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BELIEFS: ARE EQUITY RETURNS TOO  
GOOD TO BE TRUE?

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Asset Pricing with Distorted Beliefs: Are Equity  
Returns Too Good To Be True?  
Stephen G. Cecchetti, Pok-sang Lam,  
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**ABSTRACT**

We study a Lucas asset pricing model that is standard in all respects, except that the representative agent's subjective beliefs about endowment growth are distorted. Using constant-relative-risk-aversion (CRRA) utility, with a CRRA coefficient below ten, and fluctuating beliefs that exhibit, on average, excessive pessimism over expansions and excessive optimism over contractions, our model is able to match the first and second moments of the equity premium and risk-free rate, as well as the persistence and predictability of excess returns found in the data.

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# 1 Introduction

We examine the asset pricing implications of small departures from rationality in an otherwise standard representative-agent Lucas (1978), endowment-based asset pricing model. In our model, the agent know that endowment growth is subject to regime shifts, and they know whether the economy is currently in an expansion or a contraction, but their beliefs about the transition probabilities systematically deviate from the true values that govern the dynamics of real per-capita consumption. When subjective beliefs exhibit excessive pessimism about the persistence of expansion states and excessive optimism about the persistence of contraction states, under constant-relative-risk-aversion (CRRA) utility with CRRA coefficient less than ten and discount factor less than one, our model is able to match the mean and variance of the equity premium and risk-free rate, as well as the persistence and predictability of excess returns found in the data.

We examine two distinct distortions of beliefs, neither of which is without empirical foundation. First, we assume that the agent uses simple rules of thumb to form subjective estimates of the average transition probabilities governing the dynamics of the economy. These subjective values deviate systematically from the maximum likelihood estimates we obtain from data on U.S. per-capita consumption growth. Second, the agent's beliefs about the transition probabilities are allowed to exhibit random fluctuations about their subjective mean values. This is short-hand for the idea that in the real world people, on occasion, mistakenly interpret pseudo-signals as news, inducing changes in their beliefs, and generating swings in asset prices.

The importance of our contribution is to point out that the resolution of asset pricing anomalies need not rely on positing that the economy is populated by completely irrational agents, or individuals who behave in systematically suboptimal ways. Our view is that the average market participant uses simple rules of thumb in estimating the parameters of the process governing the evolution of the economy.

We contrast our approach to the more common that preserves full rationality but assumes increasingly complicated preference structures for resolving asset price puzzles. For example, Campbell and Cochrane (1995) are able to provide a fully rational unified explanation for the asset pricing anomalies mentioned above using a utility function that displays a sophisticated

form of habit persistence. We retain the simplicity of time-separable CRRA utility, but allow for what we believe to be reasonable departures from full rationality.

Our work is part of a growing literature in which local departures from full rationality hold promise in solving empirical puzzles. Examples include Cochrane's (1989) demonstration that rule-of-thumb behavior entails trivial economic costs; Hansen, Sargent and Tallarini's (1995) exploration of the implications of assuming that the social planner is excessively pessimistic; and Barsky and DeLong's (1993) examination of the impact of presuming that people believe dividend *growth* contains a unit root.

The remainder of the paper is organized as follows. The next section reports the stylized facts of the equity and bond market that we seek to explain. Section 3 presents the fully rational Lucas asset pricing model, which serves as a benchmark for results. In section 4, we endow the representative agent with a distorted belief system and discuss the solution and the computation of implied moments from the model. Section 5 reports on the model's solution to the many empirical puzzles. Section 6 contains some concluding remarks.

## 2 Stylized Facts of Asset Returns

In their original statement of the problem, Mehra and Prescott (1985) asked whether the rational, complete markets asset pricing model where the representative investor has CRRA utility with coefficient below 10 and discount factor between 0 and 1 could account for the 8 percent per annum sample mean return on the Standard and Poors index and the 2 percent per annum sample mean return on relatively risk-free short-term bonds. The resounding failure of that model has come to be known as the equity premium puzzle.

The list of challenges that asset pricing theory has taken up has grown substantially since Mehra and Prescott's original investigation. Table 1 presents the list of asset pricing anomalies around which our investigation is organized.<sup>1</sup> In addition to understanding the

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<sup>1</sup>The data on equity and short-term bond returns is the updated version of Campbell and Shiller (1988). Specifically, the equity data are the Standard and Poors 500 Price Index and Dividends, short-term bond data are the return from 6-month commercial paper bought in January and rolled over in July, and the Producer Price Index is used to obtain real returns from nominal returns. Changes in measurement of inflation can affect these numbers somewhat. For example, in Cecchetti, Lam and Mark (1994) we use consumer rather than producer prices, resulting in a lower mean and standard deviation for the risk-free rate, and a higher

Table 1: Stylized Facts of Equity and Short-Term Bond Returns  
Annual Observations from 1871–1993.

First and Second Moments		Horizon	Slope	$R^2$	Variance Ratio
Mean equity premium ( $\mu_{eq}$ )	5.76	1	0.148	0.043	1.000
Mean Risk-free rate ( $\mu_f$ )	2.70	2	0.295	0.081	1.038
Std.dev. equity premium ( $\sigma_{eq}$ )	18.87	3	0.370	0.096	0.921
Std.dev. risk-free rate ( $\sigma_f$ )	9.02	5	0.662	0.191	0.879
Correlation ( $\rho_{eq,f}$ )	-0.25	8	0.945	0.278	0.766

Note: Slope and  $R^2$  are from regressions of the  $k$ -year ( $k=1,2,3,5,8$ ) ahead equity premium on the current consumption-price ratio. The variance ratio is calculated for the equity premium.

mean-value of returns, we are concerned with three additional features of the data. The first of these is the volatility of equity returns, the risk-free rate, and their co-movement. Second, we seek to explain why equity returns are negatively serially correlated at long horizons which results in the observed mean-reverting behavior of stock prices. In the table, we characterize the correlogram for excess equity returns by computing the  $k$ -period variance ratio. These values are generally less than 1, indicating non-zero variance of transitory components in asset prices.<sup>2</sup> Third, we investigate the ability of the model to explain findings by researchers such as Campbell and Shiller (1988) and Fama and French (1988), who report that the log dividend yield predicts long-horizon excess returns. Here, we represent the predictability puzzle by regressions of the  $k$ -period excess return on the consumption-price ratio. As can be seen from the table, the predictive regressions run on our data display the familiar pattern of slope coefficients and  $R^2$ 's that increase with the return horizon.<sup>3</sup>

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equity premium.

<sup>2</sup>The variance ratio statistic, popularized by Cochrane (1988), is the variance of the  $k$ -year return divided by  $k$  times the variance of the one-year return.

<sup>3</sup>For analyses of the second moments, see Constantinides (1990) and Cecchetti, Lam, and Mark (1993). Cecchetti, Lam, and Mark (1990) and Kandel and Stambaugh (1991) study the theory's implications for mean-reverting equity returns.

## 3 The Model

### 3.1 The Lucas Model

We employ a variant of Lucas's (1978) representative agent endowment economy that has served as the workhorse in aggregate asset pricing studies. Let  $P$  be the price of the equity, which is a claim to the future stream of the nonstorable endowment, which we call dividends,  $D$ . One perfectly divisible share of the equity trades in a competitive market. Under time-separable utility defined over consumption,  $C$ , the first-order condition that must hold if the agent behaves optimally is,

$$PU_c(C) = \beta \mathcal{E}[U_c(C')(P' + D')], \quad (1)$$

where primes denote next period values,  $\beta$  is the subjective discount factor,  $U_c(C)$  is marginal utility of consumption, and  $\mathcal{E}$  is the representative individual's subjective expectation conditioned on currently available information. In equilibrium, the endowment is consumed ( $D = C$ ) and we follow the standard practice of modelling per capita consumption as the endowment.

Let the period utility function display constant relative risk aversion with coefficient  $\gamma > 0$ . Then (1) becomes

$$PC^{-\gamma} = \beta \mathcal{E}[(C')^{-\gamma}(P' + C')]. \quad (2)$$

Define  $\omega \equiv P/C$  as the price-consumption ratio and  $c \equiv \ln(C)$ . Dividing (2) by  $C^{(1-\gamma)}$  yields the stochastic difference equation in  $\omega$ ,

$$\omega = \beta \mathcal{E}[e^{(1-\gamma)\Delta c'}(\omega' + 1)]. \quad (3)$$

### 3.2 The Endowment Process

We follow Cecchetti, Lam, and Mark (1990, 1993) by assuming that  $\Delta c'$  evolves according to the following version of Hamilton's (1989) Markov switching process,

$$\Delta c' = \alpha(S') + \varepsilon' \quad (4)$$

Table 2: Maximum Likelihood Estimates of the Endowment Process  
Per capita Consumption Growth, 1890 to 1994

Parameter	p	q	$\alpha(1)$	$\alpha(0)$	$\sigma$
Estimate	0.978	0.516	2.251	-6.785	3.127
(t-ratio)	(50.94)	(1.95)	(6.87)	(-3.60)	(13.00)

Note: Asymptotic t-ratios are in parentheses.

where  $\varepsilon \stackrel{i.i.d.}{\sim} N(0, \sigma^2)$  and  $S$  is an underlying two-point Markov state variable that assumes values of 0 or 1. We normalize  $S = 1$  to be the good (expansion) state of high consumption growth and  $S = 0$  to be the bad (contraction) state of low consumption growth, so that  $\alpha(1) > \alpha(0)$ . We specify the transition probabilities to be

$$\begin{aligned}
 Pr(S' = 1|S = 1) &= p, & Pr(S' = 0|S = 1) &= 1 - p, \\
 Pr(S' = 0|S = 0) &= q & \text{and} & Pr(S' = 1|S = 0) = 1 - q.
 \end{aligned}
 \tag{5}$$

Here,  $p$  is the probability of remaining in an expansion if currently in one and  $1 - p$  is the probability of a transition from an expansionary state to a contraction. Similarly,  $q$  is the probability of remaining in a contraction if currently in a contraction while  $1 - q$  is the probability of moving from a contraction to a expansion.

Table 2 presents maximum likelihood estimates of the Markov switching model estimated using U.S. per-capita consumption data extending from 1890 to 1994. In order for the economy to evolve according to the objectively true process, we calibrate the endowment by setting its parameter values equal to the maximum likelihood estimates.<sup>4</sup>

Several features of the ‘truth’ are worth noting. First, expansions are highly persistent, with a probability of termination less than 0.03. In fact, the unconditional probability of being in the good state,  $Pr(S = 1)$  is 0.96. Second, contractions are moderately persistent,

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<sup>4</sup>These data are updated from Cecchetti, Lam, and Mark (1990). Cecchetti, *et. al.* also perform a number of diagnostic tests on the specification and conclude that this estimated Markov-switching model is a plausible representation of the data generating process.



with more than an even chance of continuing once they start. Beyond this, though, the bad state is very bad, as mean per capita consumption growth in this state is  $-6.8$  percent.

### 3.3 The Rational Economy

Since we will examine the implications of departures from rationality, it is useful to provide a benchmark for our investigation by summarizing the properties of the fully rational model. In the benchmark model, the subjective expectations of investors  $\mathcal{E}(\cdot)$  coincide with the mathematical expectations  $E(\cdot)$  taken with respect to the truth. The complete markets rational economy is the environment studied by Cecchetti, Lam and Mark (1990, 1993) and is closely related to the economic environments of Kandel and Stambaugh (1990), and Mehra and Prescott (1985).

Due to the independence of the shock  $\varepsilon$ , the fully rational environment is governed by the single state variable,  $S$ . This allows us to write the stochastic difference equation for the price-consumption ratio equation (3) as

$$\omega(S) = \beta e^{(1-\gamma)^2 \frac{\sigma^2}{2}} E([1 + \omega(S')] e^{(1-\gamma)\alpha(S')} | S) \quad (6)$$

Because  $S$  can take on only the values 0 or 1, equation (6) is a system of two linear equations in  $\omega(0)$  and  $\omega(1)$ ,

$$\begin{bmatrix} q\tilde{\beta}(0) + (1-q)\tilde{\beta}(1) \\ p\tilde{\beta}(1) + (1-p)\tilde{\beta}(0) \end{bmatrix} = \begin{bmatrix} 1 - q\tilde{\beta}(0) & -(1-q)\tilde{\beta}(1) \\ -(1-p)\tilde{\beta}(0) & 1 - p\tilde{\beta}(1) \end{bmatrix} \begin{bmatrix} \omega(0) \\ \omega(1) \end{bmatrix}, \quad (7)$$

where  $\tilde{\beta}(S) \equiv \beta e^{(1-\gamma)^2 \frac{\sigma^2}{2} + (1-\gamma)\alpha(S)}$ ,  $S = 0, 1$ . Solving equation (7) yields

$$\omega(0) = \frac{q\tilde{\beta}(0) + (1-q)\tilde{\beta}(1) - (p+q-1)\tilde{\beta}(0)\tilde{\beta}(1)}{\Delta} \quad (8)$$

$$\omega(1) = \frac{p\tilde{\beta}(1) + (1-p)\tilde{\beta}(0) - (p+q-1)\tilde{\beta}(0)\tilde{\beta}(1)}{\Delta} \quad (9)$$

where  $\Delta = 1 - q\tilde{\beta}(0) - p\tilde{\beta}(1) + pq\tilde{\beta}(0)\tilde{\beta}(1)$ .

The next step is to characterize the solution for one period returns. Gross equity returns,

Table 3: The fully rational model.

$\beta$	$\gamma$	First and second moments	Horizon	Slope	$R^2$	Variance Ratio	
0.980	0.0	$\mu_{eq}$	0.0	1	0.207	0.032	1.000
		$\mu_f$	2.0	2	0.274	0.034	0.987
		$\sigma_{eq}$	4.4	3	0.311	0.032	0.977
		$\sigma_f$	0.0	5	0.356	0.026	0.959
		$\rho_{eq,f}$	0.000	8	0.420	0.023	0.933
0.989	0.5	$\mu_{eq}$	0.1	1	0.332	0.025	1.000
		$\mu_f$	2.0	2	0.442	0.027	0.987
		$\sigma_{eq}$	3.9	3	0.502	0.026	0.975
		$\sigma_f$	0.5	5	0.571	0.023	0.956
		$\rho_{eq,f}$	-0.007	8	0.665	0.021	0.929
1.015	2.0	$\mu_{eq}$	0.2	1	0.008	0.007	1.000
		$\mu_f$	2.0	2	0.011	0.010	0.992
		$\sigma_{eq}$	3.2	3	0.013	0.012	0.982
		$\sigma_f$	1.9	5	0.014	0.014	0.965
		$\rho_{eq,f}$	0.002	8	0.015	0.016	0.940
1.031	3.0	$\mu_{eq}$	0.2	1	0.078	0.018	1.000
		$\mu_f$	2.0	2	0.106	0.021	0.976
		$\sigma_{eq}$	3.9	3	0.120	0.021	0.958
		$\sigma_f$	2.8	5	0.135	0.020	0.931
		$\rho_{eq,f}$	0.035	8	0.152	0.020	0.899

$R^e$ , are given by

$$R^e(S', S) = \left[ \frac{\omega(S') + 1}{\omega(S)} \right] e^{\alpha(S') + \varepsilon'}. \quad (10)$$

Since the price of a one-period risk-free asset  $P^f$  is the intertemporal marginal rate of substitution,

$$P^f(S) = \beta e^{\gamma^2 \frac{\sigma^2}{2}} E[e^{-\gamma \alpha(S')} | S], \quad (11)$$

the implied gross risk-free rate  $R^f$  is

$$R^f(S) = \frac{1}{P^f(S)}. \quad (12)$$

In examining the model, we restrict our attention to values of  $\gamma$  less than ten, and

associated values of  $\beta$  less than one that match the mean risk-free rate of 2 percent, subject to meeting the transversality condition of the model.<sup>5</sup> Table 3 displays the implied behavior of asset prices when the representative agent is fully rational for selected values of the preference parameters. As is well known, this model fails on many dimensions: there is virtually no equity premium, the volatility of equity returns is far below its sample value, and excess returns have neither the persistence nor the predictability found in the data.

## 4 Distorted Beliefs

Full rationality imposes discipline on a researcher. Modelling departures from unbounded rationality is largely uncharted territory in aggregate asset pricing research. A large part of the difficulty lies in the lack of guidance generally available and the absence of an acceptable methodology.

One way to motivate our approach is to view the agent in our economy as being boundedly rational as described by Simon (1957):

“The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world—or even for a reasonable approximation to such objective rationality.”

We assume that individuals find it costly to acquire the skills to do maximum-likelihood estimation or to perform integration with respect to multivariate densities in order to make decisions about every day life. Their response is to resort to rules of thumb that give approximately the right answer. We use the following simple calculations to guide our modelling of subjective beliefs.

Suppose that agent realizes that consumption growth is governed by a two-state Markov process, she just doesn't know the transition probabilities. We consider two strategies by

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<sup>5</sup>The transversality condition requires that the power series for the price level implied by the difference equation (2) yield finite values. Practically, this requirement can be checked by verifying that implied values for  $\omega(S)$  are always positive.

which a sensible person might go about ‘estimating’  $p$  and  $q$ . First, a relatively sophisticated individual obtains a copy of the NBER reference cycle chronology (available on the NBER’s home page at <http://nber.harvard.edu>). After converting the chronology to an annual frequency, the agent uses two moments to exactly identify the parameters of interest:  $q$  is estimated by the proportion of years in which contractions are followed by contractions, and  $p$  is then estimated from the unconditional probability of being in an expansion state (which is  $\frac{1-q}{2-p-q}$ ). The results of this simple exercise are an estimate for  $q$  of 0.273 and an estimate for  $p$  of 0.667. Notice that these values are both well below the maximum-likelihood estimates reported in Table 2.

A second, and equally simple method, uses per-capita consumption growth data and yields similar results. Begin by assigning all years in which per-capita consumption contracted to the bad state and the remainder, those with positive per-capita consumption growth, to the good state. Then use the same simple technique based on the persistence of contractions and the unconditional probability of expansions to estimate the transition probabilities. The result is an estimate for  $p$  equal to 0.760 and an estimate for  $q$  equal to 0.269.

These simple calculations lead us to consider cases in which the representative agent’s subjective beliefs about  $p$  and  $q$  systematically deviate from those we have estimated using sophisticated econometric methods. Specifically, we will examine the implications of assuming that the agent’s subjective beliefs exhibit excessive pessimism about the persistence of both expansion and contraction states — their subjective  $p$  and  $q$  estimates are lower than the ‘true’ ones.

We will study this model in the remainder of the paper. In sections 4.1, 4.2 and 4.3 we introduce the model, and characterize the solution for asset returns. Section 5 follows with empirical results.

## 4.1 Modelling Distorted Beliefs

We now describe the parameterization of the representative agent’s subjective beliefs about the transition probabilities governing endowment growth. We begin by defining the subjective counterparts to the transition probabilities  $p$  and  $q$  as  $\tilde{p}$  and  $\tilde{q}$ , respectively. As-

sume that  $\tilde{p}$  evolves according to

$$\tilde{p}(\eta) = \frac{1}{1 + e^{(g+\eta)}} , \quad (13)$$

where  $g$  is a parameter and  $\eta$  is an independent random variable distributed uniformly over the interval  $[B, A]$ . Taking expectations over the distribution of  $\eta$  yields the mean value of  $\tilde{p}$ ,

$$\mu_{\tilde{p}} = E_{\eta}[\tilde{p}(\eta)] = \frac{1 - \ln[1 + e^{(g+A)}] - \ln[1 + e^{(g+B)}]}{A - B} . \quad (14)$$

By varying the parameters  $(g, A, B)$  the formulation in equation (14) allows  $(\tilde{p} - p)$  to have both nonzero mean and nonzero variance. Subjective beliefs can be biased and move randomly in ways that are not explained by fundamentals.

Next, we assume that subjective beliefs about the persistence of contractions, i.e., the probability of remaining in the bad state conditional on being in it, evolves according to the two-state Markov process,

$$\tilde{q} = \tilde{q}(\tilde{S}) , \quad (15)$$

where  $\tilde{S}$  takes on the values of 0 or 1 with the symmetric transition probabilities

$$\begin{aligned} Pr(\tilde{S}' = 1 | \tilde{S} = 1) &= Pr(\tilde{S}' = 0 | \tilde{S} = 0) = \phi \\ &\text{and} \\ Pr(\tilde{S}' = 0 | \tilde{S} = 1) &= Pr(\tilde{S}' = 1 | \tilde{S} = 0) = 1 - \phi . \end{aligned} \quad (16)$$

We adopt the normalization that  $\tilde{q}(1) > \tilde{q}(0)$ . Thus, when  $\tilde{S} = 1$  investors are excessively pessimistic in thinking that the persistence of a contraction is high. When  $\tilde{S} = 0$ , investors are excessively optimistic. Again, this formulation allows for subjective beliefs to be biased, as  $\tilde{q}$  fluctuates between high and low values and the mean value of  $\tilde{q}$ ,  $\mu_{\tilde{q}}$  need not equal  $q$ . Note that the rational model described in section 3.3 is nested within the distorted beliefs model. The traditional formulation is obtained by setting  $\tilde{q}(1) = \tilde{q}(0) = q$ ,  $A = B = 0$ , and  $g = \ln[(1/p) - 1]$ .

We interpret the state variables associated with the subjective distortions,  $\eta$  and  $\tilde{S}$ , as

pseudo-signals. As suggested by Black (1986), the economic environment is complex and noisy, and so individuals may not be able to fully distinguish noise from information. They may believe that the ramblings of the Chairman of the Federal Reserve System or a rise in the popularity of a particular political candidate is information and incorporate these events into their beliefs. As these pseudo-signals are acquired, beliefs change.

It bears emphasizing that, while subjective beliefs may be distorted, the endowment—which evolves according to the truth—is not. This is a subtle, but important point that distinguishes our approach from Reitz’s (1988), in which the endowment evolved according to distorted beliefs.<sup>6</sup> The agent in our economy knows whether it is presently in the expansion state and condition their expectations on this information.

The structure we propose is only one of a large number of possibilities. An obvious alternative would be to model both  $\tilde{p}$  and  $\tilde{q}$  symmetrically as two-point Markov chains, evolving in tandem. But, as we will discuss in Section 5.4, such a model does not allow us to solve all of the asset pricing puzzles simultaneously. In the end, the asymmetry will be a crucial part of our story.

## 4.2 Solving the Distorted Beliefs Model

Characterizing returns with distorted beliefs is made complicated by the fact that the state vector has grown from  $S$  to  $\lambda \equiv (S, \tilde{S}, \eta)$ . The solution method is dramatically simplified by the assumption that the elements of  $\lambda$  are stochastically independent. As was the case in the rational model, computation of the implied returns process simply requires that we solve for the price-consumption ratio.

We begin by writing the equilibrium price-consumption ratio as a function of the state vector,  $\omega(\lambda)$ . The resulting stochastic difference equation, the analog to equation (6) constructed by combining equations (3), (4), (5), (13) and (15), is

$$\omega(\lambda) = \beta e^{(1-\gamma)^2 \frac{\sigma^2}{2}} \mathcal{E}([1 + \omega(\lambda')] e^{(1-\gamma)\alpha(S')} | \lambda) \quad (17)$$

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<sup>6</sup>Our model is closely related to the so-called ‘peso-problem’ models, that have been applied in the international finance literature. In the original peso-problem, the agent rationally attaches a nonzero probability to the event in which the monetary authorities would devalue the currency, even though the monetary history contained no such devaluations.

Taking expectations with respect to  $S$  and  $\tilde{S}$ , and using the law of iterated expectations, yields a set of four linear equations, since there are four combinations of  $(S, \tilde{S})$ . We write this as

$$w(\eta) = G\bar{w}_\eta + f(\eta) , \quad (18)$$

where the four-dimensional vector  $\bar{w}_\eta \equiv E_\eta(w)$  is the expected value of the vector of price-consumption ratios taken with respect to the uniform distribution for  $\eta$ .<sup>7</sup>

The solution for  $w$  is obtained in two additional steps. First, solve for the vector of price-consumption ratios  $w$  evaluated at  $\mu_{\tilde{p}} = E_\eta[\tilde{p}(\eta)]$ , for the four possible realizations of  $(S, \tilde{S})$  by taking expectations on both sides of (18) with respect to the uniform distribution over  $[B, A]$ . This yields,

$$\bar{w}_\eta = (I - \bar{g}_\eta)^{-1} \bar{f}_\eta \quad (19)$$

where  $(\bar{w}_\eta, \bar{g}_\eta, \bar{f}_\eta) \equiv \mathcal{E}_\eta(w, G, f)$ , and  $I$  is a 4-dimensional identity matrix. Finally, substituting  $\bar{w}$  into (18) yields the vector of price-consumption ratios  $w$  for any realization of  $\eta \in [B, A]$ .

Using these results, we can now write the implied one-period gross equity return as

$$R^e(\lambda, \lambda', \varepsilon') = \left[ \frac{\omega(\lambda') + 1}{\omega(\lambda)} \right] e^{[\alpha(S') + \varepsilon']} . \quad (20)$$

The price of the one-period risk-free bond is

$$\begin{aligned} P^f(\lambda) &= \beta \mathcal{E}(e^{-\gamma[\alpha(S') + \varepsilon']} | \lambda) \\ &= \begin{cases} \beta e^{\gamma^2 \frac{\sigma^2}{2}} \{ \tilde{p}(\eta) e^{-\gamma\alpha(1)} + [1 - \tilde{p}(\eta)] e^{-\gamma\alpha(0)} \} & \text{if } S = 1 \\ \beta e^{\gamma^2 \frac{\sigma^2}{2}} \{ \tilde{q}(\tilde{S}) e^{-\gamma\alpha(0)} + [1 - \tilde{q}(\tilde{S})] e^{-\gamma\alpha(1)} \} & \text{if } S = 0 \end{cases} , \end{aligned} \quad (21) \quad (22)$$

and the associated gross risk-free rate of return is  $R^f(\lambda) = 1/P^f(\lambda)$ . Let  $R^p(\lambda, \lambda', \varepsilon') = R^e(\lambda, \lambda', \varepsilon') - R^f(\lambda)$  be the excess return on equity. Then we evaluate the  $k$ -th moments, and cross moments, of the equity premium and the risk-free rate by integrating the appropriate expression over the density functions for the random variables on which it depends. For example,  $R^f$  depends on  $(S, \tilde{S}, \eta)$ , and so computation of  $E([R^f(\lambda)]^k)$  requires integration

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<sup>7</sup>The details for these calculations are contained in an appendix which is available from the authors upon request.

over the density of these three random variables. Since  $S$ ,  $\tilde{S}$  and  $\eta$  are all assumed to be independent, this calculation is straightforward.

## 5 Properties of the Distorted Beliefs Model

To evaluate the value-added of various aspects of the model, we proceed in several steps. First, we restrict  $\tilde{p}$  and  $\tilde{q}$  to be fixed, but not necessarily equal to  $p$  and  $q$ . We refer to this as the ‘fixed distortions’ case. Next, we introduce variation to  $\tilde{p}$ , by allowing fluctuations in  $\eta$ . This is the case in which the agent’s beliefs about the persistence of expansions are subject to stochastic variations. Finally, randomness in  $\tilde{q}(\tilde{S})$  is introduced to incorporate time-varying beliefs about the persistence of both contractions and expansions.

### 5.1 Fixed Distortions

To what extent do fixed but distorted perceptions of the persistence parameters explain the asset pricing puzzles set forth above? To answer this question, we study the case in which the agent’s beliefs over  $(\tilde{p}, \tilde{q})$  are the fixed values

$$\tilde{p} = \frac{1}{1 + e\bar{g}} \neq p = 0.978 \quad \text{and} \quad \tilde{q} = \tilde{q}(0) = \tilde{q}(1) \neq q = 0.516.$$

It is now well known that asset returns data can be explained using the standard model by choosing either large values of  $\gamma$  (e.g., 57) or  $\beta > 1$ .<sup>8</sup> The challenge then, as articulated by Mehra-Prescott (1985) and Kocherlakota (1996), is to produce a model that explains the data with values of  $\beta$  between 0 and 1, and positive values of  $\gamma$  below 10. While we searched over all admissible values of  $(\tilde{p}, \tilde{q})$ , that is to say all combinations between zero and one that obey the transversality condition, we use the rule-of-thumb calculations at the beginning of Section 4 to narrow our focus.

Table 4 reports parameter values that yield a resolution of the equity premium puzzle. These are combinations of  $(\tilde{p}, \tilde{q}, \beta, \gamma)$  for which the model implied mean equity premium and mean risk-free rate are 6% and 2%. The region of the parameter space that accomplishes this

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<sup>8</sup>See Kandel and Stambaugh (1991), Kocherlakota (1990), and Cecchetti, Lam, and Mark (1993).



Table 4: Fixed Distorted Beliefs: Implied Second Moments for Parameter Values that Solve the Equity Premium Puzzle

Beliefs		Preferences		Second Moments		
$\tilde{p}$	$\tilde{q}$	$\beta$	$\gamma$	$\sigma_{eq}$	$\sigma_f$	$\rho_{eq,f}$
0.5	0.2	0.928	2.215	4.4	1.3	-0.44
	0.4	0.903	3.058	4.1	0.6	-0.32
	0.8	0.979	0.053	4.1	0.0	0.12
0.6	0.2	0.917	3.665	4.6	1.5	-0.43
	0.4	0.780	8.626	3.9	0.0	0.00
	0.9	0.977	0.226	4.4	0.2	-0.02
0.7	0.1	0.926	5.223	5.1	2.4	-0.49
	0.2	0.900	6.490	4.6	1.4	-0.39
0.8	0.1	0.927	8.991	5.1	2.3	-0.45

is quite large since it includes values of  $\tilde{p}$  that range from 0.5 to 0.8 and values of  $\tilde{q}$  varying from 0.1 to 0.9. Notice that relatively mild degrees of relative risk aversion are required to match the first moments of returns.

The distortions in beliefs presented in the table can be divided into two categories. The first are those in which a moderately risk-averse agent is excessively pessimistic about the persistence of expansions ( $\tilde{p} < p$ ), but are excessively optimistic about the persistence of contractions ( $\tilde{q} < q$ ). For example, when  $(\tilde{p}, \tilde{q}) = (0.7, 0.2)$ , roughly our rule-of-thumb values,  $(\beta, \gamma) = (0.90, 6.4)$  solves the equity premium puzzle.

The second category of distorted beliefs that yield solutions to the equity premium puzzle are those in which nearly risk-neutral individuals are uniformly pessimistic. Here, the economic agent believes that expansions are less persistent ( $\tilde{p} < p$ ) and contractions more persistent ( $\tilde{q} > q$ ) than they truly are. For example,  $(\tilde{p}, \tilde{q}) = (0.6, 0.9)$  requires  $(\beta, \gamma) = (0.98, 0.23)$  to match mean returns.

The job of matching the first moments of the equity premium and the risk-free rate is accomplished primarily by judicious choice of two parameters —  $\tilde{p}$  and  $\gamma$ . To see how, compare two economies in which the agent has different assessments of the persistence of the current expansion — differing values for  $\tilde{p}$ . The higher  $\tilde{p}$ , the higher the agent's subjective estimate of the average growth rate in the economy, since everyone's estimate of state-

contingent average endowment growth rates is the same. But with higher consumption growth, the risk-free rate is higher. In the context of the distorted beliefs model,  $R^f$  is increasing in  $\tilde{p}$ , and so, the more pessimistic economy will have a lower risk-free rate. The immediate implication is that, for given values of  $\beta$ ,  $\gamma$ , and  $\tilde{q}$ , we can choose  $\tilde{p}$  to match  $\mu_f$ .

Not surprisingly, economies with different attitudes toward risk will have divergent equity returns. For the usual reasons, a higher value of  $\gamma$  will produce a higher average equity return. Because of the CRRA form of the utility function, changes in  $\gamma$  also change the agent's willingness to substitute consumption intertemporally. Again, higher values of  $\gamma$  result in higher levels of the average risk-free rate. In the usual case, this causes  $R^f$  to be too high — the 'risk-free rate puzzle.' But here, we can simply reduce  $\tilde{p}$  to balance the impact of higher  $\gamma$ , and match the first moments.

We now turn our attention to the last 3 columns of table 4, which display the implied volatility of and correlation between the equity premium and the risk-free rate. While the model can account for the fact that the equity premium is substantially more volatile than the risk-free rate (Table 1 shows the ratio of standard deviations to be around two), the levels are much too low. The data show asset return volatility that is between two and four times what we can produce using the fixed beliefs model. Evidently, to match the volatilities and correlation, we will need to introduce time variation in distorted beliefs.

## 5.2 Changing Beliefs of Expansion Persistence

We introduce changing beliefs in two steps. First, we allow  $\tilde{p}$  to vary according to equation (13) and maintain  $\tilde{q}$  fixed. Since the endowment spends most of its time in the good state, it seems reasonable to conjecture that changes in pessimism about the persistence of this state could induce higher volatility in asset returns.

Unfortunately, this intuition is wrong. Table 5 reports the implied means, volatilities, and correlation between the equity premium and the risk-free rate for the random  $\tilde{p}$  model. In evaluating the model, we chose  $A$  to provide the maximum implied standard deviation of the risk-free rate subject to the constraints that this standard deviation be less than 9 percent and that the implied standard deviation of  $\tilde{p}$  be less than 0.25. Given  $A$ ,  $B$  is then chosen so that the mean of  $\tilde{p}$ ,  $\mu_{\tilde{p}}$ , corresponds to the values in column 1 of table 5. A nontrivial

Table 5: Mean, Standard Deviations, and Correlation with Random  $\tilde{p}$ .

$\mu_{\tilde{p}}$	$\sigma_{\tilde{p}}$	$\tilde{q}$	$\beta$	$\gamma$	$\mu_{eq}$	$\mu_f$	$\sigma_{eq}$	$\sigma_f$	$\rho_{eq,f}$
0.5	0.25	0.2	0.928	2.215	5.82	2.24	5.63	5.15	-0.554
	0.25	0.4	0.903	3.058	5.73	2.46	6.39	6.92	-0.389
	0.25	0.8	0.979	0.053	6.10	2.02	6.08	0.12	-0.507
0.6	0.25	0.2	0.917	3.665	5.59	2.68	7.52	8.48	-0.437
	0.11	0.4	0.780	8.626	5.89	2.73	8.94	9.00	-0.059
	0.25	0.9	0.977	0.226	6.12	1.99	6.86	0.55	-0.549
0.7	0.18	0.1	0.926	5.223	5.61	2.74	8.19	9.00	-0.381
	0.14	0.2	0.900	6.490	5.68	2.75	8.46	9.00	-0.240
0.8	0.09	0.1	0.927	8.991	5.70	2.76	8.79	9.00	-0.240

Table 6: Response of Asset Returns and the Price-Consumption Ratio to a Shock in  $\tilde{p}$ .

$\tilde{p}$	$\omega$	Equity Return	Risk-free rate
0.5	18.686	-1.2	-10.0
0.6	17.860	3.3	-4.5
0.7	17.034	8.3	1.6
0.8	16.209	13.9	8.6
0.9	15.383	20.0	16.7

Note: Today's asset prices computed assuming that  $\tilde{q} = 0.2, \beta = 0.90, \gamma = 6.49$ , and that the economy is in an expansionary state. Returns computed assuming that the economy continues in the expansionary state next period and beliefs of the persistence of the expansion reverts to its mean value  $\tilde{p}' = \mu_{\tilde{p}} = 0.7$ .

subset of the admissible parameter space allows the model to match mean returns and the volatility of the risk free rate, in addition to producing a small negative correlation between the asset returns. But as is clear from the column labeled  $\sigma_{eq}$  in the table, time-variation in  $\tilde{p}$  does not raise the implied volatility of the equity premium by a sufficient magnitude, as it is still one-half to one-third the level of the standard deviation in the sample.

To understand why changing perceptions of expansion persistence alone fail to match the second moments, we examine the following experiment. Begin by letting  $\tilde{q} = 0.2, \beta = 0.90, \gamma = 6.49$ , and the mean of  $\tilde{p}$ ,  $\mu_{\tilde{p}} = 0.7$ . Now consider a one-period shock to  $\tilde{p}$ . The effect of this transitory change in pessimism, reported in Table 6, is to move the return on both the risky and risk-free asset in the same direction and by roughly the same amounts.

Table 7: Implied First and Second Moments with Random  $\tilde{p}$  and  $\tilde{q}$ .

$\mu_{\tilde{p}}$	$\sigma_{\tilde{p}}$	$\phi$	$\tilde{q}(1)$	$\beta$	$\gamma$	$\mu_{eq}$	$\mu_f$	$\sigma_{eq}$	$\sigma_{eq}$	$\rho_{eq,f}$
0.7	0.073	0.90	0.431	0.839	9.157	5.9	2.5	19.0	9.0	-0.121
0.7	0.094	0.95	0.401	0.866	8.054	5.8	2.6	19.0	9.0	-0.120
0.7	0.107	0.98	0.380	0.880	7.466	5.8	2.6	19.0	9.0	-0.127

Transitory shocks to  $\tilde{p}$  create volatility in gross asset returns, but not in excess returns.

### 5.3 Changing Beliefs of Expansion and Contraction Persistence

Finally, we introduce random shocks to beliefs about the persistence of contractions. Here,  $\tilde{q}(\tilde{S})$  follows a two-state Markov process fluctuating between  $\tilde{q}(0)$  and  $\tilde{q}(1)$  with symmetric transition probability  $\phi$ , as described in section 4.1. The agent's beliefs alternate between the excessively optimistic view that the next contraction will last only one period,  $\tilde{q}(0) = 0$ , and some relatively pessimistic belief,  $\tilde{q}(1) > 0$ .

Table 7 reports three cases in which the model is able to match the means and standard deviations of the equity premium and risk-free rate, as well as their slightly negative correlation. Again, we are guided by our rule-of-thumb calculations in choosing values of  $\mu_{\tilde{p}} = 0.7$  and values of  $\mu_{\tilde{q}} \approx 0.2$ . Note that the model is able to match the five sample statistics when the agent has a discount rate less than 0.9, and risk aversion greater than 7.

In the first line of table 7, the representative agent discounts the future at a rate of 16 percent per year and has a relative risk aversion coefficient of 9.16. In an expansion, she believes the probability that the expansion will continue fluctuates randomly about  $\mu_{\tilde{p}} = 0.7$  with standard deviation 0.07. Her beliefs about the persistence of the next contraction alternates stochastically between the excessively optimistic view that it will last one period and that it has a probability  $\tilde{q}(1) = 0.43$  of persisting another year. The agent's beliefs about the persistence of the next contraction are themselves persistent. Once the agent makes up her mind, there is only a  $1 - \phi = 0.1$  probability that it will change. When a change from  $\tilde{q}(0)$  to  $\tilde{q}(1)$  takes place, the effects on equity prices are large, resulting in a jump in the price-consumption ratio from 17.5 to 28.0 and an annual return on equity of

about 60 percent. By contrast, if the economy is currently in an expansion, the risk-free rate is unchanged.

Why does adding time-variation in distorted beliefs about contraction persistence allow us to generate sufficiently volatile equity returns, without increasing the volatility of risk-free returns? After all, since the economy is normally in an expansion, this means that beliefs about  $\tilde{q}$  change while the agent is not observing the state itself. The key lies in observing that the risk-free rate depends only on what happens between the current and the next period, and so changes in  $\tilde{q}$  affect  $R^f$  only when the economy is actually in the bad state, which is infrequent. Equity returns, on the other hand, depend on expectations of events over all future periods. Changes in  $\tilde{q}$  alter expected marginal utility in the distant but not immediate future, affecting equity returns.

## 5.4 Persistence and Predictability of Returns

Recall the two commonly observed facts about persistence and predictability reported in table 1: i) variance ratio statistics decline from above 1.0 at a two year horizon to 0.8 at an eight year horizon, implying mean reversion in equity prices; and ii) the coefficients and  $R^2$ 's of regressions of long-horizon excess returns on the current log consumption-price ratio increase with horizon, implying improved predictability at longer horizons. We now examine whether the time-varying distorted beliefs model of section 5.3 can explain these phenomena.

The dynamics of the consumption-price ratio process,  $\omega$ , provide the key to understanding the evolution of equity returns, and so we begin by presenting the model implied moments of  $\omega$  in panel A of table 8. The sample values are provided for comparison. Not surprisingly, since these parameter values match the mean equity return, the implied means are close to the sample value. As noted earlier, changes in  $\omega$  are driven predominantly by changes in  $\tilde{q}$ , and not by the transitions between expansions and contractions. Consequently, the persistence of  $\omega$  is largely governed by the persistence of  $\tilde{q}$ . In fact, the serial correlation in  $\omega$  of the economy where  $\phi = 0.9$  roughly matches the persistence found in the data.

Panel B of table 8 presents the model implied values of the variance ratios, the regression slope coefficients and  $R^2$ 's at horizons of one, two, three, five and eight years. We present

Table 8: Persistence and Predictability of Excess Returns

	$\phi$	$\tilde{q}(1)$	$\mu_{\tilde{p}}$	$\sigma_{\tilde{p}}$	$\beta$	$\gamma$
I:	0.90	0.431	0.7	0.073	0.839	9.157
II:	0.95	0.401	0.7	0.094	0.866	8.054
III:	0.98	0.380	0.7	0.107	0.880	7.466

A. Consumption-Price Ratio Summary Statistics				
	Data	I	II	III
Mean	21.902	22.749	22.979	24.430
Std. Dev.	5.782	5.653	7.358	10.700
$\rho_1$	0.783	0.743	0.853	0.930
$\rho_2$	0.590	0.593	0.767	0.892
$\rho_3$	0.542	0.474	0.689	0.856
$\rho_5$	0.360	0.303	0.555	0.788
B. Measuring predictability				
Configuration	Horizon	Slope	$R^2$	Variance Ratio
The Data	1	0.1484	0.0434	1.000
	2	0.2945	0.0810	1.038
	3	0.3702	0.0959	0.920
	5	0.6618	0.1913	0.878
	8	0.9453	0.2777	0.765
I	1	0.1900	0.0694	1.000
	2	0.3538	0.1252	0.987
	3	0.4883	0.1678	0.958
	5	0.6854	0.2229	0.888
	8	0.8592	0.2582	0.791
II	1	0.1261	0.0496	1.000
	2	0.2492	0.0947	1.030
	3	0.3625	0.1344	1.035
	5	0.5567	0.1983	1.015
	8	0.7774	0.2627	0.961
III	1	0.0582	0.0498	1.000
	2	0.1405	0.0819	1.072
	3	0.2216	0.1134	1.107
	5	0.3729	0.1691	1.132
	8	0.5687	0.2354	1.129

results for three cases, labelled I, II and III, with parameter configurations reported at the top of the table (and identical to those in table 7). In order to account for the small-sample bias in these statistics, we generate the results from a Monte Carlo experiment. In each trial of the experiment, an artificial sequence of 123 equity and risk-free returns and consumption-price ratios are generated from the model, corresponding to the 123 annual observations that we have in the data. A regression of the  $k$ -period excess return on the log consumption-price ratio,  $1/\omega$ , for  $k = 1, 2, 3, 5, 8$ , is then estimated to obtain a slope coefficient and an  $R^2$ . The slope coefficients and  $R^2$ 's that we report are the mean values from 1,000,000 trials.

The simulated data match the sample values nearly exactly. The performance of the model follows directly from the mean reversion of the consumption-price ratio. When such a process is away from its mean, the predicted deviation from its current level is increasing in the horizon. This deviation is increasingly predictable as well. Excess returns inherit the stochastic properties of  $\omega$ . It is in this manner that a mean-reverting consumption-price ratio implies both predicted returns and predictability that increase with the horizon. Again,  $\phi = 0.9$  matches the data most closely. When  $\phi$  is larger than 0.9, the mean reversion in the consumption-price ratio is too slow to generate the short-horizon predictability observed in the data.

The results for the variance ratio statistics are nearly identical to that for the regressions. This is not surprising, since both are functions of the same underlying autocorrelations of the consumption-price ratio.

## 5.5 Changing Beliefs of Expansion and Contraction Persistence: The Symmetric Case

The asymmetry in modelling  $\tilde{p}$  and  $\tilde{q}$ , while perhaps inelegant, is essential for generating predictable long-horizon excess returns, while matching the means, standard deviations and correlation of equity returns and the risk-free rate. In particular, the results rely on the lack of persistence in  $\tilde{p}$ . In a set of unreported experiments, we examined an alternative formulation in which switches in the agent's beliefs about the transition probabilities are perfectly correlated. That is to say, we specified  $\tilde{p}(\tilde{S})$  and  $\tilde{q}(\tilde{S})$  both to evolve according to the two-state Markov chain governed by  $\tilde{S}$ . Periods of excessive optimism are characterized

by high  $\tilde{p}$  and low  $\tilde{q}$ , while periods of pessimism are characterized by low  $\tilde{p}$  and high  $\tilde{q}$ .

The critical insight again is that  $\tilde{q}$  largely governs the behavior of the equity return, while  $\tilde{p}$  strongly influences the behavior of the risk-free rate. In the symmetric  $[\tilde{p}(\tilde{S}), \tilde{q}(\tilde{S})]$  model, the persistence of  $\tilde{q}$  generates predictability in the equity return but because  $\tilde{p}$  is just as persistent, the equity premium, while variable, is not serially correlated, and so it is no longer predictable. Returning to the model in which  $\tilde{p}$  is i.i.d., the equity premium inherits the serial correlation of the equity return generated by fluctuations in  $\tilde{q}$ .

## 6 Conclusion

We have shown that a simple aggregate asset pricing model in which agents have distorted beliefs can replicate a number of features of observed asset returns data. We are able to match the means, standard deviations and correlation of the equity premium and the risk-free rate, as well as the persistence and predictability of long-horizon excess equity returns. Our investigation finds that beliefs must be distorted in two ways. First, people must believe that both expansions and contractions will be less persistent than implied by careful econometric analyses of the data. And second, agents' views of these transition probabilities must exhibit stochastic variation.

We accomplish all of this within the framework of a representative agent endowment economy, with CRRA preferences, and a two-state Markov process governing per-capita consumption growth. The only deviation from the standard rational expectations paradigm is that we model the transition probabilities using a simple rule-of-thumb procedure that is easy to replicate. Our approach provides an attractive alternative to the strategy of introducing either increasingly complex preference specifications, heterogeneity, or market incompleteness, to address the failures of the standard model.



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