NBER WORKING PAPER SERIES

DURABLE GOODS CYCLES

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Working Paper 6987
http://www.nber.org/papers/w6987

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 1999

We are grateful to Chris Foote, David Laibson, Grew Mankiw, Torsten Persson, and Christopher Sims for helpful comments and discussions. We gratefully acknowledge support from the NSF. Leahy thanks the Sloan Foundation for support. The views expressed in this paper are those of the authors and do not reflect those of the National Bureau of Economic Research.

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JEL No. E21, E32

ABSTRACT

We show that a straight forward approximation of the distribution of durable goods holdings gives rise to a tractable equilibrium (S,s) model of durable demand. We analyze both competitive and monopoly supply. We show that equilibrium interactions lead to elongated impulse responses in demand, to procyclical markups in response to demand shocks, and to countercyclical markups in response to cost shocks.

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1 Introduction

Consumers generally purchase durable goods, such as automobiles and furniture, infrequently. Fluctuations in aggregate demand therefore reflect both fluctuations in the number of agents making purchases, and fluctuations in the demand of any single agent. It is very likely that the ability to time purchases contributes to the volatility and cyclicality of these pruchases. It is therefore important to develop models that incorporate this feature of demand.

The classical microeconomic model of discrete and infrequent adjustment is the (S,s) model of Arrow, Harris and Marschak (1951).[1] The problem with (S,s) behavior is that it is notoriously difficult to aggregate. The difficulty is that the cross-sectional distribution of agents' holdings becomes a state variable. This distribution determines how many agents are near their purchase triggers and therefore determines short-run demand. The high dimension of this distribution makes solving for equilibria extremely difficult. Current models either ignore equilibrium altogether, limit the analysis to special cases, or resort to computational methods.

The goal of this paper is to construct a workable model of the market for a durable good that incorporates infrequent purchases. Our approach is to simplify the equilibrium dynamics by abstracting from the echoes of previous cycles. High demand today creates a lump in the cross-sectional distribution at "big S." As the lump passes through the (S,s) bands it gets smoothed out by heterogeneity in depreciation, tastes, income and wealth. Incomplete smoothing produces an echo in demand as the lump reaches "little s." It is this echo that we assume away in what follows. The resulting model is very flexible, and we use it to analyze the equilibrium dynamics of price and the number and size of purchases in response to a variety of shocks and in a variety of market structures.

One important finding is that, if marginal costs are increasing or producers have monopoly power, then durables sales look more like the AR(1) found in the data than the ARIMA(0,1,1) predicted by Mankiw (1982). The reason is that endogenous price movements tend to spread fluctuations in demand over time. As a result there is a trade-off between the size and persistence of fluctuations. If marginal costs are flat, recessions will be sharp and deep. If marginal costs are rapidly increasing, they will be long and shallow.

A second important result is that markups are naturally procyclical in response to

[1] Grossman and Laroque (1990) apply the (S,s) model to the theory of an individual's consumption of a durable good, and Eberly (1994) and Attanasio (1995) test its empirical implications on panel data.

demand shocks and counter-cyclical in response to cost shocks. This is because the ability of consumers to time purchases places limits on producers' monopoly power. One implication is that costs shocks must be sufficiently large and procyclical, in order for markups to be counter-cyclical on average.

The next section places the paper in the context of the literature on (S,s) aggregation. We present the model in Section 3 and solve for the equilibrium in Section 4. In Section 5, we present a linearization of the model and analyze its properties. Section 6 extends the model to the case of a monopoly producer. Section 7 contains the theoretical and empirical justification for the main assumptions of the model. Section 8 concludes.

2 Literature Review

In recent years, a large body of research has developed to examine the relationship between microeconomic frictions and aggregate dynamics. Three general responses to the difficulty of placing (S,s) models in equilibrium settings have emerged.[2] First, many authors have made great progress in understanding the aggregate dynamics by ignoring equilibrium issues altogether. Work by Blinder (1981), Bertola and Caballero (1990), Caballero (1993), Eberly (1994), Adda and Cooper (1996) and Carroll and Dunn (1997) proceeds by assuming that shocks to an individual's desired consumption are exogenous, and that each individual agent follows an (S,s) policy consistent with the dynamic properties of these shocks and a fixed cost of purchasing the durable.[3] Aggregates result from summing over individual behavior given some assumption about the correlation of shocks across individuals. This literature focuses on the dynamics of the distribution of agents' holdings within the (S,s) bands, not on the equilibrium determination of the bands themselves.

The second branch of the (S,s) literature searches for settings which simplify the distributional dynamics, making it possible to include equilibrium considerations. Caplin and Spulber (1986), Benabou (1988, 1992), and Caplin and Leahy (1991, 1997) construct (S,s) models of pricing in which the cross-sectional distribution of relative prices is uniform. This

[2]There are several equilibrium models with representative agent consumers, most notably Blanchard and Melino (1986), Murphy, Shleifer and Vishny (1989), and Baxter (1992). The representative agent provides a microfoundation for the consumption of durable goods, but not one that accords with observed microeconomic behavior.

[3]In some of these models agents choose the (S,s) triggers optimally. In others the authors choose the policy in order to fit the data. Carroll and Dunn include a precautionary savings motive. Adda and Cooper solve for a competitive equilibrium but with constant marginal cost, so price is effectively exogenous. In both Carroll and Dunn and Adda and Cooper the policies may shift in response to exogenous shocks.

allows them to construct equilibrium models and, in the case of Caplin and Leahy (1987), a model with aggregate dynamics. In none of these models, however, does the timing or magnitude of the adjustment respond to the state of the cycle. The bands respond to changes in the parameters of the model, but they are constant through time.

The final branch of the literature makes assumptions that reduce the size of the state space, making possible computational solutions in the spirit of real business cycle models. Fisher and Hornstein (1996) solve an (S,s) inventory model by discretizing firms' inventory holdings. Dotsey, King and Wolman (1996) solve a model of pricing by considering a setting in which there are only a small number of different prices at any point in time. Following Krusell and Smith (1995) a number of papers consider various approximations of the distribution of holdings within the (S,s) bands. While these approaches are all promising, it is also useful for the purpose of developing our intuition to have simple analytic models of durable goods cycles.

The approach taken in this paper is a cross between the second and the third outlined above. As in the third branch, we show that our approximation yields dynamics that are a first order approximation of the actual dynamics. As in the second branch, we show that our assumption implies that the distribution of holdings in the neighborhood of "little s" is uniform. In our model, however, the (S,s) bands adjust in response to shocks. Our approach is also complementary to the first class of models. Where those papers fix the policies and focus on the distributional dynamics, we, in a sense, fix the distribution and focus on the equilibrium determination of the policies.

3 Modeling Durables

Our goal is to introduce equilibrium considerations into a model of infrequent adjustment. We focus on the direct effects of supply and demand on market equilibrium and make assumptions that limit the indirect effects of durable purchases through wealth and capital accumulation.

We begin with a general model of a market for a durable good and show how the cross-sectional distribution of holdings makes solving for an equilibrium difficult. We then solve for a steady state in which the cross-sectional distribution is constant over time. We use this steady state to motivate our main simplifying assumption: the absence of echo effects.

A General Model

Consider a competitive market for a durable good. Time is discrete and indexed by $t \geq 0$. There is a continuum of consumers indexed by i who are identical except that at any given date t they may possess different quantities of a durable good, $K_{it} > 0$. Let $\varphi_t(K)$ denote the cumulative distribution of agents' holdings of the durable good at date t. We take the initial distribution φ_0 to be exogenous to the model. In the absence of new purchases the durable depreciates geometrically at a rate Δ, so that $K_{it} = (1-\Delta)K_{i,t-1}$.[4]

Consumers must decide when to scrap their old durables and purchase a new ones. Scrapping is a type of fixed cost of adjustment. Agents will therefore tend to purchase durables infrequently, allowing each purchase to depreciate for some time before making a new one. Given that consumers are homogeneous, those with the lowest stocks will make purchases. They derive the greatest benefit from adjusting their holdings of the durable good.[5] There will be some cutoff value $s(\omega_t)$, which will depend on the state of the market ω_t, such that all agents with $K_{it} < s(\omega_t)$ will make purchases. We have chosen the notation $s(\omega_t)$ in reference to "little s" in the (S,s) model.

The number of agents that purchase the durable in period t, n_t, will be a function of $s(\omega_t)$ and the distribution φ_t:

$$n(\omega_t) = \varphi_t\left(s(\omega_t)\right).$$

Given their homogeneity, all consumers who make purchases will purchase the same quantity of the durable good. We will denote this quantity $S(\omega_t)$ in reference to "big S" in the (S,s) model. In general S will also depend on the state of the market ω. The mass of durables purchased is therefore nS. The dynamics of the distribution of holdings follows immediately from the policies $S(\omega)$ and $s(\omega)$.

On the supply side we assume production is competitive. We normalize the price of non-durable goods to one and let p_t denote the relative price of the durable good at date t. Note p is the price of a unit of the durable; a durable of size S will cost pS. With competition, this price is equal to marginal cost $\phi(n_t, S_t, c_t) \equiv \pi(n_t)S_t^{\gamma}c_t$, where c is a cost

[4]At times it will be useful to introduce an idiosyncratic shock to depreciation so that the depreciation rate is $x\Delta$ where x is is an idiosyncratic shock with mean one. We will introduce this shock in a way that does not affect the aggregate dynamics. For this reason we suppress it here.

[5]With scrapping agents will only adjust from a smaller durable to a larger one so long as the marginal utility of durable consumption is positive. Hence scrapping implies a one-sided (S,s) policy. Work by Caballero (1993) and Eberly (1994) suggests that most of the dynamics in the durable goods markets that they study come from the behavior of agents who get rid of a durable to buy a larger durable, which suggests that the one-sided assumption is appropriate. Theoretically, the high depreciation rates of most durables also suggests that the one-sided assumption is appropriate.

shock. This production function allows a separate role for the number and size of durable production. We assume that $\pi'(n) \geq 0$ and $\gamma \geq 0$, so that industry-wide marginal costs are weakly increasing in both the number and the size of the durables produced. We will assume that producers do not hold inventories so that the number and size produced is simply the number and size demanded by consumers.

Consumers choose when and how much to purchase in order to maximize the present value of utility. Each consumer's utility in each period is additively separable between the consumption of the durable and the consumption of other goods. Let $U(K) = K^\alpha/\alpha$, denote the utility that a consumer derives from K units of the durable in any given period.[6] We assume that purchases of the durable are small relative to total lifetime consumption, so that the marginal utility of wealth is exogenous to the outcomes and decisions in this market.[7,8,9]

Let λ_t denote the marginal utility of wealth. Consumer i chooses when to purchase the durable and how much of the durable to purchase in order to maximize the present value of utility from durable services:

$$V(K_{it}, \omega_t) = \max_{\{T_j, S_{T_j}\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} U(K_s) - \sum_{j=1}^{\infty} \beta^{T_j} p(\omega_{T_j}) \lambda(\omega_{T_j}) S_{T_j},$$

where,

$$K_s = \begin{cases} K_t & \text{if } s = t \text{ and } T_1 > t; \\ S_{T_j} & \text{if } s = T_j; \\ (1-\Delta)K_{s-1} & \text{otherwise;} \end{cases}$$

[6]The constant elasticity utility function and the constant elasticity of marginal cost with respect to S are needed to construct a steady state with a constant growth rate.

[7]With perfect capital markets there is a sense in which all durables, except perhaps housing, are small relative to lifetime earnings. This is especially the case here since the decision is not whether or not to purchase, but when to purchase the durable. This would not be a good assumption in the presense of liquidity constraints.

[8]We need the assumption because the staggering of durables purchases will tend to introduce heterogeneity in agents' wealth even if their labor income is identical. This introduces a second level of aggregation issues, which we avoid by making the market small relative to the economy as a whole. It is not clear that optimal insurance contracts would solve this problem. Given the discrete nature of the durable decision, an optimal insurance contract may not equalize the marginal utility of wealth.

[9]We could partially endogenize the marginal utility of wealth by making it a function of total sales or individual expenditure, reflecting the effect of the durables on individual's budgets or of the market on the aggregate economy. This would not alter the dynamics of the model, but it would alter the interpretation of some of the reduced form parameters.

E_t is the mathematical expectation conditional on date t information. The first summation represents the utility that the agent receives from the durable. β is the consumer's discount rate. The T_j represent the successive dates at which the consumer purchases a new durable. On these dates the consumer purchases S_{T_j} units of the durable good at a unit price of p_{T_j}. λ_{T_j} is the marginal utility of wealth and translates the purchase price into utility terms.

Rather than specifying the rest of the economy in detail, we will specify the dynamics of c_t and λ_t directly. In this way we can think of placing this market into a variety of macroeconomic settings by altering the dynamics of cost and the marginal utility of wealth. We want to allow c and λ to be non-stationary in order to capture the effects of technological progress and of innovations in permanent income. To analyze cycles, we use Beveridge-Nelson decompositions to detrend variables. In what follows we will use bars over variables to denote trend or steady state values and hats to denote deviations from trend. We assume $c_t = \hat{c}_t\bar{c}_t$ and $\lambda_t = \hat{\lambda}_t\bar{\lambda}_t$, where trends $\bar{c}$ and $\bar{\lambda}$ follow random walks with drift in logs, $\bar{c}_{t+1} = \rho_c\eta_{ct}\bar{c}_t$ and $\bar{\lambda}_{t+1} = \rho_\lambda\eta_{\lambda t}\bar{\lambda}_t$. Here η_c and η_λ are a random shocks, that are independently and identically distributed across time, positive and mean one. Drift ρ_c may reflect technical progress or increasing labor costs, while ρ_λ reflects discounting and the steady-state interest rate. The cycles $\hat{c}$ and $\hat{\lambda}_t$ follow an AR(1).[10,11] Note that innovations in these processes may be correlated with each other or with innovations in trend.

The state of the market includes the cyclical and trend components of marginal utility of wealth λ and the cost shock c, as well as the distribution of agents' holdings of the durable good φ: $\omega_t = (\varphi_t, \hat{\lambda}_t, \bar{\lambda}_t, \hat{c}_t, \bar{c}_t)$. Equilibrium in this market is a fixed point in p. Given their expectations of the price process, agents choose decision rules $s(\omega_t)$ and $S(\omega_t)$ to maximize expected utility. These decision rules in conjunction with the distribution of holdings give rise an actual price process. In equilibrium the expected price process must correspond with the actual price process.

While it is easy to define an equilibrium, it is more difficult to solve for one. The primary reason is that the distribution of holdings φ appears as a state variable. It matters how many people have new durables and how many have old ones, since those with old durables are more likely to purchase new ones. Changes in the distribution of holdings therefore lead

[10]It is trivial to extend the analysis to a finite-order ARMA process.

[11]It is useful to think of the evolution of the marginal utility of wealth in terms of the Euler equation for non-durables:

$$\lambda_t = \beta R_t E_t \lambda_{t+1}.$$

Here R is the gross real interest rate in terms of non-durables. Our assumptions on λ amount to assuming that $R_t = \hat{R}_t\bar{R}$, where $\hat{R}$ follows a stationary Markov process with mean one, and $\rho_\lambda = 1/\beta\bar{R}$.

to changes in demand which alter the price process. Since this distribution is potentially of large dimension, it complicates both analytic and computational solutions to the model.

Our aim is to reduce the number of state variables in order to arrive at a more tractable representation. Before presenting our assumptions, however, it is useful to consider how much easier it is to solve for equilibrium in steady state.

Steady State

In steady state there are no shocks to trend and all cyclical variables take values of one. State variables may, however, still evolve deterministically according to their rate of drift. We take $\bar{\lambda}_0$ and $\bar{c}_0$ as given.

Suppose that in steady state $\bar{\lambda}_t \bar{p}_t = b \bar{\lambda}_{t-1} \bar{p}_{t-1}$. We require that $\beta b^{\frac{\alpha}{\alpha-1}} < 1$, in order that the present value of utility is finite. In this case, Proposition 1 solves for the optimal time between purchases and the optimal purchase size. Together these define the optimal purchase trigger as well. All proofs are contained in the appendix.

Proposition 1: The optimal purchase size in period t is:

$$\bar{S}_t = \left(\frac{(1-\beta(1-\Delta)^{\alpha})\bar{\lambda}_t \bar{p}_t}{1-\beta^T (1-\Delta)^{\alpha T}} \right)^{\frac{1}{\alpha-1}} . \tag{1}$$

where T is the optimal time between purchases. Ignoring integer constraints, T is defined implicitly by:

$$\frac{(1-\alpha)\ln\beta - \alpha \ln z}{\ln\beta + \alpha\ln(1-\Delta)} = \frac{\beta^T (1-\Delta)^{\alpha T}}{1-\beta^T(1-\Delta)^{\alpha T}} \frac{1-\beta^T T b^{\frac{\alpha T}{\alpha-1}}}{\beta^T b^{\frac{\alpha T}{\alpha-1}}} . \tag{2}$$

Equation (2) implies that agents purchase the durable at fixed intervals of time in steady state. This is consistent with a constant fraction of agents making purchases each period. We will need an assumption on the distribution of initial holdings in order to generate constant purchases. Normalizing the total number of agents in the economy to unity, then $n_t = \bar{n} = 1/T$. We assume that the distribution φ_0 places weight $1/T$ on each of the points $\bar{S}(1-\Delta)^i b^i$, $i = 1, \ldots, T$.

It remains to solve for p and b. Given that price is equal to marginal cost:

$$\bar{p}_t = \pi(\bar{n}) \bar{S}_t^{\gamma} \bar{c}_t = (\pi(\bar{n})\bar{c}_t)^{\frac{\alpha-1}{\alpha-1-\gamma}} \left(\frac{(1-\beta(1-\Delta)^{\alpha})\bar{\lambda}_t}{1-\beta^T(1-\Delta)^{\alpha T}} \right)^{\frac{\gamma}{\alpha-1-\gamma}} \tag{3}$$

By definition:

$$b = \frac{\bar{\lambda}_t \bar{p}_t}{\bar{\lambda}_{t-1}\bar{p}_{t-1}} = \left(\frac{\rho_c}{\rho_\lambda}\right)^{\frac{\alpha-1}{\alpha-1-\gamma}} \tag{4}$$

Equations (1) through (4) pin down the model's steady state. In order to gain further insight into the nature of the steady state it is useful to detrend variables. Let $\bar{V}(K, \bar{\omega}_t)$ denote the value of an optimal policy in the steady state version of the model, where the trend state $\bar{\omega}_t = (\bar{\lambda}_t, \bar{c}_t)$. It is easy to show that $\bar{V}$ is homogeneous of degree α in K and $(c\lambda)^{\frac{1}{1-\alpha-\gamma}}$. This allows us to make the problem stationary by dividing through by $\bar{S}^\alpha$. Let $v(K/\bar{S}_t) = \bar{V}(K, \bar{\omega}_t)/S_t^\alpha$ and define $\hat{K} = K/\bar{S}_t$ to be the detrended component of an agent's stock of the durable.[12] The optimal purchase policy is stationary in $\hat{K}$. Each time agents adjust in steady state, they adjust to the detrended target $\hat{S} = 1$ which is independent of time. Together with the fact that adjustment occurs every T periods, we can also deduce the time-independent adjustment trigger:

$$\hat{s} \equiv s/\bar{S} = (1-\Delta)^T b^{\frac{T}{1-\alpha}}.$$

The first term on the right-hand side reflects depreciation over T periods. The second term reflects trend growth in $\bar{S}$ over this same interval of time.

Let $\hat{\varphi}$ denote the cross-sectional distribution of agents holdings of the durable good normalized by $\bar{S}_t$. This distribution is constant in steady state. The support of $\hat{\varphi}$ is a fixed set of T points:

$$\left\{1, (1-\Delta)b^{\frac{1}{1-\alpha}}, (1-\Delta)^2 b^{\frac{2}{1-\alpha}}, \ldots (1-\Delta)^{T-1} b^{\frac{T-1}{1-\alpha}}\right\}.$$

There are $1/T$ agents occupying each of these positions. If the time between periods is small, so that T is large and Δ and b are near unity, then these positions are close together and this distribution is essentially log uniform. In this sense this steady state equilibrium in this model is like the steady-state equilibria in Caplin and Spulber (1986) and Benabou (1988). Agents rotate positions within a time-invariant cross-sectional distribution.

This completes the presentation of the steady state.

[12] Note that $\bar{p}\lambda\bar{S}/\bar{S}^\alpha$ is a constant so that v does not depend on $\bar{\omega}$.

4 A Simplified Setting

Our aim is to develop a tractable model of durable goods cycles in which the number of durables that agents' purchase and the size of the durable that agents' purchase respond to shocks to the economy. It is possible to construct a steady state equilibrium because it is easy to predict the future state of the economy in that case. The cross-sectional distribution does not change and the all variables drift at constant rates. As a consequence, price changes at a constant rate.

In what follows we make two approximations that greatly simplify the dynamics out of steady state. We argue that both appoximations become more accurate, the longer is the time between an individual's purchases.

No Echo Effects

One of the properties that simplifies the steady-state analysis is that the distribution of agents near the adjustment is always the same. This is not always the case with (S,s) models. If a large number of consumers were to purchase the durable at date t, then it is possible that there will be a large cohort of consumers returning to the market T years later. A shock that leads to abnormally high purchases today therefore may lead to fluctuations in the distribution of holdings near the adjustment trigger in the future. We refer to such fluctuations as echo effects, since they are echoes of previous disturbances to demand.

We see echo effects in many situations. Most prominently the tendency to have children at a certain age leads to echo effects in population dynamics. Baby boomlets tend to follow baby booms. The echoes, however, are usually smaller than the original disturbance. Idiosyncratic behavior smooths the echo. In demographic cycles heterogeneity in the age of child bearing causes the boomlet to be smaller than the boom.

We show in Section 7 that if the time between purchases is sufficiently long, heterogeneity will practically erase the echoes. Any lumps that may appear in the distribution of holdings of the durable good near the target "big S" will be smoothed out before reaching the purchase trigger. We therefore assume that echo effects are of secondary importance to durable goods cycles. In Section 7 we consider the validity of this assumption by searching for echo effects in the data.

Assumption 1 (No Echo Effects): Suppose that agents purchase a new durable in period t if their holding is less then s_t. Then the density of log holdings is uniform in logs with density μ in a neighborhood to the right of s_t. Moreover, this neighborhood is large enough that the number of purchases in period $t+1$, n_{t+1}, is equal to

$$n_t \cong \mu(\ln s_{t+1} - \ln s_t + \delta) \tag{5}$$

where $\delta = -\ln(1-\Delta)$.

While we are ruling out one class of distributional dynamics, it is important to note that another class of distributional dynamics remains, namely the dynamics associated with movements in the purchase trigger s. When s is below its trend potential sales are abnormally high since the increase in s will trigger sales. In this sense there is "pent up demand." When s is abnormally high sales will tend to be abnormally low in the near future and there is "exhausted demand." Most of the literature on (S,s) aggregation has focused on the first kind of distributional dynamics (changes in φ at given values of $\hat{K}$) and ignored the second (changes in s).[13]

The Unpredictability of the Future

When agents purchase the durable good they must look forward to the circumstances of their next purchase. In particular they must forecast the price process, which will depend on the future state of the market. If the time between purchases is sufficiently long, then the current state will provide no information concerning the future state of the economy, except for the information it contains about the trend. We assume that this is the case: the distribution of the cyclical variables will converge to the unconditional distribution within the time between purchases. Let $f_\tau(\hat{c}_{t+\tau}, \hat{\lambda}_{t+\tau}, \hat{s}_{t+\tau} | \hat{c}_t, \hat{\lambda}_t, \hat{s}_t)$ denote the distribution of $\hat{c}$, $\hat{\lambda}$, and $\hat{s}$, τ years hence conditional on their period-t values.

Assumption 2 (Unpredictability of the Future): $f_T(\hat{c}, \hat{\lambda}) \approx f_\infty(\hat{c}, \hat{\lambda})$.

Together Assumptions 1 and 2 make the value of holding a *new* durable independent of the current state of the cycle. We can decompose the value of a new durable into two terms. The first is the utility derived form the durable over the period that it is held. This is independent of the state of the cycle because of the separability of the utility function. The second is the value of an optimal policy at the date at which the agent replaces the durable. This is independent of the current state of the cycle only if the state of the economy at that date is independent of current state. Assumptions 1 and 2 ensure this independence.

[13] Carroll and Dunn (1997) analyze cyclical fluctuations in the purchase trigger, but ignore equilibrium. Parker (1997) allows for changes in s, but the driving process behind his dynamics is the evolution of φ.

It is important to note that these assumptions do not imply that the size choice is independent of the current state of the cycle. This choice will still depend on the current price and marginal utility. The assumptions only imply that the value of a given size choice is independent of the cycle. The benefits of a given choice are independent of the cycle, the costs are not.

Let $\tilde{V}(K,\bar{\omega})$ denote the value of a new durable of size K and let $\tilde{v}(\hat{K}) = \tilde{V}(K,\bar{\omega})/\bar{S}^{\alpha}$. Given Assumptions 1 and 2, $\tilde{V}$ is independent of the cycle. We will therefore take $\tilde{V}$ as given in the analysis that follows. The only theoretical properties that we need are that in the neighborhood of $K = \bar{S}$, $\tilde{V}_1 > 0$ and $\tilde{V}_{11} < 0$. These properties follow immediately from the first and second order conditions of optimal choice.

The Model

We now summarize the main components of the model. The data of the model are:

- the initial values of the exogenous variables, $\hat{c}_0$, $\bar{c}_0$, $\hat{\lambda}_0$, $\bar{\lambda}_0$, and the processes for $\hat{c}_t$, $\bar{c}_t$, $\hat{\lambda}_t$, $\bar{\lambda}_t$.
- An initial distribution for durable goods holdings which is characterized by a minimum holding s_{-1} and a density of log holdings equal to μ on $K > s_{-1}$.
- A flow utility $U(K)$ and a value function $\tilde{V}(K,\bar{\omega})$ which captures the after purchase of a durable of size K given the trend state $\bar{\omega}$.
- A cost curve $\phi(n, S, c)$.

5 Solution to the Model

Given that the distribution of durable goods holdings is uniform in logs near the purchase trigger, it is useful to express equilibrium in terms of log durable holdings. Moreover, in order to make the model stationary, we will work with log durable holdings relative to steady state holdings. Let $k_i = \ln K_i - \ln \bar{S}_t$ denote log holdings of agent i relative to the size of the steady state purchase. Then $U(K_i) = K_i^{\alpha}/\alpha = \bar{S}_t^{\alpha} e^{\alpha(k_i)}/\alpha \equiv \bar{S}_t^{\alpha} u(k_i)$. Geometric depreciation implies:

$$k_{it} = k_{i,t-1} - \delta - \ln \bar{S}_t + \ln \bar{S}_{t-1}.$$

Demand

Consumers make two decisions: when to purchase the durable and how much to purchase. We begin with the size decision. Given Assumption 1, consumers maximize: $\tilde{V}(S, \bar{\omega}_t) - p_t \lambda_t S$. Equivalently, in terms of the normalized value function, the consumer chooses $\hat{S}$ to maximize $\tilde{v}(\hat{S}) - \hat{p}\hat{S}$, where $\hat{p}$ is the cyclical component of price, $\hat{p} = p_t \lambda_t / \bar{S}_t^{\alpha-1}$ and $\hat{S}$ is the cyclical component of the size decision, $\hat{S} = S/\bar{S}$. For notational convenience we have used $\hat{p}$ to capture cyclical movements in both p_t and λ_t. Note $\hat{p}$ has mean $\left(1 - \beta^T (1-\Delta)^{\alpha T}\right) / (1 - \beta(1-\Delta)^{\alpha})$. Let $\hat{S}(\hat{p})$ denote the result of this optimization. Given $\tilde{v}'(\cdot) > 0$ and $\tilde{v}''(\cdot) < 0$, both of which will hold in the neighborhood of the optimal purchase, we have $\hat{S}'(\hat{p}) < 0$. It is convenient to define $f(\hat{p}) = \tilde{v}(\hat{S}(\hat{p})) - \hat{p}\hat{S}(\hat{p})$. f is the cyclical part of the net value of purchasing the durable. Note that $f'(\hat{p}) < 0$ and $f''(\hat{p}) > 0$.

We now turn to the decision of when to purchase the durable. The optimal policy is a cutoff rule. Consumers make purchases if $K_i < s_t$. Let $\kappa_t = \ln s_t - \ln \bar{S}_t$, then consumers make purchases if $k_{it} < \kappa_t$. The consumers with $k_i = \kappa_t$ are indifferent between purchasing the durable in periods t and $t+1$.[14] The payoff from purchasing in period t is $\tilde{V}(S, \bar{\omega}_t) - p_t \lambda_t S$. The gain to delay is that consumers receive the services of their current durable and purchase a new one in the next period. The indifference condition is therefore

$$\tilde{V}(S_t, \bar{\omega}_t) - p_t \lambda_t S_t = U(s_t) + \beta E_t \left\{ \tilde{V}(S_{t+1}, \bar{\omega}_{t+1}) - p_{t+1} \lambda_{t+1} S_{t+1} \right\}$$

Let $\bar{e}_t = \bar{S}_t / \bar{S}_{t-1}$. We can use the homogeneity of $\tilde{V}$ to express the indifference in terms of the cyclical variables as follows:

$$f(\hat{p}_t) = u(\kappa_t) + \beta E_t \left\{ \bar{e}_{t+1}^{\alpha} f(\hat{p}_{t+1}) \right\}. \tag{6}$$

The left-hand side of the equation captures the amount a purchase today differs from trend, and the right-hand side reflects how much the return to delay differs from trend. $\bar{e}_t$ corrects for shifts in trend between t and $t+1$.

Given the form of the optimal policy it is easy to calculate the quantity sold in each period. Given the absence of echo effects, the distribution of the k_i is uniform with a lower bound of κ_{t-1} at the end of period $t-1$. All of these k_i depreciate by δ upon entering

[14] To avoid notational complications, we will assume that someone makes a purchase in each period. It is trivial to extend the analysis to handle periods of zero purchase.

period t. In addition the shift in the trend causes a further reduction of $\ln \bar{e}_t$. The number of agents that make purchases in period t is therefore

$$n_t = \mu[\kappa_t - \kappa_{t-1} + \delta + \ln \bar{e}_t],$$

where μ is the density of the uniform distribution. Each agent purchases S_t.

Competitive Equilibrium

Given the behavior of purchasers, the price of the durable is:

$$p_t = \pi\,(n_t)\,(S_t)^{\gamma}\,c_t.$$

The cyclical component of price is therefore:

$$\hat{p}_t = \zeta\pi\,(n_t)\,\hat{S}(\hat{p}_t)^{\gamma}\hat{c}_t\hat{\lambda}_t \tag{7}$$

where $\zeta = \pi(\bar{n})(1 - \beta^T(1 - \Delta)^{\alpha T})/(1 - \beta(1 - \Delta)^{\alpha})$.

Equations (6) and (7) summarize the model. The state variables in the general model were $\omega_t = (\varphi_t, \hat{\lambda}_t, \bar{\lambda}_t, \hat{c}_t, \bar{c}_t)$. Assumption 1 has allowed us to replace φ_t with κ_{t-1}. The innovations in $\bar{\lambda}_t$ and $\bar{c}_t$ enter (6) and (7) through $\bar{e}_t$. Note that $\hat{c}_t$ and $\hat{\lambda}_t$ enter equations (6) and (7) together as a product. Define $\hat{e}_t = \hat{\lambda}_t\hat{c}_t$. The stationary formulation of the model represented in equations (6) and (7) has two shocks: an innovation in trend $\bar{e}_t$ and a stationary shock $\hat{e}_t$. Consumers care only about the combined effect of price and marginal utility. λ and c affect n and S as a product. Individually they determine how the much adjustment is reflected in prices.

Let $\hat{\omega}_t = (\kappa_{t-1}, \hat{e}_t, \bar{e}_t)$. An equilibrium is a pair of functions $\hat{p}$ and $g(\hat{\omega}_t)$ such that $\hat{p}_t = \hat{p}(\hat{\omega}_t)$ and $\kappa_t = g(\hat{\omega}_t)$ satisfy (6) and (7) for all t.

Equations (6) and (7) look very much like a supply curve and a demand curve. As in Figure 1, place $\hat{p}_t$ on the y-axis and κ_t on the x-axis, the supply curve (7) depends on the past choices κ_{t-1}, the cyclical shock $\hat{e}_t$, and the shock to trend $\bar{e}_t$. Given the state, the set of $\hat{p}_t$ for which this equation holds is increasing in κ_t. This curve shifts to the right with increases in κ_{t-1}, and reductions in $\hat{e}_t$ and $\bar{e}_t$. Demand (6) depends on future price $\hat{p}_{t+1}$. Given $\hat{p}_{t+1}$, the set of $\hat{p}_t$ for which this equation holds is decreasing in κ_t. An increase in $\hat{p}_{t+1}$ increases demand at any given price, so that the curve shifts to the right. The intersection of the two curves determines the functions $\hat{p}(\hat{\omega}_t)$ and $g(\hat{\omega}_t)$.

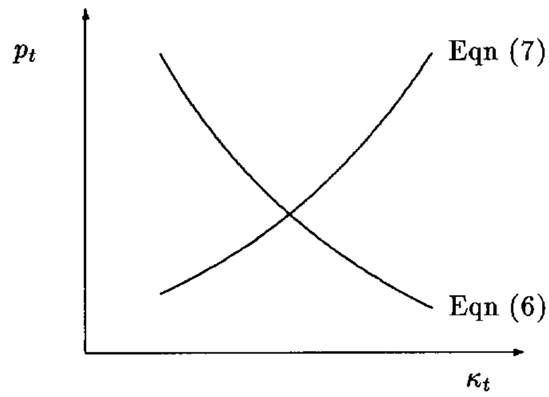

Figure 1:

The existence of an equilibrium follows from standard dynamic programming arguments.

Proposition 2: A competitive equilibrium exists.

6 A Linearization

A linear example will make the dynamics of the model clear. We linearize equations (6) and (7) around the steady state. We also assume log utility ($\alpha = 0$) so that,

$$u(k) = k.$$

Let $\tilde{v}(1) \equiv v$ denote the mean cyclical value function and let $\bar{p} = (1 - \beta^T(1-\Delta)^{\alpha T})/(1 - \beta(1-\Delta)^{\alpha})$ denote the mean of the cyclical component of price. A linearization of (6) yields,

$$v - \bar{p} - \hat{p}'(\bar{\psi})(\psi_t - \bar{\psi}) = \kappa_t + \beta E_t \left\{ (v - \bar{p} - \hat{p}'(\bar{\psi})(\psi_t - \bar{\psi})) \right\}. \tag{8}$$

where $\psi_t = (\kappa_t, \kappa_{t-1}, \ln \bar{e}_t, \hat{e}_t)$ and $\bar{\psi}$ represents the steady-state mean value of ψ. We can use (7) to calculate $\hat{p}'(\bar{\psi})(\psi_t - \bar{\psi})$. Substituting the result into (8) and keeping first order terms yields:

$$\begin{aligned} C_1 - C_2\left(\hat{e}_t - \beta E_t \hat{e}_{t+1}\right) & \quad -C_3 \ln \bar{e}_t \\ &= \quad -C_3 \kappa_{t-1} + \left[1 + C_3\left(1+\beta\right)\right]\kappa_t - \beta C_3 E_t \kappa_{t+1}, \end{aligned}$$

where the C_i are positive constants that depend on the parameters of the model.[15]

We assume that $\hat{e}_t$ follows a stationary autoregressive processes and that $\ln \bar{e}_t$ follows a white noise process.

$$\begin{aligned} \hat{e}_t &= \theta \hat{e}_{t-1} + (1-\theta) + \eta_t \\ \ln \bar{e}_t &= \epsilon_t . \end{aligned}$$

We look for a solution of the form:

$$\kappa_t = w + x\kappa_{t-1} + y\hat{e}_t + z \ln \bar{e}_t .$$

Matching coefficients:

$$\begin{aligned} w &= \frac{C_1 + C_2(1-\theta) + C_3\beta(y(1-\theta) + zE\bar{e})}{1 + C_3(1-\beta x)} \\ x &= \frac{1 + C_3(1+\beta)}{2\beta C_3} - \sqrt{\left(\frac{1 + C_3(1+\beta)}{2\beta C_3}\right)^2 - \frac{1}{\beta}} \in [0,1] \\ y &= \frac{-C_2(1-\beta\theta)}{1 + C_3[1 + \beta(1 - x - \theta)]} < 0 \\ z &= -x \end{aligned}$$

We can now analyze the dynamics of the model. The evolution of the cutoff is given by:

$$\kappa_t - \bar{\kappa} = x(\kappa_{t-1} - \bar{\kappa}) - x\epsilon_t + y\sum_{j=0}^{\infty} \theta^j \eta_{t-j} . \tag{9}$$

Hence the number of purchases is:

$$n_t - \bar{n} = \mu(1-x)\sum_{i=0}^{\infty} x^i \epsilon_{t-i} + \mu y \sum_{j=0}^{\infty} \frac{x^{j+1} - \theta^{j+1} - x^j + \theta^j}{x - \theta} \eta_{t-j} . \tag{10}$$

The dynamics of the model follow directly from equation (10) and the cost curve.

We focus on a number of special cases. Suppose first that (1) $\pi' = 0$, so that $p_t = c_t$ and price is insensitive to the number of consumers who purchase durables in any given period, and (2) all shocks are permanent: $\eta_t = 0$ and $\hat{e}_t = 1$. Then $x = 0$ and n fluctuates

[15] $C_1 = (1-\beta)(v - \bar{p})$; $C_2 = \zeta\bar{\pi}\hat{s}^\gamma / [1 - \zeta\gamma\bar{\pi}\hat{s}^{\gamma-1}\hat{s}']$; $C_3 = \mu\zeta\bar{\pi}'\hat{s}^\gamma / [1 - \zeta\gamma\bar{\pi}\hat{s}^{\gamma-1}\hat{s}']$.

randomly about its mean:

$$n_t = \bar{n} + \mu\epsilon_t$$

This case reproduces Mankiw's (1982) result. Mankiw finds that with fixed prices and interest rates and durables purchases should follow an ARIMA(0,1,1) in response to changes in permanent income in which the lagged MA term is $1 - \Delta$. Here the lagged MA term is one, but we are looking only at the number of agents who purchase the durable rather than the total value of the durables that agents purchase. If, as in Mankiw, the innovation ϵ reflects a shock to permanent income, then this shock leads to a permanent increase in S, so that durables purchases nS follow an ARIMA(0,1,1).

Now suppose that $\pi' > 0$, but that shocks are still permanent. In this case (10) shows that the number of purchases follows an AR(1) in response to a positive shock. So long as the price elasticity of the size choice is not unrealistically large the value of purchases rises with the shock and falls gradually back to its original level. This is essentially what Caballero (1990) found when he fit a high order MA process to durable consumption.

The intuition for this dynamic in the present case is straight forward. When a positive demand shock hits there would be a spike in purchases if price did not adjust. This spike, however, causes price to rise, which creates an incentive for agents to delay purchases in order to take advantage of lower prices in the future. This delay smooths out the response to the shock. The greater $\pi' > 0$, the greater is the price response and the greater is the period over which the shock is spread. Note that x governs both the impact of the shock and the rate of convergence; persistent responses are associated with muted impact effects.

It is also interesting to consider temporary shocks. If a shock is temporary, then the adjustment trigger κ must eventually return to the same long run position. Therefore, a positive innovation η, which leads to above normal purchases in the short run as κ rises, must also eventually lead to below normal purchases at some future date when κ falls. In some sense the increased demand is stolen from the future and a boom-bust cycle ensues. The position of the κ relative to its long run average captures such concepts as "pent up demand" or "exhausted demand."[16]

This analysis reinforces the point that ruling out echo effects does not eliminate all distributional dynamics. The dynamics associated with the cutoff remain. In this sense the model is a complementary to Caballero (1993). Caballero explains the slow adjustment of

[16]Chrysler includes a proxy for pent up demand in its forecasting equations (see Greenspan and Cohen [1998]).

durable consumption to shocks by the distributional dynamics within the (S,s) band. Here the focus is on the movements in the bands themselves.[17]

7 Durable Goods Monopoly

So far we have considered a competitive market. It is easy to extend the linearized model to the case of a monopolist who chooses price subject to a per period cost of production of $c_t + \xi q_t$ per unit. For simplicity we fix the size of purchases at $S = 1$ and assume that c and λ follow stationary AR(1)'s.

We look for a perfect Bayesian equilibrium to the game between the monopolist and the consumers.[18] There exists an equilibrium in which the consumer's cutoff and the monopolists pricing strategy take the following form:

$$\kappa_t = w + x\kappa_{t-1} + yc_t + z\lambda_t. \tag{11}$$

$$p_t = m + n\kappa_{t-1} + \theta c_t + \tau\lambda_t. \tag{12}$$

The monopolist maximizes:

$$J(k_t, c_t, \lambda_t) = \max_{p_t} (p_t - c_t - \bar{p}q_t)q_t + \beta E_t J(\kappa_{t+1}, c_{t+1}, \lambda_{t+1}).$$

The first order condition for this problem is:

$$q_t + (\xi p_t - c_t - 2\xi q_t)\frac{dq_t}{dp_t} + \beta E_t J_\kappa(\kappa_{t+1}, c_{t+1}, \lambda_{t+1})\frac{d\kappa_{t+1}}{dp_t} = 0. \tag{13}$$

Calculating the derivatives:[19]

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[17]This linearized version of the model shares many similarities with Parker (1996). Both models consider a distribution of consumers holding different amounts of a durable good and emphasize the timing of the decision to purchase durable goods. Both solve for the dynamics of a cutoff rule. Both treat the value of a new durable as exogenous. Parker, however, focuses on the consequences of fluctuations in the density of durable goods holdings. Such fluctuations are ruled out here by the assumption that there are no echo effects and that the density of holdings is log uniform. Parker's model is non-linear, so he solves a perfect foresight version using computational methods. The uniformity assumption will allow us to solve for the stochastic implications of a variety of shocks in closed form.

[18]This model is a stochastic version of Sobel and Takahashi (1983) with some elements of Kahn (1996).

[19]The only derivative that is not straightforward is dk_t/dp_t. This is an out of equilibrium thought experiment: how will the consumer react to a one-shot deviation in the monopolist's strategy. To make this calculation totally differentiate equation (8) under the assumption that the monopolist plays its equilibrium strategy (12) in period $t + 1$.

$$\frac{dq_t}{dp_t} = \mu \frac{d\kappa_t}{dp_t}; \qquad \frac{d\kappa_{t+1}}{dp_t} = x\frac{d\kappa_t}{dp_t}; \qquad \frac{d\kappa_t}{dp_t} = \frac{-\lambda_t}{1-\beta n E_t \lambda_{t+1}};$$

and

$$J\kappa(\kappa_{t+1}, c_{t+1}, \lambda_{t+1}) = -\mu\left(p_{t+1} - c_{t+1} - 2\xi q_{t+1}\right).$$

With these substitutions the first order condition (13) becomes:

$$\left(1-\beta n E_t \lambda_{t+1}\right) q_t = \mu\left[\left(p_t - c_t - 2q_t\right) - \beta x \lambda_t E_t\left(p_{t+1} - c_{t+1} - 2\xi q_{t+1}\right)\right]. \qquad (14)$$

This equation replaces the cost curve in the competitive model. We can solve for the optimal strategies, (11) and (12), by matching coefficients in (8) and (14).

This formulation allows us to study markup dynamics. With constant costs ($\xi = 0$), positive demand shocks (reductions in λ) increase the markup of price over marginal cost as the monopolist attempts to take advantage of the greater abundance of buyers. Positive cost shocks (increases in c) reduce markups. In both cases the monopolist would like to raise price by more but intertemporal substitution in consumers prevents this. The ability to time purchases smooths the price process. Note that monopoly power provides another explanation for the persistence in durable goods demand. In response to a positive demand shock, the monopolist raises its price. Consumers delay purchases, spreading demand over several periods.

Increasing costs ($\xi > 0$) make the response of the markup to shocks ambiguous, since demand and cost considerations mix. A positive demand shock increases sales. This causes costs to rise and leads the monopolist to raise the price of the durable. The scope for price increases, however, is limited by consumers' intertemporal substitution.

The model yields conditions under which the markup is countercyclical. If costs are sufficiently increasing or if cost shocks are of sufficient amplitude and positively correlated with demand shocks, then increases in demand will be associated with reductions in the markup.[20] Rotemberg and Woodford (1997) provide some indirect evidence for this mech-

[20]This mechanism is suggested by Carlton (1996). There are many other theories of countercyclical markups. Phelps and Winter (1970), Bils (1989), and Parker (1997) provide reasons why demand might be more elastic during booms. Rotemberg and Saloner (1986) and Rotemberg and Woodford (1991) argue that the incentive to deviate from collusive arrangements is greater during booms. Sobel (1990) shows that countercyclical markups may arise from periodic sales to the low valuation consumers. In Chevalier and Scharfstein (1996) credit market imperfections lead firms to increase margins when cash flow is low. The theory in this proposal is most closely related to the Keynesian view that countercyclical markups arise

anism. They argue that marginal costs increase by more than price during booms, so that real marginal cost is procyclical.

8 Discussion of Assumption 1

Theory

We now present a lemma that provides some justification for ruling out echo effects. The first lemma presents conditions under which heterogeneity in depreciation smooths the distribution of holdings so that it is eventually log uniform, ignoring, for the moment the fact that purchases truncate the distribution at "little s." The second lemma presents conditions under which this truncation preserves uniformity.

Lemma 1 Suppose that on average $\bar{n}$ agents make purchases each period, that the number of agents making purchases is bounded above, and that they purchase a durable that lies in $[\underline{S}, \bar{S}]$, where $S_1 > 0$ and $S_2 < \infty$. Suppose further that each period durables depreciate at a rate $x\Delta$, where x is a random shock that is independent across consumers and across time, mean one, and diffuse. In addition, we assume that $x \in [\underline{x}, 1/\Delta]$ where $\underline{x} > 0$. Then given $\varepsilon > 0$, there exists $s(\varphi) \in (0, \underline{S})$ such that the density of log holdings on $(0, s)$ is within ε of $-1/\ln(1-\Delta)$ according to the variation norm.

Lemma 1 says that given enough time heterogeneity in depreciation will smooth the density of holdings to the point that it is arbitrarily close to log uniform. The intuition behind Lemma 2 is simple. Consider water flowing through a pipe. The average flow at any point in the pipe must be the same as the average flow into the pipe. If there is some mixing of the distribution of water within the pipe and if the pipe is sufficiently long then eventually the flow will be fairly constant even if the inflow tends to fluctuate.[21]

Lemma 1 ignores the effect that truncating the distribution at s has on the shape of the distribution. If we truncate the density from Lemma 1 at s then it will be log uniform to the right of s. Now suppose that we have a distribution that is log uniform to the right

from price stickiness. Here it is intertemporal substitution by consumers rather than nominal rigidity which prevents the monopolist from adjusting prices.

[21] Lemma 1 is related to Theorem 1 in Caballero and Engel (1991). Caballero and Engel show that if agents follow one-sided (S,s) rules and there is a non-stationary idiosyncratic shock to their position between the bands, then the cross-sectional distribution converges to the uniform distribution. Lemma 1 states that this convergence can occur between adjustment if the adjustment period is long enough.

of s. After depreciation it will be log uniform in the neighborhood of s, but for some s' sufficiently less than s it will cease to be uniform since truncation has removed mass from these points. Lemma 2 presents a condition on the size of the idiosyncratic shocks and the change in the purchase trigger that ensures that such successive truncations preserve uniformity. Intuitively, the purchase trigger must fall by less then the uniform portion of the distribution, which is $\bar{x}\Delta$.

Lemma 2 Suppose that the distribution of log holdings in period t is uniform for $k > \ln s_t$. Let x be distributed with mean 1 on $[\underline{x}, \bar{x}]$ where $\underline{x} < 1 < \bar{x}$ and suppose that $\ln \bar{x} < \ln s_{t+1} - \ln s_t + \delta$. Then the distribution of log holdings in period $t+1$ is uniform for $k > \ln s_{t+1}$.

Together Lemmas 1 and 2 show when the uniformity approximation is a good one. It is accurate when there is enough time between purchases for idiosyncratic depreciation to smooth the distribution of holdings and when the purchase trigger does not fall so far as to cut the density of holdings at a region that is not uniform.

Some Suggestive Empirical Evidence

Whether or not Assumptions 1 and 2 are good approximations is an empirical matter. While a detailed empirical analysis is beyond the scope of this paper, we present some suggestive evidence in favor of our assumptions.

In order to evaluate the assumptions, we must first get a sense of the timing between purchases. Bils and Klenow (1995) present evidence on the expected life of new durables. They report that autos, televisions and refrigerators all have expected lives of over ten years. This data probably provide good estimates of the holding period for refrigerators and television sets. Autos, however, are frequently traded in secondary markets, so the holding period is likely to be shorter. Surveys of car buyers show that 90% plan to keep their new cars for at least three years, 76% for at least four years, and 61% for five years.[22] Attanasio (1995) estimates a holding period of four years from the CEX. This evidence suggests that in the case of autos we should be considering a holding period of four or five years.

Given these numbers Assumption 2 strikes us as a good approximation. There should be very little predictability in the state of the cycles four or more years in the future. For

[22]Source: *1990 Buyers of New Cars*, Newsweek Inc.

example, in Hamilton's (1989) switching model almost all of the predictive power of the current state disappears after two years.

Assumption 1 is a more complicated matter. We consider monthly data from the BLS on the number of autos sold, the average sales price and the price index for autos. Figure 2 shows the univariate impulse response of the number of autos sold. The regression is done on quarterly data from 1965 through 1990 and eight autoregressive lags. The impulse response rises initially, falls below zero at about two years and then rises above zero again at about five years. The area below zero is about 1/2 to 2/3 of the area above zero. This suggests the presence of both permanent and transitory elements; permanent shocks would lead to a response that was everywhere positive, whereas transitory shocks would lead to a response whose integral was zero. The size of the hump at five years is about 10% of the initial increase, suggesting that there may be a small echo.[23]

Although this evidence is supportive of Assumption 1, it is by no means conclusive. First, we do not know the process that these shocks follow; it may be that the boom-bust pattern or the echo is in the shock, not in the response of durables. Second, the echo is not isolated; it may be the case that the echo is in fact large and that the impulse without the echo would be well below zero.

9 Conclusions

We have constructed the simplest possible model of the dynamics of a market for a durable good that distinguishes between the number and size of purchases. We have used this model to address a number of issues including the time-series behavior of durables and the behavior of markups. We have shown that if prices rise with demand then purchases will exhibit a prolonged response to wealth shocks. We have also shown that markups are only procyclical if cost shocks are important or marginal cost rises significantly with demand.

[23]The shape of the response over the first five years was robust to alternative specifications of lag length. The echo, however, failed to apper for lag lengths less than eight.

10 Appendix

Proof of Proposition 1

Consider a consumer making a purchase in period zero. Given the constant elasticity utility function and the trend in prices and the marginal utility of wealth, the consumer's maximization problem may be written as:

$$\max_{\{S_j,T_j\}_{j=1}^{\infty}} \sum_{j=1}^{\infty} \beta^{\sum_{i=0}^{j} T_i} \sum_{k=0}^{T_j - T_{j-1} - 1} \frac{(S_j(1-\Delta)^k)^{\alpha}}{\alpha} - \lambda_0 p_0 b^{\sum_{i=0}^{j} T_i} S_j.$$

where S_j is the jth purchase, T_j is the time between the jth and the $(j+1)$st purchase, and $T_0 = 0$. Summing over k yields:

$$\max_{\{S_j,T_j\}_{j=1}^{\infty}} \sum_{j=1}^{\infty} \beta^{\sum_{i=0}^{j} T_i} \frac{S_j^{\alpha}}{\alpha} \left(\frac{1-\beta^n(1-\Delta)^{\alpha n}}{1-\beta(1-\Delta)} \right) - \lambda_0 p_0 b^{\sum_{i=0}^{j} T_i} S_j.$$

The first order condition for the jth purchase, S_j, is:

$$S_j^{\alpha-1} \left(\frac{1-\beta^{T_i}(1-\Delta)^{\alpha T_i}}{1-\beta(1-\Delta)} \right) = \lambda_0 p_0 b^{\sum_{i=0}^{j} T_i}. \tag{15}$$

Taking the first order condition with respect to an arbitrary T_k, substituting for all of the S_j's, and then setting all of the T_j's equal to T_k yields equation (2). The fact that the solution for T in equation (2) is a constant justifies setting the T's equal. With the T's equal (15) yields equation (1).

Proof of Proposition 3

Define $h(\hat{\omega}) = f(\hat{p}(\hat{\omega}))$. Since f is monotonically decreasing in $\hat{p}$, we can recover $\hat{p}$ from h. In terms of h equations (6) and (7) become:

$$\begin{aligned} h(\hat{\omega}_t) &= f\left(\zeta\pi\left(\mu\left[g(\hat{\omega}_t) - \kappa_{t-1} + \delta + \ln \bar{e}_t\right]\right) \hat{S}_t(f^{-1}(h(\hat{\omega}_t))^{\gamma}\hat{e}_t\right) \\ h(\hat{\omega}_t) &= u(g(\hat{\omega}_t)) + \beta E\bar{e}_t h(g(\hat{\omega}_t), \hat{e}_{t+1}, \bar{e}_{t+1}). \end{aligned}$$

We need to find h and g such that these hold for all $\hat{\omega}$.

Define the mapping $T: \hat{h} \rightarrow h$ as the solution to the pair of equations:

$$h(\hat{\omega}_t) = f\left(\zeta\pi\left(\mu\left[g(\hat{\omega}_t) - \kappa_{t-1} + \delta + \ln \bar{e}_t\right]\right) \hat{S}_t(f^{-1}(h(\hat{\omega}_t))^{\gamma}\hat{e}_t\right) \tag{16}$$

$$h(\hat{\omega}_t) = u(g(\hat{\omega}_t)) + \beta E\bar{e}_t\hat{h}(g(\hat{\omega}_t),\hat{e}_{t+1},\bar{e}_{t+1}).$$

The set of h that satisfy (16) for a given g, κ, $\hat{e}$, and $\bar{e}$ are increasing in g. Assuming that $\hat{h}_\kappa \geq 0$, it is easy to show that the set of h that satisfy (17) for a given g, κ, $\hat{e}$, and $\bar{e}$ are decreasing in g. Hence, given $\hat{h}_\kappa \geq 0$, T is well defined.

Given $\hat{h}_\kappa \geq 0$, it is easy to show graphically that an increase in κ leads to a rise in h. Therefore T preserves the monotonicity of $\hat{h}$. We therefore look for a fixed point in the space of bounded continuous non-decreasing functions with the sup norm.[24]

Given $\beta E_t\bar{e}_{t+1} < 1$, it is easy to show graphically that an increase in $\hat{h}$ to $\hat{h}+a$ increases h by less than $\beta a E_t\bar{e}_{t+1}$. Also $\hat{h}_1 > \hat{h}_2$ implies $h_1 > h_2$. The mapping therefore satisfies Blackwell's conditions for a contraction mapping, and there exists a unique fixed point h^* by the Contraction Mapping Theorem. From h^* we can deduce the equilibrium $\hat{p}$ and g. This establishes that there exists a unique equilibrium in the market.

Proof of Lemma 1

The statement follows from the properties of the diffusion equation.

Proof of Lemma 2

Immediate.

[24]Since our utility function is not naturally bounded. We impose a bound a some very high level of $\hat{K}$ that is unlikely to be reached in equilibrium.

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