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TESTS OF MICROSTRUCTURAL
HYPOTHESES IN THE FOREIGN
EXCHANGE MARKET

Richard K. Lyons

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ABSTRACT

This paper introduces a three-part transactions dataset to test various microstructural hypotheses about the spot foreign exchange market. In particular, we test for effects of trading volume on quoted prices through the two channels stressed in the literature: the information channel and the inventory-control channel. We find that trades have both a strong information effect and a strong inventory-control effect, providing support for both strands of microstructure theory. The bulk of equity-market studies also find an information effect; however, these studies typically interpret this as evidence of inside information. Since there are no insiders in the foreign exchange market, this finding suggests a broader conception of the information environment, at least in this context.

Richard K. Lyons
350 Barrows Hall
U.C. Berkeley
Berkeley, CA 94720
and NBER

Tests of Microstructural Hypotheses
in the Foreign Exchange Market

Empirical microstructure work to date focuses primarily on the New York Stock Exchange and its specialist structure. At least two reasons account for this focus on the specialist: theory is better developed in this area, and the concentration of the market provides more readily available data. Current interest, however, is shifting towards various non–specialist markets. This paper is representative of that shift. It examines a market with structural features that differ substantially from the New York Stock Exchange: the interbank spot foreign exchange (FX) market, which is a *decentralized*, quote–driven dealership market. The objective is to test models of intraday price movements using a dataset that is constructed to accord with recent FX theory.

The literature on microstructure highlights two channels through which trading volume generates price movements. First, inventory costs create incentives for marketmakers to use prices to control fluctuations in their positions.[1] Second, the existence of traders with private information implies that rational marketmakers adjust their beliefs, and prices, in response to order flow.[2] Though the reasons differ, both channels predict that buyer–initiated trades push prices up — and vice versa. Disentangling the two is the main challenge of empirical work in this area.

The models in this literature are built around a monopoly specialist, so they have to be adapted to the FX market. Of course, the inventory–control and

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[1] See Amihud and Mendelson (1980), Zabel (1981), Ho and Stoll (1983), and O'Hara and Oldfield (1986), among others.

[2] See Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987), Glosten (1989), and Admati and Pfleiderer (1989), among others.

information channels are still relevant since both markets share the dealership and quote–driven features of a specialist market. Because the FX market is decentralized, however, order flow is not consolidated as it is under the specialist. Nevertheless, even in the FX market there is a degree of consolidation of order flow — and the information therein: this consolidation occurs as a result of brokered interdealer trading.[3] Roughly 40% of total volume in the spot market is mediated by brokers. These brokers, by providing marketmakers verbally with transaction prices and quantities, serve as a partial clearinghouse for order flow information. The dataset and model we employ here takes these important institutional features into account.

Our dataset has significant advantages over intraday FX data used in the past. Until 1989, in fact, intraday spot data permitting systematic examination of the spread and transaction activity were not available. In that year an important dataset was introduced that provided 13 weeks of continuous bid and offer "quotes" as expressed on the Reuters screen [See Goodhart (1990) and Goodhart and Figliuoli (1991)]. The main shortcomings of this dataset are two: first, these "quotes" are only indications, not firm prices at which marketmakers can deal; and second, there is no measure of order flow or transaction prices. In contrast, the transactions dataset used here includes three interlocking components, each of which spans the same five trading days: (i) the time–stamped direct (bilateral) quotes and trades of the D–mark/dollar *marketmaker* of a major New York Bank, (ii) the same marketmaker's indirect (brokered) trades, and (iii) the time–stamped prices and quantities for transactions mediated by one of the major New York *brokers* in the same market. These features of the data permit an empirical model that can

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[3] Brokers cannot take positions; they act only to match buyers with sellers (anonymously). For all major currencies, the brokerage business is dominated by 4 to 6 firms. See Lyons (1992, 1993) for a more detailed account.

capture the institutional features of this market. In particular, components (i) and (ii) provide an exact measure of our marketmaker's inventory over time, while component (iii) provides a measure of the informativeness of third–party trading. Moreover, since trading between other marketmakers via the broker will have no impact on our marketmaker's inventory, this dataset provides additional power for disentangling the information effects of trading from the inventory–control effects.

In the end, the estimates sharply distinguish the two effects. We find that trading volume affects quoted prices through *both* an information channel and an inventory–control channel, providing support for both strands of microstructure theory. Our finding of a strong inventory–control effect in the FX market, as compared to the NYSE, is particularly striking since adjusting prices is the only way for a stock specialist to adjust inventory, whereas an FX marketmaker can simply trade away undesired inventory at another marketmaker's prices.[4] As for the strong information effect, previous empirical work on the NYSE interprets information effects as evidence of private information, interpreted typically as inside information. Since there are no insiders in the FX market, this finding suggests a broader conception of the information environment, at least in the context of FX.

The paper is organized as follows: Section I develops the model; Section II describes the dataset; Section III presents the model's results; and Section IV concludes.

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[4] For example, in a study on an NYSE specialist Madhavan and Smidt (1991) find an information effect but no inventory–control effect.

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I. A Bayesian Model of Pricing Behavior

The following model is closest in spirit to Lyons (1992) and Madhavan and Smidt (1991). There are two assets in a pure exchange economy: one riskless (the numeraire) and one with a stochastic liquidation value — representing FX. The FX market is organized as a decentralized dealership market with $2n$ marketmakers. Here, we focus on the pricing behavior of a representative marketmaker, denoted marketmaker i. A period is defined by a transaction effected against marketmaker i's quoted bid or offer, with periods running from t=1,2,...,T. Let j denote the marketmaker requesting i's quote. In addition, in each period the remaining $2n-2$ marketmakers are paired off and transactions are effected; thus, each period there are n marketmakers requesting quotes, n marketmakers providing them, and n transactions. We assume that transactions are ex–post regret–free — in the sense of Glosten and Milgrom (1985) — for the quoting marketmaker, a property standard among models in this literature. Thus, a quote must take the form of a schedule relating quantity to a distinct price; if a single bid–offer quote were good for any quantity then large purchases or sales initiated by potentially–informed marketmakers would leave the quoting marketmaker regretting her quote ex–post.

The rest of the model's specification is presented in three blocks: first, the information environment; then, the formation of expectations conditional on the information environment; and finally, the determination of bid/offer quotes as a function of expectations and current inventory.

Information Environment

The full information price of FX at time T is denoted by $\bar{V}$, which is composed of a series of increments — e.g. interest differentials — so that $\bar{V} = \sum_{i=0}^{T} \bar{r}_i$, where r_0 is

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a known constant. The increments are i.i.d. mean zero. Each increment r_t is realized immediately after trading in period t. Realizations of the increments can be thought to represent the flow of public information over time. The value of FX at t is thus defined as $V_t = \sum_{i=0}^{t} r_i$. At the time of quoting and trading in period t, i.e. before $\bar{r}_t$ is realized, $\bar{V}_t$ is a random variable. In a market without private information or transaction costs the quoted price of FX at time t, denoted P_t, would be equal to V_{t-1}, which is the expected value of the asset price conditional on public information available at t.

At the beginning of period t, each of the marketmakers receives a public signal of the value of FX:

$$(1) \qquad\qquad \bar{S}_t = V_t + \bar{\eta}_t$$

where η is distributed normally with mean zero. Additionally, at the beginning of period t each of the n marketmakers that will *request* a quote in period t receives a private signal of the value of FX:

$$(2) \qquad\qquad \bar{C}_{jt} = V_t + \bar{\omega}_{jt}$$

where the ω_j are i.i.d. normal with mean zero. As an example, in Lyons (1992) the private signals correspond to information derived from trades with non–marketmaker customers, which are not observed by other marketmakers.[5]

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[5] In the words of Mr Chris Deuters, Citibank's head of foreign exchange and derivatives in Europe: "If you don't have access to the end user your view of the market will be severely limited" (reported in the Financial Times, 4/29/91).

Beyond these two exogenous signals, the information environment includes two informative endogenous variables. First, we assume that each period some number m of the $n-1$ transactions not involving marketmaker i occur before marketmaker i trades. A signal of this order flow is publicly observable. More specifically, the signal is a noisy measure of net early-in-period volume:

$$(3) \qquad Q_t = \sum_{k \neq i,j}^{m} Q_{kt} + \nu_t$$

where $m < n$, ν is distributed normally with mean zero, and ν is uncorrelated with any other random variable in the model. The empirical counterpart of the signal Q_t is the inter-marketmaker transaction information communicated by brokers via intercoms at the marketmakers' desks. Only trades done indirectly through brokers are included in this signal. It is assumed here that the share of trades done indirectly is known; in fact, about half of inter-marketmaker trading is arranged through brokers.[6]

The second endogenous variable in marketmaker i's information environment is the quantity marketmaker j chooses to trade at marketmaker i's quoted prices, denoted Q_{jt}. This quantity will be a signal of the $\bar{C}_{jt}$ received by marketmaker j. As is standard (under exponential utility), the quantity marketmaker j chooses to trade is linearly related to the deviation between marketmaker j's expectation and the transaction price, plus a quantity representing liquidity demand X_{jt} that is uncorrelated with V_t (e.g. inventory-adjustment trading):

$$(4) \qquad Q_{jt} = \theta(\mu_{jt} - P_{it}) + X_{jt}$$

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[6] See Lyons (1993) for a model addressing why the equilibrium share of brokered trading in the FX market is about one-half.

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where μ_{jt} is the expectation of $\bar{V}_t$ conditional on information available to marketmaker j at t, and the value of X_{jt} is known only to marketmaker j. Note that Q_{jt} can take either sign.

Figure 1 summarizes the timing of the model in each period:

Figure 1

Timing in each period
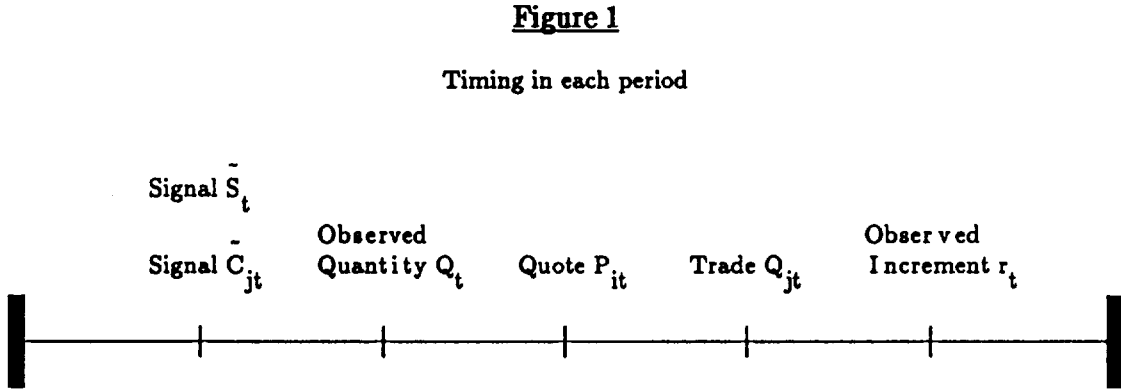
The Formation of Expectations

Marketmaker i's quotes will be a function of his expectation of V_t at the time of quoting, which we denote μ_{it}. These expectations, in turn, are conditioned on three of the variables described above: S_t, Q_t, and Q_{jt}; the fourth variable described above, $\bar{C}_{jt}$, is communicated (noisily) to marketmaker i via Q_{jt}. We now address the determination of these expectations.

Marketmaker i's prior belief regarding V_t is summarized by the first public signal S_t. After observing the second public signal Q_t, marketmaker i's posterior belief, denoted μ_t, can be expressed as a weighted average of S_t and a statistic Z_t, which is derived from Q_t (see appendix):

(5) $$\mu_t \equiv \rho S_t + (1-\rho)Z_t$$

where $\rho \equiv \sigma_Z^2/(\sigma_Z^2+\sigma_\eta^2)$ and:

(6) $\qquad Z_t \equiv [m\theta(1-\lambda)\rho]^{-1}Q_t + S_t$

$$= V_t + m^{-1}\sum_{k\neq j}^{m}\bar{\omega}_{kt} + [m\theta(1-\lambda)]^{-1}\sum_{k\neq j}^{m}\bar{X}_{kt} + [m\theta(1-\lambda)]^{-1}\bar{\nu}_t$$

with $\lambda \equiv \sigma_\omega^2/(\sigma_\omega^2+\sigma_\mu^2)$. By construction, Z_t is normally distributed with mean V_t and variance σ_Z^2 equal to the variance of the last three terms, all of which are orthogonal to V_t and S_t. These posterior beliefs μ_t are normally distributed with mean V_t and variance $\sigma_\mu^2 \equiv \rho^2\sigma_\eta^2+(1-\rho)^2\sigma_Z^2$.

Similarly, from any Q_{jt} marketmaker i can form the statistic Z_{jt} (see appendix):

(7) $\qquad Z_{jt} \equiv \dfrac{Q_{jt}/\theta + P_{it} - \lambda\mu_t}{1-\lambda} = V_t + \bar{\omega}_{jt} + [1/\theta(1-\lambda)]\bar{X}_{jt}.$

This statistic is also normally distributed, with mean V_t and variance equal to the variance of the last two terms, both of which are orthogonal to V_t. Let σ_{Zj}^2 denote this variance. Note that Z_{jt} is statistically independent of μ_t since Z_{jt} is orthogonal to both S_t and Z_t. Thus, marketmaker i's posterior μ_{it}, expressed as a function of any Q_{jt}, takes the form of a weighted average of μ_t and Z_{jt}:

(8) $\qquad \mu_{it} = \kappa\mu_t + (1-\kappa)Z_{jt}$

where $\kappa \equiv \sigma_{Zj}^2/(\sigma_{Zj}^2+\sigma_\mu^2)$. This expectation has a direct impact on marketmaker i's quotes, to which we now turn.

The Determination of Bid/Offer Quotes

Consider a prototypical inventory-control model, where price is linearly related to the marketmaker's current inventory:[7]

$$(9) \qquad P_{it} = \mu_{it} - \alpha(I_{it} - I_i^*) + \gamma D_t$$

Here, μ_{it} is the expectation of $\bar{V}_t$ conditional on information available to marketmaker i at t, I_{it} is marketmaker i's current inventory position, and I_i^* is i's desired position. The undesired-inventory effect, governed by α, will in general be a function of relative interest rates, firm capital, and carrying costs. The variable D_t is a direction-indicator variable with a value of one when a buyer-initiated trade occurs, and a value of -1 when a seller-initiated trade occurs. The term γD_t then picks up (half of) the effective spread: if marketmaker j is a buyer then the transaction price P_{it} will be on the offer side, and therefore a little higher, ceteris paribus. This term can be interpreted as compensation resulting from execution costs, price discreteness, or rents.

Following Glosten and Milgrom (1985), marketmaker i takes into account the effect of various Q_{jt}'s on his beliefs and quotes a schedule of prices that will be ex-post regret-free conditional on any realized Q_{jt}. Accordingly, substituting the value of μ_{it} in equation (8) into equation (9) yields:

$$(10) \qquad P_{it} = \kappa\mu_t + (1-\kappa)Z_{jt} - \alpha(I_{it} - I_i^*) + \gamma D_t$$

which can be shown to be equivalent to (see appendix):

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[7] Linear decision rules are optimal in a number of inventory control models.

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(11) $\qquad P_{it} = S_t + \left[\frac{1-\rho}{m\theta(1-\lambda)\rho}\right]Q_t + \left[\frac{1-\phi}{\phi\theta}\right]Q_{jt} - \left[\frac{\alpha}{\phi}\right](I_{it}-I_i^*) + \left[\frac{\gamma}{\phi}\right]D_t$

where the parameter $\phi \equiv (\kappa-\lambda)/(1-\lambda)$ and $0<\phi<1$ since $0<\kappa<1$, $0<\lambda<1$, and $\kappa>\lambda$. This equation is not estimable since S_t is not observable. However, given our assumptions governing the signals available and the evolution of the value V_t, we can express the period t prior as equal to the period $t-1$ posterior from equation (9) lagged one period, plus an expectational error term ϵ_{it}:

(12) $\qquad S_t = \mu_{it-1} + \epsilon_{it} = P_{it-1} + \alpha(I_{it-1}-I_i^*) - \gamma D_{t-1} + \epsilon_{it}.$

Substituting this expression for S_t into equation (11) yields:

$$P_{it} = \left[P_{it-1}+\alpha(I_{it-1}-I_i^*)-\gamma D_{t-1}+\epsilon_{it}\right] + \left[\frac{1-\rho}{m\theta(1-\lambda)\rho}\right]Q_t + \left[\frac{1-\phi}{\phi\theta}\right]Q_{jt}$$

$$- \left[\frac{\alpha}{\phi}\right](I_{it}-I_i^*) + \left[\frac{\gamma}{\phi}\right]D_t.$$

Therefore:

(13) $\qquad \Delta P_{it} = \left[\frac{\alpha}{\phi}-\alpha\right]I_i^* + \left[\frac{1-\rho}{m\theta(1-\lambda)\rho}\right]Q_t + \left[\frac{1-\phi}{\phi\theta}\right]Q_{jt} - \left[\frac{\alpha}{\phi}\right]I_{it} + \alpha I_{it-1}$

$$+ \left[\frac{\gamma}{\phi}\right]D_t - \gamma D_{t-1} + \epsilon_{it}$$

which corresponds to a reduced form estimating equation of:

(14) $\qquad \Delta P_{it} = \beta_0 + \beta_1 Q_t + \beta_2 Q_{jt} + \beta_3 I_{it} + \beta_4 I_{it-1} + \beta_5 D_t + \beta_6 D_{t-1} + \epsilon_{it}.$

The model thus predicts that $\{\beta_1,\beta_2,\beta_4,\beta_5\}>0$, $\{\beta_3,\beta_6\}<0$, $|\beta_3|>\beta_4$, and $\beta_5>|\beta_6|$, where the latter inequalities derive from the fact that $0<\phi<1$.

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Two comments regarding empirical implementation of the model are warranted. First, note that with a slight re-interpretation the above model can accommodate variability in desired inventories, that is, an I_i^* that varies through time. Consider the following model: $I_{it}^*=I_i+\delta(\mu_{it}-\mu_t)$, which is consistent with the linear demands arising from negative exponential utility, where μ_t represents the market price away from marketmaker i. Further, Q_{jt} is the only information available to marketmaker i that is not reflected in μ_t. It can be shown under the assumptions of our model that $(\mu_{it}-\mu_t)$ is proportional to Q_{jt}. Accordingly, we write $(\mu_{it}-\mu_t)=\pi Q_{jt}$. Hence, we can express the desired inventory as follows: $I_{it}^*=I_i+\delta\pi Q_{jt}$. In estimation, I_i will be absorbed in the constant. The estimate of β_2 now represents $\left[\frac{1-\phi}{\phi\theta}\right]+\left[\frac{\alpha}{\phi}-\alpha\right]\delta\pi$, whose significance still evinces an information effect, though we have to be more careful in interpreting its magnitude.

A second issue relevant to empirical implementation is simultaneity. To see that simultaneity is not a problem here recall that, though quoting precedes trading, a quote is a schedule from which marketmaker j, by selecting Q_{jt}, determines the transaction price P_{it}. By design there is no intrinsic–value information available to marketmaker i that is not available to marketmaker j. Hence, there is no feedback here from P_{it} to the *information content* of Q_{jt}, which is what the coefficient β_2 measures. Suppose, to think of it a different way, a public announcement occurs this period which raises the expectation of V_t conditional on public information. Though P_{it} will be higher, this does not help to predict the trade of marketmaker j this period since j's trade will reflect only the orthogonal component of his beliefs relative to P_{it}, i.e. his C_{jt}.

To recap, disentangling the information effect of trading from the inventory–control effect requires breaking the perfect collinearity in a centralized specialist market between incoming trades Q_{jt} and inventory changes ΔI_{it}. Here, the collinearity is broken in two ways, both of which derive from the richness of the

data available: (i) a period in our pricing model is defined by an *incoming* trade effected at our marketmaker's quote; but in reality, the inventory of our marketmaker also changes from period to period as a result of her *outgoing* trades at other marketmakers' quotes, which is not relevant in a specialist setting; (ii) third–party brokered trades, about which marketmakers get signed volume information, obviously have no effect on own–inventory.

II. Data

The dataset introduced in this paper is qualitatively different from any yet employed in the exchange rate literature. The main difference is that it contains time–stamped transaction prices, quantities, and quotes. The existing alternative is constructed from what are called "indicative" quotes, which are input to Reuters by trading banks.[8] Some of the shortcomings of the indicative quotes include the following. First, they are not transactable prices. Second, while it is true that the indicated spreads usually bracket actual quoted spreads, they are typically about twice as wide as quoted spreads in the interbank market (documented below). Third, the indications are less likely to bracket true spreads when the intensity of trading is highest: marketmakers can get too busy dealing to update their indications. And finally, my experience sitting next to marketmakers at major banks indicates that they pay no attention to the current indication; rather, marketmakers garner most of their high–frequency market information from the signals transmitted via intercoms connected to brokers and the IMM futures market [see Lyons (1992)]. In reality, the main purpose of the indications is to provide non–marketmaker participants with a gage of where the market is trading.

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[8] Recent work relying on the Reuters indications include Goodhart (1990), Goodhart and Figliuoli (1991), Bollerslev and Domowitz (1992), and Bessembinder (1993).

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The dataset employed here consists of three interlinked components, covering the five trading days of the week August 3-7, 1992, from the informal start of trading at 8:30 EST to roughly 2:00 EST. The first component includes the time–stamped direct (bilateral) quotes, plus the prices and quantities for all direct transactions for a single DM/$ marketmaker at a major New York bank. The second component includes the same marketmaker's indirect (brokered) trades. The third component includes the time–stamped prices and quantities for transactions mediated by one of the major New York *brokers* in the same market.

II.1. Marketmaker Data: Direct Quotes and Trades

The first component of the dataset includes the marketmaker's direct quotes, plus the prices and quantities for all direct transactions. The availability of this component is due to a recent change in technology in this market: the Reuters Dealing 2000 system. This system — very different from the system that produces the Reuters indications — allows marketmakers to communicate bilateral quotes and trades via computer rather than verbally over the telephone. Among other things, this allows marketmakers to request up to four quotes simultaneously, whereas phone requests are necessarily sequential. Another advantage is that the computerized documentation reduces the paperwork required of the marketmakers. Though use of this technology differs by marketmaker and is currently diffusing more widely, our marketmaker uses Dealing 2000 for nearly all of his direct interbank communications: over the week of August 3-7 only 5 transactions were done over the phone.[9]

Each record of the data covering the marketmaker's DM/$ activities includes the first 5 of the following 7 variables, the last 2 requiring a transaction to have taken place:

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[9] This statistic is available from the position card component of the dataset.

(1) The time the communication is initiated (to the minute).

(2) Which marketmaker is requesting the quote.

(3) The quote quantity.

(4) The bid quote.

(5) The offer quote.

(6) The quantity traded (Q_{jt}, I_t).

(7) The transaction price (P_{it}).

This component of the dataset includes 1237 transactions amounting to $5.4 billion.

Figure 2 provides two examples of marketmaker communications as recorded by the conversation printout [see Reuters (1990) for more details]. Example 1 is a conversation in which no trade occurred. The first word identifies whether the call went "To" another marketmaker, or came "From" another marketmaker. Then comes the institution code and name of the counterparty, followed by the time (Greenwich Mean), the date (day first), and the number assigned to the communication. On line 3, "DMK 10" identifies this as a request for a DM/$ quote for up to $10 million. Line 4 provides the quoted bid and offer price: typically, marketmakers only quote the last two digits of each price, the rest being superfluous in such a fast-moving market. These two quotes correspond to a bid of 1.58<u>88</u> DM/$ and an offer of 1.58<u>92</u> DM/$. Whenever a trade occurs, the communication record provides the first three digits, as is the case in example 2. Here, the marketmaker that calls to request a quote decides to buy $10 million at the D–mark offer price of 1.5891. Every time a transaction occurs the communication record confirms the exact price and quantity.

II.2. Marketmaker Data: Position Cards

The second component of the dataset is composed of the marketmaker's position cards over the same five days covered by the direct-transaction data, August 3–7, 1992. In order to track their positions, marketmakers in the spot

market record all their transactions on hand–written position cards as they go along. An average day consists of approximately 20 cards, each with about 15 transaction entries.

There are two key benefits to this component of the dataset. First, it provides a very clean measure of the marketmaker's inventory I_t at any time since it includes both direct trades *and* any brokered trades. Second, it provides a means of error–checking the first component of the dataset.

Each card includes the following information for every trade:

(1) The quantity traded (I_t),

(2) The transaction price, and

(3) The counterparty, including whether brokered.

Note that the bid/offer quotes at the time of transaction are not included so this component of the dataset alone is not sufficient for estimating our model. Note also that each entry is not time–stamped; at the outset of every card, and often within the card too, the marketmaker records the time to the minute. Hence, the exact timing of some of the brokered transactions is not pinned down since these trades to not appear in the first component. Nevertheless, this is not a drawback for our purposes: the observations for our empirical model are the direct transactions that occur on our marketmaker's quoted prices; since the timing of these is pinned down by the first component of our dataset, and since these transactions appear sequentially in both components, the intervening changes in inventory due to brokered trades can be determined exactly.

II.3. Broker Data

The data covering the broker's activities includes time–stamped transaction prices and quantities over the same five days covered by the other two components of the dataset. These data are transcribed from transaction tickets, which are filled out by the brokers themselves in the dealing room. The tickets are collected by an

attendant and are then time–stamped. This component of the dataset includes 1172 transactions amounting to roughly $6.7 billion.

The data on brokered transactions does not include the bid and offer at the time of each transaction, so it is necessary to infer the trade direction from the transaction prices. We use the method recommended by Lee and Ready (1991). The resulting series provides a measure of Q_t for the model. To insure that information in Q_t is indeed prior to the transaction Q_{jt}, we construct Q_t by summing the signed trades from the two minutes prior to the minute in which Q_{jt} is effected.

II.4. Descriptive Statistics

Table 1 provides some descriptive statistics in the form of daily averages. This is masking some daily variation in the sample: the heaviest day (8/7/92) is a little less than twice as active as the lightest day (8/5/92). Note that this marketmaker averages well over $1 billion of interbank trading daily. With respect to quoting, because our marketmaker is among the larger players in this market, he has $10 million "relationships" with many other marketmakers; that is, quote requests from other high–volume marketmakers that do not specify a quantity are understood to be good for up to $10 million. Note the tightness of the median spread. For comparison, the median spread in the Reuters indications dataset is DM 0.001, more than twice as large. The value of one pip at current exchange rates on a $10 million transaction is about $700; a bid/offer spread of 4 pips corresponds to about 0.03% of the spot price.[10]

Figure 3 presents two plots. The first is the transaction price over the the full sample, Monday August 3 to Friday August 7, 1992. The second plot provides the magnitude of position (inventory) variability over the week; the maximum long

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[10] Note that discreteness is a non–issue in the spot FX market; unlike equity markets, there is no minimum tick rule, and the size of one pip in percentage terms is very small.

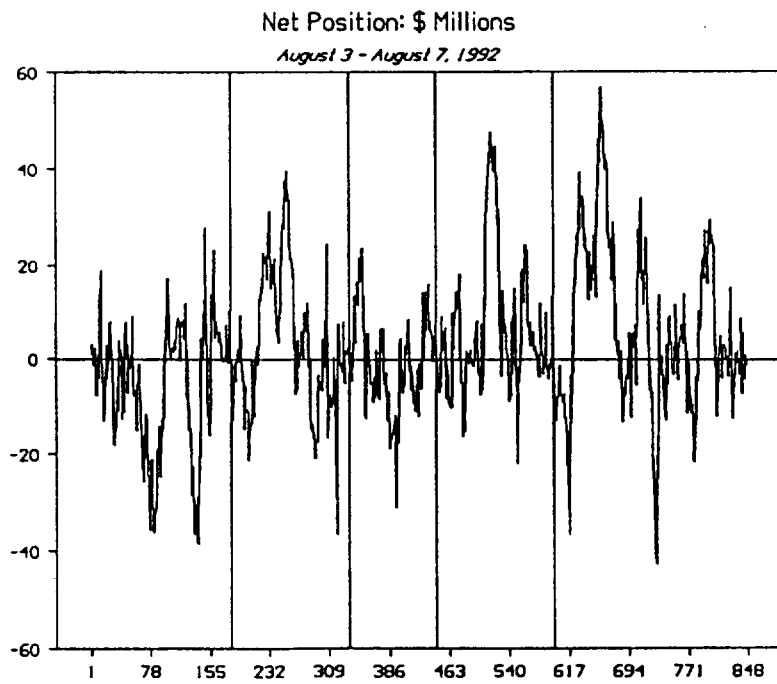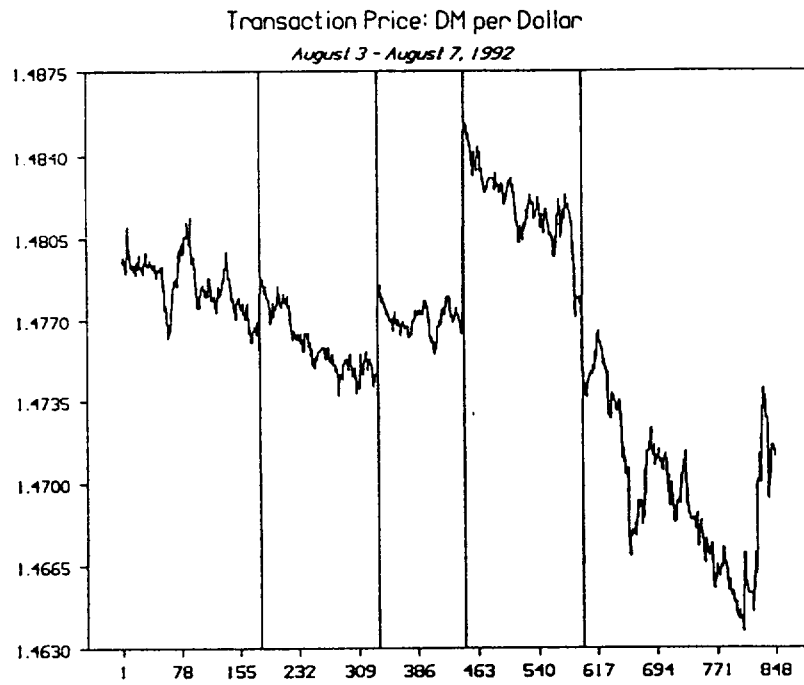dollar position was $56.8 million, the maximum short dollar position $42.7 million.

Table 1

Descriptive Statistics
Daily Averages
August 3–7, 1992

| | Marketmaker | | Broker |
	Direct	Brokered	
(1) # transactions	237	103	234
(2) Value transactions	$1.0 B	$0.4 B	$1.3 B
(3) Median trans. size	$3 M	$4 M	$5 M
(4) Median spread size	DM 0.0003		

A natural concern is whether our marketmaker is representative of the core of the interbank spot market. While we cannot answer this definitively, we offer a few relevant facts. First, he has been trading in this market for many years and is well-known among the other major marketmakers. Second, in terms of trading volume he is without a doubt one of the key players, trading well over $1 billion per day and maintaining $10 million quote relationships with a number of other marketmakers. Though this would probably not put him in the top five in terms of volume, he is not far back, possibly in the 5th to 15th range somewhere. In the end, our view is that he is representative, at least with respect to the issues addressed here. There is no doubt, however, that different trading styles exist.

Figure 3

Transaction Price: DM per Dollar
August 3 – August 7, 1992

Net Position: $ Millions
August 3 – August 7, 1992

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III. Model Estimation

A. The Core Model

Table 2 presents the OLS results over the full five day period. The sample is composed of all direct transactions initiated against marketmaker i's quoted prices, for a total of 843 observations.[11] The first row presents estimates of the full reduced form, i.e., including the information effects of brokered order flow. The standard errors are heteroskedasticity- and autocorrelation-consistent (first-order).

Table 2

Model Estimates

$$\Delta P_{it} = \beta_0 + \beta_1 Q_t + \beta_2 Q_{jt} + \beta_3 I_{it} + \beta_4 I_{it-1} + \beta_5 D_t + \beta_6 D_{t-1} + \epsilon_{it}$$

	β_0	β_1	β_2	β_3	β_4	β_5	β_6	R^2
Est.	−1.51 (−1.19)	0.73 (3.40)	1.24 (2.75)	−0.98 (−3.27)	0.78 (2.68)	11.08 (6.23)	−9.00 (−5.99)	0.23
Est.	−1.56 (−1.22)		1.20 (2.68)	−0.97 (−3.25)	0.76 (2.63)	11.37 (6.40)	−9.22 (−6.10)	0.22
Pred.		>0	>0	<0	>0	>0	<0	

* T-statistics in parentheses. Q_t is the net quantity of third-party brokered trading and Q_{jt} is the quantity transacted directly at marketmaker i's quoted prices, where both are positive for buyer initiated trades (i.e. effected at the offer) and negative for seller initiated trades (at the bid). I_t is i's position at the end of period t. D_t is an indicator variable with value 1 if the trade is buyer-initiated, and value −1 if seller-initiated. All quantity variables are in $ millions. All coefficients are multiplied by 10^5.

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[11] We have excluded overnight price changes from the sample; the model is intended to explain intraday quoting dynamics, not price changes over periods through which the marketmaker is not active.

The second row excludes the brokered order flow variable and is therefore more directly comparable to results from work on the stock market, for example Glosten and Harris (1988), Foster and Vishwanathan (1990), and Madhavan and Smidt (1991). Row three indicates the predicted signs of the coefficients under the null that both information and inventory–control effects are present.

The central results from Table 2 are the very significant and properly–signed coefficients on the information (order flow) variables, Q_t and Q_{jt}, and the inventory variables, I_{it} and I_{it-1}. The magnitude of β_2 implies that information asymmetry induces a price increase (decrease) of one pip for every \$10 million purchase (sale) at quoted prices. The magnitude of the inventory–control coefficient β_4, which equals α in equation (9), implies that to motivate dollar purchases our marketmaker lowers (raises) his quoted DM price of dollars by 3/4 of one pip for every \$10 million of long position in dollars.

The indicator variables reflecting the effective spread are the right sign, are very significant, and have the predicted relative magnitude as well $(\beta_5 > |\beta_6|)$. The coefficient β_5 suggests that once the information and inventory effects are controlled for, the baseline spread in this market over this particular period was just over 2 pips, or 2 times β_5. Finally, the level of the R^2's are reflective of the fact that Q_{jt} and Q_t together are a small fraction of the total trading activity in this market.

As an additional test of the model's fit, we now examine a testable implication of the information environment specified in section I. Specifically, our model imposes a particular structure on the error term ϵ_{it}. To see this note that equation (12) allows us to write: $\epsilon_{it} = S_t - \mu_{it-1}$. These two components reduce to:

$$S_t = V_t + \eta_t,$$

and

$$\mu_{it-1} = \kappa \mu_{t-1} + (1-\kappa) Z_{jt-1}$$

using equations (1) and (8), respectively. The latter expression for μ_{it-1}, after making some substitutions, can be expressed as:

$$\mu_{it-1}= V_{t-1}+ \kappa\rho\eta_{t-1}+ (1-\kappa)\left[\bar{\omega}_{jt-1}+[1/\theta(1-\lambda)]\bar{X}_{jt-1}\right]$$

$$+ \kappa(1-\rho)\left[m^{-1}\sum_{k\neq j}^{m}\bar{\omega}_{kt-1}+[m\theta(1-\lambda)]^{-1}\sum_{k\neq j}^{m}\bar{X}_{kt-1}+[m\theta(1-\lambda)]^{-1}\bar{\nu}_{t-1}\right]$$

Now, to determine $E[\epsilon_{it},\epsilon_{it-1}]$ recall that $\bar{\nu}$ and the increments to V are independent across time, and that $\bar{\omega}$, $\bar{X}$, and $\bar{\nu}$ are independent across marketmakers and time. Accordingly, $E[\epsilon_{it},\epsilon_{it-1}]=E[-\kappa\rho\eta_{t-1},\eta_{t-1}]= -\kappa\rho\sigma_\eta^2$; hence, the error term ϵ_{it} should follow an MA(1).

Table 3 presents estimates of the full model under an MA(1) error structure.[12] The moving–average coefficient β_7 is significant and properly–signed. Note too that the volume and inventory variables are now more precisely estimated.

Table 3

MA(1) Error Structure Estimates

$$\Delta P_{it}= \beta_0 + \beta_1 Q_t + \beta_2 Q_{jt} + \beta_3 I_{it} + \beta_4 I_{it-1} + \beta_5 D_t + \beta_6 D_{t-1} + \psi_{it} + \beta_7 \psi_{it-1}$$

	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
Est.	−1.49	0.69	1.31	−1.02	0.82	10.68	−9.04	−0.09
	(−1.10)	(2.23)	(2.97)	(−3.80)	(3.17)	(5.20)	(−6.22)	(−2.54)
Pred.		>0	>0	<0	>0	>0	<0	<0

* T–statistics in parentheses. All coefficients except β_7 are multiplied by 10^5.

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[12] We use the Hildreth–Lu method of iteration.

B. Additional Considerations

The remainder of this section addresses two additional considerations. First, are there (simple) non–linearities in the information and inventory–control effects of trading activity? And second, are there important time of day effects in the data, such as increased sensitivity to inventory control near the end of the trading day?

First we test for non–linearities in the information effect of received trades by introducing a piece–wise linear specification. We create a dummy variable L_t that equals one when the absolute value of Q_{jt} is above its median absolute value (\$2.5 million), and equals zero otherwise. We then reestimate the MA(1) model in Table 3 with the additional explanatory variable $L_t Q_{jt}$ and test whether its coefficient is significant. The p–value of the test is 49%. Defining L_t with the 25th and 75th percentiles produces similar results. Hence, we find no evidence for a piece–wise linear specification of the impact of Q_{jt}.

We turn now to non–linearities in the inventory–control effect. In a decentralized dealership market inventory control effects *must* exhibit a non–linearity at some level: arbitrage can occur if the quoting marketmaker's offer falls below another's bid, or if the bid rises above another's offer. Recall that the inventory–control effect corresponds to the magnitude of α in equation (9), and is estimated at $\beta_4 \approx 0.75$ pips per \$10 million net open position. Thus, a \$40 million open position translates into prices that are adjusted about 3 pips to induce decumulation. To examine more formally the incremental inventory–control effects at higher inventory levels we effect the following test. In lieu of I_t and I_{t-1} in the core MA(1) model, we include I_t^S, I_{t-1}^S, I_t^L, and I_{t-1}^L where (i) I_t^S is defined equal to I_t if $|I_t| < \bar{I}$, and $\bar{I}R$ otherwise, with R an indicator variable equal to 1 when $I_t > 0$ and -1 when $I_t < 0$, and (ii) I_t^L is defined equal to 0 if $|I_t| < \bar{I}$, and $I_t - \bar{I}R$ otherwise. Table 4 presents the results in the form of the marginal significance of the test that the coefficient on $I_{t-1}^L = 0$, i.e., that the α for these incremental positions is zero.

Table 4

Null Hypothesis: No Incremental Inventory–Control Effect Beyond $|I_t| = \bar{I}$

	I=\$30 million	I=\$40 million	I=\$50 million
P–Value	0.5%	1.3%	7.4%

* P–values correspond to the following test. In lieu of I_t and I_{t-1} in the core model, include I_t^S, I_{t-1}^S, I_t^L, and I_{t-1}^L; I_t^S is defined equal to I_t if $|I_t| < \bar{I}$, and $\bar{I}R$ otherwise, with R an indicator variable equal to 1 when $I_t > 0$ and -1 when $I_t < 0$; I_t^L is defined equal to 0 if $|I_t| < \bar{I}$, and $I_t - \bar{I}R$ otherwise. The table reports the marginal significance of the test that the coefficient on $I_{t-1}^L = 0$.

Incremental effects are clearly present up to the \$40 million level. At the \$50 million level and beyond, however, the coefficient is no longer significant; this is likely due to low power of the test, however, since trades with net open positions this large are rare in the sample. Figure 3 provides prima facie evidence that open positions above \$40 million are reduced rapidly. At these levels, it is typically faster and less costly to reduce unwanted inventory by trading on broker or other marketmaker prices; there is some evidence of this shift from our marketmaker's position cards.

We turn now to time–of–day effects. It is well known that currency marketmakers typically bring their net open positions to zero at the end of their trading day, and this was the case for each of the five days in our sample (see Figure 3). This may influence the inventory–control parameters, among others. Accordingly, we conduct chow tests to determine whether coefficients are stable through the day. We cut our sample into an early and late period — for each day — three different ways: (i) at the median transaction time, (ii) at the time when 25%

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of the transactions remain, and (iii) at the time when 10% of the transactions remain. We find no evidence at conventional significance levels for instabilities in any of the coefficients at any of the breaks *except* the inventory–control coefficients. Table 5 presents the results in that case:

Table 5

Null Hypothesis: Inventory–control coefficients β_3 and β_4 are constant through the day

	Median Trade	Final 25%	Final 10%
P–Value	79.7%	10.8%	3.8%

* P–values correspond to the marginal significance level of the Chow test that both β_3 and β_4 in the core model are the same value late in the day as early, where late in the day is defined these three ways.

Interestingly, in the case of the final 10% the coefficients β_3 and β_4 are *muted* at the end of the day, rather than amplified. One possible explanation of this is that it is precisely at the end of the trading day that marketmakers least want to signal their positions via quotes, preferring to trade away from positions through brokers or other marketmakers' prices.

IV. Conclusions

The model of this paper is formulated to accord with a number of institutional features relevant to the FX market. Among them are the following facts: (i) Major currencies are traded in decentralized dealership markets; (ii) Over 80% of the trading volume is between marketmakers; (iii) Market net volume is only partially

observable; And (iv) customer — or non–marketmaker — order flow is described by marketmakers as an important source of private information. While in theory the inventory–control and information effects of volume on prices are still relevant, the proper specification of order flow information is different from that in the prototypical specialist model.

The dataset we use to test the model has important advantages over Reuters indications data. The main shortcomings of the indications are that (i) the indications are not firm prices, and (ii) there is no direct measure of order flow or transaction prices. In contrast, the dataset used here includes three interlocking components: the direct quotes and trades of a marketmaker from a major New York Bank, the position cards of the same marketmaker, and the prices and quantities for third–party transactions mediated by a major New York broker. These features not only permit estimation of a more realistic empirical model, they also provide additional power for for discriminating between the information effects of trading and the inventory–control effects.

In the end, the estimates sharply distinguish the two effects: trading volume affects quoted prices through *both* an information channel and an inventory–control channel, providing support for both strands of microstructure theory. The estimated coefficient on the trade–quantities our marketmaker receives — β_2 — suggests that information asymmetry induces a price increase (decrease) of one pip for every \$10 million purchase (sale) at quoted prices. Moreover, since inside information in the usual sense is unlikely to be significant in the FX market, this finding calls for a broader conception of what constitutes private information.

There is an important hardship in focusing on a decentralized dealership market like FX that warrants recognition. Empirical work on the specialist structure has the luxury of describing the behavior of a lone marketmaker. It is much more difficult to argue that by documenting the behavior of a single

marketmaker in the FX market we have similarly captured the FX market. The data required to generate a more complete picture are out of the question given current availability. Nevertheless, the marketmaker we have tracked is without a doubt one of the key players in this market, trading well over $1 billion per day and maintaining $10 million quote relationships with a number of other marketmakers. Is he representative of marketmakers in the core of the wholesale spot market? We would argue yes, at least with respect to the issues addressed here. But, there is no doubt that different marketmakers have different trading styles.

A number of avenues for further research present themselves. For example, with data covering more trading days one could test for time–of–day effects to determine whether the perceived informativeness of trades depends on: (i) the end of the trading day in London, (ii) the closing of the IMM futures market, (iii) the timing of information releases, or (iv) the timing of the NY Fed's open market operation "go–around" for treasury quotes.[13] Another avenue involves the time spans between trades: are trades that follow in rapid succession given added weight in the updating process? Finally, there might be considerable information in prices that do *not* induce a transaction. For example, if a marketmaker provides a quote that is very competitive on one side of the market and the calling marketmaker chooses not to trade, that gives the quoting marketmaker a good indication of the direction the caller wants to go.

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[13] Usually, this is around 11:35 to 11:45 AM for repos and matched/sale purchases, and around 1 PM for outright transactions.

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Figure 2

2 Examples of Reuters Dealing 2000-1 Communications

Example 1: No Trade

```
To   CODE   FULL NAME HERE   * 1250GMT  030892 */1077
Our Terminal : CODE   Our user   : DMK
#    DMK 10
  8892
#    NOTHING THERE THANKS
#
#   #END LOCAL#

(   93 CHARS)
```

Example 2: Trade

```
From  CODE   FULL NAME HERE   * 1250GMT  030892 */1080
Our Terminal : CODE   Our user   : DMK
    SP DMK 10
# 8891
    BUY

#   10 MIO AGREED
#   VAL 6AUG92
#   MY DMK TO FULL NAME HERE
#   TO CONFIRM AT 1.5891 I SELL 10 MIO USD
#
      TO CONFIRM AT 1.5891 I SELL 10 MIO USD
      VAL 6AUG92
      MY USD TO FULL NAME HERE  AC 0-00-00000
      THKS N BIFN
#
#   #END LOCAL#
#
^   ## WRAP UP BY DMK DMK 1250GMT 3AUG92
^
^  #END#

(  265 CHARS)
```

Appendix

<u>Derivation of the Statistic Z_t</u>

Beginning with equation (3):

$$(3) \quad Q_t = \sum_{k=1}^{m} Q_{kt} + \nu_t = \sum_{k=1}^{m} \left[\theta(\mu_{kt} - \mu_t) + \bar{X}_{kt} \right] + \bar{\nu}_t$$

But:

$$\mu_{kt} = \lambda \mu_t + (1-\lambda) C_{kt} = \lambda[\rho S_t + (1-\rho) Z_t] + (1-\lambda) C_{kt} \qquad \text{where } \lambda \equiv \sigma_\omega^2 / (\sigma_\omega^2 + \sigma_\mu^2)$$

Substituting this value for μ_{kt} into equation (3) yields:

$$Q_t + m\theta(1-\lambda)[\rho S_t + (1-\rho) Z_t] = \theta(1-\lambda) \sum_{k=1}^{m} C_{kt} + \sum_{k=1}^{m} \bar{X}_{kt} + \bar{\nu}_t$$

$$\Rightarrow \quad Q_t + m\theta(1-\lambda)[\rho S_t + (1-\rho) Z_t] = \theta(1-\lambda) \sum_{k=1}^{m} [V_t + \bar{\omega}_{kt}] + \sum_{k=1}^{m} \bar{X}_{kt} + \bar{\nu}_t$$

And now defining the statistic Z_t we have:

$$\Rightarrow \quad Z_t \equiv \left[Q_t + m\theta(1-\lambda)[\rho S_t + (1-\rho) Z_t] \right] [m\theta(1-\lambda)]^{-1}$$
$$= V_t + m^{-1} \sum_{k=1}^{m} \bar{\omega}_{kt} + [m\theta(1-\lambda)]^{-1} \sum_{k=1}^{m} \bar{X}_{kt} + [m\theta(1-\lambda)]^{-1} \bar{\nu}_t$$

$$\Rightarrow \quad Z_t = \left[Q_t + m\theta(1-\lambda)\rho S_t \right] [m\theta(1-\lambda)]^{-1} + [m\theta(1-\lambda)(1-\rho)][m\theta(1-\lambda)]^{-1} Z_t$$

$$\Rightarrow \quad Z_t[1 - (1-\rho)] = \left[Q_t + m\theta(1-\lambda)\rho S_t \right] [m\theta(1-\lambda)]^{-1}$$

$$\Rightarrow \quad Z_t = [m\theta(1-\lambda)\rho]^{-1} Q_t + S_t$$

And since: $\quad Z_t = V_t + m^{-1} \sum_{k=1}^{m} \bar{\omega}_{kt} + [m\theta(1-\lambda)]^{-1} \sum_{k=1}^{m} \bar{X}_{kt} + [m\theta(1-\lambda)]^{-1} \bar{\nu}_t$

$$\Rightarrow \quad \sigma_Z^2 = (1/m)\sigma_\omega^2 + (1/m)[\theta(1-\lambda)]^{-2} \sigma_X^2 + [m\theta(1-\lambda)]^{-2} \sigma_\nu^2.$$

Derivation of the Statistic Z_{jt}

Beginning with equation (4):

(4) $\qquad Q_{jt} = \theta(\mu_{jt} - P_{it}) + X_{jt}$

$\Rightarrow \qquad Q_{jt}/\theta + P_{it} = \mu_{jt} + X_{jt}/\theta$

$\Rightarrow \qquad Q_{jt}/\theta + P_{it} = \lambda\mu_t + (1-\lambda)C_{jt} + X_{jt}/\theta$

$\Rightarrow \qquad Q_{jt}/\theta + P_{it} - \lambda\mu_t = (1-\lambda)(V_t + \bar{\omega}_{jt}) + \bar{X}_{jt}/\theta \qquad$ since $C_{jt} = V_t + \bar{\omega}_{jt}$

$\Rightarrow \qquad Z_{jt} \equiv \dfrac{Q_{jt}/\theta + P_{it} - \lambda\mu_t}{1-\lambda} = V_t + \bar{\omega}_{jt} + [1/\theta(1-\lambda)]\bar{X}_{jt}.$

Derivation of Estimating (11)

Beginning with equation (9):

(9) $\qquad P_{it} = \mu_{it} - \alpha(I_{it} - I_i^*) + \gamma D_t$

we can write:

$$\mu_{it} = \kappa\mu_t + (1-\kappa)Z_{jt} \qquad\qquad \text{where } \kappa \equiv \sigma_{zj}^2/(\sigma_{zj}^2 + \sigma_\mu^2)$$

$$= \kappa\mu_t + \left[\frac{1-\kappa}{1-\lambda}\right]\left[Q_{jt}/\theta + P_{it} - \lambda\mu_t\right]$$

$$= \kappa\mu_t - \left[\frac{\lambda(1-\kappa)}{1-\lambda}\right]\mu_t + \left[\frac{1-\kappa}{1-\lambda}\right]\left[Q_{jt}/\theta + P_{it}\right]$$

$$= \left[\kappa - \frac{\lambda(1-\kappa)}{1-\lambda}\right]\mu_t + \left[\frac{1-\kappa}{1-\lambda}\right]\left[Q_{jt}/\theta + P_{it}\right]$$

$$\equiv \phi\mu_t + (1-\phi)\left[Q_{jt}/\theta + P_{it}\right] \qquad \text{since } \left[\kappa - \frac{\lambda(1-\kappa)}{1-\lambda}\right] + \left[\frac{1-\kappa}{1-\lambda}\right] = 1.$$

Note also that $0 < \phi < 1$ since $0 < \kappa < 1$, $0 < \lambda < 1$, and $\kappa > \lambda$ — each of which follows from the definitions of κ and λ above and the fact that $\sigma_{zj}^2 = \sigma_\omega^2 + [\theta(1-\lambda)]^{-2}\sigma_X^2$.

Substituting this expression for μ_{it} into equation (9) yields:

$$P_{it} = \phi\mu_t + (1-\phi)\left[Q_{jt}/\theta + P_{it}\right] - \alpha(I_{it}-I_i^*) + \gamma D_t$$

$$\Rightarrow \quad P_{it} = \mu_t + \left[\frac{1-\phi}{\phi\theta}\right]Q_{jt} - \left[\frac{\alpha}{\phi}\right](I_{it}-I_i^*) + \left[\frac{\gamma}{\phi}\right]D_t$$

but μ_t can be written as:

$$\mu_t = \rho S_t + (1-\rho)Z_t = \rho S_t + (1-\rho)\left[[1/m\theta(1-\lambda)\rho]Q_t + S_t\right]$$
$$= S_t + \left[\frac{1-\rho}{m\theta(1-\lambda)\rho}\right]Q_t.$$

Hence, we can write:

$$(11) \qquad P_{it} = S_t + \left[\frac{1-\rho}{m\theta(1-\lambda)\rho}\right]Q_t + \left[\frac{1-\phi}{\phi\theta}\right]Q_{jt} - \left[\frac{\alpha}{\phi}\right](I_{it}-I_i^*) + \left[\frac{\gamma}{\phi}\right]D_t.$$

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