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BILATERAL SEARCH AS AN EXPLANATION FOR LABOR MARKET SEGMENTATION AND OTHER ANOMALIES

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ABSTRACT

Since applying for jobs is costly, workers prefer applying where their employment probability is high and, therefore, to jobs attracting fewer higher quality applicants. Since creating vacancies is expensive, firms create more vacancies when job-seeking is high. Our model captures these ideas and accounts for worker heterogeneity by assuming three types of nearly identical workers. These infinitesimal quality differences generate a discrete wage distribution. For some parameter values lower quality workers have discretely lower wages and higher unemployment than better workers. Moreover, increasing the number of the lowest quality workers can make all workers better off.

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This paper develops a simple model of bilateral search which can resolve anomalies in both the labor market segmentation and more mainstream literatures. Our model generates discrete wage differentials in response to infinitesimal productivity differences and can thus be considered a model of labor market segmentation. However, in contrast with most models of labor market segmentation, in our model both average wage and employment rates may be lower for less preferred workers.

Moreover, our model can explain a number of recent studies that have produced results which conflict with basic labor demand theory. Notably Card (1990) finds almost no effect of the Mariel boatlift on wages and employment in Miami. In related work, Kahn (1992) finds that, if anything, employment rose in response to comparable-worth-generated wage increases in San Jose while minimum wage laws do not appear to have any disemployment effects (Card, 1992a&b; Katz and Krueger, 1992).

This paper addresses the effect of increased labor supply on employment and wages. There are parameter values for which increasing the number of the least preferred workers can make all workers better off. The effect of minimum wage laws is discussed in a separate paper (Lang, 1993).

This paper pursues our research (Lang and Dickens, 1992) on employer search. Our model is similar to bilateral search models (Diamond, 1982; Mortensen, 1982; Howitt, 1988; Pissarides, 1990), in that the benefits of opening a vacancy are greater when more workers are searching for work. However, in contrast with most bilateral search models,[1] we assume that, everything else equal, workers are more likely to apply where wages are

[1] Sattinger (1990) is an exception.

higher. Thus firms can use wages to influence the probability of filling a vacancy.[2]

Finally, we add an element of worker heterogeneity to the model by allowing initially for three different types of workers. The model is structured so that, everything else equal, lower quality workers prefer to apply for jobs to which higher quality workers are less likely to apply. We believe that this reflects an important element of reality.

The three types are modelled as being infinitesimally different; formally, employers have a lexicographical preference for types with a lower index number. While it might appear natural to model the three types as having discrete differences in their productivity, the lexicographic preference ordering captures an idea, present in the literature on labor market segmentation literature, that differences in labor market success may be only weakly related to productivity differences.

The equilibrium of the model depends on the ratio of type 3 to type 1 workers. What is striking is that the well-being of type 1 and type 2 workers is nondecreasing in the number of type three workers. Moreover, there are parameter values for which increases in the number of type 3s makes all types better off.

From the viewpoint of standard theory, this result is extremely surprising. Increases in the supply of complements can, of course, make workers better off, but in our model, workers are, in effect, perfect substitutes. The intuition behind the model is that since they always beat type 3s when they are in direct competition, type 1s and type 2s cannot be made worse-off by their presence. However, since the supply of jobs increases

[2]The efficiency wage literature has seen the development of a series of papers (Weitzman, 1989; Montgomery, 1991; Lang, 1991) in which firms use high wages to affect the probability that they fill vacancies.

in response to the presence of type 3s, the preferred types are made better off. Moreover, type 3s can be made better off by an increase in the number of type 3s because the supply of jobs increases, reducing the cost of increased competition from similar workers. Moreover, the presence of more type 3s dilutes the competition with preferred workers.

Although formally quite different and lacking much of the institutional richness of that literature, our model is reminiscent of institutional and radical writing on job scarcity, most notably Piore (1975) and Doeringer and Piore (1971) on labor market segmentation and Thurow's (1975) work on job queues. Workers in our environment recognize the existence of job scarcity and a hierarchical ordering among job applicants. On the other hand, the model also captures the existence of worker scarcity. Vacancies and unemployment co-exist. Moreover, in contrast to much of the earlier work on labor market segmentation, the supply of jobs ("demand for labor") responds to the supply of workers. As a result, an increased supply of low quality workers can generate a labor demand response which makes all workers better off.

Before turning to the model, we must address the issue of whether it is sensible to model firms as using wages to increase the supply of applicants. It is sometimes argued that vacancies are not a serious problem for firms. Estimates of the duration of vacancies are on the order of two to four weeks. Therefore, firms would not expend significant amounts of money to shorten the duration of vacancies. This argument is deficient on both theoretical and empirical grounds. Short vacancies may well be evidence that vacancies are very costly. We would not infer from the low levels of inventories of inputs under the Japanese "just in time" system that Japanese firms take no steps to hold inventories down. Similarly, in the U.S., there are many industries in which inventories average less than two weeks of sales. We would not conclude that in these industries firms do not use price to affect inventory levels. Empirically, higher wages are associated with more applications (Holzer, Katz and Krueger, 1991) and with lower vacancy rates (Holzer, 1990). From a modelling perspective, it is clear that the firms in models in which wages are determined by bargaining after the firm and worker meet would like to commit to a wage ex ante. In general, by committing to a particular wage, they can attract more workers and make more profit.

The next section describes the model and provides the general intuition. The following three sections develop the equilibria for the cases where the ratio of the number of least preferred to the number of most preferred workers is small, moderate and large. The fifth section examines the effect of increasing the number of type 3 workers on the well-being of different types of workers.

I. The Basic Model

We model the labor market as a one-shot game with multiple stages in which firms set wages in order to attract applicants. Modelling the labor market as a one-shot game is somewhat unnatural. As discussed briefly in the conclusion, it is fairly easy to transform the one-shot game into a dynamic game provided that workers must be unemployed in order to search and firms must have a vacancy in order to recruit workers. Whether the results are robust to allowing for workers and firms to search even while in an employment relation remains to be determined, but we conjecture that they are.

The structure of the game is as follows.

(A1) There are N workers and mN potential firms where both N and m are large.

Assumption (A1) serves to ensure that we can use asymptotic approximations to the binomial distribution and that it will never be an equilibrium for all firms to enter the market.

(A2) There are three types of workers denoted type 1, type 2 and type 3. The number of workers of each type is large. Firms have lexicographical preferences over worker types. They maximize profits but for equal profits prefer type 1 workers to type 2 workers who they prefer, in turn, to type 3 workers.

It may be easier, and it would certainly be more conventional, to view type 3s as infinitesimally less productive than type 2s and type 2s as infinitesimally less productive than type 1s. At times we will slip into this interpretation. However, treating preferences as lexicographic simplifies presentation.

(A3) The firms are indexed 1,...,mN. They decide sequentially whether or not to enter.

Assumption (A3) is essentially a simple way of assuring zero-profit equilibrium given that m is large but which allows us to make use of game-theoretic concepts such as sub-game perfection.

(A4) Firms which enter the market or stay in business pay a capital rental fee of d.

Assumption (A4) captures the idea that it is costly for firms to open a vacancy. As a consequence their willingness to create a vacancy will depend on their expectations about the likelihood of filling the vacancy. Instead of a capital rental fee, d could be interpreted as a search cost.

(A5) After all firms have entered, firms simultaneously announce the wage they will offer to any worker to whom they make an offer. The wage offer cannot be dependent on the type of worker hired.

Assumption (A5) represents the major departure from most of the literature on bilateral search. Together with (A6) it makes it possible for firms to use wages to influence the probability that they fill their vacancies.

(A6) After observing all the wage offers, workers may apply to at most one firm.

Having workers apply to only one firm does not appear to be essential to the model provided that workers apply to only a small number of firms. However, equilibria with multiple applications are considerably more complex.

(A7) Firms hire at most one worker. This worker is chosen at random from the applicants belonging to the most preferred type from which the firm receive applicants.

Assumption (A7) serves three roles. First it eliminates strategies in which firms always hire "Jane Smith" if she applies. Second, it means that the firm cannot commit to hiring a less preferred type if a more preferred type shows up. Without a commitment mechanism such strategies will fail the test of sub-game perfection. Finally, and most importantly, it ensures that workers will care about the probability that a more preferred type applies for the same job.

(A8) Workers who are hired produce v and receive the contracted wage.

Assumption (A8) ensures that there are no contract enforcement problems.

These assumptions are designed to capture the idea that workers prefer to apply for jobs where they will face less competition, particularly from more qualified workers. In this model, because wages are set by the firm in advance, workers who apply to the same firm compete on the basis of quality rather than under-bidding each other. As a consequence, lower quality workers will avoid jobs where higher quality workers are likely to apply. Low-wage jobs can actually attract low-quality workers because they do not attract applications from high-quality workers.

The model also captures the idea that firms will care about the probability of filling a vacancy because it is costly to them to open a vacancy. Moreover, they can use their wage policy to affect the probability that the vacancy is filled.

In all of the equilibria we describe workers will use strategies that generate an equilibrium which is a natural extension of Harris-Todaro to heterogeneous workers. In equilibrium all workers of a given type will get the same expected wage (wage multiplied by employment probability) at all firms to which that type of worker applies. While it is obvious that these strategies are an equilibrium of the workers' sub-game, it is important to note that the following paragraph does provide a complete description of workers' strategies.

Type 1s randomize among jobs so that if all other type 1 workers followed the same random strategy, the expected wage would be the same at all jobs among which they randomize with non-zero probability and lower if they were the only applicant to any jobs to which they apply with probability zero. Type 2s randomize among jobs so that if type 1s followed the strategy just described and if all type 2s followed the same random strategy, the expected wage for type 2s would be the same at all jobs among which they randomize with non-zero probability and lower if they were the only applicant to any jobs among to which they apply with probability zero. Type 3s randomize among jobs so that if type 1s and type 2s followed the strategies just described and if all type 3s followed the same random strategy, the expected wage for type 3s would be the same at all jobs among which they randomize with non-zero probability and lower if they were the only applicant to any jobs among to which they apply with probability zero.

Although in the equilibrium described in this paper, there will be only two or three wages, the strategies described in the previous paragraph are complete for any set of wage offers including out-of-equilibrium wage distributions.

Before describing the equilibrium, it is useful to introduce some approximations to the binomial. Given that all workers of given type use the same strategy, each will apply to a given firm with the same probability. Denote the probability that a worker of type i applies to a particular firm as p_i and the number of workers of type i by N_i. Denote the expected number of applicants of type i by $z_i = p_i N_i$. Then the probability that the firm does not get any applicants is

$$(1) \qquad \prod_i (1-p_i)^{N_i} = \prod_i (1-(z_i/N_i))^{N_i} \to e^{-\Sigma z_i}$$

where the limit is obtained by letting all N_i go to infinity while holding z_i constant.

In addition, the probability of a random applicant of type i obtaining employment is

$$(2) \quad \{\prod_{j<i}(1-p_j)^{N_j}\}\{\sum_{k=0}^{N_i-1}\binom{N-1}{k}p_i^k(1-p_i)^{N_i-k-1}/(k+1)\}=\{\prod_{j<i}(1-p_j)^{N_j}\}\,(1-(1-p_i)^{N_i})/z_i$$

Again letting all the Ns go to infinity while holding z_i constant, the probability of a random applicant of type i obtaining employment is given by

$$(3) \qquad E_i \to e^{-\Sigma z_j}(1-e^{-z_i})/z_i \qquad j<i$$

where the first term is the probability that the firm received no preferred applicants and the remainder of the expression is the probability of the worker getting the job conditional on there being no preferred applicants.

The approximations in (1) and (3) are quite precise for low numbers of expected applicants and modest values of N. For example, with a single type, if the expected number of applicants is 2 and there are 1000 workers, the true probability of a vacancy is .1351. The Poisson approximation used here is .1353. We use the Poisson approximation throughout this paper.

The basic intuition underlying all the results is captured in the following lemma:

Lemma 1: In equation (3) $\partial\Sigma z/\partial\Sigma z_j$ $(j<i) < 0$ and $\partial\Sigma z/\partial w$ is independent of Σz_j $(j<i)$ where Σz denotes the total number of applicants of type i or lower.

Proof: Application of the implicit function theorem.

The first part of the lemma says that raising the number of preferred applicants lowers the total number of applicants. Thus if a wage offer would attract two types of applicants, it will be profitable to lower the wage in order to attract fewer of the preferred applicants. This will generate a higher expected number of expected applicants and thus a lower probability of a vacancy as well as lowering cost.

When there are only two types of workers as in Lang and Dickens (1992), it follows immediately that the equilibrium must involve the following. High-productivity workers apply to high-wage jobs while only infinitesimally less productive workers apply to low-wage jobs. The wages in the jobs to which the less productive workers apply are just sufficiently low to deter the preferred workers from applying. Preferred workers have both higher wages and higher expected wages than their slightly less productive counterparts.

It might appear that the equilibrium with more than two types would be a trivial extension of this equilibrium. As we successively add new inferior types, we would expect each to receive a lower wage just sufficient to deter higher quality types from applying. A moment's reflection makes it clear that this cannot be the case. No matter how low the wage, firms will only enter if there is a non-infinitesimal probability of filling their vacancy which, in turn, implies that workers have a non-infinitesimal probability of being unemployed. This, in turn, implies that as we get more and more types, the wage would get lower and lower and asymptote towards zero. However, if, as postulated, only type 1s apply to the highest wage jobs, the value of applying there is the probability of a vacancy in the highest wage jobs multiplied by the wage. This product is non-zero. Therefore for a sufficiently large number of types, the postulated equilibrium cannot, in fact, be an equilibrium. It turns out that three types is sufficient to ensure that the simple ordering between type and wage breaks down.

Thus we need not, and indeed we do not, get complete separation of types. However, the lemma does ensure that any mixing which does occur will include type 1s and type 3s. If an offer would attract only types 2 and 3 or only types 1 and 2, lowering the wage would be more profitable. However, if type 1s and 3s are mixed together, lowering the wage would attract fewer type 1s. Provided this does not attract more type 2s, this will be profitable. However, if lowering the wage does attract more type 2s, lowering the wage will reduce the expected number of applicants because the decrease in type 3s will more than offset the increase in type 2s.

We should note further that when all three types are mixed, a given increase in the wage increases the number of applicants by more than if there were only type 1 applicants. This together with the second part of the lemma implies that any mixing of all three types must occur at a wage above the equilibrium wage for the case where there is only one type of worker.

The next three sections, examine the nature of the mixing which does occur depending on the relative number of type 3s and type 1s.

II. Equilibrium in the Model with Few Type 3s

The following theorem states that if there are relatively few type 3 workers, the equilibrium is given by a three-wage distribution. The high wage and the low wage are the same as in the absence of any type 3 workers and attract type 1 and type 2 workers as they would in the two-type equilibrium. There is, however, an intermediate wage which is just sufficiently high to deter type 2 workers from applying which attracts both type 1 and type 3 workers.

Theorem 1: If the number of type 3s is sufficiently small relative to the number of type 1s, there is a sub-game perfect equilibrium given by a three-wage distribution with the following characteristics — a high wage which attracts only type 1 applicants, a low wage which attracts only type 2 applicants and a third wage which lies between them and attracts both type 1 and type 3 applicants. The equilibrium conditions are given by

(4) $z_1 = -\log(k_1/v)$

(5) $w_2 = w_1(1-e^{-z_1})/z_1 = k_1$

(6) $w_3(1-e^{-z_1^*})/z_1^* = k_1$

(7) $e^{-z_1^*}w_3 = w_2(1-e^{-z_2})/z_2$

where z_1* is the expected number of type 1 applicants to the jobs paying w_3, k_1 is the expected wage for type 1s applying to the high-wage job, and z_1 and z_2 are the expected number of type 1 and type 2 applicants to high-wage and low-wage jobs, respectively. Equations (4)-(7) along with the three zero-profit conditions, and the constraints $z_1*m_3+z_1*m_1=N_1$, $z_2*m_2=N_2$, $z_3*m_3=N_3$, fully determine z_1, z_2, z_3, $z*_1$, w_1, w_2, and w_3.

The first m_1 firms enter and offer a wage of w_1; the next m_2 firms enter and offer a wage of w_2; the next m_3 firms enter and offer a wage of w_3. The number of entrants is such that the expected profit from all strategies played is zero.

Proof: (see appendix)

Table 1 gives a clearer indication of the implications of this equilibrium. In the table the rental cost of capital, d, is the numeraire. VMP is therefore v/d. The second column gives the employment rate for individuals applying to high-wage jobs. The third column gives the employment rate for type 1s applying to jobs paying w_3. The fourth column gives the employment rate for workers applying to the low-wage jobs which is also the employment rate for type 2s. The fifth column gives the employment rate for type 3s while the fifth column gives the overall employment rate for those applying to jobs offering w_3. The last three columns give the equilibrium wages.

The zero-profit condition requires that the vacancy rate decline as the wage rises. Therefore it is not surprising that employment rates fall progressively as we go from the low-wage (w_2) to medium-wage (w_3) to high-wage (w_1) jobs. What is more interesting is that employment rates may be negatively related to type. The least preferred type (type 3s) always has the lowest employment rate. Employment rates are ranked in descending order, type 1s applying to w_3 jobs, type 2s, type 1's applying to w_1 jobs, type 3s. Whether the overall employment rate for type 1s is greater or less than the employment rate for type 2s depends on the ratio of type 3s to type 1s. When this ratio is very small, few type 1s apply to jobs offering w_3, and the employment rate for type 1s is close to the rate for type 1s applying to jobs offering w_1. As this ratio approaches z_3/z_1*, almost all type 1s apply to w_3 jobs, and the employment rate for type 1s exceeds the employment rate for type 2s.

The other point which is apparent is that there are substantial wage differentials despite the absence of any real productivity differentials. Moreover, the medium wage falls closer to the low wage than to the high wage.

It is worth pointing out that a social commentator might be concerned that the least preferred types apply to the medium-wage jobs rather than to the lowest-wage jobs. He might also note that the type 3s also have very high unemployment, because they frequently lose out to type 1s who also apply to these jobs. In contrast, the type 2s, although they apply to the lowest wage jobs, do better than type 3s because the type 2s have lower unemployment. The social commentator might conclude that the aspirations of the type 3s are inconsistent with their job opportunities and might blame the high unemployment on these excessive aspirations. In fact, however, if the type 3s applied to the low-wage jobs, they would still have high unemployment. Indeed, for all of the parameter values in table 1 except v equal to 1.5, a type 3 who applied to a low-wage job would actually have a lower employment probability than if he applied to a job offering w_3.

III. Equilibrium with a Moderate Number of Type 3s

If the ratio of type 3s to type 1s exceeds $z_3/z_1{*}$, then the equilibrium described in theorem 1 is not feasible. For the case where the ratio of type 3s to type 1s exceeds $z_3/z_1{*}$, but is not "too" large, we have the following equilibrium.

Theorem 2: If the ratio of type 3s to type 1s exceeds $z_3/z_1{*}$, but is not "too" large, there is a sub-game perfect equilibrium given by a two-wage distribution with the following characteristics — a high wage, w_1, which attracts type 1 and type 3 applicants and a low wage which attracts only type 2 applicants. The equilibrium conditions are given by

$$(8) \qquad w_2 = w_1(1-e^{-z_1})/z_1$$

$$(9) \qquad e^{-z_1}w_1 = w_2(1-e^{-z_2})/z_2$$

$$(10) \qquad (1-e^{-z_2})(v-w_2) - d = 0$$

$$(11) \qquad (1-e^{-(1+a)z_1})(v-w_1) - d = 0$$

where a is the ratio of type 3 to type 1 workers.

Proof: (see appendix)

Equation (8) ensures that type 1 workers are just indifferent between applying to a low-wage job and being hired with probability 1 and applying to a high-wage job. Equation (9) ensures that type 2 workers are just indifferent between being the only type 2 applicant to a high-wage job and applying to low-wage jobs. Equations (10) and (11) are the zero-profit conditions.

IV. Equilibrium When the Number of Type 3 Workers Is Large

As the ratio of type 3 to type 1 workers rises, both wages and the number of applicants rises, a point to which we return in the next section. Eventually the increased competition for high-wage jobs becomes sufficiently intense that it is profitable for firms to offer low-wages to attract type 3 workers only.

Theorem 3: When a is sufficiently large, the equilibrium is given by the solution to (8)-(11) and

$$(12) \qquad w_3 = w_2(1-e^{-z_2})/z_2$$

$$(13) \qquad (1-e^{-z_3})(v-w_3) - d = 0.$$

Proof: (see appendix)

Table 2 gives the employment rate for different types of workers and jobs as well as wages for different values of v. As in table 1, d is the numeraire so that v can be viewed as v/d.

One striking result is that type 1 workers are clearly better off than other workers. They both receive higher wages and have higher employment rates than other workers. Although, employment rates are highest at the low-wage jobs and lowest at the high-wage jobs, the type 1 workers have sufficiently greater access to high-wage jobs compared with type 3 applicants that their employment rate is higher than for other workers.

Employment rates for type 2 workers (applying to medium wage jobs) fall between the employment rates of type 3 workers applying to low-wage jobs and those applying for high-wage jobs. As a consequence if the fraction of type 3s applying to low-wage jobs is sufficiently low, type 3s can have the lowest employment rate. For all values of v presented in the table, type 3s will have lower employment rates than type 2s if at least one-sixth apply to high-wage jobs.

On the other hand, type 3s will have lower average wages than type 2s only if a sufficient proportion apply to low-wage jobs. Since type 3s will have low average wages when relatively few apply to high-wage jobs but low employment rates when relatively few apply to low-wage jobs, this raises the question of whether it is possible for type 3s to have both lower average wages and lower employment rates than type 2 workers.

There turns out to be quite a large range of values for which this is possible. When v equals 1.5, type 3 workers wages will, on average, be lower than type 2 workers if fewer than 40% of employed type 3 workers are in high-wage jobs. Since employment rates are lower for applicants to high-wage jobs are lower than for applicants for low-wage jobs, this in turn requires that fewer than 70% of type 3 workers apply for high-wage jobs. At the other end if the range covered in the table, if v equals 6, wages will be lower on average for type 3s than for type 2s if fewer than 57% of type 3s apply for high-wage jobs. Thus for all values of v covered in the table, there is a considerable range over which type 3s can be disadvantaged with respect to both wages and employment.[3]

V. Increasing Numbers of Low Quality Workers in the 3-Type Equilibrium

In a paper which challenges conventional economic theory, Card (1990) examines the effect of the Mariel boatlift of 1980 on the Miami labor market. The Mariel immigration entailed a huge increase in the unskilled labor force in Miami and raised the total labor force by 7 percent. Conventional theory suggests that the wages and employment rates of less-skilled workers should have declined. Models in which wages are rigid suggest a sharp increase in unemployment among this group. However, Card finds the Mariel immigration had virtually no effect on the wages or unemployment rates of less-skilled workers, even among Cubans who had immigrated earlier and were presumably close substitutes.

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[3]The fraction of type 3s who apply for each type of job depends on the ratio of type 3s to type 1s. For the values presented in the table, the critical ratio of type 3s to type 1s at which type 3s start applying to low-wage jobs is around 1. For all values in this table, if the ratio of type 3s to type 1s were 1.4, type 3s would have both the lowest wages and lowest employment rates.

These results, while difficult to reconcile with standard theory, arise naturally in the model in the previous section. Note that until the number of type 3 workers becomes large relative to the number of type 1 workers, the wages of both type 2 workers and type 3 workers increase as the number of type 3 workers increases, holding the number of type 1 workers constant. Over this range the average wage received by type 1 workers is initially decreasing and then increasing.

It follows that the welfare of type 1 and type 2 workers is nondecreasing in the number of type 3 workers and increasing in some ranges. Given that the 3 types of workers are near perfect substitutes, this result is surprising. The fact that welfare is strictly nondecreasing is, in part, due to the constant returns to scale assumptions which have been used to simplify the model. In a sense demand is infinitely elastic. However, this is insufficient to explain why increasing the population of close substitutes makes workers <u>better</u> off.

The intuition appears to be that the behavior of workers in the model is affected by a sense of job scarcity while the behavior of firms is affected by a sense of worker scarcity. Workers want to avoid unemployment while firms want to avoid vacancies. Firms will not enter markets where there are few job applicants. Increasing the number of type 3 workers generates more jobs, and for some parameter values these are jobs which are attractive to type 1 workers and make them better off. Opportunities for type 2 workers are limited by their need to avoid competition with type 1 workers. As conditions for type 1 workers improve, they therefore also improve for type 2 workers. It is worth noting that the intuition here is similar to that underlying the coordination failure literature and suggests that if labor supply were endogenous there might be pareto-ranked multiple equilibria.

Table 3 gives the relation between the ratio of type 3s to type 1s and wages and employment rates for type 3s. As already discussed, the initial impact of the increased number of type 3s is to raise wages and lower employment rates. As the number of type 3s gets very large, we shift to having most type 3s apply to low wage jobs for which their employment rates are relatively high. The model cannot predict the overall effect on either employment or wages since the impact depends on the parameters of the model. Nevertheless, examination of table 3 reveals that at relatively low values of the ratio of type 3s to type 1s, increases in the number of type 3s increase their expected wage.

The somewhat startling conclusion is therefore that for some parameter values increases in the number of type 3 workers are Pareto-improving in the sense that all workers have higher expected earnings. All types of workers are made better off even though workers are nearly perfect substitutes. We have already explained the intuition behind the improvement for type 1 and type 2 workers. The intuition for type 3 workers is that they prefer minimal competition with both type 1 and type 2 workers. Improvements in job availability for type 1 workers make type 3 workers better off by diluting competition from type 1 workers. However, the reduced competition from type 1 workers attracts more competition from type 2 workers. Initially the improvement in conditions for both types of workers makes them better off. When the number of type 3 workers is already fairly large, the second effect seems to dominate.

VI. Summary and Conclusions

The results in this paper have been developed within the framework of a one-shot game. It would not be difficult to extend them to a dynamic model although the math and proofs would be considerably more complex. Provided that both worker and firm must end the relation before engaging in search and provided that the firm can make a binding commitment to a wage path contingent on the worker's continued employment, there is no reason for employment relations to end endogenously. Both worker and firm receive quasi-rents from the existence of the employment relation. Therefore a constant wage will be sufficient to prevent either the firm or worker from renewing search. Within a range determined by the size of the quasi-rents the parties receive, the wage path could be positively or negatively sloped.

Although solidly grounded in modern economic theory, the model developed in this paper generates results which diverge sharply from those of standard supply and demand analysis. The results seem to us much more reminiscent of work on labor market segmentation. Indeed our initial interest in this model stemmed from our desire to develop a more formal model of unemployment in the context of labor market segmentation.

In the model, it is possible to identify distinct segments paying different wages in the sense that identical or nearly identical workers are employed at different wages. Less favored workers find it more difficult to obtain employment and, for some parameter values, choose between applying for a high-wage chance with only a low prospect of employment and applying for a low-wage job. Segments in the model, however, are not immutable. Both the number of jobs and the wages they pay respond to the distribution of types of workers in the population.

We do not offer this model as the sole explanation for labor market segmentation. We recognize the existence of other important determinants. Our model also lacks the richness of models which pay more attention to the fact that the labor market is a social institution. This is a price we pay for increased formalism.

Nevertheless, we find the model very promising as an explanation for a number of anomalies. In addition to the issues raised by the Mariel boatlift, our model provides an explanation for interindustry wage differentials and can generate increasing employment in response to increases in the minimum wage (Lang, 1993). Moreover, it is a model in which vacancies and unemployment exist simultaneously. The predictions of the model diverge sharply from those of standard models and suggest that it is worthy of continued investigation.

TABLE 1

EMPLOYMENT RATES AND WAGES WHEN THE RATIO OF TYPE 3'S TO TYPE 1'S IS SMALL

	Employment Rates					Wage Rates		
VMP	High	1's Med	Low	Type 3's	Medium	w1	w2	w3
1.5	.39	.61	.55	.26	.50	.39	.15	.25
2	.48	.68	.64	.37	.59	.77	.37	.55
2.5	.54	.73	.70	.44	.65	1.16	.63	.87
3	.58	.76	.73	.50	.68	1.56	.91	1.21
3.5	.62	.78	.76	.54	.71	1.97	1.21	1.56
4	.64	.80	.78	.57	.73	2.38	1.53	1.92
4.5	.66	.81	.80	.59	.75	2.80	1.86	2.29
5	.68	.82	.81	.62	.77	3.22	2.19	2.67
5.5	.70	.83	.82	.63	.78	3.64	2.54	3.06
6	.71	.84	.83	.65	.79	4.07	2.89	3.45

TABLE 2

EMPLOYMENT RATES AND WAGES WHEN THE RATIO OF TYPE 3'S TO TYPE 1'S IS HIGH

	Employment Rates					Wage Rates		
VMP	High	1's	3's High	3's Low	Medium	w1	w2	w3
1.5	0.36	0.57	0.16	0.56	0.50	0.41	0.24	0.12
2	0.46	0.64	0.25	0.66	0.60	0.81	0.52	0.31
2.5	0.51	0.69	0.32	0.71	0.65	1.22	0.84	0.55
3	0.56	0.72	0.37	0.75	0.69	1.63	1.17	0.81
3.5	0.59	0.74	0.41	0.77	0.72	2.05	1.52	1.09
4	0.62	0.76	0.45	0.79	0.74	2.47	1.88	1.39
4.5	0.64	0.78	0.47	0.81	0.76	2.89	2.24	1.70
5	0.66	0.79	0.50	0.82	0.77	3.32	2.62	2.02
5.5	0.67	0.80	0.52	0.83	0.78	3.76	3.00	2.35
6	0.69	0.81	0.54	0.84	0.79	4.19	3.38	2.68

TABLE 3

EQUILIBRIUM WAGES AND EMPLOYMENT FOR DIFFERENT VALUES OF "a"

(v=2)

	Wage Rates		Employment Rates		
a	1s and 3s	2s	1s	2s	3s
0	0.77	0.37	0.48	0.64	–
.41	0.55	0.37	0.68	0.64	0.37
.51	0.62	0.42	0.67	0.63	0.34
.61	0.68	0.45	0.66	0.62	0.32
.71	0.73	0.48	0.66	0.61	0.30
.81	0.77	0.50	0.65	0.61	0.28
.91	0.80	0.52	0.65	0.60	0.26
.94	0.81	0.52	0.64	0.60	0.25
∞	0.31[a]	0.52	0.64	0.60	0.66

[a]Wage for type 3s. Type 1 wage is .81.

REFERENCES

Card, David, "The Impact of the Mariel Boatlift on the Miami Labor Market," Industrial and Labor Relations Review, 43 (January 1990):245-57.

______, "Using Regional Variation in Wages to Measure the Effects of the Federal Minimum Wage," Industrial and Labor Relations Review, 46 (October 1992a): 22-37.

______, "Do Minimum Wages Reduce Employment? A Case Study of California, 1987-89?" Industrial and Labor Relations Review, 46 (October 1992b): 38-54.

Doeringer, Peter and Piore, Michael, Internal Labor Markets and Manpower Analysis, Lexington, MA: D.C. Heath, 1971.

Holzer, Harry J., "Hiring Procedures in the Firm: Their Economic Determinants and Outcomes," in Human resources and the performance of the firm, Edited by Morris M. Kleiner et al., Madison, Wis.: Industrial Relations Research Association, 1987, 243-74.

Holzer, Harry J., Katz, Lawrence F. and Krueger, Alan B., "Job Queues and Wages," Quarterly Journal of Economics, 106 (August 1991): 739-68.

Howitt, Peter, "Business Cycles with Costly Search and Recruiting," Quarterly Journal of Economics, 103 (February 1988): 147-65.

Kahn, Shulamit. "Economic Implications of Public-Sector Comparable Worth: The Case of San Jose, California," Industrial Relations, 31 (Spring 1992): 270-91.

Katz, Lawrence F. and Krueger, Alan B., "The Effect of the Minimum Wage on the Fast-Food Industry," Industrial and Labor Relations Review, 46 (October 1992): 6-21.

Lang, Kevin, "Persistent Wage Dispersion and Involuntary Unemployment." Quarterly Journal of Economics, 106 (February 1991): 181-202.

______, "The Effect of Minimum Wage Laws on the Distribution of Employment: Theory and Evidence," Boston University, unpublished, 1993.

Lang, Kevin, and Dickens, William T., "Labor Market Segmentation, Wage Dispersion and Unemployment," NBER Working Paper No. 4073, 1992.

Montgomery, James D., "Equilibrium Wage Dispersion and Interindustry Wage Differentials," Quarterly Journal of Economics, 106 (Feb. 1991): 163-180.

Mortensen, Dale T., "Property Rights and Efficiency in Mating, Racing, and Related Games," American Economic Review, 72 (December 1982): 968-79.

Piore, Michael, "Notes for a Theory of Labor Market Segmentation," in Richard C. Edwards et al., eds., Labor Market Segmentation, Lexington, MA: D.C. Heath, 1975.

Pissarides, Christopher A., Equilibrium unemployment theory, Oxford: Blackwell, 1990.

Sattinger, Michael, "Unemployment, the Market for Interviews, and Wage Employment," Journal of Political Economy, 98 (April 1990): 356–71.

Thurow, Lester, Generating Inequality, New York: Basic Books, 1975.

Weitzman, Martin L., "A Theory of Wage Dispersion and Job Market Segmentation," Quarterly Journal of Economics, 104 (February 1989): 121–37.

APPENDIX

Proof of theorem 1:

We begin by showing that if firms offer wages as claimed in the theorem, type 1 workers will randomize between jobs paying w_1 and w_3, type 2 workers will apply only to jobs offering w_2 and types 3s will apply only to jobs offering w_3.

Types 1s: Equation (5) states that w_2 equals the expected wage received by type 1 workers applying to jobs paying w_1. Therefore type 1 workers must apply to jobs paying w_2 with probability zero. If z_1* equalled zero (no type 1 workers applied to jobs paying w_3, then from (7) and (6), $w_3=w_2=k_1<w_3$ which is impossible. If no type 1 workers applied to jobs paying w_1, then from (5) and (6) $w_1=k_1<w_3$ which contradicts w_1 being the high wage. In words, w_3 lies below w_1 but above the expected wage for type 1 workers applying to jobs paying w_1, and therefore, both jobs must attract type 1 workers.

Type 2s: Equation (7) ensures that the expected wage received by a type 2 workers who is the sole type 2 applicants to jobs paying w_3, has the same expected wage as type 2 workers applying to jobs paying w_2. Therefore type 2 workers must apply to jobs paying w_3 with probability zero. From the lemma, we know that higher wage jobs such as those paying w_1 which attract even more type 1 applicants cannot attract more type 2 applicants than do jobs paying w_3. Therefore neither jobs paying w_3 nor jobs paying w_1 receive any type 2 applicants.

Type 3s: If jobs paying w_1 attracted type 3 workers, then by lemma 1, w_3 jobs would be more profitable than jobs paying w_1 which contradicts the fact that all offers make zero profits. If jobs paying w_2 attracted type 3 workers, then by lemma 1, it would be profitable to make a lower wage offer which would attract only type 3 workers. We will show shortly than no such deviation is profitable.

We now consider deviations from the offer distribution claimed in the theorem. We consider the following possible deviations, a wage other than w_1 which is sufficiently high that it would attract only type 1 workers (type-1 only deviation); a wage sufficiently below w_1 to attract type 3 workers but above w_3 (type-1 and type-3 deviation); a wage between w_3 and w_2 (all-type deviation); a wage below w_2 that would attract both type 2s and type 3s (type-2 and type-3 deviation); a wage sufficiently below w_2 that it would attract only type 3s (type-3 deviation).

Type-1 only deviation: Equation (4) is derived by maximizing profits subject to labor market equilibrium for type 1s:

(A1) $$\text{Max } \pi = (1-e^{-z_1})(v-w_1) - d$$

s.t.

(A2) $$w_1(1-e^{-z_1})/z_1 = k_1$$

Together with the zero-profit constraint for high-wage firms, it determines w_1. Therefore no other wage which would attract only type 1 workers can be a profitable deviation. This applies to any wage sufficiently above w_3.

Type-1 and type-3 deviation: A wage not too much above w_3 will attract both type 1 and type 3 workers. From lemma 1, this will make less profit than w_3.

<u>All-type deviation</u>: A wage between w_2 and w_3 will attract all three types of workers. From lemma 1, in this range, raising the wage will increase applications and therefore profits by more than if there were only type 1 applicants. We know that since w_1 is optimal when there are only type 1 applicants, a wage between w_2 and w_3 must make lower profits than w_3 and must therefore not be profitable.

<u>Type-2 and type-3 deviation</u>: From lemma 1, if there are just two types of applicants, it will always be more profitable to lower the wage until either one of the types no longer applies or a third type begins to apply. Therefore we need consider only type-3 only deviations.

<u>Type-3 only deviation</u>: Since when there is only one type of applicant, w_1 is optimal, the most profitable type-3 only deviation will be the one which is just sufficiently low to deter type 2s from applying. We show that this deviation is not profitable. To prove that this wage does not make a profit suppose that the deviation is profitable so that

$$\text{(A3)}\quad (1-e^{-(z_1^*+z_3^*)})(v-w_3^*) < (1-e^{-z_3})(v-w_3)$$

where * denote the values in the claimed equilibrium and w_3 refers to the proposed deviation.

Note that workers will arrange themselves so that

$$\text{(A4)}\quad e^{-z_1^*} w_3^*(1-z_3^*)/z_3^* = w_3(1-z_3)/z_3$$

where z_3 is the expected number of applicants to the deviating firm. Now w_3 is the expected wage for type 2 workers and w_3* is chosen so that $\exp(-z_3^*)w_3^*$ equals this expected value. Therefore z_3 equals z_3* and (A3) can be rewritten

$$\text{(A5)}\quad (1-e^{-(z_1^*+z_3)})(v-w_3^*) < (1-e^{-z_3})(v-e^{-z_1^*}w_3^*)$$

Rearranging terms gives:

$$\text{(A6)}\qquad e^{-(z_3+z_1^*)}v < w_3.$$

But,

$$\text{(A7)}\qquad e^{-(z_3+\ z_1^*)}v > e^{-z_1}v - w_2 > w_3$$

where the equality uses (4) and the fact that $w_2 = k_1$. So (A3) is contradicted and the proposed deviation cannot be profitable.

Given that all entrants make zero profit, no entering firm has an incentive to deviate. Any firm which does not enter the market would make negative profits if it were to enter the market.

While we have shown that no deviation from (4)-(7) is profitable, we have not shown that these equations can be satisfied simultaneously with the zero-profit conditions. The only difficulty which can arise is that if the number of type 1s is sufficiently small relative to the number of type 3s, it may not be possible to satisfy $z_1{*}m_3 \le N_1$ and $z_3{*}m_3 = N_3$ simultaneously. For N_3/N_1 sufficiently small, it will be possible to satisfy these constraints.

QED

Proof of Theorem 2:

We begin by showing there is always a solution to equations (8)-(11). Since inspection of (10) and (11) demonstrates that positive values of z_1 and z_2 will generate positive wages, we eliminate w_1 and w_2 from the system to get

$$(A8)\qquad (1-e^{-z_1})/(z_1 e^{-z_1}) = z_2/(1-e^{-z_2})$$

$$(A9)\qquad (1-e^{-z_2})v - z_2 e^{-z_1}(v - d/(1-e^{-(1+a)z_1}) - d = 0.$$

In both equations, when z_1 goes to 0, z_2 also goes to zero. However, in equation (A8), z_2 tends to ∞ as z_1 tends to ∞ while in equation (A9), z_2 tends to a finite positive value as z_1 tends to ∞. The key to showing that (A8) and (A9) can be solved simultaneously for positive values of z_1 and z_2 is therefore establishing the behavior of the relations around 0. For (A8), $dz_2/dz_1 = 1$ at zero. For (A9) we have that

$$(A10)\qquad z_2 e^{-z_1}/(1-e^{-(1+a)z_1}) \text{ tends to 1 as } z_1 \text{ tends to 0.}$$

Taking derivatives and applying l'Hopital's rule yields that $dz_2/dz_1 = (1+a)$ at $z_1 = 0$. This in turns implies that the two lines cross and that a solution to the system exists.

Given that a solution exists, it is straightforward to show that this is an equilibrium. Again, we consider first deviations by workers given the wage distribution and then deviations by firms.

Equation (8) ensures that type 1 workers apply only to firms paying w_1. Equation (9) ensures that type 2 workers apply only to firms paying w_2. If type 3 workers applied to firms paying w_2, a deviation by firms which attracted only type 3 workers would make more profit than w_2. Therefore, if no such deviation exists (a point to which we return), it cannot be the case that type 3s apply to firms offering w_2.

Now, let us consider deviations by firms.

Type-1 only deviation: A sufficiently high wage would attract only type 1 workers. We know that the most profitable such offer made zero profit when the expected wage for type 1s was k_1. In the present equilibrium, the expected wage must exceed k_1. Since expected profits are declining in k_1, no such offer can be profitable.

Multiple-type deviations: Wages in a range not too in excess of w_1 and not too below w_2 will attract more than one type of worker. The arguments that these deviations are not profitable are identical to those in the proof of theorem 1.

Type-3 only deviation: By continuity, if the ratio of type 3s to type 1s is not too large, the argument from the previous theorem that no deviation designed to attract only type 3s will be profitable is still applicable.

The arguments about firm entry are identical to those in the previous theorem.

QED

Proof of theorem 3:

We begin by showing that for "a" sufficiently high, if the equilibrium described in claim 2 were in place, a low-wage offer would be profitable, and therefore, that an equilibrium of the type described in theorem 3 exists for "a" sufficiently high. Since (8)-(10) cannot have a solution with z_1 equal to zero, as "a" gets large, the expected number of applicants to jobs offering w_1 gets large and the probability that a type 3 worker gets a job tends towards zero. Hence their expected wage equals zero, and a low-wage offer would attract sufficient applicants. Given that a low-wage offer is profitable, the proof that no workers or firms have an incentive to deviate parallels those of theorems 1 and 2.

To prove that this is an equilibrium, we consider deviations by workers given the wage distribution. The arguments concerning deviations by firms parallel those used in the previous two proofs and are not presented here.

Equations (8) and (12) ensure that type 1 workers apply only to firms paying w_1. Equations (9) and (12) ensure that type 2 workers apply only to firms paying w_2. If type 3 workers applied to firms paying w_2, from lemma 1 firms which attracted only type 3 workers would make more profit than those offering w_2. Therefore, it cannot be the case that type 3s apply to firms offering w_2. Equations (9) and (12) ensure that type 3 workers will apply to both offering w_1 and to firms offering w_3.

QED