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ENDOGENOUS NONTRADABILITY AND MACROECONOMIC IMPLICATIONS

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ABSTRACT

This paper proposes a new way of thinking about nontraded goods in an open economy macro model. It develops a simple method for analyzing a continuum of goods with heterogeneous trade costs, and it explores the endogenous decision by a seller of whether to trade a good internationally. This way of thinking is appealing in that it provides a natural explanation for a prominent puzzle in international macroeconomics, that the relative price of nontraded goods tends to move much less volatilely than the real exchange rate. Because nontradedness is an endogenous decision, the good on the margin forms a linkage between the prices of traded and nontraded goods, preventing the two price indexes from wandering too far apart. The paper goes on to find that this mechanism has implications for other macroeconomic issues that rely upon the presence of nontraded goods.

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1. Introduction

While open economy macroeconomics by definition analyzes trade across national borders, the field has long found it useful to allow for the fact that some portion of goods tend not to be traded internationally. The idea of nontraded goods has played the central role in some important models in the field over time. Balassa (1964) and Samuelson (1964) used nontraded goods to help explain why real exchange rate levels differ between countries. Dornbusch (1983) used them to show how such real exchange rate movements over time may limit intertemporal trade and shape the current account. And Stockman and Tesar (1995) used them to help explain some key features of international business cycles. But in all these models, the share of nontraded goods is taken to be exogenously determined; a good is by nature either tradable in the international market, or it is by nature not tradable.

This paper proposes a different way of thinking about nontraded goods, which builds upon the idea that whether a good is traded in the international market is an endogenous decision of the domestic seller. The paper proposes a very simple approach for dealing with a continuum of goods with heterogeneous trade costs in the context of a general equilibrium macro model. This way of thinking is appealing, in that it is found to help to explain certain puzzles in the international data.

This research is related to advances in the international trade literature dealing with heterogeneity in goods. Beginning with Dornbusch, Fisher and Samuelson (1977), there has been an interest in seeing how trade patterns along a continuum of goods are determined endogenously, including a range of goods that remain untraded due to trade costs. Recent work has proposed clever ways of parameterizing such firm heterogeneity. But in all this work, goods are ranked by their productivities, while the size of trade costs are assumed to be uniform across goods. Those goods with the greatest comparative advantage in one country or the other are traded, while those goods with small gains from trading relative to the uniform trade costs remain nontraded.

In contrast to this convention, we think that when the issue of primary interest is nontradedness rather than comparative advantage, it makes more sense to focus on the variation of trade costs among goods[1]. Clearly some goods are much more difficult to trade than others, and the identity of a good as traded or nontraded is likely to be determined by this factor more strongly than any other. For example, the reason that many types of services are nontraded is not because countries are so similar in their productivities in these sectors; rather, they remain nontraded primarily because such services are particularly difficult to trade over long distances. So to analyze the particular issue of nontradedness, our model focuses on this dimension of variation, ranking goods in the continuum by the cost of transporting them. We then develop a simple method to solve and analyze a model in the context of such a continuum.

Empirical work by Hummels (1999, 2001) has emphasized that trade costs -- including tariff and nontariff barriers, shipping costs, and other associated costs of marketing and distribution -- vary greatly across classes of goods and play an important role in trade decisions. Collecting detailed trade data for individual goods, he finds that freight costs alone can range from more than 30 percent of value for raw materials and mineral fuels down to 4 percent for some manufactures. Depending on factors such as weight, distance, and the time sensitivity of demand, trade costs can be high and variable for many manufactured goods as well. Hummels (2001) documents that in 1998 a substantial proportion

[1] The macro model here will differ also in several other respects from the related trade literature. The model describes a small open endowment economy where world price levels are exogenously given. We abstract from production and entry decisions. We also abstract from monopolistic competition and markup pricing by firms. In this context, we do not need fixed costs of trade to induce some firms to forgo international trade, but iceberg costs alone are sufficient. See section 2 for details of the model.

of U.S. trade was airshipped with air-freight costs typically amounting to 25 percent of transported good value in some cases.[2]

Empirical work has also found support for the idea that some goods do switch over time between status as traded and nontraded. Using a panel of U.S. manufacturing plants from 1987 to 1997, Bernard and Jensen (2001) find that year to year transition rates are noteworthy: on average 13.9% of non-exporters begin to export in any given year during the sample, and 12.6% of exporters stop. It should be noted that the results of this paper in no way rely upon implausibly large numbers of firms switching between traded and nontraded status, but rather upon the simple fact that firms have the ability to make such a switch. In fact, we show that the results of the model here are the strongest in those cases where only a small number of firms actually do switch in equilibrium.

The approach we develop to incorporate trade cost heterogeneity is remarkably simple. It takes a standard intertemporal open economy model, and adds one endogenous variable, the share of nontraded goods. The equilibrium value of this variable is pinned down by one additional equilibrium condition, relating the share of nontraded goods to the price of nontraded goods. We posit a particular distribution for trade costs, which implies that the share of nontradeds has a very simple and tractable relationship to nontraded prices.

This way of looking at things is appealing in that it provides a very natural explanation for a puzzling stylized fact in international economics. The relative price of the nontraded goods aggregate to traded goods tends to move much less volatilely than the real exchange rate. Empirical measures in Betts and Kehoe (2001a) indicate that movements in the relative price of nontraded goods are only

[2] Even these measured trade cost margins may be severely biased downwards. Average transportation cost measures that weight costs of individual goods by the value of observed trade flows underestimate costs to the extent that goods with higher costs are traded less. Second, if vertical production arrangements imply transshipment of raw materials and intermediate goods, the cumulative transportation costs can be much higher than those on the exports of the final product alone.

about 37% as large as movements in the real exchange rate. Empirical work by Engel (1993, 1999) indicates this ratio may be a good deal smaller yet. This fact stands in contrast to many standard theoretical models, such as that used by Balassa-Samuelson, which presume traded goods are constrained by the law of one price and explain movements in the real exchange rate primarily in terms of movements in the relative price of nontraded goods.

Our paper proposes a very simple but powerful explanation for this empirical regularity. Because trading a good on the international market is endogenous, on the margin there is a seller who is indifferent between selling his good domestically only or branching out into the international market. As a result, this marginal nontraded good forms a link between the prices of goods that are traded and other similar goods that are nontraded. In the aggregate, this linkage prevents the two price indexes of traded and nontraded goods from wandering too far apart.

More precisely, the model considers a small open economy with a continuum of home goods with a distribution of trade costs. The country will tend to export those goods with low trade costs, but depending on domestic demand conditions, the cutoff along the continuum between traded goods and nontraded goods can shift. We find that a rise in demand will tend to push more goods into the nontraded category. This means that a rise in demand will not raise the price of nontraded goods as much as found in earlier research, because the quantity of nontraded goods will increase in response. Simultaneously, as the number of traded goods falls to include those with lower trade costs, the domestic price of traded goods rises. As a result, the price of nontradeds rises less and the price of tradeds moves more. In other words, the prices of traded and nontraded goods will tend to move together when they move.

This research is related to other recent work on trade costs in macroeconomic models, notably Obstfeld and Rogoff (2000), Betts and Kehoe

(2001a, 2001b), and Bergin and Glick (2002). However, we find an extraordinarily tractable way of introducing trade costs, which allows us to consider a continuum of goods and still have discrete changes of status of goods between being traded and fully nontraded. This is not true of the previous papers. Obstfeld and Rogoff (2000) only consider the case of one home good that switches between traded and nontraded status; Bergin and Glick (2002) extend this to two goods. By integrating over a continuum of goods, our approach allows us to avoid the difficulty implied by the Kuhn-Tucker conditions, whereby the set of relevant equilibrium conditions for a good switches discontinuously as the good switches between traded and nontraded status. Betts and Kehoe (2001b) allow heterogeneous trade costs and varying degrees of tradability to play a role in explaining relative goods prices, as we do. But unlike their model, ours allows a range of goods that take on the status of being fully nontraded, and thus permits us to derive and analyze the share of nontraded goods.

The mechanism developed here is sufficiently simple, that we think it has the potential for being applied to a wide range of models to analyze a wide range of macroeconomic issues. For example, we also explore the implications of endogenous nontradability for the issue of intertemporal trade. Previous work assuming exogenously nontraded goods (Dornbusch, 1983) found that the presence of nontraded goods strongly discourages intertemporal trade and current account imbalances. We find that if nontradedness is endogenous, the share of nontradeds will tend to adjust so as to minimize this friction. Further, in contrast to recent models that allowed for one or two endogenously nontraded goods (Obstfeld and Rogoff, 2000; Bergin and Glick, 2002), trade costs that vary along a continuum of goods imply that the cost of intertemporal trade rises smoothly in relation to the size of the current account imbalance. That is, there are no sudden jumps in the cost of intertemporal trade.

2. Model

To focus on the issue of nontradedness, we follow Obstfeld and Rogoff (2000) in considering a very simple small open endowment economy. This country is endowed with a continuum of goods indexed by i on the unit interval, where y_i represents the level of endowment, c_i is the level of consumption, and p_i is the price level of this good. All of these home goods have the potential of being exported, but some endogenously determined fraction of the goods, n, will be nontraded in equilibrium. The small open economy may also import foreign goods for consumption purposes, with consumption level c_F and price level p_F. We initially omit time subscripts in the notation, but introduce them when extending the framework to two periods.

The aggregate consumption index is specified as:[3]

$$c = \frac{c_H^{\theta} c_F^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}. \tag{1}$$

Here c_H is an index of home goods consumption:

$$\begin{aligned} c_H^{(\phi-1)/\phi} &= \int_0^n \left(c_i\right)^{(\phi-1)/\phi} di + \int_n^1 \left(c_j\right)^{(\phi-1)/\phi} dj \\ &= n\left(\frac{c_N}{n}\right)^{(\phi-1)/\phi} + (1-n)\left(\frac{c_T}{1-n}\right)^{(\phi-1)/\phi} \end{aligned} \tag{2}$$

where c_N and c_T are consumption indexes of nontraded and traded goods, and n is the share of goods on the continuum $\{0,1\}$ that are nontraded.[4] Price indexes are

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[3] For simplicity we limit ourselves here to a Cobb-Douglas specification, implying a unitary elasticity of substation between home and foreign goods. Empirical work on this elasticity suggests a value between 0.5 and 1.5, with our value of 1 in the middle; e.g., see Pesenti (2002). In the present case, the Cobb-Douglas specification has the added benefit of making the algebraic results more easily interpretable; the appendix shows results for the more general CES case.

[4] Equation (2) implicitly defines

$$c_N \equiv n\left(\frac{1}{n}\int_0^n c_i^{(\phi-1)/\phi} di\right)^{\phi/(\phi-1)}, \; c_T \equiv (1-n)\left(\frac{1}{1-n}\int_n^1 c_j^{(\phi-1)/\phi} dj\right)^{\phi/(\phi-1)}.$$

defined as usual for each category of goods, in correspondence to the consumption indexes above:

$$p = {p_H}^{\theta} {p_F}^{1-\theta} \tag{3}$$

$$\begin{aligned} {p_H}^{1-\phi} &= \int_0^n \left(p_i\right)^{1-\phi} di + \int_n^1 \left(p_j\right)^{1-\phi} dj \\ &= n{p_N}^{1-\phi} + (1-n)\,{p_T}^{1-\phi} \end{aligned} \tag{4}$$

where p is the aggregate price level, p_H is the price index of all home goods, p_N is the price index of nontraded goods, and p_T is the price index of traded goods.[5] Note that because the world prices are normalized to unity, p may be interpreted as the reciprocal of the country's real exchange rate.

The home goods are distinguished from each other by the presence of good-specific iceberg costs, (τ_i) where a certain fraction of the good disappears in transport. We assume that the home country pays for this cost, so that normalizing the world price of each good i to unity, the domestic price will be $p_i = \dfrac{1}{1+\tau_i}$ if the country exports good i.[6] These trade costs are specified to follow the distribution:

$$\tau_i = \alpha i^{-\beta} - 1;\ \alpha > 0, \beta \geq 0$$

which implies the following distribution of export prices

$$p_i = \frac{1}{1+\tau_i} = \frac{i^{\beta}}{\alpha}. \tag{5}$$

The parameter β controls the curvature of the distribution, while α controls the level.[7] Figure 1 illustrates how the distribution of export prices varies with

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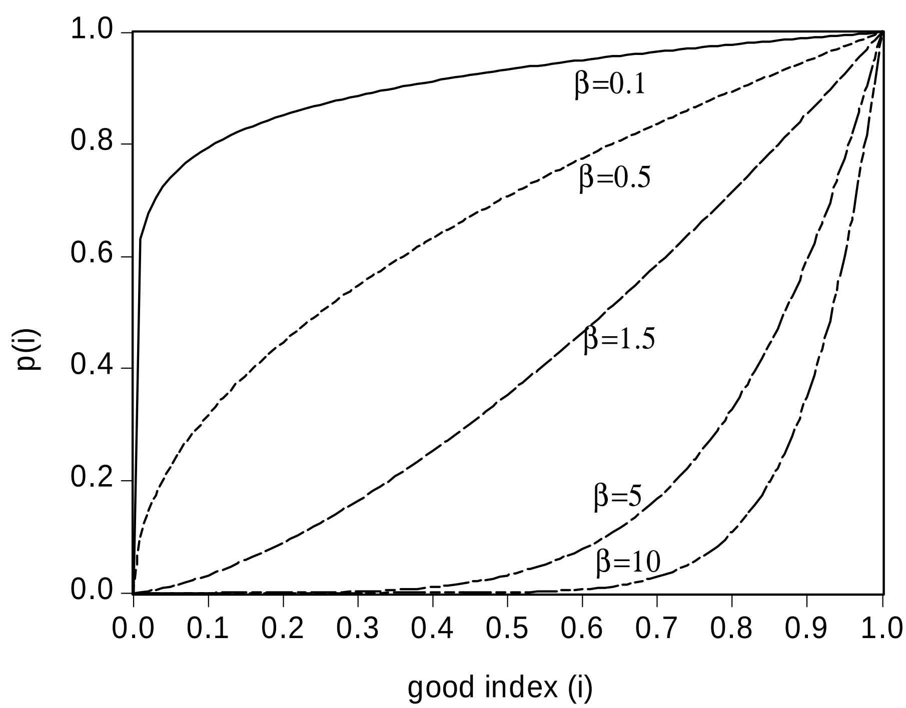
[5] Equation (4) implicitly defines $p_N \equiv \left(\frac{1}{n}\int_0^n {p_i}^{(\phi-1)/\phi} di\right)^{\phi/(\phi-1)}$, $p_T \equiv \left(\frac{1}{1-n}\int_n^1 {p_j}^{(\phi-1)/\phi} dj\right)^{\phi/(\phi-1)}$.

[6] The presence of trade costs (obviously) implies segmentation between domestic and foreign markets.

[7] This cost distribution is related to the Pareto function, where α is the "scale" parameter and β is the "shape" parameter.

β (assuming $\alpha = 1$). The goods at the left end of the continuum (i near 0) tend to have lower prices when exported because the trade cost is large; these goods are less tradable. Goods toward the right end of the continuum (i near 1) have higher prices because the trade cost is low; they are more tradable. β characterizes how quickly the price of an individual good rises with the goods index -- in fact, it can be viewed as an elasticity. For example, for a high β, the percent change in costs is high for a given percent change in the index.

Fig. 1: Price of good if traded $(\alpha = 1)$



Given the cutoff between traded and nontraded goods at index n, it is straightforward to compute the price index for traded goods:

$$p_T = \left(\left(\frac{1}{1-n} \right) \left[\int_n^1 \left(\frac{i^\beta}{\alpha} \right)^{1-\phi} di \right] \right)^{1/(1-\phi)}$$

$$= \left(\left(\frac{1}{1-n} \right) \alpha^{\phi-1} \left[\frac{1}{1-[\beta(\phi-1)]} \left(i^{1-[\beta(\phi-1)]} \right) \right] \Bigg|_n^1 \right)^{1/(1-\phi)} \tag{6}$$

$$= \left(\frac{1}{\alpha} \right) \left(\left(\frac{1}{1-n} \right) \left(\frac{1}{\omega} \right) \left\{ \left(\frac{1}{n} \right)^{\omega} - 1 \right\} \right)^{1/(1-\phi)}$$

where we define $\omega \equiv \beta(\phi-1)-1$, $\omega \geq -1$(since $\beta \geq 0$ and $\phi > 1$).[8] Keep in mind that this n is itself an endogenous variable that will be solved as part of the general equilibrium system. Equation (6) expresses the price of traded goods as a function of the share of traded goods n, the elasticity of substitution across domestic goods ϕ, and the trade cost parameters, β and α. It is straightforward to establish that $\frac{\partial p_T}{\partial n} > 0$; i.e. the price of traded goods increases with the share of nontraded goods. The reason is that, as the proportion of home goods that are nontraded rises, it is no longer profitable to export goods with marginally higher trade costs; as these goods are withdrawn from export markets, the average price of the remaining export goods rises.

The price index of nontraded goods is even easier to determine. As usual, intratemporal optimization implies relative demands for each pair of home goods *i* and *j*:

$$\frac{c_i}{c_j} = \left(\frac{p_i}{p_j} \right)^{-\phi}.$$

Since consumption must equal the endowment of nontraded goods, and since we assume uniform endowments for all goods here, (i.e. $y_i = y$ for all *i*) we can

[8] Note $p_T \geq 0$ with our specification of trade costs, since for $0 > \omega \geq -1$ and $\omega > 0$, it follows that

conclude that for any pair of nontraded goods it will be true that $c_i = c_j = y$, and so $p_i = p_j$.[9] In other words, the price of each nontraded good will be identical, because they each are by definition not affected by the trade costs which vary by good. This logic applies equally well to the home good that is just on the margin between being traded and nontraded (*i*=*n*). But because this good is on the margin of being traded, the domestic price must be the same as that as if it were sold in the world market: $p_n = n^{\beta}/\alpha$. As a result, the price index of nontraded goods is pinned down as the price of the marginal traded good:

$$p_N = \left(\left(\frac{1}{n}\right)\left[\int_0^n (p_i)^{1-\phi}\, di\right]\right)^{1/(1-\phi)} = \left(\left(\frac{1}{n}\right)\left[\int_0^n (p_n)^{1-\phi}\, di\right]\right)^{1/(1-\phi)} = p_n$$
$$= \frac{n^{\beta}}{\alpha}. \tag{7}$$

This equilibrium condition will be important in the analysis to follow, and it will be referred to as the "marginal nontraded condition." It implies that the price of nontraded goods rises with the share of nontraded goods with elasticity β . Figure 2 below illustrates how this equilibrium price level varies with the share of nontraded goods (still assuming $\alpha = 1$).

It is easily verified that there can be no discontinuous jump in price either up or down between the last nontraded good and the first traded good. Note that the iceberg trading costs for adjacent goods are essentially identical and that there is no fixed cost to trade. Suppose that the price of the first traded good jumped discontinuously above the price of the last nontraded good; then it would be profitable for the last nontraded good to become traded instead. Similarly, suppose

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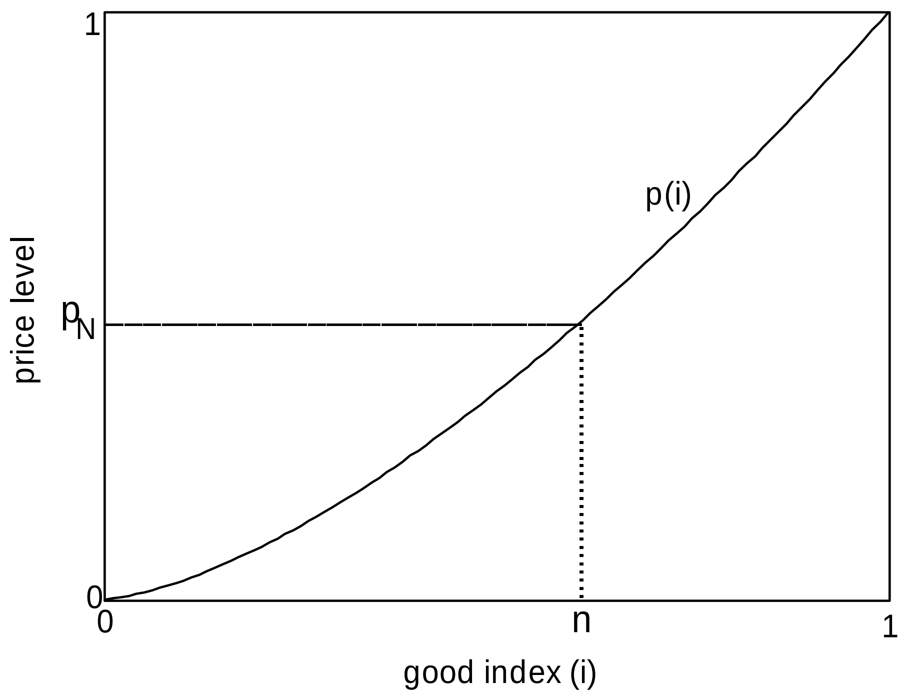
$(1/\omega)\left\{n^{-\omega} - 1\right\} > 0$ for $1 \geq n \geq 0$; for $\omega = 0$, $p_T = \left(\frac{-\log(n)}{\alpha(1-n)}\right) \geq 0$ as well.

[9] This assumption can be relaxed without undermining our ability to compute a price index for nontraded goods; the only difference is that the distribution of productivities and endowments would have to be included in the integral, making the resulting price index more complicated. Because our focus here is on the role of heterogeneous trade costs, we utilize the assumption of uniform endowments to make the results more transparent.

that the price of the first traded good jumped discontinuously below the price of the last nontraded good; then it would be profitable for the first traded good to become nontraded instead.

Fig. 2: Aggregate price level of nontraded goods

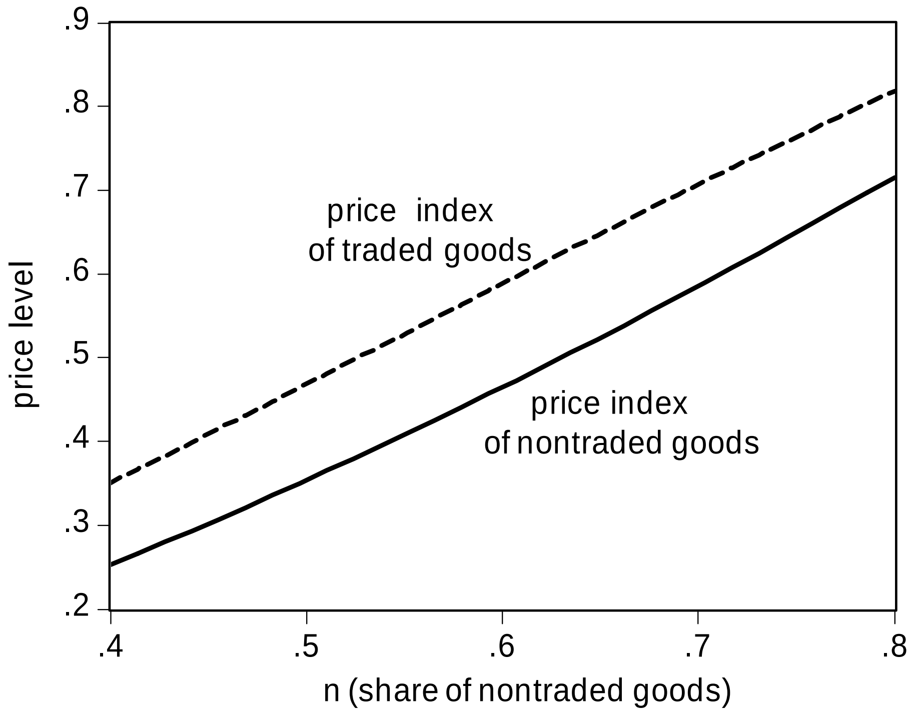
(shown for $\beta = 1.5, \quad \alpha = 1$)



The price indices of traded and nontraded goods are related to each other. Figure 3 shows their relationship as the share of nontraded goods varies. Observe that (i) p_T is everywhere higher than p_N, since traded goods are less costly to transport, (ii) both p_N and p_T rise with n, and (iii) p_N rises at a (slightly) faster rate than p_T, implying that the relative price of nontradeds rises.

Fig. 3: Price indexes of traded and nontraded goods as a function of *n*

(shown for $\beta = 1.5, \quad \alpha = 1, \quad \phi = 10$)



This last point can be seen more formally by using equations (6) and (7) to obtain

$$\frac{p_N}{p_T} = \left[\left(\frac{n}{1-n} \right) \left(\frac{1}{\omega} \right) \left(1 - n^{\omega} \right) \right]^{\frac{1}{\phi - 1}}$$

where once again $\omega \equiv \beta(\phi - 1) - 1$. This expression captures how the relative price structure is pinned down by the parameters of our model: the share of nontraded goods *n*, the elasticity of transportation costs β, and the elasticity of substitution of home goods ϕ.[10]

[10] Note that the absence of trade cost heterogeneity ($\beta = 0$) implies $\omega = -1$ and $p_N = p_T$

As additional equilibrium conditions, intratemporal optimization implies the demand functions:

$$c_N = n\left(\frac{p_N}{p_H}\right)^{-\phi} c_H \tag{8}$$

$$c_T = (1-n)\left(\frac{p_T}{p_H}\right)^{-\phi} c_H \tag{9}$$

$$c_H = \theta\left(\frac{p_H}{p}\right)^{-1} c \tag{10}$$

$$c_F = (1-\theta)\left(\frac{p_F}{p}\right)^{-1} c \tag{11}$$

It is assumed that residents of the small open economy must pay the cost of transport for imports of foreign goods. The price of foreign goods is normalized to unity in the world market, so its domestic price is set exogenously as

$$p_F \equiv 1+\tau_F = \alpha_F$$

for some given τ_F representing iceberg trade costs for imported goods. Given that the world price level is normalized to unity, the reciprocal of the aggregate home price level p may be interpreted as the real exchange rate for this small open economy.

Market clearing for nontraded goods requires

$$c_N = \int_0^n y_i \frac{p_i}{p_N} di = ny \tag{12}$$

given our assumption $y_i = y$ for all *i* and that $p_i = p_n = p_N$ for all $i \in \{0, n\}$.

The goods market described above will be analyzed in the context of a two-period model with a representative consumer. The consumer maximizes two-period utility

$$\delta U(c_1) + U(c_2)$$

subject to the intertemporal budget constraint.

$$\left(\frac{p_{H2}}{p_2}y_{H2}-c_2\right)=-\left(\frac{p_1}{p_2}\right)(1+r)\left(\frac{p_{H1}}{p_1}y_{H1}-c_1\right). \tag{13}$$

Here r is the world interest rate. The term δ is an exogenous discount factor that can change, thereby allowing us to consider shifts in demand from one period to the next. Intertemporal optimization implies the usual intertemporal Euler equation:

$$U'_{c1}=\frac{1}{\delta}(1+r)U'_{c2}. \tag{14}$$

Equilibrium here determines values each period for the variables c_t, c_{Ht}, c_{Tt}, c_{Nt}, c_{Ft}, p_t, p_{Ht}, p_{Tt}, p_{Nt}, n_t, satisfying equations (3-4, 6-12) for each period as well as the intertemporal budget constraint (13) and the intertemporal consumption Euler equation (14). This system is identical to a standard two-period model, with the addition of one extra endogenous variable, n, which is pinned down by one additional equilibrium condition, the marginal nontraded condition (7).

3. Results

A) Solution for the share of nontraded goods under balanced trade

Consider first a static version of the model where δ is constant at a value of unity. We will refer to this version as a steady state of the model, in that consumption and all other variables are constant across the two periods. According to the intertemporal budget constraint, the value of domestic production equals the value of domestic consumption in this case, and trade is balanced: $p_{H1}y_{H1}-p_1c_1=p_{H2}y_{H2}-p_2c_2=0$. In this balanced trade equilibrium, the equilibrium conditions above can be solved together to find a condition determining the steady state share of nontraded goods, $\bar{n}$:

$$\frac{1}{\omega}\left[n^{\beta+1}(1+\omega)-n^{\beta\phi}\right]=\theta\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right] \tag{15}$$

where $\omega \equiv \beta(\phi-1)-1$ (and time subscripts are still omitted). See the appendix for the derivation. It is easily verified that there exists a unique solution for the share of nontraded goods in condition (15) which lies within the permissible range of zero to one (see the appendix).

One observation is that the curvature parameter in the distribution of trade costs (β) plays an important role in determining the share of nontraded goods. Table 1 reports numerical simulations for a benchmark calibration of $\phi=10$, $\alpha=1$, $\theta=0.5, \tau_F=0.1$. Column 2 shows that a rise in β progressively raises the share of home goods that are nontraded. This result is fairly intuitive: if trade costs rise very quickly as one exports more classes of goods, it is optimal to export a smaller number of classes of goods. A country should then concentrate its exports in those commodities for which international trade is so much less costly.

Another important determinant of nontradedness is the elasticity of substitution between home goods (ϕ). Table 2 shows in column 2 that as this elasticity rises, $\bar{n}$ rises gradually. The intuition is that if home goods are highly substitutable in consumption, one can conserve on trade costs by concentrating one's exports in the goods that are easiest to trade. This means there will be a smaller quantity of these particular classes of goods to consume, but under a high elasticity, it is easy to compensate for this by consuming a greater quantity of other types of goods. On the other hand, if home goods were less substitutable with each other, one would want to consume a more even distribution of home goods, thereby requiring the country to export a smaller portion of a larger number of goods to pay the bill for imports.

The scale parameter in the distribution of trade costs, α, does not appear in equation (15) above. When one considers the effects of trade costs here, it is their relative levels between goods (summarized in β), not their overall level (summarized in α) which determines the varieties of goods that are nontraded. In part, this last implication results from the assumption of Cobb-Douglas preferences over home and foreign goods, which is a common assumption in this literature, known to have certain implications that help simplify analytical solutions.[11] Some intuition can be found in the fact that a unitary elasticity of substitution between home and foreign goods implies that a constant share of consumption expenditure goes toward foreign goods, regardless of the relative price between goods, and hence regardless of the size of transport costs. A sufficient quantity of home goods then must be traded and exported to pay for these imports under balanced trade.[12]

However, if we consider a more general CES specification between home and foreign goods, the scale of trade costs does affect the share of nontraded goods. See the appendix for derivation of the counterpart to equation (15) for the CES case (equation 15'). For an elasticity of substitution between home and foreign goods greater than unity, it can be confirmed that a rise in the scale of trade costs (α) raises the share of nontraded goods, $\bar{n}$, as one might expect. But this result can as easily be reversed: if the elasticity between home and foreign goods is less than unity, a rise in α lowers $\bar{n}$. For a unitary elasticity, as shown here for the Cobb-Douglas case, α has no effect. Empirical work on this elasticity suggests a value between 0.5 and 1.5, with our value of 1 in the middle (see Pesenti, 2002).

[11] See Corsetti and Pesenti (2001) for an example.

[12] Condition (A6) in the appendix shows that under balanced trade and Cobb-Douglas preferences, a constant fraction of home goods will be consumed domestically and a constant fraction will be exported, without any regard for the relative price of home to foreign goods. Because the scale parameter of transport costs enters only through price terms, it does not enter in this condition.

B) Implications for the relative price of nontraded goods

If we wish to solve for the dynamics of the model when trade is not restricted to be balanced, the equilibrium conditions cannot be summarized in a single equation as in (15); instead there is a system of three equations that must be solved numerically for n_1, n_2, and c_2, given a value of c_1.

$$y_1 n_1^{\beta\phi} \left\{ \frac{1}{\omega} \left[n_1^{-\omega} (1+\omega) - 1 \right] \right\}^{\frac{1-\phi-\theta}{1-\phi}} = \theta \alpha^{1-\theta} p_{F1}^{1-\theta} c_1 \tag{16}$$

$$y_2 n_2^{\beta\phi} \left\{ \frac{1}{\omega} \left[n_2^{-\omega} (1+\omega) - 1 \right] \right\}^{\frac{1-\phi-\theta}{1-\phi}} = \theta \alpha^{1-\theta} p_{F2}^{1-\theta} c_2 \tag{17}$$

$$\begin{aligned} c_2 = & \left[(1+r) \left(\frac{y_1 \left[1 + n_1^{\beta+1} \beta \right]}{\beta+1} - \left(\left\{ \left[n_1^{-\omega} \left(\frac{1+\omega}{\omega} \right) - \frac{1}{\omega} \right]^{\frac{1}{1-\phi}} \right\}^{\theta} (\alpha p_{F1})^{1-\theta} c_1 \right) \right) \right. \\ & \left. + \frac{y_2 \left[1 + n_2^{\beta+1} \beta \right]}{\beta+1} \right] \bullet \left\{ \left[n_2^{-\omega} \left(\frac{1+\omega}{\omega} \right) - \frac{1}{\omega} \right]^{\frac{1}{1-\phi}} \right\}^{-\theta} (\alpha p_{F2})^{\theta-1} \end{aligned} \tag{18}$$

See the appendix for derivations. The intertemporal Euler equation (14) allows one to interpret percent changes in the level of c_1 as taste shocks to the parameter δ.

Columns (3-6) of Table 1 show the dynamics for the model for various values of β. This is done for the case of a shock to δ that raises period-one consumption by 1.5 percent relative to its steady state level under balanced trade. (This is the standard deviation of U.S. consumption typically used in calibration studies.) The benchmark calibration will be used again here: $\phi = 10, \alpha = 1, \theta = 0.5, \ \tau_F = 0.1, r = 0$.

Table 1: Role of β

			Endogenous n model:			Exogenous n*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
β	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$	$\frac{sdev(p_N/p_T)}{sdev(p)}$
0.1	0.1966	3.5350	1.5243	0.0023	0.0008	5.5679
0.5	0.4507	0.8988	0.7970	0.0059	0.0038	2.5509
1.5	0.5802	0.3810	0.5542	0.0124	0.0103	2.1689
5	0.7184	0.1623	0.2813	0.0209	0.0193	2.0500
10	0.7963	0.1109	0.1651	0.0246	0.0232	2.0251

Benchmark parameter values: $\phi = 10, \alpha = 1, \theta = 0.5, \tau_F = 0.1, r = 0$.
Computed for a taste shock that leads to a 1.5% rise in period one consumption.
The volatility of variables, reported as 'sdev,' is computed as the percent deviation of the value in period 1 from the steady state value.
*Computed for the corresponding level of $\bar{n}$, to facilitate comparison with the endogenous n case.

Column (3) shows that the volatility of the relative price of nontraded goods depends a great deal on the curvature parameter β. As there is only one intertemporal shock in this two-period model, this column reports the percentage "standard deviation" of $\frac{p_N}{p_T}$ as $\log\left(\frac{p_{N1}/\overline{p_N}}{p_{T1}/\overline{p_T}}\right)$, where overbars indicate levels in the balanced trade steady state. This volatility is reported as a ratio to the percentage standard deviation in the real exchange rate for p computed in the same manner. This relative volatility falls dramatically as the curvature of trade costs rises, and for a value of $\beta = 1.5$, the model is able to approximately replicate the value of 0.37 found in the empirical study by Betts and Kehoe (2001a).[13] Empirical work by Engel (1999) finds that the volatility of nontraded prices may yet be lower than this, but the table shows that the model is capable of replicating even very low values of volatility as the curvature parameter β is assumed to be progressively larger.

This result stands in sharp contrast to the standard result of open economy models in the literature, where the share of nontraded goods is taken to be exogenous. For example the classic Balassa-Samuelson model explains real exchange rate levels exclusively in terms of shifts in the relative price of nontraded goods. The same is true for the well-known two-period model of Dornbusch (1983), which is very similar to the model considered here, except for the assumption that the share of nontraded goods is fixed. Under such an assumption, a rise in consumption demand will tend to push up the price of consumption goods, but this will be expressed only for nontradeds, because the price of traded goods is pinned down to the world price level by arbitrage. A rise in the relative price of nontraded goods is necessary for equilibrium, to convince households to take their extra consumption in the form of additional imports of tradable goods, given that the consumption of nontraded goods is limited by definition to the domestic supply of such goods.

This conclusion is illustrated in column (7) of Table 1, where the movement in the relative price of nontraded goods is solved for a version of the

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[13] The traded goods included in the aggregate price index include only home traded goods and exclude imported foreign goods. This is in part a matter of technical necessity: the model is designed to avoid an a priori demarcation between different types of home goods, so there is no clear way to define a price index combining imported foreign goods together with a subset of goods in the home goods CES index, while excluding other goods in this CES index. Very fortunately, the stylized fact which the model is trying to replicate is defined in precisely the same manner. When Betts and Kehoe (2001a) compute the relative price of nontraded to traded goods, they likewise define p_T in terms of the prices of goods in traded sectors that are produced at home (using either gross output deflators by sector or a domestic producer price index). In addition, the statistic we report for our model likewise reflects Betts and Kehoe by using the full consumer price index for the domestic price level, *p*.

model here where n is taken to be exogenous. The model is identical to the one reported in the earlier columns, except that the "marginal nontraded condition" (equation 7) is dropped. To maintain comparability with the earlier columns of the table the exogenous value of the nontraded share, n, is set at the level of $\bar{n}$ found for the corresponding endogenous nontraded model reported in the preceding columns. Note that it is true for all the cases in the table, that the relative price of nontraded goods moves much less under the assumption of endogenous nontradedness than for the standard assumption of exogenous nontradedness. In fact, it is easy to demonstrate that the ratio of volatilities reported in column (7) must always be greater than unity when n is exogenous. Since the aggregate price level p is a weighted average of nontraded prices (p_N), traded home goods prices (p_T), and import prices (p_F), where the latter two are fixed by world levels, the movement in the first component must always be larger than the movement in the overall average that it induces. This explains why a small open economy model with exogenously determined nontraded goods has such difficulty explaining a low volatility in the price of nontraded goods relative to the overall real exchange rate.

A comparison of columns (3) and (7) makes clear that the one change of making n endogenous has a very dramatic effect on the ability of the model to explain this empirical regularity. The chain of events characteristic of standard models, explained above, no longer applies. Now, as a rise in demand starts to push up the relative price of nontraded goods, some traded goods sellers on the margin will find it profitable to sell more in the home market, to the point of abandoning attempts to market their good abroad where they need to deal with costs of trade. This endogenous rise in the share of nontraded goods allows the supply of nontraded goods to rise, despite the fact that the endowment of each individual good is fixed. This rise in supply reduces the pressure for the relative price of nontradeds to rise in the face of the higher demand.

The marginal condition (equation 7) also helps in understanding this result. Recall that this equation states that the price index of nontraded goods will equal the price of the marginal traded good. This linkage between nontraded and traded prices prevents one price index from straying too far from the other, and thus helps dampen the volatility in their ratio.

It is important to note that this dampened volatility in the relative price of nontradeds does not rule out volatility in the overall price index or real exchange rate here. Columns (5) and (6) in Table 1 show that for high levels of β, the price of nontraded and traded goods tend to move more volatility and in a synchronized fashion. Given that these two prices are important components in the overall CPI, this overall price index moves a good deal. But because the two components are moving in synchronization, the relative price of one in terms of the other is not moving significantly. This explains why the ratio reported in column (3) is able to take on such small values under endogenous nontradedness, whereas it can never take a value less than unity under the assumption of exogenous nontradedness.

Why does this mechanism work best for high values of β? Looking at the marginal condition (equation 7), it becomes clear that β is the elasticity of the nontraded price index with respect to changes in n. It is at high values of β where the demand shock induces a small change in n and a large change in the price of nontraded goods. But this also requires a larger change in the price index of traded goods, so the overall price index changes more. One interesting implication of this logic, is that the mechanism outlined here to explain the stylized fact does not require an implausible degree of movement in the share of nontraded goods. In fact, inspection of column (4) of Table 1 confirms that the mechanism is at its most potent when n moves the least between the two periods.

The curvature parameter is not the only parameter to play an important role in this mechanism. Table 2 shows that a higher elasticity of substitution between home goods (ϕ) also plays an important role. Column (3) shows that as ϕ rises, the volatility in relative nontraded prices as a ratio to that of the real exchange rate falls. Intuitively, if the last nontraded good and the marginal traded good are highly substitutable, this makes the link between their two prices stronger. This in turn strengthens the linkages between the price indexes of traded and nontraded goods.

Table 2: Role of ϕ

Endogenous *n* model:						Exogenous *n**
(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϕ	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$	$\frac{sdev(p_N/p_T)}{sdev(p)}$
5	0.5503	0.7787	0.5470	0.0124	0.0083	2.3895
10	0.5802	0.3810	0.5542	0.0124	0.0103	2.1689
20	0.5937	0.1801	0.5630	0.0126	0.0115	2.0815

Benchmark parameter values: $\beta = 1.5$, $\alpha = 1$, $\theta = 0.5$, $\tau_F = 0.1$, $r = 0$.
Computed for a taste shock that leads to a 1.5% rise in period one consumption.
The volatility of variables, reported as 'sdev,' is computed as the percent deviation of the value in period 1 from the steady state value.
*Computed for the corresponding level of $\bar{n}$, to facilitate comparison with the endogenous *n* model.

C) Implications for the intertemporal price and intertemporal trade

Now that it has been demonstrated that endogenous nontradedness is relevant, inasmuch as it helps explain a puzzling empirical regularity, it is interesting to see what implications this feature has for other issues of interest to international macroeconomics. One such issue is intertemporal trade, the ability of a country to borrow in world financial markets to finance a current account deficit in a given period. It has long been thought that the presence of nontraded goods can be important for intertemporal trade. Dornbusch (1983) demonstrated that when nontraded goods are present, a change in their relative price can discourage intertemporal trade. Looking at the intertemporal budget constraint (equation 13), one sees that the cost of borrowing in foreign markets includes not only the world rate of interest, r, but also the change in the price level or real exchange rate over time. Since borrowing takes place in units of the world consumption index, a change in the relative price of home to foreign goods affects the cost of repaying the loan. In particular, if a temporary rise in consumption induces a temporary rise in the domestic price level, the expected fall in price for the next period implies that repayment of the loan will be larger in units of the home consumption index than implied by the interest rate alone. This rise in the "intertemporal price" can discourage such intertemporal trade.

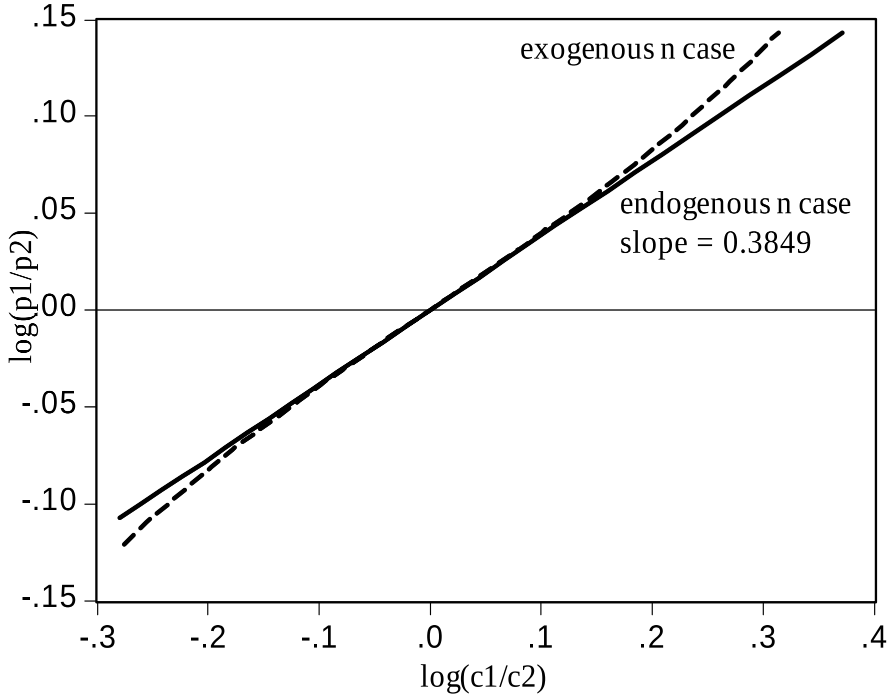
This theory was extended in a limited but important way to endogenously nontraded goods by Obstfeld and Rogoff (2000). In a model with one home good that can switch into and out of being nontraded, they showed that changes in the intertemporal price may be highly nonlinear, and may come into effect only for large current account imbalances. Bergin and Glick (2002) showed in the case of two home goods, that the nonlinear nature of the intertemporal price can lead to other interesting cases, and that the intertemporal price may rise more rapidly for a given current account imbalance than implied by exogenously nontraded goods in the model of Dornbusch (1983).

A significant disadvantage of the two models above is that they are extremely difficult to work with, given that Kuhn-Tucker conditions imply discrete changes in equilibrium conditions for various ranges of variable realizations. The model in the present paper reformulates the equilibrium conditions for the case of a continuum of goods. Rather than making the solution yet more complex, this permits us to eliminate the discrete changes and discontinuities in the prices of individual goods, and instead focus on smoothly changing levels of various integrals over regions of the continuum. As shown above, this method of dealing with endogenous nontradedness is much easier to work with, and has the promise of being incorporated into a wide range of international macro models.

To gauge the effect of endogenous nontradedness on intertemporal trade, we use our model to compute the intertemporal price (p_1/p_2) for various levels of intertemporal borrowing. Figure 4 plots this intertemporal price against various levels of intertemporal reallocation of consumption (c_1/c_2). The solid line represents the benchmark model, where we find that the log of the intertemporal price rises with consumption with a nearly constant elasticity of 0.385. It is interesting that these variables follow an approximately log-linear relationship. The dashed line represents the intertemporal price for the exogenous nontraded case defined above. The exogenous share of nontraded goods for this case is calibrated to equal the share of the endogenous model in its balanced-trade steady state.

Several conclusions emerge. First, the intertemporal price rises smoothly in the endogenously nontraded model, in contrast to the earlier papers with only one or two home goods. The absence of price changes for small shocks to the current account and the dramatic kinks and sudden price rises for large imbalances characteristic of the earlier models disappear here in the more realistic case of many goods.[14] This smooth rise in intertemporal price indicates that there is no special cost that kicks in to discourage only large current account deficits**.** The smoothing effect of endogenous tradability operates for small as well as large imbalances.

Fig 4: Intertemporal price: log(p1/p2)



Plotted for $\beta = 1.5$, $\alpha = 1$, $\theta = 0.5$, $\tau_F = 0.1$, Cobb-Douglas preferences, where $\bar{n}$ set to 0.5802 for the exogenous n case.

A second conclusion is that the intertemporal price rises less steeply when nontradedness is endogenous, compared to the standard model with exogenous nontradedness. The general insight of Dornbusch (1983) is still correct, that the rise in nontraded prices implied by the presence of nontraded goods drives up the intertemporal price. However, when goods can switch in and out of being nontraded, they will tend to do so in a way to minimize this cost. When

[14] In these models the kinks in the price response occur because there are a finite number of domestic goods with discontinuously differing trade costs. Hence, as goods shift from being traded or nontraded, export prices jump suddenly.

consumption rises in period 1 and falls in period 2, the share of nontraded goods rises in period 1 to free up more domestic goods for home consumption, and the share of nontraded goods falls in period 2 as the country needs to export more goods to repay its debt. In each case, the endogenous movement in the quantity of nontraded goods partly insulates the price of nontraded goods and thereby the intertemporal price from the shock. The difference between the two models is small for small current account imbalances, where the share of nontraded goods is about the same for both models. But the difference grows for larger current account imbalances, as the share of nontraded goods in the endogenous model deviates more from the steady state level, which is the nontraded share imposed on the exogenous model.

The fact that the key relationships here are approximately log-linear in form suggests that the endogenous nontradedness mechanism advocated here has the potential to be incorporated into a wide range of models, including those of the "New Open Economy Macroeconomics" type (initiated by Obstfeld and Rogoff, 1995), which often need to be log-linearized for analysis.

4. Conclusions

This paper has proposed a new way of thinking about nontraded goods in a macro model, focusing on nontradedness as an endogenous decision in the face of good-specific trading costs. The paper develops a very tractable way of dealing with this endogeneity, and explores its implications in the context of a simple general equilibrium macro model. This way of thinking about nontradedness proves to be quite appealing, in that it helps the model replicate a puzzling stylized fact: the relative price of nontraded goods tends to move much less volatilely than the real exchange rate. This fact stands in contrast to standard theoretical models such as Belassa-Samuelson, which rely almost entirely on such relative price movements.

The paper then shows that the endogeneity of nontradedness can have implications for other macroeconomic issues. In particular, the ability of nontraded goods to discourage international trade will be less severe than in past models, which assumed goods were exogenously nontraded. Goods will tend to switch categories in a manner that minimizes the costs of intertemporal trade.

We should emphasize that we do not view endogenous nontradability as the sole explanation for the many puzzles in international macroeconomics. Rather we view our mechanism as complementary to other explanations that suggest roles for sticky prices, nontraded distributive services, vertical production arrangements, etc. In fact, we view the incorporation of our approach into models with these other features as a fruitful line of research. Further, because the key relationships in our formulation are approximately log-linear in form, we suspect that it even will be possible to incorporate this mechanism into quite complex business cycle models, which typically require log-linear approximation for analysis. As a result, we suspect that this approach will be employed fruitfully in a wide range of models to analyze a wide range of issues in international macroeconomics.

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Appendix: Derivation of equilibrium conditions

Cobb-Douglas case:

Combine (8) and (12) to solve out for c_N:

$$c_H = y\left(p_N / p_H\right)^{\phi} \tag{A1}$$

Substitute in (A1) for p_N with (7):

$$p_H c_H = y\alpha^{-\phi} n^{\beta\phi} p_H{}^{1-\phi} \tag{A2}$$

Substitute in (4) for p_T with (6):

$$p_H{}^{1-\phi} = \frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right] \tag{A3}$$

where $\omega \equiv \beta(\phi-1)-1$. Combine (A3) with (A2) to obtain

$$p_H c_H = \frac{y}{\alpha}\left(\frac{n^{\beta\phi}}{\omega}\right)\left[n^{-\omega}(1+\omega)-1\right] \tag{A4}$$

Note next that the domestic value of aggregate home production can be derived as

$$\begin{aligned} p_H y_H &= \int_0^n p_i y_i d_i + \int_n^1 p_i y_i d_i = \int_0^n p_N y d_i + \int_n^1 p_i y d_i \\ &= \left(n^{\beta}/\alpha\right) ny + y\int_n^1 \left(\frac{i^{\beta}}{\alpha}\right) d_i \\ &= \frac{y}{\alpha} n^{\beta+1} + \frac{y}{\alpha}\left(\frac{1}{\beta+1}\right)\left(1-n^{\beta+1}\right) \end{aligned}$$

implying

$$p_H y_H = \frac{y}{\alpha}\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right] \tag{A5}$$

With balanced trade, $p_H y_H = pc$. Noting that (10) implies $p_H c_H = \theta pc$ and combining this with the balanced trade condition gives

$$p_H c_H = \theta p_H y_H \tag{A6}$$

Substituting in (A6) on the lefthand side for $p_H c_H$ with (A4) and on the righthand side for $p_H y_H$ with (A5):

$$\frac{y}{\alpha}\left(\frac{n^{\beta\phi}}{\omega}\right)\left[n^{-\omega}(1+\omega)-1\right]=\theta\left(\frac{y}{\alpha}\right)\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right]$$

Canceling y/α from both sides and gives equation (15) in the text, the equilibrium condition for *n* in the balanced trade case:

$$\frac{1}{\omega}\left[n^{\beta+1}(1+\omega)-n^{\beta\phi}\right]=\theta\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right] \tag{15}$$

To show a unique solution exists for condition (15), define

$$\mathrm{Z}\equiv\frac{1}{\omega}\left[n^{\beta+1}(\omega+1)-n^{\beta\phi}\right]-\theta\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right].$$

It is straightforward to see that for $n=0$, $Z=-\theta<0$, and for $n=1$, $Z=1-\theta>0$. Showing that $\partial Z/\partial n>0$ implies that *Z* crosses the 0 axis only once and is sufficient to establish the existence of a unique solution for *n*:

$$\begin{aligned}\frac{\partial Z}{\partial n}&=\frac{1}{\omega}\left[(\beta+1)n^{\beta}(\omega+1)-\beta\phi n^{\beta\phi-1}\right]-\frac{\theta(\beta+1)n^{\beta}\beta}{1+\beta}\\&=\beta n^{\beta}(1-\theta)+\frac{n^{\beta}\beta\phi}{\omega}(1-n^{\omega})>0\end{aligned}$$

since $\theta<1$ and $(1/\omega)(1-n^{\omega})>0$ for $0<n<1$.

Given the level of *n* that implicitly solves condition (15), it is straightforward to solve for the other endogenous variables: first the prices, p_T and p_N through (6) and (7), p_H through (A3), p through (3); and then the quantities, c_N and c_T through (8) and (9), c_H and c_F through (10) and (11), c through (1).

For the multiperiod case, we introduce time subscripts and solve out for c_{Ht} with (A2) and (10) together to get

$$\frac{y_t}{\alpha^{\phi}}n_t^{\beta\phi}p_{Ht}^{\;1-\phi}=\theta p_t c_t \tag{A7}$$

Substitute in (3) for p_{Ht} with (A3) to get

$$p_t=\frac{1}{\alpha^{\theta}}\left\{\frac{1}{\omega}\left[n_t^{-\omega}(1+\omega)-1\right]\right\}^{\theta/(1-\phi)}p_{Ft}^{\;1-\theta} \tag{A8}$$

Substitute in (A7) for p_{Ht} with (A3) and for p_t with (A8):

$$\frac{y_t}{\alpha} n_t^{\beta\phi} \frac{1}{\omega}\left[n_t^{-\omega}(1+\omega)-1\right] = \theta\frac{1}{\alpha^{\theta}}\left\{\frac{1}{\omega}\left[n_t^{-\omega}(1+\omega)-1\right]\right\}^{\theta/(1-\phi)} p_{Ft}^{1-\theta} c_t$$

Rearranging gives the equations (16) and (17) that express the relation between c_t and n_t that holds for each period t =1,2:

$$y_t n_t^{\beta\phi}\left\{\frac{1}{\omega}\left[n_t^{-\omega}(1+\omega)-1\right]\right\}^{\frac{1-\phi-\theta}{1-\phi}} = \theta\alpha^{1-\theta} p_{Ft}^{1-\theta} c_t \tag{16,17}$$

Lastly, we rearrange the intertemporal budget constraint (13) to get

$$c_2 = \left[(1+r)\left(p_{H1}y_{H1} - p_1 c_1\right) + p_{H2}y_{H2}\right] / p_2 \tag{A9}$$

Substituting in (A9) for $p_{Ht}y_{Ht}$ with (A5) and for p_t with (A8), t =1,2 gives (18):

$$\begin{aligned} c_2 = &\left[(1+r)\left(\frac{y_1\left[1+n_1^{\beta+1}\beta\right]}{\beta+1} - \left(\left\{\left[n_1^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{1}{1-\phi}}\right\}^{\theta}(\alpha p_{F1})^{1-\theta} c_1\right)\right)\right. \\ &\left. + \frac{y_2\left[1+n_2^{\beta+1}\beta\right]}{\beta+1}\right] \bullet \left\{\left[n_2^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{1}{1-\phi}}\right\}^{-\theta} (\alpha p_{F2})^{\theta-1} \end{aligned} \tag{18}$$

The system of three equations – (16), (17), and (18) -- can be solved numerically for n_1, n_2, and c_2, given a value of c_1.

<u>CES case:</u>

In the CES case the aggregate consumption and price equations (1) and (3) are replaced (with time subscripts omitted) by

$$c = \left(\theta^{1/\gamma} c_H^{(\gamma-1)/\gamma} + (1-\theta)^{1/\gamma} c_F^{(\gamma-1)/\gamma}\right)^{\gamma/(\gamma-1)} \tag{1'}$$

$$p = \left(\theta p_H^{1-\gamma} + (1-\theta) p_F^{1-\gamma}\right)^{1/(1-\gamma)} \tag{3'}$$

where $\gamma > 1$ is the elasticity of substitution between home and foreign goods, and $\theta > 0$ reflects the degree of bias for home goods. The home and foreign good allocation conditions (10) and (11) are replaced by

$$c_H / c = \theta\left(p_H / p\right)^{-\gamma} \tag{10'}$$

$$c_F / c = (1-\theta)\left(p_F / p\right)^{-\gamma} \tag{11'}$$

(A1) through (A5) continue to apply in the CES case. Inserting (A3) into (3') gives

$$p = \left(\theta\left(\frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right]\right)^{\frac{1-\gamma}{1-\phi}} + (1-\theta)\, p_F^{\;1-\gamma}\right)^{1/(1-\gamma)} \tag{A8'}$$

implying

$$\left(p_H / p\right)^{1-\gamma} = \frac{\left(\frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right]\right)^{\frac{1-\gamma}{1-\phi}}}{\theta\left(\frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right]\right)^{\frac{1-\gamma}{1-\phi}} + (1-\theta)\, p_F^{\;1-\gamma}} \tag{A10}$$

With balanced trade, $p_H y_H = pc$. Noting that (10') implies $p_H c_H = \theta pc\left(p_H / p\right)^{1-\gamma}$ and combining this with the balanced trade condition gives

$$p_H c_H \left(p_H / p\right)^{\gamma-1} = \theta p_H y_H \tag{A6'}$$

Note that this differs from the condition for the Cobb-Douglas case (A6), in that it includes on the lefthand side a relative price term

$$\left(p_H / p\right)^{\gamma-1} = \left(p_H / \left(\theta p_H^{\;1-\gamma} + (1-\theta)\, p_F^{\;1-\gamma}\right)^{1/(1-\gamma)}\right)^{\gamma-1} = \theta + (1-\theta)\left(\frac{p_H}{p_F}\right)^{\gamma-1},$$

which represents the effect of the relative price between home and foreign goods on consumption allocation and export decisions. Substituting in (A6') on the lefthand side for $p_H c_H$ with (A4) and for $\left(p_H / p\right)^{\gamma-1}$ with (A10) and on the righthand side for $p_H y_H$ with (A5) and gives

$$\frac{y}{\alpha}\left(\frac{n^{\beta\phi}}{\omega}\right)\left[n^{-\omega}(1+\omega)-1\right]\left[\frac{\theta\left(\frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right]\right)^{\frac{1-\gamma}{1-\phi}} + (1-\theta)\, p_F^{\;1-\gamma}}{\left(\frac{1}{\alpha^{1-\phi}\omega}\left[n^{-\omega}(1+\omega)-1\right]\right)^{\frac{1-\gamma}{1-\phi}}}\right] = \theta\frac{y}{\alpha}\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right]$$

Canceling y/α from both sides, multiplying both the numerator and denominator of the last square bracketed term on the lefthand side by $\alpha^{1-\gamma}$, and rearranging gives the analogue to equilibrium condition (15) for *n* in the balanced trade case:

$$n^{\beta\phi}\left(\theta\left[n^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{1-\gamma}{1-\phi}}+(1-\theta)\left(\alpha p_F\right)\right)^{1-\gamma}\Bigg)$$
$$=\theta\left[\frac{1+n^{\beta+1}\beta}{1+\beta}\right]\left[n^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{\phi-\gamma}{1-\phi}} \tag{15'}$$

Note that the trade cost level parameter (α) now enters into the equilibrium expression for n through the effect on the relative price of home goods. Note also that in the case of Cobb-Douglas preferences for the home and foreign good, $\gamma = 1$, the relative price effect disappears, and (15') reduces to (15).

To derive the analogues to (16), (17), and (18) in the multiperiod case, we reintroduce time subscripts and solve out for c_{Ht} with (A2) and (10') together to get

$$\frac{y_t}{\alpha^{\phi}} n_t^{\beta\phi} p_{Ht}^{1-\phi} = \theta p_t c_t \left(p_t / p_{Ht}\right)^{\gamma-1}$$

Substituting on the left hand side for p_{Ht} with (A3) and on the right hand side for p_t with (A8') and for $\left(p_H / p\right)^{\gamma-1}$ with (A10) yields for t = 1,2:

$$y_1 n_1^{\beta\phi}\left[\theta\left\{n_1^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right\}^{\frac{1-\gamma}{1-\phi}}+(1-\theta)\left(\alpha p_{F1}\right)\right)^{1-\gamma}\Bigg]^{\frac{\gamma}{\gamma-1}} = \theta\left[n_1^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{\phi-\gamma}{1-\phi}} c_1 \tag{16'}$$

$$y_2 n_2^{\beta\phi}\left[\theta\left\{n_2^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right\}^{\frac{1-\gamma}{1-\phi}}+(1-\theta)\left(\alpha p_{F2}\right)\right)^{1-\gamma}\Bigg]^{\frac{\gamma}{\gamma-1}} = \theta\left[n_2^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right]^{\frac{\phi-\gamma}{1-\phi}} c_2 \tag{17'}$$

Substituting in the intertemporal budget constraint (13) for $p_{Ht} y_{Ht}$ with (A5) and for p_t with (A8'), t =1,2 gives (18'), the analogue to (18):

$$c_2 = \left[(1+r)\left(\frac{y_1\left[1+n_1^{\beta+1}\beta\right]}{\beta+1} - \left(\theta\left\{n_1^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right\}^{\frac{1-\gamma}{1-\phi}} + (1-\gamma)\left(\alpha p_{F1}\right)\right)^{1-\gamma}\right)^{1/(1-\gamma)} c_1\Bigg) + \frac{y_2\left[1+n_2^{\beta+1}\beta\right]}{\beta+1}\right]$$
$$\bullet\left[\theta\left\{n_2^{-\omega}\left(\frac{1+\omega}{\omega}\right)-\frac{1}{\omega}\right\}^{\frac{1-\gamma}{1-\phi}}+(1-\theta)\left(\alpha p_{F1}\right)\right)^{1-\gamma}\Bigg]^{1/(\gamma-1)}$$