NBER WORKING PAPER SERIES

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Working Paper 5549

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1996

We thank Elhanan Helpman, Gene Grossman, Elena Seghezza, Phillippe Martin, Gianmarco Ottaviano and Jarek Wieczoreck for comments, suggestions and inspiration. The authors acknowledge financial support from the Swiss National Science Foundation (Subsidy #1214-043580.95/1). Rikard Forslid also acknowledges the Humanistisk-Samhallsvetenskapliga Forskningsradet (HSFR) (Subsidy #F 59/95). This paper is part of NBER's research programs in International Trade and Investment, and International Finance and Macroeconomics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper has two purposes. It introduces a direct approach to policy analysis in endogenous growth models - the q-theory approach - and uses this to illustrate several new openness-and-growth links that appear when we enrich the economic content of the early trade and growth models. The approach - inspired by Tobin's q - is merely a change of state variables and re-interpretation of steady-state conditions. The main difference is its focus on investment, which is after all, the heart of growth models. The approach's simplicity permits us to complicate the early models in interesting directions and to explicitly include trade barriers. The latter allows study of incremental policy reform rather than mere shifts from autarky to free trade (or small deviations from free trade) as in early literature.

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Trade Liberalization and Endogenous Growth: A q-Theory Approach

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April 1996

I. Introduction

Policy makers have long asserted that trade liberalization is good for growth, yet economists have only recently developed tools to evaluate these claims. The seminal theoretical literature - e.g., Rivera-Batiz and Romer (1991a,b), Krugman (1988), Feenstra (1990), Segerstrom, Anant and Dinopoulos (1990), and Grossman and Helpman (1991) - greatly advanced our understanding of the links between trade and endogenous growth.

Building on the early literature, this paper presents a direct approach to analyzing the growth implications of trade liberalization and uses it to illustrate several novel openness-and-growth links. The approach's strength is its simplicity and this permits us to enrich the early models in ways that open up novel openness-and-growth links. Additionally, the approach's directness facilitates explicit inclusion of trade policy variables. Liberalization can thus be viewed as an incremental policy reform rather than a shift from autarky to free trade (or small deviations from free trade) as in most of the early literature. This is a merit since formal policy analysis and econometric studies require continuous openness-and-growth relationships.[i]

The approach is inspired by Tobin's q-theory of investment (Tobin 1969). In all endogenous growth models, output growth is driven by the ceaseless accumulation of physical, human or knowledge capital (depending upon model details). The rate of real investment determines the rate of capital accumulation and this is where the q-theory comes in. Tobin's approach focuses on the

*We thank Elhanan Helpman, Gene Grossman, Elena Seghezza, Phillippe Martin, Gianmarco Ottaviano and Jarek Wieczoreck for comments, suggestions and inspiration. The authors acknowledge financial support from the Swiss National Science Foundation (Subsidy #1214-043580.95/1). Rikard Forslid also acknowledges the Humanistisk-Samhallsvetenskapliga Forskningsradet (HSFR) (Subsidy#F 59/95).

[i] For instance, some of the openness-growth links emphasised in the early literature - eg elimination of redundant research - appears to be 'discontinuous' (the effect occurs in full force for any positive level of openness).

ratio of a firm's stock market value to the replacement cost of its capital. The stock market value of a unit of capital is the present value of its income stream; Its replacement cost is the marginal cost of new capital. In exogenous growth models, q determines the steady-state capital-labor ratio. In endogenous growth models, it determines the level of real investment. Now because endogenous growth models all make assumptions implying that a constant level of real investment yields constant growth, changes in q (or more precisely q-1) predict growth rate changes.

This feature means that Tobin's q provides a direct approach to analyzing the growth implications of trade liberalization (or any policy reform). The point is that q-1 is the shadow value of moving more resources into the capital-accumulation sector. Thus any trade liberalization that boosts a country's q, draws more resources to its accumulation sector until q is restored to unity. This raises the countries' rate of capital accumulation and raises output growth as long as the policy does not lower the foreign q by too much.

The rest of the paper is organized in three sections. Section II presents the basic model, which can be thought of as a simplified version of the Romer product innovation model. It also introduces the q-theory approach to analysis in endogenous growth models, including welfare and policy analysis. Note that there is really nothing new about the approach - it is simply a switch of state variables and re-interpretation of equilibrium conditions. However as is often the case, the correct choice of state variables can greatly simplify analysis of a dynamic model. Section III illustrates six trade policy and growth links. Section IV presents a summary and concluding remarks.

II. A Basic Trade and Endogenous Growth Model

To lighten the need for superscripts and subscripts, we work with symmetric countries. Most of our results hold for a more general model which is presented in the appendix.

Consider a world economy with two symmetric countries (home H and foreign F) with identical technology and preferences, two factors (labor L and capital K) and two sectors (X and I). The labor force is fixed, but K (which can be interpreted as physical, human or knowledge capital) is the accumulated output of the I sector (which can be interpreted as a capital-goods sector, an education sector, or an innovation sector). The X sector consists of differentiated goods and is modeled as the Flam-Helpman version of the Dixit-Stiglitz monopolistic-competition model.[i] Namely, production of each X variety requires one unit of K plus "a" units of labor per unit of output, so π+wax is the cost function where π is K's rental rate, w is the wage and x is per-firm output. By choice of units a=1. X-sector goods are traded while factors are not. Home and foreign impose frictional (i.e. 'iceberg') import barriers $\tau \geq 1$.[ii]

With ρ as the time-preference parameter, representative consumer preferences are:

$$\int_{t=\tau}^{\infty} e^{-\rho t} \ \ln(C)dt, \quad C=\left(\sum_{i=1}^{K_H+K_F} c_i^{1-1/\sigma}\right)^{\frac{1}{1-1/\sigma}}, \quad \sigma > 1 \qquad \text{(2-1)}$$

where C is the consumption aggregate, c_i is consumption of variety i, and full employment (with one-unit of capital per variety) implies that K_H+K_F is the global number of varieties. A typical nation's income (denoted as Y) equals wL+πK. Utility optimization yields standard CES demand functions for X varieties and the Euler equation:

$$\dot{E} \ / \ E = r - \rho \qquad \text{(2-2)}$$

where E is consumer expenditure and r is the rate of return on savings.

Calculation of the equilibrium is greatly facilitated by several special features of the Dixit-Stiglitz monopolistic competition (derivation of these properties is well-known, so we merely list them).[iii] [1] Each monopolistically competitive X-sector firm: (i) produces a unique variety, (ii) charges $w/(1-1/\sigma)$ in the local market and $\tau w/(1-1/\sigma)$ in the export market, and (iii) earns operating profit equal to $(\Sigma_i s^i E^i)/\sigma$, where s^i is the firm's share of market i expenditure. Given the specificity of knowledge capital, capital's reward is the Ricardian surplus, i.e. the operating profit. With symmetry $\Sigma_i s^i = 1/K$, so:

$$\pi = E \ / \ \sigma K \qquad \text{(2-3)}$$

The I sector employs L to produce new capital under constant private returns and perfect competition. Following Lucas (1988), Romer (1990), and Grossman and Helpman (1991), we assume external scale economies such that the I-sector production and marginal cost functions are: where L_I is sectoral employment, F is the average (and marginal) cost of K, $0 \leq \lambda \leq 1$ and K* is the

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[i] See Flam and Helpman (1987) and Dixit and Stiglitz (1977).

[ii] For example, home firms must ship $\tau^F = 1 + t^F$ units to sell one unit in the foreign market; t^F is the tariff equivalent of the frictional barriers.

[iii] Numbered endnotes refer to the "Supplemental Guide to Calculations", which is attached

nonlocal capital stock.[i] The extent to which externalities occur internationally is regulated by the parameter λ ($\lambda=1$ or 0 indicates perfect or zero international spillovers). From (2-4):

$$g \equiv \dot{K} / K = L_I(1+\lambda) \qquad \textbf{(2-4)}$$

where g is the growth rate of K.[ii] Note that a constant L_I yields constant capital stock growth. The dual of this is that F falls at the rate of capital stock growth (see 2-4). By perfect competition, F is the supply price (replacement cost) of capital.

A. *Dynamic Analysis with Tobin's q*

All endogenous growth models, including the simple one presented above, make assumptions such that constant real investment (i.e. constant application of resources to capital-formation) yields constant capital-stock growth and therefore constant output growth. It seems natural, therefore, to take real investment as the main state variable, even though the key dynamic equation - the Euler equation - involves expenditure. Fortunately, expenditure is easily eliminated from the Euler equation. Nominal factor income Y equals E+I, where I is nominal investment (equal to wL_I in our model), so $E=wL+K\pi-wL_I$ and using $\pi=E/K\sigma$, we have:

$$E = \frac{w(L - L_I)}{1 - 1/\sigma} \qquad \textbf{(2-5)}$$

Taking L as numeraire (w=1) and the home and foreign L_I's as the state variables, we see $\dot{E} = 0$ in steady state because $\dot{L}_I = 0$ by definition of a steady state. From the Euler equation, $\dot{E} = 0$ means $r=\rho$ in both countries. This simplifies calculation of K's stock market value.[i]

The appendix presents the system equations in more detail. Here we focus on the steady-state growth rate. The stock market value of a unit of capital, which we call J, is the present value of the π stream, and in steady state[2]:

$$J(t) = \int_{s=t}^{\infty} e^{-r(s-t)} \pi(s)\, ds = \frac{\pi(t)}{\rho+g} \qquad \textbf{(2-6)}$$

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[i] A variety of well-known stories are used to justify these externalities; The most convincing ones take K to be knowledge or human capital.

[ii] As illustrated in the appendix $\lambda>0$ will imply identical growth rates in the two countries

In steady state q equals unity, so using $\pi=(L-L_I)/K(\sigma-1)$ and (2-5), the home steady-state condition is:

$$q[g] = \frac{\pi/(g+\rho)}{F} = \frac{L(1+\lambda)-g}{(\sigma-1)(\rho+g)} = 1 \qquad \textbf{(2-7)}$$

Clearly Tobin's q is a very simple monotonically decreasing function of g. Solving, we have $g=(L(1+\lambda)+(1-\sigma)\rho)/\sigma$. Since $g=L_I(1+\lambda)$, q is also a simple function of real investment L_I.

With asymmetries (see appendix), we get two conditions that must be solved for the two g's and the relative K stocks. This is possible since $g_H=g_F$ in steady state (as long as $\lambda\neq0$). This commonality of steady-state g's is an important feature of all endogenous growth and trade models.[ii] The appendix demonstrates this, but intuitively, (2-4) shows that if K_F/K_H limits to zero - which would happen if $g_H>g_F$ forever - the productivity of foreign L_I grows without bound while that of home L_I falls to unity. These shifts attract labor to the foreign I sector, repel labor from the home I sector, and thereby narrow growth rates differences. Similarly, g_F cannot be greater than g_H in steady state.

Note that it is perfect competition in the I-sector that forces q to equal unity. To see this, note that an I-sector firm producing $\dot{K}_i$ designs per period earns pure profits of:

$$(J - F)\,\dot{K}_i = (q - 1)\,L_{Ii} \qquad \textbf{(2-8)}$$

where L_{Ii} is the firm's employment and the second expression follows from (2-4). Free entry ensures that L_I always jumps to the point where pure profits are eliminated. This also formally shows why q-1 is the shadow value of moving resources into the I sector.

Calculating output growth is simple. Nominal income is constant at $L+\pi K$, since π falls at the rate K rises. Real income rises since the aggregate price index (the standard CES price index) falls with K accumulation. Specifically, P_X equals $K^{-1/(\sigma-1)}$ times $h[\tau]$, where h is increasing in τ, but is time invariant in steady state.[iii] Thus real consumption and output grow at $g/(\sigma-1)$.

B. ***Steady State Welfare Analysis***

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[i] In this model all state variables are 'jumpers' so the model is always in steady state.

[ii] This rules out the possibility of beggar-thy-neighbour growth policy. For instance, a home country pro-growth policies speed growth in the foreign country, even if the policy shifts I-sector activity from foreign to home.

[iii] $h[\tau]$ equals $(1+\tau^{1-\sigma})^{1/(1-\sigma)}/(1-1/\sigma)$.

The q-approach permits simple demonstration of the fact that laissez-faire growth is suboptimal due to I-sector externalities. The appendix shows the formal argument; Here we provide intuition.

Private agents devote resources to capital formation up to the point where the present value of capital's income equals its marginal cost, i.e. J[g]=F. However, creating a new unit of capital also lowers the cost of all future capital formation due to the externality in (2-4). Private investors do not perceive this, so the laissez-faire economy under invests and grows too slowly. To calculate the value of the labor-saving externality, note that since $\dot{K} = L_I(K + \lambda K^*)$, adding a unit of K means it takes L_I/K less labor to produce the old $\dot{K}$ path. The social present value of this is $L_I/K(\rho+g)$ since ρ reflects pure time preference and marginal utility declines at g given (2-1). Employing (2-4) and (2-5), the social optimum therefore requires J to be equal to the social marginal cost, viz. F less the value of the externality, thus:

$$q[g^*] = (1 - \frac{g^*}{\rho + g^*}) \quad \text{(2-9)}$$

The optimal capital growth g* is clearly larger than the laissez-faire g since q is declining in g. Given this, we know incremental trade policy changes that are pro-growth must be welfare improving when there are no transitional dynamics. Obviously an *ad valorem* I-sector subsidy equal to g*/(g*+ρ) would restore optimality. Note that welfare calculations are much more difficult with transitional dynamics (e.g. when countries are asymmetric *ex post*), as Baldwin (1992a) shows.

C. *A q-Theoretic Approach to Policy Analysis*

The q approach is useful since capital growth is proportional to I-sector employment and q-1 is shadow value of moving resources into the I sector. Consequently, a policy reform that forces an incipient increase in a country's q, induces a jump in L_I that raises the national g. If the reform does not lower other country's q too much, then global output growth rises. To calculate the incipient increase in q we hold the state variables at their pre-liberalization levels and study the policy's impact on q at a moment in time (so the K's, which do not jump, are fixed).

III. Six Openness and Growth Links

The early literature discusses only a few openness and growth links. Here we turn to six newer links, at least five of which are, to our knowledge, novel. Each trade-and-growth link is illustrated with the simplest possible model.

A. *Growth Effects of Import Competition: Imperfect Competition in the I Sector*

A popular interpretation of the basic model views K as knowledge capital and the I sector firms as R&D labs. Under this interpretation, however, the assumption of constant private return in the I sector is rather objectionable. After all, developing a new product or process is not like driving a taxi. Developers must invest a good deal of time in learning about the state of the art before being able to come up with their own advancements. To allow for this, we enrich the basic model by introducing scale economies and imperfect competition into the I sector. For simplicity's sake, we continue to assume - as in the all the early-literature models - that new units of knowledge capital (designs) are sold at arm's-length to single-variety manufacturing firms. As we shall see, this enrichment of the model generates an openness-and-growth link that is novel, although it is akin to that of Baldwin (1992b).

To illustrate the link, consider a symmetric-country version of the basic model with $\lambda=1$ and the I-sector cost function generalized to include an overhead cost (i.e. a flow of fixed costs) of G units of labor in addition to the variable cost F. I-sector firms (think of them as a research laboratories) sell designs in the home and foreign markets. We assume, however, that trade in innovations is hindered by a range of cost-raising barriers. The cost is intended to capture a wide range of common real-world barriers, but standards provides a concrete example. Most new product needs to be certified as meeting industrial, health, safety and/or environmental standards. The certifying boards are typically influenced by local industries (directly, when the board has industry representatives, or indirectly via political pressure on the national government) for whom the new product constitutes a threat. It is quite common, therefore, for standards to provide *de facto* discrimination against foreign varieties. In keeping with the standards-certification example, we model these barrier as posing a one-time cost.

Designs can be thought of as a homogenous good. That is to say, when sold by a lab, they are perfect substitutes (given the symmetry of varieties) even though they produce a unique variety after they are sold. In other words, they are putty-clay capital. With this homogeneity, the most natural market structure assumption for the I sector is a Cournot oligopoly with segmented markets. The profit maximization problem of an I-sector firm is:

$$\max_{\dot{K}_i,\dot{K}_i^*} \; (V - F)\dot{K}_i + (V^* - \Gamma F)\dot{K}_i^* - G \tag{3-1}$$

where V and V* are the prices of designs sold in the local and export markets, dK_i/dt and dK_i^*/dt are a typical I-sector firm's sales to the local and nonlocal markets, and Γ indicates the severity of the cost-raising barriers (Γ=1 for nondiscrimination in certification). Using symmetry, the first-order conditions are[3]:

$$V\left(1 - \frac{s}{\varepsilon}\right) = F, \quad V\left(1 - \frac{s^*}{\varepsilon}\right) = \Gamma F \tag{3-2}$$

where s* is defined as $\dot{K}_i^* \,/\, m(\dot{K}_i^* + \dot{K}_i)$, s is defined as $\dot{K}_i \,/\, m(\dot{K}_i^* + \dot{K}_i)$, ε is equal to $(\rho+2L_I)(L-L_I-mG)/L_I(\rho+2L-2mG)$, and m is the number of I-sector firms per country. Notice that: (i) V and F both fall at g, so the Nash innovation rates are time invariant, and (ii) two-way trade in designs occurs as long as Γ is not too high.

With symmetry, the state variables are L_I and m, so to close the model we determine m with free entry. I-sector firms enter up to the point where pure profits disappear i.e.[4]:

$$G = 2\left(\frac{s^2 + s^{*2}}{\varepsilon}\right)VKL_I \tag{3-3}$$

where from (4-9), we see that the s's are functions of m, i.e. $s = (m\varepsilon((\Gamma - 1) + 1)/(m(1 + \Gamma))$ and $s^* = (1/m) - s$.[5] Again the time dimension disappears because the product VK is constant in steady-state. Since both state variables can jump, the system has no transitional dynamics.

This trade in intellectual property rights is analogous to the reciprocal dumping trade of Brander and Krugman (1983). Once this analogy has been established, the outcome of reciprocal liberalization is obvious given the analysis of the procompetitive effect by Smith and Venables (1988). Heuristically, reciprocal liberalization (i.e. $d\Gamma < 0$) defragments the markets, thereby raising the degree of competition (e.g., as s minus s* shrinks the Herfindahl index of concentration falls in both markets). This reduces the average markup of V over F, thereby lowering prices and creating incipient I-sector losses. The incipient losses force exit as per (4-10), partially offsetting the competition increase. With a lower m, however, the remaining firms are better able to exploit scale economies, making a lower equilibrium V possible.

Since V is the replacement cost of capital as far as the X-sector is concerned, the procompetitive effect in the I sector leads to an incipient rise in Tobin q's in both economies. Higher output growth is the results.

B. *Tariff-Revenue Effects*

OECD tariffs are low, so remaining trade barriers are frequently modeled as frictional barriers.[i] Despite this, the analysis of tariff liberalization contains some interest. Indeed, tariff liberalization can have a growth effect even in situations where frictional-barrier liberalization does not. Moreover, the exact nature of the tariff matters.

In the basic model, we saw that trade policy had no long-run growth effects when the τ's represented frictional barriers. This is obvious from inspection of (2-8) for the symmetric case. Here we modify the basic model by: (i) assuming that τ reflects a specific tariff, i.e. $\tau=1+t$ where t is specific tariff, and (ii) assuming that tariff revenue R is returned lump-sum to consumers. With this, E equals $L-L_I+K\pi+R$. Since $R=t(Ks^*E/p^*)$ where s* is a typical firm's share in its export market[6]:

$$E = \frac{L-L_I}{1-\Delta-1/\sigma}; \quad \Delta \equiv \frac{t\tau^{-\sigma}(\sigma-1)}{\sigma(1+\tau^{1-\sigma})} \tag{3-4}$$

It is easy to show that Δ and therefore R is a bell-shaped function of t (similar to the Laffer curve), so liberalization lowers E for low initial t's, increasing it for high t's. The inflection point decreases as σ increases. Taking $\sigma=5$ or $\sigma=10$, for instance, inflection is at $t=0.44$ or $t=0.16$ respectively. It is easy to show that an auctioned quota would have the same impact interpreting t as the Lagrangian multiplier on an X-firm's quota constraint.

[i] For instance, WTO (1995) estimates that once the Uruguay Round in fully implemented, 43% of MFN trade among developed nations will be duty free. Since about 1/3 of world trade is intra-EU trade (and thereby duty free), tariffs will soon be zero on more than two-thirds of world trade.

This nominal expenditure effect of the tariff implies that liberalization changes π without altering F because $\pi = E/\sigma K$. Since t has no direct impact on F, Tobin's q changes with t and so liberalization has a growth effect. The growth effect is not, however, monotonic. The sign of the growth effect depends upon the initial level of the tariff; $dt < 0$ lowers g when t is initially small (below the inflection point of the R) and raise it when it when t is initially large. This effect has not, to our knowledge, been explored in the literature.

As a proviso, we note that the tariff revenue must alter consumer expenditure for the link to work. Such is the case when revenue is returned lump-sum to consumers (the classic assumption). It might not be the case if R was spent on, say, public goods.

As discussed in Baldwin and Forslid (1996), there is an interesting contrast with the Rivera-Batiz and Romer (1991b) finding that an *ad valorem* tariff also implies a non-monotonic openness-and-growth relationship. Their relationship, however, is U-shaped, while our specific-tariff yields an inverse U-shape.

C. *Traded Intermediate Inputs in the I-Sector*

When the I-sector uses traded intermediate inputs, trade policy affects Tobin's q via its direct impact on F, as pointed out by Lee (1994). The easiest way to illustrate this is with the basic model modified by supposing that K is produced using only intermediates - specifically a CES composite (elasticity of substitution equal to σ) of all X varieties. As before, a externalities are assumed, i.e. $\dot{K} = X_I K^{\Omega}(1+\lambda)$ and the replacement cost is:

$$F = \frac{P_X}{K^{\Omega}(1+\lambda)} \qquad (3\text{-}5)$$

where P_X is the CES price index. As Lucas (1988) shows, endogenous growth models produce constant steady-state growth only under knife-edge parameter assumptions. For instance, K must enter (3-5) with an elasticity of exactly unity. In this model, we require Ω to be exactly $(2-\sigma)/(1-\sigma)$. Of course, the assumption of unitary elasticity in the basic model has a certain elegance that $(2-\sigma)/(1-\sigma)$ lacks and elegance matters in theory. But in any case, both assumptions are equally arbitrary, equally unfounded empirically and equally necessary to ensure that F to falls at the rate K rises.

Evaluating P_X, we see that K's replacement cost rises with τ. Specifically, we have that F equals $(1+\tau^{1-\sigma})^{1/1-\sigma}/(1+\lambda)K(1-1/\sigma)$.[7] Given markup pricing, imported X varieties costs τ times more than local varieties, so I-sector firms will employ τ^{σ} times more of local varieties, thus the I-sector production function simplifies to:[8]

$$g = \frac{L_I(1+\lambda)}{(1+\tau^{1-\sigma})^{1/(1-\sigma)}} \tag{3-6}$$

Of course, all labor is employed in the X sector, but we reinterpret L_I as the X-sector labor whose output is earmarked for sale to the I sector. Notice that protection lowers I-sector labor productivity.

Calculating J requires consideration of the sales to both nation's X and I sectors. Since the X and I demand functions have the same elasticity, X firms earn $(E+I)/K\sigma$. Using the usual manipulation, this means that π equals $L/(\sigma-1)K$.[9] Thus J is $\pi/(\rho+g)$ and q is[10]:

$$q = \frac{L(1+\lambda)}{\rho\sigma(1+\tau^{1-\sigma})^{1/(1-\sigma)} + \sigma L_I(1+\lambda)} \tag{3-7}$$

Plainly reciprocal trade liberalization (i.e. $d\tau<0$) leads to an incipient rise in q, thereby forcing a positive growth effect. Growth rises since L_I rises to restore $q=1$ and because liberalization boosts I-sector labor productivity directly. This link between trade policy and growth is easy to understand intuitively. On impact reciprocal liberalization lowers replacement cost of capital without affecting π. This creates incipient I-sector profits that draw resources to capital formation.

D. *Imperfect Competition in Financial Intermediation*

One of the many simplifications adopted in the early trade and endogenous growth literature was to assume costless intermediation between savers and investors. While convenient, this assumption is not particularly realistic. Moreover, given the rapid expansion of international trade in financial services, it seems appropriate to investigate a model in which financial services trade is allowed to play a role. As we shall see, the enrichment of the model in this direction establishes a novel openness-and-growth link.

To illustrate this simply, we use the basic model with $\lambda=1$, enriching the financial sector by assuming riskless bonds (in fixed supply) and assuming investors (firms that wish to introduce a new X variety) borrow from banks. Banking involves an overhead cost B (i.e., flow of fixed costs) that is not related to the volume of loans. This noncovexity creates imperfect competition.[i]

Cournot oligopoly is the natural market structure assumption since banks "sell" a homogenous product that they "produce" at a constant variable cost. The constant variable cost is the interest rate that they must pay to savers (note that due to competition from bonds, banks are price-takers in the market for savings). In short, the banking sector cost function is B+rI, where r is the return on savings and I is the volume of loans.

Trade in financial services (loans) is possible, but the home and foreign markets are assumed to be segmented and borrowing abroad involves a proportional cost. This cost is meant to reflect real and regulatory barriers to trade in financial services.[ii] We measure it with the parameter ψ (ψ =1 indicates free trade in financial services). The objective function for a typical bank in a typical country is:

$$\max_{I_i, I_i^*} \ (R - r)I_i + (R^* - \psi r)I_i^* - B \qquad \textbf{(3-8)}$$

where $I_i = \dot{K}_i F$ and $I_i^* = \dot{K}_i^* F$ are the flow value of loans made in the local and nonlocal markets, and R and R* are the local and nonlocal lending rates. Given (2-4), we see that F(dK_i/dt) and F(dK_i*/dt) are time-invariant, so Nash equilibrium lending will also be time-invariant (recall L is numeraire). This means R and R*, will be constant through time. With constant lending rates, Tobin's q in a typical country will be π/(R+g)F. Due to perfect competition in the I sector, the steady-state level of investment is given by q=1. Solving this, we get a linear inverse demand function for loans[11]:

$$R = 2\frac{(L - nB) - \sigma I}{\sigma - 1} \qquad \textbf{(3-9)}$$

where (for a typical country) I is the sum of loans made and n is the number of banks. The first order conditions for a typical bank are:

$$R(1 - \frac{s}{\theta}) = \rho, \quad R(1 - \frac{s^*}{\theta}) = \psi\rho \qquad \textbf{(3-10)}$$

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[i] Since savers are atomistic, it never pay them to cut out the banks by incurring B themselves.

[ii] We suppose that the cost is per-loan. An interesting extension would be to consider a per-bank market-entry cost.

where $\theta = (L\text{-}nB\text{-}\sigma I)/\sigma I$, s and s* are the local and export market shares, and we use the fact that $r=\rho$ in steady state (because the two state variables - L_I and n - can jump, the economy is always in steady state). The formula for s and s* are as in Section III.A, with θ and n substituted for ε and m.

Given this set up it is obvious that "reciprocal dumping" of financial services occurs. It is also obvious that reciprocal liberalization of financial services trade (i.e., $d\psi<0$) will have a procompetitive effect that will lower the equilibrium mark up of R over ρ. This will lead to an incipient rise in both countries' Tobin q, thereby stimulating growth worldwide. To our knowledge, this openness-and-growth link is novel to the literature.

E. *Trade, Location and Growth: The Dynamic Home Market Effect*

When international externalities are imperfect ($\lambda<1$), non-reciprocal trade liberalization can have growth effects by encouraging agglomeration via production shifting. Demonstration of this effect require unbalanced trade in X, so we introduce a second final-good sector Z. For simplicity we assume it is marked by perfect competition and constant returns (L is the only input), and that it is freely traded. We adopt a Cobb-Douglas upper tier utility function, so $C=C_X^{\mu}C_Z^{1-\mu}$.[i] Countries are initially symmetric, but we study a unilateral policy reform by the home country. Unfortunately, the introduction of policy asymmetries greatly complicates the model. The appendix works out the general asymmetric case, showing that the model has four state variables - the L_{Ij}'s, the ratio of wage rates (equal to w_H with $w_F=1$), and the ratio of the number of foreign and home firms, namely $K_F/K_H=\phi$. The four system equations are the two transformed Euler equations, the g_F-g_H equation and the time differentiated trade balance equation. Since ϕ cannot jump, the model has transitional dynamics. Here we focus on the steady state.

Free trade in Z equalizes wages internationally, but allows X-sector trade imbalances. This eliminates w_H as a state variable, but introduces a new one $\zeta=E^F/E^H$ that appears in the rental rate[12]:

$$\pi_H = \frac{\mu E^H}{\sigma K_H}\Theta^H ; \quad \Theta^H \equiv \left(\frac{1}{1+\phi(\tau^H)^{1-\sigma}} + \frac{\zeta}{1+\phi(\tau^F)^{\sigma-1}}\right) \tag{3-11}$$

[i] See the appendix for more details.

where subscripts indicate the location of firms and superscripts indicate the location of markets. With E^H equal to $(L-L_{IH})/(1 - \mu\Theta^H/\sigma)$, home q is:

$$q_H = \frac{(L-L_I)}{(\frac{\sigma}{\mu\Theta^H}-1)(\frac{\rho}{(1+\lambda\phi)}+L_I)} \tag{3-12}$$

A similar expression holds for the foreign q with λ/ϕ substituted for $\lambda\phi$.

Now because Θ^H depends on τ^H and τ^F, trade policy can affect long-run growth. To find the exact impact we solve four equations ($r_H=\rho$, $r_H=\rho$, $g_H=g_F$ and the definition of ζ) for L_{IH}, L_{IF}, ϕ, and ζ, and then plug the results into the I-sector production function. Because the equations are highly nonlinear in ϕ and ζ, we rely on numerical solutions.

Figure 1: The Dynamic Home Market Effect

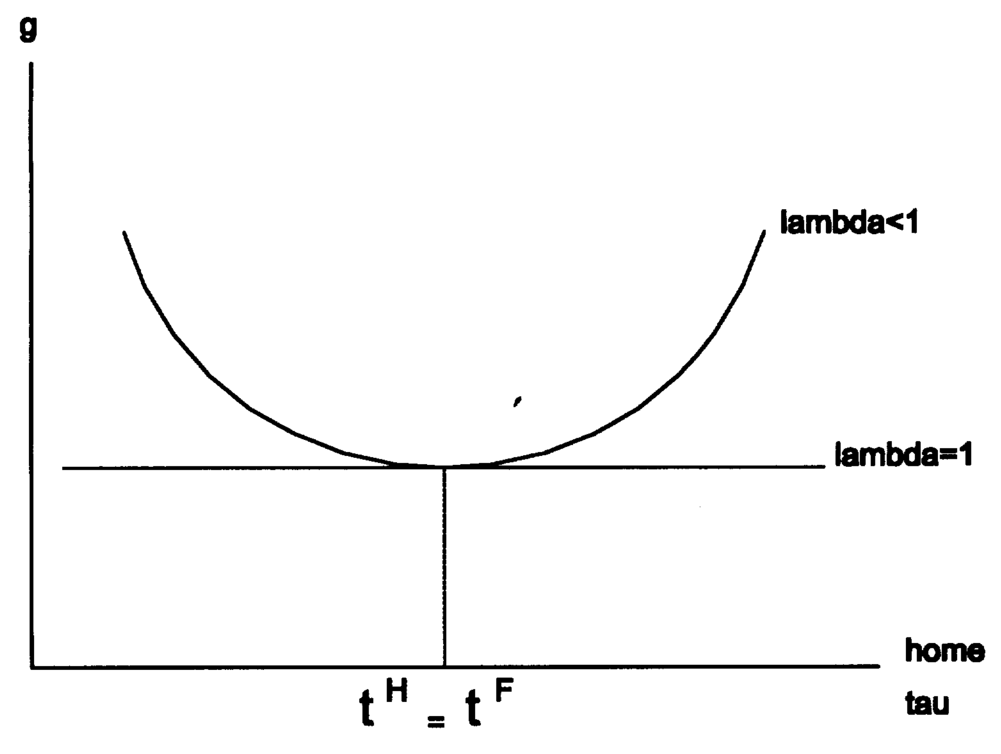

Figure 1 qualitatively illustrates the numerical results for small changes in τ^H starting from $\tau^H=\tau^F=1.25$ (also $\sigma=5$, $\rho=0.05$, $L=10$ and $\lambda=0.75$). Interestingly, raising or lowering τ^H raises long-run growth in both countries. This surprising result stems from the interaction between location and the I-sector externalities.

With $\lambda < 1$, global I-sector efficiency is maximized when $\phi = 0$ or ∞, i.e. all K production is and always has been concentrated in one country. It is minimized when $\phi = 1$ as is the case in the initial symmetric equilibrium. Because trade policy leads to X-sector 'production shifting' (see Baldwin and Venables 1995), changing τ^H moves ϕ away from unity and this raises global I-sector productivity. More specifically, raising τ^H favors the introduction of home X-sector varieties (so ϕ falls) and lowering τ^H favors foreign variety introduction (so ϕ rises), but in either case the resulting agglomeration boosts the global efficiency of capital-formation. Thus even though home protection raises L_{IH}, while home liberalization lowers it, growth always rises. Note that E^F/E^H shifts in the same direction as ϕ. If we instead started with asymmetric countries any change in τ which increased agglomeration would speed growth, and any change which decreased agglomeration would decrease growth.

This is the dynamic version of what Helpman and Krugman (1989) call the home market effect. In the static model (K's fixed), raising τ^H favors location by X-sector firms in the home market, thus raising the equilibrium number of home-based firms and lowering the number of foreign-based firms. Depending upon parameters, the home market price index may fall (since consumers pay the higher τ^H on fewer varieties) implying a real expenditure gain for the home country. With endogenous growth, the static home market effect permanently raise g since the agglomeration raises global I-sector efficiency.

The flat schedule in Figure 1 depicts the case of perfect international spillovers (i.e. $\lambda = 1$). In this case, the static home market effect occurs but it has no growth implications. Given this logic, it is clear that reciprocal liberalization has no growth implications, although it does have the usual positive impact on the prices.

This openness-and-growth link - which is clearly nonmonotonic - appears to be new to the literature. In related work, Martin and Ottaviano (1996) investigate links between trade, location and growth in a model where the externalities are associated with the location of manufacturing, rather than the location of product development.

F. *Interaction between Medium-Run and Long-Run Growth Effects*

Trade policy can have medium-run growth effects by altering the steady-state level of factor supplies, as in Baldwin (1992a). As it turns out, this type of trade-induced investment-led growth lead to a novel long-run growth effect. The key link depends upon the fact that scale economies create a link between the economic size of a country and the long-run growth rate.

The easiest way to illustrate this is to maintain the second final-good sector Z from the previous discussion, but to revert to the symmetry of the basic model. Additionally, we need to introduce a second accumulating capital stock, which we call machines (M),and a machine producing sector (the M sector). Machines are used only in the Z sector and are made with labor under perfect competition and constant returns. Choosing units appropriately, the replacement cost for machines is w, i.e. unity.

What we need is for liberalization to raise the steady-state M, since this will raise the spending on X-sector goods and thereby lead to an incipient increase in q's. A wide range of economic mechanisms can lead to such effects, as Baldwin and Seghezza (1996) show. One of the simplest (since it does not involve interaction among the Z and X sectors) is a procompetitive effect in Z. For this, we assume that Z is marked by increasing returns and a Cournot oligopoly with international market segmentation. The Z-sector cost function is $w+RMz$, where z is the output of a typical firm and R is the rental rate on machines. Finally, both nations impose frictional barriers on Z trade, with ϑ measuring the severity of the barrier. As usual, this market structure leads to reciprocal dumping. Trade in both Z and X sectors are balanced due to the symmetry of the countries.

Assuming free entry in the Z-sector (and ignoring the integer constraint as usual), national income is $L+\pi K+RM$, and investment is wL_I plus wL_M, where L_M is M-sector employment. Since we assume away depreciation, $L_M=0$ in steady state, so steady-state expenditure on the X sector is $E_X=\mu(RM+L-L_I)/(1-\mu/\sigma)$. Since π is proportional to E_X and q is proportional to π, it is easy to see how raising M leads to a permanent growth effect. We turn now to showing that M rises when ϑ is reciprocally lowered.

Lowering ϑ defragments the two Z markets and this creates pressure to lower Z prices and raise Z sales. Yet since the Z-sector variable costs involves only machines and the M stock cannot jump, this pressure is initially translated into a rise in the rental rate on machines. This change leads to an immediate jump in the q for M, triggering machine accumulation. As M and output rise, R and p_Z fall. In the new steady state, M's q is returned to unity and the M stock is higher. Of course, this procompetitive effect will also lead to industry restructuring in the Z sector (i.e., a drop in the number of Z firms) and the realization of greater scale economies.

As we saw above, the higher steady-state M in both countries increases the size of both nations and therefore stimulates growth.

IV. Summary and Concluding Remarks

Using the q-theory approach, this paper illustrates six openness-and-growth links, including five links that are, to our knowledge, new to the literature.[i] To summarize the links (not in the same order as in the text), we distinguish between links that operate via an impact on the stock market value of capital (q's numerator) and those that operate via the marginal cost of capital (q's denominator). We first summarize the three links that operate via the replacement cost of capital.

The first link rests on the fact that trade barriers can raise the marginal cost of capital - and thereby lower q - when traded intermediates are an input in the I sector. The second link arises in a model that is generalized to include the possibility of imperfect competition in the I sector. As is well known, trade liberalization can - via the procompetitive effect - alter market structure and equilibrium markups. Thus liberalization may reduce I sector markups, thereby lowering capital's replacement cost. The resulting increase in q leads to faster growth. The third link stresses the connection between localized spillovers in the I sector, agglomeration and trade policy. In a model where unilateral trade policy has the usual production-shifting effect, localized education or knowledge spillovers lead to agglomeration that lowers capital's replacement cost.

The paper also considers three links that work via the numerator of Tobin's q. The first of these exists in models in which factor supplies have steady-state levels that are a function of trade policy. By altering the level of, say, physical capital in a product innovation model, trade policy can expand expenditure and thereby raise capital's stream of income and q. In essence, this plays on the fact that most endogenous growth models assume some sort of external economies of scale. Expansion of factor supplies augments countries' economic size and thus accelerates steady-state growth. The next link is present only when trade barriers generate government revenue that is returned to consumers (e.g., tariffs). In this situation, the level of tariff revenue affects the level of expenditure, thereby affecting operating profit. This, in turn, can alter q by altering capital's stock market value. The third such link allows for imperfect competition in financial

[i] The link in III.C was discovered by Lee (1994).

intermediation. Via the procompetitive effect, reciprocal liberalization of trade in financial services lowers the mark up between savers' and investors' interest rates. By lowering the cost of borrowing, this increases the stock market value of a constant capital income stream, thereby raise Tobin's q and growth in both countries.

The simplicity of the q-theory approach has several merits. First, it makes it much easier to explicitly include of trade policy variables, so we can consider incremental trade liberalization rather than autarky-to-free trade shifts as in most of the early literature. This permits us to generate continuous trade-and-growth relationships for policy and empirical analysis. Interestingly, we often find that the relationships are far from linear, or log-linear. In some cases (e.g., the tariff-revenue link and the dynamic home market effect) the openness-growth relationship is not even monotonic. The sign of the growth effect depends upon the initial level of the trade barrier.

The ease of this approach and the simplicity of the mainstream models suggests that many other openness-and-growth links could be modeled. For instance, the modeling of the innovation process in endogenous growth models is extremely rudimentary, yet innovation is at the heart of one main strand of growth models. The fact that a wide range of effects have been capture in partial equilibrium innovation models (patent races, etc.) suggests that new trade-and-growth links would arise from enriching the modeling of the innovative sectors in growth models.

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APPENDIX

This appendix studies a more general model. The assumed preferences are:

$$\int_{t=\tau}^{\infty} e^{-\rho t} \ln\left(C_X^{\mu} C_Z^{1-\mu}\right) dt, \quad C_X = \left(\sum_{i=1}^{K_H+K_F} c_i^{1-\frac{1}{\sigma}}\right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad \sigma > 1 \qquad \textbf{(A-1)}$$

where Z is a nontraded, homogenous good produced from labor with the cost function wbZ (b=1 by choice of units). Home and foreign impose frictional (i.e. 'iceberg') import barriers on X trade $\tau^j \geq 1$ (j=H,F). Countries may differ in endowments (L) and initial conditions (K) but share identical tastes and technology.

This economy is described by a dynamic system with four state variables: the two L_I's, w_H (foreign L as numeraire w_H is the relative wage) and the relative number of varieties $\phi = K_F/K_H$. With X as the only traded good, trade balance requires $K_H s_H^F \mu E^F = K_F s_F^H \mu E^H$, so[13]:

$$\pi_H = s_H^H \frac{\mu E^H}{\sigma} + s_H^F \frac{\mu E^F}{\sigma} = \frac{\mu E^H}{\sigma}(s_H^H + \phi s_F^H) = \frac{\mu E^H}{\sigma K_H} \qquad \textbf{(A-2)}$$

Likewise, $\pi_F = \mu E^F/\sigma K_F$. Employing (2-2) and the time derivative of the formula for π, the transformed Euler equation is:

$$\frac{\dot{L}_{Ij}}{L_{Ij}} = \left(\rho - r_j + \frac{\dot{w}_j}{w_j}\right)\left(\frac{L_j - L_{Ij}}{L_{Ij}}\right), \quad j = H,F \qquad \textbf{(A-3)}$$

where[14]

$$r_H = (\mu L_H - \sigma L_{IH})\frac{1+\lambda\phi}{\sigma-\mu} + \frac{\dot{w}_H}{w_H} - \frac{\lambda\phi}{1+\lambda\phi}\left(\frac{\dot{\phi}}{\phi}\right), \quad r_F = (\mu L_F - \sigma L_{IF})\frac{1+\lambda/\phi}{\sigma-\mu} + \frac{\lambda/\phi}{1+\lambda/\phi}\left(\frac{\dot{\phi}}{\phi}\right)$$

The system equations are (A-3), the differential equation for ϕ:

$$\dot{\phi}/\phi = g_F - g_H = L_{IH}(1+\lambda\phi) - L_{IF}(1+\lambda/\phi) \qquad \textbf{(A-3)}$$

and the time-differentiated trade balance condition. Inspection of (A-3), shows $g_H = g_F$ in steady state as mentioned in the text.

Transitional dynamics with four nonlinear differential equations exceed our analytic capabilities. Yet when countries are symmetric before and after liberalization, there are no transitional dynamics because the only non-jumping variable, ϕ, never needs to move from unity. We therefore follow the standard practice of focusing on steady states.

Welfare The steady-state welfare maximization problem with symmetric countries, taking (A-1) as the planner's objective, involves the Hamiltonian $e^{-\rho t}\ln(E/P) + \zeta F$, where ζ is the usual dynamic Lagrangian multiplier.[15] Using (2-6) and the definition of the aggregate price index, the two standard necessary conditions (i.e. $\partial H/\partial L_I=0$ and $\partial H/\partial K=-\dot{\zeta}$) are:

$$\frac{e^{-\rho t}}{L-L_I} = \zeta K(1+\lambda), \qquad \frac{\mu e^{-\rho t}}{(\sigma-1)K} + \zeta L_I(1+\lambda) = -\dot{\zeta} \tag{A-4}$$

Some manipulation (involving the time derivative of the first condition), and (2-3), (2-4) and (2-5) allow us to write (for j=H,F)[16]:

$$\rho + g = -\frac{\dot{\zeta}}{\zeta} = (\frac{\sigma-\mu}{\sigma-1})\frac{\pi}{F} + g \tag{A-5}$$

When $\mu=1$, there is no static distortion between the imperfectly and perfectly competitive sectors (X and Z), so the planner's growth path simplifies to $\rho=\pi/F$. Knowing that π/F is declining in L_I and therefore g, we see that the steady-state market growth path (where $\rho+g=\pi/F$, regardless of μ) involves growth that is too slow. Intuition is served by reformulating this in terms of J and F. Using (2-5), we can write (A-5) as $F(\rho+g)=\pi(\sigma-\mu)/(\sigma-1)+Fg$, so $F=J(\sigma-\mu)/(\sigma-1)+Fg/(\rho+g)$. This explains (2-10).

Supplemental Guide to Calculations

1. Facts 2 and 3 follow from the first-order conditions $p(1-1/\sigma)=w$ and $p^*(1-1/\sigma)=w\tau^*$, where "*" indicates export-market variables. Rearranging the local first order condition, $(p-w)c=pc/\sigma=s\mu E/\sigma$, where s is the local market share. Fact 3 follows from this and a similar rearrangement of the export market first-order condition. Fact 1 comes from the fact that it is always more profitable to be a monopolist in a new product than to be a duopolist in an existing product. For more details see Helpman and Krugman (1985).

2. Here we use the facts that in steady state, $r=\rho$, E is time invariant and K grows at g.

3. The intermediate steps for the first-order condition are as follows. Taking dK^*_i/dt and dK_i/dt to be output of a typical I-sector firm that is earmarked for sale to the local and export markets (respectively) and L_{Ii} and L^*_{Ii} as the labor needed to produce these output flows, we have:

$$\dot{K}_i = L_{Ii}(1+\lambda)K =$$

The oligopolistic innovators face an inverse demand of:

$$V = \frac{\pi}{\rho+g} = \frac{E}{\sigma K(\rho+g)} = \frac{(L-L_I-mG)}{K(\rho+2L_I)(\sigma-1)}$$

the typical firm's problem can therefore be written as:

$$\max_{(L_{Ii},\, L^*_{Ii})} \left(\frac{2(L-L_I-mG)}{(\rho+2L_I)(\sigma-1)}-1\right)L_{Ii} + \left(\frac{2(L-L^*_I-mG)}{(\rho+2L^*_I)(\sigma-1)}-\Gamma\right)L^*_{Ii} - G$$

where each firm takes as given the L_{Ii} of other firms. At a typical point in time, the first order condition for local sales is:

$$\frac{2(L-L_I-mG)}{(\rho+2L_I)(\sigma-1)} + \frac{-2}{(\sigma-1)}L_{Ii}\left(\frac{(\rho+2L_I) + 2(L-L_I-mG)}{(\rho+2L_I)^2}\right) = 1$$

This rearranges to give:

$$\frac{(L-L_I-mG)}{K(\rho+2L_I)(\sigma-1)}\left(1 - \frac{sL_I\,(\rho+2L-2mG)}{(L-L_I-mG)(\rho+2L_I)}\right) = \frac{1}{2K} = F;\quad s \equiv \left(\frac{L_{Ii}}{L_I}\right)$$

Using the definition of V generates the expression in the text. The analysis is similar for the export sales condition.

4. The intermediate steps are:

$$G = \dot{K}_i\,(V-F) + \dot{K}^*_i\,(V-\Gamma F) = \frac{Vs}{\varepsilon}\dot{K}_i + \frac{Vs^*}{\varepsilon}\dot{K}^*_i$$

but since $s=(d_iK/dt)/(dK/dt)$, and $dK/dt = 2KL_I$,

$$G = 2\left(\frac{Vs}{\varepsilon}s + \frac{Vs^*}{\varepsilon}s^*\right)KL_I$$

5. We find this share as usual in an oligopoly model. Defining $S=ms$ and using $S^*=1-S$

$$\frac{V(1-s^*/\varepsilon)}{V(1-s/\varepsilon)} = \frac{\Gamma F}{F} \quad <=> \quad (1-S^*/m\varepsilon) = \Gamma(1-S/m\varepsilon)$$

$$<=> \quad (m\varepsilon - 1 + S) = \Gamma(m\varepsilon - S) \quad <=> \quad S = \Gamma m\varepsilon - \Gamma S + 1 - m\varepsilon$$

$$<=> \quad ms = S = \frac{m\varepsilon((\Gamma-1)+1}{(1+\Gamma)}$$

6. Given the definitions for p* and Ks*,

$$t\frac{Ks^*}{p^*} = t\frac{E(p^*)^{-\sigma}}{p^{1-\sigma}+(p^*)^{1-\sigma}} = \frac{tE\tau^{-\sigma}}{1+\tau^{1-\sigma}}(1-1/\sigma)$$

so

$$E = L - L_I + \frac{E}{\sigma} + \frac{Et\tau^{-\sigma}(\sigma-1)}{\sigma(1+\tau^{1-\sigma})} = \frac{L-L_I}{1-\Delta-1/\sigma};$$

$$\Delta \equiv \frac{t\tau^{-\sigma}(\sigma-1)}{\sigma(1+\tau^{1-\sigma})}$$

7. The intermediate steps are:

$$F = \frac{P_X}{K^{\Omega}(1+\lambda)} = P_X\left((1+\lambda)K^{1-\frac{1}{\sigma-1}}\right)^{-1}$$

Noting that local varieties cost $1/(1-1/\sigma)$, imported varieties cost $\tau/(1-1/\sigma)$ and expanding the CES price index, we have:

$$F = \frac{P_X}{(1+\lambda)K^{1+\frac{1}{1-\sigma}}} = \frac{\left(K(1+\tau^{1-\sigma})\right)^{\frac{1}{1-\sigma}}/(1-1/\sigma)}{(1+\lambda)K^{1+\frac{1}{1-\sigma}}} = \frac{(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}}{(1+\lambda)K(1-1/\sigma)}$$

8. Defining x_I and x_I* as the amounts of typical local and nonlocal X-varieties used in K production, Q_I as the total amount a typical X-sector firm ships to the I-sectors, and Ψ as the fraction of this shipped to the nonlocal I sector, we have:

$$X_I = \left(K(x_I^{1-1/\sigma} + (x_I^*)^{1-1/\sigma}\right)^{\frac{1}{1-1/\sigma}}$$

$$= K^{\frac{\sigma}{\sigma-1}}\left(((1-\Psi)Q_I)^{\frac{\sigma-1}{\sigma}} + (\frac{\Psi Q_I}{\tau})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Since $Q_I = L_I/K$ where L_I is defined as the amount X-sector L whose output is earmarked for sale to the I-sector, we have:

$$X_I = L_I K^{\frac{\sigma}{\sigma-1}}\left((\frac{1-\Psi}{K})^{\frac{\sigma-1}{\sigma}} + (\frac{\Psi}{K\tau})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$= L_I K^{\frac{1}{\sigma-1}}\left((1-\Psi)^{\frac{\sigma-1}{\sigma}} + (\frac{\Psi}{\tau})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Now manipulating the X-sector first order conditions:

$$\frac{p^*}{p} = \tau = (\frac{x^*}{x})^{-1/\sigma} \quad <=> \quad x^* = x\tau^{-\sigma} \quad <=>$$

$$Q_I = x + \tau x^* = x(1+\tau^{1-\sigma}) \;,=> \; 1-\Psi = \frac{1}{1+\tau^{1-\sigma}}$$

we see $\Psi = \tau^{1-\sigma}/(1+\tau^{1-\sigma})$, so:

$$X_I = \frac{L_I K^{\frac{1}{\sigma-1}}}{1+\tau^{1-\sigma}}\left(1+(\tau^{-\sigma})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \frac{L_I K^{\frac{1}{\sigma-1}}}{1+\tau^{1-\sigma}}\left(1+\tau^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}$$

$$= L_I K^{\frac{1}{\sigma-1}}\left(1+\tau^{1-\sigma}\right)^{\frac{-1}{1-\sigma}}$$

The formula in the text is found by plugging this into the I-sector production function.

9. Since $Y=E+I=L+\pi K$, and $\pi = Y/\sigma K$, $Y = L/(1-1/\sigma)$.

10. Using F from endnote (7) we have:

$$q = \frac{\pi}{(\rho+g)F} = \frac{L(1+\lambda)}{\sigma(\rho+g)(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}} = \frac{L(1+\lambda)}{\sigma(\rho + \frac{L_I(1+\lambda)}{(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}})(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}}$$

which after simplification gives the expression in the text.

11 $$\frac{\pi}{(R+g)F}=1 \Leftrightarrow \pi=\frac{R+2L_I}{2K}$$

Now $\pi = E/\sigma K$ and $E = L - L_I - nB + \pi K$ so that

$$\pi=\frac{L-L_I-nB}{K(\sigma-1)}$$

Combining the two expressions for π gives the expression in the text.

12. The intermediate steps are:

$$\pi_H = \frac{\mu E^H s_H^H}{\sigma} + \frac{\mu E^F s_H^F}{\sigma}$$

$$s_H^H = \frac{p^{1-\sigma}}{K_H\left(p^{1-\sigma}+\phi(p\tau^H)^{1-\sigma}\right)} = \frac{1/K_H}{1+\phi(\tau^H)^{1-\sigma}}$$

$$s_H^F = \frac{(\tau^F p)^{1-\sigma}}{K_H(\phi p^{1-\sigma}+(p\tau^F)^{1-\sigma})} = \frac{1/K_H}{1+\phi(\tau^F)^{\sigma-1}} ht)$$

$$\pi_H = \frac{\mu E^H}{\sigma K_H}\left(\frac{1}{1+\phi(\tau^H)^{1-\sigma}} + \frac{\zeta}{1+\phi(\tau^F)^{\sigma-1}}\right)$$

13. Here s_H^H is - from the CES demand function - is equal to: $pc/E=(p)^{1-\sigma}/(K(p)^{1-\sigma}+K^*(\tau p^*)^{1-\sigma})=(p)^{1-\sigma}/K((p)^{1-\sigma}+\phi(p^*)^{1-\sigma})$, where "*" indicates foreign variables and s_F^H is $(\tau p^*)^{1-\sigma}/K((p)^{1-\sigma}+\phi(p^*)^{1-\sigma})$, so $(s_H^H+\phi s_F^H)=1$.

14. Using that $E^j=\dfrac{w_j(L_j-L_{Ij})}{1-\mu/\sigma}$ the log time derivative of E is:

$$\frac{\dot{E}^j}{E^j} = \frac{\dot{w}_j}{w_j} - \frac{L_{Ij}}{L_j - L_{Ij}}\left(\frac{\dot{L}_{Ij}}{L_{Ij}}\right)$$

The expression for r is given by:

$$r_j = \frac{\pi_j}{J_j} + \frac{\dot{J}_j}{J_j}$$

and comes from differentiating the general definition of J using Liebnitz's rule. That is, at $t=0$, J(0) equals $\int_{t=0}^{\infty} e^{-R(t)}\pi(t)dt$, where the discount factor R(t) equals $\int_{s=0}^{t} r(s)ds$. Log differentiation of J(0) with respect to time yields $\pi(0)-r(0)J(0)$. By perfect competition in the I sector, J=F, so using (2-4), (A-2) and (2-6) we get the expressions for the r's that are in the text.

15. The Hamiltonian can be written as $e^{-\rho t}\ln(E) - e^{-\rho t}\ln(P) + \zeta(dK/dt)$. Since with symmetry:

$$\ln E^H = \left(\ln w + \ln(L - L_I) - \ln(1 - \mu / \sigma)\right)$$

$$\ln P = (1-\mu)\ln p_z + \mu \ln\left(K^{\frac{1}{1-\sigma}} \frac{(w_H + \phi \tau^{1-\sigma})^{\frac{-1}{1-\sigma}}}{1 - 1/\sigma} \right)$$

Since w=1, p_z =1, so simplification and substitution imply:

$$\ln E = \ln(L - L_I) - \ln(1 - \frac{\mu}{\sigma}), \quad \ln P = \frac{\mu \ln K}{1-\sigma} - \mu \ln(1 - \frac{1}{\sigma}) + \frac{\mu \ln(1 + \phi \tau^{1-\sigma})}{1-\sigma}$$

So

$$H[L_I, K, \zeta] = e^{-\rho t}\left(\ln(L - L_I) - \frac{\mu \ln K}{1-\sigma}\right) + \zeta L_I K(1+\lambda) + constants$$

It is useful to note that the last term equals $\zeta L_I F$, where F is a function of K.

16. Rearranging, the necessary conditions are:

$$\zeta = \frac{e^{-\rho t}}{K(1+\lambda)(L - L_I)}, \quad \frac{\mu e^{-\rho t}}{K(\sigma - 1)} + \zeta L_I (1+\lambda) = -\dot{\zeta}$$

The first necessary condition defines ζ. Log differentiating this condition with respect to time (noting that only ζ and K move on the steady-state growth path), we have the first necessary condition:

$$\frac{\dot{\zeta}}{\zeta} = -\rho - \frac{\dot{K}}{K}$$

Using the definition of ζ and the expression for $g=L_I(1+\lambda)$ in the second necessary condition, we have:

$$K(1+\lambda)(L - L_I)(\frac{\mu}{K(\sigma-1)}) + L_I(1+\lambda) = -\frac{\dot{\zeta}}{\zeta}$$

$$<=> \frac{\mu(L - L_I)/K(\sigma-1)}{1/K(1+\lambda)} + g = -\frac{\dot{\zeta}}{\zeta}$$

$$<=> (\frac{\sigma-\mu}{\sigma-1})\frac{\mu(L - L_I)/K(\sigma-\mu)}{1/K(1+\lambda)} + g = -\frac{\dot{\zeta}}{\zeta}$$

Expression (2-3) and (2-4) are used to simplify the second condition, so we have:

$$-\frac{\dot{\zeta}}{\zeta} = \rho + g, \quad (\frac{\sigma-\mu}{\sigma-1})(\frac{\pi}{F}) + g = -\frac{\dot{\zeta}}{\zeta}$$