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INTERNATIONAL ASSET ALLOCATION WITH TIME-VARYING CORRELATIONS

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ABSTRACT

It is widely believed that correlations between international equity markets tend to increase in highly volatile bear markets. This has led some to doubt the benefits of international diversification. This article solves the dynamic portfolio choice problem of a US investor faced with a time-varying investment opportunity set which may be characterized by correlations and volatilities that increase in bad times. We model the state dependance of US, UK, and German equity returns using a regime-switching model and find evidence for the existence of a high volatility regime, in which returns are more highly correlated and have lower means. Solving the dynamic asset allocation problem for a CCRA investor, we show international diversification is still valuable with regime changes. Currency hedging imparts further benefit. The costs of ignoring the regimes are small for moderate levels of risk aversion, and the intertemporal hedging demands induced by time-varying correlations are negligible.

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1 Introduction

An argument often heard is that correlations between international equity returns are higher during bear markets than during bull markets, and bear market moves are greater than bull markets. This would suggest that the benefits of international diversification are less impressive than conventional wisdom predicts.[1] This argument is potentially very important since it may help explain the "home bias puzzle", arguably one of the most important puzzles in international finance. If the diversification benefits from international investing are not forthcoming at the time that investors need them the most (when their home market experiences a downturn), international investing may not be worth the trouble.

The existing literature typically documents empirical facts about correlations but has not formalized the link with diversification benefits. Absent such a link, the argument is incomplete, vague and potentially incorrect. The gap in the literature is not so surprising. The standard benchmark when thinking about the benefits of international diversification is a static one-period mean-variance framework (French and Poterba (1991) and Tesar and Werner (1995)).[2] In this article, we analyze the impact of time-varying correlations on asset allocation in a dynamic portfolio allocation problem.

We use regime switching (RS) models to model international equity returns. Since Hamilton (1989)'s seminal work on RS models, a large literature has developed applying RS models to many financial time series where there is evidence of changing behavior of the series across business cycles or where there is other periodic change. Recent estimates of RS models for stock returns appear in Ramchand and Susmel (1998a and b) and Hamilton and Lin (1996). RS models are able to capture changing conditional means, changing conditional covariances, and the higher moments of equity returns by using only one state variable, the regime, which can take on two values. This makes the portfolio allocation solution surprisingly simple and intuitive. The effect of stochastic volatility, albeit of different forms, on asset allocation has only been considered in a few papers so far (Das and Uppal (1998), and Liu (1998)). We also consider a case where the data generating process (DGP) involves a stochastic interest rate and a case where there is a state variable (earnings yields) predicting equity returns.

Our ambition is not to explain the home bias puzzle *per se* but to provide a formal evaluation of the claim made in the first paragraph in a relatively simple portfolio choice setting. More specifically, our contribution consists of four parts. First, we numerically solve and develop intuition on the dynamic asset allocation problem in the presence of regime switches for investors with Constant Relative Risk Aversion (CRRA) preferences. Here our contribution extends beyond international finance. There has recently been a resurgence of interest in dynamic portfolio problems where investment opportunity sets change over time.[3] In most of these papers, the investment opportunity set is indexed by a set of state

[1] Among the many authors documenting this include Longin and Solnik (1998, 1995), Das and Uppal (1996), De Santis and Gerard (1997), King, Sentana and Wadhwani (1994), and Erb, Harvey and Viskanta (1994).

[2] Recent papers such as Das and Uppal (1998) have considered dynamic settings for international portfolio choice.

[3] See Balduzzi and Lynch (1999), Liu (1998), Barberis (1996), Campbell and Viceira (1998a and b), and Brennan, Schwartz and Lagnado (1997).

variables linearly affecting expected returns. Compared to these papers, in some of our examples below, expected returns vary only with the regime, rather than with state variables.

Second, we specify several RS models for international equity returns that naturally allow formal tests for different correlations, volatilities and means across different regimes. Although very different from the majority of the papers in this literature, our work here is closely related to recent work by Ramchand and Susmel (1998a). Ramchand and Susmel model covariances using a switching ARCH process. Interestingly, we find evidence of a regime which exhibits higher volatility but find weaker evidence of higher correlations and lower conditional means in that regime with monthly data. We find no evidence of RS ARCH effects on covariances of the US and UK.

Third, we estimate the portfolio choice of the investor for a number of different RS DGP's, horizons, and preference parameters. To characterize the uncertainty in the portfolio allocations resulting from the uncertainty in the parameters of the DGP, Barberis (1996) and Kandel and Stambaugh (1996) use a Bayesian setting, and Brandt (1998) estimates portfolio weights using an Euler equation approach and instruments. Instead, we characterize the uncertainty in the portfolio choices from a classical econometric perspective, using the delta-method, as do Campbell and Viceira (1998a). Our approach allows us to formally test for the presence of intertemporal hedging demands (the difference between the investor's one period ahead and long-horizon portfolio choice) and for the presence of regime-dependent asset allocation for investors with different horizons. It is quite conceivable that long-horizon investors need not worry about an occasional episode of high correlation, either because the effect on utility is minor or because they can temporarily re-balance away from international stocks, if these states of the world are somewhat predictable. In the latter case their safe haven may be US stocks or it may be cash.

Finally, we investigate the economic significance of our results and the claim in the initial paragraph. We attempt to quantify the utility cost (using the certainty equivalent notion) of: (a) not being internationally diversified and (b) ignoring the occurrences of periods of higher volatility with higher correlations across all countries. A by-product of one of the set-ups we consider is that we can put an economic value on the ability to hedge foreign exchange rate risk. In most models, we preclude this ability.

Our work is closely related to Das and Uppal (1998) who consider portfolio selection when perfectly correlated jumps across countries affect international equity returns. Our RS processes produce a "normal" regime with low correlations, low volatilities and a "down-turn" regime with higher correlations, higher volatilities and lower conditional means. However, both regimes are persistent and such persistence cannot be captured by transitory jumps independent of equity returns. In fact, Das and Uppal (1996) empirically document that the higher correlations associated with large equity shocks are persistent.[4] Furthermore we consider the effect of regime changes on portfolio choice when short rates and yields predict returns, and we examine currency hedging demands.

Our work, and some of our results, is also closely related to Brandt (1998). Brandt uses a non-

[4]Das and Uppal also conduct a static asset allocation exercise, contrasting the optimal portfolios of an investor who recognizes the dependence of the disturbance on the size of the shock with those of an investor who does not.

parametric approach to estimate portfolio weights in a domestic asset allocation problem. Even though they are more general, non-parametric approaches may have large small sample biases and our parametric models may fare better in small samples and lead to more powerful statistical tests. We also face the problem, shared with most papers in the literature, that the noisiness in expected returns may lead to noisy and unrealistic asset allocations (Best and Grauer (1991)). To partially mitigate this, we consider a restricted version of each model designed to limit sampling error in the means by restricting the means across regimes to be equal. These models cannot be statistically rejected by the data and in most cases offer better regime classification. Constraining the conditional means across regimes to be equal also allows a sharper focus on the effect of time-varying correlations.

To make the analysis tractable, we leave out many aspects of international asset allocation that may be equally or more important but may blur the focus of the paper. Examples include transaction costs (see Balduzzi and Lynch (1999) in a domestic and Cooper and Kaplanis (1994) in an international context); inflation risk (Glassman and Riddick (1996)); cross-country informational differences (Brennan and Cao (1997)); and human capital and labor (Viceira (1998)).

The outline of the paper is as follows. We start by formulating the general asset allocation problem in Section 2, and show how to numerically solve the problem with regime switching. In Section 3, we present the RS models which we use as our DGP's. In Section 4 we describe the data, test for the presence of regimes, and present the estimation results of the RS processes. We present the main results of asset allocation under regime switching in Section 5 and examine the robustness of these results in Section 6. Section 7 concludes.

2 Asset Allocation with Changes in Regimes

2.1 The General Problem

Consider the following asset allocation problem. A US investor facing a T month horizon who rebalances her portfolio over N assets every month maximizes her expected end of period utility. The problem can be stated more formally as:

$$\max_{\alpha_0,\ldots,\alpha_{T-1}} \mathrm{E}_0[U(W_T)] \tag{1}$$

subject to the constraint that the portfolio weights must sum to 1, $\alpha_{t-1}'\mathbf{1} = 1$, where W_T is end of period wealth and $\alpha_0, \ldots, \alpha_{T-1}$ are the portfolio weights at time 0 (with T periods left), ..., to time $T-1$ (with 1 period left). There are no costs for short-selling or rebalancing. Wealth W_t at time t is given by $W_t = R_t(\alpha_{t-1})W_{t-1}$ where R_t is given by:

$$R_t = \sum_{i=1}^{n} \exp(y_t^i)\alpha_{i,t-1} \equiv \exp(y_t)'\alpha_{t-1}, \tag{2}$$

where y_t^i is the return on asset i in USD at time t. We use CRRA, or iso-elastic, utility:

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma} \tag{3}$$

where γ is the investor's coefficient of risk aversion.

We concentrate on the investment problem of the investor and ignore intermediate consumption (or the investor is assumed to consume end of period wealth W_T). In effect, we take the savings decision to be exogenously specified. We choose the CRRA family of utility as it is a standard benchmark and enables comparison to earlier literature. In common with most empirical dynamic asset allocation papers in the literature, this approach does not address market equilibrium, so the investor is not necessarily the representative agent in the US economy. We also do not consider the asset allocation problem faced by foreign agents.

Using dynamic programming we can obtain the portfolio weights at each horizon t by maximizing the (scaled) indirect utility:

$$\alpha_t^* = \arg\max_{\alpha_t} \mathrm{E}_t[Q_{t+1,T} W_{t+1}^{1-\gamma}] \tag{4}$$

where

$$Q_{t+1,T} = \mathrm{E}_{t+1}\left[(R_T(\alpha_{T-1}^*) \dots R_{t+2}(\alpha_{t+1}^*))^{1-\gamma}\right], \tag{5}$$

and $Q_{T-1,T} = 1$. The first order conditions (FOC) of the investor's problem are:

$$\mathrm{E}_t\left[Q_{t+1,T} R_{t+1}^{-\gamma}(\alpha_t)\begin{pmatrix}(\exp(y_{t+1}^1) - \exp(y_{t+1}^N)\\ (\exp(y_{t+1}^2) - \exp(y_{t+1}^N)\\ \vdots \\ (\exp(y_{t+1}^{N-1}) - \exp(y_{t+1}^N))\end{pmatrix}\right] \equiv \mathrm{E}_t[Q_{t+1,T} R_{t+1}^{-\gamma}(\alpha_t)\lambda_{t+1}] = 0 \tag{6}$$

where λ_{t+1} are returns of assets 1 to $N-1$ in excess of asset N. The optimal portfolio weights α_t^* solve equation (6). Note that α_t has effectively $N-1$ degrees of freedom, as the weight in the N-th asset will make the portfolio weights sum to 1.

2.2 Introducing Simple Regime Switching

Up to this point, no specific DGP has been assumed for the asset returns y_t and the set-up of the problem is entirely general. In the special case of y_t IID across time, Samuelson (1969) shows that for CRRA utility the portfolio weights are constant ($\alpha_t^* = \alpha^*$), and the T horizon problem becomes equivalent to solving the myopic 1 period problem in equation (1). When returns are not IID then the portfolio weights can be broken down into a myopic and a hedging component (Merton (1971)). The myopic component is the solution from solving the 1 period problem. The hedging component results from the investor's desire to hedge against unfavorable changes in the investment opportunity set.

Suppose we now introduce regimes $s_t = 1, \ldots, K$ into the DGP. At each time t, y_t will be drawn from a different distribution, depending on which regime s_t is prevailing at time t. Following Hamilton (1989), the regimes s_t follow a Markov chain where the transition probabilities of going from state i to state j at time t are denoted by $p_{ij,t} = p(s_{t+1} = j | s_t = i, \mathcal{I}_{t-1})$. Let $F(\cdot, s_t)$ denote the cumulative density function of y_t conditional on regime s_t. In our simple RS models, we let $F(\cdot, s_t)$ be a multivariate normal distribution, with constant transition probabilities $p_{ij,t} = p_{ij}$. Conditional on the regime at time t, the distribution of y_t is a mixture of normals. This allows the distribution to capture fat tails, persistent volatility and other properties of equity returns.

We assume that the regimes are known by the agent at time t.[5] With K regime states the random variable $Q_{t,T} = Q_{t,T}(s_t)$ may take on one of K values, one for each regime state $s_t = 1, \ldots, K$, at each time t. Even without instrument predictability of y_t, the asset allocation implications of regime switching are potentially important. The optimal portfolio weights now become functions of the state, $\alpha_t^* = \alpha_t^*(s_t)$. Moreover, the investor will want to hedge herself against regime switches. The intertemporal hedging demands will cause $\alpha_T^*(s_t)$ to differ from $\alpha_1^*(s_t)$.

Except for special discrete distributions for $F(\cdot, s_t)$, the FOC's in equation (6) do not have a closed form solution. To our knowledge, the current state of analytical tools in continuous time also does not permit a solution for both state-dependent conditional means and covariances, and discretely sampled observations.[6] Following Tauchen and Hussey (1991), a numerical solution to equation (6) may be obtained by quadrature. An M-point quadrature rule for the function $g(u)$, $u \in \mathbb{R}^n$, over the cumulative density $F(u)$ is a set of points $\{u_k\}$, $k = 1 \ldots M$ and corresponding weights $\{w_k\}$ such that

$$\int g(u) dF(u) \doteq \sum_{k=1}^{M} g(u_k) w_k \tag{7}$$

For example, for the asset returns y_t at time t in regime $s_t = i$, we use a M_i quadrature rule with points $\{y_{ik,t}\}$, $k = 1 \ldots M_i$ and corresponding weights $\{w_{ik,t}\}$. Now consider the one-period problem at $T-1$. For $s_t = i$ the FOC can be approximated by:

$$\begin{aligned} \mathrm{E}_{T-1}[R_T^{-\gamma}(\alpha)\lambda_T | s_{T-1} = i] &= \sum_{j=1}^{K} p_{ij,T-1} \mathrm{E}[R_T^{-\gamma}(\alpha)\lambda_T | s_T = j] \\ &\doteq \sum_{j=1}^{K} \left(p_{ij,T-1} \sum_{k=1}^{M_j} (\exp(y_{jk,T})'\alpha)^{-\gamma} \lambda_{jk,T}\, w_{jk,T} \right) = 0 \end{aligned} \tag{8}$$

[5]If this assumption is weakened the problem becomes considerably more difficult. All possible sample paths must be considered, so the state space increases exponentially, as agents must update their probabilities of being in a particular state at each time in a Bayesian fashion. For a one period horizon, if investors have uncertainty about the regimes, the regime-dependent solutions deviate less from the IID solution without regimes, weakening the regime-dependent effects. In this sense, the assumption of observable regimes is a worst-case scenario.

[6]The case of switching conditional means in continuous time with complete observation of the Brownian motion paths has been solved in closed-form by Honda (1997).

where

$$y_{jk,T} = \begin{pmatrix} y_{jk,T}^{1} \\ y_{jk,T}^{2} \\ \vdots \\ y_{jk,T}^{N} \end{pmatrix} \quad \text{and} \quad \lambda_{jk,T} = \begin{pmatrix} \exp(y_{jk,T}^{1}) - \exp(y_{jk,T}^{N}) \\ \exp(y_{jk,T}^{2}) - \exp(y_{jk,T}^{N}) \\ \vdots \\ \exp(y_{jk,T}^{N-1}) - \exp(y_{jk,T}^{N}) \end{pmatrix}.$$

The optimal portfolio weights $\alpha_{i,T-1}^{*} \equiv \alpha_{T-1}^{*}(s_{T-1} = i)$ are the solution to equation (8) which can be obtained by a non-linear root solver.

We define $Q_{i,T-1,T} \equiv Q_{T-1,T}(s_{T-1} = i)$ as:

$$\begin{aligned} Q_{i,T-1,T} &= \mathrm{E}_{T-1}[R_{T}^{1-\gamma}(\alpha_{T-1}^{*})|s_{T-1} = i] \\ &\doteq \sum_{j=1}^{K} \left(p_{ij,T-1} \sum_{k=1}^{M_j} (\exp(y_{jk,T})'\alpha_{i,T-1}^{*})^{1-\gamma} w_{jk,T} \right) \end{aligned} \tag{9}$$

Then the $T-2$ problem for each state $s_{T-2} = i$ may be obtained by finding the root of:

$$\begin{aligned} &\mathrm{E}_{T-2}[Q_{T-1,T}R_{T-1}^{-\gamma}(\alpha)\lambda_{T-1}|s_{T-2} = i] \\ &\doteq \sum_{j=1}^{K} p_{ij,T-2} \left(\sum_{k=1}^{M_j} Q_{j,T-1,T}(\exp(y_{jk,T-1})'\alpha)^{-\gamma}\lambda_{jk,T-1}\, w_{jk,T-1} \right) = 0 \end{aligned} \tag{10}$$

We may continue this process for $t = T-3$ onto $t = 0$.

For the case of Gaussian distributed returns y_t IID across time, Gaussian-Hermite weights can be used and the approximations are very accurate for small choices of M_i (Press et al. (1992)). This makes the asset allocation solution surprisingly simple for switching multivariate normal returns with constant transition probabilities. In effect, for K regimes, we have only K state variables which must be tracked at each horizon. When the return distributions depend on instruments z_t, the distribution of the returns y_t will be conditional on both the regime and the realization of the instrument at time t, so $F = F(\cdot, s_t, z_t)$. In this case we construct a discrete Markov chain in each regime to approximate $F(\cdot, s_t, \cdot)$, the distribution of z_t, and then combine them to approximate the unconditional distribution. In this setting the portfolio weights now become a function both of the regime s_t and the instruments z_t, so $\alpha_t^{*} = \alpha_t^{*}(s_t, z_t)$. Further details are provided in the Appendix.

2.3 How Important is Regime Switching?

Introducing regimes into the asset allocation problem has the potential to cause investors to wildly alter their portfolio allocations across regimes, and to induce intertemporal hedging demands making the investor facing a T-period horizon hold substantially different portfolio weights from the myopic investor conditional on regime s_t. We wish to test statistically and economically whether these effects are large under RS when realistic RS DGP's have been fitted to real data. These tests are more than interesting empirical exercises: if the asset allocations are similar across regimes, then in practice investors may not go to the trouble of rebalancing, especially if transactions costs are high. If intertemporal hedging demands are small, then investors may lose very little in solving a simple one-period problem at all horizons rather than solving the rather more complex dynamic problem. If there is a bad regime where international equity returns provide fewer diversification benefits, investing overseas may not be of benefit for investors.

2.3.1 Economic Significance

We wish to calculate the utility loss, or monetary compensation required for an investor to use non-optimal weights $\{\alpha^+\}$ instead of the optimal weights $\{\alpha^*\}$ for our RS DGP. For example, an investor may have to use non-optimal weights as she may not be allowed by external constraints to use forward derivatives to hedge currency risk, or even invest internationally. Another example is if the investor chooses to ignore RS and uses portfolio weights thinking the returns are IID when in fact the true DGP is RS. We would like to see the economic loss that results from holding these non-optimal portfolios instead of using the optimal one.

We can find the amount of wealth $\bar{w}$ required to compensate an investor for using $\{\alpha^+\}$ in place of $\{\alpha^*\}$ for a T-period horizon. Formally, this is given by the value of $\bar{w}$ which solves:

$$\mathrm{E}_0[U(W_T^*|W_0 = 1)] = \mathrm{E}_0[U(W_T^+|W_0 = \bar{w})]. \tag{11}$$

Since CRRA utility is homogeneous in initial wealth and since $U(W_T^\dagger|W_0 = 1) = Q_{0,T}^\dagger/(1-\gamma)$ for $\dagger = *, +$, it follows that:

$$\bar{w} = 100 \times \left(\frac{Q_{0,T}^*}{Q_{0,T}^+}\right)^{\frac{1}{1-\gamma}}. \tag{12}$$

We express the compensation required in cents per dollar of wealth $w = 100(\bar{w} - 1)$. Equivalently, w is the percentage increase in the certainty equivalent from moving from strategy $\{\alpha^+\}$ to the the optimal strategy $\{\alpha^*\}$. In the context of asset allocation analysis, changes in certainty equivalents have been considered by many recent authors, see for example Campbell and Viceira (1998a and b) and Kandel and Stambaugh (1996).

2.3.2 Statistical Tests

To formulate statistical tests we need to derive standard errors on the portfolio weights. Suppose that the parameters of the RS process are given by $\hat{\theta}$ and have an asymptotic distribution $N(\theta_0, \Omega)$ where θ_0 is the vector of the true population parameters. The portfolio weights $\alpha_t^*(s_t)$ are implicitly defined by the FOC's in equation (6). We will suppress the dependence on $s_t = 1 \ldots K$. Denote these FOC's for horizon t as $G_t(\theta, \alpha)$ where $G_t : \theta \times \alpha \rightarrow \mathbb{R}^{N-1}$.[7] We consider α_t to be an $N-1$ vector.

[7] In the case of regime switching and predictability then $\alpha_t^* = \alpha_t^*(s_t, z_t)$ and the FOC's become an implicit function dependent on z_t, that is, $G = G_{t,z_t}(\theta, \alpha)$.

The FOC's implicitly define α_t^* as the solution to $G_t(\hat{\theta}, \alpha_t^*) = 0$. Let α_{t0} satisfy $G_t(\theta_0, \alpha_{t0}) = 0$, so α_{t0} are the portfolio weights at the population parameters. Assume the determinant

$$\Delta = \det \left(\left. \frac{\partial G_t}{\partial \alpha} \right|_{(\theta_0, \alpha_{t0})} \right) \neq 0. \tag{13}$$

The Implicit Function Theorem now guarantees the existence of a function g such that $G_t(\theta_0, g(\theta_0)) = 0$ where

$$D = \left. \frac{\partial g}{\partial \theta} \right|_{\theta = \theta_0} \tag{14}$$

is well defined. Now the standard delta-method can be used to obtain the asymptotic distribution of α_t^* as:

$$\alpha_t^* \overset{a}{\sim} N(g(\theta_0), D\Omega D'). \tag{15}$$

In practice, numerical gradients are calculated. Hence the delta-method allows us to obtain standard errors for α_t^* (See also the working paper version of Campbell and Viceira (1998a)). For a given t, we test if the portfolio weights for $s_t = i$ and $s_t = j$ are statistically different. To test hedging demands for horizon T, we may define an implicit function $G = (G_1' \, G_T')'$ which stacks the FOC's for the myopic problem and the horizon T problem. This allows a test of $\alpha_1(s_t) = \alpha_T(s_t)$. Joint tests may be similarly performed.

3 Regime Switching Models

3.1 General Model

The most general regime switching model we consider can be written as:

$$\begin{aligned} y_t - \xi r_{t-1} &= g(s_t, y_{t-1}, z_{t-1}) + u_{yt} \\ z_t &= c(s_t) + A(s_t) z_{t-1} + u_{zt} \\ u_t &= (u_{yt} \, u_{zt})' \\ u_t | s_t &\sim N(0, \Omega(s_t, y_{t-1}, z_{t-1})), \end{aligned} \tag{16}$$

where y_t ($N \times 1$) are the equity returns, z_t ($M \times 1$) are the predictive instruments, and r_{t-1} is the monthly US short rate. To consider the case of nominal returns we set $\xi = 0$ and in excess return models $\xi = 1$. We refer to excess equity returns with a tilde, so $\tilde{y}_t \equiv y_t - r_{t-1}$. The distribution of equity returns depends on the regime s_t at time t and the previous realization of the instruments z_{t-1}. The instruments z_t themselves follow an autoregressive process, of which the coefficients can vary with the regime.

The regimes s_t follow a two-state Markov chain with transition matrix:

$$\begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}$$

which can vary through time. The transition probabilities are given by:

$$
\begin{aligned}
P &= p(s_t = 1 | s_{t-1} = 1; \mathcal{I}_{t-1}) = f_1(z_{t-1}) \\
Q &= p(s_t = 2 | s_{t-1} = 2; \mathcal{I}_{t-1}) = f_2(z_{t-1}).
\end{aligned}
\tag{17}
$$

There are two sources of predictability for returns. First, the conditional mean within each regime of the equity return may be predictable which is captured by $g(s_t, y_{t-1}, z_{t-1})$. Second, the transition probabilities may be non-linear functions of the instruments, captured through $f_i(z_{t-1})$, $i = 1, 2$. Such a model can potentially capture long-horizon predictability of returns by instruments.

We concentrate on the set of equity returns $y_t = (y_t^{us}\ y_t^{uk,uh}\ y_t^{uk,h}\ y_t^{ger,uh}\ y_t^{ger,h})'$ where uh denotes unhedged USD returns and h denotes currency-hedged returns. The set of instruments we consider is $z_t = (ey_t^{us}\ r_t^{us}\ y_t^{w} - r_{t-1}^{us})'$, where ey_t denotes the log earnings yield, r_t the short rate and $y_t^w - r_{t-1}^{us}$ is the excess return on the world portfolio. We do not consider past equity returns as instruments as we will see in Section 4 that the autocorrelations of the equity returns are insignificant.[8] Heteroskedasticity can be included by appropriate parameterizations of $\Omega(s_t, y_{t-1}, z_{t-1})$. We will test for the presence of RS ARCH for the case of the US-UK.

We restrict the number of regimes to two. This number of states may be restrictive, but including more regimes poses extreme computational problems. Two states should capture the main effects of higher order moments in equity returns. The regimes of each country are also assumed to be perfectly correlated. Weakening this assumption by increasing the state space along the lines of Ang and Bekaert (1998) and Ramchand and Susmel (1998a) makes the number of parameters infeasible for estimation. Nevertheless, we will consider one formulation which allows for non-perfectly correlated regime states for the US-UK. We specify the transition probabilities as logistic functions of the instruments:

$$
f_i(z_{t-1}) = \frac{\exp(a_i + b_i' z_{t-1})}{1 + \exp(a_i + b_i' z_{t-1})}. \tag{18}
$$

If $b_i = 0$ then the transition probabilities are constant.

Given the large number of parameters in the full system, we restrict attention to subsets of the full model. Estimation of the RS models proceeds by using the Bayesian algorithm of Gray (1996). When we estimate restricted models, assumptions must be made about the RS DGP's to ensure consistent estimation. We list sufficient conditions as we present each restricted model.

We first focus on Simple RS Models with US-UK nominal returns and extend to include German nominal returns. These models have no instrument predictability. In a previous version of this paper we carefully considered the evidence of linear predictability using standard instruments such as short rates, dividend yields and earnings yields, and found it to be quite weak. In this model, we exclude such predictability and changes in the investment opportunity set are solely driven by regime changes. To build models of currency hedging we use US equity excess returns and currency hedged and unhedged

[8]See Table (1). Past equity returns can theoretically enter the regime probabilities P and Q. We estimated such models but none of the probability coefficients of the lagged equity returns were significant and the models seemed over-parameterized.

returns on UK and German equity. Here we model the world excess return as a predictor and the mean conditional on a regime of an asset will depend on the conditional covariance of that asset with the world excess return.

To allow comparison with the literature on dynamic asset allocation and because of the evidence of time-varying risk premiums, we consider a number of models accommodating instrument predictability. The first such model uses the US short rate ($z_t = r_t$) as a predictor. In our empirical analysis, it was the most powerful univariate predictor at short horizons. The inclusion of the short rate is also important since the set-up then allows investors to hold cash in addition to equities and changes in the cash/equity proportions may be important for intertemporal hedging (Balduzzi and Lynch (1999)). To include the US short rate as a predictor we use excess returns with a RS square root process for the short rate.

Since both the predictability literature and a number of asset allocation studies have focused on yield variables to track time-variation in expected returns, we also estimate a yield model. The evidence for linear predictability using both dividend and earnings yields is stronger at longer horizons, but is generally weak. We focus on the earnings yield ($z_t = ey_t$) as a (noisy) indicator of future expected returns. Earnings also vary with the business cycle and RS models can potentially capture this cyclicality. Our last model uses the US earnings yield as a predictor modeled as a RS AR(1) process. We now discuss these models in turn and present the empirical results in Section 4.

3.2 Simple RS Models

The Simple RS Model can be written as

$$y_t = \mu(s_t) + \Sigma^{\frac{1}{2}}(s_t)\epsilon_t. \tag{19}$$

We let $y_t = (y_t^{us}\, y_t^{uk,uh})'$ and $y_t = (y_t^{us}\, y_t^{uk,uh}\, y_t^{ger,uh})'$. The transition probabilities are constant, so $f_i(z_{t-1}) = k_i$.

To obtain the Simple RS Model from equation (16) we set $\xi = 0$ and parameterize $g(s_t, y_{t-1}, z_{t-1}) = \mu(s_t)$. To ensure consistent estimation we must impose restrictions so that the instruments z_t do not affect the ex-ante probabilities $p(s_t = i|\mathcal{I}_{t-1})$. Sufficient conditions in this setting are $c(s_t) = c$, $A(s_t) = A$, and the covariance Ω can be partitioned as:

$$\Omega(s_t) = \begin{pmatrix} \Sigma(s_t) & 0 \\ 0 & \Sigma_z \end{pmatrix} \tag{20}$$

where Σ_z corresponds to the covariance matrix of z_t.[9]

The Simple RS Model, as well as the General Model, assumes that the regimes in each country are perfectly correlated. To investigate if the UK undergoes regime switches different from the US we introduce an extension, Model II, with two regime variables s_t^{us} and s_t^{uk}. Model II has the feature that

[9]More specifically, we require that $f(z_t|y_t, s_t) = f(z_t|y_t)$. If the covariances of y_t and z_t are non-zero then the conditional distribution of z_t conditional on y_t will depend on s_t. See Ang and Bekaert (1998).

the regimes in the US and UK do not have to be perfectly correlated. Generally, there would be $2^2 = 4$ states for the bivariate system for two states of each country implying a 4x4 probability transition matrix. To preserve parsimony we assume that conditional on the US state, the UK process is a simple mixture of normals. That is, we let:

$$
\begin{aligned}
p(s_t^{us} = 1 | s_{t-1}^{us} = 1) &= P \\
p(s_t^{us} = 2 | s_{t-1}^{us} = 2) &= Q \\
p(s_t^{uk} = 1 | s_t^{us} = 1) &= A \\
p(s_t^{uk} = 2 | s_t^{us} = 2) &= B
\end{aligned} \tag{21}
$$

This parameterization implies that the US transition probabilities P and Q are still the driving variables of the system and allows the US and UK states to be dissimilar with only two extra parameters. Further, the correlation of the US and UK depends only on the state of the US. The estimation of this model is outlined in the Appendix.

Finally we test for the presence of RS ARCH effects for the US-UK. This specifies the covariance $\Sigma(s_t)$ as:

$$
\begin{aligned}
\Sigma(s_t) &= C(s_t)'C(s_t) + B(s_t)'u_{t-1}u_{t-1}'B(s_t) \\
u_t &= y_t - \mathrm{E}_{t-1}(y_t) \\
\mathrm{E}_{t-1}(y_t) &= \sum_{j=1}^{2} p(s_t = j | \mathcal{I}_{t-1})\mu(s_t = j).
\end{aligned} \tag{22}
$$

This model can be estimated following a special case in Gray (1996). Related models have been estimated by Ramchand and Susmel (1998a and b) and Hamilton and Susmel (1994).

3.3 Currency Hedging Beta Models

The Simple RS Model does not provide any explicit link between the conditional means and variances. The number of parameters rapidly increases with the introduction of more than 3 assets, as covariance matrices must be estimated. To facilitate the inclusion of more assets our currency hedging Beta Model imposes restrictions linking the conditional means and volatilities. We use this model to primarily focus on the additional benefits of currency hedging to international diversification for a US investor.

We derive this model from equation (16) by setting $\xi = 1$ and use the world excess return as the predictor $\tilde{y}_t^w \equiv \bar{z}_t = y_t^w - r_{t-1}^{us}$ with conditional mean $c(s_t)$. The conditional mean of the excess equity returns $\tilde{y}_t$ is given by $g(s_t, y_{t-1}, z_{t-1}) = \zeta(s_t)c(s_t)$ where $\zeta(s_t)$ is a $N \times 1$ vector for N equity returns. We consider US-UK models where $y_t = (y_t^{us}\ y_t^{uk,uh}\ y_t^{uk,h})'$ and a model for the US-UK-Germany where $y_t = (y_t^{us}\ y_t^{uk,uh}\ y_t^{uk,h}\ y_t^{ger,uh}\ y_t^{ger,h})'$. The transition probabilities are constant, $f_i(z_{t-1}) = k_i$.

To examine sufficient conditions for consistent estimation under this setting, partition the instruments $z_t = (\bar{z}_t\ z_t^*)'$ where z_t^* are the instruments which are omitted from estimation and $\bar{z}_t = \tilde{y}_t$ the included instrument. We assume that z_t^* does not affect the ex-ante probabilities and does not vary across regimes. We assume that the companion matrix $A(s_t)$ of the conditional mean of z_t can be partitioned as:

$$A(s_t) = \begin{pmatrix} \bar{A}(s_t) & 0 \\ 0 & A_{z^*} \end{pmatrix} \tag{23}$$

with $\bar{A}(s_t)$ (A_{z^*}) corresponding to $\bar{z}_t$ (z_t^*). In the Beta Model, we set $\bar{A}(s_t) = 0$. The covariance matrix Ω for assets and instruments is given by:

$$\Omega(s_t) = \begin{pmatrix} \bar{\Omega}(s_t) & 0 \\ 0 & \Omega_{z^*} \end{pmatrix} \tag{24}$$

where $\bar{\Omega}(s_t)$ ($N+1 \times N+1$) is modeled to reflect the presence of the world factor and takes the form:

$$\bar{\Omega}(s_t) = \begin{pmatrix} \zeta_1^2(s_t)\sigma_w^2(s_t) + \sigma_1^2(s_t) & \cdots & & & \\ \zeta_1(s_t)\zeta_2(s_t)\sigma_w^2(s_t) & \zeta_2^2(s_t)\sigma_w^2(s_t) + \sigma_2^2(s_t) & & \cdots & \\ \vdots & & & & \\ \zeta_1(s_t)\zeta_N(s_t)\sigma_w^2(s_t) & \cdots & & \zeta_N^2(s_t)\sigma_w^2(s_t) + \sigma_N^2(s_t) & \cdots \\ \zeta_1(s_t)\sigma_w^2(s_t) & \cdots & & \zeta_N(s_t)\sigma_w^2(s_t) & \sigma_w^2(s_t) \end{pmatrix} \tag{25}$$

We may interpret this model as a CAPM-inspired DGP conditional on the regime s_t. Introducing the notation β_i for the factor loading of asset i on the conditional mean of the excess world portfolio, and denoting excess returns by $\tilde{y}_t^i = y_t^i - r_{t-1}^{us}$ for asset i, we may rewrite the model as:

$$\begin{aligned} \tilde{y}_t^w &= \mu(s_t) + \sigma_w(s_t)\epsilon_t^w \\ \tilde{y}_t^i &= \beta_i(s_t)\mu(s_t) + \beta_i(s_t)\sigma_w(s_t)\epsilon_t^w + \sigma_i(s_t)\epsilon_t^i \end{aligned} \tag{26}$$

where

$$\beta_i(s_t) = \frac{\text{cov}(\tilde{y}_t^i, \tilde{y}_t^w | s_t)}{\sigma_w^2(s_t)}. \tag{27}$$

Here we have abused the notation so that $\mu(s_t)$ is the conditional mean of $\tilde{y}_t^w$ rather than the conditional means of equity y_t as in the Simple RS Models. Later when we refer to μ in the context of the RS Beta Model we will mean the conditional mean of $\tilde{y}_t^w$, since the conditional means of $\tilde{y}_t^i$ are functions of $\mu(s_t)$.

Alternatively, we may consider this to be a one-factor model, where the factor is the excess world return, and the conditional means of each asset are given by factor loadings ($\beta_i(s_t)$) on the conditional mean of the factor. The factor loadings compensate for the risk of the asset being linked with the factor: higher covariances demand higher risk premiums. In addition to being subject to the world portfolio shocks ϵ_t^w, each asset is subject to idiosyncratic risk ϵ_t^i. In this model the covariance of asset i and asset j depends on the extent to which each asset is linked, through the β's, to the world portfolio. The model is parsimonious: the introduction of an extra asset means only 4 additional parameters to estimate, fewer if some of the parameters are imposed to be equal across regimes.

3.4 Short Rate Model

To introduce the short rate we let $z_t = r_t^{us}$. For convenience we drop the country superscript on r_t, understanding this refers to the US short rate. We use a regime-switching discretized square root process of the Cox, Ingersoll and Ross (1985) form:[10]

$$r_t = c(s_t) + \rho(s_t)r_{t-1} + v(s_t)\sqrt{r_{t-1}}u_{zt} \tag{28}$$

In this model we work with excess returns so ξ is set equal to 1 and we let $\tilde{y}_t = (\tilde{y}_t^{us}\ \tilde{y}_t^{uk})'$.

The introduction of the square root term makes $\Omega(s_t)$ heteroskedastic. It still has the form of equation (24) except $\bar{\Omega}(s_t)$ is given by:

$$\bar{\Omega}(s_t) = \begin{pmatrix} \sigma_1^2(s_t) & \lambda_3(s_t)\sigma_1(s_t)\sigma_2(s_t) & \lambda_1(s_t)\sigma_1(s_t)v(s_t)\sqrt{r_{t-1}} \\ \lambda_3(s_t)\sigma_1(s_t)\sigma_2(s_t) & \sigma_2^2(s_t) & \lambda_2(s_t)\sigma_2(s_t)v(s_t)\sqrt{r_{t-1}} \\ \lambda_1(s_t)\sigma_1(s_t)v(s_t)\sqrt{r_{t-1}} & \lambda_2(s_t)\sigma_2(s_t)v(s_t)\sqrt{r_{t-1}} & v^2(s_t)r_{t-1} \end{pmatrix} \tag{29}$$

where $\lambda_1(s_t)$ ($\lambda_2(s_t)$) is the correlation between the short rate and US (UK) equity, and $\lambda_3(s_t)$ is the correlation between US and UK equity.

The conditional mean of the equity returns may embed predictability from the short rate:

$$\tilde{y}_t = g(s_t, y_{t-1}, z_{t-1}) + u_{yt} = \mu(s_t) + \beta(s_t)r_{t-1} + u_{yt}. \tag{30}$$

We will refer to the specification of $\beta(s_t) \neq 0$ in equation (30) as the Full Short Rate Model, and the case of $\beta(s_t) = 0$ as the Basic Short Rate Model.

To complete the model we specify the transition probabilities for $s_t = 1, 2$ as state-dependent:

$$p(s_t = i | s_{t-1} = i; \mathcal{I}_{t-1}) = \frac{\exp(a_i + b_i r_{t-1})}{1 + \exp(a_i + b_i r_{t-1})} \tag{31}$$

For consistent estimation, we assume that like the Beta Model, the omitted instruments z_t^* do not affect the ex-ante probabilities and the companion matrix $A(s_t)$ of $z_t = (r_t\ z_t^*)'$ takes the form of equation (23).

3.5 Earnings Yield Model

This model is very similar to the Short Rate Model, except we take the US earnings yield as a predictor $z_t = ey_t^{us} \equiv ey_t$ and work with nominal returns, so $\xi = 0$. We use a RS AR(1) for ey_t:

$$ey_t = c(s_t) + \rho(s_t)\, ey_{t-1} + v(s_t)u_{zt} \tag{32}$$

and also employ logistic specifications for the transition probabilities as in equation (31) except with ey_t replacing r_t. Similarly, the conditional mean for equity returns is given by equation (30) with ey_t

[10] Note that this process allows short rates to be negative which is inconsistent with the square root of the short rate appearing in the conditional volatility. In continuous time parameter restrictions rule out negative interest rates (Cox, Ingersoll and Ross (1985).) However, in all our simulations (upwards of 100,000 observations) we did not encounter any negative interest rates.

replacing r_t. The conditional covariance is given by the homoskedastic version of equations (24) and (29) and the parameter assumptions on omitted instruments are the same as in the Short Rate Model.

One advantage of the yield model is the rich dynamics it can generate in bear markets. One potential reason for a downturn in the market is a period of high volatility in news which increases discount rates. Such news may cause a shift to a higher volatility regime with higher expected returns. This mechanism can be accomplished in the Yield Model by the dynamics of the earnings yield. As prices decrease, the earnings yield increases as it is driven in the short-term by price in the denominator.

4 Data and Estimation Results

Section 4.1 describes the equity returns and the short rate and yield instruments for predicting future returns. Section 4.2 formally tests for the presence of regime switches in international equity returns. Section 4.3 discusses the estimation results for our various RS models.

4.1 Data Description

Our core data set consists of equity total return (price plus dividend) indices from Morgan Stanley Capital International (MSCI) for the US, UK and Germany. The instruments we consider for predicting future returns are short term interest rates and earnings yields for the US. The short rate is the US 1 month LIBOR rate and earnings yields are from MSCI.[11] Our sample period is from January 1970 to December 1997 for a total of 335 monthly return observations.[12] The focus on the US, UK and Germany arises from our desire to select the major equity markets that can be considered to be reasonably integrated during our sample period. This is definitely the case for the US and UK markets which currently (as of 31 July 1998) represent 49.4% and 10.5% of total market capitalization respectively in the world MSCI index. Since Japan underwent a gradual liberalization process in the 1980's we exclude it from our analysis. Adding Germany brings the total market capitalization represented to 65.5%.

Table (1) produces sample moments for the equity returns (all expressed in US dollars). These are monthly returns expressed as a continuously compounded rate. Whereas the means appear insignificantly different from one another, foreign equity returns are distinctly more variable. One culprit is currency risk, as we can decompose the foreign market return y_{t+1} into $y_{t+1} = y_{t+1}^{LC} + e_{t+1}$ where y_{t+1}^{LC} is the return in local currency and e_{t+1} is the log difference of the exchange rate. The returns show insignificant autocorrelations. Of particular interest is the correlation matrix produced in Panel C. Unconditionally, correlations are positive and range from 36% for the US and Germany to 51% for the US and UK.

The RS Beta Models of currency hedging use excess returns over the 1 month US EURO rate from January 1975 to July 1997. We define excess unhedged foreign equity returns as $\tilde{y}_{t+1}^{uh} = y_{t+1}^{USD} - i_t$ where

[11]The earnings yields use earnings summed over the last 12 months. For further details see the *EAFE and World Perspective* publication from MSCI.

[12]In the Short Rate Models our sample period is from January 1972 to December 1997.

y_{t+1}^{USD} are returns in US dollars, and i_t is the US short rate. The excess hedged foreign equity return is defined as $\tilde{y}_{t+1}^h = y_{t+1}^{LC} - i_t^*$ where i_t^* is the foreign short rate (the 1 month foreign EURO rate).[13] In Table (2) we present sample moments of excess returns for the MSCI world portfolio, US, UK hedged and unhedged, and German hedged and unhedged equity returns. Note that unhedged returns for the UK (Germany) are on average larger (smaller) than the hedged returns indicating the existence of a currency risk premium.

As we saw in Section 3 dimensionality issues make it hard to include all instruments as state variables in our asset allocation analysis. A report on an extensive analysis of linear predictability regressions using short rates, dividend yields and earnings yields is available from the authors upon request. Since there is some evidence that US instruments predict UK returns, but UK instruments have very weak predictive power for US returns we only use US instruments. Sample moments for the US instruments are presented in Table (3).

4.2 Are there Regimes in International Equity Returns?

The asymmetric correlation pattern in equity returns described in the Introduction can potentially be captured by a RS model. While previous evidence suggests that there are regimes in the data generating process for equity returns, formal tests have rarely been conducted in past literature. What follows here is a formal test of the presence of regime switching for US, UK and German equity returns.

We wish to test the following model of one regime:

$$y_t = \mu + \Sigma^{\frac{1}{2}} \epsilon_t \tag{33}$$

where $y_t = (y_t^{us}\ y_t^{uk}\ y_t^{ger})'$ are the nominal monthly equity returns, and $\epsilon_t \sim$ IID N(0,I) against the following regime-switching model:

$$y_t = \mu(s_t) + \Sigma^{\frac{1}{2}}(s_t)\epsilon_t \tag{34}$$

with $s_t = 1, 2$ and Markov transition probabilities $P = p(s_t = 1|s_{t-1} = 1)$ and $Q = p(s_t = 2|s_{t-1} = 2)$. This is the simple RS US-UK-GER Model being tested against its one regime counterpart.

[13]We derive the excess hedged foreign equity return as follows. Consider \$1 converted into foreign currency at rate E_t at time t, where E_t is the exchange rate expressed in dollars per foreign currency. This earns $\exp(y_{t+1}^{LC})/E_t$ in foreign currency at time $t+1$. Suppose a forward contract can be written on this amount at time t, where $F_t/E_t = \exp(i_t - i_t^*)$. Then the hedged gross return on foreign equity will be $\exp(y_{t+1}^{LC})F_t/E_t = \exp(y_{t+1}^{LC} + i_t - i_t^*)$. The continuously compounded excess hedged return is $\tilde{y}_{t+1}^h = y_{t+1}^{LC} - i_t^*$. In discrete time the hedged return can be approximated by the unhedged return plus the proceeds of selling \$1 forward:

$$1 + y_{t+1}^h \doteq (1 + y_{t+1}^{LC})(1 + e_{t+1}) - [e_{t+1} + i_t^* - i_t]$$

where $e_{t+1} = \log(E_{t+1}/E_t)$. If we ignore the second-order cross-term, then $y_{t+1}^h \doteq y_{t+1}^{LC} + i_t - i_t^*$ and the excess hedged return becomes $\tilde{y}_{t+1}^h = y_{t+1}^{LC} - i_t^*$. This method for hedged returns is used by Tesar and Werner (1995) and a similar method without continuous compounding by Black and Litterman (1991).

In testing regime-switching models the usual χ^2 asymptotic tests do not apply because of the presence of nuisance parameters under the null. In our situation, if $P = 1$ and the process starts in the first regime, then the parameters $\gamma = (Q, \mu(s_t = 2)', \text{vech}(\Sigma(s_t = 2))')'$ are unidentified under the null. The likelihood function will be flat with respect to γ, and these parameters also enter the Hessian making the standard likelihood ratio test, Wald test and Score test have non-standard distributions. While certain testing techniques have been developed for these conditions, these are asymptotic tests which are difficult to implement empirically.[14]

Here we focus on the empirical likelihood ratio test. The likelihood ratio statistic of the regime-switching model against the null is 103.738, but to test if this value rejects the null of one regime we use Monte Carlo simulation to find its small sample distribution. Using the one-regime model (equation (33)) we simulate a sample of 335 returns (the same length as the sample on which the model was estimated), and estimate the regime-switching model (equation (34)). The likelihood ratio for the simulated sample is then recorded and the process is repeated 500 times. Estimation of the regime-switching model for every simulated sample makes this computationally intensive.[15] The results are listed in Table (4) which shows that the largest likelihood ratio statistic generated under the null is 49 while the sample likelihood ratio statistic is 104. The data overwhelmingly reject the null of one regime.

That the null is rejected in favor of a RS model is not surprising given the significant higher order moments (Table (1)) which can be captured by the RS model. Other stochastic volatility models able to produce fat tails may also reject the null. However, we later test for the presence of ARCH effects for the US-UK beyond the RS model in equation (34) and find no evidence of RS ARCH.

4.3 Regime Switching Estimation Results

We present general results in Section 4.3.1 and discuss specific estimation results for individual models in Sections 4.3.2 to 4.3.5.[16]

4.3.1 General Patterns

Across our RS models we find the following pattern in international equity returns. In one regime the equity returns have a lower conditional mean, much higher volatility and are more highly correlated. We shall refer to this regime as "regime 1". In the second regime, equity returns have higher conditional means, lower volatility and are less correlated. Our regimes definitely correspond to periods of low and high volatility, but the evidence for significantly different conditional means and correlations across regimes is not as strong.

Table (5) presents p-values of Wald Tests for parameter equality across regimes for the various RS

[14] In particular see Hansen (1996) and the discussion in Ang and Bekaert (1998).

[15] We ignored 3 samples where no convergence was attained.

[16] The parameter estimates for all models are contained in a supporting Table Appendix available from the authors.

Models.[17] In models where the means are constant conditional on the regime, there is some evidence that the means differ across the regimes for the US in the case of the Simple US-UK Model and for the Short Rate Model. However, joint tests fail to reject the null of constant conditional means for both countries. The Wald test for the Earnings Model rejects that the parameters of the conditional mean $(\mu(s_t), \beta(s_t))$ are jointly equal across regimes.

The equality of volatilities across regimes is rejected at any significance level for the Simple RS Models, the Short Rate Model and the Earnings Yield Model. Evidence is also extremely strong for different regime-dependent volatilities for the Beta Models. The evidence of different correlations across the regimes is not particularly strong. The US-UK correlations are borderline significantly different in the Simple US-UK-GER model and the Short Rate Model. We cannot reject that correlations for the UK and Germany are constant across regimes. Similarly, for the Beta Models, where covariances are implied by the β's of the individual assets, and for the Earnings Model, we cannot reject equality of the correlation across regimes.

To concentrate on the effect of changing covariances we estimate models where μ_1 and μ_2 are imposed to be equal across states (but different across assets). Table (6) shows that these models cannot be rejected when comparing them against their unconstrained specification. Moreover, regime classification, as measured by the Ang-Bekaert (1998) Regime Classification Measure (RCM), generally improves slightly when this restriction is imposed.[18] In Section 5 we will concentrate our analysis on models with μ_1 imposed equal to μ_2.

4.3.2 Simple US-UK Model

The US and UK have been the largest most integrated markets over our sample. Being the simplest and most parsimonious RS model, we will discuss the estimation results for the Simple RS US-UK Model. For intuition we specifically discuss this model in Section 5. The results for the US-UK-GER Model are qualitatively similar.

Figure (1) shows the ex-ante probabilities $p(s_t = 1|\mathcal{I}_{t-1})$, implicitly given by construction of the likelihood function, associated with the log equity index levels for the US and unhedged UK equity. The turbulent equity returns of the OPEC oil shocks in the mid-70's are picked up, as is the 1987 crash. From the mid-90's onwards, the implied ex-ante probabilities place the economy almost definitely in the second regime. The expected duration of the first regime is 6.9 months, while the expected duration of the second regime is 4.25 years. The stable probabilities implied by the transition matrix are 0.1194 and 0.8806 for regimes 1 and 2 respectively.

Figure (2) shows the implied conditional means, volatilities and correlation of the US and UK equity

[17] The Basic Short Rate Model refers to the case where $\beta(s_t) = 0$ in equation (30)). In a test of the Basic Model versus the Full Model we cannot reject the Basic Model.

[18] The RCM is given by $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values indicate better regime classification.

returns. The implied conditional mean is given by:

$$\hat{\mu}_t = p_t\mu_1 + (1 - p_t)\mu_2 \tag{35}$$

where $p_t = p(s_{t+1} = 1|\mathcal{I}_t)$ is the ex-ante probability of being in the first regime. The implied covariance matrix at time t is given by:

$$\hat{\Sigma}_t = p_t(\Sigma_1 + \mu_1\mu_1') + (1 - p_t)(\Sigma_2 + \mu_2\mu_2') - \hat{\mu}_t\hat{\mu}_t'. \tag{36}$$

Dashed lines in the plots represent 95% confidence intervals calculated by the delta-method. We see in the top panel of Figure (2) that as p_t increases, the conditional mean of the US and UK become negative, but the standard error of the estimates also increases markedly. This reflects the increased uncertainty about the parameter estimates in the first regime. The conditional volatilities in the middle panel likewise increase substantially when p_t increases. The plots also show that the conditional means and variances of the US and UK move in tandem, as is true in any one-factor model. In our Simple RS Models this factor is the ex-ante regime probability. The bottom panel shows the conditional US-UK correlation. The plots clearly show that the higher volatility regime is also associated with higher correlations.

Table (7) shows likelihood ratio tests of the Basic Model versus Model II (the model allowing non-contemporaneous regimes of the US and UK) and the RS ARCH model. In both cases we fail to reject the Simple Model. Moreover, the parameters A and B in equation (21) are estimated to be 1. This lends support to the simple, but parsimonious DGP of the Simple RS Model: the US and UK face the same regime shifts and the stochastic volatility generated by the simple RS Model suffices to capture the time variation in monthly equity return volatilities.

4.3.3 Beta Model

We now describe the qualitative results of the RS Beta Models. The higher volatility in the first regime is driven by three parameters in this model. First, world volatility is higher in the first regime. Second, the β's are invariably higher in the first regime. Third, the idiosyncratic volatilities are higher in the first regime. It is never possible to reject that the β's are significantly different from 1 in the first regime, but they are often significantly below 1 in the second regime, which is more influenced by the idiosyncratic shocks. The β's of the unhedged returns are larger than the β's of the hedged returns, reflecting a positive currency return β. The difference between the unhedged and hedged excess equity returns in the RS Beta Models is the currency return cr_{t+1}, which is the excess return from investing in the foreign money market and is given by $cr_{t+1} = e_{t+1} + i_t^* - i_t$.

The expected value of the currency return, the currency premium $cp_t = \mathrm{E}_t(cr_{t+1})$ is the topic of a large empirical and theoretical literature. Our model implies that, conditional on the regime, the currency premium is constant. In Table (8) we report the state-dependent and unconditional currency premiums and the volatilities of the currency returns. The unconditional premium is approximately 1.5% to 2% per annum for both the pound and the deutschemark. The magnitude is similar to the recent estimates of De

Santis and Gerard (1998), but the sign is different. De Santis and Gerard use a CAPM-based model with GARCH volatility and find large time variation in currency premiums. In our model, the actual premium varies over time with the regime probability and can potentially change signs. However, we estimate the premiums to be positive in both regimes, implying that US investors are always compensated for taking foreign exchange risk. For regime 1, the currency premium is very small and insignificant. The smaller currency premium, combined with the larger volatility may contribute to home bias in this regime.

4.3.4 Short Rate Model

The short rate behavior in the two regimes is characterized by high conditional means with lower autocorrelation (higher mean reversion) and higher conditional volatility in the first regime, and low conditional means with higher autocorrelation and low volatility in the second regime. Economically, the second regime corresponds to "normal" periods where interest rates are low and equity excess returns are positive. Interest rates behave like a random walk, perhaps because of the monetary policy smoothing efforts of the US Fed. The first regime corresponds to "turbulent" periods of high monetary uncertainty with very volatile negative equity returns.[19]

In Table (7) we see that a restricted model (the Basic Model) with no within-regime predictability of the equity return by the short rate cannot be rejected from the unconstrained Full Model. The parameter estimates of $\beta(s_t)$ themselves in equation (30) have very large standard errors in both regimes. The standard errors on other parameters are also much larger than in the Basic Model. The probability coefficients in equation (31) are significant, and a constant probability version is rejected by the Basic Model. In particular, b_2 is negative and highly significant, so in normal periods, as the short rate increases a transition to the first regime becomes increasingly likely.

Comparing the ex-ante probabilities of the Basic Model in Figure (3) to the Simple US-UK Model, we see that the Short Rate Basic Model classifies the early 1980's as regime 1, which is driven by the highly turbulent short rates during the monetary targeting period. There are only a few spikes during this period for the US-UK model (Figure (1)). The implied conditional correlations for the Basic Short Rate Model are presented in the bottom panel of Figure (3). Interestingly, short rates and equity returns are more negatively correlated in regime 1 than regime 2. In addition to the low conditional means of equity, this implies that holding equity is even more unattractive in the first regime. As short rates are high during this period risk-averse investors will want to hold mostly cash. In this model, although mean excess returns are mostly positive (top panel), the conditional mean in the first regime remains hard to pin down, motivating a focus on the $\mu_1 = \mu_2$ model.

[19] Similar patterns for RS models applied to interest rates have been documented by Ang and Bekaert (1998), Bekaert, Hodrick and Marshall (1998), and Gray (1996).

4.3.5 Earnings Yield Model

In the Earnings Yield Model, US earnings yields predict US and UK equity returns both through the transition probabilities and through the conditional means. In particular, in the second regime, b_2 in equation (31) is significant and negative, so as the earnings yield increases a transition to the first regime becomes more likely. The predictability coefficients β in the conditional mean of equity returns are significant in both regimes for the US, with a strong effect in regime 1, and significant for the UK in the first regime. In Table (7), a likelihood ratio test for this model versus the null of no predictability gives a p-value of 0.0149. There is borderline significance for each case of predictability through the conditional means and through the transition probabilities.

In the first regime, earnings yields have higher conditional means, are more mean-reverting and have higher conditional volatility. The average earnings yield conditional on regime 1 (2) is 10.44% (6.56%). Lower earnings yields on average are associated with normal periods as higher prices relative to earnings push down the earnings yield. The Earnings Yield Model, like the other RS models, has average equity returns conditional on the regime being lower and more volatile in the first regime. For the US the average equity returns in regime 1 (2) are -0.6093 (1.6159) with respective standard deviations 5.7484 (3.3645). Similarly for the UK, the average equity returns in regime 1 (2) are -0.0631 (1.5058) with respective standard deviations 9.9298 (4.9110).

5 Asset Allocation Empirical Results

Under the RS DGP's estimated in Section 4.3, we will attempt to answer the following questions raised in the Introduction: (a) are there still benefits in international diversification in regimes of global financial turbulence? (b) how do these regimes affect asset allocations? (c) does currency hedging help? (d) how costly is ignoring regime switching? and (e) how large are the intertemporal hedging demands induced by regime switching? We will first discuss general results across all the models and tabulate results for risk aversion levels of $\gamma = 5$ and 10. Unless otherwise mentioned we present results for models with $\mu_1 = \mu_2$ imposed. We will examine in detail the US-UK Simple RS Model for intuition and also discuss the Short Rate and Earnings Yield Model. The portfolio weights, with statistical tests, are presented in Tables (9) to (12). Tables (13) to (17) present the economic compensation required under various sub-optimal strategies.

5.1 International Diversification under Regime Changes

Portfolio weights, along with standard errors, for the all-equity portfolios are listed in Tables (9) to (11).[20] Across these models, the proportion held in the US rises in the first regime, but the standard

[20]For the Beta Models, a risk-free rate of 6% is specified, as the model is formulated in terms of excess returns. The portfolio weights are highly insensitive to the choice of the risk-free rate except for very large (>100%) rates. This is because the

errors associated with the portfolio weights are large. The US, because of its lower volatility in the first regime compared to overseas equity, becomes a "safer" asset. Risk averse investors choose to hold more of the US at the expense of international equity during the downturn state. Portfolio weights as a function of γ are shown in Figure (4). The more risk-averse the investor, the greater the proportion of the US held in both states.

Table (13) presents the "cents per dollar" compensation required for an investor with an all-equity portfolio to hold only the US instead of investing optimally with overseas holdings. Table (13) shows that at a 1 month horizon, the costs of holding only US equity are small and, as expected, grow with the horizon. At one year we need a compensation of 1.19 cents (0.97 cents) in state 1 (2) with $\gamma = 5$ to hold no UK or German equity under the Simple RS Model. The addition of Germany brings considerable economic benefit for international diversification, especially at longer horizons where costs can exceed 10 cents for $\gamma = 10$ with a 5 year horizon. This is because of the particular covariance structure in regime 1. Although US holdings increase, Table (10) shows that German holdings also increase at the expense of UK equity.

We might expect that as correlations are higher in state 1, the costs of no international diversification in that state will be less than in state 2. This is generally not true. For $\gamma = 5$ and 10 for the US-UK this is true, but this is not the case for the US-UK-GER system. In the three country model the optimal holdings of both US and Germany rise, making diversification more valuable in this regime. Figure (5) shows that even for the US-UK, the benefits of diversification for state 1 may be greater than for state 2 for small γ. The bottom panel of Figure (5) shows that because of the benefits of holding Germany in state 1, the costs of no international diversification are uniformly higher in state 1 than in state 2.

In the bottom panel of Table (13), costs for various levels of the earnings yield for not holding overseas equity are presented. In regime 1, where the average earning yield conditional on the regime is around 10%, an investor with $\gamma = 5$ with a one year horizon needs 1.25 cents of compensation. In the second regime, the cost of not diversifying is on average smaller, being only 0.30 cents at a conditional average earnings yield level of 6%. As the earnings yield increases the cost of not diversifying increases in both regimes.

5.2 Benefits of Currency Hedging

Confirming previous evidence in Glen and Jorion (1993), being able to hedge currency risk imparts further benefit to international diversification. In Table (14) the economic compensation for not diversifying internationally under the RS Beta Models is higher than under the pure unhedged Simple RS Models in Table (13). In this model no international diversification refers to holding neither hedged nor unhedged foreign equity. To obtain a measure of the benefits of currency hedging, we need to obtain the optimal portfolios under the restriction that only investment in unhedged equity can be made.

nominal returns approximately cancel each other out (to a first-order Taylor approximation) in the FOC's in equation (6).

The economic compensation required for holding such portfolios is listed in the second panel of Table (14). This shows that the costs of not using currency hedging, like the costs of not internationally diversifying, are relatively large. For a one year horizon with $\gamma = 5$, around 70 basis points are required to not engage in currency hedging. We can compare the two panels in Table (14). Currency hedging contributes about half of the total benefit of no international diversification under the RS Beta Models.

Table (11) shows the asset allocation weights for the RS Beta Models. Like the Simple RS Models, the proportion of US equity increases in the first regime. We also list the proportion of the portfolio covered by a forward contract position, which is unrestricted. In the RS Beta Models, short positions in the forward contracts hedge the currency risk of the foreign equity position. These positions are statistically significant. The Tables also list hedge ratios, which are the value of the short forward position as a proportion of the foreign equity holdings. Our models produce hedge ratios of about 50%, and are fairly similar across regimes.

5.3 Costs of Ignoring Regime Switching

In the absence of predictability, there are two implications of regime switching for portfolio weights: (a) portfolio weights become regime-dependent, and (b) since regime switching generates intertemporal hedging demands, portfolio weights become horizon-dependent. We will look at the effects of regime-dependent weights first.

The weights reported in Tables (9) to (11) sometimes differ substantially across regimes. For example, the US weight is anywhere between 7 and 29% higher in regime 1 compared to regime 2 for the Simple RS Models and the US-UK Beta Model. The differences in weights across regimes for the US-UK-GER Model are not large for the US but more substantial for the UK and Germany (7 to 9%). Nevertheless, the standard errors are often large and we cannot reject the null that the portfolio weights are constant across regimes for many cases. Table (12) summarizes Wald tests with $\gamma = 5$ which are joint for horizons $T = 1, 12, 36, 60$ months. The only rejection occurs on the RS Beta Models for the US weights. In the Simple RS Models in Tables (9) and (10) we cannot reject that portfolio weights do not differ across regimes for $\gamma = 5$, but for $\gamma = 10$, we can reject at the 5% level for the US-UK equity portfolios.

The economic costs of ignoring RS range from fairly small to substantial at high levels of risk aversion. Table (15) shows that for a one year horizon, investors with $\gamma = 5$ in the Simple RS US-UK-GER Model lose only 14 (5) basis points for ignoring regime switching in state 1 (2). When investors ignore regime switching they are assumed to hold myopic weights implied by fitting an IID multivariate normal distribution to the equity returns. These weights are an approximate average of the regime-dependent weights. (For the US-UK and US-UK-GER portfolios, they are listed in Tables (9) and (10).) The IID weights give a reasonable approximation to the optimal weights in each regime, especially in regime 2 which has the longest duration. Treating the IID weights as a constant, we cannot reject that the optimal regime-dependent weights are different from the IID weights at the 95% level. The costs are substantially higher when γ is increased to 10, in which case regime 1 requires effectively holding all US equity in the US-UK portfolio.

Finally note that the cost of ignoring RS is higher in state 1 than state 2. This is in accordance with intuition, since in the normal regime, conditional means and variances will be closer to their unconditional counterparts, than they are in state 1. The markedly different behavior in state 1, which can persist for several periods, makes the costs of ignoring RS higher in this regime. Figure (5) plots the costs of ignoring regime switching for the Simple RS Models as a function of γ. The plots confirm that the cost of ignoring RS is higher in regime 1 for all levels of risk aversion and is robust across the Simple US-UK and US-UK-GER Models. Whereas for the US-UK, only at low levels of risk aversion do the costs of failing to diversify internationally dominate the costs of ignoring RS, this holds for all γ in the US-UK-GER model. This is because for the US-UK the optimal portfoliio for regime 1 becomes the domestic US equity portfolio when γ is high, whereas in the US-UK-GER system positive German equity holdings remain optimal in the first regime.

5.4 Intertemporal Hedging Demands

Tables (9) to (10) present Wald Tests for intertemporal hedging demands. Hedging demands are never significant, and the p-values are generally very large. Brandt (1998) also cannot reject myopia in his non-parametric estimate of domestic asset allocation weights.

The Tables also show that the convergence of the portfolio weights is extremely fast. After 3 years, the portfolio weights are constant. The convergence is even faster than in Brandt (1998), who finds convergence after 15 years. His setting however, incorporates instrument predictability and rebalancing at intervals greater than 1 month. With only regime changes and monthly rebalancing horizon effects become even smaller.

The economic costs of myopia are effectively zero. Table (16) lists the compensation required for an investor to hold myopic portfolio weights instead of the optimal T horizon weights. The numbers are astoundingly small for all models. This evidence suggests that investors lose almost nothing by solving a myopic problem at each horizon, rather than solving the more complex dynamic programming problem for longer horizons.

5.5 Simple RS US-UK Model

In this Section we will develop more intuition about the asset allocation behavior under the simple RS US-UK Model which carries over to more general cases.

Let us examine the portfolio weights in Table (9). In regime 1, the point estimates show that investors hold more US equity. The US acts as a "safer" asset in this regime. Under this model when $\mu_1 = \mu_2$, the equity returns $y_t = (y_t^{us}\, y_t^{uk})$ are distributed as $N(\mu, \Sigma_i)$, $i = 1, 2$, where $\mu = (1.1613\ 1.2488)'$ and

$$\Sigma_1 = \begin{pmatrix} \underset{(0.9515)}{7.5064^2} & \underset{(0.1032)\times(0.9515)\times(1.8432)}{0.6181 \times 7.5064 \times 14.0748} \\ \underset{(0.1032)\times(0.9515)\times(1.8432)}{0.6181 \times 7.5064 \times 14.0748} & \underset{(1.8432)}{14.0748^2} \end{pmatrix} \tag{37}$$

with standard errors in parentheses, and

$$\Sigma_2 = \begin{pmatrix} \underset{(0.1654)}{3.7917^2} & \underset{(0.0491)\times(0.1654)\times(0.2409)}{0.4480 \times 3.7917 \times 5.2470} \\ \underset{(0.0491)\times(0.1654)\times(0.2409)}{0.4480 \times 3.7917 \times 5.2470} & \underset{(0.2409)}{5.2470^2} \end{pmatrix}. \tag{38}$$

The lower volatility of the US in the first regime makes the US relatively more attractive to risk-averse investors at the expense of international holdings.[21] This pattern is repeated across all the models, including the RS Beta Models which allow currency hedging. The large standard errors, though, mean that statistically there is weak evidence that the true portfolio weights change across regimes.

Looking at Table (9), notice that as the horizon is increased the point estimates of the holdings of US equity increase with horizon. That is, with increasing horizon, investors want to hold *more* of the less risky asset.[22] It is the persistence of the regimes which lies behind this result, as can be seen by applying the intuition from Samuelson (1991).

Samuelson works with two assets, cash and and a risky asset. The risky asset follows a Markov chain where the returns can be "low" or "high". He defines a "rebound" process, or mean-reverting process, as having a transition matrix which has a higher probability of transitioning to the alternative state than staying in the current state. An example of a symmetric rebound transition matrix is

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}. \tag{39}$$

Samuelson's theorem is that with a rebound process, risk-averse investors *increase* their exposure to the risky asset as the horizon increases. That is, under rebound, long horizon investors are more tolerant of risky assets than short horizon investors.

Our setting is the opposite of a rebound process. Our transition matrix is:

$$\begin{pmatrix} \underset{(0.0698)}{0.8546} & 0.1454 \\ 0.0182 & \underset{(0.0100)}{0.9818} \end{pmatrix}, \tag{40}$$

with standard errors in parentheses. Samuelson calls such a process a "momentum" process: it is more likely to continue in the same state, rather than transition to the other state. Under a momentum process,

[21] A full list of parameter estimates is shown in Table (A-1) as part of the supporting Table Appendix. The Basic Model, where $\mu_1 \neq \mu_2$ produces covariance estimates which are approximately the same, but μ_1 = (-1.2881 -0.6921)' and μ_2 = (1.2829 1.3040)'. Even with the much more negative conditional mean for the US in state 1, the asset allocation results for the full equity portfolio are similar, because they are driven mainly by the lower volatility of the US in that state.

[22] This same effect is shared across all the RS models (Table (9) to (11)). In the case of the Short Rate Model (Figure (6)) the "less risky" asset is cash. Note that in the Earnings Yield Model, the change in US equity holdings depends on the prevailing earnings yield.

risk-averse investors will want to *decrease* their exposure to risky assets as horizon increases. Intuitively, the long-run volatility is smaller under a rebound process than under a momentum process (with the same short-run volatility). In our setting, the risky asset is overseas equity, and the safer asset is US equity. The persistence of our regime probabilities means that investors with longer horizons hold *less* foreign equity, so long-horizon investors are less tolerant of holding more risky overseas equity than short-horizon investors. However, Section 5.4 shows that this effect is economically and statistically insignificant.

5.6 Basic Short Rate Model

Introducing a predictor instrument makes the portfolio weights a function of the instrument as well as the regime. Here we focus on the asset allocation results given by the Basic Short Rate Model (equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30)). The excess conditional mean for equity returns is imposed equal across regimes ($\mu_1 = \mu_2$). From Section 4.3.4, the predictability in the conditional mean is overwhelmingly statistically insignificant but the short rate does enter significantly in the probability coefficients (b_i in equation (31)).

Portfolio weights as a function of the short rate and regime are presented in Figure (6). The top two plots show the asset allocation weight for US and UK equity in regime 1 and 2 (and the remainder of the portfolio is held in cash). The Figure shows that the hedging demand is small, and is only visible for the first regime. In regime 2, as the short rate increases investors hold less equity, but in regime 1 there is almost no effect of the short rate on the portfolio allocations. This is driven by the non-linear predictability in the probability coefficients. The portfolio holdings in state 1 are flat because the excess returns are constant and no significant short rate predictability (b_1 is insignificant) drives the transitions from this regime. In the second regime b_2 is highly significant and negative. As the short rate increases, a transition to regime 1 becomes increasingly likely. As the first regime has much higher equity volatility, investors seek to hold less equity to mitigate the higher risk.

The effect of predictability seems much weaker in our model than in the predictability models analyzed by Brennan, Schwartz and Lagnado (1997), Kandel and Stambaugh (1996), and Barberis (1996). These models have linear predictability ($\beta \neq 0$) in the conditional mean rather than the non-linear predictability in the probability coefficients and much longer rebalancing intervals than 1 month.

Table (17) presents the economic compensation required for an investor not to hold the UK. To obtain the first panel in the Table a constrained optimization problem must be solved where investors are only permitted to hold cash and US equity. In this setting, the cost of not holding the UK is only very modest, and higher in regime 2 where US-UK correlations are lower. This is consistent with the pure US-UK equity portfolios examined in Section 5.1. The main effect of introducing the short rate as a predictor is the benefit of holding cash. The bottom panel of Table (17) shows that the costs of holding only equity and ignoring regime switching is substantial.

5.7 Earnings Yield Model

Figure (7) shows the US portfolio weights for an all-equity US-UK portfolio from the Earnings Yield Model in each regime. In the top panel portfolio weights for different horizons are presented, which shows that the intertemporal demands from this model are very small. In regime 1, as the earnings yield increases, US investors seek to hold more risky UK equity. In regime 2, this same effect is repeated at higher earnings yields, but a small hump is seen at lower earnings yields. The Samuelson (1991) effect of a small increasing exposure to the safer US asset with increasing horizon can also be seen.

The second panel of Figure (7) shows the 95% standard error bands of myopic portfolio weights. These are large, but are smallest at the average value of the earnings yield conditional on each regime. In regime 1 (2), the confidence bands are smallest at 10.4% (6.6%) and then increase like a funnel in both directions for higher and lower earnings yield levels. Myopic weights and the null of constant portfolio weights across regimes can definitely not be rejected.

Table (18) presents some economic cost computations under the Earnings Yield Model for an investor with risk aversion $\gamma = 5$. The first panel lists the costs of ignoring RS and predictability where an investor holds Samuelson (1969) IID portfolio weights. The middle columns for each regime list the costs associated with the average earnings yield conditional on the regime. The other numbers are representative "high" and "low" earnings yields conditional on the regime. In regime 1 (2), with an average earnings yield of 10% (6%), the costs are only 0.06 cents (0.04 cents) with a 1 year horizon. The IID weights provide a good approximation to the optimal weights at most earnings yield levels making the costs to ignoring both RS and predictability small.

The bottom panel of Table (18) lists the costs of ignoring predictability but taking into account regime-switching. In this case, the constrained portfolio weights are those implied by the Simple RS US-UK Model (with $\mu_1 \neq \mu_2$), and are quite dissimilar from the RS Earnings Yield weights in the first regime at conditional average yield levels. Generally, this produces higher costs in the first regime relative to the IID case, which ignores both predictability and changing regimes. For example, for a 1 year horizon in regime 1, the costs are 0.33 cents compared to the IID weight cost of 0.06 cents. The reason for this is because at average values of the earnings yield conditional on each regime, the IID portfolio weights are better approximations to the optimal weights while the portfolio weights implied by the Simple RS Model tend to over (under) state in the optimal weights in regime 1 (2). The optimal myopic US weights at the conditional average earnings yield in regime 1 (2) are 0.7262 (0.8422), while the weights from the Simple US-UK Model are 0.8614 (0.7356). The IID portfolio weights 0.7642 deviate less from the optimal earnings yield weights in each regime. In fact, it is striking that at the average earnings yields the home bias in the first regime disappears. Looking at the estimated moments conditional on the regime it is clear why this happens. Compared to the Simple RS Model, in the Earnings Yield Model UK equity is relatively more attractive since the correlation with US equity is lower (0.5551 versus 0.6097 for the Simple RS Model), and its volatility ratio relative to the US is less extreme (1.7274 versus 1.9492). In the second regime the opposite effect is true but it is brought about by differences in the conditional means and volatility, whereas the correlation is estimated to be about 0.44 in both models.

6 Robustness Experiments

In this Section we conduct several experiments to determine the robustness of our results. In Section 6.1 we check the sensitivity of our results to the specification of the conditional means. In Section 6.2 we gain further intuition on optimal asset allocation under regime changes by examining how optimal portfolio weights change as a function of one changing parameter in the RS Simple US-UK Model. In Section 6.3 we investigate whether our conclusions about the costs of ignoring RS and the benefits of international diversification remain robust to alternative parameter values. Finally, in Section 6.4 we consider an out of sample experiment.

6.1 Regime-Dependent Conditional Means

One disappointing aspect of our RS model estimation is that we fail to find strong evidence that highly volatile periods coincide with bear markets. Although the point statistics suggest this relationship, the standard errors on the conditional means in regime 1 are large. This in turn may dampen the potential asset allocation effects of the high volatility regime. In order to examine this further, we re-estimate the Simple RS models constraining the conditional means to be equal across countries, but different across regimes. These models cannot be rejected in favor of the alternative of unconstrained means (p-value = 0.8415 (0.4884) for the US-UK (US-UK-GER) model). In these models, the means in each regime (equal across countries) are also not significantly different (p-value = 0.1422 (0.1927) for the US-UK (US-UK-GER) model). The quality of the regime classification measured by the Ang-Bekaert RCM statistic is largely unchanged for the US-UK-GER model, but is much worse for the US-UK. The resulting portfolio weights are largely unchanged, with almost the same economic costs and significance levels for the statistical tests. Consequently, our focus on time-varying covariances seems justified.

6.2 Changing Parameters in the Simple US-UK Model

Figure (8) shows the effect on the portfolio weights of changing various parameter values. The base-line case is the unconstrained μ case. We alter one parameter while holding all the others constant and hold the horizon fixed at $T = 12$ months. From the top plot going downwards in Figure (8) we show the effect of altering the transition probability $P = p(s_t = 1|s_{t-1} = 1)$ of staying in the first regime conditional on being in the first regime, the correlation ρ_1 of the US-UK in regime 1, the conditional mean μ_{11} of the US in regime 1, and the volatility σ_{11} of the US in regime 1.

The plots are very intuitive. As P increases, holdings of the safer US asset increase in both states as the expected duration of regime 1 increases. The largest difference between the state-dependent weights is at values around $P = 0.5$ (the sample estimate is $\hat{P} = 0.8552$), but this has only a minor effect on the regime dependence. As ρ_1 increases the diversification benefits of holding UK equity decrease, so holdings of the US increase. Note that it is only for ρ_1 greater than 0.8 that the weights in each regime become substantially different. Our estimated $\hat{\rho}_1 = 0.6181$ is far less than this. As μ_{11} increases the US becomes even more attractive relative to the UK so holdings of the US increase. (The sample estimate is $\hat{\mu}_{11} = -1.2881$). Finally, as σ_{11} increases the US becomes less "safe" and the proportion allocated to the UK increases. There is some larger regime-dependent effect for values smaller than the sample estimate of $\hat{\sigma}_{11} = 7.0376$, but for values of σ_{11} greater than 9 the portfolio weights in each state are almost identical. Overall, since the correlations are similar across regimes there is little difference in the regime-dependent portfolio weights and the main effect is to alter the amount of the US held in each regime. Figure (8) suggests that of the parameters affecting the conditional distribution of returns in regime 1, the biggest effects on the regime-dependent weights come from conditional correlations and the relative difference in means.

6.3 Asymptotic Distributions of Economic Costs

The previous Section conveys intuition on which parameters have the largest effect on regime-dependent optimal asset allocation but does not tell us whether our main conclusions are affected by these different parameters. Here we focus on the RS Simple US-UK and US-UK-GER Models and re-compute the economic costs of no international diversification, the economic costs of ignoring RS and the economic costs of myopic strategies for 1,000 alternative parameter values drawn from the asymptotic normal parameter distributions implied by the estimation. We take the sample estimates to be "population values" and use the estimations where the conditional means are constrained to be equal across regimes. We then look at the 5% and 95% tail estimates of the various costs.[23]

The results of this exercise are presented in Table (19) for a risk aversion of $\gamma = 5$ and for horizons $T = 1$, 12, 36 and 60 months. The distributions of the economic costs have means which are larger than their population values in Tables (13) and (15). The median values of the economic costs are much closer to the population values. The economic cost computations can be viewed as non-linear transformations of the parameters. The transformations result in economic costs which are skewed to the right, especially for the costs of not diversifying internationally which are far more right skewed than the costs of holding IID weights. This means that if we draw a particular set of realistic parameter values, we may likely find costs for not diversifying internationally that are substantially larger than the population values. For example for $T = 60$ for the US-UK-GER Model the cost of no international diversification is 26 cents at the 95th percentile.

For the Simple US-UK Model, for $T = 1$ and 12 months, the costs of ignoring RS are slightly higher than the costs of no international diversification, but for the longer horizons, failing to diversify

[23]For the costs of ignoring RS, the IID weights are calculated using a multivariate normal with the unconditional mean and covariance matrix implied by the simulated parameters.

internationally is much more costly than ignoring RS. In the case of the Simple US-UK-GER Model, failing to hold overseas equity is always more costly than using IID weights. For $T = 12$ months the 95% tail estimate of the cost of no diversification is 4.47 cents (4.86 cents) in regime 1 (2), while the cost of ignoring RS is 1.01 cents (0.45 cents) in regime 1 (2). Finally, Table (19) shows the costs of using myopic weights are effectively zero when drawing from the asymptotic parameter distribution.

6.4 Out of Sample Experiment

To examine the relative effectiveness of the RS strategies to ignoring RS, we run an out of sample experiment. We take the out of sample period to be from January 1986 to December 1996 (11 years), which includes the 1987 crash. We consider the performance of all equity portfolios for the simple RS models for US-UK and US-UK-GER relative to the portfolios an investor would hold for IID weights ignoring RS. We use a fixed horizon of December 1996 (time T) and for each month t in the out of sample period we record the accumulated wealth from each strategy. At a given time t, we estimate the model up to time t. Using smoothed probabilities $p(s_t = 1|\mathcal{I}_t)$ (which use all information up to time t) implied by the RS model, we then infer the regime at time t. To find the appropriate portfolio weights we solve the dynamic programming problem for a horizon of $T - t$. We also find the portfolio weights for an investor using IID portfolio weights and using RS myopic weights. In the former case, these weights are estimated using the multivariate normal distribution with means and covariances estimated from data up to time t. At time $t + 1$ we calculate the actual accumulated wealth from holding these portfolio weights.

Table (20) lists the accumulated amounts at December 1997 of $1 invested at January 1986 in all equity portfolios. The Table confirms that over this period there is very little difference from using RS weights and ignoring RS. In Table (20) we see that the myopic RS strategies are almost identical (since the intertemporal hedging demands implied by the Simple RS Models are very weak) to the optimal RS strategy. As we increase γ, our returns become larger because more risk averse investors hold more of the "safer" US asset. The US produced the best returns over this period, which also explains the higher performance of holding only the US and UK relative to holding all three countries. Of course, this sample includes the bull market of the 1990's and, apart from the few months following the 1987 crash, the regime classification infers we are always in the normal second regime. As the IID weights are much closer to the RS weights for regime 2, this may represent a very biased draw.

7 Conclusions

In this article, we introduce regime-switching into a dynamic international asset allocation setting. We look at a US investor with Constant Relative Risk Aversion (CRRA) utility who dynamically rebalances and maximizes end of period wealth. Regime-switching can potentially have a large impact in this setting by producing state-dependent portfolio weights and intertemporal hedging demands.

Consistent with much of the empirical evidence on integrated equity markets, we find evidence of the presence of a high volatility-high correlation regime which tends to coincide with a bear market. However, the evidence on higher volatility is much stronger than the evidence on higher correlation. The regime-dependence of the means also has weak statistical significance, although the point estimates suggest that the high-volatility regime is associated with lower, and possibly negative, conditional means than the "normal" regime.

We consider a number of different settings from simple regime-switching multivariate normals to a model where short rates predict equities through their effect on the regime transition probabilities, but our main conclusions are robust across these models. First, the existence of this high volatility regime does not negate the benefits of international diversification. When currency hedging is allowed these benefits are even greater.

Second, the costs of ignoring regime switching are small for moderate levels of risk aversion. The optimal behavior of a US investor is to switch towards US equity (or cash, if available), at the expense of overseas equity when the high volatility regime is reached. It is the much higher volatility of overseas equity compared to the "safer" US equity which drives this result. However, it is not very costly not to switch, if an investor were to use IID portfolio weights even if the true data generating process were regime-switching. Although the portfolio weights may be significantly different across regimes, the IID weights act as an "average" portfolio weight which diversifies risk well in both regimes. This result continues to hold when currency hedging is allowed.

Third, in common with the non-parametric results obtained by domestic dynamic allocation studies such as Brandt (1998), the intertemporal hedging demands under regime switches are economically negligible and statistically insignificant. Investors have little to lose by acting myopically instead of solving a more complex dynamic programming problem for horizons greater than one period.

Our results are remarkably robust. When we draw random parameters from the estimated parameter distribution, the conclusions remain: for all equity portfolios, failing to diversify internationally is typically much more costly than ignoring the regimes, which, in turn, is more costly than ignoring the intertemporal hedging demands. However, our results remain premised on our assumptions, which include CRRA preferences, the absence of transactions costs and full knowledge on the part of the investors of the data generating process. With transactions costs, or learning about the regime, it is even less likely to be worthwhile for investors to change their allocations when the regime changes. However, using different utility functions, for example First Order Risk Aversion (Epstein and Zin (1991)) could potentially cause regime switching to have much bigger effects than in the traditional CRRA utility case, and such preferences can be treated in the same dynamic programming framework considered in this paper.

Appendix A: Markov Discretization Under Regimes and Predictability

Under the case of regime switching and predictability we follow Tauchen and Hussey (1991) by calibrating an approximating Markov chain to the RS DGP. We will discuss the calibration of the Short Rate Model, as the Yield Model is similar. We first fit a discrete Markov chain to the predictor instrument z_t. For the Short Rate Model, $z_t = r_t^{us} \equiv r_t$ which follows the process:

$$r_t = c(s_t) + \rho(s_t) r_{t-1} + v(s_t)\sqrt{r_{t-1}} u_{rt}, \tag{A-1}$$

with $u_{rt} \sim N(0,1)$. The transition probabilities are state-dependent:

$$p(s_t = i | s_{t-1} = i; \mathcal{I}_{t-1}) = \frac{\exp(a_i + b_i r_{t-1})}{1 + \exp(a_i + b_i r_{t-1})}. \tag{A-2}$$

We first fit a Markov chain to short rates for regime 1, then to regime 2, and then combine the chain. From hereon, we use the word "state" to refer to the discrete states of the Markov chain which approximate the continuous distribution in each "regime state", or "regime". The equity return shocks are correlated with the short rate, but the short rate states are the only driving variables in the system. We will show how to easily incorporate equity without expanding the number of states beyond those needed to approximate the distribution of r_t.

The idea behind Markov discretization is to choose points $\{r_i\}$ and a transition matrix Π which approximates the distribution of r_t. Tauchen and Hussey recommend choosing $\{r_i\}$ from the unconditional distribution of r_t. We can then find the transition probabilities p_{ij} from r_i to r_j by evaluating the conditional density of r_j (which is Normal from equation (A-1)) and then normalizing the densities so that they sum to unity, that is

$$\sum_j p_{ij} = 1. \tag{A-3}$$

For any highly persistent process such as short rates, discretization is difficult because p_{ij} are computed from a conditional distribution, and there is a different conditional distribution at each r_i and these may differ substantially from the unconditional distribution of r_t. The high persistence requires a lot of states for reasonable accuracy. When a square root process is introduced, the asymmetry of the distribution and the requirement that the states be non-negative introduce further difficulties.

To aid us in picking an appropriate grid for r_t in each regime we first simulate out a sample of length 200,000 from equations (A-1) and (A-2), with an initial pre-sample of length 10,000 to remove the effects of starting values. During the simulation we record the associated regime with each interest rate. We record the minimum and maximum simulated points in each regime. For regime 1, which is the less persistent higher conditional mean regime, we take a grid over points 2.5% higher (lower) than the simulated maximum (minimum). For regime 2, the "normal regime" with very low mean reversion, the persistence leads us to take a grid starting close to zero, to 2.5% higher than the simulated maximum. We use 50 points for regime 1, and 100 points for regime 2 to take into account the stronger persistence in this regime. We also employ a strategy of "over-sampling" from the over-lapping range of the regimes. This is to aid in picking points where the discretized Markov chain is more likely to have non-zero probabilities in switching from one regime to another. We place 95% (90%) of the points in regime 1 (2) in the overlap.

Let $\{r_i^k\}$ denote the states in regime k. We create the following partial transition matrices by the method outlined above: from $\{r_i^1\}$ to $\{r_i^1\}$, from $\{r_i^1\}$ to $\{r_i^2\}$, from $\{r_i^2\}$ to $\{r_i^1\}$ and from $\{r_i^2\}$ to $\{r_i^2\}$. Denote these by $\Pi_{j\to k}$ for $j,k = 1,2$. The rows of each $\Pi_{j\to k}$ will sum to 1. The total states for the Markov chain consist of $\{\{r_i^1\}\{r_i^2\}\}$.

Denote $P_{jk}(r) = p(s_t = k|s_{t-1} = j, r_{t-1} = r)$, which is given by equation (A-2). To mix the $\Pi_{j\to k}$ matrices to obtain Π for each r_i^k we calculate $P_{jk}(r_i^k)$ and then weight the appropriate row of each $\Pi_{j\to k}$ to combine into Π. For example, for a state in the first regime, r_i^1, we calculate $P_{11}(r_i^1)$ and $P_{12}(r_i^1)$. Then the appropriate row in Π corresponding to r_i will consist of $P_{11}(r_i^1)$ times the appropriate row corresponding to $\Pi_{1\to 1}$, and $P_{12}(r_i^1)$ times the appropriate row corresponding to $\Pi_{1\to 2}$.

This Markov chain is an accurate approximation of the RS process in equations (A-1) and (A-2). In particular, following Bekaert, Hodrick and Marshall (1998), when a sample of 100,000 is simulated from the Markov chain and the RS process re-estimated, all the parameters are well within 1 standard error of the original parameters. Also, the first two moments of the chain match the population moments of the RS process to 2-3 significant digits.

The Markov chain for r_t now consists of the states $\{r_i\}$ with transition matrix Π which is 150×150. To introduce equity into the chain we introduce the triplets $\{(r_i,\ y_i^1\ y_i^2)\}$ where y_i^m are the equity points for country m. We choose the points $\{y_i^m\}$ approximating country m by Gauss-Hermite weights for the conditional normal distribution for each regime. In our setup the equity returns for country m are given by:

$$y^m = \mu^m(s_t) + \sigma(s_t)u_{mt} \tag{A-4}$$

where cross-correlations between u_{mt}, $m = 1,2$ and u_{rt} are state-dependent. In a given regime, a Cholesky decomposition can be used to make a transformation from the uncorrelated normal errors $(u_1\, u_2\, u_3)'$ into the correlated errors $(e_1\, e_2\, e_3)'$, with ρ_{ij} denoting the correlation between e_i and e_j.

Note that in this formulation only the short rate is the driving process, and is the only variable we need to track at each time t. To accomodate the equity states we can expand Π column-wise. We choose 3 states per equity, making an effective transition matrix of 150×1350 where the rows sum to 1. (Note, a full 1350×1350 transition matrix could also be constructed, but the 9 rows corresponding to a particular r_i would be exactly the same.) Each short rate state is associated with 9 possible equity states. The only modification we need in the method outlined above is to construct new partial transition matrices so $\Pi_{1\to 1}$ becomes 50×450, $\Pi_{1\to 2}$ becomes 50×900, $\Pi_{2\to 1}$ becomes 150×450, and $\Pi_{2\to 2}$ becomes 100×900. These partial transition matrices can be mixed in the same manner as outlined before.

We find that there is a systematic downward bias when the implied moments conditional on the regime, and the unconditional moments are calculated from the Markov chain. This results from the regime-dependent distributions not being exactly unconditionally normally distributed in each regime from the presence of the square root term in the volatility of r_t, so Gaussian-Hermite weights will not be optimal in this setting. We make a further adjustment of scaling the volatility of the US (UK) by 4% (5%) upwards. Our final Markov chain matches means, variances and correlations to 2-3 significant figures.

When we solve the FOC's in equation (6) we find that strong persistence in r_t causes some instability at very low (<1.5%) and very high (>28%) interest rates. In these ranges the portfolio weights are not as smooth as the plots that appear in Figure (6). At very high interest rates the portfolio weights also start rapidly increasing for regime 2. These do not affect any solutions in the middle range. The inaccuracies arise because at the end of the chains, the Markov chain must effectively truncate the conditional distributions on the left (right) at low (high) interest rates. With experimentation we found that the inaccuracies at the end of the chain decrease as the persistence decreases.

Appendix B: Estimation of Model II

Let $y_t = (y_t^{us}\, y_t^{uk})'$ and $\tilde{y}_T = (y_0'\, y_1' \cdots y_T')'$. To construct the sample likelihood $f(\tilde{y}_T)$, we first expand the state space to $s_t = 1, \ldots, 4$ where the states correspond to all possible combinations of s_t^{us} and s_t^{uk}:

s_t	US	UK
1	1	1
2	2	1
3	1	2
4	2	2

The distribution of y_t conditional on s_t is $\mathrm{N}(\mu(s_t), \Sigma(s_t))$. The conditional means for each s_t are:

$$\mu_1 = \begin{pmatrix} \mu_1^{us} \\ \mu_1^{uk} \end{pmatrix} \quad \mu_2 = \begin{pmatrix} \mu_2^{us} \\ \mu_1^{uk} \end{pmatrix} \quad \mu_3 = \begin{pmatrix} \mu_1^{us} \\ \mu_2^{uk} \end{pmatrix} \quad \mu_4 = \begin{pmatrix} \mu_2^{us} \\ \mu_2^{uk} \end{pmatrix} \tag{B-1}$$

where subscripts indicate the appropriate corresponding regime. The conditional covariances are given by:

$$\Sigma_1 = \begin{pmatrix} (\sigma_1^{us})^2 & \rho_1\sigma_1^{us}\sigma_1^{uk} \\ \rho_1\sigma_1^{us}\sigma_1^{uk} & (\sigma_1^{uk})^2 \end{pmatrix} \qquad \Sigma_2 = \begin{pmatrix} (\sigma_2^{us})^2 & \rho_2\sigma_2^{us}\sigma_1^{uk} \\ \rho_2\sigma_2^{us}\sigma_1^{uk} & (\sigma_1^{uk})^2 \end{pmatrix}$$

$$\Sigma_3 = \begin{pmatrix} (\sigma_1^{us})^2 & \rho_1\sigma_1^{us}\sigma_2^{uk} \\ \rho_1\sigma_1^{us}\sigma_2^{uk} & (\sigma_2^{uk})^2 \end{pmatrix} \qquad \Sigma_4 = \begin{pmatrix} (\sigma_2^{us})^2 & \rho_2\sigma_2^{us}\sigma_2^{uk} \\ \rho_2\sigma_2^{us}\sigma_2^{uk} & (\sigma_2^{uk})^2 \end{pmatrix} \tag{B-2}$$

The correlations between the US and UK depend only on the state of the US.

We can write the likelihood over the sample $\tilde{y}_T = (y_1' \ \ldots y_T')'$ as:

$$
\begin{aligned}
f(\tilde{y}_T) &\equiv \prod_{t=1}^{T} f(y_t|\mathcal{I}_{t-1}) \\
&= \prod_{t=1}^{T} \left(\sum_{j=1}^{2} f(y_t|s_t^{us} = j, \mathcal{I}_{t-1}) p(s_t^{us} = j|\mathcal{I}_{t-1}) \right)
\end{aligned} \tag{B-3}
$$

We condition on $f(y_t|s_t^{us} = j, \mathcal{I}_{t-1})$ to write:

$$
\begin{aligned}
f(y_t|s_t^{us} = 1, \mathcal{I}_{t-1}) &= p(s_t^{uk} = 1|s_t^{us} = 1) f(y_t|s_t^{us} = 1, s_t^{uk} = 1, \mathcal{I}_{t-1}) \\
&\quad + p(s_t^{uk} = 2|s_t^{us} = 1)) f(y_t|s_t^{us} = 1, s_t^{uk} = 2, \mathcal{I}_{t-1}) \\
f(y_t|s_t^{us} = 2, \mathcal{I}_{t-1}) &= p(s_t^{uk} = 1|s_t^{us} = 2) f(y_t|s_t^{us} = 2, s_t^{uk} = 1, \mathcal{I}_{t-1}) \\
&\quad + p(s_t^{uk} = 2|s_t^{us} = 2)) f(y_t|s_t^{us} = 2, s_t^{uk} = 2, \mathcal{I}_{t-1})
\end{aligned} \tag{B-4}
$$

Equation (B-4) can be simplified to:

$$
\begin{aligned}
f(y_t|s_t^{us} = 1, \mathcal{I}_{t-1}) &= A f(y_t|s_t = 1, \mathcal{I}_{t-1}) + (1 - A) f(y_t|s_t = 3, \mathcal{I}_{t-1}) \\
f(y_t|s_t^{us} = 2, \mathcal{I}_{t-1}) &= (1 - B) f(y_t|s_t = 2, \mathcal{I}_{t-1}) + B f(y_t|s_t = 4, \mathcal{I}_{t-1})
\end{aligned} \tag{B-5}
$$

from equation (21).

Conditioning on $p(s_t^{us} = i|\mathcal{I}_{t-1})$ gives:

$$
p(s_t^{us} = i|\mathcal{I}_{t-1}) = \sum_{j=1}^{2} p(s_t^{us} = i|s_{t-1}^{us} = j, \mathcal{I}_{t-1}) p(s_{t-1}^{us} = j|\mathcal{I}_{t-1}) \tag{B-6}
$$

where the transition probabilities $P = p(s_t^{us} = 1|s_{t-1}^{us} = 1, \mathcal{I}_{t-1})$ and $Q = p(s_t^{us} = 2|s_{t-1}^{us} = 2, \mathcal{I}_{t-1})$ are constant. The ex-ante probability $p(s_t^{us} = j|\mathcal{I}_{t-1})$ can be computed recursively using the technique of Gray (1996) and Hamilton (1994):

$$
\begin{aligned}
p(s_{t-1}^{us} = j|\mathcal{I}_{t-1}) &= \frac{f(y_{t-1}, s_{t-1}^{us} = j|\mathcal{I}_{t-2})}{f(y_{t-1}|\mathcal{I}_{t-2})} \\
&= \frac{f(y_{t-1}|s_{t-1}^{us} = j, \mathcal{I}_{t-2}) p(s_{t-1}^{us} = j|\mathcal{I}_{t-2})}{\sum_{m=1}^{2} f(y_{t-1}|s_{t-1}^{us} = m, \mathcal{I}_{t-2}) p(s_{t-1}^{us} = m|\mathcal{I}_{t-2})}
\end{aligned} \tag{B-7}
$$

Note that we could also construct the likelihood by using a 4x4 restricted transition matrix and conditioning on s_t rather than s_t^{us} in equation (B-3). However, the approach given here is computationally more tractable.

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Table 1: Sample Moments of Nominal Equity Returns

Panel A: Central Moments

	US	UK	GER
Mean	0.9813	1.0670	0.9668
	(0.2382)	(0.3784)	(0.3158)
Std dev	4.3695	6.8338	5.8613
	(0.3107)	(0.6110)	(0.3257)
Skewness	-0.5545	0.4803	-0.3693
	(0.4576)	(0.6909)	(0.1936)
Kurtosis	6.2009	8.3336	4.1674
	(1.9056)	(2.8572)	(0.3269)

Panel B: Autocorrelations

Lag	US	UK	GER
1	0.0107	0.0585	-0.0278
	(0.0699)	(0.0669)	(0.0634)
2	-0.0174	-0.0867	-0.0129
	(0.0534)	(0.0705)	(0.0594)
3	-0.0007	0.0577	0.0607
	(0.0654)	(0.0651)	(0.0586)

Panel C: US, UK and GER Equity Correlations

	US	UK
UK	0.5100	
	(0.0498)	
GER	0.3628	0.4372
	(0.0701)	(0.0521)

US, UK, GER refer to equity returns. The sample period is from January 1970 to November 1997 (335 return observations). Returns are expressed as monthly continuously compounded rates. Standard errors are in parentheses and are estimated using Generalized Method of Moments with 3 Newey-West (1987) lags. The standard errors are calculated setting up moment conditions for each country separately for each of the central moments in Panel A and the autocorrelations in Panel B. In Panel C, all the moment conditions for the US, UK and Germany are used simultaneously to calculate the standard errors.

Table 2: Sample Moments of Excess Equity Returns

	world	US	UK uh	UK h	GER uh	GER h
Mean	0.4723	0.5443	0.7083	0.6218	0.3893	0.4386
	(0.2369)	(0.2452)	(0.3494)	(0.2994)	(0.3508)	(0.3287)
Std dev	3.9512	4.1912	6.2737	5.4212	5.8899	5.2371
	(0.2586)	(0.3350)	(0.4084)	(0.4750)	(0.3801)	(0.4495)
Skewness	-0.6496	-0.8139	-0.1843	-0.6651	-0.4213	-0.7996
	(0.3486)	(0.5862)	(0.2814)	(0.6140)	(0.2158)	(0.4170)
Kurtosis	5.3019	7.2077	4.6908	7.5485	4.3490	6.4398
	(1.3725)	(2.7195)	(0.6220)	(2.3031)	(0.3607)	(1.2801)

Monthly excess returns over the US Euro 1 month short rate. Sample period 75:01 to 97:07. World is the MSCI world index in USD, uh refers to unhedged returns in USD, hedged refers to hedged returns in USD, defined as $y_t^{LC} + i_t - i_t^*$ where LC denotes local currency returns, i_t the US short rate, and i_t^* the foreign short rate (EURO 1 month short rate) Standard errors are in parentheses and are estimated using Generalized Method of Moments with 3 Newey-West (1987) lags. The standard errors are calculated setting up moment conditions for each country separately.

Table 3: Sample Moments of Instruments

Panel A: Central Moments

	ey	$\ln(ey)$	r^f
Mean	7.9208	2.0122	7.8366
	(0.2937)	(0.0365)	(0.3651)
Std dev	2.7222	0.3381	3.3034
	(0.1643)	(0.0174)	(0.3091)
Skewness	0.6033	0.1398	1.1187
	(0.1643)	(0.1489)	(0.1931)
Kurtosis	2.2792	1.9529	4.2628
	(0.3080)	(0.1531)	(0.6953)

Panel B: Autocorrelations

Lag	ey	$\ln(ey)$	r^f
1	0.9873	0.9896	0.9688
	(0.0109)	(0.0089)	(0.0239)
2	0.9714	0.9740	0.9247
	(0.0200)	(0.0163)	(0.0443)
3	0.9562	0.9586	0.8874
	(0.0273)	(0.0228)	(0.0611)

ey and $\ln(ey)$ denote the MSCI earnings yield and log earnings yield respectively of the US. r^f denotes the short rate, which is the US 1 month LIBOR rate, expressed as a continuously compounded annual rate. The sample period is from January 1970 to November 1997 for the earnings yield, and from January 1972 to November 1997 for the short rate. Standard errors are in parentheses and are estimated using Generalized Method of Moments with 3 Newey-West (1987) lags. The standard errors are calculated setting up moment conditions for each instrument separately for each of the central moments in Panel A and the autocorrelations in Panel B.

Table 4: Test for Regime-Switching

Sample LR statistic = 103.738

Small Sample Distribution

		quantiles	
mean	18.253	5%	0.0089
stdev	9.434	10%	4.6613
max	49.449	50%	17.9527
min	0.000	90%	30.0468
		95%	42.0977

Test of the presence of regime-switching. Under the null of no-regime switching $y_t = \mu + \Sigma^{\frac{1}{2}}\epsilon_t$ with $y_t = (y_t^{us}\, y_t^{uk} y_t^{ger})'$ are the nominal monthly equity returns, and $\epsilon_t \sim$ IID N(0,I) against a regime-switching alternative $y_t = \mu(s_t) + \Sigma(s_t)^{\frac{1}{2}}\epsilon_t$ with $s_t = 1, 2$ and $P = p(s_t = 1|s_{t-1} = 1)$ and $Q = p(s_t = 2|s_{t-1} = 2)$. Samples of length 355 from the estimated no-regime switching model are generated, the regime-switching model is estimated on the simulated data and the sample likelihood ratio statistic is recorded. The procedure is repeated 500 times.

Table 5: Wald Tests for Parameter Equality Across Regimes

	Simple US-UK	Simple US-UK-GER	Beta US-UK	Beta US-UK-GER	Short Rate (Basic Model)	Earnings Yield Model
			Means $\mu_1 = \mu_2$			
US	0.0351	0.0747	0.0843	0.2635	0.0351	0.0000
UK	0.3858	0.8180	0.1140	0.2924	0.0803	0.0191
GER		0.5559		0.3559		
Joint	0.0975	0.2285	0.1856	0.3770	0.0861	0.0001
			Volatilities $\sigma_1 = \sigma_2$			
US	0.0002	0.0000	0.0146	0.0001	0.0000	0.0000
UK	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
GER		0.0000		0.0000		
Joint	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
			Correlations $\rho_1 = \rho_2$			
US-UK	0.1556	0.0586	0.5676	0.5745	0.0627	0.4316
US-GER		0.1709		0.5355		
UK-GER		0.8246		0.9138		
Joint		0.2340		0.6825		

The Table lists p-values of Wald Tests of parameter equality across regimes $s_t = 1, 2$. The Simple RS models refer to equation (19), and the Basic Model for the Short Rate refers to equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30). For the Beta Models UK and GER refer to unhedged asset returns. The Joint Tests refer to a Wald test of parameter equality across regimes for all countries listed in the entries for that model. For the Earnings Yield Model, the $\mu_1 = \mu_2$ test refers to a test of equality for the conditional mean parameters $\mu(s_t)$ and $\beta(s_t)$ in equation (30) with ey_t replacing r_t.

Table 6: Likelihood Ratio Tests for Constraining $\mu_1 = \mu_2$

		P-value Test $\mu_1 = \mu_2$	unconstrained RCM	constrained RCM
Basic Model	US-UK	0.1165	24.7	23.6
	US-UK-GER	0.2289	52.9	48.4
Beta Model	US-UK	0.0644	56.0	40.0
	US-UK-GER	0.2435	64.0	64.4
Short Rate Basic Model		0.0973	29.9	29.9

The likelihood ratio test refers to a test for constraining the conditional mean μ across regimes. For the Basic Model, this is the conditional mean for each country (constrained separately) in equation (19). For the Beta Model, this is the conditional mean for the excess world return in equation (26). The Short Rate Basic Model refers to equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30). RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values denote better regime classification.

Table 7: Likelihood Ratio Tests of Restricted Models

US-UK Simple RS Model	
Basic Model vs Model II	0.9950
Basic Model vs RS ARCH	0.9853
Short Rate Model	
Basic vs Full Model	0.9145
Constant Probabilities vs Basic	0.0065
Earnings Yield Model	
No Predictability vs Full Model	0.0149
No Conditional Mean Predictability vs Full Model	0.0697
Constant Probabilities vs Full Model	0.0542

P-values are listed. The nested model is always listed first. The Basic Model for the US-UK Simple RS Model refers to equation (19), and Model II to the extension in equation (21). The RS ARCH model is presented in equation (22). The Full Model for the Short Rate Model refers to equation (28)-(31) and the Basic Model where we impose $\beta(s_t) = 0$ in equation (30). The Constant Probabilities Model refers to setting $\beta(s_t) = 0$ in equation (30) and $b_i = 0$ in equation (31), so it is the Basic Short Rate Model with P and Q constant. For the Earnings Yield Model, no predictability refers to a test of $\beta(s_t) = 0$ and $b_i = 0$, no conditional mean predictability to $\beta(s_t) = 0$, and constant probabilities to $b_i = 0$.

Table 8: Implied Currency Premiums from the RS Beta Models

US-UK Beta Model

US-UK Exchange Rate

state 1		state 2		Stable Probs	
cp	vol	*cp*	vol	*cp*	vol
0.0294	11.0317	0.1572	4.8514	0.1362	6.2980
(0.1927)	(1.2170)	(0.0845)	(0.2677)	(0.0791)	(0.4978)

US-UK-GER Beta Model

US-UK Exchange Rate

state 1		state 2		Stable Probs	
cp	vol	*cp*	vol	*cp*	vol
0.0801	9.5450	0.1980	4.5925	0.1647	6.3912
(0.1270)	(0.6583)	(0.0964)	(0.2715)	(0.0840)	(0.3788)

US-GER Exchange Rate

state 1		state 2		Stable Probs	
cp	vol	*cp*	vol	*cp*	vol
0.0610	9.7088	0.1643	5.1022	0.1351	6.7299
(0.1280)	(0.6864)	(0.0892)	(0.2436)	(0.0773)	(0.3652)

The currency return cr_{t+1} is the is the excess return in investing in the foreign money market: $cr_{t+1} = e_{t+1} + i_t^* - i_t$ where e_{t+1} is the log difference of the exchange rate, i_t^* is the foreign country's short rate, and i_t is the domestic short rate. In the RS Beta Models this is the difference between unhedged and hedged returns. The currency premium cp_t is the expected value of the currency return $cp_t = \mathrm{E}_t(cr_{t+1})$. "Vol" refers to the volatility of the currency return. We report regime-dependent and unconditional currency premiums and volatilities of currency returns. We impose the same conditional means for the world excess return across states ($\mu_1 = \mu_2$).

Table 9: Simple US-UK Model: Weight of the US in All-Equity Portfolios

	Risk Aversion $\gamma = 5$				Risk Aversion $\gamma = 10$			
	Basic Model		Restricted $\mu_1 = \mu_2$		Basic Model		Restricted $\mu_1 = \mu_2$	
Horizon	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
US Weight								
1	0.8587	0.7171	0.9348	0.6726	0.9652	0.7666	0.9999	0.7405
	(0.3662)	(0.2238)	(0.0977)	(0.2230)	(0.1739)	(0.1139)	(0.1072)	(0.1169)
12	0.8609	0.7297	0.9362	0.6769	0.9697	0.8585	1.0048	0.7769
	(0.1919)	(0.2067)	(0.0997)	(0.2203)	(0.1773)	(0.1229)	(0.1010)	(0.1063)
36	0.8614	0.7352	0.9365	0.6779	0.9699	0.8744	1.0057	0.7954
	(0.3645)	(0.2022)	(0.0989)	(0.2198)	(0.1754)	(0.1242)	(0.1013)	(0.1050)
60	0.8614	0.7356	0.9365	0.6779	0.9699	0.8744	1.0057	0.7965
	(0.2495)	(0.2224)	(0.1006)	(0.2192)	(0.1767)	(0.1240)	(0.1012)	(0.1049)
IID weights	0.7642		0.7642		0.8275		0.8275	
Intertemporal Hedging Demand Tests								
12	0.9932	0.9736	0.9260	0.9804	0.2971	0.3877	0.9605	0.6707
36	0.8701	0.9609	0.2091	0.2334	0.2091	0.2334	0.9533	0.5377
60	0.9848	0.9547	0.9619	0.9757	0.2358	0.2333	0.9529	0.5294
Tests for Equality with IID Weights								
1	0.7964	0.8334	0.0808	0.6813	0.4287	0.5926	0.1076	0.4569
12	0.6142	0.8677	0.0845	0.6921	0.4224	0.8009	0.0793	0.6344
36	0.7897	0.8862	0.0815	0.6948	0.4168	0.7055	0.0786	0.7597
60	0.6968	0.8978	0.0867	0.6941	0.4202	0.7050	0.0782	0.7677
Tests for Regime Equality								
1	0.7465		0.1448		0.2977		0.0028	
12	0.6087		0.1630		0.3141		0.0446	
36	0.7337		0.1691		0.3085		0.0493	
60	0.6181		0.1952		0.3116		0.0496	
Joint	0.9844		0.2237		0.8047		0.0102	

Asset allocation weights for the US from the Simple RS US-UK model. The coefficient of risk aversion γ is fixed at 5 and 10. Standard errors in parentheses calculated using the delta-method with 5 quadrature points for each country. Table shows weights for an all equity portfolio (so UK weight is 1 - US weight). The Intertemporal Hedging Demand Test is a Wald Test to test if the horizon T portfolio weights are different from the myopic portfolio weights within each regime state, expressed as a p-value. The Regime Equality is a Wald Test for equality of the US portfolio weights across regimes expressed as a p-value.

Table 10: Simple US-UK-GER Model: Weight of the US and UK in All-Equity Portfolio

	Restricted $\mu_1 = \mu_2$ Model							
	Risk Aversion $\gamma = 5$				Risk Aversion $\gamma = 10$			
	State 1		State 2		State 1		State 2	
Horizon	US	UK	US	UK	US	UK	US	UK
Portfolio Weights								
1	0.6836	0.0341	0.6144	0.1590	0.7332	-0.0162	0.6355	0.1234
	(0.1551)	(0.0990)	(0.2703)	(0.2591)	(0.1312)	(0.1136)	(0.1320)	(0.1311)
12	0.6839	0.0337	0.6153	0.1572	0.7352	-0.0186	0.6432	0.1104
	(0.1532)	(0.0978)	(0.2716)	(0.2601)	(0.1771)	(0.1458)	(0.1428)	(0.1227)
36	0.6839	0.0336	0.6154	0.1570	0.7354	-0.0188	0.6442	0.1088
	(0.1533)	(0.0990)	(0.2701)	(0.2579)	(0.1371)	(0.0929)	(0.1376)	(0.1295)
60	0.6839	0.0336	0.6154	0.1570	0.7354	-0.0188	0.6442	0.1088
	(0.1585)	(0.0969)	(0.2697)	(0.1585)	(0.1457)	(0.1148)	(0.1258)	(0.1231)
IID Weights	US		UK		US		UK	
	0.5889		0.1449		0.6491		0.0800	
Intertemporal Hedging Demands								
12	0.9774	0.9733	0.9040	0.8717	0.9907	0.9793	0.8385	0.7455
36	0.9948	0.9793	0.6989	0.6102	0.9783	0.9711	0.8291	0.4409
60	0.9862	0.9500	0.8622	0.8075	0.9735	0.9850	0.7740	0.7602
Tests for Equality with IID Weights								
1	0.5416	0.2632	0.9249	0.9565	0.5216	0.3974	0.9185	0.7405
12	0.5353	0.2553	0.9226	0.9623	0.6265	0.4994	0.9673	0.8038
36	0.5355	0.2613	0.9219	0.9625	0.5288	0.2879	0.9717	0.8237
60	0.5489	0.2508	0.9218	0.9626	0.5534	0.3900	0.9691	0.8148
Tests for Regime Equality								
	US	UK	Joint US,UK		US	UK	Joint US UK	
1	0.6681	0.5127	0.8064		0.4083	0.3144	0.5309	
12	0.6643	0.5057	0.8009		0.5782	0.4158	0.6980	
36	0.6974	0.5225	0.8151		0.5782	0.4158	0.6980	
60	0.6695	0.5220	0.8141		0.5541	0.1926	0.4022	
Joint across T	0.9925	0.9649			0.9440	0.7673		

Asset allocation weights for the US and UK from the Simple US-UK-GER model with $\mu_1 = \mu_2$ imposed. The coefficient of risk aversion γ is fixed at 5 and 10. Standard errors in parentheses calculated using the delta-method with 5 quadrature points for each country. The Table shows weights for an all equity portfolio (so GER weight is 1 - US - UK weight). The Intertemporal Hedging Demand Test is a Wald Test to test if the horizon T portfolio weights are different from the myopic portfolio weights within each regime state, expressed as a p-value. The Regime Equality is a Wald Test for equality of the portfolio weights across regimes expressed as a p-value.

Table 11: Currency Hedging Beta Models: Asset Allocation Weights

RS US-UK Beta Model with $\mu_1 = \mu_2$

	State 1			State 2		
Horizon	US	UK for	Hedge Ratio	US	UK for	Hedge Ratio
1	0.7766	-0.1164	0.5211	0.4874	-0.2565	0.5004
	(0.0980)	(0.0611)		(0.0395)	(0.0497)	
12	0.7775	-0.1160	0.5211	0.4886	-0.2560	0.5006
	(0.1095)	(0.0642)		(0.0411)	(0.0478)	
36	0.7775	-0.1160	0.5211	0.4886	-0.2560	0.5006
	(0.1095)	(0.0642)		(0.0411)	(0.0478)	

Restricted RS US-UK-GER Beta Model with $\mu_1 = \mu_2$

	state 1						state 2					
Horizon	US	UK	UK for	Hedge Ratio UK	GER for	Hedge Ratio GER	US	UK	UK for	Hedge Ratio UK	GER for	Hedge Ratio GER
1	0.3989	0.2826	-0.1497	0.5298	-0.1667	0.5235	0.3721	0.3721	-0.1502	0.4036	-0.1056	0.4129
	(0.0621)	(0.0559)	(0.0318)		(0.0377)		(0.0592)	(0.0595)	(0.0245)		(0.0813)	
12	0.3990	0.2824	-0.1497	0.5302	-0.1669	0.5237	0.3722	0.3716	-0.1502	0.4042	-0.1060	0.4137
	(0.0622)	(0.0559)	(0.0318)		(0.0376)		(0.0591)	(0.0595)	(0.0245)		(0.0811)	
36	0.3990	0.2824	-0.1497	0.5302	-0.1669	0.5237	0.3722	0.3716	-0.1502	0.4042	-0.1060	0.4137
	(0.0622)	(0.0559)	(0.0318)		(0.0376)		(0.0591)	(0.0595)	(0.0245)		(0.0811)	

Asset allocation weights for the RS Beta Models with the risk-free rate fixed at 6%. The coefficient of risk aversion γ is fixed at 5. Standard errors in parentheses calculated using the delta-method with 5 quadrature points for each asset. The Table shows weights for an all equity portfolio (so for the US-UK model, UK weight is 1 - US and for the US-UK-GER model, GER weight is 1 - US - UK weight) with a position in Pound forward contracts (denoted by UK for) and Deutschmark forward contracts (denoted by GER for). The hedge ratio is the value of the short forward position as a proportion of foreign equity holdings.

Table 12: Joint Wald Tests for Equality of Portfolio Weights Across Regimes

All equity portfolios

Model	US	UK	UK for	GER for
Simple US-UK	0.2237			
Simple US-UK-GER	0.9925	0.9649		
US-UK Beta	0.0000		0.2790	
US-UK-GER Beta	0.6778	0.0826	0.9997	0.7763

Joint Wald Tests to test for significantly different portfolio weights across regimes for horizons $T = 1, 12, 36, 60$ months. The Table lists p-values. The risk aversion $\gamma = 5$. For the RS Beta Models, "for" denotes the forward currency position. In all models $\mu_1 = \mu_2$.

Table 13: Economic Cost of No International Diversification: All Equity Nominal Portfolios

Simple RS Models

	US-UK Model				US-UK-GER Model			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
1	0.01	0.07	0.00	0.09	0.12	0.07	0.22	0.14
12	0.44	0.78	0.26	0.80	1.19	0.97	2.35	1.90
36	1.83	2.24	0.90	1.51	3.31	3.06	6.84	6.32
60	3.29	3.70	1.47	2.07	5.45	5.20	11.51	10.96

Earnings Yield Model with $\gamma = 5$

	$s_t = 1$			$s_t = 2$		
T	$ey = 5.5$	$ey = 10.4$	$ey = 15.2$	$ey = 2.7$	$ey = 6.0$	$ey = 11.0$
1	0.03	0.11	0.25	0.02	0.01	0.05
12	0.52	1.25	2.53	0.21	0.30	1.07
36	1.91	3.47	5.70	0.85	1.40	3.27
60	3.46	5.41	7.95	1.86	2.80	5.21

The Table presents the cost in "cents per dollar" compensation required for an investor to only hold US equity (so the portfolio weight is 1 on US equity and zero on all other assets) instead of the optimal weights. The Simple RS Models have $\mu_1 = \mu_2$ imposed.

Table 14: Economic Costs of the Currency Hedging Beta Models

Cost of Not Diversifying Internationally

	US-UK Model				US-UK-GER Model			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
1	0.04	0.09	0.01	0.14	0.17	0.10	0.25	0.19
12	0.74	0.97	0.71	1.36	1.53	1.37	2.65	2.50
36	2.58	2.83	2.84	3.55	4.43	4.26	7.97	7.81
60	4.46	4.72	5.04	5.76	7.42	7.24	13.56	13.40

Cost of Not Currency Hedging

	US-UK				US-UK-GER			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
1	0.02	0.03	0.00	0.08	0.06	0.06	0.11	0.10
12	0.26	0.32	0.38	0.47	0.72	0.74	1.27	1.23
36	0.88	0.95	1.50	1.87	2.22	2.23	3.82	3.77
60	1.50	1.57	2.64	3.02	3.73	3.74	6.42	6.38

The first panel presents the cost in "cents per dollar" compensation required for an investor to hold only the US. The second panel presents the costs required for an investor to only hold US and unhedged foreign equity instead of the optimal weights. In this case we solve an optimal asset allocation problem with restricting holdings only to US and unhedged foreign equity and find the compensation required to hold these weights instead of the optimal weights, which allow currency hedging. All models have $\mu_1 = \mu_2$ imposed.

Table 15: Economic Cost of Ignoring Regime Switching

Simple RS Models

	US-UK				US-UK-GER			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
1	0.08	0.01	0.16	0.01	0.02	0.00	0.03	0.00
12	0.58	0.13	1.65	0.44	0.14	0.05	0.26	0.11
36	1.09	0.54	4.84	3.16	0.29	0.20	0.66	0.48
60	1.53	0.97	8.20	6.46	0.44	0.35	1.05	0.87

The Table presents the cost in "cents per dollar" compensation required for an investor to ignore regime-switching and use Samuelson's (1969) myopic portfolio weights in an IID multivariate normal setting with CRRA utility instead of the optimal portfolio weights. The models have $\mu_1 = \mu_2$ imposed.

Table 16: Economic Cost of Using Myopic Strategies

Simple RS Models

	US-UK				US-UK-GER			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
12	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
36	0.00	0.00	0.03	0.06	0.00	0.00	0.00	0.00
60	0.00	0.00	0.07	0.11	0.00	0.00	0.01	0.01

Basic RS Short Rate Model with $\gamma = 5$

	$s_t = 1$			$s_t = 2$		
T	$r = 5.1$	$r = 9.9$	$r = 14.8$	$r = 5.1$	$r = 10.1$	$r = 15.1$
12	0.00	0.00	0.00	0.00	0.00	0.00
36	0.01	0.01	0.01	0.01	0.01	0.01
60	0.02	0.02	0.02	0.02	0.02	0.02

Earnings Yield Model with $\gamma = 5$

	$s_t = 1$			$s_t = 2$		
T	$ey = 5.5$	$ey = 10.4$	$ey = 15.2$	$ey = 2.7$	$ey = 6.0$	$ey = 11.0$
12	0.00	0.00	0.00	0.00	0.00	0.00
36	0.00	0.00	0.00	0.00	0.00	0.00
60	0.00	0.01	0.01	0.00	0.00	0.00

The Table presents the cost in "cents per dollar" of compensation required for an investor to use the myopic 1-month horizon weights for all horizons instead of the optimal weights. The Basic Short Rate Model refers to equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30). The Simple RS, and Basic RS Short Rate Model have $\mu_1 = \mu_2$ imposed.

Table 17: Economic Costs under the Basic Short Rate Model

Cost of Not Holding UK Equity

	$s_t = 1$			$s_t = 2$		
T	$r = 5.1$	$r = 9.9$	$r = 14.8$	$r = 5.1$	$r = 10.1$	$r = 15.1$
1	0.01	0.01	0.01	0.05	0.03	0.01
12	0.37	0.29	0.22	0.57	0.36	0.20
36	1.35	1.07	0.89	1.64	1.09	0.84
60	2.33	1.94	1.71	2.64	1.92	1.65

Cost of Ignoring RS and Holding Purely Equity

	$s_t = 1$			$s_t = 2$		
T	$r = 5.1$	$r = 9.9$	$r = 14.8$	$r = 5.1$	$r = 10.1$	$r = 15.1$
1	0.38	0.39	0.40	0.01	0.01	0.11
12	2.28	2.71	3.41	0.32	1.09	3.10
36	3.83	5.63	7.39	1.36	4.22	7.38
60	5.33	7.96	10.13	2.68	6.78	10.22

The Table presents the cost in "cents per dollar" compensation required for the Basic Short Rate Model, which refers to equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30). We also impose $\mu_1 = \mu_2$ for excess requity returns and set $\gamma = 5$. The first panel refers to the compensation required to hold only US equity and cash. For this we need to solve a restricted optimization with zero weight on the UK. The bottom panel refers to the compensation required for an investor to ignore regime-switching and predictability and hold an all-equity portfolio without any cash balances in her portfolio. The equity portfolio weights are Samuelson's (1969) myopic portfolio weights in an IID setting with CRRA utility.

Table 18: Economic Costs under the Earnings Yield Model

Cost of Ignoring RS and Predictability

T	$s_t = 1$ $ey = 5.5$	$ey = 10.4$	$ey = 15.2$	$s_t = 2$ $ey = 2.7$	$ey = 6.0$	$ey = 11.0$
1	0.01	0.00	0.04	0.00	0.00	0.00
12	0.03	0.06	0.39	0.03	0.04	0.05
36	0.11	0.21	0.71	0.11	0.11	0.19
60	0.21	0.34	0.88	0.18	0.20	0.31

Cost of Ignoring Predictability

T	$s_t = 1$ $ey = 5.5$	$ey = 10.4$	$ey = 15.2$	$s_t = 2$ $ey = 2.7$	$ey = 6.0$	$ey = 11.0$
1	0.00	0.03	0.10	0.00	0.00	0.01
12	0.06	0.33	1.02	0.01	0.02	0.27
36	0.35	0.95	2.07	0.08	0.22	0.88
60	0.60	1.30	2.51	0.22	0.44	1.22

The Table presents the cost in "cents per dollar" compensation required for the Earnings Yield Model. We set $\gamma = 5$. The first panel refers to the compensation required to hold IID portfolio weights (Samuelson (1969)) which ignore regimes and predictability. The second panel refers to the compensation required to ignore predictability, but take into account regimes. In this case, the restricted portfolio weights are those implied by the Simple RS US-UK Model with μ_1 not constrained to be equal to μ_2, which ignores earnings yield predictability.

Table 19: Asymptotic Distributions of Economic Costs

		Simple US-UK Model						Simple US-UK-GER Model					
		No Diversification		IID Weights		Myopic Weights		No Diversification		IID Weights		Myopic Weights	
Horizon		$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
	Mean	0.04	0.10	0.13	0.01			0.17	0.15	0.04	0.01		
	stdev	0.05	0.11	0.16	0.01			0.12	0.13	0.05	0.01		
$T = 1$	Quantiles												
	5%	0.00	0.00	0.00	0.00			0.02	0.01	0.00	0.00		
	50%	0.02	0.07	0.07	0.00			0.14	0.11	0.02	0.00		
	95%	0.16	0.32	0.46	0.03			0.38	0.41	0.14	0.03		
	Mean	0.82	1.17	0.91	0.20	0.00	0.00	1.92	1.83	0.29	0.14	0.00	0.00
	stdev	0.80	1.21	1.17	0.24	0.00	0.00	1.39	1.50	0.39	0.16	0.00	0.00
$T = 12$	Quantiles												
	5%	0.04	0.03	0.01	0.00	0.00	0.00	0.30	0.24	0.01	0.01	0.00	0.00
	50%	0.55	0.78	0.49	0.12	0.00	0.00	1.59	1.43	0.16	0.08	0.00	0.00
	95%	2.45	3.61	3.27	0.71	0.00	0.00	4.47	4.86	1.01	0.45	0.00	0.00
	Mean	2.96	3.42	1.82	0.86	0.00	0.00	5.81	5.71	0.68	0.49	0.00	0.00
	stdev	2.98	3.57	2.51	1.09	0.00	0.00	4.52	4.67	0.86	0.58	0.00	0.00
$T = 36$	Quantiles												
	5%	0.13	0.09	0.02	0.01	0.00	0.00	0.85	0.80	0.03	0.02	0.00	0.00
	50%	1.92	2.20	0.99	0.51	0.00	0.00	4.61	4.45	0.39	0.29	0.00	0.00
	95%	8.73	10.38	6.19	2.90	0.00	0.00	14.81	15.19	2.33	1.69	0.00	0.00
	Mean	5.25	5.74	2.62	1.60	0.00	0.00	9.94	9.83	1.04	0.86	0.00	0.00
	stdev	5.42	6.09	3.74	2.15	0.01	0.01	8.05	8.22	1.31	1.03	0.00	0.00
$T = 60$	Quantiles												
	5%	0.20	0.16	0.02	0.02	0.00	0.00	1.40	1.36	0.05	0.04	0.00	0.00
	50%	3.34	3.64	1.40	0.92	0.00	0.00	7.82	7.57	0.60	0.51	0.00	0.00
	95%	16.06	17.77	8.76	5.61	0.01	0.01	25.88	26.48	3.62	2.89	0.00	0.01

We draw parameter values for the Simple RS US-UK and US-UK-GER Models with constrained means across regimes implied by their asymptotic distribution. Economic Costs of no international diversification (holding only the US), ignoring RS (using IID weights from a multivariate normal distribution with the same implied unconditional mean and covariance from the simulated parameter values), and using myopic weights are calculated. The exercise is repeated 1,000 times with a risk aversion level of $\gamma = 5$. The Table lists "cents per dollar" of wealth compensations.

Table 20: Out of Sample Experiment

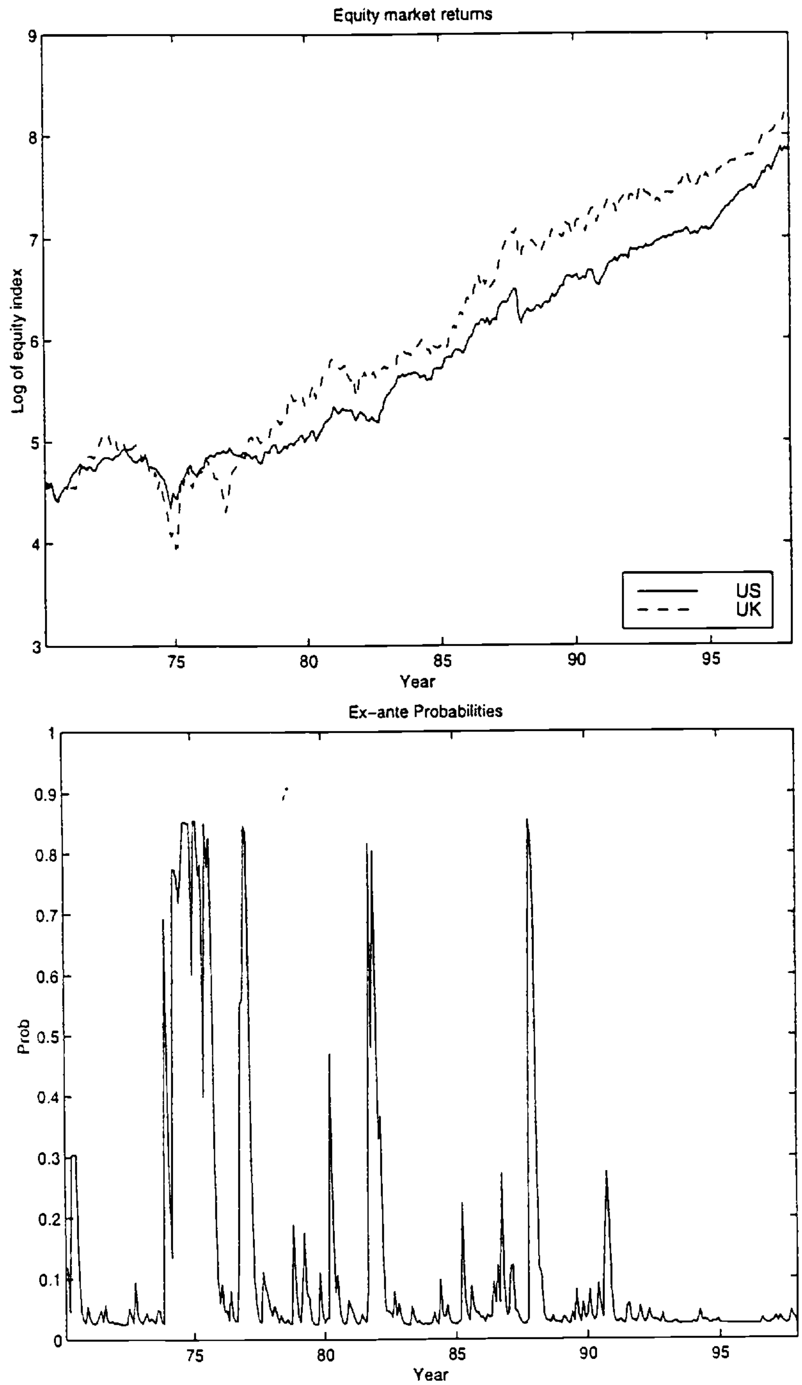
US-UK

	IID	RS optimal	RS myopic
$\gamma = 5$	6.4424	6.5929	6.5827
	(15.41%)	(15.61%)	(15.60%)
$\gamma = 10$	6.5005	6.6450	6.6557
	(15.49%)	(15.68%)	(15.70%)

US-UK-GER

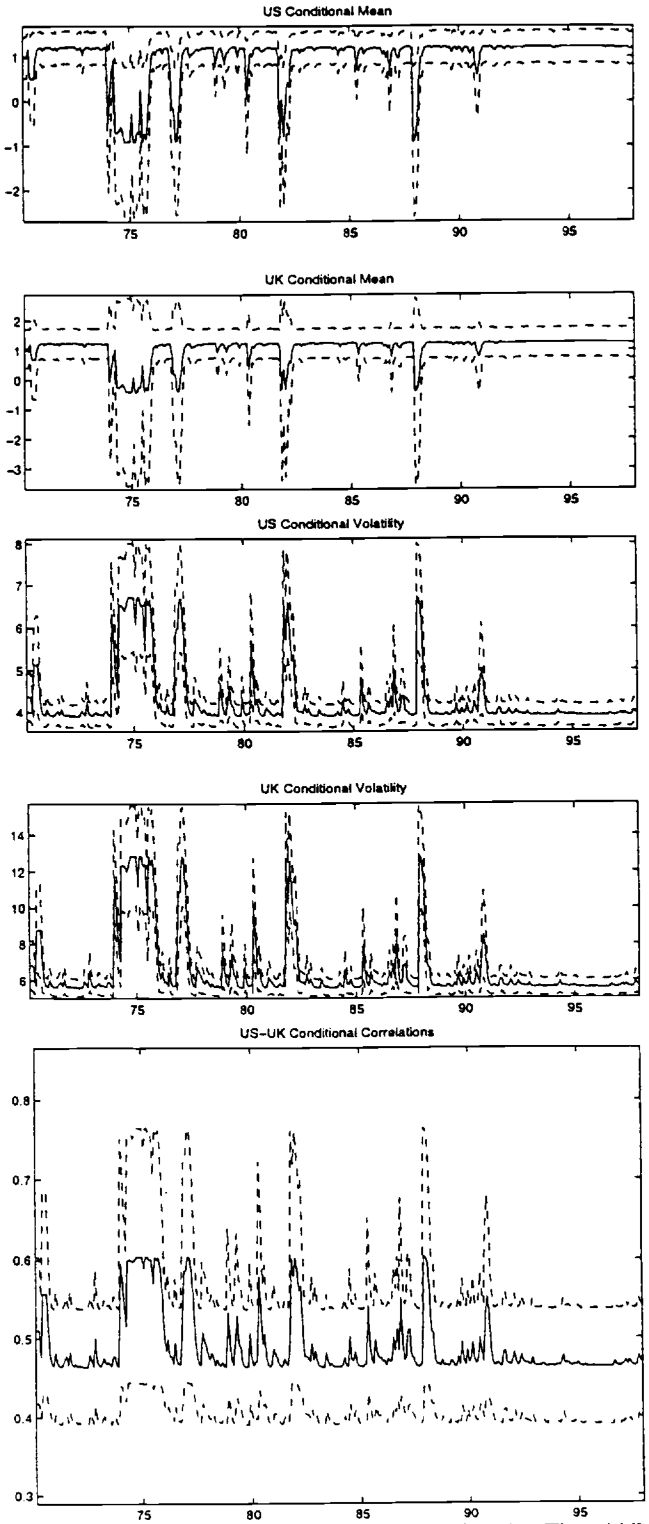
	IID	RS optimal	RS myopic
$\gamma = 5$	5.3772	5.1462	5.1399
	(13.81%)	(13.43%)	(13.42%)
$\gamma = 10$	5.6128	5.5639	5.5047
	(14.19%)	(14.11%)	(14.02%)

In the top row we list accumulated amounts of $1 in January 1986 at December 1997 where the portfolio weights are calculated from models estimated with data up to time t, and finding the actual accumulated wealth at time $t + 1$. Portfolios are all-equity. Numbers in parentheses are the annual percentage return over the entire period (11 years). IID refers to the Samuelson (1969) myopic strategy using IID multivariate normal distributions. The RS strategies use dynamic programming solutions from the Simple RS US-UK and US-UK-GER Models. RS optimal refers to the strategy using optimal portfolio weights assuming the horizon remains fixed at December 1997 through the whole sample. RS myopic refers to using myopic 1-period weights of the Simple RS Model.



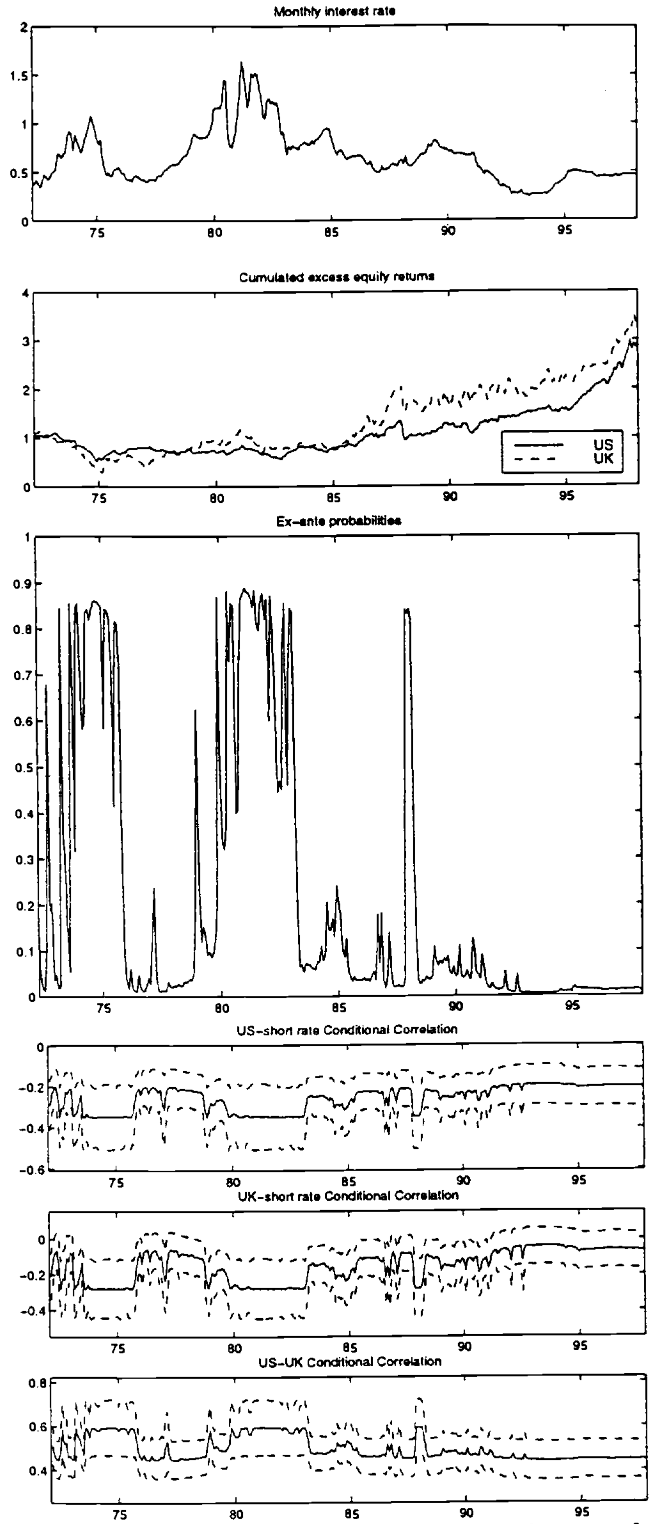
The top panel gives the log of the equity index levels for the US and UK. The bottom panel lists the ex-ante probability $p(s_t = 1|\mathcal{I}_{t-1})$.

Figure 1: Simple RS Model of US-UK Equity Returns



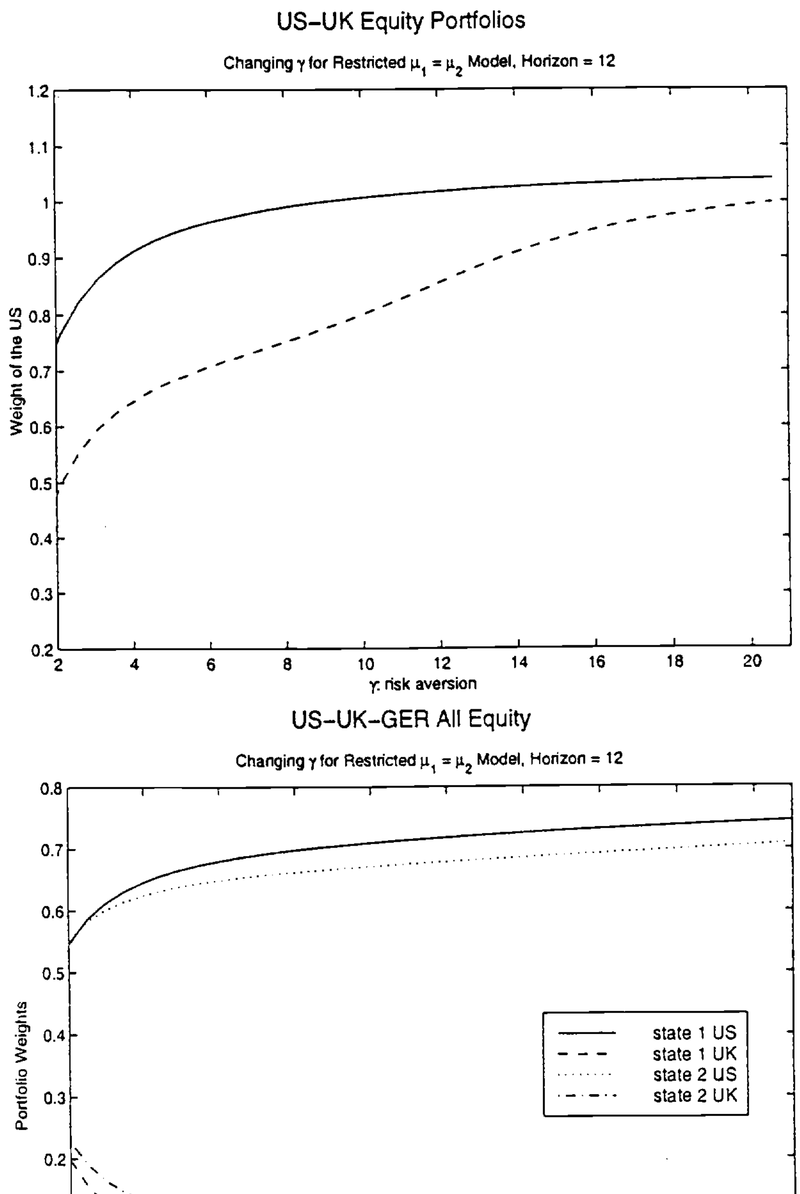
The top panel gives the implied conditional means of US and UK equity. The middle panel shows the implied conditional volatilities and the bottom panel shows the implied conditional correlation. The dashed lines are 95% confidence intervals.

Figure 2: Implied conditional moments from the Simple RS US-UK Model



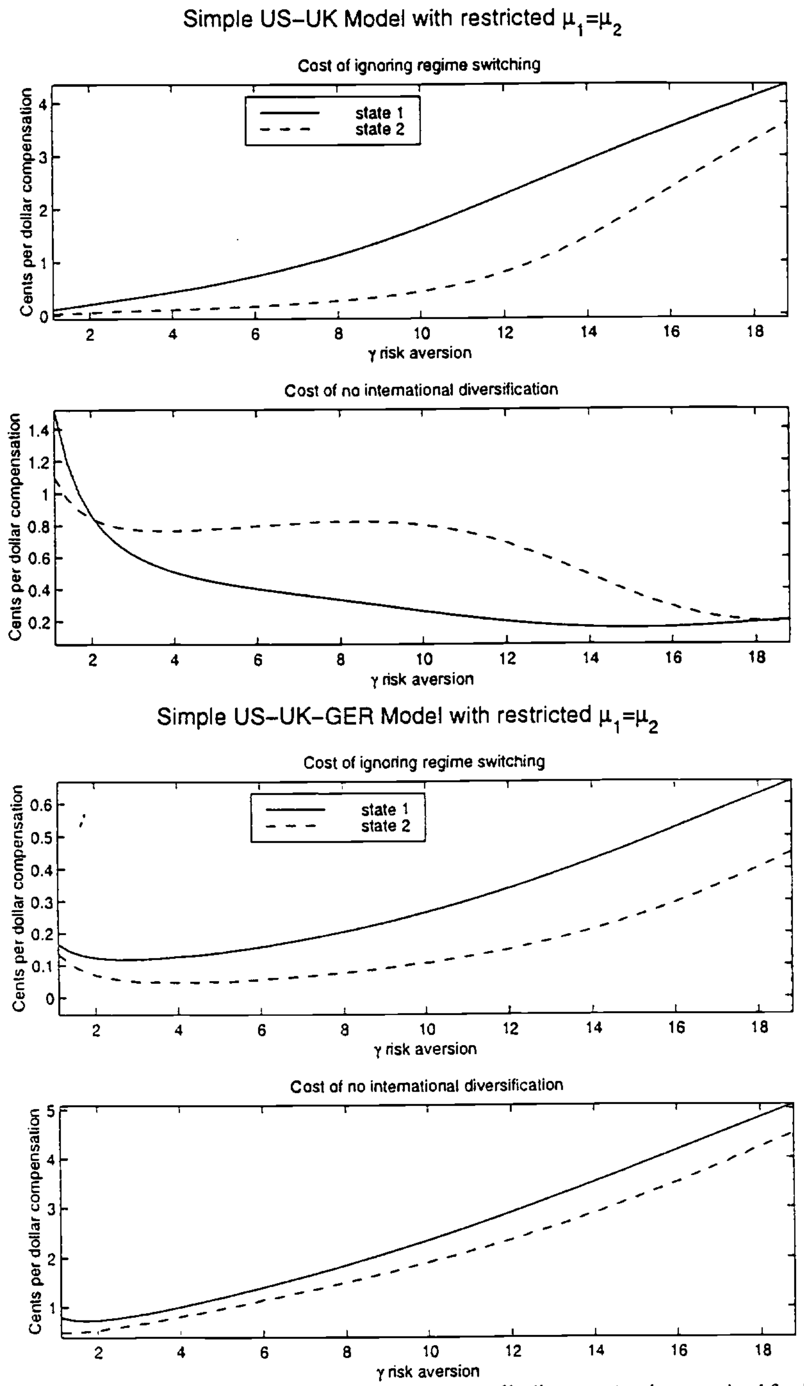
The top panel gives plots of the short rate and the cumulated excess equity returns for the US and UK. The middle panel lists the ex-ante probability $p(s_t = 1|\mathcal{I}_{t-1})$. The bottom panel shows the implied correlations between US, UK equity and the short rate.

Figure 3: Short Rate-US-UK Equity Model



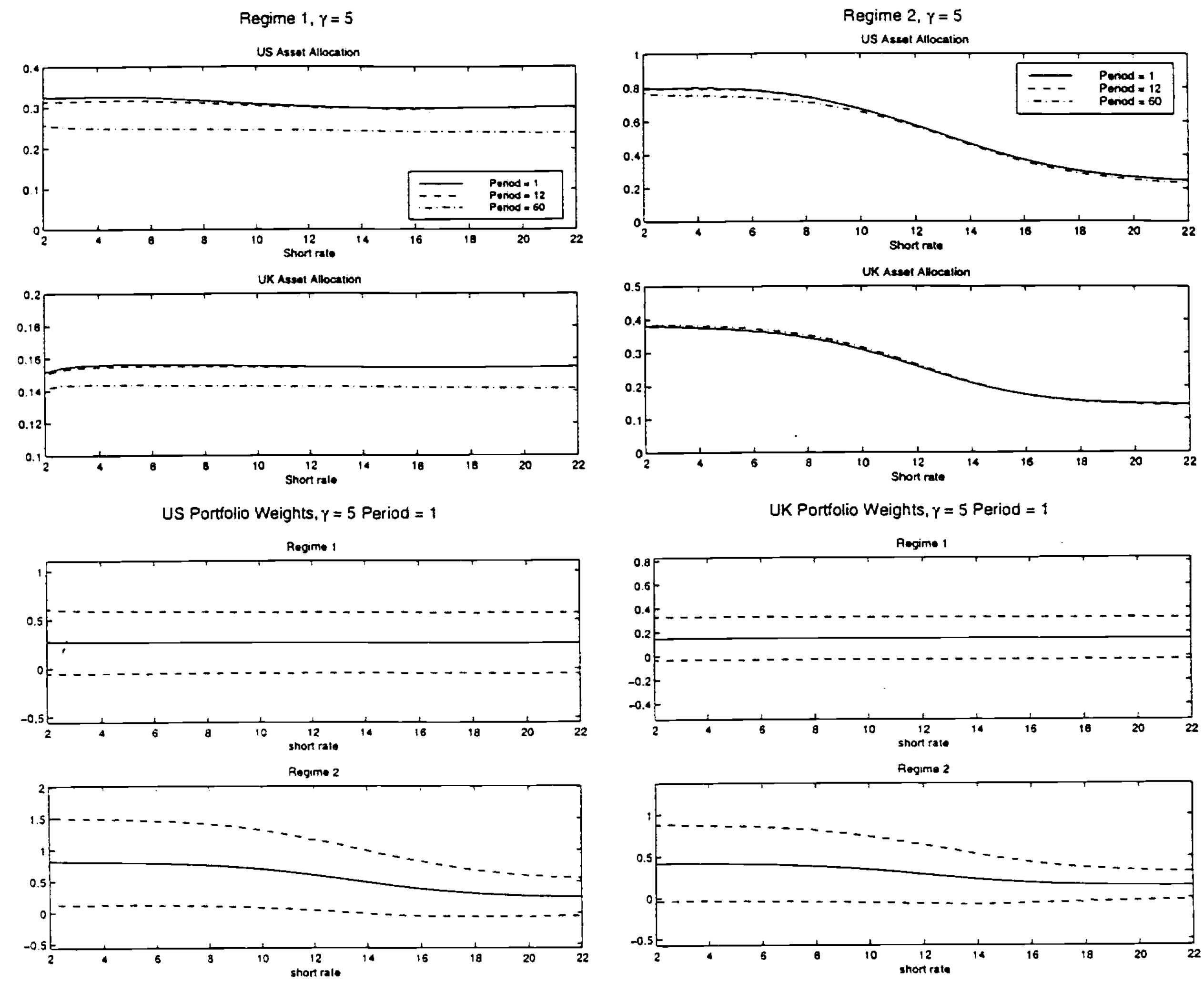
We fix the horizon at 12 months and plot out the portfolio weights as the risk aversion γ changes. The top panel gives the weights of the US in state 1 and state 2 for the Restricted $\mu_1 = \mu_2$ Simple US-UK RS Model. The bottom panel shows the Restricted $\mu_1 = \mu_2$ model for the Simple US-UK-GER RS Model.

Figure 4: Portfolio Weights when changing γ in All Equity Portfolio Models



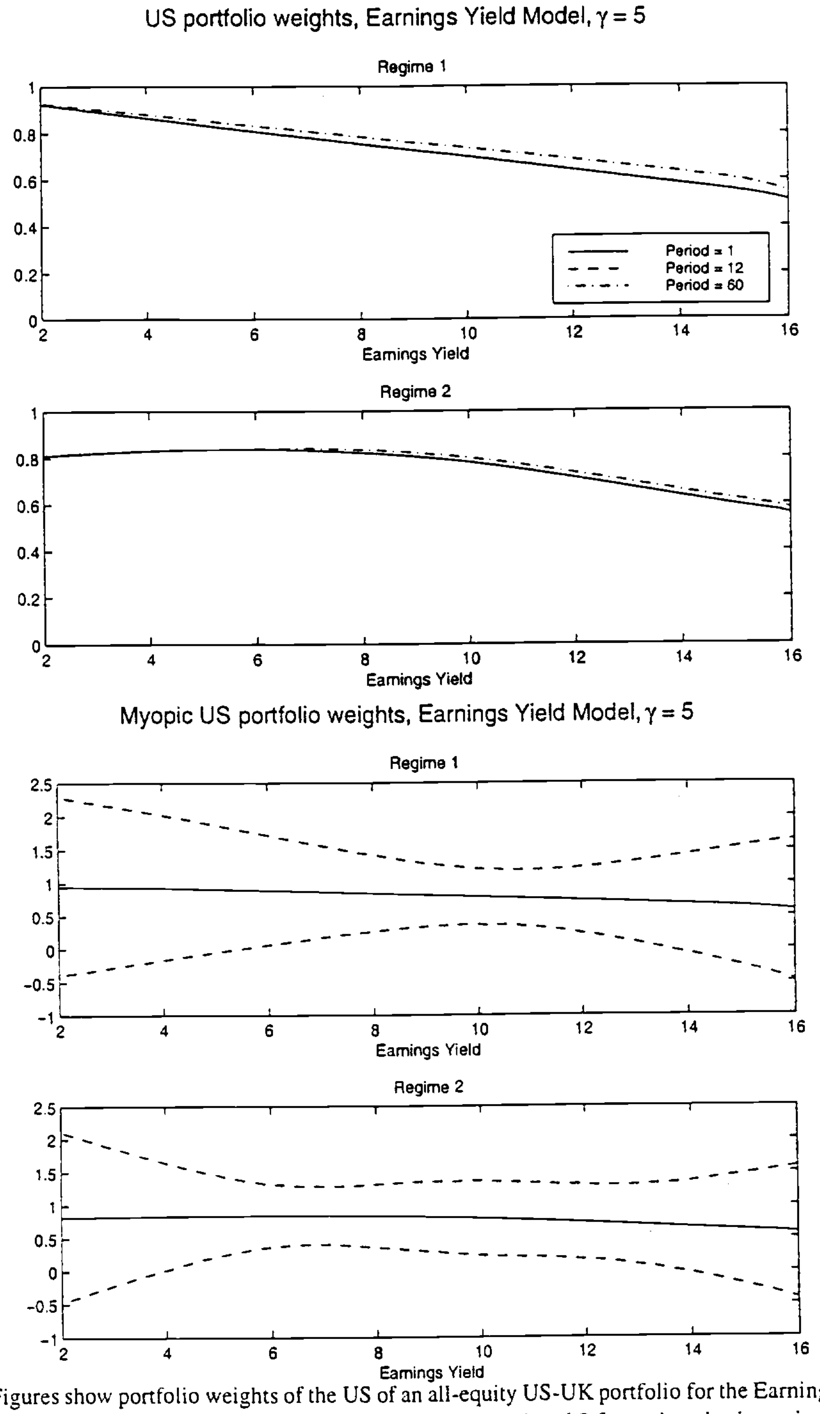
We fix the horizon at 12 months and plot out the "cents per dollar" compensation required for ignoring regime switching (holding Samuelson (1969) IID portfolio weights) and no international diversification (holding only the US) as the risk aversion γ changes. The top panel shows the Simple US-UK Model, and the bottom panel the Simple US-UK-GER Model. We restrict $\mu_1 = \mu_2$.

Figure 5: "Cents per dollar" compensation required as a function of γ



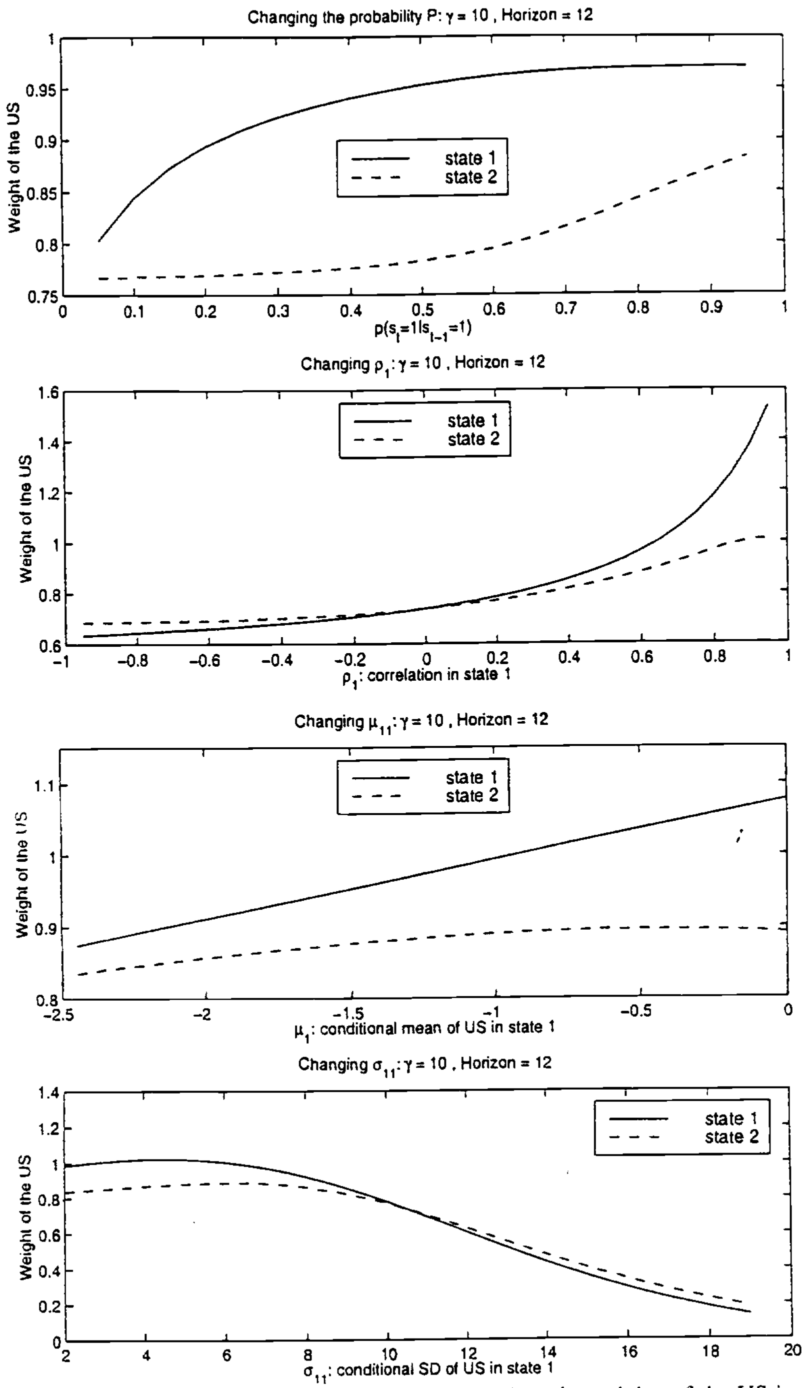
We fix $\gamma = 5$ and plot weights of the US and UK versus the short rate for various horizons in the top two plots on the first line. The bottom two plots show the 95% confidence intervals of the 1 period weights. From the top clockwise, we have: (1) the weights of the US and UK equity in state 1, (2) the weights of the US and UK equity in state 2, (3) UK portfolio weights for 1 month horizon with 95% standard error bounds, (4) US portfolio weights for 1 month horizon with 95% standard error bounds. Parameter estimates are from the Restricted $\mu_1 = \mu_2$ Basic Short Rate Model, which refers to equations (28)-(31), where we impose $\beta(s_t) = 0$ in equation (30).

Figure 6: Portfolio Weights Using the Basic Short Rate Model with $\mu_1 = \mu_2$.



The Figures show portfolio weights of the US of an all-equity US-UK portfolio for the Earnings Yield Model. The top panel gives the weights of the US in regime 1 and 2 for various horizons in months. The bottom panel gives 95% confidence bounds (dashed lines) for myopic weights (solid lines).

Figure 7: US Portfolio Weight in the Earnings Yield Model



We fix $\gamma = 10$ and horizon at 12 months. The top panel gives the weights of the US in regime 1 and 2 for changing $P = p(s_t = 1|s_{t-1} = 1)$ and ρ_1, the correlation between the US and UK in regime 1. In the bottom panel, the conditional mean and standard deviation of the US (μ_{11} and σ_{11} respectively) are altered. All other parameters are held fixed at the estimated values for the Simple US-UK Model with unrestricted μ.

Figure 8: Weight of US when changing parameters in the Simple US-UK Model

Table Appendix

These Tables constitute the supporting Table Appendix mentioned in the main text.

Table A-1: US-UK Equity Models

		Basic Model		Restricted $\mu_1 = \mu_2$		Model II		RS ARCH		
		estimate	std error	estimate	std error	estimate	std error		estimate	std error
	P	0.8552	0.0691	0.8546	0.0698	0.8556	0.0690	P	0.8555	0.0702
	Q	0.9804	0.0108	0.9818	0.0100	0.9804	0.0107	Q	0.9808	0.0108
US	μ_1	-1.2881	1.1874	1.1613	0.2198	-1.2880	1.1902	μ_1^{us}	-0.6439	1.5659
	μ_2	1.2829	0.2287	$= \mu_1$		1.2828	0.8629	μ_2^{us}	1.3668	0.2353
	σ_1	7.0376	0.8629	7.5064	0.9515	7.0374	0.8629	μ_1^{uk}	-1.3287	2.5763
	σ_2	3.7689	0.1677	3.7917	0.1654	3.7691	0.1677	μ_2^{uk}	1.3341	0.3191
	α					1.0000	0.0023	$C_1[1,1]$	4.3372	1.5229
	β					1.0000	0.0005	$C_1[1,2]$	1.5160	2.9402
UK	μ_1	-0.6921	2.2627	1.2488	0.3090	-0.7253	2.2696	$C_1[2,2]$	11.3426	4.4303
	μ_2	1.3040	0.3141	$= \mu_1$		1.3043	0.3141	$C_2[1,1]$	3.6269	0.1712
	σ_1	13.7177	1.7558	14.0748	1.8432	13.7184	1.7560	$C_2[1,2]$	0.9876	0.1532
	σ_2	5.2194	0.2376	5.2470	0.2409	5.2197	0.2375	$C_2[2,2]$	5.0538	0.2560
	ρ_1	0.6097	0.1022	0.6181	0.1032	0.6096	0.1022	$B_1[1,1]$	-1.2763	0.7584
	ρ_2	0.4455	0.0496	0.4480	0.0491	0.4455	0.0496	$B_1[1,2]$	-1.5202	1.9194
								$B_1[2,1]$	0.3915	0.2907
								$B_1[2,2]$	0.7403	0.7284
								$B_2[1,1]$	0.0839	0.1773
								$B_2[1,2]$	0.2426	0.2565
								$B_2[2,1]$	0.0591	0.1601
								$B_2[2,2]$	-0.0658	0.1775
	RCM	24.7041		23.5889		24.6741			25.7015	
	log llk	-1992.31		-1994.46		-1992.30			-1990.46	

US, UK refer to monthly equity returns with the subscripts indicating which regime. $P = p(s_t = 1|s_{t-1} = 1)$, $Q = p(s_t = 2|s_{t-1} = 2)$. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values denote better regime classification. Log llk denotes the log likelihood value. The Basic Model is a simple bivariate RS model. The Restricted $\mu_1 = \mu_2$ model sets the conditional mean constant across regimes. Model II uses transition probabilities specified in equation (21). The RS ARCH model parameterizes the conditional volatility as in equation (22). The $A[i, j]$ notation refers to the element in row i, column j of matrix A.

Table A-2: US-UK-GER Equity Models

		Basic Model estimate	std error	Restricted $\mu_1 = \mu_2$ estimate	std error
	P	0.8305	0.0760	0.8375	0.0714
	Q	0.9444	0.0269	0.9503	0.0258
US	μ_1	-0.1751	0.7966	1.1467	0.2177
	μ_2	1.3546	0.2399	$= \mu_1$	
	σ_1	6.2463	0.6185	6.4124	0.6490
	σ_2	3.4655	0.1879	3.5086	0.1909
UK	μ_1	0.8124	1.3480	1.1412	0.3143
	μ_2	1.1492	0.3476	$= \mu_1$	
	σ_1	10.9400	1.1577	11.0689	1.1928
	σ_2	4.7864	0.2736	4.8285	0.2716
GER	μ_1	0.3473	1.2073	1.0863	0.3040
	μ_2	1.1667	0.3735	$= \mu_1$	
	σ_1	8.3056	0.7395	8.3744	0.7670
	σ_2	4.7819	0.3206	4.8250	0.3131
	$\rho_1(us, uk)$	0.5994	0.0751	0.5996	0.0778
	$\rho_2(us, uk)$	0.4056	0.0607	0.4024	0.2669
	$\rho_1(us, ger)$	0.4540	0.1009	0.4627	0.1050
	$\rho_2(us, ger)$	0.2620	0.0742	0.2669	0.0726
	$\rho_1(uk, ger)$	0.4523	0.0917	0.4522	0.0940
	$\rho_2(uk, ger)$	0.4261	0.0622	0.4285	0.0609
	RCM	52.9089		48.3632	
	log llk	-3011.36		-3013.52	

The Basic Model is a RS simple trivariate model of US, UK, GER monthly equity returns with the subscripts indicating which regime. $P = p(s_t = 1|s_{t-1} = 1)$, $Q = p(s_t = 2|s_{t-1} = 2)$. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values denote better regime classification. Log llk denotes the log likelihood value. The Restricted $\mu_1 = \mu_2$ model imposes the same conditional means across regimes.

Table A-3: US-UK Beta Model

		Basic Model		Restricted $\mu_1 = \mu_2$	
		estimate	std error	estimate	std error
	P	0.6672	0.1223	0.7243	0.1140
	Q	0.9055	0.0530	0.9457	0.0275
world	μ_1	-0.8055	0.8480	0.5139	0.2381
	μ_2	0.8144	0.2822	$= \mu_1$	
	σ_1	5.1683	0.6991	4.8137	0.7146
	σ_2	3.4703	0.2180	3.7742	0.1838
US	β_1	0.8584	0.1008	1.1016	0.0878
	β_2	0.8407	0.0499	0.7791	0.0444
	σ_1	3.5021	0.4348	2.3988	0.3798
	σ_2	2.2210	0.1605	2.5054	0.1244
UK uh	β_1	1.2702	0.2034	1.5525	0.2870
	β_2	0.9891	0.0756	0.9681	0.0636
	σ_1	7.5668	1.0552	8.2472	1.1312
	σ_2	3.3013	0.3096	3.4319	0.2431
UK h	β_1	1.1436	0.2022	1.4952	0.2684
	β_2	0.6630	0.0664	0.6622	0.0581
	σ_1	6.9136	0.9075	7.3216	0.9934
	σ_2	3.1240	0.2668	3.2288	0.1919
	RCM	56.0276		39.9952	
	log lik	-2912.22		-2910.51	

The estimated model is $y_t^w = \mu^w(s_t) + \sigma^w(s_t)\epsilon_t^w$ where y_t^w is the world MSCI excess return. All returns are excess over the US 1 month EURO rate. Sample period 75:01 to 97:07. For asset j's excess return y_t^j, $y_t^j = \beta^j(s_t)\mu^w(s_t) + \beta^j(s_t)\sigma^w(s_t)\epsilon_t^w + \sigma^j(s_t)\epsilon_t^j$ with the errors $\epsilon_t = (\epsilon_t^w\, \epsilon_t^1 \ldots \epsilon_t^3)' \sim$ IID N(0,I). The Restricted Model sets $\mu_1^w = \mu_2^w$. $P = p(s_t = 1|s_{t-1} = 1)$, $Q = p(s_t = 2|s_{t-1} = 2)$. US refers to returns on US equity, UK unhedged (UK uh) returns are in USD, UK hedged returns (UK h) refer to UK returns in pounds plus the US 1 month EURO rate less the UK 1 month EURO rate. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values denote better regime classification. Log llk denotes the log likelihood value.

Table A-4: US-UK-GER Beta Model

		Basic Model		Restricted $\mu_1 = \mu_2$	
		estimate	std error	estimate	std error
	P	0.6722	0.0839	0.6758	0.0849
	Q	0.8730	0.0381	0.8723	0.0389
world	μ_1	-0.1777	0.6762	0.5654	0.2271
	μ_2	0.7078	0.2546	$= \mu_1$	
	σ_1	5.2087	0.4892	5.2459	0.4982
	σ_2	3.3378	0.1833	3.3389	0.1893
US	β_1	0.8787	0.0828	0.8821	0.0829
	β_2	0.8205	0.0497	0.8168	0.0510
	σ_1	3.4525	0.3264	3.4475	0.3243
	σ_2	2.1157	0.1352	2.1094	0.1433
UK uh	β_1	1.1877	0.1674	1.1833	0.1671
	β_2	1.0099	0.0796	1.0116	0.0815
	σ_1	7.0768	0.6444	7.0599	0.6437
	σ_2	3.1873	0.2394	3.1756	0.2386
UK h	β_1	1.0454	0.1513	1.0417	0.1508
	β_2	0.6623	0.0728	0.6614	0.0741
	σ_1	6.4140	0.5828	6.3807	0.5796
	σ_2	3.0988	0.2065	3.1048	0.2048
GER uh	β_1	1.0079	0.1700	0.9873	0.1714
	β_2	0.6954	0.0926	0.7100	0.0970
	σ_1	6.9992	0.6439	7.0009	0.6386
	σ_2	3.7240	0.2285	3.7163	0.2341
GER h	β_1	0.8907	0.1603	0.8795	0.1611
	β_2	0.4150	0.0792	0.4194	0.0814
	σ_1	6.7035	0.6325	6.7029	0.6297
	σ_2	3.3704	0.1943	3.3586	0.1957
	RCM	64.0151		64.3764	
	log lik	-4486.74		-4487.42	

The estimated model is $y_t^w = \mu^w(s_t) + \sigma^w(s_t)\epsilon_t^w$ where y_t^w is the world MSCI excess return. All returns are excess over the US 1 month EURO rate. Sample period 75:01 to 97:07. For asset j's excess return y_t^j, $y_t^j = \beta^j(s_t)\mu^w(s_t) + \beta^j(s_t)\sigma^w(s_t)\epsilon_t^w + \sigma^j(s_t)\epsilon_t^j$ with the errors $\epsilon_t = (\epsilon_t^w\, \epsilon_t^1 \ldots \epsilon_t^3)' \sim$ IID N(0,I). The Restricted Model sets $\mu_1^w = \mu_2^w$. $P = p(s_t = 1|s_{t-1} = 1)$, $Q = p(s_t = 2|s_{t-1} = 2)$. US refers to returns on US equity, UK unhedged (UK uh) returns are in USD, UK hedged returns (UK h) refer to UK returns in pounds plus the US 1 month EURO rate less the UK 1 month EURO rate, GER unhedged (GER uh) returns are in USD, and GER hedged (GER h) returns are German returns in DM plus the US 1 month EURO less less the GER 1 month EURO rate. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1|\mathcal{I}_{t-1})$. Lower RCM values denote better regime classification. Log llk denotes the log likelihood value.

Table A-5: Short Rate and US, UK Equity Model

Basic Model

	$s_t = 1$ Param	$s_t = 1$ Std Err	$s_t = 2$ Param	$s_t = 2$ Std Err
	Regime Prob Coefficients			
a	1.4239	1.6568	6.8465	1.7330
b	0.4033	1.8480	-5.0766	1.8696
	Short Rate Coefficients			
c	0.0743	0.0417	0.0047	0.0051
ρ	0.9158	0.0477	0.9939	0.0094
v	0.1294	0.0123	0.0362	0.0021
	US Equity Coefficients			
μ	-1.0583	0.8050	0.7260	0.2365
σ	6.3966	0.6038	3.5677	0.1683
λ_1	-0.3409	0.1091	-0.1897	0.0642
γ	0.5958	0.0813	0.4371	0.0539
	UK Equity Coefficients			
μ	-1.5589	1.3493	0.9010	0.3567
σ	10.7179	1.0082	5.4275	0.2667
λ_2	-0.2756	0.1146	-0.0282	0.0660
RCM	29.9194			
log lik	-1283.38			

Restricted $\mu_1 = \mu_2$ Model

	$s_t = 1$ Param	$s_t = 1$ Std Err	$s_t = 2$ Param	$s_t = 2$ Std Err
	Regime Prob Coefficients			
a	1.4921	1.6075	6.8574	1.6632
b	0.2793	1.7792	-5.0521	1.7962
	Short Rate Coefficients			
c	0.0702	0.0428	0.0050	0.0050
ρ	0.9078	0.0492	0.9937	0.0093
v	0.1314	0.0126	0.0364	0.0020
	US Equity Coefficients			
μ	0.5860	0.2228	$= \mu_1$	
σ	6.6597	0.6294	3.5753	0.1673
λ_1	-0.3537	0.1122	-0.1887	0.0640
γ	0.6189	0.0788	0.4371	0.0535
	UK Equity Coefficients			
μ	0.7452	0.3410	$= \mu_1$	
σ	11.0749	1.0441	5.4307	0.2650
λ_2	-0.2891	0.1171	-0.0296	0.0653
RCM	29.8647			
log lik	-1285.71			

Full Model

	$s_t = 1$ Param	$s_t = 1$ Std Err	$s_t = 2$ Param	$s_t = 2$ Std Err
	Regime Prob Coefficients			
a	1.4563	1.7245	6.7552	1.8248
b	0.3314	1.9305	-4.9886	1.9463
	Short Rate Coefficients			
c	0.0653	0.0447	0.0043	0.0051
ρ	0.9260	0.0512	0.9945	0.0095
v	0.1294	0.0125	0.0362	0.0021
	US Equity Coefficients			
μ	-0.0868	2.6799	0.9962	0.7363
σ	6.3939	0.6078	3.5666	0.1688
β	-1.0483	2.7469	-0.4695	1.2313
λ_1	-0.3426	0.1090	-0.1873	0.0646
γ	0.5939	0.0822	0.4394	0.0544
	UK Equity Coefficients			
μ	1.5428	4.4677	0.5907	1.1063
σ	10.7204	1.0113	5.4065	0.2845
β	-3.3483	4.5724	0.5125	1.8243
λ_2	-0.2790	0.1143	-0.0280	0.0657
RCM	30.6103			
log lik	-1282.90			

The short rate process is given by $r_t = c_i + \rho_i r_{t-1} + v_i \sqrt{r_{t-1}} u_t^r$ where the subscript indicates $s_t = i$. The US and UK excess equity returns are given by $y_t^j = \mu_i^j + \beta_i r_{t-1} \sigma_i^j u_t^j$ for $j = us, uk$. The errors $u_t = (u_t^r \, u_t^{us} \, u_t^{uk})'$ are IID N(0,1). For the Basic Model $\beta(s_t) = 0$ so there is no conditional mean predictability. The Restricted Model sets μ to be constant across regimes for each country. The parameter λ_j refers to the correlation between the short rate and equity for country j, and γ the correlation between US and UK stock returns. Both λ_j and γ are state-dependent. The state transition probabilities for $s_t = 1, 2$ are given by $p(s_t = i | s_{t-1} = i) = \exp(a_i + b_i r_{t-1})/(1 + \exp(a_i + b_i r_{t-1}))$. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T} \sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1 | \mathcal{I}_{t-1})$. Lower RCM values denote better regime classification.

Table A-6: Earnings Yield Model

		estimate	std error
Probability Coefficients	a_1	-0.1546	1.8474
	b_1	0.1584	0.1908
	a_2	5.9361	1.4186
	b_2	-0.4522	0.1705
Earnings Yield	c_1	0.9982	0.3187
	c_2	0.1720	0.0830
	ρ_1	0.9176	0.0298
	ρ_2	0.9648	0.0127
	v_1	0.6619	0.0531
	v_2	0.2892	0.0165
US Equity	μ_1	-10.4981	2.8295
	μ_2	0.2120	0.8452
	σ_1	5.2634	0.4208
	σ_2	3.3338	0.1687
	β_1	0.9579	0.2574
	β_2	0.2140	0.1245
UK Equity	μ_1	-11.2389	4.6477
	μ_2	0.1276	1.2157
	σ_1	9.5265	0.7619
	σ_2	4.8899	0.2533
	β_1	1.0763	0.4298
	β_2	0.2096	0.1783
Correlations	$\rho_1(ey, us)$	0.0752	0.1038
	$\rho_2(ey, us)$	0.1498	0.0744
	$\rho_1(ey, uk)$	-0.0369	0.1016
	$\rho_2(ey, uk)$	0.0063	0.0869
	$\rho_1(us, uk)$	0.5060	0.0775
	$\rho_2(us, uk)$	0.4273	0.0586

RCM	44.3850
log llk	-2146.83

The earnings yield process is given by $ey_t = c_i + \rho_i ey_{t-1} + v_i u_t^e$ where the subscript indicates $s_t = i$. The US and UK nominal equity returns are given by $y_t^j = \mu_i^j + \beta_i ey_{t-1} + \sigma_i^j u_t^j$ for $j = us, uk$. The errors $u_t = (u_t^r \, u_t^{us} \, u_t^{uk})'$ are IID N(0,Ω) where Ω is the correlation matrix of u_t. The state transition probabilities for $s_t = 1, 2$ are given by $p(s_t = i | s_{t-1} = i) = \exp(a_i + b_i r_{t-1})/(1 + \exp(a_i + b_i r_{t-1}))$. RCM refers to the Ang-Bekaert (1998) regime classification measure $RCM = 400 * \frac{1}{T} \sum_{t=1}^{T} p_t(1 - p_t)$, where p_t is the ex-ante regime probability $p(s_t = 1 | \mathcal{I}_{t-1})$. Lower RCM values denote better regime classification.