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ABSTRACT

In a model where *all* factors of production are *imperfectly mobile*, we argue that the Dixit-Norman scheme of commodity taxes may not lead to strict Pareto gains from trade. Rather, this scheme must be augmented by policies which give factors an incentive to move: hence, the role for trade adjustment assistance (TAA). We demonstrate that by knowledge of the distribution of adjustment costs across individuals, the government can offer a single TAA subsidy to all individuals willing to move between industries, and maintain a non-negative budget. The TAA subsidy, combined with the Dixit-Norman pattern of commodity taxes, can lead to Pareto gains from trade under the conditions we identify.

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1. *Introduction*

While adjustment costs have received substantial attention in the positive theory of trade, as in models of partially-mobile factors,[1] they have received less attention in the normative theory, such as the gains from trade.[2] Perhaps the reason is that many forms of adjustment costs are fully consistent with an Arrow-Debreu framework (the adjustment is a "production function" transforming a factor from one industry to another), and literature on the gains from trade is meant to deal with these general equilibrium settings. In particular, we know that by using lump-sum transfers all individuals in an economy can be compensated for changes in their income due to opening trade, including compensation for any adjustment costs they face (since these are part of the Arrow-Debreu framework).[3]

Recently, literature on the gains from trade has shifted away from the use of lump-sum transfers, because of their high informational requirements.[4] Dixit and Norman (1980, p. 79; 1986) have proposed a scheme of commodity taxation which can achieve Pareto gains from trade with much less information. In brief, the Dixit-Norman (DN) scheme is to arrange commodity taxes such that consumers face autarky prices for goods and factors, while producers face free trade prices. It turns out that the government raises non-negative revenue from this tax scheme, and strictly positive revenue so long as production

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1 Grossman (1983) uses a model where each unit of capital loses some efficiency as it moves between sectors. This model is consistent with our framework, where we emphasize the differences across individuals in their adjustment costs of each factor. Since our model is static, however, we do not incorporate the dynamic adjustment analysed by Mayer (1974), Mussa (1974) and Neary (1978).

2 Exception to this statement include the work on on tariff reductions by Leamer (1980), and on trade adjustment assistance by Diamond (1982).

3 Recent surveys of the literature on the gains from trade are provided by Chipman (1987) and Kemp (1987).

4 The informational requirements are discussed in footnote 10.

changes from its autarky position, which can then be distributed as a poll subsidy.[5] Kemp and Wan (1986) have emphasized that production will not necessarily shift in response to free trade prices, in which case positive revenue is not obtained, and have critiqued other aspects of the DN scheme.[6]

Our point of departure from Dixit and Norman is that we consider a model where *all* factors of production are *imperfectly mobile*. As outlined in sections 2 and 3, we shall assume that at autarky wages no factor of production is voluntarily willing to move to another industry. Then as in Kemp and Wan (1986), we argue that the DN plan will not raise positive revenue in this setting. Rather, it must be augmented by policies which give factors an incentive to move: hence, the role for trade adjustment assistance (TAA), which is introduced in section 4.

We demonstrate that by knowledge of the *distribution* of adjustment costs across individuals (but not the costs faced by any particular person), the government can offer a single TAA subsidy to all individuals willing to move between industries, while maintaining a non-negative budget. As clarified in section 4, moving industries means that the individual forgoes any of the DN factor subsidies in the new location. We argue that the TAA subsidy, combined with the Dixit-Norman pattern of commodity taxes, can lead to Pareto gains from opening trade. It should be noted that this result is not completely general, but relies on certain assumptions and special features of our model (such as no labor-leisure choice and a continuum of individuals). A brief discussion of TAA in the U.S. and conclusions are given in section 5.

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5 Dixit and Norman (1986) discuss how Pareto gains can be achieved using just commodity taxation, without the poll subsidy. See section 3.

6 Brecher and Choudhri (1990) have explored whether the DN plan can be extended to cover internationally mobile factors.

2. *The Model*

There are N final goods indexed by i, produced using M primary inputs indexed by j, under constant returns to scale.[7] The production function for each good is given by $f^i(x^i)$ where x^i is the M-dimensional vector of inputs, and the unit-cost function is $g^i(w^i)$ where w^i denotes the M-dimensional vector of wages in that industry; these functions are concave and are treated as twice continuously differentiable. The presence of adjustment costs (modelled below) means that the equilibrium wage of each factor will generally differ across industries. In equilibrium, unit-costs will equal the price p^i in each industry:

$$p^i = g^i(w^i), \qquad i=1,\ldots,N. \tag{1}$$

Given output y^i, employment of factors in each industry is:

$$x^i = y^i \nabla g^i(w^i), \qquad i=1,\ldots,N, \tag{2}$$

where $\nabla g^i(w^i) \equiv (\partial g^i/\partial w_1^i,\ldots,\partial g^i/\partial w_M^i)$ denotes the vector of partial derivatives.

There are H individuals with utility functions $u^h(c^h)$, where $c^h \geq 0$ is an N-dimensional vector of consumption. Each individual is endowed with $\bar{v}_j^h$ amount of factor $j=1,\ldots,M$. We assume that mobility costs prevent factors from being costlessly transferred between industries. To model the mobility costs in a general way, suppose there is a transformation function ϕ_j^h depending on the individual supplies of factor j to each industry, $v_j^h = (v_j^{h1},\ldots,v_j^{hN}) \geq 0$. These inputs are measured in efficiency units. The transformation function is increasing and convex in it arguments, but is constrained by the individual endowment of each factor:

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[7] We follow Dixit and Norman (1980, 1986) in assuming the underlying production functions are concave with constant returns to scale; decreasing returns can be avoided by defining artificial factors which earn pure profits.

$$\phi_j^h(v_j^h) \leq \overline{v}_j^h , \quad j = 1,\ldots,M. \tag{3}$$

For example, if factors were perfectly mobile then the transformation function would simply be the *sum* of that factor supplied to each industry,

$$\phi_j^h(v_j^h) = \sum_{i=1}^{N} v_j^{hi} \leq \overline{v}_j^h , \quad j = 1,\ldots,M, \tag{3'}$$

which cannot exceed its endowment. Alternatively, if a factor varies in its efficiency across industries, then the transformation function would be a *weighted sum*, with the weights indicating the inverse of the skill level in each industry. This case is illustrated in Figure 1, where v_1^i is labor supplied to industries i=1,2, and the set of feasible supplies satisfying (3) is shown by AB. The steep slope of this "transformation curve" indicates a relative lack of skill in industry 2, which limits the individual's effective labor supply. With wages equal across industries, as illustrated by $\tilde{w}_1^2/\tilde{w}_1^1$, this individual would supply all labor to industry 1, at point A. Grossman (1983) has examined a model in which capital is partially mobile due to efficiency loss between industries, and this is consistent with a linear transformation curve like AB with a slope that varies across each unit of capital.

An alternative shape for the transformation curve is AB', which is strictly concave. Along AB' some wages would lead to factor supply in *both* industries, such as at C with $w_1^2/w_1^1 > 1$. This situation of an individual supplying one factor to multiple industries is less common, but may be appropriate if we think of labor supply from a household, or individual supplies of capital and land. Note that a *convex* shape for the transformation curve would be consistent with fixed costs of moving between industries, such as time lost in unemployment, but would complicate the existence of equilibrium in our model and so is not considered.

Letting p denote the N-dimensional vector of goods prices, w the MxN dimensional vector of wages across all of the industries, and $v^{hi} = (v_1^{hi},\dots,v_M^{hi})$ the individual supplies of the M factors to industry i, each consumer solves:

$$\max_{c^h, v^{hi} \geq 0} u^h(c^h) \quad \text{subject to} \quad p'c^h \leq I^h(w), \tag{4}$$

where factor income is given by:

$$I^h(w) \equiv \max_{v^{hi} \geq 0} \sum_i w^{i\prime} v^{hi} \quad \text{subject to} \quad (3), \tag{4'}$$

Note that all vectors are treated as columns unless transposed with a prime. Because we have not put factor supplies in the utility function (so there is no labor-leisure choice), to maximize utility in (4) the consumer solves the sub-problem (4') of allocating factors to maximize total income.

Let $c^h(p,w)$ and $v^{hi}(w)$ denote the solutions to (4) and (4') for person h, and let $c(p,w) \equiv \sum_h c^h(p,w)$ denote the aggregate consumption vector and $v^i(w) \equiv \sum_h v^{hi}(w)$ the aggregate factor supply vector to each industry. Then making use of (1) and (2), the *autarky equilibrium* is defined by the conditions:

$$v^i(w^a) = \nabla g^i(w^{ia}) c^i[g(w^a), w^a], \quad i=1,\dots,N, \tag{5}$$

where we define $g(w) \equiv [g^1(w^1),\dots,g^N(w^N)]$, which equals prices p from (1). The left-side of (5) is the supply of factors to industry i, while the right-side is the demand. Under free trade, the N-dimensional vector z(p) of goods are demanded from abroad, where a negative component of z denotes foreign supply. In contrast to (5), the *free trade equilibrium* is defined by the conditions:

$$v^i(w^*) = \nabla g^i(w^{i*})\{c^i[g(w^*), w^*] + z^i[g(w^*)]\}, \quad i=1,\dots,N, \tag{6}$$

where now the output of an industry $y^i = c^i(p^*, w^*) + z^i(p^*)$ is used to satisfy both domestic and foreign demand.

The autarky and free trade equilibria are illustrated in Figure 2, where ABC is the production possibilities frontier (PPF). Under our assumption that the individual transformation functions (3) are convex and the industry production functions are concave, the PPF is also concave. With prices p^a the autarky equilibrium is at B, and with prices p^* the free trade equilibrium is at D. It is possible for the PPF to have "kinks" due to the mobility costs, as when a small change in prices and wages does not induce factors to move between industries, though we do not assume that this is necessarily the case.[8]

3. *Dixit-Norman Policy*

The Dixit-Norman (DN) plan involves a system of commodity (i.e. goods and factor) taxes and subsidies such that consumers face the autarky prices for goods and factors, while producers face international prices. By construction, this plan leaves individuals in the same situation as autarky. Dixit and Norman (1980) show that the government raises non-negative revenue from the policy. If revenue is positive, and there exists one good for which some consumers are net buyers and none are net sellers (this is the Weymark Condition of Dixit and Norman, 1986), then the tax rate on that good can be adjusted so that some individuals strictly gain, while none lose and the government budget remains non-negative. Thus, Pareto gains from trade are achieved.

There are two ways to interpret the DN plan in our model. First, we could regard the transformation functions (3) as production functions, where

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[8] When all individuals are at a corner solution in the supply of all factors, such as at point A in Figure 1, then a small change in product prices and wages would not induce any movement between industries, so the PPF has a kink.

the outputs are the individual factor supplies v_j^{hi} to each industry, i=1,...,N, and the inputs are the endowments $\overline{v}_j^h$. The DN plan specifies that the outputs should not be directly taxed or subsidized, but that the inputs $\overline{v}_j^h$ should receive their autarky returns. However, the returns to the inputs are the shadow prices of the constraints (3) in problem (4'), i.e. the amount by which factor income of an individual would rise with higher endowments put to their best use. We have modelled the transformation functions as specific to individuals, reflecting loss of skills and other mobility costs. In this setting, we would not expect the government to know the shadow prices of factor endowments. For this reasons, it is not feasible to apply the DN plan to the individual transformation functions (3).[9]

An alternative interpretation of the DN policy is to apply the factor taxes or subsides at the *industry* level, giving consumers the same wages w^{ia} as in autarky. To this end, let $t=(p^a-p)$ denote the N-dimensional vector of taxes on goods (negative denotes a subsidy), where p are the international prices. Let $s^i=(w^{ia}-w^i)$ denote the M-dimensional vector of industry-specific subsidies (negative denotes a tax) on the employment of factors, i=1,...,N. By construction, consumers face autarky prices and therefore make the same consumption and factor supply choices. The *DN equilibrium* satisfies the conditions:

$$v^i(w^a) = \nabla g^i(\tilde{w}^i)\{c^i[p^a,w^a]+z^i[g(\tilde{w})]\}, \quad i=1,...,N. \tag{7}$$

Note that $\tilde{w}$ denotes the factor prices paid by firms, which are endogenously determined in (7), and that the international prices are $\tilde{p}=g(\tilde{w})$. Thus, the consumption taxes and factor subsidies required in equilibrium are $\tilde{t}=(p^a-\tilde{p})$ and

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[9] This situation is similar to a firm having specific capital, in which case we would not expect the government to observe the return to this capital; see Dixit and Norman (1980, p. 80).

$\tilde{s}^i = (w^{ia} - \tilde{w}^i)$, i=1,...,N. We shall suppose that this equilibrium exists, as has been analysed by Dixit and Norman (1986).

Letting $c^a \equiv c[p^a, w^a]$, $v^{ia} \equiv v^i(w^a)$ and $\tilde{y} \equiv c[p^a, w^a] + z[g(\tilde{w})]$, Dixit and Norman show that the government budget is non-negative:

$$\begin{aligned} B^{DN} &= (p^a - \tilde{p})'c^a - \textstyle\sum_i (w^{ia} - \tilde{w}^i)'v^{ia} \\ &= \textstyle\sum_i \tilde{w}^{i\prime}v^{ia} - \tilde{p}'c^a \\ &= \tilde{p}'(\tilde{y} - y^a) \geq 0 . \end{aligned} \qquad (8)$$

The second line of (8) follows since the value of autarky consumption $p^{a\prime}c^a$ equals factor income $\sum_i w^{ia\prime}v^{ia}$. The final line follows from $c^a = y^a$ in the autarky equilibrium, and $\sum_i \tilde{w}^{i\prime}v^{ia} = \tilde{p}'\tilde{y}$ since payments to factors equal product revenue under constant returns to scale. In the last line $\tilde{y}$ and y^a are both produced with the same inputs v^{ia}, but since $\tilde{y}$ is profit-maximizing for the prices $\tilde{p}$ we must have $\tilde{p}'\tilde{y} \geq \tilde{p}'y^a$. This establishes that (8) is non-negative, and strictly *positive* only if $\tilde{y}$ differs from y^a.[10]

Kemp and Wan (1986) have pointed out that $B^{DN} = 0$ occurs when production occurs at a "kink" in the production possiblities frontier, so that outputs do not change in response to world prices p. In our model, a related situation arises when all factors are imperfectly mobile, and are unwilling to move between industries at autarky wages. Let us define an individual to be *imperfectly mobile* between industries if (4') yields a *unique* solution for v^h at the autarky

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10 Our results can be compared to a system of lump-sum transfers. Suppose that consumers and producers face free trade prices, but each individual receives the transfer $\sum_i (p^i - p^{ia})'c^{hia} + \sum_i (w^{ia} - w^i)'v^{hia}$. Adding this to factor income in (4) we see that the autarky consumption and factor supplies are feasible, so each individual is no worse than in autarky. Moreover, summing these lump-sum transfers across individuals we obtain $-B^{DN} \leq 0$, from (8), so the transfers are feasible for the government. The information required to compute these transfers are the autarky consumption and factor supply vector for each individual, in addition to the autarky and free trade prices.

wages. This simply means that at constant wages, people are unwilling to change jobs. This occurs in Figure 1, for example, with the unique factor supply decision at point A. If the transformation curve were AB', and autarky wages led to a factor supply decision at C, then we would still call this individual imperfectly mobile. The case we are ruling out is when the transformation curve is linear, such as AB, and had a slope *just equal* to the autarky relative wages.[11] That would indicate multiple solutions for the autarky factor supplies, or perfect mobility between industries.

We shall consider the extreme case where *all* individuals are imperfectly mobile. Formally, we shall assume:

Assumption 1 $v^h(w^a)$ is unique for all $h=1,\dots,H$.

Under this assumption, the factor supply decisions in the DN equilibrium are equal to those in autarky, so that outputs are the same, and $B^{DN}=0$ from (8). Thus, we have established:

Proposition 1

Under Assumption 1, the DN plan applied to industry prices and wages yields the autarky equilibrium with zero government revenue.

This result is illustrated in Figure 2, where with the international prices $\tilde{p}$ the DN equilibrium is at B. The non-tangency between the price line $\tilde{p}$ and PPF can be understood in two ways. First, with wages facing consumers fixed at w^a, and factor allocations to each industry uniquely determined by Assumption 1, we can treat each factor as if it were fully sector-specific.

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11 The leading example is when a factor can be costlessly transferred between industries, so that AB has a slope of unity, and wages are equal across the industries.

That is, the PPF reduces to the kinked line A'BC', with the equilibrium at B. This interpretation may give the mistaken impression, however, that we are assuming large adjustment costs. In fact, Assumption 1 can hold even when individuals have only very slight costs of adjustment between industries. The sector-specific interpretation of our model comes from having the (small) adjustment costs *combined with* wages rigidly fixed at their autarky level.

A second interpretation of the nontangency in Figure 2 is obtained by arguing that the DN commodity taxes and subsidies are, in this case, equivalent to prohibitive tariffs. To see this, note that with $z(\tilde{p})=0$ in (7), the factor demands $\nabla g^i(w^{ia})$ and $\nabla g^i(\tilde{w}^i)$ are equal in (5) and (7). Since the partial derivatives $\nabla g^i(w^i)$ are homogeneous of degree zero, wages $\tilde{w}^i$ under the DN plan are therefore *proportional* to w^{ia}.[12] Since the unit-cost functions are homogeneous of degree one in (1), we then have $\tilde{w}^i = (\tilde{p}^i/p^{ia})w^{ia}$. The employment subsidy to each factor j is thus $\tilde{s}^i_j/\tilde{w}^i_j = (w^{ia}_j - \tilde{w}^i_j)/\tilde{w}^i_j = (p^{ia} - \tilde{p}^i)/\tilde{p}^i$, which is exactly the ad valorem consumption tax on good i. In other words, the factor subsidies in each industry are *uniform* across all inputs, and are therefore acting as a production subsidy, equal in value to the consumption tax. This policy is identical to a tariff on every good, which is prohibitive since trade is zero. This helps to understand there is a non-tangency between international prices $\tilde{p}$ and the PPF.

The assumption that all individuals are imperfectly mobile is obviously quite strong, and in reality we would expect that some factors would move between industries at the autarky wages. In this case Pareto gains can be achieved in the DN equilibrium. However, since the goal of theorems on Pareto gains is normally to achieve this result in *any* well-behaved economy, we find

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12 This solution for $\tilde{w}^i$ is unique if the matrix of second-derivatives of each unit-cost function is of rank equal to the number of employed factors less one.

that Assumption 1 is a valid case to consider. It is of interest that this condition directly violates Assumption 1 of Diewert, Turunen-Red and Woodland (1989), which states that for *fixed quantities* of all non-traded goods and factors, there must be a change in the initial tariffs which will raise the value of traded goods at (fixed) world prices. In our model, factors are distinguished by their industry of location, so the factor supply vector is actually a MxN-tuple specifying the quantity supplied to *each* industry. Holding this vector constant, there is no feasible change in the value of outputs, so their method of obtaining Pareto gains cannot be applied. In the next section we examine whether an additional instrument - trade adjustment assistance - can be used to obtain Pareto gains through encouraging the relocation of factors.

4. *Trade Adjustment Assistance*

We begin with the DN pattern of consumption taxes $\tilde{t} = (p^a - \tilde{p})$ and factor subsidies $\tilde{s}^i = (w^{ia} - \tilde{w}^i)$, so the economy is initially in the autarky equilibrium as in Proposition 1. It will be convenient to choose a numeraire to establish known signs on these tax instruments. Considering first the consumption taxes, for any initial choice of numeraire let i_0 denote a good with the minimum value of t^i. Then choosing i_0 as the numeraire, we divide all domestic autarky prices by $p^{i_0 a}$ and all international prices by $\tilde{p}^{i_0}$, implying that i_0 has the price of unity in both cases and all other goods have $p^{ia} \geq \tilde{p}^i$. Thus, relative to this choice of numeraire all other goods have non-negative consumption taxes. Turning to the factor subsidies, from our discussion below Proposition 1 the *ad valorem* subsidies are $\tilde{s}^i_j/\tilde{w}^i_j = (p^{ia} - \tilde{p}^i)/\tilde{p}^i \geq 0$, so that all factors receive non-negative subsidies. This means that $w^{ia}_j \geq \tilde{w}^i_j$, with equality in the numeraire industry.

With these instruments in place, we add a relocation subsidy R, which is provided to *individuals who agree to forgo the factor subsidies s^i in all industries.* However, this relocation subsidy will not be made available to all workers, since if it were then those in the numeraire industry (with zero wage subsidies) could simply take it and remain in their industry, implying a direct budgetary loss for the government. We will restrict the individuals eligible for the relocation subsidy to a set $h \varepsilon \Omega \subset \{1,...,H\}$. We can think of these individuals as employed in industries for which $p^{ia} > \tilde{p}^i$, so that under the DN plan they are receiving strictly positive wage subsidies.[13]

Consider offering a TAA subsidy of R to all individuals $h \varepsilon \Omega$. Initially - before any factors have chosen to move between industries - the *minimum* value of R at which some individual would be indifferent between taking the TAA subsidy and not is given by:

$$R_0 \equiv \min_{h \varepsilon \Omega} \{I^h(w^a) - I^h(\tilde{w})\}. \tag{9}$$

We shall make the following assumption related to this minimum subsidy:

<u>Assumption 2</u>

There is a unique individual h_0 for which $R_0 = I^{h_0}(w^a) - I^{h_0}(\tilde{w})$, and:

(a) $v^{h_0}(w)$ is unique and continuously differentiable in a neighborhood of $\tilde{w}$;

(b) $v^{h_0}(w^a) \neq v^{h_0}(\tilde{w})$.

The assumption that there is a *unique* individual who is indifferent between moving and not at the minimum subsidy rate is made for technical convenience, and does seem to have much economic content. Assumption 2(a) means that this individual is imperfectly mobile at wages around $\tilde{w}$, in the

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13 The identification of these individuals supposes that the government knows who is working in which industries.

sense that their factor supply decision is unique; this condition is analogous to Assumption 1. Assumption 2(b) is particularly important, and states that the individual's factor supply choices at the wages w^a and $\tilde{w}$ *differ*. This condition is more likely to be met if the persons h included in Ω are initially employed in industries which face *large* reductions in the international prices from p^{ia} to $\tilde{p}^i$, so that the wages $\tilde{w}^i$ are significantly below w^{ia}. In addition, if some individuals in these industries have adjustment costs which are not too great, then they would find it optimal to change (at least partially) their industry of location when faced with the wages $\tilde{w}$.

A preliminary result which will be useful is:

Lemma 1

Under Assumptions 1 and 2, the minimum subsidy satisfies $R_0 > 0$.

Proof:

We have $I^{h_0}(w^a) = w^{a\prime}v^{h_0}(w^a) > w^{a\prime}v^{h_0}(\tilde{w}) \geq \tilde{w}'v^{h_0}(\tilde{w}) = I^{h_0}(\tilde{w})$, where the first inequality holds since $v^{h_0}(\tilde{w})$ is a feasible factor supply choice for individual h_0 at the wages w^a, but cannot yield income equal to $I^{h_0}(w^a)$ from Assumptions 1 and 2(b); and the second inequality follows since $w^a \geq \tilde{w}$ from our choice of numeraire. QED

Our next task is to specify the equilibrium conditions for the economy in the presence of the relocation subsidy. For some values of R there may be many individuals who wish to accept it and forgo receiving the wage subsidies s^i. For convenience, we shall restrict our attention to an equilibrium where *only person h_0 chooses to accept the relocation subsidy.* Of course, we will need to demonstrate that no other individuals want to take the subsidy.

However, even with only this one person changing their factor supply choices, there will be a *discontinuous* change in the aggregate factor supply vector as we adjust R from slightly below R_0 to slightly above. This could lead to serious difficulties for the existence of equilibrium, which we deal with in the following way. Let us suppose that there is a *unit-mass* of each of the individuals 1,...,H. That is, we can think of a continuum of agents on a line from 0 to H, with those located between h-1 and h being of type h. This specification does not affect the way we have written our earlier equilibrium conditions. For example, in Assumption 1 $v^h(w^a)$ refers to the (unique) factor supply decision of a type-h individual at the wages w^a, and with a unit-mass of these persons their total factor supply is still $v^h(w^a)$.

The advantage of assuming a unit-mass for each type is that now only a *fraction* - which we denote by λ - of the h_0-type individuals may choose to take the TAA subsidy, while the remaining $(1-\lambda)$ choose to not do so. In this case the factor market equilibrium conditions become:

$$\lambda v^{h_0 i}(w) + (1-\lambda)v^{h_0 i}(w^a) + \sum_{h \neq h_0} v^{hi}(w^a)$$
$$= \nabla g^i(w^i)\{c^i[p^a,w^a] + z^i[g(w)]\}, \qquad i=1,\ldots,N. \tag{10a}$$

The left-side of (10a) is the aggregate factor supply, with λ of the h_0-type persons making their optimal factor supply choices at the equilibrium wages w. When $\lambda=0$ then $w=\tilde{w}$ and we are back in the DN equilibrium, but for $\lambda>0$ wages w will differ from $\tilde{w}$ due to the movement of factors between industries.

In order for (10) to describe a valid equilibrium, it must be that h_0-type individuals are *indifferent* between accepting the TAA subsidy or not, since otherwise it would not be possible for only a fraction of them to choose it. Thus, a second condition which must be satisfied in equilibrium is:

$$R = I^{h_0}(w^a) - I^{h_0}(w). \tag{10b}$$

With the TAA subsidy specified as in (10b), the total income $I^{h_0}(w^a)$ received by the h_0-type individuals who do not accept the TAA subsidy is identical to $R + I^{h_0}(w)$ received by the individuals who do take it. Since we are assuming that these are the only persons willing to accept the subsidy, a final condition which must be satisfied in equilibrium is that:

$$R < I^h(w^a) - I^h(w), \quad \text{for all } h \neq h_0,\ h \in \Omega. \tag{10c}$$

Conditions (10), with $0 < \lambda \leq 1$, define a *TAA equilibrium*. It is useful to simplify condition (10a) slightly. Of the M primary factors in the economy, many industries could use only a subset of them. So suppose that industry i has $M^i \leq M$ factors in positive demand at the wages $\tilde{w}$. We shall suppose that no input is borderline between being demanded or not, so that $g^i_j(w^i) > 0$ in a neighborhood of $\tilde{w}^i$ if and only if $g^i_j(\tilde{w}^i) > 0$. Then the MxN conditions in (10a) reduce to $K \equiv \sum_i M^i$ relevant equilibrium conditions. For $\lambda = 0$ these are simply the DN equilibrium. In order to establish the existence of equilibria for small values of $\lambda > 0$, we shall make use of the following regularity conditions:

<u>Assumption 3</u>

(a) $\nabla^2 g^i(w^i)$ has rank (M^i-1) in a neighborhood of $\tilde{w}^i$;

(b) $\partial z/\partial p$ is negative semi-definite with rank (N-1) around $\tilde{p}$.

Assumption 3(a) rules out fixed-coefficients in the production functions, since in that case the corresponding unit-cost function is linear and so $\nabla^2 g^i(w^i)$ would not have its maximum rank (M^i-1) (this matrix cannot have rank M^i since $\nabla^2 g^i(w^i)w^i = 0$). Assumption 3(b) means that income effects on foreign demand are weaker than substitution effects. We can then establish:

Lemma 2

Under Assumption 1-3, there exist values $R > 0$ and $0 < \lambda \leq 1$ satisfying the TAA equilibrium (10).

The proof of this result is in the Appendix, and is an application of the Implicit Function Theorem. That is, we show that for small values of $\lambda > 0$, the equilibrium conditions (10a) - combined with a numeraire condition - determine a continuously differentiable function $w(\lambda)$, where $w(0) = \tilde{w}$ are wages in the DN equilibrium. Substituting this function into (10b) we obtain $R(\lambda)$ with $R(0) = R_0 > 0$. By choosing $\lambda > 0$ sufficiently small, we obtain $R(\lambda) > 0$ and the strict inequalities in (10c) will continue to hold. Thus, the TAA equilibrium with $R > 0$ and $0 < \lambda \leq 1$ is obtained.

Next, we argue that output evaluated at international prices is higher in the TAA equilibrium than in autarky, so that the subsidy is causing persons to change their factor supplies in an efficient manner:

Lemma 3

Under Assumptions 1 and 2, when $\lambda > 0$ then $p'y > p'y^a$, where $p=g(w)$ and y are evaluated in the TAA equilibrium.

Proof:

We have:

$$\begin{aligned} p'y - p'y^a &\geq \textstyle\sum_i p^{i\prime}\nabla f^i(x^i)(x^i - x^{ia}) \\ &= \textstyle\sum_i w^{i\prime}[v^i(w) - v^i(w^a)] \\ &= \textstyle\sum_i w^{i\prime}[v^{h_0 i}(w) - v^{h_0 i}(w^a)] \\ &> 0, \end{aligned}$$

where the first line is from concavity of the production functions with $\nabla f^i(x^i)$ evaluated in the TAA equilibrium; the second line since wages facing firms are

$w^i = \nabla f^i(x^i)$; the third line since $v^h(w)$ differs from $v^h(w^a)$ *only for* person h_0; and the last line since $v^{h_0}(w) \neq v^{h_0}(w^a)$ is the *unique* solution to maximizing factor income at the wages w, from Assumption 2. QED

It is worth noting that Lemma 3 would not hold if the relocation subsidy R was made *conditional* on a person changing their factor supply choice from autarky. In that case, an individual such as h_0 could choose $v^{h_0}(w) \neq v^{h_0}(w^a)$ in order to receive R, even though the original supply $v^{h_0}(w^a)$ may still maximize factor income at the wages w. In other words, the factor supply choice would be influenced by the criterion imposed to receive R (i.e. changing industries), and so it need not be the case that $w'v^{h_0}(w) > w'v^{h_0}(w^a)$. Then we could not conclude, as in Lemma 3, that the relocation subsidy is causing persons to change their factor supplies in an efficient manner. For this reason we have made the relocation subsidy *unconditional* on whether an individual changes their factor supplies or not, but have ensured that this change takes place by Assumption 2(b).[14]

We can now investigate the government budget in the TAA equilibrium. Let $c^a \equiv c[p^a, w^a]$ and $v^{ia} \equiv \sum_h v^{hia} \equiv \sum_h v^{hi}(w^a)$ denote the autarky values, with $y \equiv c[p^a, w^a] + z[g(w)]$ and $v^i \equiv [\lambda v^{h_0 i}(w) + (1-\lambda) v^{h_0 i}(w^a) + \sum_{h \neq h_0} v^{hi}(w^a)]$ denoting values in the TAA equilibrium. With λ of the h_0-type individuals choosing to accept the relocation subsidy R, and the remaining $(1-\lambda)$ choosing to retain the factor subsidies $s^i = w^{ia} - w^i$ and their autarky factor supplies, the government budget is:

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[14] We discuss Assumption 2(b) again in section 5. The distinction between conditional and unconditional subsidies, and their effect on efficiency, is the focus of the paper by Brander and Spencer (1989).

$$B^{TAA} = (p^a - p)'c^a - \sum_i (w^{ia} - w^i)'[\sum_{h \neq h_0} v^{hia} + (1-\lambda)v^{h_0 ia}] - \lambda R$$

$$= (p^a - p)'c^a - \sum_i (w^{ia\prime}v^{ia} - w^{i\prime}v^i) - \lambda R + \lambda\sum_i(w^{ia\prime}v^{h_0 ia} - w^{i\prime}v^{h_0 i})$$

$$= p'y - p'y^a, \tag{11}$$

where the second line follows from the above definition of v^{ia} and v^i; and the third line since $p^{a\prime}c^a = \sum_i w^{ia\prime}v^{ia}$ in the autarky equilibrium, $p'y = \sum_i w^{i\prime}v^i$ in the TAA equilibrium, and $R = I^{h_0}(w^a) - I^{h_0}(w) \equiv \sum_i (w^{ia\prime}v^{h_0 ia} - w^{i\prime}v^{h_0 i})$ from (10b). Thus, using Lemma 3, we see that the government budget is positive in the TAA equilibrium. We have therefore established:

Proposition 2

Under Assumptions 1-3, there exists a relocation subsidy $R > 0$ such that in equilibrium all individuals receive the same utility as in autarky, and the government raises positive revenue.

This result is illustrated in Figure 1. Suppose that in the TAA equilibrium, wages in industry 1 (w_1^1) are neither taxed nor subsidized, while wages in industry 2 (w_1^2) are subsidized. Let C be the factor supply choice of an h_0-type individual at the subsidized wages $w_1^2/w_1^1 > 1$, while $\tilde{w}_1^2/\tilde{w}_1^1 = 1$ are the wages faced by firms. Since wages in industry 1 are not distorted by taxes, we shall use them as numeraire and measure income along the v_1^1 axis. Then in the TAA equilibrium an h_0-type individual earns factor income of D. However, at the undistorted wages $\tilde{w}_1^2/\tilde{w}_1^1 = 1$ the income earned at C would be E, and so the factor subsidy $s_1^2 v_1^2$ given to an h_0-type individual is measured by D-E.

If h_0-type individuals were faced with the undistorted wages $\tilde{w}_1^2/\tilde{w}_1^1 = 1$, then they would choose to supply all labor to industry 1, with factor income equal to A. They would be just indifferent between this choice and the original location at C provided that the relocation subsidy equalled $R = D - A$, in which case even after moving to A they would still receive income of D. Moreover, by giving the relocation subsidy $R = D - A$ the government saves D-E in direct factor subsidies, and so the *net budgetary gain* is $A - E > 0$. Note that this magnitude is equal to the increase in factor income measured at undistorted wages when moving from C to A. Thus, the net budgetary gain is precisely reflected in the production gain experienced by the economy, as we found in (11).

It may be useful to review the informational assumptions needed to obtain Proposition 2. We have assumed that the government can identify a set Ω of individuals who are potentially available for adjustment assistance, and that within this set the person who is willing to accept the *lowest* relocation subsidy will change their factors supplies in response to unsubsidized wages. While this assumption is a strong one, we have not assumed that the government can identify such a person. That is, the relocation subsidy of R is available to all persons in the set Ω, and the individuals of type h_0 who wish to accept this subsidy come forward and identify themselves. There is no incentive for any other agent to claim to be h_0-type individual and accept the subsidy, since this would only lower their welfare. Thus, the TAA policy we have proposed is "incentive compatible," and does not rely on the government having knowledge of the adjustment costs or other characteristics of agents.

To compute a value of R satisfying Proposition 2, the government would need to know the *distribution* of adjustment functions ϕ^h across the agents in Ω (so as to compute R_0), and the general equilibrium structure of the economy (so as to compute how wages and R depend on λ). Of course, in practice we do not imagine that this information is available, but the government could perhaps determine by trial and error the minimum relocation subsidy R_0 which persons would be willing to accept. Then by using a subsidy slightly higher than this, it could induce movement of factors out of a protected industry.

While we have argued that the relocation subsidy is positive, we have not actually established that $R > R_0$. The reason that this result does not hold in general is that as h_0-type individuals move between industries, the resulting change in product and factor prices could lower the value of R computed from (10b). It is possible to establish that $R > R_0$, however, if we are willing to add the following conditions:

Assumption 4 If $v_j^{h_0 i}(w^a) > 0$ and $v_\ell^{h_0 k}(\tilde{w}) > 0$ then $i \neq k$ and $\partial z^i / \partial p^k = 0$.

Assumption 4 is stronger than Assumption 2(b), and requires that there be no overlap between the industries i and k which h_0-type individuals supply factors under the wages w^a and $\tilde{w}$, respectively. In addition, we rule out any interaction in foreign demand between these industries. Under these strong conditions, we can restate Proposition 2 as:

Proposition 2'

Under Assumptions 1-4, there exists a relocation subsidy $R > R_0$ such that in equilibrium all individuals receive the same utility as in autarky, and the government raises positive revenue.

To prove this result, we show in the Appendix that the function $R(\lambda)$ obtained in a neighborhood of $\lambda=0$ from from Lemma 2 satisfies $R'(0) > 0$. Thus, for $\lambda > 0$ sufficiently small we will have $R(\lambda) > R_0$.

While Proposition 2 demonstrated that the government budget will improve through using the relocation subsidy R, it is desirable to pass this gain onto individuals. In our model this is easily established, since the Weymark Condition used by Dixit and Norman (1986) is automatically satisfied: all individuals 1,...,H are demanders of some goods, and none are suppliers. Then consider a slight reduction in this tax vector satisfying $dt \leq 0$ and $dt'c^a < 0$. These conditions ensure that taxes are slightly reduced on at least some goods demanded by domestic consumers, and these consumers would therefore gain. Let us assume that the aggregate consumption $c(p,w^a)$ is continuously differentiable in prices around p^a. Then as argued in the Appendix, we can establish:

Proposition 3

With a small reduction in commodity taxes from the TAA equilibrium satisfying $dt \leq 0$ and $dt'c^a < 0$, some individuals are better off than in autarky, while none are worse off and the government budget is positive.

The changes to consumption taxes used in this result leads to the question of whether Pareto gains can be secured in our model by *only* adjusting the consumption taxes, and without relying on TAA. We believe this question can be answered in the affirmative. In brief, suppose that starting in autarky the government announced the formation of a "duty free zone," where consumers could pay a single fee S in order to purchase goods at international prices. The resale of these goods to others would need to be prohibited. Then analogous to (9), we could calculate the maximum value of S at which an individual would be just indifferent between purchasing their goods duty free or not:

$$S_0 \equiv \max_{h \in \Theta} \{E^h(p^a, U^{ha}) - E^h(\tilde{p}, U^{ha})\},$$

where Θ is a set of persons eligible for the duty free zone, and $E^h(p, U^{ha})$ denotes the expenditure needed by individual h to obtain the autarky utility U^{ha} at prices p^a (autarky) or $\tilde{p}$ (international).

Let h_0 now denote the individual (assumed to be unique) for whom

$S_0 = E^{h_0}(p^a,U^{h_0a}) - E^{h_0}(\tilde{p},U^{h_0a})$. So long as the autarky consumption c^{h_0a} is not the optimal choice for person h_0 to obtain utility U^{h_0a} at the prices $\tilde{p}$, then $E^{h_0}(p^a,U^{h_0a}) - E^{h_0}(\tilde{p},U^{h_0a}) > (p^a - \tilde{p})'c^{h_0a}$, where the right-side of this expression is recognized as the consumption taxes obtained from person h_0 in the DN equilibrium. Thus, by charging S_0 as the fee for the duty free zone, person h_0 is willing to purchase all goods there and the government's budget will then improve. The intuition of this results is that by charging a flat fee rather than distorting taxes, the government can collect more revenue while keeping the individual's utility constant. This result has been established more formally in a simple, pure exchange economy by Feenstra and Lewis (1991), who allow more than one type of agent to use the duty free zone.

Thus, even when adjustment costs lead the DN equilibrium to coincide with autarky, the government could generate gains from trade by the use of consumption taxes. The scheme we have outlined requires, however, that persons buying at the duty free rates be prohibited from reselling these goods to others, and such a requirement may be very difficult to enforce. Furthermore, in order to enjoy any *production gains* the government will have to introduce policies to encourage the reallocation of factors, such as the TAA program we have analysed. In the next section we compare the assumptions of our theoretical model with the actual experience of TAA in the U.S.

5. *Discussion and Conclusions*

The Trade Adjustment Assistance program in the U.S. began in 1962 and has evolved erratically over the following decades. Initially, this program was designed to compensate workers for tariff cuts under the Kennedy Round of multilateral negotiations, and facilitate their adjustment out of import competing industries. Stiff eligibility criterion limited its use in early years.

The program was expanded in the Trade Act of 1974 and expenditures rose, reaching a record high of $2.2 billion in 1980 with a large part of these funds going to unemployed auto workers.[15] Under the 1981 Amendments to the 1974 Trade Act the program was cut back, with greater emphasis given to job search and relocation allowances, such as we have modelled. More recently, the Trade Act of 1988 has made job training a condition for receiving weekly cash benefits, and allows for a $0.8 billion annual expenditure for training financed by an import fee.[16]

The large payments to unemployed auto workers around 1980 led to dissatisfaction with the program, since many of these people went back to work in that industry when domestic demand recovered: thus, the TAA benefits were seen as simply an income transfer, without encouraging the reallocation of workers in an efficient direction.[17] This issue arises in our model through the key Assumption 2(b): that the person willing to accept the minimum subsidy will choose to move some factors into other industries. This assumption was essential in showing that the value of output (at world prices) rose due to the reallocation of factors, so that the government budget was positive.

Whether or not this assumption is satisfied in practise depends on features of the industry and workers involved. Bednarzik and Orr (1984) emphasize that TAA has worked best - in the sense of encouraging movement of workers to other industries - in industries facing large potential reductions in international prices. To make the analogy with our model, we can think of the subsidized wages as those occurring under some (temporary) import protection,

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15 Much of the information in this paragraph is drawn from Bednarzik and Orr (1984).

16 Summary of the Conference Agreement on H.R. 3, "The Omnibus Trade and Competitiveness Act of 1988," April 19, 1988, Government Printing Office.

17 Evidence on the extent to which workers return to their original industries is presented by Richardson (1982).

and the unsubsidized wages in this industry as those occurring under free trade. If these unsubsidized wages are low, and some individuals have skills transferable to other industries, this would be a setting where our Assumption 2(b) is satisfied (at the unsubsidized wages some people will move). However, when this condition does not hold so that individuals return to their former jobs after receiving TAA compensation, then we should not expect the theoretical result of Pareto gains to hold. Thus, there is a close connection between the conditions needed to obtain Pareto gains in our model, and the criterion by which TAA has been judged in practice.

The most recent revisions to the TAA program, whereby job training is a condition for receiving weekly cash benefits, can be interpreted as an attempt to self-select individuals into those which have low adjustment costs - and therefore are willing to take advantage of the job training - and those which do not. Incentive mechanisms of this type are recognized as having desirable properties, as compared with simple cash payments, in settings where the government has incomplete information about the agents involved (see Blackorby and Donaldson, 1988). In our model we have assumed that the government knows the distribution of adjustment costs across individuals, but cannot identify which person has the least difficulty in moving between industries. While the TAA subsidy we have proposed is "incentive compatible" in the sense that no person will want to misrepresent their adjustment costs, it is perhaps the simplest example of such a truth-revealing policy. More sophisticated policies - such as using job training as a condition of obtaining benefits - can lead the government to learn more about the adjustment costs faced by each individual, and this information could be used to develop other Pareto improving policies.

Appendix

Proof of Lemma

We have chosen good i_0 as the numeraire, so that $1=g^{i_0}(\tilde{w}^{i_0})$, with $\tilde{w}$ denoting wages in the DN equilibrium. Let us renumber i_0 as good N, where $w^N_{M^N}$ denotes the wage of the last input in industry N. We shall now change the numeraire and specify that this wage remains constant at $w^N_{M^N} \equiv \tilde{w}^N_{M^N}$. Using "^" to omit the last element of a vector, we let $\hat{w}$ denote the vector w with the last element omitted, and $\hat{w}^N$ denote w^N with the last element omitted. Since the K conditions (10a) are not independent (by Walras Law), we omit the last equation and obtain:

$$\lambda v^{h_0 i}(\hat{w}, \tilde{w}^N_{M^N}) + (1-\lambda) v^{h_0 i}(w^a) + \sum_{h \neq h_0} v^{hi}(w^a) - \nabla g^i(w^i) y^i = 0, \quad i=1,\ldots,N-1, \qquad \text{(A1)}$$

$$\lambda \hat{v}^{h_0 N}(\hat{w}, \tilde{w}^N_{M^N}) + (1-\lambda) \hat{v}^{h_0 N}(w^a) + \sum_{h \neq h_0} \hat{v}^{hN}(w^a) - \hat{\nabla} g^N(\hat{w}^N, \tilde{w}^N_{M^N}) y^N = 0, \qquad \text{(A2)}$$

where outputs are $y^i = c^i[p^a, w^a] + z^i[g^1(w^1), \ldots, g^{N-1}(w^{N-1}), g^N(\hat{w}^N, \tilde{w}^N_{M^N})]$, and $\hat{\nabla}$ in (A2) denotes the vector of partial derivatives with respect to $\hat{w}^N$. By the Implicit Function Theorem, these conditions determine a continuously differentiable function $\hat{w}(\lambda)$ in a neighborhood of $\lambda=0$ if the corresponding Jacobian matrix is non-singular. The Jacobian can be written as A+B+C, where:

$$A \equiv \left[\lambda \hat{\nabla} \hat{v}^{h_0}\right], \qquad B \equiv -\begin{bmatrix} \nabla^2 g^1 y^1 & 0 & \cdot & \cdot & 0 \\ 0 & \nabla^2 g^2 y^2 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & 0 & \hat{\nabla}^2 g^N y^N \end{bmatrix}.$$

where $\hat{\nabla}$ in A denotes the vector of partial derivatives with respect to $\hat{w}$, and $\hat{v}^{h_0}$ is the (K-1)-dimensional vector v^{h_0} with the last element omitted; and,

$$C \equiv -\begin{bmatrix} \frac{\partial z^1}{\partial p^1}\nabla g^1 \nabla g^{1\prime} & \cdot & \cdot & \frac{\partial z^1}{\partial p^N}\nabla g^1 \hat{\nabla} g^{N\prime} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial z^N}{\partial p^1}\hat{\nabla} g^N \nabla g^{1\prime} & \cdot & \cdot & \frac{\partial z^N}{\partial p^N}\hat{\nabla} g^N \hat{\nabla} g^{N\prime} \end{bmatrix}.$$

We shall argue that A+B+C is positive definite, which establishes that the Jacobian has full rank.

Factor income in (4') is the maximum of a linear function over a convex set, and so it is a convex function of wages. In addition, $\nabla I^{h_0}(w) = v^{h_0}(w)$. It follows that $\nabla^2 I^{h_0}(w) = \nabla v^{h_0}(w)$ is positive semi-definite, and then this property also holds for the matrix A obtained by omitting the last row and column of $\nabla^2 I^{h_0}(w)$. Similarly, B is positive semi-definite from concavity of the unit-cost functions.

Letting e denote a (K-1) dimensional column vector, partition e into $(e^1, e^2, \ldots, \hat{e}^N)$ where e^i is of dimension M^i, $i=1,\ldots,N-1$, and $\hat{e}^N$ is of dimension M^N-1. Then:

$$e'Ce = -\sum_{i=1}^{N-1}\sum_{j=1}^{N-1} \frac{\partial z^i}{\partial p^j}(e^{i\prime}\nabla g^i)(\nabla g^{j\prime} e^j) - 2\sum_{i=1}^{N-1} \frac{\partial z^i}{\partial p^N}(e^{i\prime}\nabla g^i)(\hat{\nabla} g^{N\prime}\hat{e}^N) - \frac{\partial z^N}{\partial p^N}(\hat{e}^{N\prime}\hat{\nabla} g^N)^2,$$

which is non-negative from Assumption 3(a). We conclude that C is positive semi-definite, so this property also holds for the Jacobian, and we need to check that it is positive definite.

Suppose to the contrary that $e'(A+B+C)e = 0$, $e \neq 0$, which implies that $e'Ae = e'Be = e'Ce = 0$. From Assumption 3(b) we know that $e'Be = 0$ implies that $e^i = 0$ or $e^i = w^i$ for $i=1,\ldots,N-1$, and also that $\hat{e}^N = 0$ since $\hat{\nabla}^2 g^N$ has full rank. Since $e \neq 0$ there must be some goods $i \neq N$ for which $e^i = w^i$, and let us number these goods as $i=1,\ldots,L$ with $L < N$. Then evaluating e'Ce we obtain:

$$e'Ce = -\sum_{i=1}^{L}\sum_{j=1}^{L}\frac{\partial z^i}{\partial p^j}(w^{i\prime}\nabla g^i)(\nabla g^{j\prime}w^j) = -\sum_{i=1}^{L}\sum_{j=1}^{L}\frac{\partial z^i}{\partial p^j}p^ip^j$$

which is positive from Assumption 3(a). This contradicts our hypothesis that $e'(A+B+C)e=0$ with $e\neq 0$, and so we conclude that the Jacobian is positive definite. Thus, there exists a continuously differentiable function $\hat{w}(\lambda)$ in a neighborhood of $\lambda=0$. Substituting this function into (10b) we obtain $R(\lambda)$ with $R(0)=R_0>0$. By choosing $\lambda>0$ sufficiently small, we obtain $R(\lambda)>0$ and the strict inequalities in (10c) will continue to hold. Therefore, the TAA equilibrium with $R>0$ and $0<\lambda\leq 1$ is obtained. QED

Proof of Proposition 2'

Continuing from the proof of Lemma 2, use the Implicit Function Theorem to evaluate $\partial\hat{w}/\partial\lambda$ at $\lambda=0$ as:

$$\left.\frac{\partial \hat{w}}{\partial \lambda}\right|_{\lambda=0} = -(B+C)^{-1}[\hat{\nabla}h_0(\hat{w},\tilde{w}^N_{MN}) - \hat{\nabla}h_0(w^a)],$$

since $A=0$ at $\lambda=0$. Then using (10b) we obtain,

$$\left.\frac{\partial R}{\partial \lambda}\right|_{\lambda=0} = \hat{\nabla}h_0(\tilde{w})'(B+C)^{-1}[\hat{\nabla}h_0(\hat{w},\tilde{w}^N_{MN}) - \hat{\nabla}h_0(w^a)], \tag{A3}$$

since $\hat{\nabla}_I h_0(\hat{w},\tilde{w}^N_{MN}) = \hat{\nabla}h_0(\hat{w},\tilde{w}^N_{MN})$. Let S^a denote the set of industries to which h_0-type individuals supply some factor in autarky, and $\tilde{S}$ the set of industries to which h_0 supplies some factor at the DN wages $\tilde{w}$. Then with $S^a\cap\tilde{S}=\emptyset$ and $\partial z^i/\partial p^k = 0$ for $i\in S^a$ and $k\in\tilde{S}$, the matrix C is block-diagonal between the factors supplied to industries S^a and $\tilde{S}$. Since the column vectors $\hat{\nabla}h_0(\hat{w},\tilde{w}^N_{MN})$ and $\hat{\nabla}h_0(w^a)$ do not have any row which is positive in both vectors, we can rewrite (A3) as,

$$\left.\frac{\partial R}{\partial \lambda}\right|_{\lambda=0} = \hat{\nabla} h_0(\tilde{w})'(B+C)^{-1}\hat{\nabla} h_0(\tilde{w}),$$

which is strictly positive since $B+C$ is positive definite as argued in Lemma 2. It follows that $R(\lambda)>0$ for $\lambda>0$ sufficiently small. QED

Proof of Proposition 3

The equilibrium conditions are specified as (A1) and (A2), and choose a small value of $\lambda_1>0$ such that the equilibrium exists with wages $w_1 \equiv w(\lambda_1)$. Corresponding to this equilibrium is a vector of consumption taxes given by $t_1 \equiv p^a - g(w_1)$. Then keeping λ fixed at λ_1, and using the methods in Lemma 2, we solve (A1) and (A2) for a continuously differentiable function $w(t)$ in a neighborhood of t_1. Then $w(t_1)=w_1$ by construction, and at this equilibrium the government budget is positive while (10b) and (10c) hold. Now consider a small reduction in the consumption taxes satisfying $dt \leq 0$ and $dt'c^a<0$. This will lead to a change in wages of $(\partial w/\partial t)dt$, so the new value of the relocation subsidy is calculated from (10b). By choosing dt sufficiently small, the inequalities in (10c) will continue to hold, and the government budget will remain positive. Since $dt'c^a<0$, consumption taxes have been reduced for some goods consumed in positive amounts by domestic individuals, and so these individuals experience an increase in welfare. QED

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Figure 1

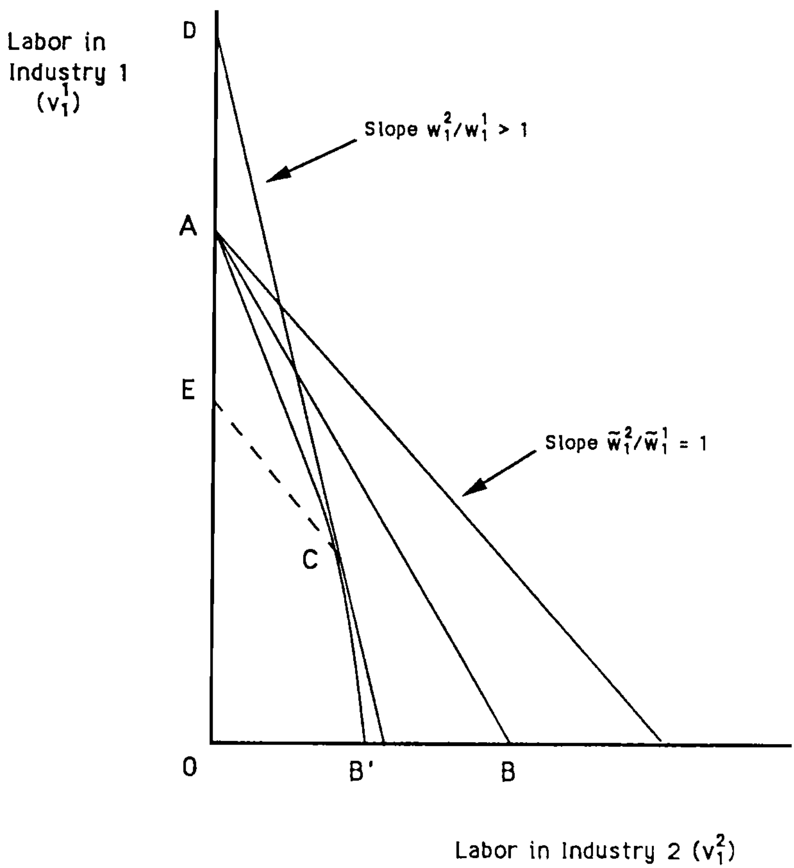
Labor in
Industry 1
(v_1^1)
D
Slope $w_1^2/w_1^1 > 1$
A
E
Slope $\tilde{w}_1^2/\tilde{w}_1^1 = 1$
C
0
B'
B
Labor in Industry 2 (v_1^2)

Figure 2