AGGLOMERATION AND ENDOGENOUS CAPITAL

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ABSTRACT

The "new" economic geography focuses on the footloose-labor and the vertically-linked-industries models. Both are complex since they feature demand-linked and cost-linked agglomeration forces. I present a simpler model where agglomeration stems from demand-linked forces arising from endogenous capital with forward-looking agents. The model's simplicity permits many analytic results (rare in economic geography). Trade-cost levels that trigger catastrophic agglomeration are identified analytically, liberalization between almost equal-sized nations is shown to entail "near-catastrophic" agglomeration, and Krugman's informal stability test is shown to be equivalent to formal tests in a fully specified dynamic model.

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1. Introduction

Much of the public debate on international integration revolves around fears that freer trade will cause industrial 'delocation', namely the shifting of manufacturing activities from one region or nation to another. These fears are many and often inconsistent. In Europe, rich nations fear delocation to low-wage nations, poor nations fear delocation to highly industrialized nations, small countries fear delocation to large countries, and nonmembers fear delocation to EU members. In the US, the same fears arise with respect to the location of 'good jobs' and are well summarized by Ross Perot's famous quip about NAFTA creating a great sucking sound of jobs going south.

The so-called economic geography literature, which permits evaluation of such concerns, focuses on two types of models. The Krugman (1989, 1991) model based on footloose labour – sometimes called the core-periphery model – and the model based on vertically-linked industries introduced by Venables (1996), and extended by Krugman and Venables (1995).* The hallmark of these models is that agglomeration forces tend to encourage concentration of industrial activity via 'circular causality.' That is to say, spatial concentration itself creates an environment that encourages spatial concentration.

The two models are complex since they feature both demand-linked and cost-linked circular causality. In the footloose-labour model, a shock to the distribution of firms triggers two distinct cycles of circular causality. First, when firms move, workers follow and this migration leads to expenditure shifting. Since firms prefer, *ceteris paribus*, to be in the big market, expenditure shifting leads to more production shifting, and the demand-linked cycle repeats. Second, production shifting lowers the price index in the 'receiving' country (via a love-of-variety effect) and raises it in the 'sending' country. Assuming migration equalizes real wages, the initial shock will lower the receiving nation's nominal industrial wage relative to that of the sending nation. This 'cost shifting' or change in industrial competitiveness encourages more delocation to the receiving nation, so the cost-linked cycle repeats.

In the vertically-linked-industries model, firms use the output of other firms as intermediate inputs. Thus, production shifting alters the international demand pattern (viewing other firms as customers) and alters the international cost pattern (viewing firms as suppliers). As in the Krugman model, two distinct but closely related processes of circular causality encourage agglomeration. Since labour mobility is negligible in Europe, often even between regions within a nation, the Venables model is generally preferred in European applications. (See Puga 1997 for a synthesis and generalization of the two model genres.)

As a consequence of this complexity, very few of the key results in the foot-loose labour and vertically-linked industry models can be derived analytically. Rather, researchers must rely on numerical simulation using specific parameter values.

This paper presents a simple model in which agglomeration is driven only by demand-linked circular causality. Firms are associated with a particular unit of capital and neither

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*Faini (1984) is a closely related model of agglomeration which assumes no trade, as opposed to costly trade, in intermediate inputs.

capital nor capital owners are internationally mobile. The fundamental agglomeration logic can be illustrated with the following thought-experiment. Starting from long-run equilibrium, consider a small rise in protection that raises home firms' operating profit and lowers that of foreign firms. Capital stocks cannot jump, so the shift in operating profits raises home's rental rate and lowers foreign's. This shift in rates of return encourages capital formation (i.e. entry of new firms) in home and capital decumulation (i.e. exit) in foreign. Income equals factor payments, so home's rising capital stock and foreign's shrinking capital stock leads to 'expenditure shifting'. As in the footloose-labour and vertically-linked-industry models, expenditure shifting itself produces a new shift in operating profits and the demand-linked cycle repeats. The model does not require labour mobility, so it seems suitable for European applications.

The Perroux (1955) notion of growth poles and growth sinks appears very clearly in this model. Indeed, growth effects and level effects are thoroughly interwoven, producing implications that line up suggestively with real-world facts. Consider, for instance, an initially stable long-run equilibrium that becomes unstable as trading costs fall. To be concrete, assume the agglomeration occurs in the home nation. Given optimal savings behaviour, home's saving rate rises above the rate necessary to sustain its initial capital stock. Consequently, home's capital stock, income and output begin to rise (we can think of this as agglomeration-induced investment-led growth). Moreover in a very concrete way, investment in the growing region is favoured exactly because expenditure in the region is growing. And expenditure is growing due to the high investment rate.

The reverse process operates in the foreign nation. The lower rate of return induces foreign consumers/savers to stop investing, so depreciation erodes the foreign capital stock, and foreign income and output begin to drop. Given the particular depreciation process assumed, foreign firms shut down one by one. In the simple model in this paper, workers displaced by the downsizing of the foreign industrial sector immediately find new jobs in the non-industrial sector. However, if finding a new job or expanding the non-industrial sector took time, the periphery's downward spiral would be associated with above-normal unemployment; the same labour market features would imply 'labour shortages' in the growing region. More colloquially, the declining region would resemble a 'rust belt' and the ascending region would resemble a 'boom belt'.

These implications seem to line up roughly with European stylized facts. That is, the hardest hit regions in Europe (eg, Southern Italy) display three traits: a low <u>level</u> of per capita income, a below-average growth <u>rate</u> and a high level of unemployment. The most favoured regions enjoy above-average growth rates, high per capita income levels and low unemployment.

The model is also a counter example to the presumption that exogenous growth models predict convergence of income levels. In this model, progressive trade liberalization between symmetric nations eventually produces the core-periphery outcome. Thus contrary to the standard assertion in the growth literature, in this neoclassical growth model, economic integration produces divergence in real per-capita income levels.*

Finally, the paper shows that capital mobility is a stabilizing force. That is, when capital is mobile but capital owners are not, production shifting is not accompanied by

*I wish to thank Philippe Martin for pointing this out.

expenditure shifting, so circular causality does not operate.

Given the simplicity of the model, many analytic results are available. For instance, as in the footloose-labour and vertically-linked-industry models, lowering trade costs between symmetric nations eventually produces a catastrophic agglomeration of industrial activity. However, unlike the two existing models, in the model presented here the critical level of trade barriers below which this catastrophe occurs can be derived analytically (without resorting to linear approximations). Moreover, the location and welfare effects of other types of liberalizations can be analytically characterized.

The plan of the paper after the introduction is as follows. Section 2 introduces the model and characterizes the equilibrium. Section 3 shows that catastrophic agglomeration of the symmetric equilibrium occurs when trade costs are low enough. Section 4 studies the agglomeration process with asymmetric-sized nations. Section 5 works out the positive and welfare effects of several forms of asymmetric liberalization. The last section contains a summary and some concluding remarks.

2. The Basic Model

This section introduces a model with geography features similar to Martin and Rogers (1995), trade features similar to Flam and Helpman (1987), and optimal savings features similar to the Ramsey model.

2.1 Basic Assumptions

Consider a world with two countries (home and foreign) each with two non-traded factors (labour L and capital K) and three sectors (X, I and Z). Countries have identical preferences and technology, but potentially different L endowments and trade costs. National K stocks are the cumulated output of national I-sectors (I is a mnemonic for investment goods). The X sector (manufacturing) consists of differentiated goods and is modelled as the Flam-Helpman version of the Dixit-Stiglitz monopolistic-competition model. Namely, manufacturing an X variety requires one unit of K plus a_X units of labour per unit of output, so the cost function is $\pi+wa_Xx_i$, where π is K's rental rate, w is the wage rate, a_X is the unit input coefficient and x_i is output. K's rental rate is the Ricardian surplus (i.e. operating profit) due to K's variety-specificity. The Walrasian (perfect competition and constant returns) Z sector produces a homogenous good employing only L with a unit input coefficient of a_Z. By choice of units $a_Z=1$ and a_X equals $(1-1/\sigma)$.

Nature of Capital and the I Sector. The Walrasian I sector produces new K using F units of L. The flow of new capital, Q_K, equals L_I/F where L_I is I-sector employment. The output of the I-sector, namely Q_K, is nontraded. Units of K depreciate according to a 'Blanchard' process in the sense that K, like individuals in Blanchard (1985), face a constant probability of 'dying' at every instant. The probability at time 't' that a unit of K will still be working at time 's' is $e^{-\delta(s-t)}$ where $0<\delta<\infty$ is the instantaneous failure rate; in continuous time δ may exceed 1.* The Dixit-Stiglitz monopolistic competition framework adopted here

*Capital is intrinsically more discrete in here than in the standard neoclassical model since it represents a fixed cost. For example, K is often viewed as a patent or design, so the standard proportional depreciation assumption is inappropriate. The Blanchard depreciation process respects the discrete-ness of K while still yielding the standard aggregate law of motion.

works with an uncountable infinity of infinitely small firms, so by the law of large numbers δ percent of the K stock depreciates each instant. Thus K evolves as:

$$\dot{K} = Q_K - \delta K \;; \qquad Q_K = L_I / F \tag{1}$$

X and Z are both traded; Z trade occurs costlessly, while X trade is impeded by frictional (i.e. 'iceberg') import barriers. Specifically $\tau=1+t \geq 1$ units of a good must be shipped in order to sell one unit abroad, where 't' is the barrier's tariff equivalent. Frictional barriers are meant to represent transport costs and so-called technical barriers to trade such as idiosyncratic industrial, safety and health regulations and standards; they generate no rents or tax revenue.

Preferences of the infinitely-lived representative consumer (in both countries) are:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \ln\left(C_{Xt}^{\alpha} C_{Zt}^{1-\alpha}\right) dt \;, \quad C_X \equiv \left(\int_{i=0}^{K+K^*} c_i^{1-1/\sigma} di\right)^{\frac{1}{1-1/\sigma}}, \quad \sigma>1 \;, \quad 0<\alpha<1 \tag{2}$$

where $\rho>0$ is the rate of pure time preference, C_Z and C_X are consumption of Z and of the CES composite of manufactured goods, c_i is consumption of X-variety i, and full employment (with one-unit of capital per variety) implies that K+K* is the global number of varieties ('*' denotes foreign variables). The representative consumer owns all the nation's L and K. A typical nation's income (denoted as Y) equals $wL+\pi K$.

2.2 Key Intermediate Results

Utility optimization implies that a constant fraction α of consumption expenditure E falls on X varieties with the rest spent on Z. It also yields a unitary elastic demand function for Z and standard CES demand functions for X varieties:

$$c_j = \frac{s_j}{p_j} \alpha E \;; \qquad s_j \equiv \frac{p_j^{1-\sigma}}{\int_{i=0}^{K+K*} p_i^{1-\sigma} di} \tag{3}$$

where s_j is variety j's share of total expenditure on X, and the p's are consumer prices. Utility optimization also implies the Euler equation $\dot{E}/E=r-\rho$ where r is the risk adjusted rate of return to savings, and a transversality condition.[1] (Numbered notes refer to the attached "Supplemental Guide to Calculations".)

On the supply side, free trade in Z equalizes nominal wage rates as long as both countries produce some Z. This is always true as long as α is not too large relative to the country-size difference.[2] When α violates this condition, all world Z may be produced in the foreign (small) nation, so while $p_Z=1$ in both nations $w>w^*=1$. For simplicity, however, this paper only considers values of α that are low enough to ensure long-run factor price equalization at all levels of trade barriers. Thus taking foreign L as numeraire, $p_Z=w=w^*=1$.

As always with monopolistic competition, optimizing X-firms engage in 'mill pricing'. Home X-firms therefore charge consumer prices 1 and τ* in the home and foreign markets respectively. Foreign firms have analogous conditions. Rearranging the first order conditions, the home rental rate is[3]:

$$\pi = \left(\frac{\alpha}{\sigma}\right)(sE + s^*E^*) \tag{4}$$

where s and s* are a typical home variety's share in the home and foreign markets; an analogous formula hold for π^*. Using mill pricing in (3), and rearranging (4) into global quantities and national share variables, national operating profits (rental rates) are[4]:

$$\pi = B\left(\frac{\alpha E^w}{\sigma K^w}\right); \qquad B \equiv \left(\frac{\theta_E}{\theta_K+\phi(1-\theta_K)} + \frac{\phi^*(1-\theta_E)}{\phi^*\theta_K+1-\theta_K}\right)$$
$$\pi^* = B^*\left(\frac{\alpha E^w}{\sigma K^w}\right); \qquad B^* \equiv \left(\frac{\phi\theta_E}{\theta_K+\phi(1-\theta_K)} + \frac{1-\theta_E}{\phi^*\theta_K+1-\theta_K}\right) \tag{5}$$

where E^w and K^w are world expenditure and world K stock, θ_K and θ_E are home shares of K^w and E^w, and $\phi \equiv \tau^{1-\sigma}$ and $\phi^* \equiv (\tau^*)^{1-\sigma}$ are measures of home and foreign openness. The ϕ's are mnemonics for the 'free-ness' of trade, with trade getting freer as ϕ rises from $\phi=0$ (prohibitive trade barriers) to $\phi=1$ (free trade). Also B and B* are mnemonics for the 'bias' in national sales, eg B measures the extent to which a home variety's sales exceed the world average of per-variety sales (which is $\alpha E^w/K^w$).

It is useful to note that the B's fully capture the impact of production and expenditure shifting on profits. Taking the symmetric case for simplicity, shifting expenditure to home ($d\theta_E>0$) raises π and lowers π^* since $dB/d\theta_E=-dB^*/d\theta_E=2(1-\phi)/(1+\phi)\geq 0$ with $\theta_K=1/2$ and $1\geq\phi\equiv\phi^*\geq 0$. Shifting production to home ($d\theta_K>0$), however, has the opposite effect on the π's since $dB/d\theta_K=-dB^*/d\theta_K=-2(1-\phi)^2/(1+\phi)^2\leq 0$ with $\theta_E=\theta_K=1/2$. The tendency for production shifting to lower profits in the receiving nation is sometimes referred to as the local competition effect. Lowering trade costs erodes the magnitude of both effects, but it erodes the local competition effect more rapidly.

Finally, I-sector competition implies K's price in both countries is F whenever K is sold. When a nation stops investing, no K is sold (since capital is international immobile and agents within a nation are identical), but the shadow price of K is less than marginal production costs F.

2.3 Long-Run Equilibrium

The long-run (i.e. steady-state) equilibrium is defined by the national capital stocks. Determining these is very much like determining the number of firms in a static model with free entry. That is, K and K* adjust up to the point where the expected present value of introducing a new variety equals the fixed set-up costs in both home and foreign. Denoting J and J* as these expected present values, the free entry conditions are $J\leq F$ and $J^*\leq F$, with strict equality for nations with nonzero steady-state capital stocks.

Calculating present values requires a discount rate. In the long-run equilibrium, $\dot{E}=0$ in both nations, so the Euler equation implies that home and foreign steady-state riskless discount rates equal ρ.[5] Given the Blanchard-depreciation[6]:

$$\bar{J} = \int_{t=0}^{\infty} e^{-(\rho+\delta)t}\,\bar{\pi}_t\,dt = \bar{\pi}\int_{t=0}^{\infty} e^{-(\rho+\delta)t}\,dt = \frac{\bar{\pi}}{\rho+\delta} \qquad (6)$$

where bars indicate steady-state values, and a similar expression holds for the steady-state J*. From (6), the long-run free-entry conditions are:

$$\bar{\pi} = (\rho+\delta)F, \qquad \bar{\pi}^* = (\rho+\delta)F \qquad (7)$$

Intuitively, this requires that the flow benefit of a unit of K (i.e. $\bar{\pi}$) equals the expected flow cost, where ρF is the borrowing cost and δF is the expected depreciation cost. Clearly (7) demands that $\bar{\pi}=\bar{\pi}^*$. Using (5), $\bar{\pi}=\bar{\pi}^*$ can be solved to yield[7]:

$$\bar{\theta}_K = \frac{-\phi}{1-\phi} + \frac{1-\phi\phi^*}{(1-\phi)(1-\phi^*)}\bar{\theta}_E \qquad (8)$$

This is the model's first key equilibrium condition. It pins down the combinations of the share variables that are consistent with long-run equilibrium profit levels.

Two aspects of (8) are worth highlighting. First, for any given free-ness of trade, shifting expenditure to the home country produces production shifting (i.e. $d\bar{\theta}_K/d\bar{\theta}_E>0$); this is one half of demand-linked circular causality. Second, when trade is perfectly free (i.e. $\phi=\phi^*=1$), the location equilibrium is undefined since any location equilibrium $\bar{\theta}_K$ satisfies (7).

Closing the model requires an expression for the steady-state θ_E and this task is facilitated by calculation of steady-state values of E^w and K^w. By definition $\bar{E}^w=\bar{Y}^w-\bar{I}^w$, where $\bar{Y}^w$ is world factor payments (equal to $L^w+(\rho+\delta)F\bar{K}^w$ since $\bar{\pi}=\bar{\pi}^*=(\rho+\delta)F$ in steady state) and $\bar{I}^w$ is world spending on replacement capital (equal to $\delta F\bar{K}^w$). Since $\bar{\pi}\bar{K}+\bar{\pi}^*\bar{K}^*=\alpha\bar{E}^w/\sigma$, $\bar{B}$ equals unity, so $\bar{\pi}=\alpha\bar{E}^w/\sigma\bar{K}^w$. Using this in $\bar{E}^w$'s definition gives us $\bar{E}^w$ in terms of L^w and parameters. Plugging this solution in $(\rho+\delta)F=\alpha\bar{E}^w/\sigma\bar{K}^w$ yields $\bar{K}^w$ in terms of L^w and parameters. Specifically[8]:

$$\bar{K}^w = \left(\frac{b}{1-b}\right)\frac{L^w}{\rho F}, \qquad \bar{E}^w = \frac{L^w}{1-b}; \qquad b \equiv \frac{\alpha\rho}{\sigma(\rho+\delta)} \qquad (9)$$

where 0<b<1 represents a group of parameters that appears frequently in the formulas.[9] Next observe that trade balance implies $\bar{E}=L-\bar{L}_I+\bar{\pi}\bar{K}$, where $\bar{L}_I=\bar{\theta}_K\delta F\bar{K}^w$ from (1). Using (9) in the definition of $\bar{E}$ yields the second key expression[10]:

$$\bar{\theta}_E = b\,\bar{\theta}_K + (1-b)\,\theta_L \qquad (10)$$

where θ_L is home share of L^w. Three results follow immediately. First, home's share of expenditure is a weighted average of home's share of world K and L. Second, (10) gives the second half of circular causality, namely, production shifting (defined as $d\theta_K>0$) implies expenditure shifting (defined as $d\theta_E>0$). Third, b measures the strength of the expenditure shifting effect.

The long-run location equilibrium is found analytically by solving (8) and (10):

$$\bar{\theta}_K = \left[\frac{-\phi}{1-\phi} + \frac{(1-\phi\phi^*)}{(1-\phi)(1-\phi^*)}(1-b)\theta_L\right]\frac{1}{\Delta}\ ; \quad \Delta \equiv 1 - \frac{(1-\phi\phi^*)b}{(1-\phi)(1-\phi^*)} \tag{11}$$

where the expression is only valid for combinations of trade barriers and θ_L that produce $0 \leq \bar{\theta}_K \leq 1$; for values of the right-hand side of (11) that are greater than unity, $\bar{\theta}_K$ equals unity and for those less than zero, $\bar{\theta}_K=0$. The equilibrium θ_E is from (11) and (10).

Observe from (8) and (10) that the symmetric equilibrium $\bar{\theta}_K=\bar{\theta}_E=\theta_L=1/2$ is always a solution when home and foreign are equally open, i.e. $\phi=\phi^*$. Finally it is worth emphasizing that, unlike the two standard economic geography models, the key equilibrium conditions of this model – (8) and (10) – are linear, so many results can be identified analytically. Of course the simplicity comes at a cost. In terms of results, there is not a range of trade costs where both the core-periphery and symmetric equilibria are stable as there is in the two standard geography models. More importantly, the simplicity requires us to ignore cost-linked agglomeration forces that are undoubtedly important in the real world.

2.4 Long-Run Equilibrium Welfare

Since no tariff revenue is collected, steady-state welfare calculations are trivial.* In steady state, the indirect utility function of a representative home agent is $\bar{\theta}_E\bar{E}^w/\bar{P}$, where $\bar{P}$ is the steady-state perfect price index. Recalling that $p_Z=p_Z{}^*=1$ and using (9) and (10), steady-state flows of utility for home and foreign agents are proportional to[11]:

$$\bar{W} = \left((b\bar{\theta}_K+(1-b)\theta_L)\left(\frac{L^w}{1-b}\right)\right)/\bar{P}\ , \quad \bar{W}^* = \left((1-b\bar{\theta}_K-(1-b)\theta_L)\left(\frac{L^w}{1-b}\right)\right)/\bar{P}^*$$

$$\bar{P} = \left(\left(\frac{bL^w}{(1-b)\rho F}\right)[\bar{\theta}_K(1-\phi)+\phi]\right)^{\frac{-\alpha}{\sigma-1}}\ , \quad \bar{P}^* = \left(\left(\frac{bL^w}{(1-b)\rho F}\right)[1-\bar{\theta}_K(1-\phi^*)]\right)^{\frac{-\alpha}{\sigma-1}} \tag{12}$$

The closed-form solution comes from plugging (11) into this, but it proves convenient to leave $\bar{W}$ and $\bar{W}^*$, in terms of $\bar{\theta}_K$. Note that $\bar{W}$ is strictly increasing in $\bar{\theta}_K$ (since $d\bar{\theta}_K>0$ raises $\bar{E}$ and lowers home's perfect price index), while $\bar{W}^*$ is strictly decreasing in $\bar{\theta}_K$.

3. Symmetric Equilibrium Instability& Catastrophic Agglomeration

Perhaps the most striking feature of the 'new' geography models is the result that reciprocal liberalization between initially symmetric nations produces catastrophic agglomeration. Namely, market opening between equal-sized nations has no impact on the location of industry until a critical level of openness is reached. Beyond this point, the symmetric equilibrium is unstable and the core-periphery outcome is the only stable equilibrium. Since this characteristic may be considered the dividing line between 'new' and 'old' economic geography, this section shows that the Section 2 model displays this feature.

*Allowing for transitional dynamics is very complex; see Baldwin (1992).

The standard procedure for investigating catastrophic agglomeration is based on Krugman (1991), and the extension by Puga (1997). In the core-periphery model, one firm is moved from foreign to home and instantaneous labour migration is assumed to equalize real wages. If this perturbation increases the profitability of home-based firms relative to that of foreign-based firms, the system is unstable since more firms would follow. If the relative profitability of home location falls, the system is stable.* The procedure adopted here is inspired by this approach. The appendix examines the model's stability properties using the more classical approach that relies on eigenvalues of the linearized system of differential equations.

3.1 Analytic Solution

In the context of the Section 2 model, the perturbation takes θ_K as a parameter and allows θ_E to fully adjust – according to (10) – to the θ_K shock. Following standard practice, consider first the symmetric case where $\phi=\phi^*$ and $\theta_L=1/2$.

A compact way of thinking about the stability investigation is to note that (5) gives π as a function of the θ's taking ϕ as a parameter, i.e. $\pi[\theta_K,\theta_E;\phi]$. The derivative we consider is $d\pi[\theta_K,\bar{\theta}_E[\theta_K,\theta_L];\phi]/d\theta_K$ where the function $\bar{\theta}_E[\theta_K,\theta_L]$, which defines the equilibrium θ_E as a function of θ_K, is given by (10). If $d\pi/d\theta_K$ is negative the equilibrium is stable. That is, if a unit of capital 'accidentally' disturbed symmetry, the 'accident' pushes capital's rental rate below its steady-state level in the 'receiving' nation (home). This induces home savers/investors to allow K to erode back to its pre-shock level. Moreover, since $d\pi/d\theta_K>0$ means $d\pi^*/d\theta_K<0$, foreign savers react in the opposite direction. In contrast, the symmetric equilibrium is unstable when $d\pi/d\theta_K$ is positive, since the 'accident' induces home savers to invest in additional capital and foreign savers to reduce their K. Of course this shift in K produces a shift in expenditure which amplifies both the increase in π and the decrease in π^*. Consequently, the net result in the unstable case is a catastrophic agglomeration, i.e. the core-periphery outcome. We turn now to signing $d\pi/d\theta_K$.

Using (5) and (9), taking θ_K as a parameter, π equals $(\rho+\delta)F$ times $\bar{B}$, where $\bar{B}$ is a function of θ_K and $\bar{\theta}_E$. Differentiating $\pi=(\rho+\delta)F\bar{B}$ at $\theta_K=1/2$ yields[12]:

$$\frac{d\pi}{d\theta_K} = (\rho+\delta)F\left(2\left(\frac{1-\phi}{1+\phi}\right)\frac{d\bar{\theta}_E}{d\theta_K} - 2\left(\frac{1-\phi}{1+\phi}\right)^2\right); \quad \frac{d\bar{\theta}_E}{d\theta_K} = b \equiv \frac{\alpha\rho}{\sigma(\rho+\delta)} \tag{13}$$

The first term in large parentheses is the pro-agglomeration expenditure-shifting effect. The second is the anti-agglomeration local-competition effect. Three points are worth making. First, reducing trade costs ($d\phi>0$) weakens both the pro- and anti-agglomeration forces, but it weakens the anti-agglomeration forces faster. Second, without circular causality between production and expenditure shifting, the θ_K shock would have no impact on the equilibrium θ_E – i.e. $d\bar{\theta}_E/\theta_K=0$ – so the symmetric equilibrium would be stable for all ϕ short of free trade. Third, with circular causality the equilibrium is stable for sufficiently high trade barriers (i.e. $\phi\approx0$), but it is unstable for sufficiently low ϕ (i.e. $\phi\approx1$).

* An exactly equivalent method is to move labour exogenously and allow the number of firms to adjust instantaneously. The stability test depends upon this shock's impact on the relative real wage.

At the critical level of ϕ (denoting this as ϕ^{cat} because beyond it catastrophic agglomeration occurs) the pro- and anti-agglomeration forces just balance. Thus:

$$\frac{1-\phi^{cat}}{1+\phi^{cat}} = \frac{d\bar{\theta}_E}{d\theta_K} \equiv b \qquad <=> \qquad \phi^{cat} = \frac{1-b}{1+b} \tag{14}$$

The set of unstable ϕ's, viz. $(\phi^{cat},1]$, expands as α and ρ rise, and σ and δ fall.

Intuition for ϕ^{cat}. The impact of σ and α on the instability set is familiar from the Krugman and the Krugman-Venables models. The novel elements here are ρ and δ. As δ falls, the expenditure shifting that comes with production shifting gets stronger, expanding the instability set. To see this, note that δ dampens the expenditure rise that comes with a higher K stock since depreciation means some additional resources must be devoted to maintenance instead of consumption. The lower is δ, the lower will be this dampening effect. Turning to ρ's impact, note that ρ rises the equilibrium $\bar{\pi}=F(\rho+\delta)$. Thus a higher ρ amplifies the expenditure shifting that accompanies a given amount of production shifting.

Comparison with Puga's Linearization. Puga (1997) introduced a technique (adopted by Fujita, Krugman and Venables, 1997) for analytically identifying ϕ^{cat} in the core-periphery and vertically-linked-industry models. By linearizing the model around the symmetric equilibrium, Puga (1997) showed that ϕ^{cat} in the footloose-labour model is (using my notation) $(1-\alpha)(\sigma(1-\alpha)-1)/[(1+\alpha)(\sigma(1+\alpha)-1)]$. The difference between ϕ^{cat} from (14) and Puga's ϕ^{cat} for the Krugman model is:

$$\frac{2\alpha\left(\rho[(\sigma-1)^2+\sigma(\sigma-\alpha^2)]+\sigma\delta(2\sigma-1)\right)}{[\sigma(1+\alpha)-1](1+\alpha)[\sigma(\rho+\delta)+\alpha\rho]} \tag{15}$$

This is positive by inspection. Thus the symmetric equilibrium in the footloose-labour model becomes unstable at a lower level of trade free-ness than ϕ^{cat} for the Section 2 model. This reflects the fact that agglomeration forces are stronger in the footloose labour model than in the Section 2 model, since the former includes cost-linked agglomeration forces.

Analytic Solution for the General Case. Next we analytically characterize ϕ^{cat} for the case of potentially asymmetric sized nations with potentially asymmetric trade barriers. Differentiating π with respect to θ_K, using $\bar{\theta}_E[\theta_K,\theta_L]$ as before, and evaluating the result at the $\bar{\theta}_K$ given by (11), we have:

$$\frac{d\pi}{d\theta_K} = (\rho+\delta)F\left(\frac{\{(1-\phi\phi^*)[\theta_L+b(1-\theta_L)]-(1-\phi^*)\}\ \{(1-\phi)(1-\phi^*)-b(1-\phi\,\phi^*)\}}{(1-\phi\phi^*)^2\left[(1-b)[(1-\phi^*)\theta_L-1]+\phi^*\right]\ [(1-b)(1-\phi)\theta_L-\phi b]}\right) \tag{16}$$

This is zero for interior θ_K's only when the second term in the numerator is zero since when the first numerator term is zero, (11) implies that $\bar{\theta}_K=1$. Thus for the general case, stability requires that $b\leq(1-\phi)(1-\phi^*)/(1-\phi\phi^*)$. Notice that this does not involve size asymmetries (i.e. θ_L) since the magnitudes of the stabilizing and destabilizing forces are unrelated to θ_L.

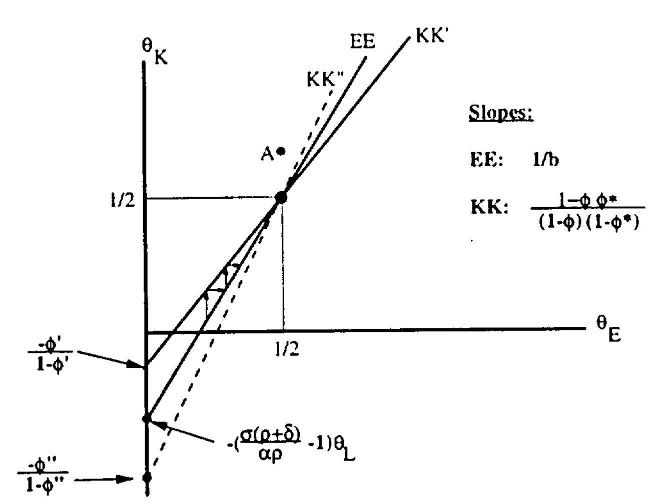
Solving $b=(1-\phi)(1-\phi^*)/(1-\phi\phi^*)$, we can express the critical level of ϕ, call this ϕ^{asy}, in terms of b and the degree of home protection asymmetry $\gamma\equiv\phi^*-\phi$. The answer, namely $\phi^{asy}=[2-\gamma(1+b)-(\gamma^2(1+b)^2+4b^2)^{1/2}]/2(1+b)$, shows that, as in the symmetric case, the critical level of home trade free-ness is decreasing in the size of the expenditure shifting effect b.[*] The novel result here is that the critical ϕ is also decreasing in γ, i.e. with the extent to which home's protection exceeds that of foreign.

Growth Poles and Growth Sinks. In this economic geography model, the Perroux (1955) notion of growth poles and growth sinks appears very clearly. Consider, for instance, an initially stable equilibrium that becomes unstable as trading cost fall and suppose the core ends up in the home nation. Given that the rates of return to investment in home and foreign are $\pi/F-\delta$ and $\pi^*/F^*-\delta$ respectively, (5) tells us that home r rises above ρ and r^* drops below ρ. Given intertemporal optimization, this means that home's saving/investment rate will rise above the rate necessary to sustain $\bar{K}$; home income and output therefore begin to rise. For similar reasons, foreign consumers/savers cease to invest, so K^* declines. In particular, foreign firms shut down one by one according to a Poisson process, and foreign income and output drop as a consequence. In home, this would appear as investment-led growth in transition to the new steady state; in foreign, it would be an investment-led recession.

3.2 Diagrammatic Analysis

The stability properties of the system are easily illustrated with Figure 1. The figure plots the two linear schedules (8) and (10) in θ_E-θ_K space as KK' and EE respectively. The heavy solid lines show the system for a level of $\phi=\phi^*$ (namely ϕ') that implies stability. The arrows show how the system is stable. Starting with a given θ_E, expression (8), i.e. KK', tells us what the equilibrium θ_K would be. This θ_K and (10), i.e. the EE schedule, tells us what the implied θ_E would be; the implied θ_E turns out to be greater than the initial θ_E, so the system converges to the $\theta_E=\theta_K=1/2$ point (assuming that $\theta_L=1/2$).

Figure 1



However, if trade becomes free enough, the relative slopes of KK and EE are reversed, so the system is unstable. Notice that only KK depends upon trade barriers and if ϕ is raised to, say ϕ'', the y-axis intercept of KK is below that of EE; this case is shown by the dashed line KK". Although the symmetric division of firms and expenditure is still an equilibrium, it is not stable. That is, if we started with a θ_E lower than 1/2, we would find that θ_K would ratchet down to zero. This is the core-periphery outcome.

From the diagram, it is obvious that ϕ^{cat} also defines the level of trade barriers below which the core-periphery outcome is sustainable. In the Fujita, Krugman and Venables (1997) terminology, the break point and the sustain point are identical in this model. Consequently, instead of the standard Fujita-Krugman-Venables 'tomahawk' diagram, the

[*]While there are two roots, the other does not equal ϕ^{cat} when $\gamma=0$.

plot of industry shares on trade costs resembles a sledgehammer.

3.3 Stability and Footloose Capital

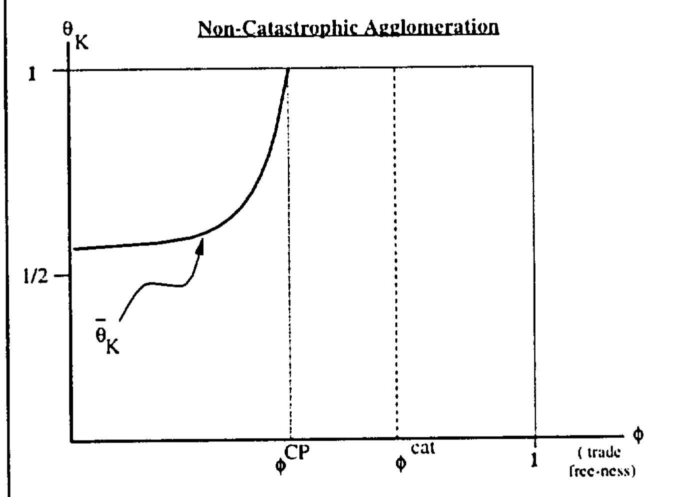
As it turns out, the immobility of capital is critical to the possibility of catastrophic agglomeration. We turn now to showing that the system is stable for any level of trade barriers when capital is mobile. The intuition for this is straightforward. When K can move internationally, but capital owners cannot, the earnings of K will be repatriated. Thus production shifting will not lead to expenditure shifting. Capital mobility, therefore, breaks the circular causality that is essential for the catastrophic agglomeration demonstrated in Section 3.

Formally, we introduce n and n* for the number of firms located in the home and foreign countries, keeping K and K* as the national capital stocks. The new (8) describes the home share of firms, namely θ_n, instead of θ_K, but θ_n has no effect on θ_E. Figure 1 is sufficient for showing the stability of this modified system, when the y-axis is interpreted as θ_n rather than θ_K. Since θ_E is unaffected by θ_n, EE becomes a vertical line. Any perturbation of θ_n would move the system to a point like A (where $\theta_E=1/2$ but $\theta_n>1/2$) where operating profits for home-based firms is below $(\rho+\delta)F$ and operating profits for foreign-based firms is above $(\rho+\delta)F$. Consequently, all perturbations are self-correcting.

4. Near-Catastrophic Agglomeration & Asymmetric-Size Nations

Krugman (1989, 1991) shows that liberalization between symmetric nations leads to location effects that are 'catastrophic'. Here the adjective catastrophic indicates that liberalization has no location effects until a critical level of trade cost is crossed but beyond that point total agglomeration is the only stable outcome. Krugman also shows that this all-or-nothing feature is a peculiarity of the symmetric case. When one nation is even slightly larger than the other, reciprocal liberalization leads to continuous delocation of firms from the small nation to the large.

Figure 2



Given that catastrophic agglomeration seems to be a knife-edge result, one might reasonably question the importance of the symmetric nation finding. This section addresses this concern by showing that the qualitative nature of catastrophic agglomeration is robust to small changes in country size. Specifically the section demonstrates that when nations are almost symmetric, the agglomeration process is near-catastrophic. Here near-catastrophic means that at high levels of trade costs, marginal liberalization has almost no location effect, but at levels of protection near some critical value, marginal liberalization has a very large location effect.

4.1 Liberalization with Size Asymmetries: Non-Catastrophic Agglomeration

In the Section 2 model, progressive reciprocal liberalization leads to progressive agglomeration when nations are unequal in size (i.e. $\theta_L>1/2$). To characterize this continuous, non-catastrophic agglomeration process analytically, two facts are established. These confirm the qualitative nature of the relationship between $\bar{\theta}_K$ and ϕ shown in Figure 2. First, progressive liberalization starting from prohibitive trade barriers produces the core-periphery outcome for levels of trade free-ness beyond a critical value (denoted as ϕ^{CP}) which is less than ϕ^{cat}. To show this note that with $\phi=\phi^*$, (11) reduces to[13]:

$$\bar{\theta}_K = \frac{-\phi + (1+\phi)(1-b)\theta_L}{(\frac{1-\phi}{1+\phi}-b)(1+\phi)}; \qquad b \equiv \frac{\alpha\rho}{\sigma(\rho+\delta)} \tag{17}$$

Solving (17) for ϕ when $\bar{\theta}_K=1$ defines ϕ^{CP} as:

$$\phi^{CP} = \frac{(1-b)(1-\theta_L)}{b+(1-b)\theta_L} \tag{18}$$

From (14), $\phi^{cat}=(1-b)/(1+b)$. Comparing the two[14]:

$$\phi^{cat} - \phi^{CP} = \frac{2\phi^{cat}}{b+(1-b)\theta_L}(\theta_L - \frac{1}{2}) \tag{19}$$

so $\phi^{CP}<\phi^{cat}$ when $\theta_L>1/2$, but the difference disappears as θ_L approaches 1/2.

Second, $\bar{\theta}_K$ is an increasing, convex function of ϕ, so agglomeration (as measured by $\bar{\theta}_K$) increases at an increasing rate as ϕ approaches ϕ^{CP}. Demonstrating this involves signing the first and second derivatives of $\bar{\theta}_K$ with respect to ϕ. Using (17)[15]:

$$\frac{d\bar{\theta}_K}{d\phi} = \frac{2(1-b)(\theta_L-1/2)}{(\frac{1-\phi}{1+\phi}-b)^2(1+\phi)^2}, \qquad \frac{d^2\bar{\theta}_K}{d\phi^2} = \frac{4(1-b^2)(\theta_L-1/2)}{(\frac{1-\phi}{1+\phi}-b)^3(1+\phi)^3} \tag{20}$$

Since $b=(1-\phi^{cat})/(1+\phi^{cat})$, both expressions are positive when $\theta_L>1/2$ and $\phi<\phi^{cat}$.

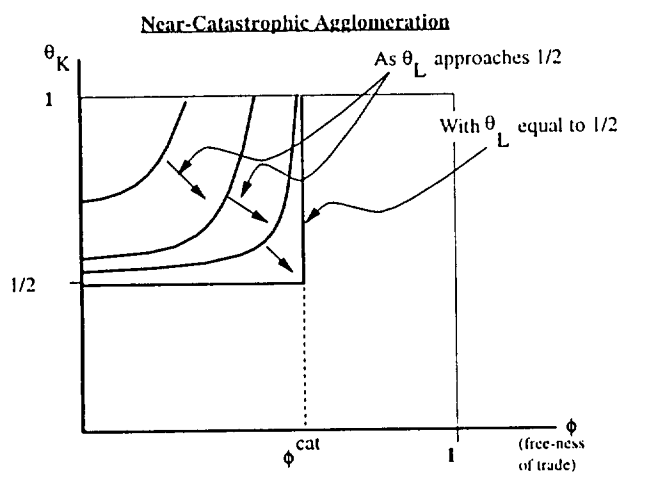
Near-Catastrophic Agglomeration. Having demonstrated that agglomeration is never catastrophic with unequal-sized nations, the next task is to show that as the countries approach the equal-sized case, agglomeration approaches catastrophic behaviour. The method is to examine the slope of the long-run equilibrium θ_K curve at its extremes – viz. $\phi=0$ and $\phi=\phi^{CP}$ – showing that as θ_L approaches 1/2, $d\bar{\theta}_K/d\phi$ evaluated at $\phi=\phi^{CP}$ limits to infinity and $d\bar{\theta}_K/d\phi$ evaluated at $\phi=0$ limits to zero. Showing these assertions to be true tells us that marginal liberalization brings about almost no agglomeration at very high levels of trade cost but brings about a great deal of agglomeration at ϕ near ϕ^{CP}. Figure 3 shows these relationships schematically.

Turning to the derivatives, observe that $d\bar{\theta}_K/d\phi$ at $\phi=\phi^{CP}$ and $\phi=0$ are:

$$\left(\frac{d\bar{\theta}_K}{d\phi}\right)\Big|_{\phi^{CP}} = \frac{[(1-b)\theta_L+b]^2}{2(1-b)(\theta_L-1/2)}\,, \qquad \left(\frac{d\bar{\theta}_K}{d\phi}\right)\Big|_0 = \frac{2}{1-b}\left(\theta_L-\frac{1}{2}\right) \tag{21}$$

so the first limits to infinity and the second limits to zero as θ_L approaches 1/2. Consequently, a small but discrete liberalization in the left-hand neighbourhood of ϕ^{CP} will produce a full agglomeration when countries are only slightly different in size. Moreover, when ϕ is near zero, small increases in ϕ lead to very little agglomeration. Finally, since ϕ^{CP} approaches ϕ^{cat} as θ_L approaches 1/2 and $b=(1-\phi^{cat})/(1+\phi^{cat})$, inspection of (20) shows that the $\bar{\theta}_K$ schedule gets infinitely convex as θ_L approaches 1/2.

Figure 3



5. Delocation and Liberalization

While the possibility of catastrophic and near-catastrophic agglomeration is startling and important for delineating the 'new' economic geography from the old, it is not much use for policy analysis. Real world nations are not identical and trade liberalizations are rarely symmetric. Combinatorics suggests that a large number of cases could be considered. This section focuses on three: Unilateral trade protection and liberalization with symmetric-sized nations, a liberalization scheme that permits unequal-sized nations to fully liberalize without delocalization, and customs union formation with initially symmetric nations.

5.1 Unilateral Protection and Liberalization: The Venables Effect Amplified

To consider the impact of asymmetric liberalization with equal-sized nations, suppose initially $\phi=\phi^*<\phi^{cat}$ and home varies its level of protection while ϕ^* is constant. Equation (11) with $\theta_L=1/2$ completely characterizes the location effects. Inspection of (11) reveals that $d\bar{\theta}_K/d\phi$ is negative over the range of ϕ's where $0<\bar{\theta}_K<1$. The end points of this range are the ϕ's where $\bar{\theta}_K=1$ and $\bar{\theta}_K=0$, namely:

$$\frac{2\phi^*-(1-b)}{\phi^*(1+b)}\,, \qquad \frac{1-b}{2-\phi^*(1+b)} \tag{22}$$

Both expressions correspond to core-periphery outcomes. The first entails the core in home and the second entails the core in foreign. Notice that if the initial trade barriers are too high – viz. $\phi=\phi^*<(1-b)/2$ – then even a prohibitive home barrier is insufficient to shift the core to home. In this case, the lower bound is $\phi=0$.

The next issues are the price and welfare implications of varying ϕ. Ignoring the difficult issue of transitional dynamics, the approach adopted here is to focus on long-run equilibrium welfare as a function of ϕ. This corresponds closely to the approach by Venables (1987), which shows – paradoxically – that home protection lowers the home price index via a location effect and therefore improves home welfare despite protection's direct effect on import price.* Venables (1987) focuses on the neighbourhood of the symmetric equilibrium. The analytic solution (11) permits extension of this analysis in three directions: to consideration of non-marginal protection changes, to consideration of asymmetric nations, and to consideration of the Venables effect in a model with agglomeration. As shown above, the production shifting that accompanies any given unilateral change in protection is amplified by circular causality, so the Venables effect is magnified by the agglomeration forces in the Section 2 model.

Consider first the price index effect. Since (9) shows $\bar{K}^w$ to be invariant to ϕ:

$$\frac{d\bar{P}}{d\phi} = \left(\frac{-\alpha}{\sigma-1}\right)\bar{K}^{w\,\frac{\alpha}{1-\sigma}}\left(\Psi^{\frac{\alpha}{1-\sigma}-1}\right)\frac{d\Psi}{d\phi}\ ; \qquad \Psi \equiv \bar{\theta}_K(1-\phi)+\phi \tag{23}$$

To extend the Venables result – i.e. to show that liberalization raises $\bar{P}$ since the location effect more than offsets the direct effect – $d\Psi/d\phi$ must be shown to be negative over the range defined in (22). Using (11), $d\Psi/d\phi$ is easily calculated, but the general expression is too cumbersome to be revealing. Instead, consider $d\Psi/d\phi$ evaluated at the minimum ϕ necessary to shift the core to home. At this ϕ, given by the first expression in (22)[16]:

$$\frac{d\Psi}{d\phi} = \frac{-(1-b)\phi^*}{2(\phi^{cat}-\phi^*)} < 0 \tag{24}$$

which is negative since ϕ^* is assumed to be low enough to ensure stability. To show that $d\Psi/d\phi$ is everywhere negative over the relevant range, it suffices to show that $d\Psi/d\phi$ gets more negative as ϕ rises, i.e. $d^2\Psi/d\phi^2$ is negative. The second derivative is:

$$\frac{d^2\Psi}{d\phi^2} = \frac{-b(1+b)(\phi^{cat}-\phi^*)}{(\Delta(1-\phi))^3(1-\phi^*)} < 0 \tag{25}$$

where Δ and ϕ^{cat} are given in (11) and (14); by inspection this is negative. Thus the home price index rises monotonically with ϕ over the range defined in (22).

The welfare aspect of the Venables effect is simple to show. From (12) $dW/d\bar{\theta}_K>0$ and when ϕ lies in the range defined by (22), $d\bar{\theta}_K/d\phi<0$ from (11), so $dW/d\phi<0$ in this range.

Given this result and the closed-form solution for $\bar{\theta}_K$ in (11), the welfare effects of varying ϕ from unity to zero can be conducted verbally. Starting at symmetric protection levels, raising home protection unambiguously lowers home's price index in a monotonic fashion (the Venables result) up to the point of total agglomeration of industry in home (the

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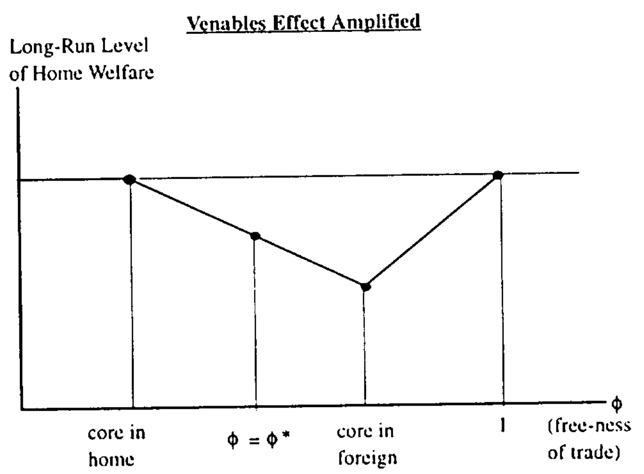
* The Venables result relies on free entry/exit; welfare during the entry/exit process is not considered. The welfare loss stems from a liberalization-induced terms of trade deterioration.

requisite level of ϕ is given by the first expression in (22)). Further protection has no impact on the home price index since the protection is applied to zero imports. In the case where $\phi^*<(1-b)/2$, the core-periphery outcome never occurs, so P falls continuously right up to $\phi=0$. Since production shifting moves $\bar{\theta}_E$ in the same direction as $\bar{\theta}_K$, welfare and price effects operate in the same direction.

Next consider the impact of home liberalization ($d\phi>0$). Again the Venables result shows up but again only up to a point. As home liberalizes, it loses firms and its price index rises. However once home protection is low enough, all X firms will be in the foreign market, so further liberalization lowers the home price index. In fact home achieves the same level of welfare with free trade as it does for levels of protection that bring the core to home. The full relationship between trade barriers and home welfare is summarized schematically in Figure 4. The highest price index (lowest utility level) is where the home is the 'periphery' and the foreign is the 'core', but home still has some protection.

Figure 4



This sort of protection might be thought of as 'strategic location policy'. That is, the positive and welfare results indicate that the lessons of the strategic trade policy literature could be applied directly to games between governments competing for industrial agglomerations. The well-known shortcomings of the strategic trade policy literature, however, suggest that such an exercise would be un-fruitful.

5.2 Liberalization without Delocation

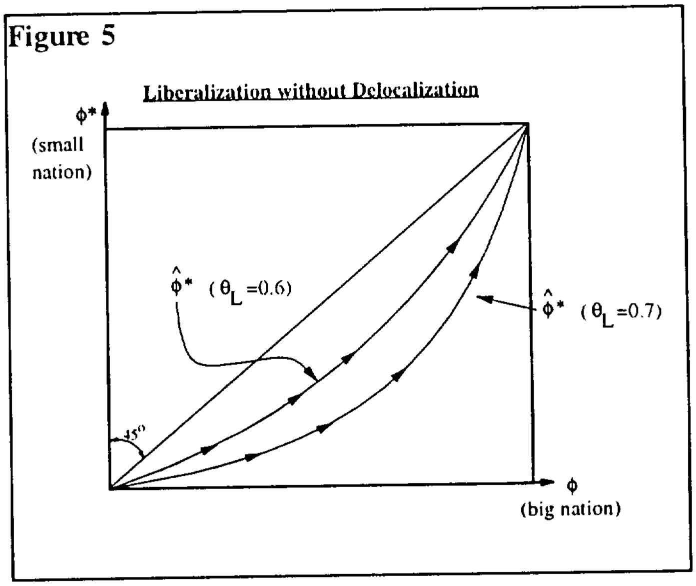
Next, consider asymmetric liberalization between unequal-sized nations. Following Robert-Nicoud (1996), the question is; "What must be the relationship between barriers of the large and small nations in order to ensure that full reciprocal liberalization leads to no delocation?" Robert-Nicoud (1996) examines the question in a setting with fixed capital stocks, and extends the analysis to the interesting case in which the large country also has a higher capital-labour ratio. The contribution here is to study the logic in a model where the long-run capital stocks are endogenous and circular causality is in operation.

The thought experiment is to start with asymmetric-sized nations, i.e. $\theta_L>1/2$, each with prohibitive trade barriers, so that initially $\bar{\theta}_K=\theta_L$. The large country progressively lowers its barriers until free trade, i.e. $\phi=1$, is reached. The task is to characterize the function $f[\phi]$ that defines the maximum ϕ^* consistent with no delocation, i.e. $\bar{\theta}_K=\theta_L$. Solving for ϕ^* from

(11) with $\bar{\theta}_K=\theta_L$:

$$\hat{\phi}^* = \frac{\phi(1-\theta_L)}{\theta_L - 2\phi(\theta_L-1/2)} \tag{26}$$

It is simple to characterize this function analytically, but Figure 5 presents the major results in a compact fashion.



In particular, the figure shows that the 'no-delocalization' liberalization path entails the small country maintaining higher barriers at all intermediate trade costs. Observe that the larger is the size differential, the larger must be the protection asymmetry. Moveover, the asymmetry must be greatest for intermediate levels of large country protection. Intuitively, this is due to two facts. At very low levels of trade barriers, delocation provided few benefits since market-access is affected only slightly by location. Consequently, slight asymmetries in trade barriers will offset the delocation tendency. At very high trade barriers, delocation is not very rewarding since the large nation's barriers make it difficult for delocating firms to supply the big market. Again, only small asymmetries are necessary to offset this slight delocation incentive.

This sort of liberalization scheme can be found in Europe. The Europe Agreements that the European Union (EU) signed with the Central and Eastern European countries (CEECs) are explicitly asymmetric, with the large nation (the EU) phasing out its tariffs faster than the small nations (the CEECs), but both going to zero tariffs.

5.3 Location Effects of Customs Union Formation

The analytic solution (11) can also be used to study the location effects of customs union (CU) formation. Take as the point of departure the classic situation of three initially symmetric nations. Presuming trade barriers are high enough for stability, each nation has a third of world industry. Labelling the two potential CU members as home and partner, this means that two-thirds of world firms are initially in the nations that will form the CU. Formation of the CU lowers trade costs to zero between home and partner without altering the common external barrier against the third nation. The question to be studied is: "Will home and partner together have more than two-thirds of the world's firms in the long-run equilibrium?"

When the CU is fully implemented, we can think of this three-country world as consisting of two unequal-sized regions, with one region having two-thirds of the world L. The equilibrium share of capital and firms in the big region is given by (11) setting θ_L=2/3.

An easy way to see the impact is to divide (11) through by θ_L to get:

$$\frac{\theta_K}{\theta_L} = \left[\frac{-\phi}{1-\phi}\frac{1}{\theta_L} + \frac{1+\phi}{1-\phi}(1-b)\right]\frac{1}{\Delta} \tag{27}$$

where we have set $\phi=\phi^*$. Since the coefficient on $1/\theta_L$ is negative (assuming stability), shifting L from one region to another shifts firms to the same region more than proportionally. Thus we know that the CU with two-thirds the world L will have more than two-thirds of the world firms.

The long-run equilibrium is only reached after a process of capital accumulation in the CU and decumulation in the rest of the world. Thus in transition, above-normal capital accumulation would imply above-normal growth in the CU and below-normal growth in the rest of the world.

6. Concluding Remarks

The 'new' economic geography literature has focused on two basic models. The model based on footloose labour by Krugman (1991), and the model based on vertically-linked industry by Venables (1996), and Krugman and Venables (1995). Both types of models are complex since they feature both demand-linked and cost-linked agglomeration forces. The model presented in this paper is much simpler because agglomeration is driven only by demand-linked circular causality.

Given the simplicity of the model, many analytic results are available. For instance, the critical level of trade barriers below which the symmetric equilibrium is unstable is identified analytically. Moreover, the paper demonstrates that the all-or-nothing nature of agglomeration in the systemic case is not as much of a knife-edge result as it appears. Symmetric liberalization between nearly equal-sized nations is shown to produce near-catastrophic agglomeration. The location and welfare effects of unilateral liberalisation and of customs union formation are also analytically derived.

The model's simplicity suggests that it may be useful for other applications such as the introduction of political economy considerations and endogenous growth. Indeed Baldwin, Martin and Ottaviano (1998), which was written after this paper, uses a version of this framework (modified to allow for ceaseless endogenous growth) to study links between income divergence, industrialization and growth takeoffs.

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Appendix

The classical approach to dynamic analysis in this sort of neoclassical-growth model takes as state variables the two E's and the two K's. Behaviour of these is described by four system equations – the two Euler equations and the two laws of motion for capital. We limit ourselves to situations where factor price equalization holds in the long-run. In this case w=w*, so, with labour as numeraire, the system equations are:

$$\dot{E} = E\left(\frac{\pi}{F} - \delta - \rho\right), \quad \dot{E}^* = E\left(\frac{\pi^*}{F} - \delta - \rho\right)$$
$$\dot{K} = \frac{Y-E}{F} - \delta K, \quad \dot{K}^* = \frac{Y^*-E^*}{F} - \delta K^* \tag{28}$$

where π is given by (5), and $Y=L+\pi K$ and $Y^*=L+\pi^*K^*$ since the symmetric case is being considered.

The standard procedure for evaluating the stability properties of a non-linear dynamic system is to linearize the system around the long-run equilibrium and to evaluate the eigenvalues of the corresponding Jacobian matrix (see Barro and Sala-i-Martin, 1995). In this case, the relevant Jacobian matrix consists of four symmetric sub-matrices, namely:

$$Jacobian = \begin{pmatrix} M1 & M2 \\ M3 & M4 \end{pmatrix} \tag{29}$$

where

$$M1 \equiv \left(\frac{\rho+\delta}{1+\phi}\right)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}, \quad M3 \equiv \left(\frac{1}{\sigma F(1+\phi)}\right)\begin{pmatrix} \alpha-\sigma(1+\phi) & \alpha\phi \\ \alpha\phi & \alpha-\sigma(1+\phi) \end{pmatrix}$$

$$M2 \equiv \left(\frac{-\sigma F(\rho+\delta)^2}{\alpha(1+\phi)^2}\right)\begin{pmatrix} (1+\phi)^2 & 2\phi \\ 2\phi & (1+\phi)^2 \end{pmatrix}, \quad M4 \equiv \left(\frac{-1}{(1+\phi)^2}\right)\begin{pmatrix} \delta(1+\phi^2)-2\rho\phi & 2\phi(\rho+\delta) \\ 2\phi(\rho+\delta) & \delta(1+\phi^2)-2\rho\phi \end{pmatrix}$$

Solving the characteristic equation yields the eigenvalues. Since this is a four dimensional system, the solution potentially requires us to solve four fourth-order polynomial equations. As it turns out, however, the block-wise symmetry of the Jacobian simplifies derivation of the eigenvalues. The resulting eigenvalues are (this and other results were derived in Maple 4; spreadsheet is available upon request):

$$\lambda_1 = \frac{\alpha\rho + \sqrt{radical_E}}{2\alpha}, \quad \lambda_2 = \frac{\alpha\rho - \sqrt{radical_E}}{2\alpha} \tag{30}$$

$$radical_E \equiv \rho^2(4\sigma-3\alpha)+4\delta(\rho(2\sigma-\alpha)+\sigma\delta)$$

and

$$\lambda_3 = \frac{real_K + \sqrt{radical_K}}{2\alpha(1+\phi)^2}, \quad \lambda_4 = \frac{real_K - \sqrt{radical_K}}{2\alpha(1+\phi)^2} \tag{31}$$

where

$$radical_K \equiv \alpha\rho^2 \left[4\sigma(1-\phi)^2(1+\phi)^2 - \alpha(3-5\phi^4-14\phi^2)\right]$$

$$+\ 4\delta(1-\phi)\left(\left[\alpha\rho[2\phi^2(1-\phi)-(1+\phi)^2]\right] + (1-\phi)\left[2\rho\sigma(1+\phi)^2+\delta(\sigma+\alpha)\phi^2+\sigma\delta(1+2\phi)\right] \right) \tag{32}$$

$$real_K \equiv 2\alpha\delta\phi(1-\phi)+\alpha\rho[(1-\phi)(1+\phi)+4\phi]$$

As usual the system's local dynamics are fully characterized by the sign and nature of the eigenvalues. If all eigenvalues are real numbers, the dynamics is non-oscillating in the sense that the system either diverges or converges in a monotonic fashion. The system is saddle path stable if and only if the number of negative eigenvalues matches the number of state variables that can jump. In this model, only E and E* can jump, so stability requires at least two negative eigenvalues. If all eigenvalues are real but fewer than two are negative, the system is locally unstable.

The analysis precedes in three steps. First we show that all four eigenvalues are real for all levels of trade costs. Then we show that for high enough trade costs (i.e. sufficiently low ϕ), the system is saddle path stable. Finally, we show that when trade becomes free enough, the system becomes unstable.

To show the system never oscillates, we must show that both terms under the radicals are non-negative. For the first two eigenvalues the task is simplified by the fact that trade costs do not enter. In fact because $\sigma>1>\alpha$, inspection of (30) is sufficient to establish the non-negativity of the term under the radical. For the last two eigenvalues the task is somewhat more involved. Nevertheless by plotting the term under the radical for a wide range of parameter values, it is possible to show that with the discount and failure rate near zero, the radical approaches zero as ϕ approaches unity. For higher discount and failure rates, the term is higher. In no case is the radical imaginary.*

The signs of the eigenvalues are established by comparing the real terms with the terms under their corresponding radical. If the square of former is less than the latter, the pair of eigenvalues have opposite signs. Turning to the first pair of eigenvalues, the condition

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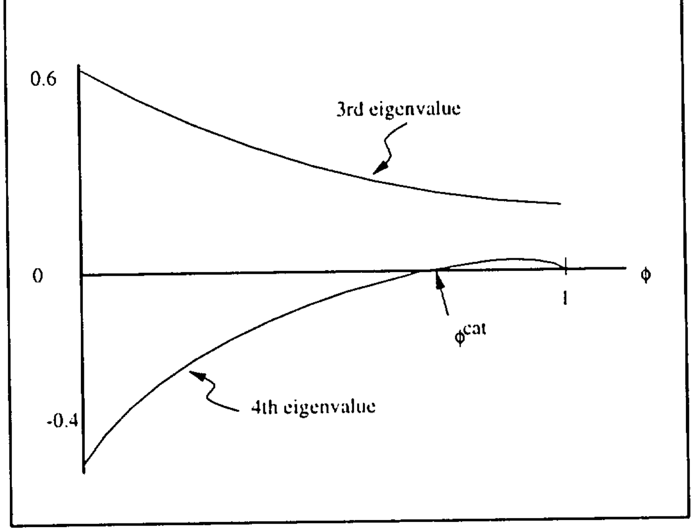
*Since the term under the radical involves only four parameters α, σ, ρ, and δ, we can search over all ϕ for any given level of ρ and δ using the 3-dimensional animation feature of Maple 4. This search was performed for ρ and δ values ranging from 0.01 to 0.5.

holds since, using the fact that $\sigma>1>\alpha$:

$$\frac{\alpha(3+\alpha)-4\sigma}{\sigma(\rho+\delta)+\rho(\sigma-\alpha)} < 0 < \frac{4\delta}{\rho^2} \tag{33}$$

Thus regardless of trade costs, the first two eigenvalues are real and of opposite sign.

Figure 6



The third eigenvalue is always positive since the radical and the real part are both positive. The fourth eigenvalue is more difficult since it depends on trade costs. To address the sign issue, we plot the third and fourth eigenvalues against trade cost using reasonable values of the parameters as shown in Figure 6 (in particular, $\delta=\rho=0.1$, $\alpha=0.3$, and $\sigma=2$). Observe three features. The third eigenvalue is always positive (as shown above), the fourth eigenvalue is negative for sufficiently low levels of trade 'free-ness' and finally, the sign of the fourth eigenvalue switches from negative to positive, exactly at the level of ϕ^{cat} that corresponds to the parameters chosen for the plot. Experimentation with other parameter values yields qualitatively identical results. Indeed, it is simple to analytically show that the fourth eigenvalue switches sign at ϕ^{cat}. At the switch-over, the fourth eigenvalue is zero, so it must be that the square of the real component equals the term under the radical. Solving the resulting equation, which is a fourth-order polynomial in ϕ, we get four solutions:

$$1\ ,\ \frac{1-b}{1+b}\ ,\ -1\ ,\ -1 \tag{34}$$

Only the first two are economically meaningful. The second is the formula for ϕ^{cat}.

To summarize, when $\phi>\phi^{cat}$, the system has three real, positive eigenvalues and one negative, real eigenvalue. While this implies that there is a one-dimensional stable manifold leading to the symmetric equilibrium, only two of the four variables can jump. Consequently, unless the perturbation disturbs K and K* in a way that puts the system exactly on the stable manifold (this is a probability zero event for random perturbations of K and K*), the system diverges to the core-periphery outcome. This divergence does not violate the transversality conditions, since the core-periphery outcome is a steady-state outcome when $\phi>\phi^{cat}$.

This illustrates the equivalence between the Krugman-inspired stability approach in Section 3 and the more traditional eigenvalue-based approach to stability analysis in models with fully specified dynamics and forward looking behaviour.

Supplemental Guide to Calculations

1. Given that preferences are intertemporally separable and consumers take the path of prices as given, we can solve the utility maximization problem in two stages. The first is to determine the optimal path of consumption expenditure E. To this end we set up the Hamiltonian, which for this problem is:

$$H[E,K,\lambda,t] = e^{-\rho t}\left(\ln(\frac{E}{P}) + \lambda(\frac{\pi K + wL - E}{P_K} - \delta K)\right)$$

where C=E/P, P is the perfect price index, P_K is the price of K, and the law of motion for the representative consumer's wealth is $\dot{K}=-\delta K+(Y-E)/F$ since K is the only store of value. The four standard necessary conditions for intertemporal utility maximization are:

$$\frac{\partial H}{\partial E} = 0 \quad <=> \quad e^{-\rho t}\left(\frac{1}{E} - \frac{\lambda}{P_K}\right) = 0$$

$$d\frac{e^{-\rho t}\lambda}{dt} = -\frac{\partial H}{\partial K} \quad <=> \quad \rho - \frac{\dot{\lambda}}{\lambda} = \frac{\pi}{P_K} - \delta$$

$$\textit{law of motion} \quad <=> \quad \dot{K} = -\delta K + (Y - E)/F$$

$$\textit{transversality condition} \quad <=> \quad lim_{t \to \infty}\ \lambda(t)K(t) = 0$$

The first three conditions characterize the optimum path at all moments in time, while the transversality condition is only an endpoint condition. The total time derivative of the first expression can be used to eliminate λ from the second expression. The result reduces to:

$$\frac{\dot{E}}{E} = (\frac{\pi}{P_K} + \frac{\dot{P}_K}{P_K} - \delta) - \rho$$

The Euler equation is found by noting that the right-hand expression in parentheses is the rate of return to K (the first term is the 'dividend' component and the second is the 'capital gains' component) and that this is the risk adjusted rate of return to savings, viz. r.

2. The formal condition is that L* is insufficient to meet global demand for Z. Since $a_Z=1$ this requires that $L^* \leq (1-\alpha)E^w$, where E^w is world expenditure. Using results derived in Section 2.3 for the long-run equilibrium E^w, this requires:

$$L^* \leq (1-\alpha)\frac{\sigma(\rho+\delta)}{\rho(\sigma-\alpha)+\delta\sigma}L^w \quad <=>$$

$$1-\theta_L \leq (1-\alpha)\frac{\sigma(\rho+\delta)}{\sigma(\rho+\delta)-\alpha\rho}$$

where θ_L is home's share of world L. Rearranging, the condition is:

$$\theta_L \geq \frac{\alpha\left(1-(\frac{\rho}{\sigma(\rho+\delta)})\right)}{1-\alpha(\frac{\rho}{\sigma(\rho+\delta)})}$$

Since the right-hand side of this is monotonically increasing in α, we know that if α is small relative to θ_L, then $w=w^*$.

3. A typical home firm's total operating profit compromises the operating profit earned on home (i.e. local) sales and foreign (i.e. export) sales. The first order condition for local sales can be rearranged into $p-a_x=p/\sigma$. Multiplying by home consumption c, the level of operating profit is $(p-a_x)c=pc/\sigma$, and $pc=s\alpha E$, where s is a home firm's share in the home market. Undertaking a similar manipulation on the first order condition for export sales we have:

$$\pi = (\frac{\alpha}{\sigma})\left(s_H^H E + s_H^F E^*\right)$$

where the s_{from}^{to} notation is used here for market shares. A similar procedure yields the expression for π^*.

4. Using the optimal pricing rules:

$$s_H^H = \frac{1}{K+K^*\phi}, \quad s_H^F = \frac{\phi^*}{\phi^*K+K^*}$$

where $\phi=(\tau)^{(1-\sigma)}$ and $\phi^*=(\tau^*)^{(1-\sigma)}$ measure the level of trade barriers. Utilizing the share formulas in the expression for operating profit, we have (with some rearrangement):

$$\pi = (\frac{\alpha E^w}{\sigma K^w})\left(\frac{\theta_E}{\theta_K+\phi(1-\theta_K)} + \frac{\phi^*(1-\theta_E)}{\phi^*\theta_K+1-\theta_K}\right)$$

where the superscript "w" indicates world totals. A similar procedure yields the expression for π^*.

5. More formally, with L as numeraire the steady state is marked by a time-invariant allocation of labour among sectors. Since we are working with a Ricardian model (only one primary factor) expenditure allocation is tantamount to resource allocation. Therefore

a time invariant allocation of resources between the production of consumption goods and investment goods is a necessary and sufficient condition for E to be time invariant. That is to say, when a time-invariant amount of labour is devoted to the production of consumption, the output of consumption goods measured in units of L is time-invariant. Supply must equal demand, so expenditure (in terms of labour) must also be time-invariant.

The representative consumer's portfolio in non-stochastic due to perfect diversification. Dixit-Stiglitz implies there is always an infinite number of firms; the chance that any firm's capital depreciates is δ. Thus the "risk-adjusted" required rate of return is $\rho+\delta$. what I mean by the "risk-less rate" is the rate that would occur if the risk that each unit of K would depreciate were zero.

6. The term $e^{-(\rho+\delta)t}=e^{-(\rho)t}e^{-(\delta)t}$ reflects the riskless discount rate and the probability that a unit of capital is still working at time t. Alternatively, we can think of $\delta+\rho$ as the risk adjusted rate (to a perfectly diversified shareholder) on the asset that pays π but is subject to the Blanchard depreciation risk.

7. To understand why these seemingly nonlinear expressions have a unique analytic solution, form a common denominator and set the two expressions equal. Since the denominators cancel and the numerators are linear in the K's, the solution for K is trivial, namely:

$$K = \alpha\frac{E(1-\phi)-E^{*}\phi(1-\phi^{*})}{\sigma\rho(1-\phi)(1-\phi^{*})}$$

Expressing this in terms of world shares yields the expression in the text. See Helpman and Krugman (1985 Chapter 10.4) for details.

8. Since in steady state $\pi=\pi^*$, the definition $E^w=L^w-L^w_I+\pi K^w$ implies:

$$\bar{E}^w = L^w-\bar{L}_I^w + \frac{\alpha\bar{E}^w}{\sigma} = \frac{L^w-\bar{L}_I^w}{1-\alpha/\sigma}$$

Because $1/\delta$ of the capital stock fails each period, the steady state L^w_I equals $\delta F\bar{K}^w$, so the expression for $\bar{E}^w$ in terms of $\bar{K}^w$ is:

$$\bar{E}^w = \frac{L^w-\delta F\bar{K}^w}{1-\alpha/\sigma}$$

Plugging this into $\bar{\pi}=\alpha\bar{E}^w/\sigma\bar{K}^w=(\rho+\delta)F$ and solving for $\bar{K}^w$ yields:

$$\bar{K}^w = \frac{\alpha L^w/F}{\rho(\sigma-\alpha)+\delta\sigma}$$

Utilizing this expression for $\bar{K}^w$ in our expression for $\bar{E}^w$ yields:

$$\bar{E}^w = \frac{\sigma(\rho+\delta)}{\rho(\sigma-\alpha)+\delta\sigma}L^w$$

Rearranging these in terms of the $b \equiv \alpha\rho/\sigma(\rho+\delta)$ notation yields to expressions in the text.

9. Since none of the parameters can be negative, $b>0$. To show that $b<1$, suppose not so

$$\alpha\rho \;>\; \sigma(\rho+\delta) \qquad <=> \qquad \rho(\alpha-\sigma) \;>\; \sigma\delta$$

The left side of the second expression is negative yet $\sigma\delta>0$, so by contradiction $b<1$.

10. The definition of steady state E is:

$$\bar{E} \;=\; L - \bar{\theta}_K \delta F \bar{K}^w + \bar{\pi}\bar{K}$$

Using (2-9) and the expression for $\bar{\pi}$, this becomes:

$$\bar{E} \;=\; L - \bar{\theta}_K \delta\left(\frac{\alpha L^w}{\sigma(\rho+\delta)-\alpha\rho}\right) + \bar{\theta}_K \alpha \frac{\bar{E}^w}{\sigma}$$

so

$$\frac{\bar{E}}{\bar{E}^w} \;=\; \frac{L}{L^w}\,\frac{\sigma(\rho+\delta)-\alpha\rho}{\sigma(\rho+\delta)} + \bar{\theta}_K\left(\frac{\alpha}{\sigma} - \frac{\alpha\delta}{\sigma(\rho+\delta)}\right)$$

Rearranging this yields the expression in the text.

11. The standard perfect price index with CES preferences nested in an upper-tier Cobb-Douglas utility function is:

$$P = p_Z^{1-\alpha} P_X^{\alpha}\;; \qquad P_X = \left(Kp^{1-\sigma} + K^* p^{*(1-\sigma)}\right)^{\frac{1}{1-\sigma}}$$

using $p=w=1$ and $p^*=\tau$ and gathering terms yields the result in the text.

12. This formulation follows Baldwin, Martin and Ottaviano (1998) and is simpler than that of earlier drafts.

To see how it is derived, we use (5), (9) and the definition of steady-state E^w:

$$\bar{\pi} \;=\; \frac{\alpha b}{\sigma\rho F}\left(\frac{\bar{\theta}_E}{\bar{\theta}_K+\phi(1-\bar{\theta}_K)} + \frac{\phi\,(1-\bar{\theta}_E)}{\bar{\theta}_K\phi+(1-\bar{\theta}_K)}\right)$$

Differentiation of π with respect to θ_K gives:

$$\frac{d\pi}{d\theta_K}\,\frac{\sigma\rho F}{\alpha b} \;=\; -\left(\frac{\theta_E}{\theta_K(1-\phi)+\phi}\,\frac{1-\phi}{\theta_K(1-\phi)+\phi} - \frac{\phi\,(1-\theta_E)}{1-\theta_K(1-\phi)}\,\frac{1-\phi}{1-\theta_K(1-\phi)}\right)$$
$$+ \left(\frac{1}{\theta_K(1-\phi)+\phi} - \frac{\phi}{1-\theta_K(1-\phi)}\right)\frac{\partial\theta_E[\theta_K,\theta_L]}{\partial\theta_K}$$

Using symmetry ($\theta_K=\theta_E=1/2$), and defining $\Delta=\theta_K(1+\phi)$ this is:

$$\frac{d\pi}{d\theta_K}\frac{\sigma\rho F}{\alpha b} = -(\frac{1-\phi}{\Delta})(\frac{\theta_E}{\Delta})(1-\phi) + (\frac{1-\phi}{\Delta})\frac{\partial\theta_E[\theta_K,\theta_L]}{\partial\theta_K}$$

$$= -(\frac{2(1-\phi)}{1+\phi})(\frac{1-\phi}{1+\phi}) + (\frac{2(1-\phi)}{1+\phi})\frac{\partial\theta_E[\theta_K,\theta_L]}{\partial\theta_K}$$

Using (10) to find the partial derivative

$$\frac{d\pi}{d\theta_K} = \frac{\alpha b}{\sigma\rho F}\left(-(\frac{2(1-\phi)}{1+\phi})(\frac{1-\phi}{1+\phi}) + (\frac{2(1-\phi)}{1+\phi})\, b\right)$$

Manipulation of the last expression yields the formula in the text.

13. Using $(1-\phi\phi^*)=(1-\phi)(1+\phi)$ when $\phi=\phi^*$, the expression is easily derived.

14. The intermediate steps are:

$$\phi^{cat} - \phi^{CP} = \frac{1-b}{1+b} - \frac{(1-b)(1-\theta_L)}{b+(1-b)\theta_L}$$

$$= (1-b)\left(\frac{1}{1+b} - \frac{(1-\theta_L)}{b+(1-b)\theta_L}\right) = (1-b)\left(\frac{b+(1-b)\theta_L - (1+b)(1-\theta_L)}{(1+b)(b+(1-b)\theta_L)}\right)$$

$$= \frac{(1-b)}{(1+b)}\left(\frac{2\theta_L-1}{(b+(1-b)\theta_L)}\right) = \frac{2\phi^{cat}}{b+(1-b)\theta_L}(\theta_L - \frac{1}{2})$$

15. With some simplification $d\bar{\theta}_K/d\phi$ can be shown to equal:

$$\frac{d\bar{\theta}_K}{d\phi} = \frac{-(1-b)+2\theta_L(1-b)}{(-1+\phi+b+b\phi)^2}$$

Rearranging the denominator, we have:

$$\frac{-(1-b)+2\theta_L(1-b)}{(-1+\phi+b+b\phi)^2} = (\frac{1-\phi}{1+\phi}-b)^2(1+\phi)^2$$

Obvious rearrangement of the numerator yields the result in the text. Similar manipulations yield the expression for the second derivative.

16. These calculations, which are rather involved, were done with Maple worksheet welf.mws, available upon request.