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ABSTRACT

Volatility in exchange rates is a prominent feature of open economies, a fact which has motivated elaborate attempts in many countries at exchange rate management. This paper analyzes quantitatively the welfare effects of exchange rate risk in a general two-country environment. It finds that the effects of uncertainty tend to be small for the types of simplified cases considered in past literature. But it identifies other cases, not considered previously, in which these effects can be significantly larger. These include habit persistence, where agents are more sensitive to risk, and also incomplete asset market structures which allow for asymmetries between countries. The latter case suggests that countries which are hosts to an international reserve currency, such as the U.S. or members of the euro zone, may accrue significant benefits because of the enhanced ability to hedge against exchange rate risk.

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1 Introduction

Exchange rate variability is one of the most prominent features of open economy macroeconomics, and the tendency for nominal exchange rates to move so volatilely and unpredictably has been blamed for limiting gains from international trade and for lowering welfare. A desire to moderate this uncertainty has been a motivation behind the managed exchange rate regimes of many countries, and recently of the monetary union in Europe. This paper conducts a quantitative examination of the welfare effects of risk in the context of a two-country general equilibrium model with sticky prices.

In Mundell's pioneering work on optimal currency areas which in large part earned him a Nobel prize in 1999, he emphasized a trade-off between the costs and benefits of exchange rate flexibility. On one hand, significant advances have been made regarding one of the two sides of Mundell's cost-benefit comparison. In particular, improved sticky price models permit a formalization of Mundell's analysis of the welfare gains of enhancing macroeconomic adjustment.[1] On the other hand, comparatively little work has been done on the second of the two parts of the tradeoff – the costs of exchange rate risk. Only recently have a small number of highly important papers begun to consider the costs of exchange rate risk in the context of the improved sticky price models, including Obstfeld and Rogoff (2001), Devereux and Engel (2000), and Bacchetta and Van Wincoop (2000).[2]

While offering tantalizing theoretical results, this literature has been limited to examining only the simplest of economic settings, as numerous simplifying assumptions are required to permit analytical solutions. For example, the papers noted above either assumed complete asset markets or special preferences that made the asset market redundant. As a result, shocks to the exchange rate can have no effects on the current account or on the net wealth of a country. While such simplifications have been necessary for model solution, they obscure factors that clearly may be important for evaluating the welfare effects of exchange rate volatility. For instance, it traditionally has been thought that among the most important implications of exchange rate movements are their effect on trade flows and the

[1] See Lane (2001) for a survey of the New Open Economy Macroeconomics literature.

[2] For a sample of other work looking at welfare analysis in micro-founded models, see Benigno (2000), Benigno and Benigno (2001), Clarida, Gali and Gertler (2001), Corsetti and Pesenti (2001a,b), Obstfeld and Rogoff (2000), Sutherland (2001).

current account. Further, if exchange rate movements affect countries asymmetrically, shifts in wealth between countries may have large effects on welfare.

This paper explores the effects of uncertainty in a general macro model that does not need to impose the simplifying assumptions discussed above. Most importantly, it will consider a two-country general equilibrium model in which asset markets are incomplete and preferences are calibrated more realistically. In addition, it includes investment in real capital and multiperiod price stickiness. As a result, the first contribution of the paper is that it can perform a quantitative analysis. Assigning a value to these welfare costs is essential if the policy implications of this literature are to be taken seriously. This is especially a concern, since work by Lucas (1987) has shown that variability of the type implied by business cycles tend to have very small welfare effects, and the same indictment might apply to this literature. The second contribution of the paper is that it uncovers and explores a number of conceptually interesting new costs associated with exchange rate variability, of types not identified in past literature.

This more general analysis is made possible by a solution algorithm created recently by Christopher Sims which can deal with second-order approximations to the equilibrium conditions of stochastic models.[3] In time, these techniques are likely to replace the currently dominant solution methods based on log-linear approximation, but they are at an early stage of development. This project will be an early application, helping to demonstrate how these powerful new methods can be usefully employed in international economics.[4]

The calibration exercise finds that the welfare effects of uncertainty are likely to be small for a wide range of cases. Even when the model incorporates the model features most emphasized by past theoretical work, they tend to imply welfare costs around only one tenth of one percent of annual steady state consumption. While the theoretical literature has emphasized certain model features in

[3] See Sims (2000) and Kim, et al (2002). Related solution algorithms have been proposed in Schmitt-Grohe and Uribe (2001), and Collard and Juillard (2001).

[4] This paper is related to Kollmann (2002a,b), which has adapted the Sims algorithm. Kollmann puts his model to a different use, finding optimal monetary policy rules and examining policy coordination, rather than examining cases where exchange rate risk is quantitativlely significant. Recently, Benigno (2001) has explored a two-country model where asset markets are incomplete. But his solution method does not allow him to solve for the welfare of countries individually; he can only consider the world aggregate, where there is no change in net assets.

its debates – such as pricing to market and substitutability between consumption and leisure – the exercise here indicates that these issues are not quantitatively significant in general.

However, the paper uncovers other features of the economy, ignored in the literature to date, which do seem to be important in a quantitative sense. One such case is where households exhibit habit persistence in consumption. This specification stipulates that utility is strongly affected by sudden consumption changes, and so it naturally amplifies the welfare effects of the risk of shocks that would require such changes. Habits have been found to be important for understanding household behavior in domestic equity markets, so if this behavior applies to international markets as well, it appears to imply exchange rate risk in fact may be quite costly.

Another case where risk has large welfare effects is where asset markets are asymmetric. In particular, if there is an international market for bonds in the currency of only one of the two countries, this country will tend to save more and have higher welfare in the stochastic steady state. But this effect comes at the direct expense of the other country, which saves less and has lower welfare in steady state. This asymmetry exists because saving in the international bond is a better hedge against exchange rate risk for a country that can save in terms of its own currency. This result indicates that the gains to countries like the U.S. or members of the euro zone, which host a reserve currency, may be substantial.

The paper then considers how these welfare costs might be offset. A government policy to fix the exchange rate is one possibility. Another is the fact that private markets allow for various assets like forward contracts to hedge against exchange rate risk. It appears that this paper is the first to analyze forward contracts in the context of a general equilibrium model of this type. The paper finds that the effectiveness of these private measures depends crucially upon which elements in the economy are generating the large welfare costs.

The next section of the paper presents the two-country model, calibration, and solution method. Section 3 presents results for a range of cases for this model. Section 4 concludes and makes suggestions for future research.

2 The Model

Consider a model of two countries, hereafter referred to as home and foreign. Suppose agents consume two final goods, where each country specializes in the production of one of these goods. Monopolistically competitive firms produce intermediate goods using capital and labor, and set prices sluggishly due to adjustment costs.

2.1 Goods Market structure

Final goods in this economy (F) are a CES index over sub-indexes of the home and foreign intermediate goods. The aggregation technology for producing final goods is:

$$F_t = \left[a^{\frac{1}{\mu}} F_{H,t}^{\frac{\mu-1}{\mu}} + (1-a)^{\frac{1}{\mu}} F_{F,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \quad \text{where} \tag{1}$$

$$F_{H,t} = \left(\int_0^1 f_{H,t}(i)^{\frac{\lambda-1}{\lambda}} di\right)^{\frac{\lambda}{\lambda-1}} \tag{2}$$

$$F_{F,t} = \left(\int_0^1 f_{F,t}(j)^{\frac{\lambda-1}{\lambda}} dj\right)^{\frac{\lambda}{\lambda-1}}. \tag{3}$$

Here F_H represents an aggregate of the home goods sold in the home country, and F_F is an aggregate of the imported foreign goods, where lower case counterparts represent outputs of the individual firms.

Final goods producers behave competitively, maximizing profit each period:

$$\pi_t = \max \left[P_t F_t - P_{H,t} F_{H,t} - P_{F,t} F_{F,t}\right], \tag{4}$$

where P is the overall price index of the final good, P_H is the price index of home goods, and P_F is the price index of foreign goods, all denominated in the home currency. These may be defined:

$$P_t = \left(a P_{H,t}^{1-\mu} + (1-a) P_{F,t}^{1-\mu}\right)^{\frac{1}{1-\mu}} \quad \text{where} \tag{5}$$

$$P_{H,t} = \left(\int_0^1 p_{H,t}(i)^{1-\lambda} di\right)^{\frac{1}{1-\lambda}} \tag{6}$$

$$P_{F,t} = \left(\int_0^1 p_{F,t}(j)^{1-\lambda} dj\right)^{\frac{1}{1-\lambda}}, \tag{7}$$

and where lower case counterparts again represent the prices set by individual firms.

Given the aggregation functions above, demand will be allocated between home and foreign goods according to

$$F_{H,t} = a\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} F_t \tag{8}$$

$$F_{F,t} = (1-a)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} F_t \tag{9}$$

with demands for individual goods:

$$f_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\lambda} F_{Ht} \tag{10}$$

$$f_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\lambda} F_{Ft} \tag{11}$$

We have analogous definitions for the foreign country.

2.2 Home household problem

The representative home household derives utility from consumption (C), real money balances ($\frac{M}{P}$), and labor (H). Households derive income by selling their labor (H) at the nominal wage rate (W), renting out capital to firms at the real rental rate (r), receiving real profits from home firms (Π), and from government transfers (T). In addition to money, households can hold a noncontingent nominal bond denominated in home currency (B_H) which pays an interest rate (i), or a bond denominated in foreign currency (B_F) which pays an interest rate (i^*). The household determines capital accumulation (K), which involves a quadratic adjustment cost that depends upon the parameter ψ_I and a constant rate of depreciation (δ).

The household optimization problem may be written:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, H_t)$$

subject to the budget constraint:

$$P_t C_t + P_t\left(K_{t+1} - (1-\delta)K_t\right) + AC_{I,t} + M_t + B_{H,t} + S_t B_{F,t} + AC_{B,t} = (1+i_{t-1})B_{H,t-1} + S_t(1+i^*_{t-1})B_{F,t-1} + M_{t-1} + W_t H_t + P_t r_t K_t + T_t + \Pi_t$$

where

$$U_t = \frac{C_t^{1-\rho}}{1-\rho} + \frac{\chi_t}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon} - \frac{H_t^{1+\psi}}{1+\psi} \tag{12}$$

and

$$AC_{I,t} = \frac{\psi_I}{2}\frac{(K_{t+1}-K_t)^2}{K_t} \tag{13}$$

$$AC_{B,t} = \frac{\psi_B}{2}\left(\frac{\left(S_t\left(B_{Ft}-\overline{B_F}\right)\right)^2}{P_{Ht}Y_t}\right)$$

Money demand shocks are represented by shifts in χ_t. There is a small adjustment cost on bond holdings, AC_B, to ensure stationarity in the net foreign asset position.[5]

Optimization implies a money demand equation:

$$\frac{M_t}{P_t} = \frac{\chi_t^{\frac{1}{\varepsilon}} C_t^{\rho/\varepsilon}}{(1-d_t)^{1/\varepsilon}}, \tag{14}$$

a trade-off between consumption and leisure:

$$\frac{W_t}{P_t C_t^{\rho}} = H_t^{\psi}, \tag{15}$$

a consumption Euler equation:

$$d_t = \beta E_t \frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}, \tag{16}$$

with the definition of d:

$$i_t = \frac{1}{d_t} - 1, \tag{17}$$

an interest parity condition:

$$E_t\left[\frac{P_t C_t^{\rho}}{P_{t+1}C_{t+1}^{\rho}}\frac{S_{t+1}}{S_t}(1+i_t^*)\left(1+\frac{\psi_B S_t\left(B_{Ft}-\overline{B_F}\right)}{P_{Ht}Y_t}\right)^{-1}\right] = E_t\left[\frac{P_t C_t^{\rho}}{P_{t+1}C_{t+1}^{\rho}}(1+i)_t\right], \tag{18}$$

and finally, capital accumulation:

$$(1+\psi_I\frac{K_{t+1}-K_t}{K_t}) = \beta E_t\left[\frac{C_t^{\rho}}{C_{t+1}^{\rho}}\left(r_{t+1}+(1-\delta)+\left(\frac{1}{2}\psi_I\frac{K_{t+2}^2-K_{t+1}^2}{K_{t+1}^2}\right)\right)\right]. \tag{19}$$

equating the benefits and costs of capital accumulation. The benefits on the right of the equation above include the return from rental of the capital plus the resale value after depreciation, as well as the last term, representing the fact that accumulating a larger capital stock today lowers the expected adjustment cost of further accumulation in the future. Analogous conditions apply to the foreign household.

[5] Home and foreign bonds are treated separately in this adjustment cost to ensure that there exists a determinate allocation between home and foreign currency bonds even in a first-order approximation to the system, as is required by the second-order accurate solution of Sims (2000).

2.3 Home firm problem

In the benchmark version of the model we assume producer currency pricing, so that firms set prices in their own currency both for sales domestically and sales abroad. They rent capital (K) at the rental rate r, and hire labor (H) at the nominal rate W. Prices are sticky because there is a quadratic cost to adjusting them[6]. The home firm maximization problem for the domestic consumer is:

$$\max \; E_0 \sum_{t=0}^{\infty} \rho_{t,t+n} \Pi_{H,t}(i) \tag{20}$$

where

$$\Pi_{H,t(i)} = (P_{H,t}(i) - MC_t(i) - AC_{P,t}\,(i)) f_{H,t}(i), \tag{21}$$

with the adjustment and marginal costs defined respectively as: $_t$

$$AC_{P,t}\,(i) = \frac{\psi_P}{2} \frac{\left(p_{H,t}\,(i) - p_{H,t-1}\,(i)\right)^2}{p_{H,t-1}\,(i)}, \tag{22}$$

$$MC_t = \frac{(r_t P_t)^{\alpha} W_t^{1-\alpha}}{\theta_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{23}$$

and subject to the demand function for $f_{H,t}(i)$ from above and the production function specifying output (Y) as:

$$Y_t(i) = \theta_t K_t^{\alpha}(i) H_t^{1-\alpha}(i). \tag{24}$$

Here θ represents technology common to all production firms in the country, and is subject to shocks. Lastly, $\rho_{t,t+n}$ is the pricing kernel used to value random date $t+n$ payoffs. Since firms are assumed to to be owned by the representative household, it is also assumed that firms value future profits according to the household's intertemporal marginal rate of substitution in consumption, so $\rho_{t,t+n} = \beta^n \left(U'_{C,t+n} / P_{t+n} \right) / \left(U'_{C,t} / P_t \right)$, where $U'_{C,t+n}$ is the household's marginal utility of consumption in period $t+n$. The optimization problem implies a trade-off between capital and labor inputs that depend on the relative cost of each:

$$P_t r_t K_t\,(i) = \frac{\alpha}{1-\alpha} W_t H_t\,(i)\,, \tag{25}$$

[6] It has been demonstrated in Rotemberg (1982) that menu costs of this type, although simple to specify and work with, generate price dynamics identical to those of Calvo random price staggering

and price setting behavior:

$$\begin{aligned} p_{Ht} &= \frac{\lambda}{\lambda-1}\left(MC_t + AC_{Pt}\right) \\ &+\frac{\psi_P}{\lambda-1}p_{Ht}\left(1-\frac{p_{H,t}}{p_{H,t-1}}\right) \\ &+\frac{1}{2}\frac{\psi_P}{\lambda-1}p_{Ht}E_t\left[\frac{\rho_{t,t+i+1}}{\rho_{t,t+i}}\left(1-\frac{p^2_{H,t+1}}{p^2_{H,t}}\right)\frac{F_{H,t+1}}{F_{Ht}}\right]. \end{aligned} \tag{26}$$

Note that in the special case of no price stickiness ($\psi_P = 0$), price-setting is set as a simple markup over marginal costs: $p_{Ht} = \frac{\lambda}{\lambda-1}MC_t$. But in the presence of price adjustment costs, price-setting will deviate from this simple markup because of several additional terms. First, the resource cost of setting a price (AC_P) should be included along with the cost of production when computing the overall price of bringing a good to market. The next term in the expression above reflects the backward looking component of price setting: firms are reluctant to make large changes in price due to the marginal adjustment cost. The final term reflects the forward-looking component of price setting. If a firm expects the need to change prices further in the next period, it will tend to change the price more today, to minimize future adjustment costs. Further, there is an additional reason to raise prices today, because a higher current price means that any future changes will be a smaller percentage change. Here we see one reason for the monopolist to set a higher price on average, as a hedge against future price changes.

Price stickiness generated through adjustment costs in this matter is different from the Calvo pricing most common in the literature, but it has distinct advantages. Perhaps the most important is that it allows all firms to reset prices if the costs of price stickiness become large. Under Calvo pricing, some firms are forced to retain prices arbitrarily far away from the optimal price, which can influence the welfare implications in a way viewed by some as unreasonable.

The optimal price for the foreign market will be:

$$p^*_{H,t} = p_{H,t}/S_t \tag{27}$$

Note that in the symmetric equilibrium $p_{H,t} = P_{H,t}$ and $p^*_{H,t} = P^*_{H,t}$.

2.4 Government

To facilitate comparison with the earlier literature (Obstfeld and Rogoff (2001), Devereux and Engel (2000), and Bacchetta and Van Wincoop (2000)), we follow them in using a money growth rule:

$$m_t = m_{t-1} + a_S (s_t - \overline{s}) \tag{28}$$

where lower case letters represent logs of upper case counterparts. This rule will permit a fixed exchange rate regime, for a_S set to a large negative value, or alternatively a flexible exchange rate regime, for a value of a_S set near zero. For most experiments the parameter a_S will be set to a value near but not exactly zero, which allows wide exchange rate variability but rules out a random walk in the exchange rate. The fact that the exchange rate is stationary allows us to examine this key variable of interest in our model simulations in nominal form, without transforming it to a real exchange rate.

The government's budget constraint is:

$$T_t = \frac{1}{P_t} (M_t - M_{t-1}) \tag{29}$$

2.5 Market clearing and equilibrium

Market clearing for the home goods market requires:

$$F_{Ht} + F^*_{Ht} = Y_t, \tag{30}$$

and for the home bond market:

$$B_{H,t} + B^*_{H,t} = 0. \tag{31}$$

Total home demand (F) may be defined:

$$F_t = C_t + (K_{t+1} - (1-\delta) K_t) + AC_{I,t} + AC_{B,t} + \int_0^1 AC_{P,t}(i)\, di \tag{32}$$

The home balance of payments condition may be written:

$$B_{H,t} - \frac{B_{H,t-1}}{d_{t-1}} - P_{H,t} Y_t + P_t C_t + P_t (K_{t+1} - (1-\delta) K_t) = 0 \tag{33}$$

Equilibrium is a set of 39 equations determining 39 sequences: C_t, C^*_t, $P_{H,t}$, $P_{F,t}$, $P^*_{H,t}$, $P^*_{F,t}$, P_t, P^*_t, S_t, W_t, W^*_t, H_t, H^*_t, Y_t, Y^*_t, F_t, F^*_t, $F_{H,t}$, $F^*_{H,t}$, F_{Ft}, F^*_{Ft}, K_t, K^*_t, r_t, r^*_t, d_t, d^*_t, MC, MC^*,

$B_{H,t}$, $B^*_{H,t}$, $B_{F,t}$, $B^*_{F,t}$, θ_t, θ^*_t, M_t, M^*_t, i_t, i^*_t. The 39 equilibrium conditions are: the definition of total demand (1), demand conditions for home and foreign goods (8 and 9), the overall price index (5), the price setting rules (26) and (27), the money supply rule (28), labor supply condition (15), capital-labor trade-off (25), money demand condition (14), the interest rate parity condition (18), production function (24), definition of marginal cost (23),definition of total demand (32), definition of d (17), consumption Euler equation (16), market clearing conditions for goods (30) and bonds (31), capital accumulation (19), along with foreign counterparts for each of the above and the balance of payments constraint (33).

The shocks, to technology and money demand in each country, will be log-normally distributed:

$$\begin{array}{cc} \left(\log\theta_t - \log\overline{\theta}\right) & = \rho_1\left(\log\theta_{t-1} - \log\overline{\theta}\right) + \varepsilon_{1t} \\ \left(\log\theta^*_t - \log\overline{\theta^*}\right) & = \rho^*_1\left(\log\theta^*_{t-1} - \log\overline{\theta^*}\right) + \varepsilon^*_{1t} \\ \left(\log\chi_t - \log\overline{\chi}\right) & = \rho_2\left(\log\chi_{t-1} - \log\overline{\chi}\right) + \varepsilon_{2_t t} \\ \left(\log\chi^*_t - \log\overline{\chi^*}\right) & = \rho^*_2\left(\log\chi^*_{t-1} - \log\overline{\chi^*}\right) + \varepsilon^*_{2_t t} \\ \multicolumn{2}{c}{[\varepsilon_{1t}, \varepsilon^*_{1t}, \varepsilon_{2t}, \varepsilon^*_{2t},]' \sim N\left(0, \Sigma\right).} \end{array} \tag{34}$$

To deal with the nonstationary nominal variables in this system, they will be transformed by dividing by their respective national price level. As noted above, this does not need to be done for the nominal exchange rate.

2.6 Solution method

The model is solved numerically to a second order approximation. This stands in contrast to the standard method relying upon log-linear approximations, which would miss many of the implications of risk. In the present context, such a method would only capture the direct effects of exchange rate variability on welfare through the fact that people dislike variance in consumption and leisure. It ignores the fact that variability in the exchange rate can have important effects on welfare through the means of variables. For example, if firms respond to exchange rate variability by producing less output, the level of consumption will be lower, and this effect on means will affect welfare. Further, if households respond to variability by precautionary saving, this rise in wealth potentially may make welfare rise rather than fall . Woodford (2002) has demonstrated in the context of a closed economy model that under certain simplifying assumptions, these mean effects will not impact welfare, and this solution approach has been employed in open economy contexts by several papers. But Benigno (2001) has shown that for this result to hold in an open economy requires many additional simplifying assumptions, such as restrictive preference specifications. In a general setting, the effects of risk working through means of variables may well be equally or more important for welfare than those effects working through variances.

The method used here is the second-order accurate solution by Sims (2000) and Kim et al. (2002). Here we present only the basic relationships. If one denotes by η the vector of all endogenous and exogenous variables, then the solution algorithm requires that the system is written in the following way

$$\Psi(\eta_t, \eta_{t-1}, \varepsilon_t) + \Pi\varrho_t = c, \tag{35}$$

where ε_t is the vector of shocks (in our case two technology and two money demand shocks) to the system and ϱ_t is a function of the shocks when the model is solved. We expand the model to a second-order around a steady state given by:

$$\Psi(\bar{\eta}, \bar{\eta}, 0).$$

The solution, then, is given by:

$$y_t = F(y_{t-1}, \varepsilon_t), \tag{36}$$

$$x_t = M(y_t), \tag{37}$$

where y_t and x_t can be interpreted in the usual sense as the predetermined and forward-looking variables, respectively, and $[y_t' \; x_t'] = Z'\eta_t$. The second-order expansion of the solution can be written as

$$\begin{aligned} dy_{it} &= F_{1ij} dy_{j,t-1} + F_{2ij}\varepsilon_{jt} + F_{3i}\sigma^2 \\ &\quad + \frac{1}{2}\left(F_{11ijk} dy_{j,t-1} dy_{k,t-1} + 2F_{12ijk} dy_{j,t-1}\varepsilon_{kt} + F_{22ijk}\varepsilon_{jt}\varepsilon_{kt}\right), \\ dx_{it} &= M_{11ijk} dy_{jt} dy_{kt} + M_{2i}\sigma^2, \end{aligned} \tag{38}$$

where the matrices Z, F and M are functions of the model parameters. Note as well that, in accordance with Sims [2000], we are using tensor notation, i.e. it is true that:

$$A_{ijk}B_{mnjq} = C_{ikmnq} \Leftrightarrow c_{ikmnq} = \sum_j a_{ijk}b_{mnjq}.$$

Taking unconditional expectations of (38) and making use of $[y_t' \; x_t'] = Z'\eta_t$, it is possible to compute the following unconditional first and second moments of the system:

$$E[d\eta_t d\eta_t'] \text{ and } E[d\eta_t]. \tag{39}$$

In this computation we disregard those terms that are higher than second order.

2.7 Computation of the welfare measure

The objective here is to compare alternative steady states of the model, one with risk and the other without.[7] In accord with this specific objective, we compute welfare measures as the unconditional expectation of utility in steady state. A second-order Taylor expansion of the utility function yields:

$$EU_t = \bar{U} + \bar{C}^{1-\rho}E(\hat{C}_t) - \frac{1}{2}\rho\bar{C}^{1-\rho}var(\hat{C}_t) - \bar{H}^{1+\psi}E(\widehat{H}_t) - \frac{1}{2}\psi\bar{H}^{1+\psi}var(\hat{H}_t). \tag{40}$$

We follow Lucas (1987) in that we represent them as the permanent shift in steady state consumption required to achieve the same expected utility, i.e. we find how much steady state consumption the household is ready to give up in order to negate the effect of the shocks. Since we use a second-order approximation, however, we can go even further. We can separate the effects of a particular shock to the dynamic system. The shock matters because it influences the expected levels of the variables and because it has a bearing on the their second moments. While the latter can be found relatively easily from a first-order solution, the former can be gleaned only from a full second-order expansion of the model. Let u^{mean} denote the permanent shift in steady state consumption that delivers the same expected utility. Then, making use of (40) we must have that

$$U\left((1+u^{mean})C, H\right) = \frac{((1+u^{mean})\bar{C})^{1-\rho}}{1-\rho} - \frac{\bar{H}^{1+\psi}}{1+\psi} = \bar{U} + \bar{C}^{1-\rho}E(\hat{C}_t) - \bar{H}^{1+\psi}E(\hat{H}_t). \tag{41}$$

[7] We agree that for experiments that consider the welfare effects of implementing alternative policies, conditional welfare is more appropriate, which would consider the discounted path of utility during transition to the new steady state. However, such an experiment is not the objective here.

Solving for u^{mean} we get:

$$u^{mean} = \left[1 + (1-\rho)E(\hat{C}_t) - \frac{(1-\rho)\bar{H}^{1+\psi}}{\bar{C}^{1-\rho}}E(\hat{H}_t)\right]^{\frac{1}{1-\rho}} - 1. \quad (42)$$

In a similar fashion we derive u^{var} which denotes the permanent shift in steady state consumption associated with the effect of the shocks on the variances of the variables. We find that:

$$U\left((1+u^{var})C, H\right) = \frac{((1+u^{var})\bar{C})^{1-\rho}}{1-\rho} - \frac{\bar{H}^{1+\psi}}{1+\psi} = \bar{U} - \rho\bar{C}^{1-\rho}var(\hat{C}_t) - \psi\bar{H}^{1+\psi}var(\hat{H}_t). \quad (43)$$

Thus, u^{var} can be found:

$$u^{var} = \left[1 - \frac{1}{2}\rho(1-\rho)var(\hat{C}_t) - \frac{1}{2}\psi\frac{(1-\rho)\bar{H}^{1+\psi}}{\bar{C}^{1-\rho}}var(\hat{H}_t)\right]^{\frac{1}{1-\rho}} - 1 \quad (44)$$

The values for u^{mean} and u^{var} reported in Table 2 are coming from these calculations.

2.8 Calibration

In the benchmark case of the model we choose the following parameterization. Empirical studies find a wide range of estimates for the interest elasticity of real money balances ($1/\epsilon$), and we follow Bergin and Feenstra (2001) in choosing an intermediate value of 0.25 ($\epsilon = 4$). Empirical studies estimate the income elasticity of real money demand (ρ/ϵ) to be about unity, so we also set $\rho = 4$. This is in the range of the estimates for the coefficient of relative risk aversion provided by Hall (1988) (as low as 1 and as high as 33) and also by Gali, Gertler and Lopez-Salido (2002) who suggest a value between 3 and 10.

The elasticity of substitution between home and foreign goods μ is a critical parameter in our experiments. According to some recent studies , such as Harrigan (1993) and Trefler and Lai (1999), a sensible assumption for this parameter is 5, and we follow this parameterization. Rotemberg and Woodford (1998) set the degree of monopolistic competition λ to be 7.66 which implies an average price mark-up of 15%. We choose $\lambda = 7$. The share of home goods in the home final goods aggregator, a, is set at 0.80, reflecting the 20% share of imports in GDP on average for the G7 countries in the 1990:1-1998:4 period.

The specification of the utility function has the convenient feature that the household has a well-defined static labor supply function, whose elasticity, $1/\psi$, is constant. The value for this elasticity is controversial. Microeconomic studies (for example, Killingsworth and Heckman (1986)) suggest values between 0.5-1.5. We follow Christiano at. al. (1997) and set $\psi = 1$. A unitary labor supply elasticity is consistent with the fact that per-capita labor supply has changed little while real wages have risen in recent decades. We calibrate $\beta = 0.99$ and interpret a period in the model as one quarter. For the depreciation rate we choose $\delta = 0.025$, and for the capital share in production $\alpha = 0.36$.

The price adjustment cost is set at $\psi_P = 50$, which implies that 95% of the price has adjusted 4 periods after a monetary shock. Investment adjustment cost, $\psi_I = 4$, is calibrated such that investment is about three times more volatile than output. Bond adjustment cost, $\psi_B = 0.000004$, is necessary in order to negate the unit root associated with the incompleteness of the asset markets. We need the monetary policy reaction parameters $a_S = 1x10^{-6}$ in order to eliminate the unit root in the monetary policy rule. It is crucial that the first-order solution does not contain unit roots, because, otherwise, in the second-order solution the variances of the variables will grow to infinity.

The variance and persistence of the technology shock is calibrated at standard values: $var(\varepsilon_1) = var(\varepsilon_1^*) = .01^2$ and $\rho_1 = \rho_1^* = 0.90$, common values in the real business cycle literature and identical to Kollmann (2002a). As will be seen in Table 2, these values help us to replicate the second moments of output, which we compute to be 1.80 for the 1973:1-2000:4 period in HP-filtered GDP data for the G7 countries on average. We calibrate the money demand shocks to help us replicate the observed second moments of the nominal exchange rate. We find that the bilateral exchange rate with the U.S. dollar of the remaining G7 countries on average is 7.81 percent for HP-filtered data in the 1973:1 - 2000:4 period, which is between 4 and 5 times as volatile as output.[8] The autoregressive coefficient indicated by this data is 0.79. Replicating these features requires the following values for the money demand shocks: $var(\varepsilon_2) = var(\varepsilon_2^*) = 0.03^2$ and $\rho_2 = \rho_2^* = 0.99$. For simplicity we assume that shocks are uncorrelated with each other.

[8] This value for exchange rate volatility is standard in quantitative business cycle studies, as in Chari et al. (1998) and in Kollmann (2002). We also considered geometrically detrending the data, even though this still leaves a unit root in the data. The standard deviation of the exchange rate then rises to 15 percent, and results of the model under calibration to this volatility will be noted later.

3 Results

Table 1 reports a summary of results from a battery of experiments, showing the effect of uncertainty on welfare, and Tables 2-4 report the details for these experiments. In particular, it reports the difference between the unconditional mean of utility in the second-order approximation of the model and that of the certainty-equivalent version of the model. To aid interpretation, this difference in utilities is presented in terms of the change in the steady state level of consumption that would be needed to change utility the same amount, as explained in section 2.7 above. As is usual in this literature, welfare is computed from the portion of the utility function excluding real money balances.

3.1 Benchmark and related cases

The first line of Table 1 refers to the benchmark model and calibration described in the section above. This case may be viewed as a more fully fleshed out version of the benchmark model used in the theoretical analysis of exchange rate risk in Obstfeld and Rogoff (2001), where the model here has been extended to include investment, multiperiod price stickiness, and a more realistic calibration of parameters like the elasticity of substitution between home and foreign goods and the money demand elasticity.

Table 1 shows that the welfare effects of risk in this benchmark case are fairly small, amounting to a fall in utility equal to 0.14% of steady state consumption. A useful comparison is the result of Lucas (1987), which measured the effect on welfare of volatility arising from business cycle fluctuations. He found that the loss in welfare was equivalent to a loss in average consumption of 0.042 percent, and he concluded that this was a trivial magnitude. The result here is somewhat larger than this, but still of a similar order of magnitude, and is far below one percent of steady state consumption.

Column 1 of Table 2 shows details behind the result for the benchmark case. First note that the standard deviations of the key variables in the model match fairly well with the average among G7 economies. Consumption is about 2/3 as volatile as output, investment is about 3 times as volatile, and the nominal exchange rate is about 4 times as volatile as output. The fall in overall welfare seems to be coming from the fact that the steady state level of consumption and production are lower under risk. This in part is due to the higher markup of the home goods price over marginal cost shown in the table. This reflects the theoretical finding in Obstfeld and Rogoff (2001) that risk averse firms will tend to set higher prices and restrict output, to hedge against uncertainty. Note also that the trade volume is higher under the presence of risk. While this stands in contrast to the usual presumption that exchange rate risk inhibits trade, the result found here has been shown to be very feasible in theoretical exercises, as in Bacchetta and van Wincoop (2000). Lastly, the breakdown of welfare effects shows that a significant fraction comes from changes in the mean variables, not just the fact that there is extra variability. This portion of the welfare effect would not be picked up by papers using a second-order approximation only to the utility function; they would miss the fact that welfare is lower because risk averse firms restrict production and consumption, and thereby alter the means of variables in the stochastic steady state.

Some additional experiments can offer further insight into the role of markups here. Recall from the price setting equation (26) that in the absence of price adjustment costs ($\psi_P = 0$), the markup ratio of price to marginal cost is a constant, $\frac{\lambda}{\lambda-1}$. Column 2 of Table 2 reports results for such a case, where prices are fully flexible. Note that the markup value in the table is zero, indicating that the markup in stochastic steady state stays constant at its value in the deterministic steady state. In other words, this case factors out the effects of risk found in the benchmark case which are working through induced increases in the markup. Since (26) is the only place in the model where ψ_P appears, the welfare effects attributable to the presence of price stickiness here are equivalent to the effects working though additional markups.[9] About half of the overall welfare effect found in column 1 is now eliminated, so this appears to be the portion that is attributable to markups and sticky prices. To be precise, markups account for a fall in welfare of 0.0644 (taking the difference between the two columns). It is worth noting that eliminating price stickiness produces results very similar to what is found when one considers the experiment of eliminating money demand shocks from the model and

[9] There is no benefit from implementing a tax a-la Woodford (2002) to offset the monopolistic distortion in deterministic steady state. In experiments we confirm that this only affects the deterministic steady state and not the gap between stochastic and deterministic steady states, so it has no effect on the values reported in the tables.

just permitting technology shocks to operate.

Much recent work in open economy macroeconomics has explored price stickiness in the local currency of the buyer (LCP) rather than the domestic currency of the producer (PCP) as assumed above. Devereux and Engel (2001) have demonstrated in an analytically solvable model that this distinction can alter the welfare effects of flexible exchange rates. Line 2 of Table 1 explores this variation on the benchmark model. This model may be regarded as a version of that used in Devereux and Engel (2001) expanded to a more realistic setting, to include investment, multiperiod price stickiness, incomplete asset markets, and more realistic parameter values as above. While LCP may make a distinction theoretically, line 2 of the table indicates that this distinction may not be significant quantitatively. The welfare effect of risk remains small, and is even a bit smaller than under the PCP benchmark specification. (See column 3 of Table 2 for more detailed results.)

Work by Bacchetta and van Wincoop (2000) also analyzed the effects of exchange rate risk in an analytically solvable model. In their case, they emphasized the role of substitutability between consumption and leisure in altering the effects of exchange rate risk. We address this issue by replacing the utility function in (12) with one that resembles that of Bacchetta and van Wincoop:[10]

$$U_t = \left(\frac{1}{1-\rho}\right)\left[(b)^{\frac{1}{\phi}}\, c_t^{\frac{\phi-1}{\phi}} + (1-b)^{\frac{1}{\phi}}\,(1-h_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}(1-\rho)} + \frac{1}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon}. \qquad (45)$$

The parameter b is calibrated so that the steady state share of time to labor is 0.37, and ϕ is adjusted to replicate the consumption-leisure substitutabilities considered in Bacchetta and van Wincoop, so ϕ is set at 0.1 and 10 for complements and substitutes respectively. Again, our more general model differs from theirs in the factors discussed above, and also in the fact that their model was not dynamic, and did not include technology shocks. As in the earlier paper, risk raises trade volume when consumption and leisure are complements, and it lowers trade volume when they are substitutes. Further, welfare is worse under substitutes than under complements, though the welfare effects are negative for both cases in our model. But while the effects in general resemble those of the earlier paper in a qualitative

[10] This utility differs from Bachetta and van Wincoop (2000) in that it includes money. Note that this utility differs from that used earlier in the paper in the way it includes labor, so that it does not collapse down to equation (12) if we assume a zero elasticity of substitution. Also note that the change in utility function requires a corresponding adjustment in the computation of u^{mean} and u^{var} in evaluating welfare effects.

sense, once again the main conclusion is that in a quantitative sense, the present model reveals that these effects are all quite small.

3.2 Sensitivity analysis

The three earlier papers cited above rely on a number of model simplifications and parameter assumptions to facilitate analytical solutions of their models. We check here what role is played by these simplifying assumptions and the fact that we relax them.

First, two of the three papers highlighted above assume a particular market structure, with Cobb-Douglas preferences that imply a unitary elasticity of substitution between home and foreign goods. This assumption promotes risk sharing between the countries and greatly simplifies the solution in certain cases. Recall that our benchmark case calibrated this elasticity at the value of 5, as suggested by empirical literature. Line 6 of Table 1 shows the result if we set this parameter to unity, indicating that there is very little effect on the outcome. It may not be surprising that the lack of risk sharing is not important for our benchmark case, given that the welfare effects appear to be quite symmetric across countries. This will not be true for some experiments to come.

The models solved analytically typically must take an approximation to the money demand function, which is exact only under the assumption of a unitary elasticity of money demand. Our benchmark model assumed the empirically more relevant value of 1/4, but line 7 of Table 1 shows that results are little changed if we were to use a value of unity.[11]

Line 8 considers if we had a model with no investment. Again the results are little changed.

We also consider some parameter values that might be expected to raise the magnitude of the very small welfare effects we are finding here. One possibility is that exchange rate variability would matter more for countries that trade more with foreign countries. Line 9 of the table shows a case where the share of imports in GDP $(1-a)$ is raised from 0.2 to 0.5. This makes little difference.

It is also possible that risk would matter if agents were more risk averse. Line 10 shows a

[11] The experiment with unitary demand elasticity required that the variance of the money demand shock be lowered to $var(\varepsilon_2) = 0.0075^2$, for the model to continue to replicate the exchange rate variability observed in the data. In the no investment case to follow, a variance of 0.01^2 was required, and the case with increased risk aversion required a variance of 0.08^2.

case where the risk aversion parameter (ρ) is set at 30 instead of 4. Again there is little effect. It appears that a wide range of cases of the two-country model imply that the welfare effects of risk are quantitatively small.

3.3 Habit persistence

Another way of enhancing the risk aversion of households is to consider preferences that exhibit habits. We consider the utility function:

$$U_t = \frac{(C_t - \gamma C_{t-1})^{1-\rho}}{1-\rho} + \frac{\chi_t}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon} - \frac{H_t^{1+\psi}}{1+\psi}, \tag{46}$$

which implies an intertemporal Euler equation:

$$d_t = \beta E_t \left[\left(\frac{P_t}{P_{t+1}}\right) \left(\frac{(C_{t+1} - \gamma C_t)^{-\rho} - \beta\gamma E_{t+1}\left(C_{t+2} - \gamma C_{t+1}\right)^{-\rho}}{(C_t - \gamma C_{t-1})^{-\rho} - \beta\gamma E_t\left(C_{t+1} - \gamma C_t\right)^{-\rho}}\right)\right].$$

As γ goes to unity, households act to smooth changes in consumption rather than the level of consumption. Habit persistence has been used extensively to explain financial anomalies such as the equity premium puzzle, and has been advocated in the macroeconomic literature as a way of better capturing consumer behavior. [12] Given that this literature has found households to be sensitive to risk in equity markets, one might also expect them to be sensitive to risk in foreign exchange markets. In particular, one might expect risk to have larger welfare effects here, as these consumers dislike large and rapid cuts in consumption, and so they are more sensitive to consumption risk than agents with time-separable utility. We calibrate the habit persistence parameter at $\gamma = 0.8$, which is approximately what Deaton (1987) and Constantinides (1990) require in order to explain aggregate consumption smoothness and the equity premium puzzle.

It is common in calibrating habit persistence models to impose a large investment adjustment cost to keep the standard deviation of consumption from falling too low. This device does not work in an open economy where households can borrow abroad, because there no longer exists a direct connection between domestic investment and saving. Instead we augment the bond adjustment cost

[12] See for example Constantinides (1990) and Campbell and Cochrane (1999) regarding the equity premium puzzle, and see Deaton (1987) and Fuhrer (2000) for a discussion in the context of consumption behavior.

in the household budget constraint to penalize large changes in asset holding as well as large levels:

$$AC_{B,t} = \frac{\psi_B}{2}\left(\frac{\left(S_t\left(B_{Ft} - \overline{B_F}\right)\right)^2}{P_{Ht}Y_t}\right) + \frac{\psi_{B2}}{2}\left(\frac{\left(S_t\left(B_{Ft} - B_{F,t-1}\right)\right)^2}{P_{Ht}Y_t}\right) \quad (47)$$

where ψ_{B2} is calibrated at 0.000004.

Line 11 of Table 1 shows that welfare now falls a significant amount due to risk. In particular, households would be willing to trade 4.6% of annual steady state consumption to eliminate this risk, which is more than an order of magnitude larger than the results found in previous cases.[13] Table 3 shows details. Surprisingly, the levels of model variables change very little due to risk here. Nevertheless the welfare of households falls significantly, both because of the changes in variance and means, when measured in terms of steady state consumption. This may be explained by the fact that under habit persistence, households do not care much about steady state levels of consumption but rather about changes in consumption between periods, so that a large amount of steady state consumption must be used to compensate for lost welfare.[14]

It is worthwhile asking the question how much of the welfare loss due to risk here is attributable to exchange rate risk in particular? While studies using analytical solutions can answer this question easily by looking at the role of the exchange rate variance term in the analytical expression for welfare, this is not possible here. One way to try to extract the effect of exchange rate variability here is to compare the results of our model with a version that assumes a fixed exchange rate regime. This amounts to setting the parameter in the monetary policy rule, a_S, equal to a large value for both countries. Line 12 of Table 1 indicates that the welfare loss falls by about one-third. Column 2 of Table 3 indicates that the standard deviations of the other variables in the model have remained constant: the variability of output and consumption are virtually unchanged, and it is only the

[13] Note that with a different utility function, the formulas for computing u^{mean} and u^{var} must be altered accordingly.

[14] Just to confirm that the large welfare effects here are not arising from grossly sub-optimal policy behavior, we compute an optimal policy rule for steady state under a Nash equilibrium. The policy rule was specified to permit responses to shocks but not a direct response to the exchange rate other than that needed for dynamic stability: $m_t = m_{t-1} + a_1\theta_t + a_2\theta_t^* + a_3\chi_t + a_4\chi_t^* - a_s\frac{s_t}{\bar{s}}$, where a_s was specified as $1x10^{-6}$ as previously. The optimal policy parameters converged to $a_1 = 0.1219$, $a_2 = -0.0296$, $a_3 = 0.0033$, and $a_4 = -0.0004$. While the signs of these coefficients coincide with those found in the simpler model of Devereux and Engel (2000), they are much smaller in magnitude. In fact, the welfare under the optimized rule improves only from -4.554 to -4.544 percent of steady state consumption. This indicates that the large welfare effects found here are not simply due to a suboptimal policy rule.

variability of the exchange rate that has been eliminated from the model. This suggests that about one third of the welfare loss in the habit persistence case is due to exchange rate risk. Note that this experiment properly should not be regarded as a comparison of flexible versus fixed exchange rate regimes, as the flexible case we consider was not an optimal policy rule.

One important means by which private markets have evolved to combat exchange rate risk is the use of forward exchange contracts. It is next worth asking to what degree such private markets can offset the welfare effects found here. Forward contracts are introduced into the model as assets F_H that can be purchased in foreign currency but pay off in home currency at a pre-determned exchange rate, f_H; likewise assets F_F can be purchased in home currency that pay off in foreign currency units at a predetermined exchange rate f_F. The home household budget constraint then becomes:

$$\begin{aligned} P_tC_t + P_t\left(K_{t+1} - (1-\delta)K_t\right) + AC_{I,t} + M_t + B_{H,t} + S_tB_{F,t} + F_{H,t} + S_tF_{F,t} + AC_{B,t} \\ = (1+i_{t-1})B_{H,t-1} + S_t(1+i^*_{t-1})B_{F,t-1} + \tfrac{S_t}{f_{H,t-1}}(1+i_{t-1})F_{H,t-1} \\ +f_{F,t-1}(1+i_{t-1})F_{F,t-1} + M_{t-1} + W_tH_t + P_tr_tK_t + T_t + \Pi_t \end{aligned} \tag{48}$$

where the adjustment cost is augmented to include all four interest bearing assets. The six new variables for home and foreign (F_H, F^*_H, f_H, F_F, F^*_F,and f_F) are determined by two market clearing conditions and four new first order conditions, including covered interest rate parity conditions. Line 13 of Table 1 shows that forward contracts can offset only about 10% of the welfare costs of risk in the flexible exchange rate case. This is significantly smaller than the welfare improvements achieved by using monetary policy to eliminate all exchange rate variability.

3.4 Asymmetric asset market

The specification of asset markets can also make a difference for welfare effects. Consider a case where there is only one nominal bond that is traded internationally, denominated in the currency of the home country. This implies the benchmark model above, except that B_{Ft} is set to zero in all periods. This version of the model is arguably quite relevant for a large number of countries in the world. The home country in our model certainly is relevant for those countries whose currencies have the status of reserve currencies, such as the U.S., Japan, and now EMU countries. And the foreign country in the model is relevant essentially for the other countries in the world, who do not enjoy reserve currency status.

Looking at line 14 in Table 1, this asymmetry in asset markets implies that risk has a fair-sized impact on welfare of the two countries.[15] While the foreign country's welfare is hurt by the presence of risk, the home country actually benefits. The effect is not as large as that found for the habit persistence case above, but it is significantly larger than for the benchmark case, especially for the foreign country. Part of the magnitude of welfare effects here is obscured by the fact that the effects of monetary shocks and technology shocks work in offsetting directions for each country. If one considers just the effects of monetary shocks alone, risk raises welfare by 1.07% at home and lowers foreign welfare by approximately the same amount. In other words, welfare effects here can exceed one full percent of steady state consumption, which is an order of magnitude larger than the case reported in rows 1-10 in the table. Table 4 indicates that the large majority of this effect comes from a change in the mean value of variables rather than the variances. In particular there is a rise in the mean of home consumption and a fall in foreign. Clearly the asymmetry of this result distinguishes it from the analytical models used in previous studies, examined in the beginning of this paper. The simplifying features of those models implied that risks were perfectly pooled between the two countries, so no asymmetry was possible.[16]

The logic for what happens in this case of asymmetric incomplete markets is as follows. The introduction of risk makes households want to engage in precautionary saving to hedge. But since this is true for both countries, the main effect is to make the world interest rate on the bond fall, as can be seen clearly in the more detailed set of results in column 3 of Table 4. Since the international asset is in the currency of the home country, the exchange rate risk makes it a less attractive instrument for saving for the foreign country than for the home country. Given the fall in the interest rate, the foreign country chooses to save less in equilibrium, while the home country saves more. This rise in home net foreign asset position is also reflected clearly in column 3 of Table 4. So the home country has greater

[15] For the model to continue producing the same level of exchange rate variability under this specification, the variance of the money demand shock needed to be increased somewhat from 0.03^2 to 0.05^2.

[16] It was not possible to compute an optimal policy rule for a Nash equilibrium here. It is always in the interest of one country to dampen exchange rate movements, while it is always in the interest of the other to undo this policy to amplify exchange rate volatility. As a result, there is no convergence in the pair of optimal policy rules.

wealth and hence greater consumption in steady state than the foreign country.[17] See the appendix for a more detailed derivation of these points.

Line 15 shows the fixed exchange rate specification applied to the asymmetric asset markets case. The majority of the welfare effects are eliminated, and virtually all of the large effects associated with monetary shocks noted above disappear. This is true, while the standard deviations in Table 4 indicate there is virtually no change at all in the model's implications other than the elimination of exchange rate volatility. It appears that exchange rate risk is an essential element of the welfare costs here. Further, these welfare costs can in large part be eliminated if forward contracts are traded, as shown in line 16. In contrast to the habit persistence case, it appears that if welfare loss is due to the inability of countries without reserve currencies to hedge effectively against exchange rate risk, private markets can effectively solve this problem by the creation of other assets to serve this hedging function.

4 Conclusions

The paper has examined quantitatively the welfare effects of exchange rate risk in a two country model. The first conclusion is that the welfare effects are likely to be small for a wide range of cases. This certainly appears to be true for the special cases considered in most of the previous literature, where a reasonable calibration indicates welfare costs around only one tenth of one percent of annual steady state consumption. While the model features and issues debated in this earlier literature do have effects on welfare, such as pricing to market and substitutability between consumption and leisure, our exercise indicates that these issues in general are not quantitatively significant.

However, the explorations here do find at least two plausible cases where risk does matter quantitatively, lowering welfare in excess of one percent of steady state consumption. One case is where households exhibit habit persistence in consumption. This specification stipulates that utility is strongly affected by sudden consumption changes, and so it naturally amplifies the welfare effects of

[17] This result helps us to better understand and qualify the result in the small open economy model of Kollmann (2002). Because the model is of a small open economy, the world interest rate is taken as exogenous and is therefore unaffected by the presence of risk. As a result, the small country saves more and ends up with higher welfare in steady state. Our analysis shows that when a two-country model takes into consideration the effects of risk on the world environment, the resulting fall in interest rate reverses the result implied above.

the risk of shocks that would require such changes.

A second case where risk has large welfare effects is where asset markets are asymmetric. If there is an international market for bonds in the currency of only one of the two countries, we find that this country will tend to save more and have higher welfare in the stochastic steady state. But this effect comes at the direct expense of the other country, which saves less and has lower welfare in steady state. This asymmetry exists because saving in the international bond is a better hedge against exchange rate risk for a country that can save in terms of its own currency. This result indicates that gains to countries like the U.S. that host a reserve currency may be substantial.

The paper has also considered how these welfare costs might be offset. A government policy to fix the exchange rate is one possibility. Another is the fact that private markets allow for various assets like forward contracts to hedge against exchange rate risk. The paper finds that the effectiveness of these private measures depends crucially upon which elements in the economy are generating the large welfare costs.

This work suggests that attention in the theoretical literature could profitably be shifted away from examining features of the economy that seem not to matter quantitatively, and we point out at least two new areas that should receive attention in this literature: habit persistence and asymmetric asset markets. In particular, a comparison of fixed versus optimal flexible exchange rate rules should be examined under these two cases. Further, our results suggest that there may be other ways of generating asymmetries between countries as yet unexplored that could be quantitatively important for welfare.

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Appendix: Asymmetric Asset Market Case

One can gain some insight into the mechanism described in the text for the asymmetric assets markets case by comparing the intertemporal Euler equations across countries. The consumption Euler equation for the foreign country is:

$$\frac{1}{1+i_t} = \beta E_t \left[\frac{S_t P_t^* C_t^{*\rho}}{S_{t+1} P_{t+1}^* C_{t+1}^{*\rho}} \frac{1}{1 + \frac{\psi_B \left(B_{Ht}^* - \overline{B_H^*}\right)}{S_t P_{Ft}^* Y_t^*}} \right] \tag{49}$$

A bar over a variable denotes its deterministic steady state. The consumption Euler for the home country is:

$$\frac{1}{1+i_t} = \beta E_t \left[\frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right] \tag{50}$$

Equalizing (49) and (50)

$$E_t \left[\frac{S_t P_t^* C_t^{*\rho}}{S_{t+1} P_{t+1}^* C_{t+1}^{*\rho}} \frac{1}{1 + \frac{\psi_B \left(B_{Ht}^* - \overline{B_H^*}\right)}{S_t P_{Ft}^* Y_t^*}} \right] = E_t \left[\frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right] \tag{51}$$

Denoting $q_t^f = \frac{S_t P_t^* C_t^{*\rho}}{S_{t+1} P_{t+1}^* C_{t+1}^{*\rho}}$ and $q_t^h = \frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}$ and taking unconditional expectations of (51) yields:

$$E \left[q_t^f \frac{1}{1 + \frac{\psi_B B_{Ht}^*}{S_t P_{Ft}^* Y_t^*}} \right] = E(q_t^h) \tag{52}$$

A second-order Taylor expansion of (52) gives:

$$\frac{\psi_B}{S P_F^* Y^*} E\left(dB_{Ht}^*\right) = E(\widehat{q}_t^f) - E(\widehat{q}_t^h) - \frac{\psi_B}{S P_F^* Y^*} cov(\widehat{q}_t^f, dB_{Ht}^*) \tag{53}$$

A hat over a variable denotes a log deviation from its deterministic steady state. Since bonds are assumed to be zero in steady state, dBh_t^* stands for the absolute deviation of bond holdings from zero. The last expression is crucial in our analysis because it demonstrates the relationship between the expected holdings of home bonds by the foreign country and the variability of the exchange rate.[18] This can be seen more clearly by further expanding q_t^f (using a second-order Taylor-series approximation). We also denote $q_t^* = \frac{P_t^* C_t^{*\rho}}{P_{t+1}^* C_{t+1}^{*\rho}}$. Then $E(\widehat{q}_t^f) - E(\widehat{q}_t^h)$ will be equal to:

$$E(\widehat{q}_t^f) - E(\widehat{q}_t^h) = var(\widehat{S}_t) - cov(\widehat{S}_t, \widehat{S}_{t+1}) + cov(\widehat{S}_t, \widehat{q}_t^*) - cov(\widehat{S}_{t+1}, \widehat{q}_t^*) \tag{54}$$

[18] Note that among the three terms on the right hand side of the equality, the last term will be quantitatively insignificant in comparison with the others, as it is scaled by the adjustment cost parameter, ψ_B, which is calibrated to be very small.

where we have made use of the fact that the unconditional expectations of $\widehat{S}_t$ and$\widehat{S}_{t+1}$, and $\widehat{q}_t^*$ and $\widehat{q}_t^h$ are the same. Looking at (54) it is clear that an increase in the variance of the exchange rate in isolation would tend to make the foreign country save more by investing in the home-currency bond. This is so because the variability of the exchange rate makes the intertemporal marginal rate of substitution regarding foreign currency (q_t^f) higher. Moreover, it is always true that $\left(var(\widehat{S}_t) - cov(\widehat{S}_t, \widehat{S}_{t+1})\right) > 0$ since $var(\widehat{S}_t) = var(\widehat{S}_{t+1})$. If this were the whole effect then we could have safely concluded that in the true (stochastic) steady state, the foreign holdings of home-currency denominated bonds are positive. However, the covariance terms at the end of the expression alter this result. In particular, $cov(\widehat{S}_t, \widehat{q}_t^*)$ and $cov(\widehat{S}_{t+1}, \widehat{q}_t^*)$ are both negative, where the first of these covariances dominates due to the stationarity of the model. Further, this covariance is sufficiently negative that it makes the overall expression $E(\widehat{q}_t^f) - E(\widehat{q}_t^h) < 0$. Notice that $q_t^* = \frac{P_t^* C_t^{*\rho}}{P_{t+1}^* C_{t+1}^{*\rho}} = \frac{C_{t+1}^{*-\rho}}{P_{t+1}^*} / \frac{C_t^{*-\rho}}{P_t^*}$ which is the ratio of the marginal utility of consumption between periods $(t+1)$ and t. Therefore, one could interpret the covariances between that ratio and the exchange rate as a risk premium associated with the investing in a foreign currency. As a result of this risk premium, the foreign agent's desire to save is less than that of the home agent, and the stochastic steady state implies a net foreign debt for the foreign country.

Table 1: Summary of Welfare Effects

CASE		SHOCK: Both	Monetary	Technology
Basic Cases:				
1) Benchmark case (comparable to Obstfeld-Rogoff 2001)		-0.144	-0.068	-0.076
2) Local Currency Pricing Case (comparable to Devereux-Engel 2001)		-0.090	-0.067	-0.023
3) consumpion-leisure complements (ϕ= 0.1) (comparable to Bachetta-van Wincoop 2000)		-0.181	-0.015	-0.167
4) consumpion-leisure substitutes (ϕ = 10)		-0.242	-0.136	-0.106
Sensitivity Analysis:				
5) Flexible Prices		-0.078	0.000	-0.078
6) Unitary elasticity of sub between home and foreign goods (μ=1)		-0.109	-0.052	-0.057
7) Unitary money demand elasticity (ε=1)		-0.127	-0.051	-0.076
8) No investment case		-0.160	-0.061	-0.092
9) Higher import share (a = 0.5)		-0.153	-0.072	-0.082
10) Higher risk aversion (ρ = 30)		-0.147	-0.009	-0.139
Habit Persistence Cases:				
11) Basic - Flexible Exchange rate		-4.554	-2.106	-1.628
12) Fixed Exchange rate		-3.133	-0.990	-2.245
13) Forward contracts		-4.132	-2.927	-1.355
Asymmetric Asset Market Cases:				
14) Basic - Flexible Exchange rate	home:	0.384	1.060	-0.648
	foreign:	-0.828	-1.083	0.266
15) Fixed Exchange rate	home:	0.012	-0.007	0.020
	foreign:	-0.434	-0.008	-0.427
16) Forward contracts	home:	0.118	-0.017	0.134
	foreign:	-0.292	-0.014	-0.278

All welfare effects are computed in terms of the change in steady state consumption that would have the equivalent effect.

Table 2: Benchmark and Related Cases

	(1) Benchmark	(2) Flexible Prices	(3) LCP	(4) Cons.-Leis. Complements	(5) Cons.-Leis. Substitutes
Standard deviations:					
consumption	1.24	1.22	1.04	1.06	2.41
output	1.87	1.37	3.91	3.45	3.12
investment	5.61	5.00	6.16	5.78	6.87
exchange rate	8.02	7.45	9.16	8.03	8.01
Stochastic steady state deviations[1]:					
consumption	-0.036	0.016	-0.053	-0.032	0.017
leisure	0.013	0.035	-0.025	-0.022	0.082
output	-0.015	0.055	-0.041	-0.040	0.062
capital stock	-0.034	0.118	-0.014	-0.070	0.276
interest rate	-0.054	-0.031	-0.103	-0.018	-0.119
markup ratio	0.341	0.000	0.214	0.652	0.391
net foreign assets	0.000	0.000	0.000	0.000	0.000
trade volume	0.301	0.293	0.488	1.183	-0.037
Welfare effects (as percentage of steady state consumption):[2]					
u-overall	-0.144	-0.078	-0.090	-0.181	-0.242
u-variance	-0.098	-0.071	-0.056	-0.174	-0.202
u-mean	-0.046	-0.008	-0.035	-0.007	-0.039

[1]Percent difference between stochastic steady state and deterministic steady state.

[2]Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.

Foreign variables are identical to home in the cases shown here.

Table 3: Habit Persistence Cases

	(1) Flexible Exch. Rate	(2) Fixed Exch. Rate	(3) Forward Contracts
Standard deviations:			
consumption	1.19	1.18	1.18
output	1.63	1.60	1.64
investment	5.54	3.91	5.63
exchange rate	8.00	0.00	7.45
Stochastic steady state deviations[1]:			
consumption	-0.019	-0.007	-0.008
leisure	0.035	0.025	0.016
output	0.008	0.015	0.004
capital stock	0.072	0.112	0.075
interest rate	-0.039	-0.028	-0.042
price markup	0.250	0.137	0.241
net foreign assets	0.000	0.000	0.000
trade volume	0.067	0.070	0.068
Welfare effects (as percentage of steady state consumption):[2]			
u-overall	-4.554	-3.133	-4.133
u-variance	-2.769	-1.800	-3.318
u-mean	-1.991	-1.443	-0.931

[1]Percent difference between stochastic steady state and deterministic steady state.

[2]Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.

Foreign variables are identical to home in the cases shown here.

Table 4: Asymmetric Asset Market Cases

	(1) Flexible Exch. Rate	(2) Fixed Exch. Rate	(3) Forward Contracts
Standard deviations:			
consumption	1.29	1.30	1.12
output	1.77	1.78	1.08
investment	5.89	5.89	5.44
exchange rate	8.58	0.00	7.00
Stochastic steady state deviations[1]:			
consumption (home)	0.192	0.092	0.068
consumption (foreign)	-0.175	-0.043	-0.057
leisure (home)	-0.505	-0.134	-0.162
leisure (foreign)	0.658	0.294	0.225
output (home)	-0.509	-0.148	-0.156
output (foreign)	0.562	0.247	0.198
capital stock (home)	-0.478	-0.148	-0.418
capital stock (foreign)	0.379	0.167	0.167
interest rate	-0.470	-0.216	-0.084
price markup	0.146	0.068	0.094
net foreign assets	74.147	27.416	18.957
trade volume	0.598	0.326	0.423
Welfare effects (as percentage of steady state consumption):[2]			
u-overall (home)	0.384	0.012	0.118
u-overall (foreign)	-0.828	-0.434	-0.292
u-variance	-0.182	-0.178	-0.071
u-mean (home)	0.570	0.191	0.189
u-mean (foreign)	-0.651	-0.259	-0.292

[1]Percent difference between stochastic steady state and deterministic steady state.

[2]Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.