RISK PREMIA AND TERM PREMIA
IN GENERAL EQUILIBRIUM

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ABSTRACT

The equity premium consists of a term premium reflecting the longer maturity of equity

relative to short-term bills, and a risk premium reflecting the stochastic nature of equity payoffs and

the deterministic nature of payoffs on reckless bills. This paper analyzes term premia and the risk

premia in a general equilibrium model with catching up with the Joneses preferences and a novel

formulation of leverage. Closed-form solutions for moments of asset returns are derived. First-order

approximations illustrate the effects of parameters and provide an algorithm to match the means and

variances of the riskless rate and the rate of return on equity.

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The average annual rate of return on equity exceeds the average annual rate of return on short-term riskless bills by several hundred basis. There are two components to this equity premium: a risk premium and a term premium. The risk premium reflects the fact that equity is claim on *stochastic* payoffs, whereas a short-term riskless bill is a fixed-income security that is a claim on a *known* payoff. The term premium reflects the longer maturity of equity relative to short-term bills.

The literature on the equity premium has typically focused on the overall equity premium rather than on the separate components.[1] In the most influential quantitative application of the Lucas (1978) fruit-tree model of asset pricing, Mehra and Prescott (1985) found that this model was incapable of producing an equity premium of more than 35 basis points when they confined attention to parameter values that they deemed plausible. With such a small overall equity premium implied by the model, it did not much matter how much was a risk premium and how much was a term premium. However, subsequent research has produced models that can account for an equity premium of several hundred basis points per year. I will show in this paper how the equity premium in a general equilibrium model can be decomposed into a risk premium and a term premium. This decomposition provides helpful insights about the source of the equity premium and provides guidance in choosing appropriate parameter values.

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[1] Recently, Jermann (1998) has examined term premia and payout uncertainty premia in a model with preferences that exhibit habit formation rather than catching up with the Joneses as in the current paper. Boldrin, Christiano, and Fisher (1995) also examine asset pricing with habit formation, and Lettau and Uhlig (1995) examine asset pricing behavior with a form of catching up with Joneses utility, but unlike Jermann (1996) neither of these papers examines term premia.

The most basic application of the Lucas fruit-tree model can be used to price unlevered equity in an economy with i.i.d. consumption growth and a representative consumer with time-separable isoelastic utility. The literature has extended this basic model in many ways, and I will adopt two of these extensions, though I will combine them in a way to obtain fresh insights. First, I will specify preferences to have the catching up with the Joneses feature introduced in Abel (1990) and used more recently by Campbell and Cochrane (1994) and Carroll, Overland, and Weil (1997). Second, I will introduce a novel tractable formulation of leverage. Despite the additional richness introduced by these extensions, I derive closed-form solutions for expected rates of return, term premia, and risk premia for a general class of assets.

The catching up with the Joneses feature of the utility function was originally introduced to help account for the high average value of the equity premium observed empirically. However, when this form of the utility function was specified to imply a realistic value of the equity premium in Abel (1990), the model produced a riskless rate of return that was far too volatile. Campbell and Cochrane (1994) developed a form of catching up with the Joneses preferences that yielded, as in actual data, a large equity premium and low variability of the riskless rate. They achieved this low variability of the riskless rate by specifying a complicated recursive function for the determination of the benchmark level of consumption. Here I adopt a simpler formulation of the benchmark level of consumption that produces, with the inclusion of leverage, low variability of the riskless rate along with a large equity premium.

The analysis of leverage arises naturally from the formulation of the canonical asset introduced in this paper. The payoff in period t on the canonical asset is specified to

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be proportional to y_t^λ, where y_t is an observable random variable and λ is a constant. I use this formulation in order to include fixed-income securities and equities as special cases. For fixed-income securities, $\lambda = 0$ so that payoffs are nonstochastic. Unlevered equity in the Lucas fruit-tree model is modeled by setting $\lambda = 1$ and y_t equal to aggregate output (which equals consumption) per capita. Although the form of the canonical asset was initially developed with only the values of $\lambda = 0$ and $\lambda = 1$ in mind, it became apparent that much of the analysis can be conducted, and closed-form solutions derived, for arbitrary values of λ. Indeed, values of λ greater than one provide a good approximation to levered equity.

Although closed-form solutions are derived for various moments of asset returns, many of these expressions are too cumbersome to clearly illustrate the effects of various parameters on the moments of returns. To make these effects transparent, I derive first-order approximations to the exact solutions. Numerical calibration demonstrates that these approximations yield values close to the values calculated using the exact solutions. More importantly, I use these approximations to develop a simple algorithm for choosing parameter values that allow the approximate unconditional means and variances of the riskless rate and the rate of return on equity to match the corresponding historical sample values.

Section I develops the model with catching up with the Joneses preferences and introduces the canonical asset. Asset pricing is discussed in Section II which includes a discussion of risk premia and term premia without restricting the distribution of growth rates. Beginning in Section III, I assume that growth rates are lognormal. The interpretation of the parameter λ as a measure of leverage is discussed in Section IV. In

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Section V, I derive closed-form expressions for the means and variances of the riskless rate and the rate of return on equity. I calibrate the model in Section VI and develop the algorithm for choosing parameter values that allow the model's predictions of the unconditional means and variances of the riskless rate and the rate of return on levered equity to match the corresponding empirical moments. Concluding remarks are presented in Section VII.

I. The Model

Consider a closed economy populated by a continuum of identical infinitely-lived consumers. Output in this economy is a homogeneous good that is completely perishable. In equilibrium, all output is consumed in the period in which it is produced so that, as in the Lucas (1978) fruit-tree model, consumption equals output. The amount of output per person in period t is $C_t > 0$, which equals the consumption of the representative consumer.

I.A. Preferences

In period t an individual consumer chooses a level of consumption c_t to maximize utility U_t which is given by the function

$$U_t \equiv E_t \left\{ \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta} \right)^j u(c_{t+j}, v_{t+j}) \right\}, \quad \text{where} \quad u(c_t, v_t) = \frac{1}{1-\alpha} \left(\frac{c_t}{v_t} \right)^{1-\alpha}, \quad \alpha > 0, \, \delta > 0 \quad (1)$$

and v_t is a benchmark level of consumption which is exogenous to an individual

consumer.[2] The curvature parameter α is the coefficient of relative risk aversion. If v_t is

a fixed constant, then the utility function is the standard time-separable utility function

with a constant coefficient of relative risk aversion α and a constant rate of time

preference δ.

The benchmark level of consumption, v_t, tends to grow over time as the standard

of living in a country rises. Specifically,

$$v_t \equiv C_t^{\gamma_0} C_{t-1}^{\gamma_1} \left(G^t\right)^{\gamma_2} \tag{2}$$

where $G \geq 1$ and $0 \leq \gamma_i \leq 1$ for $i = 0, 1,$ and 2. The dependence of the benchmark level of

consumption on current and lagged aggregate consumption per capita, C_t and C_{t-1}, is a

generalization of the "catching up with the Joneses" specification of utility introduced in

Abel (1990), and captures the notion that the benchmark level of consumption is an

increasing (homogeneous) function of current and recent levels of consumption per

capita. The special case with $\gamma_1 > 0$ and $\gamma_0 = \gamma_2 = 0$ corresponds the simple formulation of

catching up with the Joneses in Abel (1990). The case with $\gamma_0 > 0$ and $\gamma_1 = \gamma_2 = 0$

corresponds to Gali's (1994) specification of consumption externalities. The dependence

of the benchmark level of consumption on G^t allows for the possibility that the

benchmark level of consumption grows simply with the passage of time.

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[2]Campbell and Cochrane (1994) assume that the representative consumer's utility depends on the *difference*

between c_t and v_t rather than on the *ratio* of c_t to v_t. Specifically, they assume that $u(c_t, v_t) = (c_t - v_t)^{1-\alpha}/(1-\alpha)$. With the formulation in Campbell and Cochrane, care must be taken to prevent c_t from falling below

v_t. The formulation in equation (1) makes this consideration unnecessary.

Equilibrium asset prices and returns depend on the marginal rate of substitution of the representative consumer. For the utility function in equation (1) and the benchmark level of consumption in equation (2) the marginal rate of substitution between period t and period $t+1$ is

$$M_{t+1} \equiv \frac{1}{1+\delta} \frac{u_c(c_{t+1}, v_{t+1})}{u_c(c_t, v_t)} = \frac{1}{1+\delta} \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} \left(\left(\frac{C_{t+1}}{C_t}\right)^{\gamma_0} \left(\frac{C_t}{C_{t-1}}\right)^{\gamma_1} G^{\gamma_2}\right)^{\alpha-1}. \qquad (3)$$

The marginal rate of substitution can be expressed more conveniently in terms of the growth rate of consumption. Let x_{t+1} be the ratio of consumption in period $t+1$ to consumption in period t and observe that in equilibrium

$$x_{t+1} \equiv \frac{C_{t+1}}{C_t} \equiv \frac{c_{t+1}}{c_t}. \qquad (4)$$

Substituting equation (4) into equation (3) gives an expression for the equilibrium marginal rate of substitution

$$M_{t+1} = \beta x_{t+1}^{-A} x_t^{\theta} \qquad (5a)$$

where $\qquad \beta \equiv \dfrac{G^{\gamma_2(\alpha-1)}}{1+\delta} > 0,$ $\qquad\qquad\qquad (5b)$

$$A \equiv \alpha - \gamma_0(\alpha-1) > 0, \qquad (5c)$$

and $\qquad \theta \equiv \gamma_1(\alpha-1). \qquad\qquad\qquad\qquad (5d)$

Although the specification of preferences involves six parameters—δ, α, γ_0, γ_1, γ_2, and G—and all six of the parameters affect the marginal rate of substitution, there are only three independent parameters that determine the marginal rate of substitution. Thus, relative to the standard time-separable isoelastic specification of preferences, which has

two independent parameters—the rate of time preference and the coefficient of relative

risk aversion—the current specification of preferences introduces only one additional

degree of freedom. The fact that there are six parameters but only three independent

parameters allows for a variety of interpretations of preferences. However, from the

perspective of fitting empirical data on asset returns, there are only three preference

parameters: β, A, and θ.[3]

I. B. The Canonical Asset

Rather than proceed with separate derivations for the prices and rates of return for

different assets such as equity, short-term bills and long-term bonds, I will introduce a

canonical asset that includes all of these assets as special cases. The canonical asset is an

n-period asset. From the standpoint of period t, the terminal period of an n-period asset is

period $t+n$. This asset pays $a_j y_{t+n-j}^{\lambda}$ in the period that is j periods before the terminal

period for $j = 0, \ldots, n-1$, where $y_{t+n-j} > 0$ is a random variable, $a_0 > 0$ is a constant,

$a_j \geq 0, j = 1, \ldots, n-1$ are constants, and λ is a constant that indexes the variability of

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[3]These three parameters can be used to state the condition for utility in equation (1) to be finite.

Specifically, if x_t is i.i.d. over time, then utility in equation (1) is finite if and only if $\beta E\{x^{1-A+\theta}\} < 1$. To

prove this result, rewrite equation (1) as $U_t = \sum_{j=0}^{\infty} \overline{u}_{t,t+j}$ where $\overline{u}_{t,t+j} \equiv E_t \left\{ \left(\frac{1}{1+\delta} \right)^j u(c_{t+j}, v_{t+j}) \right\}$. In

equilibrium,

$$\overline{u}_{t,t+j+1} / \overline{u}_{t,t+j} = \left[G^{\gamma_2(\alpha-1)} / (1+\delta) \right] E_t \left\{ C_{t+j+1}^{1-A} C_{t+j}^{\theta} \right\} / E_t \left\{ C_{t+j}^{1-A} C_{t+j-1}^{\theta} \right\} = \beta E_t \left\{ x_{t+j+1}^{1-A} x_{t+j}^{1-A+\theta} C_{t+j-1}^{1-A+\theta} \right\} / E_t \left\{ x_{t+j}^{1-A} C_{t+j-1}^{1-A+\theta} \right\}.$$

If x_t is i.i.d. over time, then for $j \geq 1$, $\overline{u}_{t,t+j+1} / \overline{u}_{t,t+j} = \beta E_t \{x^{1-A+\theta}\}$. Therefore, $U_t = \sum_{j=0}^{\infty} \overline{u}_{t,t+j}$ is finite if and

only if $\beta E\{x^{1-A+\theta}\} < 1$.

future payoffs. Thus, for instance, in the terminal period, the asset pays $a_0 y_{t+n}^{\lambda}$; and in period $t+1$ the asset pays $a_{n-1} y_{t+1}^{\lambda}$.

The canonical asset introduced here includes equities and fixed-income securities of all maturities. Fixed-income securities such as bonds and bills are represented by $\lambda = 0$ which implies that the payoff in period $t+n-j$ is the known amount a_j . A standard coupon bond with face value F and coupon d is represented by $a_0 = F + d$, and $a_1 = \cdots = a_{n-1} = d > 0$; in this formulation, a pure discount bond is represented by $d = 0$.

Securities with risky payoffs have nonzero values of λ. For instance, in the Lucas (1978) fruit-tree model, the dividend (per capita) on unlevered equity equals consumption per capita C_t. In terms of our canonical asset, unlevered equity in the Lucas model is an infinite-period asset that pays C_t in period t. Using the notation for canonical assets, $n = \infty, a_j = 1$ for all $j \geq 0$, $y_t \equiv C_t$, and $\lambda = 1$. As discussed in section IV, levered equity can be represented by values of λ greater than one.

Let $p_t(n,\lambda)$ be the ex-payment price of the canonical n-period asset in period t. The price of this asset also depends on the sequence of constants $a_j, j = 0, \ldots, n-1$, and on the stochastic specification of y_t, but this dependence is not reflected in the notation. The gross rate of return on the canonical asset between period t and period $t+1$ is

$$R_{t+1}(n,\lambda) \equiv \frac{p_{t+1}(n-1,\lambda) + a_{n-1} y_{t+1}^{\lambda}}{p_t(n,\lambda)} , \text{ for } n \geq 1. \tag{6}$$

The moments of the rate of return on the canonical asset can be determined after specifying the stochastic structure of the economy.

I.C. The Stochastic Structure

Recall that $x_{t+1} \equiv C_{t+1} / C_t$ is the growth rate of aggregate consumption per capita.

Define $z_{t+1} \equiv y_{t+1} / y_t$ to be the growth rate of y_t. As noted earlier, in the specification of

unlevered equity in the Lucas fruit-tree model, $y_t \equiv C_t$ so that $z_{t+1} \equiv x_{t+1}$. More

generally, x_{t+1} and z_{t+1} are distinct random variables and are elements of a random vector

of growth rates W_{t+1} which is observable at the beginning of period $t+1$. I assume that

W_{t+1} is i.i.d.

II. Asset Pricing

In this section I use the marginal rate of substitution in equation (5a) to determine

the price of the canonical asset. As will become evident, the price of the canonical asset

can be expressed as

$$p_t(n,\lambda) = \omega(n,\lambda)x_t^\theta y_t^\lambda \tag{7}$$

where $\omega(n, \lambda)$ is a function to be determined. Substituting equation (7) into equation (6)

and recalling that $z_{t+1} \equiv y_{t+1} / y_t$ yields

$$R_{t+1}(n,\lambda) \equiv \frac{\omega(n-1,\lambda)x_{t+1}^\theta + a_{n-1}}{\omega(n,\lambda)x_t^\theta}z_{t+1}^\lambda, \text{ for } n \geq 1. \tag{8}$$

It is well-known that the product of the gross rate of return and the intertemporal

marginal rate of substitution has a conditional expectation equal to one. In the notation of

this paper

$$E_t\{R_{t+1}(n,\lambda)M_{t+1}\} = 1. \tag{9}$$

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Substituting equations (5a) and (8) into equation (9), and using the assumption that W_{t+1}

is i.i.d., yields the following difference equation for $\omega(n,\lambda)$

$$\omega(n,\lambda) = \kappa(\lambda)\omega(n-1,\lambda) + \beta a_{n-1} E_t\left\{x_{t+1}^{-A} z_{t+1}^{\lambda}\right\}, \text{ for } n \geq 1 \qquad (10a)$$

where $\qquad \kappa(\lambda) \equiv \beta E\left\{x_{t+1}^{\theta-A} z_{t+1}^{\lambda}\right\}.$ $\qquad\qquad$ (10b)

I will assume that β and the distribution of the vector W_{t+1} are such that $0 < \kappa(\lambda) < 1$.

This assumption guarantees that the difference equation converges as n grows.[4]

The boundary condition for this difference equation is provided by the fact that

after the asset has yielded its last payment, and is formally a zero-period asset, its price is

zero. Thus the boundary condition is $p_t(0,\lambda) \equiv 0$ which implies $\omega(0,\lambda) \equiv 0$.

Substituting this boundary condition into equation (10) yields

$$\omega(1,\lambda) = \beta a_0 E\left\{x_{t+1}^{-A} z_{t+1}^{\lambda}\right\} > 0. \qquad\qquad (11)$$

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[4] The assumption that $\kappa(\lambda) < 1$ also leads to a sufficient condition to guarantee that utility in equation (1) is

finite in the case in which $z \equiv x$ and $\ln x$ is $N(\mu,\sigma^2)$, which is widely considered in the literature and is

considered in sections V and VI. In this case, $\kappa(\lambda) \equiv \beta E\left\{x^{\lambda-A+\theta}\right\}$ which implies that

$$\beta E\left\{x^{1-A+\theta}\right\} = \beta E\left\{x^{\lambda-A+\theta}\right\} \exp\left[-(\lambda-1)\left(\mu+\left(\theta-A+\frac{1+\lambda}{2}\right)\sigma^2\right)\right] = \kappa(\lambda)\exp\left[-(\lambda-1)\left(\mu+\left(\theta-A+\frac{1+\lambda}{2}\right)\sigma^2\right)\right]$$

Therefore, the assumption that $\kappa(\lambda) < 1$ implies that $\beta E\left\{x^{1-A+\theta}\right\} < 1$ if $(\lambda-1)\left(\mu+\left(\theta-A+\frac{1+\lambda}{2}\right)\sigma^2\right) \geq 0.$

Thus, the assumption that $\kappa(\lambda) < 1$ implies that $(\lambda-1)\left(\mu+\left(\theta-A+\frac{1+\lambda}{2}\right)\sigma^2\right) \geq 0$ is a sufficient condition

for utility in equation (1) to be finite when $z \equiv x$. See footnote 3.

It is straightforward to verify that the following expression for $\omega(n,\lambda)$ satisfies the difference equation (10a) and the boundary condition (11)

$$\omega(n,\lambda) = \frac{\omega(1,\lambda)}{a_0} \sum_{i=1}^{n} a_{i-1} [\kappa(\lambda)]^{n-i}. \tag{12}$$

The function $\omega(n,\lambda)$ can be either increasing or decreasing in the maturity n.

- For a pure discount security, which is characterized by $a_1 = \cdots = a_{n-1} = 0$, equation (12) implies $\omega(n,\lambda) = \omega(1,\lambda)[\kappa(\lambda)]^{n-1}$ which is strictly decreasing in n. Thus, for given values of x_t and y_t, the price of an n-period discount security, $p_t(n,\lambda) = \omega(n,\lambda)x_t^\theta y_t^\lambda$ is strictly decreasing in maturity n.

- For a security with $a_0 = a_1 = \cdots = a_{n-1} > 0$, equation (12) implies
  $$\omega(n,\lambda) = \omega(1,\lambda) \frac{1 - [\kappa(\lambda)]^n}{1 - \kappa(\lambda)},$$ which is strictly increasing in n.

- For a coupon bond with face value F and coupon d, $a_0 = F + d$ and $a_1 = \cdots = a_{n-1} = d > 0$. In this case, equation (12) implies
  $$\omega(n,\lambda) = \frac{\omega(1,\lambda)}{F+d} \left[(\kappa(\lambda))^{n-1} F + \frac{1 - (\kappa(\lambda))^n}{1 - \kappa(\lambda)} d \right],$$ which is increasing in maturity n if $d / F > (1 - \kappa(\lambda)) / \kappa(\lambda)$ but is decreasing in n if $d / F < (1 - \kappa(\lambda)) / \kappa(\lambda)$. Thus, the price of an n-period coupon bond is an increasing, decreasing, or constant function of the maturity n depending on whether d/F is greater than, less than, or equal to $(1 - \kappa(\lambda))/\kappa(\lambda)$.

II.A. The Expected Rate of Return on the Canonical Asset

Empirical and theoretical studies of asset pricing commonly focus on the expected rates of return on various assets such as equities and fixed-income securities. Before

computing the expected rate of return on the general canonical asset, it is easiest to begin

with the expected rate of return on one-period assets. Taking the expectation of both

sides of equation (8), setting $n = 1$, and using equation (11) yields

$$E\{R_{t+1}(1,\lambda)\} = \frac{E\{z_{t+1}^{\lambda}\}}{\beta E\{x_{t+1}^{-A}z_{t+1}^{\lambda}\}x_{t}^{\theta}}.$$ (13)

It can be shown that[5]

$$E\{R_{t+1}(n,\lambda)\} = \left[\Psi + (1-\Psi)\frac{a_{n-1}}{a_0}\frac{\omega(1,\lambda)}{\omega(n,\lambda)}\right]E\{R_{t+1}(1,\lambda)\}$$ (14a)

where

$$\Psi \equiv \frac{E\{x_{t+1}^{\theta}z_{t+1}^{\lambda}\}E\{x_{t+1}^{-A}z_{t+1}^{\lambda}\}}{E\{z_{t+1}^{\lambda}\}E\{x_{t+1}^{\theta-A}z_{t+1}^{\lambda}\}}.$$ (14b)

Equations (14a,b) present the expected rate of return on the canonical asset for

any maturity n and any value of the parameter λ. I will use this equation to examine term

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[5]Taking the expectation of both sides of equation (8) for maturity equal to n and then for maturity equal to

1 yields $\dfrac{E\{R_{t+1}(n,\lambda)\}}{E\{R_{t+1}(1,\lambda)\}} = \dfrac{\omega(n-1,\lambda)E\{x_{t+1}^{\theta}z_{t+1}^{\lambda}\}+a_{n-1}E\{z_{t+1}^{\lambda}\}}{\omega(n,\lambda)}\dfrac{\omega(1,\lambda)}{a_0E\{z_{t+1}^{\lambda}\}}$ which can be rewritten as

$\dfrac{E\{R_{t+1}(n,\lambda)\}}{E\{R_{t+1}(1,\lambda)\}} = \dfrac{\omega(n-1,\lambda)E\{x_{t+1}^{\theta}z_{t+1}^{\lambda}\}\omega(1,\lambda)}{\omega(n,\lambda)a_0E\{z_{t+1}^{\lambda}\}} + \dfrac{a_{n-1}\omega(1,\lambda)}{a_0\omega(n,\lambda)}$. Now use equation (10a) to obtain

$\dfrac{E\{R_{t+1}(n,\lambda)\}}{E\{R_{t+1}(1,\lambda)\}} = \dfrac{1}{\kappa(\lambda)}\left[1-\dfrac{\beta a_{n-1}E\{x_{t+1}^{-A}z_{t+1}^{\lambda}\}}{\omega(n,\lambda)}\right]\dfrac{E\{x_{t+1}^{\theta}z_{t+1}^{\lambda}\}\omega(1,\lambda)}{a_0E\{z_{t+1}^{\lambda}\}} + \dfrac{a_{n-1}\omega(1,\lambda)}{a_0\omega(n,\lambda)}$. Next use equation (11), the

definition of $\kappa(\lambda)$ in equation (10b), and the definition of Ψ in equation (14b) to obtain

$\dfrac{E\{R_{t+1}(n,\lambda)\}}{E\{R_{t+1}(1,\lambda)\}} = \left[1-\dfrac{a_{n-1}\omega(1,\lambda)}{a_0\omega(n,\lambda)}\right]\Psi + \dfrac{a_{n-1}\omega(1,\lambda)}{a_0\omega(n,\lambda)}$ which is equivalent to equation (14a).

premia by comparing expected rates of return for different values of n, and then I will examine risk premia by comparing expected rates of return for different values of λ.

II.B. Term Premia

For any given value of λ and any given choice of the variable y_t (which determines the payoff $a_j y_{t+n-j}^{\lambda}$), there is a term structure of expected one-period rates of return on assets of various maturities. The term premium is the excess of the expected one-period rate of return on a n-period asset over the expected one-period rate of return on a one-period asset with the same value of λ. Formally, the term premium on an n-period asset is

$$TP(n,\lambda) \equiv \frac{E\{R_{t+1}(n,\lambda)\}}{E\{R_{t+1}(1,\lambda)\}} - 1. \tag{15}$$

Using the definition of the term premium in equation (15), it follows from equation (14a) that the term premium is

$$TP(n,\lambda) = (\Psi - 1)\Gamma(n,\lambda) \tag{16a}$$

where

$$\Gamma(n,\lambda) \equiv 1 - \frac{a_{n-1}}{a_0}\frac{\omega(1,\lambda)}{\omega(n,\lambda)} = 1 - \frac{a_{n-1}}{\sum_{i=1}^{n} a_{i-1}[\kappa(\lambda)]^{n-i}} \tag{16b}$$

According to equation (16a), the term premium is the product of two factors: Ψ -1 and $\Gamma(n, \lambda)$. The first factor, Ψ -1, is independent of the maturity n and of the sequence of payoff coefficients a_j, $j = 0,...,n-1$. Thus, I will call Ψ -1 the "term premium scale

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factor" because a change in this factor changes the term premia at all maturities by the same proportion. Inspection of the definition of Ψ in equation (14) reveals that when $\theta = 0$, $\Psi = 1$ and hence the term premium scale factor is zero. Since $\theta = \gamma_1(\alpha - 1)$, the term premium will be zero if utility is logarithmic ($\alpha = 1$) or if $\gamma_1 = 0$.[6] Thus, the term premium will be nonzero only if the consumption externality, i.e., the dependence of an individual's utility on aggregate consumption per capita, has a lagged component as reflected in $\gamma_1 > 0$. The contemporaneous component of the consumption externality parameterized by γ_0 does not affect whether the term premium is nonzero. Therefore, in the formulation of consumption externalities used by Gali (1994) with $\gamma_0 > 0$ and $\gamma_1 = 0$, there will be no term premium.

The second factor, $\Gamma(n, \lambda)$, captures the shape of the term structure for given λ. Notice that $\Gamma(1, \lambda) = 0$, and $\Gamma(n, \lambda) > 0$ for $n > 1$.[7]

- For an n-period security that makes no payments until the final period, such as an n-period discount bond, $a_1 = \cdots = a_{n-1} = 0$, and hence $\Gamma(n,\lambda) = 1$ for $n > 1$. Thus, the term premium is independent of the maturity n for discount securities with a given λ and with more than one period to run.

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[6]In a somewhat different model, Backus, Gregory, and Zin (1989) show (Proposition 1, p. 382) that term premia will be identically zero if the marginal rate of substitution is independent over time. In the model presented here with x_t i.i.d. over time, if $\theta = 0$, the marginal rate of substitution, $M_{t+1} = \beta x_{t+1}^{-A}$, is independent over time; the term premium will be zero as in Backus, Gregory, and Zin.

[7] Since $0 \leq a_{n-1} \leq \sum_{i=1}^{n} a_{i-1}[\kappa(\lambda)]^{n-i}$, equation (16b) implies that $0 \leq \Gamma(n,\lambda) \leq 1$.

- For a security with $a_0 = a_1 = \cdots = a_{n-1} > 0$, $\Gamma(n,\lambda) = \dfrac{\kappa(\lambda) - [\kappa(\lambda)]^n}{1 - [\kappa(\lambda)]^n}$ which is increasing in n.

- For a standard coupon bond with face value F and coupon d, $a_0 = F + d$ and

$a_1 = \cdots = a_{n-1} = d > 0$, so $\Gamma(n,\lambda) = 1 - \dfrac{d}{[\kappa(\lambda)]^{n-1} F + \dfrac{1 - [\kappa(\lambda)]^n}{1 - \kappa(\lambda)} d}$. The term premium

is decreasing in maturity n if $d / F < (1 - \kappa(\lambda)) / \kappa(\lambda)$ and is increasing in maturity n

if $d / F > (1 - \kappa(\lambda)) / \kappa(\lambda)$.

II. C. Risk Premia

The risk premium is the expected excess rate of return on a risky asset relative to

the rate of return on a riskless asset of the same maturity. I will define the risk premium

in terms of one-period assets. A riskless one-period security is characterized by $n = 1$ and

$\lambda = 0$, and has a known return $R_{t+1}(1,0)$, so the risk premium is

$$RP_t(1,\lambda) \equiv \frac{E_t\{R_{t+1}(1,\lambda)\}}{R_{t+1}(1,0)} - 1. \tag{17}$$

The rate of return on a one-period riskless security is easily calculated by setting $\lambda = 0$ in

equation (13) to obtain

$$R_{t+1}(1,0) = \frac{1}{\beta E_t\{x_{t+1}^{-A}\}x_t^\theta} . \tag{18}$$

Now substitute equations (13) and (18) into equation (17) to obtain

$$RP_t(1,\lambda) = \frac{E_t\{x_{t+1}^{-A}\}E_t\{z_{t+1}^{\lambda}\}}{E_t\{x_{t+1}^{-A}z_{t+1}^{\lambda}\}} - 1. \tag{19}$$

This risk premium in equation (19) equals $-Cov_t\{x_{t+1}^{-A}, z_{t+1}^{\lambda}\} / E_t\{x_{t+1}^{-A}z_{t+1}^{\lambda}\}$ so that the sign

of the risk premium is the opposite of the sign of the conditional covariance of x_{t+1}^{-A} and

z_{t+1}^{λ}. In the standard Lucas fruit-tree model where $z_{t+1} \equiv x_{t+1}$, this conditional covariance

is negative and the risk premium is necessarily positive.[8]

III. Lognormality

So far I have examined the expected rate of return on the canonical asset without

imposing restrictions on the distribution of growth rates (W_{t+1}) except that

$\kappa(\lambda) \equiv \beta E\{x_{t+1}^{\theta-A}z_{t+1}^{\lambda}\} < 1$. Many contributions to the asset pricing literature assume that

the distribution of growth rates is lognormal, and I will adopt this assumption for the

remainder of the paper. Specifically, I assume that

$$\begin{bmatrix} \ln x_{t+1} \\ \ln z_{t+1} \end{bmatrix} \text{ is } N\left(\begin{bmatrix} \mu_x \\ \mu_z \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \sigma_{xz} \\ \sigma_{xz} & \sigma_z^2 \end{bmatrix}\right). \tag{20}$$

Under the distributional assumption in equation (20), the term premium scale

factor Ψ defined in equation (14b) simplifies to

$$\Psi = \exp(\theta A \sigma_x^2). \tag{21}$$

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[8]This statement is based on the assumption that $\lambda > 0$. Unlevered equity in the fruit-tree model is

represented by $\lambda = 1$, and, as we show in section IV, levered equity is represented by $\lambda > 1$. Of course, if λ

$= 0$, the asset is riskless and the risk premium in equation (19) is zero.

Although Ψ is defined in terms of the joint conditional distribution of z_{t+1} and x_{t+1}, the value of Ψ depends only on the conditional variance of x_{t+1}; it is independent of the parameters of the marginal conditional distribution of z_{t+1}, and is also independent of the parameter λ which measures the variability of future payoffs. Thus, the term premium scale factor, $\Psi - 1$, depends on the preference parameters θ and A and the variance of consumption growth σ_x^2, but it is independent of the payoff characteristics of the asset.

Substituting equation (21) into equation (16a) yields

$$TP_t(n,\lambda) = \Gamma(n,\lambda)\left[\exp\left(\theta A \sigma_x^2\right) - 1\right] \cong \Gamma(n,\lambda)\theta A \sigma_x^2 \ . \tag{22}$$

Equation (22) presents an exact expression for the term premium as well as an approximation. According to the approximation, the term structure of term premia is proportional to $\theta A \sigma_x^2$, which is approximately equal to the term premium scale factor. Recall that $\Gamma(n, \lambda) \geq 0$ for $n > 1$ so that the term premium has the same sign as the term premium scale factor. The sign of the term premium scale factor is the same as the sign of θ. Thus for $\theta > 0$, the term premium is positive for $n > 1$. However, if $\theta < 0$ (for instance, if $\gamma_1 > 0$ and $\alpha < 1$), the term premium is negative for all $n > 1$.[9]

The assumption of conditional lognormality also leads to a simple expression for the risk premium. Under the distributional assumption in equation (20), the expression for the risk premium in equation (19) becomes

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[9]These findings are consistent with the result in Backus, Gregory, and Zin (1989, p. 382) that the sign of the term premium will be the opposite of the sign of the autocorrelation of the marginal rate of substitution. In the current model with x_t i.i.d, equation (5a) implies that the sign of this autocorrelation is the opposite of the sign of θ. Therefore, the term premium has the same sign as θ.

$$RP_t(1, \lambda) = \exp(A\lambda\sigma_{xz}) - 1 \cong A\lambda\sigma_{xz} \tag{23}$$

According to the approximation in equation (25) the risk premium equals A multiplied by

the covariance of $\ln x_{t+1}$ and $\lambda\ln z_{t+1}$ where $\ln x_{t+1}$ is the (logarithm of the) growth rate of

consumption and $\lambda\ln z_{t+1}$ is the (logarithm of the) growth rate of the risky payoff on the

asset. In the special case of the Lucas fruit-tree model, the payoff on unlevered equity is

simply per capita consumption, so that $z_{t+1} \equiv x_{t+1}$. In this case, $\lambda = 1$ and $\sigma_{xz} = \sigma_x^2$, so

that the risk premium is approximately equal to $A\sigma_x^2$.

IV. Leverage

Having shown in equation (23) that the (approximate) risk premium is

proportional to λ, I will now show that the parameter λ provides a convenient way to

model leverage. Recall that if $\lambda = 0$, future payoffs are deterministic and are given by

the sequence of constants $a_j, j = 0,1,...,n-1$. Thus, with $\lambda = 0$, the canonical asset is a

fixed-income security such as a bill or a bond. Alternatively, if $\lambda = 1$ and if $y_t \equiv C_t$ so

that $z_{t+1} \equiv x_{t+1}$, the canonical asset is unlevered equity in a fruit-tree model. I will show

below that levered equity can be represented approximately by values of λ greater than

one, with higher values of λ representing higher degrees of leverage.

IV.A. A One-Period Levered Asset

Consider a one-period security in period t that offers a stochastic payoff ξ_{t+1} in

period $t+1$. Call this security L (for "levered equity") and let PL_t denote the price of this

security in period t. The gross rate of return on security L is $RL_{t+1} = \xi_{t+1}/PL_t$. In order to

make security L correspond to (one-period) levered equity, specify the future payoff as

$\xi_{t+1} = \max[0, x_{t+1} - B]C_t$, where $B \geq 0$ is a constant. Observe that $x_{t+1}C_t = C_{t+1}$ is per

capita output in period $t+1$, and $B\ C_t$ is interpreted as a fixed payment in period $t+1$

promised to bondholders. Applying equation (9) along with the expression for the

equilibrium marginal rate of substitution in equation (5a) yields

$$E_t\{RL_{t+1}(B)\} = \frac{E_t\{\max[0, x_{t+1} - B]\}}{\beta x_t^\theta E_t\{x_{t+1}^{-A} \max[0, x_{t+1} - B]\}} ,\qquad (24)$$

where the notation for the expected rate of return on the left-hand side emphasizes the

dependence on B.

IV.B. An Approximation to a One-Period Levered Asset

Now I will use the canonical asset with $n = 1$ to approximate the expected return

on security L. Consider a one-period canonical asset that pays $C_{t+1}^\lambda \equiv (x_{t+1}C_t)^\lambda$ in period

$t+1$. Under the assumed lognormality of x_{t+1}, the coefficient of variation of $(x_{t+1}C_t)^\lambda$ is

$\sqrt{\exp\left(\lambda^2 \sigma_x^2\right) - 1}$. For security L, the coefficient of variation of ξ_{t+1} is

$\sqrt{\dfrac{Var\{\max[0, x - B]\}}{(E\{\max[0, x - B]\})^2}}$. Therefore, the payoffs $(x_{t+1}C_t)^\lambda$ and ξ_{t+1} will have equal

coefficients of variation if $\lambda = \lambda_1$ where

$$\lambda_1 \equiv \frac{1}{\sigma_x} \sqrt{\ln\left(1 + \frac{Var\{\max[0, x - B]\}}{(E\{\max[0, x - B]\})^2}\right)} .\qquad (25)$$

Even with the simplifying assumption of lognormality, the analytic expression for λ_1 in terms of the parameters of the distribution of x is cumbersome. This expression is relegated to Appendix A where it is also shown that for values of B that are small enough that the probability of default, $\Pr\{x < B\}$, is negligible, the coefficient of variation of ξ is approximately equal to the coefficient of variation of x^{λ_2} where

$$\lambda_2 \equiv \frac{1}{1 - \dfrac{B}{E\{x\}}} . \tag{26}$$

For values of B small enough that the probability of default, $\Pr\{x < B\}$, is negligible, the introduction of $B > 0$ is essentially a leftward translation of the distribution of payoffs in which the mean payoff is reduced to a fraction $1 - B / E\{x\}$ of its original value and the standard deviation of the payoff is unchanged. This reduction in the mean payoff increases the coefficient of variation of the payoff to λ_2 times its value when $B = 0$.

Figure 1 shows the values of λ_1 and λ_2 as functions of $B/E\{x\}$. The distributional assumption underlying Figure 1 is that $\ln x$ is $N(0.018, (0.036)^2)$ where the mean and variance of the distribution were chosen to match the historical behavior of the annual growth rate of consumption per capita in the United States as reported by Mehra and Prescott (1985). For values of $B / E\{x\}$ small enough that $\Pr\{x < B\}$, the probability of default shown in Figure 2, is close to zero, λ_2 is an excellent approximation for λ_1. As $B/E\{x\}$ increases toward 1.0, the probability of default increases noticeably and the values of λ_1 and λ_2 diverge somewhat.

To see how well the canonical asset approximates levered equity, I compute the expected rate of return on the one-period canonical asset, and compare this expected rate

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of return with that on one-period levered equity in equation (24). The rate of return on

the one-period canonical asset is easily computed from equation (13) by setting $z \equiv x$ and

using the lognormality of x to obtain

$$E\{R_{t+1}(1,\lambda)\} = \frac{1}{\beta x_t^{\theta}} \exp\left[A\mu + A\sigma^2 \left(\lambda - \frac{1}{2} A \right) \right]. \tag{27}$$

Figure 2 shows the effect of leverage on the expected rate of return and assesses

the quality of the approximation offered by the one-period canonical asset for the case in

which $A = 10$.[10] Because the purpose of the calculations reported here is to examine the

role of leverage on expected rates of return, I have simply set $\beta x_t^{\theta} = 1$ in calculating the

rates of return. Increasing βx_t^{θ} slightly would decrease the expected rates of return

reported in Figure 2, thereby making these rates look more realistic.[11] Figure 2 illustrates

two important points. First, leverage can have a substantial effect on the expected rate of

return even when the probability of default is essentially zero. In the absence of leverage

$(B = 0)$, the expected rate of return on security L is 13.67%, but when $B/E\{x\} = 0.6$, the

expected rate of return on security L increases to 15.94%, even though the probability of

default is smaller than 10^{-14}. Second, Figure 2 shows that the expected rates of return

calculated using the canonical asset with $\lambda = \lambda_1$ or $\lambda = \lambda_2$ are excellent approximations to

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[10]The calculation of the expected rate of return on security L requires the calculation of moments of a

truncated lognormal distribution. See Appendix A.

[11] In fact, in section VI, $\beta = 1.085$ and $\theta = 1.575$, so that evaluating βx_t^{θ} at the median value of $x_t = 1.018$

yields $\beta x_t^{\theta} = 1.12$, which would reduce the expected rates of return in Figure 2 by about 12 percentage

points.

the exact expected rate of return calculated from equation (24), provided that the probability of default is sufficiently small.

V. Moments of Returns

In this section I focus on the moments of returns on a one-period riskless asset and on levered equity. To simplify the resulting expressions, I assume that $z_{t+1} \equiv x_{t+1}$ as is standard in applications of the Lucas fruit-tree model.[12] This assumption implies that $\mu_x \equiv \mu_z \equiv \mu$ and $\sigma_x^2 \equiv \sigma_{xz} \equiv \sigma_z^2 \equiv \sigma^2$. In addition I assume that $a_j \equiv 1, j = 0, ..., n-1$, where n may be infinite.

The aim of this section is to understand how the unconditional means and variances of asset returns depend on the underlying preference parameters and payoff characteristics. I derive exact expressions for these moments of asset returns but some of the resulting expressions are too cumbersome to clearly reveal the effects of parameter values and payoffs on moments of returns. A common approach when facing such cumbersome analytic expressions is to calibrate the model and simulate it for various choices of parameter values. Rather than pursue a numerical strategy, I will pursue an analytical strategy by deriving first-order approximations that clearly illustrate the relationships between the underlying parameters and the moments of returns. It is worth emphasizing that I do not derive the solution to a problem in which the objective function and/or constraints have first been linearized.[13] Instead, I first obtain exact solutions to the

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[12]The Lucas fruit-tree model also restricts λ equal to one. I do not impose this restriction here.

[13]Campbell (1994) and Restoy and Weil (1995) (log) linearize the budget constraint of a representative consumer and use the approximate budget constraint to derive and analyze asset returns.

nonlinear problem, derive closed-form expressions for the moments of equilibrium

returns, and then approximate the moments. The quality of these approximations is

shown to be excellent in Table 1.

V.A. Exact Solutions

The unconditional riskless rate is calculated using equation (18) and the

lognormality of x_{t+1} to obtain

$$E\{R_{t+1}(1,0)\} = \beta^{-1} \exp\left((A - \theta)\mu - \frac{1}{2}(A^2 - \theta^2)\sigma^2 \right).$$ (28)

The unconditional expected rate of return on equity is calculated by setting $n = \infty$ in

equations (12) and (14a), and then using equations (21), (23), and (28) to obtain

$$E\{R_{t+1}(\infty, \lambda)\} =$$
$$\beta^{-1}\left[1 + \kappa(\lambda)\left(\exp(\theta A \sigma^2) - 1\right)\right]\left[\exp(A\lambda\sigma^2)\right]\exp\left((A - \theta)\mu - \frac{1}{2}(A^2 - \theta^2)\sigma^2 \right) .$$ (29)

To get an explicit expression for the term premium, set $a_i = 1$, $i = 0, 1, 2, \ldots$ in equation

(16b) and use equation (21) to rewrite equation (16a) as

$$TP_t(n, \lambda) = \frac{\kappa(\lambda) - [\kappa(\lambda)]^n}{1 - [\kappa(\lambda)]^n}\left[\exp(\theta A \sigma^2) - 1\right].$$ (30)

According to equation (30), the magnitude of the term premium increases monotonically

in n; the term premium is positive and monotonically increasing in n if $\theta > 0$ and is

negative and monotonically decreasing in n if $\theta < 0$.

Exact expressions for the unconditional variances of rates of return are more

complicated and are contained in Appendix B.

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V.B. First-Order Approximations to Moments of Returns

In this section I present first-order approximations to moments of returns.

Specifically, I treat the moments of returns as functions of μ, σ^2, and β, and linearize the

functions around $(\mu, \sigma^2, \beta) = (0,0,1)$.

The approximate unconditional mean of the riskless rate is calculated by

linearizing equation (28) to obtain

$$E\{R_{t+1}(1,0)\} \cong 2 - \beta + (A - \theta)\mu - \frac{1}{2}(A^2 - \theta^2)\sigma^2 . \tag{31}$$

The approximate unconditional mean of the rate of return on equity is calculated

by linearizing equation (29) to obtain[14]

$$E\{R_{t+1}(\infty, \lambda)\} \cong 2 - \beta + (A - \theta)\mu - \left[\frac{1}{2}A^2 - \frac{1}{2}\theta^2 - (\theta + \lambda)A\right]\sigma^2 . \tag{32}$$

It follows immediately from equations (31) and (32) that the first-order approximation to

the unconditional equity premium $E\{R_{t+1}(\infty, \lambda) - R_{t+1}(1,0)\}$ is

$$E\{R_{t+1}(\infty, \lambda) - R_{t+1}(1,0)\} \cong (\theta + \lambda)A\sigma^2 . \tag{33}$$

The approximate equity premium is the sum of the approximate risk premium $\lambda A \sigma^2$ (see

equation (23)) and the approximate term premium[15] $\theta A \sigma^2$.

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[14]Note that $\kappa(\lambda) = \beta = 1$ at the point of linearization.

[15]To calculate the term premium for levered equity, set $n = \infty$ in equation (30) to obtain

$TP(\infty, \lambda) = \kappa(\lambda)[\exp(\theta A \sigma^2) - 1]$. The approximate term premium is derived by linearizing this expression,

noting that $\kappa(\lambda) = 1$ at the point of linearization, to obtain $TP(\infty, \lambda) \cong \theta A \sigma^2$.

The approximate unconditional variances of returns are calculated in Appendix B. The approximate unconditional variance of the riskless rate (see equation B.20) is

$$Var\{R_{t+1}(1,0)\} \cong \theta^2 \sigma^2 . \tag{34}$$

and the approximate unconditional variance of the rate of return on equity (see equation B.22) is

$$Var\{R_{t+1}(\infty, \lambda)\} \cong \left[(\theta + \lambda)^2 + \theta^2\right]\sigma^2 . \tag{35}$$

VI. Matching Sample Means and Variances of Rates of Return

In this section I show how to choose values of the preference parameters A, β, and θ, and the leverage parameter λ so that the unconditional means and variances of the riskless rate and the rate of return on equity in the model match the sample values of these means and variances. The unconditional moments of returns depend on the four parameter values, A, β, θ, and λ, and on the parameters of the lognormal distribution of the growth rate of consumption x_t. I will choose values of this lognormal distribution to match the sample moments of the growth rate of aggregate consumption per capita reported by Mehra and Prescott (1985) for annual data during the period 1889-1978. Specifically, I will set the mean growth rate, μ, equal to 0.018 and the standard deviation of the growth rate, σ, equal to 0.036.

Let m_f and m_e represent the sample means of the riskless rate and the rate of return on levered equity, respectively. Similarly, let s_f and s_e represent the respective sample standard deviations of these rates of return. The values of these four sample moments, as

reported by Mehra and Prescott (1985), are shown in the last column (labeled "Empirical Values") in Table 1. I will match the four sample moments with the corresponding approximate unconditional moments implied by the model using equations (31), (33), (34), and (35). Given the sample values of the four moments to be matched, this set of equations is a recursive system of four equations in four parameters to be determined. I will use a circumflex (^) to denote the values of the parameters that solve this system of equations.

First, choose θ to match the variability of the riskless rate by setting the approximate unconditional variance of the riskless rate in equation (34) equal to s_f^2 and solving for θ to obtain

$$\hat{\theta} = \frac{s_f}{\sigma}. \tag{36}$$

Second, choose the leverage parameter λ to match the approximate variability of the rate of return on levered equity by setting the approximate unconditional variance of this rate of return in equation (35) equal to s_e^2 and solving for λ to obtain[16]

$$\hat{\lambda} = \left[\sqrt{\frac{s_e^2}{s_f^2} - 1} - 1 \right] \hat{\theta}. \tag{37}$$

Third, choose the preference parameter A to match the equity premium by setting the approximate unconditional mean of the equity premium in equation (33) equal to $m_e - m_f$ to obtain

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[16]Note that $\hat{\lambda}$ will be real if and only if $s_e \geq s_f$, and $\hat{\lambda}$ will be positive if and only if $s_e \geq s_f\sqrt{2}$. Both of these conditions are satisfied in the sample used in this paper.

$$\hat{A} = \frac{m_e - m_f}{\left(\hat{\theta} + \hat{\lambda}\right)\sigma^2} . \tag{38}$$

Finally, choose the value of β to match the average riskless rate by setting the approximate unconditional mean of the riskless rate in equation (31) equal to m_f to obtain

$$\hat{\beta} = 2 - m_f + \left(\hat{A} - \hat{\theta}\right)\mu - \frac{1}{2}\left(\hat{A}^2 - \hat{\theta}^2\right)\sigma^2 . \tag{39}$$

Given the empirical values of μ, σ, m_e, m_f, s_e, and s_f, the values of $\hat{\theta}$, $\hat{\lambda}$, $\hat{A}$, and $\hat{\beta}$ implied by equations (36) - (39) are shown in the top panel of Table 1. The leverage parameter that allows the model to match the sample moments of returns is $\hat{\lambda} = 2.74$. Interpreting $B/E_t\{x_{t+1}\}$ as the leverage ratio, and setting $\lambda_2 = 2.74$ in equation (26) implies that the leverage ratio is 0.64, which is in the middle of the range reported by Masulis (1988) for leverage ratios based on book value, but is above the values of the leverage ratio based on market value.[17] An alternative way to gauge the plausibility of λ = 2.74 is to observe that dividends in the model are proportional to $y_t^{\lambda} = c_t^{\lambda}$, where y_t is output per capita and c_t is consumption per capita. Therefore, the (gross) growth rate of dividends is x_t^{λ} where x_t is the growth rate of output and consumption, and the standard deviation of the growth rate of dividends is approximately λ times the standard deviation of the growth rate of output and consumption.[18] Using the standard deviations of growth rates reported in Cecchetti, Lam and Mark (1990, Table 1, p. 402), the standard deviation

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[17]Masulis (1988), Table 1-3, pp. 8-9 reports leverage ratios for U.S. corporations. The leverage ratio based on market value ranged from 0.13 to 0.44 during the period 1929-1986, and the leverage ratio based on book value ranged from 0.53 to 0.75 during the period 1937-1984.

of the growth rate of dividends is 2.5 times the standard deviation of the growth rate of

output, and 3.6 times the standard deviation of the growth rate of consumption. Thus the

dividend variability implied by $\lambda = 2.74$ seems consistent with the data.

The general formulation of preferences introduced in Section I has six parameters,

but this set of parameters is under-identified. In addition to the moment conditions in

equations (36), (38), and ((39), three restrictions are needed to obtain unique values of the

six parameters. I will use the following three restrictions: $\gamma_0 = 0$, $\gamma_0 + \gamma_1 + \gamma_2 = 1$, and G

$= 1+ \mu$. The values of the five nonzero preference parameters implied by these

restrictions are reported in the second panel of Table 1. The coefficient of relative risk

aversion α is slightly above 11, which is a little higher than the upper bound of the range

considered by Mehra and Prescott (1985), but is much smaller than values reported by

Kandel and Stambaugh (1990) and Campbell and Cochrane (1994). The value of γ_1 is

much smaller than the value of 1.0 used in Abel (1990) and allows the value of θ to be

relatively low even though α exceeds 10. The relatively low value of θ keeps the

implied unconditional variance of the riskless rate from being too high, while the

relatively high value of α allows for a substantial equity premium. The rate of time

preference δ is almost 7% per year, even though β exceeds 1.08. Because G is greater

than one and γ_2 is not negligible, this positive rate of time preference is consistent with

the value of β greater than one.[19] As noted by Benninga and Protopapadakis (1990) and

Kocherlakota (1990), a value of β greater than one can improve the empirical fit of fruit-

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[18]Since $\ln x_t$ is $N(\mu,\sigma^2)$, $Var(x_t^\lambda) = \exp(2\lambda\mu + \lambda^2\sigma^2)[\exp(\lambda^2\sigma^2)-1] \approx \lambda^2\sigma^2$. Therefore s.d $(x_t^\lambda) \approx \lambda$ s.d.(x_t).

tree asset-pricing models, in particular by reducing the otherwise high implied value of the riskless rate. The introduction of γ_2 in this formulation offers a rationale for $\beta > 1$.

The third panel of Table 1 reports the unconditional moments of rates of return. Two values are presented for each moment implied by the model. The first value is computed from the exact solution, and the second value is computed from the first-order approximation. The first-order approximations are generally close to the exact values— in some cases, remarkably so. The approximate values of the means and standard deviations of the riskless rate and the rate of return on equity are equal to the corresponding sample moments by construction. The exact values of these moments are generally quite close to the sample values because the approximations are so close to the actual values.

The empirical column in Table 1 also contains an estimate of the average term premium. Over the period 1926-1993 the (arithmetic) average return on long-term government bonds was 5.4% per year compared to an average return on U.S. Treasury bills of 3.7% per year,[20] implying a term premium on fixed-income securities of 170 basis points per year. With the parameter values in Table 1, the model predicts a term premium on perpetual securities of 226 basis points per year which is 56 basis points higher than the sample average term premium on long-term government bonds. However, the implied term premium in Table 1 applies to an asset of *infinite* maturity with *risky* payoffs ($\lambda = 2.74$), whereas the empirical term premium is for long-term government

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[19]Although $\beta > 1$, the value of the utility function in equation (1) is finite. Recall from footnote 3 that the utility function will be finite if $\beta E\{x^{1-A+\theta}\} < 1$. For the parameters values in Table 1, $\beta E\{x^{1-A+\theta}\} = 0.9759$.

[20]Ibbotson Associates (1994), Exhibit 9, p. 31.

bonds, which are assets of *finite* maturity with *known* payoffs ($\lambda = 0$). As it turns out, however, the quantitative impact of these differences in risk and maturity on the term premium is small. Reducing λ from 2.74 to zero reduces the annual term premium on an infinite-maturity canonical asset by 5 basis points.[21] The effect of maturity on the term premium depends on the ratio of the coupon to the face value. Reducing the maturity from infinite to 20 years has virtually no effect if the coupon-face ratio is 0.03, reduces the term premium by about 2 basis points if the coupon-face ratio is 0.06, and reduces the term premium by about 4 basis points if the coupon-face ratio is 0.09. Thus, a small part of the 56-basis point gap in Table 1 between the implied term premium on equity and the sample term premium on long-term government bonds is due to the finite maturity of bonds and to the variability of payoffs to equity. These effects imply that the gap between the term premium predicted by the model and the sample average term premium is smaller than 56 basis points per year.

Now consider the risk premium. The model predicts yields a risk premium of 4% per year. There is no direct empirical counterpart to the risk premium defined in equation (17). The value of 4.48% of the risk premium reported in parentheses in the final column of Table 1 is simply the empirical value of the equity premium minus the empirical value of the term premium.

The question that motivated the research in this paper was how much of the equity premium is a term premium and how much is a risk premium. According to the model, about one third of the equity premium is a term premium, and according to the empirical

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[21]The first-order approximation for the term premium (see footnote 15) indicates that λ does not have a first-order effect on the term premium.

values in Table 1, the term premium accounts for about one fourth of the equity premium. Taken together, these results suggest that the risk premium is about 2 or 3 times as large as the term premium.

VII. Concluding Remarks

I have developed a simple model of asset pricing that can account for the unconditional means and variances of the rates of return on the riskless asset and on levered equity. In addition to providing closed-form expressions for the exact values of these moments, I have derived first-order approximations to these moments. These approximations permitted the development of an algorithm that allows the values of these unconditional moments in the model to match historical sample moments. In addition, these approximations provide a clear view of how the parameters of the model affect the moments of returns implied by the model. The approximate equity premium is the sum the approximate term premium $\theta A \sigma^2$ and the approximate risk premium $\lambda A \sigma^2$. The term premium and the risk premium are both proportional to $A \sigma^2$, where A is a curvature parameter in the utility function and σ is the standard deviation of consumption growth. However, there is an interesting difference in the factors that multiply $A \sigma^2$: in the case of the term premium, this factor is θ which depends only on preferences and is independent of the characteristics of the asset; in the case of the risk premium, this factor is λ which is independent of preferences and is directly related to the volatility of the payoff on the asset.

The equity premium in this framework is $(\theta + \lambda) A \sigma^2$ compared with $A \sigma^2$ in a standard model without catching up with the Joneses preferences ($\theta = 0$) and without

leverage ($\lambda = 1$). With $\theta = 1.58$ and $\lambda = 2.74$, the model used here yields an equity

premium that is $\theta+\lambda = 4.32$ times as high as the equity premium in a standard model

(where $\theta+\lambda = 1$) for given values of the coefficient of relative risk aversion[22] A and

consumption growth variance σ^2. Thus, the model can match the equity premium with a

value of A equal to 11.05 rather 4.32 x 11.05 = 47.74. The increase in the equity

premium relative to a standard model is due in about equal amounts to the change in

preferences, which contributes 1.58 $A\sigma^2$, and the increase in the variability of the payoff,

which contributes 1.74 $A\sigma^2$.

By using a high enough value of θ, the term premium $\theta A \sigma^2$ could be made large

enough to account for the empirical equity premium without appealing to leverage.

However, such a high value of θ would make the implied riskless rate too volatile. The

unconditional standard deviation of the riskless rate is $\theta\sigma$, so there is an intimate link

between the term premium and the standard deviation of the short-term riskless rate: both

are proportional to θ. In economic terms, it is the volatility of the short-term riskless rate

that gives rise to a term premium. If the riskless rate were constant over time, there

would be no variability in the price of fixed-income securities, and there would be no

term premium.

Because an increase in θ increases the volatility of the riskless rate, one cannot

rely on catching up with the Joneses preferences alone to match both the expected equity

premium and the volatility of the short-term riskless rate in an i.i.d. framework. Here is

where leverage plays an important role. Increasing the value of the leverage parameter λ

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[22]The parameter A is defined as $\alpha - \gamma_0(\alpha-1)$. Under the assumption that $\gamma_0 = 0$ as in Table 1, $A = \alpha$ which is

directly increases the risk premium and hence the equity premium implied by the model, without increasing the implied volatility of the short-term riskless rate.

The formulation of leverage in this paper is perhaps the most novel modeling aspect, and further research along this line is needed. Although Section IV demonstrates that λ captures the effect of leverage on the expected return on a levered asset, this analysis is restricted to one-period assets and one-period senior claims against the payoffs to these assets. Additional research is needed to determine how best to model infinite-horizon assets that have issued multi-period fixed claims. This research would shed light on the role of leverage in asset pricing and also help to determine the appropriate empirical counterpart to be used to calibrate the degree of leverage in the model.

The analysis in this paper was confined to a fruit-tree framework in which equilibrium consumption per capita is an exogenous stochastic process. This framework is a useful initial testing ground for the asset pricing implications of leverage and catching up with the Joneses preferences. Since the model is capable of producing moments of returns that match the corresponding sample moments, the next step is to extend the model to allow the economy to transfer goods across time by capital investment. This richer framework will permit analysis of the implications for business cycles and growth as well as for asset returns.

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the coefficient of relative risk aversion.

Figure 1
Values of $\lambda 1$ and $\lambda 2$
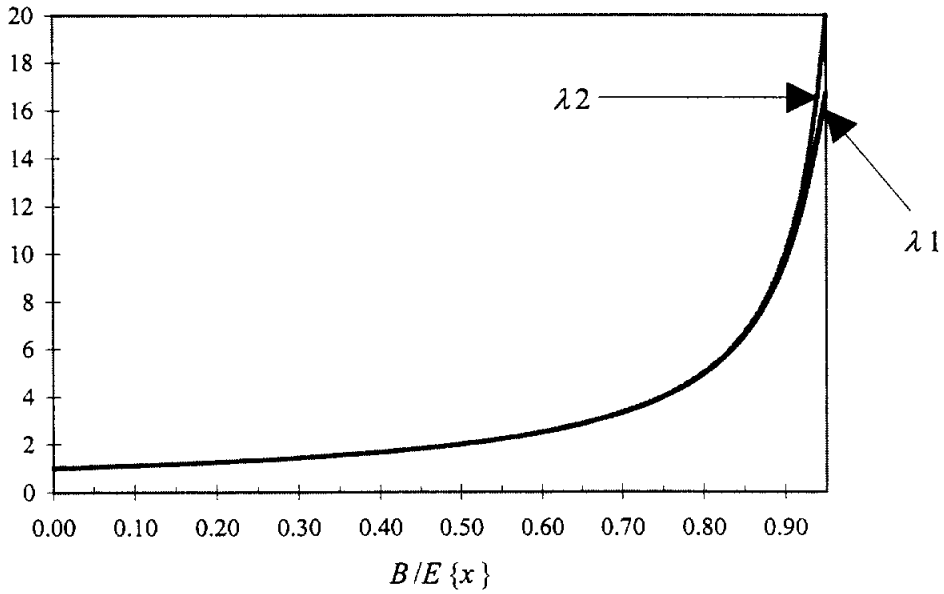
$$\lambda_1 \equiv \frac{1}{\sigma_x} \sqrt{\ln\left(1 + \frac{Var\{\max[0, x - B]\}}{\left(E\{\max[0, x - B]\}\right)^2}\right)} \qquad \lambda_2 \equiv \frac{1}{1 - \frac{B}{E\{x\}}}$$

Figure 2
Expected Net Returns on One-Period Assets and Probability of Default
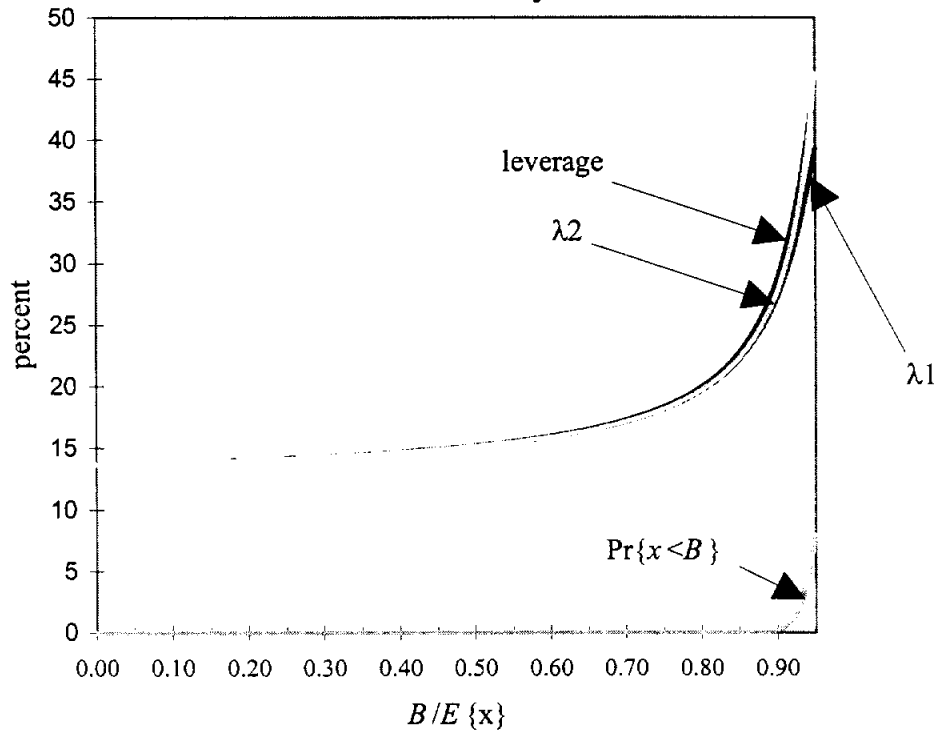

leverage

$\lambda 2$

$\lambda 1$

$Pr\{x<B\}$

percent

$B/E\{x\}$

Table 1 Matching the Moments of Returns ($\mu = 0.018$, $\sigma = 0.036$)	
Parameters Chosen to Match Sample Moments	
$\hat{\lambda}$	2.7411
$\hat{A}$	11.0483
$\hat{\theta}$	1.5750
$\hat{\beta}$	1.0888
Implied Values of Preference Parameters (with 3 additional restrictions: $\gamma_0 = 0$, $\gamma_0 + \gamma_1 + \gamma_2 = 1$, $G = 1 + \mu$)	
α	11.0483
γ_1	0.1567
γ_2	0.8433
δ	0.0683
G	1.018

Implied Unconditional Moments (percent per year)		Empirical Values
riskless rate: mean	1.15	0.80
approx	0.80	
risky rate: mean	7.57	6.98
approx	6.98	
term premium	2.26	1.70
approx	2.26	
risk premium	4.00	(4.48)
approx	3.92	
riskless rate: standard deviation	5.74	5.67
approx	5.67	
risky rate: standard deviation	17.86	16.54
approx	16.54	

Note: The period utility function is $u(c_t, v_t) = (c_t / v_t)^{1-\alpha}$ where $v_t \equiv C_t^{\gamma_0} C_{t-1}^{\gamma_1} \left(G^t \right)^{\gamma_2}$. The marginal rate of substitution is $M_{t+1} = \beta x_{t+1}^{-A} x_t^{\theta}$ where $\beta \equiv \dfrac{G^{\gamma_2(\alpha-1)}}{1+\delta} > 0$, $A \equiv \alpha - \gamma_0(\alpha - 1) > 0$, and $\theta \equiv \gamma_1(\alpha - 1)$.

Appendix A: Moments of Truncated Lognormal Distributions

<u>Proposition A.1</u>: If $x \equiv \ln y$ is $N(\mu, \sigma^2)$ and $B > 0$, then

$$E\{y^a | y > B\} = \frac{1}{\rho}\left[1 - \Phi\left(\frac{\ln B - (\mu + a\sigma^2)}{\sigma}\right)\right]E\{y^a\} \text{ where } \rho \equiv 1 - \Phi\left(\frac{\ln B - \mu}{\sigma}\right) \text{ and } \Phi(\ ) \text{ is}$$

the standard normal c.d.f.

<u>Proof</u>: Note that $e^x = y \Rightarrow e^{ax} = y^a$ and $y \geq B \Rightarrow \ln y \geq \ln B \Rightarrow x \geq \ln B$. Therefore,

$$E\{y^a | y > B\} = \frac{1}{\rho}\int_{\ln B}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right)\exp(ax)dx. \qquad (A.1)$$

Combining the terms to be exponentiated, completing the square, and rearranging yields

$$E\{y^a | y > B\} = \frac{1}{\rho}\exp\left(\mu a + \frac{1}{2}a^2\sigma^2\right)\frac{1}{\sigma\sqrt{2\pi}}\int_{\ln B}^{\infty}\exp\left(-\frac{1}{2\sigma^2}\left((x - (\mu + a\sigma^2))^2\right)\right)dx. \qquad (A.2)$$

Recognizing that $E\{y^a\} = \exp(a\mu + \frac{1}{2}a^2\sigma^2)$ yields

$$E\{y^a | y > B\} = \frac{1}{\rho}\left[1 - \Phi\left(\frac{\ln B - (\mu + a\sigma^2)}{\sigma}\right)\right]E\{y^a\}. \qquad \text{q.e.d.} \qquad (A.3)$$

<u>Corollary</u>:

$$E\{y^{-A}\max[0, y - B]\} = \left[1 - \Phi\left(\frac{\ln B - (\mu + (1-A)\sigma^2)}{\sigma}\right)\right]E\{y^{1-A}\} - B\left[1 - \Phi\left(\frac{\ln B - (\mu - A\sigma^2)}{\sigma}\right)\right]E\{y^{-A}\}.$$

<u>Proposition A.2</u>: Let $\xi \equiv \max[y - B, 0]$ where $x \equiv \ln y$ is $N(\mu, \sigma^2)$. Then

$$\frac{Var\{\xi\}}{(E\{\xi\})^2} = \frac{E\{y^2 | y > B\} - (E\{y | y > B\})^2}{\rho(E\{y | y > B\} - B)^2} + \frac{1}{\rho} - 1, \text{ where } \rho \equiv 1 - \Phi\left(\frac{\ln B - \mu}{\sigma}\right).$$

<u>Proof</u>:

$$\frac{Var\{\xi\}}{(E\{\xi\})^2} = \frac{E\{\xi^2\} - (E\{\xi\})^2}{(E\{\xi\})^2} = \frac{E\{\xi^2\}}{(E\{\xi\})^2} - 1. \qquad (A.4)$$

$$E\{\xi\} = E\{\max[0, y - B]\} = \rho\left(E\{y|y > B\} - B\right). \tag{A.5}$$

$$\left(E\{\xi\}\right)^2 = \rho^2\left(\left(E\{y|y > B\}\right)^2 - 2BE\{y|y > B\} + B^2\right). \tag{A.6}$$

$$E\{\xi^2\} = E\left\{\left(\max[0, y - B]\right)^2\right\} = \rho E\left\{(y - B)^2|y > B\right\} = \rho\left(E\{y^2|y > B\} - 2BE\{y|y > B\} + B^2\right). \tag{A.7}$$

$$E\{\xi^2\} = \rho\left(E\{y^2|y > B\} - \left(E\{y|y > B\}\right)^2\right) + \frac{1}{\rho}\left(E\{\xi\}\right)^2. \tag{A.8}$$

Substituting (A.6) and (A.8) into equation (A.4) and using equation (A.5) yields

$$\frac{Var\{\xi\}}{\left(E\{\xi\}\right)^2} = \frac{E\{y^2|y > B\} - \left(E\{y|y > B\}\right)^2}{\rho\left(E\{y|y > B\} - B\right)^2} + \frac{1}{\rho} - 1. \qquad \text{q.e.d.} \tag{A.9}$$

<u>Corollary</u>: If ρ is approximately equal to 1, then $\dfrac{Var\{\xi\}}{\left(E\{\xi\}\right)^2} \cong \dfrac{1}{\left(1 - \dfrac{B}{E\{y\}}\right)^2}\dfrac{Var\{y\}}{\left(E\{y\}\right)^2}$.

<u>Proof</u>: For ρ sufficiently close to one, realizations of $y < B$ are of negligible importance. Therefore, Proposition A.2 implies

$$\frac{Var\{\xi\}}{\left(E\{\xi\}\right)^2} \cong \frac{E\{y^2\} - \left(E\{y\}\right)^2}{\left(E\{y\} - B\right)^2} = \frac{\dfrac{E\{y^2\}}{\left(E\{y\}\right)^2} - 1}{\left(1 - \dfrac{B}{E\{y\}}\right)^2} = \frac{1}{\left(1 - \dfrac{B}{E\{y\}}\right)^2}\frac{E\{y^2\} - \left(E\{y\}\right)^2}{\left(E\{y\}\right)^2} = \frac{1}{\left(1 - \dfrac{B}{E\{y\}}\right)^2}\frac{Var\{y\}}{\left(E\{y\}\right)^2}$$

.

q.e.d.

Appendix B: Moments of Rates of Return

This appendix calculates the first and second moments of rates of return on riskless bills and levered equity under the assumption that $\ln z_{t+1} \equiv \ln x_{t+1}$ is $N(\mu, \sigma^2)$. For equity, $a_j = 1$ for $j = 0, ..., n\text{-}1$, so $\omega(n,\lambda) = \omega(1,\lambda)(1\text{-}\kappa^n)/(1\text{-}\kappa)$. Using this expression for $\omega(1,\lambda)$ in equation (8), the realized rate of return on the canonical asset can be written as

$$R_{t+1}(n,\lambda) = x_t^{-\theta} H(x_{t+1};n,\lambda) \tag{B.1}$$

where

$$H(x_{t+1};n,\lambda) \equiv \left(\frac{1}{1-\kappa^n}\right)\left[(1-\kappa^{n-1})x_{t+1}^{\theta+\lambda} + \frac{1-\kappa}{\omega(1,\lambda)}x_{t+1}^{\lambda}\right] \tag{B.2}$$

and $\kappa(\lambda)$ is written simply as κ.

Using the fact that $\ln x_{t+1}$ is $N(\mu,\sigma^2)$ it can be shown that

$$E\{H(x_{t+1};n,\lambda)\} \equiv \left(\frac{1}{1-\kappa^n}\right)\left[(1-\kappa^{n-1})\exp\left((\theta+\lambda)\mu+\frac{1}{2}(\theta+\lambda)^2\sigma^2\right)+\frac{1-\kappa}{\omega(1,\lambda)}\exp\left(\lambda\mu+\frac{1}{2}\lambda^2\sigma^2\right)\right] \tag{B.3}$$

and

$$Var\{H(x_{t+1};n,\lambda)\} = \left(\frac{1}{1-\kappa^n}\right)^2 \left[\begin{array}{l}(1-\kappa^{n-1})^2\exp\left(2(\theta+\lambda)\mu+(\theta+\lambda)^2\sigma^2\right)\left(\exp\left((\theta+\lambda)^2\sigma^2\right)-1\right) \\[2mm] +\left(\frac{1-\kappa}{\omega(1,\lambda)}\right)^2\exp\left(2\lambda\mu+\lambda^2\sigma^2\right)\left(\exp\left(\lambda^2\sigma^2\right)-1\right) \\[2mm] +2\frac{(1-\kappa^{n-1})(1-\kappa)}{\omega(1,\lambda)}\exp\left((\theta+2\lambda)\mu+\frac{1}{2}\left((\theta+\lambda)^2+\lambda^2\right)\sigma^2\right)\left(\exp\left((\theta+\lambda)\lambda\sigma^2\right)-1\right)\end{array}\right] \tag{B.4}$$

For riskless one-period bills, $n = 1$ and $\lambda = 0$. The first and second moments of $H(x_{t+1}; n$ $\lambda)$ are

$$E\{H(x_{t+1};1,0)\} = \frac{1}{\omega(1,0)} \tag{B.5}$$

and

$$Var\{H(x_{t+1};1,0)\} = 0.$$ (B.6)

For levered equity, $n = \infty$ and the first and second moments of $H(x_{t+1}; n \ \lambda)$ are

$$E\{H(x_{t+1};\infty,\lambda)\} \equiv \exp\left((\theta+\lambda)\mu + \frac{1}{2}(\theta+\lambda)^2\sigma^2\right) + \frac{1-\kappa}{\omega(1,\lambda)}\exp\left(\lambda\mu + \frac{1}{2}\lambda^2\sigma^2\right)$$ (B.7)

and

$$Var\{H(x_{t+1};\infty,\lambda)\} = \begin{bmatrix} \exp\left(2(\theta+\lambda)\mu + (\theta+\lambda)^2\sigma^2\right)\left(\exp\left((\theta+\lambda)^2\sigma^2\right)-1\right) \\ +\left(\frac{1-\kappa}{\omega(1,\lambda)}\right)^2 \exp\left(2\lambda\mu + \lambda^2\sigma^2\right)\left(\exp\left(\lambda^2\sigma^2\right)-1\right) \\ +2\frac{(1-\kappa)}{\omega(1,\lambda)}\exp\left((\theta+2\lambda)\mu + \frac{1}{2}\left((\theta+\lambda)^2 + \lambda^2\right)\sigma^2\right)\left(\exp\left((\theta+\lambda)\lambda\sigma^2\right)-1\right) \end{bmatrix}.$$ (B.8)

Unconditional moments. The assumption of i.i.d. growth rates implies that $x_t^{-\theta}$ and $H(x_{t+1}; n,\lambda)$ are independent. I will use the following identities that hold for independent random variables u and v:

$$E\{uv\} = E\{u\}E\{v\}$$ (B.9)

$$Var\{uv\} = Var\{u\}Var\{v\} + [E\{u\}]^2Var\{v\} + [E\{v\}]^2Var\{u\}$$ (B.10)

Therefore

$$E\{R_{t+1}(n,\lambda)\} = E\{x_t^{-\theta}\}E\{H(x_{t+1};n,\lambda)\}$$ (B.11)

and

$$Var\{R_{t+1}(n,\lambda)\} = Var\{x_t^{-\theta}\}Var\{H(x_{t+1};n,\lambda)\}$$
$$+[E\{x_t^{-\theta}\}]^2 Var\{H(x_{t+1};n,\lambda)\} + [E\{H(x_{t+1};n,\lambda)\}]^2 Var\{x_t^{-\theta}\}$$ (B.12)

The mean and variance of $H(x_{t+1}; n,\lambda)$ have already been calculated in equations (B.2) and (B.4). The mean and variance of $x_t^{-\theta}$ are

$$E\left\{x_t^{-\theta}\right\} = \exp\left(-\theta\mu + \frac{1}{2}\theta^2\sigma^2\right) \cong 1 - \theta\mu + \frac{1}{2}\theta^2\sigma^2 \tag{B.13}$$

and

$$Var\left\{x_t^{-\theta}\right\} = \exp\left(-2\theta\mu + \theta^2\sigma^2\right)\left[\exp\left(\theta^2\sigma^2\right) - 1\right] \cong \theta^2\sigma^2. \tag{B.14}$$

Linearization. Recall that $a_0 = 1$ so that under conditional lognormality equation (11) implies

$$1/\omega(1,\lambda) = \beta^{-1}\exp\left(-(\lambda - A)\mu - \frac{1}{2}(\lambda - A)^2\sigma^2\right). \tag{B.15}$$

Linearizing equation (B.15) around the point $(\mu, \sigma^2, \beta) = (0,0,1)$ yields

$$\frac{1}{\omega(1,\lambda)} \cong 2 - \beta - (\lambda - A)\mu - \frac{1}{2}(\lambda - A)^2\sigma^2. \tag{B.16}$$

Under normality with $x_{t+1} \equiv z_{t+1}$, equation (10b) implies

$$\kappa = \beta\exp\left((\theta + \lambda - A)\mu + \frac{1}{2}(\theta + \lambda - A)^2\sigma^2\right). \tag{B.17}$$

Linearizing equation (B.17) around $(\mu, \sigma^2, \beta) = (0,0,1)$ yields

$$\kappa \cong \beta + (\theta + \lambda - A)\mu + \frac{1}{2}\sigma^2(\theta + \lambda - A)^2. \tag{B.18}$$

The approximate unconditional mean of the riskless rate is calculated using the identity in equation (B.11) along with equations (B.5), (B.16) with $\lambda = 0$, and (B.13) to obtain

$$E\left\{R_{t+1}(1,0)\right\} \cong 2 - \beta + (A - \theta)\mu + \frac{1}{2}(\theta^2 - A^2)\sigma^2. \tag{B.19}$$

The approximate unconditional variance of the riskless rate is calculated using the identity in equation (B.1210) along with equations (B.6) and (B.13) to obtain

$$Var\{R_{t+1}(1,0)\} \cong \theta^2 \sigma^2 \ . \tag{B.20}$$

The approximate unconditional mean of the rate of return on equity is calculated by substituting equations (B.13) and (B.7) into equation (B.11), then using equations (B.15) and (B.17) and linearizing to obtain

$$E\{R_{t+1}(\infty,\lambda)\} = 2 - \beta + (A - \theta)\mu + \frac{1}{2}(\theta^2 - A^2)\sigma^2 + (\theta + \lambda)A\sigma^2 \ . \tag{B.21}$$

The approximate unconditional variance of the rate of return on equity is calculated by substituting equations (B.7), (B.8), (B.13), (B.14), into equation (B.12), and linearizing to obtain

$$Var\{R_{t+1}(\infty,\lambda)\} \cong \left[(\theta + \lambda)^2 + \theta^2\right]\sigma^2 \ . \tag{B.22}$$

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