NBER WORKING PAPER SERIES

SOLOW VS. SOLOW: MACHINE PRICES AND DEVELOPMENT

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Working Paper 5871

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 1997

We thank A. Atkeson, W. Baumol, T. Cooley, D. Dwyer, Z. Griliches, C. Jones, P. Klenow, R. Manueli, S. Parente, A. Rodriguez, and C. Tse for comments, and the NSF and the C.V. Starr Center at NYU for financial help. This paper is part of NBER's research program in Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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NBER Working Paper No. 5871
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JEL No. O31
Productivity

ABSTRACT

Machines are more expensive in poor countries, and the relation is pronounced. It is hard for a Solow (1956) type of model to explain the relation between machine prices and GDP given that in most countries equipment investment is under 10% of GDP. A stronger relation emerges in a Solow (1959) type of vintage model in which technology is embodied in machines.

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1. Introduction

Are some countries poor because their governments' policies are bad? One set of policy instruments are tariffs and other non-price distortions that affect the price that a country's producers pay for equipment. We ask how strongly output responds to permanent variation in equipment prices in a Solow (1956) type of model in which technical change is disembodied, and in a Solow (1959) type of model in which technical change is embodied in equipment. The incentive effects are bigger in the 1959 model. The reason seems to be that in the 1956 model, technological progress translates into higher output even without investment; in the 1959 model, technological progress is of no value *unless* there is investment which therefore plays a dual role -- it raises capacity *and* implements new technology. So the 1956 model includes one effect, the 1959 model includes two.

Of course, a government may be forced into bad policies because its country is poor; when machine prices are high, it is mainly because of tariffs and other trade barriers -- a thin tax base can force a government to use tariffs just to get revenue. Jones (1994) finds that in common units, prices vary among countries by a factor of about 4 to 1. Restuccia and Urrutia (1996) instead deflate the price of equipment by the local consumption goods prices, and find that the factor sometimes exceeds 10 to 1! Therefore the level of development may well explain equipment prices. However, this paper concentrates on the reverse causality: How equipment prices affect the level of development.

In a Solow (1956) type of model, a producer can not change the quality of his machines, only their quantity. In our version of Solow (1959), a producer can change the quality of his machines but not their quantity, so that we isolate the quality-enhancing role of investment (a role absent in the 1956 model). When he faces a limit on how many machines he can use, and when he can not augment their quality incrementally, a producer will periodically replace old machines with new higher quality machines. In Johansen (1959), Salter (1960) and Solow (1959), it was not optimal to ever replace equipment. Replacement is analyzed by Zeckhauser (1968), Chari and Hopenhayn (1991), Parente (1994) and Cooley, Greenwood and Yorukoglou (1995), but they do not focus on the price of equipment. Easterly, King, Levine, and Rebelo (1994) get large *growth* effects, and Romer (1994) gets large *level* effects of machine prices in models in which technologies are differentiated horizontally. Closest to what we do is Rodriguez-Clare (1996); he focuses on equipment prices in a vintage model of investment, and as we shall later explain, his results differ from ours, especially on point (2) below. Our results are:

(1) The effect of machine prices on the *level* of income is stronger if frontier machine-quality improves rapidly, and if machines do not physically depreciate too fast -- e.g. computers. There are no growth effects, in accord with evidence in Easterly, Kremer, Pritchett, and Summers (1993) who find little persistence in growth-rate differentials.

(2) A negative cross-country relation between mean efficiency and variance of efficiency -- reminiscent of the right-hand portion of the Kuznets curve. This is because an economy in which machines are taxed more heavily will have a wider (s, S) band of inaction in the adoption decision. In steady state producers will use a wider range of technologies and have a bigger dispersion of efficiencies. This may explain why in poor countries ancient methods of production coexist with modern ones Pack (1988, pp 360 - 365).

The paper proceeds as follows: We first analyze a small open economy that imports all the machines it needs to produce final goods. It takes as given the growth-rate in the quality of machines available. We contrast the 1956 model (Sec. 2) to the 1959 model (Sec. 3). We then show in Section 4 that the kind of exogenous quality improvement model that we assume at the small country level can in fact be generated in a world equilibrium with a capital goods sector that generates growth endogenously.

2. Disembodied Technology: Solow (1956)

In a 1956 type of model, the effect on output of a change in the price of machines will be biggest if the economy is open. We follow Barro, Mankiw, and Sala-I-Martin (1995) and allow borrowing at the world interest rate to finance investment in physical capital. Assume that the representative producer uses a unit of his labor (the quality of which is measured by "human capital", h) and k_t machines (of equal quality) to produce output. Technological growth is disembodied, and takes place at the rate g. Then output is

$$Y = e^{gt} k_t^{\beta} h^{\eta} .$$

The price of a machine, P, is assumed to be constant. Machines depreciate at the rate δ, and there are no costs of adjustment. The equilibrium rental for machines is then $(r + \delta)P$. The rental market need not exist, but the outcome will be the same as if it did. A producer then faces a static decision problem at date t: Choose k_t to maximize profits (we ignore payments to the nontraded factor h):

$$Y - (r + \delta)Pk_t \; .$$

The optimal number of machines is

$$k_t = \left(\frac{\beta e^{gt} h^{\eta}}{(r + \delta)P}\right)^{1/(1 - \beta)} .$$

Substituting for the optimal number of machines, output in country j is

$$(2.1) \qquad Y_{j,t} = e^{g^*t} \left(\frac{\beta h_j^{\eta}}{(r + \delta)P_j}\right)^{\beta/(1 - \beta)} ,$$

where $g^* = g/(1 - \beta)$. The elasticity of output with respect to P is $\beta/(1 - \beta)$ -- a level effect. If we introduce a second variable factor (e.g., structures) that can also be financed at world rates, with an exponent γ, then the elasticity rises to $\beta/(1 - \beta - \gamma)$. This P - elasticity of Y is small: In units of the consumption good, equipment investment is $(\delta + g^*)Pk$, which together with the first-order condition for optimality of k implies an investment rate of $\beta(\delta + g^*)/(\delta + r)$ Equipment investment tends to be under 10% of GDP in most countries (see e.g., DeLong and Summers 1991, figure 4) and the share of structures is under 5% in most places. This means that at the commonly assumed values for δ, g^*, and r, (i.e. 0.05, 0.02, and 0.04, respectively) the elasticity $\beta/(1 - \beta - \gamma)$ will be about 0.10. As δ tends to zero, the elasticity gets a bit larger, to about 0.20. If something *else* varies with P, say an unobserved human capital component, the elasticity can be raised farther still, but as we shall see, this unmeasured factor is probably not related to the level of schooling, because schooling is controlled for in the regressions below.

In a cross section of countries, equation (2.1) implies the following relation:

$$(2.2) \qquad ln(Y_{j,t}) = C_0 - \frac{\beta}{(1 - \beta)} \, ln(P_j) + \frac{\eta\beta}{(1 - \beta)} \, ln(h_j) ,$$

where C_0 is a constant. We measured the variable P as the price of machinery in 1980 also used by Jones. The variable h is average years of total schooling for people over 15 years of age, and is taken from Barro and Lee (1993) for years as close to 1980 as possible. The variable Y is real GDP per capita in 1980 from Heston and Summers (1991). All three variables are available for only 53 countries. The regression results are:

$$ln(Y_{j,t}) = 8.3 - \underset{(4.66)}{1.17}\ ln(P_j) + \underset{(2.55)}{0.34}\ ln(h_j)\,, \tag{2.3}$$

$R^2 = 0.39$, t-ratios in parentheses.

Evidently, the P-elasticity is 10 times or bigger than what the 1956 model can produce. The 1956 model fails in this "conditional" sense, when h_j is held constant.[2] Part of the discrepancy may be accountable by simultaneity bias, but since we do not know just how much, we will look to a different mechanism -- almost as simple as the one outlined above.

The difference between the two mechanisms is this: In a 1956 type of model, a producer can not change the quality of his machines, only their quantity. Our version of the 1959 model assumes the opposite: A producer can change the quality of his machines but not their quantity. It isolates the quality-enhancing role of investment (a role absent in the 1956 model). These effects can both be present of course.

3. Technology Embodied in Machines: Solow (1959)

In a 1959 type of model we can get a bigger elasticity. Before embarking on its details, let us outline the argument. Equipment investment now does two things: it increases capacity, *and* it improves the technology. Therefore the effect of a change in P should be more pronounced.

Assume that a producer uses his own labor and one machine to produce output. The price of machines affects the bands of the optimal (s, S) policies. Section 3.A describes the distribution of capital quality that results from a *given* (s, S) policy. Section 3.B. derives the

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[2] The unconditional correlation coefficient between $ln(Y_{j,t})$ and $ln(P_j)$ is -1.35 with a standard error of 0.25.

optimal (s, S) policy, and Section 3.C. connects the 1959 model with the regression in (2.2). The main implications are:

(a) A poor country's age distribution of capital first-order (stochastically) dominates that of a rich one (result is in Section 3.A., especially Figure 2).

(b) The average age of capital and the variance of the age of capital are negatively related in the cross-section of countries (Sec. 3.A., esp. Fig. 3).

(c) The oldest capital is replaced with the latest vintage capital (Sec. 3.B, esp. Fig. 6),

(d) A reinterpretation of the of the regression in (2.3) (Sec. 3.C).

3.A. <u>(s, S) Policies and the Stationary Distribution of Capital Quality</u>: Let k be the *quality* of the machine used in production. Let K be the quality of a machine embodying the frontier technology, and assume that frontier quality grows exogenously:

$$K_t = e^{gt}$$

The producer can use only one machine at a time, so a machine has a positive opportunity cost -- the profits that he can get using a frontier technology. This opportunity cost keeps growing as frontier equipment gets better and better. Machines depreciate like light bulbs -- randomly, one-hoss-shay. Machine lifetime is distributed exponentially, with a failure hazard rate of δ. Until it fails a machine is as good as new . Let T be the age at which the machine will be replaced if it has not broken down by then, and suppose that the producer then buys a frontier-quality machine. That is, he will use the (s, S) policy:

> "hold on to the machine while $k \in (uK, K]$, and replace the machine when it breaks down, or when K reaches the level at which $uK = k$, whichever comes first. Then, buy a new machine of frontier quality,"

where $u \equiv e^{-gT}$ is the relative quality of the oldest machine when it is about to be replaced. It is easier to discuss the (s, S) policy in terms of the variable $z \equiv k/K$ -- the quality of the machine relative to the frontier quality. Let z_τ denote the value of z when the machine is τ periods old so that $z_\tau = e^{-g\tau}$, and so that the policy is:

"hold on to the machine while $z_\tau \in (u, 1]$, and replace the machine when it breaks down, or when z_τ reaches u."

The distribution of machine ages: Assume that T is constant, and that as a group producers upgrade in sequence at a constant rate.[3] Let $H(\tau)$ be the fraction of machines of age τ or less in the population. This distribution is truncated exponential:

$$H(\tau) = \frac{1 - \exp\{-\delta\tau\}}{1 - \exp\{-\delta T\}}, \qquad \text{for } \tau \in [0, T].$$

When $\delta = 0$, $H(\tau)$ is the uniform distribution. Figure 1 plots two versions of $H(\tau)$:

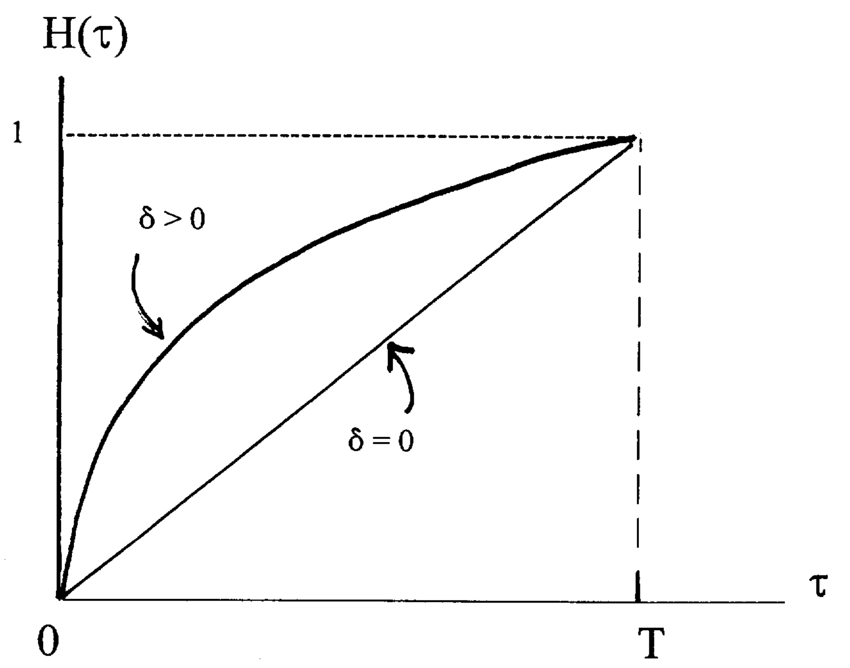

FIGURE 1: The age distribution of machines

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[3] Actually, world equilibrium requires only that the *world* age distribution of equipment be uniform. If a small country's demand for machines is too small to affect their price, its distribution need not be uniform unless there is a scarce local factor needed to help install a new machine. A non-uniform age distribution of equipment is sustainable only by a non-uniform rate of replacement, which requires that the price of the local factor respond to changes in the replacement rate. The cost of installing machines would then depend on the installation rate, producers would want to install machines when the price of the local factor was low, and this would induce a uniform rate of replacement and a uniform age distribution.

A real world example of $H(\tau)$ is for the jet plane. The data, also used by Goolsbee (1996), are from the Boeing Commercial Airplane Group (1988, 1993). They record the jet inventory by country, model, and vintage. The top panel of Figure 2 shows the cumulative age distribution ("age" ≡ current date minus the plane's vintage) predicted by the model if $\delta = 0$, and if $T_{US} = 28$, and $T_{AFRICA} = 35$. The next two panels show the actual age distributions for the

U.S. and Africa in 1988, and 1993. First-order dominance holds up in both years, mean age was less in the U.S. in both years, and it stayed roughly the same over the 5-year period: 18.3 and 19.2 years in the U.S., and 21.9 and 23 years in Africa.[4] Moreover, jets are clearly phased out earlier in the U.S. Contrary to the prediction of the model, however, age was not more dispersed in Africa: The standard deviation of jets' ages went up from 7.3 to 8.3 years in the U.S., and (remarkably), from 5.9 to 8.7 in Africa.

To see if the standard deviation of age and average age are positively related over all the world's regions, Figure 3 presents a plot of these two quantities for six major regions for 1993. The relationship is indeed positive.

The distribution of z: Let $\theta \equiv \delta/g$, and let $F(z \mid u, \theta)$ be the CDF of z. Then

$$F(z \mid u, \theta) = 1 - H[\tau(z)] = \frac{(z^{\theta} - u^{\theta})}{1 - u^{\theta}},$$

for $z \in [u, 1]$. The density of this distribution is

$$f(z \mid u, \theta) = \frac{\theta z^{\theta - 1}}{(1-u^{\theta})},$$

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[4] For machines whose frontier quality grows fast (so that g is high) mean age will be less and because u will be higher. For mainframe computers, Greenstein (1994) finds mean age to be about 7 years, and for PCS it is even less than that (Berndt *et al*, 1995).

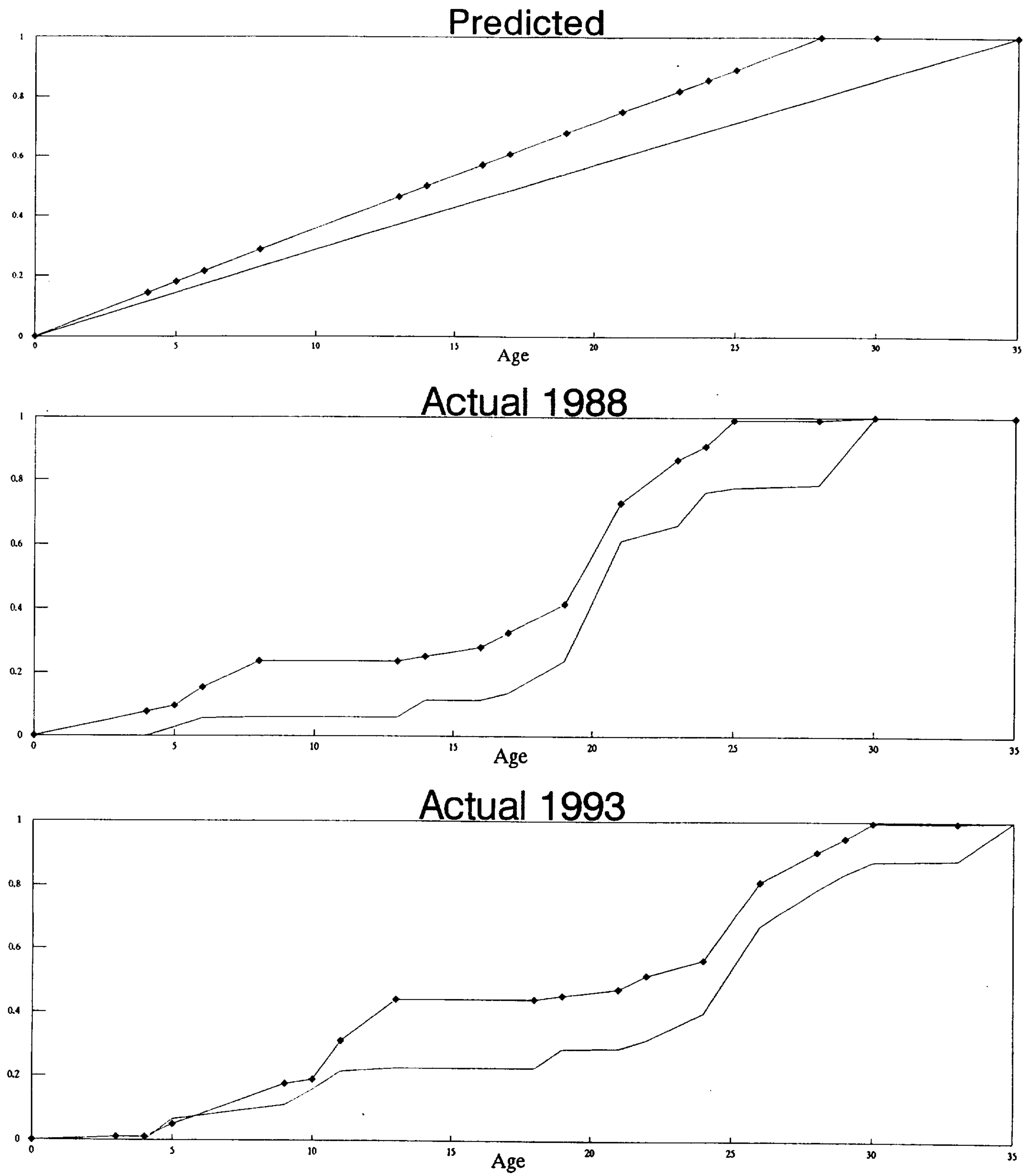


Figure 2 Cumulative Age Distribution

United States: –◆–

Africa: – – –

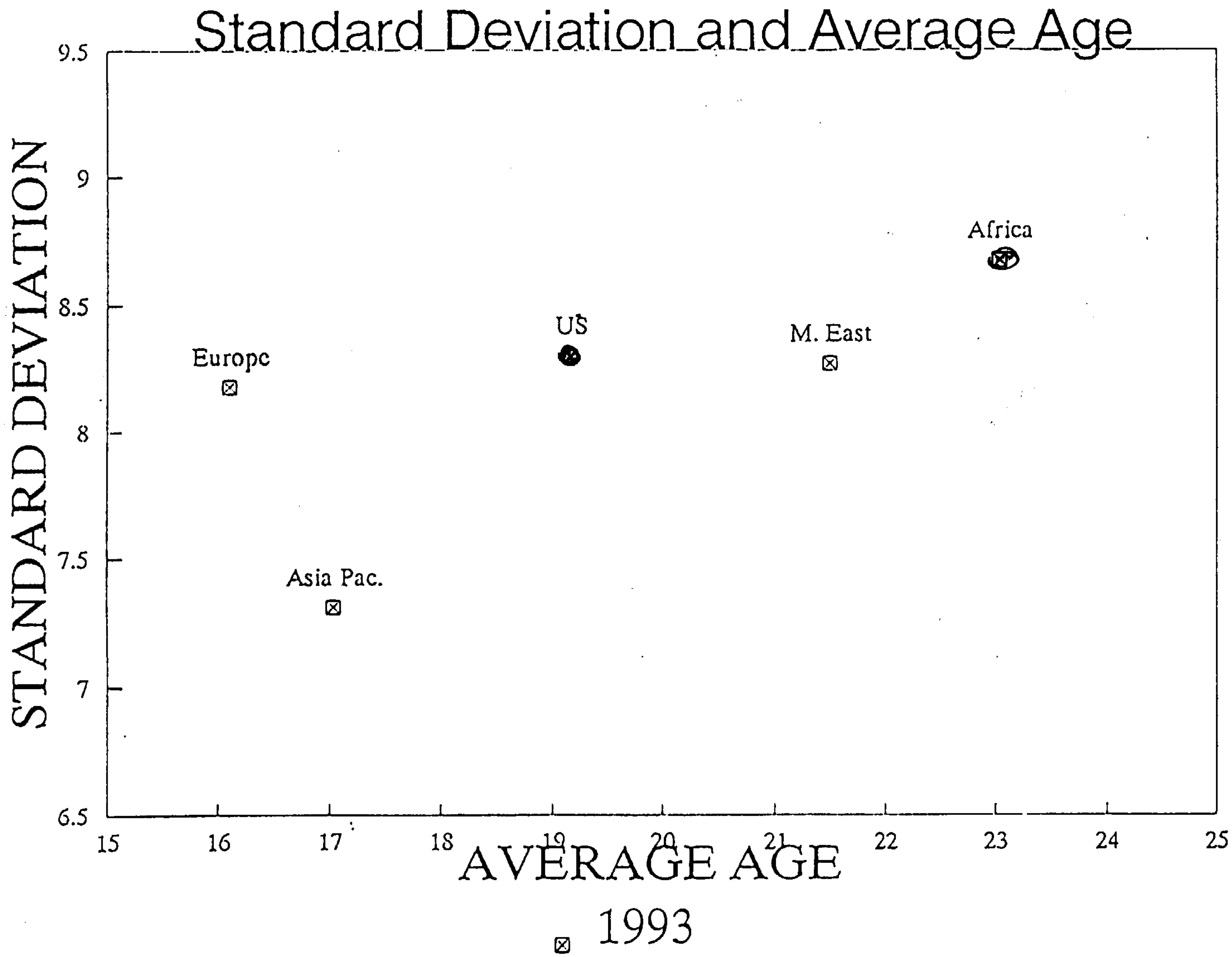

Standard Deviation and Average Age
STANDARD DEVIATION
9.5
9
8.5
8
7.5
7
6.5
15
16
17
18
19
20
21
22
23
24
25
AVERAGE AGE
Europe
Asia Pac.
US
M. East
Africa
1993

Figure 3

which we plot below:

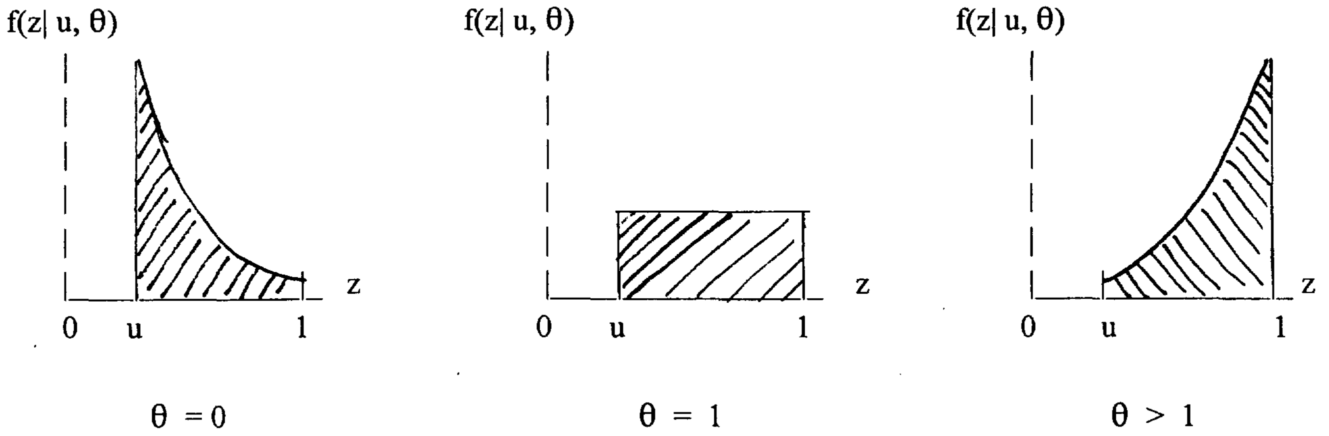


FIGURE 4: Plot of $f(z|\ u, \theta)$ for 3 different values of θ

Figure 4 shows that the effect of a change in u on the mean of z will be bigger when $\theta < 1$. When $\theta > 1$, many machines "die" before age T, and so changes in T have little effect. We have two interesting special cases plotted in the first two panels of figure 4; $f(z|u, 0) = -1/[z\,ln(u)]$, and $f(z|u, 1) = 1/(1-u)$.

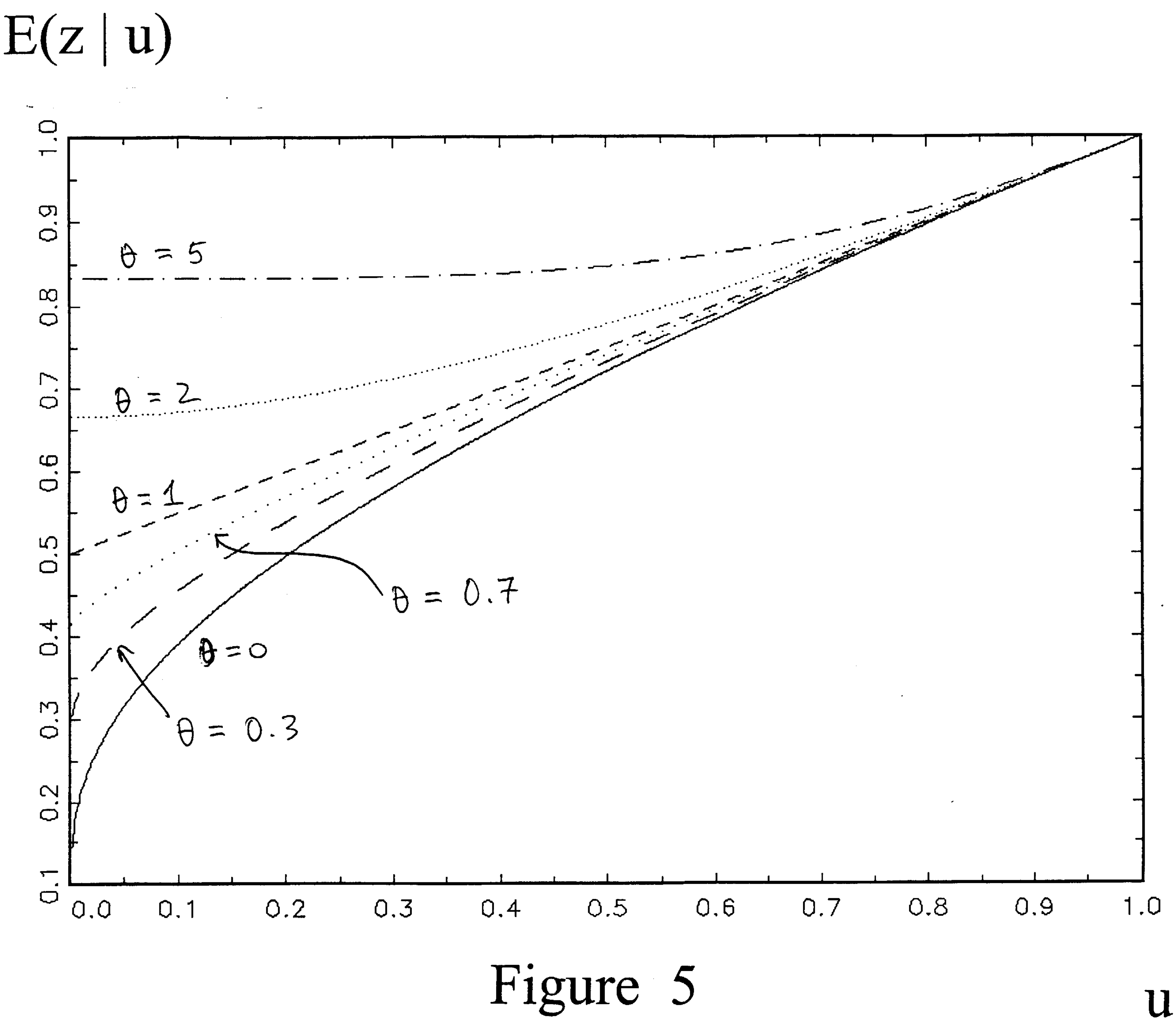
The average quality of capital is

$$e^{gt} E\{z \mid u\}, \tag{3.1}$$

where

$$E\{z \mid u\} = \frac{\theta(1 - u^{1+\theta})}{(1+\theta)(1 - u^{\theta})}.$$

$E\{z \mid u\}$ increases from $\theta/(1+\theta)$ when $u = 0$, to 1 when $u = 1$. It is plotted in figure 5 for various θ.



E(z | u)
θ = 5
θ = 2
θ = 1
θ = 0.7
θ = 0
θ = 0.3
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
u

Figure 5

3.B. Optimal Replacement: A producer uses his labor and one machine to make final goods. He can, at any time, replace his machine. Frontier quality, K, grows at the rate g. The cost of buying a machine of quality $k' \leq K$ is Pk', where P (a constant) is the price per machine-efficiency unit. A producer is a price taker. The production function for output, Y, depends on the quality of capital, k, and the quality of labor h, as follows:

$$Y = k h^{\eta} \tag{3.2}$$

Optimal behavior is unchanged if we divide revenues and costs by the constant h^{η}. Let

$$y \equiv Y/h^{\eta}, \qquad \text{and} \qquad p \equiv P/h^{\eta}.$$

Let v(k, K) be the present value of net revenues of a final goods producer when the quality of his machine is k, and the "frontier" is at K.

PROPOSITION 1: When buying a machine, it always pays to upgrade to the frontier.

PROOF: (a) Investment is the only cost, and the producer has the option of never buying another machine, and so $v(uK, K) > 0$. (b) v(k, K) is linear in k For $k \in [0, K]$. To see why, suppose that at some date, say $t = 0$, the states are (k, K). If machines of quality k_j ($j = 1, 2, \ldots$) are feasible to purchase at dates $t_1, t_2, \ldots$, then those purchases remain feasible if k were changed to some other value. From t_1 onwards, neither costs nor benefits depend on k, and for $t \in [0, t_1]$ the criterion is linear in k. (c) At the point of upgrading,

$$v(uK, K) = \max_{0 \leq k' \leq K} \{ v(k', K) - pk' \} > 0,$$

where the strict inequality follows from (a). By (b), $v(k', K) = [w(K) - p]k'$ for some constant w(K). Hence since K is fixed, the expression is positive at the maximum, and so it will be at its largest at the point $k' = K$. ■

Proposition 1 asserts that *all* equipment purchases in backward countries should involve frontier machines.[5] We pause with the analysis in order to check how this prediction fares in practice. Figure 6 shows the predicted and the actual net purchases and retirements of jets between 1988 and 1993. In both figures, the sum of the changes is normalized to one. Contrary to assumption, new vintage planes do not appear every year: the latest vintage available for purchase in 1993 was vintage 1990 planes, and the earliest vintage in inventory is 1958 in both years for which we have data. In agreement with the Proposition, the 1958 vintages were retired (although some remained in use in Africa in 1993), and frontier or close to frontier equipment was acquired in both the U.S. and Africa. But contrary to Proposition 1, earlier vintages were acquired (especially in Africa), and some younger vintages were retired (especially in the U.S.)

We now resume the analysis. By Proposition 1, when upgrading, the producer is indifferent between holding the old machine for another instant and getting $v(uK, K)$, and upgrading to a frontier machine of quality K and getting $v(K, K) - pK$. There are no other costs, and no salvage value for the old equipment, and so

$$v(uK, K) = v(K, K) - pK. \tag{3.3}$$

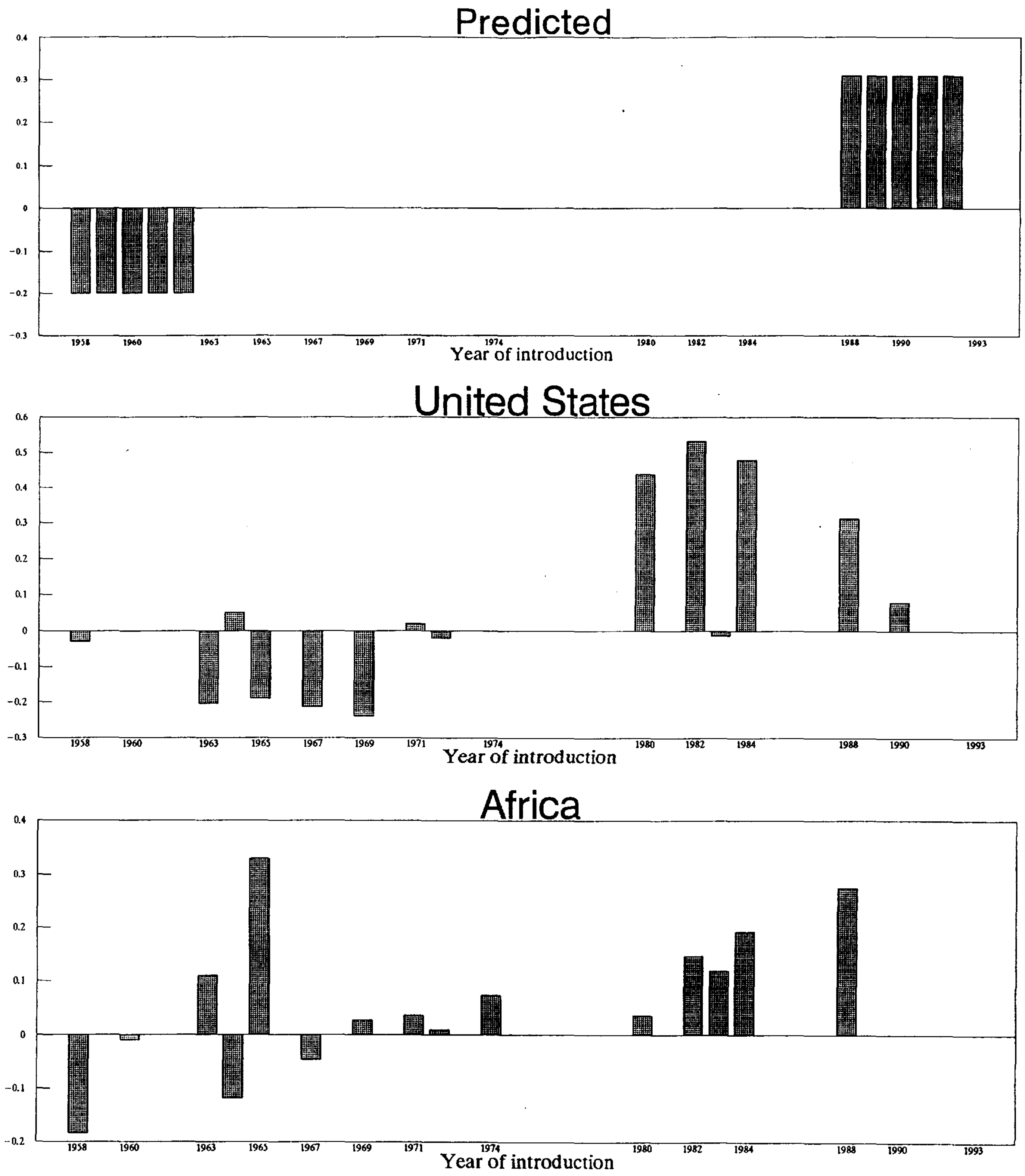
A consequence of Proposition 1 is that $v(K, K)$ is linear in K:

$$v(K, K) = \lambda K, \tag{3.4}$$

where $\lambda > 0$ is a constant. Then (3.3) and (3.4) imply that

$$v(uK, K) = (\lambda - p)K. \tag{3.5}$$

[5] Rodriguez-Clare (1996) assumes that adopters must also adapt the capital goods to local conditions. This induces producers in developing countries to buy older-vintage equipment.



Change in Equipment by Year of Manufacturing
from 1988 to 1993
(in relative terms)

Figure 6

Let t be the age at which the machine is replaced if it has not yet broken down. Then

$$\text{(3.6)} \qquad v(k, K) = \underset{t}{\text{Max}} \left\{ \int_0^t \delta e^{-\delta s}\Big([k(1-e^{-rs})/r] + e^{-rs}[Ke^{gs}(\lambda - p)]\Big)\, ds + e^{-\delta t}\Big(k[(1-e^{-rt})/r] + e^{-rt}[Ke^{gt}(\lambda - p)]\Big) \right\}.$$

Evidently, $v(k, K)$ is homogeneous of degree 1 in (k, K): $v(k, K) = \lambda(z)K$, where $z = k/K$, and $\lambda(z) = v(z, 1)$. Of course, $\lambda(1) = \lambda$. Denote the optimal t by T.

<u>LEMMA 1</u>: The optimal waiting time, T, between upgrades is such that

$$\text{(3.7)} \qquad u = (\lambda - p)(r - g).$$

<u>PROOF</u>: Differentiating in (3.6) with respect to t gives the first-order condition: $ke^{-rt} - rKe^{-(r-g)t}(\lambda - p) + gKe^{-(r-g)t}(\lambda - p) = 0$, which implies that $k + (g-r)(\lambda - p)Ke^{gt} = 0$.

(A) If $k = uK$, the maximizing value of t in (3.6) should be zero, which implies $uK + (g-r)(\lambda - p)K = 0$, i.e., (3.7).

(B) If $k = K$, the maximizing value of t in (3.6) should be T. That is, $K = Kue^{gT}$, and so, $\log u + gT = 0$, i.e., $T = -(\log u)/g$, i.e., the definition of u. ■

Aggregate output (again, in units of h^{η}), is equal to the average efficiency of capital:

$$\text{(3.8)} \qquad Y = K\int_u^1 z\, dF(z \mid u) \equiv e^{gt} E\{z \mid u\}.$$

In (3.7), u is given in terms of p <u>and</u> λ.

Equipment prices work through u. The key result here is the following:

PROPOSITION 2: u declines monotonically from $u = 1$ when $p = 0$, to $u = 0$ when $p = 1/(r + \delta)$. Moreover, for $p \in [0, 1/(r + \delta)]$,

$$\frac{du}{dp} = \frac{-(r + \delta - g)}{1 - u^{(r + \delta - g)/g}} < 0 .$$

PROOF: Evaluating (3.6) at $v(K, K) \equiv \lambda K$ we show in the appendix that

$$(r-g)\lambda + \delta p = 1 - \frac{g}{r + \delta}\left(1 - u^{(r+\delta)/g}\right). \tag{3.9}$$

Together with (3.7), this implies

$$(r + \delta - g)\, p + u = 1 - \frac{g}{r + \delta}\left(1 - u^{(r+\delta)/g}\right). \tag{3.10}$$

Taking r and g as exogenous, this allows us to solve for u in terms of p. Implicit differentiation of (3.10) yields the claim. ■

Proposition 2 and equation (3.1) show that if countries' policies cause permanent differences p, such policies induce

(a) a negative level effect (i.e., a lower $E\{ z \mid u\}$) around the trend g, and
(b) a wider dispersion in z

Since $d^2u/dp^2 < 0$, the relation between u and p is concave to the origin, as drawn in Figure 7. This suggests (and we shall later show) that the effects of p will be the biggest when p is high. Nevertheless, there are no growth effects of p so long as $p < 1/(r + \delta)$. Above this upper bound, no one *ever* upgrades, and there is no growth.

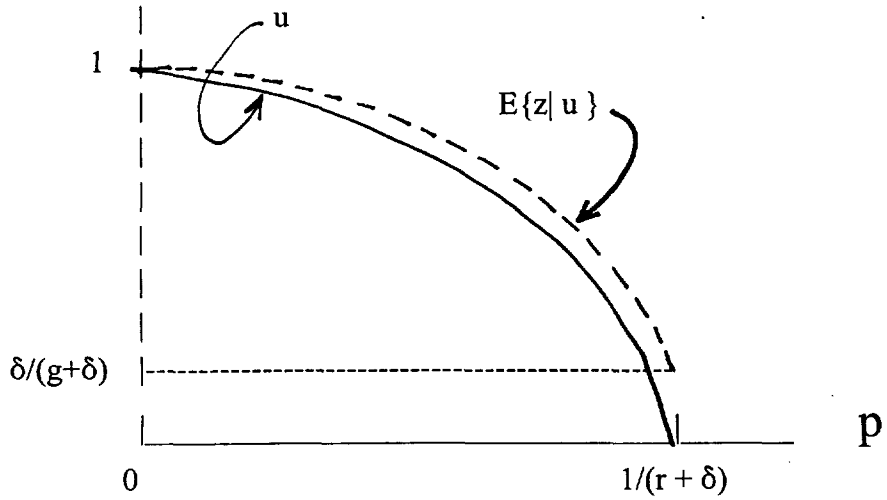


FIGURE 7: How u and E{z| u } depend on p

The largest *possible* efficiency in a rich relative to a poor economy is $(1 + \theta)/\theta = 1 + g/\delta$. So g/δ must be large (i.e., θ must be small) for the model to have a chance to explain big effects of incentives. The ratio g/δ is big for computers: Berndt, Griliches, and Rappaport (1995) find that on average quality-adjusted price indexes for computers decline at about 30% per year (g), and their physical depreciation (δ) is small.

3.C. Reinterpreting the regression in (2.2): Since $k = ze^{gt}$, and $y = Y/h^{\eta}$, the production function (3.2) implies that (interpreting y as output per worker),

$$ln(E\{z|\, u(p)\}) \;=\; ln(Y) - gt \;=\; ln(y) - gt - \eta\, ln(h)$$

Recall that $p = P/h^{\eta}$. Let $\varepsilon(p)$ denote the absolute value of the elasticity of output with respect to the price of equipment:

$$\varepsilon(p) \;=\; \frac{-\partial\, ln(E\{z|\, u(p)\})}{\partial\, ln(p)} \;.$$

(As we shall see, the model does not imply that the elasticity of output with respect to equipment prices is constant) Then the observable relation becomes

(3.11) $$ln(y) = gt - \varepsilon(p)\, ln(P) + \eta[1 + \varepsilon(p)]\, ln(h).$$

The regression (2.3), which we reproduce here, gives the result

$$\underset{}{ln(Y_{j,t})} = 8.3 - \underset{(4.66)}{1.17}\, ln(P_j) + \underset{(2.55)}{0.34}\, ln(h_j),$$

which now has a different interpretation. In a cross section, the term gt is constant. The coefficient of $ln(h)$ will exceed the true contribution of h to output by an amount $\varepsilon(p)$. So how large is $\varepsilon(p)$ likely to be? This very much depends on the value of u, and on whether $\delta < g$ or not. This is the subject of the next proposition:

<u>PROPOSITION 3</u>: The equipment price-elasticity of output is

$$\varepsilon(p) = \frac{-\left[1 - u - \frac{g}{r+\delta}\left(1 - u^{(r+\delta)/g}\right)\right]}{\left(1 - u^{(r+\delta-g)/g}\right)}\left(\frac{(1+\theta)\,u^{\theta}}{1 - u^{1+\theta}} - \frac{\theta\, u^{\theta-1}}{1 + u^{\theta}}\right).$$

<u>PROOF</u>: We can write $\varepsilon = -[pdu/dp](\partial ln E\{z|u\}/\partial u)$. The three components of this expression are gotten as follows. First, from (3.10),

$$p = \frac{1}{(r+\delta-g)}\left\{1 - u - \frac{g}{r+\delta}\left(1 - u^{(r+\delta)/g}\right)\right\}.$$

The second component is the result of proposition 2. The third is

$$\frac{\partial ln E\{z|u\}}{\partial u} = -\frac{(1+\theta)\,u^{\theta}}{1 - u^{1+\theta}} + \frac{\theta\, u^{\theta-1}}{1 - u^{\theta}}.$$

Multiplying them together verifies the claim. ■

<u>COROLLARY</u>: $$\lim_{p \to 1/(r+\delta)} \varepsilon(p) = \infty.$$

PROOF: As p tends towards $1/(r+\delta)$ and as u is driven towards zero, the term $[pdu/dp]$ is bounded away from zero. But if $\theta < 1$, that is if $\delta < g$, the term $\partial ln E\{z|u\}/\partial u$ goes to infinity as u gets small. And this means that the elasticity of u with respect to p goes to infinity. ■

The corollary implies $\varepsilon(p)$ *is highest in the very countries where machines cost the most.*[6] We conclude that if $\delta < g$, we easily get big effects of P among countries where equipment prices are high.

4. The General Equilibrium Model

Next we show that the constants P and g that we took as exogenous in our version of the 1959 model, are consistent with general equilibrium. We shall not model heterogeneous policies in this world. Instead, we shall assume that there are no taxes, and that skills are the same everywhere, so that $h = 1$, and so that machines trade at the price $p = P$ everywhere.

4.A. Machine producers. Assume that the world's agents have measure 1. Of these, a group of measure n are producers of machines. Machines are intermediate goods, and the intermediate goods sector is vertically integrated into research -- the machine producers also spend money on improving the quality of the machines they make.

Each period, a machine producer uses his own labor and his knowledge to make one machine. There are no other inputs. Let q denote the *quality* of a producer's output of machines, and let Q denote the industry-wide average quality of machines produced. A producer can raise the quality of his good by doing research, and research requires only goods, no labor. Let x be his research outlay. We assume

$$\frac{dq}{dt} = AQ^{1-\alpha} x^{\alpha} . \tag{4.1}$$

[6] An interesting analogy is inventory theory, which indeed suggests that ε would be higher: P is like a fixed reorder cost. Reordering inventory is like buying a frontier machine, and average inventory is like average productivity in our model. Baumol (1952) found that the elasticity of average cash inventory with respect to the re-order cost was -1/2.

The incentive to improve productivity depends on α. The spillover from other producers' *average* knowledge is $1 - \alpha$. There are no scale effects in (4.1).

We assume a steady-state equilibrium in which p and the rate of interest, r, are constant. Each machine seller is a price taker and maximizes discounted profits:

(4.2) $$\max_{(x_t)_0^\infty} \left\{ \int_0^\infty e^{-rt} (pq_t - x_t)\, dt \right\} .$$

LEMMA 2: Given an initial condition q_0, (4.1) implies

$$\int_0^\infty e^{-rt} q_t \, dt = \frac{1}{r} \left[q_0 + \int_0^\infty e^{-rt} A Q_t^{1-\alpha} x_t^{\alpha} \, dt \right] .$$

PROOF: $q_t = q_0 + \int_0^t \dot{q}_s ds = q_0 + \int_0^t AQ^{1-\alpha}_s x^{\alpha}_s ds$. Integrating by parts, $\int_0^\infty e^{-rt} q_t \, dt = -(1/r)\, e^{-rt} q_t \,|_0^\infty + \int_0^\infty (e^{-rt}/r)\,(dq_t/dt)\, dt$. Evaluating and substituting from (4.1) proves the lemma. ■

Therefore (4.2) has the same solution as the maximization of $\int_0^\infty e^{-rt} [(p/r)AQ_t^{1-\alpha} x_t^{\alpha} - x_t]dt$. The first order conditions for a maximum are that $\alpha(p/r)AQ_t^{1-\alpha} x_t^{\alpha-1} - 1 = 0$, i.e.,

(4.3) $$x_t / Q_t = \left(\alpha p A / r\right)^{1/(1-\alpha)} .$$

Assume that equilibrium is symmetric, so that $q = Q$. Then the growth rate of Q is

(4.4) $$g = A\left(x_t / Q_t \right)^{\alpha} = A^{1/(1-\alpha)} \left(\alpha p / r\right)^{\alpha/(1-\alpha)} .$$

Normalize q_0 to 1 so that $q_t = e^{gt}$, and $x_t = (x/Q)Q = (A\alpha p/r)^{1/(1-\alpha)} e^{gt}$, and substitute into (4.2) to get that at date 0, the value of the representative machine producer is

$$(4.5) \qquad \int_0^\infty e^{-(r-g)t}\left[p - (A\alpha p/r)^{1/(1-\alpha)}\right] dt \;=\; \frac{p - (A\alpha p/r)^{1/(1-\alpha)}}{r - g} \;\equiv\; w .$$

The value of the firm at date t then is $wQ = we^{gt}$. The rate of imitation is proportional to α -- it is finite, and so producers can appropriate a part of the value of their inventions even though as sellers of machines they are perfect competitors. Fishman and Rob (1995) study the incentive monopoly seller of a durable good to engage in research.

4. B. Equilibrium in the market for machines: In equilibrium, only frontier machines are demanded and only frontier machines are produced. Frontier quality is called K in section 2 where we described the optimal behavior of machine-demanders. In this section, in discussing the behavior of machine suppliers, we have been calling it Q. In equilibrium,

$$K_t = Q_t$$

for all t. So we proceed by replacing Q with K in all the expressions. An agent is either a producer of final goods, or a producer of machines. Since there are n machine producers, there are $1 - n$ final goods producers. The supply of machines is just n. If the rate of upgrading is a constant, world demand for machines is $(1-n)\,[\delta + H'(T)]$ which after substituting for $H'(T)$ becomes $(1-n)\delta/(1-u^{\delta/g})$. Hence in equilibrium,

$$(4.6) \qquad n = \frac{(1-n)\,\delta}{1 - u^{\delta/g}} .$$

As δ goes to zero, the right-hand side of (4.6) converges to $(1-n)/T$, because when $\delta = 0$, each of $1 - n$ machine users would be need a new machine exactly once every $1/T$ periods.

4.C. Equilibrium occupational choice: A person has the option of producing machines or goods. In a steady state no one must want to switch sectors. First, consider a machine producer, the present value of whose profits is wK. If he were to buy a frontier machine and switch to the other sector, he would get $(\lambda - p)K$. He will go on making machines if and only if $w \geq \lambda - p$.

Second, a final goods producer can upgrade to the frontier any time, so that $v(k, K) \geq (\lambda - p)K$. It suffices, then, that $(\lambda - p)K$ exceed what he could get if he switched to machine production. Suppose we assume that a new entrant into machine production can somehow inherit the accumulated knowledge of the machine producers of the day. Then switching offers him a value wK. A final goods-producer will not want to switch if and only if $(\lambda - p)K \geq wK$, i.e., if and only if $w \leq \lambda - p$.

Combining these two inequalities implies that

(4.7) $$w = \lambda - p .$$

This condition is necessary and sufficient for occupational choices to be privately optimal.

4.D. Consumers: Consumers hold shares in the two kinds of firms, and their wealth grows at the rate g. They maximize their lifetime utility

$$\frac{1}{1-\sigma} \int_0^{\infty} e^{-\rho t} c_t^{1-\sigma} \, dt ,$$

Subject to a lifetime wealth constraint. The growth rate of consumption, income and wealth is:

(4.8) $$g = \frac{r - \rho}{\sigma} .$$

4.E. Goods market equilibrium: The market for final goods clear:

(4.9) $$c + x = (1 - n) K \int_u^1 z \, dF(z) .$$

The left-hand side is demand, the right-hand side is supply. In the steady state, investment x will equal desired savings, and both will be proportional to K.

4.F. Equilibrium: A steady state equilibrium consists of six scalars: λ, p, g, u, n, and r, that satisfy (3.7), (3.9), (4.4), (4.6), (4.7), and (4.8). For the solution to have economic meaning, equilibrium must also satisfy certain other boundary constraints: $r > g > 0$, $u \in (0, 1)$, $n \in (0, 1)$, and $0 < p < \lambda$.

Equilibrium exists, but only for some parameter values. Let $H(b) \equiv -1 + b + (1/\tilde{\rho})(1 - b^{\tilde{\rho}})$, where $\tilde{\rho} = ((\rho + (1 - \sigma)\delta)/\rho) > 1$, and let B be for smaller root of the equation $H(b) = 0$ (this equation has two roots, one at $b = B \in (0, 1)$, the other at $b = 1$). Assume (as illustrated in Figure 8) that

$$B < B_0 \equiv \frac{(1 - \alpha)\rho}{\alpha(1 - \sigma)A} < 1. \tag{4.10}$$

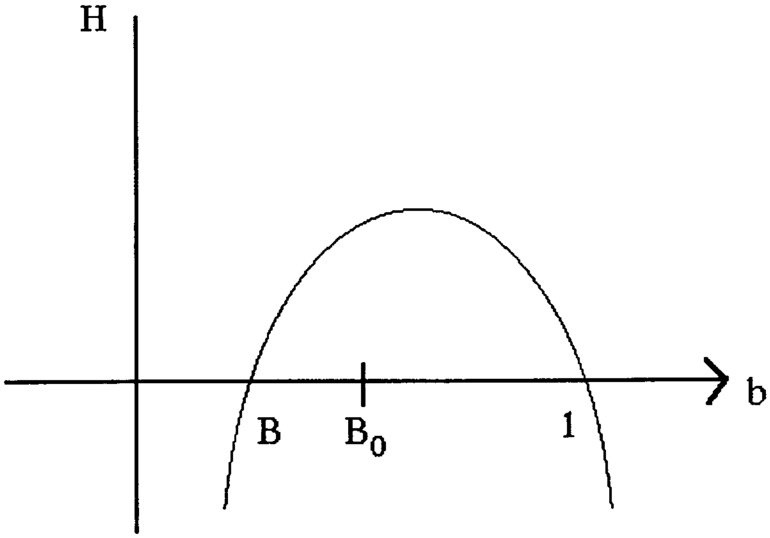


Figure 8: An illustration of the condition (4.10)

Since H(.) does not depend on α or A, neither does B. So, given the other parameters, we can always choose $\alpha A/(1-\alpha)$ (an index of the marginal product of x) so that (4.10) holds.

PROPOSITION 4: (Existence of Equilibrium): If (4.10) holds, then there exists a steady-state equilibrium.

PROOF: Appendix 3. ■

5. Conclusions

Solow (1956) is about the quantity of machines. Solow (1959) adds endogenous quality of machines -- the user of machines decides when to adopt a better machine. In our version of 1959 we shut down the quantity side and still find potentially big effects of machine prices if δ is low relative to g.

When the quantity and the quality effects are both at work, the price elasticity should be higher than what either effect would yield on its own. But the combined effect is likely to be less than the sum of the parts: If I buy a new machine but I do not scrap any, the average quality of my capital stock will rise, but not by as much as it would if I also scrapped my worst machine to make room for the new one.

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Appendix:

1. Proof of expression (3.9): From (3.6),

$$\lambda = \int_0^T \left[(\delta/r)e^{-\delta s} - (\delta/r)e^{-(r+\delta)s} + \delta e^{-(r+\delta-g)}(\lambda-p)\right]ds + e^{-\delta T}/r - e^{-(r+\delta)T}/r + e^{-(r+\delta-g)T}(\lambda-p)$$

The first term on the right-hand side reads

$$-(e^{-\delta s}/r)|^T_0 + [\delta/r(r+\delta)]\, e^{-(r+\delta)t}|^T_0 - [\delta/(r+\delta-g)]\, e^{-(r+\delta-g)s}|^T_0$$

$$= (1-e^{-\delta T})/r + [\delta/(r(r+\delta)]\,(e^{-(r+\delta)T}-1) + [(\lambda-p)\delta/(r+\delta-g)](1 - e^{-(r+\delta-g)T})$$

Adding these terms to line 2 of the above expression and collecting terms,

$$\lambda = 1/r - \delta/(r(r+\delta) + \delta(\lambda-p)/(r+\delta-g) + e^{-\delta T}(-r^{-1} + r^{-1})$$

$$+ e^{-(\delta+r)T}[-r^{-1} + \delta/r(r+\delta)] + e^{-(\delta+r-g)T}(\lambda-p)(1 - \delta/(r+\delta-g)).$$

Therefore $\lambda = 1/(r+\delta) + \delta(\lambda-p)/(r+\delta-g) - e^{-(\delta+r)T}/(r+\delta) + (r-g)(\lambda-p)\, e^{-(\delta+r-g)T}/(r+\delta-g)$ [because $-r^{-1} + \delta/r(r+\delta) = 1/(r+\delta)$]. Therefore, $\lambda = (1-e^{-(\delta+r)T})/(r+\delta) + (\lambda-p)\left[\delta+(r-g)e^{-(\delta+r-g)T}\right]/(r+\delta-g))$.

Using (4.2), $\lambda = (1-e^{-(\delta+r)T})/(r+\delta) + (\lambda-p)\delta/(r+\delta-g) + ue^{-(\delta+r-g)T}/(r+\delta-g)$. Since $ue^{gT} = 1$ and $e^{-(\delta+r)T} = (e^{gT})^{(-\delta+r)/g} = u^{(\delta+r)/g}$, $\lambda = (1-u^{(\delta+r)/g})/(r+\delta) + (u^{(\delta+r)/g})/(r+\delta-g)) + (\lambda-p)\delta/(r+\delta-g)$

Multiplying through by $r+\delta-g$ and subtracting $\delta\lambda$ from both sides yields:

$$\begin{aligned}(r-g)\lambda + \delta p &= (r+\delta-g)(1 - u^{(\delta+r)/g})/(r+\delta) + u^{(\delta+r)/g} \\ &= 1 - g/(r+\delta) + gu^{(\delta+r)/g}/(r+\delta) \\ &= 1 - g(1-u^{(r+\delta)/g})/(r+\delta) \quad \blacksquare\end{aligned}$$

2. Proof of Proposition 4: We will first show that equations (3.7), (3.9), (4.4), (4.6), (4.7), and (4.8) can be combined into the following equation for r:

$$\text{(A1)}\qquad \begin{aligned} &\big((r(\sigma-1) + \rho)/\sigma + \delta\big)(r/\alpha)\big((r - \rho)/\sigma\big)^{(1-\alpha)/\alpha} A^{-1/\alpha} - 1 + \\ &\Big[(r/\alpha)\big((r - \rho)/\sigma\big)^{(1-\alpha)/\alpha} - \big((r - \rho)/\sigma\big)^{1/\alpha}\Big] A^{-1/\alpha} + \\ &\frac{r-\rho}{(r+\delta)\sigma}\left\{1-\Big(\Big[(r/\alpha)\big((r - \rho)/\sigma\big)^{(1-\alpha)/\alpha} -\big((r-\rho)/\sigma\big)^{1/\alpha}\Big]A^{-1/\alpha}\Big)^{\sigma(r+\delta)/(r-\rho)}\right\} = 0 \end{aligned}$$

First note that (4.8) determines r given g, and (4.6) determines n given u and g. So it suffices to solve (3.7), (3.9), (4.4), and (4.7) for g, p, λ and u. Rewrite (4.7) as follows:

$$\text{(A2)}\qquad (\lambda-p)(r-g) = p- (p\alpha A/r)^{1/(1-\alpha)}$$

Combine the above with (3.7) to get

$$\text{(A3)}\qquad u = p - (p\alpha A/r)^{1/(1-\alpha)}$$

Again, combine (3.7) and (3.9) to get

$$\text{(A4)}\qquad u + (r-g)p + \delta p = 1 - \frac{g}{r+\delta}\left[1- u^{(\sigma+\delta)/g}\right]$$

Now solve (4.4) for p:

$$\text{(A5)}\qquad p = (r/\alpha)\, g^{(1-\alpha)/\alpha} A^{-1/\alpha}$$

Substitute the above into (A3) and (A4) respectively:

$$\text{(A6)}\qquad u = \big((r/\alpha)\, g^{(1-\alpha)/\alpha} - g^{1/\alpha}\big)$$

and

$$\text{(A7)}\qquad (r + \delta - g)(r/\alpha)\, g^{(1-\alpha)/\alpha} A^{-1/\alpha} = 1 - u - \frac{g}{r+\delta}\left[1 - u^{(r+\delta)/g}\right]$$

Finally, substitute (A6) into (A7) and then use (4.8) to solve for g in the resulting expression to get (A1).

If we let $r = \rho$ in the left-hand side (LHS) of (A.1), then the LHS = -1. On the other hand, we claim that if we let $r = \rho/(1-\sigma)$, the LHS is > 0. If this claim is true (which is shown below), then since the equation is continuous in r, there must be an $r^* \in (\rho, \rho/(1-\sigma))$ for which (A.1) holds.

Consider equation (4.8). Since $0 < r^* < \rho/(1-\sigma)$, $r^* > g > 0$ and from (4.4′), $1/(r+\delta) > p > 0$. This implies that the objective functions (4.5) and (3.6) of firms in the machine-producing and the output-producing sectors are bounded. Because v is strictly increasing in its first argument, (3.3) plus the fact that $p > 0$ imply that $u < 1$. Moreover, $\lambda \equiv v(1, 1) \geq 1/(r+\delta) \geq p$. Since $\lambda > p$, (3.5) implies $u \in (0, 1)$. Finally, equation (4.6) can be written as $((1-u^{\delta/g})/\delta) + 1 = 1/n$. Since $g > 0$ and $u \in (0, 1)$, this implies $n \in (0, 1)$.

If remains to be show that the LHS of (A.1) is positive at $r_0 = \rho/(1-\sigma)$. If we insert $\rho/(1-\sigma)$ into the LHS of (A.1) we obtain $H(B_0)$ where B_0 is defined in (4.10). Consider the function H(b). Then it can be verified (see the diagram below) that, $H(b) > 0$ for $b \in (B, 1)$ and since B_0 is in this range $H(B_0) > 0$. So the LHS of (A.1) is indeed positive. ■