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#### OUTSOURCING IN A GLOBAL ECONOMY

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Outsourcing in a Global Economy Gene M. Grossman and Elhanan Helpman NBER Working Paper No. 8728 January 2002 JEL No. F12, L14, L22, D23

#### **ABSTRACT**

We study the determinants of the location of sub-contracted activity in a general equilibrium model of outsourcing and trade. We model outsourcing as an activity that requires search for a partner and relationship-specific investments that are governed by incomplete contracts. The extent of international outsourcing depends inter alia on the thickness of the domestic and foreign market for input suppliers, the relative cost of searching in each market, the relative cost of customizing inputs, and the nature of the contracting environment in each country.

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# 1 Introduction

We live in an age of outsourcing. Firms seem to be subcontracting an ever expanding set of activities, ranging from product design to assembly, from research and development to marketing, distribution, and after-sales service. Some firms have gone so far as to become "virtual" manufacturers, owning designs for many products but making almost nothing themselves.<sup>1</sup>

Vertical disintegration is especially evident in international trade. A recent annual report of the World Trade Organization (1998) details, for example, the production of a particular "American" car:

Thirty percent of the car's value goes to Korea for assembly, 17.5 percent to Japan for components and advanced technology, 7.5 percent to Germany for design, 4 percent to Taiwan and Singapore for minor parts, 2.5 percent to he United Kingdom for advertising and marketing services, and 1.5 percent to Ireland and Barbados for data processing. This means that only 37 percent of the production value ... is generated in the United States. (p.36)

Feenstra (1998), citing Tempest (1996), describes similarly the production of a Barbie doll. According to Feenstra, Mattel procures raw materials (plastic and hair) from Taiwan and Japan, conducts assembly in Indonesia and Malaysia, buys the molds in the United States, the doll clothing in China, and the paints used in decorating the dolls in the United States. Indeed, when many observers use the term "globalization," they have in mind a manufacturing process similar to what Feenstra and the WTO have described.

<sup>&</sup>lt;sup>1</sup>See *The Economist* (1991) for an overview of trends toward greater outsourcing in manufacturing. Helper (1991), Gardner (1991), Bardi and Tracey (1991), Bamford (1994) and Abraham and Taylor (1996) document increased subcontracting in particular industries or for particular activities.

To us, outsourcing means more than just the purchase of raw materials and standardized intermediate goods. It means finding a partner with which a firm can establish a bilateral relationship and having the partner undertake relationship-specific investments so that it becomes able to produce goods or services that fit the firm's particular needs. Often, but not always, the bilateral relationship is governed by a contract, but even in those cases the legal document does not ensure that the partners will conduct the promised activities with the same care that the firm would use itself if it were to perform the tasks.<sup>2</sup>

Because outsourcing involves more than just the purchase of a particular type of good or service, it has been difficult to measure the growth in international outsourcing. Audet (1996), Campa and Goldberg (1997), Hummels et al. (2001) and Yeats (2001) have used trade in intermediate inputs or in parts and components to proxy for what they have variously termed 'vertical specialization', 'intra-product specialization' and 'global production sharing'. While these are all imperfect measures of outsourcing as we would define it, the authors do show that there has been rapid expansion in international specialization for a varied group of industries that includes textiles, apparel, footwear, industrial machinery, electrical equipment, transportation equipment, and chemicals and allied products. It seems safe to tentatively conclude that the outsourcing of intermediate goods and business services is one of the most rapidly growing components of international trade.

In this paper, we develop a framework that can be used to study firms' decisions about where to outsource. We consider a general equilibrium model of production and trade in which firms in one industry must outsource a particular activity. These firms can seek partners in the technologically and legally advanced North, or they can look in the low-wage South. Our model of a firm's decision incorporates what we consider to be the three essential features of a modern outsourcing strategy. First, firms must *search* for partners with the expertise that allows them to perform the

<sup>&</sup>lt;sup>2</sup>Marsh (2001, p.10) notes some of the pitfalls in outsourcing: "Outsourcers depend on others caring as much about the product as they do. If you ask someone else to make a vital component, you may lose control over the way it evolves."

particular activities that are required. Second, they must convince the potential suppliers to *customize* products for their own specific needs. Finally, they must induce the necessary relationship-specific investments in an environment with *incomplete contracting*.

Using the framework developed in Sections 2 and 3, we are able to examine in Sections 4 and 5 several possible determinants of the location of outsourcing. First, the size of a country can affect the 'thickness' of its markets. All else equal, a firm prefers to search in a thicker market, because it is more likely to be able to find a partner there with the appropriate skills that would make it able and willing to tailor a component or service for the final producer's needs. Second, the technology for search affects the cost and likelihood of finding a suitable partner. Search will be less costly and more likely successful in a country with good infrastructure for communication and transportation. Third, the technology for specializing components determines the willingness of a partner to undertake the needed investment in a prototype. Finally, differences in contracting environments can impinge on a firm's ability to induce a partner to invest in the relationship. We study the contracting environment by introducing a parameter that represents the extent to which relationship-specific investments are verifiable by an outside party.

While our model is rich in its description of the outsourcing relationship, it is limited by its focus on activities for which firms have no choice but to outsource. Elsewhere (see Grossman and Helpman, 2002) we have studied a firm's decision of whether to undertake an activity in-house or to outsource it. There, like here, we cast individual firms' choices in the context of an industry and general equilibrium. But we focus narrowly on a closed rather than a global economy. The next step in our progression would be a model in which firms have a four-way choice of whether to undertake an activity either in-house or by subcontract, and either at home or abroad. Such a model would come closer to describing the central decisions facing modern, multinational firms. But the current paper takes an important intermediate step, because it highlights the considerations that are bound to be important in a more complete analysis.

# 2 The Model

Consider a world with two countries, North and South, and two industries. Firms in either country can produce a homogeneous consumer good z with one unit of local labor per unit of output. Firms in the North also can design and assemble varieties of a differentiated consumer good y. The South lacks the know-how needed to perform these activities. Both countries are able to produce intermediate goods (business services or components), which are vital inputs into the production of good y.

The varieties of good y are differentiated in two respects. First, as is usual in models of intra-industry trade, consumers regard the different products as imperfect substitutes. Second, the varieties require different components in their production. We capture product differentiation in the eyes of consumers with the now-familiar formulation of a CES sub-utility function. On the supply side, we associate each final good with a point on the circumference of a unit circle, so that the "location" of a good represents the specifications of the input needed for its assembly.

Consumers in both countries share identical preferences. The typical consumer seeks to maximize

$$u = z^{1-\beta} \left[ \int_0^1 \int_0^{\hat{n}(l)} y(j,l)^{\alpha} \, dj \, dl \right]^{\frac{\beta}{\alpha}}, \quad 0 < \alpha, \beta < 1.$$
 (1)

where z is consumption of the homogeneous final good and y(j, l) is consumption of the  $j^{th}$  variety located at point l on the unit circle (relative to some arbitrary zero point). We shall assume that there is a continuum of goods located at each point on the circle, but (1) implies that consumers regard the various goods at the same location on the circle as differentiated. In the limit to the integral,  $\hat{n}(l)$  is the measure of varieties available to consumers that require an intermediate input with characteristics l. Note that, as usual,  $\beta$  gives the spending share that consumers optimally devote to the homogeneous good, and  $\varepsilon = 1/(1 - \alpha)$  is the elasticity of substitution between any pair of varieties of good y.

The production process is as follows. First, firms in the North enter as potential producers of a variety of good y. Entry requires an investment of  $w^N f_n$  in product

design, where  $w^N$  is the Northern wage rate and  $f_n$  is the amount of labor needed to develop a differentiated product.<sup>3</sup> The production of a differentiated product requires one unit of a customized intermediate input per unit of output. The finalgood producers cannot manufacture these inputs themselves (or, it is prohibitively costly for them to do so due to their relatively small scale of production). Rather, they must *outsource* this activity to a specialized producer of components in one country or the other. If a final producer is successful in finding a suitable partner and in convincing this firm to tailor an intermediate good for its use, it needs no additional inputs to assemble the final output.<sup>4</sup> The location on the circle of a firm's requisite component is a random element in its product design, and all locations are equally likely.

At the same time that firms in the North enter as potential final producers, firms in either country may enter as suppliers of intermediate products. Such entry involves an investment in expertise (and, perhaps, equipment). A supplier's expertise is represented by a point on the unit circle. The investment in developing expertise is large relative to the cost of designing a single final product, so there are relatively few (i.e., a finite number) of suppliers of components in any country, each of which serves many final producers. The cost of entry by a component producer in country i is  $w^i f_m^i$ , where  $w^i$  is the wage in country i for i = S, N, and  $f_m^i$  is the labor requirement there.

Figure 1 shows the location of the intermediate producers in one of the countries. For ease of visualization, we have depicted a market with only eight suppliers, although for reasons that will become clear, we have in mind equilibria with larger numbers than this. We neglect the integer "problem," and treat the finite number of input suppliers in country i,  $m^i$ , as a continuous variable. In equilibrium, the suppliers in any market will be equally spaced around the circle.

<sup>&</sup>lt;sup>3</sup>Since there is a continuum of differentiated final goods, the fixed cost of designing a single product is infinitesimally small. Of course, the total resources used in designing a positive measure of such goods is finite.

<sup>&</sup>lt;sup>4</sup>This is an inessential simplification. We could as well assume that production of final goods requires labor and components in fixed proportions.



Figure 1: Locations of input suppliers

Once entry has occurred, the next stage involves search. Northern firms that have developed designs for final products must locate suppliers for their components. Each firm must choose the market in which it will seek a supplier, because search in one geographic region is a distinct activity from search in the other. For reasons that will become clear, a firm's goal in its search is to find a partner whose expertise (as represented by its location on the circle) is close to the firm's own input requirement.

Final producers are assumed to know how many suppliers are active in each country (the "thickness" of the market) but not the precise locations of these producers in terms of their expertise. The firms know that the suppliers in a market are equally spaced around the circle, and regard all equi-spaced configurations as equally likely. Each firm chooses the intensity of its search in the market of its choice, with search costs rising quadratically in the extent of search. We assume that search must be carried out by Northern labor. A search of intensity x in market i requires  $\eta^i x^2$  units of labor, and thus costs  $w^N \eta^i x^2$ . Such a search will turn up all suppliers in an arc of length 2x. Since each firm seeks a supplier with expertise appropriate to its needs, it searches symmetrically about its own location in input space. In Figure 1 we show two firms, at h and k, that conduct searches of equal intensity x, as represented by the indicated arcs. In this case, the firm at location k is successful in finding a potential supplier (input producer 7), but the firm at location h fails in its search effort. A firm that fails to locate a supplier of components is unable to produce final output and thus forfeits its initial investment.

A successful search results in a bilateral match. This leads to a negotiation and possibly to a relationship-specific investment by the input supplier. We postpone discussion of the bargaining and contracting issues for a moment, to focus on the technological considerations. In order to produce the customized inputs needed by a particular final producer, a supplier must invest in a prototype. The greater is the distance between the supplier's expertise and the final producer's needs, the larger is the cost of customization. In particular, if a supplier in country *i* wishes to provide components to a final producer whose location is at a distance *x* from its own expertise, then it must pay a fixed cost of  $w^i \mu^i x$  to develop the prototype. Thereafter, it can produce customized components for its partner at constant marginal cost, with one unit of local labor needed per unit of output.

#### 2.1 Bargaining and Contracting

Bargaining occurs in two stages. When a final producer locates a potential supplier in a given market, the two firms first negotiate over the extent of the supplier's investment in customization and the amount of compensation for the prototype. Later, they negotiate over the quantity and price of an input order. The size of the order and the payment cannot be negotiated ahead of time, because then the input supplier might make no investment in customization but still produce inputs and demand to be paid.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>Alternatively, we might allow the two sides to negotiate a price of inputs when they first meet while giving the final producer the option to decide the size of its order after it inspects the prototype. Such a contract typically would be renegotiated ex post, but it would alter the disagreement point for the second-stage negotiation and thus could be used to induce investment by the supplier. We have investigated such contracts and find that they can stimulate investment that would not otherwise be made if and only if  $\alpha > 1/2$  (i.e.,  $\varepsilon > 2$ ). But even in cases where an initial negotiation of price with an option to buy can mitigate the hold-up problem, it can never induce the same investment choices

We shall refer to the contract that governs the supplier's investment in the prototype as the *investment contract* and that which governs the exchange of inputs as the *order contract*. Note that the prototype is valuable only inside the relationship.<sup>6</sup> Also, while the supplier's investment (or its result) can be perfectly observed by the final producer, we assume that it is only partly verifiable to outside parties. As is familiar from the work of Williamson (1985), Hart and Moore (1990), and others, the imperfect verifiability of investment constrains the contracting possibilities. Thus, the investment contract is an incomplete contract. In contrast, the order contract is a complete contract, because both quantity and price are verifiable.

We wish to allow for different contracting environments in the two countries, inasmuch as this may be an important consideration in a firm's choice of where to outsource. To this end, we extend the existing literature on imperfect contracts to incorporate intermediate cases between the familiar extremes of "no contracts" and "perfect contracts." We assume that, in country i, an outside party can verify a fraction  $\gamma^i < 1/2$  of the investment in customization undertaken by an input supplier for a potential downstream customer. The parameter  $\gamma^i$  may be given a literal interpretation: the development of the prototype requires a number of stages or subinvestments, some of which are verifiable and others are not. More figuratively, we imagine that  $\gamma^i$  captures the quality of the legal system in country i; the greater is  $\gamma^i$ , the more complete are the contracts that can be written there.

#### 2.1.1 The Investment Contract

Now consider the negotiation of an investment contract between an input producer in  $\frac{1}{2}$  country *i* and a downstream firm whose required component is at a distance *x* from as would result if customization were fully contractible. Since the flavor of the analysis with the possibility of first-stage price contracts is similar to what we describe below, we choose to preserve the simpler contracting environment.

<sup>6</sup>In other words, we assume that a firm's input requirements are unique, and in particular different from those of other firms located at the same point on the unit circle. Also, final producers may not use components that nearly fit but not precisely so. These assumptions simplify the analysis without significantly affecting the nature of the hold-up problem. the supplier's expertise. The contract can stipulate a level of investment, but not one greater than  $\gamma^i w^i \mu^i x$ . The constraint reflects the limit on verification. The contract also can specify a payment  $P^i$  for which the downstream firm will be liable if the supplier carries out the agreed investment. We assume Nash bargaining wherein the parties share equally in the surplus that accrues from the contract.<sup>7</sup>

Let  $S^i$  denote the profits that the parties will share if the supplier develops a component that fits the buyer's needs, and if the two parties subsequently negotiate an efficient order contract. The parties anticipate that if they reach a stage where a suitable prototype exists, their negotiation will lead to an equal sharing of  $S^i$ . So the parties each can expect to earn  $S^i/2$  if a first-stage bargain is reached, and if the supplier chooses to invest the full amount  $w^i \mu^i x$  needed for production to proceed.

The supplier will not make the full investment in customization unless its share of the prospective profits,  $S^i/2$ , exceeds the cost of that part of the investment that is not governed by the contract. Thus, if  $(1 - \gamma^i)w^i\mu^i x > S^i/2$ , the supplier will invest at most  $\gamma^i w^i \mu^i x$  (doing so if and only if  $P^i \ge \gamma^i w^i \mu^i x$ ), and the final producer will be left with a component that is useless for its purposes. In such circumstances, there is no surplus to be shared in the relationship, and so no deal is struck.<sup>8</sup>

Next note that, if  $S^i/2 \ge w^i \mu^i x$ , it is worthwhile for the supplier to proceed with the full investment in customization, even if there is no first-stage contract and no initial payment. Here, the supplier's share of the prospective profits covers the full cost of the requisite investment. In such circumstances, the parties' threat points in the first stage are  $S^i/2$  for the final-good producer and  $S^i/2 - w^i \mu^i x$  for the supplier

<sup>&</sup>lt;sup>7</sup>In principle, the downstream firm might search for two potential suppliers, and then hold one supplier in reserve to improve its bargaining position vis-à-vis the other. Typically, this strategy would not prove profitable in the equilibria we describe. But, in any case, we wish to avoid a taxonomic treatment with many different strategies. Accordingly, we assume that, if a downstream firm locates more than one potential supplier, it must choose one firm with which to conduct negotations. In the event the negotiation with this firm fails, there is no time to take up with another supplier. With this assumption, the outside options for both firms are zero.

<sup>&</sup>lt;sup>8</sup>Note that this condition requires  $S^i/2 \ge w^i \mu^i x$  when  $\gamma^i = 0$ ; this is the usual condition requiring half of the surplus to cover the cost of the relationship-specific investment that applies when all investment expenses are unverifiable.

of inputs.<sup>9</sup> But these options sum to exactly what the parties stand to share if they reach a first-stage agreement, which means that there is nothing to be gained from signing a contract. It follows that the investment contract is a null contract in this case, with  $P^i = 0$  and with no required investment.

In contrast, when  $S^i/2 < w^i \mu^i x$ , the input producer will not make the investment absent an first-stage agreement with its customer. Meanwhile, when  $S^i/2 \ge (1 - \gamma^i)w^i\mu^i x$ , the supplier would be willing to make the unverifiable investment provided the verifiable part is governed by a binding contract. In such circumstances, the parties can generate profits if they manage to conclude a first-stage deal, but their outside options are zero if their initial negotiation fails. The Nash bargain calls for an equal sharing of surplus, which means an initial payment by the downstream firm that provides equal net rewards to both parties. The final producer's reward net of the payment is  $S^i/2 - P^i$ , while the input producer's reward net of the investment cost is  $S^i/2 + P^i - w^i\mu^i x$ . Equating the two, we have  $P^i = w^i\mu^i x/2$ , when x is such that  $w^i\mu^i x > S^i/2 \ge (1 - \gamma^i)w^i\mu^i x$ . In other words, the two sides divide the investment cost evenly.

To summarize, the investment contract and the induced investment behavior depend upon the contracting environment in the supplier's country, on the distance between the supplier's expertise and the final producer's input requirement, and on the amount of potential profits that would be generated by an efficient order. Let  $P^{i}(x)$  be the payment that is dictated by an investment contract between a final producer in the North and an input supplier in country i when the supplier's expertise differs from the buyer's input needs by x, and let  $I^{i}(x)$  be the induced investment level. Then

$$P^{i}(x) = \begin{cases} \frac{1}{2}w^{i}\mu^{i}x & \text{for } \frac{S^{i}}{2w^{i}\mu^{i}} < x \leq \frac{S^{i}}{2w^{i}\mu^{i}(1-\gamma^{i})} \\ 0 & \text{otherwise} \end{cases}$$
(2)

<sup>&</sup>lt;sup>9</sup>Recall that, even if there is no first-stage contract, the supplier cannot commit to refrain from negotiating an order contract in the second stage. Thus, the supplier has every incentive to make the first-stage investment without any payment, which works to the benefit of its downstream producer.

and

$$I^{i}(x) = \begin{cases} w^{i}\mu^{i}x & \text{for } x \leq \frac{S^{i}}{2w^{i}\mu^{i}(1-\gamma^{i})} \\ 0 & \text{otherwise} \end{cases}$$
(3)

#### 2.1.2 The Order Contract

Once the input supplier has sunk its investment in the prototype, the partners have coincident interests regarding the production and marketing of the final good. They can write an efficient contract to govern the manufacture and sale of the intermediate inputs. The preferences in (1) imply that the producer of the  $j^{th}$  variety of good y at location l faces a demand given by

$$y(j,l) = Ap(j,l)^{-\varepsilon}, \qquad (4)$$

when it charges the price p(j, l), where

$$A = \frac{\beta \sum_{i} E^{i}}{\left[\int_{0}^{1} \int_{0}^{\hat{n}(l)} p\left(j,l\right)^{1-\varepsilon} dj dl\right]}$$
(5)

and  $E^i$  denotes spending on consumer goods in country *i*, for i = N, S. This is a constant-elasticity demand function, which means that profits are maximized by mark-up pricing. Any partnership in which the supplier resides in country *i* faces a marginal cost of output of  $w^i$ . Thus, joint profits are maximized by a price  $p^i = w^i/\alpha$ of final output. Maximal joint profits are

$$S^{i} = (1 - \alpha) A \left(\frac{w^{i}}{\alpha}\right)^{1 - \varepsilon}, \qquad (6)$$

which are independent of the distance between the supplier's expertise and the final producer's input type. The order contract that generates the maximal joint profits dictates a quantity of inputs

$$y^{i} = A\left(\frac{w^{i}}{\alpha}\right)^{-\varepsilon} \tag{7}$$

and a total payment by the final producer to the input supplier  $of^{10}$ 

$$\frac{1+\alpha}{2}A\left(\frac{w^i}{\alpha}\right)^{1-\varepsilon}.$$

#### 2.2 Search

We consider now the search problem facing a firm that has developed a variety of good y. The firm must decide where to look for a partner and the intensity of its search effort.<sup>11</sup> Suppose the firm searches at intensity x in country i. If  $x \ge 1/2m^i$ , the firm will find a potential partner with probability one. Otherwise, the probability that it will find a partner is  $2m^i x$ , since the firms are spaced at distance  $1/m^i$  in country i, and a search of intensity x covers an arc of length 2x.

There are two self-imposed bounds on the firm's search intensity in country *i*. First, the firm would never choose x greater than  $1/2m^i$ , because search is costly and in our model there is no benefit to finding a second partner whose expertise is less suited to the firm's input needs than the first. Second, the firm would never choose x greater than  $S^i/2w^i\mu^i(1-\gamma^i)$ , because even if it found a potential supplier at such

The equilibria described below with outsourcing in both countries would remain equilibria even if we were to allow firms to search in both markets. In these equilibria, some firms break even by searching only in the North and others by searching only in the South, so a firm that searched in both places would suffer an expected loss. However, if firms were free to search in both markets, there might be additional equilibria in which all firms search in both countries, and firms that find potential partners in both places choose ex post where to outsource. This choice would be based on the distance between their input requirement and the expertise of the two potential partners and on the profit opportunities that would ensue from production of intermediates in the alternative locations.

<sup>&</sup>lt;sup>10</sup>The payment is such that the input supplier's reward net of manufacturing costs is half of the joint profits. Thus, the payment is  $S^i/2 + w^i y^i$ , which, with (6) and (7), implies the expression in the text.

<sup>&</sup>lt;sup>11</sup>We assume that final producers search for an outsourcing partner in only one country. This could be justified by assuming that the entry cost  $f_n$  incorporates a component that is a fixed cost of search (independent of intensity) and that a firm choosing to search in both markets would have to bear this cost twice. If this cost element were large enough, then search in two countries would be unprofitable.

a distance, the input producer would be unwilling to make the necessary investment in customization, in view of the contracting environment in country i. Accordingly, a firm searching for a partner in country i chooses x to maximize its expected operating profits,

$$\pi_n^i(x) = 2m^i \int_0^x \left[\frac{S^i}{2} - P^i(q)\right] dq - w^N \eta^i x^2,$$
(8)

subject to

$$x \le \frac{1}{2m^i} \tag{9}$$

and

$$x \le \frac{S^i}{2w^i \mu^i \left(1 - \gamma^i\right)}.\tag{10}$$

In (8), the first term is the expected profits for the firm considering all the different partners it might find at the various distances q (and the possibility that it may find no partner at all), and the second term is the cost of the search.<sup>12</sup>

We let  $r^i$  denote the optimal intensity of search for a final producer that searches in country *i*. There are several cases to consider, depending upon whether one or the other or neither of the constraints binds in the determination of  $r^i$ . We will not say anything more about that here, but will postpone further discussion of the optimal search intensity until after we have described the remaining equilibrium conditions.

Once we have  $r^N$  and  $r^S$ , we can identify the market or markets in which the Northern firms will choose to search. If  $\pi_n^N(r^N) > \pi_n^S(r^S)$ , all search is conducted in the North and all outsourcing takes place there. Similarly, if  $\pi_n^S(r^S) > \pi_n^N(r^N)$ , all search focuses on the South and there is no domestic outsourcing. Mostly, we will study equilibria in which outsourcing occurs in both regions. This requires  $\pi_n^S(r^S) = \pi_n^N(r^N)$ .

### 2.3 Free Entry and Market Clearing

The remaining equilibrium conditions comprise a set of free-entry conditions for producers of intermediate and final goods, and a pair of market-clearing conditions for

<sup>&</sup>lt;sup>12</sup>Rauch and Trindade (2000) have also developed a trade model in which firms have to be matched. But their formulation is very different from ours.

the two labor markets.

Final-good producers must enter in positive numbers, since consumers spend a constant fraction of their income on differentiated products. All entrants earn zero expected profits in equilibrium. The expected operating profits for a typical firm that enters industry y is  $\pi_n = \max \{\pi_n^N(r^N), \pi_n^S(r^S)\}$ , and the free-entry condition is

$$\pi_n = w^N f_n. \tag{11}$$

Intermediate-good producers may enter in one or both countries.<sup>13</sup> A firm that enters in country *i* will serve a measure  $2n^i r^i$  of final-good producers, where  $n^i$  is the total measure of final-good producers that searches in country *i*. A firm's customers are spread uniformly at distances ranging from 0 to  $r^i$  in each direction from the point representing the firm's expertise. An intermediate-good producer earns profits of  $P^i(x) + S^i/2 - w^i \mu^i x$  from its relationship with a final-good producer whose input requirement is at a distance *x* from its own expertise. Thus, potential operating profits for an input producer that enters in country *i* are

$$\pi_m^i = 2n^i \int_0^{r^i} \left[ P^i(x) + \frac{1}{2}S^i - w^i \mu^i x \right] dx.$$
 (12)

We assume that the number of entrants is sufficiently large so that, in making its entry decision, each firm ignores the small effect of its own choice on  $r^i$  and  $S^i$ . Then free-entry implies

$$\pi_m^i \le w^i f_m^i$$
 and  $\left(\pi_m^i - w^i f_m^i\right) m^i = 0$  for  $i = N, S$ .

We turn next to the labor-market clearing condition in the South. We examine equilibria in which the wage rate in the North is higher than the wage rate in the South, so that  $\omega \equiv w^N/w^S > 1$ . In such equilibria, the entire world output of the homogeneous good z is produced in the South. Since aggregate profits are zero in both

<sup>&</sup>lt;sup>13</sup>The intermediate producers also choose their expertise (i.e., location). We assume that this choice is made with rational expectations about the choices of others. It is a dominant strategy for each firm to locate at a point mid-way between the expected locations of the two most-distantly-spaced adjacent producers of intermediates. This strategy gives rise to a symmetric equilibrium with equi-spaced input producers.

countries, all income is labor income. Aggregate spending equals aggregate income in country *i*, which implies that  $E^i = w^i L^i$ , where  $L^i$  is the labor supply there. A fraction  $1 - \beta$  of spending is devoted to homogeneous goods, which carry a price of  $w^S$ . This means that in equilibrium the South employs  $(1 - \beta)(\omega L^N + L^S)$  units of labor in the production of good *z*.

The South also devotes labor to entry by input producers, to investment in customization, and to the manufacture of components. Entry absorbs  $m^S f_m^S$  units of labor. Customization requires  $\mu^S x$  units of labor for a final-good producer whose needs are a distance x from the expertise of the input producer. Each of the  $m^S$ producers of intermediates undertakes such an investment for all final-good producers that search in the South and that are located within  $r^S$  to its right or to its left. Since a constant density  $n^S$  of final-final producers searches in the South, the Southern labor needed for developing prototypes is  $2\mu^S m^S n^S \int_0^{r^S} x dx = \mu^S m^S n^S (r^S)^2$ . Finally, the density  $n^S$  of Northern firms searching in the South results in a measure  $2m^S r^S n^S$ of bilateral matches. Each such match generates a demand for  $y^S$  units of Southern labor to manufacture components. Therefore, manufacturing absorbs  $2m^S r^S n^S y^S$ units of Southern labor. Summing the components of labor demand, and equating this to the fixed labor supply, we have

$$(1-\beta)(\omega L^N + L^S) + m^S f_m^S + \mu^S m^S n^S (r^S)^2 + 2m^S r^S n^S y^S = L^S .$$
(13)

In the North, labor is used in the design of final goods, in searching for outsourcing partners, and in entry, investment and manufacturing by producers of intermediate goods. The fixed costs of entry by final-good producers requires  $(n^N + n^S)f_n$  units of labor. The search for partners by these firms requires an additional  $n^N\eta^N(r^N)^2 +$  $n^S\eta^S(r^S)^2$  units of labor. The components of labor demand by intermediate-good producers in the North are analogous to those in the South. Therefore, the labormarket clearing condition in the North is given by

$$f_n \sum_{i} n^i + \sum_{i} n^i \eta^i (r^i)^2 + m^N f^N + \mu^N m^N n^N (r^N)^2 + 2m^N r^N n^N y^N = L^N.$$
(14)

This completes the description of the model.

## **3** Outsourcing with Unverifiable Investment

To gain an understanding of the workings of the model, we begin with a case in which incomplete contracting takes an extreme form: none of the supplier's investment in customization is verifiable by an outside party. We examine the case with  $\gamma^N = \gamma^S =$ 0 to shed light on the technological determinants of outsourcing behavior.

With  $\gamma^N = \gamma^S = 0$ , the first-stage negotiations are pointless. The input supplier cannot commit to undertake any investment, so the final producer will not promise any up-front payment. Instead, the supplier invests in customization if and only if its prospective share of the profits from the bilateral relationship exceeds the total cost of developing the prototype. According to (3), with  $\gamma^i = 0$ ,  $I^i = w^i \mu^i x$  if  $S^i/2 \ge w^i \mu^i x$ and  $I^i = 0$  otherwise.

We return now to the problem facing the final-good producer as regards the choice of search intensity in a given market. As we noted before, the final-good producer searching in country *i* maximizes  $\pi_n^i$  given in (8), subject to (9) and (10). There are different possible solutions to this problem depending upon which, if either, of the constraints binds. Let us suppose first that (10) binds, as illustrated in Figure 2. The figure shows the marginal benefit and marginal cost of search as functions of the search intensity. The benefit of a more intensive search is an improved prospect for finding a partner. Since the probability of finding one rises linearly with the search intensity (provided that  $x < 1/2m^i$ ) and the prospective profits are independent of x (provided that  $x \leq S^i/2w^i\mu^i$ , so that the supplier will invest in customization), the marginal benefit is constant for low values of x. The marginal cost of added search is  $2w^N \eta^i x$ , an increasing function of x. The figure depicts circumstances in which the marginal benefit is relatively high, so that it exceeds the marginal cost at  $x = S^i/2w^i\mu^i$ . In this case, the final producer would be willing to search still further than this, but for its recognition that any partner it might find at such a great distance would not be willing to undertake the investment in customization. We shall refer to this case as one of a *binding investment constraint*.

Another possibility, illustrated in Figure 3, is that neither constraint binds. This is a case where the marginal benefit of search is relatively low and the marginal



Figure 2: Choice of  $r^i$ : Binding investment constraint

cost of search rises steeply. We call this the case of *costly search*. It results in a search intensity determined by equating the (positive) marginal benefit of search to the marginal cost.

The final possibility is that constraint (9) binds, as it will if  $S^i/w^i\mu^i > 1/m^i$ and the marginal benefit of search exceeds the marginal cost at  $x = 1/2m^i$ . In this case (not shown), the final producer searches a distance equal to the space between input suppliers, thereby ensuring itself of finding an outsourcing partner. This case of *assured matching* is less interesting than the others, so we will pay little attention to it.

It is worth emphasizing that the thickness of the market has an important bearing on the search decision. If there are many input producers, then ceteris paribus the marginal benefit of search will be large. In such circumstances, the final producer is more likely to search until it surely finds a partner, or else be constrained by the unwillingness of some potential suppliers to invest in a prototype. Also, the profit opportunity affects the incentives to search. Of course, the two are related: the greater is the number of intermediate producers, the greater will be the fraction of final producers served by input suppliers, and so the smaller will be the demand for



Figure 3: Choice of  $r^i$ : Costly search

a particular variety. While  $m^i$  and  $S^i$  separately affect the search decision, these variables are jointly determined in the general equilibrium.

We are prepared now to write an equation for  $r^i$  as a function of  $m^i, S^i$ , and the wages. This equation delineates the various "regimes" to be considered. First, when neither constraint binds,  $r^i = m^i S^i / 2w^N \eta^i$ . Neither constraint will bind if  $m^i < w^N \eta^i / w^i \mu^i$  (so that  $r^i < 2S^i / w^i \mu^i$ ) and  $m^i < \sqrt{w^N \eta^i / S^i}$  (so that  $r^i < 1/2m^i$ ). Second,  $r^i = S^i / 2w^i \mu^i$  when the investment constraint binds, as it does when  $m^i$  falls between  $w^N \eta^i / w^i \mu^i$  and  $w^i \mu^i / S^i$ . Finally, assured matching implies  $r^i = 1/2m^i$ , and occurs when  $m^i$  falls outside the indicated ranges. In short,

$$r^{i} = \begin{cases} \frac{m^{i}S^{i}}{2w^{N}\eta^{i}} & \text{for} & m^{i} \leq \min\left\{\frac{w^{N}\eta^{i}}{w^{i}\mu^{i}}, \sqrt{\frac{w^{N}\eta^{i}}{S^{i}}}\right\}\\ \frac{S^{i}}{2w^{i}\mu^{i}} & \text{for} & \frac{w^{N}\eta^{i}}{w^{i}\mu^{i}} \leq m^{i} \leq \frac{w^{i}\mu^{i}}{S^{i}} & . \\ \frac{1}{2m^{i}} & \text{otherwise} \end{cases}$$
(15)

There are obviously many cases to consider, and we shall not dwell on all of them. There might be equilibria with outsourcing in both countries and any combination of binding investment, costly search and assured matching in each one. This gives nine possible combinations. And there might be equilibria in which only one country supplies all of the intermediate goods. Again, that country may be characterized by a binding investment constraint, costly search, or assured matching. In what follows we will discuss the procedure for finding all of the equilibria for a given set of parameter values, and then concentrate on the comparative statics in selected regimes.

### 3.1 Equilibria

To identify equilibria, we construct a pair of reduced-form curves, one representing the labor-market clearing condition in the North and the other representing the labormarket clearing condition in the South. In deriving these curves we incorporate the zero-profit conditions as well as the equilibrium search intensities. The curves are constructed as follows. First, we hypothesize a combination of search regimes in the North and the South, and derive the combinations of  $m^N$  and  $m^S$  that are consistent with labor-market clearing in each region under the maintained hypothesis. Then we find the region in  $(m^N, m^S)$  space in which the hypothesized combinations of regimes and "connect up" the curves at the boundaries between the regimes. The points of intersection between the curves so constructed are equilibria of the world economy.

We illustrate the construction of the reduced-form curves for the case of a binding investment constraint in both the North and the South. Other cases are discussed in Appendix A.

When the binding constraint on search is the willingness of input suppliers to undertake the necessary investment in customization, the search intensities are given by (see (15))

$$r^{i} = \frac{S^{i}}{2w^{i}\mu^{i}} \qquad \text{for } i = N, S.$$
(16)

Assuming that outsourcing takes place in both countries, the free-entry conditions imply

$$r^{i}n^{i}(S^{i} - w^{i}\mu^{i}r^{i}) = w^{i}f_{m}^{i}$$
 for  $i = N, S.$  (17)

Substituting (16) and (17) into the South's labor-market clearing condition (13)

 $gives^{14}$ 

$$(1-\beta)\left(\omega L^N + L^S\right) + 2\frac{1+\alpha}{1-\alpha}m^S f_m^S = L^S.$$
(18)

Next, we use (8) and (11), together with the recognition that  $\pi_n^N = \pi_n^S$  if outsourcing takes place in both countries, to write

$$r^{i}\left(m^{i}S^{i}-w^{N}\eta^{i}r^{i}\right)=w^{N}f_{n} \qquad \text{for } i=N,S .$$
<sup>(19)</sup>

Using this equation, (5) and (6), we derive<sup>15</sup>

$$f_n \sum_{i} n^i + \sum_{i} n^i \eta^i \left( r^i \right)^2 = \frac{1}{2} \left( 1 - \alpha \right) \beta \left( L^N + \frac{1}{\omega} L^S \right);$$

that is, the value of labor used by final-good producers for product design and search amounts to a fraction  $(1-\alpha)\beta/2$  of world income. Finally, we substitute this equation together with (16) and (17) into the North's labor-market clearing condition (14) to derive

$$\frac{1}{2}\left(1-\alpha\right)\beta\left(L^{N}+\frac{1}{\omega}L^{S}\right)+2\frac{1+\alpha}{1-\alpha}m^{N}f_{m}^{N}=L^{N}.$$
(20)

The two equations, (18) and (20), involve  $m^S$ ,  $m^N$ , and the relative wage,  $\omega$ . But the relative wage can be solved as a function of  $m^S$  and  $m^N$  using the requirement that, if  $m^S$  and  $m^N$  are both positive, search for input suppliers must be equally profitable in both countries. Substituting (16) into (19) and noting that (6) implies  $S^N = \omega^{1-\varepsilon} S^S$ , we can write an equal-profit condition for the case with binding investment constraints in both countries, namely

$$\frac{\omega^{1-2\varepsilon}}{\mu^N} \left( m^N - \frac{\eta^N}{2\mu^N} \right) = \frac{1}{\mu^S} \left( m^S - \omega \frac{\eta^S}{2\mu^S} \right) . \tag{21}$$

$$A = \frac{\beta \sum_{i} w^{i} L^{i}}{\sum_{i} 2m^{i} r^{i} n^{i} \left(\frac{w^{i}}{\alpha}\right)^{1-\varepsilon}}$$

<sup>&</sup>lt;sup>14</sup>We also use  $y^i = \alpha S^i / (1 - \alpha) w^i$ , which follows from (6) and (7).

<sup>&</sup>lt;sup>15</sup>The derivation uses the fact that  $p^i = w^i/\alpha$  for all differentiated products assembled using intermediate inputs from country *i*, and that the number of varieties of good *y* that are actually produced using intermediate inputs from country *i* is  $2m^i r^i n^i$ . Together, these considerations and (5) imply



Figure 4: Equilibria with binding investment constraints in North and South

This equation allows us to solve for the relative wage as a function of the numbers of intermediate-good producers in the two countries, which we denote by  $\omega(m^S, m^N)$ . Substituting  $\omega(m^S, m^N)$  for  $\omega$  in equation (18) yields the reduced-form SS curve that applies when the investment constraint binds in both places. Substituting  $\omega(m^S, m^N)$  for  $\omega$  in (20) yields the analogous NN curve.

Now we identify the region of  $(m^N, m^S)$  space in which the indicated equations apply. According to (15), the investment constraint binds in the South when  $m^S$ falls between  $\omega \eta^S / \mu^S$  and  $w^S \mu^S / S^S$ . But (16) and (19) imply that  $m^S \leq w^S \mu^S / S^S$ whenever  $4f_n (m^S)^2 - 2\mu^S m^S / \omega + \eta^S \leq 0$ . Thus, the SS curve has the indicated form in the region where  $m^S \geq \omega (m^N, m^S) \eta^S / \mu^S$  and  $4f_n (m^S)^2 - 2\mu^S m^S / \omega (m^N, m^S) + \eta^S \leq 0$ . Similarly, we find that the NN curve has the indicated form in the region where  $m^N \geq \eta^N / \mu^N$  and  $4f_n (m^N)^2 - 2\mu^N m^N + \eta^N \leq 0$ . Outside of these regions, the curves obey different formulas, as detailed in Appendix A.

Figure 4 shows the SS and NN curves for one set of parameter values.<sup>16</sup> The <sup>16</sup>The parameter values for the case illustrated are:  $\alpha = 1/2$ ,  $\beta = 2/5$ ,  $\mu^S = \mu^N = 1$ ,  $\eta^S = 100$ ,  $\eta^N = 60$ ,  $f_m^S = f_m^N = f_n = 1/1200$ ,  $L^S = 10.5$ , and  $L^N = 3.5$ . The curves have been drawn using Mathematica 4.1. The simulation program can be downloaded from http://www.princeton.edu/~grossman/grossman working papers.htm or obtained from either of



Figure 5: An equilibrium with a binding investment constraint in North and costly search in South

dashed line represents combinations of  $m^S$  and  $m^N$  for which  $\omega = 1$ . Only points above and to the left of this line (which have  $\omega > 1$ ) are of interest to us. The dotted horizontal lines show the boundaries between the regions of costly search and binding investment in the North (the lowermost line), and between binding investment and assured matching in the North (the uppermost line). The dotted curves show the boundaries between the regions of costly search and binding investment in the South (the left-most curve), and between binding investment and assured matching in the South (the right-most curve). Thus, the figure shows two equilibria, labelled  $E_1$  and  $E_2$ , each characterized by active outsourcing in both countries and binding investment constraints in both places.

Figure 5 shows the NN and SS curves for a different set of parameter values.<sup>17</sup> This figure shows only the boundaries between the regions of costly search and binding investment in each country, because the other boundary curves fall outside the range

the authors. We thank Yossi Hadar for writing the program.

<sup>&</sup>lt;sup>17</sup>These parameter are:  $\alpha = 1/2$ ,  $\beta = 1/2$ ,  $\mu^S = \mu^N = 1$ ,  $\eta^S = 100$ ,  $\eta^N = 60$ ,  $f_m^S = f_m^N = f_n = 1/1200$ ,  $L^S = 3.535$  and  $L^N = 1.55$ .

of values illustrated in the figure. Here, there again are two equilibria with outsourcing in both countries. The point labelled  $E_1$  has binding investment constraints in both places; that labelled  $E_2$  has a binding investment constraint in the North and a regime of costly search in the South.

Parameter values also exist for which there are four different equilibria with outsourcing in both countries. The reader can picture such a situation by reexamining Figure 5 and imagining that the NN curve were just a bit higher than the one shown there. Then the curve would intersect the SS curve twice in the region with costly search in the South and twice more in the region with a binding investment constraint in the South. In this case, all four equilibria would be characterized by a binding investment constraint in the North.

The possibility of multiple equilibria reflects an important feedback mechanism in the model. The greater is the number of input suppliers in a country, the more profitable it is for final producers to search for partners there. This is because a search of given intensity is more likely to turn up a potential partner when there are more input suppliers to be found. Moreover, when such a search does turn up a partner, that partner will be more likely to be willing to undertake the needed investment in customization. At the same time, the greater is the number of final producers that search for partners in a given country, the more profitable it is for an input producer to operate there.<sup>18</sup>

The positive feedback associated with the thick-market externality is, however, limited by a wage response. As more intermediate producers enter in a country, their demand for labor bids up the country's relative wage. This tends to dampen the incentive of final producers to search there. In our model, the general-equilibrium wage response creates the possibility of multiple equilibria with production of intermediate inputs in both countries and different patterns of outsourcing.

When several equilibria exist, it is natural to ask which ones are stable. We have

<sup>&</sup>lt;sup>18</sup>McLaren (2000) was the first to study the thick-market externality in international trade. He pointed out that this externality can give rise to multiple equilibria when firms can choose between outsourcing and in-house production.

conducted a stability analysis and report the results in Appendix B. In the analysis, we take the numbers of each type of firm (final producers, intermediate producers in the North, and intermediate producers in the South) as the state variables and assume that entry and exit respond to profit opportunities. When profits net of entry costs are positive for a typical firm of a given type, more firms of that type enter. When profits are negative, firms exit. With this adjustment process, stability requires that the NN and SS curves both be downward sloping and that the SS curve be the steeper of the two at the point of intersection. Thus, the equilibria labelled  $E_1$  in Figures 4 and 5 are stable, whereas those labelled  $E_2$  are not.

There also may be equilibria with outsourcing concentrated in one country. For example, an equilibrium with all outsourcing in the North (and  $\omega > 1$ ) always exists when  $\beta L^S > (1 - \beta)L^N$ . In such an equilibrium,  $\omega = \beta L^S / (1 - \beta)L^N$ , and the fact that there are no input suppliers in the South ( $m^S = 0$ ) discourages final producers from searching there. Given that no final producers search for partners in the South, no input suppliers have an incentive to enter there. The equilibrium can be one with costly search, a binding investment constraint, or assured matching in the North, depending upon the size of the cost and demand parameters. When an equilibrium exists with all outsourcing activity concentrated in the North, that equilibrium always is stable. We do not consider such equilibria further in this paper.

# 4 Comparative Statics

In this section, we study how the pattern of outsourcing and world trade are affected by the sizes of the two countries and by the technologies for search and customization. We begin with country size, because this allows us to illustrate some important properties of the model.

#### 4.1 Country Size

Consider growth in the resource endowment of the South, as would be reflected in an increase in  $L^S$ . An initial stable equilibrium with outsourcing in both countries is



Figure 6: Labor supply growth in the South

depicted in Figure 6 by point  $E_1$ . No matter whether the South is in a regime with costly search or a binding investment constraint an increase in  $L^S$  shifts the SS curve to right. This is because the added labor in the South more than suffices to serve the country's increased demand for homogeneous goods. The new SS curve is represented by the broken curve in the figure. In the North, the growth in Southern income means additional demand for differentiated products, and thus a greater demand for labor by final-good producers. This implies a leftward shift of the NN curve, as shown in the figure. The new equilibrium is at point  $E_2$ .

Evidently, expansion in the South induces entry by local producers of intermediate goods and exit by such producers in the North. This has immediate implications for the composition of world outsourcing activity. We define the volume of outsourcing as  $v^i = 2m^i r^i n^i y^i$ ; that is, the number of units of intermediate goods manufactured by input suppliers in country *i*. In a regime with a binding investment constraint, for example, (17), (19), and (7), together with (16), imply that

$$v^i = \frac{4\alpha}{1-\alpha} m^i f^i_m . aga{22}$$

In this case, the volume of outsourcing in a country is proportional to the number of input producers active there. In all regimes, an increase in  $L^S$  boosts outsourcing activity in the South while diminishing such activity in the North.

It is interesting to note the effect on the relative wage. Figure 6 shows the combinations of  $m^N$  and  $m^S$  that imply the same relative wage as at point  $E_1$ . These points satisfy  $\omega(m^S, m^N) = \omega_1$ , where  $\omega_1$  is the relative wage at  $E_1$  and  $\omega(m^S, m^N)$  is the wage that ensures equal profits from search in both places. When the investment constraint binds in both countries, the equal-profit condition (21) implies that the locus of points with  $\omega(m^S, m^N) = \omega_1$  is a line with positive slope. The relationship need not be linear for other combinations of regimes, as can be seen by inspecting the various equal-profit conditions given in Appendix A, but it is always upward sloping. Points above the curve correspond to a higher relative wage in the North than  $\omega_1$ , while points below the curve correspond to a higher relative wage for the South.

We see that, as long as outsourcing continues to take place in both countries, an increase in  $L^S$  must boost the relative wage of the South. The direct effect of an increase in  $L^S$  is to generate excess supply for labor in the South and excess demand in the North. But the shift in outsourcing activity has the opposite effects. Moreover, the thick-market externality implies that outsourcing is an increasing returns activity at the industry level. Only when the wage of the North falls relative to that of the South will the final producers find it to be equally profitable to search in either region in view of the now thinner market in the North and the thicker market in the South.

An increase in  $L^S$  also increases the value of world trade, the share of trade in world income, and the fraction of world trade that is intra-industry trade. The value of world trade is the sum of the value of Northern imports of homogeneous goods, the value of Southern imports of final goods, and the value of Northern imports of components. But trade balance implies that the total value of Southern imports,  $\beta w^S L^S$ , equals the value of its exports of homogeneous goods and of components. Therefore, the value of world trade is

$$T = 2\beta w^S L^S, \tag{23}$$

which rises with  $L^S$  when measured either in terms of the numeraire good (so that  $w^S = 1$ ) or in terms of Northern labor (so that  $w^S$  rises). The ratio of trade to world

income is

$$\frac{T}{GDP} = \frac{2\beta L^S}{\omega L^N + L^S} \tag{24}$$

while the fraction of trade that is intra-industry trade is<sup>19</sup>

$$\frac{T_{intra}}{T} = 1 - \frac{1 - \beta}{\beta} \frac{\omega L^N}{L^S}.$$
(25)

It is clear that both of these ratios rise with  $L^S$ , because the direct effect and the indirect effect that derives from the change in the relative wage both point in the same direction.

We will not repeat the analysis for the case of an increase in  $L^N$ . The reader may confirm that the qualitative effects on the number of intermediate producers in each country, the location of outsourcing activity, the relative wage, the ratio of trade to world income, and the share of intra-industry trade are just the opposite of those for an increase in  $L^{S,20}$ 

## 4.2 Outsourcing Technology

The technology for outsourcing is reflected in the parameters that describe the cost of search  $(\eta^i)$  and the cost of customization  $(\mu^i)$ . Arguably, improvements in transportation and communication technology have lowered the cost of search for outsourcing partners. The internet, especially, has facilitated business-to-business matching. Also, changes in production methods associated with computer-aided design may

<sup>&</sup>lt;sup>19</sup>The volume of intra-industry trade is defined as twice the smaller of the North's exports of differentiated final goods and the South's exports of intermediates. In this case, the latter quantity is smaller. Since the volume of these exports equals  $\beta L^S - (1 - \beta)\omega L^N$  (the difference between the South's imports of differentiated goods and the South's exports of homogeneous goods, the expression in (25) follows from (23).

<sup>&</sup>lt;sup>20</sup>An equi-proportionate increase in the size of both countries is not neutral with respect to the composition of outsourcing activity. To see this, suppose to the contrary that  $m^S$  and  $m^N$  were to grow by the same proportion as  $L^S$  and  $L^N$ . By (18) and (20), this can be consistent with labor clearing in both countries only if the relative wage  $\omega$  remains unchanged. But the equal-profit condition (21) implies that the relative wage cannot remain unchanged when  $m^S$  and  $m^N$  grow proportionately, except if search costs in both region are negligible.

have reduced the cost of customizing components. We investigate how improvements in the search and investment technologies affect the location of outsourcing activity.

First, consider an equi-proportionate improvement in all search and investment technologies; i.e.,  $\eta^N$ ,  $\eta^S$ ,  $\mu^N$  and  $\mu^S$  all fall by similar percentages of their initial values. From the equal-profit condition (21) for the regime with binding investment constraints in both countries, we see that this change has no effect on the relative profitability of searching in the North versus the South. The same conclusion applies for the other combinations of regimes, as can be seen in Appendix A. Thus, there is no shift in the relative wage function,  $\omega(m^S, m^N)$ . Moreover, the search and investment parameters either do not appear directly in the reduced-form SS and NN equations (as is the case when the investment constraint binds), or they appear only in ratio form. It follows that a uniform improvement in search and investment technologies leaves all of these curves in their initial locations. There is no affect on the number of intermediate-good producers in either country, on the relative wage, on the levels of outsourcing activity, or on the level and composition of international trade.

When the technologies for search and investment improve worldwide, the profitability of search rises in both locations. Final producers respond by conducting more intensive searches for outsourcing partners, irrespective of the location of their search. The more intensive search efforts generate an increased number of bilateral matches. As a result, consumers enjoy a greater variety of differentiated products, while they consume smaller quantities of each one.

Now we consider an improvement in technology in the South alone. This improvement may be reflected in a decline in search costs (fall in  $\eta^S$ ) or a decline in the cost to a Southern producer of customizing a component (fall in  $\mu^S$ ), or both. To limit repetition, we will formally investigate only the case in which the investment constraint binds in both countries, although similar results can be derived for other combinations of regimes.

When  $\eta^S$  or  $\mu^S$  falls, it becomes more profitable for final producers to search for partners in the South at the initial relative wage. The relative wage of the North must fall to restore equal profitability of search in both places (see (21)). In other



Figure 7: Technological improvement in the South

words, the technological improvements in the South cause the  $\omega(m^S, m^N)$  function to shift down. But then the SS curve shifts up and the NN curve shifts to the left, as illustrated in Figure 7. The equilibrium moves from  $E_1$  to  $E_2$ , corresponding to an increase in the number of input suppliers in the South and a fall in their number in the North. Outsourcing activity shifts from North to South (see(22)).

The fall in  $\eta^S$  or  $\mu^S$  implies, by (21), that the relative wage  $\omega_1$  can be achieved with a smaller number of input suppliers in the South (given  $m^N$ ) than was true before the technological change. Thus, the  $\omega = \omega_1$  curve shifts to the left, as drawn. At  $E_2$ , the relative wage of the North is lower than  $\omega_1$ . It follows, from (24) and (25), that an improvement in the search technology or the investment technology in the South results in an increased ratio of trade to world income and an increased share of intra-industry trade.

To summarize, a rise in international outsourcing with concomitant growth in the importance of trade and of intra-industry trade can be explained by improvements in the technologies for search and customization, but only if these improvements have occurred to a disproportionate extent in the South. It is certainly plausible that such technological catch-up has taken place in recent years.

# 5 The Contracting Environment

We return now to a setting in which the relationship-specific investments are partially verifiable. In this setting, the final producer and its potential supplier can write an incomplete contract governing investment in the prototype. The contract can require the final producer to pay an agreed amount to the input supplier if the latter carries out a subset of the investments needed for customization of the input. The specified investment can include at most the fraction  $\gamma^i$  of the tasks needed for full customization that are verifiable by the courts. We take  $\gamma^i < 1/2$  to be an exogenous measure of the contracting environment in country *i*.

To derive the reduced-form SS and NN equations, we need to revisit the optimal search problem facing a final producer in this environment. Consider a producer that searches in country *i*. According to (2) and (3), if this producer finds a partner whose expertise lies at a distance  $x \leq S^i/2w^i\mu^i$  from its input needs, the partner inevitably undertakes the necessary investment in a prototype and the contractual payment is zero. If the distance x between the partner's expertise and the producer's needs is between  $S^i/2w^i\mu^i$  and  $S^i/2w^i\mu^i(1-\gamma^i)$ , the investment takes place, but at a cost to the final producer of  $P^i = w^i\mu^i x/2$ . Finally, if the distance x exceeds  $S^i/2w^i\mu^i(1-\gamma^i)$ , the input supplier does not undertake any of the tasks associated with customizing the component that are not verifiable by the court.

As we have noted, a final producer chooses its search intensity to maximize its expected profits in (8), subject to (9) and (10). The solution is illustrated in Figure 8. In the figure, the marginal cost of search is a rising linear function of intensity, with slope  $2w^N \eta^i$ . The marginal benefit of search is constant and equal to  $m^i S^i$  for all search distances that result in only matches involving an up-front payment of zero. For distances beyond  $S^i/2w^i\mu^i$ , the final producer's gain from a match is reduced by the amount of its expected first-stage payment. This payment grows with the distance between the final producer and its partner. Thus, the marginal benefit of search falls discontinuously at  $S^i/2w^i\mu^i$ , the smallest distance for which  $P^i > 0$ , and falls linearly thereafter until  $x = S^i/2w^i\mu^i(1-\gamma^i)$ . At distances still greater than this, the input supplier would not make the full investment in customization, and so



Figure 8: Choice of search intensity with  $\gamma^i > 0$ : binding investment constraint

the marginal benefit of search is zero.

Figure 8 depicts a case in which the investment constraint binds. Here, the marginal benefit of search at  $x \leq S^i/2w^i\mu^i(1-\gamma^i)$  exceeds the marginal cost; but the final producer does not search any further than  $x = S^i/2w^i\mu^i(1-\gamma^i)$ , because a more distant potential partner would not make the full investment in customization given the prevailing contracting environment. Other possible regimes arise when the MBcurve intersects the MC curve where the former is flat and the latter is rising, when the intersection comes at the point of discontinuity of MB, when the intersection comes where the MB curve is falling, or when matching is assured.<sup>21</sup>

$$r^{i} = \begin{cases} \frac{m^{i}S^{i}}{2w^{N}\eta^{i}} & \text{for} & m^{i} \leq \min\left\{\frac{w^{N}\eta^{i}}{w^{i}\mu^{i}}, \sqrt{\frac{w^{N}\eta^{i}}{S^{i}}}\right\} \\ \frac{S^{i}}{2w^{i}\mu^{i}} & \text{for} & \frac{w^{N}\eta^{i}}{w^{i}\mu^{i}} \leq m^{i} \leq \min\left\{\frac{2w^{N}\eta^{i}}{w^{i}\mu^{i}}, \frac{w^{i}\mu^{i}}{S^{i}}\right\} \\ \frac{m^{i}S^{i}}{2w^{N}\eta^{i} + m^{i}w^{i}\mu^{i}} & \text{for} & \frac{2w^{N}\eta^{i}}{w^{i}\mu^{i}} \leq m^{i} \leq \min\left\{\frac{2w^{N}\eta^{i}}{w^{i}\mu^{i}(1-2\gamma^{i})}, \hat{m}^{i}\right\} \\ \frac{S^{i}}{2w^{i}\mu^{i}(1-\gamma^{i})} & \text{for} & \frac{2w^{N}\eta^{i}}{w^{i}\mu^{i}(1-2\gamma^{i})} \leq m^{i} \leq \frac{w^{i}\mu^{i}(1-\gamma^{i})}{S^{i}} \\ \frac{1}{2m^{i}} & \text{otherwise} \end{cases}$$

where

$$\hat{m}^{i} = \frac{1}{4S^{i}} \left( w^{i} \mu^{i} + \sqrt{(w^{i} \mu^{i})^{2} + 16w^{N} \eta^{i} S^{i}} \right) ;$$

<sup>&</sup>lt;sup>21</sup>Formally, the solution to the final producer's problem is

Changes in the contracting environment can affect the outsourcing equilibrium only when they alter the final producer's search decision in at least one country, or when they change the payment from final-good producers to their suppliers. But search intensity and expected payments by the final producer are independent of  $\gamma^i$ except when the investment constraint binds. To focus on the most interesting case, we henceforth assume that the investment constraint binds in both countries.

When the investment constraint binds in country i, producers search up to the limit at which suppliers are willing to make the full investment in customization. Then

$$r^{i} = \frac{S^{i}}{2w^{i}\mu^{i}(1-\gamma^{i})}$$
 for  $i = N, S$ . (26)

Expected operating profits for a final producer searching in country i can be calculated using (2), (19) and (26). Equating these expected profits to the fixed cost of product design, we have the zero-profit condition for firms that search in country i:

$$r^{i}\left[m^{i}S^{i} - \frac{1}{2}m^{i}w^{i}\mu^{i}r^{i}\gamma^{i}\left(2 - \gamma^{i}\right) - w^{N}\eta^{i}r^{i}\right] = w^{N}f_{n} \text{ for } i = N, S.$$
 (27)

The difference between this equation and (19) — that applied when  $\gamma^i = 0$  — is the second term in the square brackets. When multiplied by  $r^i$  his term reflects the expected first-stage payment by a final producer that searches according to (26).

Similarly, we can use (2) and (12) to calculate the operating profits for a component producer in country *i*. Equating these to the fixed cost of entry (and thereby assuming that  $m^i > 0$  for i = N, S), we have

$$r^{i}n^{i}\left[S^{i} + \frac{1}{2}w^{i}\mu^{i}r^{i}\gamma^{i}\left(2 - \gamma^{i}\right) - w^{i}\mu^{i}r^{i}\right] = w^{i}f_{m}^{i} \text{ for } i = N, S.$$
(28)

The second term in the square brackets times  $r^i n^i$  is the total amount of up-front payments received by the typical input supplier from its various customers.

We now are ready to derive the reduced-form labor market clearing conditions that apply when  $\gamma^i > 0$  and the investment constraint binds in both countries. Substituting (6), (7), (26) and (28) into (13), we find that

i.e., the largest value of  $m^i$  for which  $2(m^i)^2 S^i \leq 2w^N \eta^i + m^i w^i \mu^i$ . Note that  $\gamma^i = 0$  is a special case in which there is no discontinuity in the *MB* curve and no downward-sloping segment. Then the five possible regimes for  $r^i$  collapse to three, as given in (15).

$$(1-\beta)\left(\omega L^{N} + L^{S}\right) + \left[\frac{2\frac{1+\alpha}{1-\alpha} - \frac{1+3\alpha}{1-\alpha}\gamma^{S} - \frac{1}{2}\left(\gamma^{S}\right)^{2}}{1-\gamma^{S} - \frac{1}{2}\left(\gamma^{S}\right)^{2}}\right]m^{S}f_{m}^{S} = L^{S}.$$
(29)

Notice that (29) reduces to (18) when  $\gamma^S = 0$ .

Similarly, we use (5), (6), (7), (26), (27) and (28) to substitute for the terms in (14). This yields

$$\frac{1}{2}(1-\alpha)\beta\left(L^{N}+\frac{1}{\omega}L^{S}\right) - \left[\frac{\gamma^{N}\left(1-\frac{1}{2}\gamma^{N}\right)}{1-\gamma^{N}-\frac{1}{2}\left(\gamma^{N}\right)^{2}}\right]m^{N}f_{m}^{N} - \left[\frac{\gamma^{S}\left(1-\frac{1}{2}\gamma^{S}\right)}{1-\gamma^{S}-\frac{1}{2}\left(\gamma^{S}\right)^{2}}\right]\frac{1}{\omega}m^{S}f_{m}^{S} + \left[\frac{2\frac{1+\alpha}{1-\alpha}-\frac{1+3\alpha}{1-\alpha}\gamma^{N}-\frac{1}{2}\left(\gamma^{N}\right)^{2}}{1-\gamma^{N}-\frac{1}{2}\left(\gamma^{N}\right)^{2}}\right]m^{N}f_{m}^{N} = L^{N}.$$
(30)

The first three terms on the left-hand side of (30) represent the total demand for labor by final-good producers for entry and search, while the last term represents the labor used by intermediate producers in the North for entry, investment, and production.

To complete the construction of the reduced-form SS and NN curves, we need an equal-profit condition that will allow us to replace the relative wage  $\omega$  in (29) and (30) by a function  $\omega(m^S, m^N)$ . We substitute (6) and (26) into the free-entry condition for final producers (27), and equate the expected operating profits from search in either country, to derive

$$\frac{\omega^{1-2\varepsilon}}{\mu^N \left(1-\gamma^N\right)^2} \left\{ m^N \left[2-3\gamma^N+\frac{1}{2}\left(\gamma^N\right)^2\right] - \frac{\eta^N}{\mu^N} \right\}$$
$$= \frac{1}{\mu^S \left(1-\gamma^S\right)^2} \left\{ m^S \left[2-3\gamma^S+\frac{1}{2}\left(\gamma^S\right)^2\right] - \omega\frac{\eta^S}{\mu^S} \right\} . \tag{31}$$

The left-hand side of (31) is an increasing function of  $\gamma^N$  for all values of  $\gamma^N$  between zero and one-half while the right-hand side of (31) is an increasing function of  $\gamma^S$ for all such values of  $\gamma^S$ .<sup>22</sup> This means that — holding the relative wage and the thickness of each market constant — an improvement in the contracting environment in a country raises the relative profitability to final producers of searching there.

<sup>&</sup>lt;sup>22</sup>To substantiate this claim, we make use of the fact that  $m^i \ge 2\eta^i/\mu^i(1-2\gamma^i)$  in a regime with a binding investment constraint in country *i*, as can be seen in footnote 16.

#### 5.1 Improvements in Contracting in the North

We begin by examining improvements in the contracting environment in the North. Suppose that, initially,  $\gamma^N = \gamma^S = 0$ , and consider a marginal increase in  $\gamma^N$ . As we have just noted, this raises the relative profitability of search in the North. The relative wage  $\omega$  must rise at given  $m^S$  and  $m^N$  to equalize the expected profits from search in either market.

The upward shift in  $\omega(m^S, m^N)$  induces an inward shift of the SS curve, as we have depicted in Figure 9. This shift reflects the greater amount of Southern labor needed to produce homogeneous goods for the now better-paid Northern consumers. In the Northern labor market, there are several influences to be assessed. First, the fall in the relative wage of the South spells a reduction in Southern demand for differentiated products, which tends to reduce employment by final producers. The demand for labor by final producers at given  $m^N$  also falls for another reason: the improved contracting environment facilitates investment by intermediate producers, which means that final producers are willing to search at greater distance in the input space. Since each final producer ultimately has a better chance of finding a suitable partner, there are more final goods produced for any given number of entrants. The increased competition in the market for differentiated products means that fewer such producers enter. This effect is reflected in the second term on the left-hand side of (30), which is zero when  $\gamma^N = 0$  but turns negative as  $\gamma^N$  grows. The fall in labor demand by final-good producers is offset, however, by an increase in demand by component producers, which reflects their greater numbers of customers and their higher investment levels; the fourth term on the left-hand side of (30) grows with  $\gamma^N$ . It is easy to verify that these latter two effects exactly offset one another when  $\gamma^N$ increases slightly from zero. This leaves only the effect of the rise in  $\omega(m^S, m^N)$ , and so the NN curve shifts out, as illustrated in the figure.

The net result is an increase in the number of component producers in the North, a decline in the number of component producers in the South, and a hike in the North's relative wage. This can be seen in Figure 9, which shows the new equilibrium at  $E_2$ , above and to the left of  $E_1$ . Note that this equilibrium lies above the broken


Figure 9: Contracting improves in the North: Low initial  $\gamma^N$ 

line parallel to  $\omega = \omega_1$ , which shows the combinations of  $m^S$  and  $m^N$  that give the same relative wage as at  $E_1$  considering the change in the contracting environment that has taken place. It is also easy to show that the volume of domestic outsourcing rises while the volume of international outsourcing falls.<sup>23</sup> Since equations (24) and (25) describe the ratio of trade to world income and the share of intra-industry trade in total trade, respectively, and both decline with the relative wage in the North, it follows that the ratio of trade to world income and the share of intra-industry trade in total trade both fall.

While an initial improvement in contracting conditions in the North causes outsourcing to relocate from the South, further improvements in the contract environment need not have this effect. In fact, once  $\gamma^N$  is positive, the boost in labor demand by component producers at given  $\omega$  and  $m^N$  induced by further growth in  $\gamma^N$  outweighs the fall in such demand by final-good producers (i.e., the fourth term in (30)

$$v^{i} = rac{4lpha}{1-lpha}rac{1-\gamma^{i}}{1-\gamma^{i}-rac{1}{2}\left(\gamma^{i}
ight)^{2}}m^{i}f_{m}^{i}$$

 $<sup>^{23}\</sup>mathrm{The}$  volume of outsourcing now is given by

So  $v^N$  grows, because there are more Northern intermediate producers and each one produces more components. In the South, the fall in outsourcing results from the exit of Southern input suppliers.

grows by more than the second term shrinks). Still, there is an additional fall in demand by final-good producers owing to the decline in Southern income (and reflected in the shift in  $\omega(m^S, m^N)$ ). On net, the NN curve may shift in either direction. It is easy to find situations in which an increase in  $\gamma^N$  from an initially high level causes exit by intermediate-good producers in the North, entry by intermediate-good producers in the South, and an expansion in international outsourcing and trade.<sup>24</sup>

We have solved the model numerically for a wide variety of parameter values. In these computations, we generally took search costs to be negligible, so that the investment constraints would bind in any equilibrium without assured matching. Holding  $\gamma^S = 0$ , we varied  $\gamma^N$  gradually from zero to 0.4 and found a recurring pattern. Namely, the volume of outsourcing in the North rises then falls as  $\gamma^N$  increases, but always remains above the level for  $\gamma^N = 0$ . Meanwhile, the volume of outsourcing in the South falls and then rises, while remaining below the level for  $\gamma^N = 0$ . The relative wage of the North rises, then falls, which implies that the ratio of world trade to world income and the share of intra-industry trade in total trade do just the opposite.<sup>25</sup>

### 5.2 Improvements in Contracting Worldwide

Before we turn to the contract environment of the South, it is helpful to discuss the effects of worldwide gains in contracting possibilities. We again take an initial

<sup>24</sup>This occurs, for example, whenever search costs are low in both countries and  $\gamma^N > \bar{\gamma}^N$ , where  $\bar{\gamma}^N < 1/2$  is the unique solution to

$$\frac{\bar{\gamma}^{N}\left(1-\frac{1}{2}\bar{\gamma}^{N}\right)}{1-\bar{\gamma}^{N}-\frac{1}{2}\left(\bar{\gamma}^{N}\right)^{2}} = \frac{1-2\bar{\gamma}^{N}}{2-3\bar{\gamma}^{N}+\frac{1}{2}\left(\bar{\gamma}^{N}\right)^{2}}$$

The value of  $\bar{\gamma}^N$  has been calculated so that the downward shift in SS at the initial  $m^S$  exactly matches the downward shift in NN. With this initial value of  $\gamma^N$ , an improvement in the contracting environment in the North results in a fall in  $m^N$  and no change in  $m^S$  or the relative wage. For still larger initial values of  $\gamma^N$  than  $\bar{\gamma}^N$ , the NN curve shifts down by more than the SS curve, so  $m^S$ rises and  $m^N$  falls.

<sup>25</sup>Such a pattern obtains, for example, when  $\eta^N = \eta^S = 0$ ,  $\mu^N = \mu^S = 50$ ,  $\alpha = 0.5$ ,  $\beta = 0.75$ ,  $f_m^N = f_m^S = 0.01$ ,  $L^N = 40$  and  $L^S = 32$ .

situation with unverifiable investment in both countries ( $\gamma^N = \gamma^S = \gamma = 0$ ) but this time consider a change in the legal environment that makes some investment tasks contractible in both countries ( $d\gamma > 0$ ). We will show that, perhaps surprisingly, such a development would not be neutral with respect to the siting of outsourcing activity.

From the equal-profit condition (31) we see that an increase in a common  $\gamma$  from an initial level of  $\gamma = 0$  raises the relative profitability of outsourcing in the North (at given  $\omega$ ,  $m^S$  and  $m^N$ ) if  $m^N \eta^S / \mu^S > m^S \eta^N / \mu^N$  and raises the relative profitability of outsourcing in the South if the inequality runs in the opposite direction. If  $m^N \eta^S / \mu^S = m^S \eta^N / \mu^N$  — as would be the case, for example, were search costs to be negligible in both countries — then the change in the common contract parameter would have no direct effect on the relative profitability of outsourcing in either location. We take this as our benchmark case — with the implication that a small increase in  $\gamma$  leaves the function  $\omega(m^S, m^N)$  undisturbed.

With no shift in  $\omega(m^S, m^N)$ , an increase in  $\gamma$  must shift the SS curve to the left, as depicted in Figure 10. As can be seen from (29), an increase in  $\gamma^{S}$  increases the demand for labor (at given  $m^S$  and  $\omega$ ) by Southern producers of components. These producers need more labor, because they undertake more investment and serve more customers. The NN curve, in contrast, shifts to the right. While it is true that Northern component producers demand more labor (at given  $m^N$  and  $\omega$ ) for much the same reason as their Southern counterparts, this is more than offset by a decline in employment by final producers. As we noted previously, at  $\gamma^N = 0$ , the second term on the left-hand side of (30) decreases with  $\gamma^N$  by the same amount as the fourth term increases. But now we also have a decline in the third term of (30)due to the growth in  $\gamma^{S}$ . The additional relationships that are consummated by final producers with input suppliers in the South are an added source of intensified competition in the product market. In response, final producers exit in even greater number than they do when contracting improves only in the North. The result is an overall decline in labor demand in the North at given wages and given numbers of component producers.



Figure 10: Contracting improves worldwide

As the figure shows, a worldwide improvement in contracting possibilities is not neutral with respect to the location of outsourcing. In the new equilibrium at  $E_2$ , there are more producers of components in the North and fewer producers of components in the South than before. The improvement in the legal environment induces a shift in outsourcing activity from South to North.<sup>26</sup> The asymmetric effects of the change in  $\gamma$  come about, because the improved prospects for investment by input suppliers mitigates the need for entry by final-good producers. With the resources freed from the activity of designing differentiated products, the North can expand its outsourcing activities. Meanwhile, the improvements in contracting possibilities raise world income (evaluated in terms of the numeraire good), and with it the demand for homogeneous goods. More labor must be devoted in the South to producers.

<sup>&</sup>lt;sup>26</sup>Outsourcing activity falls in the South, despite the increase in  $\gamma^S$ , because  $m^S$  falls by a greater percentage than output per firm rises.

#### 5.3 Improvements in Contracting in the South

We are now ready to explain why improvements in the contracting environment in the South, even if achieved from a very low initial level, need not result in an expansion of outsourcing activity there. We take an initial situation with  $\gamma^N > \gamma^S = 0$  and consider a marginal increase in  $\gamma^S$ .

For reasons that are familiar by now, an increase in  $\gamma^S$  raises (at given  $\omega$ ,  $m^S$  and  $m^N$ ) the relative profitability of search in the South. To restore the equal-profit relationship, the function  $\omega(m^S, m^N)$  must shift down. The movement in the relative wage (or the terms of trade) expands the demand for differentiated goods by the South, and reduces the demand for homogeneous goods by the North. Thus, the shift in  $\omega(m^S, m^N)$  exerts rightward pressure on the SS curve and downward pressure on the NN curve, both of which tend to generate an expansion of outsourcing activity in the South and a contraction of such activity in the North.

But the effects of the change in relative profitability are offset by impacts on labor demand at the initial pattern of search activity. In the South, component producers are able to serve more customers, and so their demand for labor grows for both investment and production purposes. This alone would shift the SS curve to the left. At the same time, the intensified competition in the product market that results from the broader search efforts of firms seeking partners in the South spells the exit of some final producers in the North. This alone reduces labor demand, tending to push the NN curve upward. On net, the SS curve can shift in either direction, as can the NN curve.

Again, we resorted to numerical computations to see what outcomes are possible. Taking search costs to be small and holding  $\gamma^N$  fixed at  $\gamma^N = 0.4$ , we varied  $\gamma^S$  from 0 to 0.4 for a wide range of values of the remaining parameters. Repeatedly, we found that the volume of outsourcing in the North rises monotonically with  $\gamma^S$ , while the volume of outsourcing in the South rises at first, but then falls to a level below that for  $\gamma^S = 0.2^7$  So too does the ratio of world trade to world income and the share of intra-

<sup>&</sup>lt;sup>27</sup>If search costs are not so small, improvements in the contracting environment in the South may lead to a decline in international outsourcing for all initial values of  $\gamma^{S}$ . Take, for example, the

industry trade in total trade. In other words, the volume of international outsourcing and the volume of world trade typically are largest when the legal environment allows somewhat less complete contracts in the South than in the North.<sup>28</sup>

## 6 Conclusions

We have developed a framework for studying outsourcing decisions in a global economy. In our model, producers of differentiated final goods must go outside the firm for an essential service or component. Search is costly and firms choose whether to conduct it in one national market or the another. If a firm finds a potential partner with suitable expertise, the supplier must customize the input for the final producer's use. Such relationship-specific investments are governed by incomplete contracts, and the contracting environment may vary across national markets.

Our model features a thick-market externality: search in a market is more profitable the more suppliers are present there, while input producers fare best when they have many customers to serve. This externality creates the possibility of multiple equilibria, some of which may involve a concentration of outsourcing activity in one location. But stable equilibria need not involve complete specialization of input production in a single country. In the paper, we focus on equilibria in which some firms outsource at home while others fill their input needs abroad.

First, we studied how country size and the technologies for search and investment affect the equilibrium location of outsourcing activity. As the South expands, its share of world outsourcing grows, as does the ratio of trade to world income and the share of intra-industry trade in total world trade. A uniform worldwide improvement in search and investment technologies, as might result from technological progress in communications and computer-aided design, has no affect on the volume of outparameter values that underlie Figure 4 and suppose that  $\gamma^N = 0.2$ . Then, as  $\gamma^S$  rises from 0 to 0.1, there is a monotonic decline in outsourcing activity in the South and an increase in outsourcing activity in the North.

<sup>28</sup>These patterns obtain, for example, when when  $\eta^{N} = \eta^{S} = 0$ ,  $\mu^{N} = \mu^{S} = 50$ ,  $\alpha = 0.5$ ,  $\beta = 0.75$ ,  $f_{m}^{N} = f_{m}^{S} = 0.01$ ,  $L^{N} = 40$  and  $L^{S} = 32$ .

sourcing or its international composition. But a disproportionate improvement in the search or investment technology of the South spells a shift in outsourcing activity from North to South.

Next, we investigated the role of the contracting environment. We characterized the legal setting in a country by the fraction of a relationship-specific investment that is verifiable to a third party. An improvement in the contracting possibilities in a country raises the relative profitability of outsourcing there, given the numbers of component producers in each country and the relative wage. But changes in the contracting environment also affect the demand for labor by component producers and final-good producers at a given wage. A global increase in the fraction of contractible investment tends to favor outsourcing in the North, whereas an improvement in the legal environment of the South can raise or lower the volume of outsourcing there while raising outsourcing from the North.

Our model does not allow for in-house production of components by final producers. Therefore, it cannot be used to study the make-or-buy decision that is a central issue in the organization of a firm. In future research, we intend to enlarge the set of opportunities open to a firm, and to study the four-way choice between domestic investment, foreign investment, local outsourcing, and international outsourcing.

## 7 Appendix A: Equilibrium Conditions

In this appendix we develop equilibrium conditions for regimes with outsourcing in both countries and assured matching in neither. Thus, we examine cases in which both countries have costly search, both have a binding investment constraint, and one has costly search and the other a binding investment constraint. We also consider the existence of equilibria with all outsourcing activity concentrated in the North. Throughout this appendix we assume that none of the relationship-specific investment is verifiable; i.e.,  $\gamma^N = \gamma^S = 0$ .

### 7.1 Outsourcing in Both Countries

We begin with the labor-market clearing condition in the South. In Section 3.1 we showed that, if the investment constraint binds in the South, the number of component producers and the relative wage must be such that

$$(1-\beta)\left(\omega L^N + L^S\right) + 2\frac{1+\alpha}{1-\alpha}f_m^S m^S = L^S.$$
(A1)

When the investment constraint does not bind in the South and final producers find themselves in a regime of costly search, the optimal search intensity is  $r^{S} = m^{S}S^{S}/2w^{N}\eta^{S}$ . Substituting this expression, together with the output equation (7) and the free-entry condition (17) into (13) yields

$$(1-\beta)\left(\omega L^N + L^S\right) + \frac{1+\alpha}{1-\alpha} \frac{\omega f_m^S}{\frac{\omega}{m^S} - \frac{\mu^S}{2\eta^S}} = L^S.$$
 (A2)

This equation replaces (A1) as the Southern labor-market clearing condition in a regime with costly search in the South.

Now we turn to the labor market in the North. We have seen that when the investment constraint binds on final producers seeking a partner at home, then  $r^N = S^N/2w^N\mu^N$  and labor-market clearing requires

$$\frac{1}{2}\left(1-\alpha\right)\beta\left(L^{N}+\frac{1}{\omega}L^{S}\right)+2\frac{1+\alpha}{1-\alpha}f_{m}^{N}m^{N}=L^{N}.$$
(A3)

When, instead, there is a regime of costly search in the North, the optimal search intensity is given by  $r^N = m^N S^N / 2w^N \eta^N$ . Substituting this expression, together with the output equation (7) and the free-entry condition (21) into (14) gives

$$\frac{1}{2}\left(1-\alpha\right)\beta\left(L^{N}+\frac{1}{\omega}L^{S}\right)+\frac{1+\alpha}{1-\alpha}\frac{f_{m}^{N}}{\frac{1}{m^{N}}-\frac{\mu^{N}}{2\eta^{N}}}=L^{N}.$$
(A4)

Finally, we consider the requirement that, with outsourcing in both countries, search must be equally profitable in both places. If the investment constraint binds for search in both countries, then the equal-profit condition is

$$\frac{\omega^{1-2\varepsilon}}{\mu^N} \left( m^N - \frac{\eta^N}{2\mu^N} \right) = \frac{1}{\mu^S} \left( m^S - \omega \frac{\eta^S}{2\mu^S} \right) , \qquad (A5)$$

as we have seen before. If the investment constraint binds in the South but final producers are in a regime of costly search in the North, then  $r^S = S^S/2w^S\mu^S$  and  $r^N = m^N S^N/2w^N\eta^N$ . These expressions for the search intensities, together with the free-entry condition (19) imply an equal-profit condition of the form

$$\frac{\left(m^{N}\right)^{2}\omega^{1-2\varepsilon}}{2\eta^{N}} = \frac{1}{\mu^{S}}\left(m^{S} - \omega\frac{\eta^{S}}{2\mu^{S}}\right) . \tag{A6}$$

If the investment constraint binds for search in the North but not in the South, then the equal-profit condition is

$$\frac{\omega^{2-2\varepsilon}}{\mu^N} \left( m^N - \frac{\eta^N}{2\mu^N} \right) = \frac{\left(m^S\right)^2}{2\eta^S} . \tag{A7}$$

Finally, if final producers are in a regime of costly search in both countries, then  $r^i = m^i S^i / 2w^N \eta^i$  for i = S, N. This, together with the free-entry conditions (19) implies an equal-profit condition of the form

$$\frac{m^S}{\sqrt{\eta^S}} = \frac{m^N \omega^{1-\varepsilon}}{\sqrt{\eta^N}} \ . \tag{A8}$$

Now we can derive the reduced-form SS and NN curves that apply for each combination of regimes. First, we take the appropriate equal-profit condition to derive the relative wage that is consistent with equal profitability given the numbers of component producers in each country. This gives the function  $\omega(m^S, m^N)$  for the particular regime combination. We then substitute this function into (A1) if the investment constraint binds in the South, or into (A2) if the South has a regime of costly search. This generates the SS curve. Similarly, we substitute the applicable form of  $\omega(m^S, m^N)$  into either (A3) or (A4) to derive the NN curve, depending upon whether the North has a binding investment constraint or a regime of costly search. Finally, we use (15) to derive the boundaries between the various combinations of regimes.

We applied this procedure for the particular parameter values indicated in the text to draw the SS and NN curves that are represented in Figures 4 and 5.

### 7.2 Outsourcing in the North only

We now consider the conditions for an equilibrium with outsourcing only in the North, with production of homogeneous goods only in the South, and with a binding investment constraint that limits search by final producers.<sup>29</sup> In such an equilibrium, there is no entry by component producers in the South, and no search by final-good producers for potential partners there.

With  $m^S = n^S = 0$  the labor-market clearing condition for the South (13) implies

$$\omega = \frac{\beta}{1 - \beta} \frac{L^S}{L^N} . \tag{A9}$$

Thus, for the existence of an equilibrium of this type with  $w^N > w^S$ , we need that  $\beta L^S/(1-\beta)L^N > 1$ . In the North, the assumption of a binding investment constraint implies the labor-market clearing condition (20). Substituting (A9) into this expression and rearranging terms, we find

$$m^N = \frac{(1-\alpha) L^N}{4f_m^N}$$
 (A10)

Finally, we use the free-entry conditions (17) and (19) with i = N to derive

$$n = n^{N} = \frac{f_{m}^{N}}{f_{n}} \left[ \frac{(1-\alpha) L^{N}}{2f_{m}^{N}} - \frac{\eta^{N}}{\mu^{N}} \right]$$
(A11)

<sup>&</sup>lt;sup>29</sup>There might be other types of equilibria that we do not consider here. For example, outsourcing may be concentrated in the South, or homogeneous goods may be produced in both locations. In the latter case, the wage in the South must be equal to the wage in the North.

and

$$r^{N} = \left(\frac{f_{n}}{\mu^{N}}\right)^{1/2} \left[\frac{(1-\alpha)L^{N}}{2f_{m}^{N}} - \frac{\eta^{N}}{\mu^{N}}\right]^{-1/2}.$$
 (A12)

The expression that appears in the square brackets in both (A11) and (A12) must be positive for the existence of such an equilibrium; but this is guaranteed by the requirements for a binding investment constraint (see (15)). It follows that these conditions, together with the requirement that  $\beta L^S / (1 - \beta) L^N > 1$ , are sufficient for the existence of an equilibrium with outsourcing concentrated in the North and  $\omega > 1$ . Such an equilibrium exists, for example, for the parameter values used to draw Figures 4 and 5.

## 8 Appendix B: Stability

In this appendix, we consider the stability of equilibria with outsourcing in both countries and search limited by a binding investment constraint in both the North and the South. We also consider the stability of equilibria with outsourcing only in the North and a binding investment constraint there. The stability conditions for other combinations of regimes can be derived with similar methods.

Our procedure for conducting the stability analysis is as follows. First, we calculate a "temporary" equilibrium for a given number of final-good producers and given numbers of component producers in each country. This temporary equilibrium involves optimal search behavior by final producers given the numbers of firms and requires that both labor markets and all product markets clear. The temporary equilibrium implies levels of net profits (operating profits less entry costs) for each type of firm. We assume that the numbers of firms adjust over time, with positive profits inducing entry and negative profits inducing exit. For the purposes of this appendix, we take  $\gamma^i = 0$  for i = S, N and treat the numbers of component producers as continuous variables.

More formally, our procedure is to calculate functions  $\hat{\Pi}_n(n, m^S, m^N)$ ,  $\hat{\Pi}_m^S(n, m^S, m^N)$ and  $\hat{\Pi}_m^N(n, m^S, m^N)$ , where  $\hat{\Pi}_n(\cdot)$  is the expected profit for a typical final-good producer net of fixed costs when there is a measure n of final-good producers in the North and  $m^i$  component producers in country *i*, and  $\hat{\Pi}^i_m(\cdot)$  is the net profit of a typical component producer in country *i* under the same conditions. We measure  $\hat{\Pi}^i_m(\cdot)$  in units of the labor of country *i* and  $\hat{\Pi}_n(\cdot)$  in units of Northern labor. We then assume that the numbers of the different types of firms adjust according to

$$\dot{n} = \lambda_n \hat{\Pi}_n \left( n, m^S, m^N \right), \tag{B1}$$

and

$$\dot{m}^{i} = \lambda_{m}^{i} \hat{\Pi}_{m}^{i} \left( n, m^{S}, m^{N} \right), \quad \text{for } i = S, N, \tag{B2}$$

where  $\lambda_n$ ,  $\lambda_m^S$  and  $\lambda_m^N$  are arbitrary positive constants. An equilibrium is a triplet  $(n, m^S, m^N)$  that implies zero net profits for final producers, zero net profits for component producers in country *i* if  $m^i > 0$ , and zero or negative net profits for component producers in country *i* if  $m^i = 0$ . We consider such an equilibrium to be (locally) stable if and only if the adjustment process represented by (B1) and (B2) is stable for all positive values of  $\lambda_n$ ,  $\lambda_m^S$  and  $\lambda_m^N$ .

# 8.1 Stability of Equilibria with a Binding Investment Constraint in Both Countries

We consider first the stability of equilibria such as those depicted in Figure 4. For an equilibrium to be locally stable for all adjustment speeds, the system comprising (B1) and (B2) must be stable for all positive values of  $\lambda_m^S$  and  $\lambda_m^N$  as  $\lambda_n \to +\infty$ . To derive necessary conditions for stability, we focus on the extreme case with very fast adjustment in the number of final producers  $(\lambda_n \to +\infty)$ . In this case *n* adjusts instantaneously to the numbers of component producers  $m^S$  and  $m^N$ , and  $\hat{\Pi}_n[n(m^S, m^N), m^S, m^N] = 0$  in the temporary equilibrium.

For the limiting case with  $\lambda_n \to +\infty$ , the stability analysis can be conducted with the two equations that result from substituting  $n(m^S, m^N)$  for n in equation (B2). We write this system as

$$\begin{bmatrix} \dot{m}^{S} \\ \dot{m}^{N} \end{bmatrix} = \begin{bmatrix} \lambda_{m}^{S} \Pi^{S}(m^{S}, m^{N}) \\ \lambda_{m}^{N} \Pi^{N}(m^{S}, m^{N}) \end{bmatrix}$$
(B3)

where  $\Pi^{i}(m^{S}, m^{N}) \equiv \hat{\Pi}^{i}_{m}[n(m^{S}, m^{N}), m^{S}, m^{N}].$ 

First, we need an expression for the profit levels for component producers in a temporary equilibrium. These profits are the difference between operating profits and fixed costs (in units of local labor), or

$$\Pi^{i} = r^{i} n^{i} (\frac{S^{i}}{w^{i}} - \mu^{i} r^{i}) - f_{m}^{i} \qquad \text{for } i = N, S.$$
 (B4)

We can use (B4), together with the zero-profit condition for final-good producers (19) and the equal-profit condition (21) to derive the labor-market clearing conditions in a temporary equilibrium.<sup>30</sup> In place of (18), we have

$$D^{S}(m^{S}, m^{N}) + \left(1 - \beta + \frac{1 + 3\alpha}{1 - \alpha}\right) m^{S} \Pi^{S} + (1 - \beta)\omega(m^{S}, m^{N})m^{N} \Pi^{N} = L^{S}, \quad (B5)$$

where

$$D^{S}(m^{S}, m^{N}) = (1 - \beta) \left[ \omega \left( m^{S}, m^{N} \right) L^{N} + L^{S} \right] + 2 \frac{1 + \alpha}{1 - \alpha} f_{m}^{S} m^{S}$$

is demand for Southern labor when all profits are zero; i.e., the left-hand side of (18). The new terms in (B5) represent the demand for Southern labor that results from the profit income in each country.

To derive a reduced-form equation for labor-market clearing in the North, we combine (6), (19) and the expression for A in footnote 26 to obtain

$$f_n \sum_{i} n^i + \sum_{i} \eta^i (r^i)^2 n^i = \frac{1}{2} (1 - \alpha) \beta \left( L^N + \frac{1}{\omega} L^S + m^N \Pi^N + \frac{1}{\omega} m^S \Pi^S \right).$$

Substituting this equation, (16) and (B4) into (14) yields the new labor-market clearing condition for the North,

$$L^{N} = D^{N} \left( m^{S}, m^{N} \right) + \frac{\frac{1}{2} \left( 1 - \alpha \right) \beta}{\omega \left( m^{S}, m^{N} \right)} m^{S} \Pi^{S} + \left[ \frac{1}{2} \left( 1 - \alpha \right) \beta + \frac{1 + 3\alpha}{1 - \alpha} \right] m^{N} \Pi^{N}$$
(B6)

<sup>30</sup>Note that, in place of the expression for A in footnote 14, we have

$$A = \frac{\beta \sum_{i} w^{i} (L^{i} + m^{i} \Pi^{i})}{\sum_{i} 2m^{i} r^{i} n^{i} \left(\frac{w^{i}}{\alpha}\right)^{1-\varepsilon}}$$

The numerator in this expression is the fraction  $\beta$  of *total* world income, including the profits or losses of component producers.

where

$$D^{N}(m^{S}, m^{N}) = \frac{1}{2}(1-\alpha)\beta\left[L^{N} + \frac{1}{\omega(m^{S}, m^{N})}L^{S}\right] + 2\frac{1+\alpha}{1-\alpha}f_{m}^{N}m^{N}$$

is demand for Northern labor when all profits are zero (i.e., the left-hand side of (20)).

The labor-market clearing conditions (B5) and (B6) can be used to solve for the profit levels. The solutions are

$$\Pi^{S}\left(m^{S}, m^{N}\right) = \frac{\frac{1}{2}\left(1-\alpha\right)\beta + \frac{1+3\alpha}{1-\alpha}}{\Gamma m^{S}} \left[L^{S} - D^{S}\left(m^{S}, m^{N}\right)\right] - \frac{\left(1-\beta\right)\omega\left(m^{S}, m^{N}\right)}{\Gamma m^{S}} \left[L^{N} - D^{N}\left(m^{S}, m^{N}\right)\right] \quad (B7)$$

and

$$\Pi^{N}\left(m^{S}, m^{N}\right) = -\frac{\frac{1}{2}\left(1-\alpha\right)\beta}{\Gamma\omega\left(m^{S}, m^{N}\right)m^{N}}\left[L^{S} - D^{S}\left(m^{S}, m^{N}\right)\right] + \frac{1-\beta + \frac{1+3\alpha}{1-\alpha}}{\Gamma m^{N}}\left[L^{N} - D^{N}\left(m^{S}, m^{N}\right)\right], \quad (B8)$$

where

$$\Gamma = \left(1 - \beta + \frac{1 + 3\alpha}{1 - \alpha}\right) \left[\frac{1}{2}\left(1 - \alpha\right)\beta + \frac{1 + 3\alpha}{1 - \alpha}\right] - \frac{1}{2}\left(1 - \alpha\right)\beta\left(1 - \beta\right) > 0.$$

Finally, we are ready to examine the stability of system (B3) at an equilibrium point, say  $(\tilde{m}^S, \tilde{m}^N)$ , at which  $\Pi^S(\tilde{m}^S, \tilde{m}^N) = 0$  and  $\Pi^N(\tilde{m}^S, \tilde{m}^N) = 0$ , and thus  $D^i(\tilde{m}^S, \tilde{m}^N) = L^i$  for i = S, N. Stability requires  $\Pi_S^S < 0, \Pi_N^N < 0$ , and  $\Pi_S^S \Pi_N^N >$  $\Pi_N^S \Pi_S^N$ , where  $\Pi_j^i = \partial \Pi^i(m^S, m^N) / \partial m^j$ . Since  $D_j^i > 0$  for  $i \neq j$ , it follows from (B7) that  $\Pi_S^S < 0$  requires  $D_S^S > 0$ . Similarly, it follows from (B8) that  $\Pi_N^N < 0$  requires  $D_N^N > 0$ . Therefore, both the SS curve (defined by  $D^S(m^S, m^N) = L^S$ ) and the NN curve (defined by  $D^N(m^S, m^N) = L^N$ ) must be downward sloping at a locally stable equilibrium. Also,  $\Pi_S^S \Pi_N^N > \Pi_N^S \Pi_S^N$  requires  $D_S^S D_N^N > D_N^S D_S^N$ , which in turn requires that the SS curve be steeper than the NN curve at a stable point of intersection. We conclude that point  $E_2$  in Figure 4 is not a stable equilibrium point.

# 8.2 Stability of Equilibria with a Binding Investment Constraint in the North and Costly Search in the South

In this case, the equal-profit condition is given by (A7). We use it to solve for the relative wage function  $\omega(m^S, m^N)$ . Since we assume that final-good producers enter and exit very rapidly in response to profit opportunities, we have  $\Pi_n = 0$  at every moment in time, which implies that (19) holds also at every moment in time. Therefore, the labor-market-clearing condition for the North is the same as in (B6), except that the relative wage function now is derived from (A7).

For the South, we need to derive a new labor-market-clearing condition that accounts for the profits (or losses) of input suppliers there. In place of (13), we have

$$(1-\beta)(\omega L^N + L^S + \omega \Pi^N + \Pi^S) + m^S f_m^S + \mu^S m^S n^S (r^S)^2 + 2m^S r^S n^S y^S = L^S .$$
(13')

Now, using the expression for the profits of intermediate-good producers given in (B4), together with the facts that  $y^S = \alpha S^S / (1 - \alpha) w^S$  and that the equilibrium search distance is  $r^S = m^S S^S / 2w^N \eta^S$ , the labor-market-clearing condition for the South becomes

$$D^{S}(m^{S},m^{N}) + \left[ \left( \frac{1+\alpha}{1-\alpha} \right) \frac{\frac{\omega(m^{S},m^{N})}{m^{S}}}{\frac{\omega(m^{S},m^{N})}{m^{S}} - \frac{\mu^{S}}{2\eta^{S}}} - \beta \right] m^{S} \Pi^{S} + (1-\beta)\omega(m^{S},m^{N})m^{N} \Pi^{N} = L^{S},$$
(B5')

where

$$D^{S}(m^{S},m^{N}) = (1-\beta) \left[ \omega \left( m^{S},m^{N} \right) L^{N} + L^{S} \right] + \left( \frac{1+\alpha}{1-\alpha} \right) \frac{\omega \left( m^{S},m^{N} \right) f_{m}^{S}}{\frac{\omega (m^{S},m^{N})}{m^{S}} - \frac{\mu^{S}}{2\eta^{S}}}$$

is demand for Southern labor when all profits are zero. Here too the relative wage function  $\omega(m^S, m^N)$  is the one derived from (A7).

The labor-market-clearing conditions (B5') and (B6) can be used to solve for the profit levels. The solutions are

$$\Pi^{S}\left(m^{S}, m^{N}\right) = \frac{\frac{1}{2}\left(1-\alpha\right)\beta + \frac{1+3\alpha}{1-\alpha}}{\Gamma'm^{S}} \left[L^{S} - D^{S}\left(m^{S}, m^{N}\right)\right] - \frac{\left(1-\beta\right)\omega\left(m^{S}, m^{N}\right)}{\Gamma'm^{S}} \left[L^{N} - D^{N}\left(m^{S}, m^{N}\right)\right]$$
(B7')

and

$$\Pi^{N}\left(m^{S}, m^{N}\right) = -\frac{\frac{1}{2}\left(1-\alpha\right)\beta}{\Gamma'\omega\left(m^{S}, m^{N}\right)m^{N}}\left[L^{S}-D^{S}\left(m^{S}, m^{N}\right)\right] + \frac{\left(\frac{1+\alpha}{1-\alpha}\right)\frac{\frac{\omega\left(m^{S}, m^{N}\right)}{m^{S}}-\frac{\mu^{S}}{2\eta^{S}}}{\frac{\omega\left(m^{S}, m^{N}\right)}{m^{S}}-\frac{\mu^{S}}{2\eta^{S}}} - \beta}{\Gamma'm^{N}}\left[L^{N}-D^{N}\left(m^{S}, m^{N}\right)\right], \quad (B8')$$

where

$$\Gamma' = \left[ \left(\frac{1+\alpha}{1-\alpha}\right) \frac{\frac{\omega(m^S, m^N)}{m^S}}{\frac{\omega(m^S, m^N)}{m^S} - \frac{\mu^S}{2\eta^S}} - \beta \right] \left[ \frac{1}{2} \left(1-\alpha\right)\beta + \frac{1+3\alpha}{1-\alpha} \right] - \frac{1}{2} \left(1-\alpha\right)\beta \left(1-\beta\right).$$

Note that  $\Gamma' > 0$ , because the term in the first square bracket is larger than  $1 - \beta$  and so its product with  $\frac{1}{2}(1-\alpha)\beta$  is larger in absolute value than the negative term.

Finally, we are ready to examine the stability of system (B3) at an equilibrium point, say  $(\tilde{m}^S, \tilde{m}^N)$ , at which  $\Pi^S(\tilde{m}^S, \tilde{m}^N) = 0$  and  $\Pi^N(\tilde{m}^S, \tilde{m}^N) = 0$ , and thus  $D^i(\tilde{m}^S, \tilde{m}^N) = L^i$  for i = S, N. Stability requires  $\Pi_S^S < 0$ ,  $\Pi_N^N < 0$ , and  $\Pi_S^S \Pi_N^N >$  $\Pi_N^S \Pi_S^N$ , where  $\Pi_j^i = \partial \Pi^i(m^S, m^N) / \partial m^j$ . In view of the fact that  $\Gamma' > 0$ , however,  $\Pi_S^S \Pi_N^N > \Pi_S^S \Pi_N^N$  if and only if  $D_S^S D_N^N > D_N^S D_S^N$ .

Since  $D_S^N > 0$ , it follows from (B7') that  $\Pi_S^S < 0$  requires  $D_S^S > 0$ . There are now two possibilities: either  $D_N^S \ge 0$  or  $D_N^S < 0$ . If  $D_N^S \ge 0$ , it follows from (B8') that  $\Pi_N^N < 0$  requires  $D_N^N > 0$ . In this case, the NN and SS curves both slope downward and  $D_S^S D_N^N > D_N^S D_S^N$  requires that the SS curve is the steeper of the two. This is similar to what we found in the previous section.

Alternatively, if  $D_N^S < 0$ , the SS curve slopes upward, because  $D_S^S > 0$  at a stable equilibrium. Now  $\Pi_N^N < 0$  does not imply that  $D_N^N > 0$ . If it happens that  $D_N^N > 0$ , then the equilibrium is unstable, because the requirement that  $D_S^S D_N^N > D_S^S D_S^N$  will be violated. Thus, an equilibrium at which the NN curve slopes downward and the SS curve slopes upward – such as the one depicted by point  $E_2$  in Figure 5 – is not stable. The only remaining possibility is that  $D_N^N < 0$ , in which case both the NNand SS curves slope upward. Then the requirement that  $D_S^S D_N^N > D_N^S D_S^N$  implies that the NN curve must be the steeper. We have not been able to rule out the existence of a stable equilibrium of this sort, but nor have we been able to construct one in our numerical simulations.

# 8.3 Stability of Equilibria with Outsourcing Concentrated in the North

We examine next the stability of an equilibrium in which all intermediate goods are produced in the North and a binding investment constraint limits the search of final producers there. The conditions for such an equilibrium were derived in Appendix A.

When  $m^S$  is close to zero, no final-good producer searches for an outsourcing partner in the South. With  $n^S = 0$ , the operating profits of a Southern component producer are zero, and the total profits are negative; see (B4) with i = S. It follows that if a small number of Southern component producers do happen to be in the market, they will gradually exit according to the dynamics given in (B2).

The profits of Northern component producers are given by (B4) with i = N. The profits of final-good producers are given by

$$\Pi_n = r^N \left( m^N \frac{S^N}{w^N} - \eta^N r^N \right) - f_n .$$
(B9)

In the South, the labor-market clearing condition in a temporary equilibrium becomes

$$L^{S} = (1 - \beta) \left( \omega L^{N} + L^{S} + \omega n \Pi_{n} + \omega m^{N} \Pi^{N} + m^{S} \Pi^{S} \right) + f_{m}^{S} m^{S}$$
(B10)

while in the North it is

$$L^{N} = \frac{1}{2} (1 - \alpha) \beta \left( L^{N} + \frac{1}{\omega} L^{S} + n\Pi_{n} + m^{N} \Pi^{N} + \frac{1}{\omega} m^{S} \Pi^{S} \right) - n\Pi_{n} + 2 \frac{1 + \alpha}{1 - \alpha} f_{m}^{N} m^{N} + \frac{1 + 3\alpha}{1 - \alpha} m^{N} \Pi^{N}.$$
 (B11)

Now we can use (B4), (B9), (B10), (B11) and  $\Pi^S = -f_m^S$  to solve for the expected profits of final-good producers and the profits of the typical Northern component producer, both as functions of n and  $m^N$ . The solutions are

$$\Pi_{n}\left(n,m^{N}\right) = \frac{2m^{N} - \frac{\eta^{N}}{\mu^{N}}}{n\Omega\left(m^{N}\right)} \left(L^{N} - \frac{4}{1-\alpha}f_{m}^{N}m^{N}\right) + \frac{m^{N}\left(\frac{1-\alpha}{1+\alpha} + 2\frac{1+3\alpha}{1-\alpha^{2}}\right)}{n\Omega\left(m^{N}\right)} \left[\left(2m^{N} - \frac{\eta^{N}}{\mu^{N}}\right)f_{m}^{N} - nf_{n}\right]$$

and

$$\Pi^{N}(n,m^{N}) = \frac{1}{\Omega(m^{N})} \left( L^{N} - \frac{4}{1-\alpha} f_{m}^{N} m^{N} \right) + \frac{1}{\Omega(m^{N})} \left[ \left( 2m^{N} - \frac{\eta^{N}}{\mu^{N}} \right) f_{m}^{N} - n f_{n} \right],$$

where

$$\Omega\left(m^{N}\right) = \frac{\eta^{N}}{\mu^{N}} + \left(\frac{1-\alpha}{1+\alpha} + 2\frac{1+3\alpha}{1-\alpha^{2}} - 2\right)m^{N} > 0$$

The necessary and sufficient conditions for stability are  $\partial \Pi_n / \partial n < 0$ ,  $\partial \Pi^N / \partial m^N < 0$ , and  $(\partial \Pi_n / \partial n) (\partial \Pi^N / \partial m^N) > (\partial \Pi_n / \partial m^N) (\partial \Pi^N / \partial n)$ . At an equilibrium point with  $m^N = (1 - \alpha) L^N / 4f_m^N$  and  $n = (2m^N - \eta^N / \mu^N) f_m^N / f_n$ , these conditions are satisfied. Therefore, when there exists an equilibrium with outsourcing concentrated in the North and a binding investment constraint, that equilibrium is stable.

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