NBER WORKING PAPER SERIES

INVESTMENT WITH UNCERTAIN TAX POLICY: DOES RANDOM TAX POLICY DISCOURAGE INVESTMENT?

Kevin Hassett
Gilbert E. Metcalf

Working Paper No. 4780

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1994

We appreciate helpful comments from Alan Auerbach, Avinash Dixit, Steve Oliner, and Bob Pindyck. The National Science Foundation has provided support for this research on grant SES-9210407. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the authors and not those of the Board of Governors of the Federal Reserve System or the National Bureau of Economic Research.

NBER Working Paper #4780
June 1994

INVESTMENT WITH UNCERTAIN TAX POLICY: DOES RANDOM TAX POLICY DISCOURAGE INVESTMENT?

ABSTRACT

In models with irreversible investment, increasing uncertainty about prices has been shown to increase the required rate of return (hurdle rate) and delay investment (e.g., Pindyck, 1988). One serious form of uncertainty faced by firms, a form that policy makers could conceivably control, is tax uncertainty. In this paper, we show that it does not follow from past work that tax policy uncertainty increases the expected hurdle price ratio and delays investment. This is because tax uncertainty has an unusual form that distinguishes it from price uncertainty: tax rates tend to remain constant for many years, and then change in large jumps. When tax policy follows a jump process, firms' expectations of the likelihood of the jump occurring have important effects on investment. Indeed, as we show below, while price uncertainty increases the hurdle rate and slows down investment, tax uncertainty has the opposite effect.

Kevin Hassett
Board of Governors of the Federal Reserve
Federal Reserve System
20th Street and Constitution Avenue, NW
Washington, D.C. 20551

Gilbert E. Metcalf
Department of Economics
Fisher Hall
Princeton University
Princeton, NJ 08544-1021
and NBER

"The certainty of what each individual ought to pay is, in taxation, a matter of so great importance, that a very considerable degree of inequality it appears ... is not near so great an evil as a very small degree of uncertainty."

- Adam Smith, The Wealth of Nations

I. Introduction

It is often said that there is nothing certain in life except death and taxes. While death is undoubtedly certain, there is, in fact, considerable uncertainty with respect to taxes. Tax policy provides a key source of uncertainty about the cost of capital to U.S. firms, for example. The investment tax credit (ITC) was first introduced in 1962, and subsequently, has been changed on numerous occasions.[1] That the random ITC dice are still being tossed is evidenced by recent events in Washington D.C. President Bush advocated a modified ITC, known as the "investment tax allowance" in 1992, and President Clinton proposed an incremental ITC in early 1993, but neither of these measures was enacted.

Adam Smith thought that uncertainty in taxation was so injurious that a great deal of tax inequality would be justifiable if it was associated with reduced uncertainty in tax

[1] Cummins, Hassett and Hubbard (1994) provide a thorough review of post-war tax changes. Since 1962, the mean duration of a typical state in which a specific ITC is in effect is 3.67 years. The mean duration of the "no-ITC" state is 3.00 years.

levies. The view that uncertainty is harmful is echoed by findings that randomness in output prices retards investment (Pindyck (1988)). Smith's views on uncertainty are probably not uncommon. Those views, combined with Pindyck's findings, suggest that policy uncertainty will discourage investment. The purpose of this paper is to consider tax policy uncertainty explicitly and evaluate whether that is indeed the case.[2] Specifically, does an increase in uncertainty discourage investment?

Uncertainty has typically been introduced in previous work by assuming that some parameter follows a continuous time random walk (Brownian Motion or Geometric Brownian Motion). When prices follow a random walk, one's rational-expectations forecast for the price at any time in the future is today's price (perhaps adjusted for some trend). Tax parameters, unlike most prices, tend to remain constant for a few years, and then jump to new values. The expected value of tomorrow's effective tax rate is not equal to today's value. What makes the investment decision especially intriguing under these conditions is the possibility that the firm might invest today, only to see an ITC introduced, or might delay investment today, only to see an existing ITC repealed. Armed with the knowledge of the expected frequency with which tax policy changes, firms will delay or speed up

[2]We focus in this paper on a model with irreversible investment, as these models generally provide the biggest negative effects of uncertainty on investment. In a companion paper, we show that tax policy uncertainty in a model with convex adjustment costs (e.g. that of Abel (1983)), has similar effects to those shown here.

investment depending on their perceptions of the probability of tax changes occurring. Below, we show that this behavior is crucial to understanding the effects of uncertain tax policy. To summarize, we find the following. In the traditional model of random tax policy, increasing uncertainty slows down investment. This follows naturally and directly from previous work (e.g. Pindyck (1988)). However, when tax policy is modeled as a jump process, we find that increasing uncertainty has the opposite effect.

For specificity in this paper, we will consider random policy toward an investment tax credit for new capital; the effect of tax policy is to alter the marginal price of capital. We begin in the next section with some background on previous work on tax policy uncertainty. We turn in the following section to modeling policy uncertainty as a continuous time random walk in logs. For this model, increasing uncertainty takes the form of increases in the instantaneous variance of the capital price process. The results from this section provide a benchmark for the following section. In Section IV we turn to the main contribution of this paper: discontinuous policy changes. Here, tax parameters periodically "jump" between fixed values. In this model, there are a couple of ways to think about uncertainty and we consider each in turn. Finally, we conclude in Section V.

II. Background

To date, there has been little work addressing the issue of investment behavior and tax policy uncertainty. That tax policy is uncertain is not a new concept; indeed, the notion that investment tax credits may randomly switch on and off was a key argument in Lucas (1976). More recently, Auerbach and Hines (1988) attack this same problem in a discrete time model in which there is a probability each year that tax policy (investment tax credit and depreciation allowances) will change. To obtain a tractable solution, they must make linear approximations around steady-state values of the capital stock. This turns out to be a critical assumption as the use of a first order approximation around the steady state means that the information in the second moment of the distribution of tax policies is eliminated. Thus mean preserving spreads of the distribution of the random tax variable will have no effect on the measures of effective tax rates that they construct.

Bizer and Judd (1989) develop and solve numerically a general equilibrium model that includes random taxation. In this closed-economy model, investment equals saving and saving follows from utility maximization. Thus the results they derive, that fluctuations in output attributable to random taxation lower welfare, are intricately related to the curvature of the utility function. In this paper, we follow Pindyck (1988) and Abel (1983) and focus on the investment decision in isolation. This allows us to adopt a more complicated investment model that includes irreversibility and, at the same time, solve explicitly for the effects of uncertainty. As it turns out, this last step helps motivate the intuition for our results considerably.

Before turning to the model with jump processes in investment costs, we provide a brief discussion of results from a model in which the price of capital follows Geometric Brownian Motion. In the subsequent section, we will model changes in the price of capital as following a Poisson Process in which the expected duration of a tax regime is known but in which the actual duration is uncertain. We take this approach given the historical experience documented in the introduction.

III. Investment with Uncertainty Modeled as Geometric Brownian Motion

III. A. The Model

We begin with a simple model in which firms choose when to undertake a project. The amount of capital employed in the project (K) and the time at which the project is initiated are at the discretion of the investor. The capital can be used to produce F(K) units of output forever which can be sold at price p_t. The production function has the standard properties ($F' > 0$, $F'' < 0$)[3]. The price p_t is an after tax return and is stochastic. We assume it follows Geometric Brownian Motion:

---

[3] Second order conditions will place additional restrictions on the production function. In particular the elasticity of substitution must be less than 1. See Dixit (1992).

(1) $$\frac{dp_t}{p_t} = \mu_p \, dt + \sigma_p \, dz_p$$

where dz_p is an increment to a Wiener process. The return follows a continuous time random walk with volatility σ_p and drift μ_p. We model p as stochastic to allow for supply shocks which affect the price level of corporate output as well as the possibility of randomness in the rate of tax on corporate output. In the discussion below, we focus on the first source of randomness for p and consider randomness in the cost of capital through changes in the purchase price of the investment.

In addition to taxes on the output of the firm, there may be tax incentives which reduce the cost of capital. Accelerated depreciation and investment tax credits are the two most common forms of cost reduction. Specifically, we assume that the price of capital evolves according to the process:

(2) $$\frac{dp_{k_t}}{p_{k_t}} = \mu_k \, dt + \sigma_k \, dz_k .$$

We assume that the correlation between the Wiener Processes for output price and capital price equals ν. Given the randomness in output and capital prices, the firm wishes to choose an optimal size of the project (K^*) and time to invest (T^*) to maximize the expected discounted value of the stream of profits from the investment (net of the cost of the investment). This value can be written as

(3) $$V(p,p_k) = \max_{K,T} E \left\{ \int_T^\infty p_s F(K_T) e^{-\rho s} ds - p_{k_T} K_T e^{-\rho T} \right\}.$$

Homogeneity of degree 1 in prices allows us to rewrite the value function in terms of p/p_k and p_k. That is $V(p,p_k) = p_k v(p/p_k)$. Since both prices evolve according to a Geometric Brownian Motion, the ratio p/p_k evolves according to Geometric Brownian Motion as well:

(4) $$d(p/p_k) = \underbrace{(\mu_p - \mu_k + \sigma_k^2 - \nu\sigma_p\sigma_k)}_{\alpha}(p/p_k)dt + (\sigma_p dz_p - \sigma_k dz_k)(p/p_k)$$

The ratio p/p_k has trend α and variance (σ_0^2) equal to $(\sigma_p^2 + \sigma_k^2 - 2\nu\sigma_p\sigma_k)$. Thus, this model reduces to the canonical model with a single source of uncertainty. The firm's problem is to find the ratio $(p/p_k)^*$, which describes the optimal time to invest. When the ratio p/p_k is equal to $(p/p_k)^*$ the firm invests, otherwise the firm waits. We obtain a differential equation in one variable that can be easily solved to give the following investment conditions (see Dixit and Pindyck (1994) for more details):

(5a) $$pF(K) \geq (\rho-\mu_p)\left(\frac{\beta}{\beta-1}\right) p_k K$$

(5b) $$pF'(K) = p_k(\rho-\mu_p).$$

where $$\beta = \frac{.5\sigma_0^2 - \alpha + \sqrt{(.5\sigma_0^2 - \alpha)^2 + 2(\rho-\mu_k)\sigma_0^2}}{\sigma_0^2}.$$

Equation (5a) merely states that the firm will not invest unless the expected revenue $\frac{pF(K)}{(\rho-\mu_p)}$ is greater than the cost of purchasing the machine, where the cost includes the markup factor, $\left(\frac{\beta}{\beta-1}\right)$, which accounts for the cost of giving up the option to invest in the future. Equation (5b) indicates that once the firm has decided to invest, the loss of the option value of investing in the future is a sunk cost, and the firm should choose the level of capital so that the marginal revenue is equal to marginal cost.

In this example, tax policy uncertainty enters via an increase in the variance of the capital price, and we begin by considering a mean preserving spread in this price ($d\sigma_k^2 > 0$).[4] We derive the effect of changing the variance in three steps:

i) Some simple algebra shows that $\frac{\partial\left(\frac{\beta}{\beta-1}\right)}{\partial\sigma_k^2} > 0$; thus the amount by which expected revenue exceeds the cost of purchasing capital increases with the variance of capital.

ii) From equation (5b), it is clear that K increases with the price ratio p/p_k.

iii) Combining equations (5a) and (5b) yields the relationship

---

[4] Strictly speaking, this is an increase in scale in the terminology of Abel (1985). As Abel notes, if the correlation among all Wiener Processes is zero, then an increase in scale is equivalent to a mean preserving spread.

$$(6) \qquad \frac{F(K)/K}{F'(K)} = \frac{\beta}{\beta-1} .$$

Second order restrictions on the production function (see footnote 3) can be invoked to show that the ratio of the average to marginal product is increasing in K (alternatively, that the elasticity of output with respect to K is decreasing in K).

Thus, increases in the variance of p_k increase $\frac{\beta}{\beta-1}$, as documented in step i. This leads to firms choosing a higher level K^* , because of step iii. Since the hurdle price ratio, is monotonic in K (step ii), increases in the variance of p_k increase the hurdle price ratio $(p/p_k)^*$. This result is not very surprising given previous work by Pindyck (1988). Given homogeneity of degree 1 in prices, investment is triggered by the ratio of the output to capital price. As equation 4 shows, increasing the variance of capital prices produces the same effect on the variance of the price ratio p/p_k as does an increase in the variance of output prices[5]. In both models, the value function is concave in the price ratio. We can invoke Jensen's Inequality to show that increasing σ will decrease the expected value of the project. Hence increasing σ will lead to a higher required hurdle price ratio - or delayed investment.

---

[5] Changing the variance of p_k affects the trend of p/p_k slightly differently than changing the variance of p. This can be seen in the expression for α in equation 4.

III.B Simulation Results.

In the last section we showed that increasing uncertainty leads to an increase in the hurdle price ratio. The increase in this ratio leads one to expect that time to investment will rise. However, with increasing variance in capital costs, it becomes increasingly likely that capital costs will fall sharply in a short period of time - raising the chances of the hurdle price ratio being hit in a shorter time. Thus whether time to investment goes up or down as the variance of capital costs increases is ambiguous. Therefore we turn to simulations to determine which of these effects dominate. Simulations also allow us to assess the economic importance of changes in uncertainty.

Table 1 illustrates the importance of mean preserving spreads on hurdle prices and investment times. We present results from a Monte Carlo experiment in which we simulate 1500 price paths and consider the investment behavior of a firm facing these prices. The firm has a production function of the form $F(K) = \ln(1+K)$ and uses a discount rate of .05 for investment. Output prices are GBM with no trend and variance equal to .01. Capital costs also follow GBM with no trend and variance ranging from 0 to .06. The covariance between the output and capital price series is set to 0.[6] Column 2 in Table 1 presents the

---

[6] We start the simulations at time 0 with output price equal to 1.75 and capital cost equal to 20. Thus the ratio of prices at time zero equals 8.75%.

hurdle rates from equation 5. As noted above, the hurdle price ratios increase as σ_k^2 increase. If capital costs are certain, the optimal time to invest is when output price exceeds 9.7% of the price of capital. The ratio increases to 16.6% when the instantaneous variance of p_k rises to .02. The hurdle price ratio rises with σ_k^2 reaching .344 at σ_k^2 equal to .06. The next column of Table 1 gives the median time to investment. If there is no uncertainty in capital costs, the median time is 2.81 years[7]. The average output price at investment equals 1.95, an increase from time zero of 11%.

Increasing σ_k^2 from 0 to .01 delays investment dramatically. The required price ratio has increased from 9.7% to 13.1% and the median time to investment is now 15.9 years. Investment is triggered roughly equally by an increase in output price and a decrease in capital costs. The average output price is 19% higher than at time 0 and the average capital cost is 20% lower than at time 0. As volatility in capital costs increase, investment is increasingly driven by decreases in capital costs rather than increases in output price. If the variance of

---

[7] When running the simulations, we stop the process if t exceeds 50 and no investment has occurred. Thus we have not estimated mean time to investment. Mean time to investment will exceed median time as there is significant positive skew in the distribution of time to investment. Since there is no trend in the two price variables, truncating the price process at year 50 should not bias our estimates of the mean prices and investment level at time of investment. Standard errors of the median time to investment are based on Koenker and Bassett (1982) as implemented in STATA.

capital costs equals .06 average output price increases by 11% while average capital costs fall by over 70% at investment.

Table 1 shows that the the effect of raising the hurdle price ratio outweighs the increase in volatility in capital costs in determining time to investment. The median time to investment rises monotonically with σ_K^2 from a minimum of 2.81 with no uncertainty. The increase is very rapid at first, with the median time increasing to nearly 16 years at σ_k^2 equals .01 and to over 24 years at σ_k^2 equals .02. Beyond σ_k^2 equals .02 the median time increases more slowly reaching 27.7 years at σ_k^2 equals .06.

Summing up, when policy uncertainty is modeled as the capital cost following Geometric Brownian Motion, increasing uncertainty both increases the hurdle price ratio and lengthens the time to investment. These results provide a benchmark for considering the next model in which tax changes are discrete and infrequent.

IV. Investment with Tax Policy Jumps.

IV.A. The Model

As noted above, tax policy changes typically are discrete changes. Thus we now turn to a model of tax policy in which tax incentives change randomly and in discrete amounts. Without loss of generality, we consider an investment tax credit $\pi_t \in [0,1)$ which reduces the price of capital from p_k to $(1-\pi_t)p_k$.[8] Unlike

[8] Without loss of generality, we now treat the capital price p_k as fixed and let the output price follow Geometric Brownian Motion.

the tax on corporate output, the tax credit is assumed to follow a Poisson Process. The actual duration of a credit is unknown although the expected duration is known. Conversely, if there is no credit in effect, the actual time to a credit is unknown although the expected time is known. Specifically, the tax process follows the equation of motion:

$$d\pi_t = \begin{cases} \Delta\pi & \lambda_1 dt \\ 0 & 1-\lambda_1 dt \end{cases} \quad \pi = \pi_0 \qquad \begin{cases} -\Delta\pi & \lambda_0 dt \\ 0 & 1-\lambda_0 dt \end{cases} \quad \pi = \pi_1 \tag{7}$$

where $\Delta\pi \equiv \pi_1 - \pi_0 > 0$. The tax credit randomly switches between π_0 and π_1 with transition parameters λ_1 and λ_0. The transition parameters are informative on a number of counts. The expected duration of a regime with a high ITC (π_1) is given by $\frac{1}{\lambda_0}$ while the expected duration of a tax regime with low ITC (π_0) is given by $\frac{1}{\lambda_1}$. In addition, the expected fraction of the time that a high ITC will be in effect is given by $\frac{\lambda_1}{\lambda_0 + \lambda_1}$.

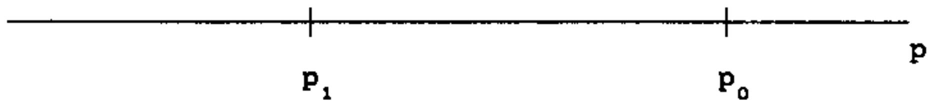
Given the randomness in output price and capital costs, the firm wishes to choose an optimal size of the project (K) and time to invest (T) to maximize the expected discounted value of the stream of profits from the investment (net of the cost of the investment). Normalizing by the price of capital, this value can be written as

$$V = \max_{K,T} E \left\{ \int_T^\infty p_s F(K_T) e^{-\rho s} ds - (1-\pi_T) K_T e^{-\rho T} \right\}. \tag{8}$$

There are three regions in the output price space of importance: In region 1 ($p < p_1$), there is no investment, regardless of the value of π. In region 2 ($p_1 < p < p_0$) there is investment if the high ITC is in effect and in region 3 ($p_0 < p$) there is investment regardless of the level of the ITC. The following picture illustrates the investment decision:

Investment Rule

$\pi=\pi_1$	No	Yes	Yes
$\pi=\pi_0$	No	No	Yes



Consider first region 1 below p_1. In this region, no investment is made regardless of the level of the ITC. Let V^1 represent the value function when the tax credit is in effect and V^0 the value when the tax credit is zero. An arbitrage argument can be made that[9]

$$\rho V^0 dt = E(dV^0) \tag{9}$$

$$\rho V^1 dt = E(dV^1) \tag{10}$$

Using Ito's Lemma, $E(dV^0) = (.5\sigma^2 p^2 V^0_{pp} + \mu p V^0_p + \lambda_1 (V^1 - V^0))dt + A(dt)^2$. Similarly, $E(dV^1) = (.5\sigma^2 p^2 V^1_{pp} + \mu p V^1_p - \lambda_0 (V^1 - V^0))dt +$

9 This also follows from the Bellman equation.

$B(dt)^2$. Substituting these expressions into (9) and (10), dividing through by dt and letting dt go to zero yields two differential equations in p:

(11) $$\rho V^0 = .5\sigma^2 p^2 V^0_{pp} + \mu p V^0_p + \lambda_1 (V^1 - V^0)$$

(12) $$\rho V^1 = .5\sigma^2 p^2 V^1_{pp} + \mu p V^1_p - \lambda_0 (V^1 - V^0).$$

The expectation on the right hand sides of equations 11 and 12 are composed of two parts. The first is the expected gain in V^0 as prices evolve. The second part reflects the capital gain (loss) if the tax credit is put in place (eliminated) which occurs with probability λ_1 (λ_0).

Equations 11 and 12 are solved in appendix A. The solutions are

(13) $$V^0 = \frac{1}{\lambda_1 + \lambda_0} \left\{ A_1 p^\beta - \lambda_1 A_2 p^\gamma \right\}$$

(14) $$V^1 = \frac{1}{\lambda_1 + \lambda_0} \left\{ A_1 p^\beta + \lambda_0 A_2 p^\gamma \right\}$$

where β is the positive root to the quadratic $Q(x) = \frac{1}{2}\sigma^2 x(x-1) + \mu x - \rho$, γ the positive root to the quadratic $R(x) = \frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + \lambda_1 + \lambda_0)$ and A_1 and A_2 are constants of integration (see appendix for details).

In region 2, the firm only invests if the high level of the tax credit is in place. The arbitrage argument used above is modified slightly to account for the investment in the presence of the tax credit:

(15) $$\rho V^0 dt = E(dV^0)$$

(16) $$V^1 = \frac{p_T F(K_1)}{\rho - \mu} - (1 - \pi_1)K_1$$

Equation (16) says that the value of the project if the investment is made equals the expected present discounted value of the flow of profits from the investment (discounted at rate ρ) less the cost of the project. $E(dV^0)$ in equation (15) is the same as above so that the differential equation for V^0 is still given by equation (11). But now we have only one differential equation to solve since V^1 is given explicitly in equation (16):

(17) $$V^0 = C_1 p^{\eta_1} + C_2 p^{\eta_2} + \frac{\lambda_1 p_T F(K_1)}{(\rho-\mu)(\rho+\lambda_1-\mu)} - \frac{\lambda_1(1-\pi_1)K_1}{\rho + \lambda_1}$$

where η_1 and η_2 are roots to the quadratic $S(x) = \frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + \lambda_1)$ and C_1 and C_2 are constants of integration.

Finally, in region 3 investment is made regardless of the ITC value. The value functions are given by

(18) $$V^0 = \frac{p_T F(K_0)}{\rho - \mu} - (1-\pi_0)K_0$$

(19) $$V^1 = \frac{p_T F(K_1)}{\rho - \mu} - (1-\pi_1)K_1$$

Value matching and smooth pasting arguments can be invoked to complete the system. Value matching implies that the value function V^0 and V^1 must be equal at the boundaries of the regions. These imply the following three equations:

$$(20) \qquad \frac{1}{\lambda_1 + \lambda_0}\left\{A_1 p_1^{\beta} - \lambda_1 A_2 p_1^{\gamma}\right\} = C_1 p_1^{\eta_1} + C_2 p_1^{\eta_2} + \frac{\lambda_1 p_1 F(K_1)}{(\rho-\mu)(\rho+\lambda_1-\mu)} - \frac{\lambda_1 (1-\pi_1) K_1}{\rho + \lambda_1}$$

$$(21) \qquad \frac{1}{\lambda_1 + \lambda_0}\left\{A_1 p_1^{\beta} + \lambda_0 A_2 p_1^{\gamma}\right\} = \frac{p_1 F(K_1)}{\rho - \mu} - (1 - \pi_1)K_1$$

$$(22) \qquad C_1 p_0^{\eta_1} + C_2 p_0^{\eta_2} + \frac{\lambda_1 p_0 F(K_1)}{(\rho-\mu)(\rho+\lambda_1-\mu)} - \frac{\lambda_1 (1-\pi_1) K_1}{\rho + \lambda_1} = \frac{p_0 F(K_0)}{\rho - \mu} - (1-\pi_0)K_0$$

In addition to the value matching conditions, smooth pasting conditions must be met. These imply that the derivatives of V^0 and V^1 at the points p_0 and p_1 with respect to p must be equal and the the derivatives with respect to K must be equal to zero in cases where the investment is actually made. These conditions yield the following 5 equations:

$$(23) \qquad \frac{1}{\lambda_1 + \lambda_0}\left\{A_1 \beta p_1^{\beta-1} - \lambda_1 \gamma A_2 p_1^{\gamma-1}\right\} = C_1 \eta_1 p_1^{\eta_1-1} + \eta_2 C_2 p_1^{\eta_2-1} + \frac{\lambda_1 F(K_1)}{(\rho-\mu)(\rho+\lambda_1-\mu)}$$

$$(24) \qquad \frac{1}{\lambda_1 + \lambda_0}\left\{A_1 \beta p_1^{\beta-1} + \lambda_0 \gamma A_2 p_1^{\gamma-1}\right\} = \frac{F(K_1)}{\rho - \mu}$$

$$(25) \qquad 0 = \frac{p_1 F'(K_1)}{\rho - \mu} - (1 - \pi_1)$$

$$(26) \qquad C_1\eta_1 p_0^{\eta_1 - 1} + C_2\eta_2 p_0^{\eta_2 - 1} + \frac{\lambda_1 F(K_1)}{(\rho-\mu)(\rho+\lambda_1-\mu)} = \frac{F(K_0)}{\rho - \mu}$$

$$(27) \qquad 0 = \frac{p_0 F'(K_0)}{\rho - \mu} - (1 - \pi_0)$$

Equations 20-27 are 8 equations in the 8 unknowns: A_1, A_2, C_1, C_2, p_0, p_1, K_0, and K_1. The prices p_0 and p_1 are the trigger prices at which time investment is made in the absence of a tax credit and in the presence of the credit, respectively. The quantities K_0 and K_1 are the corresponding investment levels.

At this point we could do comparative statics to determine the effects of increasing uncertainty on the hurdle price ratio. However, there are two issues to consider. First, there are several ways in which we can define "increasing uncertainty" and it is important to clarify which definition we are using. In the next sub-section, we discuss this issue. Second, there exists the same problem in moving from changes in hurdle price ratios to time to investment as in the model with Geometric Brownian Motion. Thus we will need to turn to simulations as in section III. In the following sub-section, we consider the relationship between the hurdle price ratio and the parameters of the Poisson process and then turn to simulations to consider the effects of increasing uncertainty in tax policy on investment.

IV.B. Defining Uncertainty for the Jump Process.

Before proceeding, we need to define what we mean by increasing uncertainty. Unlike the GBM model there are a number of ways to change the level of uncertainty in the model. One way to generate a mean preserving spread is to fix the values of λ and change π_0 and π_1 such that $E(\pi)$ is unaffected. For example, consider the case where $\lambda_0 = \lambda_1$. A mean preserving spread results as (π_0, π_1) progresses from (.10, .10) to (.05, .15) to (0.0, .20). In the first case, there is no uncertainty in tax policy. The ITC always equals 10%. In the second case, it randomly switches between 5% and 15% while in the third case it switches on and off with its value equaling 20% when in effect. Results from a simulation in which we vary the ITC in this way can be compared to results from Table 1.

It is also possible to increase uncertainty by altering the λ's; however, for the Poisson Process, increasing λ does not necessarily imply an increase in uncertainty. Unlike the GBM process, there is not a monotonic relationship between a key parameter (e.g. σ) and uncertainty. To see this, let us take the case where $\lambda_0 = \lambda_1 = \lambda$ and consider extreme values for λ. If λ equals 0, there is clearly no uncertainty over future tax policy; whatever policy is in effect now will be in effect forever. Now suppose that λ is a very large number; the instantaneous probability of switching between the tax and no-tax states is very close to 1. In that case, the credit will switch on and off every instant. The variation will be extraordinarily high, but there will be almost no uncertainty. At each point in time, you know with great confidence what the credit will be at the next point in time. While there is no uncertainty at boundary values for λ, there is clearly uncertainty at intermediate values of λ.

The problem of defining uncertainty in this type of case was first studied in statistical thermodynamics, and the application to information theory is described at length by Theil (1967), where uncertainty, or "entropy" is shown to be highest in a discrete time Markov process when the probability of transiting to any of n states of the world is 1/n. The continuous time analogue to Theil's discrete time maximal uncertainty case is a probability of transiting out of a state over the course of a year equaling 1/2. It is a simple matter of integration to show that the probability of at least one transition occurring in one year equals 1/2 when λ equals .69. In tables 3 and 4 below, we range λ from 0 to .5 to consider how increases in uncertainty of the form of increases in λ affect investment.[10]

An obvious variant on this second approach is to increase one of the λs. Doing so however changes the mean and the variance.[11] Thus we do not have a clean experiment on the effects

---

10 Using this approach, one could also examine increasing uncertainty starting with the case where the ITC is changed with certainty every instantaneous moment. We do not believe that this would be interesting however, as that region of the parameter space has not been--and is not likely to ever be--inhabited.

11 One could also vary one of the λs and adjust the values of π to fix the expected value of the tax credit. This would combine the "frequency" effect with the "spread between rates" effect. We do not report results from this type of experiment, since it adds

of changing uncertainty alone. However, it is worth considering this case as it provides some intuition for results from the other experiments, especially for how changing the odds of tax regime shifts can lead to anticipatory behavior on the part of firms. Thus we begin by examining how the optimal trigger prices are altered as one or both of the λs change. We then turn to further Monte Carlo results in the Poisson Process model. What we will find is that in general the hurdle price ratio increases with increases in uncertainty. As noted above, knowing that the hurdle price ratio has increased does not mean that the average time to investment will also increase. Increasing volatility of the tax policy will lead to a delay in investment but it will also make it more likely that a favorable investment period will occur at a time when the investor would like to act. Thus the average ex post price at which investment occurs can fall. It turns out that in the Poisson model, this second effect dominates and that time to investment actually falls with increasing uncertainty.

nothing to the intuition gained from analyzing these two types of increased uncertainty separately.

IV.C The Effect of Changes in Frequency on Trigger Prices

In this section, we calculate the optimal trigger prices for investment for the model developed in section IV.A.[12] As in the GBM model, we assume that the project uses capital according to the production function F(K) = ln(1+K). The trend in output price is set to zero ($\mu_p = 0$), the volatility of price is 10% (σ_p) and the discount rate is .05. Finally, the price of capital (exclusive of any tax credit) equals 20. We set the low tax credit value at .05 and the high tax credit at .15.

Figure 1 illustrates random tax policy where we vary the frequency of tax changes. We set $\lambda_0 = \lambda_1 \equiv \lambda$ and range the parameters from 0 to 1. This means that there is a 50-50 chance of there being an ITC at any given moment and that the expected duration of a tax policy ranges from a lower bound of 1 year to an upper bound of forever. In cases where there is no uncertainty over tax policy ($\lambda = 0$), the trigger price in the absence of an ITC is nearly 2.0 - double the trigger price in the absence of output price uncertainty. Similarly the trigger price if an ITC is in place is nearly 1.8. As it becomes more likely that existing tax regimes will switch, the trigger price in the absence of an ITC increases while the trigger price when an ITC

---

12 We focus on this section on the output price rather than the ratio of the output to capital prices to highlight the issue of delay (higher output prices take longer to reach). The capital price (exclusive of tax credit) is fixed at 20 so looking at the ratio or the output price is equivalent.

is in place initially decreases. This is intuitive: if there is no ITC, firms will delay investment if the probability of an ITC being implemented in the future increases. Conversely, if an ITC is in effect and the probability of it being removed increases, firms will speed up investment.

Note though that after λ exceeds .20, the trigger price when an ITC is in effect gradually begins to rise again. This suggests we should consider the effects of varying λ_1 and λ_0 separately. Figure 2 fixes λ_0 at .33 (the expected duration of an ITC is 3 years) and varies λ_1. Increasing the probability of an ITC being enacted has a dramatic effect on investment when there is no ITC (p_0). Firms delay investment in hopes that the ITC will soon be enacted. Interestingly, the trigger price if an ITC is currently in effect (p_1) also increases (though less dramatically). It is now less urgent for firms to invest to take advantage of an existing ITC. That the credit may be eliminated before they invest is less costly since the probability that it will be reenacted is increasing.

Figure 3 holds the probability of an ITC being enacted fixed at .33 and ranges λ_0 from 0 to 1.0. Now we are increasing the probability that an existing ITC will be repealed. When the credit is in effect the trigger price falls (firms speed up investment). In addition, the cost to investing without the credit goes down since you may not have to wait as long for the credit to be reenacted during a favorable price realization. Hence p_0 also falls.

Changing both λ_0 and λ_1 thus has offsetting effects. As λ_1 goes up the "cost" of investing without a tax credit increases since the expected wait to an increased tax credit has fallen. Hence p_0 goes up. Similarly, the cost to waiting to invest with a tax credit goes down (since it will soon come back on again). So you are willing to hold out for a higher return (p_1). As λ_0 goes up, the "cost" to waiting goes up as you may lose the tax credit. Hence you invest sooner (lower p_1) if the credit is in effect. Similarly, the cost to investing without the credit goes down since you might have to wait a very long time for the credit to be in effect during a favorable price realization. As figure 1 suggests, these secondary effects are of little importance in regimes where there is no ITC (p_0). However, they are important in the case where the ITC is in effect and more than offsets the primary effect as λ exceeds .2.

This model allows us to capture in a simple way the "announcement effects" that asset pricing models (e.g. Summers (1981)) suggest should occur. Asset pricing models indicate how prices and investment change as policies change and suggest how investment <u>should</u> change as policy changes are announced. One of the attractive features of the jump process model of uncertain tax policy is that we can model anticipations of changes in government policy as simply a change in λ_1. For example, a shift to a more pro-business administration might lead policy makers to expect that an ITC might be enacted in the near future. In the context of our model, that is equivalent to an increase in λ_1.

Similarly, the election of a president that has campaigned on the "inequities" of an ITC might be modeled as an increase in λ_0. Interestingly, there are additional announcement effects. Changes in the probability of an ITC being implemented have an effect on investment even when the ITC is currently in effect (and conversely, a change in the probability of the ITC being removed affects investment even when there is no ITC). This secondary effect is negligible in the case where there is no ITC but it is significant when there is an ITC.

IV.D. Investment Simulations

The previous section showed how optimal trigger prices respond to changes in various parameters of the model. The actual average price at which investment occurs will turn out to be some weighted average of the two trigger prices p_0 and p_1. In this section we present simulations based on the model described in the last section to calculate the average price at which investment occurs. The model is the same as the one used in the Monte Carlo experiment for Table 1 except that capital costs are now fixed at 20 and there is a Poisson Process for the tax credit. We begin the output price process at 1.40 and assume that $\pi = \pi_0$ at time zero[13].

Table 2 considers how mean preserving spreads in the level of the credit will affect investment. The first row shows

[13] Our conclusions are unaffected by starting the process with π equal to π_1.

results from the model in which there is a constant 10% ITC. The hurdle price equals 1.75 which corresponds to a hurdle price ratio of 9.7% and the median time to investment equals 7.64 years. Standard errors are reported in parentheses. We then construct a mean preserving spread of this credit by allowing the ITC to randomly switch from a low value of .05 to a high value of .15 (row 2 of Table 2). The instantaneous probability of switching in either direction is .35. This corresponds to an expected duration of the current tax regime of about 3 years. In this case investment is now delayed in the low ITC state until the hurdle price ratio reaches 12.2%. This represents an increase of roughly 26% over the hurdle price ratio in the certainty case. In the high ITC state the hurdle price ratio falls to 9.2%, a drop of about 5% from the certainty price. The table also reports that on average the ITC switched states 1.78 times before an investment occurred. Most noteworthy is the result that median time to investment falls from 7.6 years to 5.5 with the mean preserving spread.

The final row increases the variance in the ITC variable by setting (π_0, π_1) equal to (0, .20). In the low ITC world, investment is now delayed until the hurdle price ratio reaches 13.4% while in the high ITC world, investment occurs when the ratio reaches 9.1%. More importantly, the median time to investment continues to fall. The median time is now 4.3, a fall of 44% from the certainty case.

It is interesting to consider why the results in Table 2 are so different from those in Table 1. A clue to the difference lies in the expected price of capital (net of the tax credit). When the ITC vector equals (.05, .15) the net price of capital can equal 19 or 17. The average net price of capital equals 17.14 which implies that 93% of the time the investment occurred when the high tax credit was in effect. Recall that the hurdle price ratio fell by 5% in the high ITC state and increased by 26% in the low ITC state. This means that the expected hurdle price ratio should fall slightly.[14] When the ITC vector is (0, .20) investment occurs in the high ITC state 98% of the time. Now the hurdle price ratios change by +38% and -6% for the low and the high ITC states respectively. The expected change in hurdle price ratio in this case equals -5.1%, nearly twice the drop in the (.05, .15) ITC world. Because investment is driven more and more by the hurdle price ratio in the high ITC state, it becomes more likely that output price will exceed p_1 sooner since p_1 falls with mean preserving spreads. Hence the time to investment falls.

We turn next to considering how investment behavior changes as λ ($= \lambda_0 = \lambda_1$) changes. Table 3 sets the ITC values at 0 and .20 and ranges λ from .10 to .50. As we discussed before, the hurdle price ratio increases monotonically as λ increases when π equals π_0 while the ratio first decreases and then increases when

14 The expected change in hurdle price ratio equals (.93)(-5)+(.07)(26) = -2.8%.

π equals π_1. In Table 3, the ratio in the high ITC state ranges between .090 and .092. The real movement occurs in the ratio in the low ITC state. Here the ratio increases by 12% as λ increases from .1 to .5. As λ increases, it becomes increasingly likely that ratio of output to capital prices will exceed the hurdle price ratio when the high ITC is in effect. The fraction of times that investment occurs in the high ITC state increases from 78% when λ equals .1 to 99.5% of the time when λ equals .5. The fall in the hurdle price ratio in the high ITC state more than offsets the increase in the hurdle price ratio in the low ITC state so that the expected hurdle rate falls throughout. The median time to investment decreases from 9.6 years to 3.1 as λ increases from .1 to .5. Table 4 presents the same results for an ITC configuration of (.05, .15). The qualitative results are the same as in Table 3. However, as we showed in Table 2, the reduced spread between the low and high ITC values delays investment. Median times to investment for all values of λ are higher in Table 4 than in Table 3.

Summing up, we find that unlike the model in which capital prices move according to Geometric Brownian Motion, the ex post average hurdle price ratio (as distinguished from the ex ante ratio, conditional on the tax state) falls with increases in uncertainty in the Poisson Process model. Moreover, the median time to investment falls in contrast to the model in Section II.

V. Conclusion

In this paper, we construct a simple model of investment in a world in which there is demand uncertainty (uncertainty in output price) and also uncertainty in tax policy. We model the uncertainty first in a continuous fashion, assuming that the capital cost follows Geometric Brownian Motion. We next model the uncertainty in a discontinuous fashion, positing an investment tax credit as following a Poisson Process, randomly switching between a low and a high value.

We begin with some simple historical statistics on the ITC in the U.S. These data suggest that there is considerable uncertainty with respect to an ITC. We next turn to some Monte Carlo experiments. When tax policy uncertainty leads to capital costs following a continuous time random walk in logs, increasing uncertainty delays investment. This result follows directly from work by Pindyck (1988) and others. However, when tax policy follows a Poisson Process, we find that increasing uncertainty speeds up investment. In the latter case, investment "piles up" in periods in which the high ITC is in effect. If the frequency with which the ITC is altered is high, then the odds of reaching the high-ITC state sooner, wherein investment usually occurs, are higher.

Our results show that whether random tax policy delays or speeds up investment depends critically on the form of policy uncertainty. While modeling uncertainty by Geometric Brownian

Motion can often provide tractable and useful results for modeling uncertainty in many applications, it is probably not a very realistic model for thinking about investment incentives in the U.S. tax code. The added complexity in moving from GBM to a jump process for policy uncertainty proves worthwhile.

Appendix A

We seek a solution to the set of differential equations (11) and (12). Let $Z = V^1 - V^0$ and $X = \lambda_1 V^1 + \lambda_0 V^0$. Making this change of variables yields the two independent differential equations in Z and X:

(A1) $$\rho Z = .5\sigma^2 p^2 Z_{pp} + \mu p Z_p - (\lambda_1 + \lambda_0) Z$$

(A2) $$\rho X = .5\sigma^2 p^2 X_{pp} + \mu p X_p + (\lambda_1 + \lambda_0) X$$

We try a solution of the form Ap^{γ} for Z. Substituting in to equation A1 yields

(A3) $$Ap^{\gamma}(.5\sigma^2\gamma(\gamma-1) + \mu\gamma - (\rho + \lambda_1 + \lambda_0)) = 0$$

A general solution to the differential equation in A1 is given by $Z = A_1 p^{\gamma_1} + A_2 p^{\gamma_2}$ where the γ's are the roots the quadratic equation $R(x) = \frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + \lambda_1 + \lambda_0)$ and the A's are constants to be determined. The limiting behavior or Z as p approaches zero provided information about A_1. Since zero is an absorbing state for p, Z must equal zero if p ever goes to zero. Since $\gamma_1 < 0$, the first term in the expression for Z would explode unless A_1 equals 0.

Using a similar approach for equation A2, we get a general solution for $X = B_1 p^{\beta_1} + B_2 p^{\beta_2}$ where the β's are the positive and negative roots to the quadratic equation $Q(x) = \frac{1}{2}\sigma^2 x(x-1) + \mu x - \rho$. Again we use a limiting argument as p approaches zero to determine that $B_1 = 0$. Substituting the expressions for Z and X into their definitions and solving for V^0 and V^1 gives us equations 13 and 14 in the paper.

A similar approach is used to solve for the value function V^0 in region 2. We plug in the value function V^1 in equation 16 and obtain a general solution to the differential equation as above. However, we cannot use limiting arguments to eliminate either of the constants C_1 or C_2 since p is bounded above and below by p_0 and p_1.

References

Abel, A. "Optimal Investment under Uncertainty," American Economic Review, 73(1983): 228-33.

Abel, A. "A Stochastic Model of Investment, Marginal q and the Market Value of the Firm," International Economic Review, 26(1985): 305-322.

Auerbach, A. and J. Hines, "Investment Tax Incentives and Frequent Tax Reforms," American Economic Review, 78(1988): 211-216.

Bizer, D. and K. Judd, "Taxation and Uncertainty," American Economic Review, 79(1989): 331-336.

Cummins, J, K. Hassett, and R.G. Hubbard, "A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments," mimeo, Columbia Business School, 1994.

Dixit, A. "Choosing Among Alternative Discrete Investment Projects Under Uncertainty," mimeo, Princeton University, 1992.

Dixit, A. and R. Pindyck, Investment Under Uncertainty, Princeton University Press: Princeton NJ, 1994.

Hartman, R. "The Effects of Price and Cost Uncertainty on Investment," Journal of Economic Theory, 5(1972): 258-66.

Koenker, R. and G. Bassett, "Robust Tests for Heteroskedasticity Based on Regression Quantiles," Econometrica, 50(1982): 43-61.

Lucas, R. "Econometric Policy Evaluation: A Critique," in K. Brenner and A. Meltzer, eds., The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy, Vol 1, Journal of Monetary Economics (1976): 19-46.

Pindyck, R. "Irreversible Investment, Capacity Choice, and the Value of the Firm," American Economic Review, 78(1988): 969-85.

Pindyck, R. "A Note on Competitive Investment under Uncertainty," American Economic Review, 83(1993): 273-277.

Summers, L. "Taxation and Corporate Investment: A q Theory Approach," Brookings Papers on Economic Activity (1981): 67-127.

Theil, H. Economics and Information Theory, Amsterdam: North Holland Press, 1967.

Table 1. Mean Preserving Spreads With
Continuous Policy Uncertainty

σ_K^2	$\left(\frac{p}{p_k}\right)^*$	t_{med}	$E(p)$	$E(p_k)$
0.00	.097	2.81	1.95	20.0
		(.16)	(0.0)	(0.0)
0.0025	.106	7.28	2.03	19.12
		(.40)	(.01)	(.08)
0.0050	.114	10.97	2.07	18.09
		(.59)	(.01)	(.10)
0.01	.131	15.93	2.09	15.91
		(.76)	(.02)	(.12)
0.02	.166	24.39	2.06	12.37
		(.93)	(.02)	(.13)
0.03	.204	25.44	2.05	10.04
		(.87)	(.03)	(.12)
0.04	.246	27.23	2.00	8.11
		(.71)	(.03)	(.11)
0.06	.344	27.71	1.95	5.65
		(.69)	(.03)	(.08)

This table presents results from a simulation in which output prices (p) and capital costs (p_k) follow Geometric Brownian Motion. The series have zero trend and are uncorrelated. The instantaneous variance of the output price series equals .01. The discount rate equals .05. The second column of the table presents the optimal ratio of output to capital prices for investment (the hurdle ratio). The next three columns present averages for key variables while t_{med} is the median time to investment. There are 1500 replications for each simulation.

Table 2. Mean Preserving Spreads in Jump Processes

π_0	π_1	$\left(\frac{p_0}{(1-\pi)p_k}\right)^*$	$\left(\frac{p_1}{(1-\pi)p_k}\right)^*$	t_{med}	E(p)	$E((1-\pi)p_k)$	$E(\Delta\pi)$
.10	.10	.097	.097	7.64 (.30)	1.75 (0.0)	18.00 (0.0)	-
.05	.15	.122	.092	5.48 (.30)	1.69 (.01)	17.14 (.02)	1.78 (.05)
.00	.20	.134	.091	4.29 (.17)	1.60 (.01)	16.08 (.03)	1.66 (.03)

This table presents hurdle prices and expected prices for output and capital cost (net of tax credit) at which investment takes place. In addition, it reports the expected number of changes in the ITC before investment occurs and the median time to investment. The probability of switching an ITC from a low (high) value to a high (low) value is fixed at .35. The index zero (one) indicates the low (high) ITC state. There are 1500 replications for each simulation.

Table 3. The Effect of Policy Frequency on Investment
$\pi_0 = 0 \quad \pi_1 = .20$

λ	$\left[\frac{p_0}{(1-\pi)p_k}\right]^*$	$\left[\frac{p_1}{(1-\pi)p_k}\right]^*$	t_{med}	$E(p)$	$E((1-\pi)p_k)$	$E(\Delta\pi)$
.10	.122	.090	9.59 (.34)	1.79 (.01)	16.89 (.05)	.88 (.02)
.20	.129	.090	6.07 (.22)	1.68 (.01)	16.29 (.03)	1.27 (.03)
.35	.134	.091	4.29 (.17)	1.60 (.01)	16.08 (.02)	1.66 (.04)
.50	.136	.092	3.09 (.14)	1.58 (.01)	16.02 (.01)	1.87 (.05)

See notes for table 2 for explanation of the table.

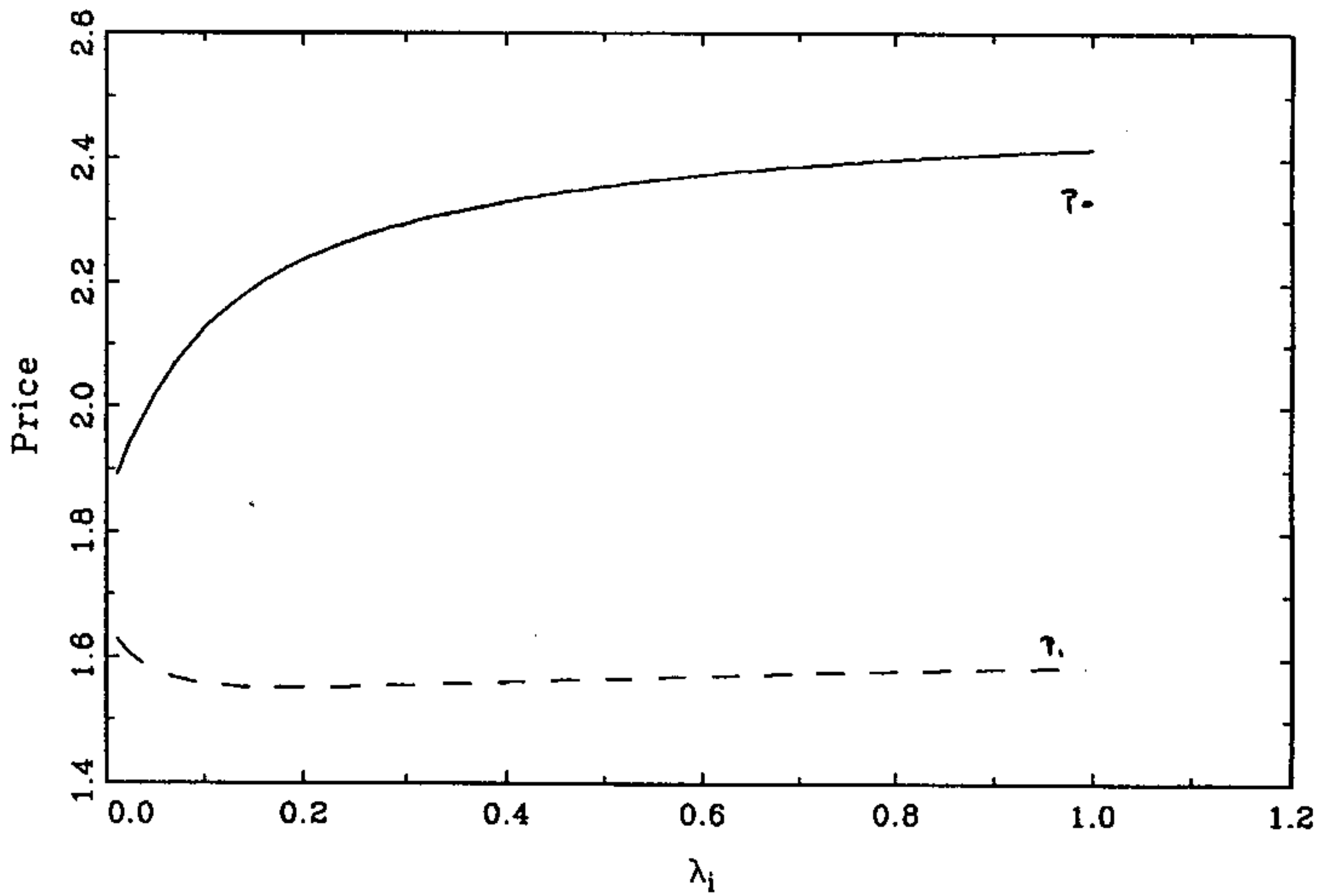
Table 4. The Effect of Policy Frequency on Investment
$\pi_0 = .05 \quad \pi_1 = .15$

λ	$\left(\frac{p_0}{(1-\pi)p_k}\right)^*$	$\left(\frac{p_1}{(1-\pi)p_k}\right)^*$	t_{med}	E(p)	$E((1-\pi)p_k)$	$E(\Delta\pi)$
.10	.112	.092	11.69 (.40)	1.81 (.01)	17.70 (.03)	.84 (.03)
.20	.118	.091	7.37 (.31)	1.76 (.01)	17.39 (.02)	1.24 (.03)
.35	.122	.092	5.48 (.30)	1.69 (.01)	17.14 (.02)	1.78 (.05)
.50	.124	.092	4.86 (.22)	1.66 (.01)	17.07 (.01)	2.31 (.07)

See notes for table 2 for explanation of table.

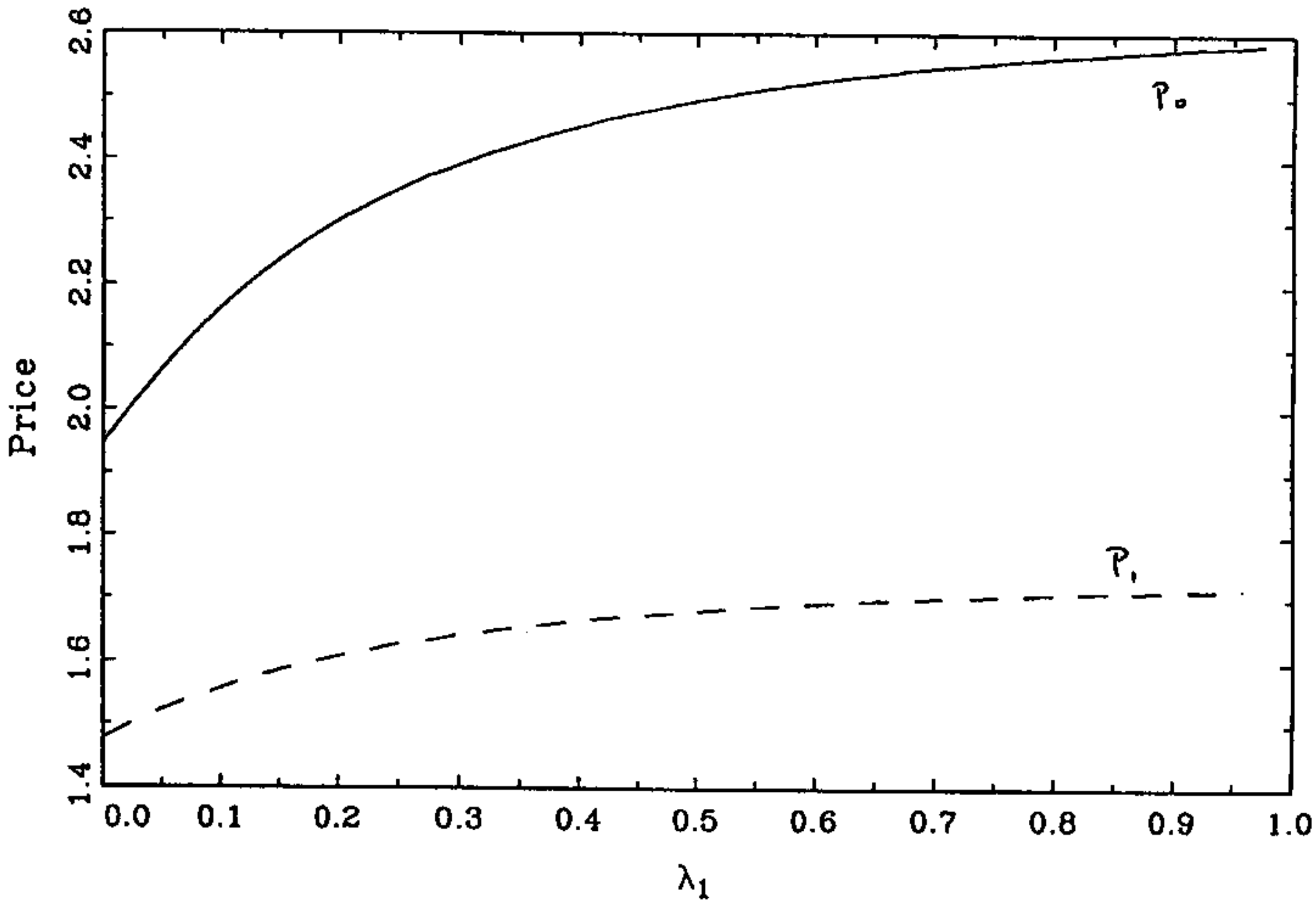
GAUSS Thu Apr 21 13:53:38 1994

Figure 1. Trigger Prices: $\lambda_1 = \lambda_0$



GAUSS Thu Apr 21 14:01:36 1994

Figure 2. Trigger Prices: $\lambda_0 = 0.33$



GAUSS Thu Apr 21 14:17:52 1994

Figure 3. Trigger Prices: $\lambda_1 = 0.33$