Do Business Cycles Really Have Permanent Effects? Using Growth Theory to Distinguish Trends from Cycles†

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Abstract

We draw on growth theory to identify variables to help separate trends from cycles in aggregate data. We treat one stochastic trend in aggregate data as a common factor with two "regimes," high-growth and low-growth. We also allow for a second nonstationarity in per capita work effort. This aspect of the analysis turns out to be important, as we find little evidence of a permanent component to business cycle fluctuations once we control for low-frequency movements in work effort. Thus we find that aggregate data incorporate three distinct and more or less independent components. One is a transitory "business cycle" component. The other two are permanent: a roughly piecewise linear "technology trend" component and a "demographic" component that tracks low frequency movements in per capita work effort.

Keywords: Productivity Growth, Regime-Switching, Neoclassical Growth Model, Factor Model

JEL Codes: O4, O51, C32

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1. Introduction

Does a recession permanently lower expected future output? Over the last two decades a large body of research has examined this question. Initially a number of papers pointed out that GDP and other aggregate variables appear to have unit roots, with the implication that business cycle fluctuations may alter expected future levels of these variables. These findings contrasted with a long tradition of treating economic aggregates as stationary fluctuations around deterministic trends. Then came the counter-revolution, which argued that the apparent unit roots were only the consequence of constraining the time series to be linear processes, and that allowing for regime shifts and other nonlinearities could eliminate at least some of the apparent permanent impact of business cycles.

This question is a particular example of the broader challenge of distinguishing permanent and transitory components in time series data, and of quantifying their relative importance. Even the early literature, which focused on the question of whether particular series had unit roots (e.g. Nelson and Plosser, 1982, Campbell and Mankiw, 1987) recognized that the mere presence of a unit root does not imply any particular long-term impact of a current innovation. Quah (1992) demonstrated a more fundamental impossibility theorem for quantifying the relative importance of permanent and transitory components in a univariate setting, and further argued that in practice such computations depend on arbitrary identifying restrictions.

Another related approach has been to characterize structural disturbances by their respective long-run properties implied by theory, and to use these properties as identifying restrictions.[1] Typically this involves the assumption that "demand" shocks have only a transitory impact on output, whereas "supply" shocks can have a permanent effect. Once so identified, the relative contributions of these disturbances at business cycle frequencies can be computed.

In this paper we use growth theory together with a multivariate approach to identify permanent and transitory components of aggregate output. First, we note that while a number of studies equate the (common) permanent component of per capita output and consumption with that of labor productivity, this can be problematic if per capita work effort exhibits secular changes—which it does. We then derive implications from growth theory for common permanent components that are robust to nonstationarity in hours of work. In our empirical implementation we incorporate three features that go beyond the univariate linear time-series approach that

[1] See, for example, Blanchard and Quah (1989) and Shapiro and Watson (1988).

characterized the original contributions to this literature, the first two of which are motivated by earlier findings in the literature. First, we adopt a multivariate approach, in the spirit of King, Plosser, Stock, and Watson (1991), among others, to gain additional information about components that may be shared by more than one variable. Second, we allow for regime changes, both in levels and growth rates, the omission of which others have argued can lead erroneously to a finding of unit root behavior.[2] Third, we take into account the possibility that aggregate quantities may have two distinct permanent components, one connected to productivity growth, and the other connected to demographics or preferences.

It turns out that all three elements of our approach contribute significantly to our finding that aggregate output is made up of three distinct and more or less independent components: a transitory "business cycle" component, a permanent "technology trend" component and a permanent "demographic" component. The business cycle component is well characterized by a symmetric, stationary, second-order autoregressive process. The asymmetry found by Kim et al. and Beaudry and Koop (1993) and others is statistically significant but does not appear to be economically important, and moreover appears to have diminished over time. The technology component, on the other hand, is dominated by regime changes, and appears close to piecewise linear. We show that failure to take the demographic component into account can confound efforts to isolate the trend in productivity, and consequently mislead researchers into attributing permanence to cyclical fluctuations. Thus Hamilton's (1989) regime-switching model applied to aggregate real output failed to find changes in long-run growth. Instead he found high- and low-growth regimes associated with business cycles, and concluded that business cycles have primarily permanent effects on the level of output. In contrast, our methods find striking evidence of long-term growth regimes, with distinct switches in the early 1970s and late 1990s, while business cycle fluctuations are mostly transitory.

2. Implications of the Neoclassical Growth Model

2.1. Background

Over forty years ago, Nicholas Kaldor (1961) established a set of stylized facts about economic growth that have guided empirical researchers ever since. His facts are: (1) labor and capital's income shares are relatively constant; (2) growth rates and real interest rates are relatively constant; (3) the ratio of capital to labor grows over time, and at roughly the same rate as output per hour, so that the capital-output ratio is roughly constant. To these facts, more recent research has added another: (4) measures of work effort show no clear tendency to grow or shrink over time on a per capita basis. The important implication of this additional fact is that wealth and substitution effects roughly offset each other. This means, for example, that a permanent change in the level of labor productivity has no permanent impact on employment.

Of course, closer inspection suggests that none of the above "stylized facts" is literally true. Indeed the premise of much work on U.S. productivity is that productivity growth was systematically higher from 1948-1973 than it was over the subsequent 20-plus years. We will also see that work effort per capita has been anything but stationary since World War II, and that there have been large shifts in capital-output ratios. But Kaldor's facts still provide a starting point for modeling economic growth, particularly since there may be reasonable explanations for departures from those facts that do not require discarding the framework that they inspired. We begin in this section with a neoclassical growth model consistent with the Kaldor facts, but then relax all but the first. We then examine the implications of the generalized model for empirical efforts to assess growth trends. In Section 3 we propose a statistical model that incorporates these implications, and in Section 4 we present the results. Section 5 concludes.

2.2. A Growth Model with Nonstationary Labor Supply

In our analysis we allow for exogenous changes in preferences between consumption and leisure to account for long-term movements in work effort (as measured by hours) that show up in the data. Specifically, let C denote aggregate consumption, Y aggregate output, N population (measured in person-hours and growing at rate n), K capital, X effective labor per unit of labor input, and L aggregate labor input (in hours). We also assume that there is a production function

$$Y_t = A_t K_{t-1}{}^{\alpha} \left(L_t X_t\right)^{1-\alpha} \tag{1}$$

where A denotes a transitory shock to the production function, while X represents permanent technological progress and has a unit root. Preferences are defined in terms of a present discounted value of single-period utility

$$U(C_t / N_t, \ell_t) = \ln\left(C_t / N_t\right) + \Lambda_t v\left(1-\ell_t\right), \tag{2}$$

where $\ell \equiv L/N$ represents the proportion of available hours devoted to work. The term $\Lambda v(1-\ell)$ represents the utility of leisure, where v is a concave differentiable function, v' is strictly decreasing, and Λ is a taste parameter that

[2] See, for example, Perron (1989), or Kim, Morley, and Piger (2002).

can shift over time. Note that while Λ is modeled as a preference shock, it could reflect taxation or other labor market distortions (see Mulligan, 2002), as well as demographic shifts.

We will also allow Λ to be non-stationary. For the sake of exposition we will specify it as a unit root process, though it could also be a deterministic function of time, or a combination. This is in recognition of the fact that there is significant low-frequency variation of work effort in postwar U.S. data. To illustrate this, Figure 1 displays per capita[3] hours in the non-farm sector since 1947. Apart from the large middle frequency fluctuations associated with business cycles, there are clear secular changes. There was a decline of roughly 15 percent between the end of World War II and the early 1960s, followed by an increase of about 20 percent from the mid-1960s to the present. Previous studies (e.g. Bai, Lumsdaine, and Stock, 1998) have assumed that aggregate per capita output, along with consumption and investment, have the same permanent component as (i.e. are cointegrated with) labor productivity. We argue below that the trend in per capita output is better described as two separate components, one demographic, the other technological, and that, moreover, failure to take account of this distinction leads to the confounding of permanent and transitory disturbances to output and productivity.

We assume that the economy evolves as if a planner solves the following problem:

$$\max E_t\left\{\sum_{t=0}^{\infty}\beta^t U(C_t/N_t, L_t/N_t)\right\}, \tag{3}$$

subject to

$$C_t + \Delta K_t + \delta K_{t-1} \leq A_t F\left(K_{t-1}, L_t X_t\right), \tag{4}$$

where β is a discount factor. Since the aggregate quantities are growing over time, a normalization is required for stationarity. Moreover, since L is not assumed to be stationary, the usual normalization by X or NX will not suffice. Let $c_t \equiv C_t/\left(L_t X_t\right)$, $y_t \equiv Y_t/\left(L_t X_t\right)$ and $k_{t-1} \equiv K_{t-1}/\left(L_{t-1}X_t\right)$. Also, let $d_t \equiv L_t/L_{t-1}$, $G_t \equiv X_t/X_{t-1}$, and $\zeta_t \equiv \Lambda_t/\Lambda_{t-1}$. The first-order conditions for the above maximization problem can be expressed in terms of these transformed variables as follows:

$$A_t k_{t-1}{}^{\alpha} d_t^{-\alpha} - c_t - [G_{t+1}k_t - k_{t-1}d_t^{-1}(1-\delta)] = 0 \tag{5}$$

$$c_t h_t - (1-\alpha) A_t k_{t-1}^{\alpha} d_t^{-\alpha} = 0 \tag{6}$$

$$\beta E_t\left\{\left(\frac{c_t}{c_{t+1}G_{t+1}}\right)\left[\alpha A_{t+1}k_t^{-(1-\alpha)}d_{t+1}^{1-\alpha}) + (1\text{-}\delta)\right]\right\} = 1 \tag{7}$$

$$d_t - \phi\left(h_t/\Lambda_t\right)/\phi\left(h_{t-1}/\Lambda_{t-1}\right) = 0 \tag{8}$$

[3] Per capita variables are obtained by dividing by the total resident population, interpolated from annual to quarterly.

where $h_t \equiv h(\ell_t, \Lambda_t) = \Lambda_t v'(1-\ell_t)\ell_t$, and the function ϕ is defined implicitly from the definition of h.[4] This system has a stationary solution for the transformed endogenous variables (c_t, h_t, d_t) and state variable k_t given k_{t-1} and the processes governing the stochastic variables (A_t, ζ_t, G_t), under the assumption that A_t, ζ_t, and G_t ,are stationary. The idea here is that *h* and *d* (along with *c*, *y*, and *k*) will be stationary even if ℓ and Λ are not (the mapping $h(\ell_t, \Lambda_t)$ is analogous to a cointegrating vector).

Given this solution, we can then "back out" the behavior of variables of direct interest. Moreover, since $K_{t-1}/L_t = k_{t-1}X_t$, $C_t/L_t = c_tX_t$, $Y_t/L_t = A_tk_{t-1}^{\alpha}d_t^{-\alpha}X_t$, and $W_t/L_t = (1-\alpha)A_tk_{t-1}^{\alpha}X_t$ (where *W* refers to labor compensation), these four ratios have a common stochastic trend X_t. Moreover, this trend is unrelated to the stochastic trend in $L_t = \phi(h_t/\Lambda_t)N_t$, which only depends on Λ_t. Output, consumption, and other aggregates, whether in levels or per capita terms, thus are composed of two distinct stochastic trends—one involving technology and the other related to labor supply.

The result that *K*/*L*, *C*/*L*, *Y*/*L*, and *W*/*L* have a common permanent component is robust to other generalizations of the model as well, provided they are consistent with the same balanced growth path, i.e. so long as they result in only transitory deviations from the steady state implied by (5)-(8). For example, the variables may be measured with error, or may have transitory dynamics that reflect imperfect information, adjustment costs, or other rigidities. So long as such deviations (which we will allow for in the estimation) are transitory, the four ratios should be cointegrated.

It turns out that for both theoretical and empirical reasons, the proposition that the capital-labor ratio has the same permanent component *K*/*L* as the other series (*Y*/*L*, *C*/*L*, *W*/*L*) is problematic. First, theory suggests that its cointegration with these other series is less robust to reasonable generalizations of the model, such as allowing for changes in the relative price of capital (as in models with embodied technical progress[5]), or in the openness of international capital markets. Second, as an empirical matter it is clearly not cointegrated with the other three ratios, as will be documented later in the data section of the paper.

One final issue: Can per capita hours (ℓ) really be non-stationary? Certainly a bounded variable cannot follow a simple linear process with a unit root. In our model, however, it is log(Λ), not ℓ, that follows such a

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[4] For example, if v(*x*)= log (*x*), then $h = \Lambda\ell/(1-\ell)$, and ϕ has the form $\phi(x)=x/(1+x)$. Note also that with this specification, ℓ remains in the interval [0,1] even if log(Λ) has a unit root.

[5] With embodied technical progress of the sort considered by Greenwood, Hercowitz, and Krusell (1997), *KP*/*LX* would be stationary, where *P* is the price of capital in terms of consumption goods.

process. Consequently, only a transformation of ℓ does so (for example, if $v(\ell) = \log(\ell)$ then $\log(\ell/(1+\ell))$ is cointegrated with $\log \Lambda$ and ranges over the entire real line). In any case, the long-run implications of non-stationary ℓ are not the issue. Rather, if ℓ exhibits non-stationary behavior in the sample we have, i.e. post-war U.S. data, then treating it as stationary may give misleading results, as we shall see.

2.3. Summary

The upshot of this foray into a neoclassical stochastic growth model is that under plausible assumptions, labor compensation per hour, consumption divided by hours, and output per hour should have a common permanent component. It is natural to think of this component as "technology," i.e. that its driving force is technological progress. Hours of work itself may have its own permanent component that is unrelated to technology, but instead is driven by policy and demographics. To the extent that either the technology or demographic component has regime shifts, lumping the two components together could blur the shifts and make them difficult to identify.

Since the three "per hour" variables may have some independent sources of measurement error, there may be some gain to using all three to extract the best estimate of the common permanent component. This is not to say that the information is completely independent, and indeed the source of errors in one series may be present in the other series as well. For example, an inaccurate price deflator could result in common mismeasurement across multiple series. Nonetheless, the theory suggests that considering these series together may provide better information about underlying trends than consideration of any of them in isolation.

3. Econometric Specification

3.1. The Regime-Switching Dynamic Factor Model

Our estimation strategy draws upon the regime-switching dynamic factor model recently proposed by Kim and Murray (2002) and Kim and Piger (2002), among others. The essence of this approach is to examine a number of related economic time series and to use their comovements to identify two shared factors: a common permanent component and a common transitory component. In addition, we follow these authors in allowing for regime changes in both components. The regime-switching aspect of the model has several attractive features. First, in the permanent component it allows us to account for sustained changes in trend growth without making the growth process itself nonstationary. Second, in the transitory component it allows for asymmetries in business cycles that others using this methodology have found significant.[6] These regime changes in the transitory component capture the idea proposed in Friedman's (1964, 1993) "plucking model" model that economic fluctuations are largely permanent during expansions and transitory during recessions. Third, Perron (1989) has argued that regime-changes can cause testing procedures to indicate non-stationarity, i.e. failure to allow for them could lead one erroneously to infer the presence of a unit root. Finally, the regime-switching specification is a straightforward way of estimating both the timing and expected duration of periodic changes in the processes generating the two components.

Following Kim and Murray (2002), we can describe the regime-switching dynamic factor model as follows.[7] Suppose we consider a number of time series indexed by *i.* Let Q_{it} denote logarithm of the *i*th individual time series. It is assumed that the movements in each series are governed by the following process:

$$Q_{it} = \gamma_i X_t + \lambda_i x_t + z_{it} , \qquad (9)$$

where X_t denotes a permanent component that is common to all series, x_t denotes a common transitory component, and z_{it} is an idiosyncratic error term. The parameter γ_i (the permanent "factor loading") indicates the extent to which the series moves with the common permanent component. Similarly, the parameter λ_i indicates the extent to which the series is affected by the transitory component.

The common permanent component is assumed to be difference stationary, but subject to the type of regime-switching proposed by Hamilton (1989) in which there are periodic shifts in its growth rate:

$$\Delta X_t = \mu(S_{1t}) + \phi_1 \Delta X_{t-1} + ... + \phi_p \Delta X_{t-p} + \upsilon_t, \upsilon_t \sim iidN(0,1) \qquad (10)$$

$$\mu(S_{1t}) = \begin{cases} \mu_0 & if \quad S_{1t} = 0 \\ \mu_1 & if \quad S_{1t} = 1 \end{cases} , \qquad (11)$$

$$\Pr[S_{1t} = 0 \mid S_{1,t-1} = 0] = q_{1,} \qquad \Pr[S_{1t} = 1 \mid S_{1,t-1} = 1] = p_1 \qquad (12)$$

where S_{1t} is an index of the regime for the common permanent component. The transition probabilities p_1 and q_1 indicate the likelihood of remaining in the same regime. Under these assumptions, the common permanent component X_t grows at the rate $\mu_0 /(1-\phi_1 - ... - \phi_p)$ when S_{1t} = 0, and at the rate $\mu_1 /(1-\phi_1 - ... - \phi_p)$ when S_{1t} =1.

The common transitory component x_t is stationary in levels, but also subject to regime-switching:

$$x_t = \tau(S_{2t}) + \phi_1^* x_{t-1} + \phi_2^* x_{t-2} + ... + \phi_p^* x_{t-p} + \varepsilon_t, \varepsilon_t \sim iidN(0,1) \qquad (13)$$

[6] See, for example, Kim and Piger (2002), Kim, Piger, and Startz (2002), and Beaudry and Koop (1993).
[7] Additional details are provided in the Appendix.

$$\tau(S_{2t}) = \begin{cases} 0 & if \quad S_{2t} = 0 \\ \tau & if \quad S_{2t} = 1 \end{cases}, \tag{14}$$

$$\Pr[S_{2t} = 0 \mid S_{2,t-1} = 0] = q_2 \qquad \Pr[S_{2t} = 1 \mid S_{2,t-1} = 1] = p_2 \tag{15}$$

where S_{2t} is an index of the regime for the common transitory component, with transition probabilities p_2 and q_2. The permanent and transitory regimes are assumed to be independent of each other. While the two regimes are not directly observable, it is nevertheless possible to estimate the parameters of the model and to extract estimates of the common components.[8] An important byproduct from the estimation procedure is that we can draw inferences about the likelihood that each common component is in a specific regime at a particular date. The restriction of unit variance for the error terms of the two processes is an identifying restriction, since *X* and *x* are of indeterminate scale. The parameter τ represents the size of the "pluck," with $\tau < 0$ implying that the common transitory component is plucked down during a recession.

Finally, the idiosyncratic components are assumed to have the following structure:

$$z_{it} = \psi_{i1} z_{i,t-1} + \psi_{i2} z_{i,t-2} + \ldots + \psi_{ip} z_{i,t-p} + \eta_{it}, \qquad \eta_{it} \sim iidN(0, \sigma_i^2), \qquad i = 1, \ldots, 4 \tag{16}$$

where all innovations in the model are assumed to be mutually and serially uncorrelated at all leads and lags. We assume that z_{it} is stationary, i.e. the roots of $1 - \Psi_i(L)$ all lie inside the unit circle, so these are transitory shocks to the levels of the variables.

To relate all of this back to the growth model from Section 2 of the paper, the permanent component *X* corresponds to the stochastic trend term from the growth model, which we saw is common to *Y*/*L*, *C*/*L*, and *W*/*L*. The theory implies, therefore, that the factor loadings on the permanent component should satisfy $\gamma_1 = \gamma_2 = \gamma_3$. The transitory component *x* reflects the direct impact of transitory disturbances as well as transition dynamics from all shocks, but only to the extent they are linearly related across all four series. The idiosyncratic component includes what is left of the transitory movements after the common component is subtracted. It will include measurement error and model noise. For example, a literal reading of the model is that $W/L = (1-\alpha)Y/L$, so the two series should be perfectly correlated. But *W* and *Y* are measured (to some extent) independently, and with error, and moreover the assumption of Cobb-Douglas production with constant parameters is undoubtedly not literally accurate. These factors will also contribute to idiosyncratic variation.

3.2. Data

Our data consist of quarterly observations of non-farm sector output, labor productivity, real compensation per hour (nominal compensation relative to the nonfarm output deflator), and hours of work. The non-farm sector was chosen because of the availability of consistent data for all of these series. We also use aggregate data on real consumption expenditures. While a series that converted expenditures on durables to service flows would be preferable, at the time of these computations such a series was not available for the whole sample period, and where it was it exhibited very similar behavior to total consumer expenditures. Another issue with using this consumption series is that it is for the entire U.S. economy, whereas the other series represent the nonfarm sector only. This will only create a problem if there are significantly different trends, but the results did not appear to be sensitive to these choices. Unless stated otherwise, all variables are in logarithms, multiplied by 100, with first differences interpreted as quarterly growth rates in percent. For calculating per capita figures we used the resident population (interpolated from annual to quarterly).

As we have seen, the growth model from Section 2 implies that C/L, Y/L, K/L,and W/L have a common permanent component that is unrelated to the permanent component of L (or L/N). Rather than try to estimate both permanent components, since the other variables do not provide information about the demographic component, we simply detrend the hours series using the Hodrick-Prescott (1980) filter. Using the first difference of hours (in logs) rather than the H-P filtered series yielded very similar results. Thus our benchmark specification has $Q_1 = Y / L$, $Q_2 = W / L$, $Q_3 = C / L$, and $Q_4 = \hat{L}$, where the "^" indicates the use of Hodrick-Prescott (1980, hereafter "HP") filtering to take out the effects of low frequency movements in N and Λ.

That Y/L, W/L,and C/L are cointegrated is illustrated by Figure 2, which plots Y/C, Y/W and Y/K. Y/C and Y/W appear stationary, which supports the notion that Y, C, and W have a single common permanent component, from which it would follow that Y/L, C/L, and W/L do as well (to the extent they have any permanent component at all). As mentioned earlier, it is equally clear that Y/K is not stationary, and therefore K/L does not have a permanent component in common with Y/L. Hence in what follows we drop it from our system.

[8]Kim and Murray (2002) discuss how the regime-switching dynamic factor model can be cast in a state-space representation and estimated using the Kalman filter. More details of how we applied their methods to the present study are in the Appendix.

To examine the cointegration properties of *Y*/*L*, *W*/*L*, and *C*/*L* more rigorously we conducted multivariate unit root tests based on the procedure developed by Johansen (1991, 1995). Table 1 describes the results, based on quarterly data over the sample period 1947:Q2 to 2002:Q2, including a constant in the cointegrating relationship. The combination of trace and maximal eigenvalue tests suggests that there are two cointegrating equations, meaning a single common trend, as the theory suggests. Letting $Q = (Q_1\, Q_2\, Q_3)$, and letting $\beta^i = (\beta_1^i\, \beta_2^i\, \beta_3^i)$, i = 1,2 denote the cointegrating vectors, we estimated the two cointegrating relationships $Q'\beta^1$ and $Q'\beta^2$ and found:

$$\beta^1 = (1.0 \quad 0.0 \quad -1.027)'$$
$$\beta^2 = (0.0 \quad 1.0 \quad -0.986)',$$

where we normalized the equations with respect to β_1^1 and β_2^2. Since the estimates of β_3 differ from -1, we also conducted tests of the null hypothesis that the two cointegrating equations are $(1.0 \quad 0.0 \quad -1.0)$ and $(0.0 \quad 1.0 \quad -1.0)$. The calculated value of the χ^2 statistic with two degrees of freedom is 8.92, with an associated *p*-value of 0.0115. Thus although the results confirm the theory qualitatively, the quantitative implications that $\beta_3 = -1$ in both vectors is rejected. This does not necessarily mean, however, that we will reject $\gamma_1 = \gamma_2 = \gamma_3$ in (9), which is the real implication of the theory, and one that we will test when we estimate the model.

Initial estimates of more general specifications suggested that the common permanent component should include one lagged value of ΔX_t, the common transitory component should include two lagged values of x_t, and that the idiosyncratic component should include one lagged values of z_{it} for each series. We restricted the estimated factor loadings on the permanent component for productivity, real compensation per hour, and consumption per hour to be equal (i.e. we set $\gamma_1 = \gamma_2 = \gamma_3$), and set the value of the permanent factor loading for detrended hours, γ_4, equal to zero.

We also considered two alternative specifications. In one, we eliminated the regime shift in the transitory component, thereby imposing symmetry on business cycles. We will refer to this as the “no pluck” specification. In the other, we used per capita rather than per hour variables, i.e. we set $Q_1 = Y/N$, $Q_2 = W/N$, $Q_3 = C/N$. We will refer to this as the “per capita” model, as opposed to the benchmark or “per hour” specification. If L/N were stationary, the per capita and per hour specifications would yield similar estimates of the permanent component.

One final modeling issue relates to the synchronization of the data. The three trending variables were selected primarily on the basis of their having a common permanent component. They are not necessarily "coincident indicators" with respect to the transitory component. In theory it would be possible to allow for a more general lead/lag structure in our system, but this would greatly increase the number of parameters to estimate. As an alternative, we first examined the cross-correlations of the four series. We found that the first two variables (productivity and labor compensation per hour) both tended to lead the other two series by about three quarters. To capture this asynchronization in the estimation we lagged variables 1 and 2 by three quarters in the system described above.[9]

4. Results

4.1. Parameter Estimates

The first column of Table 2 provides the parameter estimates for our benchmark model with the four variables as described above.[10] The data cover 1947:Q1-2002:Q2, though because the first two variables are lagged three quarters, their growth rates cover the period 1947:Q2-2001:Q3, while the growth rates of consumption and detrended hours variables run from 1948:Q1-2002:Q2 . The results indicate that the model yields precise estimates of most of the parameters of interest: The factor loadings on both the permanent and transitory components, the transition probabilities, and the shift parameters associated with the regimes (μ_0, μ_1, τ) all enter significantly. The difference between the low and high permanent regimes works out to be $\gamma(\mu_0 - \mu_1)/(1-\phi) = 0.367$. This corresponds to approximately 1.46 percent on an annualized basis, very close to the difference between the 1948-73 and 1973-96 growth rates of productivity.

The transition probabilities for the permanent regimes imply an expected duration of $1/(1-p_1) = 25$ years for the high-growth regime, and 17.9 years for the low-growth. They also imply that the unconditional probability of being in the high growth regime is $(1-q_1)/(2-p_1-q_1) = 0.593$, suggesting that the economy was in the high growth regime on the order of 60 percent of the time between 1947 and the present. The AR coefficient on the permanent component is estimated to be –0.381, suggesting that growth innovations in one quarter tend to get partially offset in the following one.

The transitory process is estimated to be a "hump-shaped" autoregressive process typical of the business cycle (see, e.g., Blanchard, 1981), but with a statistically significant negative pluck, i.e. a relatively short-lived reduction in the level of the transitory component presumably associated with recessions. The magnitude of this

[9] A minor drawback to this procedure is that the last three observations of variables 1 and 2 are not used in estimating the parameters of the model. We do incorporate them in the period-by-period assessments of the state variables, as described below.

downward shift is related to $\lambda_i \tau$. Productivity, for example, would decline by $\lambda_1 \tau = 0.620$, which corresponds to about 2.5 percent annualized. The expected duration of the pluck regime is given by $1/(1-q_2)$, or 2.34 quarters. The transition probabilities imply that the pluck regime occurs less than four percent of the time. Since about one-sixth of the quarters since 1947 have been in recessions, and the average recession has lasted roughly 3.5 quarters, these results suggest that not all recessions are associated with transitory regime shifts.

The four λ_i coefficients, which represent the factor loadings on the transitory process, are all estimated very precisely, and have the expected signs. Note that λ_3, the loading factor for consumption/hours, is negative, reflecting the fact that hours are more cyclical than consumption. The positive estimates for the factor loadings on real hourly compensation and productivity indicate that those two variables are positively related to the transitory component, albeit leading by three quarters relative to the other two variables (since they enter the system lagged by three quarters).

Finally, we test the restriction $\gamma_1 = \gamma_2 = \gamma_3$ by estimating the unrestricted model and conducting a likelihood ratio test. The test statistic has a value of 0.525, and is asymptotically distributed as a χ^2 random variable with 2 degrees of freedom. The critical value for rejecting the hypothesis at a 10 percent significance level is 4.61. This suggests that the basic theoretical implications with respect to the common permanent component are confirmed by the data, notwithstanding the estimates of the cointegrating vectors described in Section 3.2.

4.2. Growth Regime Assessments

Before further describing the permanent and transitory components of productivity growth, we will examine the inferred probability of being in the high- or low-growth state over time. There is a technical issue related to the timing of the data series. Since ΔQ_1 and ΔQ_2 are lagged by three periods, if we were simply to assess the state variable given data through a given observation number, we would be ignoring the most recent three data points for those two series. For example, a standard application of the Kalman filter at date *t* would yield an assessment of the state given data through period *t*,

$$\hat{\xi}_{t|t} = E(\xi_t \mid Q_{1t}, Q_{2t}, Q_{3t}, Q_{4t}; Q_{1t-1}, Q_{2t-1}, Q_{3t-1}, Q_{4t-1}; \ldots) . \qquad (17)$$

[10] Estimates of the idionsyncratic variances are omitted from Table 2 for the sake of brevity.

In terms of the actual underlying data, however, it would really be providing

$$E(\xi_t \mid \tilde{Q}_{1t-3}, \tilde{Q}_{2t-3}, \tilde{Q}_{3t}, \tilde{Q}_{4t}; \tilde{Q}_{1t-4}, \tilde{Q}_{2t-4}, \tilde{Q}_{3t-1}, \tilde{Q}_{4t-1}; ...), \tag{18}$$

where the "~" denotes the variable indexed by its true time period. Fortunately it is relatively straightforward to "partially update" the state and regime assessments. This involves conditioning on three subsequent observations of Q_1 and Q_2 at each point in time to get

$$\begin{aligned} \hat{\xi}_{t|t+3'} &= E(\xi_t \mid Q_{1t+3}, Q_{2t+3}, Q_{3t}, Q_{4t}; Q_{1t+2}, Q_{2t+2}, Q_{3t-1}, Q_{4t-1}; ...) \\ &= E(\xi_t \mid \tilde{Q}_{1t}, \tilde{Q}_{2t}, \tilde{Q}_{3t}, \tilde{Q}_{4t}; \tilde{Q}_{1t-1}, \tilde{Q}_{2t-1}, \tilde{Q}_{3t-1}, \tilde{Q}_{4t-1}; ...) \end{aligned} \tag{19}$$

Here we use t+3′ to denote Q_1 and Q_2 observed through t+3, and Q_3 and Q_4 observed through time t. This simply undoes the staggering so that the information set is appropriately aligned. (Additional details on the partial updating procedure are provided in the Appendix.)

The regime assessments are plotted in Figure 3. The vertical axis is the probability of being in the high-growth regime. The figure presents a striking picture: The economy was in a high-growth state until the early 1970's, followed by a roughly 20-year low-growth regime, followed by a clear switch back to high-growth in the second half of the 1990's. Perhaps the only surprise here is how unambiguous the current assessment is. The probability that the economy was in the high-growth regime surpassed 0.95 in 1998:Q4, and has not subsequently fallen below 0.97. Given the 3-quarter lead for productivity, this dates the acceleration of productivity growth at 1998:Q1. If we use a more lenient 0.5 threshold, we would date the acceleration at 1997:Q3.

It is worth noting that the partial updating methodology described earlier can make a substantial difference in assessing current conditions. For example, without it (that is, ignoring the last three observations of productivity and real labor compensation per hour), the estimated probability that the economy is in the high-growth regime declines from 0.9997 in 2000:Q4 to 0.9057 in 2002:Q2. Adding back the information contained in the most recent three observations results in a 2002:Q2 estimate of 0.9740.

We can also examine the assessment of the transitory regimes. The probability of being in the "plucked down" state is plotted in Figure 4. Here the probability assessments are a little more ambiguous. While the more prominent spikes all coincide with NBER-defined recessions, in only two cases does the probability of a negative pluck exceed 0.5. Moreover, several recessions (most notably the 1990-91 recession) are missed entirely. It is perhaps instructive that the 1990-91 recession does not register in this picture. The idea of a pluck is a sharp downturn followed by an equally sharp recovery sufficient to get the economy back to trend. The 1990-91 recession was characterized by a relatively mild downturn followed by an unusually slow and gradual recovery.

4.3. The Permanent Component

If we examine the permanent component, in comparison to labor productivity, for example (Figure 5), we see it clearly indicates changes in trend in the early 1970's and mid-1990's, with little apparent cyclical residue. This contrasts with other recent estimates of the permanent component of output. For example, Kim and Murray's (2002) estimated permanent aggregate component shows substantial downward movement during recessions. Kim and Piger (2002) equate the permanent component of output to consumption of nondurables and services, which is much more cyclical and volatile (the standard deviation of its growth rate is 2.8 times larger) than our permanent component. Kim, Piger, and Startz (2002) estimate a common permanent component for output and consumption, with regime-switching, and find that the permanent component contributes meaningfully to business cycle fluctuations.

Other work, notably Beaudry and Koop (1993) and Kim, Morley, and Startz (2002), has argued that if asymmetries in business cycles are properly taken into account, the permanent impact of business cycles (and recessions in particular) is greatly diminished. Our Figure 4, however, suggests that asymmetries may not be very important once we account for low frequency movements in hours of work. To examine this further, we estimated our model without the transitory regime-switching, and obtained results that are virtually indistinguishable from our first set of estimates, particularly with regard to the estimated permanent component. The parameter estimates from this specification are given in second column of Table 2.

If, on the other hand, we ignore the low-frequency changes in hours of work per capita, and estimate the model with output, consumption, and compensation on a per capita (instead of per hour) basis, we get very different results, as indicated in the third set of results in Table 2. The growth regime shift disappears (note that the estimated values of μ_0 and μ_1 are identical, so the transition probabilities are indeterminate). To understand where these results are coming from, note that the dependent variables in the per capita specification differ from those of the per hour and no pluck models. We can see how the models are related if we begin with the equation for output per hour from the per hour specification, and then add the equation for detrended hours:

$$
\begin{array}{rl}
Y_t - L_t = \gamma X_t + \lambda_1 x_t + z_{1t} & (20) \\
\hat{L}_t = \lambda_4 x_t + z_{4t} & (21) \\
\hline
Y_t - \left(L_t - \hat{L}_t\right) = \gamma X_t + \left(\lambda_1 + \lambda_4\right) x_t + z_{1t} + z_{4t} & (22)
\end{array}
$$

The resulting dependent variable is output relative to the trend in hours. That trend in turn can be divided into population growth and the low-frequency movement in Λ (i.e. the trend in *L*/*N*) depicted earlier in Figure 1, which we will denote by $\overline{\Lambda}_t$. Consequently output per capita should be

$$Y_t - N_t = \overline{\Lambda}_t + \gamma X_t + \left(\lambda_1 + \lambda_4\right) x_t + z_{1t} + z_{4t} \tag{23}$$

where $\overline{\Lambda}_t$ is just $L_t - \hat{L}_t + N_t$. And just as $W_t - L_t$ and $C_t - L_t$ have the same form as (20), $W_t - N_t$ and $C_t - N_t$ have the same form as (23).

Thus the permanent component in the per capita specification is $\overline{\Lambda}_t + \gamma X_t$ rather than just γX_t. In other words, the permanent component in the per capita model is the sum of the demographic and technology trends. The inability to detect the regime switch in the permanent component seems to be the result of combining the two components. The secular movements in $\overline{\Lambda}_t$ apparently mask the regime switches in *X*. Also note that the transitory factor loadings in the per capita specification include both the response of output per hour and (detrended) hours to x_t. Thus the estimate of λ_1 in the per capita specification is very close to the sum of the estimates of λ_1 and λ_4 from the per hour specification. This suggests that the per capita model gets qualitatively similar results with respect to the transitory component even though it finds a very different permanent component.

Further indication of the effects of estimating (23) instead of (20) is displayed in Figures 6 and 7, which compare filtered permanent and transitory components from the per capita and per hour specifications. Clearly the per capita specification implies a much more substantial contribution of the permanent component to the business cycle than does our benchmark model.

We next examine the relative volatilities of the estimated permanent and transitory components of the three specifications. We filtered the components in two ways in order to emphasize cyclical volatility. We looked at first differences (i.e. growth rates); and we used the band-pass filtering methodology described by Baxter and King

(1999). In both cases we also first removed the permanent regime subcomponent $\mu(S_{1t})$, since it represents low frequency behavior, though the results were not sensitive to this.

The results are given in Table 3. For the growth rate volatilities, the permanent component contributes less than five percent of the total contribution of both components to the variance of output per capita under the two per hour specifications, whereas it contributes more than 10 percent in the per capita model.[11] The results are more dramatic with the band-pass filtered series: The per hour specifications have the permanent component contributing only about 0.7 percent of the combined contributions, versus more than 9 percent in the per capita model. These results demonstrate that the biggest impact on quantifying the cyclicality of the permanent component comes from moving from the per capita to the per hour specification, i.e. from taking low frequency movements in hours into account. The asymmetry of business cycles turns out to be inconsequential for this question.

These findings also shed light on Perron's (1989) argument that failures to reject unit roots can be a consequence of series with occasional structural breaks. Hamilton (1989), Kim and Murray (2002), and others have found permanent components in business cycles even after allowing for structural breaks. Our results suggest that the treatment of hours of work is crucial. When we treat hours in a manner that is robust to nonstationarity (i.e. valid whether or not the hours per capita series is stationary), the business cycle virtually disappears from our estimated permanent component. This is presumably at least in part because it enables us to isolate the regime shift in the (technological) permanent component.

As a final illustration, we combine the demographic trend (an HP filter) with the estimated technology trend from the per hour model, and compare it with the estimated permanent component from the per capita model. These are more directly comparable, in the sense that they are both estimates of $\overline{\Lambda}_t + \gamma X_t$. These are plotted against per capita output in Figure 8. The permanent component from the per hour model is much smoother and less cyclical. While we cannot "prove" that it is a better estimate of the permanent component, it seems at least plausible that using the implications of growth theory to separate the two kinds of permanent components will lead to better results than estimating a hodgepodge of two completely different concepts.

Thus in the end we are able to characterize aggregate output as made up of three distinct and more or less independent components: a transitory and essentially symmetric "business cycle" component, a permanent "technology trend" component and a low frequency "demographic" component. The business cycle component

[11] For the two per hour specifications, "output" refers to non-farm output deflated by the HP trend from hours of work. For the per capita specification, "output" is output per capita. The behavior of the two series in growth rates or band-pass filtered is virtually identical.

exhibits the standard hump-shaped behavior emphasized by Blanchard (1982). The technology component is close to piecewise linear, with two breaks, one in around 1973, the other around 1996. The demographic component is J-shaped, declining in the post-war era until the early 1960s and then rising from the mid-1960s till the present.

4.4. The Relative Importance of Additional Variables

Finally, we have also stressed the importance of information gleaned from several series in identifying the common permanent component. It is reasonable to ask how important this actually is. To answer this, we estimated the same econometric model, but with only two series, nonfarm output per hour and detrended hours (variables 1 and 4 from the previous analysis). Thus we are looking to estimate trend growth with productivity data alone, using the detrended hours series to control for the business cycle. The result of this exercise is the final set of estimates in Table 2. Note first that the estimates of the transition probabilities are very similar to the earlier estimates, suggesting that the fundamental properties of the regime-switching dimension of the model are similar.

We also estimated two additional specifications, adding back alternately either C/L or W/L to the 2-equation system. The results are illustrated in Figure 9, in which all four specifications' retrospective regime assessments are plotted against each other. There appears to be a big impact from adding either variable to the system, with the labor compensation variable clearly outperforming consumption/hours in terms of approaching the 4-equation results. The change in going from three to four equations is relatively modest, especially if the fourth variable is consumption/hours. Note, however, that in the more recent data it is the consumption/hours variable that is giving the stronger indication that the economy is in the high-growth regime, and comes closest to matching the assessment based on all four series.

5. Conclusions

In this paper we have used growth theory to help sort out the permanent and transitory components of aggregate output. The theory supports distinguishing two independent permanent components in aggregate output, one attributable to technology, the other to demographics. Moreover, it suggests that normalizing aggregates by hours of work rather than by population effectively isolates the demographic component from the technological. We treat the latter as a stochastic process whose mean growth rate has two "regimes," high and low, with some probability of switching between the two at any point in time. We also allow for low frequency variation in the demographic component in our empirical specification.

Controlling for the low frequency demographic component turns out to be surprisingly important for the question of the permanence of business cycle fluctuations. This is because doing so makes it easier to spot the regime shifts in the technological component, which, once accounted for, leave a business cycle that is largely transitory. Thus we find that aggregate output is made up of three distinct and more or less independent components: a transitory "business cycle" component, a permanent "technology trend" component and a permanent "demographic" component. The business cycle component is well characterized by a stationary, second-order autoregressive process. The technology component, on the other hand, is dominated by regime changes, and appears close to piecewise linear. Moreover, we show that failure to take the demographic trend into account can confound efforts to isolate the productivity component, and consequently mislead researchers into attributing permanence to cyclical fluctuations.

Apart from the econometric benefit of obtaining a cleaner estimate of the productivity trend, the bifurcation of the permanent component of output into what we have called the technological and demographic components arguably has other benefits as well. Since growth theory provides some reason to believe the two components are largely independent of each other, examination of each separately should prove more fruitful than looking at their joint behavior. The demographic trend presumably depends on factors such as the distribution of age and education in the population, immigration patterns, distinct underlying trends in labor supply across different subgroups of the population, and so forth. The technology trend, on the other hand, depends on the rate of underlying technical progress as reflected in total factor productivity, embodied technical progress as indicated by declines in the relative price of capital equipment, and human capital accumulation. There would seem to be nothing to gain from analyzing the sum of these two components even if they were measured accurately (which looking at them together seems to hinder).

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Appendix

A1. State Space Model

We employ the following state-space representation for our model:

Measurement Equation: $\Delta Q_t = H'\xi_t$, $\Delta Q_t \equiv (\Delta Q_{1t},...,\Delta Q_{4t})'$

Transition Equation: $\xi_t = \alpha(S_t) + F\xi_{t-1} + V_t$,

with $E(V_t V_t^{'}) = \Sigma$,

and where (after we restrict $\gamma_1 = \gamma_2 = \gamma_3 \equiv \gamma$, and set $\gamma_4 = 0$)

$$H' = \begin{bmatrix} \gamma & \lambda_1 & -\lambda_1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & \lambda_2 & -\lambda_2 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ \gamma & \lambda_3 & -\lambda_3 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & \lambda_4 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xi_t = \begin{bmatrix} \Delta X_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \\ z_{4t} \\ z_{4t-1} \end{bmatrix}, \qquad \alpha(S_t) \equiv \alpha(S_{1t}, S_{2t}) = \begin{bmatrix} \mu_0(1-S_{1t}) + \mu_1(S_{1t}) \\ \tau S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad V_t = \begin{bmatrix} \upsilon_t \\ \varepsilon_t \\ 0 \\ \eta_{1t} \\ 0 \\ \eta_{2t} \\ 0 \\ \eta_{3t} \\ 0 \\ \eta_{4t} \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1^* & \phi_2^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{411} & \psi_{412} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We next provide a brief overview of a filter developed by Kim (1994) that can be used for approximate maximum likelihood estimation of the state-space model with Markov switching. We focus our attention on the issue of drawing inferences about the unobserved regimes. For further details, interested readers are referred to Kim and Murray (2002).

To facilitate the discussion, we will represent the two unobserved Markov-switching variables S_{1t} and S_{2t} by a single Markov-switching variable defined such that:

$$S_t = 1 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 0$$

$$S_t = 2 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 1$$

$$S_t = 3 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 0$$

$$S_t = 4 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 1$$

with

$$\Pr[S_t = j \mid S_{t-1} = i] = p_{ij}$$

and

$$\sum_{j=1}^{4} p_{ij} = 1$$

Conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter equations are given by:

$$\begin{aligned}
\xi_{t|t-1}^{(i,j)} &= \alpha(S_j) + F\xi_{t-1|t-1}^{i} + V_t \\
P_{t|t-1}^{(i,j)} &= FP_{t-1|t-1}^{i}F' + \Sigma \\
\eta_{t|t-1}^{(i,j)} &= \Delta Q_t - H'\xi_{t|t-1}^{(i,j)} \\
f_{t|t-1}^{(i,j)} &= HP_{t|t-1}^{(i,j)}H' \\
\xi_{t|t}^{(i,j)} &= \xi_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1}\eta_{t|t-1}^{(i,j)} \\
P_{t|t}^{(i,j)} &= (I - P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1})HP_{t|t-1}^{(i,j)}
\end{aligned}$$

where $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{((i,j)}$ are, respectively, an inference on ξ_t based on information through time period t (Ω_t) and t-1 (Ω_{t-1}), given $S_t = j$ and $S_{t-1} = i$; $P_{t|t}^{(i,j)}$ and $P_{t|t-1}^{(i,j)}$ are, respectively, the mean squared error matrix of $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{(i,j)}$, given $S_t = j$ and $S_{t-1} = i$; $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of ΔQ_t based on information through time period t-1, given $S_t = j$ and $S_{t-1} = i$; and $f_{t|t-1}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{(i,j)}$.

To keep the Kalman filter from becoming computationally infeasible, the following approximations are introduced to collapse the posteriors terms $\xi_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ into the posterior terms $\xi_{t|t}^{j}$ and $P_{t|t}^{j}$:

$$\xi_{t|t}^{j} = \frac{\sum_{i=1}^{4}\Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\xi_{t|t}^{(i,j)}}{\Pr[S_t = j \mid \Omega_t]}$$

and

$$P_{t|t}^{j} = \frac{\sum \ \Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\{P_{t|t}^{(i,j)} + (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)})(\xi_{t|t}^{j} - \xi_{t|t}^{j})'\}}{\Pr[S_t = j \mid \Omega_t]}$$

The approximations result from the fact that $\xi_{t|t}^{(i,j)}$ does not calculate $E[\xi_t \mid S_{t-1} = i, S_t = j, \Omega_t]$ and $P_{t|t}^{(i,j)}$ does not calculate $E[(\xi_t - \xi_{t|t}^{(i,j)})(\xi_t - \xi_{t|t}^{(i,j)})' \mid S_{t-1} = i, S_t = j, \Omega_t]$ exactly. This is because

ξ_t conditional on Ω_{t-1}, $S_t = j$, and $S_{t-1} = i$ is a mixture of normals for t > 2.

To obtain the probability terms necessary to construct the approximations, the following three-step procedure is employed.

Step 1:

At the beginning of the t^th iteration, given $\Pr[S_{t-1} = i \mid \Omega_{t-1}]$, we can calculate:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}] = \Pr[S_t = j \mid S_{t-1} = i] \times \Pr[S_{t-1} = i \mid \Omega_{t-1}]$$

where $\Pr[S_t = j \mid S_{t-1} = i]$ is a transition probability.

<u>Step 2:</u>

We can then consider the joint density of $\Delta Q_t, S_t$ and S_{t-1}:

$$f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1}) = f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

and then obtain the marginal density of ΔQ_t as:

$$f(\Delta Q_t \mid \Omega_{t-1}) = \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1})$$

$$= \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

where the conditional density $f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1})$ is obtained using the prediction error decomposition:

$$f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) = (2\pi)^{\frac{T}{2}} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\eta_{t|t-1}^{(i,j)}{}' f_{t|t-1}^{(i,j)-1}\eta_{t|t-1}^{(i,j)}\}$$

A byproduct of this step is that we can obtain the log likelihood function:

$$\ln L = \sum_{t=1}^{T} \ln(f(\Delta Q_t \mid \Omega_{t-1}))$$

which can be maximized with respect to the parameters of the model.

<u>Step 3:</u>

We can then update the probability terms after observing ΔQ_t and the end of period t:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_t] = \Pr[S_t = j, S_{t-1} = i \mid \Delta Q_t, \Omega_{t-1}]$$

$$= \frac{f(S_t = j, S_{t-1} = i, \Delta Q_t \mid \Omega_{t-1})}{f(\Delta Q_t \mid \Omega_{t-1})}$$

$$= \frac{f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]}{f(\Delta Q_t \mid \Omega_{t-1})}$$

with

$$\Pr[S_t = j \mid \Omega_t] = \sum_{i=1}^{4} \Pr[S_t = j, S_{t-1} = i \mid \Omega_t]$$

The last term provides the “real-time” inference about the unobserved regimes conditional on only contemporaneously available information.

We can also derive smoothed values of ξ_t and S_t using all available information through period T. That is, we can construct $\xi_{t|T}$ as well as $\Pr[S_t = j \mid \Omega_T]$ which represent the "retrospective" assessments of the state vector and unobserved regimes. Because the inferences about the unobserved regimes do not depend on the state vector, we can first calculate smoothed probabilities. The smoothed probabilities can then be used to generate the smoothed estimates of the state vector.

The smoothing algorithm for the probabilities will involve the application of approximations similar to those introduced in the basic filtering. The procedure can be understood by considering the following derivation of the joint probability that $S_{t+1} = k$ and $S_t = j$ conditional on full information:

$$\begin{aligned}
\Pr[S_{t+1} = k, S_t = j \mid \Omega_T] &= \Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid S_{t+1} = k, \Omega_T] \\
&\approx \Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid S_{t+1} = k, \Omega_t] \\
&= \frac{\Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_{t+1} = k, S_t = j \mid \Omega_t]}{\Pr[S_{t+1} = k \mid \Omega_t]} \\
&= \frac{\Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid \Omega_t] \times \Pr[S_{t+1} = k \mid S_t = j]}{\Pr[S_{t+1} = k \mid \Omega_t]}
\end{aligned}$$

and

$$\Pr[S_t = j \mid \Omega_T] = \textstyle\sum_{i=1}^{4} \Pr[S_{t+1} = k, S_t = j \mid \Omega_T]$$

The actual construction of the smoothed probabilities requires running through the basic filter and then storing the sequences $P_{t|t-1}^{(i,j)}, P_{t|t}^{j}, \Pr[S_t = j \mid \Omega_{t-1}]$ and $\Pr[S_t = j \mid \Omega_t]$. For $t = T$ - 1, T - 2, . . . , 1, the above formulas define a backwards recursion that can be used to derive the full-sample smoothed probabilities. It should be noted that the starting value for the smoothing algorithm is $\Pr[S_t = j \mid \Omega_T]$, which is given by the final iteration of the basic filter.

A2. Partial Updating

Suppose that additional observations become available, but only for some subset of the four data series represented by *Q*. Specifically, suppose that for the subset Q^1, data are available for periods 1 through *T*+3, whereas for Q^2, observations are only available through *T*. Let $T+1'$ denote the augmented information available through $T+1$, i.e. including Q_{T+1}^1 but not Q_{T+1}^2.

Through T the standard Kalman updating algorithm applies (ignoring the regime-related term $\alpha(S_t)$ for brevity's sake), i.e.

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H)^{-1}(\Delta Q_t - H'\hat{\xi}_{t|t-1})$$
$$P_{t+1|t} = F[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H)^{-1}H'P_{t|t-1}]F' + \Sigma$$

for $t \leq T$.

To update further, we simply recognize that

$$\Delta Q_t^i = H^i{}'\xi_t, \qquad i = 1, 2$$

where H^i is the appropriate submatrix of H. We then iterate beginning at T+1 according to

$$\hat{\xi}_{T+1|T+1'} = \hat{\xi}_{T|T-1} + P_{T|T-1}H^1(H^1{}'P_{T+1|T}H^1)^{-1}(\Delta Q_{T+1}^1 - H^1{}'\hat{\xi}_{T+1|T})$$
$$P_{T+1|T+1'} = P_{T+1|T} - P_{T+1|T}H^1(H^1{}'P_{T+1|T}H^1)^{-1}H^1{}'P_{T+1|T}$$

and then

$$\hat{\xi}_{T+2|T+1'} = F\hat{\xi}_{T+1|T+1'}$$
$$P_{T+2|T+1'} = FP_{T+1|T+1'}F' + \Sigma \, .$$

The iteration can then proceed forward if subsequent observations on Q^1 become available.

In our case we essentially have three additional observations on two of the variables, since they appear lagged by three quarters. To take them into account at any point in time t, we can compute $\hat{\xi}_{t|t+3'}$, that is, the assessment of the state vector given the three additional observations of the two series that would otherwise be ignored because they are lagged by three quarters. To obtain $\hat{\xi}_{t|t+3'}$, we iterate forward to get $\hat{\xi}_{t+3|t+3'}$ and $P_{t+3|t+3'}$, and then iteratively "smooth" backwards using, e.g.

$$\hat{\xi}_{t+2|t+3'} = \hat{\xi}_{t+2|t+2'} + P_{t+2|t+2'} H^1 (H^1{}' P_{t+3|t+2'} H^1)^{-1} (\hat{\xi}_{t+3|t+3'} - \hat{\xi}_{t+3|t+2'}) \,.$$

The result is an improved estimate of the state vector, one that incorporates the most recent values of all of the variables.

Table 1: Unrestricted Cointegration Rank Test

Hypothesized Number of Cointegrating Equations	Eigenvalue	Trace Statistic†	5 Percent Critical value	1 Percent Critical value
None**	$\hat{\lambda}_1 = 0.172154$	70.85945	29.68	35.65
At most 1**	$\hat{\lambda}_2 = 0.109881$	29.10624	15.41	20.04
At most 2	$\hat{\lambda}_3 = 0.015186$	3.381938	3.76	6.65

Hypothesized Number of Cointegrating Equations	Eigenvalue	Maximal Eigenvalue Statistic††	5 Percent Critical value	1 Percent Critical value
None**	$\hat{\lambda}_1 = 0.172154$	41.75320	20.97	25.52
At most 1**	$\hat{\lambda}_2 = 0.109881$	25.72431	14.07	18.63
At most 2	$\hat{\lambda}_3 = 0.015186$	3.381938	3.76	6.65

† $\lambda_{trace} = -T \sum \ln(1 - \hat{\lambda}_i)$

†† $\lambda_{\max} = -T \sum \ln(1 - \hat{\lambda}_{r+1})$

** Denotes rejection of the hypothesis at the 1 percent level.

Table 2: Estimation of Model

Coefficient	4 equation system	No pluck system	Per capita system	2 equation system
p_1	0.990 (0.011)	0.991 (0.010)	***	0.994 (0.007)
q_1	0.986 (0.013)	0.985 (0.013)	***	0.994 (0.008)
p_2	0.985 (0.012)	—	0.976 (0.017)	0.953 (0.026)
q_2	0.573 (0.227)	—	0.579 (0.236)	0.621 (0.130)
ϕ	-0.440 (0.132)	-0.450 (0.151)	0.577 (0.097)	-0.746 (0.133)
ϕ_1^*	1.416 (0.089)	1.488 (0.067)	1.400 (0.077)	1.253 (0.097)
ϕ_2^*	-0.552 (0.080)	-0.605 (0.066)	-0.538 (0.068)	-0.443 (0.085)
ψ_{11}	0.879 (0.042)	0.871 (0.046)	0.922 (0.040)	-0.905 (0.045)
ψ_{21}	0.876 (0.055)	0.891 (0.055)	0.897 (0.043)	—
ψ_{31}	-0.562 (0.087)	-0.565 (0.109)	-0.242 (0.144)	—
ψ_{41}	1.454 (0.055)	1.461 (0.055)	1.705 (0.174)	1.585 (0.140)
ψ_{42}	-0.528 (0.040)	-0.533 (0.040)	-0.726 (0.148)	-0.628 (0.111)
γ	0.262 (0.042)	0.260 (0.044)	0.416 (0.045)	0.314 (0.122)
λ_1	0.207 (0.046)	0.235 (0.051)	0.662 (0.058)	0.128 (0.041)
λ_2	0.121 (0.030)	0.133 (0.034)	0.240 (0.037)	—
λ_3	-0.483 (0.042)	-0.547 (0.043)	0.596 (0.045)	—
λ_4	0.443 (0.044)	0.497 (0.047)	0.770 (0.046)	0.429 (0.120)
μ_0	0.906 (0.218)	0.889 (0.226)	0.0002 (0.359)	0.851 (0.417)
μ_1	-1.099 (0.254)	-1.117 (0.270)	0.0002 (0.911)	-0.873 (0.441)
τ	-2.883 (0.749)	—	-2.539 (0.578)	-3.507 (1.099)

Note: The estimation also produces estimates of the variances of the idiosyncratic errors, not reported here.
*** indicates parameters that could not be estimated because they are not identified.

Table 3: Variances of Permanent versus Transitory Component

Growth Rates

Variances				
Specification	Output per capita	Permanent component (1)	Transitory component (2)	Percent (1) vs. ((1)+(2))
per hour	1.963	0.036	0.793	4.35
per hour /no pluck		0.038	0.785	4.57
per capita		0.211	0.964	17.98

Band-Pass Filtered

Variances				
Specification	Output per capita	Permanent component (1)	Transitory component (2)	Percent (1) vs. ((1)+(2))
per hour	4.797	0.017	3.122	0.55
per hour /no pluck		0.019	2.981	0.64
per capita		0.891	3.547	20.08

Figure 1: Hours of Work Per Capita in the Postwar U.S.

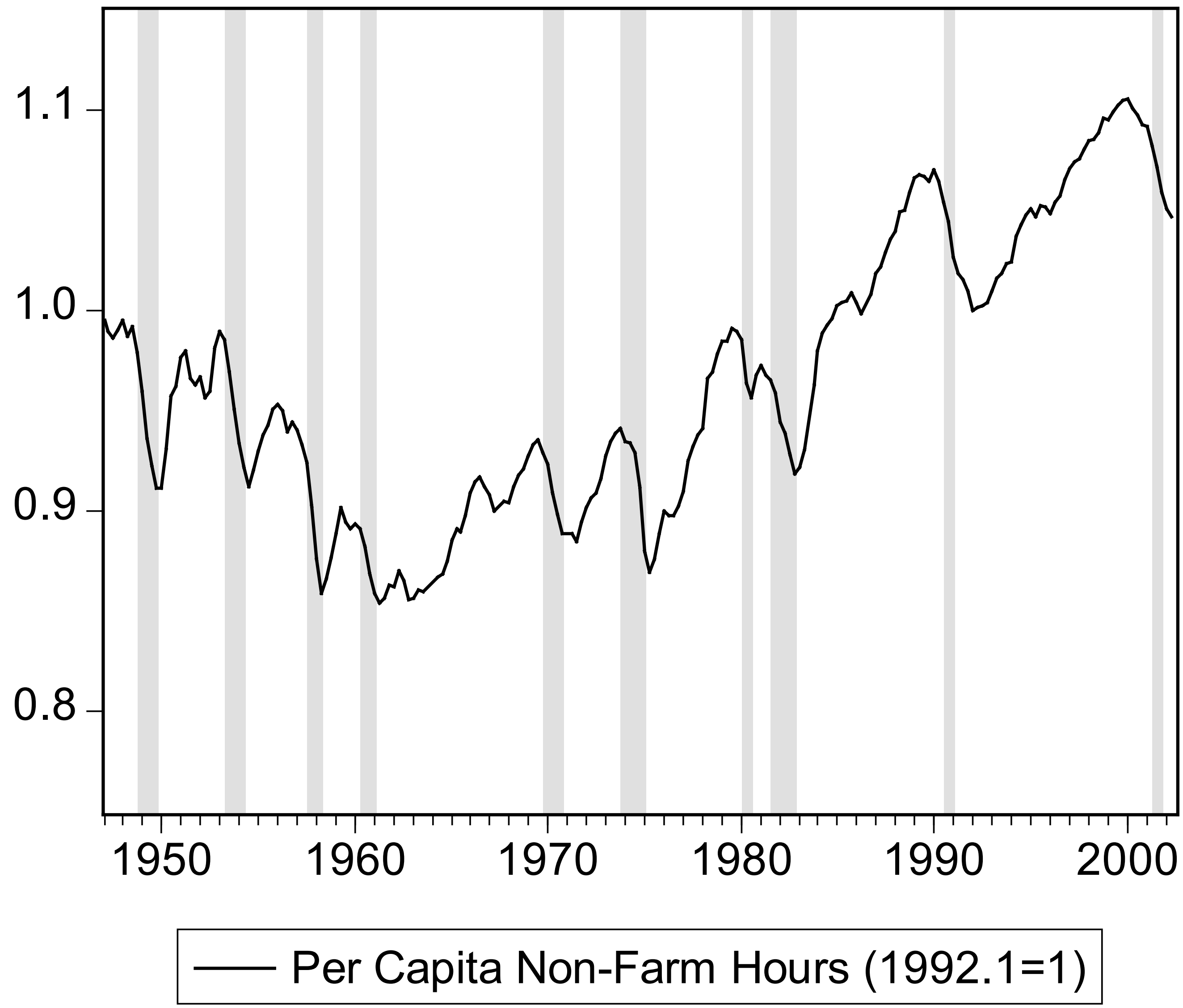

Figure 2: Cointegration Properties of Y, W, C, and K

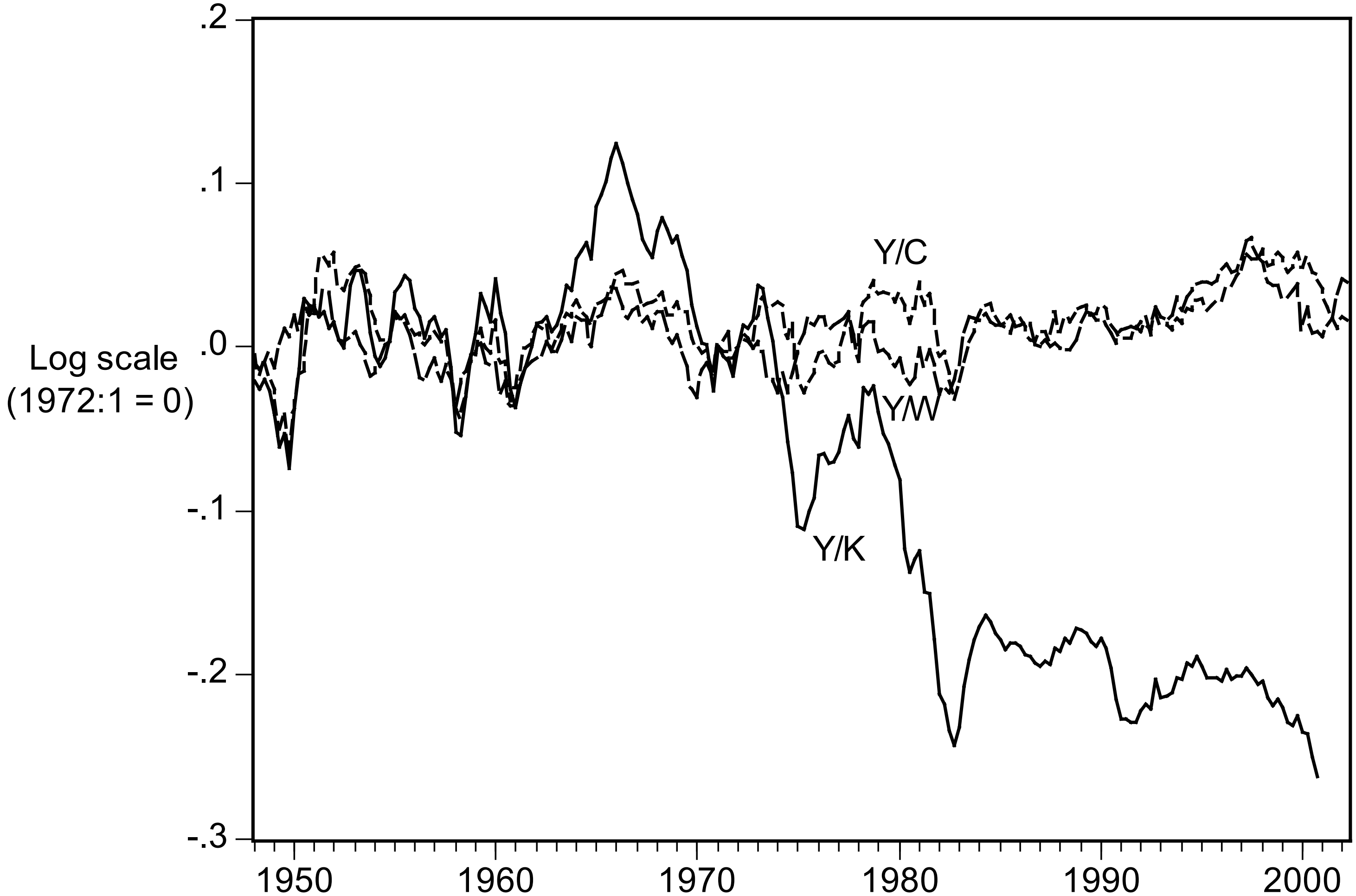

Figure 3: Full-Sample Assessment of Growth Regimes

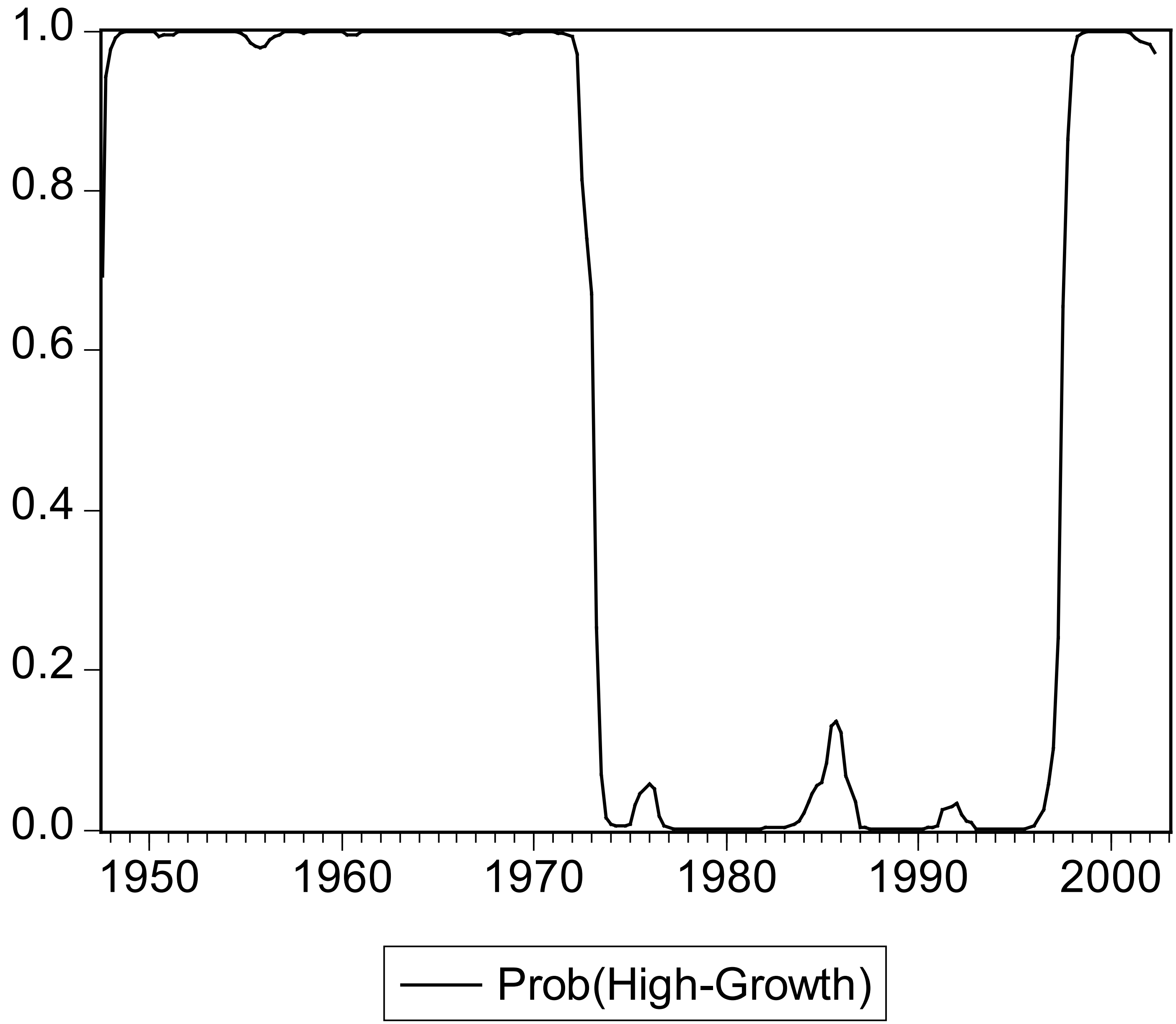

Figure 4: Full-Sample Assessments of Transitory Regimes

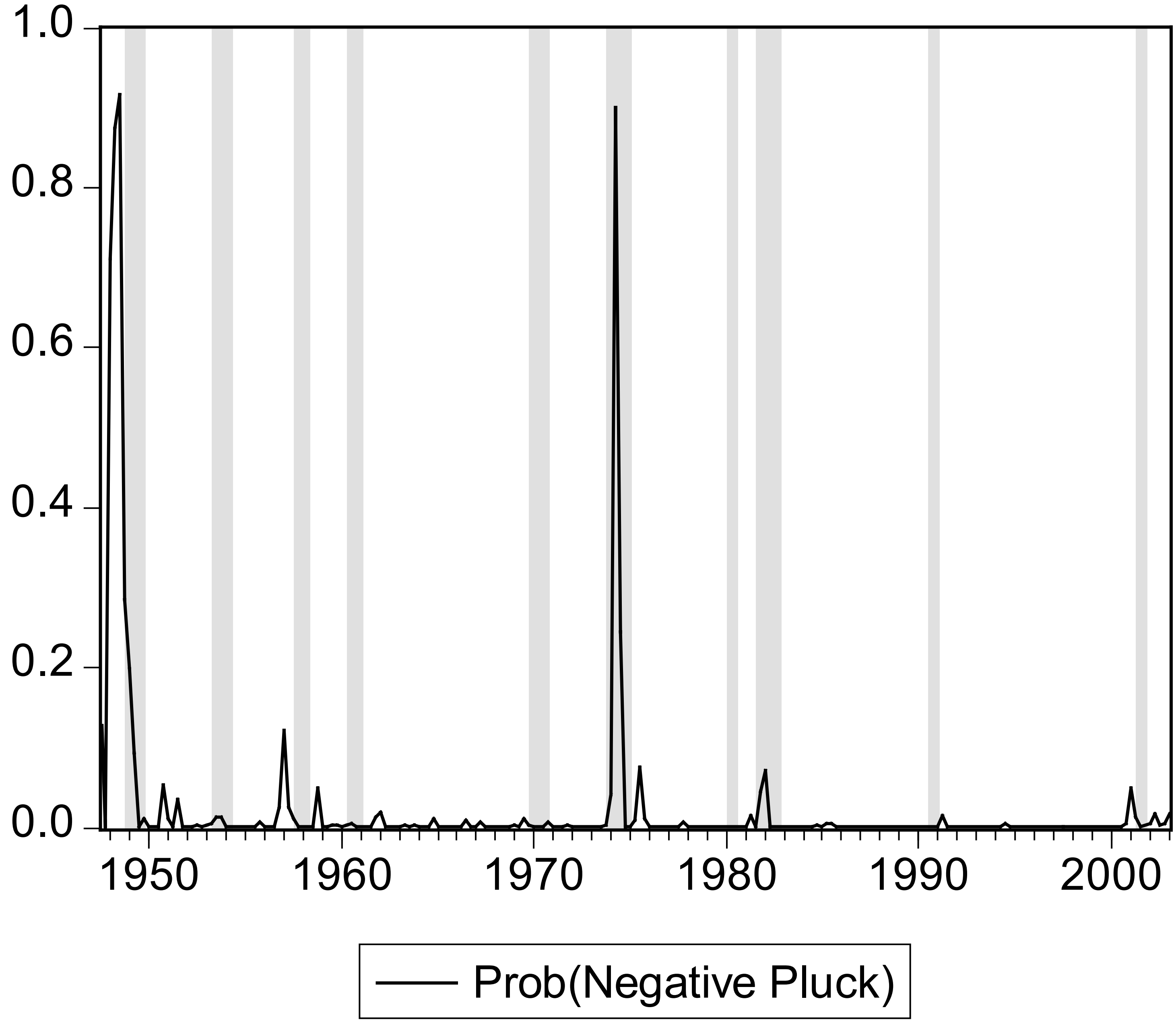

Figure 5: Trend Productivity

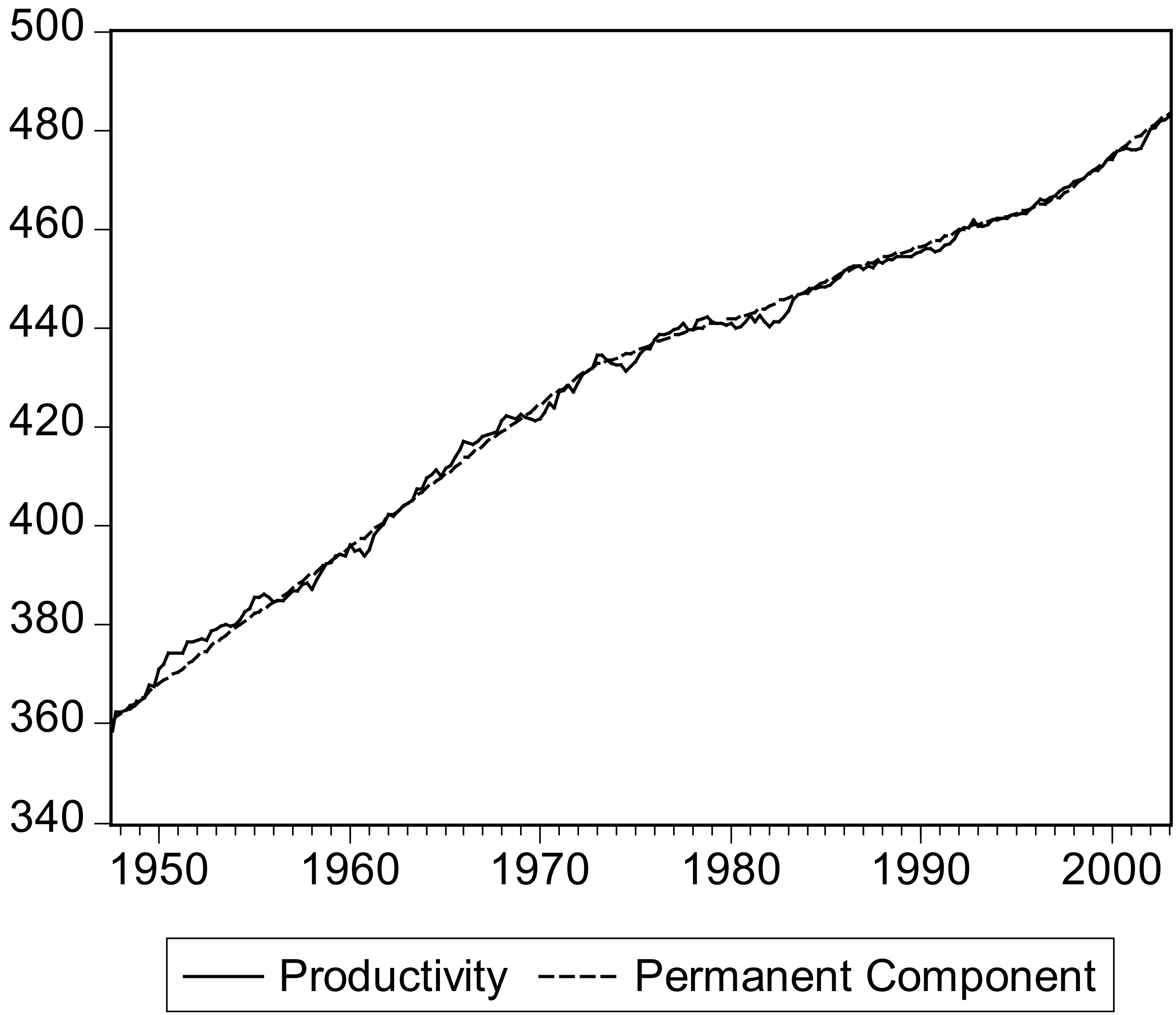

Figure 6: The Cyclicality of the Permanent Component

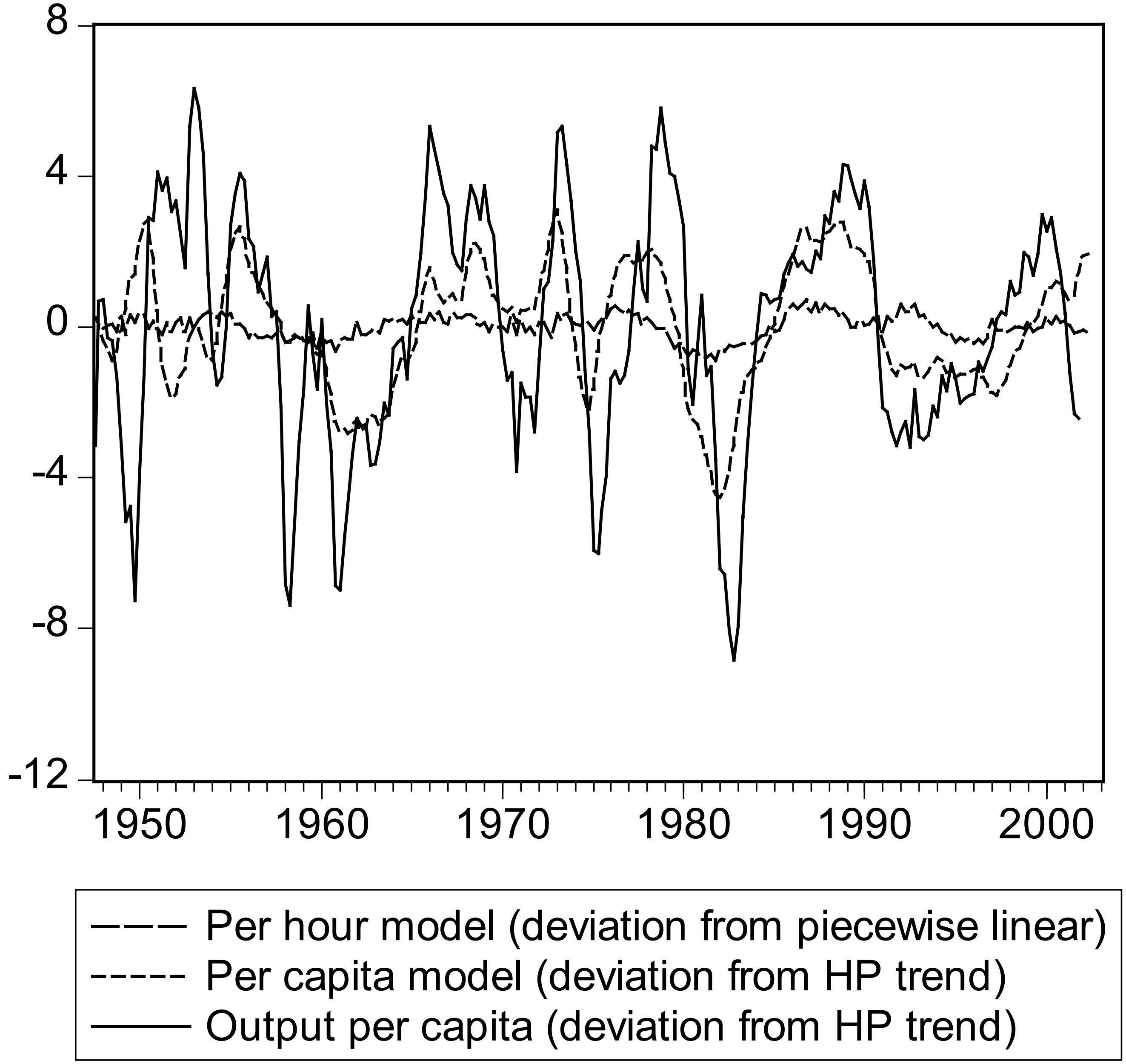

Figure 7: A Comparison of Permanent and Transitory Components across Specifications

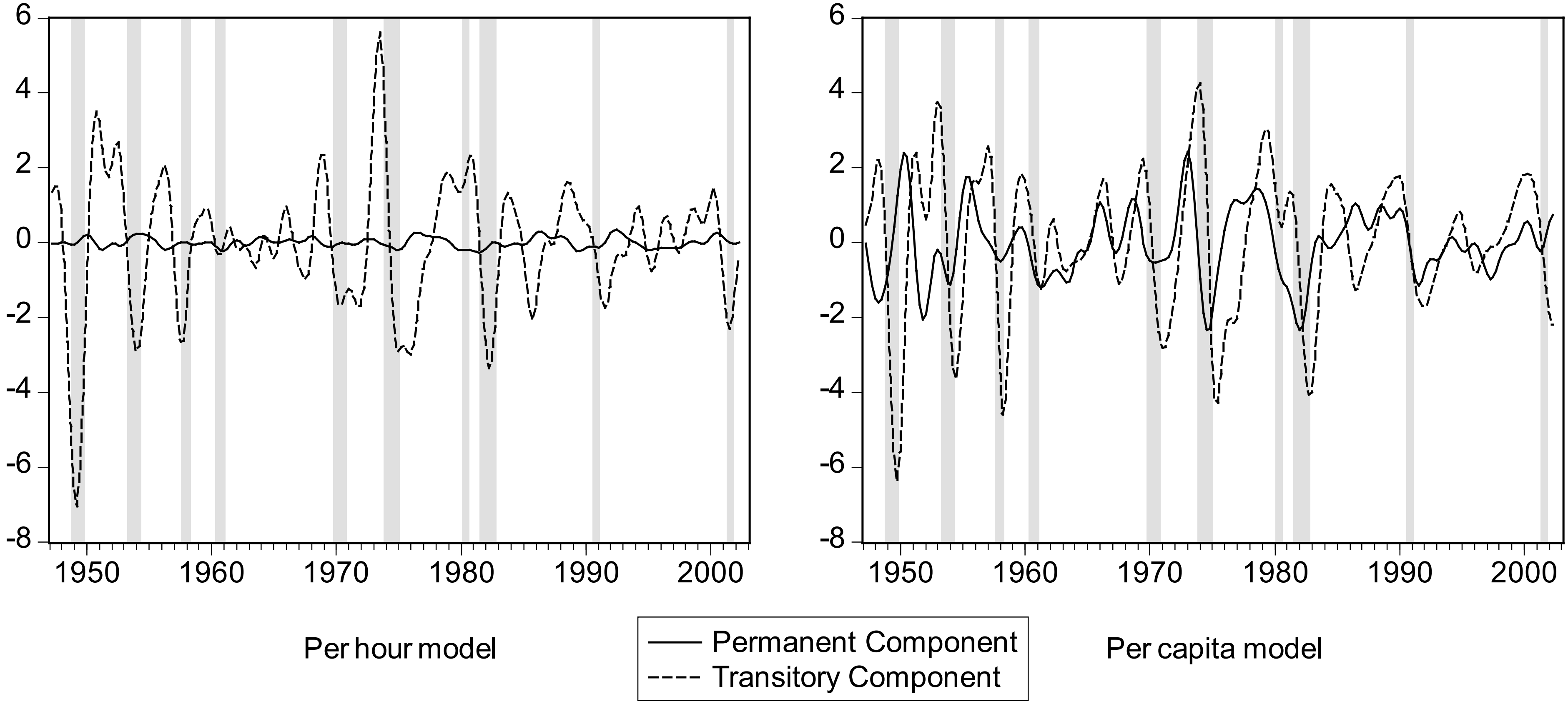

Figure 8: Comparing the Estimated Combined Trends

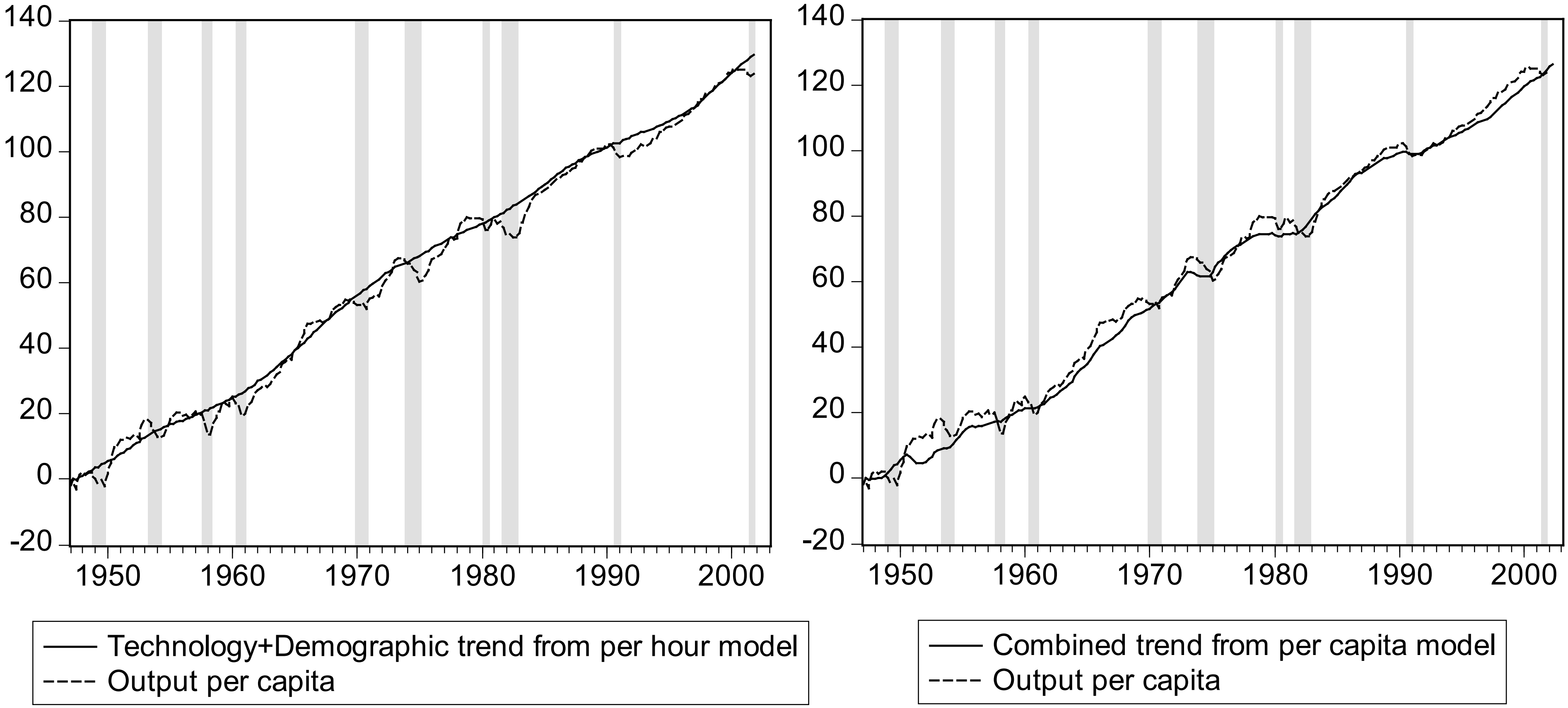

Figure 9: Relative Importance of Variables
for Regime Assessments