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ON THE REGULATION OF FEE STRUCTURES IN MUTUAL FUNDS

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ABSTRACT

We offer an alternative framework for the analysis of mutual funds and use it to examine the rationale behind existing regulations that require mutual fund advisor fees to be of the "fulcrum" variety. We find little justification for the regulations. Indeed, we find that asymmetric "incentive fees" in which the advisor receives a flat fee plus a bonus for exceeding a benchmark index provide Pareto-dominant outcomes with a lower level of equilibrium volatility.

Our model also offers some insight into fee structures actually in use in the asset-management industry. We find that when leveraging is not permitted and the fee structure must be of the fulcrum variety, the equilibrium fee in our model is a flat fee with no performance component; if asymmetric incentive fees are allowed and leveraging is permitted the equilibrium fee is an incentive fee with a large performance component. These predictions match observed fee structures in the mutual fund industry and the hedge fund industry, respectively.

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1 Introduction

Permissible fee structures in the US mutual fund industry are laid out in the 1970 Amendment to the Investment Advisers Act of 1940. The Act, which is reviewed in Appendix A, allows mutual funds and their investment advisers to enter into performance-based compensation contracts only if the fees are of the "fulcrum" variety, that is, ones in which the adviser's fee is symmetric around a chosen index, decreasing for underperforming the index in the same way in which it increases for outperforming it. Thus, while the Act does not rule out "fraction of funds" fees in which advisers are paid a fixed percentage of the total funds under management, it does prohibit so-called "incentive fee" contracts in which advisers receive a base fee plus a bonus for exceeding a benchmark index.[1]

The rationale offered for the prohibition of incentive fee contracts is theoretical rather than empirical in nature; that is, the ban has more to do with concerns about the inherent nature of incentive-fee contracts, rather than any actual evidence of abuse. Supporters of the prohibition, both in the SEC and in Congress, have argued that a fee structure which rewards advisers for outperforming a benchmark index without penalizing them for underperforming it provides advisers with an incentive to take excessive risk. Effectively (so the argument goes), such advisers hold an option that gives them the right to exchange a fraction of their portfolio for the benchmark portfolio. The value of this option can be increased by increasing the spread between the standard deviations of the two portfolios, leading to the concern about increased risk.

A little reflection suggests that this line of reasoning is incomplete. By linking choices of risk levels solely to fee structures, it implicitly invokes a "partial equilibrium" assumption that investors are passive and will not change their portfolio allocations in reaction to the altered environment. If, however, investors are active and choose portfolio allocations as optimal responses to fee structures and fund risk levels, it is not obvious that admitting incentive fee structures will lead to increased levels of risk in equilibrium.

Indeed, once we move away from the partial equilibrium framework and explicitly model investor reactions, it becomes apparent that equilibrium risk levels are not the only—or even the primary—quantity with which we should be concerned. Since the objective of the legislation is to protect investors, the relevant question should be: does the prohibition of incentive fees lead to an increase in investor *welfare*? The answer is not immediate. Certainly, it is not apparent that admitting incentive fees will necessarily make the investor worse off.

This paper examines, in an equilibrium model, the extent to which the prohibition on

[1] The terminology is standard but unfortunate. It does not make clear that incentive fees (or "performance fees" as they have also been called) are necessarily *asymmetric*, rewarding good performance without penalizing poor performance.

incentive fees can be justified at a theoretical level. We find that the regulatory concern may be misplaced: we describe a set of robust models in which incentive fee structures dominate fulcrum fee structures on *all* fronts, providing Pareto-superior outcomes with lower equilibrium volatility levels. The two subsections immediately following discuss special features of the framework we employ and provide a more detailed description of our main results. A review of the related literature follows in Section 2.

1.1 Comments on Our Framework

A central question facing the modeller in constructing a useful framework for analysis of mutual funds is: who makes the decision on the choice of contract form? Following the lead of the vast literature on principal/agent problems, the majority of papers in the literature on mutual fund compensation have assumed that this power rests with the principal (i.e., the investor in his role as fund shareholder). Our model breaks with this tradition. We make no distinction between the fund and its investment advisers, and assign the decision on fee structure to the fund. Two considerations guide our choice in this matter.

First, while an assumption that investors choose compensation structures may be apposite in dealing with the relationship between a large client and an investment adviser, it is, perhaps, a little less suitable in the context of mutual funds. In principle, a mutual fund is controlled by its shareholders (and, indeed, is required to have "outsiders" comprise at least 40% of its board). In practice, nonetheless, the relationship between a fund and its advisers tends to be extremely close. Indeed, most management companies are responsible for establishing the funds that they advise. It appears appropriate, therefore, to explore the consequences of allowing the adviser to choose not only the risk-characteristics of the fund portfolio, but also its fee structure.

Second, our decision to have the fund's investment advisers choose the fee structure is especially appropriate from the narrow point of view of the questions motivating this paper. Indeed, if we were to adopt the standard paradigm and have the investor (in his role as principal) choose the form of the compensation contract, restricting the set of permissible contracts could end up lowering investor welfare, but certainly can never increase it. On the other hand, it makes perfect sense to ask (as we do) whether restrictions on the fund's ability to set fees can enhance investor welfare.

Our model also differs from the standard principal/agent approach in not taking the amount invested in the fund as exogeneous. Rather, investors in our model choose portfolio allocations as optimal responses to funds' choice of risk levels and fee structures. Endogeneizing investment choices and solving for it as part of the equilibrium appears to us to be an important consideration. Among other things, it captures the point that fee structures and risk characteristics of the fund portfolio are not chosen in isolation, but in a competitive environment for the investment dollar.

Also in contrast to much of the existing literature, our model makes explicit and central use of a "benchmark" portfolio. Since mutual funds are restricted by law to using fulcrum fees and a fulcrum fee cannot be meaningfully defined without reference to a benchmark, it appears that benchmarking should be an important ingredient in a study of mutual fund fee structures.[2] The use of a benchmark introduces a complication that is not present in typical principal-agent models: for each action available to the fund/investment adviser, the model must now specify not only a distribution of returns on the fund portfolio, but also the relationship between this distribution and that of the benchmark returns.

Finally, one aspect of our model merits further comment. As the description provided above suggests, our model may be thought of as a principal/agent model in which, in addition to the usual considerations, the agent chooses the fee structure, and the principal responds by choosing the amount of resources to be invested with the agent. To our knowledge, such a framework has not been investigated in the literature. It appears to us that it may be useful in analyzing compensation structures in a variety of other settings as well.

1.2 Main Results

Our analysis begins in Section 3 with a simple two security framework in which the intuition underlying our results is easily captured. Sections 4 and 5 enrich this framework in two directions: by allowing for fund "effort" to enhance return distributions and by allowing for multiple risky securities. In each case, we characterize the equilbrium solutions first when the fund is restricted to using only fulcrum fees, and then when it may use only incentive fees. A comparison of the equilibria under the two regimes reveals that, in each case, incentive fees dominate fulcrum fees on *all* fronts: (a) the equilibrium utility levels of the fund and the investor are both higher under an incentive fee structure than a fulcrum fee structure; and (b) the volatility of net-of-fees returns to the investor is lower under incentive fees than under fulcrum fees.

These results may not appear entirely intuitive. *Ceteris paribus*, for instance, risk-averse investors such as those in our model would prefer fulcrum fees since this would lower their returns' variance. Thus, one may expect investors to fare better in equilibrium under a

[2] The existence of fee regulations (and, therefore, benchmarking) may play a role in explaining differences in fee structures *across* the asset management industry as well. For example, mutual funds overwhelmingly tend to use flat fees with no performance-adjustment component (see Appendix B). However, hedge funds, which are not subject to the fulcrum-fee requirement, typically employ incentive fees with a substantial performance component. To our knowledge, no paper has attempted to examine the differences in existing fee structures from such a perspective.

fulcrum fee regime (although the fund may still prefer incentive fees). In fact, our results derive in most part from risk-sharing arguments. Since investors in our model react adversely to increases in the level of fees or of portfolio risk, selecting a mix that maximizes total fees is a delicate task. The symmetric nature of fulcrum fees makes them relatively inflexible in this direction, while incentive fees, with their asymmetric patterns enable better risk-sharing between the investor and the fund. We elaborate further on the intuitive underpinnings of our results in Section 3.5.

Our model also offers some insights into the fee structures commonly found in mutual funds and hedge funds. In the mutual fund industry, the overwhelming fee of choice is a flat fraction-of-funds fee with no explicit performance-adjustment component (see Appendix B for more details). In hedge funds, which are not subject to the fulcrum fee requirement and which tend to use leveraged strategies, the most popular fee structure is an incentive fee with a large performance component. These are exactly the equilibrium fee structures that are predicted by our model. In the absence of leveraging, we find that the equilibrium fulcrum fee is always a flat fee with no performance component. When incentive fees are allowed and leveraging is permitted, we find that the equilibrium fee is an incentive fee with a large performance component. The intuitive arguments underlying these results again derive from risk-sharing considerations and are discussed further in Section 3.5.

2 The Related Literature

This section provides a brief discussion of the theoretical literature on compensation structures in the mutual fund industry. The presentation here is meant to be indicative of the work that has been done in this area and not as a survey of the field.

Broadly speaking, there are two branches to the literature on mutual fund compensation. On the one hand are the papers that take a partial equilibrium approach and examine the reaction of managers to a *ceteris paribus* change in the fee structure. On the other hand are the papers that adopt a "full" equilibrium approach, solving for compensation structures as part of an equilibrium. Papers falling into the first group include Davanzo and Nesbit [3], Ferguson and Lestikow [4], Goetzmann, Ingersoll and Ross [7], Grinblatt and Titman [8], Grinold and Rudd [9], and Kritzman [12]. Those falling into the second group include Heinkel and Stoughton [10], Huddart [11], and Lynch and Musto [15]. Finally, there is the recent paper of Admati and Pfleiderer [1] which combines aspects of both approaches. We discuss some of these papers in more detail below.

Of the first category of papers, a comprehensive analysis is carried out in Grinblatt and Titman [8]. Grinblatt and Titman assume that managers can risklessly capture the value of any options implicit in their payoff structure by hedging in their personal portfolios. This enables the use of results from option pricing theory in characterizing the optimal (i.e., fee maximizing) level of risk for any given contract structure. Among other things, Grinblatt and Titman show that for certain classes of portfolio strategies, adverse risk-sharing incentives are avoided when the penalties for poor performance outweigh the rewards for good performance.

Heinkel and Stoughton [10] aim to explain the predominance of fraction-of-funds fee arrangements in the asset-management industry (including, but not only, mutual funds). They employ a two-period model with heterogeneous types of managers, in which moral hazard is also present. Under some assumptions, the authors show that the optimal initial set of contracts features a smaller performance-based fee in the first period than in a first-best contract. They suggest that this reduced emphasis on the performance component in the first period is analogous to the lack of a performance-based fee in many parts of the asset-management industry.

Huddart [11] builds on the Heinkel-Stoughton model by dropping the assumption that managers are risk-neutral, and by introducing competing fund managers. He examines the problem in which the investor must decide which fund to invest in under the assumption that fees are exogeneously fixed at some proportion of assets under management. However, Huddart does show that the adoption of a performance fee can mitigate undesirable reputation effects and result in investors being ex-ante better off.

Lynch and Musto [15] aim to explain the fee-structures commonly found in mutual funds and hedge funds. They employ a moral hazard model in which the manager's effort is observable by the investor, but is not contractable (i.e., cannot be used as legal evidence). The manager commits to an effort level; observing this, the investor then decides on the amount of money to be invested in the fund. Lynch and Musto identify conditions in this model in which different fee structures predominate.

Our objectives evidently differ from those of Heinkel and Stoughton [10] and Lynch and Musto [15]. Our model is also different in several respects. For example, the choice of fee structure in our model is made exclusively with the fund/investment adviser. Secondly, in contrast to both papers, our model involves the central use of a "benchmark" portfolio; as we have already mentioned, we regard this as an important part of our approach. However, unlike Heinkel and Stoughton [10], our model is a static one and does not involve adverse selection considerations.

Admati and Pfleiderer [1] consider a scenario where the fund manager has superior information to the investor and faces a fulcrum fee structure. Their aim is to examine whether there are any conditions under which the manager would pick the investor's most desired portfolio (i.e., the portfolio that the investor would have chosen had he been possessed of the same information as the manager). There are superficial similarities between this question and that motivating our paper, but there are some fundamental differences in the analyses. First, the issue studied by Admati and Pfleiderer is the desirability of benchmarking within a fulcrum fee structure; they do not, for instance, consider incentive fee structures. We, on the other hand, take benchmarking as a given, and compare the effects of different fee structures on equilibrium payoffs. Second, Admati and Pfleiderer are not explicitly concerned with determining equilibrium fee structures and portfolio allocations. Thus, for example, they take the amount invested with the manager as exogeneous; they also compute the investor's most desired portfolio by using gross returns rather than returns net of the manager's fees. Finally, the presence of asymmetric information is central to the Admati–Pfleiderer paper, while our paper, as mentioned above, involves a symmetric information setting.

The empirical literature on the impact of different fee structures on fund performance and equilibrium risk levels is somewhat limited. Baumol, et al [2] and Lakonishok, Shleifer, and Vishny [13] have each documented the prevailing payoff structures and the extent of variation in these structures. There have also been a few direct econometric studies of the performance-fee issue, including Golec [5], Golec [6], and Lin [14]. All three of these studies find that fulcrum fees are typically used only by large (well-capitalized) firms, and, more importantly, that funds with fulcrum fees on average outperform those without such fees. However, while Golec [5] finds a significant performance differential, Lin [14] does not.

3 The Model

There are two players in our model, an investor and a fund. The investor, a representative stand-in for a large number of identical investors, has an initial wealth of $w > 0$. The investor's objective is to choose an allocation of this amount between a riskless asset and the fund[3] so as to maximize his utility $U(\widetilde{w}_T)$ from his wealth $\widetilde{w}_T$ at the end of the model's single period.[4] We assume throughout that $U(\cdot)$ has a mean-variance form:

$$U(\widetilde{w}_T) \;=\; E(\widetilde{w}_T) - \frac{1}{2}\gamma V(\widetilde{w}_T), \tag{3.1}$$

where $E(\widetilde{w}_T)$ and $V(\widetilde{w}_T)$ denote, respectively, the expectation and variance of $\widetilde{w}_T$, and $\gamma > 0$ is a parameter signifying the investor's aversion to variance. We also assume that the amount x invested in the fund must be non-negative, i.e., that the investor cannot short the fund.

[3]We are making an implicit assumption here that the investor does not have the same investment opportunities as the fund and must go through the fund to obtain these returns. This may be justified by an appeal to, for example, transactions costs.

[4]The assumption of a one-period horizon may be a limitation of our setting. However, while we remain curious about the impact of a multi-period investment horizon, we do not believe our results will be substantially altered.

The net return on the riskless asset is normalized to zero throughout the paper. The return to the investor from the fund depends on two factors: the fee structure adopted by the fund (which determines the after-fees returns to the investor), and the return characteristics of the fund portfolio. The investor takes both of these factors as given in determining his portfolio allocation; we discuss each factor in more detail below.

Fees

The fees charged by the fund may depend on the realized returns r_p on the fund portfolio, as well as on the realized returns r_b on a "target" or "benchmark" portfolio.[5] The fees, denoted $F(r_p, r_b)$, are received at the end of the period, and are stated as fees per dollar invested in the fund. Thus, if the amount invested in the fund is x, the dollar fees received by the fund are $x \cdot F(r_p, r_b)$, and the amount received by the investor is $x \cdot (r_p - F(r_p, r_b))$.

Return Distributions

The return characteristics of the fund portfolio depend on the fund's action a. Let $\widetilde{r}_p(a)$ denote the returns per dollar invested in the fund portfolio given a. Throughout the paper, returns are stated in gross terms, i.e., as one plus the net return.

The description of the returns distributions must encompass two aspects: (i) for each action a, the relationship between the returns on the fund portfolio $\widetilde{r}_p(a)$ and the benchmark portfolio $\widetilde{r}_b$, and (ii) the distribution for the benchmark returns $\widetilde{r}_b$. Concering the former, we adopt in this section the setting of Grinblatt and Titman [8] and take the action a to denote the fraction of initial asset value invested in the benchmark portfolio, with the remainder invested at the riskless rate. Thus, the returns $\widetilde{r}_p(a)$ on the fund portfolio under the action a are given by

$$\widetilde{r}_p(a) \;=\; a\widetilde{r}_b + (1 - a). \tag{3.2}$$

This specification of the fund portfolio returns is, of course, a simple one. In Sections 4 and 5, we enrich it in two directions: by allowing for fund "effort" to improve return distributions and by allowing for multiple risky securities. For two important reasons, nonethless, it makes sense to begin with the model (3.2). First, the intuitive arguments underlying the results of our paper are transparent in this simple framework (see Section 3.5). Second, we are able to obtain closed-form solutions for equilibrium strategies in this setting to confirm the intuitive arguments; this is difficult in the more complex settings of Sections 4 and 5.

[5] The choice of benchmark may itself be a strategic issue. We do not model this choice here.

The set A of all actions is taken to be an interval $[0, a^{\max}]$; we allow the possibility that $a^{\max} \geq 1$. Finally, throughout this section, we assume that the benchmark returns $\tilde{r}_b$ follow a trinomial distribution:[6]

$$\tilde{r}_b = \begin{cases} 1 + \pi_h, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1 - \pi_l, & \text{with probability } 1/3 \end{cases} \tag{3.3}$$

where $\pi_h, \pi_l \in (0, 1)$ are constants satisfying $\pi_h > \pi_l$. We will refer to the three states as the high, middle, and low states, subscripted wherever necessary by h, m, and l, respectively. Note that the expected net return from the benchmark portfolio is $(\pi_h - \pi_l)/3$ which is strictly positive. From (3.2) and (3.3), we obtain

$$\tilde{r}_p(a) = \begin{cases} 1 + \pi_h a, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1 - \pi_l a, & \text{with probability } 1/3 \end{cases} \tag{3.4}$$

To ensure that portfolio values remain non-negative, we must have $a^{\max} \leq 1/\pi_l$. For later analytical convenience, we make the stronger assumption that that $a^{\max} \leq 1/2\pi_l$.

Equilibrium

Let F_h, F_m, and F_l denote the fee charged by the fund in the three states per dollar of asset value in the fund at that state, and let $F = (F_h, F_m, F_l)$. Then, the distribution of returns to the investor (per dollar invested in the fund) net of the fund's fees is given by

$$Y = \begin{cases} Y_h = 1 + \pi_h a - F_h, & \text{with probability } 1/3 \\ Y_m = 1 - F_m, & \text{with probability } 1/3 \\ Y_l = 1 - \pi_l a - F_l, & \text{with probability } 1/3 \end{cases} \tag{3.5}$$

Let $E(Y)$ denote the expectation of these returns, and $V(Y)$ their variance. Since the riskless asset has a net return of zero, the expected terminal wealth of the investor from investing x in the fund is simply $(w - x) + xE(Y)$, and the variance of this terminal wealth is $x^2 V(Y)$. Thus, the investor chooses x to solve

$$\max_x \left\{ [(w - x) + xE(Y)] - \frac{1}{2}\gamma x^2 V(Y) \right\}.$$

[6] The linear structure of fulcrum fees (see below) makes it posssible to analyze them under other distributions also (e.g., normal), but this is, unfortunately, not the case with incentive fees. Nonetheless, the inutition underlying our results appears robust, and we do not believe they depend in an essential way on the trinomial assumption.

The objective function is a strictly concave function of x. First-order conditions are, therefore, necessary and sufficient for a maximum, and this yields the solution[7]

$$x^* = \frac{E(Y) - 1}{\gamma V(Y)}. \tag{3.6}$$

Observe that x^* depends on the choice of action a taken by the fund as well as the fee structure F it adopts, through the dependence of Y on these quantities. When necessary, we shall write $x^*(a, F)$ to emphasize this.

The fund is assumed to be risk-neutral.[8] Given a choice of (a, F), the expected fee EF received by the fund is given by

$$EF = \frac{1}{3}[F_h + F_m + F_l] \cdot x^*(a, F). \tag{3.7}$$

The fund picks a and its fee structure F so as to maximize this expected fee. These optimal choices determine the equilibrium payoffs to the two players.

Before proceeding, we should note here the change in perspective here from the usual principal/agent framework. In the latter, the investor in his role as principal plays the role of the "large player" (i.e., the Stackelberg leader) and maximizes his payoffs taking into account the reactions of the agent (the investment adviser). In our model, the fund/investment adviser is the Stackelberg leader and maximizes its payoffs taking into account the investor's reactions.[9] We note also that unlike the standard approach, there is no exogeneously specified reservation utility criterion that must be met. In a sense, this role is played in our model by the alternative investment opportunity available to the investor, but the analogy is not quite perfect. In a principal/agent model, the agent typically receives his reservation utility in equilibrium, while we show that the investor in our model receives strictly higher expected utility in equilibrium than he could obtain by investing all his funds in the riskless asset.

Fee Structures of Special Interest

There are two fee structures of special importance for the material that follows. The first, the so-called *fulcrum fees*, have the property that the fees per dollar invested in the fund

[7] We ignore the constraints $x \geq 0$ and $x \leq w$. The former is without loss of generality; it can never arise as an equilibrium outcome in the models we study. The latter is potentially more important. It turns out, however, that our results are qualitatively unaffected if we include this constraint; moreover, for most reasonable parameter values, the constraint is not binding in the numerical examples we develop.

[8] This assumption is made in the interests of analytical simplicity, but it does not appear to make a qualitative difference to the main results; see Appendix D.

[9] Note that in both cases the player identified with the fund is the leader. The difference, of course, is that we identify the fund with the adviser, the standard approach identifies it with the investor.

are symmetric in the fund's performance relative to the benchmark: they increase for outperforming the benchmark in the same way that they decrease for underperforming it. We consider only linear fulcrum fees; these are by far the most common types used in practice. Such fees are described by

$$F(r_p, r_b) \;=\; b_1 r_p + b_2(r_p - r_b), \tag{3.8}$$

where b_1 and b_2 are non-negative constants denoting, respectively, the base fee and the performance adjustment component. When $b_2 = 0$, the fees are simply a constant fraction b_1 of the total returns r_p; such fees are called "flat fees" or "fraction-of-funds" fees.

In practice, when fulcrum fees are employed, it is almost invariably the case that a floor (and, by the symmetry requirement, a corresponding cap) are placed on the size of the performance adjustment component (see Appendix B). We adopt such a restriction; to simplify notation, we use zero as the floor value, and require that the realized fees F always be non-negative. The floor of zero implies, by symmetry, a cap of $2b_1 r_p$ for fees, and so our final form for fulcrum fees is:

$$F(r_p, r_b) \;=\; \max\{0, \min\{b_1 r_p + b_2(r_p - r_b), 2b_1 r_p\}\}. \tag{3.9}$$

The second class of fees of importance are (asymmetric) *incentive fees*. Like fulcrum fees, incentive fees are described by two parameters b_1 and b_2, with b_1 denoting the base fee level, and b_2 the performance adjustment component. However, unlike fulcrum fees, the performance adjustment component must remain non-negative, and the total fee is given by

$$F \;=\; b_1 r_p + b_2 \max\{r_p - r_b, 0\}. \tag{3.10}$$

In the analysis that follows, we compare equilibrium outcomes under three settings: when the fund is limited to using only fulcrum fees, when it is limited to using only incentive fees, and when it is unrestricted in its fee choices. Our decision to compare the outcomes under a fulcrum fee regime to not just the unrestricted model, but also to a restricted model in which only incentive fees are used is based on two considerations. First, the existing legislation on mutual fund fees is motivated explicitly by fear of the consequences of *incentive* fee structures. Second, from a practical standpoint, incentive fee structures are commonly used in relationships between investors and investment advisers where they are legal (e.g., in hedge funds). In contrast, as is well known, unrestricted equilibrium contracts in principal/agent models often take on unrealistically complex and unintuitive forms. Consequently, we view the unrestricted fee structures more as an idealized benchmark, than as a feasible alternative. Nonetheless, the properties of the unrestricted fees in equilibrium are of considerable use in understanding the intuitive underpinnings of our results, as we explain in Section 3.5.

3.1 Equilibrium under Fulcrum Fees

Given the fund's choice of (b_1, b_2) and its portfolio choice a, a little algebra shows that the fees F_h, F_m, F_l received by the firm in the three states are given by

$$\begin{aligned} F_h &= \max\{0, \min\{2b_1(1+\pi_h a), b_1 + b_2\pi_h(a-1)\}\} \\ F_m &= b_1 \\ F_l &= \max\{0, \min\{2b_1(1-\pi_l a), b_1 - b_2\pi_l(a-1)\}\} \end{aligned} \tag{3.11}$$

Using this together with (3.5), the net-of-fees return distribution to the investor may be computed for any choice of (b_1, b_2, a). In principle, solving for the equilibrium is now a straightforward process. Using the distribution of net-of-fees returns, we can identify, via (3.6), the optimal x^* for the investor as a function of (a, b_1, b_2). Then, we use this optimal choice and expression (3.11) for the fees to obtain the expected fee in terms of (a, b_1, b_2). Maximizing this expected fee then yields the optimal choices of (a, b_1, b_2), and thereby the players' equilibrium payoffs.

In practice, because $a^{\max}$ is unspecified, the last step is tricky. We use, therefore, a two-step procedure. First, we hold the portfolio choice a fixed at some arbitrary level, and identify the equilibrium payoffs for the fund and the investor for this fixed a. Then, for any possible value of $a^{\max}$, we identify the value of $a \in [0, a^{\max}]$ that maximizes the fund's equilibrium payoffs.

The following proposition summarizes equilibrium outcomes when a is held fixed. It may also be of some independent interest insofar as it relates equilibrium fee structures to the ability to leverage.

Proposition 3.1 *For fixed a, the equilibrium fulcrum fee structure is as follows:*

1. *If $a \leq 1$, then the equilibrium fulcrum fee contract is a flat fee contract, i.e., we have $b_1 > 0$ and $b_2 = 0$. A closed-form expression for b_1 is given by (C.7) in Appendix C.1.*

2. *If $a > 1$, then b_1 and b_2 are both positive. Closed-form expressions for b_1 and b_2 are given by (C.20) and (C.21), respectively, in Appendix C.1.*

Proof See Appendix C.1. □

An intuitive explanation of all the results of this section is provided in Subsection 3.5. Using the equilibrium values of (b_1, b_2), we can derive closed-form expressions for the investor's equilibrium expected utility and the fund's equilibrium expected fees for a fixed value of a. Letting $\xi = \pi_h^2 + \pi_h\pi_l + \pi_l^2$, these expressions are given by the following. When $a \leq 1$:

$$EU = \frac{(1+16\gamma)\xi - 3\pi_h\pi_l}{16\gamma\xi} \tag{3.12}$$

$$EF = \frac{[3 + a(\pi_h - \pi_l)]\,(\pi_h - \pi_l)^2}{24\gamma\xi} \tag{3.13}$$

If $a > 1$, then the expressions are

$$EU = \frac{(1+16\gamma)\xi - 3\pi_h\pi_l}{16\gamma\xi} \tag{3.14}$$

$$EF = \frac{(\pi_h - \pi_l)^2(2a\pi_h^2 - \pi_l + 2\pi_h(2 - a\pi_l))}{24\gamma\pi_h\xi} \tag{3.15}$$

Observe that the investor's equilibrium payoffs do not depend on a in any way. Some simple calculation also reveals that the fund's equilibrium payoffs are strictly increasing in a for both $a \leq 1$ as well as for $a > 1$. Moreover, a comparison between the equilibrium expected fee when $a \leq 1$ and $a > 1$ reveals that the latter is always strictly larger. It follows immediately that

Proposition 3.2 *In a fulcrum fee regime, the optimal portfolio choice for the fund is always* $a = a^{\max}$.

1. *If* $a^{\max} \leq 1$, *the equilibrium fee is a flat fee with* b_1 *given by (C.7). The investor's expected utility level and the fund's expected fees in equilibrium are given by (3.12) and (3.13), respectively.*

2. *If* $a^{\max} > 1$, *the equilibrium fee structure is given by (C.20)-(C.21). The investor's expected utility level and the fund's expected fees in equilibrium are given by (3.14) and (3.15), respectively.*

Proof See Appendix C.2. □

3.2 Equilibrium under Incentive Fees

Under incentive fees, the fund picks a base fee b_1 and a performance-dependent fee b_2. Given the action a, the fee to the fund is

$$F = b_1 \tilde{r}_p(a) + b_2 \max\{0, \tilde{r}_p(a) - \tilde{r}_b\}. \tag{3.16}$$

Using the expressions (3.3) and (3.4) for the benchmark and fund portfolio returns, respectively, we can see that whether the fund portfolio outperforms the benchmark portfolio in a particular state depends on the level of a. If $a < 1$, then the fund's fees are distributed as

$$\begin{aligned} F_h &= b_1(1 + \pi_h a) \\ F_m &= b_1 \\ F_l &= b_1(1 - \pi_l a) - b_2 \pi_l (a - 1) \end{aligned} \tag{3.17}$$

On the other hand, if $a > 1$, then the fund's fees are distributed as

$$\begin{aligned} F_h &= b_1(1 + \pi_h a) + b_2 \pi_h (a - 1) \\ F_m &= b_1 \\ F_l &= b_1(1 - \pi_l a) \end{aligned} \tag{3.18}$$

The net-of-fees returns to the investor may be obtained from these fees using expression (3.5). To identify the equilibrium, we proceed now as in the fulcrum fee case. We use the distribution of net-of-fees returns to identify the optimal amount invested in the fund for each (a, b_1, b_2). Using this, we solve for the optimal (a, b_1, b_2) by first identifying equilibrium payoffs for a fixed a, and then solving for the optimal a. The following result describes the equilibrium fee structure for each fixed a:

Proposition 3.3 *For fixed a, the equilibrium incentive fee structure is as follows:*

1. *When $a \leq 1$, the equilibrium incentive fee contract is a flat-fee contract with $b_1 > 0$ and $b_2 = 0$. The base fee b_1 is the same as in the fulcrum-fee case, and is given by expression (C.7) in Appendix C.1.*

2. *When $a > 1$, the equilibrium incentive fee contract is a pure performance fee, i.e., $b_1 = 0$ and $b_2 > 0$. A closed-form expression for b_2 is given by (C.25) in Appendix C.3.*

Proof See Appendix C.3. □

These fee structures may be used to solve for the equilibrium payoffs to the players for each fixed a. Since the equilibrium fee structure under incentive fees is the same as that which obtains under fulcrum fees when $a \leq 1$, equilibrium levels of expected utility and expected fees in this case are given by (3.12) and (3.13), respectively. When $a > 1$, equilibrium levels of expected utility and expected fees are given by

$$EU = \frac{\left(\xi(1+8\gamma) + 3\pi_l^2 - 2\sqrt{3\xi}(\pi_l - 2\pi_h)\right)}{4\gamma\left(2\xi + \pi_h\sqrt{3\xi}\right)} \tag{3.19}$$

$$EF = \frac{\left(\sqrt{3\xi} - 3\pi_l\right)\left(\xi - \pi_l\sqrt{3\xi}\right)}{6\gamma\pi_l\left[2\xi + \pi_h\sqrt{3\xi}\right]} \tag{3.20}$$

Observe that in this case the equilibrium payoffs to both the investor and the fund are independent of a. Comparing the expected fees for the fund when $a > 1$ to that which obtains when $a < 1$ establishes that the former is strictly larger, and therefore, that:

Proposition 3.4 *In an incentive fee regime:*

1. *When $a^{\max} \leq 1$, the optimal portfolio choice for the fund is $a^* = a^{\max}$. The equilibrium fee structure and payoffs are the same as in the corresponding fulcrum fee case, and are given by (C.7), (3.12) and (3.13), respectively.*
2. *When $a^{\max} > 1$, any action $a > 1$ is an optimal portfolio choice for the fund. The optimal fee structure is a pure performance fee with $b_1 = 0$ and b_2 given by (C.25) in Appendix C.3. The equilibrium levels of expected utility and expected fees are given by (3.19) and (3.20), respectively.*

Proof See Appendix C.4. □

3.3 Comparison of the Equilibrium Outcomes

Having identified the equilibrium outcomes under both regimes, we turn now to a comparison of these outcomes. We employ three criteria: (i) the investor's expected utility in equilibrium, (ii) the fund's expected fees in equilibrium, and (iii) the standard deviation, or "volatility," of the net-of-fees returns to the investor in equilibrium.

We have already described closed-form expressions for the expected utility and expected fees in equilibirum under either regime for any fixed a. We now do the same for equilibrium volatilities. As above, let $\xi = \pi_h^2 + \pi_h\pi_l + \pi_l^2$. In a fulcrum fee regime, when $a < 1$, equilibrium volatility is given by

$$\sigma(R) = \frac{2a\xi\sqrt{2}}{6 + a(\pi_h - \pi_l)}, \tag{3.21}$$

while if $a > 1$, we have

$$\sigma(R) = \frac{2\sqrt{2}a\pi_h\xi}{7\pi_h + 2a\pi_h^2 - \pi_l - 2a\pi_h\pi_l}. \tag{3.22}$$

Equilibrium outcomes under an incentive-fee regime coincide with those under a fulcrum fee regime if $a < 1$. Consequently, volatility of returns under incentive fees when $a < 1$ are also given by (3.21). When $a > 1$, the equilibrium volatility under an incentive fee regime is

$$\sigma(R) = \frac{a\pi_l(4\xi + \pi_h\sqrt{6\xi})}{\sqrt{3}(\pi_h + 2\pi_l)}. \tag{3.23}$$

With these closed-form solutions in hand, we are in a position to prove the main result of this section:

Proposition 3.5 *If $a^{\max} \leq 1$, then equilibrium outcomes under incentive fees are identical to those under fulcrum fees. If $a^{\max} > 1$:*

1. *The investor's equilibrium expected utility is strictly higher in an incentive fee regime than in a fulcrum fee regime.*
2. *Equilibrium volatility of gross returns (i.e., of the fund portfolio) is never higher under an incentive-fee regime than under a fulcrum fee regime.*
3. *Provided a mild parameter restriction is satisfied:*
   (a) *The fund's expected fee is also strictly higher in an incentive fee regime.*
   (b) *The volatility of net-of-fees returns to the investor is strictly smaller under an incentive fee regime.*

Remark The required restriction on parameter values is stated precisely in the proof of the proposition. The restriction is a very mild one, and is effectively just a requirement that the ratio π_h/π_l not be too close to unity. Of course, if $\pi_h/\pi_l = 1$, then the benchmark portfolio would be dominated by the riskless asset, since it would then provide the same expected return at a higher volatility. Note that the investor is always better off under incentive fees whether or not this parameter restriction is met. □

Proof See Appendix C.5. □

Tables 1 and 2 provide a numerical illustration of Propositions 3.1–3.5. We take a specific parametrization of the benchmark returns, and present the equilibrium outcomes for various values of a ranging between $a = 0.25$ and $a = 2$. Each table presents six quantities of interest: (i) the values of b_1 and b_2 in the equilibrium fee structure, (ii) the expected fee EF received by the fund, (iii) the amount of wealth x allocated to the fund, (iv) the expected net-of-fees returns EY to the investor, (v) the volatility $\sigma(Y)$ of this return, and (vi) the investor's expected utility EU.[10]

Since the equilibria under the two regimes coincide when $a^{\max} \leq 1$, we focus in this paragraph on the case $a^{\max} > 1$. The tables illustrate the Pareto improvement that takes place under incentive fees in this case. For the parameterizations used, the investor's utility is a little bit higher under incentive fees than under fulcrum fees; the fund's payoff is substantially improved. For any fixed $a > 1$, the amount x invested in the fund is also larger under an incentive fee than under a fulcrum fee. Finally, the expected net-of-fees return to the investor is smaller under an incentive fee; but the volatility is lower, sufficiently lower, in fact, that the investor's overall welfare is improved.

3.4 Equilibrium when Fee Structure is Unrestricted

Several other fee structures than the fulcrum- and incentive-fee mechanisms may be envisaged. The most general of these is one in which the fund elects to charge a different fee in each state of the world, i.e., in which the choices of F_h, F_m, and F_l are totally unrestricted. Such unrestricted fee structures often lead to unrealistically complex state-dependence in contracting problems. Nonetheless, it is worthwhile considering this case for completeness, and also to be able to relate the earlier outcomes to it.

Table 3 presents the solutions obtained in the unrestricted case for the same parametrization as used in Tables 1 and 2. These solutions were obtained using an optimization package.

[10]It should be evident that we mean these tables to be illustrative; the numbers they present should be viewed qualitiatively rather than literally.

As the table reveals, the unrestricted optimal fee structure involves positive fees in only a single state of the world, with zero fees in the other two states. In particular, base or guaranteed fees are not optimal in this case.

More interesting is the comparison with the restricted fee structures. When $a \leq 1$, the unrestricted equilibrium fee structure differs from the equilibrium fee structure under both the fulcrum-fee and incentive-fee regimes. However, when $a > 1$, outcomes under the unrestricted fee coincide with those under incentive-fees. We examine the possible reasons for this in the next subsection.

3.5 An Intuitive Explanation of the Results

The results of this section may be understood from an intuitive standpoint without recourse to the specific trinomial model we have examined. Of course, such an informal explanation will necessarily be incomplete, but it serves to highlight the driving factors behind the model.

Consider first of all the model in which fee structures are unrestricted. In this case, one would expect that the fees taken by the fund in equilibrium would be increasing—or, at least, non-decreasing—in the total returns r_p on the fund portfolio. This would reduce the variance of net-of-fees returns to the risk-averse investor, improving risk-sharing and causing a larger amount to be invested with the fund. Section 3.4 confirms this intuition in the trinomial model.

In the presence of benchmarking, however, the fees depend not only on the fund returns r_p, but also on the *difference* between r_p and the benchmark returns r_b. Since the two returns are related by $r_p = ar_b + (1-a)$, this difference is given by[11]

$$r_p - r_b = (a-1)(r_b - 1) \tag{3.24}$$

Now, r_p and r_b are strictly monotonically related: as one increases, so does the other. However, as (3.24) shows, if $a < 1$, the difference $(r_p - r_b)$ *decreases* with an increase in r_p. Thus, the performance adjustment component of the fees (i.e., the term $b_2(r_p - r_b)$) also *decreases* as r_p increases. This is obviously inefficient in view of the arguments of the previous paragraph, so it becomes optimal for the fund to simply set the term b_2 to zero. Thus, flat fees result under benchmarking when $a \leq 1$. Note that this argument holds for both fulcrum and incentive fees.

When $a \geq 1$, however, (3.24) implies that the difference between the returns also increases as r_p increases. Thus, with a positive b_2, the fund can achieve a fee structure qualitatively

[11] It suffices for the intuitive arguments that follow that (3.24) hold in expectation. That is, the model of returns could be of the form $r_p = ar_b + (1-a) + \epsilon$, where r_b and ϵ are uncorrelated and ϵ has mean zero.

similar to the unrestricted one. Under incentive fees, this process is further facilitated by the asymmetric structure of the performance-adjustment component; in the trinomial model of this section, in fact, an exact correspondence becomes possible between the unrestricted solution and the incentive fee solution. With fulcrum fees, however, the symmetric nature of the fulcrum limits the extent to which the gap may be narrowed, except in models where the unrestricted fee is itself symmetric around the benchmark (the current model is evidently not one such). Hence, fulcrum fee outcomes are inferior to incentive fee outcomes when $a > 1$, while both are inferior to the unrestricted outcome when $a \leq 1$.

4 Introducing Effort into the Model

We now build on the model of the previous section by allowing for the possibility that, apart from the choice of portfolio, the fund can also affect return characteristics by expending costly effort. This effort leads to information about the likely state of the world (high medium, or low) concerning the return on the benchmark portfolio. Based on this information, the fund chooses the appropriate portfolio mix. Greater effort leads to more accurate information but at a higher cost.

The details of the framework we study are as follows. The prior distribution of the three states is as used in Section 3: that is, the benchmark portfolio returns are given by

$$\tilde{r}_b = \begin{cases} 1 + \pi_h, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1 - \pi_l, & \text{with probability } 1/3 \end{cases}$$

If the fund takes the effort level $e \in [0, 1]$, then with probability $p(e)$, it identifies which state of the world (high, medium, or low) will actually occur. With the complementary probability $(1 - p(e))$ it receives no information at all, so the prior distribution is also the posterior distribution. Finally, expending an effort of e has a cost to the fund of $\kappa(e)$. Throughout this section, we will presume that $p(\cdot)$ and $\kappa(\cdot)$ are given by

$$p(e) = \sqrt{e}, \qquad \kappa(e) = ke^2, \tag{4.1}$$

where $k > 0$ is a given constant. Under (4.1), the functions $p(\cdot)$ and $\kappa(\cdot)$ are both increasing, reflecting the fact that higher effort reveals more information but at a higher cost. Moreover, the the strict concavity of $p(\cdot)$ ensures that effort has a declining marginal benefit, while the convexity of $\kappa(\cdot)$ implies an increasing marginal cost of effort.

The order of moves is as follows. The firm announces (i.e., precommits to) a fee structure F and an effort level e. Based on these, the investor decides on the amount x to be invested with the fund. Upon observing the outcome of its effort decision, the fund then chooses a portfolio mix, and final rewards are realized. As earlier, we assume that the fraction a of initial asset value that the fund invests in the risky asset must satisfy $a \in [0, a^{\max}]$, where $a^{\max}$ is a given constant.[12]

This model is a stylized one, but it has a number of attractive features. It captures the fact that there are two possible inputs into returns generation: portfolio choice (which could be passive) and active management such as gathering information that could improve portfolio choice. In our model, the fund can choose to simply generate the benchmark returns by setting $a = 1$ and taking zero effort. Alternatively, it could take positive effort ($e > 0$), which would lead to valuable information for portfolio allocation a fraction e of the time. Indeed, under the specification (4.1), an effort level of $e = 1$ would lead to perfect information that would enable the fund to tie or beat market returns *all* of the time (although the cost of doing so could make this a suboptimal action).

Once again, we restrict attention to only fulcrum and incentive fee structures. Our aim is to obtain and compare equilibria under the two structures using the same criteria as used earlier: the equilibrium expected utility of the investor, the equilibrium expected fees received by the fund (now measured net of the cost of effort), and the volatility of equilibrium returns to the investor.

Solving this problem involves a backwards induction argument. We first identify, for given values of the fee structure (b_1, b_2), the effort level e, and investment in the fund x, the portfolio choice of the fund for each possible realization of the information event. Then, taking these choices and the fee structure and effort level as given, we solve for the investment level that maximizes the investor's expected utility. Finally, taking into account the investor's reaction to different fee structures and effort levels, we solve for the levels that maximize the fund's expected fees.

The large number of decision variables and the various non-linearities in the model make this a non-trival problem. The solution is, however, simplified by the following observations. If the firm receives the information that the high state is going to occur, then its optimal action is evidently to shift all its resources into the benchmark portfolio, so we will have $a = a^{\max}$ in this case. If it receives information that the medium or low states will occur, then it is optimal to invest entirely in the riskless asset, so we will now have $a = 0$. Finally, if it receives no information at all, then it is easy to see from the fund's risk-neutrality that

[12] Note that this model differs slightly from the model of the previous section in that the fund is no longer required to precommit to a portfolio choice. Rather, it precommits to an effort level and waits for the realization of the information event before deciding on its asset mix.

$a = a^{\max}$ is always an optimal choice under either fee structure.

Unfortunately, even with this simplification, the model remains too complex to solve in closed form. We solved it numerically for a number of different parameter configurations. The results for two of these are reported in Tables 4 and 5. The tables reveal that the important qualitiative results of our earlier model survive essentially intact:

1. Concerning the structure of equilibrium contracts:
   (a) When $a^{\max} \leq 1$, the equilibrium fee contract under both fulcrum and incentive fees is a flat fee without a performance adjustment component.
   (b) When $a^{\max} > 1$, a positive performance-adjustment component arises in both cases.
2. Concerning the equilibrium outcomes:
   (a) When $a^{\max} \leq 1$, equilibrium outcomes under the two regimes coincide along all dimensions.
   (b) When $a^{\max} > 1$, outcomes under an incentive fee structure Pareto-dominate those under a fulcrum fee structure.

Finally, for any fixed $a^{\max}$, equilibrium payoffs for both parties decline as the cost of effort increases. In the limit, when effort becomes infinitely costly, the fund never expends any effort, so the optimal payoffs are exactly those obtained in Section 3 with $a = a^{\max}$.

Our numerical analyses also revealed an interesting trade-off between leveraging and effort levels. When a fund employs incentive fees, it is already protected from downside risk, so the benefits from expending effort accrue mostly from the increased upside it can derive from knowledge of the true state. This upside benefit is limited in turn by the extent of leveraging available to the fund (i.e., by the size of $a^{\max}$). When $a^{\max}$ is only slightly larger than unity, the upside benefit is restricted, so for some parameter values the fund simply takes no effort at all. This is not the case under a fulcrum fee, where extra effort not only increases the upside but also limits downside losses. Thus, for some parameter values involving low leveraging abilities, we found that the investor was slightly better off under fulcrum fees, though the fund remained significantly better off under incentive fees.

5 Introducing Additional Sources of Risk

A second possible criticism of the model of Section 3 is that under the strategies available to the fund, the return on the fund portfolio is always perfectly correlated with the return on the benchmark portfolio. In this section, we consider a generalization of the model that admits the possibility of imperfect correlation. We consider a four-state model with two risky securities and one riskless security. The riskless security will, as usual, be assumed to have a net return of zero. Letting (μ_1, μ_2) denote the net expected returns on the two risky securities and (σ_1, σ_2) their volatilities, the gross return on the two risky securities in the four states are assumed to be given by:

	Security 1	**Security 2**
State 1	$1 + \mu_1 + \sigma_1$	$1 + \mu_2 + \sigma_2$
State 2	$1 + \mu_1 + \sigma_1$	$1 + \mu_2 - \sigma_2$
State 3	$1 + \mu_1 - \sigma_1$	$1 + \mu_2 + \sigma_2$
State 4	$1 + \mu_1 - \sigma_1$	$1 + \mu_2 - \sigma_2$

Finally, the probabilities q_i of the four states are taken to be

$$q_1 = q_4 = \frac{1+\rho}{4} \qquad q_2 = q_3 = \frac{1-\rho}{4}$$

where $\rho \in (-1, 1)$. It is easily checked under this specification of returns and probabiltiies that the expected return and volatility of security i are, indeed, given by μ_i and σ_i; and that ρ is the correlation between the returns on the two securities.

Throughout this section, we concentrate on a symmetric version of this model where it is assumed that $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma$. (The correlation ρ remains unrestricted.) The benchmark portfolio is defined to be an equally weighted portfolio of the two securities. Thus, denoting by $\tilde{r}_1$ and $\tilde{r}_2$ the returns on the two securites, the benchmark returns $\tilde{r}_b$ are given by

$$\tilde{r}_b = \frac{1}{2}(\tilde{r}_1 + \tilde{r}_2).$$

Finally, it remains to specify the portfolio strategies available to the fund. Let a_1 and a_2 denote the fractions of initial asset value invested by the fund in each of the two risky securities, with the balance $(1 - a_1 - a_2)$ denoting the fraction invested in the riskless asset. We assume that a_1 and a_2 must be non-negative, and that there is a maximum leverage available to the fund, i.e., there is some $a^{\max} \geq 1$ such that any feasible pair (a_1, a_2) must satisfy

$$0 \leq a_1 \leq a_1 + a_2 \leq a^{\max}.$$

Note that given any choice of (a_1, a_2), the returns $\widetilde{r}_p(a)$ on the fund portfolio are given by

$$\widetilde{r}_p(a) \;=\; a_1\tilde{r}_1 + a_2\tilde{r}_2 + (1 - a_1 - a_2).$$

Fulcrum and incentive fee structures are defined as in Section 3. Given any feasible choice of portfolio (a_1, a_2) for the fund, and any specific fee structure, a distribution is generated in the obvious way for the net-of-fees return Y received by the investor per dollar invested in the fund. Using the expectation $E(Y)$ and the variance $V(Y)$ of Y, (3.6) then defines the optimal amount invested in the fund as a function of the portfolio choice (a_1, a_2) and the fee structure. Taking this dependence into account, the fund chooses a portfolio mix and parameters for its fee structure that maximize its expected fees. All of this is conceptually straightforward but notationally cumbersone, so we avoid the details here.

The introduction of a second risky security makes this model significantly more complicated than the one studied in Section 3; it no longer looks amenable to analytical solution. We investigated it numerically, therefore, for a wide variety of possible parameters. In solving for the equilibrium, we use a two-step procedure. We first fix a fraction $y \in [0, a^{\max}]$ of initial asset value that may be invested in the two risky securities combined (i.e., we assume that a faction $(1 - y)$ must be invested in the riskless asset). Then, we maximize expected fees subject to the constraint that $a_1 + a_2 = y$. Using these maxima, we finally identify the optimal value of y for the fund.

Our most important finding is that although the fund in this model may choose a portfolio that is imperfectly correlated with the benchmark, it never does so: for any given value of y, and under either fee regime, the values of (a_1, a_2) that maximize the fund's payoffs subject to the constraint $a_1 + a_2 = y$ are always $a_1 = a_2 = y/2$.[13] Intuitively, this result appears driven by the investor's variance aversion. Under benchmarking, the investor's net-of-fees returns depend on two distributions: that of the fund portfolio r_p and of the benchmark return r_b. Since the investor is risk-averse, he reacts favorably to any *ceteris paribus* reduction in uncertainty. By choosing weights so that the fund and benchmark portfolios are perfectly correlated, the two (correlated) sources of noise are effectively reduced to a single source.

Under the choice of equal weights, the multiple risky securities model of this section reduces to a one risky security model analogous to the one studied in the previous section. Unsurprisingly, then, the welfare properties of the equilibria under the two regimes are very similar to those obtained in the previous section. Table 6 presents equilibrium values of all relevant quantities for a specific parametrization of the problem. The table shows, in particular, that

[13] It is possible to prove this result analytically in the fulcrum fee model, but an analogous proof in the incentive fee case seems very difficult.

1. When $a^{\max} \leq 1$, a flat fee with no performance component is arises as the equilibrium fee under both regimes. Thus, equilibrium outcomes coincide completely.

2. When $a^{\max} > 1$, the incentive fee regime Pareto-dominates the fulcrum fee regime, providing higher payoffs for both the fund and the investor. Volatility of net-of-fees returns are also strictly lower under incentive fees.

Thus, even under the richer set of portfolio strategies available to the fund, it continues to be the case that the investor is weakly (and, if $a^{\max} > 1$, strictly) better off under incentive fees.

6 Conclusions

Existing analyses of mutual funds have mostly been conducted within a classical principal/agent framework. In this paper, we propose an alternative model for the study of these institutions, and use this model to study the existing regulations that require fee structures used to compensate mutual fund advisers to be of the "fulcrum" variety, i.e., that decrease in the same way for underperforming an index as they increase for outperforming it.

We find little justification for the legal restrictions. In particular, we find that asymmetric "incentive-fee" structures—in which the adviser receives a base fee plus a bonus for exceeding a benchmark index—Pareto-dominate fulcrum fees, providing a higher utility to all participants with, in fact, a lower level of equilibrium volatility. These results contrast with those obtained using a partial equilibrium framework in which the investor's reactions are not explicitly modelled.

Our model also provides some insight into existing fee structures in the asset-management industry. In the mutual fund industry, the most commonly used fee structure observed in practice is a flat "fraction-of-funds" fee with no explicit performance component. In hedge funds, which are not subject to the fulcrum fee requirement and which also tend to use leveraged strategies, the most common fee found in practice is an incentive fee with a large performance component. These are exactly the structures that arise as equilibria in our model. In the absence of leveraging, we find that the equilibrium fulcrum fee is always a flat fee with no performance component. When incentive fees are allowed and leveraging is permitted, we find that the equilibrium fee is an incentive fee with a large performance component.

A A Brief History of the Investment Advisers Act

The Investment Advisers Act of 1940 lays out compensation strucutures that are impermissible for investment advisers. The act prohibits a registered investment adviser from receiving compensation on the basis of a share of capital gains in, or capital appreciation of, a client's account. In particular, performance-based compensation structures such as those which pay a flat fee plus a bonus for outperforming an index are disallowed.

The Act was prompted more by concerns about the inherent nature of such "incentive fees," than any evidence of actual abuse. Nonetheless, the prohibition in the original act was not absolute. Incentive fees were allowed in contracts between investment advisers and investment companies (including mutual funds), as long as the chosen basis of compensation was adequately disclosed to the shareholders.

In the 1960's, this situation was challenged by the SEC, which recommended that the prohibition on incentive-fee contracts be extended to cover investment company contracts also. The commission furnished Congress with the information that of 137 registered investment companies that then had fee arrangements based in some measure on performance, 48 allowed the investment adviser to earn a bonus for good performance without a penalty for bad performance, while a further 45 had arrangements in which the rewards for superior performance far outweighed the penalties for inferior performance. Although the commission did not present Congress with any actual evidence of abuse, Congress nevertheless accepted the commission's recommendation in 1970, and amended the 1940 Act to include investment company contracts also.

At the same time, however, Congress provided for one important exemption to the prohibition of performance-based fees. Contracts with registered investment companies were allowed to have compensation based on performance if they were of the "fulcrum" variety, that is if mangerial compensation were computed symmetrically around a chosen benchmark, decreasing for underperforming the benchmark in the same way in which it increased for outperforming it.

Since 1970, there has been only one major change to the regulation of performance-based compensation. In 1985, the SEC allowed the unlimited use of performance-based fees in contracts in which the client had either (i) at least $500,000 under the adviser's management, or (ii) a net worth of at least $1,000,000. This amendment has not, however, affected mutual funds in any important way, since for a mutual fund to qualify for the exemption, *every* single shareholder in the fund would have to meet one of the two specified criteria.

B Existing Patterns of Fees

The single most prominent (and perhaps most intriguing) fact concerning compensation structures in the mutual fund industry is the overwhelming popularity of "fraction of funds" fees, in which the investment adviser receives as compensation a fixed fraction of the total funds under management. A recent article in the *New York Times* reported that out of 5,400 stock and bond funds, only 75 (or 1.4%) use fulcrum fees that depend non-trivially on performance.[14] Although small, the list does include some prominent names, such as Fidelity's Magellan Fund, and Vanguard's Windsor Fund.

Within each of these two categories, a number of variants may be found in the mutual fund industry. In the use of fraction-of-funds fees, for instance, some funds tend to use a fixed percentage of assets under management, while others tend to use a sliding scale, with the percentage declining as the assets under management increases.

A typical fulcrum fee in the industry takes on the form of a base fee plus a "performance adjustment" for exceeding or falling short of a chosen benchmark. For equity funds, the benchmark is usually, though not always, the S&P 500 index. In most cases, a cap and floor are also placed on the fulcrum fee, that is, the performance adjustment component of the total fee is limited to some maximum amount (often $\pm$ 20 basis points). A variant on these themes is offered by Vanguard's Windsor fund in which the final fee is calculated as the base fee *times* an adjustment factor, where the adjustment factor varies from 0.50 to 1.50 depending on the fund's performance vis-a-vis the S&P 500 index. Finally, it is not uncommon for funds using a fulcrum fee to base the performance adjustment component not just on peformance over the last year, but over a longer period (say, the preceding three years).

C Proofs

C.1 Proof of Proposition 3.1

For ease of reference, we begin with a statement of the problem. To this end, recall that the lower bound on fees in any state is zero. Taking this into account, for any portfolio choice a

[14]See Carole Gould, "Paying Fund Managers with Carrots and Sticks," *New York Times*, February 9, 1997. The article attributed these statistics to Lipper Analytical Services.

and any choice of $b_1 \geq 0$ and $b_2 \geq 0$, the fund's fees in the three states are given by

$$\begin{aligned} F_h &= \max\{0, \min\{2b_1(1+\pi_h a), b_1(1+\pi_h a) + b_2(a-1)\pi_h\}\} \\ F_m &= b_1 \\ F_l &= \max\{0, \min\{2b_1(1-\pi_l a), b_1(1-\pi_l a) - b_2(a-1)\pi_l\}\} \end{aligned} \tag{C.1}$$

Given the fee distribution (F_h, F_m, F_l), the net-of-fees returns Y to the investor are realized as

$$\begin{aligned} Y_h &= 1 + a\pi_h - F_h \\ Y_m &= 1 - F_m \\ Y_l &= 1 - a\pi_l - F_l \end{aligned} \tag{C.2}$$

The investor's optimal action now is to invest an amount x^* with the fund, where

$$x^* = \frac{E(R)-1}{\gamma V(R)}. \tag{C.3}$$

Note that x^* is a function of only γ and the three parameters (a, b_1, b_2) chosen by the fund. The expected fees received by the fund are

$$EF = \frac{1}{3}[F_h + F_m + F_l] \cdot x^*. \tag{C.4}$$

For any fixed a, the fund's objective is to choose (b_1, b_2) so as to maximize these expected fees. The tricky part of this maximization exercise is ensuring that all the constraints are met, i.e., that (i) b_1 and b_2 are non-negative, and (ii) equilibrium fees in any state do not fall below the floor level or exceed the ceiling level.

We begin by checking for the existence of an "unconstrained" solution. That is, we ignore the non-negativity constraints on b_1 and b_2, as well as the floor and ceiling levels, and simply maximize the expected fees function with respect to (b_1, b_2). Taking the first-order condition with respect to b_1 results in two possible values for b_1 in terms of b_2:

$$b_1 = 1 - b_2\left(\frac{a-1}{a}\right) \tag{C.5}$$

$$b_1 = \frac{(a - b_2(a-1))(3a - 6b_2(a-1) + a^2(\pi_h - \pi_l)(\pi_h - \pi_l)}{(6a - 6b_2(a-1) + a^2(\pi_h - \pi_l))(3 + a(\pi_h - \pi_l))} \tag{C.6}$$

Under the first case, we have $b_2 = a(1-b_1)/(a-1)$. Substituting this into the expression for the fees in each state yields

$$\begin{aligned} F_h &= (b_1 + a\pi_h) \\ F_m &= b_1 \\ F_l &= (b_1 - a\pi_l) \end{aligned}$$

which, in turn implies, that we must have $R_h = R_m = R_l = (1-b_1)$. As long as $b_1 > 0$, this ensures that the investment with the fund will never be a positive amount, ruling this out as a candidate solution.

The second candidate (C.6) may also be ruled out as a solution. If we use this expression to substitute for b_1 in terms of b_2 in the fees, and then maximize expected fees (ignoring the constraints), the only value of b_2 that meets the first-order conditions implies an infinite value for b_1. Thus, no unconstrained solutions exist.

We turn to constrained solutions. There are four constraints that could hold with equality in a solution: (i) $b_1 = 0$, (ii) $b_2 = 0$, (iii) the floor of zero could hold for fees in some state, and (iv) the ceiling of $2b_1$ could hold for fees in some states. It helps in the sequel to consider the cases $a < 1$ and $a > 1$ separately.

C.1.1 The Case $a < 1$

When $a < 1$, it is the case that in state h the fund portfolio underperforms the benchmark, while in state l the fund portfolio outperforms the benchmark. This means for any non-negative values of b_1 and b_2, the fees are effectively given by

$$\begin{aligned} F_h &= \max\{0, b_1(1 + a\pi_h) + b_2(a-1)\pi_h\} \\ F_m &= 1 - b_1 \\ F_l &= \min\{2b_1(1 - a\pi_l), b_1(1 - a\pi_l) - b_2(a-1)\pi_l\} \end{aligned}$$

To identify the optimal choice of (b_1, b_2), we proceed in several steps, identifying at each step the set of candidate solutions that arise when only a subset of the constraints holds with equality. Comparing the expected fees in the candidate solutions then yields the optimal choice of (b_1, b_2). We first examine the candidate solutions that arise when either $b_1 = 0$ or $b_2 = 0$. Then, we will identify the candidate solutions that arise if the lower-bound constraint on F_h holds with equality. Third, we will repeat this exercise when the upper-bound constraint on F_l holds with equality.

So consider first the case $b_1 > 0$, $b_2 = 0$. Then, we have $F_h/(1 + a\pi_h) = F_m = F_l/(1 - a\pi_l) = b_1$. We use this to obtain first the investor's optimal response as a function of b_1, and thereby the fund's expected fees as a function of b_1. Maximizing the expected fees over b_1 results in only a single possibility, namely

$$b_1 = \frac{a(\pi_h - \pi_l)}{6 + a(\pi_h - \pi_l)} \tag{C.7}$$

It is easily checked that all the relevant constraints are met when b_1 is given by (C.7) and $b_2 = 0$. Thus, the expected fees under the choice (C.7) are a candidate solution to the fund's optimization problem. Letting $\xi = \pi_h^2 + \pi_h\pi_l + \pi_l^2$, these fees are given by

$$EF = \frac{(3 + a(\pi_h - \pi_l))(\pi_h - \pi_l)^2}{24\gamma\xi}. \tag{C.8}$$

The case $b_1 = 0$, $b_2 > 0$ is easily eliminated from consideration: in this case, fees must violate the non-negativity condition in state h and the upper bound of $2b_1$ in state l.

We turn now to the case where the lower bound on F_h holds with equality. If $F_h = 0$, then

$$b_1 = -b_2(a-1)\left(\frac{\pi_h}{1 + a\pi_h}\right).$$

We can use this expression to obtain the entire fee structure in terms of b_2. Thus, we can identify the investor's optimal action as a function solely of b_2, and thereby the fund's expected fees. Taking the first-order condition with respect to b_2 of these expected fees results in only one non-negative value for b_2. This value of b_2, and the corresponding value of b_1 are:

$$b_1 = \frac{a\pi_h(\pi_h - \pi_l}{5\pi_h + \pi_l} \tag{C.9}$$

$$b_2 = \frac{-a(1 + a\pi_h)(\pi_h - \pi_l)}{(5\pi_h + \pi_l)(a-1)}. \tag{C.10}$$

It is easy to check that under these values of b_1 and b_2, the non-negativity requirements as well as the upper-bound condition for F_l are satisfied. Thus, (C.9) (C.10) are also candidate solutions to the optimization problem. Letting $\xi = (\pi_h^2 + \pi_h\pi_l + \pi_l^2)$ as above, the expected fees they generate is given by

$$EF = \frac{(\pi_h - \pi_l)^2(2\pi_h + \pi_l)}{24\pi_h\gamma\xi}. \tag{C.11}$$

Next, we turn to the case where the upper-bound constraint in F_l holds with equality. If $F_l = 2b_1(1 - api_l)$, then we must have

$$b_1 = -b_2(a-1)\left(\frac{\pi_l}{1 - a\pi_l}\right).$$

Once again, we can use this expression to obtain the entire fee structure in terms of b_2. Thus, we can identify the investor's optimal action as a function solely of b_2, and thereby the fund's expected fees. Taking the first-order condition with respect to b_2 of these expected fees results in only one non-negative value for b_2. This value, and the corresponding value it implies for b_1, are

$$b_1 = \frac{a\pi_l(\pi_h - \pi_l)}{7\pi_l - 2a\pi_l^2 + 2a\pi_h\pi_l - \pi_h} \tag{C.12}$$

$$b_2 = \frac{a(1 - a\pi_l)(\pi_h - \pi_l)}{(1-a)(7\pi_l - 2a\pi_l^2 + 2a\pi_h\pi_l - \pi_h)} \tag{C.13}$$

Expressions (C.12)–(C.13) offer a third pair of candidate solutions. Defining ξ as above, the expected fees they imply is given by

$$EF = \frac{(\pi_h - \pi_l)^2(4\pi_l - 2a\pi_l^2 + 2a\pi_h\pi_l - \pi_h)}{24\gamma\pi_l\xi}. \tag{C.14}$$

As the last step in the proof, we compare the expected fees under the three candidate solutions. Consider first (C.8) and (C.11). Eliminating the common terms, it is seen that the former expression is larger than the latter only if

$$3\pi_h + a\pi_h(\pi_h - \pi_l) > 2\pi_h + \pi_l,$$

which always holds since $\pi_h > \pi_l > 0$. Now comparing (C.8) with the expected fee (C.14) under the third candidate solution, the former is seen to be larger if and only if

$$(1 - a\pi_l)(\pi_h - \pi_l) \geq 0,$$

which always holds under our assumptions on π_h and π_l. Thus, the largest of the three candidate values for the expected fee is given by (C.8), and it follows that when $a < 1$, the unique optimal fee structure for the fund is to set $b_2 = 0$ and have b_1 given by (C.7).

C.1.2 The case $a > 1$

When $a > 1$, the fund portfolio always does worse than the benchmark portfolio in the state l and always does better in the state h. Therefore, the lower bound of zero can have an impact only in state l and the higher bound of $2b_1$ can have an impact only in state h; the effective constrained fee structure is given by

$$\begin{aligned} F_h &= \min\{2b_1(1 + a\pi_h), b_1(1 + a\pi_h) + b_2(a-1)\pi_h\} \\ F_m &= b_1 \\ F_l &= \max\{0, b_1(1 - a\pi_l + b_2(a-1)\pi_l\}. \end{aligned}$$

We proceed in the same manner that we did above by identifying candidate solutions when some constraints hold with equality. The first case we consider, that of $b_1 > 0$ and $b_2 = 0$, again results in the candidate solution (C.7) with an expected fee given by (C.8). The second case, that of $b_1 = 0$ and $b_2 > 0$, can be eliminated from consideration for the same reason as above.

Consider next the case where the lower bound on F_l holds with equality. If $F_l = 0$, then we must have

$$b_1 = b_2(a-1)\left(\frac{\pi_l}{1 - a\pi_l}\right).$$

We can use this to eliminate b_1 from the expression for expected fees in the usual manner. Taking the first-order conditions of the expected fees with respect to b_2 now results in two solutions for b_2. The first solution, together with its corresponding value of b_1, is:

$$b_1 = a\pi_l \tag{C.15}$$

$$b_2 = \frac{a(1 - a\pi_l)}{a - 1} \tag{C.16}$$

Under these values for (b_1, b_2), a simple calculation reveals that the net-of-fees returns to the investor satisfy $R_h = R_m = R_l = (1 - a\pi_l)$. It is apparent that optimal investment in the fund cannot be positive under these circumstances, ruling this out as a candidate solution.

The second solution when $F_l = 0$ is given by

$$b_1 = \frac{a\pi_l(\pi_h - \pi_l)}{\pi_h + 5\pi_l} \tag{C.17}$$

$$b_2 = \frac{a(\pi_h - \pi_l)(1 - a\pi_l)}{(a-1)(\pi_h + 5\pi_l)}. \tag{C.18}$$

This pair of values also satisfies the upper-bound on F_h if $(\pi_h - \pi_l) \leq 2a\pi_h\pi_l$. We retain it, therefore, as a candidate solution subject to this condition being met. Defining ξ as above, the expected fee in this case is

$$EF = \frac{(\pi_h - \pi_l)^2(\pi_h + 2\pi_l)}{24\gamma\pi_l\xi} \tag{C.19}$$

Next, we turn to the case where the upper bound on F_h holds with equality. If $F_h = 2b_1(1 + \pi_h a)$, then we must have

$$b_1 = b_2(a-1)\left(\frac{\pi_h}{1 + a\pi_h}\right).$$

We use this in the usual manner to eliminate b_1 and obtain an expression for the expected fees in terms of b_2. Taking the first-order conditions of these expected fees with respect to b_2 gives us two possible values for b_2. One of these, together with its corresponding value for b_1 results in the net-of-fees returns to the investor of $R_h = R_m = R_l = (1 + a\pi_h)/(1 + 2a\pi_h) < 1$. This eliminates it as a candidate solution. The other value for b_2, together with its corresponding value for b_1, is

$$b_1 = \frac{a\pi_h(\pi_h - \pi_l)}{7\pi_h + 2a\pi_h 2 - \pi_l - 2a\pi_h\pi_l} \tag{C.20}$$

$$b_2 = \frac{a(\pi_h - \pi_l)(1 + a\pi_h)}{(a-1)(7\pi_h + 2a\pi_h^2 - \pi_l - 2a\pi_h\pi_l)}. \tag{C.21}$$

This is a third pair of candidate solutions with an expected fee of

$$EF = \frac{(\pi_h - \pi_l)^2(2a\pi_h^2 - \pi_l + 4\pi_h - 2a\pi_h\pi_l)}{24\gamma\pi_h\xi}. \tag{C.22}$$

To complete the proof, we compare the expected fees (C.8), (C.19), and (C.22). The last of these exceeds the first if and only if $(\pi_h - \pi_l)(1 + a\pi_h) > 0$ which always holds. It also exceeds the second if $2a\pi_h\pi_l - (\pi_h - \pi_l) > 0$. But the second fee is a candidate solution only if this condition holds (see above). Therefore, the third candidate solution dominates the others, and the equilibrium when $a > 1$ is given by (C.20)–(C.21) with an expected fee of (C.22).

C.1.3 The Case $a = 1$

We have not so far considered the case $a = 1$, but this is easily accomodated within the developments so far. When $a = 1$, the fund portfolio and the benchmark portfolio have identical returns. Thus, b_2 is irrelevant, and it may be set to zero without loss. Maximizing expected fees with respect to b_1 results in the solution (C.7) for b_1 with an expected fee of (C.8). This completes the proof of Proposition 4.1. □

C.2 Proof of Proposition 3.2

When $a \leq 1$, we have seen that the expected fee is given by (C.8), which is clearly increasing in a. Thus, $a^* = a^{\max}$ is the optimal action if $a^{\max} \leq 1$. If $a > 1$, the expected fee is given by (C.22) which is also increasing in a. Moreover, we have shown that for any fixed $a > 1$, (C.22) is larger than (C.8). Since (C.8) is increasing in a, it is the case that for any $a > 1$, the maximum expected fee (C.22) under a dominates the expected fee (C.8) for any $a \leq 1$. It is immediate now that the optimal action for any $a^{\max} > 1$ is $a^* = a^{\max}$, completing the proof. □

C.3 Proof of Proposition 3.3

The maximization problem here is the same as was outlined at the top of section C.1, with the difference being that the fee structure in effect is an incentive fee structure. Thus, the non-negativity constraints on the realized fees, or the corresponding upper-bounds, play no role here. The only constraints facing the fund in its choice of parameters for the fees are that b_1 and b_2 must be non-negative.

The proof provided in section C.1 concerning the absence of unconstrained solutions evidently continues to apply to this case also. Thus, only two forms of constrained solutions are possible: $b_1 > 0$, $b_2 = 0$ and $b_1 = 0$, $b_2 > 0$. It helps, in identifying the optimal solution, to handle the cases $a < 1$ and $a > 1$ separately.

C.3.1 The Case $a < 1$

For any $a < 1$, the fund portfolio outperforms the benchamrk in state l and underperforms it in state h. Therefore, given any choices of b_1 and b_2, the fund's fees in the three states are determined as $F_h = b_1(1 + \pi_h a)$, $F_m = b_1$, and $F_l = b_1(1 - \pi_l a) + b_2(a - 1)\pi_l$.

Consider first the case where $b_1 > 0$ and $b_2 = 0$. In this case, solving for the optimal value of b_1 evidently results in the same solution as in the corresponding fulcrum fee case:

the optimal b_1 is given by (C.7) and the expected fee is given by (C.8).

Now consider $b_1 = 0$. Solving for the expected fee in terms of b_2 using the procedure we have described many times above, and then taking the first-order conditions with respect to b_2 results in only a single non-negative solution for b_2. Defining ξ once again as $(\pi_h^2 + \pi_h\pi_l + \pi_l^2)$, this solution is given by:

$$b_2 = \frac{a(\xi - \pi_h\sqrt{3\xi}}{(a-1)\pi_l(2\pi_h + \pi_l)} \tag{C.23}$$

It is easily checked that b_2 so defined is strictly positive and is therefore a candidate solution. The expected fee it generates is

$$EF = \frac{(\sqrt{3\xi} - 3\pi_h)(\xi - \pi_h\sqrt{3\xi})}{6\gamma\pi_h(2\xi + \pi_l\sqrt{3\xi})} \tag{C.24}$$

To complete the proof for the case $a < 1$, we compare expected fees under the two candidate solutions (C.8) and (C.24). Let π_h/π_l be denoted by k, and define $d = 1 + k + k^2$. Note that $k > 1$. Dividing (C.8) by (C.24), we now obtain:

$$\text{ratio} = \frac{k(k-1)^2(3 + a\pi_l(k-1))(2d + \sqrt{3d})}{4d(\sqrt{3d} - 3k)(d - k\sqrt{3d})}.$$

Both numerator and denominator are positive; since $k > 1$, this ratio is an increasing function of a, and reaches its minimum when $a = 0$. When $a = 0$, a plot of the ratio reveals that it is strictly increasing in k and greater than unity whenever $k \geq 1$. Thus, the ratio is also greater than unity for $a > 0$ when $k > 1$. It follows that the optimal incentive fee structure when $a < 1$ is to have $b_1 > 0$ and $b_2 = 0$, with b_1 given by (C.7).

C.3.2 The Case $a > 1$

When $a > 1$, the fund portfolio outperforms the benchmark in state h and underperforms it in state l. Therefore, for any choice of (b_1, b_2), the fund's fees are given by

$$\begin{aligned} F_h &= b_1(1 + a\pi_h) + b_2(a-1)\pi_h \\ F_m &= b_1 \\ F_l &= b_1(1 - a\pi_l) \end{aligned}$$

Once again, there are two cases to consider: when $b_1 = 0$ and when $b_2 = 0$. In the former case, we clearly get the solution (C.7) for b_1 with expected fees given by (C.8). In the latter case, obtaining the expected fees in terms of b_2 and taking first-order conditions results in two possible values for b_2:

$$b_2 = \frac{a(\xi - \pi_l\sqrt{3\xi})}{(a-1)\pi_h(\pi_h + 2\pi_l)} \tag{C.25}$$

$$b_2 = \frac{a(\xi + \pi_l\sqrt{3\xi})}{(a-1)\pi_h(\pi_h + 2\pi_l)} \tag{C.26}$$

The expected fees in the two solutions are, respectively:

$$EF = \frac{(\sqrt{3\xi} - 3\pi_l)(\xi - \pi_l\sqrt{3\xi})}{6\gamma\pi_l(2\xi + \pi_h\sqrt{3\xi})} \tag{C.27}$$

$$EF = \frac{(-\sqrt{3\xi} - 3\pi_l)(\xi + \pi_l\sqrt{3\xi})}{6\gamma\pi_l(2\xi - \pi_h\sqrt{3\xi})} \tag{C.28}$$

The first of these expressions is positive, but the second can be shown to be negative, so we discard it as a possible solution.

To complete the proof, it remains to compare the expected fees (C.27) to the expected fees (C.8) under the two solutions. Once again, we define $k = \pi_h/\pi_l$ and $d = 1 + k + k^2$, and take the ratio of (C.28) to (C.8). Cancelling common terms, this ratio is seen to be

$$\text{ratio} = \frac{4d(d - \sqrt{3d})(\sqrt{3d} - 3)}{(k-1)^2(3 + k(a-1)\pi_l)(2d + k\sqrt{3d})}$$

The ratio is decreasing in a and reaches its minimum value at the maximum feasible value for a of $1/\pi_l$. A plot of the ratio shows that even when $a = 1/\pi_l$, the ratio is strictly greater than unity whenever $k > 1$. It follows that the optimal structure of incentive fees when $a > 1$ has $b_1 = 0$ and $b_2 > 0$, with the optimal b_2 given by (C.25), and the expected fees under this optimal choice given by (C.27).

C.3.3 The Case $a = 1$

As with fulcrum fees, the case $a = 1$ is easily handled.The fund portfolio and the benchmark portfolio have identical returns in this case. Thus, b_2 is irrelevant, and it may be set to zero without loss. Maximizing expected fees with respect to b_1 results in the solution (C.7) for b_1 with an expected fee of (C.8). This completes the proof of Proposition 4.3. □

C.4 Proof of Proposition 3.4

When $a \leq 1$, expected fees in equilibrium are given by (C.8) which is clearly increasing in a. Thus, if $a^{\max} \leq 1$, the optimal value of a for the fund is $a^{\max}$. If $a > 1$, we have shown that equilibrium fees are given by (C.8), which is independent of a. In the course of establishing this result, we also showed that for any fixed $a > 1$ the expected fees (C.27) are greater than (C.8); the latter, in turn, is greater than (C.8) for any $a \leq 1$. It follows that when $a^{\max} > 1$, any value of $a > 1$ is an optimal action for the fund, completing the proof of Proposition 4.4. □

C.5 Proof of Proposition 3.5

When $a^{\max} \leq 1$, the equilibrium strategies under fulcrum and incentive fees coincide, so the equilibrium payoffs to the investor and fund are identical under the two regimes, as are equilibrium risk levels. To complete the proof we will consider the case $a^{\max} > 1$ and show that

1. The equilibrium expected utility under incentive fees is always greater than under fulcrum fees.
2. The volatility of the fund portfolio is never lower under fulcrum fees than under incentive fees.
3. Provided π_h/π_l is sufficiently large (precise bounds are given below):
   (a) The expected fee under an incentive fee regime dominates that under a fulcrum regime.
   (b) The volatility of net-of-fees returns is strictly lower under incentive fees than under fulcrum fees.

We begin with a comparison of the equilibrium expected utilities under the two regimes. Fix any $a > 1$. Recall that for $a > 1$, the equilibrium expected utility is independent of a in both cases. Using the closed-forms described in the text, the ratio of the expected utility under an incentive fee to that under a fulcrum fee is given by

$$\text{Ratio of EU} = \frac{4\xi[(1+8\gamma)\xi + 3\pi_l^2 - 2\sqrt{3\xi}(\pi_l - 2\pi_h)]}{(1+16\gamma)\xi - \pi_h\pi_l)(2\xi + \pi_h\sqrt{3\xi})} \tag{C.29}$$

Let $k = \pi_h/\pi_l > 1$, and let $d = 1 + k + k^2$. Substituting this into (C.29) and eliminating common terms, we obtain:

$$\text{Ratio of EU} = \frac{4d\left[(4 + k + k^2 + 8\gamma d) + 2\,(2\gamma k - 1)\,\sqrt{3d}\right]}{\left[(k-1)^2 + 16\gamma d\right]\left[2d + k\sqrt{3d}\right]} \tag{C.30}$$

The right hand-side is a function of just the single variable k. Plotting this right-hand side reveals that the ratio is an increasing frunction of k and is greater than unity for all values of $k > 1$. This completes the proof that the investor's welfare is higher under an incentive fee.

Next, note that when $a^{\max} > 1$, the fund's uniquely optimal choice of a under a fulcrum fee is $a = a^{\max}$, but any $a > 1$ is optimal for the fund under an incentive fee. Since the volatility of the fund portfolio is given by a^2, it follows immediately that this volatility is never higher (and could be strictly lower) under an incentive fee than under a fulcrum fee.

Turning to the fund's payoffs, fix any $a > 1$. Recall that the fund's expected fee for $a > 1$ is independent of a in an incentive fee regime, but not in a fulcrum fee regime. The ratio of the expected fee under incentive fees to that under fulcrum fees is given by

$$\text{Ratio of EF} = -\frac{4\pi_h\xi(-3\pi_l + \sqrt{3\xi})(\xi - \pi_l\sqrt{3\xi})}{(\pi_h - \pi_l)^2\pi_l(\pi_l + 2a\pi_h\pi_l - 4\pi_h - 2a\pi_h^2)(2\xi + \pi_h\sqrt{3\xi})} \tag{C.31}$$

Using, once again, the substitutions $k = \pi_h/\pi_l$ and $d = 1 + k + k^2$, and eliminating common terms, we obtain

$$\text{Ratio of EF} = \frac{4kd(d - \sqrt{3d})(\sqrt{3d} - 3)}{(k-1)^2(4k - 1 - 2ak\pi_l(1-k))(2d + k\sqrt{3d})}. \tag{C.32}$$

The right-hand side is greater than unity whenever

$$a\pi_l > \left(\frac{4kd(d - \sqrt{3d})(\sqrt{3d} - 3) - (k-1)^2(4k-1)(k\sqrt{3d} + 2d)}{2(k-1)^3 k(2d + k\sqrt{3d})}\right) \tag{C.33}$$

This restriction holds in "most" reasonable cases, failing only when k is too close to unity and $a\pi_l$ is large (close to its upper bound). Of course, whenever it holds, the fund's equilibrium payoffs are strictly higher under incentive fees than udner fulcrum fees.

Finally, consider the volatility of the net-of-fees returns to the investor. Fix any $a > 1$. The ratio of this volatility under a fulcrum fee to that under an incentive fee is

$$\text{Ratio of Vol} \;=\; \frac{2a\pi_h(\pi_h + 2\pi_l)\sqrt{3\xi}}{(2a\pi_h^2 - \pi_l + \pi_h(7 - 2a\pi_l))(a\pi_l\sqrt{2\xi + \pi_h\sqrt{3\xi}})} \tag{C.34}$$

Expressing this ratio in terms of k and d defined as above, we have

$$\text{Ratio of Vol} \;=\; \frac{2k(2+k)\sqrt{3d}}{(6k + (k-1)(1 + 2ak\pi_l))\sqrt{2d + k\sqrt{3d}}} \tag{C.35}$$

This ratio is greater than unity whenever

$$a\pi_l \;>\; \frac{(1-7k)(\sqrt{2d + k\sqrt{3d}}) + 2k\sqrt{3d}(2+k)}{2k(k-1)(\sqrt{2d + k\sqrt{3d}})}. \tag{C.36}$$

As with condition (C.33), this condition also holds in "most" reasonable cases, failing only when k is too close to unity and $a\pi_l$ is large. This completes the proof of Proposition 3.5. □

D A Risk-Averse Fund

Throughout this paper, we have assumed that the fund is risk-neutral. This assumption was made in part for analytical tractability, since it enabled us to obtain closed-form solutions in Section 3. However, it turns out that, from a qualititative standpoint, the assumption is not very important: the same features of the equilibria highlighted in Section 3 also obtained when the fund's utility function was taken to be $u(x) = x_f^{\gamma}$ for $\gamma_f \in (0,1]$. We present some examples of the new equilibria for this version of the model of Section 3. The results presented here were obtained by solving the problem numerically.

So fix any value of $\gamma_f \in (0,1]$. As in Section 3, we found that whenever $a^{\max} \leq 1$, the equilibrium fee under both a fulcrum fee regime and an incentive fee regime is a flat fee. Thus, equilibrium outcomes coincide completely in this case. Table 7 presents numerical values of equilibrium outcomes under the two regimes for two values of γ and three values of $a^{\max} > 1$. As the table shows, the qualitative nature of these figures is the same as that which obtained in Section 3, with the outcomes under an incentive fee regime dominating those under a fulcrum fee regime on all counts. The only additional feature of interest is that as the value of γ_f falls, the difference in outcomes between the regimes narrows.

Table 1: Optimal Contracts in the Fulcrum Fee case

This table presents the equilibrium values in a fulcrum-fee regime of six quantities for various given values of the fund's portfolio choice a: (i) the fee structure b_1 and b_2, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns EY to the investor, (iv) the volatility of these returns $\sigma(Y)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the parameters of the benchmark portfolio returns are fixed at $\pi_h = 0.15$ and $\pi_l = 0.05$.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0041	0.0083	0.0123	0.0164	0.0181	0.0215	0.0283
b_2	0.0000	0.0000	0.0000	0.0000	0.5723	0.3517	0.2453
x	4.6346	2.3269	1.5577	1.1731	1.0641	0.8932	0.6795
EY	1.0041	1.0083	1.0123	1.0164	1.0181	1.0215	1.0283
$\sigma(Y)$	0.0212	0.0421	0.0630	0.0836	0.0922	0.1098	0.1443
EF	0.0194	0.0196	0.0197	0.0199	0.0251	0.0254	0.0261
EU	1.0096	1.0096	1.0096	1.0096	1.0096	1.0096	1.0096

Table 2: Optimal Contracts in the Performance Fee case

This table presents the equilibrium values of in an incentive-fee regime of six quantities for various given values of the fund's portfolio choice a: (i) the fee structure b_1 and b_2, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns EY to the investor, (iv) the volatility of these returns $\sigma(Y)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the parameters of the benchmark portfolio returns are fixed at $\pi_h = 0.15$ and $\pi_l = 0.05$.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0041	0.0083	0.0123	0.0164	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.0000	2.2517	1.3510	0.9007
x	4.6346	2.3269	1.5577	1.1731	1.4508	1.2090	0.9067
EY	1.0041	1.0083	1.0123	1.0164	1.0135	1.0162	1.0216
$\sigma(Y)$	0.0212	0.0421	0.0630	0.0836	0.0683	0.0819	0.1092
EF	0.0194	0.0196	0.0197	0.0199	0.0408	0.0408	0.0408
EU	1.0096	1.0096	1.0096	1.0096	1.0098	1.0098	1.0098

Table 3: Optimal Contracts in the Unrestricted Fee case

This table presents the optimal values of the percentage fees F_h, F_m, and F_l for various choices of a by the fund. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the benchmark return parameters are fixed at $\pi_h = 0.15$, $\pi_l = 0.05$. The table reports the equilibrium values of: (i) the fees in each state of the world, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns EY to the investor, (iv) the volatility of these returns $\sigma(Y)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
F_h	0.0163	0.0314	0.0455	0.0587	0.0711	0.0827	0.1039
F_m	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F_l	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x	7.2538	3.6269	2.4179	1.8135	1.4508	1.2090	0.9067
EY	1.0027	1.0054	1.0081	1.0108	1.0135	1.0162	1.0216
$\sigma(Y)$	0.0137	0.0273	0.0410	0.0546	0.0683	0.0819	0.1092
EF	0.0408	0.0408	0.0408	0.0408	0.0408	0.0408	0.0408
$E(U)$	1.0098	1.0098	1.0098	1.0098	1.0098	1.0098	1.0098

Table 4: Equilibrium in the Model with Fund Effort

This table presents the equilibrium values under fulcrum and incentive fee regimes of various quantities for given values of the fund's maximum leveraging ability $a^{\max}$: (i) the equilibrium values of b_1 and b_2, (ii) the equilibrium effort level e, (iii) the volatility $\sigma(Y)$ of the net-of-fees equilibrium returns to the investor, (iv) the fund's expected fees EF, and (v) the investor's expected utility EU. The cost-of-effort parameter is fixed at $k = 0.05$. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the benchmark return parameters are fixed at $\pi_h = 0.15$ and $\pi_l = 0.05$.

Equilibrium under Fulcrum Fees

$a^{\max}$	0.50	0.75	1.00	1.25	1.50	2.00
b_1	0.0108	0.0161	0.0214	0.0213	0.0260	0.0368
b_2	0	0	0	1.5941	0.2682	0.1716
e	0.3850	0.3898	0.3945	0.3604	0.4376	0.4629
$\sigma(Y)$	0.0380	0.0567	0.0751	0.0863	0.1030	0.1344
EF	0.0338	0.0343	0.0347	0.0423	0.0420	0.0438
EU	1.0202	1.0203	1.0204	1.0200	1.0211	1.0217

Equilibrium under Incentive Fees

$a^{\max}$	0.50	0.75	1.00	1.25	1.50	2.00
b_1	0.0108	0.0161	0.0214	0	0	0
b_2	0	0	0	1.3509	1.1370	1.1370
e	0.3850	0.3898	0.3945	0.2969	0.4615	0.4615
$\sigma(Y)$	0.0380	0.0567	0.0751	0.0842	0.0848	0.0828
EF	0.0338	0.0343	0.0347	0.0446	0.0632	0.0632
EU	1.0202	1.0203	1.0204	1.0201	1.0230	1.0230