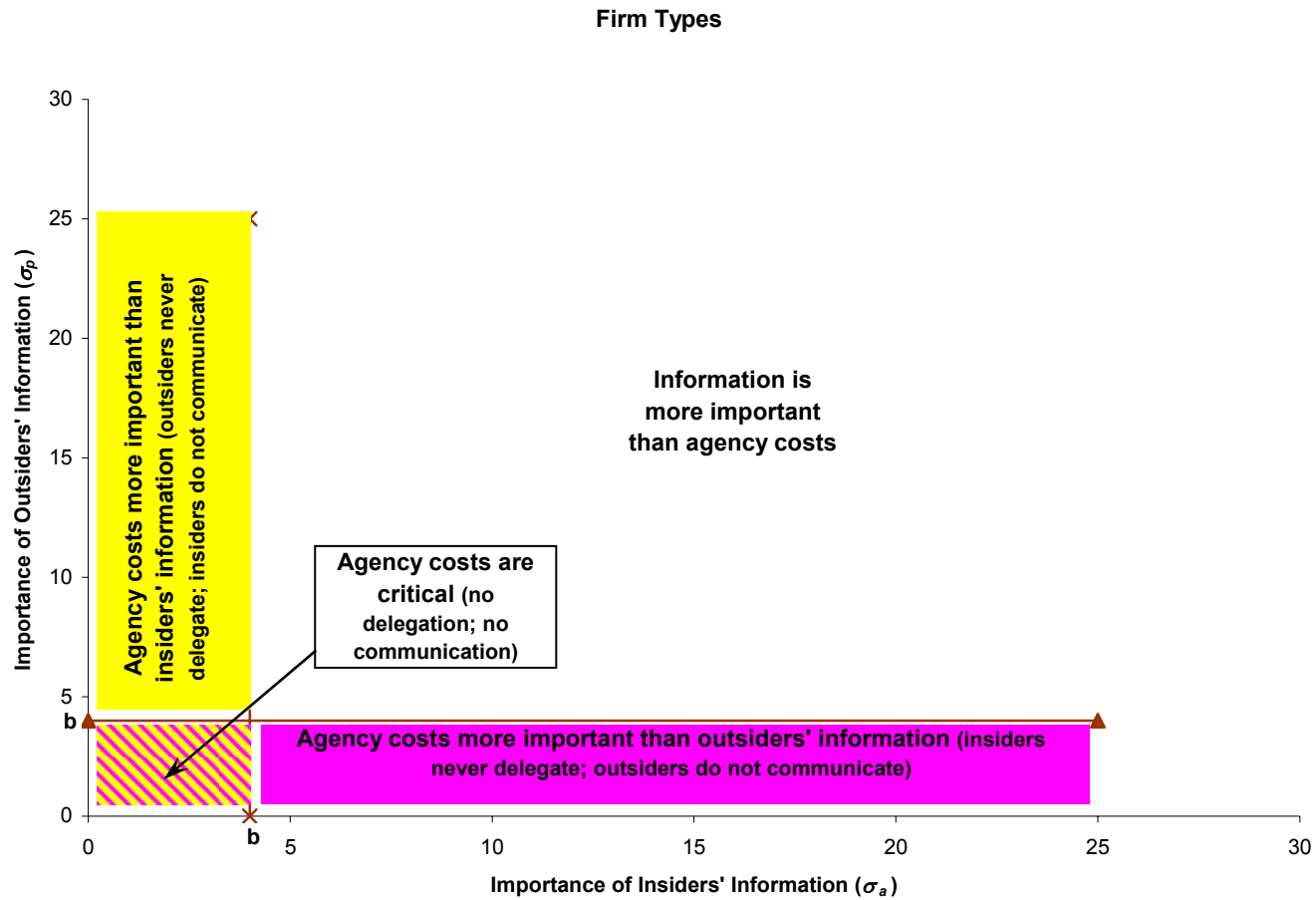
^c Assumes effort cost is prohibitive.

^d Assumes insiders' information is less important than agency costs.

^e Assumes insiders' information is critical or is more important than agency costs, and effort cost is prohibitive.

^f Assumes insiders' information is more important than agency costs but not critical and effort cost is not prohibitive.

^g Results are numerical.



This graph shows the definitions of the terms “Agency costs are more important than insiders’ (outsiders’) information,” $b \geq \sigma_a (\sigma_p)$, “Agency costs are critical,” $b \geq \max \{ \sigma_a, \sigma_p \}$, and “Information is more important than agency costs,” $b < \min \{ \sigma_a, \sigma_p \}$.

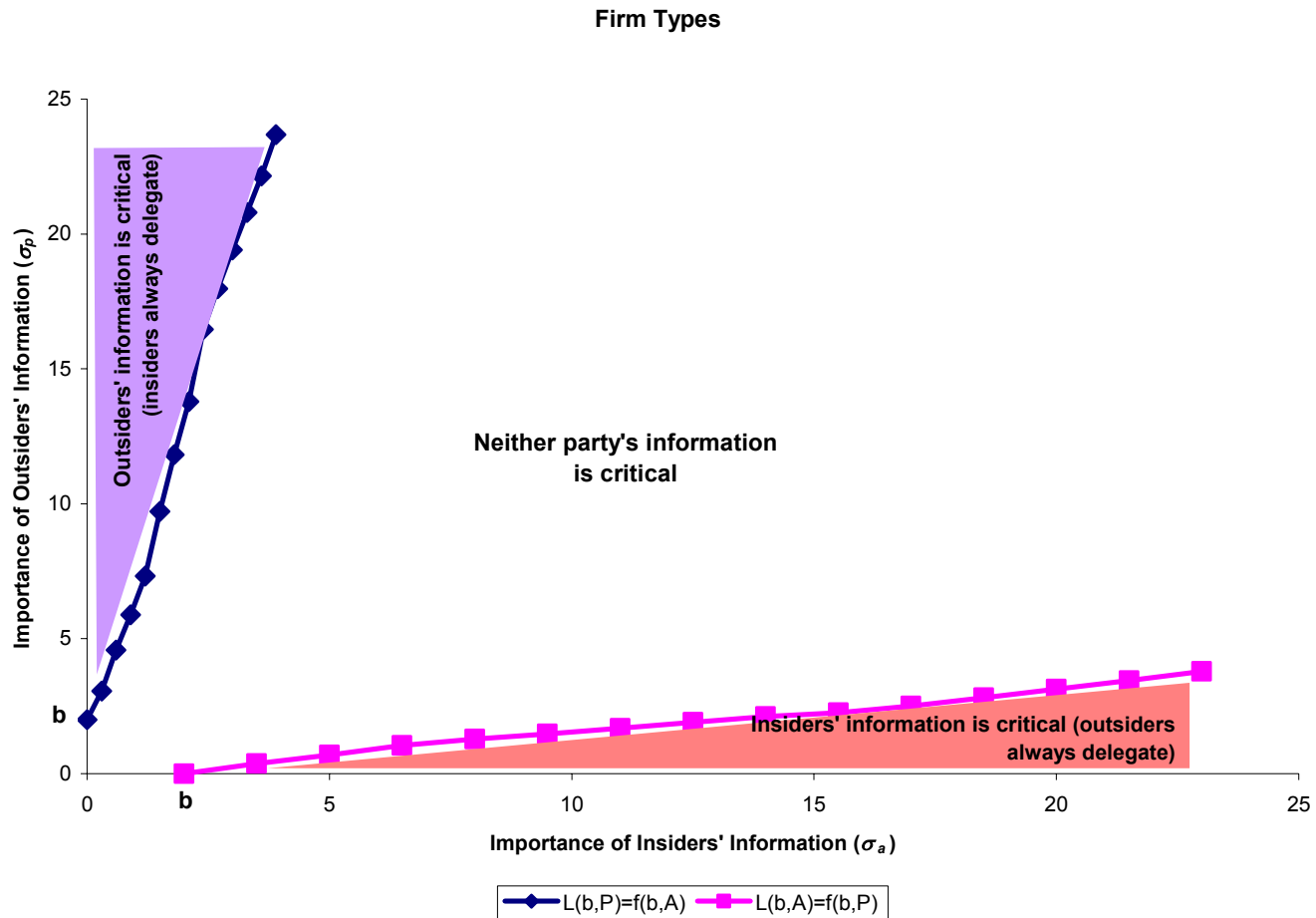


Figure 2

This graph illustrates the definitions of the terms “Insiders’ (Outsiders’) information is critical,” $f(b, P) \leq L(b, A)$ ($f(b, A) \leq L(b, P)$), and “Neither party’s information is critical,” i.e., neither of the previous two inequalities is satisfied.

Equilibrium Effort and the Optimal Number of Outsiders

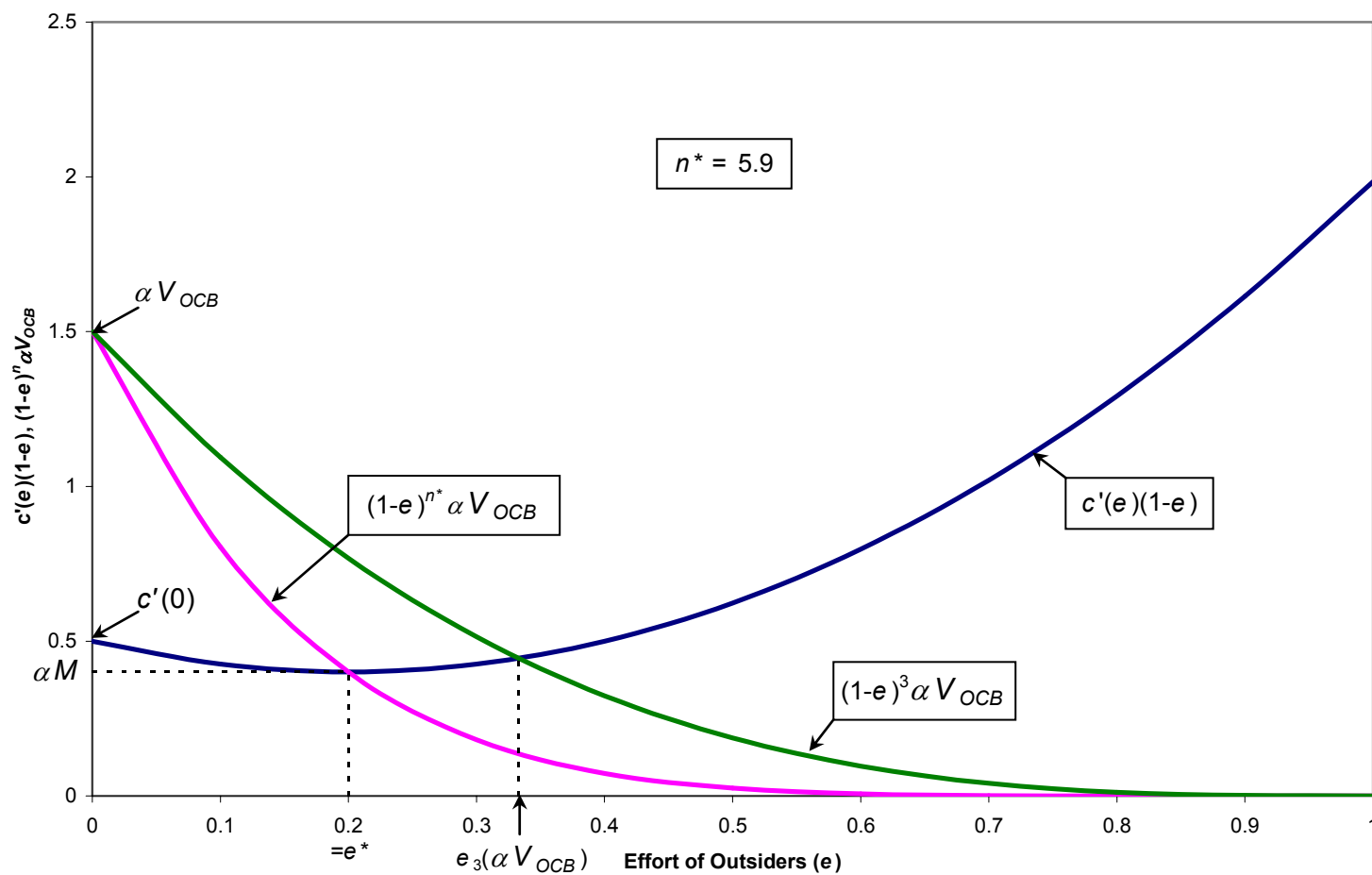


Figure 3

The figure shows the determination of equilibrium effort for an outsider-controlled board with three outsiders, $e_3(\alpha V_{OCB})$. It also shows the determination of the optimal number of outsiders, n^* . For this figure, $c'(e)(1-e) = 0.5 - 2.475e(0.4 - e)$, and $\alpha V_{OCB} = 1.5$.

Optimal Board Control by Firm Informational Characteristics, Numerical Example

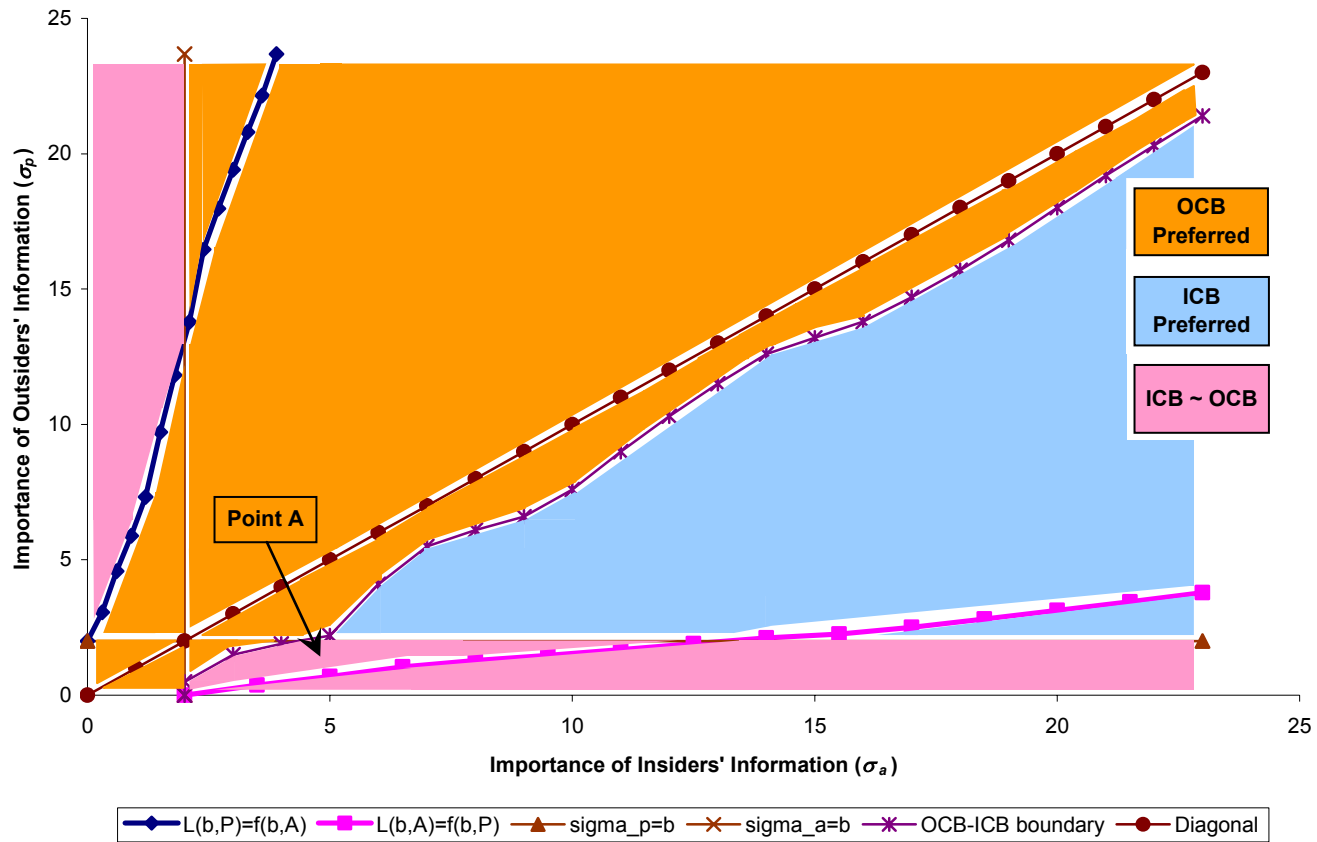


Figure 4

This figure shows, for various combinations of the two parties' information importance, whether shareholders prefer an outsider-controlled board, an insider-controlled board or are indifferent. For this example, $b = 2$, $\bar{U} = 1$, $\pi_0 = 600$, and $c(e) = -\gamma \ln(1-e) - \varepsilon \left[\frac{1}{2} e^2 + (1-2\delta)(e + \ln(1-e)) \right]$, with $\delta = e^* = 0.2$, $\varepsilon = 0.0012375$, and $\gamma = c'(0) = 0.0005$.

Firm Types

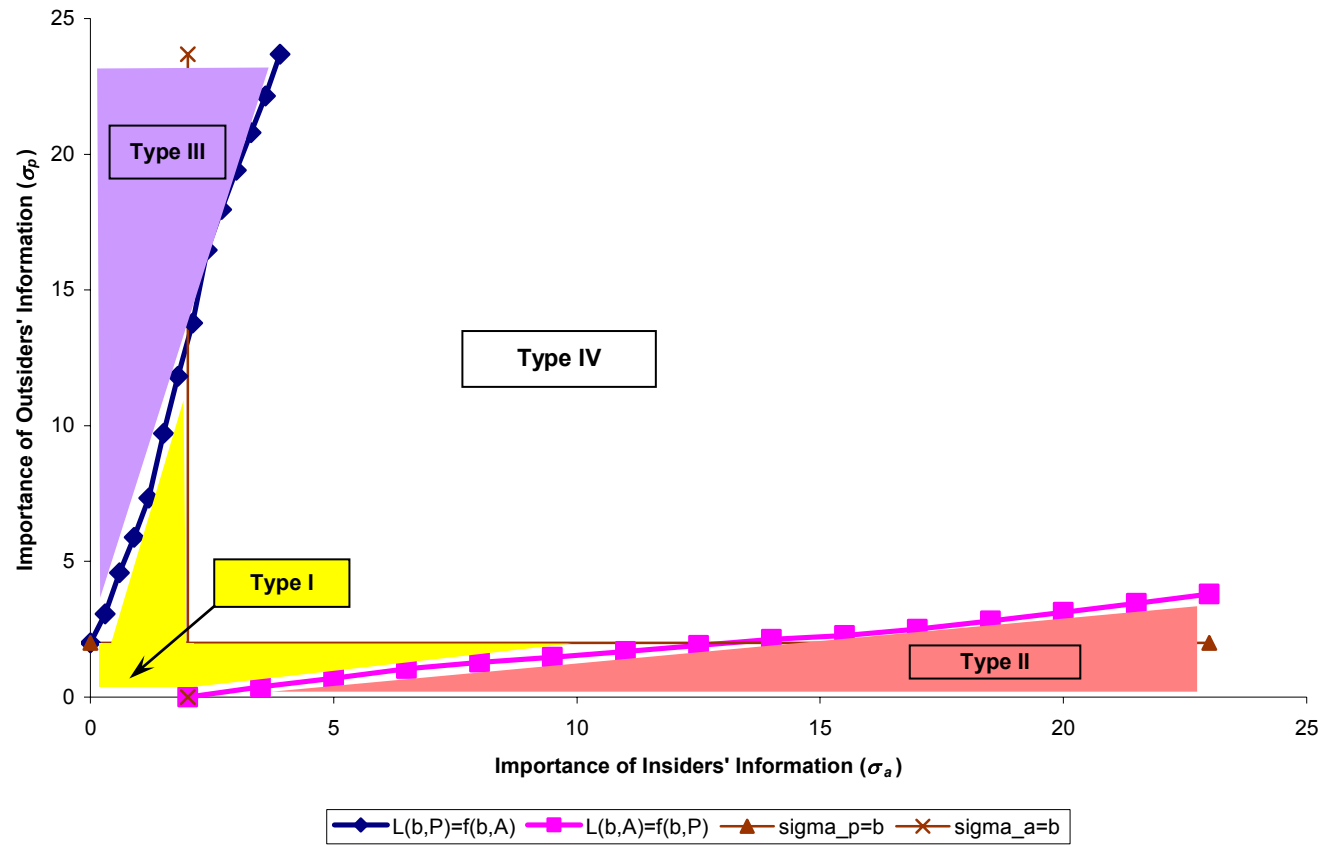


Figure 5

This figure shows the four firm types as defined in section 5. For this diagram, $b = 2$.