NBER WORKING PAPER SERIES

THE GENDER GAP, FERTILITY, AND GROWTH

Oded Galor
David N. Weil

Working Paper No. 4550

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November, 1993

We are grateful to Roland Benabou and Harl Ryder for their valuable comments and to seminar participants at Brown University, Harvard University, Hebrew University, Kobe University, and the NBER Summer Institute for helpful discussions. This paper is part of NBER's research program in Growth. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #4550
November 1993

THE GENDER GAP, FERTILITY, AND GROWTH

ABSTRACT

This paper examines a novel mechanism linking fertility and growth. Household fertility is determined by relative wages of women and men. Increasing women's wages reduces fertility by raising the cost of children relatively more than household income. Lower fertility raises the level of capital per worker which in turn, since capital is more complementary to women's labor input than men's, raises women's relative wages. This positive feedback leads to the possibility of multiple steady-state equilibria. Countries with low initial capital may converge to a development trap with high fertility, low capital, and low relative wages for women.

Oded Galor
Department of Economics
Box B
Brown University
Providence, R.I. 02912

David N. Weil
Department of Economics
Box B
Brown University
Providence, R.I. 02912
and NBER

1. Introduction

Changes in fertility have long been recognized as important correlates of economic growth. Indeed, the relationship between the level of fertility and the level of income per capita is one of the strongest observable correlations in cross country data. The nature of the relationship between development and fertility has been studied from the perspective of the theory of growth as well as from the perspective of family economics. Growth theory has focussed on the negative effect of population growth on the level of capital per worker, and thus on the level of output per worker, whereas family economics has focussed on the changes in the economic environment that lead families to reduce fertility as countries become wealthier.

This paper integrates these two strains of literature. It combines a model of the household's fertility/labor supply choice with a growth model in which the wages of men and women are endogenously determined. The main concern of the study is with how growth, via changes in relative factor prices, affects household decisions about the level of fertility and women's labor force participation, and how these decisions in turn feed back through the aggregate production mechanism to affect output growth. The analysis demonstrates that the effects of growth on fertility and of fertility on growth are mutually reenforcing.

Several recent studies of fertility and growth have focussed on various mechanisms by which the two variables are related. Becker and Barro (1988) consider fertility in the context of a model of intergenerational altruism, in which the discount applied to future consumption depends negatively on the number of descendants in future generations. In their model, increased technological progress will lead to a higher growth rate of consumption and to a lower rate of fertility. Becker, Murphy, and Tamura (1990) examine a model in which a high societal level of human capital raises the return to individual investments in human capital. In economies with high levels of human capital, families find it optimal to have few children, and to provide each child with a high level of human capital. The high level of human capital also leads to a high rate of economic growth, and thus economic growth is negatively correlated with fertility. Azariadis and Drazen (1990) explain the decline in fertility in the face of economic growth in a model where fertility is driven by individuals' desire to provide for their old age: an increase in market wages worsens the bargaining positions of parents whose principal asset is a family farm, leading to a reduction in the value of children.[1]

In this paper we examine a different mechanism linking fertility and growth, one that is more rooted in pre-existing models of fertility: the effect of economic growth, and in particular of capital deepening, on the relative wages of women and men. The model we present has three important components: First, the fertility decision of the household is taken to be a function of the relative wages of women and men. Higher wages for women raise the cost of children relatively more than they raise household income, and lead to a reduction in the number of children that couples choose to have. Second, the rate of population growth affects the level of capital per worker. Finally, the level of capital per worker affects the relative wages of men and women. Higher capital per worker raises women's relative wages.

The first part of our story – the analysis of fertility in terms of men's and women's relative wages – dates back to Becker (1960) and Mincer (1963). Children are considered durable goods that appear in the parents' utility function. The pure effect of an increase in household income holding the price of children constant is to raise the demand for children. If all childrearing is done by women, an increase in men's wages will have such a pure income effect. Increases in women's wages raise both household income and the price of children, and so have offsetting income and substitution effects on the demand for children. The overall effect on fertility of a proportional increase in men's and women's wages is theoretically ambiguous. One way to draw the link between economic

[1] In the model of Kremer (1993), the growth rate of output is indirectly related to fertility via the effect of the size of the population on the growth rate of output.

growth and fertility declines is to simply assume that the utility function is such that the substitution effect dominates and so fertility falls as countries become richer. We take a more restrictive approach in this paper, choosing a utility function under which proportional increases in men's and women's wages will keep fertility constant. Instead, we focus on a theoretically less ambiguous channel: the effect of an increase in women's *relative* wage in lowering fertility. An example of an application of this model is Heckman and Walker's (1990) finding of a negative effect of women's wages and a positive effect of male income on birth rates. Similarly, Schultz (1985), using world changes in the prices of agricultural commodities as an instrument to overcome the endogeneity of income and labor supply, finds that an increase in the relative wages of women played an important role in Sweden's fertility transition.

The second part of our story – the effect of population growth on the level of capital per worker – is a standard part of almost all growth models. Barro (1991) and Mankiw, Romer, and Weil (1992), among many others, cite the effect of capital dilution to explain the negative coefficient on the rate of population growth in cross-country regressions of either the level or the growth rate of income.

The final piece of our story is that an increase in the capital intensity of the economy raises the relative wages of women. An increase in women's relative wages seems to be part of the process of economic development. In the U.S., full time earnings of women rose from 46% to 67% of men's earnings over the period 1890-1988 (Goldin, 1990; Blau and Kahn, 1992). Although data is not available for all sectors of the economy, Goldin reports that women's relative wages rose significantly over the course of the Nineteenth Century in both agriculture and manufacturing. Schultz (1981) reports that, although the data are of uneven quality, a similar increase is present in the sample of countries he examines over the period 1938-78. One explanation for this rise in women's wages is that as economies develop, they are more prone to reward the attributes in which women have a comparative advantage: while women and men have equal quantities of brains, men have more brawn. And, the more developed is an economy, the higher the rewards of brains relative to brawn. For example, Goldin reports that industrialization at the beginning of the 19th century was responsible for a dramatic increase in the relative wages of women.

The three pieces of our model lead to a positive feedback loop: the aggregate effect of women substituting out of childrearing and into labor force participation is to increase the level of capital and output per worker and to enhance those conditions which induce further substitution away from fertility. This positive feedback mechanism, along with a discontinuous change in the *rate* of output growth that occurs as women join the labor force, leads to the possibility that the economy with a given set of conventional non-altruistic preferences and CRS technology may exhibit multiple stable steady state equilibria.[2] In one steady state, fertility is high, output and capital per worker are low, and women's wages relative to those of men are low. In the other steady state, fertility is low, output and capital per worker are high, and women's relative wages are high. Thus initial conditions may determine a country's long-run steady state equilibrium. Countries with a low initial level of capital per-worker may converge to a development trap where high fertility induces lower per-worker capital and output which in turn induces women, who confront low relative wages, to maintain their high fertility rate and low labor supply.

Regardless of whether the economy displays multiple equilibria, the dynamics of the model provide important insights. Even when the economy is characterized by a unique, globally stable steady-state equilibrium, the dynamical path towards this steady-state equilibrium exhibits non-monotonic behavior as a result of the feedback between fertility and the gender gap in wages. In particular, as long as women's participation in the labor

[2]The existence of multiple steady-state equilibria in a one-sector overlapping-generations model is consistent with the neoclassical assumptions concerning preferences and technology (e.g., Galor and Ryder (1989)). The multiplicity of equilibria in the current model, however, can occur even under a set of parameters that would guarantee uniqueness in the conventional one-sector overlapping-generations model. In such a case, the multiplicity is due to the discontinuity in the rate of growth that is associated with women joining the labor force.

force is relatively low, the level of output grows at a decreasing rate while fertility remains high. However, once the per-worker capital stock is sufficiently high so as to support an attractive relative wage to women, the economy will experience an acceleration of the growth rate that is associated with a decline in fertility. Ultimately, the growth rate declines as output converges to a steady-state equilibrium with a lower fertility rate and higher labor force participation by women.

The rest of this paper is organized as follows. In Section 2, we formalize the assumptions about the determinants of fertility and relative wages presented above, and incorporate them into an overlapping generations model. We derive the dynamical system implied by the model, and show that the model can have multiple steady states. Section 3 considers the dynamics of the model in the case where the state of technology is constant, and where it is growing exogenously over time. In Section 4, we discuss an extension of the model to incorporate the accumulation of human capital. Section 5 concludes.

2. Structure of the Model

We consider an overlapping generations model in which people live for three periods. In the first period of life, people are children: they consume a fixed quantity of time from their parents. In the second period of life, people raise children and supply labor to the market, earning a wage. For convenience, we assume that they do not consume in this period. In the third period of life people do not work, and they consume their wages from the previous period along with accrued interest. The capital stock in each period is equal to the aggregate savings in the previous period.

We model the economy as being made up of two kinds of people: men and women. In childhood and old age, the men and women are identical. In adulthood, however, men and women differ in terms of their ability to earn wages in the labor market. Men and women are endowed with different proportions of two kinds of labor input. Workers can supply both raw physical strength and mental input. We assume that men and women have equal endowments of mental input to contribute, but that men have more physical strength than women.

Although we are concerned with the differences between men and women, our basic unit of analysis is the couple, which is composed of one man and one woman. Couples are taken to have joint consumption and joint utility. There is no heterogeneity within a generation. Rather than model the matching of men and women, we assume that couples are "born" as such. Explicitly modelling couples as matches between men and women from different families would not change the results presented here.

2.1 Production

There are three factors of production: physical capital, K, physical labor, L^p, and mental labor L^m. Physical labor is the kind of labor in which men have superior abilities to women, that is, work requiring strength. Mental labor is labor in which men and women have equal abilities. To simplify matters we will assume that women have *no* physical strength, but the results presented below will follow as long as women have less strength than men.

Our key assumption will be that, the richer in physical capital is an economy, the more highly rewarded is mental labor relative to physical labor.[3] To give a simple example, if the only form of capital is a shovel, then men will be far more productive in digging ditches than will be women. If there is more capital available — in the form of a tractor, for example — then the relative productivity of men and women will be more nearly equal. The reason for this effect is that, at least so far, physical capital does a better job replacing human strength than it does replacing human thinking.[4]

[3] For example, Goldin (1990) writes "The labor market's rewards for strength, which made up a large fraction of earnings in the nineteenth century, ought to be minimized by the adoption of machinery, and its rewards for brain power ought to be increased."

[4] This idea — that the reward to physical labor is falling relative to the reward to mental labor — has appeared in labor economics in discussions of growth of the wage premium to educated workers. See Katz and Murphy (1991) and Mincer (1991).

We do not explicitly include the accumulation of human capital in our basic model, but in Section 4 we extend the model to include it. There we assume that the accumulation of human capital enhances the input of mental labor, the kind that men and women have in common. Our key assumption that physical capital raises the return to mental labor more than it raises the return to physical labor thus becomes equivalent to assuming that physical capital is more complementary to human capital than it is to physical labor.

Other work in this area has placed far more severe restrictions on the production function. For example, Becker, Murphy, and Tamura (1990) assume that, holding other factors constant, there are increasing returns to human capital over some range of the production function. By contrast, we make the standard assumption that all factors have non-increasing marginal products.

Technically, our assumption is that an increase in physical capital input raises the marginal product of mental labor proportionally more than it raises the marginal product of physical labor. In other words, physical capital complements mental labor more than it complements physical labor. Griliches (1970) proposes just such an assumption to explain the failure of the relative wage of educated workers to fall in the face of growth in the stocks of physical capital and educated labor. Whether physical capital actually reduces the marginal product of physical labor we consider an open question, but the answer is not essential for our results.

The production function that we use incorporates the above assumptions in a simple way: we assume that physical capital and mental labor exhibit complementarity in production, whereas physical labor is neither a complement nor a substitute for either of the other factors of production. Specifically, the production function is:

$$Y_t = a[\alpha K_t^\rho + (1-\alpha)(L_t^m)^\rho]^{1/\rho} + bL_t^p, \tag{1}$$

where $a, b > 0$, $\alpha \in (0,1)$ and $\rho \in (-\infty, 1)$.[5] Exogenous technological progress is considered in section 3.2.

Since only men supply physical labor and, as will be justified below, men supply this labor inelastically, the total amount of physical labor input, L^p, is equal to the number of working-age couples. We can thus rewrite the production function in per-couple terms as

$$y_t = a[\alpha k_t^\rho + (1-\alpha) m_t^\rho]^{1/\rho} + b \tag{2}$$

where $k_t \equiv K_t / L_t^p$ is the per-couple capital stock at time t and $m_t \equiv L_t^m / L_t^p$ is the per-couple input of mental labor. Since the man will always supply one unit of physical and one unit of mental labor, and the woman will supply between zero and one units of mental labor, the variable m will take values between one and two.

All factors of production are assumed to earn their marginal products. Given the structure of the production technology, the return to a unit of physical labor at time t, w_t^ρ, and the return to mental labor at time t, w_t^m, are

$$w_t^p = b \tag{3}$$

$$w_t^m = a(1-\alpha) m_t^{\rho-1} [\alpha k_t^\rho + (1-\alpha) m_t^\rho]^{\frac{1-\rho}{\rho}} \tag{4}$$

Men earn a wage of $w_t^p + w_t^m$, while women earn a wage of w_t^m. Increases in the amount of physical capital, holding m_t constant, raise the return to mental labor thus reducing the proportional wage gap between men and women.

2.2 Couples' Decision Problem

Couples receive utility from the number of children that they have and from consumption in the last period of life. There is no uncertainty and no bequest motive. The utility function is

$$u_t = \gamma \, ln \, n_t + (1-\gamma) \, ln \, c_{t+1} \tag{5}$$

[5] $\rho \in (-\infty, 0)$ implies that the elasticity of substitution between capital and mental labor is smaller than one. As ρ increases in absolute value the complementarity between capital and mental labor rises.

where n_t is the number of children that the couple has.[6] Note that since the basic unit of counting that we are using in this model is the couple, n_t is in fact the number of *couples* that each couple has as children.

We follow the standard "demand" model of household fertility behavior (see Birdsall, 1988, for a summary) in assuming that the household chooses the number of children to have in the face of a constraint on the total amount of time that can be devoted to child-raising and labor market activities. Time spent raising children cannot be spent working, and so the opportunity cost of children is proportional to the market wage. We ignore any issues of child quality, focusing only on the quantity of children. We do not assume that women are better at raising children than are men but, given the differences in factor endowment between men and women, the opportunity cost of raising children is higher for a man than for a woman. Thus, as in Becker (1985), a small difference in endowments can lead to specialization and to large differences in earnings.[7] Adding an assumption that women are superior to men in their child-rearing abilities would not affect our results.

The household's income in the first period is $w_t^p + 2w_t^m$ if the family does not have any children. Let z be the cost in time of raising one child, that is, z is the fraction of the time endowment of one parent that must be spent in order to raise one child.[8] If the wife spends time raising children, then the marginal cost of a child is $z \cdot w_t^m$. If the husband spends time raising children, then the marginal cost of a child is $z \cdot (w_t^m + w_t^p)$. Consequently if $zn_t \leq 1$ only the wife raises children, while if $zn_t > 1$ the wife will spend full-time and the husband part time raising children. Since the couple

[6] Utility from consumption in period t could have been incorporated into the analysis without altering the qualitative nature of this paper's results.

[7] In the model presented in Section 4, such specialization will also lead to a large difference in wage rates.

[8] The existence of economies of scale in raising children will not affect the analysis of division of labor within the household presented here. In such a case, however, the budget constraint relating household consumption to the number of children will curve inward in a manner similar to that analyzed in Section 4 of this paper.

does not generate utility from consumption at time t, the couple's income is divided between expenditure on child rearing and savings for future consumption, s_t, so as to maximize their intertemporal utility function. The couple faces in the first period the budget constraint:[9]

$$\begin{aligned} w_t^m z n_t &+ s_t \leq w_t^p + 2w_t^m && \text{if} \quad zn_t \leq 1 \\ w_t^m &+ (w_t^m + w_t^p)(zn_t - 1) + s_t \leq w_t^p + 2w_t^m && \text{if} \quad zn_t \geq 1 \end{aligned} \tag{6}$$

In the second period, the couple simply consumes the value of their savings with accrued interest:

$$c_{t+1} = s_t(1 + r_{t+1}). \tag{7}$$

The only decision that the household makes is how many children to have — alternatively, the household can be seen as deciding what fraction of the household's time should be spent working, and thus saving for future consumption, and what fraction raising children.

Figure 1 shows the kinked budget constraint facing the couple. There are three possible optima: first, if an indifference curve is tangent to the lower portion of the budget constraint, at a point like A, the woman will work part time and raise children part time, while the man works full time. Second, if an indifference curve is tangent to the upper portion of the budget constraint, at a point like B, then the man will work part time and raise children part time, while the woman raises children full time. It is obvious that both of these two conditions cannot hold at the same time. Finally, if neither of these conditions hold, then the couple's optimum will be at the kink point C, where men and women are completely specialized; women raising children full time and the men working full time.

Maximizing (5) with respect to n_t subject to the constraint (6) and (7) it follows

[9] Alternatively, the budget constraint can be written as: $zn_t w_t^m + w_t^p \max[0, zn_t - 1] + s_t \leq w_t^p + 2w_t^m$.

that the time spent by parents on raising children is,

$$zn_t = \begin{cases} \gamma\{2+(w_t^p/w_t^m)\} & \text{if} \quad \gamma\{2+(w_t^p/w_t^m)\} \leq 1 \\ 2\gamma & \text{if} \quad 2\gamma > 1 \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

For a sufficiently low relative wages of mental labor, (8) implies that women raise children full time. As the relative wage of mental labor increases, women may join the labor force and increase gradually the fraction of their time devoted to market labor. In the limit, as the wage of mental labor rises, women spend a fraction $\min(1, 2\gamma)$ of their time raising children. Note that if $\gamma > 1/2$, then women will not supply labor and will devote themselves for raising children, no matter how high the wage of mental labor. Since we observe that women do supply labor when their wages are sufficiently high, we will restrict γ to be less than $1/2$. This assumption guarantees that for some low enough ratio of (w_t^p/w_t^m) women will supply labor. Furthermore, as follows from (8) this restriction implies that zn_t is necessarily bounded from above by one and consequently men allocate their entire time endowment to work and do not participate in raising children. Figure 2 shows the effect of an increase in the relative wage of women on the couples choice of fertility and saving. For simplicity, it is drawn for the case where women's wages move from being half of men's wages to being equal to men's wages, while men's wages remain constant.

Thus, given that $\gamma < 1/2$,

$$zn_t = \min[1, \gamma\ \{2+(w_t^p/w_t^m)\}], \tag{9}$$

and the couple's saving is[10]

[10]Note that if $zn_t = 1$ it follows from (9) that $w_t^p/w_t^m = (1-2\gamma)/\gamma$. Consequently, for $zn_t = 1$, as suggested by (10), $(1-\gamma)[w_t^p + 2w_t^m] = w_t^p + w_t^m$.

$$s_t = \begin{cases} (1-\gamma)[w_t^p + 2w_t^m] & \text{if} \quad zn_t \leq 1 \\ w_t^p + w_t^m & \text{if} \quad zn_t = 1. \end{cases} \tag{10}$$

Since

$$m_t = \frac{L_t^m}{L_t^p} = \frac{L_t(2-zn_t)}{L_t} = 2 - zn_t \tag{11}$$

it follows from (9) and (11) that, for $\gamma < \frac{1}{2}$,

$$zn_t = \min[1, \gamma\{2 + b/\{a(1-\alpha)(2-zn_t)^{\rho-1}[\alpha k_t^\rho + (1-\alpha)(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}\}]. \tag{12}$$

Let $G(zn_t, k_t) \equiv zn_t - \gamma\{2 + b/\{a(1-\alpha)(2-zn_t)^{\rho-1}[\alpha k_t^\rho + (1-\alpha)(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}\} = 0$. Following the implicit function theorem since $\partial G(zn_t, k_t)/\partial n_t$ is strictly monotonic and non-vanishing $\forall k_t \geq 0$, there exists a differentiable and invertible function $\psi(k_t)$ such that

$$zn_t = \min[1, \psi(k_t)], \tag{13}$$

where $\psi'(k_t) < 0 \ \ \forall k_t \geq 0$.

Since $zn_t = 1$ if and only if $k_t \leq k^*$, where

$$k^* = \psi^{-1}(1), \tag{14}$$

it follows that

$$zn_t = \begin{cases} \psi(k_t) & \text{for} \quad k_t \geq k^* \\ 1 & \text{for} \quad k_t \leq k^* \end{cases} \tag{15}$$

where $\psi(k_t) \in (0,1] \ \ \forall k_t \geq k^*$.

2.3 The Dynamical System

The stock of capital at time $t+1$ is determined by the aggregate supply of savings at time t:

$$K_{t+1} = L_t s_t. \tag{16}$$

The number of working age households at time $t+1$ is

$$L_{t+1} = n_t L_t. \tag{17}$$

Thus, following (9), (10) and the definition of k^*[11]

$$k_{t+1} = \frac{s_t}{n_t} = \begin{cases} z\,\frac{1-\gamma}{\gamma}\,w_t^m & \text{if } k_t \geq k^* \\ z[w_t^p + w_t^m] & \text{if } k_t \leq k^* \end{cases} \tag{18}$$

The dynamical system is governed by the evolution of the per-couple capital stock from an historically given initial stock k_0. Using (3), (4), (11), (15) and (18), the dynamic equilibrium sequence $\{k_t\}_{t=0}^{\infty}$ is determined by

$$k_{t+1} = \phi(k_t) = \begin{cases} za(1-\alpha)\frac{1-\gamma}{\gamma}\frac{\{\alpha k^\rho + (1-\alpha)[2-\psi(k_t)]^\rho\}^{\frac{1-\rho}{\rho}}}{[2-\psi(k_t)]^{1-\rho}} & \text{if } k_t \geq k^* \\ z\{b + a(1-\alpha)[\alpha k_t^\rho + (1-\alpha)]^{\frac{1-\rho}{\rho}}\} & \text{if } k_t \leq k^* \end{cases} \tag{19}$$

where the initial level of per-couple capital stock, k_0, is historically given.

Along the dynamic path k_t evolves monotonically. Namely,

$$\phi'(k_t) = \begin{cases} \alpha\frac{1-\gamma}{\gamma}Ak_t^{\rho-1}\frac{[2-\psi(k_t)+k_t\psi'(k_t)]}{[2-\psi(k_t)]^{2-\rho}\{\alpha k_t^\rho + (1-\alpha)[2-\psi(k_t)]^\rho\}^{2-\frac{1}{\rho}}} > 0 & \text{if } k_t \in (k^*, \infty) \\ \alpha A k_t^{\rho-1}[\alpha k_t^\rho + (1-\alpha)]^{\frac{1}{\rho}-2} > 0 & \text{if } k_t \in (0, k^*) \end{cases} \tag{20}$$

where $A \equiv za(1-\alpha)(1-\rho)$.

Furthermore, as obtained from (19) and (20)

$$\begin{aligned} \phi(0) &= z[b + a(1-\alpha)^{1/\rho}] > 0; \\ \lim_{k_t \to \infty} \phi'(k_t) &= 0; \end{aligned} \tag{21}$$

---

[11]Note that as follows from (10) $k_t = k^*$ (and thus $pn_t = 1$) if $w_t^p / w_t^m = (1-2\gamma)/\gamma$. Thus, for $k_t = k^*$, as suggested by (18) $z[(1-\gamma)/\gamma]w_t^m = z[w_t^p + w_t^m]$.

and,

$$\phi''(k_t) = \frac{\alpha A k_t^{\rho-2}[(1-\alpha)(\rho-1) - \alpha\rho k_t^{\rho}]}{[\alpha k_t^{\rho} + (1-\alpha)]^{3-\frac{1}{\rho}}} \quad \forall k_t \in (0, k^*). \tag{22}$$

Consequently,

$$\lim_{k_t \to 0} \phi''(k_t) \begin{cases} < 0 & \text{if} \quad \rho \in [0,1) \\ > 0 & \text{if} \quad \rho \in (-\infty, 0) \end{cases} \tag{23}$$

and

$$\phi''(k_t) < 0 \quad \forall k_t \in (0, k^*) \quad \text{if} \quad \rho \in [0,1). \tag{24}$$

Thus, as long as ρ is non-negative (i.e., the degree if complementarity between mental labor and capital is relatively small), $\phi(k_t)$ is strictly concave over the internal $(0, k^*)$, whereas as long as ρ is negative (i.e., the degree of complementarity between mental labor and capital is relatively large), $\phi(k_t)$ is strictly convex over the interval $[0, \tilde{k})$ where $\tilde{k} \in (0, k^*)$.

2.4 Steady-State Equilibria

The evolution of the per-couple capital stock $\{k_t\}_{t=0}^{\infty}$ determines the evolution of the fertility rate $\{n_t\}_{t=0}^{\infty}$ as well as that of the level of per-couple output $\{y_t\}_{t=0}^{\infty}$. A steady-state equilibrium is a stationary level of the per-couple capital stock $\overline{k}$, such that

$$\overline{k} = \phi(\overline{k}). \tag{25}$$

Corresponding to $\overline{k}$, are a stationary fertility rate $\overline{n}$ and a stationary level of the per-couple output $\overline{y}$.

Since $\phi(k_t)$ is a continuous function of k_t, a steady-state equilibrium exists if $\phi(0) > 0$ and there exists k_t such that $\phi(k_t) < k_t$. As established in (23) $\phi(0) > 0$, $\lim_{k_t \to \infty} \phi'(k_t) = 0$, and therefore $\phi(k_t) < k_t$ for some $k_t > 0$. Thus, a steady-state equilibrium exists. However, the steady-state equilibrium need not be unique.

Given the strict monotonicity of $\phi(k_t)$ and given that $\phi(0) > 0$, multiple nontrivial stable steady-state equilibria exist if $k^* > 0$, $\phi(k^*) < k^*$, $\exists k_t > k^*$ such that $\phi(k_t) > k_t$, and $\lim_{k_t \to \infty} \phi'(k_t) = 0$. Noting (19) - (24) and noting that $\phi(k^*) < k^*$ implies that $k^* > 0$, it follows that for some range of parameter values the system is characterized by multiple steady-state equilibria. In particular, for any feasible set of values for the parameters, a, b, α, γ, and z there exists a sufficiently large negative value of ρ such that multiple steady-state equilibria exists.

Furthermore, as can be verified using equation (20) the slope of the dynamical system in a close neighborhood to the right of k^* is more than that in a close neighborhood to the left of it. Namely,

$$\lim_{k_t \to k^*_-} \phi'(k_t) < \lim_{k_t \to k^*_+} \phi'(k_t). \tag{26}$$

Figure 3 describes the dynamical system in the case where there exists a unique steady-state equilibrium. Figure 4 describes the dynamical evolution of the economy in the case where the dynamical system is characterized by multiple stable nontrivial steady-state equilibria; a low output and high fertility steady-state and a high output and a low fertility steady-state.

The existence of multiple steady-state equilibria in a one-sector overlapping-generations model is consistent with the neoclassical assumptions concerning preferences and technology (e.g., Galor and Ryder (1989)). The multiplicity of equilibria in the current model occurs because of the discontinuity in the rate of growth that is associated with women joining the labor force, even under a set of parameters that would guarantee uniqueness in the conventual one-sector overlapping-generations model.

3. The Joint Evolution of Fertility and Output

3.1 Constant Technology

The joint evolution of fertility and per-worker output is governed by the dynamical system explored in section 2. As is established in the course of this analysis, the dynamical system may exhibit multiple stable nontrivial steady-state equilibria. Regardless of whether multiple steady states exist, however, the model generates important insights into the dynamic behavior of output per worker and fertility. The dynamical path towards a steady-state equilibrium is consistent with the inverse relationship between per-capita output and fertility. Furthermore the model makes novel predictions about the acceleration of fertility decline and output growth during a demographic transition.

Consider Figure 3, where the economy is characterized by a unique globally stable steady-state equilibrium. The pace of the evolution of the per-couple capital stock is not monotonic. The pace declines as the capital stock grows towards k^*, accelerates once k^* is passed, and declines once again as the economy approaches the steady-state equilibrium $\overline{k}$. Thus, as long as women do not participate in the labor force (i.e., $k_t < k^*$), the rate of growth of output declines over time, the level of output remains relatively low and the level of fertility remains relatively high. However, once the per-worker capital stock is sufficiently high so as to support an attractive relative wage to women (i.e., once the level of per-couple capital stock exceeds k^*), the economy experiences an accelerated growth that is accompanied by a declining fertility rate. Ultimately, growth slows down and the economy converges to a high output, low fertility, steady-state equilibrium.

Consider Figure 4 which describes the evolution of an economy that is characterized by multiple steady-state equilibria. In one steady state, fertility is relatively high, output and capital stock per worker are relatively low, and women's wages relative to those of men are low. In the other steady state, fertility is lower, output and capital per worker are higher, and women's relative wages are higher. The two steady states differ in their levels of female labor force participation: women spend all of their time raising children in the lower steady state, while in the higher steady-state women work part-time and raise children part-time. In this case initial conditions determine a country's long-run steady state equilibrium. Countries with a relatively low initial level of capital per worker may converge to a development trap where high fertility induces lower per-capita capital, which in turn induces women, who confront low relative wages, to maintain their high fertility rate and low labor supply.

3.2 Technological Progress

Suppose that in every period the economy experiences an exogenous technological change:

$$a_t = a_0\lambda^t; \quad b_t = b_0\lambda^t; \quad \lambda > 0. \tag{27}$$

The technological change is neutral with respect to the different factors of production (mental labor, physical labor, and capital), that is, it raises all of their marginal products equally. Modifying the analysis in the previous sections, it follows from (12)-(14) and (19)-(24) that such technological change will shift the function $\phi(k_t)$ upward in a proportional manner. However, the value of k^*, the point at which the $\phi(k_t)$ function kinks upward, will not change. In the case where there is a single steady state equilibrium, the qualitative nature of the dynamical system will not change. However, if multiple steady-state equilibria exist, this possibility will ultimately disappear. In particular, the lower of the two stable steady states will no longer exist for sufficiently productive technology. A country which is at the lower steady state at the point in time when multiple equilibrium is not longer possible will eventually experience a fertility transition and a period of rapid output growth as it moves to the unique steady state.

4. The Model with Human Capital

This section extends the couple's fertility choice problem presented in Section 2 to include the choice of accumulating human capital. We model the accumulation of human capital as a third potential use of time – along with raising children and working. Accumulation human capital is assumed to raise a person's level of mental labor input.[12] Although we do not re-consider the full dynamic system, we show that the response of

[12] Although they take place during the same period of life, we are implicitly we assume that all accumulation of human capital is done before market labor is supplied.

fertility to increases in the accumulation of physical capital will be the same as that considered above.

For an individual, the market wage received will depend on the level of human capital accumulated. Human capital accumulation will depend, in turn, on the amount of time left over for work: a person who spends a large amount of time raising children will optimally accumulate less human capital than one who spends only a little time raising children. Put another way, a person who spends a high fraction of his or her time raising children will have a low marginal cost of children in terms of the market wage. Figure 5A shows the budget constraint that would face an individual who divided his or her time between raising children, acquiring human capital, and working. The fact that it is bowed inward reflects the fact that the marginal cost of time spent raising children increases as the number of children decreases.

To simplify our analysis, we maintain the assumption that γ, the weight of children in the couple's utility function, is less than one half, and we add a further assumption: that the return to physical labor is not so high that the couple will choose to have the man work full time and not accumulate any human capital. These assumptions guarantee that the man will not spend any time raising children, and that the man's accumulation of human capital will be invariant to how much time is spent raising children by the woman. The couple's budget constraint will thus have the odd shape — two bowed-inward segments, connected at a kink — depicted in Figure 5B.

Given the budget constraint in Figure 5B, the couple will choose fertility as in Section 2. The slopes of the different line segments which connect the bowed sections of the budget constraint (that is, segments AB and BD in Figure 5) are determined by the relative wages of physical and mental labor. If the wage of physical labor is zero, then a woman who devotes no time to childrearing will earn the same wage as a man, and the lower part of the budget constraint will be an arc between points B and C.

The curvature of the budget constraint will mean the properties noted in Section

2 — that as the wage for mental labor increases, fertility is at first constant, but then falls — will not only be present, but may also be exacerbated. As the relative wage for mental labor rises, the slope of the line segment connecting points B and D will rise. At a high enough relative wage for mental labor, fertility will fall. Because of the curvature of the budget constraint, the change in fertility (and in women's human capital investment) may not be continuous: a small increase in the wage for mental labor may lead to a jump down in fertility and a jump up in the level of women's human capital accumulation.

5. Conclusion

We have presented a general equilibrium model in which there is a positive feedback from low fertility to higher capital and output per worker, higher relative wages for women, and back to low fertility. The model is stripped down to highlight the effect in which we are interested, but even in its simple form it presents fairly rich dynamics. In this last section we discuss extensions of the model to incorporate more realistic descriptions of the determinants of the key variables.

The two key effects in our model are the positive effect of capital accumulation on women's relative wages and the negative effect of women's relative wage on fertility. In each case we have used a simple model of the causal relationship. For the effect of capital accumulation, we have posited a production function in which capital is more complementary to the factors with which women are endowed than it is to the factors with which men are endowed. For the effect of relative wages, we have examined a simple "demand" model of the fertility choice, in which the higher relative wages for women raise the price of children by proportionally more than they raise the couple's full income, and thus lead to a reduction in fertility. In both these cases, however, it seems likely that the models we consider are proxies for more general tendencies which go in the same direction as the models we have used. Although difficult to incorporate into a model, these broader effects are important to note.

In attributing changes in women's relative wages over time to an increase in the level of capital, we do not intend to deny the importance of legal and social changes that have accompanied the emancipation of women. At the same time, we would suggest that many of these legal and social changes are in turn, at least partially, consequences of economic growth. Two examples: First, a capital-rich economy may require a more sophisticated mechanism for the enforcement of property rights and law generally than will arise in an economy with low capital accumulation, and the existence of a law-centered society may make it easier for women to overcome discrimination in the labor market. Second, education — the accumulation of human capital — may inevitably lead to the introduction of ideas that make it more difficult to sustain the oppression of part of a country's population. In either of these cases, the key effect that we have examined in this paper would still function: a higher capital stock would raise women's relative wages.

The determinants of fertility are similarly more complex than those embodied in the model we use. Among the factors that we have abstracted from are issues of child quality vs quantity, the effect of the children's labor income on the household budget, and the effect of contraceptive availability. Once again, however, some of these factors may in themselves depend of women's relative wages. For example, women's ability to influence the couple's use of contraceptives will be higher in families where women bring home a larger share of total income.

Finally, while the model presented here suggests a positive, monotonic relationship between income per capita and women's labor force participation, empirical evidence indicates that the relationship between the two is U-shaped. The source of this discrepancy is the assumption in our model that time spent raising children cannot also contribute to income. Goldin (1990), discussing the reduction of women's labor force participation during the 19th century, writes, "Early industrialization and the expansion of cities rapidly led to the specialization of tasks within the home and within the lives of women. Married women in an era of high fertility could be engaged in family labors only if work were done at home, and the progressive separation of home and work made their paid and unpaid labor less feasible." Thus our model applies only to the stage of development at which home production is no longer possible. An extension of the model in which women have access to home production technology that is not rival with child rearing and is not complemented by capital could potentially match the observed U-shaped relationship between income per capita and women's labor force participation.

Bibliography

Azariadis, Costas, and Allan Drazen, "Demographic Transition in a Dual Economy," mimeo, University of Pennsylvania, 1990.

Barro, Robert J., 1991, "Economic Growth in a Cross-Section of Countries," *Quarterly Journal of Economics,* 106, 467-444.

Becker, Gary S., 1960, "An Economic Analysis of Fertility" in Ansley J. Coale, ed., Demographic and Economic Change in Developed Countries, Princeton University Press, 1960.

Becker, Gary S., 1985, "Human Capital, Effort, and the Sexual Division of Labor," *Journal of Labor Economics,* 3, s33-58.

Becker, Gary S. and Robert J. Barro, 1988, "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics,* 103, 1-26.

Becker, Gary S., Kevin M. Murphy and Robert Tamura, 1990, "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy,* 98, S12-S37.

Birdsall, Nancy, 1988, "Economic Approaches to Population Growth," in Hollis Chenery and T. N. Srinivasan, eds., *The Handbook of Development Economics,* North Holland.

Blau, Francine and Lawrence Kahn, 1992, "The Gender Earnings Gap: Some International Evidence," NBER Working Paper 4224.

Galor, Oded, and Harl E. Ryder, 1989, "Existence, Uniqueness and Stability of Equilibria in an Overlapping-Generations Model with Productive Capital," *Journal of Economic Theory,* 49, 360-375.

Goldin, Claudia, 1990, *Understanding the Gender Gap: An Economic History of American Women.* New York: Oxford University Press.

Griliches, Zvi, 1970, "Notes on the Role of Education in Production Functions and Growth Accounting," in W. Lee Hansen, ed., *Education, Income, and Human*

Capital, Columbia University Press.

Heckman, James J. and James R. Walker, 1990, "The Relationship between Wages and Income and the Timing and Spacing of Births: Evidence from Swedish Longitudinal Data," *Econometrica* 58:6, 1411-1442.

Katz , and Kevin M. Murphy, 1991, "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," NBER Working Paper 3927.

Kremer, Michael, 1993, "Population Growth and Technological Change: One Million B.C. to 1990." *Quarterly Journal of Economics,* 108, 681-716.

Mankiw, N. Gregory, David Romer, and David N. Weil, 1992, "A Contributions to the Empirics of Economic Growth," *Quarterly Journal of Economics,* 107, 407-437.

Mincer, Jacob, 1963, "Market Prices, Opportunity Costs, and Income Effects," in Carl F. Christ, ed., *Measurement in Economics: Studies in Mathematical Economics and Econometrics in memory of Yehuda Grunfeld*, Stanford University Press.

Mincer, Jacob, 1991, "Human Capital, Technology, and the Wage Structure: What do Time Series Show?" NBER Working Paper 3581.

Schultz, T. Paul, 1985, "Changing World Prices, Women's Wages, and the Fertility Transition: Sweden, 1860-1910, *Journal of Political Economy*, 93:6, 1126-1154.

Schultz, T. Paul, 1981, *The Economics of Population*, Addison-Wesley: Reading, MA.

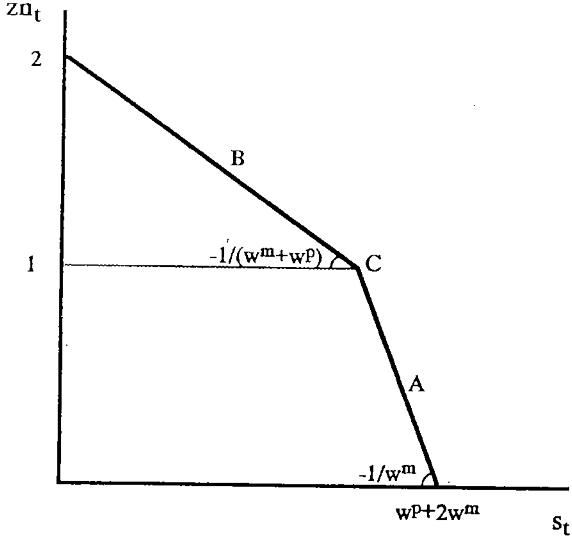

Figure 1. The couple's kinked budget constraint and three possible optima.

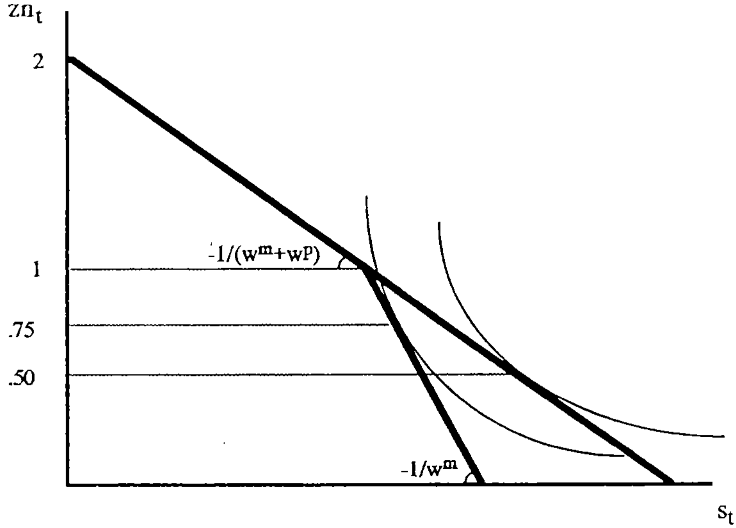

Figure 2. The effect of an increase in the women's relative wage on fertility
In this example $\Upsilon=1/4$ and women's wages rise from half of men's wages to be equal to men's wages.

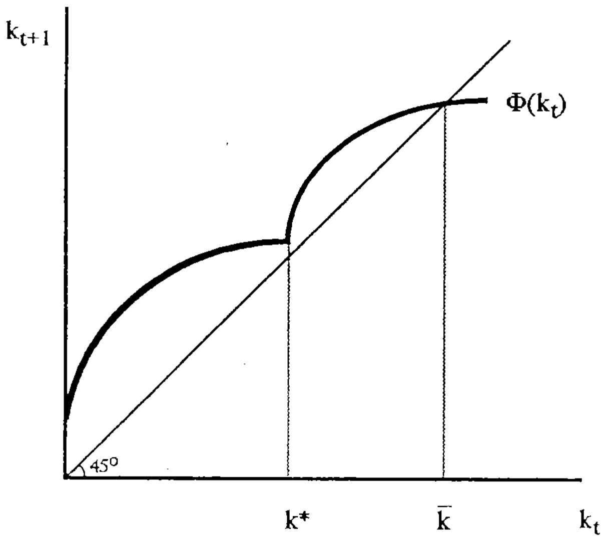

Figure 3. A Globally Stable Steady-State Equilibrium.

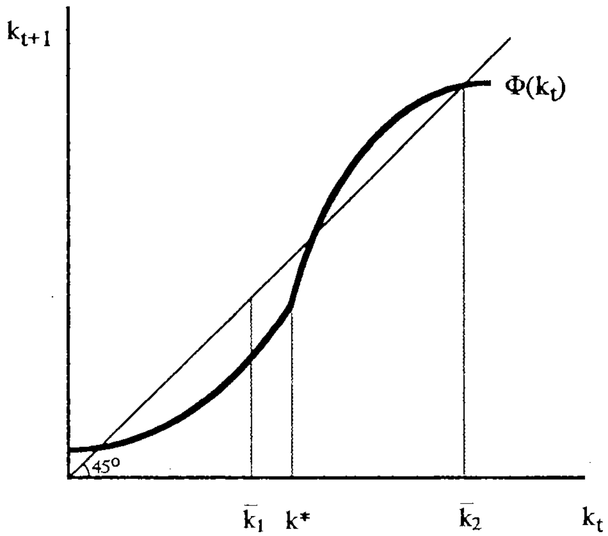

Figure 4. Multiple Locally Stable Steady-State Equilibria.

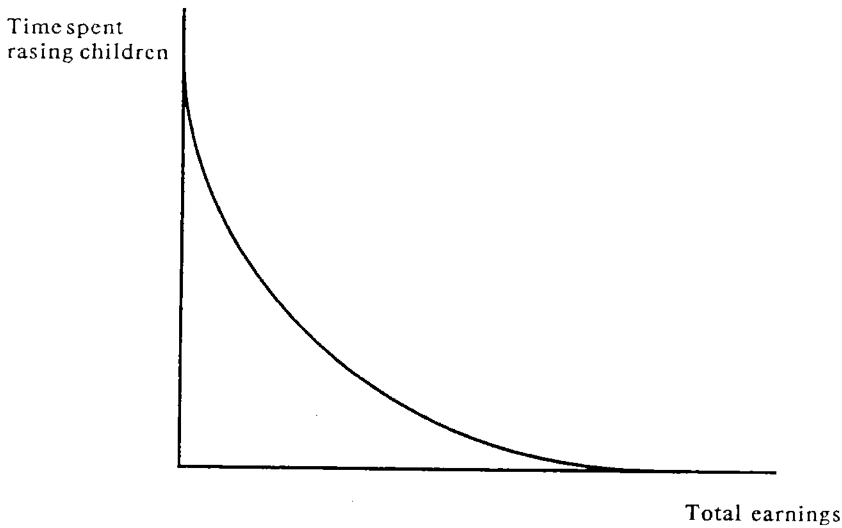

Figure 5a. Budget constraint for a single individual who can accumulate human capital.

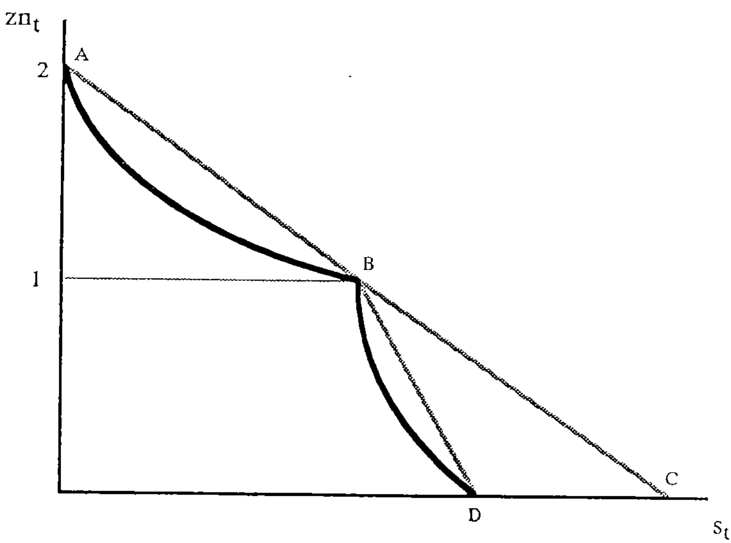

Figure 5b. The couple's budget constraint incorporating of human capital