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A SIMPLE APPROACH FOR DECIDING WHEN TO INVEST

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ABSTRACT

A straightforward generalization of the simple net present value rule that correctly predicts when to invest in two classes of projects that can be delayed is derived. The first class consists of projects for which the option to delay derives its value exclusively from uncertainty about interest rates. It is shown that the optimal rule for investing in such projects is to simply multiply the discount rate of the project by the ratio of the mortgage rate to the riskless rate and then use this new rate as the discount rate in a standard net present value analysis. The other class of investment opportunities that is considered is the firm's option to expand. It is shown that it is only optimal for the firm to expand when a particular call option on the firm's stock has no time value. The fact that mortgage bonds (in the form of GNMAs) and stock options are actively traded implies that these rules have potentially important practical and empirical value. Besides their simplicity, the rules have the added advantage that they do not depend on a maintained assumption on the dynamics of interest rates in the economy.

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1 Introduction

It has long been recognized that the simple net present value (NPV) rule for deciding whether to invest in a project provides an unambiguously correct answer only under certain conditions. In particular, if the investment decision can be delayed, the investment is not fully reversible and there is some anticipated resolution of uncertainty that might affect the value of the project, it might not be optimal to invest even if the NPV is positive.[1]

Few people would argue that many real investments are fully reversible. Consequently, because reversibility is only relevant if the investment decision can be delayed, research on the question of when standard NPV analysis can be applied has centered on the delayability of the investment decision. Since the option to delay is only valuable if there is an anticipated resolution of uncertainty that affects value, the earlier papers in this literature concentrated on projects for which there is an anticipated resolution of uncertainty about the project's *cash flows*.[2] For example, Brennan and Schwartz (1985) show how the decision of a copper mining company on when to open a copper mine depends on the stochastic nature of copper prices. Although this approach has yielded many important economic insights, there is a drawback. The value of the option to delay depends idiosyncratically on the nature of the uncertainty and its effect on the project's cash flows. Thus, deriving a simple rule to replace the NPV rule that could be universally applied to all projects has proved difficult. Consequently, although the economic significance of the option to delay is widely appreciated, the ability of corporate managers to explicitly take it into account in making investment decisions is limited.

The anticipated resolution of cash flow uncertainty is not the only condition under which the option to delay has value. There is another determinant of project value for which the anticipation of the arrival of new information is important, namely, the level of interest rates. Since the present value calculation depends on interest rates (which are stochastic), *every* project, regardless of its cash flow dynamics, must have the feature that there will be an anticipated resolution of uncertainty that will affect its value. Thus the option to delay will have value even when project cash flows are riskless. This implies two things. On the one hand, since interest rates affect the value of all projects, the simple NPV rule cannot be applied to any investment that can be postponed regardless of the nature of the cash flows. On the other hand, since the effect of interest rates on project value is similar across projects, it should, in principle, be possible to derive a practical investment rule that, like simple NPV analysis, would apply to a wide range of projects.

The object of this paper is to derive a such a rule for two important classes of investment decisions that can be postponed. The first class isolates the effect of the interest rate option by restricting attention to projects for which no resolution of uncertainty about the distribution of

[1]There is an extensive literature on the implications of reversibility and the anticipated resolution of uncertainty on the investment decision. An interested reader can consult Robert Pindyck's survey in the *Journal of Economic Literature* (1991), Dixit and Pindyck (1994) or Trigeorgis (1996).

[2]See Cukierman (1980), Bernanke (1983), Brennan and Schwartz (1985), McDonald and Siegel (1986), Titman (1985) and Pindyck (1988).

the cash flows is expected. In this case the simple NPV rule needs only a small adjustment to incorporate the option to delay. The correct procedure, in a nutshell, is to multiply the discount rate by the ratio of the callable riskless rate to the non-callable riskless rate and then apply the rule as before. Since callable riskless rates (in the form of mortgage backed securities (GNMAs)) are almost as actively quoted as riskless rates themselves, this rule is as tractable as the standard NPV rule for projects that cannot be delayed.

As we have already pointed out, obtaining a widely applicable and simple rule outside this class is difficult because of the idiosyncratic nature of the resolution of cash flow uncertainty. Nevertheless, we derive such a rule for a particular class of investments — the option to expand. Specifically, we show that a firm should choose to expand whenever the time value of a particular (American) call option on its stock is zero.[3] Since stock options are widely traded, this rule can be easily implemented. It also provides a theoretical basis for arguing, at least in this narrow context, for the importance of not ignoring the option component of value. Generally, only deep in the money stock options have zero time values. Since we show that the present value of the expansion itself is proportional to the intrinsic value of the option, the result implies that the option to expand will only be exercised early if the NPV of the expansion itself is very large. We also derive a simple rule, using stock options with differing strike prices, for deciding on the optimal size of the expansion.

In both cases, the rules that are derived have the advantage that they provide an unambiguous relation between an easily observable variable and firm investment. This is important from an empirical standpoint because the fact that no such relation had been identified hampered the ability to test the theory. For instance, the ambiguous effect of change in interest rates on investment is often cited at an important implication of the theory. What this paper does is provide a particular interest rate — the mortgage rate — that is unambiguously related to investment. Similarly, a decrease in the time value of the firm's stock options increases the likelihood of undertaking an expansion. Thus far, the literature on real options has emphasized the ambiguous nature of how macroeconomic policy changes might affect investment. The time value of the firm's options can be used as an unambiguous measure of the theoretical effect of a policy change. Since this time value can be measured on the firm level for different time horizons, this research opens the possibility of not only assessing the effect of a policy change on the economy as a whole, but also assessing the differential effect across sectors and time horizons.

The class of investment decisions that are the focus of this paper have been analyzed before. Ingersoll and Ross (1992) have derived the appropriate rule for deciding when to invest in projects with riskless cash flows (a subclass of the first class) in the presence of interest rate uncertainty. Their solution relies on the fact that the option to wait can be valued as an option on interest rates. It therefore depends on the model of interest rates that is used. Since the value of this option must be calculated explicitly, the solution also lacks the principal advantage of standard NPV analysis — simplicity. Although it is not clear what fraction of investment decisions are delayable, it would be

[3] The time value of an option is the difference between the option price and the intrinsic value (the value if exercised immediately).

hard to argue that this fraction is insignificant. Ingersoll and Ross (1992, p. 27) therefore conclude that to correctly make a non-trivial fraction of corporate financing decisions, corporate decision makers should not use the simple NPV rule. The implication is that corporate managers would not only need to have a knowledge of finance far beyond the level that is currently standard but would also have to take a stand on what model of interest rates is “correct.” By showing that the hurdle rate derived in that paper corresponds to the yield on a actively traded bond, a mortgage backed security, we demonstrate how both of these difficulties can be avoided.

The option to expand has been analyzed, implicitly, in the literature on delayable investment decisions in the context of capital budgeting. This literature has its roots in papers by McDonald and Siegel (1986) and Brennan and Schwartz (1985). Recently, Abel, Dixit, Eberly and Pindyck (1996) have explicitly analyzed the expandability option. Much like the interest rate case, the optimal rule for investment that is derived is not simple enough to be easily implementable by corporate decision makers nor is the relation to observable parameters direct enough to provide empirically testable predictions. In this paper we show how these rules can be simply implemented and empirically tested by using call options on the firm’s stock. It is also possible to extend our methodology to derive straightforward and testable rules for other real options studied in that paper, for instance, the option to reduce the scale of the firm.

The rest paper is organized as follows. In Section 2 we derive the appropriate simple NPV rule for the type of delayable projects were the option value derives exclusively from interest rate uncertainty. Section 3, provides a simple investment rule for a firm with an option to expand. Section 4 concludes the paper. All non-trivial proofs can be found in the appendix.

2 The Interest Rate Option

In this section we restrict attention to projects for which the value of the option to delay derives solely from the stochastic nature of interest rates. We first demonstrate the main result of this section in the context of a simple example. We then prove the result.

2.1 An Example

Before we derive the general result, it is illustrative to demonstrate the underlying intuition with a simple example. We take a positive NPV investment opportunity that can be postponed. First, in the spirit of Ingersoll and Ross (1992), we compute whether or not the investment should be undertaken by explicitly valuing the option using state prices. We then show that the same result can be obtained by using the callable interest rate as the single discount rate in a simple NPV analysis.

Consider the decision, which can be delayed indefinitely, to invest \$1 in a project that yields a riskless cash flow of \$0.18 per period in perpetuity. Assume that the interest rate dynamics are such that the short rate one period from now will be either 10% or 20% and will stay at that level forever. Thus, after one period all interest rates with either be 10% or 20%. Assume that each state is equally likely under the risk neutral probability measure, or equivalently, that the Arrow-Debreu state price of each pure state claim is the same. If r is the current risk free short rate and r_c is the current risk free consol rate, then, under the risk neutral measure, the value today of the consol, $\frac{1}{r_c}$, is just its discounted expected value in one period plus one coupon payment:

$$\frac{1}{r_c} = \frac{1}{1+r}\left(\frac{1}{2}\frac{1}{0.10} + \frac{1}{2}\frac{1}{0.20} + 1\right).$$

Solving for r_c,

$$r_c = \frac{2(1+r)}{17}.$$

Now, the NPV of the project if it is undertaken is

$$\frac{0.18}{r_c} - 1 = \frac{3.06}{2(1+r)} - 1.$$

Note that the project has positive NPV whenever the current short rate is below 53%. However, if the investment decision is delayed, then one of two possibilities will occur. Either the short rate goes to 20% and the project is never taken on, or the short rate drops to 10% and the project is taken on immediately. Therefore, the NPV of the investment opportunity, if it is delayed, is the expected discounted value of the investment opportunity in each state:

$$\frac{1}{(1+r)}\frac{1}{2}\left(\frac{0.18}{0.10} - 1\right).$$

The investment opportunity should be taken today only if it has positive NPV *and* is worth more dead than alive:

$$\frac{3.06}{2(1+r)} - 1 > \frac{1}{2(1+r)}\left(\frac{0.18}{0.10} - 1\right).$$

Solving this inequality for r shows that the project will be taken on today if $r < 13\%$. As Ingersoll and Ross (1992) argue, the simple NPV rule can provide the wrong answer over a wide range of interest rate environments, in this case for short rates between 53% and 13%.

Now the rate on a consol bond that can be called at par any time, r_m (hereafter, the mortgage rate), solves,

$$\frac{1}{r_m} = \frac{1}{1+r}\left(\frac{1}{2}\frac{1}{r_m} + \frac{1}{2}\frac{1}{0.20} + 1\right).$$

That is, the left hand side of the above expression is the current price of the callable consol (mortgage) bond, while the right hand side is the discounted expected value of this bond one period hence. Note that we have used the fact that since the mortgage will be called if rates drop to 10%, it is worth its par value in that state. Solving for r_m,

$$r_m = \frac{1+2r}{7}.$$

The NPV of the investment opportunity, discounted at the mortgage rate is

$$\frac{0.18}{r_m} - 1 = \frac{1.26}{1+2r} - 1,$$

which is positive whenever $r < 13\%$. Thus, applying the NPV rule using the mortgage rate as the discount rate provides the correct rule for deciding when to invest in this project in this interest rate environment.

This example illustrates the underlying intuition in this paper. The reason why it is optimal to delay taking on the positive NPV investment is that by not taking it on, the firm does not give up the opportunity to capture an even bigger NPV should rates drop to 10%. Ideally, though, the firm would like to have it both ways, that is, take on the project today without giving up the option to capture the bigger NPV should rates drop to 10%. The way the firm can do this is to finance the investment by borrowing in a mortgage bond. If rates drop, then the larger NPV is captured by simply calling the bond — exercising the prepayment option by returning the principle. Of course, this strategy can only work if financing the investment at the mortgage rate does not involve an additional expense, or to put this another way, if the investment has non-negative NPV when discounted at the mortgage rate. In the next section we demonstrate that this result holds generally.

2.2 A General Rule

The example demonstrates that by simply replacing the riskless rate with the mortgage rate, the simple NPV rule can be used to determine when to invest. Since mortgage interest rates are as easily observable as riskless rates themselves, this result has the potential to be as useful as the simple NPV rule itself.

Consider the decision, which can be costlessly delayed, to take on a project at date t which, for expositional simplicity, is assumed to provide a perpetual cash flow stream. Let $r(t)$ be the (date-t) yield of a riskless bond that pays \$1 forever (hereafter, the consol rate), and let $r_m(t)$ be the yield of an equivalent consol bond that is callable at par at any time, that is, at any time it allows the issuer (i.e., the borrower) the option to absolve himself of all future interest obligations by returning the face value of the bond. At time τ, the callable consol bond issued at time t with current price $P_m(\tau)$ that is callable at $P_m(t)$ is defined to have yield

$$r_m(\tau) \equiv \frac{1}{P_m(\tau)}.$$

United States mortgage obligations such as GNMAs all contain such a call feature and have yields defined in this way. Furthermore, they are guaranteed by the United States government so, like treasury bonds, they contain no risk of default. These obligations are therefore (finitely lived[4])

[4]GNMAs are 30 year bonds.

examples of callable riskless bonds. In light of this we will refer to $r_m(t)$ as the *mortgage consol rate*, or more simply as the *mortgage rate*.

Let $z(t)$ be the pricing operator (kernel) in the economy. Using standard finance arguments (see Duffie (1996)) this operator can be used to derive the price of any asset. Explicitly, the (date-t) price of any cash flow $c(\tau)$, where $\tau > t$, is given by $E_t[\frac{z(\tau)}{z(t)}c(\tau)]$. So, for example, the price of the consol bond is $\frac{1}{r(t)} = \sum_{\tau=t}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\right]$.

We will limit attention to potential projects that require an investment of \$1 at time t and, if this investment is made, deliver a risky cash flow stream $c(\tau) = \bar{c} + \epsilon(\tau)$ for all $\tau > t$, where $\bar{c}$ is constant and $E_t[\epsilon(\tau)] = 0$. As was pointed out in the introduction, to derive a general result that can be applied to all projects an assumption on the nature of the resolution of cash flow uncertainty is required. What is needed is that certainty equivalent of every cash flow be constant. This assumption requires that for some constant π and for all $\tau > t$,

$$E_t[\epsilon(\tau)z(\tau)] = -\pi E_t[z(\tau)]. \tag{1}$$

Under this assumption, the project can be valued by discounting the certainty equivalent of each cash flow at the consol rate. To see why, let $P(t)$ be the time-t value of the cash flow stream. Then, using the pricing operator to value the cash flows and (1),

$$\begin{aligned} P(t) &= \sum_{\tau=t}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}c(\tau)\right] \\ &= \sum_{\tau=t}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\left(\bar{c} + \epsilon(\tau)\right)\right] \\ &= \bar{c}\sum_{\tau=t}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\right] - \pi\sum_{\tau=t}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\right] \\ &= \frac{\bar{c} - \pi}{r(t)}. \end{aligned} \tag{2}$$

We will henceforth take (1) to be the definition of π, the certainty equivalent of the uncertain part of the project's cash flows.

Since the certainty equivalent of any cash flow is determined endogenously, it is important that we establish that assumptions on primitives exist that will provide cash flows with constant certainty equivalents. Clearly, as was pointed out in the introduction, attention must be restricted to projects for which waiting provides no new information about the distribution of the cash flows. Perhaps the most straightforward example of such projects that also satisfy (1) are projects with riskless cash flows ($\epsilon(t) = 0,\ \forall t$). Another example would be projects in which all cash flow uncertainty is idiosyncratic ($\text{cov}\left(\epsilon(t), z(t)\right) = 0,\ \forall t$). In both cases the assumption is satisfied trivially because π is always zero. A non-trivial example of a set of primitives that provides (1) with π non-zero is if the $\{\epsilon(t)\}$ are independently and identically distributed and $z(t)$ is defined such that

$$z(t) = z(t-1)\ e^{-i(t-1)} Y(t)$$

with $z(0) = 1$, where $i(t)$ is the one period riskless interest rate and $\{Y(t)\}$ are strictly positive and independently and identically distributed with $E[Y(t)] = 1$. In this case $\pi = -\text{cov}\,(\epsilon(t), Y(t))$.

The following proposition shows that the NPV rule can be used to correctly decide when to take on the project by simply discounting the certainty equivalent cash flows at the mortgage rate rather than the consol rate.

Proposition 2.1 *The investment opportunity to undertake a project with risky cash flow* $c(\tau) = \bar{c} + \epsilon(\tau), \tau > t$, *that requires an initial investment of \$1 should be undertaken at time t if, and only if,*

$$\frac{\bar{c} - \pi}{r_m(t)} \geq 1. \tag{3}$$

To understand, intuitively, the proof strategy, consider the case in which the project cash flows are riskless, so $\pi = 0$ and so (3) becomes

$$\frac{\bar{c}}{r_m(t)} \geq 1. \tag{4}$$

Let τ_m be the first time (4) is satisfied, that is, the first time it is optimal to invest in the project. It is suboptimal to wait longer, say to time τ_a. To see why, assume that when the investment is made at time τ_m, it is fully financed by borrowing \$1 in a mortgage. Because (4) is satisfied, the cash flows of the project exceed the interest owed on the mortgage, so this strategy generates a non-negative cash flow stream. The key insight is to notice that by calling (repaying) the mortgage at time τ_a, the firm can duplicate exactly the strategy of investing at time τ_a (in both cases the firm will need to pay \$1 and will receive $\bar{c}$ forever). However, by investing at time τ_a the firm is worse off because it misses the non-negative cash flow stream from $\tau_m + 1$ to τ_a, which implies that it is suboptimal to wait beyond τ_m.

The reason why it is suboptimal to invest in the project before τ_m is that before τ_m, the interest paid on the mortgage always exceeds the cash flow of the project (because at any time before τ_m, (4) is not satisfied). So before τ_m, the firm is always better off investing the \$1 in the mortgage rather than the project.

Another way to understand why this proposition holds is to recognize that the callable bond or mortgage is a portfolio which consists of a long position in a non-callable consol bond and a short position in an American call option on the bond. Thus a portfolio that consists of a long position in the non-callable consol and a short position in the mortgage is exactly a long position in the call. The investment should be made when the NPV exceeds the value of this call or equivalently, the portfolio of a long position in the consol and a short position in the mortgage. However, since the underlying project is essentially a non-callable consol bond, the non-callable consol appears in both expressions. Canceling leaves the simpler expression that price of the mortgage must exceed the net investment, which is (4).

In the general case with uncertain cash flows, the above proposition derives the optimal investment rule using certainty equivalents. The next proposition derives the appropriate discount rate if the simple NPV rule is used instead (i.e., the expected cash flows rather than their certainty equivalents are discounted). Let the *risk adjusted discount rate* (i.e., the discount rate that is used in a simple NPV analysis) be defined to be the rate that discounts the expected cash flows to give the current price.

Corollary 2.1 *An investment opportunity with risky cash flow* $c(\tau) = \bar{c} + \epsilon(\tau), \tau > t$, *that requires an initial investment of \$1 and has a risk adjusted discount rate of* $R(t)$ *should be taken on at time* t *if, and only if, it has positive NPV when the expected cash flows are discounted at* $R(t) \times \frac{r_m(t)}{r(t)}$.

This proposition provides the simple rule for deciding whether to invest in a risky project that can be delayed — apply the standard NPV rule, but discount the expected cash flows of the project at the discount rate multiplied by the ratio of the mortgage rate to risk free rate. Note that for projects with riskless cash flows (or cash flows with only idiosyncratic risk), $R(t) = r(t)$, and so the rule provides the result obtained in the example — simply discount at the mortgage rate.

2.3 Implications

Even in the case when there is no option to delay investment, the simple NPV rule gives a theoretically correct answer only under certain conditions. For instance, when the riskiness and expected value of a project's cash flows varies stochastically the simple NPV rule fails because the present value of the cash flows is obtained by discounting the current expected cash flow at a single discount rate that is not a function of the cash flow (or interest rate) dynamics. To apply the simple NPV rule, such dynamics must be ruled out. One way to do this is to assume that the certainty equivalent of each cash flow is constant. Since our object is to provide an adjustment to the simple NPV rule it is not surprising that we have made assumptions at least as restrictive as in the standard case.

The constant certainty equivalent assumption is also a standard, if implicit, assumption in many applications in the real options literature. For example, in their analysis of the same option Ingersoll and Ross (1992) impose the far more restrictive assumption of riskless cash flows. Furthermore, the single period CAPM pricing relation is often used, in this literature, to specify the risk premium.[5] To apply this single period result in a multiperiod setting an auxiliary assumption is required to ensure that risk premia are constant.

In spite of the widespread use of the constant certainty equivalent assumption, its restrictiveness undoubtedly limits the applicability of this paper. However, it is worth noting that this assumption nevertheless provides a theory — the simple NPV rule — that is the *de facto* standard for making

[5]see Dixit and Pindyck (1994, p. 148), for example.

corporate financing decisions. Presumably, this is because the theory works better than any other theory. This paper further refines this theory by presenting a simple method for managers to explicitly take into account the option to delay.

The fact that Proposition 2.1 identifies an observable threshold rate means that the theory is testable. It is therefore possible to determine, empirically, the importance of the assumptions that underly the results. Although it has long been recognized that changes in interest rates should affect investment, designing a test based on this relation has been hampered by the fact that the relation between non-callable rates and investment is ambiguous. As Ingersoll and Ross (1992) demonstrate, a decrease in non-callable rates can either increase or decrease investment because although such a decrease increases the present value of future cash flows it also decreases the cost of waiting. No such ambiguity exists when callable rates are used. *Ceteris paribus,* a drop in mortgage rates unambiguously decreases the hurdle rate and so should increase investment.

The above analysis could also be used to gauge the likely effect of a change in monetary policy. Assume a central bank lowered interest rates for the purpose of stimulating investment. As many researchers have pointed out, in the case of investments which can be delayed such a change could have the opposite effect, not only for the aforementioned reason, but also because the change itself might increase uncertainty about future monetary policy which provides a further incentive to delay investment. Callable interest rates provide an unambiguous answer to what the theoretical effect of the policy change will be — if mortgage interest rates react to the change by decreasing, the policy will have the intended effect, otherwise it will not.

Although the analysis in this section has been restricted to projects with infinitely lived cash flows, it is possible to derive equivalent results for projects with finitely lived cash flows.[6] In this case the appropriate mortgage rate to use is the rate on a mortgage with maturity of the last cash flow. In a recent paper, Dunn and Spatt (1997) show that unlike the non-callable yield curve, the callable yield curve can never slope downwards. This fact implies that when evaluating a decision to invest in a project when a mortgage contract of exactly comparable maturity does not exist, other mortgage contracts can be used to provide bounds on when to invest. A project should be undertaken if it has positive NPV when discounted at the yield of the closest, longer, maturity mortgage contract. Similarly, the project should not be undertaken if it has negative NPV when discounted at the yield of the closest, shorter, maturity mortgage contract.

3 The Option to Expand

In this section we will turn our attention to another class of real options — the option to expand production. We will show that, like the previous case, this investment decision can be simplified by using the price of a traded security, in this case, a call option on the firm.

[6]The details are available on the author's home page.

3.1 The Simple Rule

Assume that at time t, an all equity firm with N shares of stock outstanding with stock price $P(t)$ has an opportunity, which can be postponed until date T (were T need not be finite) to invest αNI which will increase its scale by a factor $\alpha > 0$ so that all future cash flows of the firm would be multiplied by $1 + \alpha$.[7] Assume that financial call options on the firm's stock are traded. Let the price of an American[8] call option with strike K and exercise date T on one share of stock be $C(P(t), K, T)$. The following proposition shows that the firm should expand whenever the difference between its stock price and I exceeds the value of an American call option, with strike I and exercise date T, on the firm's stock.

Proposition 3.1 *A firm should exercise the real option to expand by α if and only if*

$$P(t) - I \geq C(P(t), I, T). \tag{5}$$

The proof follows the same logic as the proof of Proposition 2.1 with the call option on the stock playing a similar role to that of the mortgage. In a nutshell, the American call option is precisely the same option as the option to invest. When the investment is made, the firm must give up this option, so it is only optimal to make the investment when the benefits of the investment, the NPV, exceeds the cost of giving up the option, which is (5).

By using the well know fact that an American call cannot sell for less than its immediate exercise value, $C(P(t), I, T) \geq P(t) - I$, and (5) the following corollary is immediate:

Corollary 3.1 *A firm will only exercise the real option to expand by α when*

$$C(P(t), I, T) = P(t) - I.$$

A firm only expands when a particular option on its stock loses all its time value (the difference between the immediate exercise value and the option price). This result is perhaps the most surprising result in the paper. Although it is theoretically possible for an American stock option to be worth no more than its intrinsic value, this condition only rarely occurs. When it does it is for deep in the money options on dividend paying stocks that are close to their exercise dates. Now the intrinsic value of the option (i.e., P(t)-I) is directly proportional to the NPV of the expansion.

[7] Note that this assumption does not restrict the production technology nor does it constrain the market effects of the expansion. The investment, αNI, is simply the cost to the firm of increasing its cash flows to shareholders by α. There is no assumption that twice that investment would produce a twice as large an increase in the cash flows, nor is it assumed that the change in scale will not have an impact in the firm's product market.

[8] An American option is defined to be an option that can be exercised on any date up to and including the exercise date.

The implication is that any expansion that can be appreciably delayed will only be undertaken if the NPV of the expansion is very high (i.e., $\alpha(P(t) - I) >> 0$). The corollary therefore provides a theoretical justification for the general consensus in the real options literature that "the simple NPV rule is not just wrong; it is often *very* wrong"[9] and the observation that firms tend to set hurdle rates well above the internal rate of return of the investment.

Since the proof of Proposition 3.1 is based solely on arbitrage arguments, the proposition holds quite generally. In particular no assumption is needed on the interest rate process in the economy, the production function of the firm or even the pricing process in the economy. Of course, the result does rely on the existence of actively traded American options with maturities comparable to the last possible delay date. In fact, American options with maturities of up to nine months are traded on most actively traded stocks (practically all traded options are American in nature). For a significant fraction of these stocks, long maturity options of up to two years (LEAPS) are traded. Since the time premium of an option cannot exceed that of an otherwise identical option of longer maturity, such options can also be used as a bound for an investment opportunity that can be delayed longer than two years.

In the case of an investment opportunity that cannot be delayed, the firm invests to the point that the marginal cost of each unit of investment equals the present value of the revenues it is expected to generate — on the margin the project has zero NPV. It is well known that this is no longer true when the opportunity can be postponed (see Dixit and Pindyck (1994, Ch. 11)). In the case that the investment can be delayed to time T the optimal level of investment maximizes the NPV *subject to* the constraint that Corollary 3.1 is satisfied. This investment rule can be operationalized as follows. First the price of all American call options that expire on the last possible date the decision to expand can be made are observed. Then all option prices that contain a time value greater than zero are removed. If any options remain, define $\bar{I}$ as the highest remaining strike price, otherwise it is 0. The decision maker then decides on the optimal level of the expansion in the usual way except that it is recognized that the per share cost of the expansion cannot exceed $\alpha\bar{I}$.

The solution illustrates nicely the well known result that the firm should expand in stages. As time passes and the set of stock options with zero time values varies, $\bar{I}$ varies. When there is an increase in $\bar{I}$, the choice set expands and opens the possibility for further expansion. The time value of an option is decreasing in the time to expiration, so the likelihood of further expansion increases as the last possible date the decision can be made nears. When this date occurs, the time value disappears altogether and the investment decision reduces to the non-delayable solution.

3.2 Implications

What Proposition 3.1 and Corollary 3.1 provide is a link between the real option to expand and financial options *traded on the firm's stock*. Thus, a problem that would otherwise require strong

[9] Dixit and Pindyck (1994, p. 136, their italics)

assumptions and an advanced level of sophistication to solve from first principles can be solved simply by just using the market price of these options.

Although the above methodology applies just to the option to expand, in principal, it could be used in other contexts as well. In many cases capital budgeting decisions are made by comparing the proposed investment to similar investments already undertaken at other companies. For instance, a common method for estimating the discount rate of a project in an industry outside of the firm's core business, is to use the average cost of capital for firms in the new industry. A similar principal could be used for making an investment decision for a project that can be postponed. For example, take the decision that is analyzed in Brennan and Schwartz (1985) about whether to open a copper mine. If there are other copper mining companies with traded stock options, then the decision on whether to develop the mine can be made by simply observing the time premia on the stock options of the other companies.

It is frequently argued that increased uncertainty decreases investment because it increases the value of the option to wait. In fact it is conceivable that a change in economic policy that is designed to stimulate investment could produce exactly the opposite effect because of the increased uncertainty about future macroeconomic policy that might result from the change itself. Thus far the economic importance of this effect has been difficult to gauge. The corollary provides an unambiguous measure of the effect of a change in macroeconomic policy on firm expansion — the time premia of the firm's stock options. Since the time premia of all options (with the same exercise date) on the firm's stock are monotonically related to each other, in principle one needs only to observe the time premium of one option to infer the effect of a policy change on the investment decision. Furthermore, since changes in time premia can be monitored on the firm level, it is theoretically possible to also measure the differential importance the option across economic sectors.

A well known result in option pricing theory is that it is never optimal to exercise an American call on a non-dividend paying stock early. The implication in this context is that it is never optimal to exercise the option to expand early if the firm chooses to retain earnings. To understand, intuitively, why this is the case, consider why a dividend paying firm might exercise the option to expand early. By exercising the expansion option all future dividends are increased. Dividend payments made before the expansion are not subject to this increase, so by not exercising the option to expand, this increase in current dividends is lost. In the case were there are no current dividend payments, there is no growth in current dividends to loose so there is no reason to exercise early.[10]

[10]This intuition does not imply that there is a relation between the financial policy of the firm and its investment policy. The Modigliani-Miller proposition guarantees that the decision about whether or not to pay a dividend cannot affect the firm's investment policy. The key point is that the above intuition does not hold investment policy constant. To understand the effect of change in financial policy alone, the unpaid dividend must still be distributed to stockholders, say by using it to buy back stock. In this case, since the cost of the expansion is specified on a per share basis, from the perspective of an individual shareholder, the strike price of the relevant traded option will change. This amounts to replacing one option with another one. Such a swap is not necessarily in the interests of the option holder. The cases in which it is optimal to preempt this swap by exercising early are the set of states that it is optimal to exercise the option on the dividend paying stock early.

Finally, although we have limited attention to the option to expand, there is no reason why the same methodology cannot be applied to the option to reduce the scale of the firm. In this case the optimal exercise policy can be deduced from the time value of an American put option (on the firm's stock) with strike equal to the liquidation value of the unused assets.

4 Conclusion

The theory in this paper provides two simple rules for deciding when to invest in class of projects that can be delayed. These rules have the added advantage that they do not depend on a maintained assumption on interest rates. They therefore provide a systematic method for differentiating between investment opportunities that can be delayed and ones that cannot.

The fact that both callable risk free rates exist in the U.S. market in the form of mortgage-backed securities and American call options are traded on many firms' stock is the principal advantage of the methodology outlined in this paper. However, it is well known that, because of the existence of transaction costs, U.S. home owners do not prepay their mortgages optimally.[11] As a result, actual mortgage rates may not precisely reflect the value of the refinancing option and so might depart from the theoretical rate considered in this paper. The extent of this departure and how it affects the decision rule developed in this paper is an empirical question. One thing to note is that in recent years refinancing costs have dropped substantially. Consequently, as any mortgage trader will attest, homeowners have become far more likely to exercise prepayment options that appear in the money when transaction costs are ignored.

Thus far, surprisingly little empirical work with the objective of providing evidence in favor of the modern theory of investment under uncertainty has been undertaken. Dixit and Pindyck (1994, p. 423) attribute this, at least partially, to the fact that the threshold that triggers investment cannot be observed directly. Because our results provide unambiguous measures of this threshold, they have the potential of providing insight into the importance of the option to delay investment by stimulating more empirical research in this area.

[11]See Stanton (1995) and LeRoy (1996) for two excellent analysis of optimal mortgage prepayment.

A Lemmas

The following lemmas are required for the proof of Proposition 2.1.

Lemma A.1 *A security with a (risky) cash flow of $\pi + \epsilon(\tau)$ for all $\tau > t$ always has a price of zero.*

Proof: Denote the price of such a security as $P_\epsilon(t)$. Then,

$$\begin{aligned} P_\epsilon(t) &= \sum_{\tau=t+1}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\pi\right] + \sum_{\tau=t+1}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\epsilon(\tau)\right] \\ &= \pi \sum_{\tau=t+1}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\right] - \pi \sum_{\tau=t+1}^{\infty} E_t\left[\frac{z(\tau)}{z(t)}\right] = 0. \end{aligned}$$

■

Lemma A.2 *Let S denote a security that has a face value of \$1, pays a dividend $r_m(t) + \pi + \epsilon(\tau)$ for all $\tau > t$ and is callable at par. If $P_S(\tau)$ is the price of this security at time τ, then $P_S(t) = 1$.*

Proof: Consider forming a portfolio at time t by taking a long position in security S and simultaneously shorting a callable consol bond (with a \$1 face value) that pays a coupon of $r_m(t)$ and shorting a security that has a risky payoff $\pi + \epsilon(\tau)$ for all $\tau > t$. Note that so long as the call on S is not exercised, this portfolio's cash flows are exactly zero. However, whenever S is called the portfolio can always be unwound at no cost by calling the consol and buying back the remaining asset (for nothing, by Lemma A.1). Thus the value of the portfolio at time t can never be less than zero. A similar argument implies that a portfolio with the above long and short positions reversed also cannot have a time t value less than zero. This implies that the portfolio's value at time t must be exactly zero. By Lemma A.1, the value of the security with payoff $\pi + \epsilon(\tau)$ is always zero, so at time t, security S must have the same price as the callable consol. At time t the callable consol is a par bond, so $P_S(t) = 1$. ■

B Proof of Proposition 2.1

Proof: To prove this proposition we must show that any other rule is suboptimal. Let τ_m be the first time that (3) is satisfied, that is, τ_m is the optimal time to invest in the project. We need to show that it is suboptimal to use a rule that prescribes investing at any other time. There are two possibilities to consider — rules that prescribe investing before τ_m and rules that prescribe investing after τ_m.

First consider a rule that prescribes investing later than τ_m. Let τ_a be the first time under this rule that it is optimal to invest in the project, so $\tau_a > \tau_m$. We will compare the cash flows generated by investing at time τ_m (the *time-τ_m investment strategy*) to the cash flows generated by investing at time τ_a (the *time-τ_a investment strategy*). Assume the investment at time τ_m is finance by issuing (or shorting) one unit of security S (by Lemma A.2) which, since the market is complete, either exists or can be constructed. Thus, so long as security S is not called, this investment strategy generates the certain cash flow $\bar{c} - r_m(\tau_m) - \pi$ for all $t > \tau_m$. Assume that the firm calls security S at time τ_a (by paying \$1 at that time) so that after time τ_a the cash flows generated by both investment strategies are identical and the initial investment of \$1 is paid at τ_a in both cases. At time τ_m, the cash flow of both strategies is zero. This just leaves the time interval from $\tau_m + 1$ to τ_a to consider. The difference in the cash flows of the time-τ_m and time-τ_a investment strategies over this time interval is the certain cash flow stream $\bar{c} - r_m(\tau_m) - \pi$. From (3) and the definition of τ_m,

$$\bar{c} \geq r_m(\tau_m) + \pi,$$

so this cash flow difference is greater than zero. Thus the firm is better off investing at time τ_m rather than τ_a.

To complete the proof we must show that a rule that prescribes investing before τ_m is also suboptimal. Let τ_b be the first time that it is optimal to invest in the project under such a rule, so $\tau_b < \tau_m$. To show that such a rule is suboptimal we will compare the cash flows generated from investing \$1 in the project at time τ_b to the cash flows generated from investing \$1 in security S. At any time, t, up until security S is called, its cash flows are $r_m(\tau_b) + \pi + \epsilon(t)$. This cash flow strictly exceeds the cash flows of the project, $\bar{c} + \epsilon(t)$, because, since $\tau_b < \tau_m$,

$$\frac{\bar{c} - \pi}{r_m(\tau_b)} < 1 \;\Rightarrow\; r_m(\tau_b) + \pi > \bar{c}.$$

When security S is called the firm can always choose to reinvest the \$1 it receives in the project thereby guaranteeing that the cash flows under the two strategies are identical after the call date. Investing in security S at time τ_b therefore dominates investing in the project at that time. Thus the rule that prescribed investing in the project at time τ_b is suboptimal. ■

C Proof of Corollary 2.1

Proof: Let $\gamma(t)$ be the corrected discount rate for the expected cash flows so that the simple NPV rule can be used to determine when to invest. By Proposition 2.1, $\gamma(t)$ must satisfy

$$\frac{\bar{c} - \pi}{r_m(t)} = \frac{\bar{c}}{\gamma(t)},$$

so solving provides,

$$\gamma(t) = \frac{\bar{c}}{\bar{c} - \pi} r_m(t). \tag{6}$$

Now $R(t)$ satisfies

$$P(t) = \frac{\bar{c}}{R(t)}.$$

Substituting this relation into (2) and solving for π provides

$$\pi = \bar{c}\left(1 - \frac{r(t)}{R(t)}\right).$$

where it is implicitly assumed that $\bar{c} > 0$ and $P(t) > 0$. Substituting this relation into (6) completes the proof. ■

D Proof of Proposition 3.1

Proof: The proof of this proposition uses the same logic as the proof of Proposition 2.1. First define τ_c as the first time (5) is satisfied. We first show that it is suboptimal to delay the investment decision beyond this point. As before, let $\tau_a > \tau_c$ and compare the cash flows from a strategy of expanding at time τ_c (the τ_c-*investment strategy*) to the cash flows from a strategy of expanding at τ_a (τ_a-*investment strategy*).

Assume that, at time τ_c, the firm simultaneously expands production by α, purchases αN American call options with exercise date T and strike I on its own stock and shorts (or issues) αN shares of its own stock. The net cash flow at time t is,

$$-\alpha NI - \alpha NC(P(t), I, T) + \alpha NP(t) = \alpha N\Big(P(t) - I - C(P(t), I, T)\Big)$$

which is non-negative by the definition of τ_c and (5). Furthermore, so long as the option is not exercised, the net cash flow (to existing shareholders) of this strategy for all $t > \tau_c$ is zero, since the extra cash flow generated by the expansion is exactly offset by the cash flow owed on the short position in the stock. Assume that the firm exercises the option at time τ_a. This entails paying αNI for αN shares of stock which are used to close out the short position in the stock. Thus, the net cash flow to existing shareholders is multiplied by $1 + \alpha$. Since the τ_a-investment strategy produces the same cash flows from τ_a onwards (requires an investment of αNI and multiplies the future net cash flow by $1 + \alpha$), the difference in the cash flows from the two strategies is zero for all $t > \tau_c$. However, at τ_c the τ_c-investment strategy yielded a non-negative cash flow, while the τ_a-investment strategy yielded nothing, so the firm is weakly better off investing at time τ_c.

The last step of the proof requires showing that it is suboptimal to expand before time τ_c. Let $\tau_b < \tau_c$ and compare the resulting cash flows from undertaking the expansion at time τ_b to the following stock repurchase at time τ_b. Assume that the firm repurchases αN shares of its own stock and simultaneously sells αN American call options with strike I and exercise date T. The net cash flow at time τ_b is,

$$-\alpha NP(t) + \alpha NC(P(t), I, T) = \alpha N\left(C(P(t), I, T) - P(t)\right)$$

which is strictly greater than $-\alpha NI$ by the definition of τ_b and (5). So the stock repurchase costs strictly less to implement than the expansion. Note, however, that so long as the call is not exercised the *per share* cash flow to shareholders is identical under both the stock repurchase and the expansion (in the one case the number of shareholders is unchanged but the net cash flow is increased by fraction α while in the other the net cash flow is unchanged but the number of shareholder is reduced by α fraction). If the call is exercised the firm can reissue the αN shares of its own stock and deliver this stock to the option holder. In return the firm receives αNI which it can then use to expand production by α. The firm is now identical to what it would be under the time τ_b expansion strategy. Thus the two strategies have identical per share cash flows, but the stock repurchase is strictly cheaper to implement, which implies that it is suboptimal to expand at time τ_b. ■

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