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EXPECTATION TRAPS AND DISCRETION

ABSTRACT

We argue that discretionary monetary policy exposes the economy to welfare-decreasing instability. It does so by creating the potential for private expectations about the response of monetary policy to exogenous shocks to be self-fulfilling. Among the many equilibria that are possible, some have good welfare properties. But others exhibit welfare-decreasing volatility in output and employment. We refer to the latter type of equilibria as *expectation traps*. In effect, our paper presents a new argument for commitment in monetary policy because commitment eliminates these bad equilibria. We show that full commitment is not necessary to achieve the best outcome, and that more limited forms of commitment suffice.

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1. Introduction

The classic literature on discretionary monetary policy attributes persistent episodes of inflation to central bankers' willingness to purposefully initiate and pursue inflationary policies. This willingness is said to reflect the appeal of inflation as a source of revenue and as a tool for exploiting perceived short-run Phillips curve trade-offs. This paper offers a new argument about why inflationary episodes can arise under discretion. Our argument is that, under discretion, policymakers can also be *pushed* into pursuing inflationary policies. This can happen when the private sector, for whatever reason, expects high inflation. Under these circumstances, the central banker may find it optimal to accommodate private agents' expectations if the cost of not doing so is a recession. When the monetary authority does accommodate, private agents' expectations are self-fulfilling and we say the economy has fallen into an *expectation trap*.[1] The expectation traps emphasized in this paper arise from agents' expectations about how monetary policy will respond to exogenous shocks. We argue that the notion of a policy maker caught in an expectations trap is potentially helpful in accounting for inflationary episodes like the one endured by the U.S. in the mid 1960s and 1970s.

The expectation traps that we analyze are of two types. The first arises when agents expect that monetary policy will react to shocks which do not affect preferences or technology. If these expectations about monetary policy are self-fulfilling, non-fundamental shocks con-

[1] See William Fellner (1976, pp.116-118) for an early discussion of the idea that expectations about monetary policy could be self-fulfilling by forcing the hands of benevolent policy makers. For a more recent discussion of these issues, see Ball (1991, pp. 448-449).

stitute an additional set of impulses to the economy. The second type of expectation trap can arise when agents' expectations of how monetary policy will respond to fundamental shocks are self-fulfilling. This can cause actual monetary policy to overreact to fundamental shocks. Here, expectation traps amplify volatility by modifying the impact and propagation mechanisms from fundamental shocks to the economy. They could, for example, cause a real shock to lead to a persistent change in the inflation rate even though it would only produce a change in the level of prices under a non-accommodative monetary policy. In either case, expectation traps are a source of welfare-decreasing instability in the economy.

We articulate the preceding arguments formally using a general equilibrium monetary model which is similar to that of Ireland (1995). The model economy is populated by a representative household and three types of firms: a competitive producer of final goods, a continuum of monopolists, each of whom produces an intermediate good, and a financial intermediary. The household purchases the final consumption good, supplies labor to intermediate good firms, and lends funds to the financial intermediary. The financial intermediary combines funds received from the households with lump-sum injections of money from the government, and makes loans to firms. Firms need loans because they must pay labor before they sell their output. There is a monetary authority which chooses monetary transfers to the financial intermediary with the objective of maximizing the expected utility of the representative household. We model discretion by assuming that the policy maker acts sequentially through time and cannot commit to any future action.

The sequence of events within a period is as follows. First, all exogenous shocks, including both fundamental and non-fundamental shocks, are realized. Then, intermediate goods producers set their prices based on their expectation of the current-period money growth rate. Finally, the monetary authority's action is realized, and all other model variables are determined, with the output of the intermediate good being demand determined.

As in Kydland and Prescott (1979), Barro and Gordon (1983) and Ireland (1995), our model is one in which high expected money growth is socially costly while unexpected money growth confers social benefits. High anticipated money growth is costly because it leads to higher inflation which acts as a tax on employment. Unexpected money growth raises output in the model because intermediate goods producers post prices before the realization of the monetary policy action and output is demand determined. This rise in output is socially beneficial because output is inefficiently low reflecting the existence of monopoly power in the intermediate goods industry. This inefficiency cannot be eliminated, in equilibrium, by monetary policy.

To establish that expectation traps can occur without commitment, we characterize the space of sustainable outcomes for our model economy. We do this by adapting a characterization theorem due to Chari and Kehoe (1990,1993a, 1993b), based on Abreu (1988). Specifically, we establish a set of restrictions on a given outcome for it to be the outcome of some sustainable equilibrium. We then show, by way of two examples, that both types of expectation traps discussed above are elements of the space of sustainable outcomes. In the first example, there are no fundamental shocks. Money growth follows a first order autoregression, with the disturbance term determined by a non-fundamental shock. We find this example interesting for two reasons. First, it demonstrates in a particularly simple way, the possibility of instability under discretion. Second, it offers an alternative perspective on the empirical literature on monetary policy shocks. This literature seeks to identify an exogenous component in monetary policy. The conventional interpretation of these shocks is that they represent exogenous randomness inherent within the policy making process. Our results suggest that this randomness may also originate from outside the policy making process and reflect the sort of self-fulfilling expectations that can arise under discretion.

In our second example of an expectations trap, there are only fundamental shocks. We show that there is a sustainable outcome in which money growth is typically low. Occasionally, following a decline in the state of technology, money growth and, hence, inflation rise for an extended period of time before returning to their typically low levels. We find this example interesting because it illustrates another sense in which discretion makes possible volatile equilibria. Also, the example may form the basis for a simple explanation of the prolonged rise in inflation starting in the mid-1960's. Specifically, it articulates the notion that the US fell into an expectation trap in which a benevolent, sequentially optimizing monetary authority pursued a policy which exacerbated the loss in output caused by various fundamental shocks. Economic theory, certainly of the standard monetarist variety, is at a loss to explain how a real shock can trigger a long term rise in inflation. It is also at a loss to explain why the monetary authority would have responded to such shocks by shifting for a period of years to a higher money growth regime which most contemporary observers deplored as being destructive to the economy.

The literature does contain discussions about the link between real shocks and long lived episodes of inflation. Many of these discussions involve ideas related to those lying at the heart of our second example of an expectation trap. For example, these ideas play an important role in discussions of the wage price spiral, as well as the notion that an 'overheating' economy, i.e. one with high sustained rates of capacity utilization, can trigger a rise in the core inflation rate. They are also implicit in Blinder's (1982) discussion of how supply shocks could have contributed to the persistent inflation of 1970s:

> 'Inflation from special factors can "get into" the baseline rate if it causes an acceleration of wage growth. At this point policymakers face an agonizing choice—the so-called accommodation issue. To the extent that aggregate nominal demand is *not* expanded to accommodate the higher wages and prices, unemployment and slack capacity will result. There will be a recession. On the other hand, to the extent that aggregate demand *is* expanded (say, by raising the growth rate of money above previous targets), inflation from the special factor will get built into the baseline rate.' (Blinder (1982, p. 264))

These ideas were not confined to academics. Our reading of Arthur Burns' speeches suggests that his views about the genesis of the inflation of the mid 1960's and 1970's have much in common with the notion that the economy was caught in an expectation trap, triggered by transitory real shocks. The principal real shock, in his view, was the expansionary fiscal policy ('the forces of excess demand') associated with the Great Society programs of the 1960s and the Vietnam war.[2] As Burns put it around the time he became Chairman of the Federal Reserve in 1970:

> 'The forces of excess demand that originally led to price inflation disappeared well over a year ago. Nevertheless, strong and stubborn inflationary forces, emanating from rising costs, linger on...' (Burns (1978, p.124).)

A key factor fueling rising costs, according to Burns, was the widespread expectation that the inflation which started in the mid 1960s, would continue. He described his vision of how inflationary expectations drive up costs, and hence prices, in a statement before the Joint Economic Committee of the U.S. Congress in 1971:

[2] Burns believed that crop failures and oil shocks were other real disturbances that contributed to the inflation of the 1970s

'Consumer prices have been rising steadily since 1965 - much of the time at an accelerating rate. Continued substantial increases are now widely anticipated over the months and years ahead...in this environment, workers naturally seek wage increases sufficiently large...to get some protection against future price advances...thoughtful employers...recon, as they now generally do, that cost increases probably can be passed on to buyers grown accustomed to inflation.' (Burns (1978, p.126.)

Like the central banker in our model, Burns clearly understood that this upward pressure on prices could not be transformed into persistent, high inflation without monetary accommodation. As he put it in a speech in 1977:

'Neither I nor, I believe, any of my associates would quarrel with the proposition that money creation and inflation are closely linked and that serious inflation could not long proceed without monetary nourishment. We well know–as do many others–that if the Federal Reserve stopped creating new money, or if this activity were slowed drastically, inflation would soon either come to an end or be substantially checked.' (Burns (1978, p. 417).)

Burns chose to accommodate agents' inflationary expectations for the same reason that monetary authority in our second example does so: to do otherwise would generate a costly recession. As he put in testimony before the Committee on Banking and Currency of the House of Representatives on July 30, 1974:

'...an effort to use harsh policies of monetary restraint to offset the exceptionally powerful inflationary forces in recent years would have caused serious financial disorder and economic dislocation. That would not have been a sensible course for monetary policy.' (Burns (1978, p. 171).)

Burns clearly understood the accommodation dilemma alluded to by Blinder, and judged that the best response was to accommodate. In our terminology, he acted like a benevolent central banker caught in an expectation trap. What Burns did not understand was the critical role the institution of discretion played in setting that trap.

After our formal analysis of discretionary monetary policy, we investigate alternative institutional arrangements that can eliminate the possibility of expectation traps. One solution is full commitment on the part of the monetary authority. An obvious practical problem with this solution is that it is hard to imagine any monetary policy authority committing to a sequence of policy actions infinitely far into the future. More limited forms of commitment are practical. We consider a situation in which the monetary policy authority commits at time t to its time $t+1$ state contingent growth rate of money. We establish that expectation traps cannot occur in this regime and that the equilibrium quantities in this economy coincide with those in the Ramsey equilibrium.

The result that one period ahead limited commitment reduces the set of sustainable equilibria to a singleton - the Ramsey equilibrium - depends on particular features of our model economy. The intuition underlying the proof gives some insight into how it could be extended to accommodate different environments, say, those with multiperiod wage and price contracts.

The next section presents a formal description of our model. Section 3 discusses equilibrium under full commitment. Section 4 characterizes the set of sustainable outcomes under discretion, and section 5 presents our two examples. In section 6 we discuss equilibrium under limited commitment. Finally, we offer some concluding remarks.

2. The Model

We consider an infinite horizon, monetary economy with uncertainty. In each period, $t = 0, 1, 2, \ldots$, the economy experiences an exogenously determined event, s_t, drawn from a finite set. Let $s^t = (s_0, s_1, \ldots, s_t)$ denote the history of exogenous events up to and including time t. The probability of s^t is given by $\mu(s^t)$. We denote the probability of s^t conditional on s^r by $\mu(s^t \mid s^r)$. The monetary authority chooses an action, A_t (a lump sum monetary transfer to financial intermediaries) in period $t = 0, 1, 2, \ldots$. Let $h_t = (s^t, A_0, \ldots, A_t)$ denote the history of exogenous events up to date t, and of policy actions up to time t. Throughout we assume that private agents view A_t as being generated according to the policy rule

$$A_t = X_t(h_{t-1}, s_t).$$

We say that a set of histories h_{t+1}, $h_{t+2}, \ldots$ is induced by $X_t(\cdot, \cdot)$ from $h_t = (h_{t-1}, s_t, A_t)$ if $h_{t+1} = (h_t, s_{t+1}, X_t(h_t, s_{t+1}))$, $h_{t+2} = (h_{t+1}, s_{t+2}, X_{t+1}(h_{t+1}, s_{t+2}))$, ... To conserve on notation, from here on we delete the subscript, t, on X. This should not cause confusion: that the functions $X(h_{t-1}, s_t)$ and $X(h_{r-1}, s_r)$, $r \neq t$ are different is evident from the fact that the number of elements in h_{t-1} and h_{r-1} are different. We adopt this notational convention for all functions of histories.

The commodity space in this economy consists of history contingent functions. That is, allocations, prices and policy actions are expressed as functions of history. In standard Arrow-Debreu economies, rules of this type are a function only of the history of exogenous events. The extension considered here is necessary to accommodate the fact that the government in our model optimizes sequentially and that, being a 'large' agent, its actions have a non-trivial impact on private allocations and prices. At every date the government considers the consequences of all the current and future state-contingent actions that it could possibly take, and selects the set of actions that it prefers. Since its objective is a function of current and future allocations, these must be well-defined for every possible history for the government problem to be well-posed. Histories, h_t, can exclude past household actions because, though individual households optimize sequentially like the government, they are small and have no impact on aggregate allocations and prices (see Chari and Kehoe (1990) for a further discussion.)

To reiterate from the introduction, the sequence of events within a period is as follows. First, the exogenous event, s_t, is realized. Then, intermediate goods producers set the prices of their goods. Finally, the monetary authority's action is realized and all other model variables are determined. We now describe the problems of the agents in our economy in detail.

2.1. Firms

Final Good Firms

The final good, $c(h_t)$, is produced by a perfectly competitive firm that combines a continuum of intermediate goods, indexed by $i \in (0,1)$, using the following technology:

$$c(h_t) = \left[\int_0^1 (y_i(h_t))^\lambda di \right]^{\frac{1}{\lambda}}. \tag{2.1}$$

Here $0 < \lambda < 1$ and $y_i(h_t)$ denotes the time t input of intermediate good i. Let $P_i(h_{t-1}, s_t)$ denote the time t price of intermediate good i. This price is not a function of the time t

policy action because intermediate goods producers set their prices before A_t is realized.

The final good producer's problem is:

$$\max_{c(h_t),\{y_i(h_t)\}} P(h_{t-1}, s_t)c(h_t) - \int_0^1 P_i(h_{t-1}, s_t)y_i(h_t)di, \tag{2.2}$$

subject to (2.1). Here, $P(h_{t-1}, s_t)$ denotes the time t price of the final good. We explain below why this price is not a function of A_t. Problem (2.2) gives rise to the following input demand functions:

$$y_i(h_t) = c(h_t)\left(\frac{P(h_{t-1}, s_t)}{P_i(h_{t-1}, s_t)}\right)^{\frac{1}{1-\lambda}}. \tag{2.3}$$

In conjunction with (2.1), this implies:

$$P(h_{t-1}, s_t) = \left[\int_0^1 P_i(h_{t-1}, s_t)^{\frac{\lambda}{1-\lambda}}di\right]^{\frac{1-\lambda}{\lambda}}, \tag{2.4}$$

which expresses the time t price of the final good as a function of the time t prices of the intermediate goods.

Intermediate Good Firm

Intermediate good i is produced by a monopolist using the following technology:

$$y_i(h_t) = \theta(s^t)n_i(h_t). \tag{2.5}$$

Here $n_i(h_t)$ denotes time t labor used to produce the i^{th} intermediate good and $\theta(s^t)$ denotes the stochastic economy wide level of technology at time t. Recall that $P_i(h_{t-1}, s_t)$ is chosen before the date t government policy action, A_t, is realized. We assume that the producer must supply all of the goods demanded by the final goods producers as determined by (2.3). Thus, at time t, producer i's problem, which we refer to as *Problem F*, is to maximize profits:

$$\max_{P_i(h_{t-1},s_t)} P_i(h_{t-1}, s_t)y_i(h_t) - W(h_t)R(h_t)n_i(h_t), \tag{2.6}$$

subject to (2.3), (2.5) and taking as given the time t wage rate, $W(h_t)$, the gross nominal interest rate, $R(h_t)$, as well as $P(h_{t-1}, s_t)$, and $c(h_t)$. The firm's unit labor costs are $W(h_t)R(h_t)$ because the firms need to pay workers before production, and must borrow these funds from the financial intermediary at interest rate $R(h_t)$.

Note that from the firm's perspective (2.6) is a function of the information set (h_{t-1}, s_t). This is because they view A_t as being generated according to $A_t = X(h_{t-1}, s_t)$ and $h_t = (h_{t-1}, s_t, X(h_{t-1}, s_t))$.

Financial Intermediary

A perfectly competitive financial intermediary receives deposits, $I(h_t)$, from households, and a transfer, A_t, from the government. These funds are lent to intermediate good producers at the gross interest rate $R(h_t)$. At the end of the period, the intermediary pays $R(h_t)I(h_t)$ to households in return for deposits, and distributes $R(h_t)A_t$ to households in the form of profits.

2.2. Household

At time r the representative household's expected present discounted utility is given by:

$$\sum_{t=r}^{\infty} \beta^{t-r} \sum_{s^t} \mu(s^t \mid s^r) U(c(h_t), n(h_t)). \tag{2.7}$$

Here $n(h_t)$ denotes total hours time t hours worked, and

$$U(c,n) = \left[c(1-n)^{\psi}\right]^{(1-\sigma)}/(1-\sigma), \tag{2.8}$$

where $\psi > 0$ and $\sigma > -1$. The household faces the following cash constraint on its purchases:

$$P(h_{t-1}, s_t)c(h_t) \leq W(h_t)n(h_t) + M(h_{t-1}) - I(h_t). \tag{2.9}$$

Here $M(h_{t-1})$ denotes the household's end-of-period $t-1$ holdings of cash.

The household's money holdings evolves according to:

$$M(h_t) = W(h_t)n(h_t) + M(h_{t-1}) - I(h_t) - P(h_{t-1}, s_t)c(h_t) \tag{2.10}$$

$$+R(h_t)\left[I(h_t) + A_t\right] + D(h_t), \tag{2.11}$$

where $D(h_t)$ denotes total time t profits from the intermediate goods producers.

Let

$$Z(h_t) = [c(h_t), n(h_t), I(h_t), M(h_t)].$$

The household's problem, which we refer to as *Problem H*, is then to choose a non-negative contingency plan for $Z(h_t)$ to maximize (2.7) subject to (2.8)-(2.10), $n(h_t) \leq 1$ and $I(h_t) \leq M(h_{t-1})$. In solving its problem, household views A_t as being generated by the policy rule $X(h_{t-1}, s_t)$, so that $h_{r+1} = (h_r, s_{r+1}, X(h_r, s_{r+1}))$ for all r.

2.3. Government

At time t, given history (h_{t-1}, s_t), the government chooses a current action, A_t, and a sequence of state contingent policy rules for each future period, to maximize the representative household's utility, (2.7). It takes as given, $M(h_{r-1}) + A_r \geq 0$, $r \geq t$, and that future histories will be induced by its future policy rules from $h_t = (h_{t-1}, s_t, A_t)$. The government also takes as given the household's contingency plan, $\{Z(h_r); r \geq t\}$. We refer to this problem as *Problem G*.

2.4. Sustainable Equilibrium

In order to characterize the set of equilibrium outcomes, it is useful to define a sustainable equilibrium for our model economy.

Definition 2.1. *A sustainable equilibrium is a collection of history-contingent allocation rules $Z(h_t)$, pricing functions, $P_i(h_{t-1}, s_t)$, $P(h_{t-1}, s_t)$, $W(h_t)$, $R(h_t)$, and a government policy rule, $X(h_{t-1}, s_t)$, such that:*

1. for all histories, h_t, the allocation rules $Z(h_t)$, solve *Problem H*,
2. for all histories, (h_{t-1}, s_t), the pricing function $P_i(h_{t-1}, s_t)$ solves *Problem F*,
3. for all histories, (h_{t-1}, s_t), producers of the final good make zero profits so that $P(h_{t-1}, s_t)$ satisfies (2.4),
4. for all histories, h_t, the goods market clears, i.e., $c(h_t) = \theta(s^t)n(h_t)$, the loan market clears, i.e., $W(h_t)n(h_t) = I(h_t) + A_t$, and the money market clears, i.e., $M(h_t) = M(h_{t-1}) + A_t$.
5. for all histories, (h_{t-1}, s_t), the policy rule, $X(h_{t-1}, s_t)$ solves *Problem G*.

To proceed we find it convenient to scale all time t nominal variables by the end of time $t-1$ stock of money, $M(h_{t-1})$. Let

$$p_i(h_{t-1}, s_t) = \frac{P_i(h_{t-1}, s_t)}{M(h_{t-1})},\ p(h_{t-1}, s_t) = \frac{P(h_{t-1}, s_t)}{M(h_{t-1})},\ d(h_t) = \frac{D(h_t)}{M(h_{t-1})},$$

$$w(h_t) = \frac{W(h_t)}{M(h_{t-1})},\ i(h_t) = \frac{I(h_t)}{M(h_{t-1})},\ x(h_{t-1}, s_t) = \frac{X(h_{t-1}, s_t)}{M(h_{t-1})}, a_t = \frac{A_t}{M(h_{t-1})}.$$

In addition, we define the variable $m(h_{t-1})$ as the ratio of the household's time $t-1$ stock of money to the aggregate stock of time $t-1$ money. Of course in equilibrium $m(h_{t-1}) = 1$. Finally, let

$$z(h_t) = [c(h_t), n(h_t), i(h_t), m(h_t)]. \tag{2.12}$$

In what follows, we proceed in terms of these lower case scaled variables.

3. Private Sector Equilibrium Under Commitment

A key objective of our paper is to characterize the set of allocations that are the outcome of some sustainable equilibrium in the model economy. To do this it is useful to first analyze the behavior of the economy when the government can commit to a particular policy rule. Specifically we consider the case of commitment, by which we mean that the government never deviates from the policy rule, $x(h_{t-1}, s_t)$. Under these circumstances, the set of all possible date t histories can be indexed by s^t alone. This is because, under commitment, h_t can be expressed as a function of s^t using the following recursion:

$$h_{-1} = \emptyset,\ h_0 = (s^0, x(h_{-1}, s_0)),\ h_1 = (s^1, x(h_{-1}, s_0), x(x(h_{-1}, s_0), s_1)),$$

and similarly for $h_2, h_3, ..., h_t$.

Let $z^*(s^t) = [c^*(s^t),\ n^*(s^t),\ i^*(s^t),\ m^*(s^t)]$ denote the state contingent allocations under commitment. In addition, let $[w^*(s^t),\ R^*(s^t),\ p_i^*(s^t),\ p^*(s^t)]$, $x^*(s^t)$ and $d^*(s^t)$ denote the pricing functions, government policy rule, and intermediate good firm profit function under commitment, respectively.

Definition 3.1. *A private sector equilibrium is a collection of state contingent allocation rules, $z^*(s^t)$, pricing functions, $[w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t)]$, and a government policy rule, $x^*(s^t)$, such that (1) the allocation rules $z^*(s^t)$ solve the household's problem, (2) the pricing function $p_i^*(s^t)$ solves the intermediate good producers' problem, (3) producers of the final good make zero profits, (4) the goods market clears, i.e., $c^*(s^t) = \theta(s^t)n^*(s^t)$, the loan market clears, i.e., $w^*(s^t)n^*(s^t) = i^*(s^t) + x^*(s^t)$, and the money market clears, i.e., $m^*(s^t) = 1$.*

Note that a private sector equilibrium satisfies all the conditions for a sustainable equilibrium, with two exceptions. First, it does not require optimality by the government. Second, private sector allocation rules and pricing functions solve the private sector optimization problems and satisfy market clearing only for histories induced by $x^*(s^t)$.

We now characterize the private sector equilibrium. The first order condition of the intermediary goods firm's problem is:

$$\lambda\theta(s^t)\left(\frac{c^*(s^t)}{y_i^*(s^t)}\right)^{1-\lambda} = \frac{w^*(s^t)}{p^*(s^t)}R^*(s^t).$$

In symmetric equilibria, $y_i^*(s^t) = c^*(s^t)$ for all i, so that,

$$\lambda\theta(s^t) = \frac{w^*(s^t)}{p^*(s^t)}R^*(s^t). \tag{3.1}$$

The Lagrangian representation of the household's problem at time 0 is:

$$\sum_{t=0}^{\infty}\beta^t\sum_{s^t}\mu(s^t|s^0)\{u(c^*(s^t), n^*(s^t))$$
$$+\nu(s^t)\left[m^*(s^{t-1}) - i^*(s^t) + w^*(s^t)n^*(s^t) - p^*(s^t)c^*(s^t)\right]$$
$$+\eta(s^t)[\Big(w^*(s^t)n^*(s^t) + m^*(s^{t-1}) - i^*(s^t) - p^*(s^t)c^*(s^t)\Big)$$

$$R^*(s^t)(i^*(s^t) + x(s^t)) + d^*(s^t) - (1 + x(s^t))m^*(s^t)]\},$$

where $\nu(s^t), \eta(s^t)$ are the non-negative Lagrange multipliers on the cash constraint and the budget constraints, respectively. In addition to the budget and cash constraints, the first order conditions for $c^*(s^t)$, $n^*(s^t)$, $m^*(s^t)$ and $i^*(s^t)$ are:

$$u_c(s^t) = (\nu(s^t) + \eta(s^t))p^*(s^t), \tag{3.2}$$

$$u_n(s^t) = -(\nu(s^t) + \eta(s^t))w^*(s^t), \tag{3.3}$$

$$(1 + x^*(s^t))\eta(s^t) = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t)\eta(s^{t+1})R^*(s^{t+1}), \tag{3.4}$$

$$\nu(s^t) + \eta(s^t) = \eta(s^t)R^*(s^t), \tag{3.5}$$

where $s^{t+1}|s^t$ denotes the set of time $t+1$ histories of exogenous events s^{t+1} that are consistent with s^t. Also, $u_c(s^t)$ and $u_n(s^t)$ denote the marginal utility of consumption and labor at date t. The transversality condition for the household problem is

$$\lim_{T\to\infty} \sum_{s^T} \beta^T \mu(s^T) u_c(s^T) \frac{1 + x^*(s^T)}{p^*(s^T)} = 0. \tag{3.6}$$

These conditions can be simplified as follows. From (3.2) and (3.3), we obtain:

$$\frac{w^*(s^t)}{p^*(s^t)} = \frac{\psi c^*(s^t)}{1 - n^*(s^t)}, \tag{3.7}$$

according to which the household equates the real time t wage rate to the marginal rate of substitution between consumption and leisure. Non-negativity of the Lagrange multipliers and (3.5) imply

$$R^*(s^t) \geq 1. \tag{3.8}$$

Relations (3.2), (3.4) and (3.5) imply

$$\frac{u_c(s^t)}{p^*(s^t)} = \beta \frac{R^*(s^t)}{1+x^*(s^t)} \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{u_c(s^{t+1})}{p^*(s^{t+1})}, \tag{3.9}$$

According to (3.9), the household equates the marginal utility of spending an additional dollar on consumption to the expected marginal utility associated with investing an additional dollar with the financial intermediary.

Recall that loan market clearing requires

$$w^*(s^t)n^*(s^t) = i^*(s^t) + x^*(s^t). \tag{3.10}$$

Substituting (3.10) into the scaled version of the household's cash constraint, (2.9) yields:

$$p^*(s^t)c^*(s^t) = 1 + x^*(s^t). \tag{3.11}$$

Goods market clearing implies:

$$c^*(s^t) = \theta(s^t)n^*(s^t). \tag{3.12}$$

Using our assumed functional form for the utility function, equations (3.1), (3.9)-(3.12) reduce to:

$$\sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) v\left(n^*(s^t), n^*(s^{t+1}), \theta(s^t), \theta(s^{t+1}), x^*(s^{t+1})\right) = 0, \tag{3.13}$$

where

$$v(n, n', \theta, \theta', x') = \frac{n}{1-n}\left[\theta n(1-n)^{\psi}\right]^{1-\sigma} - \frac{\lambda\beta}{\psi(1+x')}\left[\theta' n'(1-n')^{\psi}\right]^{1-\sigma}. \tag{3.14}$$

Finally, substituting (3.11) and (3.12) into the transversality condition, (3.6), we obtain:

$$\lim_{T\to\infty} \sum_{s^T} \beta^T \mu(s^T)\left[\theta(s^T) n^*(s^T)(1-n^*(s^T))^{\psi}\right]^{1-\sigma} = 0. \tag{3.15}$$

We are now in a position to characterize a private sector equilibrium.

Proposition 3.2. *The allocation rule $n^*(s^t)$ is part of a private sector equilibrium if and only if the following five conditions are satisfied: $n^*(s^t) \geq 0$, (3.13), (3.15),*

$$\frac{\lambda}{\psi}\frac{1-n^*(s^t)}{n^*(s^t)} \geq 1, \tag{3.16}$$

and

$$\psi\frac{n^*(s^t)}{1-n^*(s^t)} \geq \frac{x^*(s^t)}{1+x^*(s^t)}. \tag{3.17}$$

Proof. Suppose $n^*(s^t)$ satisfies the five conditions. Then we can construct the remaining objects in a private sector equilibrium as follows. The consumption allocation rule is $c^*(s^t) = \theta(s^t)n^*(s^t)$, the price rule is obtained from the cash constraint, (3.11). The nominal wage is obtained from (3.7), the nominal interest rate from (3.1) and, finally, $i^*(s^t)$ is obtained from (3.10). The only conditions that remain to be checked are (3.8) and the constraint on the household's problem that $0 \leq i^*(s^t) \leq 1$. The first is verified by noting that the expression to the left of the inequality in (3.16) is the equilibrium rate of interest. To verify the constraints on $i^*(s^t)$ notice that $i^*(s^t) = (1+x^*(s^t))\frac{\psi n^*(s^t)}{1-n^*(s^t)} - x^*(s^t)$. The requirement that $i^*(s^t) \leq 1$ is guaranteed by (3.16) and the fact, $\lambda \leq 1$. Inequality (3.17) guarantees that $0 \leq i^*(s^t)$.

Suppose we have an equilibrium. The proof follows immediately from necessity of (3.13), (3.15), $R^*(s^t) \geq 1$, and $0 \leq i^*(s^t) \leq 1$. ■

In general, the set of private sector equilibria under commitment in our economy is not unique. The expectation traps that we focus on in this paper have nothing to do with this form of multiplicity. Consequently, it is useful to proceed under assumptions which guarantee the uniqueness of a private sector equilibrium under commitment. One such assumption is $\sigma = 1$, in which case, equation (3.13) reduces to

$$\frac{\psi}{\lambda}\frac{n^*(s^t)}{1-n^*(s^t)} = \beta\sum_{s^{t+1}}\mu(s^{t+1}|s^t)\frac{1}{1+x^*(s^{t+1})}. \tag{3.18}$$

Consequently, if an equilibrium exists, then it is unique. This equilibrium has standard properties: employment is increasing in λ, i.e., decreasing in the degree of market power; employment is decreasing in money growth; and it is decreasing in ψ, which governs the marginal utility of leisure.

To see the potential for non-uniqueness when $\sigma \neq 1$, consider the case $\theta(s^t) \equiv 1$ and $x^*(s^t) \equiv x$. In this case, there exists an equilibrium in which employment is given by the deterministic analog of (3.18). However, there may be other equilibria as well, even sunspot equilibria (see Mathene (1995) and Woodford (1986).) To eliminate these possibilities, henceforth we restrict ourselves to the case $\sigma = 1$.[3]

[3]To see how sunspot equilibria may arise with $\sigma \neq 1$, consider the case in which the monetary growth rate is a constant, equal to x, and $\theta \equiv 1$. Then (3.13) requires $v = 0$. This implicitly defines n' as a function (possibly a correspondence) from n. The implicit function theorem guarantees that there exists a differentiable function relating n' to n in a neighborhood of steady-state: $n = n' = \lambda\beta/[\psi(1+x)]$, with:

$$\frac{dn'}{dn} = \frac{2-\sigma-(\psi(1-\sigma)-1)\frac{\lambda\beta}{\psi(1+x)}}{(1-\sigma)\left[1-\frac{\lambda\beta}{1+x}\right]}.$$

Here, $dn'/dn = 0.77$ when $\sigma = 7$, $\lambda = 0.87$, $\psi = 3$, $\beta = 1/1.03$. The fact that this derivative is less than one makes it possible to construct multiple deterministic equilibria and sunspot equilibria.

3.1. The Ramsey Equilibrium

As a benchmark, it is useful to consider the equilibrium of the model under the assumption that the government had access to a commitment technology at $t = 0$. To this end we now define a Ramsey equilibrium.

Definition 3.3. *A Ramsey equilibrium is a collection of allocation rules for private agents, $z^*(s^t)$, pricing functions $[w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t)]$, and a government policy rule, $x^*(s^t)$, that yields the highest utility for households over the set of private sector equilibria.*

Proposition 3.4. *The Ramsey equilibrium satisfies*

$$\frac{w^*(s^t)}{p^*(s^t)} = \lambda\theta(s^t),\ n^*(s^t) = \frac{\lambda}{\psi + \lambda},\ R^*(s^t) = 1, \tag{3.19}$$

$$\sum_{s^{t+1}} \mu(s^{t+1}|s^t)\frac{1}{1 + x^*(s^{t+1})} = \frac{1}{\beta},\ \lambda \geq \frac{x^*(s^t)}{1 + x^*(s^t)}. \tag{3.20}$$

Proof. ¿From Proposition 1, the Ramsey equilibrium maximizes discounted utility subject to $n^*(s^t) \geq 0$, (3.13), (3.15), (3.16), and (3.17). With $\sigma = 1$, equation (3.13) reduces to (3.18). It follows from this that solving the Ramsey problem is equivalent to solving a sequence of static problems subject to the constraints (3.15) - (3.17). The first expression in (3.19) follows from the first order condition of intermediate good firms. Condition (3.16), which requires a positive interest rate, implies

$$\frac{\psi}{\lambda}\frac{n^*(s^t)}{1 - n^*(s^t)} \leq 1. \tag{3.21}$$

But as long as (3.21) is satisfied, the period utility function is strictly increasing in $n^*(s^t)$. Because the left side of (3.18) is increasing in $n^*(s^t)$, the solution to the Ramsey problem is to set $n^*(s^t)$ so that (3.21) holds with equality. This establishes that $R^*(s^t) = 1$, $n^*(s^t) = \frac{\lambda}{\psi + \lambda}$ and that the right hand side in (3.18) equals one. This last result implies the first expression in (3.20). Since $n^*(s^t)$ is constant, the transversality condition is satisfied. Finally the last expression in (3.20) follows by substituting (3.16) with equality into (3.17). ■

Notice that even absent uncertainty in technology, there are many rules for $x^*(s^t)$ that satisfy (3.20), which only requires that the time t conditional expectation of $1/[1 + x^*(s^{t+1})]$ be equal to $1/\beta$. So a constant growth rate of money equal to $\beta - 1$ is sufficient but not necessary to implement the Friedman rule. Finally, note that even in the Ramsey equilibrium, as long as there is monopoly ($\lambda < 1$), employment is less than the social optimum, $1/(1+\psi)$. The monetary authority would like to increase employment beyond the Ramsey, but this is not achievable by monetary policy because of the non-negativity constraint on $R(s^t)$.

4. Characterizing Sustainable Outcomes

We now turn to the task of characterizing the set of sustainable outcomes for our model economy. To do this we must allow for allocation rules, pricing functions and government policy rules to be contingent on both the history of exogenous events and the history of policy actions. Throughout we impose the restriction that $x(h_{t-1}, s_t) \leq \bar{x}$ for all (h_{t-1}, s^t), i.e. we impose an upper bound on the growth rate of the money supply. See the conclusion for an extended discussion of this assumption.

We begin by characterizing the worst sustainable equilibrium which we refer to as the 'high inflation' equilibrium. Consider the following candidate sustainable equilibrium in which the government policy rule is given by $x(h_{t-1}, s_t) = \bar{x}$ for all possible (h_{t-1}, s^t). Let x_t denote the actual time t policy action. For all histories (h_{t-1}, s^t), the candidate pricing functions are,

$$p_i(h_{t-1}, s_t) = p(h_{t-1}, s_t) = \frac{1+\bar{x}}{\theta(s^t)} \frac{\psi(1+\bar{x}) + \lambda\beta}{\lambda\beta}. \tag{4.1}$$

These candidate pricing functions reflect our assumptions that (i) firms set prices in period t prior to the realization of the period t policy action, and (ii) they do so under the expectation that the policy action will be $x(h_t, s_{t+1}) = \bar{x}$.

The candidate allocation rules, interest rate and wage functions are

$$c(h_t) = \frac{(1+x_t)}{p(h_{t-1}, s_t)},\ n(h_t) = \frac{c(h_t)}{\theta(s^t)},\ R(h_t) = \frac{\lambda}{\psi}\frac{(1-n(h_t))}{n(h_t)}, \tag{4.2}$$

$$w(h_t) = \frac{p(h_{t-1}, s_t)\psi\theta(s^t)n(h_t)}{1-n(h_t)},\ i(h_t) = 1-(1+x_t)\left[1-\frac{\psi n(h_t)}{1-n(h_t)}\right], \tag{4.3}$$

respectively.

By construction, these candidate rules satisfy private agent optimality and market clearing for all possible histories, h_t. So to establish that the candidate equilibrium is sustainable, we need only establish that it is consistent with optimization on the part of the government. Consider a one-shot deviation at period t, given some history, (h_{t-1}, s^t). Given our assumed upper bound on money growth, the only feasible one-shot deviation is for the government to set money growth at $x_t < \bar{x}$, and then to return to $\bar{x}$ thereafter. From (4.2), this reduces current consumption and employment. Since utility is strictly increasing in levels of equilibrium employment below $1/(1+\psi)$, this reduces current utility. Future outcomes are unaffected. Thus, the government has no incentive to pursue a one-shot deviation for any history. Since this includes histories in which there have been any number of deviations, it follows that no deviation raises welfare (see Abreu (1988) and Whittle (1983).) This establishes the sustainability of the candidate equilibrium.

To see that the candidate equilibrium is the worst sustainable equilibrium, note that in any other equilibrium, employment at some date is necessarily higher because of (3.18). Thus, utility must be higher too. This establishes the following proposition:

Proposition 4.1. *The high inflation equilibrium is the worst sustainable equilibrium.*

We denote by $u^d(s^t)$ the highest one period utility level associated with a deviation by the government:

$$u^d(s^t) = \max_{-1\le x\le \bar{x}} u\left(\frac{1+x}{p^*(s^t)}, \frac{1+x}{\theta(s^t)p^*(s^t)}\right).$$

Let $x^d(s^t)$ denote the value of x that achieves the optimum. In addition, let $\overline{u}(s^t)$ denote the one period utility level in the high inflation equilibrium. We are now ready to state our main result. The following proposition establishes a set of restrictions on allocations and prices which are necessary and sufficient for them to be the outcomes of some sustainable equilibrium. In what follows, it is useful to define a class of policy rules characterized by a particular trigger strategy, a *grim* trigger. Such a rule specifies that in all histories in which there has been some deviation, $x(h_{t-1}, s_t) = \bar{x}$. We refer to an equilibrium associated with such a policy rule as a *grim trigger equilibrium.*

Proposition 4.2. *Let $z^*(s^t)$, $w^*(s^t)$, $R^*(s^t)$, $p_i^*(s^t)$, $p^*(s^t)$ be an arbitrary set of allocations and prices. Let $x^*(s^t)$ be an arbitrary sequence of government policies. Then, $\{z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t)\}$ is the outcome of some sustainable equilibrium if and only if:*

1. *$(z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t))$ is a private sector equilibrium;*
2. *$(z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t))$ satisfies*

$$\sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^r} \mu(s^r \mid s^t) U(c^*(s^r), n^*(s^r)) \geq u^d(s^t) + \sum_{r=t+1}^{\infty} \beta^{r-t} \sum_{s=r} \mu(s^r \mid s^t) \bar{u}(s^r) \quad (4.4)$$

for all s^t, all $t = 0, 1, 2, \ldots$.

Proof. We first consider sufficiency. Suppose the allocations and prices satisfy conditions 1 and 2. We construct a particular grim trigger sustainable equilibrium which supports these outcomes. Consider the following candidate equilibrium. For all histories with no deviation, let $x(h_{t-1}, s_t) = x^*(s^t)$, let $p_i(h_{t-1}, s_t) = p_i^*(s^t)$, $p(h_{t-1}, s_t) = p^*(s^t)$, $w(h_t) = w^*(s^t)$, $R(h_t) = R^*(s^t)$ and let $z(h_t) = z^*(s^t)$. For all other histories, let $x(h_{t-1}, s_t) = \bar{x}$, and let $p_i(h_{t-1}, s_t)$, $p(h_{t-1}, s_t)$ be defined by (4.1). Also, let $w(h_t)$, $R(h_t)$ and $z(h_t)$ be defined by (2.12) and (4.1)-(4.2) with x_t replaced by $\bar{x}$. For all histories with no deviation, these constitute a private sector equilibrium by assumption. For all other histories, they constitute a private sector equilibrium by the discussion leading up to Proposition 4.1. To show government optimality, note that since the right hand side of (4.4) is the discounted utility associated with a deviation by the government, no deviation can raise welfare. Thus, the candidate equilibrium is a sustainable equilibrium, which establishes sufficiency.

We now consider necessity. Suppose $(z(h_t), w(h_t), R(h_t), p_i(h_{t-1}, s_t), p(h_{t-1}, s_t), x(h_{t-1}, s_t))$ is a sustainable equilibrium with outcomes $(z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t))$. Condition 1 is satisfied by the definition of a sustainable equilibrium. We establish condition 2 by contradiction. Suppose 2 is violated for some s^t. Consider the following deviation: the government sets $x_t = x^d(s^t)$, and specifies future policies according to $x(h_{r-1}, s_r)$, $r > t$. The expected present discounted value of utility under this deviation is $u^d(s^t) + \sum_{r=t+1}^{\infty} \beta^{r-t} \sum_{s=r} \mu(s^r \mid s^t)\tilde{u}(s^r)$, where $\tilde{u}(s^r)$ denotes utility in state s^r. From the discussion preceding Proposition 4.1, $\bar{u}(s^r)$ is a lower bound on period utility when policy is set according to $x(h_{r-1}, s_r)$. It follows that $\tilde{u}(s^r) \geq \bar{u}(s^r)$. Therefore, the return associated with the deviation is greater than, or equal to, the right hand side of equation (4.4). Given our supposition that condition 2 is violated, this means that the proposed deviation will be implemented at s^t. Consequently, the equilibrium is not consistent with government optimization. This establishes the contradiction. ■

We conclude this section with a brief discussion of the proposition. The proposition says that a particular outcome is sustainable if, and only if, it is the outcome of the associated grim trigger equilibrium. The proposition does *not* say that a particular outcome is sustainable *only* by a grim trigger equilibrium. In general, an outcome may be sustainable by a variety of equilibria. For example, the equilibria could involve less extreme trigger strategies, or no trigger strategy at all. The proposition is silent on the nature of the equilibria that support a given outcome. It only provides conditions under which a particular outcome is sustainable by *some* equilibrium. In the next section, we make these observations concrete through a series of examples.

5. Examples

This section reports two examples which illustrate the types of expectation traps discussed in the introduction. In both cases, we establish that the expectation trap outcome is the outcome of some sustainable equilibrium. Our first example illustrates the type of expectation trap that can arise when agents expect that monetary policy will react to non-fundamental shocks. In addition, we discuss three equilibria that can sustain this outcome. In the second example, agents expect monetary policy to react to technology shocks in a particular way. The example is constructed to articulate in a stylized way an interpretation of the US inflation experience since the mid-1960s.

Example 1: Non-Fundamental Shocks

The parameter values for the model economy are given by $\psi = 3$, $\lambda = 0.95$, $\theta_t \equiv 1$, $\sigma = 1$, $\beta = 1/1.03$, $\bar{x} = 0.30$. In each period there is a shock, $s \in \{s(1), s(2)\}$, which is drawn from a highly persistent, symmetric, two state Markov chain, where the probability of switching states is 0.10. The shock is non-fundamental, in that it has no impact on preferences or technology. Still, there is a sustainable outcome, in which money growth responds to s. When $s = s(1)$ the money growth rate is 0 percent, and when $s = s(2)$, the growth rate of money is 3 percent. The Wold representation for x_t is given by

$$x_t = 0.15 + .9x_{t-1} + \epsilon_t, \tag{5.1}$$

where ϵ_t is uncorrelated with past variables, and has standard deviation 0.0092.

Several features of this example are worth emphasizing. First, the model is observationally equivalent to one in which policy is set according to (5.1). In this respect, the model is formally identical to a standard, monetized real business cycle model (see, for example, the 'cash-in-advance' model in Christiano and Eichenbaum (1996)). The interpretation of the policy shock, however, is very different. In the literature, ϵ_t is assumed to reflect the effects of fundamental disturbances to the policy making process, e.g., shocks to preferences of policy makers. In our environment, the shocks originate outside the policy making process and reflect the non-fundamental disturbances that can generate an expectation trap.

Second, consistent with the previous observations, the dynamic response of real variables to changes in x_t resemble those in standard monetary real business cycle models. For example, Table 1 indicates that employment, consumption and beginning-of-period real balances (M/P) are low when x_t is high, while R_t and inflation are high.

Third, Table 1 contains information relevant for verifying that the outcome in this example is sustainable. The table reports, for both states of the world, the current period utility, u, of the household and the expected present value of its utility from tomorrow on, v, along the equilibrium path. In addition, the table reports, for both states of the world, the current period utility, u^d, and expected value of utility from tomorrow on, v^d, associated with a one-period deviation from the equilibrium path. In the example, u^d turns out to be the level of utility associated with the socially efficient level of employment, $1/(1+\psi) = 0.25$. The deviation growth rate of money, x^d, needed to achieve this level of employment is larger in the $s = s(2)$ state of the world. To see why, note that achieving $n = 0.25$ requires setting the real value of end-of-period money balances, $(1+x)/p$, equal to 0.25. But, in the high state of the world, intermediate goods producers anticipate a high (3 percent) growth rate of money. Consequently, the price level is high. With p high, x must be high too. Since the deviation induces a higher level of consumption and employment, $u < u^d$. The outcome is nevertheless sustainable because $u + v > u^d + v^d$ in both states of the world.

Table 1: Non-Fundamental Expectation Trap		
variable	$x(h_{t-1}, s_t) = 0.0$	$x(h_{t-1}, s_t) = 0.03$
c	1	0.9709
n	0.2346	0.2304
M/P (beg. of period)	0.2346	0.2237
W/P	0.9196	0.8962
u	-2.2519	-2.2536
v	-75.0892	-75.0950
u^d	-2.2413	-2.2413
u^a	-2.2910	-2.2910
v^a	-76.3650	-76.3650

Fourth, we briefly consider what sort of equilibria can sustain this outcome. One such equilibrium is the grim trigger equilibrium used in the proof to Proposition 4.2. In results not reported here, we verified that a milder trigger strategy equilibrium that also sustains the outcome is one in which a deviation triggers a shift to the maximum growth rate of money, $\bar{x}$, for one period. Finally, we discuss a particular non-trigger equilibrium which sustains the expectations trap outcome. Loosely speaking, in this equilibrium the monetary policy rule specifies that deviations in which money growth is high in a particular state persist forever, but deviations down have no impact on future policy. Formally, for histories, (h_{t-1}, s_t), in which there has never been a deviation, $x(h_{t-1}, s_t) = \nu(s_t)$, where

$$\nu(s) = \begin{cases} 0.0, \text{ if } s = s(1) \\ 0.03, \text{ if } s = s(2) \end{cases} . \tag{5.2}$$

Next, consider histories, (h_{t-1}, s_t), in which there was: (i) one deviation in the past, (ii) the deviation money growth rate was a, (iii) the growth rate called for by the policy rule was $x < a$,and (iv) the state was $s = s(k)$. In histories like this, the policy rule specifies that $x(h_{t-1}, s_t) = \tilde{\nu}(s_t)$, where $\tilde{\nu}(s(k)) = a$ and $\tilde{\nu}(s(j)) = \nu(s(j))$ for $j \neq k$. That is, the rule specifies that if the policy authority deviates up in a particular state, the deviation action will be followed whenever that state recurs in the future. Histories in which there has been more than one deviation are handled in the same way. Deviations down have no impact on the policy rule. We verified, using numerical methods, that the outcome in example 1 is sustainable by this equilibrium.

Taken together, these three equilibria make clear the technical role played by the grim strategy equilibrium in the proof of the characterization result. Expectation trap outcomes can be supported by variety of equilibria.

Example 2: Fundamental Shocks.

Apart from the stochastic process governing the state of the world, s, and the state of technology, θ_t, the parameter values used in this example coincide with those in Example 1. The state of the world can take on values numbered 1 through 7. In state one, $\theta_t = 1$, while $\theta_t = 0.90$ in the other six states. The Markov chain governing s has the following properties. If the economy is in state 1, it stays with probability 0.99, and it moves to state

2 with probability 0.01. If the economy is in states 2 through 6, the economy moves to the next higher state with probability 0.95. It stays in the current state with the complementary probability. If the economy is in state 7 it moves to state 1 with probability 0.95. In this example, there is a sustainable expectation trap outcome in which money growth is 0.0, 0.05, 0.10, 0.15, 0.20, 0.15, and 0.10 in each of states 1 through 7. This outcome has the property that money growth is typically low. Occasionally, following a decline in the state of technology, money growth and, hence, inflation rise for an extended period of time before returning to their typically low levels. Figure 1 illustrates a typical realization in which the economy is in state 1 for 5 periods, then switches to state 2 for 8 periods, and finally returns to state 1 for 2 periods. Presumably, it is easy to construct similar examples in which expectation traps like this are triggered by other fundamental shocks, like changes in government purchases.

We find Example 2 interesting because it illustrates another sense in which discretion makes possible volatile equilibria. Also, this type of example may form the basis for a simple explanation of the prolonged rise in inflation during the mid-1960s and 1970s. Authors, such as Fellner (1976) have argued that this rise in inflation was triggered by the simultaneous increase in expenditures due to the expansion of the Vietnam war and Great Society Programs. Other authors such as Blinder (1982) argue that the inflation of the 1970s was triggered by the supply shocks of that decade. Our analysis raises the possibility that the long lived inflation is consistent with sequential rationality on the part of private agents and a benevolent monetary authority. It simply reflects the type of expectation trap that can arise under discretion.

6. Limited Commitment

We now show that a simple institutional change in the way monetary policy is conducted eliminates the expectation traps associated with discretionary monetary policy. The change is that we impose a form of limited commitment upon the monetary authority, in which it is required to commit to a state-contingent action one period in advance. We formally model this by specifying the time t sequence of events as follows. First, the exogenous event, s_t, is realized. Second, intermediate goods producers set the time t prices of their goods. Then, the monetary authorities' date $t+1$ action, contingent upon s_{t+1}, is realized. Finally, the other date t model variables are determined. In this modified version, the history at the end of date t is $h_t = (h_{t-1}, s_t, x(s^{t+1}))$, where $x(s^{t+1})$ denotes the monetary authority's time $t+1$ state-contingent action.

Proposition 6.1. *Under limited commitment, the set of sustainable outcomes coincides with those in the Ramsey equilibrium.*

Remark 1. *Recall that in a Ramsey equilibrium, employment, n^R, is given by $n^R = \lambda/(\psi + \lambda)$ and the growth rate of the money satisfies $\sum_{s^{t+1}} \mu(s^{t+1} \mid s^t)\frac{1}{1+x(s^{t+1})} = \frac{1}{\beta}$. Thus, in a Ramsey equilibrium, allocations and prices are uniquely determined and monetary policies are determined only in an expected value sense.*

Proof. In the Ramsey equilibrium, employment is independent of the state of nature. The first step in the proof is to show that in any sustainable equilibrium, the date t consumption and employment allocations depend only upon the date $t+1$ state-contingent monetary policy actions. To see this, first recall that, in a sustainable equilibrium, for all histories, the continuation outcome induced by the sustainable equilibrium must solve the consumer's optimization problem and satisfy market clearing. The analog of (3.18) is:

$$\frac{\psi}{\lambda}\frac{n(h_t)}{1-n(h_t)} = \beta \sum_{s^{t+1}} \mu(s^{t+1}|s^t)\frac{1}{1+x(s^{t+1})}. \tag{6.1}$$

It follows that, for all histories, the employment allocation at date t depends only upon the state-contingent actions of the monetary authority at date $t+1$. The second step in the proof is by contradiction. Consider a sustainable outcome such that $n(h_t) \neq n^R$ for some h_t.

Consider the following sequence of deviations by the government: $x(s^{r+1}) = \frac{1}{\beta} - 1$ for $r \geq t$. Under this deviation, $n(h_r) = n^R$ for $r \geq t$. But, equations (3.16) and (6.1) imply that $n(h_t)$ is strictly less than n^R. Since utility is strictly increasing for levels of employment less than $1/(1+\psi)$, this deviation raises utility. This establishes the contradiction. ■

To understand this result, it is useful to recall why expectation traps are possible when there is no commitment. Two conditions that are necessary for expectation traps to arise are: (i) private agents' actions depend upon their expectations about future monetary policy, and (ii) private agents suffer a loss if their expectations are not validated. When these conditions are satisfied and the monetary authority is benevolent, then it is possible for expectations to drive monetary policy. Many equilibria can arise, because there is nothing in the environment to uniquely pin down agents' expectations. Limited commitment eliminates expectation traps by forcing the monetary authorities to commit before private agents make their decisions. In this way, expectations are uniquely pinned down.

We conclude this section by discussing how the previous proposition would have to be modified to accommodate different environments. For example, suppose agents committed to nominal wages or prices K periods in advance. To eliminate expectation traps in this environment, we anticipate that monetary policy would have to commit $K+1$ periods in advance. In environments in which decisions depend on the entire future history of monetary policy, for example, when agents have the option to invest in physical capital, then it may not be possible to eliminate all expectation traps with limited commitment.

7. Concluding Remarks

This paper studied the operating characteristics of monetary policy when the monetary authority cannot commit to future policies. Our main finding is that discretion exposes the economy to expectation traps. In addition, we argue that alternative institutional arrangements for the conduct of monetary policy which impose limited forms of commitment on policy makers can eliminate the possibility of expectations traps.

The existing literature emphasizing the benefits of commitment has not recognized that expectation traps of the sort analyzed in this paper can occur can occur under discretion.[4] Perhaps the paper closest in spirit to ours is Cole and Kehoe (1995), who construct a model of sovereign debt and show that, under discretion, default can occur in response to a non fundamental shock. In the debt default literature, Calvo (1988) and Chari and Kehoe (1993a,b) show that, under discretion there can be multiple equilibria, even with a finite horizon. Neither of these papers discusses the possibility of excessive volatility.

The sunspot literature has discussed extensively the possibility of excessive volatility.[5] This literature shows that there are economies in which private agents respond to nonfundamental shocks and also shows that private agents can respond excessively to fundamental shocks (see Beaudry and Devereux, 1994). In this literature the excessive volatility arises from the structure of preferences and technology of private agents. In our environment, it arises from the structure of policy making.

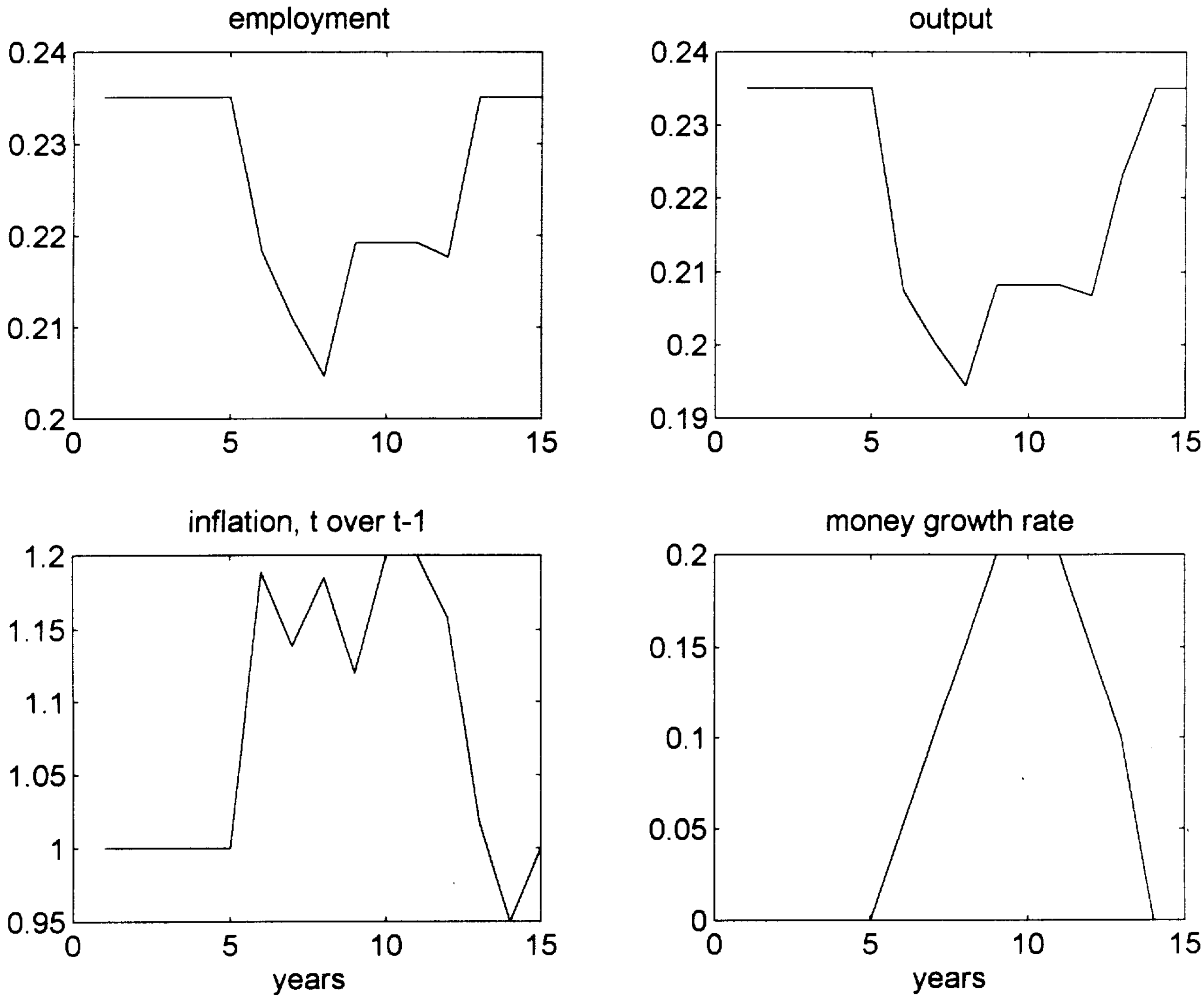
We conclude by discussing a shortcoming of our model, and then arguing that the basic findings of this paper are likely to be robust to a variety of plausible remedies. In our model we assumed there is an exogenously given upper bound, $\bar{x}$, on the money growth

[4]See for example, Kydland and Prescott (1977), Calvo (1978), Fischer (1980), Barro and Gordon (1983), Stokey (1991), Chari and Kehoe (1990,1993a,b) and Ireland (1995). A reference to the possibility of expectations traps in which monetary policy responds to nonfundamental shocks appears in Rogoff (1989, p. 245-46). However, Rogoff refers to this in the context of trigger-strategy equilibria, while we show that these equilibria can be supported without trigger strategies.

[5]See Azariadis (1981), Benhabib and Farmer (1994,1995), Bryant (1983), Cass and Shell (1983), Cooper and John (1988), Farmer and Guo (1994), Gali (1994a,b), Matsuyama (1991), Shleifer (1986) and Woodford (1986,1991).

rate. Our characterization theorem requires the existence of a worst sustainable equilibrium. In our model economy there is no worst equilibrium absent an exogenous upper bound on the money growth rate. This reflects the fact that, in our environment, there is only a contemporaneous benefit, and no contemporaneous cost, from an unexpected increase in money growth. To avoid the need to specify an exogenous upper bound on money growth, we must introduce features which associate contemporaneous costs with high money growth. One such cost that has been widely emphasized is that increases in the inflation rate induce resource misallocation by changing the cross-sectional distribution of relative prices. One simple way to capture this resource misallocation is to introduce staggered price setting along the lines of Taylor (1980). To see this, consider the case where each period, half the intermediate goods producers set a price for their goods which will prevail for two periods. Under these circumstances, the higher is inflation, the greater the difference is between the two sets of prices. This results in an inefficient distribution of production among the intermediate goods producers. If we assume that the firms setting prices in any particular period do so after the current period realization of the monetary action, then this obviously creates a contemporaneous cost to increasing the money growth rate, and this cost is greater at higher inflation rates. We expect that expectation traps can arise under discretion in this environment too: suppose the subset of firms setting prices in a particular period expect money growth to be high in the next period. They will find it in their interest to set prices high. Then, in the next period the monetary authority faces a choice: either validate the expectations of the previous periods' price setters, or incur a recession. As in the environment of this paper, it may choose to validate the expectations, thus ensnaring the economy in an expectations trap.

FIGURE 1


employment
output
inflation, t over t-1
money growth rate
years
years

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