

## HIGH WATER MARKS

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### **ABSTRACT**

Incentive fees for money managers are frequently accompanied by high water mark provisions which condition the payment of the incentive upon exceeding the maximum achieved share value. In this paper, we show that these high water mark contracts are valuable to money managers, and conversely represent a claim on a significant proportion of investor wealth. We provide a closed-form solution to the high water mark contract under certain conditions. This solution shows that managers have an incentive to take risks.

We conjecture that the existence of high water mark compensation is due to decreasing returns to scale in the industry. Empirical evidence on the relationship between fund return and net money flows into and out of funds suggests that successful managers, and large fund managers are less willing to take new money than small fund managers.

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## **I. Introduction**

The growth of the hedge fund industry over this past decade has brought an unusual form of incentive contract to the attention of the investment community. Hedge fund managers typically receive a proportion of the fund return each year in excess of the portfolio's previous high water mark, i.e. the maximum share value since the inception of the fund. These incentive fees generally range from 15% to 25% of the return of new profits each year and managers also charge an additional fixed fee of 1% to 2% of portfolio assets. For example, George Soros' Quantum Fund charges investors an annual fixed fee of 1% of net asset value, and a high water mark based incentive fee of 20% of net new profits earned annually. As a result, the Quantum Fund returned 49% (pre-fee) in 1995 on net assets of \$3.7 billion resulting in an estimated total compensation of \$393 million for that year, most of which was due to the incentive terms.<sup>†</sup> Of course, when the high water mark is not achieved, manager returns are substantially reduced. In 1996, the Quantum fund lost 1.5%, and thus earned only their fixed fee on \$5.4 billion — \$54 million. While the Quantum fund stands out as an unusually good performer over the past decade, its compensation terms are typical of the hedge fund industry. High water mark contracts have the appealing feature of paying the manager a bonus only when the investor makes a profit, and in addition, requiring that the manager make up any earlier losses before becoming eligible for the bonus payment. On the other hand, their option-like characteristics clearly induce risk-taking behavior when the manager is below the high watermark, and the large bonus of 20% above the benchmark clearly reduces long-run asset growth.

In this paper we examine the costs and benefits of high-water mark compensation to the investor. To do so, we develop a valuation equation which allows us to estimate the division of wealth that an investor implicitly makes with the portfolio manager, upon entering into a such a contract. We find that the manager receives as much as 40% of investor wealth for reasonable parameters of the valuation equation. A significant proportion

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<sup>†</sup> Figures from the U.S. Offshore Funds Directory, 1995 and 1996 editions for the Quantum Fund N.V. Returns assume re-investment of income, and manager fees calculated from reported changes in net asset value.

of this compensation is due to the incentive feature of the contract, however the tradeoff between fixed fees and high water mark fees depends upon the volatility of the portfolio and the investor withdrawal policy. We find that this proportion is high when money is “hot” i.e. when the probability of investors leaving the fund is high, and when the volatility of the assets is high. In contrast, when investors are likely to remain for the long term, and when volatility is low, the fixed-fee portion of the contract provides the greatest value to the manager.

We also consider why high water mark contracts exist, and in particular, why they are used by hedge fund managers as opposed to, say, mutual funds. While their prevalence in the hedge fund industry might be an accident of history, the high water mark compensation contract may have features particularly suited to the types of investment strategies employed by hedge funds. The role of volatility and investor withdrawal, for example, may account for why we find high water mark incentives are used in “speculative” asset classes such as hedge funds, commodity funds and venture capital funds. In these asset classes, investor payoff is presumably based more upon expectations of superior manager skill and less upon the expected returns to an undifferentiated or passively managed portfolio of assets. Given that hedge fund investment is, in a sense, a pure bet on manager skill, our analysis provides a framework for considering how much skill a hedge fund manager must have to justify earning such high fees.

In addition to valuation of the high water mark contract, we explore the question of whether the high water mark compensation is due to the fact that hedge fund technology may have diminishing returns to scale. Most hedge fund managers are engaged in some form of “arbitrage in expectations,” in the domestic and global debt, equity, currency and commodities markets. By its very nature, arbitrage returns may not be scaled up as investors purchase more fund shares. To test whether the high water mark contract may be a substitute for increasing compensation through fund growth, we examine the empirical relationship between hedge fund investor cash-flows and performance. In contrast to similar studies in the mutual fund industry, we find that large funds, and funds with superior performance, do not issue new shares — indeed we find evidence that they experience net reclamations. This

is consistent with the hypothesis that the hedge fund industry itself has important limits to growth. This has implications for investors seeking alternative investments to equities and debt. While hedge fund performance over the past seven years has been strong on a risk-adjusted basis, this performance may be in part due to the relatively small size of the hedge fund sector. The unwillingness of successful funds to accept new money may be indicative of diminishing returns to the industry as a whole as investment dollars flow in. We conjecture that the option-like fees commanded by hedge funds exist because managers cannot expect to trade on past superior performance to increase compensation through growth.

The paper is structured as follows. Section II develops a valuation equation for determining the cost of the manager's contract. Section III provides some comparative statics and discusses the implications of our results. Section IV estimates parameters for the model, using empirical data on hedge funds. Section V presents evidence on hedge fund performance, size and fund flows. Section VI concludes.

## **II. The cost of the management contract**

The hedge fund management contract has interesting option-like characteristics. It is a potentially perpetual contract with a path-dependent payoff. The payoff at any point in time depends on the high-water mark which is related to the maximum asset value achieved. As such the contract can be valued using option-pricing methods. We begin our analysis under the simple null hypothesis that the manager provides no additional component of return; i.e., there is no manager skill in predicting excess returns or timing the market.

We work in a continuous-time framework and assume that, in the absence of payouts, the assets of the fund,  $S$ , follow a lognormal diffusion process with expected rate of return  $\mu$  and variance  $\sigma^2$ .  $H$  is the current high-water mark; it is the highest level the net asset value has reached subject to certain adjustments. The net asset value is reduced when the client

makes withdrawals or receives distributions from the fund.<sup>†</sup> There may be regular or periodic withdrawals or distributions which we approximate as continuous occurring at the rate  $W(S, H, t)$ . Also the fund may be liquidated by a complete withdrawal or distribution of the assets at any time. The probability of liquidation at any point is  $\Lambda(S, H, t)$ .

The fund has operating expenses, including, a regular management fee which must be paid from the assets. We assume these expenses are proportional to the value of the fund,  $cS$  per unit time. When the asset price moves above the high-water mark, the manager also collects an extra or incentive fee equal to the fraction  $k$  of this return. In the stylized setting of the model, the incentive fee is earned continuously. In practice, the incentive fee is usually accrued on a monthly basis with  $H$  being reset annually or quarterly.  $F(S, H, t)$  is the present value of future fees and operating costs.

The evolution of the assets of the fund (in the absence of liquidation) is

$$dS = [\mu S - W(S, H, t) - cS]dt + \sigma S d\omega \quad (1)$$

The high-water mark is the highest level the asset value has reached net of any withdrawals and certain expenses allocated to its reduction. If the withdrawals and allocated expenses are a fraction  $a$  of the asset value, then the high-water mark is also adjusted down by the fraction  $a$ , that is,  $dH = -aHdt$ . So the evolution of  $H$  is

$$dH = -\frac{W(S, H, t) + c'S}{S}Hdt \quad (2)$$

where  $c'S$  is the costs and fees allocated to reducing the high-water mark.

While the fund's assets are below the high-water mark, *i.e.*,  $S < H$ , the cost function satisfies the option-like partial differential equation

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<sup>†</sup>Since hedge funds generally have no “maturity date” and frequently pay no dividends, a perpetual investment from which only the manager withdraws cash would be worthless to the investor. The withdrawal of funds ensures the contract has some value to the investor.

$$\begin{aligned}
0 = & \frac{1}{2}\sigma^2 S^2 F_{SS} + [rS - W(S, H, t) - cS]F_S \\
& - \frac{W(S, H, t) + cS}{S}HF_H + F_t - \hat{\Lambda}(S, H, t)F - rF + cS .
\end{aligned} \tag{3}$$

This equation has the standard Black-Scholes interpretation.

The first four terms are the expected risk-neutral change in the value of the fees due to the change in  $S$ ,  $H$ , and  $t$ . The expected rate of return on  $S$  has been risk-neutralized to  $r$ . The change in  $H$  requires no similar adjustment since it is locally deterministic and therefore free of risk.

The function  $\hat{\Lambda}(\cdot)$  is the risk-neutral probability of liquidation. The term  $-\hat{\Lambda}F$  is the risk-neutral expected change in the value of future fees due to a liquidation. The value drops from  $F$  to zero with a risk-neutral probability of  $\hat{\Lambda}$ .

The term  $cS$  is the flow rate of costs whose present value (along with that of the incentive fees) we are determining. It is like a dividend paid to the derivative asset in the Black-Scholes model.

Four boundary conditions are required to solve this equation. Three of the boundary conditions for the problem are

$$F(0, H, t) = 0 , \quad F_H(S, \infty, t) = 0 , \quad \text{and} \quad F(S, H, T) = \Xi(S, H) \tag{4}$$

The first condition says if the asset value falls to a zero, then there are no further costs. The second condition says if the high-water mark is very high (relative to the asset value), then there is little chance of ever receiving an incentive payment so a change in the high-water mark will not affect the value. The third condition applies a contractual sharing rule at the maturity of the management contract. Invariably these hedge funds have no contractual termination so such a boundary condition would not apply.

The third condition applies along the boundary  $S = H$ . When the asset value rises from  $S = H$  to  $S = H + \epsilon$ , the high water mark is reset to  $H + \epsilon$ , and an incentive fee of  $k\epsilon$  is



paid reducing the asset value to  $H + \varepsilon(1 - k)$ . Therefore,  $F(H + \varepsilon, H) = k\varepsilon + F(H + \varepsilon - k\varepsilon, H + \varepsilon)$  and

$$\begin{aligned} k\varepsilon &= F(H + \varepsilon, H, t) - F(H + \varepsilon - k\varepsilon, H + \varepsilon, t) \\ &\approx \left[ k\varepsilon \frac{\partial F}{\partial S} - \varepsilon \frac{\partial F}{\partial H} \right] \Big|_{S=H} . \end{aligned} \quad (5)$$

In the limit as  $\varepsilon \rightarrow 0$  this is exact giving our third boundary condition

$$\left[ k \frac{\partial F}{\partial S} - \frac{\partial F}{\partial H} \right] \Big|_{S=H} = k . \quad (6)$$

We consider the special cases when withdrawals are proportional to asset value  $W(S, H, t) = wS$ , and the risk-neutral probability of liquidation is constant,  $\Lambda(S, H, t) = \lambda$ . Since withdrawals and liquidation were the only time-dependent features of the problem, the cost function  $F$  no longer depends on time under this assumption, and  $F_t = 0$ . Furthermore, it is clear by inspection and the economics of the problem that  $F$  is now homogeneous of degree one in  $S$  and  $H$ , so the solution has the form  $F(S, H, t) = HG(x)$  with  $x \equiv S/H$ . Substituting this into (3) gives an ordinary differential equation

$$\frac{1}{2}\sigma^2 x^2 G_{xx} + (r + c' - c)xG_x - (r + c' + w + \lambda)G + cx = 0 . \quad (7)$$

The solution to this equation is  $G(x) = c/(w + c + \lambda)x + Ax^\gamma$  where  $A$  is a constant of integration and  $\gamma$  is the positive root of the quadratic equation<sup>†</sup>

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<sup>†</sup>The boundary conditions for  $G$  corresponding to those in ? are  $G(0) = 0$  and  $\lim_{x \rightarrow 0} G(x) - xG_x(x) = 0$ . The solution corresponding to the negative root of the quadratic equation can be eliminated since it gives an unbounded value for  $G(0)$ .

$$\begin{aligned}
Q(\gamma) &\equiv \frac{1}{2}\sigma^2\gamma^2 + \left(r+c'-c-\frac{1}{2}\sigma^2\right)\gamma - (r+c'+w+\lambda) = 0 \\
\text{i.e.,} \\
\gamma &\equiv \frac{\frac{1}{2}\sigma^2+c-r-c'+\sqrt{\left(\frac{1}{2}\sigma^2+c-r-c'\right)^2+2\sigma^2(r+c'+w+\lambda)}}{\sigma^2} .
\end{aligned} \tag{8}$$

Note that  $\gamma$  must be bigger than 1 since the quadratic form,  $Q(\gamma)$ , is convex and  $Q(1) < 0$  implying the positive root must exceed 1.

In terms of the original variables,  $S$  and  $H$ , the solution for the time-invariant function is

$$F(S,H) = \frac{c}{w+c+\lambda}S + AH^{1-\gamma}S^\gamma . \tag{9}$$

The only remaining task is to determine the constant of integration  $A$ . Applying the third boundary condition (6) gives

$$A = \frac{k}{\gamma(1+k)-1} \cdot \frac{w+\lambda}{w+c+\lambda} . \tag{10}$$

So the present value of all future fees is

$$F(S, H) = \frac{c}{c+w+\lambda}S + \frac{w+\lambda}{c+w+\lambda} \cdot \Psi(k, \gamma) H^{1-\gamma} S^\gamma$$

where

$$\Psi(k, \gamma) \equiv \frac{k}{\gamma(1+k)-1} \quad (11)$$

$$\gamma \equiv \frac{\frac{1}{2}\sigma^2 + c - r - c' + \sqrt{\left(\frac{1}{2}\sigma^2 + c - r - c'\right)^2 + 2\sigma^2(r + c' + w + \lambda)}}{\sigma^2}$$

### III. Interpretation

There are seven parameters in the valuation function:  $r$  and  $\sigma$  are environmental;  $k$ ,  $c$ , and  $c'$  are contractual; and  $w$  and  $\lambda$  are actually endogenous choices which we have assumed here to be constant. These “choice” variables enter the solution in a symmetric fashion; an increase in the withdrawal rate,  $w$ , is exactly the same as an increase in the probability intensity of liquidation. In other words withdrawing funds at the constant rate  $w$  has exactly the same effect on the cost as a probability  $w$  per unit time of withdrawing all the funds.

Similarly the parameters  $r$  and  $c'$  have symmetric effects. An increase in the interest rate is equivalent to an increase in the fees which are allocated to reducing the high-water mark. Note that an increase in  $c'$  is an accounting change that does not directly increase costs. It does, however, indirectly increase the present value of future costs by lowering the high-water mark and thereby shortening the time until the incentive fee is charged. An increase in  $c'$  is therefore like an increase in  $r$  which increases the risk-neutral expected rate of growth in the assets which shortens the expected time until the incentive fee is charged.

The first term of the solution  $cS/(c+w+\lambda)$  is the present value in perpetuity of the regular fees. Since the investor withdraws funds at the rate  $w$  and costs occur at the rate  $c$ , the present value of all future costs is the proportion  $c/(w+c)$  of the asset value corrected for

the probability that the flows will be terminated with a liquidation. The second term is the present value of the incentive fees. It can be expressed as a product of three factors

$$\text{PV}[\text{Incentive}] = F(S, H) - \frac{c}{w + c + \lambda} S = \frac{w + \lambda}{w + c + \lambda} S \cdot \Psi(k, \gamma) \cdot (S/H)^{\gamma-1} \quad (12)$$

The first factor,  $(w + \lambda)S/(w + c + \lambda)$ , is the present value of the assets net of the future regular costs. The factor  $\Psi(k, \gamma)$  measures the present value of the incentive fees as a fraction of this “remaining” value at the inception of the contract (or whenever the asset value is at the high-water mark). The final factor  $(S/H)^{\gamma-1}$  is the reduction in the present value of future incentive fees due to the extra time required before the asset value hits the high-water mark again.

Figure 1 plots the present value of the management costs as a fraction of the asset value; that is

$$\frac{F(S, S)}{S} = \frac{c}{w + \lambda + c} + \frac{w + \lambda}{w + \lambda + c} \Psi(k, \gamma) \quad (13)$$

for a range of values for the incentive fee,  $k$ , when the high water mark is equal to the asset value, for the parameter values  $\sigma = 20\%$ ,  $r + c' = 5\%$ ,  $w + \lambda = 5\%$ , and  $c = 1\%$ . Even without an incentive fee, the fraction of wealth given to the manager is surprisingly high. For instance, in the simple case where the incentive fee is zero, the manager claims a percentage of the assets in proportion to his claim on the future payouts of the fund. With a 5% payout and a 1% fixed fee, this is 16.7%. Presumably, this fixed fee is not all profit to the manager. It must cover management expenses. For active managers, these costs may be high. Even a low-cost equity index fund may have expenses of 40 basis points. With the payout rule of 5%, index fund fees translate into a 7½% fraction of investor wealth. With a payout ratio equal to current dividend yields, this fraction increases to 13%. Thus, our analytical framework demonstrates that even low fixed fees claim a non-trivial proportion of investment assets. As the incentive fee rises, the proportion of assets it represents increases

— rapidly at first. Given a volatility of 20% and incentive fees of 20%, the incentive fees amount to a fraction of assets between 10% and 15%.

Due to the perpetual nature of the investment problem, the cost fraction is very sensitive to the withdrawal policy,  $w + \lambda$ . Figure 2 shows the ratio of the fixed fee value to the incentive fee value for ranges of asset volatility  $\sigma$  and the withdrawal policy  $w + \lambda$ . Notice that for low withdrawal rates and low asset volatility, the fixed fee portion of the compensation is the dominant source of value. This is not surprising, since the option value is increasing in  $\sigma$  and the present value in perpetuity of the regular fees is decreasing in  $w$ . For asset volatility over 10% and withdrawal policies over 20%, the high water mark compensation has the greatest manager value. This suggests that manager compensation contracts may separate according to the volatility of the strategies and investment outflows.

#### *IV. Model parameters*

What are reasonable parameter values for the valuation equation? To address this we turn to the database of hedge fund returns used in Brown, Goetzmann and Ibbotson (1997) [BGI]. The data are annual returns and fund characteristics gathered from the 1990 through 1996 volumes of the *U.S. Offshore Funds Directory*, the only publicly available source of information about hedge funds that includes defunct as well as surviving funds. Offshore funds in the directory represent a substantial portion of the hedge funds in operation, and include most of the major managers.<sup>†</sup>

##### *IV.1 Fund volatility*

To estimate the fund volatility, we calculate the sample standard deviation for all funds. Of 610 hedge funds in the sample, 229 have return histories exceeding two years. Of this group, the median and mean sample standard deviation are 18.7% and 23.0% per year, respectively. There are two reasons why such a small percentage of funds have enough data to calculate volatility. First, many funds have started recently, so a large number of the

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<sup>†</sup> See Brown, Goetzmann and Ibbotson (1997) for a complete discussion of the coverage of the database.

extant funds have only a short track record. Second, the attrition rate for funds is relatively high — about 20% of these funds fail each year. Since we are effectively conditioning upon fund survival we are presumably losing the funds which had such poor returns that they failed in their second year. This may bias our volatility estimate downward.

#### *IV.2 Withdrawal rate, $w$ and $\lambda$ the probability of liquidation*

In our model, the payout policy  $w$  is a flow, however it is unlikely that all hedge fund investors conceive of it that way. A constant payout ratio is a reasonable assumption for certain institutional investors such as university endowments and charitable foundations which choose payout ratios as a matter of policy, however it may not be a reasonable assumption for the most common type of hedge fund investor — traditionally a high net worth individuals. Life-cycle issues are potentially important, and in addition, modeling the conditional probability of withdrawal may be useful in determining a realistic value for  $w$ .

The annual attrition rate of about 20% in the BGI data suggest that  $\lambda$ , the probability of fund liquidation may be high. The empirically estimated attrition rate in fact conditions upon the fund appearing originally in the annual volume. Consequently, a fund must survive through the end of the year of its inception to be counted. This would suggest that new funds have a probability of liquidation greater than 20%. By the same token, some of the largest funds, such as the Quantum fund, are long-lived. The 20% is not a dollar-weighted estimate of fund disappearance. Consequently, the probability of liquidation might be lower than 20% on a dollar-weighted basis.

#### *IV.3 Incentive fee, $k$ and fixed fee $c$*

Figure 3 is a histogram of the incentive fees and the fixed fees for the BGI data. 15% is the most common incentive fee, and Figure 4 indicates that 2% is the most common fixed fee. A natural question is what factors differentiate funds on the basis of fees. We tried volatility, past performance and fund size as predictors, and found none to explain differences in incentive fees.

#### IV.4 Fixed fee vs. performance fee

How does a high water mark contract compare to a simple fixed fee contract? Absent any incentive differences implied by the contracts, it is possible to characterize the trade-off between a higher fixed fee and the incentive fee, conditional upon a given value,  $F$ . To do this, we assume that investors are indifferent among contracts that cost the same in terms of the manager's fraction. Solving for different values of the incentive fee in terms of the contract value as a fixed point provides a measure of the trade-off. Figure 5 shows the tradeoffs for a representative set of parameters. For a benchmark contract of 20% performance fee and 1% fixed fee,  $w+\lambda=5\%$ ,  $\sigma=20\%$  the figure shows that the manager fraction would be preserved at the same value by a fixed fee of 3% with no incentive fee. This trade-off is dramatically affected by the volatility of the assets, but not so much by the withdrawal/liquidation policy  $w+\lambda$ . With asset volatility at 50%, the investor is willing to pay a 6% fixed fee to eliminate the incentive fee of 20%.

### V. Positive risk-adjusted returns

#### V.1 Required alphas

Thus far we have not addressed the question of managerial ability. By how much does the manager's rate of return have to exceed that of passively managed assets in order to justify the fee structure? All the analysis has been based upon a model in which the manager has no extra information. Indeed, nowhere in the valuation equation does the expected rate of return on the asset value appear. Nevertheless, the simple perpetuity framework with fixed payout policies in fact allows us to explore this question in a limited but potentially interesting way.

The beneficial ownership interest in the fund is worth  $S - F(S, H)$  so the owners "claim" represents a fraction

$$\frac{S - F(S, H)}{S} = \frac{w + \lambda}{c + w + \lambda} \left[ 1 - \Psi(k, \gamma) (H/S)^{\gamma-1} \right] \quad (14)$$

of the assets. For investors to be indifferent between investing in the hedge fund and investment in a passive index (with no costs), the managers must provide expertise which increases the value of the assets by a factor equal to the reciprocal of this ratio.

Suppose the managed portfolio has an opportunity cost of capital  $\mu$ . What active rate of return  $\mu + \alpha$  is implicitly required to offset the cost discount? Assume for the moment there are no withdrawals. Then if the fund's assets grow at the rate  $\mu + \alpha$ , each dollar at time 0 will be worth  $e^{(\mu+\alpha)t}$  at time  $t$ . The fund will be liquidated with probability  $\lambda e^{-\lambda t}$  at time  $t$ . The appropriate discount rate is  $\mu$ , so one dollar under management is worth

$$\int_0^{\infty} \lambda e^{-\lambda t} e^{(\mu+\alpha)t} e^{-\mu t} dt = \frac{\lambda}{\lambda - \alpha} \quad (15)$$

When there are withdrawals,  $\lambda$  is replaced by the effective withdrawal rate  $\lambda + w$  as before. So for an investor to be indifferent between the hedge fund investment and an investment in a similar passive index (with no costs) the following condition should be satisfied

$$1 = \frac{S - F \left[ \frac{\lambda + w}{\lambda + w - \alpha} \right]}{S} \quad (16)$$

$$\alpha = (\lambda + w) \frac{c + (\lambda + w) \Psi(S, H) (H/S)^{\gamma-1}}{c + \lambda + w}$$

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Thus, the alpha required by the investor is a product of the payout ratio times the manager's fraction of assets. Figure 6 shows the required alpha for varying asset volatility and varying withdrawal/liquidation policies. The increasing value in  $\sigma$  follows from the option characteristic of the valuation equation. Notice that for asset volatilities around 20%, the required alpha is about 4%, depending upon the withdrawal policy, and/or the probability of fund liquidation. When the probability of liquidation is high, the alpha required to compensate the investor increases dramatically. The increasing required alpha in  $w$  follows from equation (16). [ Jon -- this seems strange. Maybe I did the analysis wrong?]



Although it seems natural to identify the manager's contribution in terms of a positive additional rate of return — an alpha — this might not be the appropriate way of considering the benefits to investing in a hedge fund. The benefits expressed by alpha are linear in the capitalization of the fund, but hedge funds might in fact provide decreasing returns to scale. An alternative way of thinking of hedge funds is that they are firms that can capture a fixed amount of “arbitrage” profits in the economy. In other words, they have a limited net present value. The choice of how to finance this venture is a capital structure decision. From this perspective, the issuance of additional shares has a diluting effect on the outstanding claims — investors simply divide a fixed pie of arbitrage gains. In this framework, new money, i.e. a positive flow of funds into the account from new investors, has only limited attraction to the hedge fund manager. It benefits him only to the extent that he is unable to borrow fully what his activities require, or to the extent that he fears bankruptcy through a margin call.

## **VI. Incentives and new money**

Do hedge funds take new money when they do well? If the manager's technology were linear, then on balance, more money would be welcome. If not, then new money, at least for large funds, would be accepted when the fund decreased in scale, rather than when it grew. To test the hypothesis that hedge fund managers do not accept new money when they do well, we examine the relationship between flow of funds and past performance for hedge funds by regressing net fund growth on lagged return in cross section. If managers accept new money after a good year, and/or investors pull out of poorly performing funds, we would expect to find a positive regression coefficient. On the other hand, if managers refuse new money after a good year, and seek additional funding after a bad year, then we would expect to find negative a regression coefficient on past returns. We define net fund growth as the increase in net asset value of the fund due to the purchase of new shares, as opposed to the investment return of the fund. This requires us to make the simplifying assumption that new shares are purchased only at the beginning of the year — purchases

during the year will be interpreted as investment return.<sup>†</sup> Another problematic issue is survivorship. Although we have defunct fund data, we must make some assumption regarding the fund outflow in the year of its disappearance. We address survival issues by assuming a 100% outflow for the year a fund closes. We control for year effects by performing the regression separately for each year, and also by including year dummies for the stacked regression.

### *V.1 New money regression results*

Besides estimating a single linear response, we also consider how the response differs depending upon past fund performance. Following Sirri and Tufano (1996) and Goetzmann and Peles (1997) we examine the differential response of new money to past returns via a piecewise linear regression. We separate fund return in cross section into quintiles each year, and allow the coefficients to differ across quintile. We test for the equality of the coefficients across quintiles via a Chow test. The results for the single response regression are reported in panel 1 of Table I and the results for the piecewise regression are reported in panel 2 of Table 1. The year-by year results for the piecewise regression are reported in Table 2.

The results from panel 1 indicate that new money responds negatively to past positive performance. The response differs across quintile of lagged returns, however. The best and worst performers have quite different coefficients. Panel 2 shows that new money responds by flowing out of poor performers, but does not flow into good performers as one might expect. These results are quite different from the pattern observed in mutual funds. Sirri and Tufano (1992), Chevalier and Ellison (1995) and Goetzmann and Peles (1997), for example, all find a positive response to superior performance. The negative response to top performance we find in the hedge fund universe provides some support for the hypothesis that good performers may not readily accept new money.

### *V.2 Sorting on size*

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<sup>†</sup> When we assumed that money flowed in at the end of the period, the results were essentially the same.

Another approach to the issue of whether the technology of hedge funds is linear is to test whether larger funds continue to take new money. We can address this question simply by sorting on size, and then averaging a measure of new money. Table 3 reports the results of this exercise. We break funds into size quintiles in the first period, and then we average the net growth of the fund in the following period for each quintile. We define growth slightly differently, under the assumption that money flows in at the end of the period. As in the previous test, we find this change make no difference our results. Table 3 shows that the largest size funds have net cash outflows, while the smallest performers have net cash inflows. This pattern is consistent throughout the period, with negative flows for large funds and positive flows for small funds each year. The second panel of the table shows the results of t-tests for each group, annually as well as in the aggregate — the extreme quintiles have means different from 0. As in the previous test, this pattern is consistent with the story that well-capitalized funds avoid taking new money. It differs in that it is also consistent with the hypothesis that smaller funds raise capital. Since we did not sort on performance, many of the funds in the first quintile may be good performers, and thus able to raise new money, or stop funds from flowing out.

Taken together, the empirical tests suggest that hedge fund managers behave differently than mutual fund managers with respect to accepting new money. While mutual funds demonstrate dramatic positive inflows into superior performers, this appears not to be the case with hedge funds. In addition, large funds do not seem to grow at a rate as high as smaller funds — even when growth is measured in dollar terms rather than percentage terms. We conjecture that this may be due to the limits of the investment strategies employed by hedge fund managers. To the extent that they engage in “arbitrage in expectations,” success creates its own limitations. Million dollar winning positions may not be possible when the assets grow to billions of dollars.

## **VII. Conclusion**

Hedge funds are an interesting new investment class with an unusual form of manager compensation. In this paper, we provide a closed-form expression for the value of

a hedge fund manager contract. We also provide estimates of the typical parameter values for the equation, and we examine its implications to both the manager and the investor. The high water mark provision creates a distinct option-like feature to the contract. As such, it is clear that the value of the contract to the manager increases in the variance of the portfolio. As a result, the manager has an incentive increase risk. Depending upon the variance, the incentive fee effectively “costs” the investor 10% to 15% of the portfolio. With fixed fees, the total percentage of wealth claimed by the hedge fund manager can be between 30% and 40%. Investing with a hedge fund manager would only appear to be rational if he or she provided a large positive risk-adjusted return in compensation. When we consider the possibility that managers are able to create value, i.e. provide a positive alpha in return for the incentives, we find that investors would accept 200 to 500 basis points of additional fixed fee per year to forego the incentive feature of the contract. Put another way, if managers are able to provide positive alphas, we find that rational investors would expect 200 to 900 basis point is additional risk-adjusted return when they enter into a hedge fund contract. Interestingly, BGI report that alphas for hedge funds over the 1989 through 1995 period are positive, and range from 4% to 8% annually. Consequently, hedge fund contracts may be priced about right.

The closed-form valuation equation demonstrates the crucial role that the withdrawal policy plays in the valuation of the manager contract. The most common type of manager fee is a fixed percentage of assets. When assets are placed with a manager (or a class of managers with the same fee structure) for the long term, then the implicit cost to the investor can be high, when the withdrawal policy is low. The manager’s percentage fees are like an additional discount applied to the future cash flows from the fund.

In considering why high water mark contracts exist in the hedge fund industry, we considered how hedge funds differ in terms of the product they offer. An analysis of the relative benefits of the fixed fee vs. the incentive fee to the manager suggests that high variance strategies, and strategies for which the investor may pull out soon, lend themselves to high water mark contracting. The relative value of the fixed fee portion on the contract decreases as the time until the investor withdraws decreases. Empirical evidence on the short

half-life of hedge funds may explain why hedge fund managers choose to use high water mark contracts.

It has become nearly axiomatic in studies of the investment management industry that managers seek to increase the size of assets under management. This presumes, however, that the benefits to investment in the fund can be scaled up with the growth in net asset value. Hedge fund strategies are fundamentally different from “long” asset portfolio strategies, however. Large sectors of the hedge fund industry have nearly zero “beta” exposure. Many hedge funds use the invested money as margin for maintaining offsetting long and short positions. Hedge fund managers are made up of event arbitrageurs, global macro market and debt market speculators, pairs traders and opportunistic managers exploiting “undervalued” securities. They use leverage of all types to exploit these opportunities — from short-selling equities to sophisticated debt repurchase agreements. In this context, the dollar investment benefits the manager only to the extent that he is credit constrained in his strategy. By their very nature, arbitrage in expectations are not infinitely exploitable.

Since it is not possible to directly investigate the relationship between scale and strategy payoff, we use flow of fund, return and size data from the hedge fund industry over the period 1989 through 1995 to explore the issue of linear vs. non-linear returns to scale. Regression of net growth in fund assets on lagged returns indicates that, unlike the mutual fund industry, the hedge funds show a net decrease in investment, conditional upon past performance. We conjecture that this is due to the manager’s unwillingness to increase the fund size. A sort on fund size, however shows that small funds tend to grow (net of returns), while large funds tend to shrink.

This pattern may help explain the usefulness of the high water mark compensation to the hedge fund manager. While mutual fund managers and pension fund managers can increase their compensation by growing assets under management, hedge fund managers cannot. Thus, they must explicitly build in benefits conditional upon positive returns, since they appear to resist net growth.

The implications of these results extend beyond the issue of the cost of compensation within an unusual sector of the investment industry. The existence of high water mark contracts may in fact be a signal to investors that the returns in the industry are diminishing in scale. Option-like incentive contracts are scarce in the mutual fund industry and pension fund management industry, but are prevalent in the real estate sector, the venture capital sector and the hedge fund sector. Perhaps the compensation structure itself is telling us that future returns in these asset classes depend crucially upon how much money is chasing a limited set of unique opportunities.

## References

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Goetzmann, William N. and Nadav Peles, 1997, "Cognitive Dissonance and Mutual Fund Investors," *Journal of Financial Research*, Summer.

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**Table 1: Net Fund Growth and Lagged Returns, 1990 - 1995**

The table reports the results of two linear regressions of net fund growth on previous year returns. The growth in net asset value of fund  $i$  in year  $t$ ,  $N_{it}$ , is defined as the new dollar cash flow into the fund (in millions) in the year following the return observation. It is calculated as  $N_{it} = \text{NAV}_{i,t-1}[(1+G_{it})/(1+R_{it})-1]$  where  $\text{NAV}_{it}$  is the fund net asset value in year  $t$ ,  $R_{it}$  is the total return for fund  $i$  in year  $t$ , and  $G_{it}$  is the percent change in net asset value for fund  $i$  in the year  $t$ . This assumes that money is only invested at the beginning of the year, and that reinvested dividends are defined as growth. The form of the regressions are:

Year effects are captured by dummies  $I_j$  defined as differing from 1990. Coefficients on returns are allowed to differ according to quartiles each year:  $R_{i,t-1,q}$  where coefficients 6 through 10 capture quartiles 1 through 4. The null hypothesis is that flows are independent of returns, i.e.,  $\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$ .

$$(2) \quad N_{it+1} = \beta_0 + \sum_{j=1}^5 \beta_j I_j + \sum_{q=6}^9 \beta_q R_{i,t-1,q} + e_{it}$$

**Regression 1 Results**

	coef	std.err	t.stat	p.value
Intercept	-1.71	19.4	-0.08	0.92
1990	0.01	24.7	0.00	0.99
1991	10.56	23.4	0.45	0.65
1992	39.75	21.9	1.81	0.06
1993	-18.37	21.0	-0.87	0.38
1994	-16.83	21.1	-0.79	0.42
Net Growth	-62.28	24.0	-2.60	0.00

Multiple R-Square = 0.0306 N = 934

**Regression 2 Results**

Intercept	-7.104	20.4	-0.3484	0.7276
1990	14.357	25.1	0.5709	0.5682
1991	18.254	23.4	0.7815	0.4347
1992	45.338	21.9	2.0695	0.0388
1993	-19.063	20.9	-0.9116	0.3622
1994	-0.585	22.4	-0.0261	0.9792
Quintile 1	127.621	65.6	1.9450	0.0521
2	42.556	130.1	0.3272	0.7436
3	60.703	92.0	0.6601	0.5094
4	9.453	61.4	0.1541	0.8776
Quintile 5	-112.260	27.9	-4.0292	0.0001

Multiple R-Square = 0.0491

Chow test of coefficient equality: F = 5.23, 3,872 p-value = .998



**Table 2: Year by Year Regression Results**

This table reports the results of year-by-year regressions analogous to those described in Table 1. These are cross-sectional regressions in which new money [ $N$ ] in period  $t+1$  is regressed on period  $t$  fund return. Coefficients are allowed to vary by the quartile of return. New money is denominated in millions of dollars. The year indicates  $t$ , thus the 1990 column shows 1990 new money regressed on 1989 returns.

**Coefficients**

	1990	1991	1992	1993	1994	1995
Int	-22.0	15.3	46.2	47.4	-19.3	-1.1
Q1	-48.8	56.6	163.6	666.4	-21.5	199.9
Q2	-229.3	245.2	-603.5	-788.1	107.2	146.7
Q3	77.5	-416.3	-279.6	197.3	26.0	843.6
Q4	67.9	-96.1	-151.1	29.0	-14.6	-149.6
Q5	88.3	-164.9	-162.2	-92.7	-164.1	-40.7

**Standard Errors**

	1990	1991	1992	1993	1994	1995
Int	12.1	18.7	36.2	31.6	23.5	12.1
Q1	105.2	127.5	168.6	377.8	243.9	85.3
Q2	161.4	419.2	606.7	942.7	266.1	196.4
Q3	87.9	594.3	293.9	404.8	160.4	572.4
Q4	69.1	212.8	176.4	239.8	111.7	390.9
Q5	45.2	73.1	89.1	108.5	52.4	66.7

**t-Statistics**

	1990	1991	1992	1993	1994	1995
Int	-1.823	0.820	1.277	1.500	-0.819	-0.091
Q1	-0.464	0.444	0.970	1.764	-0.088	2.344
Q2	-1.421	0.585	-0.995	-0.836	0.403	0.747
Q3	0.882	-0.701	-0.951	0.487	0.162	1.474
Q4	0.982	-0.452	-0.857	0.121	-0.131	-0.383
Q5	1.954	-2.257	-1.821	-0.855	-3.132	-0.610

**Table 3: Fund Growth Sorted on Size**

For each year, funds are sorted on size into quintiles, and the average net growth for each quintile in the following year is reported. Net growth is defined as the new money, in millions, measured for each fund, assuming dollar flows at the end of the period. The last row on each panel reports the results for the aggregate across years. A t-test is performed for each quintile separately and the t-statistic is reported in the second panel. The null hypothesis is that the net growth is different from zero.

**Net Fund Growth by Size Quintiles**

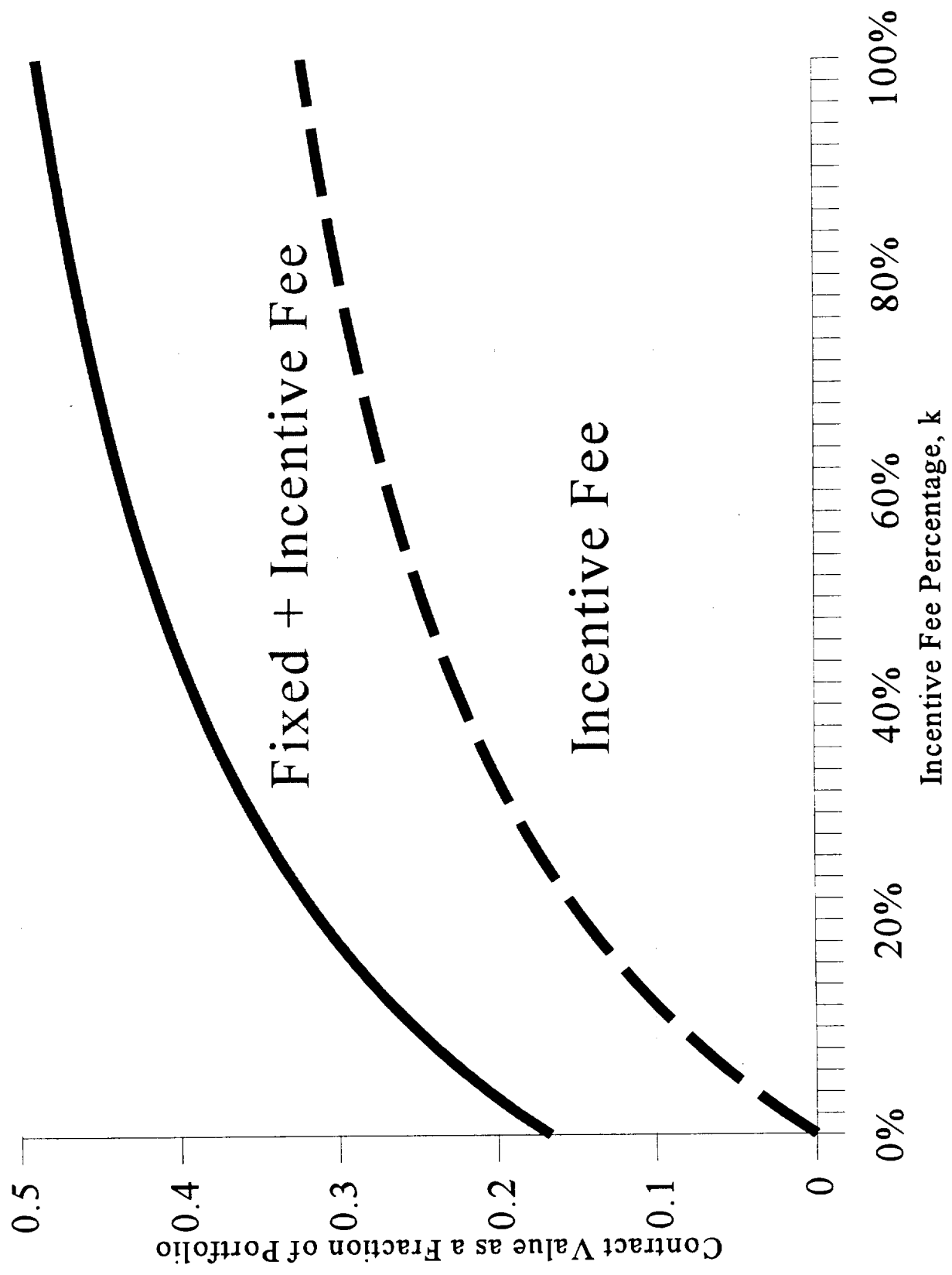
	Q1	Q2	Q3	Q4	Q5	Yr. Avg
90	6.93	1.32	-7.461	2.100	-26.8	-4.69
91	2.32	1.99	0.434	3.378	-58.8	-10.82
92	42.98	2.75	3.444	1.838	-85.1	-9.97
93	5.12	3.97	14.399	4.211	-13.4	2.34
94	7.29	5.82	3.676	-0.667	-148.2	-31.10
95	5.61	1.05	-3.905	-9.523	-120.6	-29.51
aggregated	10.87	3.03	2.351	-1.475	-92.1	

**T-statistic for Fund Growth Different From 0**

	Small Q1	Q2	Q3	Q4	Q5
90	1.69	0.844	-2.408	0.246	-1.044
91	2.38	1.061	0.215	0.373	-0.911
92	1.05	1.981	0.796	0.493	-0.993
93	1.92	1.504	2.476	0.405	-0.175
94	1.23	2.191	1.888	-0.168	-2.614
95	2.05	0.877	-3.048	-2.231	-2.810
aggregated	1.86	3.375	1.663	-0.539	-3.494

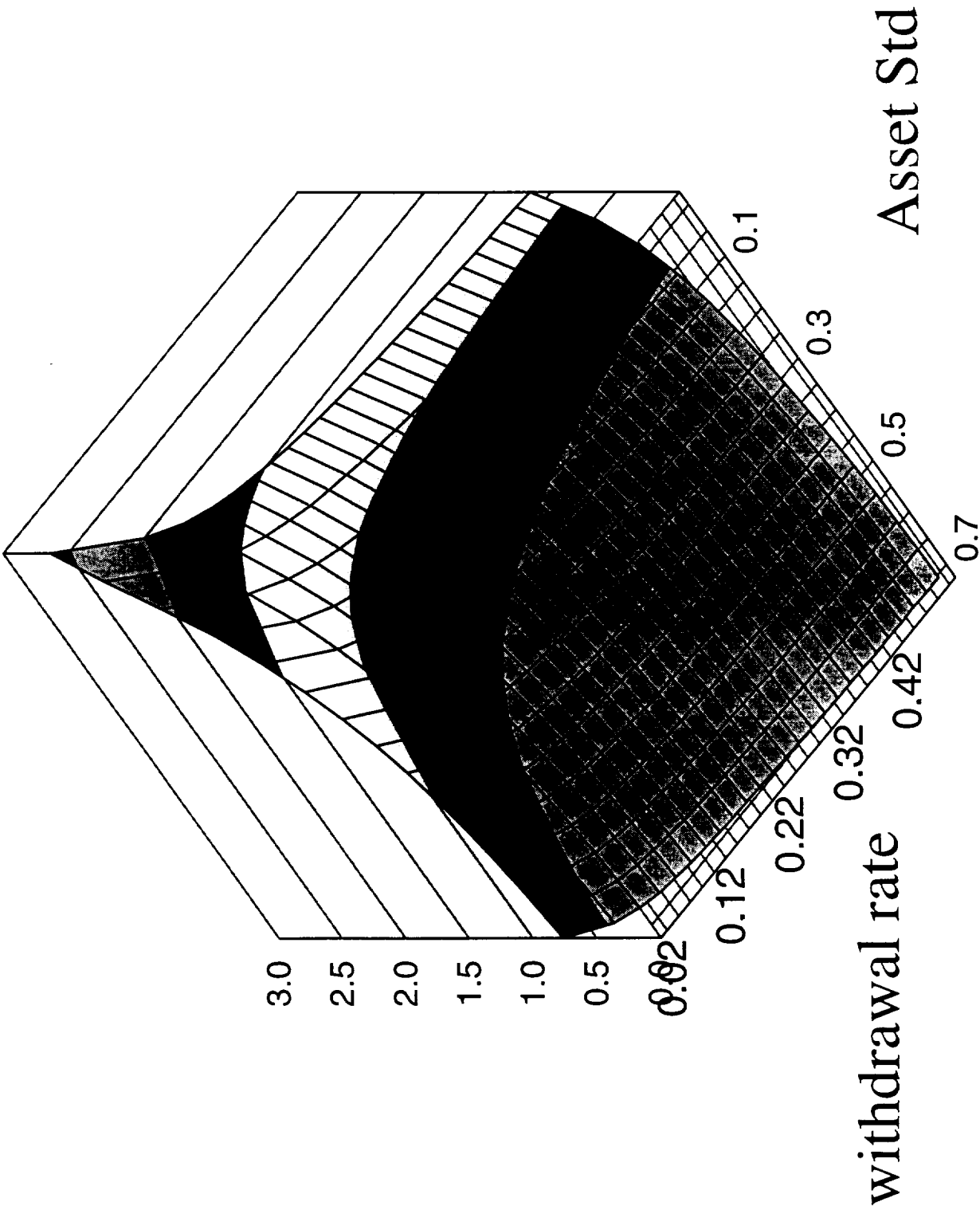
# 1. Fixed and Incentive Fee for Values of k

For std.= .2, w=.05, r=.05 c=.01



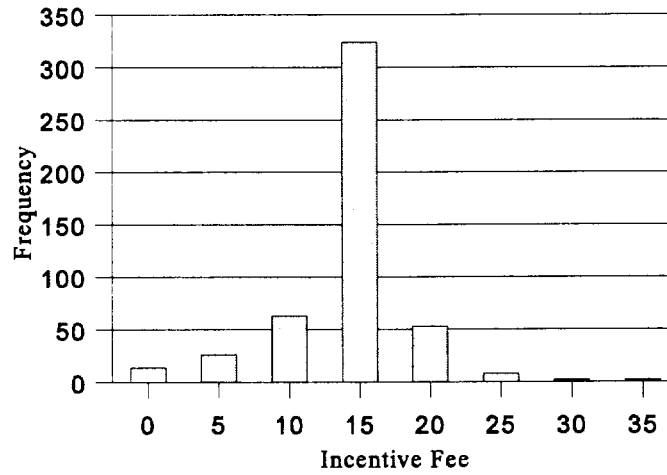
## 2. Fixed Fee NPV/Incentive Fee NPV

for ranges of std. and withdrawal



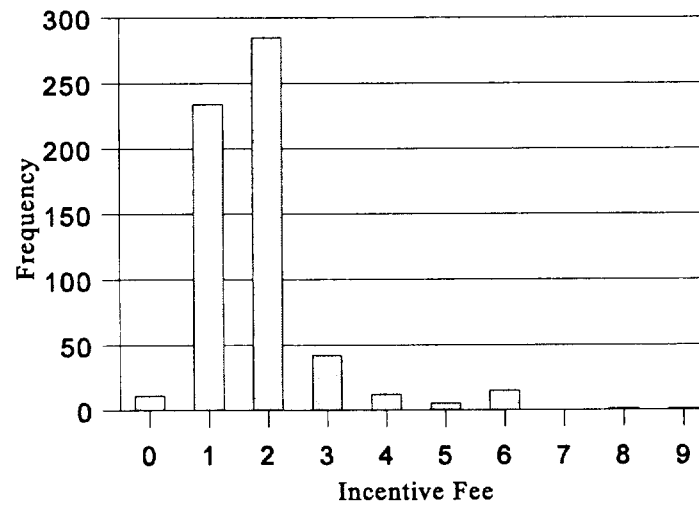
### 3: High Water Mark Incentive Fees

Offshore Funds 1990 - 1996



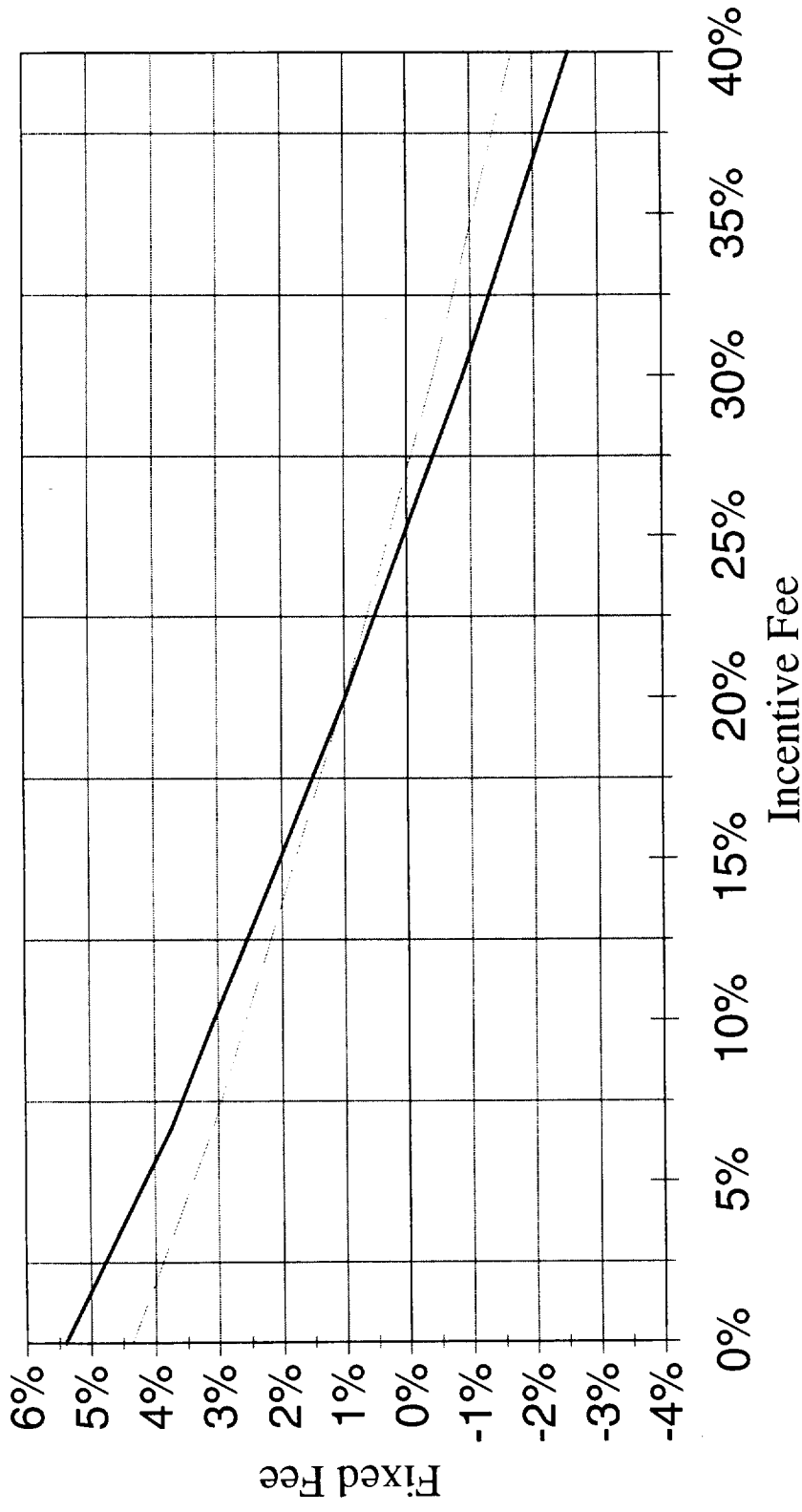
### 4: Hedge Fund Annual Fees

Offshore Funds 1990 - 1996



## 5. Fixed vs. Incentive Fee Tradeoff

For Different Values of  $w$  and  $\text{Std}$ .



$\text{std} = .2, w = .05, F = .31$        $\text{std} = .5, w = .05, F = .47$

$\text{std} = .2, w = .2, F = .12$        $\text{std} = .5, w = .2, F = .21$

**C = 1%**

