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ABSTRACT

Complementarities can have important welfare consequences, but quantifying these effects would seem to require identifying and valuing the underlying mechanisms separately. This paper shows that this need not be the case. I provide necessary and sufficient conditions under which baseline subsidized activity and the direct and total responses to a subsidy determine aggregate willingness to pay. A sufficient condition for valuing the equilibrium adjustment is that it has the same effect on utility as an additional subsidy common to all agents and relevant actions. The formula is easy to apply to existing estimates and guides the design of new experiments.

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1 Introduction

Complementarities arise in many settings where policy affects participation, investment, or production. The value of an action may depend on what others do or on equilibrium conditions shaped by their choices. These forces can have important welfare consequences, but they also make policy evaluation difficult: a policy changes both the incentive faced directly by an agent and the environment in which the agent makes decisions. When several complementarities operate at once, modeling and valuing each one separately can be demanding.

In this paper, I characterize when the welfare contribution of complementarities can be recovered from the response of the subsidized activity itself, without having to identify and value each equilibrium channel separately. The key objects are the direct and total responses to the same subsidy. A subsidy offered to a small number of individuals in a large market changes their own incentive while leaving the equilibrium conditions they face largely unaffected, giving the direct response. A broader change in the same subsidy also changes equilibrium conditions, giving the total response. The difference captures the additional response generated by complementarities. I ask when this additional response is enough to value the welfare effect of the equilibrium adjustment. When it is, the direct and total responses provide sufficient statistics that can be estimated empirically.

First, I derive *necessary and sufficient* conditions for valuing the equilibrium adjustment from these responses. The response of subsidized activity is sufficient for aggregate willingness to pay exactly when any two admissible local changes that generate the same response also generate the same willingness to pay. Equivalently, any admissible local change that leaves subsidized activity unchanged must also leave aggregate willingness to pay unchanged. This characterization makes clear when the direct and total responses are enough for welfare analysis and when additional information is needed: if two equilibrium changes generate the same activity response but different willingness to pay, the response alone cannot value them.

I next give a simple and economically interpretable condition under which the equilibrium adjustment induced by the subsidy can be valued from these responses. Suppose the adjustment affects utility at every relevant action in the same way as a common additional subsidy. A larger payments network, for example, may raise the value of adoption just as a larger adoption subsidy would. Under this subsidy equivalence condition, the equilibrium adjustment can be valued in units of the subsidy, so baseline subsidized activity and the direct and total responses are sufficient for welfare analysis. The condition applies to the combined equilibrium adjustment, so the individual sources of complementarity do not have to be subsidy equivalent separately. Price incidence provides a familiar special case: when the equilibrium adjustment operates through a common change in the transaction price, the

price response measures it directly in the same units as the subsidy. The results here also cover settings in which no observed price summarizes the equilibrium adjustment and its welfare value must instead be recovered from the response of subsidized activity.

These results yield a simple formula for welfare per dollar of government spending. Let τ denote the subsidy rate and A baseline subsidized activity. The direct response is $D = \partial A/\partial \tau$, holding equilibrium conditions fixed; the total response is $T = dA/d\tau$, allowing them to adjust. When activity responses are sufficient for willingness to pay, T/D gives total marginal willingness to pay relative to the direct subsidy benchmark. For a positive baseline subsidy, let ϵ^D and ϵ^T denote the corresponding elasticities.[1] If government expenditure, G, consists only of subsidy payments, net welfare, W, per dollar of government spending is

$$\frac{dW}{dG} = \text{MVPF} - 1 = \frac{\epsilon^T/\epsilon^D}{1+\epsilon^T} - 1,$$

where MVPF is the marginal value of public funds, defined as aggregate willingness to pay per dollar of marginal government expenditure. The ratio ϵ^T/ϵ^D captures the amplification of the direct response generated by complementarities. Together with baseline subsidized activity, these responses form sufficient statistics for this component of welfare, so the separate sources of complementarity do not need to be modeled or valued individually.

The framework also extends beyond the scalar benchmark. When the value of the equilibrium adjustment differs across predetermined types, welfare combines type-specific direct and total responses using baseline subsidized activity as weights. When several subsidized margins are affected, multiple policy instruments can provide the variation needed to recover welfare. The framework also allows the subsidy and its equilibrium effects to arrive at different dates.

Moreover, the results also clarify the policy variation needed to estimate these responses. Localized subsidy variation can identify D when it changes agents' incentives without materially changing the equilibrium conditions they face, while broader variation can identify T by allowing those conditions to adjust. The estimates can come from different experiments, quasi-experiments, or policy changes, provided they refer to the same subsidy, subsidized activity, population, baseline, and horizon. When broader policy variation is unavailable, T can be recovered by combining D with exogenous variation in equilibrium feedback.

I illustrate the results using estimates from existing experiments. In the latrine adoption experiment of Guiteras, Levinsohn and Mobarak (2015), I use the reported ownership effects to approximate the ratio of total to direct elasticities. The comparison implies that equilibrium feedback makes a marginal subsidy increase worth about 53 percent more to voucher

[1] That is, $\epsilon^D = \frac{\tau}{A}\frac{\partial A}{\partial \tau} = \frac{\tau D}{A}$ and $\epsilon^T = \frac{\tau}{A}\frac{dA}{d\tau} = \frac{\tau T}{A}$.

recipients than the direct incentive alone. I use estimates from the sanitation experiment of Deutschmann, Lipscomb, Schechter and Zhu (2024) to show how the framework can account for health benefits even among households that make no subsidized purchase. The framework also captures equilibrium feedback that offsets, rather than amplifies, the direct incentive. Using the price pass-through estimate from the maize-market experiment of Bergquist and Dinerstein (2020), I calculate that, at a zero baseline subsidy, price adjustment reduces consumers' willingness to pay to about 22.4 cents per additional dollar of subsidy spending. Experiments on peer effects and information provide further applications, using responses to individual rewards or premiums as benchmarks for willingness to pay. These examples show how estimates collected for other empirical questions can support welfare analysis. For new studies, the results guide the incentive comparisons and outcomes needed to measure the welfare consequences of complementarities.

This paper contributes to the sufficient-statistics approach to welfare analysis. Chetty (2009) emphasizes that welfare can often be expressed using a small number of reduced-form objects rather than a fully specified structural model. Arkolakis, Costinot and Rodríguez-Clare (2012) and Alvarez, Le Bihan and Lippi (2016) provide prominent examples in which different models have the same welfare implications conditional on a small set of empirical statistics. I characterize when the response of a subsidized activity is sufficient to determine aggregate willingness to pay in environments with potentially many sources of complementarity, establishing when the direct and total responses contain all the information needed to value the equilibrium adjustment.

Hendren (2016), Hendren and Sprung-Keyser (2020), and Finkelstein and Hendren (2020) show how reduced-form policy effects can be combined with economic restrictions to measure welfare, while Dávila and Schaab (2025) provide a general treatment of welfare comparisons in heterogeneous environments. The problem considered here is that a policy may induce equilibrium changes whose welfare value is not directly observed. The subsidy provides a benchmark: the envelope theorem gives willingness to pay for a direct subsidy change, and the characterization establishes when that same benchmark can be used to value the equilibrium adjustment from its effect on subsidized activity.

The ratio of total to direct responses also connects the paper to the social-multiplier literature (Glaeser et al., 2003). Kroft (2008) uses this ratio in a sufficient-statistics formula for optimal unemployment insurance with endogenous takeup costs. I characterize when these responses and baseline subsidized activity are sufficient for welfare across models, without having to specify or estimate the underlying complementarities. Finally, Bhattacharya, Dupas and Kanaya (2024) study demand and welfare identification in binary-choice settings with social interactions, deriving welfare bounds and conditions for point identification. I

focus on marginal policy changes and characterize when their welfare effects can be recovered from a few causal responses, without estimating a demand system. The results cover both discrete and smooth choices and allow several sources of complementarity. The extensions show how to combine responses across subsidized activities.

2 Welfare Accounting

Consider a policy indexed by a scalar τ. Agent i has indirect utility $v_i(y_i, \tau, \mathbf{z}(\tau))$, where y_i is disposable income and $\mathbf{z}(\tau)$ collects equilibrium variables that may respond to the policy. Let $\lambda_i \equiv \partial v_i / \partial y_i > 0$ denote the marginal utility of income. Aggregate willingness to pay for a marginal policy change is

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\tau} = \int \frac{1}{\lambda_i} \left(\frac{\partial v_i}{\partial \tau} + \sum_{j=1}^{J} \frac{\partial v_i}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau} \right) \mathrm{d}i.$$

Dividing by λ_i expresses utility changes in dollars. The first term values the direct policy change; the second values the induced equilibrium adjustment. Denote the latter by

$$E_\tau \equiv \int \sum_{j=1}^{J} \frac{1}{\lambda_i} \frac{\partial v_i}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau} \, \mathrm{d}i.$$

Knowing how choices respond is not generally enough to recover E_τ: the same change in choices can be associated with different changes in utility. I focus on a subsidy, whose direct effect provides a benchmark for willingness to pay.

Let τ denote the subsidy rate. Agent i chooses an action a_i, and $Q_i(a_i)$ denotes the expected present discounted amount of activity on which the subsidy is paid.[2] This may be discounted adoption, discounted time spent participating, or the number or value of subsidized transactions.[3] The same object is eligible earnings under a wage subsidy, acres under an agricultural subsidy, or output under a production subsidy. The expected present value of the transfer is $\tau Q_i(a_i)$. The agent chooses

$$a_i^*(y_i, \tau, \mathbf{z}) \in \arg\max_{a_i} U_i\big(a_i, y_i + \tau Q_i(a_i), \mathbf{z}\big).$$

[2]For uncertain or delayed payments, I assume that their expected present value enters income and is calculated using the same probabilities and discounting as government spending.

[3]For a one-time subsidy paid upon adoption at stopping time T_i, $Q_i = \mathbb{E}_i[e^{-\rho T_i} \mathbf{1}\{T_i < \infty\}]$. For a subsidy paid per unit of a flow $q_i(t)$, $Q_i = \mathbb{E}_i[\int_0^\infty e^{-\rho t} q_i(t) \, \mathrm{d}t]$.

The envelope theorem gives $\frac{1}{\lambda_i}\frac{\partial v_i}{\partial \tau} = Q_i(a_i^*)$.[4] Thus, holding equilibrium variables fixed, agent i's willingness to pay for a marginal subsidy increase is exactly the amount of subsidized activity associated with the optimal action. Define aggregate subsidized activity as

$$A(\tau, \mathbf{z}) = \int Q_i\big(a_i^*(y_i, \tau, \mathbf{z})\big)\,\mathrm{d}i.$$

Thus $A(\tau, \mathbf{z})$ is also aggregate willingness to pay for a marginal subsidy increase holding equilibrium variables fixed. I assume that $A(\tau, \mathbf{z})$ and the equilibrium path $\mathbf{z}(\tau)$ are differentiable at the baseline. Writing $A(\tau)$ for $A(\tau, \mathbf{z}(\tau))$, total willingness to pay is

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\tau} = A + E_\tau.$$

The direct and total responses of subsidized activity are

$$D = \left.\frac{\partial A}{\partial \tau}\right|_{\mathbf{z}}, \qquad T = \frac{\mathrm{d}A}{\mathrm{d}\tau}.$$

The direct response holds equilibrium variables fixed, while the total response allows them to adjust. Their difference, $T - D$, is the additional response generated by the equilibrium adjustment.[5]

If subsidy payments are the only fiscal cost, government expenditure is $G(\tau) = \tau A(\tau)$, so $\mathrm{d}G/\,\mathrm{d}\tau = A + \tau T$. The first term is the cost of increasing the subsidy on baseline activity; the second is the additional spending induced by the response. Let $W(\tau) = \mathcal{V}(\tau) - G(\tau)$. The welfare gain per dollar of government spending and the marginal value of public funds are

$$\begin{aligned} \frac{\mathrm{d}W}{\mathrm{d}G} &= \frac{\mathrm{d}\mathcal{V}/\,\mathrm{d}\tau - \mathrm{d}G/\,\mathrm{d}\tau}{\mathrm{d}G/\,\mathrm{d}\tau} = \frac{(A + E_\tau) - (A + \tau T)}{A + \tau T} = \frac{E_\tau - \tau T}{A + \tau T}, \\ \text{MVPF} &= \frac{\mathrm{d}\mathcal{V}/\,\mathrm{d}\tau}{\mathrm{d}G/\,\mathrm{d}\tau} = \frac{A + E_\tau}{A + \tau T}. \end{aligned} \tag{1}$$

This is the standard MVPF accounting. In the absence of equilibrium effects, $E_\tau = 0$, and the expressions reduce to the usual formulas. The fiscal cost is determined by A and T; the remaining problem is to recover E_τ.

[4] For example, substituting the budget constraint $c_i + C_i(a_i) = y_i + \tau Q_i(a_i)$ into utility gives this formulation after incorporating the cost $C_i(a_i)$ into U_i.

[5] By the chain rule, $T - D = \sum_{j=1}^{J} (\partial A/\partial z_j)(\,\mathrm{d}z_j/\,\mathrm{d}\tau)$.

3 Responses as Sufficient Statistics

The difference $T - D$ measures how much the equilibrium adjustment changes subsidized activity, while E_τ measures willingness to pay for that adjustment. This section characterizes when the first determines the second.

Consider a local change indexed by ϵ, with subsidy and equilibrium variables $(\tau(\epsilon), \mathbf{z}(\epsilon))$. At the baseline, $\tau(0) = \tau$ and $\mathbf{z}(0) = \mathbf{z}(\tau)$. Derivatives with respect to ϵ are evaluated at $\epsilon = 0$. By the chain rule,

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = D\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon} + \sum_{j=1}^{J} \frac{\partial A}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\epsilon}.$$

Aggregate willingness to pay for the same local change is $\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} \equiv \int \frac{1}{\lambda_i}\frac{\mathrm{d}v_i}{\mathrm{d}\epsilon}\,\mathrm{d}i$. The chain rule and the envelope theorem give

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = A\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon} + \int \sum_{j=1}^{J} \frac{1}{\lambda_i}\frac{\partial v_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\epsilon}\,\mathrm{d}i.$$

A unit direct subsidy increase has $\mathrm{d}\tau/\,\mathrm{d}\epsilon = 1$ and $\mathrm{d}z_j/\,\mathrm{d}\epsilon = 0$ for every j. It generates a response D and willingness to pay A. To isolate the equilibrium adjustment, hold the subsidy fixed and let each equilibrium variable change by the amount induced by the marginal subsidy increase. Then $\mathrm{d}\tau/\,\mathrm{d}\epsilon = 0$ and $\mathrm{d}z_j/\,\mathrm{d}\epsilon = \mathrm{d}z_j/\,\mathrm{d}\tau$, giving

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = \sum_{j=1}^{J} \frac{\partial A}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau} = T - D, \qquad \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \int \sum_{j=1}^{J} \frac{1}{\lambda_i}\frac{\partial v_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau}\,\mathrm{d}i = E_\tau.$$

The direct subsidy therefore gives the known pair (D, A), while the equilibrium adjustment gives $(T - D, E_\tau)$. The question is whether the response $T - D$ is enough to determine the unknown willingness to pay E_τ. More generally, if two local changes generate the same activity response but different aggregate willingness to pay, the response alone cannot be sufficient for welfare. Equal responses must therefore imply equal willingness to pay. The next theorem shows that this condition is both necessary and sufficient.

THEOREM 1. Suppose $D \neq 0$.[6] The following statements are equivalent:

(i) Any two local changes that generate the same response $\mathrm{d}A/\,\mathrm{d}\epsilon$ generate the same aggregate willingness to pay $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon$.

[6]The comparisons include all direct subsidy changes holding equilibrium variables fixed, the equilibrium adjustments being valued, and first-order differences between the changes considered.

(ii) Any local change that leaves subsidized activity unchanged also leaves aggregate willingness to pay unchanged: $\mathrm{d}A/\mathrm{d}\epsilon = 0$ implies $\mathrm{d}\mathcal{V}/\mathrm{d}\epsilon = 0$.

(iii) For every local change,

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \frac{A}{D}\frac{\mathrm{d}A}{\mathrm{d}\epsilon}. \tag{2}$$

When these conditions hold, the response of subsidized activity is a sufficient statistic for aggregate willingness to pay, and equation (2) is the unique relationship consistent with the direct subsidy benchmark.

The intuition for the theorem comes from the direct subsidy benchmark. A unit increase in the subsidy, holding equilibrium conditions fixed, generates willingness to pay A and changes subsidized activity by D. The ratio A/D therefore gives willingness to pay per unit of response to the direct subsidy. The theorem establishes when this benchmark can also value another local change: changes that generate the same activity response must generate the same willingness to pay, whether they operate through the subsidy or through equilibrium conditions.

To apply the result to the policy-induced equilibrium adjustment, hold the subsidy fixed. The adjustment generates a response $T - D$ and willingness to pay E_τ. Under the theorem's conditions, a direct subsidy change generating the same response must have the same willingness to pay. This gives the sufficient-statistics result below.

Corollary 1. Suppose the conditions of Theorem 1 hold. Then

$$E_\tau = \frac{A}{D}(T - D). \tag{3}$$

Total aggregate willingness to pay for the policy change is thus $A + E_\tau = A(T/D)$. For $\tau > 0$, define the direct and total elasticities of subsidized activity with respect to the subsidy as

$$\epsilon^D = \frac{\tau D}{A}, \qquad \epsilon^T = \frac{\tau T}{A}.$$

Then

$$\frac{T}{D} = \frac{\epsilon^T}{\epsilon^D}.$$

If subsidy payments are the only fiscal cost, substituting equation (3) into equation (1) gives

$$\frac{\mathrm{d}W}{\mathrm{d}G} = \frac{\epsilon^T/\epsilon^D}{1 + \epsilon^T} - 1, \qquad \mathrm{MVPF} = \frac{\epsilon^T/\epsilon^D}{1 + \epsilon^T}.$$

Equation (3) is the sufficient-statistics result. The direct subsidy generates willingness to pay A and a response D, providing a benchmark for valuing the additional response $T - D$. Under the theorem's conditions, A, D, and T therefore recover the combined welfare effect of complementarities without having to separately identify or value the equilibrium variables responsible for the additional response.

The characterization also shows when additional welfare information is needed. Suppose an equilibrium change raises utility by the same amount regardless of the action an individual chooses. It leaves the relative value of the available actions unchanged, so choices and subsidized activity do not change even though willingness to pay rises. This violates condition (ii) in Theorem 1: the activity response cannot capture this welfare gain. The next section gives a sufficient condition on utility under which the equilibrium adjustment can be valued as an additional subsidy, yielding $E_\tau = A(T/D - 1)$.

4 Subsidy Equivalence

I now give a sufficient condition on the individual problem for valuing the policy-induced equilibrium adjustment using the direct subsidy benchmark.[7] Recall that agent i has utility $U_i\left(a_i, y_i + \tau Q_i(a_i), \mathbf{z}\right)$. Holding the action and the equilibrium variables fixed, a marginal increase in the subsidy changes utility at rate $(\partial U_i/\partial y_i)Q_i(a_i)$. A marginal increase in the subsidy also changes the equilibrium variables. Holding the action fixed, the utility effect generated by this equilibrium adjustment is $\sum_{j=1}^{J}(\partial U_i/\partial z_j)(\,\mathrm{d}z_j/\,\mathrm{d}\tau)$. The policy-induced equilibrium adjustment satisfies *subsidy equivalence* if there exists a scalar γ such that, for every agent i and every relevant feasible action a_i,

$$\sum_{j=1}^{J} \frac{\partial U_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau} = \gamma \frac{\partial U_i}{\partial y_i} Q_i(a_i). \tag{4}$$

The equilibrium adjustment must have γ times the utility effect of the direct subsidy change, with the same γ across agents and actions.

A familiar example is when the equilibrium adjustment affects consumers only through a common price change for the subsidized good. A higher price has the same effect as a lower per-unit subsidy, so the condition follows from the budget constraint.[8]

Importantly, the adjustment need not operate through prices. In a technology-adoption

[7] For smooth interior choices, this condition can be weakened considerably (see Proposition 2).

[8] For a price-taking consumer with budget constraint $c_i = y_i - (p - \tau)q_i$ and $Q_i = q_i$, $\partial U_i/\partial p = -(\partial U_i/\partial y_i)q_i$ at every fixed q_i. Thus, equation (4) holds with $\gamma = -\,\mathrm{d}p/\,\mathrm{d}\tau$. See Weyl and Fabinger (2013) for a general treatment of pass-through and incidence under imperfect competition.

setting, a subsidy may expand the network. If a larger network raises the payoff from adoption in the same proportion as the subsidy, without changing the payoff from non-adoption, subsidy equivalence holds. Similarly, when usage is subsidized, the equilibrium adjustment may raise the payoff from use in the same proportion as an additional subsidy. The next proposition shows that the additional response and willingness to pay are then governed by the same factor γ.

PROPOSITION 1. Suppose subsidy equivalence in equation (4) holds.[9] Then

$$T - D = \gamma D, \qquad E_\tau = \gamma A. \tag{5}$$

If $D \neq 0$, then $\gamma = T/D - 1$, and therefore

$$E_\tau = A\left(\frac{T}{D} - 1\right). \tag{6}$$

The two equalities in equation (5) connect the effect of the equilibrium adjustment on choices to its welfare value. A marginal increase in the subsidy, holding equilibrium variables fixed, generates the direct response D. Subsidy equivalence implies that the induced equilibrium adjustment changes the incentives across actions in the same proportion γ as the subsidy itself. It therefore generates an additional response γD. The total response combines the direct response and this equilibrium effect, so $T = D + \gamma D$, or equivalently $T - D = \gamma D$.

The same condition determines willingness to pay. At the optimal action, the envelope theorem gives willingness to pay $Q_i(a_i^*)$ for a marginal subsidy increase. The equilibrium adjustment has γ times its utility effect, so willingness to pay for that adjustment is $\gamma Q_i(a_i^*)$. Aggregating across agents gives $E_\tau = \gamma A$, while the response ratio identifies $\gamma = T/D - 1$ when $D \neq 0$.

4.1 Interpretation and Scope

Subsidy equivalence is most natural when the subsidy and the equilibrium adjustment affect the same choice margin. Consider a subsidy for school attendance. The subsidy raises the payoff from attending school, while higher attendance by other students may also make attendance more valuable through peer effects or learning. If the improvement in the peer environment raises the payoff from attendance by an amount proportional to the change

[9] I assume throughout that changes without a first-order utility effect for any agent at any relevant action have no first-order effect on the aggregate amount of any subsidized activity.

generated by the subsidy, while leaving the payoff from nonattendance unchanged, the equilibrium effect is subsidy equivalent.

The condition can accommodate several equilibrium effects at once. An increase in the subsidy may change networks, information, complementary inputs, prices, or other equilibrium variables. These effects need not satisfy subsidy equivalence separately; only their combined effect must satisfy equation (4).

Subsidy equivalence is sufficient, but not necessary, for the welfare formula in equation (3) to hold for the policy-induced equilibrium adjustment. It requires proportional utility effects across agents and relevant actions. Theorem 1 characterizes when the direct subsidy benchmark applies to all the local changes under consideration.

For smooth interior choices, this requirement can be weakened. To make the local argument transparent, consider the case in which a_i is one-dimensional. Subsidy equivalence imposes proportionality at every relevant action, but the response and welfare calculation depend only on what happens locally around the optimal action. To value the equilibrium adjustment, it is enough that its utility effect at the optimum be proportional to the effect of the subsidy. To determine how the optimal action responds, the derivatives of these utility effects with respect to the action must also be proportional at the optimum. The equilibrium adjustment and the subsidy may differ away from the optimal action without affecting either calculation.

PROPOSITION 2. Suppose each agent makes a smooth interior choice satisfying a strict second-order condition. Assume that differentiation and integration can be interchanged when computing D and T. Suppose there exists a common scalar γ such that, at $a_i = a_i^*$,

$$\sum_{j=1}^{J} \frac{\partial U_i}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau} = \gamma \frac{\partial U_i}{\partial y_i} Q_i(a_i), \qquad \frac{\partial}{\partial a_i} \left[\sum_{j=1}^{J} \frac{\partial U_i}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau} \right] = \gamma \frac{\partial}{\partial a_i} \left[\frac{\partial U_i}{\partial y_i} Q_i(a_i) \right]. \tag{7}$$

Then $T - D = \gamma D$ and $E_\tau = \gamma A$. If $D \neq 0$, then $E_\tau = A(T/D - 1)$.

The two conditions in equation (7) have different roles. The first determines willingness to pay for the equilibrium adjustment because it holds at the action actually chosen. The second determines how the equilibrium adjustment changes the incentives around that action and therefore how the optimal action responds. Full subsidy equivalence imposes these relationships at every relevant action. For a smooth interior choice, these local conditions are enough for the welfare calculation.

5 A Simple Example

Consider a participation problem with network effects and social learning. Appendix B gives a smooth-choice example in which the weaker local condition applies even though full subsidy equivalence fails.

5.1 Participation with Network Effects and Social Learning

Let agents have unit mass, indexed by $i \in [0,1]$, and let $p_i \in \{0,1\}$ indicate whether agent i participates. An agent who participates receives a benefit b_i and a subsidy τ, and incurs a cost c_i. Without complementarities, agent i participates whenever $c_i - b_i \leq \tau$. Since $Q_i(p_i) = p_i$, aggregate subsidized activity is the participation rate, $A = N \equiv \int_0^1 p_i \, \mathrm{d}i$. In this case, $D = T$ and $E_\tau = 0$.

Now suppose participation becomes more valuable when more people participate and when their experience generates useful information. Let I denote the information generated by participants and let $\Theta(N, I)$ denote the additional value of participation. The payoff from not participating is unaffected, so agent i participates whenever $c_i - b_i \leq \tau + \Theta(N, I)$. A larger N may raise the value of participation through a network or peer effect, while a larger I may raise it through learning from others. The two channels may also interact through Θ.

Holding N and I fixed, the response of participation to the subsidy is D.[10] Because τ and $\Theta(N, I)$ enter the participation payoff additively, a change in Θ affects participation in the same way as an equal change in the subsidy. When the subsidy changes, N and I also adjust. Since $A = N$ and $T = \mathrm{d}N/\mathrm{d}\tau$,

$$T = D\left(1 + \frac{\mathrm{d}\Theta}{\mathrm{d}\tau}\right), \qquad \frac{\mathrm{d}\Theta}{\mathrm{d}\tau} = \frac{\partial\Theta}{\partial N}\frac{\mathrm{d}N}{\mathrm{d}\tau} + \frac{\partial\Theta}{\partial I}\frac{\mathrm{d}I}{\mathrm{d}\tau}.$$

If $D \neq 0$, then $T/D - 1 = \mathrm{d}\Theta/\mathrm{d}\tau$. Proposition 1 therefore gives

$$E_\tau = A\frac{\mathrm{d}\Theta}{\mathrm{d}\tau} = A\left(\frac{T}{D} - 1\right).$$

Therefore, the network and learning effects do not need to be valued separately. Their combined welfare effect is summarized by A, D, and T.

[10] If $c_i - b_i$ has cumulative distribution function F with density f, then $N = F(\tau + \Theta(N, I))$ and, holding N and I fixed, $D = f(\tau + \Theta(N, I))$.

6 Extensions

The approach remains useful when the value of the equilibrium adjustment differs across predetermined groups, when several subsidized margins are affected, or when benefits arrive at different dates from subsidy payments.

6.1 Predetermined Heterogeneity

The common γ condition can be relaxed when heterogeneity is determined before the policy and responses can be measured separately across groups. Let k index a predetermined type with distribution F, and let A_k, D_k, and T_k denote its baseline subsidized activity and direct and total responses. Aggregate activity is $A = \int A_k \, \mathrm{d}F(k)$. The scalar result can then be applied within each type and aggregated.

PROPOSITION 3. Suppose $D_k \neq 0$ for every relevant type and the conditions of Theorem 1 hold within each type. Then

$$E_\tau = \int A_k \left(\frac{T_k}{D_k} - 1 \right) \mathrm{d}F(k), \tag{8}$$

and total aggregate willingness to pay is

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\tau} = \int A_k \frac{T_k}{D_k} \, \mathrm{d}F(k).$$

When T_k/D_k differs across types, applying the scalar formula to the pooled responses $D = \int D_k \, \mathrm{d}F(k)$ and $T = \int T_k \, \mathrm{d}F(k)$ need not give equation (8).[11]

6.2 Several Subsidized Margins

A technology may have a fixed adoption benefit and a separate per-use benefit, or a platform may subsidize both users and suppliers. One subsidy may then provide too little variation to value the full equilibrium adjustment.

Let ϵ index a policy change, with $\epsilon = 0$ at the baseline. Let $\boldsymbol{\tau}(\epsilon) = (\tau_1(\epsilon), \ldots, \tau_M(\epsilon))'$ denote the subsidy rates and $\boldsymbol{\tau}_\epsilon = \mathrm{d}\boldsymbol{\tau}(\epsilon)/\mathrm{d}\epsilon|_{\epsilon=0}$ their change with the policy. Let $\mathbf{A} =$

[11] The type-specific formula implies $E_\tau/A = \int (A_k/A)(T_k/D_k - 1) \, \mathrm{d}F(k)$, so each group's equilibrium effect is weighted by its share of baseline subsidized activity. Pooling first instead gives $T/D - 1 = \int (D_k/D)(T_k/D_k - 1) \, \mathrm{d}F(k)$.

$(A_1, \ldots, A_M)'$ collect the corresponding subsidized activities and assume it is differentiable in $\boldsymbol{\tau}$ and $\mathbf{z}$ at the baseline. Define

$$\mathbf{D} = \left.\frac{\partial \mathbf{A}}{\partial \boldsymbol{\tau}'}\right|_{\mathbf{z}}, \qquad \mathbf{T} = \left.\frac{\mathrm{d}\mathbf{A}(\epsilon)}{\mathrm{d}\epsilon}\right|_{\epsilon=0},$$

where D_{mn} is the direct response of activity m to subsidy n, holding equilibrium variables fixed, while $\mathbf{T}$ gives the total response to the policy.

Subsidy equivalence now requires the first-order utility effect of the policy, including the equilibrium adjustment, to be reproduced by the same vector $\boldsymbol{\beta}$ of direct subsidy changes for every agent and relevant action.[12] Under this condition, $\mathbf{T} = \mathbf{D}\boldsymbol{\beta}$, and the envelope theorem gives total aggregate willingness to pay $\mathbf{A}'\boldsymbol{\beta}$. Let E_ϵ denote willingness to pay for the equilibrium adjustment.

PROPOSITION 4. Suppose the policy change is equivalent, for every agent and relevant action, to the same vector $\boldsymbol{\beta}$ of direct subsidy changes, and suppose $\mathbf{D}$ is invertible. Then

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \mathbf{A}'\mathbf{D}^{-1}\mathbf{T}, \qquad E_\epsilon = \mathbf{A}'\left(\mathbf{D}^{-1}\mathbf{T} - \boldsymbol{\tau}_\epsilon\right). \tag{9}$$

Invertibility is sufficient but stronger than necessary. Even when $\mathbf{D}$ is singular, welfare may still be identified if the components of $\boldsymbol{\beta}$ that cannot be recovered from the responses do not affect aggregate willingness to pay. Proposition 6 in the appendix gives the exact weaker condition.

6.3 Differences in Timing

The framework allows a_i and $\mathbf{z}$ to describe entire dynamic paths. The relevant measure of subsidized activity must then reflect the timing of the subsidy change. Let ϵ index a change in the subsidy path and let $\ell(t) = \partial\tau(t,\epsilon)/\partial\epsilon|_{\epsilon=0}$ denote the marginal change at date t. With a per-unit subsidy on a flow $q_i(t)$, the relevant activity is $Q_i(a_i) = \mathbb{E}_i[\int_0^\infty e^{-\rho t}\ell(t)q_i(t)\,\mathrm{d}t]$. Define A using this discounted activity, and let D and T denote its direct and total responses to ϵ. The results above apply whenever the response of A is sufficient for willingness to pay.

Suppose the subsidy and the equilibrium effect both enter as per-unit returns to $q_i(t)$, with their expected present values added to y_i in U_i. Let $\Theta_\epsilon(t)$ denote the change in the equilibrium return induced by the policy.

[12] Formally, $\sum_{m=1}^M (\partial U_i/\partial\tau_m)\tau_{\epsilon,m} + \sum_{j=1}^J (\partial U_i/\partial z_j)(\,\mathrm{d}z_j/\,\mathrm{d}\epsilon) = \sum_{m=1}^M \beta_m(\partial U_i/\partial\tau_m)$. Equivalently, $\boldsymbol{\beta} - \boldsymbol{\tau}_\epsilon$ is the multidimensional analogue of γ: it expresses the equilibrium adjustment alone in units of the M subsidies.

LEMMA 1. Suppose there is a common scalar γ such that

$$\Theta_\epsilon(t) = \gamma \ell(t) \tag{10}$$

at every relevant date. Then subsidy equivalence holds over the relevant dynamic activity paths.

The condition requires the equilibrium benefit and the subsidy change to have the same time profile, up to the factor γ. If the profiles differ, one response ratio is generally not enough. A temporary production subsidy, for example, may raise activity today while learning by doing lowers costs mainly in the future (Hahn et al., 2026). If several independently varied subsidy profiles span the relevant equilibrium effect, Proposition 4 can instead be applied using those profiles as separate policy margins.[13]

7 Empirical Implementation

Individual subsidy variation can identify D when it does not materially affect the equilibrium conditions agents face. Coupons, eligibility rules, or firm-specific incentives can provide this variation, while broader changes in the same subsidy can identify T. The estimates may come from different designs, but must refer to the same subsidy, subsidized activity, population, baseline, and horizon.[14] Effects on utility that leave subsidized activity unchanged must be valued separately.

7.1 Recovering the Total Response from Equilibrium Feedback

When a broad subsidy change is not observed, the total response can sometimes be recovered from the direct response and equilibrium feedback. Suppose individual choices depend on aggregate subsidized activity. Let $\Phi(\tau, A)$ denote the activity generated by those choices at subsidy τ and a conjectured level of aggregate activity A. Equilibrium requires $A = \Phi(\tau, A)$. Holding A fixed, the direct response is $D = \partial\Phi/\partial\tau$, while $\partial\Phi/\partial A$ measures the feedback from aggregate activity into individual choices.

PROPOSITION 5. Suppose the equilibrium is locally differentiable and $1 - \partial\Phi/\partial A \neq 0$.

[13] Appendix C uses studies of vaccination, health insurance, and land leveling to illustrate how the welfare calculation accounts for differences across individuals, substitution between subsidized alternatives, and the timing of benefits.

[14] A diffusion path reflects endogenous propagation and other changes in the environment, so it does not by itself identify D or T.

Then

$$T = \frac{D}{1 - \partial\Phi/\partial A}. \tag{11}$$

Randomized saturation can help estimate this feedback by varying offers to others while holding an agent's own subsidy fixed. The offers must change A (e.g., the fraction of people using a technology) and affect the agent's choice only through that change. Aggregating individual marginal activity responses to A, using the population weights in Φ, gives $\partial\Phi/\partial A$. The variation must shift the same equilibrium variable that would respond to the policy being evaluated. Together with D, the estimated feedback then gives T through equation (11).

8 Applications

The sufficient statistic is simple to calculate from estimates already reported in empirical studies. I illustrate its use for equilibrium amplification, benefits requiring additional welfare information, and price adjustments that offset the direct incentive. I then consider peer participation and information, using responses to individual incentives as benchmarks for willingness to pay under the conditions of Theorem 1.

8.1 Equilibrium Amplification from Existing Estimates

In Guiteras, Levinsohn and Mobarak (2015), eligible poor households in rural Bangladesh were offered vouchers covering 75 percent of the cost of hygienic-latrine components, with households paying for delivery and installation. Communities were randomly assigned to voucher coverage rates of 25, 50, or 75 percent, with recipients randomly selected among eligible households within each community. A separate group received the same sanitation-promotion program but no vouchers. The voucher terms are unchanged across coverage arms. The reported outcome is hygienic-latrine ownership at the end of the study. For a change in the voucher discount rate, Q_i is expenditure on components purchased with the voucher, measured before the discount.

At 25 percent coverage, latrine ownership among voucher recipients is 13.7 percentage points higher than among eligible households in communities receiving promotion without vouchers.[15] The corresponding increase among eligible nonrecipients is only 0.9 percentage points. This small response is consistent with limited equilibrium feedback, so I use the

[15] The reported effects subtract the promotion-only coefficient from the corresponding coefficients in Table S4, column (8); for example, $0.153 - 0.016 = 0.137$ at 25 percent coverage.

recipient effect at 25 percent coverage as a benchmark against which to compare the effects at higher coverage.

The effect on recipients rises to 20.9 percentage points at 50 percent coverage and is similar at 75 percent coverage. These effects reflect both the recipient's own voucher and the changes induced by vouchers offered to others. While Guiteras, Levinsohn and Mobarak (2019) study how subsidies affect adoption in a demand model with complementarities, they do not calculate the welfare effects of those complementarities. Under subsidy equivalence, the theory developed above allows the equilibrium adjustment to be valued using the direct and total responses. For illustration, I compare the effect at 50 percent coverage with the low-coverage benchmark: $20.9/13.7 \simeq 1.53$. I use this ratio to approximate $T/D = \epsilon^T/\epsilon^D$ for a marginal increase in the voucher discount rate, holding the set of recipients fixed.[16] By Corollary 1, the same marginal subsidy increase is worth about 53 percent more to voucher recipients once equilibrium feedback is included.

8.2 Accounting for Additional Welfare Effects

I use the sanitation experiment of Deutschmann, Lipscomb, Schechter and Zhu (2024) in Dakar to illustrate how the sufficient statistic can be combined with a separate measure of willingness to pay for health benefits. A neighbor's sanitation investment may improve a household's health even if that household makes no subsidized purchase. The program offered discounts on a mechanized service that empties latrine pits, randomly varying households' own discounts and those offered to their neighbors separately. Households could also hire an operator independently or use manual desludging. The subsidized activity Q_i consists of services purchased through the program, so sanitation could improve without an increase in subsidized activity.

Neighbors' discounts have little effect on whether households make a subsidized purchase but increase the use of mechanized desludging more broadly. Giving one more neighbor a large rather than a small discount changes an offered household's probability of a first subsidized purchase by only -0.2 percentage points (s.e. 0.3), while raising its probability of any mechanized desludging by 0.9 percentage points (s.e. 0.5). Among households receiving no offer, the increase in mechanized desludging is 2.0 percentage points (s.e. 0.8). Across all households, the point estimate also suggests an improvement in health.

For households offered the program, the subsidy-equivalent component of the adjustment

[16] I assume that ownership and voucher-covered expenditure respond proportionally, feedback is limited at low coverage, and the reported ratio approximates T/D for the same marginal subsidy change at a common baseline. An alternative calculation under the saturation conditions in Section 7.1 does not require the low-feedback assumption. Using the 25–50 percent coverage comparison to approximate the local slope at 25 percent gives a similar result: $T/D \simeq 1 + (20.9 - 13.7)/(13.7 - 0.9) \simeq 1.56$.

can be valued using A, D, T, as in Corollary 1. This value may already include health gains that make the subsidized service more attractive. If the remaining benefits leave relative payoffs across choices unchanged, they do not affect these responses, and their willingness-to-pay value can be added separately. Welfare effects on households receiving no offer also require separate information. The framework thus shows how to combine the welfare effect recovered from subsidy responses with the value of benefits those responses do not capture.

8.3 Price Incidence and Subsidy Equivalence

When the equilibrium adjustment operates through prices, the price response directly shows how much it offsets or reinforces the direct incentive. The experiment of Bergquist and Dinerstein (2020) in local maize markets in Kenya illustrates this case. Traders received a per-unit subsidy on eligible sales, and the authors estimated how the subsidy changed the prices paid by consumers.

Although the experiment paid traders, I focus on consumer welfare. Consider paying consumers the same subsidy on the same purchases. A posted price higher by the subsidy amount would leave consumers' net payments and traders' receipts unchanged. Traders therefore face the same pricing problem under either payment arrangement, even with market power.[17] Consumer purchases are then the subsidized activity Q_i, and the seller price is the equilibrium variable.

The estimated pass-through of the trader subsidy to consumer prices is 22.4 percent. Paying consumers instead would produce the same net price reduction. Using this estimate as the marginal pass-through rate in each market, the posted price would rise by approximately 0.776 shillings per additional shilling of consumer subsidy. This increase offsets 77.6 percent of the direct incentive, giving $\gamma \simeq -0.776$ and $T/D = 1 + \gamma \simeq 0.224$.

For a small increase in the consumer subsidy at $\tau = 0$, marginal government spending is $\mathrm{d}G/\,\mathrm{d}\tau = A$, while consumer willingness to pay is $A(T/D)$ by Corollary 1. The consumer component of the MVPF is therefore

$$\text{MVPF} = \lim_{\tau \to 0^+} \frac{\epsilon^T/\epsilon^D}{1+\epsilon^T} = \frac{T}{D} \simeq 0.224. \tag{12}$$

Consumers value each additional dollar of subsidy spending at about 22.4 cents. In the latrine adoption example, equilibrium feedback instead raises willingness to pay above the direct subsidy benchmark. Prices reveal the adjustment here, while direct and total adoption

[17] Let $q(p)$ denote demand at consumer price p and c unit cost. When traders receive the subsidy, they choose p to maximize $(p+\tau-c)q(p)$. When consumers receive it, traders choose $\tilde{p}$ to maximize $(\tilde{p}-c)q(\tilde{p}-\tau)$. Substituting $\tilde{p} = p + \tau$ shows that traders face the same profit-maximization problem under either payment arrangement.

responses reveal it in the latrine example. The framework thus extends the logic of price incidence to settings in which no observed price summarizes the equilibrium adjustment.

8.4 Valuing Peer and Information Effects

The direct subsidy response can also help value greater peer participation or additional information. Under the conditions of Theorem 1, willingness to pay for either change equals willingness to pay for an additional reward that produces the same activity response.

Valuing peer effects. In Thornton (2008), participants in rural Malawi received randomly assigned cash vouchers for returning to a counseling center to learn their HIV test results. The subsidized activity Q_i indicates whether an individual collects the results needed to redeem the voucher. Using the same experiment, Godlonton and Thornton (2012) estimate how collection responds to neighbors' collection, instrumenting the fraction collecting with the fractions randomly assigned to different voucher amounts. Their specification includes the individual's own reward, allowing the peer effect and the direct reward response D to be estimated together.

Let ϵ denote the fraction of nearby respondents who collect their results. Holding the individual's own reward fixed, dividing $\mathrm{d}A/\,\mathrm{d}\epsilon$ by D gives the additional reward that produces the same collection response per unit increase in ϵ. Multiplying by the baseline collection rate A gives its first-order value per respondent. Under Theorem 1, $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon = (A/D)(\,\mathrm{d}A/\,\mathrm{d}\epsilon)$.

Among positive-voucher recipients, the estimates are $\mathrm{d}A/\,\mathrm{d}\epsilon = 0.070$ and $D = 0.119$ per dollar.[18] The percentages reported in Thornton (2008) imply a collection rate of about 79 percent among positive-voucher recipients, which I use to approximate the baseline rate in Godlonton and Thornton (2012)'s sample.[19] Using $A \simeq 0.79$ gives $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon \simeq 0.79(0.070/0.119) = 0.465$ dollars: a one percentage point increase in nearby collection is worth approximately 0.47 cents per positive-voucher recipient. Shared travel offers one interpretation: if greater peer collection lowers the cost of obtaining results by a common amount without changing the payoff from not collecting, it acts like an additional voucher. The calculation values this cost reduction without measuring it directly. Other mechanisms may contribute, provided their combined effect satisfies subsidy equivalence.

Valuing information. Cai, de Janvry and Sadoulet (2015) study how information available through friends affects demand for rice insurance in China. Farmers were randomly

[18] Table 6, Panel B, column (3).

[19] Thornton (2008) reports that 69 percent collected, 34 percent of those receiving no incentive collected, and 22 percent received no incentive (Table 2, Panel C; Section I.B). Thus, $(0.69 - 0.22 \times 0.34)/(1 - 0.22) \simeq 0.79$.

offered different premiums, and some friends were randomly assigned to an early, intensive information session. One additional informed friend raises insurance take-up by about as much as a 13 percent premium reduction. If information raises the payoff from being insured in the same proportion as a premium reduction, without changing the payoff from remaining uninsured, subsidy equivalence holds. Its effect on insured activity can then be valued using the direct subsidy response and baseline insured area.[20] The theory thus turns a comparison of demand responses into a welfare calculation.

9 Conclusion

This paper provides necessary and sufficient conditions for using direct and total subsidy responses to value equilibrium effects in settings with complementarities. The key requirement is that, among the local changes considered, any change with no first-order effect on subsidized activity must generate zero aggregate willingness to pay. One sufficient condition is subsidy equivalence: the equilibrium adjustment affects utility as a common additional subsidy would, across individuals and relevant choices. This condition can hold for complementarities operating through prices, networks, learning, or other channels, allowing their combined welfare effect to be recovered without separately estimating each mechanism.

Applications to adoption, peer effects, and incidence show how estimates collected for other purposes can support welfare analysis. The results also guide the design of new studies aimed at measuring welfare. I show how to adapt these welfare comparisons when differences across individuals, substitution between subsidized alternatives, or the timing of benefits matter.

[20] Premiums are paid per unit of insured land, so the welfare calculation requires knowing how much rice-growing land each farmer insures when purchasing insurance.

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APPENDIX

A Proofs

Proof of Theorem 1. We prove the equivalence in steps and then establish uniqueness.

Step 1: (i) implies (ii). Consider a local change that leaves subsidized activity unchanged, so $\mathrm{d}A/\,\mathrm{d}\epsilon = 0$. Compare it with making no change at all. If neither the subsidy nor any equilibrium variable changes, both the response of subsidized activity and willingness to pay are zero. The two changes therefore have the same response of subsidized activity. By (i), they must also have the same willingness to pay. Hence the original change must satisfy $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon = 0$. This proves (ii).

Step 2: (ii) implies (i). Consider two local changes, labeled 1 and 2, that generate the same response of subsidized activity:

$$\frac{\mathrm{d}A_1}{\mathrm{d}\epsilon} = \frac{\mathrm{d}A_2}{\mathrm{d}\epsilon}.$$

Consider the local change obtained by subtracting, to first order, change 2 from change 1. Because the chain-rule expressions for both $\mathrm{d}A/\,\mathrm{d}\epsilon$ and $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon$ are linear in the marginal changes in the subsidy and equilibrium variables, the response of subsidized activity for this difference is

$$\frac{\mathrm{d}A_1}{\mathrm{d}\epsilon} - \frac{\mathrm{d}A_2}{\mathrm{d}\epsilon} = 0.$$

By (ii), its willingness to pay must also be zero. Hence

$$\frac{\mathrm{d}\mathcal{V}_1}{\mathrm{d}\epsilon} - \frac{\mathrm{d}\mathcal{V}_2}{\mathrm{d}\epsilon} = 0.$$

It follows that $\mathrm{d}\mathcal{V}_1/\,\mathrm{d}\epsilon = \mathrm{d}\mathcal{V}_2/\,\mathrm{d}\epsilon$, which proves (i).

Step 3: (ii) implies (iii). Consider any local change. Its response of subsidized activity is $\mathrm{d}A/\,\mathrm{d}\epsilon$, and we want to determine the willingness to pay associated with that response.

Recall the two chain-rule expressions derived in the main text. For a general local change,

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = D\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon} + \sum_{j=1}^{J} \frac{\partial A}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\epsilon},$$

while aggregate willingness to pay is

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = A\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon} + \int \sum_{j=1}^{J} \frac{1}{\lambda_i}\frac{\partial v_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\epsilon}\,\mathrm{d}i.$$

For a direct subsidy change, equilibrium variables are held fixed, so the two chain-rule expressions derived in the main text reduce to

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = D\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon}, \qquad \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = A\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon}.$$

Since $D \neq 0$, eliminating $\mathrm{d}\tau/\,\mathrm{d}\epsilon$ gives

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \frac{A}{D}\frac{\mathrm{d}A}{\mathrm{d}\epsilon}.$$

Thus, for a direct subsidy change, A/D gives willingness to pay per unit of response of subsidized activity.

Because $D \neq 0$, the size of the direct subsidy change can be chosen so that it generates the same response of subsidized activity as the original local change. Step 2 established that, under (ii), two local changes that generate the same response of subsidized activity must also generate the same willingness to pay. The willingness to pay associated with the original local change must therefore be A/D times its response. Hence

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \frac{A}{D}\frac{\mathrm{d}A}{\mathrm{d}\epsilon},$$

which proves (iii).

Step 4: (iii) implies (i) and (ii). If a local change leaves subsidized activity unchanged, then $\mathrm{d}A/\,\mathrm{d}\epsilon = 0$, and equation (2) immediately gives $\mathrm{d}\mathcal{V}/\,\mathrm{d}\epsilon = 0$. Thus (iii) implies (ii).

Now consider two local changes that generate the same response of subsidized activity. Equation (2) assigns willingness to pay equal to A/D times that response. The two changes must therefore have the same willingness to pay. Thus (iii) also implies (i).

Step 5: Uniqueness. Finally, consider any local change with response $x = \mathrm{d}A/\,\mathrm{d}\epsilon$. For a direct subsidy change, holding equilibrium variables fixed,

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = D\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon}, \qquad \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = A\frac{\mathrm{d}\tau}{\mathrm{d}\epsilon}.$$

Because $D \neq 0$, there is a direct subsidy change that generates response x, obtained by setting $\mathrm{d}\tau/\,\mathrm{d}\epsilon = x/D$. For that change,

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = A\frac{x}{D} = \frac{A}{D}x.$$

Any relationship that determines willingness to pay from the response of subsidized activity alone must therefore assign willingness to pay $(A/D)x$ to response x. Since x was arbitrary, this pins down the relationship for every response that can be generated by a local change. Hence equation (2) is the unique relationship consistent with the direct subsidy benchmark.

■

Proof of Corollary 1. For the equilibrium adjustment, the subsidy is held fixed while the equilibrium variables change by the amount induced by a marginal increase in the subsidy. As shown in Section 3, this local change generates a response of subsidized activity

$$\frac{\mathrm{d}A}{\mathrm{d}\epsilon} = T - D$$

and willingness to pay

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = E_\tau.$$

Under the conditions of Theorem 1, equation (2) therefore implies

$$E_\tau = \frac{A}{D}(T - D) = A\left(\frac{T}{D} - 1\right),$$

which gives equation (3). Adding the direct willingness to pay from the subsidy, A, gives total willingness to pay

$$A + E_\tau = A\frac{T}{D}.$$

For $\tau > 0$, the definitions $\epsilon^D = \tau D/A$ and $\epsilon^T = \tau T/A$ imply

$$\frac{T}{D} = \frac{\epsilon^T}{\epsilon^D}, \qquad \frac{\tau T}{A} = \epsilon^T.$$

Substituting these expressions and equation (3) into equation (1) gives

$$\mathrm{MVPF} = \frac{\epsilon^T/\epsilon^D}{1 + \epsilon^T}, \qquad \frac{\mathrm{d}W}{\mathrm{d}G} = \frac{\epsilon^T/\epsilon^D}{1 + \epsilon^T} - 1.$$

■

Proof of Proposition 1. The proof has two parts. We first derive the response of subsidized activity and then value the equilibrium adjustment.

Step 1: Response of subsidized activity. We first show how subsidy equivalence affects the incentives that determine choices. Holding an action a_i fixed, the effect of the subsidy on utility, holding equilibrium variables fixed, is

$$\frac{\partial U_i}{\partial \tau} = \frac{\partial U_i}{\partial y_i} Q_i(a_i).$$

When the subsidy changes under the actual policy, the equilibrium variables also adjust. Holding the action fixed, the chain rule gives

$$\frac{\mathrm{d}U_i(a_i)}{\mathrm{d}\tau} = \frac{\partial U_i(a_i)}{\partial \tau} + \sum_{j=1}^{J} \frac{\partial U_i(a_i)}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau}.$$

Using subsidy equivalence in equation (4),

$$\frac{\mathrm{d}U_i(a_i)}{\mathrm{d}\tau} = (1+\gamma) \frac{\partial U_i(a_i)}{\partial \tau}.$$

Because the same γ applies to every agent and every relevant action, rearranging this equality gives

$$\frac{\mathrm{d}U_i(a_i)}{\mathrm{d}\tau} - (1+\gamma) \frac{\partial U_i(a_i)}{\partial \tau} = 0.$$

The expression on the left is the first-order utility effect of combining the actual policy change with $-(1+\gamma)$ times the direct subsidy change, starting from the same baseline. This combined change therefore has no first-order utility effect for any agent at any relevant action. We assume that any local change with no first-order utility effect for any agent at any relevant action also has no first-order effect on aggregate subsidized activity. The combined change satisfies this condition, so the assumption requires its activity response to be zero. The chain rule applied to $A(\tau, \mathbf{z})$ expresses this same response as $T - (1+\gamma)D$. Setting it equal to zero therefore gives $T - (1+\gamma)D = 0$, which proves the first equality in equation (5).

Step 2: Willingness to pay. Evaluate subsidy equivalence in equation (4) at the optimal action a_i^* and divide by λ_i:

$$\sum_{j=1}^{J} \frac{1}{\lambda_i} \frac{\partial U_i}{\partial z_j} \frac{\mathrm{d}z_j}{\mathrm{d}\tau} = \gamma \frac{1}{\lambda_i} \frac{\partial U_i}{\partial y_i} Q_i(a_i^*).$$

At the optimum, $\lambda_i = \partial U_i/\partial y_i$, so the right-hand side is $\gamma Q_i(a_i^*)$. By the envelope theorem, the left-hand side is agent i's willingness to pay for the equilibrium adjustment. Integrating across agents gives

$$E_\tau = \gamma \int Q_i(a_i^*)\,\mathrm{d}i = \gamma A.$$

Finally, if $D \neq 0$, $T - D = \gamma D$ implies $\gamma = T/D - 1$. Substituting into $E_\tau = \gamma A$ gives

$$E_\tau = A\left(\frac{T}{D} - 1\right),$$

which proves equation (6). ■

Proof of Proposition 2. The proof again separates the response of subsidized activity from willingness to pay. Unlike Proposition 1, however, the conditions now hold only at the optimum, so the response result is obtained from the first-order condition.

Step 1: Response of subsidized activity. Derivatives with respect to a_i below apply to the composite objective $U_i(a_i, y_i + \tau Q_i(a_i), \mathbf{z})$. At the optimum,

$$\frac{\partial U_i}{\partial a_i} = 0.$$

First consider a change in the subsidy holding the equilibrium variables fixed. Differentiating the first-order condition with respect to τ gives

$$\frac{\partial^2 U_i}{\partial a_i^2}\left.\frac{\partial a_i^*}{\partial \tau}\right|_{\mathbf{z}} + \frac{\partial}{\partial a_i}\left[\frac{\partial U_i}{\partial y_i} Q_i(a_i)\right] = 0.$$

This equation determines the response of the optimal action when only the subsidy changes.

Now differentiate the same first-order condition when the equilibrium variables are allowed to adjust:

$$\frac{\partial^2 U_i}{\partial a_i^2}\frac{\mathrm{d}a_i^*}{\mathrm{d}\tau} + \frac{\partial}{\partial a_i}\left[\frac{\partial U_i}{\partial y_i} Q_i(a_i) + \sum_{j=1}^{J} \frac{\partial U_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau}\right] = 0.$$

The second condition in equation (7) implies, at $a_i = a_i^*$,

$$\frac{\partial}{\partial a_i}\left[\sum_{j=1}^{J} \frac{\partial U_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau}\right] = \gamma \frac{\partial}{\partial a_i}\left[\frac{\partial U_i}{\partial y_i} Q_i(a_i)\right].$$

Substituting this into the first-order condition for the actual policy change gives

$$\frac{\partial^2 U_i}{\partial a_i^2}\frac{\mathrm{d}a_i^*}{\mathrm{d}\tau} + (1+\gamma)\frac{\partial}{\partial a_i}\left[\frac{\partial U_i}{\partial y_i}Q_i(a_i)\right] = 0.$$

Multiply the first-order condition for the subsidy change holding equilibrium variables fixed by $1+\gamma$ and subtract it from the previous equation. The second terms cancel, leaving

$$\frac{\partial^2 U_i}{\partial a_i^2}\left[\frac{\mathrm{d}a_i^*}{\mathrm{d}\tau} - (1+\gamma)\left.\frac{\partial a_i^*}{\partial \tau}\right|_{\mathbf{z}}\right] = 0.$$

The strict second-order condition implies

$$\frac{\mathrm{d}a_i^*}{\mathrm{d}\tau} = (1+\gamma)\left.\frac{\partial a_i^*}{\partial \tau}\right|_{\mathbf{z}}.$$

Recall that $A = \int Q_i(a_i^*)\,\mathrm{d}i$. Differentiating A under the two policy changes gives

$$D = \int \frac{\partial Q_i(a_i^*)}{\partial a_i}\left.\frac{\partial a_i^*}{\partial \tau}\right|_{\mathbf{z}}\mathrm{d}i, \qquad T = \int \frac{\partial Q_i(a_i^*)}{\partial a_i}\frac{\mathrm{d}a_i^*}{\mathrm{d}\tau}\,\mathrm{d}i.$$

Using the relationship between the two action responses,

$$T = (1+\gamma)D,$$

and therefore $T - D = \gamma D$.

Step 2: Willingness to pay. The first condition in equation (7), evaluated at $a_i = a_i^*$, gives

$$\sum_{j=1}^{J}\frac{\partial U_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau} = \gamma\frac{\partial U_i}{\partial y_i}Q_i(a_i^*).$$

Dividing by λ_i and using $\lambda_i = \partial U_i/\partial y_i$ at the optimum gives

$$\sum_{j=1}^{J}\frac{1}{\lambda_i}\frac{\partial U_i}{\partial z_j}\frac{\mathrm{d}z_j}{\mathrm{d}\tau} = \gamma Q_i(a_i^*).$$

By the envelope theorem, the left-hand side is agent i's willingness to pay for the equilibrium adjustment. Integrating across agents gives

$$E_\tau = \gamma\int Q_i(a_i^*)\,\mathrm{d}i = \gamma A.$$

If $D \neq 0$, $T - D = \gamma D$ implies $\gamma = T/D - 1$. Substituting into $E_\tau = \gamma A$ gives

$$E_\tau = A\left(\frac{T}{D} - 1\right).$$

■

Proof of Proposition 3. Apply Theorem 1 separately within each predetermined type k. For type k, baseline subsidized activity is A_k, the direct response is D_k, and the total response is T_k. Therefore, the willingness to pay generated by the equilibrium adjustment for that type is

$$E_{\tau,k} = A_k\left(\frac{T_k}{D_k} - 1\right),$$

and total willingness to pay for the marginal policy change is

$$A_k + E_{\tau,k} = A_k\frac{T_k}{D_k}.$$

Aggregating across predetermined types gives

$$E_\tau = \int E_{\tau,k}\,\mathrm{d}F(k) = \int A_k\left(\frac{T_k}{D_k} - 1\right)\mathrm{d}F(k),$$

which proves equation (8). Similarly,

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\tau} = \int A_k\frac{T_k}{D_k}\,\mathrm{d}F(k).$$

Dividing aggregate willingness to pay by the fiscal derivative $\mathrm{d}G/\,\mathrm{d}\tau$ gives

$$\mathrm{MVPF} = \frac{\displaystyle\int A_k(T_k/D_k)\,\mathrm{d}F(k)}{\mathrm{d}G/\,\mathrm{d}\tau}.$$

■

Proof of Lemma 1. Fix an agent i and any feasible dynamic activity path a_i. Holding the equilibrium path fixed, the utility effect of the marginal subsidy change is

$$\frac{\partial U_i}{\partial y_i}\mathbb{E}_i\left[\int_0^\infty e^{-\rho t}\ell(t)q_i(t)\,\mathrm{d}t\right] = \frac{\partial U_i}{\partial y_i}Q_i(a_i).$$

At the same fixed activity path, the utility effect generated by the induced change in equi-

librium returns is

$$\frac{\partial U_i}{\partial y_i}\mathbb{E}_i\left[\int_0^\infty e^{-\rho t}\Theta_\epsilon(t)q_i(t)\,\mathrm{d}t\right].$$

Using equation (10), this becomes

$$\gamma\frac{\partial U_i}{\partial y_i}\mathbb{E}_i\left[\int_0^\infty e^{-\rho t}\ell(t)q_i(t)\,\mathrm{d}t\right]=\gamma\frac{\partial U_i}{\partial y_i}Q_i(a_i).$$

Thus, for every agent and every feasible dynamic activity path, the equilibrium adjustment has γ times the utility effect of the direct subsidy change. This is subsidy equivalence. ■

Proof of Proposition 4. Under subsidy equivalence, the difference between the actual policy change and the direct subsidy change $\boldsymbol{\beta}$ has no first-order utility effect for any agent at any relevant action. By assumption, this difference also has no first-order effect on any component of $\mathbf{A}$. The chain rule applied to $\mathbf{A}(\boldsymbol{\tau},\mathbf{z})$ implies

$$\mathbf{T}=\mathbf{D}\boldsymbol{\beta}.$$

Since $\mathbf{D}$ is invertible,

$$\boldsymbol{\beta}=\mathbf{D}^{-1}\mathbf{T}.$$

At the optimal actions, the envelope theorem implies that aggregate willingness to pay for the equivalent direct subsidy change $\boldsymbol{\beta}$ is

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon}=\mathbf{A}'\boldsymbol{\beta}=\mathbf{A}'\mathbf{D}^{-1}\mathbf{T}.$$

The policy itself directly changes the subsidies by $\boldsymbol{\tau}_\epsilon$. The remaining equivalent subsidy change, $\boldsymbol{\beta}-\boldsymbol{\tau}_\epsilon$, corresponds to the equilibrium adjustment. Hence

$$E_\epsilon=\mathbf{A}'(\boldsymbol{\beta}-\boldsymbol{\tau}_\epsilon)=\mathbf{A}'\left(\mathbf{D}^{-1}\mathbf{T}-\boldsymbol{\tau}_\epsilon\right).$$

This proves equation (9). ■

PROPOSITION 6. Suppose the policy change is equivalent, for every agent and relevant action, to the same vector $\boldsymbol{\beta}$ of direct subsidy changes. Aggregate willingness to pay is identified from $\mathbf{A}$, $\mathbf{D}$, and $\mathbf{T}$ if and only if

$$\mathbf{A}\in\mathrm{Range}(\mathbf{D}').$$

When this condition holds, any vector $\boldsymbol{\ell}$ satisfying $\mathbf{D}'\boldsymbol{\ell} = \mathbf{A}$ gives

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \boldsymbol{\ell}'\mathbf{T}, \qquad E_\epsilon = \boldsymbol{\ell}'\mathbf{T} - \mathbf{A}'\boldsymbol{\tau}_\epsilon.$$

Proof of Proposition 5. Differentiate the equilibrium condition $A = \Phi(\tau, A)$ with respect to τ along the equilibrium path:

$$T = \frac{\partial \Phi}{\partial \tau} + \frac{\partial \Phi}{\partial A} T.$$

Using $D = \partial\Phi/\partial\tau$ and rearranging gives

$$T = \frac{D}{1 - \partial\Phi/\partial A}.$$

■

Proof of Proposition 6. As in the proof of Proposition 4, the complete policy change is represented by a vector $\boldsymbol{\beta}$ satisfying

$$\mathbf{T} = \mathbf{D}\boldsymbol{\beta}, \qquad \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \mathbf{A}'\boldsymbol{\beta}.$$

Suppose first that $\mathbf{A} \in \text{Range}(\mathbf{D}')$. Then there exists a vector $\boldsymbol{\ell}$ such that $\mathbf{D}'\boldsymbol{\ell} = \mathbf{A}$. It follows that

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\epsilon} = \mathbf{A}'\boldsymbol{\beta} = \boldsymbol{\ell}'\mathbf{D}\boldsymbol{\beta} = \boldsymbol{\ell}'\mathbf{T}.$$

Thus aggregate willingness to pay is identified even if $\boldsymbol{\beta}$ itself is not. Subtracting the willingness to pay associated with the direct subsidy change gives

$$E_\epsilon = \boldsymbol{\ell}'\mathbf{T} - \mathbf{A}'\boldsymbol{\tau}_\epsilon.$$

For necessity, suppose instead that $\mathbf{A} \notin \text{Range}(\mathbf{D}')$. Then $\mathbf{A}$ cannot be written as a linear combination of the rows of $\mathbf{D}$. By a standard result in linear algebra, there must therefore exist a nonzero vector $\mathbf{n}$ such that $\mathbf{D}\mathbf{n} = 0$ but $\mathbf{A}'\mathbf{n} \neq 0$.

Now let $\boldsymbol{\beta}$ be any vector satisfying $\mathbf{T} = \mathbf{D}\boldsymbol{\beta}$, and consider another vector $\widetilde{\boldsymbol{\beta}} = \boldsymbol{\beta} + \mathbf{n}$. Since $\mathbf{D}\mathbf{n} = 0$,

$$\mathbf{D}\widetilde{\boldsymbol{\beta}} = \mathbf{D}\boldsymbol{\beta} + \mathbf{D}\mathbf{n} = \mathbf{T}.$$

Thus $\boldsymbol{\beta}$ and $\widetilde{\boldsymbol{\beta}}$ imply exactly the same observed total response. They imply different willing-

ness to pay, however, because

$$\mathbf{A}'\widetilde{\boldsymbol{\beta}} - \mathbf{A}'\boldsymbol{\beta} = \mathbf{A}'\mathbf{n} \neq 0.$$

Hence the same observed responses are consistent with different levels of aggregate willingness to pay. Aggregate willingness to pay therefore cannot be uniquely recovered from $\mathbf{A}$, $\mathbf{D}$, and $\mathbf{T}$. ■

B An Example without Full Subsidy Equivalence

In the participation example, subsidy equivalence holds for both participation and nonparticipation. With a smooth interior choice, Proposition 2 requires less. The equilibrium adjustment need only have the same effect as the subsidy, in level and slope, at the action actually chosen.

To illustrate this, consider a continuum of identical firms of unit mass. Firm i chooses R&D expenditure a_i. The government pays a subsidy τ per unit of R&D, so $Q_i(a_i) = a_i$. Let $A = \int a_i^* \, \mathrm{d}i$ denote aggregate R&D expenditure. Because firms are identical and have unit mass, $A = a_i^*$ in a symmetric equilibrium.

Let π_i denote the profit of firm i. We write

$$\pi_i(a_i; \tau, A) = \Pi(a_i) + \tau a_i + \Theta G(a_i, A).$$

The function $\Pi(a_i)$ is the firm's private profit net of the cost of R&D. The function $G(a_i, A)$ captures the contribution of aggregate knowledge to the firm's profit. In particular, $G_A(a_i, A)$ measures how an increase in aggregate R&D affects the profit of a firm choosing a_i, while $G_{aA}(a_i, A)$ measures how it affects the marginal return to the firm's own R&D. The scalar Θ measures the strength of the knowledge spillover. Assume profit is twice continuously differentiable and the firm's optimum remains interior, with $\partial^2 \pi_i / \partial a_i^2 < 0$. Holding aggregate R&D fixed, a marginal increase in the subsidy changes profit at rate $\partial \pi_i / \partial \tau = a_i$. When the subsidy changes in equilibrium, aggregate R&D also changes. Holding the firm's own R&D fixed, this equilibrium adjustment changes profit at rate $\Theta G_A(a_i, A)(\, \mathrm{d}A / \, \mathrm{d}\tau)$.

Full subsidy equivalence would require these two effects to be proportional for every possible value of a_i. There is no reason for this to hold when knowledge spillovers affect firms differently at different levels of R&D. For a smooth choice, however, Proposition 2 only requires proportionality at $a_i = a_i^*$ and proportionality of their slopes at the same point. The first condition requires $\Theta G_A(a_i^*, A)(\, \mathrm{d}A / \, \mathrm{d}\tau) = \gamma a_i^*$, while the second requires

$\Theta G_{aA}(a_i^*, A)(\,\mathrm{d}A/\,\mathrm{d}\tau) = \gamma$. Both conditions hold if

$$G_A(a_i^*, A) = a_i^* G_{aA}(a_i^*, A).$$

This condition has a direct interpretation. At the firm's optimal R&D expenditure, the effect of aggregate R&D on the level of profits and on the marginal return to R&D must line up in the same proportion as the level and marginal effect of the subsidy. Nothing requires this relationship to hold at other values of a_i. Thus $G(a_i, A)$ can be nonlinear in a_i, and full subsidy equivalence can fail.

When this local condition holds, Proposition 2 implies

$$T - D = \gamma D, \qquad E_\tau = \gamma A = A\left(\frac{T}{D} - 1\right).$$

Full subsidy equivalence would require knowledge spillovers to mimic the subsidy across all R&D choices. With a smooth interior choice, it is enough that they do so locally at the firm's optimum.[21]

C Illustrating the Extensions

The next three studies illustrate the role of heterogeneity, substitution between subsidized alternatives, and timing in applying the extensions from Section 6.

Heterogeneity. Sato and Takasaki (2019) randomly offer women cash incentives for tetanus vaccination and estimate how their decisions respond to both their own rewards and vaccination by friends. Women respond differently to their friends' vaccination depending on their initial beliefs about vaccine benefits and risks. Proposition 3 allows the welfare calculation to be carried out separately for these groups, using each group's baseline participation A_k, direct response D_k, and total response T_k. Under its conditions, the responses determine the reward that would reproduce the equilibrium adjustment within each group. Baseline participation determines its value, and the values are then added across groups.

Margins. Tebaldi (2025) studies subsidy design in a marketplace where households choose among health insurance plans. A subsidy can lead households to switch plans while remaining insured, so total coverage may change little even when many households choose a different plan. The welfare calculation therefore needs to account for which plans gain or lose

[21] This follows directly from the proof of Proposition 2, with $\gamma = \Theta G_{aA}(a_i^*, A)T$.

enrollment. Proposition 4 shows when responses to the subsidies for individual plans provide enough information to value the adjustment, recovering welfare information that total coverage alone would miss.

Timing. Magnan, Spielman, Lybbert and Gulati (2015) study a program that offered farmers discounted land-leveling services to improve irrigation. An auction established which farmers were willing to pay, and a lottery selected which of them could purchase the service. Other farmers could then learn from their experience. The authors measured demand again one year later. The example shows why the timing of benefits matters: a discount offered today can generate gains from learning after the discount has ended. Lemma 1 establishes when those gains can be valued using a single statistic based on discounted subsidized activity. When subsidy payments and the returns to learning do not have proportional time profiles, the timing extension identifies the additional variation in subsidy schedules needed for valuation.