NBER WORKING PAPER SERIES

ALTERNATIVE STRATEGIES FOR AGGREGATING PRICES IN THE CPI

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Working Paper 5980

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1997

This paper was prepared for the conference on Measuring Inflation and Real Growth, Federal Reserve Bank of St. Louis, October 16-17, 1996. The views expressed are not necessarily shared by the members of the Board of Governors of the Federal Reserve System, nor by the other members of its staff. We are grateful to Erwin Diewert, John Greenlees, Brent Moulton, and Marshall Reinsdorf for comments and helpful discussions, and to Dwight Bibbs for excellent research assistance. We are also grateful to the Bureau of Labor Statistics for compiling the dataset used in this paper, and to Anna Aizcorbe, Patrick Jackman, and Robert Cage for discussion of the data and related issues. Shapiro acknowledges the financial support of the Alfred P. Sloan Foundation. This paper is part of NBER's research programs in Economic Fluctuations and Growth, Monetary Economics and Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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NBER Working Paper No. 5980
March 1997
JEL Nos. C82, E31
Economic Fluctuations and Growth,
Monetary Economics and Productivity

ABSTRACT

The Consumer Price Index does not take into account the fact that consumers alter the composition of their purchases in response to changes in relative prices. This substitution effect will cause the CPI to grow faster than the cost of living. This paper presents new estimates showing that this bias in the CPI averaged 0.3 percentage points per year between December 1986 and December 1995. This bias could be eliminated by using a superlative index to aggregate prices across the item-area strata of the CPI. The paper discusses the practical difficulties in implementing such a calculation and suggests a method for overcoming them. In particular, it shows how to construct an accurate approximation to a superlative price index that can be published with the same timeliness as the CPI.

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There is a growing body of evidence that the Consumer Price Index overstates the true rate of increase in the cost of living. One of the best understood and best documented sources of bias in the CPI as a measure of the cost of living is its failure to take into account the fact that consumers alter the composition of their purchases in response to changes in relative prices. In our recent examination of the evidence on the biases in the CPI, we labeled the bias arising from this failure the "across-strata" effect.[1]

In contrast to the CPI, the members of the so-called "superlative" class of price indexes do allow for substitution among goods and services in response to changes in relative prices. Given certain assumptions, superlative indexes can be shown to provide second-order approximations to the true cost-of-living index [see Diewert (1976)], whereas fixed-weight indexes including the Laspeyres index provide only first-order approximations. This difference in degree of approximation has led many researchers (including us) to interpret the discrepancy between a Laspeyres-type index and a superlative index as an estimate of the across-strata effect in the CPI.

The BLS has taken many steps over the years to improve the CPI. Of the various remaining sources of bias in the CPI as a measure of the cost of living, the

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[1]See Shapiro and Wilcox (1996a). A "stratum" in the CPI is an item-area pair (e.g., apples in St. Louis or dental services in Denver). We refer to the effect arising from substitution among goods and services as "across" strata to distinguish it from the substitution within strata (e.g., among stores selling apples).

across-strata effect is probably the easiest to address. To borrow a phrase from Vice President Gore's National Performance Review, the across-strata effect is the "low hanging fruit" of the remaining CPI biases, because the economic theory of superlative prices indexes is well understood and relevant data on prices and quantities are available.

This paper has two main purposes. The first is to present a new estimate of the magnitude of the across-strata effect in the CPI. The new estimate differs from its predecessors in two key respects. First, it is derived from a new dataset recently constructed at our request and released by the Bureau of Labor Statistics. As we discuss below, the new dataset builds on the pioneering work of Aizcorbe and Jackman (1993).[2] Second, the new estimate is derived from a direct comparison between the CPI itself and its superlative counterparts; previous researchers used either a 1982-based or a 1986-based Laspeyres-type index in place of the CPI, and assumed that the results were relevant for the CPI, which is calculated using expenditure shares from the three-year period 1982-1984. We find a noticeable difference between the actual CPI and either the 1982-based or the 1986-based Laspeyres index. This difference leads us to estimate a bigger across-strata substitution effect than have other researchers.

The second major purpose of the paper is to propose a method for picking the low-hanging fruit—that is, for publishing a real-time index that is substantially free of across-strata bias. In doing so, we confront—and propose resolutions

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[2]The new dataset was constructed by Robert Cage of the BLS.

of—two practical difficulties. First, the data on expenditures that are required for computation of either Fisher's Ideal index or the Törnqvist index are available only at the quarterly frequency. Moreover, the sample size in the underlying survey is such that considerable averaging across quarters is required before the resulting estimates of expenditure shares at the stratum level are deemed sufficiently reliable to be used in the CPI. Indeed, the BLS currently calculates the aggregation weights in the CPI using three years' worth of expenditures data. The unavailability of monthly data on expenditures implies that neither the Törnqvist index nor Fisher's Ideal index can be implemented exactly, even retrospectively.

The second major difficulty we confront is that the relevant data on expenditures become available only with a lag of about nine or ten months. This implies that the best possible approximation to a superlative index can be produced only after a lag at least that long (and possibly longer, depending on the details of any averaging of expenditure shares). The BLS could deal with this lag in data availability in any of several ways: It could delay publication of a superlative-type index until final expenditures data are available; it could publish a preliminary version of the superlative-type index, based on expenditures data available in real time, and make this preliminary version susceptible to revision; or it could publish two separate indexes, one using the latest available expenditures data and always susceptible to revision, the other using only the expenditures data available in real time and not susceptible to revision. The

Boskin Commission recommended that the BLS pursue the third option (Boskin *et al.*, 1996). This paper can be seen as helping to lay the groundwork for implementing that option.

The rest of this paper is organized as follows: In the first section of the paper, we review the formulas for computing price indexes with special attention to what is feasible to compute given the data currently collected. In the second section, we provide the new estimate of the magnitude of the across-strata effect. In the third section, we propose a method for alleviating the across-strata effect on a real-time basis. The final section discusses the implications of our findings.

1. The CPI and Superlative Price Indexes in Practice

We begin with a short, practical review of some basic price index formulas and their relation to the CPI. The basic ingredients of a price index are the price of an item i in year y and month m, denoted $p_{i,y,m}$, and the quantity of this item purchased q.[3] If price and quantity were both observed at the monthly frequency and on a timely basis, it would be straightforward to calculate a superlative version of the CPI. As we noted in the introduction, however, expenditures (and hence quantities) are observed neither monthly nor on a timely basis. In this section, we concentrate only on the implications of the frequency-re of the

[3]In this section, we suppress the geographical dimension of the CPI structure and refer only to "items" rather than "item-area pairs."

expenditure data, and leave considerations related to timeliness for later in the paper.

The BLS obtains information on nominal expenditures from the Consumer Expenditures Survey (CEX), which provides data at the quarterly frequency. These data are sufficiently noisy as to require averaging over many quarters before they can be used in the CPI.[4] At present, the weights in the CPI are calculated using average expenditures over the three-year period from 1982 through 1984; the BLS plans to introduce new weights at the end of 1997 based on average expenditures during the three-year period from 1993 through 1995.

The mismatch in frequency between the price and expenditure data creates an ambiguity as to how one might best approximate the index formulas prescribed by theory. In constructing the CPI, the BLS deals with this problem by taking the following "modified" Laspeyres index as its statistical target for months after December 1986, when the 1982-84 marketbasket was introduced:

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[4]In part, the need to average the CEX data over time reflects the current practice of producing area-specific price indexes. If, instead, a determination were made that only regional indexes or a single national index was to be produced, the need for time-averaging of expenditures data might be substantially reduced. Even in the unlikely event that such a move alleviated any need for time-averaging, the expenditures data would still be available at only the quarterly frequency, given the current structure of the CEX.

$$\frac{P^{CPI}_{y,m}}{P^{CPI}_{1986,12}} = \frac{\sum_i q_{i,b}\, p_{i,y,m}}{\sum_i q_{i,b}\, p_{i,1986,12}} \tag{1}$$

where b denotes the base period, currently 1982-84. Note that the CPI can be interpreted as the ratio of the cost of purchasing the base-period quantities at current-period prices to the cost of purchasing those same quantities at link-period prices.

The BLS does not observe quantities, so a more accurate representation of the method it actually uses to calculate the CPI for months after December 1986 is given by:

$$\frac{P^{CPI}_{y,m}}{P^{CPI}_{1986,12}} = \frac{\sum_i e_{i,b} \frac{p_{i,y,m}}{p_{i,b}}}{\sum_i e_{i,b} \frac{p_{i,1986,12}}{p_{i,b}}} = \frac{\sum_i e_{i,b} \frac{p_{i,1986,12}}{p_{i,b}} \frac{p_{i,y,m}}{p_{i,1986,12}}}{\sum_i e_{i,b} \frac{p_{i,1986,12}}{p_{i,b}}} \tag{1'}$$

$$= \sum_i RIW_{i,1986,12} \frac{p_{i,y,m}}{p_{i,1986,12}}$$

where $e_{i,b}$ is the nominal expenditure devoted to item i during the base period b, and $RIW_{i,1986,12}$ is known as the "relative importance weight" of item i in December 1986. The relative importance weight of item i in December 1986 can be interpreted as the ratio of outlays for that item to total outlays in that month, assuming the quantities are the same then as they were in the base period.[5] Although the BLS measures base-period expenditures $e_{i,b}$ over a three-year period, it proxies for the base-period price $p_{i,b}$ using the June 1983 reading on the relevant item-area price index—that is, the reading from approximately the middle of the base period (see Bureau of Labor Statistics, 1992, page 180).[6]

Just as it is not possible with existing source data to calculate the textbook version of the Laspeyres index, so it is not possible to calculate the textbook versions of either Fisher's Ideal index, or the Törnqvist index, or a geometric means index. Following Aizcorbe and Jackman (1993), we calculate *chained* versions of the two superlative indexes, with the links in the chain occurring each December. Specifically, we calculate the growth of a Fisher's-Ideal-type index from December in year y-1 to month m in year y as the geometric mean of the

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[5]Each stratum-level price index is calculated based on the arithmetic average of results from a probability sample of establishments. The probability of selection for any given outlet is proportional to that outlet's share in total expenditures. As we discuss in Shapiro and Wilcox (1996a), the BLS's algorithm for aggregation of prices *within* strata leads to another departure from the textbook Laspeyres formulation. Moreover, this algorithm involves chaining, as outlets are rotated into the sample and as items that disappear are replaced with new items. These aspects of the CPI create a number of measurement problems, but our best guess is that these measurement problems are uncorrelated with the across-strata effect that is the focus of this paper. See also Shapiro and Wilcox (1996b).

[6]If stratum-level price indexes were easily available for the entire base period, it would make sense to identify $p_{i,b}$ with the arithmetic average of the 36 monthly readings for stratum i during the base period, rather than with any single monthly reading. Such may not have been the case, however, when the 1982-84 marketbasket was being introduced because the BLS in January 1983 changed its method for measuring homeowners' costs from an asset-value-based approach to the current owners'-equivalent-rent-based approach.

growth of a Laspeyres-type index based in December of year y-1 and a Paasche-type index based in December of year y:

$$\frac{P^F_{y,m}}{P^F_{y-1,12}} = \left(\frac{P^L_{y,m}}{P^L_{y-1,12}} \cdot \frac{P^P_{y,12} / P^P_{y-1,12}}{P^P_{y,12} / P^P_{y,m}} \right)^{1/2} . \qquad \textbf{(2)}$$

The representation of the growth of the Paasche index is necessarily cluttered because neither December of year y-1 nor month m of year y is the base period for the index (unless m happens to be December).

The formula for the Laspeyres-type index is given by:

$$\frac{P^L_{y,m}}{P^L_{y-1,12}} = \sum_i \omega_{i,y-1} \frac{p_{i,y,m}}{p_{i,y-1,12}}, \qquad \textbf{(3)}$$

where $\omega_{i,y-1}$ is the expenditure share of stratum i in year y-1. Similarly, the formula for the Paasche-type index is given by:

$$\frac{P^P_{y,12}}{P^P_{y,m}} = \left(\sum_i \omega_{i,y} \frac{p_{i,y,m}}{p_{i,y,12}} \right)^{1} , \qquad \textbf{(4)}$$

where $\omega_{i,y}$ is the share of expenditures falling on stratum i in year y.[7]

[7]Equations (2)-(4) follow Aizcorbe and Jackman (1993) exactly insofar as calculation of December-to-December changes are concerned; they go beyond Aizcorbe and Jackman in proposing a specific method for calculating a Fisher-type index at the monthly frequency. The method we propose could usefully be further refined by (a) centering the price relatives within the period over which the expenditure shares are calculated (for example, by measuring the price in the denominator on the right-hand-side of equation (3) as the arithmetic average of

We compute a Törnqvist-type index as follows:

$$\frac{P^{T}_{y,m}}{P^{T}_{y-1,12}} = \prod_{i} \left(\frac{p_{i,y,m}}{p_{i,y-1,12}} \right)^{\frac{\omega_{i,y-1}+\omega_{i,y}}{2}} . \qquad (5)$$

This index is calculated as the weighted geometric mean of the price relatives from December in year y-1 to month m in year y, where the weights are the arithmetic averages of the expenditure shares in the years to which t-1 and t belong.[8]

Finally, we calculate a fixed-base geometric-means price index as:

$$\frac{P^{G}_{y,m}}{P^{G}_{y_0,12}} = \prod_{i} \left(\frac{p_{i,y,m}}{p_{i,y_0,12}} \right)^{\omega_{i,y_0}} . \qquad (6)$$

This index is exact for the cost of living if the utility of the representative consumer is given by the Cobb-Douglas function. In that case, expenditure shares are constant across time, so any averaging that might be done is strictly an issue

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the June and July values for stratum i in year y-1), and (b) updating (and "downdating") the expenditure shares to the period over which the denominators of the price relatives are measured. A further possible refinement might use appropriately aligned quarterly expenditures data; we have limited ourselves here to use of expenditures data at the annual frequency.

[8]The Törnqvist index can be written in the same form as equation (2), but because it is multiplicative in structure rather than arithmetic, it can be simplified to the form given in equation (5). The calculation of this index could also be refined by centering the denominators of the price relatives within the period over which expenditure shares are calculated. We doubt that updating of expenditure shares is appropriate in the case of the Törnqvist index, and we have not performed any updating here.

of noise reduction and does not introduce any approximation. Previous investigators' results suggest that using the geometric-means formula to aggregate the stratum-level indexes would induce a downward bias in the overall index.[9]

Following Aizcorbe and Jackman, we present results only for December-over-December changes in price; we reiterate, however, that the formulas presented above could be used to calculate results at the monthly frequency.

2. **Assessing the Across-Strata Effect**

This section updates the work of Aizcorbe and Jackman (1993) in assessing the magnitude of the across-strata effect, using a newly assembled dataset made available to us by the BLS in December 1996. Aizcorbe and Jackman constructed their original dataset relatively soon after the 1987 revision to the CPI. Although the easiest thing by far for them to have done would have been to include only post-December-1986 data in their dataset, doing so would have left them with only a few years' worth of observations. Accordingly, they extended the dataset back to December 1982, which was when the BLS introduced the current rental equivalence approach to measuring homeowners' costs. A consequence of this decision to extend the dataset backward in time was that Aizcorbe and Jackman then had to deal with several discontinuities that were

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[9]For example, unpublished tables provided by the BLS and calculated from the Aizcorbe-Jackman dataset show the geometric means index growing more slowly than either the Tornqvist price index or the Fisher's Ideal price index. Consistent with this finding, Braithwait (1980) shows compensated own-price elasticities by category of item; most of these elasticities are less than 1. We are grateful to Brent Moulton for bringing the evidence in Braithwait to our attention.

introduced with the 1987 revision. For example, the previously unified New York area was split into three areas, and a new item was introduced for information processing equipment. Aizcorbe and Jackman handled these situations by painstakingly constructing three concordances linking the pre- and post-1987 data.

In comparison with the original Aizcorbe and Jackman dataset, the new one is conceptually straightforward because it includes only post-December-1986 data. The new dataset contains 9522 monthly time-series on price relatives—one for each possible combination of 46 areas and 207 items. For reasons explained in the data appendix, we consolidate four of these areas into two. The version of the dataset we use, therefore, has 9108 monthly time-series on prices. This dataset was merged with corresponding annual data from the CEX on expenditures.

2.1. Retrospective Estimates of the Across-Strata Effect

The top panel of Table 1 shows December-over-December rates of price change, calculated according to a variety of different methods. These estimates could not have been calculated in real time. In Section 3, we address the problem of ameliorating the across-strata effect in real time.

The first column in the top panel of Table 1 shows inflation as measured using the CPI aggregation formula and the official CPI weights—that is, according to equation (1′). The second column shows inflation as measured by a different Laspeyres-type aggregator, this one based on expenditure shares for 1986. The third column shows the Fisher index (equation 2), while the fourth shows the Törnqvist index (equation 5), and the fifth shows the index based on geometric means (equation 6). The bottom panel gives the differences of the non-CPI indexes from the CPI. In both the top and bottom panels, the last two rows give the means and standard deviations of the annual figures.

The difference between either the Törnqvist or Fisher measure of price growth and the CPI represents an estimate of the across-strata effect in the CPI. Over the nine years from 1987 through 1995, the across-strata effect by either measure averaged an estimated 0.30 percentage point per year with a standard deviation of 0.12 percentage point per year.

These results cause us to revise up our estimate of the average magnitude of the across-strata effect between December 1986 and December 1995 to 0.3 percentage point per year.[10] Looking *pro*spectively, however, our best guess remains that the across-strata effect will average about 0.2 percentage point per year once the updated marketbasket based on expenditure shares for 1993-95 is introduced. This guess is based on the fact that the 1986-based Laspeyres index shown in column 2 of Table 1 increases about 0.2 percentage point per year faster than the Tornqvist index, and that this bias appears to have no trend.[11] Our best

[10]In Shapiro and Wilcox (1996a), we used an updated version of the original Aizcorbe-Jackman dataset, and estimated the across-strata effect at 0.2 percentage point per year. This result was consistent with the finding of Aizcorbe and Jackman in their 1993 article. These estimates were calculated using a 1982-based Laspeyres index rather than the official CPI.

[11]Laspeyres indexes with base years later than 1986 also exhibit about the same differences from the Tornqvist.

guess is that the difference between the CPI and the Laspeyres indexes reported in Table 1 owes to idiosyncracies in the 1982-84 weights which may not be replicated in the 1993-95 weights.

We see little evidence linking the magnitude of the across-strata effect to the age of the marketbasket. Accordingly, we are not particularly optimistic that updating the marketbasket every five years rather than every ten years would noticeably reduce the average magnitude of the across-strata effect. In any event, the solution for the across-strata effect seems to us clearly to involve moving away from a fixed-marketbasket concept, not a more rapid turnover of the marketbasket.

As we noted above, the geometric means index provides an estimate of the cost of living for a Cobb-Douglas representative consumer. As expected, the geometric price index increases less rapidly than does either the Törnqvist index or Fisher's Ideal index. That is, the unit elasticity of substitution implicit in the geometric formula appears to overstate the extent to which consumers respond to changes in relative prices at the upper level of aggregation.

Before concluding this section, we digress briefly to consider how well we can explain the year-to-year variation in the across-strata effect. In particular, we would like to be able to explain the fact that—contrary to widespread presumption—the across-strata effect does not exhibit any tendency to increase in size as the marketbasket becomes more outdated. One possible explanation for the absence of any trend along these lines is that inflation declined over the period studied here. If the decline in inflation was associated with a reduction in the variability of relative prices, consumers might have had less scope for substitution later in the sample period. We tested this hypothesis by constructing an index of the cumulative change in relative prices, as follows:

$$J_{y,m} = \sum_i RIW_{i,1986,12}\left(\ln(\frac{p_{i,y,m}}{p_{i,1986,12}}) - \ln(\frac{P^{G^*}_{y,m}}{P^{G^*}_{1986,12}}) \right)^2 \qquad (7)$$

Thus, $J_{y,m}$ is an index of cumulative relative price change since December 1986, when the current marketbasket in the CPI was introduced. As in equation (1'), *RIW* denotes the official CPI relative importance weights as of December 1986. The aggregate index $P^{G^*}_{y,m}/P^{G^*}_{1986,12}$ is calculated according to equation (6) except for the fact that here, we use the official relative importance weights rather than the expenditure shares for 1986.

The index $J_{y,m}$ would equal zero if no net change in relative price had taken place since December 1986. A decline in the value of *J* would indicate that relative prices had moved back more into line with their constellation as of December 1986.

The scope for substitution in any given year should be a function of the drift in relative prices during that year. Accordingly, we compare the December-to-December changes in *J* to the estimates of the across-strata effect derived by comparing the CPI to the Törnqvist index. The results are shown in Chart 1. We see these results as mixed: Beginning in 1989, our index of relative price change appears to have some explanatory power for the size of the across-strata effect, but the first two years remain a puzzle: there, the pace of relative price drift was increasing, but the size of the across-strata effect was declining.

2.2. Imposing a Zero Elasticity of Substitution Across Areas

The price index calculations reported in Table 1 assume that consumers may substitute any item-area stratum in the CPI marketbasket for any other. In this section, we contemplate placing restrictions on the allowable range of substitution. In particular, we explore the implications of imposing the restriction that the elasticity of substitution *across areas* is zero. The results just presented strongly suggest that the elasticity of substitution among at least some CPI item-area strata is not zero. But we think it likely that the evident sensitivity to relative price changes arises from substitution across strata *within* areas, not *across* areas. Indeed, the CPI's assumption of zero elasticity of substitution across areas strikes us as more plausible than the geometric means index's assumption of a unit elasticity of substitution across areas as well as across items within areas. Moreover, while the Törnqvist index will handle the case of zero elasticity of substitution, there might be a case for imposing it to reduce the impact of noise in the estimated expenditure shares.

Table 2 shows the effect of imposing a zero elasticity of substitution across areas. The first two columns repeat the results for the Törnqvist and geometric indexes from Table 1 with one extra decimal point of precision. The next two columns show the results that we obtain when we rule out the possibility of substitution across areas. Specifically, we apply the Törnqvist and geometric indexes within areas, and then aggregate the area-level indexes using the Laspeyres formula based on 1986 weights. The last two columns give the differences between the restricted and unrestricted estimates.

There is very little difference between the rates of price growth as measured by the unrestricted and restricted versions of the Törnqvist index. Indeed, at first glance, this result appears to be a powerful victory for superlative index numbers: The standard Törnqvist index appears to be capable of tracking zero-elasticity behavior remarkably well.

But the geometric index also differs little when a zero elasticity of substitution across areas is imposed. Given that the geometric index assumes unit elasticity, we would have expected a more substantial difference if relative price variation across areas were substantial. Hence, it may be that Table 2 is best interpreted as showing that relative price variation across areas is insubstantial, so there is little bite to imposing a zero elasticity in response to such variation.[12,13]

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[12]Two of the differences for the geometric means estimator are negative. If the estimator were chained, this would not be possible. However, because we use a fixed-base estimator, there is no restriction on the sign of the difference.

[13]We also tried inverting the order of operations involved in imposing the zero-elasticity index; specifically, we tried applying the Laspeyres aggregator across areas first, and then applying the Törnqvist aggregator across items. The resulting index can be interpreted as a "national" version of the CPI, in the sense that it treats the entire country as one area. This index increased about 0.2 percentage point per year faster, on average, between December 1986 and December 1995 than did the standard Törnqvist index. That the "national" index grows faster is a consequence of Jensen's inequality. A true national index would

3. Real-time Feasible Price Indexes

Using the Törnqvist or Fisher indexes presented in the previous section, one can estimate an index that is free of the across-strata effect currently afflicting the CPI. But owing to delays in the availability of the required expenditures data, these superlative indexes cannot be computed with the same timeliness that is possible with the current CPI. In this section, we explore the performance of an alternative index formula that is implementable in real time.

Our aim is to produce an approximation to the Törnqvist index that is feasible using the data available to the BLS when it computes the CPI. The BLS has the price relatives with a lag of less than one month, but the expenditure shares for year y are not available until about September or October of year y+1.

The evidence presented in the previous section suggests that the CPI—which is a Laspeyres-type index—grows more quickly than either the Törnqvist index or Fisher's Ideal index, while an index based on geometric means grows more slowly. Accordingly, we look for an intermediate case from within the family of utility functions exhibiting a constant elasticity of substitution (CES) across items.[14]

We construct the real-time index as follows:

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be based on a probability sample drawn for that purpose. Our results suggest that research on appropriate national sampling weights is required before the Boskin Commission's recommendation that some items be priced on a national basis could be implemented.

[14]We are grateful to Brent Moulton for suggesting that we investigate the CES family of utility functions as a basis for a real-time index.

$$\frac{P^{RT}_{y,m}}{P^{RT}_{y-1,12}} = \frac{P^{CES}_{y,m} / P^{CES}_{y-2,12}}{P^{CES}_{y-1,12} / P^{CES}_{y-2,12}}. \tag{8}$$

We calculate the numerator of equation (8) as follows:

$$\frac{P^{CES}_{y,m}}{P^{CES}_{y-2,12}} = \left[\sum_i \omega_{i,y-2} \left(\frac{p_{i,y,m}}{p_{i,y-2,12}} \right)^{1-\sigma} \right]^{1/(1-\sigma)}, \tag{9}$$

where σ is the elasticity of substitution between items (assumed to be identical for all possible pairs of goods). Because we use expenditure shares from year y-2, and those expenditure shares are available by the end of year y-1, equations (8) and (9) are implementable in real time, at the monthly frequency.[15]

Table 3 compares annual rates of change in the cost of living calculated using five different aggregation formulas. Column 1 uses the CPI aggregation formula and weights, and thus repeats the first column from Table 1. (We drop the observation for 1987 from Table 3 because we are using lagged expenditure shares in the computation of the CES indexes.) Columns 2 through 4 present the results of implementing equations (8) and (9), for three different assumptions

[15]Moulton (1996) shows that equation (9) would be exact for the cost of living if the utility of the representative agent came from the CES family and if the expenditure shares were measured over the same period as the denominator of the price relative. Just as we suggested might be done with the other formulas we presented earlier, equation (9) might be further refined by centering the price relative within the period over which expenditure shares are measured. In addition, the expenditure shares in year y-2 could be updated to the period over which the denominator of the price relative is measured. Finally, the underlying quarterly data on expenditures could be employed.

about the elasticity of substitution: σ=0.6, σ=0.7, and σ=0.8. Finally, column 5 uses the Törnqvist aggregation formula (repeating the fourth column from Table 1). The top panel of the table shows the rates of change in the various indexes, while the bottom panel shows the differences between the non-Törnqvist and Törnqvist indexes.

As we noted in the previous section, the CPI grows more rapidly than the Törnqvist, and the same turns out to be true for the CES index with σ set equal to 0.6. With σ set equal to 0.7, however, the average growth in the CES index is almost exactly the same as the average growth in the Törnqvist index of this period. With σ set equal to 0.8, the CES grows a bit too slowly. The standard deviation from zero of the discrepancy between the CES with σ equal to 0.7 and the Törnqvist rounds down to 0.04 percentage point per year. Under the assumption that these discrepancies are distributed normally, this implies that 90 percent of them should lie between -0.07 percentage point per year and 0.07 percentage point per year. In fact, with only one exception, all discrepancies are 0.05 percentage point per year or smaller in absolute value. By way of comparison, we note that the *smallest* discrepancy for the CPI aggregator is 0.12 percentage point per year, and the standard deviation of the discrepancies for the CPI formula is also 0.12 percentage point per year.

These results suggest that the CES utility function can be used to substantially eliminate the across-strata effect in the CPI; that is, it can deliver estimates with the same timeliness of the current CPI without adding substantial noise. In common with the conventional Laspeyres and geometric means formulas, the CES functional form we have implemented makes the strong assumption that the elasticity of substitution is the same across all pairs of items. This restriction is unlikely to hold: The elasticity of substitution between chicken and beef is surely greater than that between beef and, for example, children's clothing. The approximation of equal elasticities of substitution will cause changes in the relative price of close substitutes to contribute too much to the change in the cost of living, and changes in the relative price of poor substitutes to contribute too little. The elasticity of 0.7 makes such errors cancel *in the sample.* If the BLS were to implement a procedure like ours, it would be well-advised to monitor that such cancellation continues to occur in the future. In particular, the BLS should make adjustments to σ so that the average difference between the index using the projected expenditure shares and the Törnqvist index once the actual shares become available remains close to zero.

Our calibration exercise can be viewed as involving a crude form of estimation. One way to refine the procedure would be to make the estimation explicit rather than implicit. This could be done by estimating a demand system for the items in the CPI marketbasket, and then calculating the exact cost-of-living index for the estimated system; indeed, we intend to pursue this approach. In effect, this effort will represent a throwback to some of the earlier literature on the substitution bias. Of course, like the earlier authors in this genre, we will have to take a stand on various issues including the choice of functional form.

Nonetheless, this seems to us to represent a promising avenue if the goal is to create a timely price index free of the across strata bias.

4. **Discussion**

One of the aims of this paper is to explore the practicalities of computing a timely price index that does not suffer from systematic across-strata bias. Lags in availability of expenditure data make superlative price indexes feasible only after a certain passage of time. The results of the previous section show that it is possible to produce an approximation to the Törnqvist index that is both feasible in real time and quite accurate.

There are a number of issues that deserve further exploration. First, we have only looked at year-to-year inflation rates, but month-to-month rates are of great interest as well, and we have not examined monthly data. We do, however, give a formula for a feasible approximation to the cost of living index that can be implemented in real time—that is, with the same timeliness as the CPI is now published. Second, in our consideration of annual inflation rates, it made sense to use calendar-year expenditure shares. But the underlying expenditure data are available on a quarterly basis. A more refined procedure than the one we have proposed here might be based on a moving average of four quarters' worth of data. The empirical properties of an index constructed along these lines should be investigated. Moreover, it is not obvious that four quarters is the optimal span of time over which to average the expenditure data. The tradeoffs involved with shorter and longer periods of averaging should be explored. Third, it would be useful to develop an algorithm for updating the elasticity of substitution in the context of the formula we propose for a real-time index, and useful as well to test that algorithm in Monte Carlo simulation to see, for example, how quickly the statistical agency is likely to be able to identify a change in the elasticity of substitution, and change the assumed parameter value accordingly.

We have intentionally hamstrung our analysis in this paper by accepting the current item-area structure of the CPI. In particular, the area structure of the CPI requires a substantially richer dataset than is needed to produce a national price index. While area-level indexes might be useful for some purposes, they need not be the building block of a national index. The results in Section 2.2 suggest that it is worth investigating the building of a consumer price index from national indexes of the component goods and services. Indeed, with a national index, it might be possible to use other sources (e.g., the Retail Trade Survey) to develop more timely estimates of expenditure shares than are provided by the CEX.

Finally, there remain some elusive empirical puzzles. Why is there not a more pronounced tendency for the estimates of the across-strata effect to drift up as the base period grows more remote? The failure of this to occur is all the more puzzling in light of the fact that it does seem to have been a prominent feature of various price indexes in the national income and product accounts before those indexes were shifted to a Fisher's Ideal basis. Also, why does imposing zero elasticity of substitution across areas make so little difference in the estimated rate of growth of the cost of living? These remaining puzzles demonstrate that there is more fruitful research to be done even in this most-thoroughly examined area of CPI bias.

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Data Appendix

The dataset that forms the basis for this paper was prepared by Robert Cage of the Bureau of Labor Statistics and made available to us in December 1996. The dataset includes monthly price indexes for 9522 item-area strata (46 areas and 207 items), annual expenditures by stratum, and official CPI relative importance weights as of December 1986 by stratum (see text for the definition of a relative importance weight).

We modified the dataset in the following manner. First, we identified and adjusted four outliers in the price dataset, all having to do with rebates of utility charges. Three of these pertained to natural gas charges (area 17, April 1990; area 45, July 1990; and area 45, March 1992). In these cases, a rebate from the natural gas utility to its customers was sufficiently large as to nearly offset the typical customer's normal monthly bill. The BLS interpreted this as implying that the "price" for the month had been nearly zero. We interpret the "price" for the month to be the marginal cost of a unit of natural gas, which remained positive. (If anything, the rebates reflected news about the marginal cost in some previous month or months.) Lacking any measure of this marginal cost, we impute in all three cases, using the reading from the preceding month. The same situation arises once with respect to local phone services (area 16, December 1989). Only the phone observation is relevant for our results in this paper, because we use only December price readings.

The other modification we made to the dataset was to consolidate the data for Columbus and Milwaukee, and likewise for Phoenix and Portland. When the 1982-84 weights were originally introduced in 1987, the BLS was collecting prices in 46 areas, including Columbus and Phoenix. Budgetary pressures forced them, however, to reduce their coverage from 46 areas to 44, so they ceased collecting price and expenditures data in Columbus and Phoenix. In order to maintain representation of those cities in the official index, the BLS increased the weight given to price changes observed in Milwaukee and Portland. So that we can work with a uniform panel, we effectively extended this imputation procedure backward from the spring of 1988 to the beginning of the dataset. We implemented this modification by attributing the sum of the expenditures shown in the dataset for Columbus and Milwaukee to Milwaukee, and dropping Columbus from the dataset. Similarly, we attributed the sum of the expenditures shown for Phoenix and Portland to Portland, and dropped Phoenix.

Table 1

Alternative Measures of Annual Inflation,
and Alternative Estimates of the Across-Strata Effect
(Percentage Points Per Year)

	CPI*	Laspeyres**	Fisher	Törnqvist	Geometric**
1987	4.44	4.54	4.27	4.27	4.22
1988	4.41	4.25	4.05	4.07	4.04
1989	4.65	4.43	4.21	4.21	4.20
1990	6.16	6.05	5.64	5.63	5.49
1991	3.00	2.61	2.77	2.77	2.58
1992	2.96	2.78	2.61	2.61	2.46
1993	2.74	2.57	2.45	2.45	2.27
1994	2.65	2.72	2.54	2.54	2.52
1995	2.57	2.39	2.32	2.32	2.13
mean	3.73	3.59	3.43	3.43	3.32
std. dev.	1.17	1.20	1.09	1.09	1.12

Differences from CPI

	Laspeyres	Fisher	Törnqvist	Geometric
1987	-.10	.17	.17	.23
1988	.16	.36	.34	.37
1989	.21	.43	.43	.45
1990	.11	.51	.53	.67
1991	.39	.24	.24	.42
1992	.18	.35	.35	.50
1993	.17	.29	.29	.47
1994	-.07	.12	.12	.13
1995	.18	.25	.25	.44
mean	.14	.30	.30	.41
std. dev.	.14	.12	.12	.15

* Expenditure shares measured over 1982-84 and updated for relative price change to the end of 1986.
** Expenditure shares measured over 1986.

Table 2

Alternative Measures of Inflation,
Calculated under the Assumption of Zero Substitution Across Areas
(Percentage Points Per Year)

	Substitutability Across Areas Not Restricted to Zero		Zero Substitutability Across Areas		Difference	
	Törnqvist	Geometric	Törnqvist	Geometric	Törnqvist	Geometric
1987	4.268	4.215	4.272	4.220	.004	.005
1988	4.068	4.040	4.066	4.046	-.002	.007
1989	4.213	4.198	4.211	4.208	-.002	.010
1990	5.629	5.488	5.636	5.494	.007	.006
1991	2.766	2.580	2.764	2.588	-.002	.008
1992	2.605	2.456	2.615	2.462	.010	.007
1993	2.445	2.270	2.446	2.267	.001	-.003
1994	2.535	2.521	2.542	2.514	.007	-.006
1995	2.324	2.132	2.320	2.134	-.004	.002
mean	3.428	3.322	3.430	3.326	.002	.004
std. dev.	1.089	1.117	1.090	1.120	.005	.005

Table 3

Alternative Estimates of the Rate of Change in the Cost of Living
(Percentage Points Per Year)

		CES			
	CPI	σ=.6	σ=.7	σ=.8	Törnqvist
1988	4.41	4.13	4.11	4.09	4.07
1989	4.65	4.23	4.21	4.19	4.21
1990	6.16	5.63	5.58	5.54	5.63
1991	3.00	2.68	2.68	2.67	2.77
1992	2.96	2.64	2.61	2.58	2.61
1993	2.74	2.51	2.49	2.46	2.45
1994	2.65	2.59	2.58	2.56	2.54
1995	2.57	2.36	2.35	2.33	2.32
mean	3.64	3.35	3.32	3.30	3.32
std. dev.	1.21	1.11	1.10	1.09	1.11
		Differences from Törnqvist			
1988	.34	.06	.04	.02	
1989	.43	.02	-.00	-.02	
1990	.53	-.00	-.05	-.09	
1991	.24	-.08	-.09	-.09	
1992	.35	.03	.00	-.02	
1993	.29	.07	.04	.02	
1994	.12	.06	.04	.02	
1995	.25	.04	.02	.01	
mean	.32	.02	.00	-.02	
std. dev.	.12	.05	.04	.05	

Chart 1

The Across-Strata Effect and the Pace of Relative Price Drift