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Basically, David Laibson is responsible for my working on this topic. I have also benefited from comments by Alberto Alesina, John Campbell, Diego Comin, Roberta Gatti, Elhanan Helpman, Ken Judd, Greg Mankiw, and Tom Sargent, and from research assistance by Jordan Rappaport. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

The neoclassical growth model is modified to allow for a non-constant rate of time preference. If the household cannot commit future choices of consumption and if utility is logarithmic, then an equilibrium is found that resembles the standard results of the neoclassical model. In this solution, the effective rate of time preference is high, but constant. Although this model has potentially important implications for institutional design and other policies—because households would benefit from an ability to commit future consumption—there is a sense in which the results are observationally equivalent to those of the conventional model. When the framework is extended to allow for partial commitment ability, some testable hypotheses emerge concerning the link between this ability and the rates of saving and growth. Steady-state results are obtained for general concave utility functions, and some properties of the dynamic paths are worked out for the case of isoelastic utility.

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In the neoclassical growth model, saving behavior derives from simple forms of household preferences. Specifically, the rate of time preference is typically assumed to be a positive constant. Laibson (1997a, 1997b), motivated partly by introspection and partly by experimental findings, argues that rates of time preference are not constant. Rather, he suggests that individuals have a tendency to act myopically in the sense that rates of time preference are very high between today and tomorrow but much lower between dates advanced some time in the future.

My purpose in this paper is not to address the question of whether these new ideas about time preference are introspectively appealing or empirically valid. Rather, I wish to explore the consequences of this type of deviation from the usual form of time preference for the standard results on consumer behavior and economic growth that emerge from the familiar and frequently relied upon neoclassical growth model (see Ramsey [1928], Cass [1965], Koopmans [1965], and the exposition in Barro and Sala-i-Martin [1995, Ch. 2]). In other words, I assess the macroeconomic consequences of non-constant rates of time preference. One issue that arises here is whether a researcher can tell from macroeconomic observations whether households' preferences exhibit the kinds of myopia that have been proposed in the recent literature. This inference is difficult because myopic preferences may be observationally equivalent to a high, but constant, rate of time preference.

1 Rates of Time Preference

In the standard neoclassical growth model, consumer preferences take the form

$$U(\tau) = \int_{\tau}^{\infty} u[c(t)] \cdot e^{-\rho(t-\tau)} dt \tag{1}$$

where τ is the current date, $u'(c) > 0, u''(c) < 0$, and $\rho > 0$ is the constant rate of time preference. Following standard practice, the time discounting for period t depends only on the distance in time, t-τ, from the current date.[1] As has been known since Strotz (1956), Pollak (1968), and Goldman (1980), non-constancy of the rate of time preference can create a time-consistency problem because the relative valuation of consumption flows at different dates changes as the date, τ, evolves.[2] In this context, committed choices of consumption will typically differ from those chosen sequentially, taking account of the way that future consumption will be determined.

For an individual consumer or family, the rationale for a constant rate of time preference is unclear.[3] Perhaps this is because the rationale for time preference is itself unclear. Ramsey (1928, p. 543) justified the exclusion of time preference in his main analysis by saying "we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible." Similarly, Fisher (1930, Ch. IV) argued that time preference—or impatience, as he preferred to call it—reflects, to a considerable extent, a person's lack of foresight and self-control.

The origin of time preference is clearer in an intergenerational context. For

[1]The utility expression could be extended to include the chronological date, t, and a household's age and other life-cycle characteristics. Such extensions would affect the analysis only if changes in t, age, etc. alter the relative values that a household attaches to consumption at future dates. For example, the analysis would not change materially if the expression inside the integral in equation (1) included a multiplicative term $\Psi(t,\text{age})$.

[2]The Strotz point is actually anticipated by Ramsey (1928, p. 439) in the part of his analysis that allows for time preference: "In assuming the rate of discount constant, I [mean that] the present value of an enjoyment at any future date is to be obtained by discounting it at the rate ρ. ... This is the only assumption we can make, without contradicting our fundamental hypothesis that successive generations are activated by the same system of preferences. For, if we had a varying rate of discount—say a higher one for the first fifty years—our preference for enjoyments in 2000 A.D. over those in 2050 A.D. would be calculated at the lower rate, but that of the people alive in 2000 A.D. would be at the higher."

[3]See Koopmans (1960) and Fishburn and Rubinstein (1982) for axiomatic derivations of a constant rate of time preference.

an extended family with altruistic linkages across generations, time preference can be viewed as a form of selfishness in which parents like their children but not as much as they like themselves (see Becker and Barro [1988]). In this context, a constant rate of intergenerational time preference can emerge if parents value their children's utility but care about grandchildren (and members of later generations) only indirectly through the children's valuation of their children. The key point here is that the relative valuation of consumption for future generations need not change as the extended family ages. Given this property, the choices of consumption by each generation will not suffer from the time-consistency problem posed by Strotz.[4]

Laibson (1997a) refers to experimental evidence that suggests that individuals have non-constant rates of time preference. (See Thaler [1981], Ainslie [1992], and Loewenstein and Prelec [1992] for discussions.[5]) Specifically, Laibson argues that rates of time preference are very high between now and the near future but much lower between periods far out in the future. I shall, for convenience, refer to this pattern of time-preference rates as myopia. However, consumers with these preferences need not be short-sighted in the sense of failing to take

[4]If parents care directly about the utility of grandchildren—or of more distant descendants—then time-inconsistency can arise. Phelps and Pollak (1968) refer to this situation as one of "imperfect altruism," but there is nothing in parental selfishness, *per se*, that necessitates this imperfection.

[5]This literature typically interprets the experimental findings as directly revealing attitudes toward different time patterns of consumption. Yet the experiments are usually posed in terms of alternative paths of money incomes. As Mulligan (1996) points out, with well-functioning credit markets and rational consumers, this kind of experiment should reveal only market interest rates, no matter what forms of time preference individuals possess. Fuchs (1982) makes an analogous point. Probably the organizers of the experiments believe that their patients typically lack much access to credit markets, but the nature of these imperfections would have to be detailed to know how to interpret the results. In any event, the macroeconomic consequences of non-constant rates of time preference are worth exploring even if the experimental literature is flawed.

account of long-term consequences, and my analysis assumes no failures of this sort.[6] The myopia is a characteristic of the preferences themselves. Consumer sovereignty suggests that these "myopic" preferences ought to be respected for welfare analyses, and I shall take this approach in the subsequent analysis. In the next section, I incorporate this type of time preference into an otherwise standard neoclassical growth model.

2 Structure of the Neoclassical Growth Model with Non-Exponential Time Preference

The basic idea is to modify an otherwise standard neoclassical growth model to incorporate a non-constant rate of time preference. Equation (1) is therefore altered to

$$U(\tau) = \int_{\tau}^{\infty} u[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \tag{2}$$

Thus, the representative household's preferences now include a term $\phi(t-\tau) > 0$, which brings in the new aspects of time preference.[7] Specifically, this term contains the elements that cannot be described by the standard exponential factor, $e^{-\rho(t-\tau)}$. The new term is assumed, as in the case of the conventional time-preference factor, to depend only on the distance in time, $t-\tau$. We can normalize to have $\phi(0) = 1$. The function $\phi(\cdot)$ is taken in the main discussion to be continuous and differentiable, although these properties are not strictly necessary. We assume $\phi' \leq 0$, so that later flows of utility do not receive a higher weight than

[6] In contrast, Akerlof (1991) looks at analogous problems but assumes that "individuals choose a series of current actions without fully appreciating how those actions will affect future perceptions and behavior."

[7] In this formulation, a person at time τ has preferences that relate directly to the path of $c(t)$. An alternative approach, as in Epstein and Hynes's (1983) recursive preferences, would have $U(\tau)$ depending only on $c(\tau)$ and on utility as evaluated at the next instant in time, $U(\tau + \epsilon)$. As in the intergenerational model mentioned before, no time inconsistency would arise under this specification.

earlier ones (all considered relative to the exponential term). We also assume, following Laibson (1997a), that the magnitude of $\frac{\phi'}{\phi}(v)$ is non-increasing in v and approaches zero as v tends to infinity. The instantaneous rate of time preference at a time distance $v = t - \tau$ in the future is then given by $\rho - \frac{\phi'}{\phi}(v) \geq \rho$. That is, ρ is the lower bound for the rate of time preference. These properties imply that the rate of time preference is particularly high in the near term but looks roughly constant at the value ρ in the distant future.

The rest of the model is standard. The production function is the usual neoclassical one, given by

$$y = f(k) \tag{3}$$

where y is output per worker and k is capital per worker, with $f'(k) > 0$ and $f''(k) < 0$. The assumption for now is that population is constant and technological progress is nil. The number of workers (and the quantity of labor input) equals the constant population. The economy is closed, so that assets per person correspond to capital per worker, k. The rate of return, $r(t)$, equals $f'[(k(t)] - \delta$, where $\delta \geq 0$ is the constant rate of depreciation on capital, and the wage rate, $w(t)$, equals $f[k(t)] - k(t) \cdot f'[k(t)]$.

3 Results under Commitment

The first-order optimization conditions for the representative household's path of consumption, $c(t)$, would be straightforward if the full path of current and future consumption could be chosen in a committed manner at the present time, τ. For example, if utility takes the iso-elastic form,

$$u(c) = (c^{1-\theta} - 1)/(1 - \theta) \tag{4}$$

where $\theta > 0$, then the usual Ramsey formula for the growth rate of consumption becomes

$$(1/c) \cdot (dc/dt) = (1/\theta) \cdot [r(t) - \rho + \frac{\phi'}{\phi}(t - \tau)] \tag{5}$$

for $t > \tau$. That is, the usual formula would be modified to add to the constant rate of time preference, ρ, the new term, $-\frac{\phi'}{\phi}(t-\tau)$, which is evaluated from the perspective of the starting (and commitment) date, τ. We can think of equation (5) as coming from usual perturbation arguments, whereby consumption is lowered at some point in time and raised at another point in time—perhaps the next instant in time—with all other values of consumption held constant.

Given the properties for $\phi(\cdot)$ assumed above, the full rate of time preference would start out at a high value and then decline toward ρ as $t-\tau$ tended toward infinity. Thus, the steady-state rate of time preference would be ρ, and the steady state of the model would coincide with that of the usual model. The new results would involve the transition, during which time-preference rates were greater than in the standard setting but falling over time. This behavior tends to generate a rising path of the saving rate, a pattern that weakens the convergence force, whereby a poor economy grows faster than a rich one.

One problem with this proposed solution is that the current time, τ, is arbitrary and, in the typical situation, the potential to commit did not suddenly arise at this time. Rather, if perpetual commitments on consumption were feasible, then these commitments would presumably have existed in the past, perhaps in the infinite past. In this case, current and future values of consumption would have been determined earlier, and τ would be effectively minus infinity, so that $\frac{\phi'}{\phi}(t-\tau)$ is zero for all $t \geq 0$. Hence, the rate of time preference is constant and equal to ρ for all $t \geq 0$, so that the standard Ramsey results apply throughout, not just in the steady state.

In any event, the more basic problem with the proposed solution is that commitment on future choices of $c(t)$ is problematic. We therefore now consider the solution of the modified Ramsey problem in the absence of any commitment technology for future consumption. That is, at time τ, the consumer can determine only the instantaneous flow of consumption, $c(\tau)$. A subsequent section considers the possibility of limited commitment, viewed as a situation in which the consumer at date τ can dictate consumption choices between dates τ and $\tau + T$, for some $T \geq 0$. (These selections are, however, subject to the constraint of having to honor past commitments.)

4 Results without Commitment under Log Utility

The first-order condition shown in equation (5) will not generally hold in the absence of commitment. The reason is that it is infeasible for the household to carry out the type of perturbation that underlies this condition. Specifically, the household cannot commit to lowering $c(\tau)$ at time τ and then increasing $c(t)$ at some future date (such as the next instant in time), while holding fixed consumption at all other dates. We therefore have to take a different approach. In particular, the household has to figure out how its setting of $c(\tau)$ at time τ will alter its stock of assets and how this change in assets will influence the choices of consumption at later dates.

The full solution without commitment is worked out here for the case of log utility, $u(c) = \log(c)$. The steady-state results for a general concave utility function, $u(c)$, are discussed in a later section. Some transitional results under iso-elastic utility, as in equation (4), are derived in a still later section. We begin with the case of log utility.[8]

Think of choosing $c(t)$ at time τ for the interval $[\tau, \tau + \epsilon]$, where ϵ will eventually approach zero. The full integral of utility flows from equation (2) can then be broken up into two pieces:

$$U(\tau) = \int_{\tau}^{\tau+\epsilon} \log[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt + \int_{\tau+\epsilon}^{\infty} \log[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt$$

$$\approx \epsilon \cdot \log[c(\tau)] + \int_{\tau+\epsilon}^{\infty} \log[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \qquad (6)$$

[8] Pollak (1968, section 2) works out results under log utility with a finite horizon and a zero interest rate.

where the approximation comes from treating $c(t)$ as constant between dates τ and $\tau+\epsilon$ and from taking $\phi(t-\tau)$ and $e^{-\rho(t-\tau)}$ as equal to unity over this interval. These approximations will become exact in the equilibrium as ϵ tends to zero.

The consumer can pick $c(\tau)$ and thereby the choice of saving at time τ. This selection influences $c(t)$ for $t \geq \tau + \epsilon$ by affecting the stock of assets, $k(\tau + \epsilon)$, available at time $\tau+\epsilon$. In order to determine the optimal $c(\tau)$, the household has to know, first, the relation between $c(\tau)$ and $k(\tau + \epsilon)$ and, second, the relation between $k(\tau + \epsilon)$ and the choices of $c(t)$ for $t \geq \tau + \epsilon$.

The first part of the problem is straightforward. The household's budget constraint is

$$dk/dt = r(t) \cdot k(t) + w(t) - c(t) \tag{7}$$

where the economy-wide prices, $r(t)$ and $w(t)$, are treated as given by the individual household. For a given starting stock of assets, $k(\tau)$, the stock at time $\tau + \epsilon$ is determined by

$$k(\tau + \epsilon) \approx k(\tau) \cdot [1 + \epsilon \cdot r(\tau)] + \epsilon \cdot w(\tau) - \epsilon \cdot c(\tau) \tag{8}$$

The approximation comes from neglecting compounding over the interval $(\tau, \tau+\epsilon)$ and from treating the variables $r(t)$, $w(t)$,and $c(t)$ as constants over this interval.[9] These assumptions will all be satisfactory in the equilibrium when ϵ approaches zero. The important result from equation (8) is that

$$d[k(\tau + \epsilon)]/d[c(\tau)] \approx -\epsilon \tag{9}$$

Hence, more consumption today means less assets at the next moment in time.

The difficult part of the model's solution involves the assessment of the effect of $k(\tau + \epsilon)$ on $c(t)$ for $t \geq \tau + \epsilon$, that is, in figuring out the propensities to consume out of assets. In the standard Ramsey model with log utility, $c(t)$ is set as the constant fraction ρ of wealth, where wealth consists of assets, $k(t)$, plus the

[9]This analysis is effectively one of discrete time, where ϵ is the length of a period.

present value of current and future wages. The fraction is constant because the income and substitution effects associated with the path of future interest rates exactly cancel under log utility. (See, for example, Barro and Sala-i-Martin [1995, Ch. 2].) Given this background, it is reasonable to conjecture that the income and substitution effects associated with future interest rates would still cancel in the present environment of log utility, even though time preference is non-exponential and commitment is absent. However, the constant of proportionality, denoted by λ, need not equal ρ. Thus, the conjecture—which turns out to be correct—is that consumption is given by

$$c(t) = \lambda \cdot [k(t) + \textit{present value of wages}] \tag{10}$$

for $t \geq \tau + \epsilon$ for some constant $\lambda > 0$.[10]

Under the assumed conjecture, it can readily be verified that $c(t)$ grows at the rate $r(t) - \lambda$ for $t \geq \tau + \epsilon$. Hence, for any $t \geq \tau + \epsilon$, consumption is determined from

$$\log[c(t)] = \log[c(\tau + \epsilon)] + \int_{\tau+\epsilon}^{t} r(v)dv - \lambda \cdot (t - \tau - \epsilon)$$

The expression for utility from equation (6) can therefore be written as

$$U(\tau) \approx \epsilon \cdot \log[c(\tau)] + \log[c(\tau + \epsilon)] \cdot \int_{\tau+\epsilon}^{\infty} \phi(t - \tau) \cdot e^{-\rho(t-\tau)}dt$$
$$+ \textit{ terms that are independent of } c(t) \textit{ path} \tag{11}$$

Define the integral

$$\Omega \equiv \int_{0}^{\infty} \phi(v) \cdot e^{-\rho v}dv \tag{12}$$

[10] Phelps and Pollak (1968, section IV) use a parallel conjecture to work out a Cournot-Nash equilibrium for their problem. They assume iso-elastic (not necessarily logarithmic) utility but also assume a linear technology so that the rate of return is constant. The last property is critical, because consumption is not a constant fraction of wealth under iso-elastic utility (except in the logarithmic case) if the rate of return varies over time. The linear technology also eliminates any transitional dynamics, that is, the economy is always in a position of steady-state growth.

This expression—which is constant over time—corresponds, as ϵ approaches zero, to the integral in equation (11).

The marginal effect of $c(\tau)$ on $U(\tau)$ can be calculated as

$$\frac{d[U(\tau)]}{d[c(\tau)]} \approx \frac{\epsilon}{c(\tau)} + \frac{\Omega}{c(\tau+\epsilon)} \cdot \frac{d[c(\tau+\epsilon)]}{d[k(\tau+\epsilon)]} \cdot \frac{d[k(\tau+\epsilon)]}{dc(\tau)}$$

The final derivative equals $-\epsilon$, from equation (9), and the next-to-last derivative equals λ, according to the conjectured solution in equation (10). Therefore, setting $d[U(\tau)]/d[c(\tau)]$ to zero implies

$$c(\tau) = \frac{c(\tau+\epsilon)}{\Omega\lambda}$$

If the conjectured solution is correct, then $c(\tau+\epsilon)$ must approach $c(\tau)$ as ϵ tends to zero. Otherwise, $c(t)$ would exhibit jumps at all points in time, and the conjectured answer would be wrong. The unique value of λ that delivers this correspondence follows immediately as

$$\lambda = 1/\Omega = 1/\int_0^\infty \phi(v) \cdot e^{-\rho v} dv \tag{13}$$

To summarize, the solution for the household's consumption problem under log utility is that $c(t)$ be set as the fraction λ of wealth at each date, where λ is the constant shown in equation (13). The solution is time consistent because, if $c(t)$ is chosen in this manner at all future dates, then it will be optimal for consumption to be set this way at the current date.[11]

[11] This approach derives equation (13) as a Cournot-Nash equilibrium but does not show that the equilibrium is unique. Uniqueness is easy to demonstrate in the associated discrete-time model with a finite horizon, as considered by Laibson (1996). In the final period, the household consumes all of its assets, and the unique solution for each earlier period can be found by working backwards sequentially from the end point. This result holds if utility takes the isoelastic form, $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, not just for log utility. The uniqueness result also holds if the length of a period approaches zero (to get continuous time) and if the length of the horizon becomes arbitrarily large. However, Laibson (1994) uses an explicitly game-theoretic approach

Inspection of equation (13) reveals that $\lambda = \rho$ in the standard Ramsey case in which $\phi(v) = 1$ for all v. Otherwise, since $\phi(v) \leq 1$ holds for all v, $\lambda \geq \rho$ applies. Thus, the introduction of the $\phi(\cdot)$ term in the utility function of equation (2) and the consequent shift to a time-inconsistent setting amounts, under log utility, to an increase in the rate of time preference.[12] The effective rate of time preference, λ, is still constant over time. Therefore, the full dynamics and steady state of the model take exactly the same form as in the standard Ramsey framework. The higher rate of time preference corresponds to a higher steady-state interest rate,

$$r^* = \lambda \tag{14}$$

and, thereby, to a lower steady-state capital intensity, k^*, which is determined from the condition

$$f'(k^*) = \lambda + \delta$$

The speed of convergence to the steady state is not much affected by an increase in the effective rate of time preference (see Barro and Sala-i-Martin [1995, Ch. 2]). The reason is that the higher rate of time preference reduces the willingness to save, but the rate of convergence depends not so much on the level of the saving rate but, rather, on whether the saving rate rises or falls during the transition to the steady state. The rate of time preference does not affect this transitional behavior of the saving rate in a clear way.

To assess quantitatively the impact on the effective rate of time preference, we can consider some alternative specifications for the term $\phi(v)$. Laibson (1997a)

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to demonstrate the possibility of non-uniqueness of equilibrium in the infinite-horizon case. The existence of multiple equilibria depends on punishments that sanction past departures of consumption choices from designated values, and these kinds of equilibria unravel if the horizon is finite. My analysis of the infinite-horizon case does not consider these kinds of equilibria.

[12]The analysis in a later section shows that the right-hand side of equation (13) can be expressed as a weighted average of future time-preference rates (as viewed from today's perspective), $\rho - \frac{\phi'}{\phi}(v)$. Thus, λ is a time invariant weighted average of these time-preference rates.

proposes a "quasi-hyperbolic" form in discrete time, whereby $\phi(v) = 1$ in the current period, $\phi(v) = \beta\gamma$ for the next period, $\phi(v) = \beta\gamma^2$ for the following period, and so on, where $0 < \beta < 1$ and $0 < \gamma < 1$. (Phelps and Pollak [1968] also use this functional form.) In this specification, the discount factor between today and tomorrow is $\beta\gamma$, which is less than that, γ, between any two adjacent future periods. Laibson also argues that, while γ is likely to be close to one on an annual basis, β would be substantially less than one—perhaps between one-half and two-thirds.

This quasi-hyperbolic case can be applied to a continuous-time setting by specifying

$$\phi(v) = 1 \text{ for } 0 \leq v \leq V, \quad \phi(v) = \beta \text{ for } v > V \tag{15}$$

for some $V > 0$, where $0 < \beta < 1$. (In this specification, $\phi(v)$ is discontinuous at $v = V$.) Laibson's suggestion is that V is small, so that the heavy rate of discounting applies over the near term. I take this operationally to mean that $\rho V << 1$ holds.

Substitution from equation (15) into the formula for the integral in equation (12) leads to

$$\Omega = (1/\rho) \cdot [1 - (1-\beta) \cdot e^{-\rho V}]$$

This expression is increasing in V, so that $\lambda = 1/\Omega$ is decreasing in V. As V approaches infinity, Ω goes to $1/\rho$, which corresponds to the standard Ramsey case. If we use the condition $\rho V << 1$, then the expression for Ω simplifies, as an approximation, to β/ρ, so that

$$\lambda \approx \rho/\beta \tag{16}$$

If we accept Laibson's suggestion that β is between one-half and two-thirds, then the effective rate of time preference, λ, is between 1.5ρ and 2ρ. Hence, if ρ is, say, 0.02 per year, then the heavy near-term discounting of future utility converts the Ramsey model into one in which the effective rate of time preference,

λ, is $0.03 - 0.04$ per year. This kind of change in the time-preference rate could have a substantial effect on the steady-state capital intensity. For example, for a Cobb-Douglas production function with capital share of 1/3, if $\rho = 0.02$ per year and δ (the depreciation rate) is 0.05 per year, then $\lambda = 0.03$ implies that the steady-state capital intensity is reduced (relative to the Ramsey case) by a factor of 0.82, whereas $\lambda = 0.04$ means that the steady-state capital intensity is reduced by a factor of 0.69.

The specification in equation (15) yields simple closed-form results, but the functional form implies an odd discrete jump in $\phi(\cdot)$ at a time V in the future. More generally, the notion from the literature on myopic preferences is that the rate of time preference, given by $\rho - \frac{\phi'}{\phi}(v)$, is high when v is small and declines, say toward ρ, as v becomes large. A simple functional form that captures this property in a smooth fashion is

$$-\frac{\phi'}{\phi}(v) = be^{-\alpha v} \tag{17}$$

where $b > 0$, $\alpha > 0$. The parameter b gives the contribution of the ϕ term to the rate of time preference at $v = 0$, and the parameter α determines the speed at which this contribution decays toward zero (so that the rate of time preference tends toward ρ).

Integration of the expression in equation (17), together with the boundary condition $\phi(0) = 1$, leads to an expression for $\phi(v)$:[13]

$$\log[\phi(v)] = (b/\alpha) \cdot (e^{-\alpha v} - 1) \tag{18}$$

This result can be substituted into the integral in equation (12) to get an expression for Ω:

$$\Omega = e^{-(b/\alpha)} \cdot \int_0^\infty e^{[-\rho v + (b/\alpha) \cdot e^{-\alpha v}]} dv$$

[13]The expression in equation (18) is similar to the "generalized hyperbola" proposed by Loewenstein and Prelec (1992, p. 580): $\phi(v) = (1 + \alpha v)^{-b/\alpha}$.

This formula involves an integral that cannot be solved in closed form but can be evaluated numerically if values are specified for the parameters ρ, b, and α.

The basic idea from Laibson (1997a) is that the addition to the rate of time preference at $v = 0$—that is, the parameter b—should be high, something like 0.25-0.50 per year. The rate of decay of this added rate of time preference—that is, the parameter α—should also be high, something like 0.50 per year, so that the rate of time preference gets close to ρ a few years in the future. With $\rho = 0.02$, $b = 0.25$, and $\alpha = 0.50$, Ω turns out to be 31.0, so that $\lambda = 1/\Omega$ is 0.032. If $b = 0.50$ and the other parameters are the same, then $\Omega = 19.3$ and $\lambda = 0.052$. Thus, the more appealing functional form in equation (18) delivers results that are similar to the simpler form assumed in equation (15).

The myopia element in households' time preference can have a major effect on saving behavior and, hence, on capital accumulation. There could also be important welfare and policy implications because the people making current consumption choices would benefit from the introduction of institutions that allowed them, fully or partially, to commit their choices of future consumption. (Laibson [1997a] discusses some possible institutional mechanisms.)

As it stands, however, the solution to the model under myopic time preference is, in a sense, observationally equivalent to that in the conventional neoclassical growth model. That is, the answers coincide with those in the standard model for a suitable specification of the rate of time preference, ρ. Since the parameter ρ cannot be observed directly, there would be a problem in inferring from macroeconomic data whether preferences included the myopia term, $\phi(\cdot)$. Some identification would be possible if different population groups—perhaps different countries—included the myopia element to varying extents. Perhaps more promising, however, is the idea that different societies have different technologies for committing their choices of future consumption. This element was missing in the model considered thus far—where no commitment ability at all existed—but is included in an extended framework developed in a later section.

5 Population Growth and Technological Progress

It is straightforward to incorporate population growth in the manner normally applied to the neoclassical growth model. If population (extended family size) grows at the constant rate n, then the utility function is modified from equation (2) to

$$U(\tau) = \int_{\tau}^{\infty} u[c(t)] \cdot e^{nt} \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \tag{19}$$

That is, the per capita flow of utils, $u[c(t)]$, is multiplied by the family size, e^{nt} (where the size at time 0 is normalized to unity). We assume, as is usual for the Ramsey model, that $\rho > n$, so that the net exponential term in equation (19) is declining in t.

The solution under log utility is similar to that from before, except that the integral Ω is now defined by

$$\Omega \equiv \int_0^{\infty} \phi(v) e^{-(\rho-n)v} dv \tag{20}$$

This expression includes the population term, e^{nv}. The relation between the propensity to consume out of wealth, λ, and the modified Ω term is given by

$$\lambda = n + (1/\Omega) \tag{21}$$

and the steady-state interest rate is still given by $r^* = \lambda$.

In the Ramsey case, where $\phi(v) = 1$ for all v, $\Omega = 1/(\rho - n)$ (in equation [20]) and $\lambda = \rho$ (in equation [21]). For the case of Laibson's quasi-hyperbolic preferences in equation (15), the result is

$$\Omega \approx \beta/(\rho - n), \quad \lambda \approx (\rho/\beta) - n \cdot (1-\beta)/\beta \tag{22}$$

Since $0 < \beta < 1$, an increase in n lowers λ and, therefore, reduces the steady-state interest rate, r^*, and raises the steady-state capital intensity, k^*.

It is also straightforward to introduce the conventional type of exogenous, labor-augmenting technological progress at the rate $x \geq 0$. The solution for λ, the propensity to consume out of wealth, is still that shown in equations (20) and (21). However, since consumption per person grows in the steady state at the rate x, the condition for the steady-state interest rate under quasi-hyperbolic preferences is

$$r^* = \lambda + x \approx (\rho/\beta) - n \cdot (1 - \beta)/\beta + x$$

Hence, r^* responds one-to-one to the rate of technological progress, x.

6 Solution under Partial Commitment with Log Utility

The assumption in the previous setting was that a household at date τ could determine only the consumption flow at the same moment in time, $c(\tau)$. We now consider the possibility that— because of personal discipline or by use of institutional commitment devices—each household has some capacity, but not infinite capacity, to commit choices of future consumption. Specifically, the assumption is that a household can select at date τ the consumption flows over the closed interval $[\tau, \tau + T]$, that is, for a period of length $T \geq 0$. However, these choices must respect any commitments on consumption that were made earlier.

Laibson (1997a) points out that the illiquidity of some assets, such as pension funds, can help to commit future consumption—in the present context by raising the commitment interval, T. In other circumstances, the interval T could represent the planning interval for a consumption activity, for example, the notice required for reservations for travel, theatre, and restaurants.

There are two contexts in which the effects of the commitment technology can be assessed. First, in an ongoing situation where the T-period commitment ability has been present for a long time (at least T periods), a household at date τ will be selecting *only* $c(\tau + T)$, the flow of consumption at date $\tau + T$. The choices of consumption in the half-open interval $[\tau, \tau + T)$ will already have been made and committed at prior dates. This problem is formally analogous to the one considered earlier, but some interesting new results emerge concerning the effect of the commitment interval, T.

Second, some shift in the commitment interval may occur at some point. For example, the representative household might initially have no commitment capacity, as in the model studied before, but the potential for T-period commitments may be created (perhaps because of some institutional change) at a particular date. Since no previous commitments existed, the household has an opportunity at the time when commitments first become available to choose consumption over the whole interval of length T. Similarly, if $T > 0$ applied initially and T then expanded, the household would have an interval of finite length over which to make choices.[14] Changes in T create transition periods, after which the situation looks like the first case of ongoing commitments, as long as no further changes in T occur.

The effect of economic development on the commitment interval, T, is uncertain. On the one hand, improvements in financial markets and in the sophistication of contracts would allow people to make more binding commitments and, in that respect, raise T. On the other hand, T would fall with enhanced access to funds via credit cards and ATM machines, increases more generally in the liquidity of assets, and improvements in transactions technologies that reduce the required length of prior reservations for various activities. Other disturbances, such as wartime, may have the effect of eliminating the commitments that were made in the past.

[14]The effects from reductions in T are not entirely symmetric if past commitments must always be respected.

6.1 Ongoing Commitments

Let s denotes the current date and τ the most advanced future date for which commitments are feasible, so that $T = \tau - s$ is the commitment interval. The household's overall utility can now be broken up into three pieces, one from s to τ, a second from τ to $\tau + \epsilon$, and a third from $\tau + \epsilon$ to infinity:

$$U(s) = \int_s^\tau \log[c(t)] \cdot \phi(t-s) \cdot e^{-\rho(t-s)} dt + \int_\tau^{\tau+\epsilon} \log[c(t)] \cdot \phi(t-s) \cdot e^{-\rho(t-s)} dt$$

$$+ \int_{\tau+\epsilon}^{\infty} \log[c(t)] \cdot \phi(t-s) \cdot e^{-\rho(t-s)} dt \tag{23}$$

The last two integrals are the ones considered before in equation (6). The first integral will already be fixed from prior commitments and, therefore, need not be considered by the household at time s. The household chooses $c(\tau)$ at time s, that is, for time $T = \tau - s$ ahead. We pretend, as before, that this consumption remains constant over the interval $[\tau, \tau + \epsilon]$, where we shall again let ϵ approach zero. Then, also as before, we need to know how the choice of $c(\tau)$ impacts on the values of $c(t)$ that will be chosen later, that is, for the interval from $\tau + \epsilon$ to infinity.

With the first integral omitted, the expression for $U(s)$ from equation (23) can be approximated as

$$U(s) \approx \epsilon \cdot \log[c(\tau)] \cdot \phi(T) \cdot e^{-\rho T} + \int_{\tau+\epsilon}^{\infty} \log[c(t)] \cdot \phi(t-s) \cdot e^{-\rho(t-s)} dt \tag{24}$$

As before, the approximation is that $c(t)$ is treated as constant at the value $c(\tau)$ between dates τ and $\tau + \epsilon$. In addition, $\phi(t)$ and $e^{-\rho t}$ are approximated, respectively, by $\phi(T)$ and $e^{-\rho T}$ in this interval. These approximations will be satisfactory in the equilibrium as ϵ approaches zero.

The rest of the analysis proceeds as before, based on the conjecture that $c(t)$ is the fraction λ of wealth, for some constant λ, for $t \geq \tau + \epsilon$. The solution is the same as equation (13), except for two considerations: the term $\phi(T) \cdot e^{-\rho T}$

multiplies the first term in equation (24), and the lower limit of integration in the second term in this equation is advanced from the current date s by $T + \epsilon$ (or by T as ϵ approaches zero). The result for λ, which depends on T, can be written as

$$\lambda(T) = \phi(T) \cdot e^{-\rho T} / \Omega(T) \tag{25}$$

where the integral $\Omega(T)$ is now given by

$$\Omega(T) \equiv \int_T^\infty \phi(v) \cdot e^{-\rho v} dv \tag{26}$$

For $T = 0$, the results in equations (25) and (26) coincide with the one discussed before in equation (13), where recall that $\lambda(0) \geq \rho$. As T approaches infinity, $\lambda(T)$ in equation (25) can be shown (by the use of L'Hopital's Rule on the indeterminate form 0/0) to approach ρ. That is, if commitment is, and always has been, feasible over an infinite horizon, then the results are the same as those of the standard Ramsey model.

Differentiation of equation (25) with respect to T yields

$$\lambda'(T) = \lambda(T) \cdot [\frac{\phi'}{\phi}(T) - \rho + \lambda(T)] \tag{27}$$

As already noted, $\lambda(T)$ falls from $\lambda(0)$ to ρ as T rises from 0 to infinity. It can be shown from equation(27) that this decrease is monotonic, that is, $\lambda'(T) \leq 0$ holds for all $T \geq 0$.[15] This result provides some potential for using macroeconomic data to isolate the effects of myopic preferences. Specifically, countries (or families)

[15]The second derivative of $\lambda(T)$ can be computed as

$$\lambda''(T) = \lambda(T) \cdot [\frac{d[\frac{\phi'}{\phi}(T)]}{dT} + \lambda'(T)] + \lambda'(T) \cdot [\frac{\phi'}{\phi}(T) - \rho + \lambda(T)]^2$$

We have been assuming that $\frac{d[\frac{\phi'}{\phi}(T)]}{dT} \geq 0$ holds for all $T \geq 0$. Therefore, if $\lambda'(T) > 0$ for any T, then $\lambda''(T) > 0$ and, hence, $\lambda'(T) > 0$ for all subsequent T. These properties are inconsistent with $\lambda(T)$ equaling $\lambda(0) \geq \rho$ at $T = 0$ and declining to ρ as T tends to infinity. Therefore, $\lambda'(T) \leq 0$ must hold for all $T \geq 0$.

with better commitment technologies, as represented in the model by higher values of T, should have lower effective rates of time preference and, therefore, lower propensities to consume and higher propensities to save and accumulate capital.

The condition $\lambda'(T) \leq 0$ in equation (27), together with $\lambda(\infty) = \rho$, implies

$$\rho \leq \lambda(T) \leq \rho - \frac{\phi'}{\phi}(T) \tag{28}$$

for all $T \geq 0$. Note that $\lambda(T)$ in the center of this inequality is the effective rate of time preference today when the household has always had the ability to commit consumption T periods ahead. The right side of the inequality would be the effective rate of time preference for T periods ahead if the household first got the ability today to commit to consumption T periods ahead (see equation [5]). For $T = 0$, the inequality implies that the effective rate of time preference, $\lambda(0)$, with zero ability to commit is *less* than that prevailing today under commitment if the household first obtained today the ability to make this commitment on future consumption (over a period of some finite length).

6.2 Changes in the Commitment Technology

As mentioned before, shifts in the ability to commit—modeled here as changes in T—create transition intervals from one ongoing commitment situation to another. To illustrate the nature of these transitions, consider a case in which the ability to commit is initially nil ($T = 0$) and in which people anticipate that the ability to commit will always remain nil. Then the effective rate of time preference is the value $\lambda(0)$, as derived before, which satisfies the inequality from (28):

$$\rho \leq \lambda(0) \leq \rho - \frac{\phi'}{\phi}(0)$$

Suppose then that a T-period commitment ability is introduced (as a surprise) at date τ and that everyone then believes that this system for commitment will remain in place forever. At the outset, the household can choose $c(t)$ over the interval $[\tau, \tau + T]$. In this context, the usual first-order condition for consumption growth, as shown in equation (5), applies, that is,

$$(1/c) \cdot (dc/dt) = r(t) - \rho + \frac{\phi'}{\phi}(t - \tau)$$

for $\tau < t < \tau + T$. This condition holds because, under the new commitment technology, the household can carry out the perturbations to the consumption path that underlie the condition. In particular, within the interval $[\tau, \tau + T]$, the household can lower consumption at one date and raise consumption at another date, while holding fixed consumption at all other dates (and also holding fixed the assets left over at date $\tau + T$).

The results imply that, at time τ, the rate of time preference shifts discretely from $\lambda(0)$ to the *higher* value $\rho - \frac{\phi'}{\phi}(0)$. The rate of time preference then declines gradually to reach $\rho - \frac{\phi'}{\phi}(T)$ at time T. At this point, the system returns to the case of ongoing commitment that has already been analyzed, and the rate of time preference shifts discretely downward to the value $\lambda(T)$. We also know that the long-term effect is a lowering of the rate of time preference, that is, $\lambda(T) \leq \lambda(0)$.

The surprise introduction of the commitment technology at date τ generally also produces a discrete shift in the level of consumption at that date. Then, because of the rise in the rate of time preference, $c(t)$ grows at a lower rate than under the initial plan, but this growth rate rises as the rate of time preference falls. Finally, at time $\tau + T$, there is a kink in the $c(t)$ path because of the discrete decline in the rate of time preference. Subsequent to this date, consumption grows at a faster rate than under the initial plan.

7 Steady-State Analysis under General Utility

In the usual neoclassical growth model, it is typical to assume iso-elastic, though not necessarily logarithmic, utility. That is, a commonly used form of the utility function, as in equation (4), is

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}$$

where $\theta > 0$. In this form, log utility corresponds to $\theta = 1$. For $\theta \neq 1$ and with the conventional exponential time preference of the Ramsey model, consumption is not a constant fraction of wealth. Rather, the propensity to consume out of wealth depends on the path of future interest rates. However, the first-order condition for consumption growth is well known to take the simple form

$$(1/c) \cdot (dc/dt) = (1/\theta) \cdot [r(t) - \rho] \tag{29}$$

A reasonable conjecture is that the form of equation (29) would still hold when the household's utility takes the form of equation (2), which includes the non-exponential time-preference factor, $\phi(t-\tau)$. The presumption—based on an extrapolation of the results under log utility—is that the constant ρ would be replaced by some other constant λ, which would represent the effective (constant) rate of time preference. This conjecture turns out to be incorrect. The problem is that the effective rate of time preference at time τ involves an interaction of the path of the future values of $\phi(t-\tau)$ with future interest rates and, hence, is not constant over time when interest rates are changing (unless $\theta = 1$). Some details of this interaction are worked out in the next section.

Although the transitional dynamics are complicated, it is straightforward to work out the characteristics of the steady state under a general concave utility function, $u(c)$. We can, as in equation (6), break up the household's utility evaluated at time τ into two pieces, one from date τ to date $\tau + \epsilon$ and the other from time $\tau + \epsilon$ to infinity:

$$U(\tau) = \int_{\tau}^{\tau+\epsilon} u[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt + \int_{\tau+\epsilon}^{\infty} u[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt$$

$$\approx \epsilon \cdot u[c(\tau)] + \int_{\tau+\epsilon}^{\infty} u[c(t)] \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \tag{30}$$

Note that we have returned here to the setting of zero population growth and technological progress.

To choose $c(\tau)$ optimally, the household has to know how the implied shift in assets available as of date $\tau+\epsilon$ will affect subsequent choices of $c(t)$. This analysis is straightforward if the economy is in a steady state. The key property of a steady state (assuming that it exists) is that the interest rate, r^*, will be such as to motivate each household to select a constant level of consumption, that is, $(1/c) \cdot (dc/dt) = 0$ will apply.[16] Therefore, if a household contemplates a perturbation to $c(\tau)$ when the steady-state interest rate prevails, then the corresponding change in assets will be used to raise or lower permanently the level of $c(t)$ in all future periods.

Let c^* be the household's constant level of consumption from date $\tau + \epsilon$ to infinity. The utility function from equation (30) can then be expressed as

$$U(\tau) \approx \epsilon \cdot u[c(\tau)] + u(c^*) \cdot \int_{\tau+\epsilon}^{\infty} \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \tag{31}$$

We know from equation (9) that the effect of $c(\tau)$ on the assets, $k(\tau+\epsilon)$, available at time $\tau + \epsilon$ is given by

$$d[k(\tau+\epsilon)]/d[c(\tau)] \approx -\epsilon$$

We also have from the budget constraint in equation (7) that the constant consumption level sustainable forever from date $\tau + \epsilon$ onward is given by

$$c^* = r^* \cdot k(\tau + \epsilon) + w^*$$

where r^* is the steady-state interest rate and w^* is the steady-state wage rate. Therefore, the effect of $c(\tau)$ on c^* is given by

$$\frac{dc^*}{dc(\tau)} = \frac{dc^*}{dk(\tau+\epsilon)} \cdot \frac{dk(\tau+\epsilon)}{dc(\tau)} \approx -r^*\epsilon \tag{32}$$

[16]The results go through if households differ with respect to their levels of assets. In this case, richer households have higher levels of consumption. But the same interest rate will motivate each household to select a constant level of consumption over time.

The first-order condition for $c(\tau)$ follows from differentiation of $U(\tau)$ in equation (31) with respect to $c(\tau)$ as

$$\frac{dU(\tau)}{dc(\tau)} \approx \epsilon \cdot u^{'}[c(\tau)] - u^{'}(c^*) \cdot r^* \epsilon \Omega = 0 \tag{33}$$

where the integral Ω is given from equation (12) (as ϵ tends to zero) by

$$\Omega = \int_0^\infty \phi(v) \cdot e^{-\rho v} dv$$

If the economy is actually in a steady state, then the chosen $c(\tau)$ must correspond to c^*. Hence, $u^{'}[c(\tau)] = u^{'}(c^*)$ applies, and the steady-state interest rate follows from equation (33) as

$$r^* = 1/\Omega = 1/\int_0^\infty \phi(v) \cdot e^{-\rho v} dv \tag{34}$$

Thus, the steady-state result under a general concave utility function coincides with that for log utility, as shown in equations (13) and (14).

The steady-state results for a general utility function with population growth are also the same as those under log utility. The condition is

$$r^* = n + 1/\Omega$$

where the integral Ω corresponds to that given in equation (20),

$$\Omega = \int_0^\infty \phi(v) e^{-(\rho - n)v} dv$$

With exogenous, labor-augmenting technological progress at the rate x, we know from the analysis of the standard neoclassical growth model that the existence of a steady state requires $u(c)$ to take the iso-elastic form shown in equation (4),

$$u(c) = (c^{1-\theta} - 1)/(1 - \theta)$$

where $\theta > 0$. In this case, the steady-state interest rate is given by

$$r^* = n + x + 1/\Omega$$

where the integral Ω is now given by

$$\Omega = \int_0^\infty \phi(v) e^{-[\rho - n - (1-\theta)v]} dv$$

In the standard Ramsey case, where $\phi(v) = 1$ for all v, we get $r^* = \rho + \theta x$. For the case of Laibson's quasi-hyperbolic utility function in equation (15), the result is

$$r^* \approx \frac{\rho}{\beta} - n \cdot \frac{(1-\beta)}{\beta} + x \cdot \frac{(\beta + \theta - 1)}{\beta}$$

where recall that $0 < \beta < 1$. Thus, for the case considered before of log utility $(\theta = 1)$, the effect of x on r^* is one-to-one. More generally, the effect of x on r^* is more or less than one-to-one depending on whether θ is greater or less than one.

8 Transitional Behavior under Iso-Elastic Utility

The objective here is to characterize some aspects of the transitional dynamics with myopic preferences when utility departs from the logarithmic form. We assume here that utility takes the iso-elastic form,

$$u(c) = (c^{1-\theta} - 1)/(1-\theta)$$

where $\theta > 0$.

As for the case of log utility $(\theta = 1)$ from equation (6), we can break up the overall utility into the part from τ to $\tau + \varepsilon$ and that from $\tau + \varepsilon$ to infinity:

$$U(\tau) \approx \varepsilon \cdot \frac{[c(\tau)^{1-\theta} - 1]}{(1-\theta)} + \int_{\tau+\varepsilon}^\infty \frac{[c(t)^{1-\theta} - 1]}{(1-\theta)} \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} dt \qquad (35)$$

The effect of $c(\tau)$ on the assets, $k(\tau + \varepsilon)$, available at date $\tau + \varepsilon$ is also still given from equation (9) by

$$d[k(\tau + \epsilon)]/d[c(\tau)] \approx -\epsilon$$

The difficult part of the problem is the assessment of the response of $c(t)$ to $k(\tau+\varepsilon)$ for $t \geq \tau+\varepsilon$. We measure this response here by modifying the form of the conjectured solution for $c(t)$ from that shown in equation (10). The conjecture now is that the growth rate of consumption is given by

$$(1/c) \cdot (dc/dt) = (1/\theta) \cdot [r(t) - \lambda(t)] \tag{36}$$

for $t \geq \tau + \varepsilon$. In the standard Ramsey model, $\lambda(t) = \rho$ holds, whereas, in the myopia model under log utility, $\lambda(t)$ equals a constant, λ, which differs from ρ (in equation [13]). The specification in equation (36) allows $\lambda(t)$ to vary over time. This equation therefore restricts the behavior of $c(t)$ only in that $\lambda(t)$ does not depend on the level of assets. Hence, the choice of consumption at date τ and, hence, of assets at date $\tau + \epsilon$ will, under this conjecture, affect the level but not the shape of the path of future consumption.

We can proceed as under log utility, but with more algebra, to work out an expression for the first-order condition for the choice of $c(\tau)$, given that the behavior of future consumption accords with equation (36). If we define time averages of $r(t)$ and $\lambda(t)$ by

$$R(t,\tau) \equiv \frac{1}{(t-\tau)} \cdot \int_\tau^t r(v)dv, \quad \Lambda(t,\tau) \equiv \frac{1}{(t-\tau)} \cdot \int_\tau^t \lambda(v)dv$$

then the optimization condition leads eventually (as ε approaches zero) to

$$\int_\tau^\infty e^{[\frac{(1-\theta)}{\theta} \cdot R(t,\tau)\cdot(t-\tau) - \frac{1}{\theta}\cdot\Lambda(t,\tau)\cdot(t-\tau)]} \left\{\phi(t-\tau)e^{[\Lambda(t,\tau)-\rho]\cdot(t-\tau)} - 1\right\} dt = 0 \tag{37}$$

This condition can be shown to imply that $\Lambda(t,\tau) = \lambda(t) = \rho$ (the standard Ramsey result) if $\phi(t-\tau) = 1$ for all $t \geq \tau$. We can also use equation (37) to get the result for λ that we found in equation (13) under log utility ($\theta = 1$). However, if $\phi(t-\tau)$ varies with $t-\tau$ and $\theta \neq 1$, then time variation in $r(t)$ will require $\lambda(t)$ to vary over time.

If we differentiate the expression on the left-hand side of equation (37) with respect to τ and set the result to zero, then we get, after simplifying,

$$\lambda(\tau) = \frac{\int_\tau^\infty e^{\frac{(1-\theta)}{\theta}\cdot[R(t,\tau)-\Lambda(t,\tau)]\cdot(t-\tau)} \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)} \cdot [\rho - \frac{\phi'}{\phi}(t-\tau)]dt}{\int_\tau^\infty e^{\frac{(1-\theta)}{\theta}\cdot[R(t,\tau)-\Lambda(t,\tau)]\cdot(t-\tau)} \cdot \phi(t-\tau) \cdot e^{-\rho(t-\tau)}dt} \tag{38}$$

Hence, $\lambda(\tau)$ is a weighted average of future rates of time preference (as viewed from the perspective of time τ), $\rho - \frac{\phi'}{\phi}(t-\tau)$. This perspective also works under log utility, $\theta = 1$, but then the interest-rate terms vanish, the numerator of the right side of equation (38) can be shown to equal unity, and, hence, λ equals the constant shown in equation (13).

If $\theta \neq 1$, then equation (38) shows that time variation in $r(t)$ and, hence, in $R(t,\tau)$, will affect the weighted averaging of the future time-preference rates, $\rho - \frac{\phi'}{\phi}(t-\tau)$, and, thereby, affect $\lambda(\tau)$. The weight on the time-preference rate for time t depends on the cumulation of interest between dates τ and t, that is, on $R(t,\tau)$ in equation (38). If $\theta > 1$—signifying that households are not very willing to substitute consumption intertemporally—then the income effect from a higher $R(t,\tau)$ dominates the substitution effect. In this case, a higher $R(t,\tau)$ shifts consumption toward time τ and away from time t. This shift away from consumption at time t means that the weight attached to time t's time-preference rate falls in equation (38).

If the capital stock begins below its steady-state value, then $r(t)$ and $R(t,\tau)$ will be high initially. If $\theta > 1$, then equation (38) implies that these high interest rates cause households to put relatively little weight on time-preference rates far in the future. Since these future time-preference rates are relatively small, the effect is to make $\lambda(\tau)$ relatively high. However, as the economy approaches its steady state and interest rates fall, $\lambda(\tau)$ tends to decline because of the greater weight attached to the low time-preference rates in the distant future. This descending path of $\lambda(\tau)$ tends to generate a rising path of saving rates, a pattern that retards convergence to the steady state. Thus, if $\theta > 1$, then the presence of the myopia term, $\phi(t-\tau)$, tends to slow down convergence in comparison with the standard Ramsey model, in which λ is constant. These effects are, however, reversed if $\theta < 1$, a situation in which households are very willing to substitute intertemporally, and the net effects from the interest-rate terms are opposite to those just examined.

The precise dynamics of the model are difficult to work out because equation (38) expresses $\lambda(\tau)$ as a function of integrals of future values of $\lambda(t)$. It would probably be feasible to use numerical methods to simulate the economy's transitional dynamics.

The form of equation (38) can be simplified if we assume that the term $\phi(\cdot)$ takes the quasi-hyperbolic form given in equation (15):

$$\phi(v) = 1 \text{ for } 0 \leq v \leq V, \quad \phi(v) = \beta \text{ for } v > V$$

If we assume, as before, that V is small, in the sense that $\rho V << 1$, then the result for $\lambda(\tau)$ from equation (38) can be approximated as

$$\lambda(\tau) \approx \rho + \frac{(1-\beta)}{\beta} \cdot \frac{1}{\int_\tau^\infty e^{\frac{(1-\theta)}{\theta}\cdot[R(t,\tau)-\Lambda(t,\tau)]\cdot(t-\tau)} e^{-\rho(t-\tau)} dt} \tag{39}$$

If $\theta = 1$, then this formula reduces to $\lambda(\tau) \approx \rho/\beta$, as in equation (16).

If $\theta \neq 1$, then $\lambda(\tau)$ depends on the path of interest rates, $r(t)$. In particular, as already discussed, if $\theta > 1$, then the decline in $r(t)$ as the economy develops generates a declining path for $\lambda(\tau)$. This effect is again reversed if $\theta < 1$.

9 Concluding Observations

In most respects, the allowance for non-constant rates of time preference leaves intact the qualitative properties of the neoclassical growth model. Consumption depends on an effective rate of time preference, which is a weighted average of future rates of time preference. Under log utility, the weights are constant, and the effective rate of time preference is constant. Therefore, the results under myopia—whereby rates of time preference are high over the near term but low in the distant future—are observationally equivalent to those in the standard model when the rate of time preference is high (but constant). Under more general specifications of utility, there are some new results that involve the interplay between the dynamics of effective rates of time preference and the dynamics of market interest rates.

Despite this correspondence in form, myopic time preference can have quantitatively important implications for saving and growth—these effects are analogous to those generated in the standard model from a higher rate of time preference. There are also potentially important welfare implications, because the outcomes in a non-commitment equilibrium can differ greatly from those that would arise if households were able fully to commit their future choices of consumption and saving. Thus, in a world in which myopic time preference is important, institutional devices that enable households to commit future consumption can have large effects on saving, growth, and welfare.

From a positive standpoint, the most important macroeconomic predictions involve the relation between commitment technologies and saving. Economies that feature a greater capacity to commit future consumption have lower effective rates of time preference and, thereby, tend to exhibit higher rates of saving and growth. This commitment capacity involves partly the state of financial markets and the legal system. However, some developments—such as a greater capacity to write enforeceable contracts—would enhance the ability to commit, whereas others—such as increased liquidity of financial assets—would go the other way.

The quantitative significance of these findings depends on, first, whether household preferences actually exhibit the myopia property in which rates of time preference are high in the short term but sharply lower thereafter and, second, whether households have serious self-control problems that hinder the commitment of future consumption and saving. Introspection and experimental evidence are, at present, inadequate for reaching clear conclusions about the significance of myopic preferences for macroeconomic outcomes. It may be that empirical studies at the macroeconomic level will prove more useful than further microeconomic experiments in reaching definitive answers.

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