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ENDOGENOUS CAPITAL UTILIZATION AND PRODUCTIVITY MEASUREMENT IN DYNAMIC FACTOR DEMAND MODELS: THEORY AND AN APPLICATION TO THE U.S. ELECTRICAL MACHINERY INDUSTRY

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ABSTRACT

Studies of the firm's demand for factor inputs often assume a constant rate of utilization of the inputs and ignore the fact that the firm can simultaneously choose the level and the rate of utilization of its inputs. In particular, the literature on dynamic factor demand models has, until recently, largely overlooked the issue of capital utilization and/or did not distinguish carefully between the distinct concepts of capital and capacity utilization. In this paper we allow for variations in the rate of capital utilization within the context of a dynamic factor demand model by adopting a modeling framework within which the firm combines its beginning-of-period stocks with other inputs to produce its outputs as well as its end-of-period stocks. We also derive measures of productivity and capacity utilization for the adopted modeling framework. Given the depreciation rate is endogenous a consistent capital stock series must be generated during estimation from the investment data. This yields, as a byproduct, a consistent decomposition of gross investment into replacement and expansion investment. As an illustration, the model is applied to U.S. Electrical Machinery data.

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1. Introduction[1]

In the last two decades a large body of literature emerged regarding the utilization of capital.[2] Still, studies of the firm's demand for factor inputs often assume a constant rate of utilization of the inputs and ignore the fact that the firm can simultaneously choose the level and the rate of utilization of its inputs. In particular, the literature on dynamic factor demand models has, until recently, largely overlooked the issue of capital utilization, and/or did not distinguish carefully between the distinct concepts of capital and capacity utilization.

In this paper we allow for variations in the rate of capital utilization within the context of a dynamic factor demand model by adopting a modeling framework in which the firm combines its beginning-of-period stocks with other inputs to produce its outputs as well as its end-of-period stocks. This modeling framework goes back to Hicks (1946), Malinvaud (1953) and Diewert (1977,1980). In the literature on dynamic factor demand models this framework was first adopted by Epstein and Denny (1980) and more recently by Kollintzas

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2 Most of this literature focused on the firm's long-run decision. Excellent summaries (including important extensions) of this literature are given in Betancourt (1987), Betancourt and Clague (1981) and Winston (1974, 1982).

and Choi (1985) and Bernstein and Nadiri (1987a,b).[3] Only the first two papers implement the model empirically. In contrast to Epstein and Denny (1980) we model and estimate not only the firm's demand for its variable factors, but also the demand for its quasi-fixed factors. Kollintzas and Choi's (1985) model differs from ours in that adjustment costs are modeled as external. In contrast to both studies we allow for more than one quasi-fixed factor. The quasi-fixed factors may become productive immediately or with a lag. (Apart from those general modeling differences, these studies also differ from the current one in terms of the actual empirical specification, implementation, and detail of the analysis of the empirical results.)

To facilitate a full interpretation of the empirical results it seems of interest to also report estimates of technical change. Consequently we develop new measures of technical and capacity utilization based on the adopted modeling framework (which allows for temporary equilibrium and for the endogenous determination of the depreciation rate). The obtained productivity measures generalize the productivity measures for multiple output technologies introduced recently by Berndt and Fuss (1989). Furthermore, we give a decomposition of the traditional measure of total factor productivity growth into technical change, scale effect, adjustment cost effect, and the variable

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3 Bischoff and Kokkelenberg (1987) adopt a related framework where the depreciation rate is modeled as a function of capacity utilization. Other contributions to the literature on dynamic factor demand models that allow for the firm to operate at different levels of utilization, but are based on alternative modeling frameworks, include papers by Nadiri and Rosen (1969), Abel (1981), Bernstein (1983), Kokkelenberg (1984), Honkapohija and Kanniainen (1985), and Shapiro (1986).

depreciation effect.

Given the depreciation rate is endogenous, existing capital stock series cannot be employed for estimation and a consistent capital stock series must be generated during estimation of the model from gross investment data. Thus, as a byproduct, we generate alternative capital stock series that can be contrasted with "official" capital stock estimates. We also obtain a consistent decomposition of gross investment into replacement and expansion investment which is important from the vantage point of public policy analysis.

To illustrate the various features of the model we have estimated the model using data from the U.S. electrical machinery industry. We compare those estimates with those obtained by Nadiri and Prucha (1989a) from a model with an exogenously given capital depreciation rate.

The paper is organized as follows. The theoretical specification of the model is presented in Section 2. Both primal and dual measures of technical change for multi-product firms, and measures of capacity utilization are developed in Section 3. In this section we also explore the traditional measure of total factor productivity growth as a measure of technical change in more detail and identify sources of (possible) bias. In Section 4 we give an empirical specification of the model and apply this model to U.S. electrical machinery data. We present estimates of the model parameters, price and output elasticities, estimates of technical change and scale, as well as the internally generated capital stock series and the decomposition of gross investment into replacement and expansion investment. We also give a decomposition of the traditional measure of total factor productivity growth. Concluding remarks are given in Section 5. Most of the underlying mathematic derivations are relegated to several appendices.

2. Theoretical Model Specification[4]

2.1 Technology and Stochastic Closed Loop Feedback Control Policy

Consider a firm that produces a set of outputs from a set of variable inputs and a set of quasi-fixed inputs. We distinguish between quasi-fixed factors whose depreciation rates can be chosen by the firm and those whose depreciation rates are exogenous to the firm. To keep the theoretical discussion general we also allow for some quasi-fixed factors to become immediately productive and for some to become productive with a lag. More specifically, we assume that the firm's technology can be represented by a factor requirement function of the form

$$(2.1) \qquad M_t = M(Y_t, L_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t),$$

with $\underline{K}_t = \lambda^K K_t + (I-\lambda^K)K_{t-1}$ and $\underline{R}_t = \lambda^R R_t + (I-\lambda^R)R_{t-1}$. Here Y_t denotes the vector of output goods. $V_t = [M_t, L_t^T]^T$ denotes the vector of the variable inputs. K_t is the vector of the end-of-period stocks whose depreciation rates can be chosen endogenously by the firm. More specifically, let K_t^o denote the vector of "old" stocks left over at the end of period t from K_{t-1}; it is then assumed that the firm can choose the level of K_t^o by e.g. choosing

4 The subsequent discussion makes use of the following notational conventions (unless explicitly indicated otherwise): Let Z_t be some kx1 vector of goods in period t, then p_t^Z refers to the corresponding kx1 price vector; Z_{ti} and p_{ti}^Z denote the i-th elements of Z_t and p_t^Z, respectively. Furthermore, in the following we often write $(p_t^Z)^T Z_t$ for $\sum_{i=1}^k p_{ti}^Z Z_{ti}$ where the superscript "T" stands for "transpose".

appropriate levels of maintenance. Of course, this is equivalent to choosing the rate of depreciation for respective stocks, since we can always write $K_t^o = (1-\delta_t^K)K_{t-1}$ and interpret δ_t^K as a diagonal matrix of depreciation rates. R_t is the vector of the end-of-period stocks whose depreciation rates are exogenous to the firm. $\underline{K}_t$ and $\underline{R}_t$ denote the vectors of productive stocks and λ^K and λ^R are diagonal matrices where the diagonal elements lie between zero and unity. If a diagonal element is one, then the corresponding quasi-fixed factor becomes immediately productive; if a diagonal element is zero, then the corresponding quasi-fixed factor becomes productive with a one-period lag.[5] The vectors $\Delta K_t = K_t - K_{t-1}$ and $\Delta R_t = R_t - R_{t-1}$ represent internal adjustment costs in terms of foregone output due to changes in the quasi-fixed factors. The variable T_t represents an index of technology.

The stocks K_t and R_t accumulate according to the following equations:

$$(2.2) \qquad K_t = I_t^K + K_t^o, \qquad\qquad R_t = I_t^R + (I - \delta_t^R)R_{t-1},$$

where I_t^K and I_t^R denote the respective vectors of gross investment and δ_t^R denotes the diagonal matrix of exogenous depreciation rates (some of which may be zero).

We assume that it is the firm's objective to minimize the expected present value of its future cost stream.[6] More specifically, we assume that

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5 For example, let K_{t1} and $\underline{K}_{t1}$ denote the first element of K_t and $\underline{K}_t$, respectively, and let λ_1^K denote the first diagonal element of the matrix λ^K. Then $\underline{K}_{t1} = \lambda_1^K K_{t1} + (1-\lambda_1^K)K_{t-1,1}$ equals K_{t1} if $\lambda_1^K=1$ and equals $K_{t-1,1}$ if $\lambda_1^K=0$.

6 We note that the subsequent theoretical discussion can be readily

the firm's objective function is given by

$$
\begin{aligned}
(2.3)\qquad & E_t\sum_{\tau=t}^{\infty}[(p_\tau^V)^T V_\tau+(p_\tau^K)^T K_\tau+(p_\tau^R)^T R_\tau+(q_\tau^K)^T I_\tau^K+(q_\tau^R)^T I_\tau^R]\prod_{s=t}^{\tau}(1+r_s)^{-1} = \\
& E_t\sum_{\tau=t}^{\infty}[M(Y_\tau,L_\tau,K_\tau^o,\underline{K}_\tau,\underline{R}_\tau,\Delta K_\tau,\Delta R_\tau,T_\tau)+(p_\tau^L)^T L_\tau-(q_\tau^K)^T K_\tau^o \\
& \qquad +(p_\tau^K)^T K_\tau+(p_\tau^R)^T R_\tau+(q_\tau^K)^T K_\tau+ (q_\tau^R)^T [R_\tau-(I-\delta_\tau^R)R_{\tau-1}]]\prod_{s=t}^{\tau}(1+r_s)^{-1} ,
\end{aligned}
$$

where the expression on the r.h.s. was obtained utilizing (2.1) and (2.2). E_t denotes the expectations operator conditional on the set of information available in period t. The information set is assumed to include all lagged stocks and all current and lagged exogenous variables. The firm is assumed to face perfectly competitive markets with respect to its factor inputs. For reasons of generality we distinguish between the price (cost) associated with operating the stocks, p_τ^K and p_τ^R, and the price of new investment goods after taxes, q_τ^K and q_τ^R, possibly normalized by $1-u_\tau$ where u_τ denotes the corporate tax rate.[7] (We assume that at least one price in each

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modified to also apply to the case of a profit maximizing firm.

[7] As an illustration, suppose R is a scalar and corresponds to the number of non-production workers; then p^R may represent total compensation per non-production worker with q^R equal to zero. As a further illustration, suppose K is a scalar and corresponds to the stock of a certain capital good; then p^K may represent the insurance cost and q^K may equal $[1-c-u(1-mc)B]p^{IK}/(1-u)$, where p^{IK} denotes the price of new investment goods, u denotes the corporate tax rate, c is the rate of the investment tax credit, m is the portion of the investment tax credit which reduces the depreciable base for tax purposes, and B is the present value of the depreciation allowances.

corresponding pair is positive.) All prices in period τ are taken to be normalized by the price of the variable factor M_τ. With r_τ we denote the real discount rate (which may possibly also incorporate variations in the corporate tax rate).

Assume that the firm follows a stochastic closed loop feedback control policy in minimizing the expected present value of its future cost stream (2.3). Then in period t the firm will choose optimal values for its current inputs L_t, K_t, R_t, and for K_t^o. At the same time the firm will choose a contingency plan for setting L_τ, K_τ, R_τ, and K_τ^o in periods $\tau=t+1,t+2,...$ optimally, depending on observed realizations of the exogenous variables and past choices for the quasi-fixed factors. Of course, for given optimal values for L_τ, K_τ, R_τ, and K_τ^o the optimal values for M_τ is implied by (2.1). Prices, output and the discount rate are assumed to be exogenous to the firm's optimization problem.

Since L_τ and K_τ^o can be changed without adjustment costs the stochastic closed loop feedback control solution can be found conveniently in two steps. In the first step we minimize the total (normalized) cost in each period τ with respect to L_τ and K_τ^o for given values of the quasi-fixed factors and the exogenous variables. Substitution of the minimized expressions into (2.3) then leads in the second step to an optimal control problem that only involves the the quasi-fixed factors K_τ and R_τ.

The part of total cost that actually depends on L_τ and K_τ^o is given by $M(Y_\tau,L_\tau,K_\tau^o,\underline{K}_\tau,\underline{R}_\tau\Delta K_\tau,\Delta R_\tau,T_\tau) + (p_\tau^L)^T L_\tau - (q_\tau^K)^T K_\tau^o$, i.e. variable cost minus the value of the "old" stocks left over at the end of the period from the beginning of period stocks. Assuming that M(.) is differentiable and that a unique interior minimum of the above expression exists, the first order conditions for that minimum are given by:

$$(2.4) \qquad \partial M_\tau/\partial L_\tau + p^L_\tau = 0, \qquad \partial M_\tau/\partial K^o_\tau - q^K_\tau = 0.$$

Let $\hat{L}_\tau$ and $\hat{K}^o_\tau$ denote the minimizing vectors and define

$$(2.5) \qquad G_\tau = G(p^L_\tau, q^K_\tau, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) = \hat{M}_\tau + (p^L_\tau)^T\hat{L}_\tau - (q^K_\tau)^T\hat{K}^o_\tau ,$$

with $\hat{M}_\tau = M(Y_\tau, \hat{L}_\tau, \hat{K}^o_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau)$. The function $G(.)$ has the interpretation of a normalized variable cost function net of the value of the "old" stocks. Technically it can be viewed as the negative of a normalized restricted profit function. For duality results between factor requirement functions and normalized variable profit functions see, e.g., Diewert (1982) and Lau (1976). We assume that the function G(.) is twice continuously differentiable in all its arguments, homogeneous of degree zero in p^L and q^K, non-decreasing in Y, $|\Delta K|$, $|\Delta R|$ and p^L, non-increasing in $\underline{K}$, $\underline{R}$ and q^K, concave in p^L and q^K, and convex $\underline{K}$, $\underline{R}$, ΔK and ΔR.

As indicated above, the stochastic closed loop optimal control solution for $\{K_\tau, R_\tau\}^\infty_{\tau=0}$ can now be found by replacing $M_\tau + (p^L_\tau)^T L_\tau - (q^K_\tau)^T K^o_\tau$ in (2.3) by G_τ and by minimizing

$$(2.6) \qquad E_t \sum^\infty_{\tau=t} [G(p^L_\tau, q^K_\tau, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p^K_\tau)^T K_\tau + (p^R_\tau)^T R_\tau + (q^K_\tau)^T K_\tau + (q^R_\tau)^T [R_\tau - (I-\delta^R_\tau) R_{\tau-1}]] \prod^\tau_{s=t} (1+r_s)^{-1}$$

with respect to the quasi-fixed factors only. Standard control theory implies that the optimal control solution for $\{K_\tau, R_\tau\}^\infty_{\tau=0}$ that minimizes (2.6) must satisfy the following set of stochastic Euler equations ($\tau=t,\ldots,\infty$):[8]

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8 Compare, e.g., Stockey, Lucas and Prescott (1989), ch.9, for a more

$$(2.7)\quad -\lambda_K\frac{\partial G_\tau}{\partial K_\tau} - (I-\lambda_K)E_\tau\frac{\partial G_{\tau+1}}{\partial K_{\tau+1}}/(1+r_{\tau+1}) = p_\tau^K+q_\tau^K + \left\{\frac{\partial G_\tau}{\partial \Delta K_\tau} - E_\tau\frac{\partial G_{\tau+1}}{\partial \Delta K_{\tau+1}}/(1+r_{\tau+1})\right\}$$

$$(2.8)\quad -\lambda_R\frac{\partial G_\tau}{\partial R_\tau} - (I-\lambda_R)E_\tau\frac{\partial G_{\tau+1}}{\partial R_{\tau+1}}/(1+r_{\tau+1}) = p_\tau^R+c_\tau^R + \left\{\frac{\partial G_\tau}{\partial \Delta R_\tau} - E_\tau\frac{\partial G_{\tau+1}}{\partial \Delta R_{\tau+1}}/(1+r_{\tau+1})\right\}$$

where $c_\tau^R = E_\tau[q_\tau^R(1+r_{\tau+1})-(I-\delta_\tau^R)q_{\tau+1}^R]/(1+r_{\tau+1})$ has the interpretation of a vector of rental prices. The optimal values for L_τ and K_τ^o can be found by differentiating (2.6) with respect to p_τ^L and q_τ^K and making use of (2.5), i.e. via Shephard's and Hotelling's lemma:[9]

$$(2.9)\quad L_\tau = \partial G_\tau/\partial p_\tau^L ,$$

$$(2.10)\quad K_\tau^o = -\partial G_\tau/\partial q_\tau^K .$$

2.2 Interpretation of Optimality Conditions

We first compare the structure of the two sets of stochastic Euler equations for the quasi-fixed factors K_τ and R_τ with endogenous and exogenous depreciation rates, respectively. For this purpose observe that upon denoting the normalized variable cost by

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detailed list of assumptions and a careful exposition of stochastic control theory, as well as for a discussion of the transversality condition. An explicit solution and a more detailed list of assumptions for the case where G(.) is linear quadratic will be given in Section 4.

[9] In case of a profit maximizing model we have furthermore the following condition for the output vector: $\partial G_\tau/\partial Y_\tau = p_\tau^Y + [\partial p_\tau^Y/\partial Y_\tau]Y_\tau$.

(2.11) $$g_\tau = g(p^L_\tau, q^K_\tau, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) = \hat{M}_\tau + (p^L_\tau)^T \hat{L}_\tau ,$$

we have $G_\tau = g_\tau - q^{K'}_\tau \hat{K}^o_\tau$. To simplify the discussion also assume temporarily that all quasi-fixed factors become productive with a lag, i.e. $\lambda_K = 0$ and $\lambda_R = 0$ and $\underline{K}_\tau = K_{\tau-1}$ and $\underline{R}_\tau = R_{\tau-1}$. We can then rewrite (2.7) and (2.8) as:

(2.7') $$-E_\tau \frac{\partial g_{\tau+1}}{\partial K_\tau}/(1+r_{\tau+1}) + E_\tau \frac{\partial \hat{K}^o_{\tau+1}}{\partial K_\tau} q^K_{\tau+1}/(1+r_{\tau+1}) = p^K_\tau + q^K_\tau + \frac{\partial G_\tau}{\partial \Delta K_\tau} - E_\tau \frac{\partial G_{\tau+1}}{\partial \Delta K_{\tau+1}}/(1+r_{\tau+1})$$

(2.8') $$-E_\tau \frac{\partial g_{\tau+1}}{\partial R_\tau}/(1+r_{\tau+1}) + E_\tau \frac{\partial \hat{K}^o_{\tau+1}}{\partial R_\tau} q^K_{\tau+1}/(1+r_{\tau+1}) + (I-\delta^R_\tau) E_\tau q^R_{\tau+1}/(1+r_{\tau+1}) = p^R_\tau + q^R_\tau + \frac{\partial G_\tau}{\partial \Delta R_\tau} - E_\tau \frac{\partial G_{\tau+1}}{\partial \Delta R_{\tau+1}}/(1+r_{\tau+1})$$

Now define $R^o_t = (I-\delta^R_t)R_{t-1}$ analogously to K^o_t as the vector of "old" stocks left over at the end of the period from R_{t-1}. We then have $(I-\delta^R_{\tau+1})E_\tau q^R_{\tau+1}/(1+r_{\tau+1}) = E_\tau[\partial R^o_{\tau+1}/R_\tau] q^R_{\tau+1}/(1+r_{\tau+1})$ and $\partial R^o_{\tau+1}/\partial K_\tau = 0$. Substitution of those expressions into (2.7') and (2.8') shows that - expectedly although not immediately obvious from (2.7) and (2.8) - the two sets of first order conditions have the same basic structure. Of course, for the special case where the depreciation rates of K are exogenous, i.e. $K^o_t = (I-\delta^K_t)K_{t-1}$ with δ^K_t given, we expect the form of the stochastic Euler equations for K to reduce to that for R. To see this observe that in this case $E_\tau[\partial \hat{K}^o_{\tau+1}/K_\tau] q^K_{\tau+1}/(1+r_{\tau+1}) = (I-\delta^K_{\tau+1}) E_\tau q^K_{\tau+1}/(1+r_{\tau+1})$ and $E_\tau[\partial \hat{K}^o_{\tau+1}/R_\tau] q^K_{\tau+1}/(1+r_{\tau+1}) = 0$.

Based on (2.7') and (2.8') we can give the following economic interpretation of the stochastic Euler equations: The optimizing firm invests in the quasi-fixed factors K and R until, at the margin (and properly discounted), the reduction in the variable cost plus the increase in the value of the "old" stocks K^o and R^o in the next period equals the price (cost) of operating the quasi-fixed factor plus the acquisition price plus current period adjustment costs minus the expected adjustment cost that would have occurred if the investment would be undertaken in the next period (rather than the current one).

We note that the above discussion can be readily extended to the general case where some of the quasi-fixed factors may become productive immediately and some with a lag. For the purpose of further interpretation of the stochastic Euler equations consider the transformation function

$$(2.12) \qquad F(Y_t, M_t, L_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t) = 0 \ ,$$

such that $F(Y_t, M(Y_t, L_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t), L_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t) \equiv 0$; for example $F(.) = M(.) - M$. We assume that the function $F(.)$ is twice continuously differentiable in all its arguments. It follows readily from (2.4), (2.5) and (2.12) that $-\partial G/\partial Z = [\partial F/\partial Z]/[\partial F/\partial M]$ for $Z = \underline{K}, \underline{R}, \Delta K, \Delta R$; cp. also Section 3. Hence we can interpret the derivatives $-\partial G/\partial Z$ appearing in the stochastic Euler equations as marginal rates of transformation between the factors Z and the variable factor M.

2.3 Certainty Equivalence Feedback Control Policy

The formulation of a stochastic closed loop control policy generally requires knowledge of the entire distribution of the exogenous variables. Alternatively one may hence postulate that the firm formulates a certainty equivalence feedback control policy, which only requires knowledge of the first moment (mean) of the exogenous variables. In that case the firms objective function is given by (2.3) or (2.6) with the expectations operator moved next to each of the exogenous variables. The firm would now devise in each period t an optimal plan for its inputs in periods t, t+1,... such that its objective function in period t is optimized and choose its inputs in period t accordingly. In each future period the firm will revise its expectations and optimal plan for its inputs based on new information. In case of a certainty equivalence policy the first order conditions for the optimal plan (in period t) for quasi-fixed factors would be given by (2.7) and (2.8) with all exogenous variables replaced by there expected values (conditional on information available at time t and the expectations operator in front of the respective derivatives suppressed). Equations (2.9) and (2.10) remain the same. If G(.) is linear-quadratic, then the well known certainty equivalence principle implies that the closed loop and the certainty equivalence feedback control policy are identical.

3. Generalized Measures of Technological Characteristics

3.1 Primal and Dual Measures of Technical Change

The traditional measure of total factor productivity assumes, in particular: (1) that producers are in long-run equilibrium, (2) that the technology exhibits constant returns to scale, (3) that output and input markets are perfectly competitive, and (4) that factors are utilized at a constant rate. The puzzle of the observed slowdown of productivity growth during the 1970's has initiated a critical methodological review of the traditional measures of productivity.[10]

The model considered here relaxes all of the above listed assumptions that correspond to the traditional measures of productivity. In the following we define, within the context of our model, appropriate measures of technical change. We first define those measures in terms of the firm's transformation function, F(.), and then show how those measures can be evaluated from the normalized variable cost function net of the value of the "old" stocks, G(.). We also discuss corresponding index number formulae.

As a byproduct of our discussion we also define measures of capacity utilization. Those measures generalize corresponding measures considered in Berndt and Fuss (1981, 1986, 1989), Morrison (1985a,b, 1986) and Nadiri and

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[10] Cp., e.g, Berndt and Fuss (1981, 1986, 1989), Bernstein and Mohnen (1988), Caves, Christensen and Swanson (1980, 1981), Denny, Fuss and Waverman (1981a), Griliches (1988), Hulten (1986), Mohnen, Nadiri and Prucha (1983), Morrison (1983, 1985a,b, 1986, 1989), Nadiri and Prucha (1983, 1984, 1989a,b) and Nadiri and Schankerman (1981a,b).

Prucha (1989b), in that the model considered here not only allows for the firm to choose its factor inputs but also its rate of factor utilization optimally.

For ease of notation we drop in the following time-subscripts whenever those subscripts are obvious from the context. Assume that the technology index T shifts by, say, Δ. Let $a = a(\Delta,Y,V,K^o,\underline{K},\underline{R},\Delta K,\Delta R,T)$ be the proportionality factor by which all outputs Y can be increased corresponding to this shift in technology such that the firm remains on its production surface, i.e. $F(aY,V,K^o,\underline{K},\underline{R},\Delta K,\Delta R,T+\Delta) = 0$. Similarly, let $b = b(\Delta,Y,V,K^o,\underline{K},\underline{R},\Delta K,\Delta R,T)$ be the proportionality factor by which all inputs and K^o can be decreased corresponding to this shift in technology such that the firm remains on its production surface, i.e. $F(Y,bV,bK^o,b\underline{K},b\underline{R},b\Delta K,b\Delta R,T+\Delta) = 0$. Furthermore let $c = c(\kappa,Y,V,K^o,\underline{K},\underline{R},\Delta K,\Delta R,T)$ be the proportionality factor by which all outputs Y can be increased corresponding to an increase in all inputs and K^o by a factor κ such that the firm remains on its production surface, i.e. $F(cY,\kappa V,\kappa K^o,\kappa\underline{K},\kappa\underline{R},\kappa\Delta K,\kappa\Delta R,T) = 0$. We can now give the following two definitions of productivity growth, λ_Y and λ_X, and returns to scale, ρ:

$$
\begin{aligned}
(3.1)\qquad \lambda_Y &= \frac{\partial a}{\partial\Delta}\Big|_{\Delta=0} = -\frac{\partial F}{\partial T}\Big/\Big[\textstyle\sum_i(\partial F/\partial Y_i)Y_i\Big],\\
\lambda_X &= -\frac{\partial b}{\partial\Delta}\Big|_{\Delta=0} = \frac{\partial F}{\partial T}\Big/\Big[\textstyle\sum_j(\partial F/\partial V_j)V_j + \sum_k(\partial F/\partial\underline{K}_k)\underline{K}_k + \sum_l(\partial F/\partial\underline{R}_l)\underline{R}_l +\\
&\qquad \textstyle\sum_k(\partial F/\partial K^o_k)K^o_k + \sum_k(\partial F/\partial\Delta K_k)\Delta K_k + \sum_l(\partial F/\partial\Delta R_l)\Delta R_l\Big],\\
\rho &= \frac{\partial c}{\partial\kappa}\Big|_{\kappa=1} = \lambda_Y/\lambda_X,
\end{aligned}
$$

We refer to λ_Y and λ_X as the rates of, respectively, output and input based productivity growth or technical change. The definitions given above are analogous to those given in Caves, Christensen and Swanson (1981) and Caves, Christensen and Diewert (1982a,b) for technologies without explicit adjustment costs and constant factor utilization rates.

We next show how the above measures can be evaluated from the cost side. Observe from (2.1) and (2.12) that $\partial F/\partial Z = -[\partial M/\partial Z][\partial F/\partial M]$ for $Z = Y, L, \underline{K}, \underline{R}, K^o, \Delta K, \Delta R, T$. Furthermore observe from (2.4) and (2.5) that $\partial G/\partial Z = \partial M/\partial Z$, and hence

$$(3.2) \qquad \partial F/\partial L = p^L[\partial F/\partial M], \quad \partial F/\partial K^o = -q^K[\partial F/\partial M], \quad \partial F/\partial Z = -[\partial G/\partial Z][\partial F/\partial M],$$

for $Z = Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T$. Therefore we can write the above expressions for technical change and returns to scale alternatively in terms of the normalized variable cost function net of the "old" stocks, G, as:

$$(3.3) \qquad \lambda_Y = -[\partial G/\partial T]/[\textstyle\sum_i(\partial G/\partial Y_i)Y_i],$$

$$\lambda_X = -[\partial G/\partial T]/[G - \textstyle\sum_k(\partial G/\partial \underline{K}_k)\underline{K}_k - \sum_l(\partial G/\partial \underline{R}_l)\underline{R}_l - \sum_k(\partial G/\partial \Delta K_k)\Delta K_k - \sum_l(\partial G/\partial \Delta R_l)\Delta R_l],$$

$$\rho = \lambda_Y/\lambda_X = [G - \textstyle\sum_k(\partial G/\partial \underline{K}_k)\underline{K}_k - \sum_l(\partial G/\partial \underline{R}_l)\underline{R}_l - \sum_k(\partial G/\partial \Delta K_k)\Delta K_k - \sum_l(\partial G/\partial \Delta R_l)\Delta R_l]/[\sum_i(\partial G/\partial Y_i)Y_i],$$

The total shadow cost is defined as

$$(3.4) \qquad C = G - \textstyle\sum_k(\partial G/\partial \underline{K}_k)\underline{K}_k - \sum_l(\partial G/\partial \underline{R}_l)\underline{R}_l - \sum_k(\partial G/\partial \Delta K_k)\Delta K_k - \sum_l(\partial G/\partial \Delta R_l)\Delta R_l$$

where $-\partial G/\partial \underline{K}_k$, $-\partial G/\partial \underline{R}_l$, $-\partial G/\partial \Delta K_k$, and $-\partial G/\partial \Delta R_l$ denote the respective shadow prices. For given shadow prices we have $\partial G/\partial T = \partial C/\partial T$ and hence it follows from (3.3) that $\lambda_X = -[\partial G/\partial T]/C = -[\partial C/\partial T]/C$.

The above expressions for output based and input based technical change and returns to scale generalize those previously given in Nadiri and Prucha (1983, 1984, 1989a,b) for single output technologies with adjustment costs, but constant factor depreciation rates:[11] To see this recall that according to (2.5) and (2.11) we have $G = g - \sum_k q_k^K \hat{K}_k^o$ where g represents the (standard) normalized variable cost function. Hence $\partial G/\partial Z = \partial g/\partial Z - \sum_k q_k^K \partial \hat{K}_k^o/\partial Z$ for $Z = Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T$ and therefore

$$(3.4') \qquad C = g - \textstyle\sum_k (\partial g/\partial \underline{K}_k)\underline{K}_k - \sum_l (\partial g/\partial \underline{R}_l)\underline{R}_l - \sum_k (\partial g/\partial \Delta K_k)\Delta K_k - \sum_l (\partial g/\partial \Delta R_l)\Delta R_l$$
$$- \textstyle\sum_k q_k^K (\hat{K}_k^o - \tilde{K}_k^o),$$
$$\tilde{K}_k^o = \textstyle\sum_s (\partial \hat{K}_k^o/\partial \underline{K}_s)\underline{K}_s + \sum_l (\partial \hat{K}_k^o/\partial \underline{R}_l)\underline{R}_l + \sum_s (\partial \hat{K}_k^o/\partial \Delta K_s)\Delta K_s + \sum_l (\partial \hat{K}_k^o/\partial \Delta R_l)\Delta R_l .$$

Suppose $\hat{K}^o$ is proportional to $\underline{K}$, as is the case if $\underline{K} = K_{-1}$ and the corresponding depreciation rates are exogenous constants, then clearly $\partial G/\partial Z = \partial g/\partial Z$ for $Z = Y, \underline{R}, \Delta K, \Delta R, T$ and $\hat{K}^o = \tilde{K}^o$. Therefore, in case of exogenously given depreciation rates, the expressions in (3.3) hold with the normalized variable cost function net of "old" stocks, G, replaced by the normalized variable cost function, g. It is readily checked that the resulting expressions correspond to those reported in Nadiri and Prucha (1983, 1984, 1989a,b); i.e., the expressions for technical change and returns to scale given in (3.3) contain those given in the latter papers as a special case.

The issue of a proper measure of technical change, given that the firm is in short-run or temporary equilibrium but not in long-run equilibrium, has

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[11] Nadiri and Prucha (1989b) also provide expressions for multiple output technologies. We note furthermore, that the algebra adopted here is completely analogous to that used by Caves, Christensen and Swanson (1981) for multiple output technologies without explicit adjustment costs and constant factor utilization rates.

also been discussed by, among others, Berndt and Fuss (1981, 1986, 1989), Hulten (1986), and Morrison (1983, 1986, 1989). Those authors discuss proper measures of technical change in terms of adjustments of traditional technical change measures by utilization rate measures. Berndt and Fuss (1981, 1986) and Hulten (1986) consider single output technologies with constant returns to scale. Morrison also considers single output technologies, but allows for (possibly) non-constant returns to scale and explicitly takes into account adjustment costs. Berndt and Fuss (1989) consider multiple output technologies with (possibly) non-constant returns to scale, but do not explicitly consider adjustment cost. None of those papers considers explicitly the effect of variable factor depreciation rates.

We next relate the productivity measures λ_Y and λ_X derived within our generalized framework to those by Berndt, Fuss, Hulten and Morrison. As shown in Appendix A

$$(3.5) \qquad -\partial G/\partial T = \sum_i (\partial G/\partial Y_i)\dot{Y}_i - [\sum_j p^V_j \dot{V}_j - \sum_k q^K_k \dot{K}^o_k - \sum_k (\partial G/\partial \underline{K}_k)\dot{\underline{K}}_k - \sum_l (\partial G/\partial \underline{R}_l)\dot{\underline{R}}_l - \sum_k (\partial G/\partial \Delta K_k)\Delta\dot{K}_k - \sum_l (\partial G/\partial \Delta R_l)\Delta\dot{R}_l],$$

where for any variable, say, Z we adopt the notation $\dot{Z} = \partial Z/\partial T$. Since $\lambda_X = -(\partial G/\partial T)/C$ and since for given shadow prices $\partial G/\partial Y = \partial C/\partial Y$, equations (3.4) and (3.5) imply the following index number formula for input based technical change:

$$(3.6) \qquad \lambda_X = \Lambda(Y,V,K^o,\underline{K},\underline{R},\Delta K,\Delta R,T) = \sum_i \varepsilon_i (\dot{Y}_i/Y_i) - [\sum_j s^V_j(\dot{V}_j/V_j) - \sum_k s^O_k(\dot{K}^o_k/K^o_k) + \sum_k s^K_k(\dot{\underline{K}}_k/\underline{K}_k) + \sum_l s^R_l(\dot{\underline{R}}_l/\underline{R}_l) + \sum_k s^{\Delta K}_k(\Delta\dot{K}_k/\Delta K_k) + \sum_l s^{\Delta R}_l(\Delta\dot{R}_l/\Delta R_l)],$$

with

$$\varepsilon_i = (\partial C/\partial Y_i)/(Y_i/C), \qquad s^V_j = p^V_j V_j/C, \qquad s^O_k = q^K_k K^o_k/C,$$
$$s^K_k = (-\partial G/\partial \underline{K})\underline{K}/C, \qquad s^R_l = (-\partial G/\partial \underline{R})\underline{R}/C,$$

$$s_k^{\Delta K} = (-\partial G/\partial \Delta K)\Delta K/C, \qquad s_l^{\Delta R} = (-\partial G/\partial \Delta R)\Delta R/C.$$

Here ε_i denotes the elasticity of the shadow cost with respect to the i-th output, s_j^V is the cost share of the j-th variable input in total shadow costs, and s_l^R, $s_k^{\Delta K}$, $s_l^{\Delta R}$ are the shadow cost shares of $\underline{R}_l$, ΔK_k and ΔR_l in total shadow cost. s_k^K is the shadow cost share corresponding to the acquisition and operating cost of $\underline{K}_k$, and s_k^O is the share reflecting the value of the k-th "old" stock, K_k^o, in total shadow cost.

Suppose now K^o is proportional to $\underline{K}$, as is implicitly assumed in Berndt and Fuss (1989): Then, as remarked above, $\partial G/\partial Z = \partial g/\partial Z$ for Z = Y, $\underline{R}$, ΔK, ΔR, T; furthermore $\underline{K}$ and K^o grow at the same rate and hence $\sum_k s_k^K(\dot{\underline{K}}_k/\underline{K}_k) - \sum_k s_k^O(\dot{K}_k^o/K_k^o) = \sum_k \sigma_k^K(\dot{\underline{K}}_k/\underline{K}_k)$ with $\sigma_k^K = (-\partial g/\partial \underline{K})\underline{K}/C$. Substituting the latter expressions into (3.6) and suppressing the terms that take adjustment costs explicitly into account then yields Berndt and Fuss' (1989) technical measure for multiple output technologies - denoted in that paper by $\dot{A}_5/A_5$. That is, the input based technical change measure λ_X generalizes the latter technical change measure of Berndt and Fuss (1989). Consequently, it also generalizes the corresponding technical change measures considered in the above listed papers by Berndt, Fuss, Hulten and Morrison for single output technologies.

In general, technical change is a function of respective inputs and outputs. In (3.6) technical change was evaluated at current input and output values. Berndt and Fuss (1989) define, within the context of their model, alternative measures that evaluate technical change at long-run equilibrium input and output values. In Appendix A we present also measures that generalize those latter measures by Berndt and Fuss within the context of the model considered here.

For ease of notation, we assume for the remainder of this section that L, K^{o}, $\underline{K}$, and $\underline{R}$ are scalars (and that all quasi-fixed factors exhibit positive growth).[12] Furthermore, to simplify the discussion we assume that all quasi-fixed factors only become productive with a lag, i.e. $\lambda_K = 0$ and $\lambda_R = 0$ and $\underline{K} = K_{-1}$ and $\underline{R} = R_{-1}$. We also assume that $p^K = 0$ and $p^R = 0$ and that all price expectations are static.

3.2 Measures of Capacity Utilization

For further interpretation of our input based and output based technical change measures consider the following measure of total cost (normalized by the price of the variable factor M):

$$(3.7) \qquad C^{+} = M + p^{L}L + \underline{c}^{K}\underline{K} + \underline{c}^{R}\underline{R}$$

$$= G(p^{L}, q^{K}, Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T) + (1+r)q^{K}\underline{K} + \underline{c}^{R}\underline{R} \ ,$$

where $\underline{c}^{K} = q^{K}(r+\delta^{K})$ and $\underline{c}^{R} = q^{R}(r+\delta^{R})$ denote respective rental prices; the second equality follows given M, L and K^{o} are chosen optimally and observing that $\underline{c}^{K}\underline{K} = (1+r)q^{K}\underline{K} - q^{K}K^{o}$. Now suppose we attempt to measure technical change in terms of the total cost function C^{+} by $\lambda_X^{+} = -(\partial C^{+}/\partial T)/C^{+}$. Observing that $\partial C^{+}/\partial T = \partial G/\partial T$ it follows immediately from (3.3) and (3.4) that

$$(3.8) \qquad \lambda_Y = \rho\lambda_X^{+}(C^{+}/C), \qquad\qquad \lambda_X = \lambda_X^{+}(C^{+}/C).$$

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12 The results generalize trivially to the case where L, K^{o}, $\underline{K}$ and $\underline{R}$ are vectors.

Clearly, in long-run equilibrium C^+ equals C and hence λ_X^+ equals λ_X. In general, however, λ_X^+ differs from λ_X and λ_Y.

We note that the above formulae generalize analogous formulae given, in particular, in Morrison (1983,1986) for adjustment cost technologies in case of a single output good and exogenously given factor depreciation rates. Analogously to Berndt, Fuss, Hulten and Morrison we can interpret C/C^+ as a measures of capacity utilization and we can therefore interpret our input and output based measures for technical change as being derived from λ_X^+ via an adjustment in terms of a capacity utilization measure to account for temporary equilibrium; cp. also Nadiri and Prucha (1989b).[13]

3.3 Sources of Bias in the Traditional Measure of TFP Growth

As indicated by the above discussion the traditional measure of total factor productivity growth, say $\dot{TFP}$, is only a proper measure for technical change under the assumption that returns to scale equal unity, that producers are in long-run equilibrium, that output and input markets are perfectly competitive and factors are utilized at a constant rate. Since the $\dot{TFP}$ measure is used widely as a measure for technical change we provide in the following a decomposition of the $\dot{TFP}$ measure under the less restrictive

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13 Berndt and Fuss (1989, p.10) use the term "input-specific utilization rates "for the ratios of long-run and short-run equilibrium values of the quasi-fixed factors. We note that within the context of our model those ratios can vary over time, although the intensity of their use, as reflected by the corresponding depreciation rates, remains constant. We hence refrain from using this terminology here.

assumptions maintained here to isolate more clearly respective sources of bias. We provide this decomposition for the case of a firm that produces a single output good and $T_t=t$.

Consider the following typical Törnquist approximation for the traditional measure of total factor productivity growth:

$$(3.9a) \qquad \Delta TFP_t = \Delta \ln Y_t - \Delta \ln N_t \ ,$$

where $\Delta \ln Y_t$ denotes the growth rate of output and $\Delta \ln N_t$ denotes the growth rate of a cost share weighted index of aggregate inputs. The index of aggregate inputs, N, is defined by

$$(3.9b) \qquad \Delta \ln N_t = \tfrac{1}{2}[\Delta \ln N_t^t + \Delta \ln N_t^{t-1}]$$

$$\Delta \ln N_t^\tau = \bar{s}^M(\tau)\Delta \ln M_t + \bar{s}^L(\tau)\Delta \ln L_t + \bar{s}^K(\tau)\Delta \ln \underline{K}_t + \bar{s}^R(\tau)\Delta \ln \underline{R}_t ,$$

$$\bar{s}^M(\tau) = M_\tau/C_\tau^+, \qquad \bar{s}^L(\tau) = p_\tau^L L_\tau/C_\tau^+,$$

$$\bar{s}^K(\tau) = \underline{c_\tau^K K_\tau}/C_\tau^+ \qquad \bar{s}^R(\tau) = \underline{c_\tau^R R_\tau}/C_\tau^+,$$

where total cost C^+ is defined in (3.7) and the $\bar{s}^Z$ denote the respective cost shares (Z = M,L,K,R). In Appendix A we show that in general ΔTFP can be decomposed as follows:[14]

$$(3.10) \qquad \Delta TFP_t = \Delta TFP_t^1 + \Delta TFP_t^2 + \Delta TFP_t^3 + \Delta TFP_t^4 + \Delta TFP_t^5 \ ,$$

where

$$\Delta TFP_t^1 = \tfrac{1}{2}\,[\lambda_X(t)+\lambda_X(t-1)] \ ,$$

$$\Delta TFP_t^2 = (1-1/\rho(t))\Delta \ln Y_t \ ,$$

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[14] For ease of notation we give the decomposition under the assumption that $\rho(t)=\rho(t-1)$. Formulae for the case where $\rho(t)$ may differ from $\rho(t-1)$ are given in Appendix A.

$$\Delta TFP_t^3 = \frac{1}{2} \sum_{\tau=t,t-1}[[-\partial G_\tau/\partial \underline{K}_\tau-(1+r)q_\tau^K]\underline{K}_\tau/C_\tau^+][\Delta \ln \underline{K}_t-\Delta \ln N_t^\tau] +$$
$$\frac{1}{2} \sum_{\tau=t,t-1}[[-\partial G_\tau/\partial \underline{R}_\tau-\underline{c}_\tau^R]\underline{R}_\tau/C_\tau^+][\Delta \ln \underline{R}_t-\Delta \ln N_t^\tau],$$
$$\Delta TFP_t^4 = \frac{1}{2} \sum_{\tau=t,t-1}[[-\partial G_\tau/\partial \Delta K_\tau]\Delta K_\tau/C_\tau^+][\Delta \ln \Delta K_t-\Delta \ln N_t^\tau] +$$
$$\frac{1}{2} \sum_{\tau=t,t-1}[[-\partial G_\tau/\partial \Delta R_\tau]\Delta R_\tau/C_\tau^+][\Delta \ln \Delta R_t-\Delta \ln N_t^\tau],$$
$$\Delta TFP_t^5 = \frac{1}{2} \sum_{\tau=t,t-1}[-q_\tau^K K_\tau^o/C_\tau^+][\Delta \ln K_t^o-\Delta \ln K_{t-1}].$$

The first term in the above decomposition of ΔTFP corresponds to actual technical change. The remaining terms decompose the difference between ΔTFP and technical change, i.e. they reflect sources of potential bias of ΔTFP as a measure of technical change. More specifically, the second term reflects scale effects. We note that under increasing returns to scale and positive output growth ΔTFP will overestimate the technical change. The third term reflects the difference in the marginal conditions for the quasi-fixed factors between short and long-run equilibrium due to adjustment cost, i.e. the difference between the shadow price and (long-run) rental price. Suppose the shadow price for a particular quasi-fixed factor exceeds the long-run price used in the computation of ΔTFP. In this case ΔTFP will, ceteris paribus, overestimate the technical change effects given the growth rate of the quasi-fixed input exceeds that of the aggregate input index. The fourth term reflects the direct effect of adjustment costs in the sense that due to the presence of ΔK_t and ΔR_t in the transformation function the growth rates of those terms also enter the decomposition of the output growth rate. The fifth term stems from the fact that the firm can choose the depreciation rate for some of its quasi-fixed factors endogenously. Clearly, in case of a constant depreciation rate K^o and K_{-1} will grow at the same rate and this latter term will be zero.

4. Empirical Application

In the following we apply the above described model to analyze the production structure, factor demand, productivity growth, capacity utilization, and the determinants of the rate of capital depreciation in the U.S. Electrical machinery industry.

4.1 Empirical Specification and Estimation Procedure

For the empirical analysis we specialize the model to two variable inputs, two quasi-fixed factors, and one output good. More specifically, in the following L_t and M_t denote, respectively, labor input and material input, and K_t and R_t denote, respectively, the end of period stocks of physical capital and R&D, and Y_t denotes gross output. The firm can determine the depreciation rate of capital endogenously, while the depreciation rate of R&D is fixed. p_t^L now denotes the price of labor, and q_t^K and q_t^R denote the after tax acquisition price for capital and R&D normalized by the price of material goods, respectively. We assume $p_t^K = p_t^R = 0$ and $r_t \equiv r$.

To model the technology we specify (dropping subscripts t) the following functional form for the normalized variable cost function net of the value of the "old" stocks:

$$
\begin{aligned}
(4.1)\quad & G(p^L, q^K, K_{-1}, R_{-1}, \Delta K, \Delta R, Y, T) = \\
& Y^{1/\rho}\{\alpha_0 + \alpha_L p^L + \alpha_{LT} p^L T + \tfrac{1}{2}\alpha_{K^oK^o}(q^K)^2 + \alpha_{LK^o} p^L q^K + \tfrac{1}{2}\alpha_{LL}(p^L)^2\} + \\
& \alpha_K K_{-1} + \alpha_R R_{-1} + \alpha_{KL} K_{-1} p^L + \alpha_{KK^o} K_{-1} q^K + \\
& \alpha_{RL} R_{-1} p^L + \alpha_{RK^o} R_{-1} q^K + \alpha_{KT} K_{-1} T + \alpha_{RT} R_{-1} T + \\
& Y^{-1/\rho}\{\tfrac{1}{2}\alpha_{KK} K_{-1}^2 + \alpha_{KR} K_{-1} R_{-1} + \tfrac{1}{2}\alpha_{RR} R_{-1}^2 + \tfrac{1}{2}\alpha_{\dot{K}\dot{K}} \Delta K^2 + \tfrac{1}{2}\alpha_{\dot{R}\dot{R}} \Delta R^2\}.
\end{aligned}
$$

For reasons of interpretation of the function G(.) we note that in general (as is not difficult to see) the normalized variable cost function net of the value of the "old" stocks corresponding to a homothetic production function is of the form

$$(4.2) \qquad G_{*}\left(p^{L}, q^{K}, \frac{K_{-1}}{H(Y)}, \frac{R_{-1}}{H(Y)}, \frac{\Delta K}{H(Y)}, \frac{\Delta R}{H(Y)}, T\right) H(Y)$$

where $H(Y)$ is a function in Y and the scale elasticity is then given by $H(Y)/[Y(dH/dY)]$. In case $H(Y) = Y^{1/\rho}$ the technology is homogeneous of degree ρ. Consequently the function G(.) defined in (4.1) can be viewed as a second order approximation of the normalized variable cost function net of the value of the "old" stocks corresponding to some general homogeneous technology (where parameter restrictions such that the marginal adjustment costs at $\Delta K = \Delta R = 0$ are zero have been imposed).[15] The convexity of G(.) in K, R, ΔK, ΔR and the concavity in p^L and q^K implies that $\alpha_{KK} > 0$, $\alpha_{RR} > 0$, $\alpha_{KK}\alpha_{RR} - \alpha_{KR}^2 > 0$, $\alpha_{\dot{K}\dot{K}} > 0$, $\alpha_{\dot{R}\dot{R}} > 0$, $\alpha_{LL} < 0$, $\alpha_{K^oK^o} < 0$, $\alpha_{LL}\alpha_{K^oK^o} - \alpha_{LK^o}^2 > 0$.

We assume that the firm determines its inputs according to a certainty equivalence feedback control policy and holds static expectations on relative prices, output and the technology. As shown in Appendix B, the firm's optimal quasi-fixed factor inputs in period t corresponding to the technology defined

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[15] We note that G(.) defined in (4.1) is a generalization of the normalized variable cost function introduced by Denny, Fuss and Waverman (1981b) and Morrison and Berndt (1981) for constant returns to scale technologies. Nadiri and Prucha (1983, 1989b) generalized the latter function to homothetic technologies. In imposing parameter restrictions such that the marginal adjustment costs are zero for zero net investment we follow that literature.

by (4.1) can then be described by the following accelerator equations:

(4.3a) $\Delta K_t = m_{KK}(K_t^* - K_{t-1}) + m_{KR}(R_t^* - R_{t-1})$,

(4.3b) $\Delta R_t = m_{RK}(K_t^* - K_{t-1}) + m_{RR}(R_t^* - R_{t-1})$,

with

$$\begin{bmatrix} K_t^* \\ R_t^* \end{bmatrix} = - \begin{bmatrix} \alpha_{KK} & \alpha_{KR} \\ \alpha_{KR} & \alpha_{RR} \end{bmatrix}^{-1} \begin{bmatrix} \alpha_K + \alpha_{KT}T_t + \alpha_{KL}\hat{p}_t^L + \hat{q}_t^K(1+r+\alpha_{KK^o}) \\ \alpha_R + \alpha_{RT}T_t + \alpha_{RL}\hat{p}_t^L + \hat{c}_t^R \end{bmatrix} \hat{Y}_t^{1/\rho} ,$$

where expectations are characterized with a carat ($\hat{}$). The accelerator coefficients $M = (m_{ij})_{i,j=K,R}$ have to satisfy the following matrix equation: $BM^2 + (A+rB)M - A = 0$ with $A = (\alpha_{ij})_{i,j=K,R}$ and where B is the diagonal matrix with elements $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ in the diagonal. The firm's demand equations for the variable factors and the firm's optimal choice for the "old" stock (to be left over from the beginning of period capital stock) can be derived from (4.1) as $M_t = G_t - p_t^L L_t + q_t^K K_t^o$, $L_t = \partial G_t/\partial p_t^L$, and $K_t^o = -\partial G_t/\partial q_t^K$:

(4.4a) $$M_t = \{\alpha_0 - \tfrac{1}{2}\alpha_{K^oK^o}(\hat{q}_t^K)^2 - \alpha_{LK^o}\hat{p}_t^L\hat{q}_t^K - \tfrac{1}{2}\alpha_{LL}(\hat{p}_t^L)^2\}\hat{Y}_t^{1/\rho} +$$
$$\alpha_K K_{t-1} + \alpha_R R_{t-1} + \alpha_{KT}K_{t-1}T_t + \alpha_{RT}R_{t-1}T_t +$$
$$\{\tfrac{1}{2}\alpha_{KK}K_{t-1}^2 + \alpha_{KR}K_{t-1}R_{t-1} + \tfrac{1}{2}\alpha_{RR}R_{t-1}^2 + \tfrac{1}{2}\alpha_{\dot{K}\dot{K}}\Delta K_t^2 + \tfrac{1}{2}\alpha_{\dot{R}\dot{R}}\Delta R_t^2\}/\hat{Y}_t^{1/\rho} ,$$

(4.4b) $$L_t = \{\alpha_L + \alpha_{LT}T_t + \alpha_{LK^o}q_t^K + \alpha_{LL}(p_t^L)^2\}\hat{Y}_t^{1/\rho} + \alpha_{KL}K_{t-1} + \alpha_{RL}R_{t-1}$$

(4.5) $$K_t^o = -\{\alpha_{K^o} + \alpha_{LK^o}\hat{p}_t^L + \alpha_{K^oK^o}\hat{q}_t^K\}\hat{Y}_t^{1/\rho} - \alpha_{KK^o}K_{t-1} - \alpha_{RK^o}R_{t-1} ,$$

Recall also from (2.2) that

(4.6) $$K_t = I_t^K + K_t^o.$$

Equation (4.5) provides an economic model for K_t^o and and hence for the depreciation rate of capital δ_t^K; recall that the depreciation rate of capital is implicitly defined by $K_t^o = (1-\delta_t^K)K_{t-1}$. Equation (4.5) explains K_t^o as a function of relative prices, output and lagged stocks. The case of a constant and exogenously given depreciation rate is contained as a special case with $\alpha_{K^o} = \alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$ and $\alpha_{KK^o} = -(1-\delta^K)$.

For purposes of estimation it proves advantageous to reparameterize the model. More specifically, instead of estimating the parameter matrices A and B it proves advantageous to estimate the matrices $C = (c_{ij})_{i,j=K,R} = -BM$ and B (and to express the elements of A as functions of the elements of B and C). This approach is explained in more detail in Appendix B. The matrix C is found to be symmetric and negative definite.

For purposes of estimation we also add stochastic disturbance terms to each of the factor demand equations in (4.3) and (4.4).[16] Analogously to the approach taken by Epstein and Denny (1980) we assume that equation (4.5) for K_t^o holds exactly. This assumption is clearly strong. It facilitates that the unobservable stocks K_t and K_t^o can, at least in principle, be expressed as functions of observable variables and the unknown model parameters. More specifically, by solving (4.5) together with the identity (4.6) recursively for K_t and K_t^o from some given initial capital stock, say K_0, we can express K_t as a function of $I_t^K, I_{t-1}^K, \ldots, K_0, R_{t-1}, R_{t-2}, \ldots$, the exogenous variables and the model parameters. Consequently, upon replacing K_t and K_{t-1} in (4.3) and (4.4) by the obtained expressions we can, at least in principle, rewrite the system of factor demand equations as a dynamic system

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16 When necessary we have corrected for first order autocorrelation of the disturbances.

of equations that determines I_t^K, R_t, M_t, and L_t, and where in the so obtained system all variables are observable. (If the initial stock is unobserved we may treat it as an additional parameter.)

Of course, for the actual numerical computation of estimators of the model parameters it is generally not necessary to solve (4.5) and (4.6) analytically for K_t (and K_t^o). Numerical algorithms for the computation of estimators that are defined as optimizers of some statistical objective function generally require the numerical evaluation of the statistical objective function for different sets of parameter values. For any given set of parameter values we can solve (4.5) and (4.6) numerically for K_t (and K_t^o). Hence, rather than to substitute the analytic solution for K_t we can, in evaluating the statistical objective function, first solve (4.5) and (4.6) numerically and then substitute the numerical solution for K_t. The statistical objective function underlying the parameter estimates reported in the next section is the Gaussian full information maximum likelihood (FIML) function. We used the subroutine VA10AD from the Harwell program library to numerically maximize this function, i.e. to calculate the FIML estimates. We note that the factor demand system (4.3) and (4.4) in conjunction with (4.5) and (4.6) may be viewed as a system of equations with implicitly defined variables.[17]

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[17] Subroutine VA10AD calculates the gradient of the objective function numerically. This is convenient but numerically expensive. For an algorithm for the computation of estimators of the parameters of a system of equations with implicitly defined variables that evaluates the gradient of the objective function from analytic expressions see, e.g., Prucha and Nadiri (1988).

4.2 Parameter Estimates and Elasticities

We have estimated two versions of model (4.3)-(4.6) from U.S. electrical machinery industry data. In one version we have imposed the parameter restrictions $\alpha_{K^o} = \alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$ which implies that $K_t^o = -\alpha_{KK^o}K_{t-1}$, i.e. that the depreciation rate of capital is constant. In the other version no parameter restrictions are imposed and thus the depreciation rate of capital is a function of relative prices, output and lagged stocks. We refer to those versions of the model as model 2 and 3, respectively. We note that for both models 2 and 3 the depreciation rate is estimated and the respective capital stock series are determined consistently with the estimated model parameters from gross investment data during estimation. To contrast these results we also report parameter estimates presented in Prucha and Nadiri (1989a) for a model with exogenous capital depreciation rate based on the capital stock series provided in the OBA data bank. We refer to this latter model as model 1. It corresponds to (4.3)-(4.4) with $\alpha_{K^o} = \alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$ and with α_{KK^o} replaced by $-(1-\delta_t^K)$ where δ_t^K is defined by the OBA capital stock series. The underlying data for the U.S. electrical machinery industry are described in Appendix C and are the same as those used in Nadiri and Prucha (1989a). Since a full discussion of the parameter estimates of model 1 and their implication is given in Nadiri and Prucha (1989a) we will focus in the following on a comparison of the three models.

The structural parameter estimates are given in Table 1. The squared correlation coefficients between actual and fitted data are quite high and very similar across models. (Fitted values are calculated from the reduced

TABLE 1: Full Information Maximum Likelihood Estimates of the Parameters for the U.S. Electrical Machinery Industry: 1960-1980*.

Parameters	Model 1 OBA Capital Stock		Model 2 Estimated Capital Stock Constant Depreciation Rate		Model 3 Estimated Capital Stock Variable Depreciation Rate	
α_0	1.83	(7.99)	1.87	(6.78)	1.86	(25.03)
$1/\rho$	0.82	(16.37)	0.86	(16.75)	0.84	(19.15)
α_K	-0.95	(2.86)	-0.82	(2.11)	-0.73	(10.93)
α_R	-0.65	(4.51)	-0.77	(1.89)	-0.81	(5.51)
α_{KT}	-0.19	(3.96)	-0.20	(4.71)	-0.17	(11.23)
α_{RT}	0.22	(5.24)	0.27	(2.26)	0.23	(8.00)
c_{KK}	-2.05	(2.73)	-1.71	(2.56)	-1.41	(9.30)
c_{RR}	-2.10	(6.49)	-2.45	(1.89)	-2.27	(5.44)
c_{RK}	0.15	(0.83)	0.15	(0.67)	0.01	(0.07)
$\alpha_{\dot{K}\dot{K}}$	8.70	(2.65)	8.01	(3.35)	7.30	(7.11)
$\alpha_{\dot{R}\dot{R}}$	13.80	(12.22)	16.20	(1.69)	15.27	(5.07)
α_L	1.91	(25.85)	1.94	(17.17)	1.88	(30.44)
α_{LL}	-0.48	(3.88)	-0.44	(3.47)	-0.52	(5.50)
α_{KL}	0.29	(2.66)	0.32	(2.82)	0.35	(7.56)
α_{RL}	-0.52	(4.75)	-0.57	(3.96)	-0.56	(10.61)
α_{LT}	-0.28	(6.88)	-0.34	(4.56)	-0.32	(6.20)
α_{KK^o}			-0.96	(41.60)	-0.998	(163.90)
$\alpha_{K^oK^o}$					-0.003	(3.08)
α_{LK^o}					0.027	(8.85)
α_{RK^o}					0.023	(9.48)
Log of likelihood	222.10		223.62		224.16	
M- Equation: R^2	0.85		0.84		0.84	
L- Equation: R^2	0.65		0.65		0.65	
I^K-Equation: R^2	0.91		0.89		0.89	
I^R-Equation: R^2	0.86		0.86		0.86	

* Absolute values of the asymptotic "t"-ratios are given in parentheses. The R^2 values correspond to the squared correlation coefficients between the actual M, L, I^K, I^R variables and their fitted values calculated from the reduced form.

form). In terms of asymptotic "t"-ratios the parameter estimates are in general statistically significant.[18] The parameter estimates also satisfy the theoretical restrictions for all models. In particular, the estimates for c_{KK}, c_{RR}, α_{LL}, and $\alpha_{K^oK^o}$ are negative and those for $\alpha_{\dot{K}\dot{K}}$, $\alpha_{\dot{R}\dot{R}}$, $c_{KK}c_{RR} - c_{KR}^2$, and $\alpha_{LL}\alpha_{K^oK^o} - \alpha_{LK^o}^2$ are positive. Some of the parameter estimates differ across models. To obtain an appropriate interpretation of the importance of those differences we will report in the following various implied characteristics for the estimated factor demand systems.

The adjustment cost coefficients $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ are (though different) in magnitude similar across models and are generally statistically different from zero. Clearly, omitting these terms would not only have resulted in a misspecification of the investment pattern, but also in inconsistent estimates of the other technology parameters. For models 1, 2 and 3 the implied accelerator coefficients m_{KK} and m_{RR} are, respectively, 0.24 and 0.15, 0.21 and 0.15, and 0.19 and 0.15. Thus for all models the accelerator coefficient of capital exceeds that of R&D. The cross-adjustment coefficients m_{KR} and m_{RK} are small and (in absolute value) less than 0.02 for all models.

Our specification does not impose a priori constant returns to scale. Rather, we estimate the scale elasticity (represented by ρ^{-1}) from the data.

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[18] It is interesting to note that while the log-likelihood only increases slightly between models 2 and 3 the asymptotic "t"-ratios corresponding to the additional parameters $\alpha_{K^oK^o}$, α_{LK^o} and α_{RK^o} appearing in model 3 are all greater than two. This suggests an increase in the local curvature of the log-likelihood function between the maxima corresponding to models 2 and 3.

TABLE 2: Short-Run, Intermediate-Run, and Long-Run Elasticities in the U.S. Electrical Machinery Industry: 1976*.

Elasticity	Model 1 OBA Capital Stock			Model 2 Estimated Capital Stock Constant Depreciation Rate			Model 3 Estimated Capital Stock Variable Depreciation Rate		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
				Price Elasticities of Materials					
ε_{Mp^M}	-0.32	-0.40	-0.64	-0.29	-0.40	-0.75	-0.26	-0.38	-0.80
ε_{Mp^L}	0.36	0.41	0.65	0.33	0.41	0.75	0.33	0.43	0.88
ε_{Mq^K}	-0.01	0.02	0.09	-0.02	-0.02	0.10	-0.04	-0.02	0.03
ε_{Mq^R}	-0.01	-0.02	-0.08	-0.01	-0.02	-0.08	-0.01	-0.02	-0.10
				Price Elasticities of Labor					
ε_{Lp^M}	0.47	0.55	0.90	0.44	0.54	0.97	0.46	0.58	1.10
ε_{Lp^L}	-0.48	-0.58	-1.12	-0.45	-0.57	-1.14	-0.52	-0.67	-1.40
ε_{Lq^K}	0.00	-0.02	-0.06	0.00	-0.02	-0.07	0.05	0.04	0.02
ε_{Lq^R}	0.00	0.04	0.27	0.00	-0.04	0.24	0.00	0.04	0.27
				Price Elasticities of Capital					
ε_{Kp^M}	0.10	0.17	0.38	0.11	0.19	0.44	0.13	0.23	0.59
ε_{Kp^L}	-0.05	-0.09	-0.17	-0.07	-0.12	-0.24	-0.10	-0.18	-0.47
ε_{Kq^K}	-0.04	-0.08	-0.18	-0.04	-0.07	-0.17	-0.03	-0.05	-0.13
ε_{Kq^R}	-0.01	-0.01	-0.04	0.00	0.00	-0.04	0.00	0.00	0.00
				Price Elasticities of R&D					
ε_{Rp^M}	-0.05	-0.09	-0.27	-0.05	-0.09	-0.28	-0.05	-0.10	-0.32
ε_{Rp^L}	0.11	0.20	0.65	0.10	0.19	0.61	0.11	0.21	0.71
ε_{Rq^K}	-0.01	-0.01	-0.03	0.00	0.00	-0.03	0.00	-0.02	-0.05
ε_{Rq^R}	-0.06	-0.10	-0.34	-0.05	-0.09	-0.30	-0.05	-0.10	-0.33

* ε_{Zs} is the elasticity of Z = materials (M), labor (L), capital (K), R&D (R), capital left over at the end of the period (K^o), and gross capital investment (I^K) with respect to s = price of materials (p^M), price of labor (p^L), price of capital (q^K), price of R&D (q^R), and output (Y). The symbols SR, IR, and LR refer to the short, intermediate and the long-run.

TABLE 2: cont.

Elasticity	Model 1 OBA Capital Stock			Model 2 Estimated Capital Stock Constant Depreciation Rate			Model 3 Estimated Capital Stock Variable Depreciation Rate		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
	Price Elasticities of Capital Left Over at End of Period								
$\varepsilon_{K^o p^M}$				0.00	0.11	0.44	0.02	0.15	0.63
$\varepsilon_{K^o p^L}$				0.00	-0.07	-0.24	-0.02	-0.13	-0.52
$\varepsilon_{K^o q^K}$				0.00	-0.04	-0.17	0.00	-0.03	-0.13
$\varepsilon_{K^o q^R}$				0.00	0.00	-0.04	0.00	0.00	0.01
	Gross Capital Investment								
$\varepsilon_{I^K p^M}$				1.69	1.49	0.44	1.78	1.50	-0.52
$\varepsilon_{I^K p^L}$				-1.05	-0.90	-0.24	-1.31	-1.06	0.83
$\varepsilon_{I^K q^K}$				-0.61	-0.54	-0.17	-0.48	-0.44	-0.12
$\varepsilon_{I^K q^R}$				-0.06	-0.07	-0.04	0.00	-0.02	-0.19
	Output Elasticities								
ε_{MY}	1.19	1.07	0.82	1.32	1.18	0.86	1.28	1.16	0.84
ε_{LY}	1.07	1.06	0.82	1.08	1.07	0.86	1.01	1.01	0.84
ε_{KY}	0.20	0.34	0.82	0.19	0.33	0.86	0.18	0.32	0.84
ε_{RY}	0.14	0.24	0.82	0.13	0.25	0.86	0.14	0.25	0.84
$\varepsilon_{K^o Y}$	0.00	0.19	0.82	0.00	0.19	0.86	0.01	0.18	0.84
$\varepsilon_{I^K Y}$	2.55	2.22	0.82	2.93	2.63	0.86	3.02	2.74	0.84

The implied scale estimates are similar, i.e. 1.22, 1.16 and 1.19 for models 1, 2 and 3.

The own- and cross-price elasticities and output elasticities of labor, materials, capital, R&D, the capital left over at the end of the period, and gross capital investment for 1976 are reported in Table 2. The elasticities are calculated for the short-run (SR), intermediate-run (IR) and long-run (LR) for each input.[19] All of the own-price elasticities are negative. The magnitudes of the own- and cross-price elasticities are generally similar across models. However due to the endogeneity of the capital depreciation rate we can recognize some important differences between model 3 and models 1 and 2: One interesting difference can be observed in comparing the long-run elasticities of labor with respect to the price of capital, ε_{Lq^K}, and the long-run elasticity of capital with respect to the price of labor, ε_{Kp^L}. For models 1 and 2 both ε_{Lq^K} and ε_{Kp^L} are negative which reflects the fact that in the long-run $\partial L/\partial q^K = (r+\delta^K)\partial K/\partial p^L$. (Recall that q^K denotes the after tax acquisition price and not the rental price.) However for model 3 the elasticity ε_{Lq^K} is positive while ε_{Kp^L} is negative. At first glance this may seem inadmissible. However, as is demonstrated in Appendix D, due to the endogeneity of the depreciation rate of capital we have the following long-run relationship for model 3: $\partial L/\partial q^K = (1+r)\partial K/\partial p^L - \partial K^o/\partial p^L$. Therefore in the case of an endogenous depreciation rate the sign of $\partial L/\partial q^K$ may differ from that of $\partial K/\partial p^L$. The sign of $\partial L/\partial q^K$ depends on the relative magnitudes of $\partial K/\partial p^L$ and $\partial K^o/\partial p^L$, as opposed to the case of an exogenous depreciation rate where $\partial K^o/\partial p^L = (1-\delta^K)\partial K/\partial p^L$ and hence $\partial L/\partial q^K = (r+\delta^K)\partial K/\partial p^L$. Another interesting difference can be observed in comparing the long-run elasticities of gross capital investment with respect to the price of labor. For model 2 this elasticity is negative .24 while for model 3 it is positive .83. Again

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[19] For $Z = M, L, K, R, K^o, I^K, I^R$ let $Z_{t,\tau}$ denote the optimal plan value for Z in period $t+\tau$ corresponding to the firm's optimization problem in period t. Short-run, intermediate-run and long-run elasticities then refer to the elasticities of $Z_{t,\tau}$ for $\tau = 0$, 1 and ∞, respectively.

this can be explained from the fact that in the case of an exogenous depreciation rate we have $\partial I^K/\partial p^L = \delta^K \partial K/\partial p^L$, while in case of an endogenous depreciation rate we have more generally $\partial I^K/\partial p^L = \partial K/\partial p^L - \partial K^o/\partial p^L$. As a consequence $\partial I^K/\partial p^L$ can be positive while both $\partial K/\partial p^L$ and $\partial K^o/\partial p^L$ are negative. The switch in the long-run elasticity of gross capital investment with respect to the price of materials from positive .44 for model 2 to negative .52 for model 3 can be explained analogously.

The pattern of the output elasticities reveals that the variable factors of production, labor and materials, respond strongly in the short-run to changes in output; in the short-run they overshoot their long-run equilibrium values. The output elasticities of the quasi-fixed factors, capital and R&D, are small in the short-run but increase over time. In general the respective output elasticities are similar across models. It is interesting to note, however, that the short-run elasticity of capital investment corresponding to models 1 and 3 differ by approximately on half of a percentage point.

4.3 Capital Stock and Capital Depreciation Rate

The model considered in this paper generates series for the capital stock and depreciation rate as a byproduct of the estimation process. In Table 3 we report those estimates for models 2 and 3. For reasons of comparison we also report the OBA capital stock and depreciation rate series that underlie the estimates of model 1. For both models 2 and 3 the estimated depreciation rate is on average 0.038 as compared to 0.055 for the OBA capital stock series. This translates, for example, into a sizable difference of 16 percent in the magnitude of the capital stock at the end of the sample period. The estimates

TABLE 3: Comparison of OBA and Estimated Capital Stock Data in the U.S. Electrical Machinery Industry: 1960-1980.

	Model 1 OBA Capital Stock			Model 2 Estimated Capital Stock Constant Depreciation Rate			Model 3 Estimated Capital Stock Variable Depreciation Rate		
Year	K	K^O	δ_K	K	K^O	δ_K	K	K^O	δ_K
1959	0.536	0.489	0.055	0.544	0.497	0.038	0.550	0.502	0.028
1960	0.561	0.506	0.055	0.576	0.523	0.038	0.588	0.533	0.030
1961	0.587	0.530	0.055	0.613	0.556	0.038	0.626	0.569	0.033
1962	0.612	0.555	0.055	0.647	0.590	0.038	0.661	0.604	0.035
1963	0.639	0.578	0.055	0.683	0.622	0.038	0.698	0.637	0.036
1964	0.669	0.604	0.055	0.722	0.657	0.038	0.737	0.672	0.038
1965	0.720	0.632	0.055	0.782	0.694	0.038	0.796	0.708	0.040
1966	0.796	0.681	0.054	0.867	0.752	0.038	0.877	0.763	0.039
1967	0.876	0.754	0.052	0.955	0.833	0.038	0.963	0.841	0.040
1968	0.943	0.831	0.051	1.030	0.918	0.038	1.036	0.924	0.041
1969	1.016	0.896	0.050	1.110	0.991	0.038	1.114	0.994	0.041
1970	1.069	0.965	0.050	1.172	1.068	0.038	1.174	1.070	0.039
1971	1.108	1.015	0.051	1.220	1.127	0.038	1.221	1.128	0.040
1972	1.143	1.050	0.052	1.266	1.173	0.038	1.263	1.170	0.041
1973	1.208	1.081	0.054	1.344	1.217	0.038	1.335	1.209	0.043
1974	1.281	1.142	0.055	1.432	1.292	0.038	1.422	1.283	0.040
1975	1.305	1.211	0.055	1.471	1.377	0.038	1.465	1.371	0.036
1976	1.337	1.230	0.057	1.521	1.415	0.038	1.516	1.409	0.038
1977	1.389	1.258	0.059	1.593	1.463	0.038	1.587	1.457	0.039
1978	1.460	1.306	0.060	1.687	1.532	0.038	1.681	1.526	0.039
1979	1.549	1.372	0.060	1.799	1.622	0.038	1.794	1.617	0.038
1980	1.678	1.456	0.061	1.952	1.730	0.038	1.952	1.729	0.036

of the depreciation rate implied by model 3 vary in the rage from .028 to .043. The pattern of the depreciation rates indicates a general increase until 1973. The pattern also shows a decline in 1974 and 1975 as well as in 1980 reflecting periods of recession and slow output growth in U.S. electrical machinery industry. (Of course, the depreciation rate depends not only on output but also on relative prices and the observed pattern reflects both effects.)

It seems of interest to discuss the magnitude of the estimated depreciation rate as it relates to the average survival time of capital in more detail. Let $K_t = \sum_{i=0}^{\infty}\phi_i I_{t-i}^K$ where $\phi_i \geq 0$ denotes the efficiency function. Assume that the ϕ_i are nonincreasing, $\phi_0=1$, $\phi_i>0$ for $i=0,\ldots,m$ and $\phi_i=0$ for $i>m$, where m is the maximal survival time (which may possibly be infinite). Given $K_t = I_t^K + (1-\delta_t^K)K_{t-1}$ it follows that

$$(4.7) \qquad \delta_t^K = \frac{\sum_{i=0}^{m}(\phi_i-\phi_{i+1})I_{t-i-1}^K}{\sum_{i=0}^{m}\phi_i I_{t-i-1}^K} .$$

The average survival time is given by $\sum_{i=0}^{m}(\phi_i-\phi_{i+1})i$.

Clearly if gross investment grows at a constant rate, i.e. $I_t^K = (1+\rho_I)^t I_0^K$, and the efficiency function does not depend on t, the depreciation rate is constant over time and given by

$$(4.8) \qquad \delta_t^K = \frac{\sum_{i=0}^{m}(\phi_i-\phi_{i+1})(1+\rho_I)^{-i}}{\sum_{i=0}^{m}\phi_i(1+\rho_I)^{-i}} .$$

That is, the depreciation rate is only a function of $\phi_0,\ldots,\phi_m$ and the growth rate of gross investment (and hence constant) regardless of the shape of the efficiency function. We consider two "limiting" cases. In case of a one-hoss shay efficiency function, i.e. $\phi_i=1$ for $i=1,\ldots,m$, the depreciation rate equals $\delta_t^K = 1/[\sum_{i=0}^m (1+\rho_I)^i]$, and the average survival time equals the maximal survival time m. In case of a geometrically declining efficiency function, i.e. $\phi_i = (1-\delta)^i$, the depreciation rate is constant regardless of the pattern of investment and given by $\delta_t^K = \delta$, and the average survival time equals $(1-\delta)/\delta$. The average growth rate of gross investment in our sample is nine percent. Corresponding to this growth rate and our estimate of an average depreciation rate of 0.038 the implied average survival times for the two "limiting" cases are approximately 13 and 25 years, respectively.

The assumption of a constant depreciation rate has a long history and has been the subject of considerable debate.[20] (As remarked above, both a geometrically declining efficiency function and a constant growth rate of gross investment imply a constant depreciation rate.) The model considered in this paper allows for quasi-fixed factors with a constant depreciation rate as well as for quasi-fixed factors with a variable depreciation rate.
Replacement investment is defined as the difference between the initial stocks and what is left over from those stocks at the end of the period, i.e. $I_t^{KR} = K_{t-1} - K_t^0$. Net investment is defined as the difference between gross investment and replacement investment, i.e. $I_t^{KE} = I_t^K - I_t^{KR}$ or $I_t^{KE} = K_t - K_{t-1}$. Within our model both K_t and K_t^0 are endogenously determined by the firm;

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20 The assumption of a constant depreciation rate has been challenged, among others, by Feldstein and Foot (1971), Eisner (1972), Eisner and Nadiri (1968, 1970), Feldstein (1974), Feldstein and Rothschild (1974) and Bitros and Kelejian (1974); and was forcefully defended by Jorgenson (1974). Recently the validity of the geometric depreciation assumption has been tested in several papers by Hulten and Wykoff (1980, 1981a,b,c) based on a sample of used asset transaction prices.

hence also I_t^{KR} and I_t^{KE} are endogenously determined. That is, as a byproduct, our specification also yields a structural model for the endogenous determination of replacement investment versus expansion investment. We repeat that at the estimation stage only gross investment enters as an observed variable. Stocks are generated internally and hence are generated consistently with replacement investment. As pointed out by Jorgenson (1974) some of the previous studies on replacement investment were not fully consistent in that they employed capital stock data that have been generated under a different set of assumptions than those maintained in those studies. Our approach is not subject to the same criticism and hence allows in principle for a proper test of the constancy of depreciation rates.

In Table 4 we present the ratio of net investment to gross investment for the period 1960 to 1980. The ratio implied by models 2 and 3 is much higher than the ratio implied by the OBA capital stock series. This implies (consistent with our previous remarks) a much higher rate of capital accumulation as compared to the OBA capital stock series. We note that the patterns of the net to gross-investment ratio over time seem quite similar across the models and the ratio generally drops in years of slow output growth.

4.4 Technical Change and Capacity Utilization

Our estimates of pure technical change as reported in Table 5 are 0.60, 0.69 and 0.66, respectively, based on models 1, 2 and 3. That is, our estimates of technical change are on average higher based on models 2 and 3 as compared to model 1. As discussed in Section 3, the traditional measure of

TABLE 4: Ratio of Net Investment to Gross Investment in the U.S. Electrical Machinery Industry: 1960-1980.

Year	Model 1 OBA Capital Stock	Model 2 Estimated Capital Stock Constant Depreciation Rate	Model 3 Estimated Capital Stock Variable Depreciation Rate
1960	0.46	0.62	0.69
1961	0.45	0.61	0.66
1962	0.43	0.59	0.62
1963	0.45	0.59	0.60
1964	0.46	0.60	0.59
1965	0.58	0.68	0.67
1966	0.66	0.73	0.72
1967	0.66	0.73	0.70
1968	0.60	0.67	0.65
1969	0.60	0.67	0.65
1970	0.51	0.59	0.57
1971	0.41	0.51	0.48
1972	0.38	0.50	0.47
1973	0.51	0.61	0.58
1974	0.53	0.63	0.61
1975	0.25	0.41	0.41
1976	0.30	0.47	0.49
1977	0.40	0.55	0.56
1978	0.46	0.60	0.61
1979	0.50	0.63	0.65
1980	0.58	0.69	0.70

total factor productivity only equals technical change if, in particular, producers are in long-run equilibrium, the technology exhibits constant returns to scale, input and output markets are perfectly competitive, and factors are utilized at a constant rate. In Table 5 we also report estimates of the traditional measure of total factor productivity. Those estimates are approximately three times larger than our estimates of pure technical change. Based on the decomposition formula (3.10) given in Section 3 and based on the estimates of the respective models we also provide a decomposition of the sources for this difference. The main source of the difference is the scale effect which represents about 40 to 50 percent of the growth in the

TABLE 5: Decomposition of Total Factor Productivity Growth in the U.S. Electrical Machinery Industry in Percentages: 1960-1980.

	Model 1 OBA Capital Stock	Model 2 Estimated Capital Stock Constant Depreciation Rate	Model 3 Estimated Capital Stock Variable Depreciation Rate
Technical Change	0.60	0.69	0.66
Scale Effect	1.04	0.83	0.93
Adjustment Cost Effects			
Temporary Equilibrium Effect	0.33	0.42	0.39
Direct Adjustment Cost Effect	0.03	0.02	0.02
Variable Depreciation Effect	0.00	0.00	0.02
Unexplained Residual	0.04	0.03	-0.03
Total Factor Productivity	2.04	1.99	1.99

traditional total factor productivity measure. The remainder of the difference is mainly due to the presence of adjustment cost. The measures of total factor productivity differ across the models since they are based on different capital stock series. Comparing the decomposition between models 2 and 3 shows that allowing for the depreciation rate to vary increases the scale effect, lowers the adjustment cost effect and decreases the estimate for pure technical change. The variable depreciation rate effect is small. (Of course, the importance of the adjustment cost and the variable depreciation rate effect will differ across applications depending on the size of respective factor shares and differences in growth rates; cp. formula (3.10).)

We note that for all models estimated pure technical change exhibits a very smooth pattern and increases over time. For example, for model 3 the estimate of pure technical change is .56 in 1960 and .88 in 1980 with a low of .51 in 1964.

TABLE 6: Capacity Utilization in the U.S. Electrical Machinery Industry: 1960-1980.

Year	Model 1 OBA Capital Stock	Model 2 Estimated Capital Stock Constant Depreciation Rate	Model 3 Estimated Capital Stock Variable Depreciation Rate
1960	1.073	1.089	1.093
1961	1.092	1.107	1.107
1962	1.100	1.109	1.107
1963	1.111	1.122	1.118
1964	1.119	1.130	1.125
1965	1.119	1.125	1.121
1966	1.116	1.119	1.116
1967	1.120	1.126	1.121
1968	1.115	1.124	1.118
1969	1.113	1.123	1.117
1970	1.098	1.112	1.104
1971	1.071	1.086	1.078
1972	1.094	1.107	1.099
1973	1.111	1.122	1.114
1974	1.099	1.112	1.104
1975	0.994	1.010	1.009
1976	1.059	1.074	1.068
1977	1.102	1.116	1.107
1978	1.113	1.126	1.117
1979	1.125	1.137	1.127
1980	1.129	1.142	1.132

In Table 6 we report estimates of capacity utilization based on the cost ratio C/C^{+}; cp. Section 3.2. Those estimates are similar across models. For all models capacity utilization drops approximately ten percent in 1975, reflecting a fourteen percent decline in gross output in the U.S. electrical machinery industry in that year. Comparing the capacity utilization estimates corresponding to models 2 and 3 we see that the estimates corresponding to the latter are generally somewhat smaller. This seems consistent with the fact that in model 3 also the depreciation rate of capital can be chosen optimally.

5. Conclusion

In this paper we have specified a general dynamic factor demand model where the firm can choose the depreciation rate of some (or all) of the quasi-fixed factors optimally. The model allows for multiple outputs, variable inputs, and for the quasi-fixed factors to become productive immediately or with a lag. Based on the model we develop primal and dual measures of technical change. Those measures extend those recently given in Berndt and Fuss (1989). We also deduce a measure of capacity utilization and explore the sources of bias for the traditional measure of total factor productivity growth.

We have estimated two version of the empirical model using data from the U.S. electrical machinery industry. The more general version of the model determines the depreciation rate of capital as a function of output and relative prices. For the other version of the model (in order to identify the implications of allowing for an endogenous determination of the depreciation rate) we have imposed parameter restrictions such that the depreciation rate of capital is constant. We note that for both version of the model the depreciation rate is estimated and the respective capital stocks are generated internally during estimation in a theoretically consistent fashion from the gross investment series. For further contrast of our model we also report the estimates obtained in Nadiri and Prucha (1989a) from a model with exogenous depreciation rate that utilizes the capital stock series published by OBA. We refer to those models as, respectively, model 3, 2 and 1.

On the whole the price and output elasticities are similar across models. However some interesting differences can be observed. In particular, as explained in more detail in the text, when the depreciation rate is endogenously determined the sign of the long-run cross price elasticities of the variable factors and capital need not be the same. In fact, we find the estimated long-run cross price elasticities of capital and labor to be of opposite sign. Related to this phenomenon we also observe that the long-run elasticity of gross investment with respect to the prices of labor and materials changes between models 2 and 3, i.e. by allowing the deprecation rate to be endogenously determined, from -.24 to .83 and .44 to -.52, respectively.

For both models 2 and 3 the depreciation rate is estimated on average to be 0.038 as compared to 0.055 for the OBA capital stock series. This translates into a sizable difference of 16 percent in the level of the capital stock at the end of the sample period. Also the ratio of net to gross investment implied by models 2 and 3 is much higher than the ratio implied by the OBA data. All these ratios show sensitivity to the growth in output.

Our estimates of pure technical change is approximately .6, which is approximately one third of the estimate implied by the traditional measure of total factor productivity growth. I.e., the traditional measure of total factor productivity growth significantly overestimates the rate of technical change.

Although the model considered here is quite general, several extensions of the theoretical model seem of interest. In particular, variations in the rate of utilization of an input can be achieved by varying the numbers of hours the input is employed and/or by changing the intensity or speed with which the input is used in the production process. An increase in the intensity or speed with which capital is operated will typically result in an increase in the rate of depreciation of capital. An increase in the length of time capital is employed will typically results in increased costs due to shift and overtime premiums and an increase in the rate of depreciation of capital. It seems of interest to incorporate both cost aspects into the model.

At the empirical stage it may be interesting to include more quasi-fixed factors by distinguishing between production and nonproduction workers and by differentiating between different types of capital. Also it seems desirable to endogenize the depreciation rate of R&D. Another extension would be to allow for more general patterns of expectations. Furthermore, the model can be reformulated in a profit maximizing setting to explore the existence of markups in different industries.

Appendix A: Background Material for Productivity Measures

For ease of notation we drop in the following time-subscripts whenever those subscripts are obvious from the context.

A.1 Derivation of Equation 3.5

Totally differentiating the transformation function (2.12) yields

$$\sum_i(\partial F/\partial Y_i)\dot{Y}_i + \sum_j(\partial F/\partial V_j)\dot{V}_j + \sum_k(\partial F/\partial K_k^0)\dot{K}_k^0 + \sum_k(\partial F/\partial \underline{K}_k)\dot{\underline{K}}_k +$$
$$\sum_l(\partial F/\partial \underline{R}_l)\dot{\underline{R}}_l + \sum_k(\partial F/\partial \Delta K_k)\Delta\dot{K}_k + \sum_l(\partial F/\partial \Delta R_l)\Delta\dot{R}_l + \partial F/\partial T = 0.$$

Equation (3.5) is then obtained by substituting (3.2) into the above expression.

A.2 Technical Change Measures Corresponding to Long Run Equilibrium Values

As remarked in the text, in general technical change is a function of respective inputs and outputs. The technical change measure $\lambda_X = \Lambda(Y,V,K^0,\underline{K},\underline{R},\Delta K,\Delta R,T)$ defined in (3.6) corresponds to current input and output values. Analogous to Berndt and Fuss (1989) we may also calculate, within the context of our model, technical change at long-run equilibrium input and output values. As in Berndt and Fuss we define two alternative long-run equilibria. The first assumes that inputs are adjusted optimally for current output levels; the second assumes that outputs are adjusted optimally for current levels of the quasi-fixed factors. Assume for a moment that prices are static. To simplify the exposition assume furthermore that $\underline{K}=K_{-1}$, $\underline{R}=R_{-1}$, $p^K=0$ and $p^R=0$. The the long-run optimal quasi-fixed factors, say $\underline{K}^*=K^*$ and $\underline{R}^*=R^*$, needed to produce the current level of outputs Y are then implicitly defined by $-\partial G(p^L,q^K,Y,\underline{K}^*,\underline{R}^*,0,0,T)/\partial\underline{K} = (1+r)q^K$ and

$-\partial G(p^L,q^K,Y,\underline{K}^*,\underline{R}^*,0,0,T)/\partial\underline{R} = \underline{c}^R$; cp. equations (2.7) and (2.8). The corresponding long-run variable factors and "old" stocks are given by $L^* = \partial G^*/\partial p^L$, $K^{0*} = -\partial G^*/\partial q^K$, $M^* = G^* - p^{L\prime}L^* + q^{K\prime}K^{0*}$ where $G^* = G(p^L,q^K,Y,K^*,R^*,0,0,T)$; cp. equations (2.5), (2.9) and (2.10). Similarly, if the number of quasi-fixed inputs equals the number of outputs, long-run optimal outputs, say $Y^\odot$, for given levels of the quasi-fixed factors can be defined implicitly by $-\partial G(p^L,q^K,Y^\odot,\underline{K},\underline{R},0,0,T)/\partial\underline{K} = (1+r)q^K$ and $-\partial G(p^L,q^K,Y^\odot,\underline{K},\underline{R},0,0,T)/\partial\underline{R} = \underline{c}^R$. The corresponding long-run variable factors and "old" stocks are given by $L^\odot = \partial G^\odot/\partial p^L$, $K^{0\odot} = -\partial G^\odot/\partial q^K$, $M^\odot = G^\odot - p^{L\prime}L^\odot + q^{K\prime}K^{0\odot}$ where $G^\odot = G(p^L,q^K,Y^\odot,\underline{K},\underline{R},0,0,T)$. Total cost for the two long-run equilibria is given by:

$$C^* = G(p^L,q^K,Y,K^*,R^*,0,0,T) + \textstyle\sum_k(1+r)q^K_kK^*_k + \sum_l\underline{c}^R_lR^*_l ,$$
$$C^\odot = G(p^L,q^K,Y^\odot,\underline{K},\underline{R},0,0,T) + \textstyle\sum_k(1+r)q^K_k\underline{K}_k + \sum_l\underline{c}^R_l\underline{R}_l .$$

Using (3.6) we can now define, corresponding to the two long-run equilibria, the following two measures of input based technical change:

$$\lambda^*_X = \Lambda(Y,V^*,K^{0*},K^*,R^*,0,0,T) = \textstyle\sum_i\varepsilon^*_i(\dot{Y}_i/Y_i) -$$
$$[\textstyle\sum_j s^{V*}_j(\dot{V}^*_j/V^*_j)-\sum_k s^{0*}_k(\dot{K}^{0*}_k/K^{0*}_k)+\sum_k s^{K*}_k(\dot{\underline{K}}^*_k/\underline{K}^*_k)+\sum_l s^{R*}_l(\dot{\underline{R}}^*_l/\underline{R}^*_l)],$$
$$\lambda^\odot_X = \Lambda(Y^\odot,V,K^0,\underline{K},\underline{R},0,0,T) = \textstyle\sum_i\varepsilon^\odot_i(\dot{Y}^\odot_i/Y^\odot_i) -$$
$$[\textstyle\sum_j s^{V\odot}_j(\dot{V}^\odot_j/V^\odot_j)-\sum_k s^{0\odot}_k(\dot{K}^{0\odot}_k/K^{0\odot}_k)+\sum_k s^{K\odot}_k(\dot{\underline{K}}_k/\underline{K}_k)+\sum_l s^{R\odot}_l(\dot{\underline{R}}_l/\underline{R}_l)],$$

where ε^*_i and $\varepsilon^\odot_i$ denote respective long-run total cost elasticities with respect to output, and s^{V*}_j, s^{0*}_k, s^{K*}_k, s^{R*}_l and $s^{V\odot}_j$, $s^{0\odot}_k$, $s^{K\odot}_k$, $s^{R\odot}_l$ denote respective shares in long-run total cost. We note that the input based technical change measures λ^*_X and $\lambda^\odot_X$ generalize, respectively, Berndt and Fuss' (1989) technical change measures $\dot{A}_3/A_3$ and $\dot{A}_4/A_4$. We can also define the following alternative capacity utilization measures: C/C^* and $C/C^{\odot}$.

A.3: Decomposition of the Traditional Measure of TFP Growth

In the following we give a proof for the decomposition of the Törnquist approximation of the traditional measure of total factor productivity growth, ΔTFP, presented in Section 3.3. We consider the case of a single output good and maintain that all assumptions stated in the text hold.

We shall utilize the following lemma.

Lemma A1: Let $\dot{u} = \dot{w} - \dot{v}$ where $\dot{w} = \varepsilon\sum_{i=1}^{m+k}(\alpha_i/\alpha)\dot{\eta}_i + \lambda$ and $\dot{v} = \sum_{i=1}^{m}(\beta_i/\beta)\dot{\eta}_i$ with $\alpha = \sum_{i=1}^{m+k}\alpha_i$ and $\beta = \sum_{i=1}^{m}\beta_i$. Then

$$\dot{u} = (1-1/\varepsilon)\dot{w} + \sum_{i=1}^{m}[(\alpha_i-\beta_i)/\alpha](\dot{\eta}_i-\dot{v}) + \sum_{i=m+1}^{m+k}(\alpha_i/\alpha)(\dot{\eta}_i-\dot{v}) + (1/\varepsilon)\lambda.$$

Proof: $\dot{u} = (1-1/\varepsilon)\dot{w} + (1/\varepsilon)\dot{w} - \dot{v} = (1-1/\varepsilon)\dot{w} + \sum_{i=1}^{m+k}(\alpha_i/\alpha)\dot{\eta}_i - \dot{v} + (1/\varepsilon)\lambda =$ $(1-1/\varepsilon)\dot{w} + \sum_{i=1}^{m+k}(\alpha_i/\alpha)(\dot{\eta}_i-\dot{v}) + (1/\varepsilon)\lambda$. The result now follows upon observing that $\sum_{i=1}^{m}(\beta_i/\alpha)(\dot{\eta}_i-\dot{v}) = (\beta/\alpha)\sum_{i=1}^{m}(\beta_i/\beta)(\dot{\eta}_i-\dot{v}) = (\beta/\alpha)\{\sum_{i=1}^{m}(\beta_i/\beta)\dot{\eta}_i - \dot{v}\} =$ $(\beta/\alpha)\{\dot{v} - \dot{v}\} = 0$. Q.E.D.

Now let

$$\text{(A.1)} \qquad Y_t = f(M_t, L_t, K_t^0, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t)$$

with $\underline{K}_t=K_{t-1}$, $\underline{R}_t=R_{t-1}$, and $T_t=t$ denote the firm's production function obtained by solving the transformation function (2.12) for Y_t. Assume that this solution is unique and differentiable. Implicit differentiation of the transformation function then implies $\partial Y/\partial Z = -[\partial F/\partial Z]/[\partial F/\partial Y]$ for Z = M, L, K^0, $\underline{K}$, $\underline{R}$, ΔK, ΔR. Because of (3.2) it follows further that

$$\text{(A.2)} \qquad \partial Y/\partial M = p^M/[\partial G/\partial Y], \ \partial Y/\partial L = p^L/[\partial G/\partial Y], \ \partial Y/\partial K^0 = -q^K/[\partial G/\partial Y],$$

$$\partial Y/\partial Z = -\ [\partial G/\partial Z]/[\partial G/\partial Y], \qquad Z = \underline{K},\ \underline{R},\ \Delta K,\ \Delta R.$$

(Recall that $p^M=1$.) Now consider the following decomposition of output growth based on a translog expansion of the production function:

$$\text{(A.3)} \qquad \Delta \ln Y_t = \tfrac{1}{2}[\Delta \ln Y_t^t + \Delta \ln Y_t^{t-1}]$$
$$\Delta \ln Y_t^\tau = \varepsilon_{YM}(\tau)\Delta \ln M_t + \varepsilon_{YL}(\tau)\Delta \ln L_t + \varepsilon_{YK^o}(\tau)\Delta \ln K_t^0 + \varepsilon_{YK}(\tau)\Delta \ln \underline{K}_t$$
$$+ \varepsilon_{YR}(\tau)\Delta \ln \underline{R}_t + \varepsilon_{Y\Delta K}(\tau)\Delta \ln \Delta K_t + \varepsilon_{Y\Delta R}(\tau)\Delta \ln \Delta R_t + \lambda_Y(\tau),$$

with $\tau = t,\ t-1$, and where the $\varepsilon_{YZ}(\tau) = [\partial Y_\tau/\partial Z_\tau][Z_\tau/Y_\tau]$ with $Z = M,\ L,\ K^0,\ \underline{K},\ \underline{R},\ \Delta K,\ \Delta R$ denote output elasticities, and output based technical change λ_Y is defined by (3.1). (For notational convenience we do not underline the subscripts K and R in denoting the output elasticity with respect to $\underline{K}$ and $\underline{R}$.) In light of (3.3) and (3.4) we have for the scale elasticity $\rho(\tau) = C_\tau/[(\partial G_\tau/\partial Y_\tau)Y_\tau]$ where C denotes total shadow cost. It now follows from (A.2) that

$$\text{(A.4)} \qquad \varepsilon_{YM}(\tau) = \rho(\tau)[p_\tau^M M_\tau/C_\tau], \qquad \varepsilon_{YL}(\tau) = \rho(\tau)[p_\tau^L L_\tau/C_\tau],$$
$$\varepsilon_{YK^o}(\tau) = \rho(\tau)[-q_\tau^K K_\tau^0/C_\tau], \qquad \varepsilon_{YZ}(\tau) = \rho(\tau)[(-\partial G_\tau/\partial Z_\tau)Z_\tau]/C_\tau,$$

with $Z = \underline{K},\ \underline{R},\ \Delta K,\ \Delta R$. Using the above lemma it follows readily form (A.2) and the definition of $\Delta \ln N_t^\tau$ in (3.9) that ($\tau = t,\ t-1$)

$$\text{(A.5)} \qquad \Delta \ln Y_t^\tau - \Delta \ln F_t^\tau =$$
$$\rho(\tau)\{[p_\tau^M M_\tau/C_\tau]\Delta \ln M_t + [p_\tau^L L_\tau/C_\tau]\Delta \ln L_t - [q_\tau^K K_\tau^0/C_\tau]\Delta \ln K_t^o$$
$$- [\partial G_\tau/\partial \underline{K}_\tau][\underline{K}_\tau/C_\tau]\Delta \ln \underline{K}_t - [\partial G_\tau/\partial \underline{R}_\tau][\underline{R}_\tau/C_\tau]\Delta \ln \underline{R}_t$$
$$- [\partial G_\tau/\partial \Delta K_\tau][\Delta K_\tau/C_\tau]\Delta \ln \Delta K_t - [\partial G_\tau/\partial \Delta R_\tau][\Delta R_\tau/C_\tau]\Delta \ln \Delta R_t\} + \lambda_Y(\tau)$$
$$- \{[p_\tau^M M_\tau/C_\tau^+]\Delta \ln M_t + [p_\tau^L L_\tau/C_\tau^+]\Delta \ln L_t$$
$$- [\underline{c}_\tau^K \underline{K}_\tau/C_\tau^+]\Delta \ln \underline{K}_t - [\underline{c}_\tau^R \underline{R}_\tau/C_\tau^+]\Delta \ln \underline{R}_t\} =$$

$$
\begin{aligned}
&(1-1/\rho(\tau))\Delta\ln Y_t^\tau - [q_\tau^K K_\tau^0/C_\tau][\Delta\ln K_t^o - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \underline{K}_\tau + \underline{c}_\tau^K][\underline{K}_\tau/C_\tau][\Delta\ln\underline{K}_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \underline{R}_\tau + \underline{c}_\tau^R][\underline{R}_\tau/C_\tau][\Delta\ln\underline{R}_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \Delta K_\tau][\Delta K_\tau/C_\tau][\Delta\ln\Delta K_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \Delta R_\tau][\Delta R_\tau/C_\tau][\Delta\ln\Delta R_t - \Delta\ln F_t^\tau] \quad + \quad (1/\rho(\tau))\lambda_Y(\tau) = \\
&(1-1/\rho(\tau))\Delta\ln Y_t^\tau - [q_\tau^K K_\tau^0/C_\tau][\Delta\ln K_t^o - \Delta\ln\underline{K}_t] \\
&\quad - [\partial G_\tau/\partial \underline{K}_\tau + (1+r)q_\tau^K][\underline{K}_\tau/C_\tau][\Delta\ln\underline{K}_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \underline{R}_\tau + \underline{c}_\tau^R][\underline{R}_\tau/C_\tau][\Delta\ln\underline{R}_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \Delta K_\tau][\Delta K_\tau/C_\tau][\Delta\ln\Delta K_t - \Delta\ln F_t^\tau] \\
&\quad - [\partial G_\tau/\partial \Delta R_\tau][\Delta R_\tau/C_\tau][\Delta\ln\Delta R_t - \Delta\ln F_t^\tau] \quad + \quad (1/\rho(\tau))\lambda_Y(\tau).
\end{aligned}
$$

where the last equality follows by observing that $\underline{c}^K\underline{K} = (1+r)q^K\underline{K} - q^K K^o$. A general decomposition of the Törnquist approximation of the traditional measure of total factor productivity growth is obtained by substituting the above expression into $\Delta TFP_t = \Delta\ln Y_t - \Delta\ln F_t = \frac{1}{2}\sum_{\tau=t,t-1}[\Delta\ln Y_t^\tau - \Delta\ln F_t^\tau]$. The decomposition given in (3.10) follows under the additional assumption that $\rho(t)=\rho(t-1)$.

Appendix B: Derivation of Estimated System of Factor Demand Equations

Consider the model described in Section 4.1. Given the specification of G(.) in (4.1) it follows from (2.7) and (2.8), and the discussion in Section 2.3, that the certainty equivalence feedback control policy for the quasi-fixed factors needs to satisfy the following set of Euler equations ($\tau=t,t+1,\ldots$):

(B.1a) $$-\alpha_{\dot K\dot K}K_{\tau+1} + [\alpha_{KK}+(2+r)\alpha_{\dot K\dot K}]K_{\tau} - (1+r)\alpha_{\dot K\dot K}K_{\tau-1} + \alpha_{KR}R_{\tau} = a_t^K$$

(B.1b) $$-\alpha_{\dot R\dot R}R_{\tau+1} + [\alpha_{RR}+(2+r)\alpha_{\dot R\dot R}]R_{\tau} - (1+r)\alpha_{\dot R\dot R}R_{\tau-1} + \alpha_{KR}R_{\tau} = a_t^R$$

with

$$a_t^K = [\alpha_K + \alpha_{KT}T_t + \alpha_{KL}\hat p_t^L + \hat q_t^K(1+r+\alpha_{\dot K\dot K})]\hat Y_t^{1/\rho}$$
$$a_t^R = [\alpha_R + \alpha_{RT}T_t + \alpha_{RL}\hat p_t^L + \hat c_t^R]\hat Y_t^{1/\rho}$$

Solving the above set of second order difference equations (in conjunction with the transversality condition) is a standard problem; cp. e.g. Madan and Prucha (1989). The solution is given by the accelerator model (4.3). As noted in the text, the accelerator coefficients have to satisfy the following matrix equations:

(B.2) $$BM^2 + (A+rB)M - A = 0$$

where $M = (m_{ij})_{i,j=K,R}$, $A = (\alpha_{ij})_{i,j=K,R}$ and B is the diagonal matrix with elements $\alpha_{\dot K\dot K}$ and $\alpha_{\dot R\dot R}$ in the diagonal. Furthermore, the matrix $C = (c_{ij})_{i,j=K,R} = -BM$ is symmetric and negative definite; cp. e.g. Madan and Prucha (1989). Unless we impose separability in the quasi-fixed factors, i.e., $\alpha_{KR} = 0$ which implies $m_{KR} = 0$, (B.2) cannot generally be solved for M in terms of A and B. We can, however solve (B.2) for A in terms of M and B: $A = BM(M+rI)(I-M)^{-1}$. Since the real discount rate r was assumed to be constant, M is constant over the sample. Hence, instead of estimating the elements of A and B, we may estimate those of M and B. To impose the symmetry of C we can also estimate B and C instead of B and M. Let $D = (d_{ij})_{i,j=K,R} = -MA^{-1}$ and observe that $A = C - (1+r)[B - B(C+B)^{-1}B]$ and that $D = B^{-1} + (1+r)(C-rB)^{-1}$ is symmetric.[21] It is then readily seen that we can write (4.3) equivalently as:

$$\text{(B.3a)}\quad \Delta K_t = d_{KK}[\alpha_K + \alpha_{KT}T_t + \alpha_{KL}\hat{p}^L_t + \hat{q}^K_t(1+r+\alpha_{K^\circ K^\circ})]\hat{Y}_t^{1/\rho}$$
$$+ d_{KR}[\alpha_R + \alpha_{RT}T_t + \alpha_{RL}\hat{p}^L_t + \hat{c}^R_t]\hat{Y}_t^{1/\rho} + [c_{KK}/\alpha_{\dot{K}\dot{K}}]K_{t-1} + [c_{KR}/\alpha_{\dot{K}\dot{K}}]R_{t-1},$$

$$\text{(B.3b)}\quad \Delta R_t = d_{KR}[\alpha_K + \alpha_{KT}T_t + \alpha_{KL}\hat{p}^L_t + \hat{q}^K_t(1+r+\alpha_{K^\circ K^\circ})]\hat{Y}_t^{1/\rho}$$
$$+ d_{RR}[\alpha_R + \alpha_{RT}T_t + \alpha_{RL}\hat{p}^L_t + \hat{c}^R_t]\hat{Y}_t^{1/\rho} + [c_{KR}/\alpha_{\dot{R}\dot{R}}]K_{t-1} + [c_{RR}/\alpha_{\dot{R}\dot{R}}]R_{t-1},$$

where

$$d_{KK} = 1/\alpha_{\dot{K}\dot{K}} + (1+r)[c_{RR} - r\alpha_{\dot{R}\dot{R}}]/e\ ,$$
$$d_{RR} = 1/\alpha_{\dot{R}\dot{R}} + (1+r)[c_{KK} - r\alpha_{\dot{K}\dot{K}}]/e\ ,$$
$$d_{KR} = -(1+r)c_{KR}/e\ , \quad \text{and} \quad e = (c_{KK} - r\alpha_{\dot{K}\dot{K}})(c_{RR} - r\alpha_{\dot{R}\dot{R}}) - c_{KR}^2\ .$$

Furthermore, we can express α_{KK}, α_{RR}, α_{KR} in (4.4) as

$$\alpha_{KK} = c_{KK} - (1+r)[\alpha_{\dot{K}\dot{K}} - (\alpha_{\dot{K}\dot{K}})^2(\alpha_{\dot{R}\dot{R}} + c_{RR})/f]\ ,$$
$$\alpha_{RR} = c_{RR} - (1+r)[\alpha_{\dot{R}\dot{R}} - (\alpha_{\dot{R}\dot{R}})^2(\alpha_{\dot{K}\dot{K}} + c_{KK})/f]\ ,$$
$$\alpha_{KR} = c_{KR} - (1+r)(\alpha_{\dot{K}\dot{K}}\alpha_{\dot{R}\dot{R}}c_{KR})/f\ , \quad \text{and} \quad f = (\alpha_{\dot{K}\dot{K}}+c_{KK})(\alpha_{\dot{R}\dot{R}}+c_{RR}) - c_{KR}^2.$$

---

21 The reparametrization approach was first suggested by Epstein and Yatchew (1985) for a somewhat different model with a similar algebra. It was further generalized by Madan and Prucha (1989); for application of this approach see, e.g., Mohnen, Nadiri and Prucha (1986) and Nadiri and Prucha (1989a).

Appendix C: Data Sources and Construction of Variables

Gross Output: Data on gross output in current and constant 1972 dollars were obtained from the U.S. Department of Commerce, Office of Business Analysis (OBA) database and correspond to the gross output series of the U.S. Department of Commerce, Bureau of Industrial Economics (BIE). Gross output is defined as total shipments plus the net change in work in process inventories and finished goods inventories.

Labor: Total hours worked were derived as the sum of hours worked by production workers and nonproduction workers. Hours worked by production workers were obtained directly from the OBA database. Hours worked by nonproduction workers were calculated as the number of nonproduction workers*hours worked per week*52. The number of nonproduction workers was obtained from the OBA database. Weekly hours worked of nonproduction workers were taken to be 39.7. A series of total compensation in current dollars was calculated by multiplying the total payroll series from the OBA database with the ratio of compensation of employees to wages and salaries from U.S. Department of Commerce, Bureau of Economic Analysis (1981, 1984).

Materials: Materials in current dollars were obtained from the OBA database. Materials in constant 1972 dollars were calculated using deflators provided by the U.S. Department of Commerce, Bureau of Economic Analysis.

Value Added: Value added in current and constant 1972 dollars was calculated by subtracting materials from gross output.

Capital: The net capital stock series in 1972 dollars and the current and constant 1972 dollar gross investment series were taken from the OBA database.

The method by which the OBA capital stock series is constructed is described in the U.S. Department of Labor, Bureau of Labor Statistics (1979). The user cost of capital was constructed as $c^K = q^K(r+\delta^K)$ with $q^K = p^{IK}/(1-u)$ where p^{IK} is the investment deflator, u is the corporate tax rate and $r = 0.05$. (Of course, the OBA capital stock series was only used in the estimation of Model 1. In estimating Models 2 and 3 only the investment series and q^K were utilized.)

R&D: The stock of total R&D is constructed by the perpetual inventory method with a depreciation rate $\delta^R = 0.1$. The benchmark in 1958 is obtained by dividing total R&D expenditures by the depreciation rate and the growth rate in real value added. The nominal R&D expenditures are taken from National Science Foundation (1984) and earlier issues. To avoid double counting we have subtracted the labor and material components of R&D from the labor and material inputs. The GDP deflator for total manufacturing is used as the deflator for R&D expenditures, p^{IR}. All R&D expenditures were taken to be immediately expensible. The user cost for R&D was hence constructed as $c^R = p^{IR}(r+\delta^R)$.

All constant dollar variables were normalized by respective sample means. Prices were constructed conformably and normalized by the price of materials. Expectations on prices were set equal to current prices. Expectations on gross output were calculated as follows. We first estimated a first order autoregressive model for output and then used this model to predict Y_t. Time was used for the technology index T. In estimating models 2 and 3 we used as the initial capital stock the corresponding value of the OBA capital stock.

Appendix D: Long-Run Reciprocity Relationships

In the following we show that, as claimed in the text, the following reciprocity relationship holds in the long-run: $\partial L/\partial q^K = (1+r)\partial K/\partial p^L - \partial K^o/\partial p^L$. The maintained assumptions are that $\underline{K}=K_{-1}$ and $\underline{R}=R_{-1}$, that the firm is in long-run equilibrium and that expectations are static. For reasons of notational simplicity we also assume that L, K^0, K and R are scalars and that p^K and p^R are zero.

Under the maintained assumptions equations (2.7)-(2.9) reduce to

(D.1a) $-\partial G/\partial K = (1+r)q^K$, (D.1b) $-\partial G/\partial R = q^R(r+\delta^R)$,

(D.1c) $L = \partial G/\partial p^L$, (D.1d) $K^o = -\partial G/\partial q^K$.

Let $G_{\alpha\beta}$ denote the second order partial derivatives of G with respect to α and β where $\alpha,\beta = q^K$, p^L, K, R. (Of course, since G is assumed to be twice continuously differentiable in all its arguments we have $G_{\alpha\beta} = G_{\beta\alpha}$.) Differentiating (D.1a), (D.1b), (D.1c) with respect to q^K and (D.1a), (D.1b), (D.1d) with respect to p^L yields:

(D.2a) $-G_{q^K K} - G_{KK}(\partial K/\partial q^K) - G_{RK}(\partial R/\partial q^K) = (1+r)$,

(D.2b) $-G_{q^K R} - G_{KR}(\partial K/\partial q^K) - G_{RR}(\partial R/\partial q^K) = 0$,

(D.2c) $G_{q^K p^L} + G_{Kp^L}(\partial K/\partial q^K) + G_{Rp^L}(\partial R/\partial q^K) = \partial L/\partial q^K$.

(D.3a) $-G_{p^L K} - G_{KK}(\partial K/\partial p^L) - G_{RK}(\partial R/\partial p^L) = 0$,

(D.3b) $-G_{p^L R} - G_{KR}(\partial K/\partial p^L) - G_{RR}(\partial R/\partial p^L) = 0$,

(D.3c) $G_{p^L q^K} + G_{Kq^K}(\partial K/\partial p^L) + G_{Rq^K}(\partial R/\partial p^L) = -\partial K^o/\partial p^L$.

Define

$$H = -\begin{bmatrix} G_{KK} G_{KR} \\ G_{RK} G_{RR} \end{bmatrix}, \quad h_K = \begin{bmatrix} \partial K/\partial q^K \\ \partial R/\partial q^K \end{bmatrix}, \quad h_L = \begin{bmatrix} \partial K/\partial p^L \\ \partial R/\partial p^L \end{bmatrix}, \quad j_K = \begin{bmatrix} 1+r+G_{Kq^K} \\ G_{Rq^K} \end{bmatrix}, \quad j_L = \begin{bmatrix} G_{Kp^L} \\ G_{Rp^L} \end{bmatrix}.$$

We can now rewrite (D.2c) and (D.3c) as, respectively:

(D.4a) $\quad \partial L/\partial q^K = G_{q^K p^L} + (j_L)^T h_K ,$

(D.4b) $\quad (1+r)(\partial K/\partial p^L) - \partial K^o/\partial p^L = G_{p^L q^K} + (j_K)^T h_L .$

Solving equations (D.1a), (D.1b) and (D.2a), (D.2b), respectively, yields $h_K = H^{-1} j_K$ and $h_L = H^{-1} j_L$. The claimed result now follows upon substituting those expressions into (D.4) and observing that H is symmetric.

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