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DOES THE HUMAN CAPITAL/EDUCATIONAL SORTING DEBATE MATTER FOR DEVELOPMENT POLICY?

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ABSTRACT

If education increases human capital, subsidizing education can generate economic growth and combat poverty. Estimates of its return suggest that education is a good social investment. In sorting models, the return reflects in part the information about productivity revealed by the worker's education. Thus the social and private returns diverge. It might appear that if we believe the sorting model, we should be less swayed by evidence that estimated returns to education exceed the social discount rate, and therefore less likely to support education-based development policies. This conclusion is shown to be incorrect.

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1. INTRODUCTION

The argument that subsidizing education is desirable implicitly assumes some form of market imperfection since, in the absence of an imperfection, each individual would choose the optimal level of education, and the market outcome would be Pareto efficient. Comparing estimates of the return to education with the social discount rate generates a plausible case for subsidizing education. Estimated returns to education are typically on the order of 8% in the United States and are generally higher in developing countries. Thus if the estimated returns to education are reasonably accurate measures of the social returns, education seems to be a good social investment.

A problem arises in sorting models of education because the return to education reflects not only the effect of education on productivity but also that higher levels of education reveal the worker to be innately more productive. Thus the social return, which is only the effect of education on productivity, and the private return, which includes the return to signalling innate ability, diverge.

It might appear that if we believe the sorting model, we should be less swayed by evidence that the estimated returns to education exceed the social discount rate, and therefore less likely to support development policies on the basis of this evidence.[1] This conclusion is incorrect. If anything the opposite is true. The next section presents the argument informally. Some

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[1]Stiglitz (1975) shows that if education helps to match workers to the right jobs, the social return to education can exceed the private return even when education is a pure signal of ability. In this paper I assume workers are perfect substitutes up to a scale factor. Therefore the discussion restricts itself to the case where the private return exceeds the social return in the presence of informational imperfections which generates a reasonable expectation that there will be excess investment in education.

examples, using a model which embeds perfect information as a special case, are provided in section 3.

2. AN INFORMAL ARGUMENT

The critical assumption underlying the argument in the remainder of this paper is that the distinguishing characteristic of a sorting model is that knowing an individual's education provides employers with information about that individual's productivity which would be unknown otherwise. In human capital models, education is not informative, because employers observe productivity directly.

Thus, in the following argument, human capital and sorting are not distinguished by assumptions about the human capital production function. Instead the models are distinguished by the role of education in conveying information about individual productivity. In both models, units of human capital will be generated in the same way from inputs of innate ability and schooling.

Although theoretical work on sorting models sometimes uses the simplifying assumption that education is completely unproductive, even the earliest work on these models (Spence, 1974) made it clear that they were robust to allowing education to be productive. It is also possible to develop sorting models in which productivity is at least imperfectly observed (Weiss, 1983). The third section of this paper develops a model in which productivity is observed with error and in which education is productive.

To see why the measured return to schooling may more nearly approximate the social return when information is imperfect, consider the measured relation between wages and education. Let q be the productivity (value of marginal product) of an individual with education (s) and innate ability (i) and assume that workers are paid their value of marginal product. In equilibrium, there will be some relation between levels of education and innate ability so that we can write the human capital production function as

$$(1) \qquad q = q(s, i(s)).$$

Note that the relation between i and s is the outcome of some equilibrium and does not mean that schooling affects innate ability which is assumed to be predetermined. Instead ability and education are related in equilibrium because ability affects the net benefit of investing in education.

Assuming that i is not observed by the econometrician,[2] the measured return to education is given by

$$(2) \qquad dq/ds = q_s + q_i di/ds$$

where subscripts denote partial derivatives.

Equation (2) does not depend on whether the relation between i and s is generated by sorting or by human capital or some combination of the two. Therefore in both models, the measured return to education, dq/ds, differs from the social return q_s by a factor which can be interpreted as ability bias. The fact that in the human capital model this ability bias arises solely because the econometrician does not observe ability while in the sorting model it arises because neither the econometrician nor the employer observes ability is irrelevant. The only question is in which model ability bias is likely to be greater.

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[2]Adding measured ability would not change the story providing that there is no ability which the econometrician can measure but the employer cannot.

This question cannot be answered without reference to specific sorting models. Nevertheless, it is a common (although not universal) property of sorting models that they spread out the amount of education people get relative to what would occur in the presence of perfect information.[3] In other words, in general ds/di is greater in the presence of sorting considerations. Thus di/ds is smaller. Consequently, <u>we would generally expect the measured return to education to approximate the social return more nearly when sorting considerations are more important</u>. Thus if support for subsidizing education follows from comparing the measured return to education with the social discount rate, belief in the importance of informational imperfections and in the information value of education does not speak strongly against this support. If anything, the opposite is true. Similarly, belief in the importance of sorting could increase our confidence in the usefulness of OLS estimates of the return to education for the purposes of human capital growth accounting.

In order to make this argument more rigorous, it is helpful to develop a sorting model which includes perfect information (human capital) as a limiting case. The next section develops such a model and explores the circumstances under which the measured return to education more nearly approximates the social return as informational imperfections increase.

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[3]Sufficient conditions for this appear to be that 1) in the presence of perfect information ability and schooling would be positively related, 2) that education is continuous and unbounded above and 3) the equilibrium of the sorting game is the most efficient separating equilibrium in which the lowest ability workers get the same education as they would in the presence of perfect information. The first condition is necessary for workers' indifference curves to fulfil the single—crossing condition which plays a prominent role in much of the refinements literature.

3. SORTING WITH PRODUCTIVE EDUCATION AND IMPERFECTLY OBSERVED PRODUCTIVITY

Assume that log productivity p* is equal to mean log productivity for individuals of a given education and ability level plus a normally distributed error term

$$(3) \qquad p^* = q(s,i) + e, \quad 0 < \sigma_e^2 < \infty .$$

Note that in this equation i has not been written as a function of s because productivity depends only on the individual's ability and schooling and not on the equilibrium relation between ability and schooling. It should also be noted that there are implicitly two types of ability in this model, i, which affects performance in school as well as in the market and is known to individuals before they decide how much schooling to get, and e, which does not affect performance in school. Although this does not affect the model, it is easiest to think of e as being unobserved by the worker.

Employers observe a measure of productivity p which is equal to p* plus a normally distributed error term

$$(4) \qquad p = p^* + u, \qquad 0 \le \sigma_u^2 < \infty$$

where u and e are assumed, without loss of generality, to be independent.

Assume that individuals live forever and maximize the present discounted value of their lifetime earnings. There are no direct costs of education. The only costs are foregone earnings. Workers choose their levels of education without knowledge of e or u. The signal p, as well as s, is then observed by all firms. Firms make wage offers. Workers accept the highest wage offer.[4]

I will consider three examples in order to examine how informational imperfections affect the divergence between the measured return to education and the social return. In each case, I will assume that the equilibrium is the "most efficient" separating equilibrium.[5] In the first example, this can be shown to be the only equilibrium satisfying the Cho–Kreps (1987) refinement. For the other examples, this refinement is insufficient to establish uniqueness.

The separation assumption together with the normality assumption ensures that firms will estimate productivity and pay wages according to the formula[6]

$$(5)\qquad w(s,p) = \lambda\, p + (1-\lambda)\, q + .5(1-\lambda)\sigma_e^2.$$

where

$$(6)\qquad \lambda = \sigma_e^2/(\sigma_e^2 + \sigma_u^2)$$

Then in a separating equilibrium in which s fully reveals i, λ

[4]To keep the examples simple, it is assumed that individual productivity is never revealed. For more dynamic models see Farber and Gibbons (1990) and Milgrom and Oster (1987).

[5]This is defined as the separating equilibrium in which the lowest ability group gets the level of education it would receive in full information equilibrium and in which the remaining ability groups get just sufficient education to deter those below them from copying them.

[6]To derive equation (5), recall that under the log–normality assumption $E(\exp(q+e)) = \exp(q+.5\sigma_e^2)$ which must be the expected wage under competition. The $.5(1-\lambda)\sigma_e^2$ term ensures this equality.

is a measure of how informative education is about productivity. As λ goes to 1, productivity is observed perfectly, and education provides no additional information about productivity. As it goes to zero, productivity is completely unobserved, and all information about productivity must be derived indirectly from the level of education. Thus λ equals 1 corresponds to the pure human capital model.

Example 1: Let us begin with an example in which there are two ability types, h and l and a continuum of possible education levels. This example has the advantage that the efficient separating equilibrium assumed in the example can be shown under certain conditions to be the unique (Cho—Kreps refined) equilibrium and that the reduction in bias as information becomes more imperfect can be shown quite generally. This is offset by some ambiguity about the definition of the social return to education. Since the education levels of high and low ability workers differ by a discrete amount, we will be concerned with inframarginal as well as marginal returns. Moreover, the increased productivity experienced by high ability workers as a result of their education will differ from the productivity increase low ability workers would experience if they imitated high ability workers. The example adopts the convention that the social return to schooling is the productivity increase due to schooling among those who obtain the schooling.

It is assumed that high ability workers are more productive than low ability workers at any given education level

$$(7) \qquad q(s,h) > q(s,l)$$

and the social return to education is higher for high ability types

(8) $\partial q(s,h)/\partial s > \partial q(s,l)/\partial s$

for all s. Equation (8) ensures that high ability workers always prefer more education than low ability workers in the presence of perfect information.

Finally, assume that education is continuous with no upper bound.

Theorem 1: Provided that there are no pooling equilibria which Pareto dominate the separating equilibrium, the unique equilibrium satisfying the Cho–Kreps intuitive criterion is the separating equilibrium with education level s_l for the low ability workers such that

(9) $q_s(s_l,l) = r$

and education level s_h for the high ability workers such that

(10) $s_h = \text{Max}\{s^*, s_l + [\lambda q(s_h,l)+(1-\lambda)q(s_h,h)-q(s_l,l)]/r\}$,

where s* solves $q_s(s^*,h) = r$.

Log wages are given by

(11) $w(s_l,p) = \lambda p + (1-\lambda)q(s_l,l) + .5(1-\lambda)\sigma_e^2$

and

(12) $w(s_h,p) = \lambda p + (1-\lambda)q(s_h,h) + .5(1-\lambda)\sigma_e^2$

Proof (see appendix).

There are two noteworthy points about this result. First, there may be cases where the full information equilibrium is also an imperfect information equilibrium. This will occur whenever λ is close to 1. Secondly, the intuitive criterion is insufficient to eliminate all pooling equilibria. There are cases in which pooling equilibria which are Pareto superior to the separating equilibrium exist and are robust to the use of the intuitive criterion.

For example, suppose that $q(s,l) = 0$ and $q(s,h) = 10 + s^{.5}$. Let the variance of $e+u = .001$ and $\lambda = .1$. Suppose further that $r = .1$ and that there are equal numbers of high and low ability workers in the population. Consider a pooling equilibrium at $s = 0$. In the pooling equilibrium before observing p, employers' prior that the individual is high quality is .5, the same as the proportion of high ability workers in the population. However, since the variance of e+u is small relative to the difference between $q(0,h)$ and $q(0,l)$, upon observing p, firms will almost always have a very tight posterior around 1 or 0 regarding whether the worker is low or high quality. As a consequence, in the pooling equilibrium, high quality workers have an expected log present discounted value of wages very close to 10 and low quality workers have an expected log present discounted value of wages very close to 0.[7]

If employers assumed that any out of equilibrium move was made by a high quality worker, low quality workers who deviated would have an expected log lifetime earnings of $-.1s + 0\lambda + (1-\lambda)(10+s^{.5}) + .45(.001)$. Thus the

[7]Because the variance of wages is small, we can ignore differences between the log of the expectation and the expectation of the log.

intuitive criterion requires a deviation of s equal to about 225 before it excludes the possibility that the deviation was made by a low quality workers. However, a high quality workers would not make this deviation because the expected log of the present value is only 2.5 which is less than they receive in the pooling equilibrium.

At the same time, the separating equilibrium is also an equilibrium. As noted above low quality workers would not deviate if the equilibrium were $s_l = 0$, $s_h = 225$. A high quality worker who deviated would receive an expected log present discounted value of earnings of about .9(0) + .1(10) = 1 which is less than the 2.5 associated with $s_h = 225$.

It is trivial to establish the following results. As λ tends to 1, s* equals s_h. As λ tends to 0, s* is less than s_h. Finally, s_h is nonincreasing in λ. Thus for small informational imperfections, the high ability workers obtain the same education as they would in the presence of perfect information. However beyond some point, the education they obtain increases as informational imperfections worsen.

The estimated return to education is given by

$$(13) \qquad \hat{r} = (q(s_h,h) - q(s_l,l))/(s_h - s_l)$$

while the true social return is given by

$$(14) \qquad r = (q(s_h,h) - q(s_l,h))/(s_h - s_l).$$

It is readily verified that the difference between (13) and (14) diminishes as s_h increases. It follows that as informational imperfections as measured by λ increase, the difference between the measured return to education and the social return diminishes. In effect, the ability bias consists of the difference between the productivity of high and low ability workers at the low ability education level. As information worsens, high ability workers get more education, and this bias is divided over more years of education.

It might appear that the results of this example are driven by the fact that there are only two ability types. The following example shows that a similar results arises when there is a continuum of ability types. Unfortunately, with this model, it does not appear possible to derive general conditions under which the efficient separating equilibrium is the unique equilibrium which satisfies the Cho—Kreps intuitive criterion refinement.

Example 2: Let $q(i,s) = is^b$ and let i be continuous on $[0,i^*]$ with $0<b<1$. These restrictions on b ensure that schooling increases productivity but at a declining rate so that individuals do not choose infinite education. While the choice of human capital production function is obviously arbitrary, as we will see it leads to a wage equation which has the usual log—level functional form.[8]

Under this assumption, the log of expected wages is given by

$$\text{(15)} \qquad \log E(W) = (1-\lambda)\ i^*(s)s^b + \lambda\ is^b + .5\sigma_e^2.$$

Note that in one place i^*, the equilibrium level of i associated with s, has been written as a function of s since firms use s to infer i and place weight $(1-\lambda)$ on this estimate. In the other term productivity is inferred directly from p. Although in equilibrium i equals i^*, we cannot combine the two terms, because the individual decides how much education to get by comparing expected

[8] We could replace i by any function of i with no effect on the results. For example if $q = \exp(i)s^b$, we can simply redefine the measure of innate ability as $v = \exp(i)$ without changing any of the results.

wages associated with different levels of education holding other workers' behavior constant.

Maximizing the present discounted value of lifetime earnings gives the first order condition

$$(16)\qquad [bis^{b-1} + (1-\lambda)(di/ds)s^{b}] = r.$$

Solving the differential equation (14) gives

$$(17)\qquad s = [i(b\lambda+1-\lambda)/r]^{1/(1-b)}$$

where the constant of integration has been chosen so that individuals with ability level 0 choose zero education as they would in the presence of full information.

To get the expected log wage as a function of education only, solve (17) for i as a function of s and substitute into the q equation to get

$$(18)\qquad E(w) = rs/(b\lambda+1-\lambda) + .5(1-\lambda)\sigma_e^2.$$

At each level of education, the expected wage is lower than the expected wage which would be observed in the presence of perfect information since in that case, the expected log wage expressed solely as a function of education is given by rs/b.

It is easily verified that the social return to education is given by

$$(19)\qquad \partial q/\partial s = br/(b\lambda+1-\lambda).$$

The ratio of the OLS and social returns is therefore independent of informational imperfections, but the absolute value of the bias decreases as informational imperfections increase.

We note in passing that the measured return to education can be a very poor measure of the social return to education even when information is perfect. If we take the derivative of (18) with respect to b, we see that the measured return to education falls as b rises, that is as education becomes more productive. At the extreme, even when information is perfect, the measured return to education goes to infinity as b goes to zero.

Examples 1 and 2 show that ability bias may be lower in the presence of informational imperfections than with perfect information in models with either two discrete types or a continuum of ability types. What appears to be critical to the examples is therefore not the assumptions about the number of types. Instead, the following example suggests that the critical assumption is that education is continuous and unbounded, at least from above.

Example 3: The model here is similar to that in example 1 except that there are only two possible levels of education. Thus we return to the case of two ability types with conditions (7) and (8). With two possible levels of education, there are five possible equilibria. In the first two equilibria, everyone gets the same level of education regardless of ability type. These are uninteresting since the return to education cannot be measured and ability bias is undefined. A third equilibrium separates the two types of workers so that the low ability workers get the low level of education and the high ability workers gets the high level. Two other equilibria exist, one in which low ability workers get the low level of education and the high ability workers divide themselves between the two levels, and one in which the high ability workers get the high level of education and the low ability workers divide themselves between the two levels.

The equilibrium with complete separation entails the highest level of ability bias. It is perfectly possible that with perfect information high ability workers are just indifferent between the two levels and distribute themselves randomly between them (the fourth equilibrium). Now contrast this with a situation in which there is some slight informational imperfection sufficient to make all high ability types choose the higher level of education but not sufficient to get the low ability types to imitate them (the third equilibrium). Clearly the average level of ability at the low level of education will fall while the average level of ability at the high level of education will be unchanged. Therefore ability bias will rise.

It would clearly be possible to construct examples with two levels of education in which informational imperfections caused ability bias to decline. Indeed as informational imperfections increase, more and more low ability types will choose to get the high level of education, and ability bias will decline.

The message of example 3 is that if we perceive the education system as being best modelled as one in which there are a small number of discrete outcomes, there is no strong basis for arguing that informational imperfections increase or decrease the discrepancy between measured and actual social returns.

The fact that I have used a hybrid model may give the misleading impression that my argument does not apply to the standard Spence sorting model. To show that this is not the case, I use an example based on the signaling exercise in Diamond and Rothschild (1978, p. 303—4) which is in turn based on an example developed by Spence.

Example 4. Let productivity be

(20) $P^{*}(s,i) = is^{b}$

where i is continuous over the internal [0,i*], and let the cost of education be

(21) $C(s,i) = s/i.$

Since q_i depends only on s and not on i, to show that ability bias is greater in the presence of perfect information, we need only show that di/ds is greater. It is readily verified that with perfect information the equilibrium relation between i and s is given by

(22) $i = s^{.5(1-b)}/b^{.5}.$

Somewhat more tedious calculations establish that when productivity is not observable

(23) $i = (2/(1+b))^{.5}\, s^{.5(1-b)}.$

It is readily verified that for b<1,

(24) $b^{-.5} > (2/(1+b))^{.5}$

and therefore that di/ds is greater when productivity is observed.

4. CONCLUSION

The principal conclusion of this paper is that our priors about the importance of informational imperfections should not greatly affect our willingness to rely on OLS estimates of the return to education as a guide to policy. This conclusion does not mean that the human capital/sorting debate is utterly without importance although it should greatly diminish its significance. The importance of educational sorting may have some bearing on the effects of other policies such as minimum wage laws (Lang, 1987) or compulsory school attendance (Lang, 1986), but these policies have not been a central focus of the debate over the source of the observed return to education.

We also should not conclude on the basis of this paper that the OLS return to education is a good guide to the social return. The point is that the OLS return may be a very bad or a very good measure of the social return regardless of the importance of informational imperfections. When very little that is productive is taught in school, the OLS estimate is likely to poor. I have occasionally heard economists argue that the MBA is a pure sorting device. These economists argue that very little is taught in MBA programs but that the returns appear to be large. I leave it to the reader to judge the validity of these claims. Instead I wish to assess the strength of the argument, assuming that the claims are true. If the argument is that the social return to the MBA program is below the private discount rate, then clearly we must be observing sorting or the MBA must have considerable consumption value. If instead the argument is that the measured return greatly exceeds a reasonable assessment of what is learned, this is consistent with either a human capital or sorting interpretation. In either case, only the best students find it worthwhile to get an MBA. This generates a high measured return to getting an MBA regardless of whether employers observe productivity directly or use the MBA to infer quality. Thus it is our priors about the nature of the human capital production function rather than those regarding information that should inform our assessments of the accuracy of the OLS estimate.

A corollary of this argument is that the extensive literature on ability bias is not informative about the human capital/sorting debate. As the second example showed, any level of information is consistent with any positive level of ability bias. In that example, ability bias goes to zero as b goes to 1 and goes to infinity as b goes to 0 regardless of the level of informational imperfection (λ). Therefore it is unfortunately impossible to use existing estimates of ability bias to get much sense of the importance of informational imperfections.

To see this consider what could happen if we knew the shape of the human capital production function, the true social return to education and the OLS estimate of that return. In particular, suppose that we know that the human capital production function takes the form $q = is^b$. From equations (18) and (19), under this assumption the ratio of the true return to education to the OLS return is equal to b. The difference between the two sets of estimates can also be derived from (18) and (19) and is equal to

$$(20) \qquad \beta_{ols} - \beta_{social} = (1-b)r/(b\lambda+1-\lambda).$$

Having derived b from the ratio of the two estimates, we can calculate λ as a function of r. "Ability bias corrected" estimates of the return to education vary considerably. Including measures of ability generally does not lower the estimated coefficient by more than 15% to 20% and sometimes raises it. IV estimates of the return are sometimes negative and sometimes substantially higher than the OLS returns but seem to be centered around the OLS estimate. To take a realistic example, suppose that the OLS return is .08 and the ability-bias corrected return is .072. This implies that b = .9. Using (20) we see that λ equal to zero (completely imperfect information) is consistent with r equal to .08. On the other λ equal to one (perfect information) would be consistent with r equal to .072.

In sum the human capital/sorting debate is not informative about the usefulness of the OLS estimate of the return to education for social policy, and estimates of ability bias are not informative for resolving this debate. Given the strength of feeling on both sides of the debate, it is somewhat surprising that the implications for major social policy questions are not more clearly differentiated.

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APPENDIX

Proof of theorem 1:

The case where the full information equilibrium is also an imperfect information equilibrium is uninteresting. We therefore concentrate on the case where $s_h > s^*$.

We show first that conditions (9) and (10) correspond to an equilibrium.

Under the separation assumption the log of the expected present value of lifetime earnings for low ability workers with education s_l is given by

(A1) $\log(E(PV)) = -rs_l + q(s_l,l) + .5\sigma_e^2 - \log(r)$

while for low ability workers with education s_h, the equivalent expression is

(A2) $\log(E(PV)) = -rs_h + \lambda q(s_h,l) + (1-\lambda)q(s_h,h) + .5\sigma_e^2 - \log(r)$.

Condition (10) ensures that the right hand sides of (A1) and (A2) are equal so that low ability workers do not have an incentive to imitate high ability workers.

It is obvious that high quality workers will not deviate by getting more education. We must however establish that they would not find it worthwhile to get less education even if employers inferred that they were low quality workers. Note that s_h is chosen so that low quality workers would find it optimal to deviate for any lower s if employers inferred that they were high quality workers. Therefore the intuitive criterion does not rule out employers inferring that workers with $s<s_h$ are low quality. Consider some s in (s_l,s_h). Since s_l is optimal for low quality workers

$$\text{(A3)} \qquad -rs + q(s,l) < -rs_l + q(s_l,l).$$

Combining (10) and (A3) gives

$$\text{(A4)} \qquad -rs_h + \lambda q(s_h,l) + (1-\lambda)q(s_h,h) > -rs + q(s,l)$$

or

$$\text{(A5)} \qquad -rs_h + q(s_h,h) > -rs + q(s,l) + \lambda[q(s_h,h)-q(s_h,l)].$$

But, the right-hand-side of (A5) equals

$$\text{(A6)} \quad -rs + \lambda q(s,h) + (1-\lambda)q(s,l) + \lambda[q(s,l)-q(s,h)+q(s_h,h)-q(s_h,l)] > -rs + \lambda q(s,h) + (1-\lambda)q(s,l).$$

So high quality workers will not find it worthwhile to deviate by lowering their education level.

It is obvious that no other separating equilibrium can be supported. To prove uniqueness we must therefore merely show that no pooling equilibrium exists. Suppose that a pooling equilibrium exists at education level s. Low ability workers prefer s to s_l; otherwise s would not be an equilibrium. We know that low ability workers prefer s_l to s_h even if employers infer from their choice of s_h that they are high ability. By transitivity, low quality workers will never deviate from s to s_h. By the intuitive criterion, if employers observe a worker choose s_h, they must therefore infer that he is a high quality worker. Since by assumption the pooling equilibrium at s does not Pareto dominate the separating equilibrium, high quality workers prefer s_h when employers infer from this choice that they are high quality. Therefore s cannot be a pooling equilibrium.