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ABSTRACT

We study a growth model with an environmental asset which is a source of utility and an input to consumption and production. The stock of this asset follows its own ecological dynamics, which are affected by economic activity.

We study the implications of an approach to ranking sequences of consumption and environment over time that place weight both on the characteristics of the sequence over any finite period and on its very long run or limiting characteristics. Chichilnisky [5] has called these "sustainable preferences". The criterion shows more intertemporal symmetry than the discounted utilitarian approach, which clearly emphasizes the immediate future at the expense of the long run. In this respect Chichilnisky's criterion captures some of the concerns of those who argue for sustainability and for a heightened sense of responsibility to the future. To characterize optimal paths we define the "green golden rule", the path which maximizes long-run sustainable utility from consumption and environment.

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1 Introduction

1.1 The Concept of "Sustainability"

The concept of "sustainable growth" is an appealing but elusive one. There are two factors common to most discussions and definitions of sustainability. One is the need to follow development strategies consistent with the planet's endowments. A clear statement of this can be found in the Bariloche model ([17], see also Chichilnisky [3]), whose authors remarked:

> "The underdeveloped countries cannot advance by retracing the steps followed in the past by the now developed countries: ... not only because of the historic improbability of such a retracing being feasible, ..., but ... because it is not desirable. It would imply repeating those errors that have lead to the current situation of .. deterioration of the environment .... The solution ... must be based on the creation of a society that it intrinsically compatible with its environment."

In economic terms, following development strategies consistent with the planet's endowments means recognizing the importance of resource and environmental constraints and the limitations that they may impose on growth patterns.

A second common factor is an emphasis on equity, both within and between generations. The authors of the Bariloche model emphasized this through the introduction and analysis of the concept of the satisfaction of basic needs as a development priority. The Brundtland Report's [37] reference to sustainable development includes an emphasis on needs and on intergenerational equity:

> "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs."

Both elements in the analysis of sustainability, resource and environment constraints and equity, have been analyzed previously. Daley [8], Forester and Meadows [25], the Bariloche model [17], Chichilnisky [3], Hotelling very much earlier [18], all study the importance of resource constraints. This literature is summarized in Heal [16] and [15]. The issue of intergenerational equity was addressed inter alia by Ramsey [29], Solow [31] and others: this literature is summarized in Dasgupta and Heal [10].

1.2 Equity between Generations

It is probably fair to say that of these two factors, resource constraints and intergenerational equity, the issue of intergenerational equity is the one that is more inadequately treated in the existing literature. This literature is largely built around the discounted utilitarian approach to defining an optimal path of resource use, and many authors have expressed reservations about extent to which this approach strikes an appropriate balance between present and future. Cline [7] and Broome [2] have argued for the use of a zero discount rate in the context of global warming, and Ramsey and Harrod were scathing about the ethical dimensions of this approach in a more general context (see [16]). Heal [16] has argued that a zero consumption discount rate can in fact be consistent with a positive utility discount rate in the context of environmental projects. It may be a fair summary to say that discounted utilitarianism dominates our approach to the subject more for lack of convincing alternatives than because of the conviction that it inspires.

A positive discount rate forces a fundamental asymmetry between present and future generations, particularly those of the future who are very far into the future. This asymmetry is troubling to many who are concerned with environmental matters such as climate change, species extinction and disposal of nuclear waste, as many of the consequence of these may be felt only in the very long run indeed. At any positive discount rate these consequences will clearly not loom large (or even at all) in project evaluations. In this context, one could point out that at a positive discount rate no-one would ever have built a Romanesque or gothic cathedral: however, a strict utilitarian would perhaps not be displeased with this outcome. To pursue this point further, if one discounts present world GNP over two hundred years at 5% per annum, it is worth only a few hundred thousand dollars, the price of a good apartment. On the basis of such valuations, it is clearly irrational to be concerned about global warming, nuclear waste, species extinction, and other long-run phenomena. Yet we are worried about these issues, and are actively considering devoting very substantial resources to them. There appears to be a part of our concern about the future that is not captured by discounted utilitarianism. Perhaps as much as anything it is this that is driving an interest in formalizing the concept of sustainability. The emphasis on sustainability is calling for a more temporally symmetric approach to intertemporal welfare economics, one in which the interests of generations in the very far distant future are not annihilated by discounting.

1.3 Valuing the Long Run: the Chichilnisky Criterion

Here we take a new approach to this issue, using a welfare criterion developed in Chichilnisky [5], and explore the implications of an alternative formulation of intertemporal welfare criteria which places positive weight on the very long run properties of a growth path. Technically speaking, it places positive weight on the limiting properties of a path. We feel that this formulation is in tune with the concerns of those who write about sustainability and our responsibilities to future generations.

This approach builds on the observation that selecting an objective function in intertemporal planning involves solving a social choice problem. This is the problem of deciding how to combine the preferences of different generations. The distinctive feature of this social choice problem is that there are infinitely many generations or potential "voters". It is a simple mathematical fact that in general you cannot in summing give equal weight to each of an infinite sequence of numbers and expect to derive a well-behaved function of these. To be sure of a well-behaved outcome, it is customary to give more weight to the near ones than to the far ones - i.e., to discount (see Heal [14]).

In a companion paper [5] Chichilnisky presents a set of axioms for intergenerational social choice which imply that positive weight be placed on the limiting properties of alternative utility streams, as well as on their properties over finite horizons. Her approach builds on her earlier axiomatization of the social choice problem [4] (combining continuity, equal treatment and respect of unanimity) with its application to infinite populations by Lauwers [21] and Lauwers and van Liederkierke [22]. Chichilnisky introduces two axioms which underlie our approach: these are that neither "the present" nor "the future" should be dictatorial. Nondictatorship of the present means that it should not be possible to determine the ranking of any two utility streams by looking only at finite numbers of their components. Nondictatorship of the future means that the ranking of two utility streams should not depend only on their limiting properties, but must be sensitive to their characteristics over finite horizons. These axioms suffice to characterize the valuation of utility streams as the sum of two terms, one that is a discounted integral of utilities and one that depends on the limiting properties of the stream. Technically this result builds on the Yosida-Hewitt theorem [39], which states that a continuous linear functional on a Banach space can be represented as the sum of an integral against a countably additive measure and an integral against a purely finitely additive measure[1].

1.4 Earlier Literature

Our approach is fully consistent with earlier discussions of sustainability, but is more formal and so lends itself to mathematical analysis and the derivation of precise, testable and implementable conclusions.

The basic idea of the analysis in the Bariloche model, echoed in the Brutland Report, was that development is necessary to improve the living conditions of those who are disadvantaged today, but that its environmental and resource impact has to be managed in such a way as not to damage the future generations. A similar perspective recurs in Tietenberg ([34]), according to whom "...the sustainability criterion suggests that future generations should be left no worse off than current generations", and also in Solow ([32]), who defines sustainability as "an obligation to conduct ourselves so that we leave to the future the option or the capacity to be as well off as we are" (see also Solow [33]).

Other definitions are more specific, and refer to targets for clearly measurable physical objects. Pearce et al. ([27]) suggest more precisely that "sustainability" is to be measured in terms of constancy of capital stocks. They define sustainable development by the requirement that a vector of desirable social objectives like increases in real income per-capita, improvements in health and nutritional status, etc. does not decrease over time. The key necessary condition for achieving such a goal is, in their opinion, constancy of the natural capital stock. Hammond [12], in an interesting review of the concept, traces stationarity of capital stocks as an indicator of sustainability or its precursors back to the works of Daly [8] and others in the 1970s, and relates sustainability to the Hicksian concept of income. Weitzman ([35]) relates Hicksian income to the solution of a dynamic maximization problem, and Maler ([23]) extends the framework to a model with environmental resources, defining a sustainable path as one that is compatible with a non-decreasing level of utility.

But constancy of the natural capital stock is not a necessary requirement for increasing welfare, and is not to be confused with the necessity of an infinitely-lived agent to maintain her stock of wealth constant. A neoclassical growth model may give rise to depletion of natural capital which is replaced by physical capital along a path that maximizes the utility of the decision-maker. This gives a counterexample to definitions that require constancy of natural capital. In fact Pearce et al. mention irreversibilities and low substitutability as the main factors that motivate their requirement of constancy of natural capital, while Nordhaus ([26]) is critical about the practical importance of such elements for the definition of sustainability.

1.5 Goals and Conclusions

Our application of the Chichilnisky criterion in an optimal growth model with resource and environmental constraints leads naturally to the introduction and analysis of the "Green Golden Rule", an extension to the environmental field of the Meade-Phelps-Robinson concept of the Golden Rule of Economic Growth (see Phelps [28]). Such a connections should not be surprising: Phelps described the golden rule as the growth path that gives the highest indefinitely maintainable level of consumption per head. Clearly there is an implicit concept of sustainability here: the Golden Rule path is the best sustainable path. Our Green Golden Rule gives the highest indefinitely maintainable level of instantaneous utility, in a framework where environmental goods are valued in their own rights, i.e., are a source of utility, and are used as inputs to the productive process. It is a generalization of the earlier concept. It is an easily-defined and operational concept which is an essential element in the task of making operational the concept of a sustainable optimal path. Generally, our goal is to make the concept of sustainability precise so that it can be operationalized in the manner in which the discounted utilitarian solution has so successfully been operationalized. This means inter alia deriving the implications of sustainable optimality for associated shadow prices and embodying these implications in rules for project evaluation.

2 A Model of Growth with Environment

The economic model with which we work is an extension of Dasgupta and Heal [9]. We extend that model by adding a regeneration process for the natural resource, so that it becomes renewable rather than exhaustible: we take the renewable resource to be an environmental resource, such as a rain forest, the climate, species diversity, etc. This resource serves as a source of utility, as an input to production, and may serve as a complement to consumption. We include the stock of the resource as an argument of the utility function (as in Krautkraemer [20]), and as an argument of the production function.

The significance of these extensions is as follows. The presence of a *renewable* resource means that it is possible in principle for a positive stock of the resource to be maintained indefinitely. The fact that, along with a good produced from capital and the resource, the resource is an argument of the utility function, captures the concern that environmental resources are an important determinant of the quality of life, and that our long-term successors may ultimately be deprived of this. The model also allows us to examine a trade-off between consumption of the produced good (and indirectly use of the resource) and long-run valuation of the resource stock. Higher long-run consumption of the produced good implies lower long-run levels of the resource stock.

The basic framework with which we work is the following. The social valuation of the state of the economy at time t depends on the level of consumption of a produced good and on the existing stock of an environmental good. Formally,

Assumption 1 *Instantaneous utility is given by the strictly concave continuously differentiable real valued function* $U(C_t, A_t)$ *defined on consumption* $C_t \in \Re$ *and on the stock of an environmental good* $A_t \in \Re$. *We also assume, without any loss of generality, that* $U(A, C)$ *is bounded above. We assume that* $\frac{\partial U}{\partial C} > 0$ *always:* U *may show satiation in* A *so that* $\frac{\partial U}{\partial A}$ *may have either sign*[2].

Assumption 2 *Production of the single produced good occurs according to the linear* *homogeneous production function $F(K_t, A_t)$ where K_t is the stock of produced capital at time t. Capital accumulation is therefore described by the usual equation:*

$$\dot{K}_t = F(K_t, A_t) - C_t \tag{1}$$

Note that we are specifically assuming the production process to depend on the sock rather than the flow of the environmental resource. We are deliberately not considering the optimal use of an exhaustible resource such as oil or iron ore, for which it is the flow not the stock that matters: these cases have been adequately considered elsewhere (Dasgupta and Heal [10], Krautkraemer [20]). We are interested in environmental goods whose stocks have value both in themselves and as inputs to production: examples, as already mentioned, are climate, species diversity, etc.

Assumption 3 *The stock of the environmental good has the ability to renew itself: the rate of renewal is given by the function $R(A)$, satisfying $R(0) = 0$. However, the act of consuming output may deplete the environment, so that the net rate of change of the stock of the environment is*

$$\dot{A}_t = -\alpha C_t + R(A_t),\ \alpha \geq 0 \tag{2}$$

We assume that the renewal function R is bounded above (i.e., $\exists B : R(A) \leq B$ $\forall A$). R may exhibit a threshold effect, i.e., $\exists H : R(A) = 0\ \forall A \leq H$ and $R(A)$ is strictly concave for $A \geq H$. It is possible that above a certain level of A, $R(A)$ may be *decreasing, i.e.,* $R'(A) < 0$ *for* $A > A_m$. *This is always the case for the most commonly used reproduction function, the Pearl-Verhulst logistic mode (see Dasgupta and Heal [10] and Wilen [36]). In addition it is assumed that the set of attainable values of* A *is bounded above, so that there is a limit to the amount of the environmental resource that can be accumulated. Such a situation is depicted in figure 1.*

In addition, certain initial conditions and non-negativity constraints are imposed:

$$K_0 = \bar{K},\ A_0 = \bar{A},\ K_t \geq 0\ , A_t \geq 0,\ C_t \geq 0 \tag{3}$$

Agents derive utility both from produced goods and from a reproducible environmental good. Because of (2) and the boundedness of $R(A)$, the depletion of the environment may exceed the environment's capacity to regenerate itself. It is possible to attain consumption levels that are not compatible with indefinite preservation of a positive stock of the environmental good.

Note from equations (1) and (2) and from the assumption about $R(0)$ that when the stock of environment is zero there may still be a positive production level if the environment is not essential (in the sense of Dasgupta and Heal [9]) for production. The possibility of running down the stock of capital completely cannot be excluded *a priori* if capital is not essential: it might be optimal to deplete the capital stock gradually and produce from resources only by means of the function $F(0, A)$.

The Chichilnisky criterion for evaluating alternative growth paths [5], referred to in the previous section, depends both on the sum of utilities over time and on the long-run behavior of utility values. It has the form:

$$\theta \int_0^\infty U(C_t, A_t)\mu(t)dt + (1-\theta)\lambda(\overline{U}),\ 0 \leq \theta \leq 1 \tag{4}$$

where $\mu : \Re \rightarrow \Re$ is a measurable function with $\int_0^\infty \mu(t) < \infty$. Such a function is a countably additive measure[3]. $\overline{U}$ represents the entire consumption path $U_{t,t\in(0,\infty)}$, and λ is a singular or purely finitely additive measure. A singular or purely finitely additive measure is a function of a sequence that depends only on the limiting properties of that sequence. Here without great loss of generality we take it that

$$\lambda(\overline{U}) = \lim_{T\rightarrow\infty} \inf_{t\geq T} U(C_t, A_t) \tag{5}$$

The lim inf of an infinite sequence of numbers, is the largest number such that only a finite number of elements of the sequence are less than it. The measure $\mu(t)$ is assumed to have the form $\mu(t) = e^{-\delta t}$, so that the criterion function as applied here takes the form

$$\theta \int_0^\infty U(C_t, A_t)e^{-\delta t}dt + (1-\theta) \lim_{T\rightarrow\infty} \inf_{t\geq T} U(C_t, A_t) \tag{6}$$

A ranking of this form is referred to below as a "sustainable preference". In effect what we are doing here is supplementing the conventional discounted utilitarian criterion with a term that depends only on the very long run behavior of utility sequences. The value of the term $\lim_{T\rightarrow\infty} \inf_{t\geq T} U(C_t, A_t)$ is not affected by changes in the values of C_t or A_t for any finite t. This term only depends on the very long run or limiting behavior of utility values. The use in the criterion function of terms such as lim inf, lim and long run average which depend on the very long run behavior of the instantaneous payoff function is common in dynamic programming and dynamic games (see Dutta [11] and Kannai [19]). These are conventional elements of an intertemporal criterion function in situations where the very long run matters. Returning to the perspective given by the sustainability debate, Chichilnisky's axioms allow us to capture a concern for sustainability - the capacity to generate welfare in the very long run, for our distant successors - by including in the maximand (6) a term commonly used for valuing long-run characteristics of payoff sequences in game theory and dynamic programming.

The overall optimization problem that we study is the maximization of (6) subject to the constraints on capital accumulation (1), resource renewal (2) and to initial conditions (3). Our approach to solving this problem is to note that it is solvable by conventional methods in the extreme cases of $\theta = 1$ (pure discounted utilitarianism) and $\theta = 0$ (maximizing the long run value of utility), and then base a general argument on the solution in these two cases. First we consider the pure discounted utilitarian case in which $\theta = 1$.

3 The Discounted Utilitarian Solution

Setting $\theta = 1$, our problem is

$$Max \int_0^\infty U(C_t, A_t)e^{-\delta t}dt, \ \delta > 0 \tag{7}$$

subject to (1), (2) and (3). The Hamiltonian that can generate the necessary conditions is the following:

$$H = e^{-\delta t}U(C,A) + pe^{-\delta t}[F(K,A) - C] + qe^{-\delta t}[-\alpha C + R(A)] \tag{8}$$

The first-order conditions are:

$$U_C = p + q\alpha \tag{9}$$

$$\dot{p} - \delta p = -pF_K \tag{10}$$

$$\dot{q} - \delta q = -U_A - pF_A - qR_A \tag{11}$$

$$lim_{t\rightarrow\infty} e^{-\delta t} pK = 0 \tag{12}$$

$$lim_{t\rightarrow\infty} e^{-\delta t} qA = 0 \tag{13}$$

To write the necessary conditions in a compact way which allows us to keep track of dependence of the utility and production functions on various variables we define $z \equiv K/A$ $F(K,A) = Af(z)$. A stationary solution to the necessary conditions (9) to (13) above must satisfy the following:

$$U_C(C,A) = p + q\alpha \tag{14}$$

$$\delta = F_K(z) \tag{15}$$

$$\delta q = U_A(C,A) + pF_A(z) + qR_A(A) \tag{16}$$

$$F(K,A) = C \tag{17}$$

$$\alpha C = R(A) \tag{18}$$

According to equation (14) the marginal utility of stationary consumption has to be equal to a linear combination of the prices of the two stocks, in order to take into account the fact that consuming prevents capital accumulation and depletes the environment. According to equation (15) the capital-environment ratio in production has to make the marginal productivity of capital equal to the rate of time preference. Equation (16) yields the shadow price of the environment as a present discounted value of marginal utility and marginal productivity. (18) shows that in a stationary state the consumption of goods must be proportional to the regenerative capacity of the environment.

These conditions point to the following proposition.

Proposition 1 *If $R(A)$, the regeneration function for the environmental resource, has a threshold and is strictly concave above this (as in figure 1), then there may be zero, one or two stationary solutions with a positive level of A. These are characterized as solutions to:*

$$\alpha A f[f^{-1}(\delta)] = R(A).$$

Proof. A stationary solution can be shown to exist in the following way.

Given $A = A'$, (18) allows one to choose $C'=\alpha^{-1}R(A')$. Given A' and C', (17) then determines the K' compatible with the production possibility set. So by this route we have for each value of A associated values of C and K.

In addition, (15) determines z, call it z^*, as a function of the pure rate of time preference, $\delta = F_K(K, A) = Af'(z^*)$. Given z^*, we have a second relationship between K and A. By setting $C = F(K, A) = Af(z^*)$ from (17) we have another relationship between C and A parameterized by δ. There will be a stationary solution if these two relationships are consistent.

The consistency or otherwise of these relationships is explored in figure 1, where we plot the relationship between C and A given by $C = F(K, A) = Af(z^*)$ and that given by $C = \alpha^{-1}R(A)$. When $R(A)$ is strictly concave there may be no intersection or one intersection; when $R(A)$ exhibits a threshold there may be zero, one or two intersections. This completes the proof. □

Note that $f(z) = F(K, A)/A$ is output per unit of the resource stock. If this is bounded above, as it is in the case of F a CES function with elasticity of substitution at or near zero, then the slope of the line $C = Af(z)$ in figure 1 is bounded independently of the value of the discount rate δ. If this bound is sufficiently low, then there will always be a stationary solution with a non-zero value of the resource stock. Formally, assume that $f(z) \leq \overline{f}$ for all z. Then

Corollary 1 *(a) If $R(A)$ shows a threshold effect, then there is at least one stationary solution with a positive value of A for any positive rate $\delta > 0$ if and only if $\overline{f} \leq \max_A \frac{R(A)}{A\alpha}$. (b) Otherwise, there is at least one stationary solution with a positive value of A for any discount rate $\delta > 0$ if and only if $\overline{f} \leq \alpha^{-1} R'(0)$. (c) If $f(z)$ does not satisfy these bounds[4], then there will be a stationary solution with a positive value of A if and only if $\delta \geq \beta$ for some number $\beta > 0$.*

Proof. Parts (a) and (b) of the corollary are obvious. Part (c) follows from the fact that z^* decreases with δ. □

Note that the bound on f is inversely proportional to α, the environmental impact coefficient of consumption: the lower is this impact, the higher is the permissible value of f. The corollary also shows the importance of the rate of time preference for the existence of stationary solutions: if $f(z)$ is not bounded, or not bounded tightly enough, then this must be "high enough". This might seem at first a surprising prediction: to have an equilibrium with a high level of environmental preservation one may need a high rate of time preference. The intuition is that a high δ implies

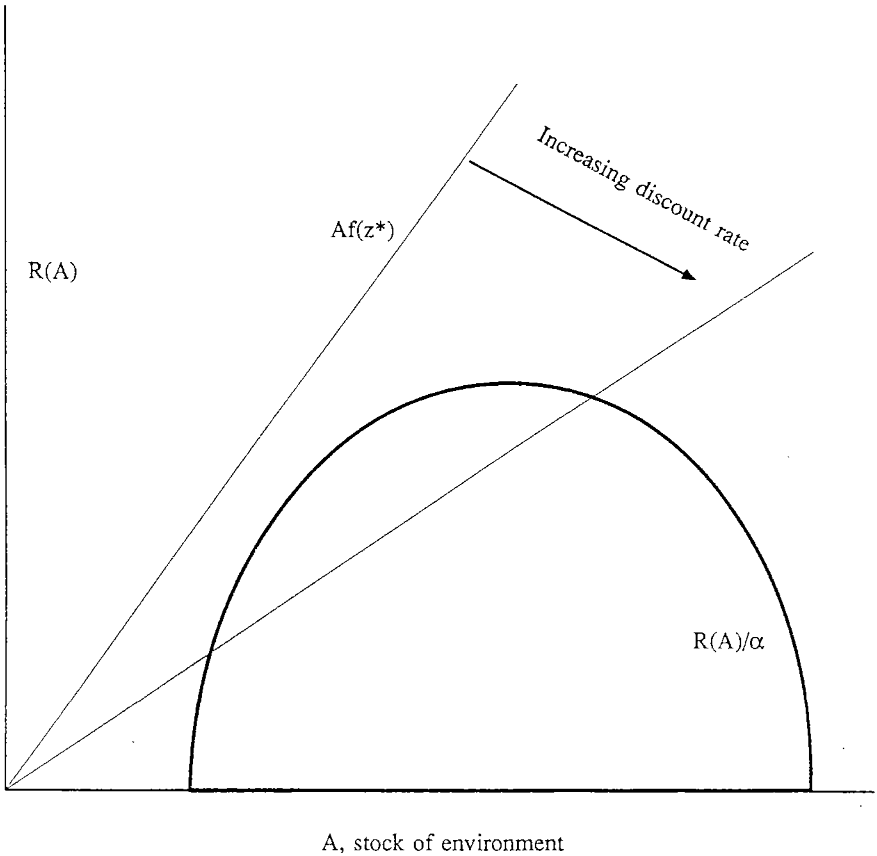

Figure 1: stationary states of the utilitarian solution are intersections of the line and the curve.

a desire for immediate consumption and a low level of capital. Given the fact that A enters the production function and given the relation between the regenerative capacity of the environment and the level of consumption it is then necessary to have a high A to be able to produce and consume. Another interpretation of this result, is to note that a high value of the discount rate δ leads to a low value of the capital-environment ratio $\frac{K}{A}$ in the long run by equation (15).

To understand the dynamics of an optimal utilitarian path, we need not only to know whether stationary solutions to the first order conditions exist, but whether they are dynamically approachable in the sense that from any initial conditions there are paths leading to a stationary. Because we are working with a system of four simultaneous differential equations, this is a complex mathematical issue. We address it in detail in [1], where we show that a stationary solution of the first order conditions is always locally approachable if $R'(A) < 0$ at that solution, provided that α and F_A satisfy certain upper bounds. In figure 1, this result implies that the right hand of the two stationary solutions will be locally approachable under the appropriate conditions.

4 The Green Golden Rule

Now we consider the case in which in (6) $\theta = 0$. In this case society is only concerned with the very long run values of consumption and environment. We seek a path of consumption and capital accumulation that maximizes $\lim_{T \to \infty} \inf_{t \geq T} U(C_t, A_t)$ over the

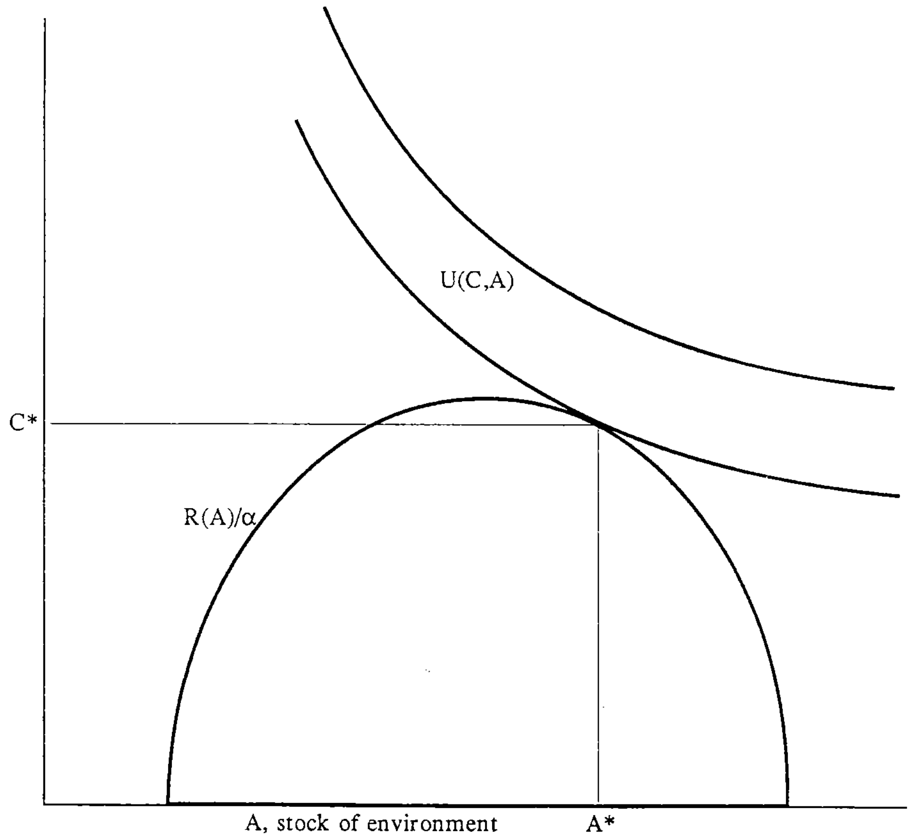

Figure 2: the path which maximizes lim inf u(C,A) approaches (A*,C*), the point giving the highest sustainable utility level.

set of feasible paths. The solution of the problem therefore requires that we find the indefinitely maintainable values of C and A which give the maximum utility level over all such levels. As indefinitely maintainable values of C and A satisfy $R(A) = \alpha C$, this means that the problem

$$\max \lim_{T\to\infty} \inf_{t\geq T} U(C_t, A_t) \text{ over feasible paths satisfying (1) to (3)}$$

reduces to:

$$\max\ U(C, A)\ \textit{subject to}\ R(A) = \alpha C \tag{19}$$

The stock of capital is not a concern in this situation because any stock of capital can be accumulated given a sufficiently long period of time. The set of $\{C, A\}$ pairs satisfying the constraint in (19) is compact, so this problem is well-defined. The maximum is characterized by the first order condition:

$$\frac{U_A}{U_C} = -\frac{R_A}{\alpha} \tag{20}$$

The solution given in the proposition amounts to equality between the marginal rate of transformation and the marginal rate of substitution between consumption and environment across steady states. It can be depicted in graphical terms as the point of tangency between the indifference curve and the renewal function (see figure 2). It is clear that the solution to the problem of maximizing the lim inf of utility values does not define a growth path for the economy: it merely defines a long-run or limiting configuration. There are many paths that will lead to this, some efficient, other inefficient. Amongst the efficient paths, some will give higher values of the integral of discounted utilities than others.

We term the configuration defined by (20) the "green golden rule", in reference to its relationship to the original Meade-Phelps-Robinson "golden rule of economic growth" [24], [28], [30]. That rule characterized the greatest indefinitely-maintainable consumption level: the green golden rule (20) characterizes the highest indefinitely-maintainable utility level. In a one-good growth model with increasing preferences the two concepts coincide: with an argument in addition to consumption in the utility function, they do not. The first order condition (20) gives the optimal trade-off between consumption and the environment. In the present model, the green golden rule not only solves the problem of maximizing the lim inf of the utility values along a plan: it also gives a maximum of any of lim, lim sup, or any increasing function of the limiting utility value.

It is worth pointing out the formal difference between the lim inf criterion and the Rawlsian criterion, which involves the maximization of the inf rather of the lim inf. This difference has a large impact on the results obtainable from the model. The Rawlsian criterion leads to unappealing solutions: in the context of a model with no natural resources Solow and others have shown that the Rawlsian criterion may require a society to remain at the initial configuration, without any accumulation of capital.

5 Optimal Sustainable Paths

The solution to the problem of maximizing the lower bound to the long-run utility level may or may not coincide with the solution to the discounted utilitarian formulation of the problem. In general the two will not coincide, unless the rate of time preference is such that the intersection between the line and the renewal function that was referred to in the previous propositions coincides with the point of tangency between the indifference curves and the renewal function. The example given in the next section illustrates such a coincidence.

We therefore have to analyze the solution implied by the use of the "sustainable preferences" introduced in section 2, i.e. we have to solve the problem:

$$\begin{gathered} \max\ \theta \int_0^\infty U(C_t, A_t)e^{-\delta t}dt + (1-\theta) \lim_{T\to\infty} \inf_{t\geq T} U(C_t, A_t) \\ \textit{subject to } (1) \textit{ to } (3) \end{gathered} \tag{21}$$

A solution to this problem will be referred to as a "sustainable optimal path". We have so far solved this for the special cases in which $\theta = 0$ and $\theta = 1$: it remains to put these together. This is a mathematically complex task, which is analyzed in its entirety in [1]. Here we note only that any path which is a solution to the overall problem must satisfy the conditions necessary of a utilitarian optimum, namely equations (9) through (13). This follows from a straightforward argument which says that unless these conditions hold, it will be possible to make a small perturbation about the proposed path over a finite interval of time, leave the path unchanged outside of this interval, and raise the value of the integral of discounted utilities. As the path has been left unchanged outside a finite interval, the second term in the maximand has not been altered, and of course the first has been increased. This tells us how a solution to the overall problem behaves locally: what remains is to establish its asymptotic behavior. This more complex task is carried out in [1]. The existence of an optimal sustainable path can be assured by mild conditions on the optimal time path of consumption c_t: this issue is also reviewed in [1].

6 An Example of a Sustainable Optimal Path

To give precision to the ideas we have introduced in the earlier sections of the paper, we illustrate them here by applying them to a simplified model in which some of the richness of the full model is lost, but which has the advantage of being readily analytically solvable. We treat the environmental resource as the only good, which may be consumed or allowed to reproduce. This formulation retains the conflict between current consumption and long-run maintenance, but gives us a model which has only one state variable. Its dynamics can therefore be studied graphically. The overall optimization problem can now be stated as:

$$Max\ \theta \int_0^\infty U(C_t, A_t)e^{-\delta t}dt + (1-\theta) \lim_{T\to\infty} \inf_{t\geq T} U(C_t, A_t)$$

subject to

$$\dot{A}_t = -C_t + R(A_t)$$

To be even more specific, we assume

$$U(C_t, A_t) = \ln C + \gamma \ln A$$

$$R(A) = rA - \frac{rA^2}{A^s}$$

This reproduction function is logistic with A^s the carrying capacity of the environment. In this case the Hamiltonian of the system is:

$$H = [\ln C + \gamma \ln A] + q[-C + rA - \frac{r}{A^S}A^2]$$

The first-order conditions for a utilitarian optimum are now:

$$\frac{1}{C} = q$$

$$\dot{q} - \delta q = -\frac{\gamma}{A} - q(r - \frac{2qr}{A^S}A)$$

$$\dot{A} = -C + rA - \frac{r}{A^S}A^2$$

$$lim_{t\to\infty} e^{-\delta t} qA = 0$$

The steady state of the two variables implied by these necessary conditions are:

$$A = \frac{A^S(\gamma r - \delta + r)}{2r + \gamma r}$$

$$C = \left[\frac{A^S(\gamma r - \delta + r)}{r(2+\gamma)}\right]\left[\frac{r+\delta}{2+\gamma}\right]$$

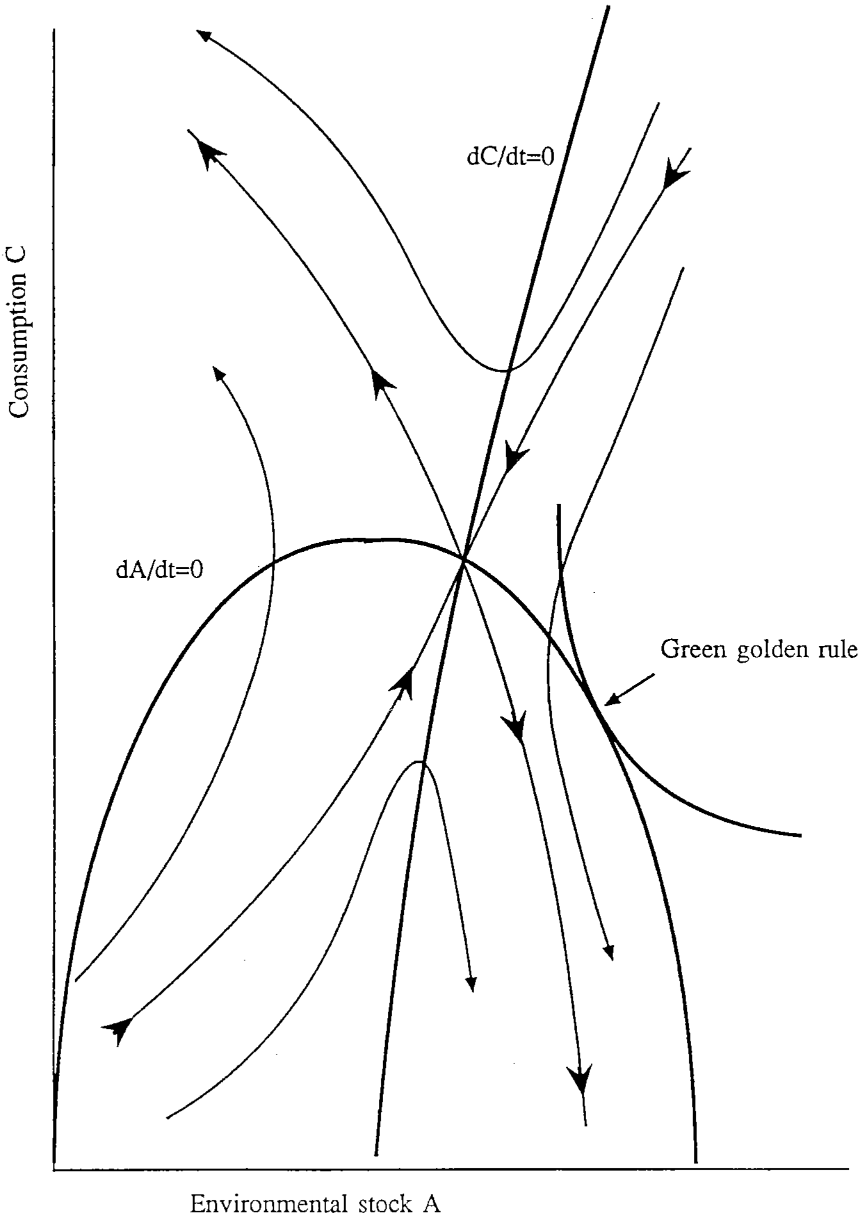

Figure 3: dynamics of the utilitarian solution
for the simple case of one state variable A

When $\gamma = 0$, corresponding to the case of no utility of the environment, we have that:

$$A = \frac{A^S}{2}(\frac{r-\delta}{r}) < \frac{A^S}{2}$$

and this means that the steady state level of environment is lower than the one the gives the maximum sustainable yield. This result resembles the one obtained in standard growth theory, in which impatience prevents society from accumulating the stock of capital that maximizes steady state consumption. It is also possible to calculate that the steady state level of the environment is larger than the one that allows maximum consumption only if:

$$\gamma > \frac{2\delta}{r}$$

The lines along which values of A and C are constant are given respectively by

$$C = R(A)$$

$$R'(A) = \delta - \frac{\gamma C}{A}$$

The characterization of the dynamics of the discounted utilitarian solution is given in figure 3, which shows that the utilitarian stationary solution is a saddle point.

We can also characterize the lim inf solution to find that in that case the steady state stock of environment is

$$A = \frac{rA^s + \gamma r}{2rA^s + \gamma r}$$

This is in general larger than the value at the utilitarian stationary solution, although the two stationary states are equal if the discount rate is zero. Maximizing an undiscounted sum of utilities also maximizes the long-run utility level.

7 Conclusions

We have developed the implications of Chichilnisky's axiomatization [5] of the ranking of intertemporal utility sequences, an axiomatization that places weight both on the characteristics of the sequence over any finite period and its very long run or limiting characteristics. The criterion shows more intertemporal symmetry than the discounted utilitarian approach, which clearly emphasizes the immediate future at the expense of the long run. In this respect our criterion captures some of the concerns of those who argue for sustainability and for a heightened sense of responsibility to the future.

The characterization of optimal paths that emerges from this criterion is eminently intuitive. Their long-run characteristics are a mixture of utilitarianism and the green golden rule: locally, they always satisfy the utilitarian first-order conditions familiar from optimal growth theory.

Our objective function (6) has some point of contact with the Rawlsian approach to optimal resource use. The key distinction is that the Rawlsian approach ranks paths only by the lowest of their utility values: we replace the lowest value of utility by the limiting lowest value (lim inf) and supplement this by the discounted integral of utilities along the path. This has two great advantages. One is that it avoids trapping an economy into low consumption levels because it has poor initial endowments (see Dasgupta and Heal [10] and Solow [31]). The other is that it ensures that any solution path is dynamically locally optimal because the path satisfies the local optimality conditions (see Heal [13]).

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Notes

[1] A measure is countably additive if the measure of a countable family of disjoint sets is the sum of their measures. For a purely finitely additive measure, this property holds only for finite families of disjoint sets. In the theory of general equilibrium with infinitely many commodities, trouble is taken to ensure that only countably additive measures occur naturally - see Chichilnisky and Heal [6].

[2] One can conceive of having too much rainforest.

[3] For definitions see Yosida [38]. Most measures used by economists are countably additive.

[4] That is, if it is not bounded or if the bounds are too great.