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ABSTRACT

This paper challenges the view that a system of "competing currencies," if sufficiently substitutable for one another, would lead to stable exchange rates and hence to stable price levels in terms of the various currencies. A theoretical frame work for the analysis of the consequences of increasing substitutability of currencies is proposed, in a multiple-currency "cash-in-advance" model. High (though imperfect) substitutability is shown to make more likely indeterminacy of equilibrium exchange rates and the failure of learning dynamics to converge to rational expectations, and to make the management of a fixed exchange rate system considerably more difficult.

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The United Kingdom (HM Treasury, 1989) has recently proposed an "evolutionary" approach to European monetary union, intended as an alternative to the recommendations of the Delors Report. In addition to generally urging a more gradual and cautious approach to fundamental institutional reforms, the UK proposal argues in particular that it is undesirable to move to a single European currency, primarily because of the extent to which this would place both monetary and fiscal policy in the hands of supra-national agencies (such as the proposed European System of Central Banks). Instead, it argues, the objective of "stable prices and currencies" can be achieved, while preserving national control over monetary and fiscal policy, by "allow[ing] currencies to compete to provide the non-inflationary anchor in the European Monetary System." The existing European Monetary System's "reliance on many currencies" is said to be its "strength" rather than a liability, because of the need for "competition" between currencies to force convergence of member states' monetary policies on low rates of money growth; "by eliminating both competition and accountability from members' monetary policies" it is argued that "the Delors Report version of monetary union risks producing a higher inflation rate in Europe (p. 9)."

The UK statement argues that the removal of barriers to trade in financial services, as part of Stage 1 of the process of economic and monetary union agreed to by the European Council, will greatly further the extent of competitive pressures toward coordination on non-inflationary policies, by "increas[ing] the amount of currency substitution. Greater use will be made of low inflation currencies at the expense of high inflation ones in both transactions and deposits (p. 3).[1]" However, it also calls for additional steps to further this process. These include "the complete removal of all unnecessary restrictions ... on the use of all currencies throughout the Community," such as regulations requiring German pension funds to invest mainly in Deutschmark assets, and the removal of legal impediments to the development and diffusion of information-processing technologies that would reduce "the costs and inconvenience of changing between Community currencies," such as simplified check-clearing systems and electronic funds transfer networks (p. 5). The outcome of this "evolutionary" process is envisioned as a Europe in which "all Community currencies would become effectively interchangeable" and "the European Monetary System could evolve into a system of more or less fixed exchange rates (p. 5)," although it would have come about through the invisible hand of the market rather than through "administrative fiat" (p. 7).

It is not clear how seriously the UK proposal should be taken as an actual prescription for "practical monetary union" in the near future, given the rather insignificant degree to which currency substitution seems to occur in Europe, even in the absence of legal prohibitions on the use of foreign currencies in many kinds of transactions. If, as experience tends to suggest, the cost advantages of using a local currency for payments within local markets are such as to deter currency substitution except in the case of extremely large (Latin American-style) inflation differentials, it is not clear how much discipline upon Community monetary policies can be expected from this channel. Still, an investigation of what the effects of increased currency substitutability ought in principle to be would seem to be timely.[2] In this paper I argue that it is not at all clear that movement toward markedly greater substitutability of currencies would favor stability of nominal price levels and exchange rates. Instead, it would seem likely to increase the scope for speculative instabilities in exchange rates and corresponding speculative fluctuations in price levels. Insofar as "competition" between currencies resulted in lower rates of money growth in all countries, as argued in the UK proposal, this fact in itself could produce greater exchange rate and price level instability, owing to the greater scope for the existence of "sunspot equilibria" in the case of contractionary policies. And finally, rather than resulting in a natural, unplanned evolution toward a system of fixed exchange rates, as suggested in the UK proposal, too great a degree of currency substitutability seems likely to make a fixed exchange rate system more difficult to manage, if not impossible. Hence insofar as Stage 1 of economic and monetary union were expected already to bring with it greatly increased currency substitution, this might be a reason to regard the adoption of a single currency as not only not unnecessary, but actually essential to the preservation of even the degree of monetary stability presently achieved by the European Monetary System.

1. Currency Substitution in a Cash-in-Advance Framework

The model I describe here generalizes in some respects the cash-in-advance models of Lucas (1982) and Lucas and Stokey (1987).[3] I assume the existence of a finite number of infinite lived households. Household h seeks to maximize an infinite horizon objective of the form

$$V^h = \Sigma_{t=0}^{\infty} \beta_h^t U^h(c_t^{h1}, \ldots, c_t^{hn}; n_t^{h1}, \ldots, n_t^{hn}) \tag{1}$$

Here c_t^{hi} denotes consumption by household h of goods of type i in period t, and n_t^{hi} denotes that household's supply of the same type of goods. The single period utility function U^h is a concave function, increasing in its first n arguments, and decreasing in its last n; the discount factor β_h lies between zero and one. Here the n "types" of goods represent goods that must be purchased with each of n distinct currencies. It may be wondered why it is necessary to introduce endogenous supply as opposed to simply positing an exogenous endowment of goods as in Lucas (1982). An important reason is that in a framework like that of Lucas (1982), the exogenous supply of goods of the various types implies an exogenous demand for real balances of each of the types of currencies, so that interest rate variations do not affect the demand for the various currencies. Use of such a simple framework would accordingly prevent us from discussing the way in which currency substitution should be related to the way in which foreign as well as domestic interest rates should enter money demand functions -- an approach that has often been used in empirical investigations of the phenomenon (Thomas and Wickens, 1989). The model presented here is still too simple to be empirically realistic; money demand can vary only insofar as a household varies the value of its transactions, rather than through variations in cash management practices that would result in variation in the "velocity" of money. Nonetheless, it allows for at least one way in which money demand can vary in response to market conditions, which is a minimum requirement for a useful framework. Furthermore, as is discussed below, I propose to model currency substitution not as a change in the degree to which goods of the same type are purchased using different currencies, but instead as substitution between the "types" of goods purchased; this would not be possible, in equilibrium, unless substitution in supply were possible.

Household h seeks to maximize V^h subject to the sequence of budget constraints

$$\Sigma_i \, [M_t^{hi} + (B_{t+1}^{hi}/R_t^i)]\, e_t^i \;\leq\; \Sigma_i \, [W_t^{hi} - p_t^i \tau_t^{hi}]\, e_t^i \tag{2}$$

$$p_t^i c_t^{hi} \;\leq\; M_t^{hi} + \alpha^i p_t^i n_t^{hi} \tag{3}$$

$$W_{t+1}^{hi} \;=\; (M_t^{hi} - p_t^i(c_t^{hi} - n_t^{hi}))I_t^i + B_{t+1}^{hi} \tag{4}$$

$$\lim_{T\to\infty} \left[e_T^1 \Pi_{s=0}^{T-1} R_s^1 \right]^{-1} \Sigma_i \, W_T^{hi} e_T^i \;\geq\; 0 \tag{5}$$

as well as the non-negativity constraints $c_t^{hi}, n_t^{hi} \geq 0$, where (2) - (4) hold for each period $t = 0, 1, 2, \ldots$, and where W_0^{hi} is given as an initial condition. We suppose, as in Lucas and Stokey, that each period t consists of two sub-periods in which different markets are open. In the first sub-period, the n different currencies can be exchanged for one another and for one-period riskless bonds denominated in any of the n currencies, which bonds mature in the first sub-period of the following period. Condition (2) represents household h's budget constraint for these financial exchanges in period t. Here M_t^{hi} denotes the quantity of currency i held at the end of the first sub-period of period t, and B_{t+1}^{hi} denotes the nominal value at maturity of the bonds denominated in currency i that are held at the same time. The price of currency i in the period t foreign exchange market is denoted e_t^i, where we adopt the normalization

$$\Sigma_i \, e_t^i \;=\; 1 \tag{6}$$

and R_t^i denotes one plus the nominal interest rate on currency-i bonds. Finally, W_t^{hi} denotes the nominal value of wealth denominated in terms of currency i (whether in the form of currency or bonds) carried into period t, τ_t^{hi} denotes the real lump sum taxes paid by household h to government i in period t (assumed to be collected during the financial market sub-period), and p_t^i denotes the period t price of type i goods in terms of currency i.

In the second sub-period, goods are purchased using currency, in n distinct markets, each with a separate budget constraint (3). It is the presence of the "cash-in-advance" constraints (3) that results in a demand for the various currencies in this model. As noted earlier, "type i" goods may be purchased only using currency i. The parameters α^i, satisfying $0 \leq \alpha^i < 1$, indicate the relative efficiency of the payments systems using the n different currencies. If one sets $\alpha^i = 0$, one has the standard cash-in-advance constraint, according to which only currency held at the end of the financial market sub-period can be used to purchase goods, and currency received for goods supplied in that same sub-period cannot be used until the financial market sub-period of the following period. This implies that currency can be used to purchase goods only once per period. An undesirable feature of this specification for our present purposes is that the assumed synchronization in time of all financial market trading and goods market trading implies that the payments lag (the time between when money is spent and when it can be spent again) must be of equal length whichever means of payment is used. I wish to be able to consider differences in the efficiency with which different payments systems are operated (in terms of, for example, the amount of time taken for checks to clear) as one of the grounds for currency substitution, and so one would like to be able to impose different payments lags for the different currencies. In order to stay within the discrete-time framework that is proposed here (and that is standard in the cash-in-advance literature), I substitute for a variable payment lag the notion that a fraction α^i of the currency i received from sales can be spent again immediately, while fraction $1-\alpha^i$ cannot be spent until the following period.

Larger values of α^i correspond to more efficient payments systems, and to higher transactions "velocities" of the currencies in question.

Equation (4) indicates how the wealth denominated in terms of currency i at the beginning of period t+1 is determined, as the sum of currency not spent in the previous period, currency received from sales in the previous period, interest payments on currency held at the end of the previous period, and bonds purchased in the previous period. The innovation here relative to standard cash-in-advance models is the allowance for interest on currency holdings; I_t^i denotes one plus the nominal interest payments (by government i) on holdings of currency i at the end of period t. Interest payments on currency are introduced to allow consideration of differential interest rates on monetary assets as another motive for currency substitution, and indeed one that has been frequently discussed in previous analyses, both theoretical and empirical. The timing chosen here for the interest payments (interest payments at the end of period t rather than at the end of the financial markets sub-period) is selected so as to preserve the simple and familiar form of first order condition (8) below, which allows the real exchange rate to be interpreted as a marginal rate of substitution on the part of households between goods that are sold for different types of currency, and which makes "purchasing power parity" a consequence of perfect substitutability between currencies. Finally, condition (5) rules out "Ponzi schemes" on the part of households, which are otherwise allowed to borrow an unlimited amount (i.e., hold unboundedly negative quantities of the various types of bonds). The choice of currency 1 as the one in terms of which present discounted values are defined in (5) is arbitrary.

Now this is formally identical to a standard multi-country cash-in-advance model, if one identifies household types with countries and goods types with goods produced in different countries, except that our notation allows a given household to supply many different goods. Given that interpretation, it would appear that the model assumes a rigid cash-in-advance constraint (a given country's goods can be purchased only using that country's currency) that makes no allowance for currency substitution whatsoever. But the same formalism can be given an alternative interpretation. In the interpretation intended here, the different "types" of goods do not differ except in the kind of means of payment used to purchase them, so that the fact that it is possible for both consumers and producers to shift from one "type" of goods to another indicates the possibility of changing the degree to which one currency is used rather than another for transactions.[4] If one "type" of good supplies a greater marginal utility to a given household than another, this indicates that it is more convenient for that household to use the first currency in transactions than the other, and if different "types" of goods are imperfectly substitutable, this indicates the costs involved in currency substitution. The limiting case of perfect substitutability among currencies corresponds to a utility function (1) of the form

$$V^h = \Sigma_{t=0}^{\infty} \beta_h^t U^h(\Sigma_i c_t^{hi}; \Sigma_i n_t^{hi}) \qquad (1')$$

Optimization by households with perfect foresight can be characterized by the following conditions. Since households are assumed to be able to sell the various types of bonds short, a solution to any household's optimization problem exists only if

$$\frac{R_t^i e_{t+1}^i}{e_t^i} = \frac{R_t^j e_{t+1}^j}{e_t^j} \qquad (7)$$

for every two currencies i, j. This is the usual interest rate parity condition that follows from perfect capital mobility.[5] Optimization then requires the first order conditions

$$\frac{\lambda_t^{hi}}{\lambda_t^{hj}} = \frac{p_t^i e_t^i}{p_t^j e_t^j} \qquad (8)$$

$$\frac{\nu_t^{hi}}{\nu_t^{hj}} = \frac{p_t^i e_{t+1}^i [\alpha^i R_t^i + (1-\alpha^i) I_t^i]}{p_t^j e_{t+1}^j [\alpha^j R_t^j + (1-\alpha^j) I_t^j]} \qquad (9)$$

$$\frac{\lambda_t^{hi}}{\nu_t^{hi}} = \frac{R_t^i}{\alpha^i R_t^i + (1-\alpha^i) I_t^i} \qquad (10)$$

$$p_t^i c_t^{hi} = M_t^{hi} + \alpha^i p_t^i n_t^{hi} \qquad \text{if } R_t^i > I_t^i \qquad (11)$$

$$p_t^i c_t^{hi} < M_t^{hi} + \alpha^i p_t^i n_t^{hi} \qquad \text{only if } R_t^i = I_t^i \qquad (11')$$

$$\frac{\lambda_t^{hi}}{\beta_h \lambda_{t+1}^{hi}} = \frac{p_t^i R_t^i}{p_{t+1}^i} \qquad (12)$$

Here λ^{hi}_t denotes the value (in utility units) to household type h in the securities trading subperiod of period t of an additional amount of currency i sufficient to allow purchase of one more unit of type i consumption goods, and ν^{hi}_t denotes the value of the same quantity of currency i if received in the goods market subperiod of period t. Condition (10) is the relation that must exist between these quantities given the possibility of borrowing currency i during the securities trading period and repaying the loan out of revenues received in the goods market trading period. The shadow prices λ^{hi}_t and ν^{hi}_t must furthermore be positive, and satisfy

$$U_c^{hi}(t) = \lambda_t^{hi} \qquad \text{if } c_t^{hi} > 0$$

$$U_c^{hi}(t) < \lambda_t^{hi} \qquad \text{only if } c_t^{hi} = 0$$

$$U_n^{hi}(t) = -\nu_t^{hi} \qquad \text{if } n_t^{hi} > 0$$

$$U_n^{hi}(t) < -\nu_t^{hi} \qquad \text{only if } n_t^{hi} = 0$$

where $U_c^{hi}(t)$ denotes $(\partial/\partial c^{hi})U^h(c_t^{h1}, \ldots, c_t^{hn}; n_t^{h1}, \ldots, n_t^{hn})$, and so on. In the case of equations (11) - (11'), as in the similar pairs of complementary slackness conditions involving the marginal utilities, it is to be understood that the two cases listed exhaust the possibilities consistent with optimization, so that one or the other must apply. Finally, assuming non-satiation, optimization requires that the household completely exhaust its budget, so that

$$\Sigma_i [M_t^{hi} + (B_{t+1}^{hi}/R_t^i)] e_t^i = \Sigma_i [W_t^{hi} - p_t^i \tau_t^{hi}] e_t^{hi} \qquad (13)$$

$$\lim_{T\to\infty} \left[e_T^1 \Pi_{s=0}^{T-1} R_s^1 \right]^{-1} \Sigma_i W_T^{hi} e_T^i = 0 \qquad (14)$$

A perfect foresight equilibrium is then a state of affairs in which each household optimizes in the manner just described, and in addition all markets for goods, currencies, and bonds clear, so that

$$\Sigma_h c_t^{hi} + g_t^i = \Sigma_h n_t^{hi} \quad (15)$$

$$\Sigma_h M_t^{hi} + p_t^i g_t^i = M_t^i \quad (16)$$

$$\Sigma_h B_{t+1}^{hi} = B_{t+1}^i \quad (17)$$

for t = 0, 1, 2, ..., and for i = 1, ..., n. Here g_t^i denotes goods purchased by government i in period t; it is assumed for the sake of simplicity that governments purchase goods only with their own currencies. M_t^i denotes the supply of currency i in period t (to be precise, at the end of the financial markets sub-period), which is equated in (16) to the sum of private demands and government i's demand for currency to be used in making its purchases. Finally, B_{t+1}^i denotes the supply of bonds by government i maturing at time t+1; again, it is assumed for the sake of simplicity that governments only issue bonds denominated in their own currencies. The quantities of goods purchased by the governments and the evolution over time of the money supplies and stocks of government debt depend upon governments' monetary and fiscal policies. Various assumptions will be made below about these, but they must always satisfy the government budget constraints

$$(M_t^i - M_{t-1}^i I_{t-1}^i) + ((B_{t+1}^i / R_t^i) - B_t^i) = p_t^i (g_t^i - \Sigma_h \tau_t^{hi}) \quad (18)$$

$$\lim_{T\to\infty} \left[\Pi_{s=0}^{T-1} R_s^i \right]^{-1} (M_{T-1}^i I_{T-1}^i + B_T^i) = 0 \quad (19)$$

(Note that only n-1 of each of these sets of equations represent additional, independent, equilibrium conditions, since the last of each set is implied by the other n-1 and the household budget constraints (13) and (14).) An equilibrium is then a set of sequences for the endogenous variables satisfying (4) and (6) - (17), given some specification of the monetary and fiscal policy variables of the governments that is consistent with (18) - (19).

It will often be convenient to simplify the model by considering the case of a world representative household, by which I mean the case of a single household type that is taxed by all n governments and that both supplies and purchases all n types of goods. One might interpret such households as being made up of a large number of extended family members who are linked by altruistic transfers and as a result act so as to jointly maximize a single household utility function, with different members of the family living in different jurisdictions, and working in regions where different currencies are most conveniently used. Lest this seem an extravagant fantasy even at a conference concerned with post-1992 Europe, I might point out that the common use of representative households for individual countries is most plausibly defended, not on the ground that authors believe that most households in a single country have roughly similar economic circumstances, but rather on the ground that the effects of the distribution of wealth (as opposed to the aggregate wealth of the country) on aggregate consumption and labor supply are believed to be small and so need not be modeled. A similar hypothesis regarding the insignificance of the international wealth distribution for world aggregate consumption and labor supply ought equally well to justify consideration of a world representative consumer.

The assumption of a world representative consumer greatly simplifies all of the above equilibrium conditions, eliminating the need for separate state variables and separate first-order conditions and budget constraints for the separate households. (I will still, however, keep a superscript h on the household's demands for currencies, to distinguish these from the corresponding money supplies.) This furthermore immediately allows the variables c_t^i to be eliminated from all equilibrium conditions, using the substitution $c_t^i = n_t^i - g_t^i$, because of (15). Equations (10) are then a set of n equations that can be solved for the n unknowns $\{n_t^i;\ i = 1,\ \ldots,\ n\}$; let us suppose that there is a unique solution[6] and let it be denoted

$$n_t^i = n^i(R_t^1/I_t^1,\ \ldots,\ R_t^n/I_t^n;\ g_t^1,\ \ldots,\ g_t^n) \qquad (20)$$

Then using this and also (16), equations (11) and (11') become[7]

$$M^i_t/p^i_t = (1-\alpha^i)^{-1} n^i(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t) \qquad \text{if } R^i_t > I^i_t \tag{21}$$

$$M^i_t/p^i_t > (1-\alpha^i)^{-1} n^i(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t) \qquad \text{only if } R^i_t = I^i_t \tag{21'}$$

Substitution of (20) for the arguments of the representative household's utility function allows us to define

$$U_c^{hi}(t) = \lambda^i(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t)$$

Substitution of this in turn into (8) and (12) then yields

$$\frac{\lambda^i(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t)}{\lambda^j(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t)} = \frac{p^i_t e^i_t}{p^j_t e^j_t} \qquad \text{if } (R_t, I_t, g_t) \in C \tag{22}$$

$$\frac{\lambda^i(R^1_t/I^1_t, \ldots, R^n_t/I^n_t; g^1_t, \ldots, g^n_t)}{\beta\, \lambda^i(R^1_{t+1}/I^1_{t+1}, \ldots, R^n_{t+1}/I^n_{t+1}; g^1_{t+1}, \ldots, g^n_{t+1})} = \frac{p^i_t R^i_t}{p^i_{t+1}} \tag{23}$$

where R_t, I_t, and g_t denote n-vectors giving the corresponding quantities for each currency, and where C is the set of values of (R_t, I_t, g_t) for which (20) implies that $c^i_t > 0$ for all i. Given a specification of monetary and fiscal policies consistent with (18) - (19), a perfect foresight equilibrium (in which all n currencies are used at all times to purchase private consumption goods) is then a set of sequences $\{p_t, R_t, e_t\}$ for t = 0, 1, 2, ..., that satisfy (6), (21) - (21'), (22) and (23) for t = 0, 1, 2, ... The remaining equilibrium conditions are all implied by these, in the single household case. Thus the assumption of a world representative household allows us to reduce the equilibrium conditions to a system of 3n-1 difference equations for 3n-1 state variables (eliminating one variable using (6)). The equations are only slightly more complex in the case of equilibria in which not all currencies are used by the representative household in some periods.

It will be observed that this system bears a certain resemblance to the sort of *ad hoc* equation systems postulated in previous studies of currency substitution. Equations (21) - (21') can be interpreted as a set of money demand functions, in which the demand for each currency is shown to depend upon the interest rate spreads between bonds and currency for all of the currencies.[8] (Equation (21') indicates that money demand becomes infinitely elastic when the interest rate spread is set to zero, since currency and bonds then become perfect substitutes, in the deterministic environment assumed here.) Equations (22) equate real exchange rates to marginal rates of substitution between the goods purchased with the different currencies (which marginal rates of subsitution the theory determines as a function of the other state variables), while equations (23) are Fisher equations for the various currencies (with the real interest rates identified with marginal rates of substitution that again the theory determines).

This derivation may shed some light upon the relation between the various formulations employed in previous analyses, contrasted in Thomas and Wickens (1989). Some, such as Girton and Roper (1981), have assumed that the demand for currency i should be an increasing function of the currency i interest rate and a decreasing function of other currency interest rates, because the interest rates being discussed are I^i versus I^j. Others, such as Miles (1978), have assumed that the demand for currency i should be a *decreasing* function of the level of interest rates in country i and an *increasing* function of the level of interest rates in other countries, because the interest rates in question are R^i versus R^j.[9] Both sorts of effects of foreign and domestic interest rates are consistent with the model presented here, where the demand for currency i depends upon the spreads R^i/I^i and R^j/I^j, and where, if substitution effects dominate, it will be a decreasing function of R^i/I^i and an increasing function of R^j/I^j, for all $j \neq i$.[10]

Of course this derivation is adequate only if one cares only about the determination of a world demand for each currency. Determination of separate national demands would require heterogeneous households, the simplest possible case being that of a representative household for each country. In this case one cannot derive money demand functions as simple as (21). However, equations (8), (10), and (15) represent in this case a system of

$2n^2$ equations that can be solved for the $2n^2$ variables representing consumption and output of each "type" of good in each country. Let us denote the solutions

$$c_t^{ij} = c^{ij}(R_t^1/I_t^1, \ldots, R_t^n/I_t^n; p_t^2e_t^2/p_t^1e_t^1, \ldots, p_t^ne_t^n/p_t^1e_t^1; g_t^1, \ldots, g_t^n)$$
$$n_t^{ij} = n^{ij}(R_t^1/I_t^1, \ldots, R_t^n/I_t^n; p_t^2e_t^2/p_t^1e_t^1, \ldots, p_t^ne_t^n/p_t^1e_t^1; g_t^1, \ldots, g_t^n)$$

Substituting these into (11) and (11'), we obtain equilibrium conditions of the form

$$M_t^{ij}/p_t^j = m^{ij}(R_t^1/I_t^1, \ldots, R_t^n/I_t^n; p_t^2e_t^2/p_t^1e_t^1, \ldots, p_t^ne_t^n/p_t^1e_t^1; g_t^1, \ldots, g_t^n) \qquad \text{if } R_t^j > I_t^j$$

and a corresponding generalization of (21'). Note that in general the demand for each currency will depend not only upon the interest rates in all countries but also upon all countries' real exchange rates.

2. Effects of Currency Substitution on Price Level and Exchange Rate Determinacy

In this section I wish to consider whether a particular set of monetary and fiscal policies on the part of the n governments result in a determinate perfect foresight equilibrium path for the various price levels, exchange rates, and interest rates, and in particular to consider how the degree of substitutability between the various currencies (as indicated by the degree of substitutability between different "types" of goods in households' utility functions) affects the answer to this question. In order to simplify the analysis, it is useful to restrict our attention to the case of a two-currency world with a world representative household. I also consider only monetary policies that involve fixing an exogenous constant rate of growth, and an exogenous constant rate of interest payments on money holdings, for each of the currencies, and allowing exchange rates to float. There is also assumed to be no government debt outstanding, and government purchases are assumed to be constant. Finally, the number of parameters entering the equations below is reduced by restricting attention to the case of perfect symmetry between the two currencies.

That is, I assume the same money supply growth rates and rates of interest on money in the case of both currencies, the same constant levels of goods purchases by the two governments, perfect symmetry of the utility functions between the two currencies, and equal values for α^i.

Let us consider only equilibria in which both currencies are used to purchase consumption goods by the representative household. (This does not rule out the possibility of equilibria in which one currency ceases to be used asymptotically.) Then combining (10) and (12) yields

$$-U_n^i(c_t; n_t) - \alpha U_c^i(c_t; n_t) = (1-\alpha)\beta I U_c^i(c_{t+1}; n_{t+1}) \frac{p_t^i}{p_{t+1}^i} \qquad (24)$$

for i = 1, 2, where c_t denotes the vector (c_t^1, c_t^2), and likewise for n_t.

Let g denote the constant level of purchases by both governments, and define

$$\hat{n}_1(n_2) = \arg\max_{n_1} U(n_1\text{-}g, n_2\text{-}g; n_1, n_2)$$

$$\hat{n}_2(n_1) = \arg\max_{n_2} U(n_1\text{-}g, n_2\text{-}g; n_1, n_2)$$

$$\bar{n} = \arg\max_{n} U(n\text{-}g, n\text{-}g; n, n)$$

Note that $\hat{n}_1(\bar{n}) = \hat{n}_2(\bar{n}) = \bar{n}$. Also note that $n_1, n_2 \geq \bar{n}$ imply that at least one of the inequalities $n_1 \geq \hat{n}_1(n_2)$ or $n_2 \geq \hat{n}_2(n_1)$ must hold, and that $n_1, n_2 \leq \bar{n}$ imply that at least one of the inequalities $n_1 \leq \hat{n}_1(n_2)$ or $n_2 \leq \hat{n}_2(n_1)$ must hold. Then (11), (11'), and (15) imply

$$n^1_t = m^1_t,\ n^2_t = m^2_t \qquad \text{if } m^1_t \leq \hat{n}_1(m^2_t),\ m^2_t \leq \hat{n}_2(m^1_t) \qquad (25a)$$

$$n^1_t = m^1_t,\ n^2_t = \hat{n}_2(m^1_t) \qquad \text{if } m^1_t \leq \bar{n},\ m^2_t \geq \hat{n}_2(m^1_t) \qquad (25b)$$

$$n^1_t = \hat{n}_1(m^2_t),\ n^2_t = m^2_t \qquad \text{if } m^1_t \geq \hat{n}_1(m^2_t),\ m^2_t \leq \bar{n} \qquad (25c)$$

$$n^1_t = n^2_t = \bar{n} \qquad \text{if } m^1_t, m^2_t \geq \bar{n} \qquad (25d)$$

where $m^i_t = M^i_t/p^i_t$, i = 1, 2. Equations (25a) - (25d) show that the complete allocation of resources in a given period t (recalling that $c^i_t = n^i_t - g$) can be determined as a function of the levels of real money balances (m^1_t, m^2_t) alone. The four different cases correspond to whether one, the other, both, or neither of the cash-in-advance constraints are binding in the period in question. Let the pair of continuous functions defined in (25a) - (25d) be denoted $n^i_t = n^i(m^1_t, m^2_t)$, for i = 1, 2.

Then, substituting (25a) - (25d) into (24), we obtain a pair of difference equations for the perfect foresight equilibrium dynamics of the two price levels of the form[11]

$$F^i(m^1_t, m^2_t) = (\beta I/\mu)\, G^i(m^1_{t+1}, m^2_{t+1}) \qquad (26)$$

for i = 1, 2, where

$$F^i(m) =$$
$$m^i[-U_n{}^i(n^1(m)\text{-}g, n^2(m)\text{-}g;\; n^1(m), n^2(m)) - \alpha U_c{}^i(n^1(m)\text{-}g, n^2(m)\text{-}g;\; n^1(m), n^2(m))]$$
$$G^i(m) = (1-\alpha)\, m^i[U_c{}^i(n^1(m)\text{-}g, n^2(m)\text{-}g;\; n^1(m), n^2(m))]$$

and where μ is the common growth rate of the two currencies (i.e., M^i_{t+1}/M^i_t). Conditions (26) describe the determination of the demand for both types of real balances (and hence of the price levels) in period t as a function of the expected price levels in the following period.

Finally, substitution of (12) allows (14) to be put in the form

$$\lim_{T\to\infty} \beta^T \Sigma_i G^i(m_T) = 0 \qquad (27)$$

Any pair of sequences $\{m^1_t, m^2_t\}$ satisfying (26) - (27) represent a perfect foresight equilibrium; that is, given such sequences, one can find unique sequences of values for the price levels, interest rates, and so on, that satisfy all of the other equilibrium conditions. Hence it is the uniqueness of solutions to (26) - (27) with which we are concerned. Since there are no initial conditions for the difference equations (26), it is evident that there could be as large a set of equilibria as a two-dimensional continuum, corresponding to different possible initial levels of real balances (m_0^1, m_0^2).

I propose to interpret the effects of switching to a single, common currency for all transactions as corresponding to a utility function for the representative household of the form

$$U^*(c; n) = U(c, c; n, n)$$

where c and n represent consumption goods purchased and output sold using the common currency. In this case it is easily seen that perfect foresight equilibria of the economy with a common currency correspond exactly to the those equilibria of the symmetric two-currency economy in which $m_t^1 = m_t^2$ for all t. As a result, *requiring the use of a common currency cannot increase the number of equilibria.* Furthermore, if there exists a contimuum of equilibria even in the case of a common currency, there must be a continuum of equilibria in the case of multiple currencies, *regardless of the degree of substitutability of the different currencies.* Hence the only interesting case to analyze is the one in which perfect foresight equilibrium is unique in the case of a common currency; we can then consider whether equilibrium may nonetheless be indeterminate in the case of multiple currencies, and we can consider how currency substitution affects this.

In the case of a common currency, the model presented here reduces to the model of Lucas and Stokey (1987); the determinacy of equilibrium in this model is analyzed in Woodford (1988). It is shown in the latter paper that a sufficient condition to rule out the existence of multiple equilibria in which the level of real balances goes neither to zero nor to infinity asymptotically is to assume that the functions $F(m) = F^1(m, m) = F^2(m, m)$ and $G(m) = G^1(m, m) = G^2(m, m)$ satisfy

$$F' > 0, \quad G' > 0, \quad F'/F > G'/G \tag{28}$$

for all m. The first and third inequalities hold in the case, for example, of additive separability of consumption and leisure within periods. The second holds in the case of sufficient intertemporal substitutability of consumption. This is also an assumption that implies that the demand for real balances will be a monotonically decreasing function of expected inflation, which is surely the most realistic case. It is also shown that a sufficient condition to rule out the existence of equilibria in which real balances asymptotically approach zero is for G to remain bounded above F as m approaches its lowest feasible value. If $g > 0$, the lowest feasible value is $m = g$, and the property must hold.[12] In the case that $g = 0$, it is necessary to assume that

$$\lim_{m \to 0} G(m) > 0 \tag{29}$$

This means that the marginal utility of consumption rises sufficiently steeply as the level of consumption goes to zero, a not implausible stipulation. Finally, it is shown that equilibria in which the level of real balances asymptotically becomes infinite exist if and only if the rate of money growth μ satisfies $\beta I < \mu < I$; there exists a continuum of solutions of this kind to the difference equation (26) for any $\mu > \beta I$, but the solutions (in which real balances grow asymptotically at the rate $\mu/\beta I$, and so in which G(m) grows asymptotically at that same rate) also satisfy the transversality constraint (27) only if $\mu < I$. Hence (28) and (29), or just (28) if $g > 0$, and $\mu \geq I$ suffice to insure the existence of a unique rational expectations equilibrium, in which the price level grows at the same rate as the growth rate μ of the common currency.

I wish to maintain assumptions (28) and (29) in the case of two currencies, but consider whether there can exist additional equilibria in which $m_t^1 \neq m_t^2$. Again, these assumptions (together with $\mu > \beta I$) imply the existence of a unique symmetric stationary equilibrium in which $m_t^1 = m_t^2 = m^*$ for all t, and in which both cash-in-advance constraints always bind. Can there also exist asymmetric stationary equilibria in which both currencies are used for consumption purchases? A further reasonable assumption on preferences is that

$$-\frac{U_c^{\,i}}{U_n^{\,i}} > -\frac{U_c^{\,j}}{U_n^{\,j}} \qquad \text{if } c^i < c^j,\, n^i < n^j \tag{30}$$

for i, j = 1, 2. This is also a property that necessarily holds, for example, if U is strictly concave and additively separable between consumption and output. It then follows that $m^i < m^j$ implies $F^i/G^i < F^j/G^j$, from which it follows that one cannot have $F^i/G^i = F^j/G^j = \beta I/\mu$. Hence assuming (30), there can be no stationary equilibria in which both currencies are used, other than the symmetric one (m*, m*).

A further simple question to address is whether there exist other perfect foresight equilibria in which both m_t^1 and m_t^2 remain near m* forever (e.g., non-stationary equilibria that converge asymptotically to the stationary values). Linearizing (26) around the stationary equilibrium, we obtain a pair of equations of the form

$$\Delta m_t^i = \Sigma_j M_{ij} \Delta m_{t-1}^j$$

for i = 1, 2, where Δm_t^i denotes $(m_t^i - m^*)/m^*$. The two eigenvalues of the matrix M are

$$\xi^1 = \frac{\mu}{\beta I} \frac{F_1^1 + F_2^1}{G_1^1 + G_2^1}$$

corresponding to an eigenvector whose components are equal, and

$$\xi^2 = \frac{\mu}{\beta I} \frac{F_1^1 - F_2^1}{G_1^1 - G_2^1}$$

corresponding to an eigenvector whose components are of equal magnitude but opposite sign, where the derivatives of F^1 and G^1 are evaluated at (m*, m*). It follows from (28) that $\xi^1 > 1$. Furthermore, if we interpret a change in the degree of substitutability of currencies as a change in U that does not affect the reduced form utility U*, then changes in the degree of substitutability of the two currencies will not affect the size of ξ^1.

Changes in the degree of substitutability will not affect the location of the stationary equilibrium (i.e., the value of m*) either, but they will affect the size of ξ^2. In particular, it is easily seen that in the case of perfect substitutability (1'), $F_1^1 - F_2^1 = F^1/m^1$ and $G_1^1 - G_2^1 = G^1/m^1$, so that $\xi^2 = 1$, whereas in the case that purchases and sales using one currency are not at all substitutable for those using the other, so that the marginal utility of purchases or sales of one kind is independent of the quantity of purchases and sales of the other type, $F_2^1 = G_2^1 = 0$, so that $\xi^2 = \xi^1 > 1$. In general we are interested in cases intermediate between these two, so that we should expect $1 < \xi^2 < \xi^1$. In fact, (30) implies

$$\frac{F_1^1 - F_2^1}{F^1} > \frac{G_1^1 - G_2^1}{G^1} \tag{31}$$

If we assume in addition (as just suggested) that we are interested in the case of partial substitutability, so that $U_{c^1c^2}$, $U_{n^1n^2} < 0$, then $F_2^1 > 0$, $G_2^1 < 0$, and (28) then implies that the right hand side of (31) is positive. Then (31) together with these latter inequalities implies that $1 < \xi^2 < \xi^1$.

It follows that regardless of the degree of substitutability of currencies, as long as it is less than perfect, both eigenvalues are greater than one, and there will exist no equilibrium other than the symmetric stationary equilibrium that does not eventually diverge from values of real balances near (m^*, m^*). In this regard the case of *perfect* substitutability is qualitatively different from that of even very great (but still imperfect) substitutability. For with perfect substitutability, every pair (m^1, m^2) satisfying $m^1 + m^2 = 2m^*$ is equally a stationary equilibrium, so that perfect foresight equilibrium is indeterminate even if only equilibria remaining forever near the symmetric stationary equilibrium are considered.[13] This may well correspond to a difference as respects global uniqueness of equilibrium as well, since, as explained below, solutions to (26) that involve diverging levels of real balances for the two currencies may after some number of periods reach extreme values that prevent solution of the difference equations for further periods. Hence analyses of exchange rate determinacy (such as Kareken and Wallace, 1981) that assume perfect substitutability may be misleading as guides to the consequences of increased substitutability.

On the other hand, there is a sense in which the analysis just given indicates that increased substitutability may increase the volatility of exchange rates and prices. For increased substitutability will move ξ^2 closer to one. And a value of ξ^2 only slightly above one implies that a change in expectations regarding fundamentals far in the future (future tastes, future transactions costs, future money growth rates) can have a significant effect upon the current equilibrium price levels and exchange rate. In the model analyzed formally here, I have assumed perfect foreknowledge of fundamentals into the indefinite future, and this may lead to a unique perfect foresight equilibrium. But in reality there will certainly be uncertainty and constant changes of opinion about the future, and if even (presumably quite volatile) expectations regarding the very distant future have a large impact upon equilibrium, one must suppose that this will increase instability. With less substitutability of currencies, the smallest eigenvalue may be significantly above one, so that only expectations about fundamentals over a relatively short horizon have much effect upon equilibrium, in which case speculative instability should be less of a problem.

Another class of non-stationary equilibria that are easily analyzed are those in which real balances of one currency decline to their minimum possible value, g (i.e., the private sector ceases eventually to use that currency for purchases), while real balances of the other currency approach a constant positive long run level, m**.[14] In the case of equilibria which may involve zero private sector purchases using one currency, equations (26) must be generalized to

$$F^i(m_t^1, m_t^2) \geq (\beta I/\mu)\, G^i(m_{t+1}^1, m_{t+1}^2) \qquad (32)$$

with an equality if $m_t^i, m_{t+1}^i > g$, for i = 1, 2. (The derivation is the same as for (26), but now allowing for a possible discrepancy between the marginal utilities and the shadow values of real balances, and I assume here that $g > 0$, so that necessarily $n_t^i > 0$.) Let us suppose, without loss of generality, that currency 1 is the one that eventually ceases to be used for private consumption purchases. Then the asymptotic level of real balances of currency 2, m**, must satisfy

$$F^1(g, m^{**}) \geq (\beta I/\mu)\, G^1(g, m^{**}) \qquad (33a)$$

$$F^2(g, m^{**}) = (\beta I/\mu)\, G^2(g, m^{**}) \qquad (33b)$$

However, if $0 < g < n^*$, then (33b) and (30) imply that (33a) cannot hold. Hence in this case no equilibria of this kind are possible, even in the case of preferences of a sort that would allow such equilibria when g = 0.[15]

When g = 0, (33a) can hold, in fact with equality, even though, because of (33b) and (30),

$$-U_n^1 < [\alpha + (1-\alpha)\beta I/\mu]\, U_c^1$$

In this case, there necessarily exists an m** satisfying (33b), and sufficient conditions for the existence of a continuum of perfect foresight equilibria in which $m_t^1 \to 0$, $m_t^2 \to m^{**}$ (and similarly, a continuum of equilibria in which $m_t^2 \to 0$, $m_t^1 \to m^{**}$) are

$$\lim_{m \to 0} F^1(m, m^{**}) = \lim_{m \to 0} G^1(m, m^{**}) = 0 \qquad (34)$$

together with the condition that $F^1(m, m^{**})$ be positive for all small enough m > 0. On the other hand, a sufficient condition to rule out such equilibria is that

$$\lim_{m \to 0} G^1(m, m^{**}) > 0 \qquad (35)$$

This rules out equilibria in which one currency asymptotically ceases to be used for the same reason that (29) rules out equilibria in which both currencies asymptotically cease to be used. Condition (35) requires that the marginal utility from purchases of goods using a particular currency becomes very large fast enough as the quantity of goods purchased using that currency goes to zero. It is thus a condition expressing a sense in which the use of that currency, for at least some purchases, is essential.

Now an increase in the substitutability of currencies need not imply that (35) should cease to hold. In particular, (35) might hold despite the existence of an arbitrarily high degree of substitutability between currencies, up until the point where one currency is used for only a very small number of purchases. Hence, again, analysis of the case of perfect substitutability is a poor guide to what must happen in the case of highly but not perfectly substitutable currencies. But, on the other hand, it should be pointed out that substitutability of currencies can result in (34) holding, despite the fact that (29) holds, as indeed must be the case if there is perfect substitutability; while in the case that U is additively separable between transactions involving the two different currencies, (29) implies (35).

Finally, we can consider the possibility of equilibria in which real balances of one currency become unboundedly large asymptotically. It is easily seen that in any solution of (26) of this form, real balances of the currency in question grow asymptotically at the rate $\mu/\beta I$, so that (27) is satisfied only if $\mu < I$. Hence, assuming that $\mu \geq I$, equilibria of this kind can be ruled out, regardless of the degree of substitutability of the two currencies. If $\beta I < \mu < I$, equilibria of this kind do exist, but again this is independent of the degree of substitutability of currencies.

However, it is arguable that the rate of money growth should not be independent of the degree of substitutability of currencies; indeed, this is the main argument of the proponents of "competing currencies" for the desirability of increased substitutability. The usual argument (see, e.g., Girton and Roper, 1981) is some variant of the following.[16] Suppose that each government chooses a path $\{\mu^i_t\}$ for the rate of growth of its currency, and a path $\{I^i_t\}$ for the rate of interest on holdings of its currency, taking as given the rate of growth of and rate of interest on the other currency, so as to maximize its seignorage income from money creation.[17] To simplify the analysis, let us restrict attention to strategies for each country in which money growth is held constant in all periods $t \geq 1$, and in which the interest rate on money is held constant in all periods $t \geq 0$, although μ^i_1 may differ from μ^i_0 owing to the existence of a special constraint on the period zero choice. The most important advantage of this restriction is that it allows us to make a natural choice regarding what equilibrium prices the countries expect to result from a given choice of money growth rates and interest payments. Specifically, we may assume that each country expects that from period one onward, the equilibrium that will exist will be the unique stationary equilibrium in which both currencies are used.[18] That is, each government assumes that given (μ^1, μ^2, I^1, I^2), the constant values expected to prevail for all $t \geq 1$, an equilibrium will result in which real balances of each currency are constants (m^1, m^2) for all $t \geq 0$, satisfying

$$F^i(m^1, m^2) = (\beta I^i/\mu^i)\, G^i(m^1, m^2) \qquad (36)$$

in which nominal interest rates in each country are constants (R^1, R^2) for all $t \geq 0$, satisfying $R^i = \mu^i/\beta$, and in which inflation in each country is equal to μ^i for all $t \geq 1$, for $i = 1, 2$. This results in a well-defined non-cooperative game between the two countries.

Now the government budget constraints (18) - (19) may equivalently be written as a single infinite horizon constraint for each government, which in the case of a stationary equilibrium of the kind just described reduces to

$$\Sigma_{t=0}^{\infty} \beta^t [g^i_t - \tau^i_t] = (1-\beta)^{-1} m^i [1 - \beta(I^i/\mu^i)] - \Sigma_h W^{hi}_0/p^i_0 \qquad (37)$$

where $\Sigma_h W^{hi}_0$ is an initial condition. It is reasonable then to take the right hand side of (37) to be the quantity that government i wishes to maximize. The second term on the right hand side of (37) can also be expressed $(\Sigma_h W^{hi}_0/M^i_{-1})(m^i/\mu^i_0)$. Then, using (36) to define m^i, both governments' objectives are specified as functions of their joint strategies and the initial conditions $\Sigma_h W^{hi}_0/M^i_{-1}$.

It is immediately obvious that given any choice of I^i/μ^i by the two countries, country i can increase the right hand side of (37) by increasing μ^i_0. Hence, in the absence of any further constraint, the optimization just described should lead to choice of an unboundedly large rate of money growth in the initial period. Apart from the fact that this makes equilibrium prices undefined, this result is undesirable because, as is well known, the optimizing behavior described is not time-consistent. One solution to this problem is to suppose that each government must commit itself a period in advance to an inflation target for each period, and is in the subsequent period constrained to choose a policy that results in an equilibrium price level no higher than the pre-announced target level.[19] With such a constraint in addition to the initial condition $\Sigma_h W^{hi}_0$, country i's optimal policy will be to choose I^i/μ^i so as to maximize $(1-\beta)^{-1} m^i [1 - \beta(I^i/\mu^i)]$, given the other country's choice (which affects the determination of m^i through the pair of equations (36)), and then to choose μ^i_0 so as to make p^i_0 equal to the pre-announced target. These policies not only lead to finite price levels, but are time-consistent.

The optimal choice of I^i/μ^i, taking as given the policy of the other country, will then satisfy a first-order condition[20]

$$[1 - \beta(I^i/\mu^i)] \quad \partial m^i/\partial(I^i/\mu^i) \; - \; \beta m^i \; = \; 0 \qquad (38)$$

where $m^i(I^1/\mu^1, I^2/\mu^2)$, for i = 1, 2, denote the solutions to the pair of equations (36).[21] Considering again the case of a symmetric two-country economy, the symmetric Nash equilibrium of the monetary policy game will involve a common choice for I/μ, and a common level of real balances m^*, that satisfy

$$F(m^*, m^*) \; = \; (\beta I/\mu)\, G(m^*, m^*) \qquad (39)$$

$$[1 - (\beta I/\mu)] \; = \; (2m^*/G(m^*, m^*))\, [Z_+^{-1} + Z_-^{-1}]^{-1} \qquad (40)$$

where

$$Z_+ \; = \; (F^1_1 + F^1_2) - (\beta I/\mu)(G^1_1 + G^1_2)$$

$$Z_- \; = \; (F^1_1 - F^1_2) - (\beta I/\mu)(G^1_1 - G^1_2)$$

with the derivatives evaluated at (m^*, m^*). For the reasons discussed above, an increase in currency substitutability will reduce Z_-, for any given m^*, with Z_- approaching zero (regardless of the value of m^*) as substitutability becomes perfect. As a result, the Nash equilibrium value of I/μ must approach the value β^{-1} from below. From this one may conclude that a sufficient degree of substitutability will imply a Nash equilibrium with

$\mu < I$. On the other hand, in the case of a common currency (again interpreted as implying the utility function U*), the optimizing policy choice by the single central bank will correspond to a stationary equilibrium in which

$$F(m^*, m^*) = (\beta I/\mu) G(m^*, m^*) \qquad (39')$$

$$[1 - (\beta I/\mu)] = (m^*/G(m^*, m^*)) Z_+ \qquad (40')$$

A change in the degree of currency substitutability has no effect upon the form of the function $Z_+(m, m)$ or of the function G(m, m), and so has no effect upon the solution to (39') - (40'). It follows that the solution may well involve $\mu > I$ even in the case of a transactions technology that would imply extremely high substitutability of currencies.

This is, of course, exactly the sort of reasoning behind the usual arguments for "competing currencies"; multiple currencies with a high degree of substitutability lead to a deflationary equilibrium while a single currency controlled by a single supra-national central bank may well lead to an inflationary equilibrium (as is warned of in the UK Treasury paper). But the result that one should expect $\mu < I$ in the case of sufficiently great substitutability of currencies has other implications besides the existence of a low-inflation steady state. Another consequence, as was shown above, is that equilibrium price levels and exchange rates will be indeterminate; not only will there exist a two-dimensional continuum of possible perfect foresight equilibria (in which one or the other or both kinds of real balances asymptotically grow without bound), but there will exist a large class of rational expectations equilibria in which both the price levels and the exchange rates vary stochastically in response to "sunspot" events. In this sense, too high a degree of currency substitutability may lead to the adoption of monetary policies that actually increase the degree of instability of price levels and exchange rates, even under exactly the sort of assumptions about the nature of policy interaction between governments that are behind the arguments for "competing currencies".

3. Currency Substitution and Stability of Learning Dynamics

Another question relevant to the likely stability of exchange rates in the case of increased currency substitution concerns not the uniqueness of rational expectations equilibrium, but instead how likely it is that the expectations of traders will in fact converge to those that characterize such an equilibrium. That is, if traders must learn what to expect about future exhange rate movements on the basis of their experience, can one be confident that even relatively crude adaptive learning rules would imply that a rational expectations equilibrium (and in particular, the rational expectations equilibrium in which all currencies are used and their relative values change only at a rate dependent upon their relative growth rates) will eventually be reached? Or will disequilibrium beliefs produce exchange rate movements that cause beliefs to diverge even farther from the rational expectations equilibrium beliefs, so that the latter are never reached even though they are theoretically determinate?

As in the previous section, I propose to address this question in the context of a world representative household and monetary policies that involve fixing an exogenous constant rate of growth, and an exogenous constant rate of interest payments on money holdings, for each of the currencies, and allowing exchange rates to float. Again, for simplicity, there is assumed to be no government debt outstanding, and government purchases are assumed to be constant, and again I specialize to the case of only two currencies with the same money supply growth rates and rates of interest on money, and the same constant levels of goods purchases by the two governments, and to the case of perfect symmetry of the utility functions between the two currencies and equal values for α^i. Finally, it will simplify the range of possible parameter variations that need to be considered if attention is also restricted to utility functions that are additively separable between consumption and work, i.e., that have the form $U(c^1, c^2; n^1, n^2) = u(c^1, c^2) + v(n^1, n^2)$.

First-order condition (10) can then be rewritten

$$U_c(c_t^1, c_t^2) = \beta R_t^i v_t^i \tag{41}$$

where

$$v_t^i = \left[\frac{p_t^i}{p_{t+1}^i} U_c(t+1)\right]^e$$

for i = 1, 2. Here the superscript indicates that what matters is the representative household's expectation at time t regarding the value of the quantity inside the brackets, which we will no longer assume must coincide with what actually happens in period t+1. I assume in order to simplify the analysis of learning that these expectations are a simple point value for each of the quantities in question, rather than functions describing how the quantities in the brackets are expected to vary depending upon the household's decisions in period t. This is doubtless unrealistic, but I do not believe that the instability result below depends upon this aspect of the proposed learning rule.[22] Equations (41), together with (10) and (15), also for i = 1, 2, comprise a set of six equations[23] that can be solved for the six variables (c_t^i, n_t^i, R_t^i) as functions of the variables (v_t^i), given values for $g^1 = g^2 = g$, $I^1 = I^2 = I$, and $\alpha^1 = \alpha^2 = \alpha$. Let the solutions for the variables (n_t^i) be denoted

$$n_t^i = n^i(v_t^1, v_t^2) \tag{42}$$

Then equations (11) and (16) imply

$$\frac{M_t^i}{p_t^i} = (1-\alpha)\, n^i(v_t^1, v_t^2) \tag{43}$$

These equations together with (8) allow us to determine the *temporary equilibrium* values for the complete set of state variables $(c_t^i, n_t^i, R_t^i, p_t^i, e_t^i)$ as functions of the money supplies at time t and the state of expectations (v_t^i).

We wish to analyze the evolution of a sequence of temporary equilibria of this sort, in response to exogenously growing money supplies and to changes in the state of expectations in response to observed rates of price change and marginal utilities. A simple but not entirely unrealistic rule for updating expectations is the *adaptive expectations* rule used by authors such as Cagan (1956). That is, expectations are adjusted in proportion to observed expectational errors:

$$v_t^i - v_{t-1}^i = \lambda \left[\frac{p_{t-1}^i}{p_t^i} U_c(n_t^1 - g, n_t^2 - g) - v_{t-1}^i \right] \tag{44}$$

Here $0 < \lambda \leq 1$ is a parameter indicating the speed of adjustment of expectations (the case of greatest interest being one where "periods" are short but $\lambda << 1$). This sort of simple rule has at least the desirable property of consistency with the stationary rational expectations equilibrium. That is, suppose households happen to start out with exactly the expectations characteristic of that equilibrium, i.e., $v_0^1 = v_0^2 = v$, where v satisfies

$$v = \mu^{-1} U_c(n^1(v, v), n^2(v, v))$$

and where μ is the common growth rate of the two currencies. (This condition for v is the same regardless of the value of i, because of the assumed symmetry of the utility function.) Then under the learning rule specified, the temporary equilibrium dynamics will generate constant (and hence rational expectations equilibrium) expectations $v_t^1 = v_t^2 = v$ for all t.

The temporary equilibrium dynamics are defined by the system of difference equations (42) - (44). We are interested in the stability under these dynamics of the stationary equilibrium. Substituting (43) into (44) we obtain

$$v_t^i - v_{t-1}^i = \lambda \left[\frac{n_t^i}{\mu n_{t-1}^i} U_c(n_t^1 - g, n_t^2 - g) - v_{t-1}^i \right] \tag{45}$$

Linearizing (42) and (45) around the stationary equilibrium values $v_t^i = v_{t-1}^i = v$, $n_t^i = n_{t-1}^i = n = n^i(v, v)$, we obtain

$$\Delta n_t^1 = \rho \, \Delta v_t^1 - \sigma \, \Delta v_t^2 \tag{46a}$$

$$\Delta n_t^2 = \rho \, \Delta v_t^2 - \sigma \, \Delta v_t^1 \tag{46b}$$

$$\Delta v_t^1 = (1-\lambda) \, \Delta v_{t-1}^1 + \lambda \, [(1-\gamma) \, \Delta n_t^1 - \eta \, \Delta n_t^2 - \Delta n_{t-1}^1] \tag{47a}$$

$$\Delta v_t^2 = (1-\lambda) \, \Delta v_{t-1}^2 + \lambda \, [(1-\gamma) \, \Delta n_t^2 - \eta \, \Delta n_t^1 - \Delta n_{t-1}^2] \tag{47b}$$

where Δn_t^i denotes $(n_t^i - n)/n$ and Δv_t^i denotes $(v_t^i - v)/v$, and using the notation

$$\gamma = -n \, U_{c^1c^1}^1 / U_{c^1}^1 = -n \, U_{c^2c^2}^2 / U_{c^2}^2$$

$$\delta = -n \, U_{n^1n^1}^1 / U_{c^1}^1 = -n \, U_{n^2n^2}^2 / U_{c^2}^2$$

$$\eta = -n \, U_{c^1c^2}^1 / U_{c^1}^1$$

$$\phi = -n \, U_{n^1n^2}^1 / U_{c^1}^1$$

$$\rho = \frac{\alpha\gamma + \delta}{(\alpha\gamma+\delta)^2 - (\alpha\eta+\phi)^2} \, \beta(1-\alpha) \, \frac{I}{\mu}$$

$$\sigma = \frac{\alpha\eta + \phi}{(\alpha\gamma+\delta)^2 - (\alpha\eta+\phi)^2} \, \beta(1-\alpha) \, \frac{I}{\mu}$$

where the derivatives of the utility function are evaluated at the stationary equilibrium allocation (n-g, n-g; n, n). Substituting (46a-b) into (47a-b), we obtain a pair of coupled difference equations for the variables (Δv_t^i), of the form

$$\Delta v_t^i = \Sigma_j \, M_{ij} \, \Delta v_{t-1}^j$$

The stationary rational expectations equilibrium is then locally stable under the learning dynamics if and only if both eigenvalues of the matrix M are of modulus less than one.

One eigenvalue, corresponding to an eigenvector whose two components are equal, is

$$\xi^1 = \frac{1 - \lambda(1+\rho-\sigma)}{1 - \lambda(1-\gamma-\eta)(\rho-\sigma)}$$

The other eigenvalue, corresponding to an eigenvector whose two components are equal in magnitude but opposite in sign, is

$$\xi^2 = \frac{1 - \lambda(1+\rho+\sigma)}{1 - \lambda(1-\gamma+\eta)(\rho+\sigma)}$$

We wish to consider the effects upon stability of varying the degree to which currencies are substitutable. Again, the natural experiment to consider is to fix the values of $\gamma+\eta$ and $\delta+\phi$, while varying the individual parameters; this means keeping constant the effect upon the marginal utility of one or another type of consumption or work of a proportional change in both types of consumption, or of a proportional change in both types of work, while varying the extent to which variation in one type of good by itself has the same effect as variation in the other type by itself. Concavity of the utility function requires that we fix values $\gamma+\eta$, $\delta+\phi > 0$. Imperfect substitutability exists as long as $\gamma > \eta$ or $\delta > \phi$. Currency substitutability is increased by increasing η (and correspondingly reducing γ) and by increasing ϕ (and correspondingly reducing δ); the limit of perfect substitutability is reached when $\eta = \gamma$ and $\phi = \delta$.

Under this experiment, if the value of λ is held fixed, the eigenvalue ξ^1 does not vary. Let us suppose that its modulus is less than one. (This is necessarily true for small enough positive λ, given any positive values for $\gamma+\eta$ and $\delta+\phi$.) Then stability depends upon the size of $|\xi^2|$. Noting that

$$\rho + \sigma = \frac{\beta\,(1-\alpha)\,(I/\mu)}{\alpha(\gamma-\eta) + (\delta-\phi)}$$

we see that $|\xi^2| < 1$ if and only if

$$-1 < \frac{(2-\gamma+\eta)\,\beta\,(1-\alpha)\,(I/\mu)}{\alpha\,(\gamma-\eta) + (\delta-\phi)} < \frac{2-\lambda}{\lambda} \qquad (48)$$

It is then obvious that if $\gamma-\eta$ and $\delta-\phi$ are both made small enough, the right hand inequality in (48) is eventually violated. Hence given a rate of adjustment of expectations λ, a great enough degree of substitutability between currencies implies instability of the stationary rational expectations equilibrium under the learning dynamics.[24] On the other hand, a low enough degree of substitutability can result in stability. For example, $\eta = \phi = 0$ (the case of preferences additively separable between the transactions involving different currencies) would imply $\xi^2 = \xi^1$, so that under the assumption made above both eigenvalues would then have a modulus less than one. Clearly it will in general not be necessary to reduce substitutability to this extent to insure that $|\xi^2| < 1$. Hence this result provides an example of a case in which insitutional barriers to currency substitution could improve exchange rate stability.[25]

It also provides an example of a case in which price level stability could be improved through adoption of a common currency. Let us assume, as in the previous section, that adoption of a common currency would result in preferences for the representative household of the form

$$U^*(c; n) = U(c, c; n, n)$$

as well as a value of α for the single currency equal to the previous common value. Then we can analyze, in the common currency world, the temporary equilibrium dynamics resulting from the representative household's adjusting its beliefs about the value of $U_c(t+1)p_t/p_{t+1}$ according to a rule of the form (45). In this case, the matrix M has a single element, equal to the quantity ξ^1 defined above. Hence, under the assumptions made above, adoption of a common currency would result in local stability of the stationary rational expectations equilibrium under the learning dynamics. This would bring about a steady and fully anticipated rate of world inflation. Contrastingly, with multiple currencies and in the unstable case, the rates of inflation in terms of the two currencies would diverge from the stationary rational expectations equilibrium value, along with the exchange rate.

4. Feasibility of Fixed Exchange Rates with Currency Substitution

Up to this point, I have analyzed the consequences of increased substitutability of currencies in the case of a system of fully flexible exchange rates. There are some obvious reasons for analysis of the doctrine of "competing currencies" within that context; in particular, the view that competition will result in pressure for lower rates of money growth presumes a world in which individual countries are free to choose their own rates of money growth, while a fixed exchange rate regime in the full sense of that term implies subordination of individual countries' monetary policies (in the case of all but one country, probably Germany for the EMS) to the need to maintain exchange rate parities. Nonetheless, Europe has made substantial progress toward fixed exchange rates within the EMS, and adoption of the proposal to stimulate increased competition between currencies would presumably be within a context of continued commitment to that goal; and the UK proposal explicitly reaffirm's the UK's commitment to eventual participation in the current Exchange Rate Mechanism (HM Treasury, 1989, p. 2). Hence it is also of interest to ask what the consequences of increased substitutability of currencies would be within the context of an attempt to maintain fixed exchange rates.

Let us consider the character of a perfect foresight equilibrium in which exchange rates are fixed forever, and known in advance to be so fixed with certainty. Without loss of generality, we may suppose that the fixed exchange rates are $e_t^i = 1/n$ for all i and t. Condition (7) then implies the existence of a common world nominal interest rate on bonds each period, $R_t^i = R_t$ for all i. Let us now repeat the derivation of (32), again assuming a world representative consumer, but allowing for n currencies, not assuming symmetry, and not assuming a constant exogenous growth rate for the various money supplies. We obtain

$$F^i(m_t) = \frac{\beta M_t^i I_t^i}{M_{t+1}^i} \gamma_{t+1}^i \tag{49}$$

$$G^i(m_t) \leq \gamma_t^i \tag{50}$$

with an equality in (50) if $m_t^i > g$. Here m_t denotes $(m_t^1, \ldots, m_t^n)$ and

$$\gamma_t^i = (1-\alpha^i) m_t^i \lambda_t^i$$

I have also assumed that $g > 0$, for simplicity, so that $n_t^i > 0$ is assured. In terms of this notation, (12) becomes

$$\frac{\gamma_t^i}{\gamma_{t+1}^i} = \beta R_t \frac{M_t^i}{M_{t+1}^i} \tag{51}$$

The aspects of monetary policy that remain to be exogenously specified, given the fixed exchange rates, are a set of n+1 variables per period, one (either M_t^i or I_t^i or some function of the two) for each country that is responsible for maintaining its exchange rate, and two (both M_t^i and I_t^i, or one of these and the world interest rate R_t) for the country that is not so constrained.[26] Given a full specification of world monetary policy in this sense, equations (49) - (51) are then a system of 3n equations per period to solve for the remaining 3n of the variables

$$\{M_t^1, \ldots, M_t^n, I_t^i, \ldots, I_t^n, \gamma_t^i, \ldots, \gamma_t^n, m_t^1, \ldots, m_t^n, R_t\}$$

Now (49) - (51) taken together imply

$$F^i(m_t) \geq (I_t^i/R_t)\, G^i(m_t) \qquad (52)$$

with an equality if $m_t^i > g$. These n equations taken together determine the demands for real balances of the n currencies as a function of the n spreads I_t^i/R_t.[27] These are again just the money demand functions derived in section 1. I have here written these relationships in the form (52) to make it clear what determines whether there is a positive private demand for all of the currencies. If

$$\lim_{m^i \to g} \frac{F^i(m)}{G^i(m)} > 0 \qquad (53)$$

where the m^j, $j \neq i$, are fixed at values greater than g, then for given interest rate spreads for the currencies $j \neq i$, there will exist a maximum spread θ such that $I_t^i/R_t > \theta^{-1}$ is necessary in order for currency i to be used for consumption purchases. Condition (53) necessarily holds if $g > 0$, and even if $g = 0$ (or if government purchases do not require cash in advance, or governments do not insist on using their own currency), currency substitutability may result in such a condition holding, as discussed in connection with condition (34) above. Without fixed exchange rates, this would simply imply that there is a limit to the extent to which nominal interest rates on bonds can be raised in a given country if that country's currency is not to be driven out of circulation. With fixed exchange rates, a given country has no possibility of controlling nominal interest rates on bonds, so that now the condition can be insured only by making the nominal interest payments on money high enough. Hence it may not be possible for a country to insure that its currency continues to be used unless interest payments on money are available as an instrument of monetary policy.

This is especially a problem in the case of a high degree of substitutability between currencies. In the case of perfect substitutability (1'), one has for all currencies i

$$\frac{F^i(m)}{G^i(m)} = \frac{\rho(m) - \alpha^i}{1 - \alpha^i}$$

where $\rho(m)$ denotes the common marginal rate of substitution between output and consumption. It follows that (52) can hold with equality for each of two currencies (so that both can be used for consumption purchases in equilibrium) only if $\alpha^i + (1-\alpha^i)(I^i_t/R_t)$ is identical for the two currencies. Hence variations in α^i across payments systems must be compensated for by variations in I^i_t. If it is not possible to pay interest on money balances, then *only the currency with the highest value of* α^i *can be used for consumption purchases in equilibrium, under a fixed exchange rate regime.* Since it is reasonable to suppose that payments systems must differ to some extent in the time lags associated with transactions,[28] this means that all but one currency should be driven out of use, through a sort of reverse Gresham's Law. Nor is such an outcome dependent upon exactly perfect substitutability. If subsitutability is very great (although not perfect), the marginal rates of substitution between consumption purchases and output sales in the case of the different currencies will still be close to one another, even if the quantities transacted in terms of the various currencies are very different, and so it may not be possible to satisfy (52) with equality in the case of the low-α^i (i.e., high transaction cost) currencies, given that it holds with equality for the high-α^i currencies, without having differential interest payments upon cash balances of the different kinds.

Nor is this problem solved if it can be arranged that α^i be the same for all currencies, due to adoption of a common technology or even perhaps a common payments system. For there would remain the problem of speculation about future adjustments of the exchange rate parities, that would continue to be a possibility as long as one did not move to a common currency. Perfect capital mobility implies that expectations of the possibility of such an adjustment must be reflected in differentials across currencies of the nominal interest rate on bonds. Then even with a common value for α^i and a common (institutionally fixed) value for I^i, it will not in general be possible for $\alpha^i + (1-\alpha^i)(I^i/R^i_t)$ to be identical for any two currencies, with the result, in the case of perfect substitutability, that only the lowest-R^i_t currency (i.e., the currency thought least likely to be devalued in the future) will be used for consumption purchases.

Again, the problem persists in the case of high but not perfect substitutability of currencies. In this case, *the greater the degree of substituability of currencies, the smaller the change in expectations required to drive currencies out of use by the private sector.* Even an arbitrarily small positive probability of even an arbitrarily small realignment suffices to drive out all but one currency, in the event of great enough substitutability, although in the event of a given probability and of a given size of realignment, the degree of substitutability needed for such an outcome is less than perfect. Hence in the event of a great increase in currency substitutability, a fixed exchange rate regime will become extremely unstable (in terms of the volatility of the rates of growth of the various currencies that will be required to maintain the exchange rate parities), and quite possibly unmanageable as a practical matter, unless the nominal interest rate on money balances is available as a policy instrument in each country that is responsible for maintaining the exchange rates. And this instrument would have to be able to be adjusted very precisely and very rapidly in response to speculation regarding future exchange rate realignments. Since no such independent instrument of monetary policy is currently used in the European Community or elsewhere, pursuit of the U.K.'s program of stimulating "competition" between currencies, were it possible, would likely be incompatible with maintenance of the sort of Exchange Rate Mechanism currently in force.[29]

For this reason, as well as those summarized above, increased currency substitutability would likely be a source of greater instability. And insofar as reduced barriers to currency substitution are regarded as either desirable (for reasons unrelated to the supposed stabilizing consequences of competition between currencies) or inevitable (as a consequence of the removal of barriers to trade in financial services), one should regard the substitution of a common currency for the current system of multiple currencies as a development that would favor price level and exchange rate stability.

FOOTNOTES

[1] Giovannini (1989) also argues that the process of European integration will lead to increasing substitutability of currencies, quite apart from the Delors Report proposals.

[2] Some especially influential prior analyses of competition between currencies, of the sort that presumably provide the reasoning behind the U.K. proposal, are collected in Salin (1984).

[3] It is also closely related to the models of Svensson (1985a, 1985b). For an overview of uses of this general modeling strategy in international monetary economics, see Stockman (1989).

[4] This suggestion is analogous to Lucas and Stokey's use of substitution between "cash goods" (subject to a cash-in-advance constraint) and "credit goods" (subject to no such constraint) by consumers and producers as a way to represent the possibility of substitution away from the use of money to carry out transactions.

[5] I assume perfect capital mobility throughout the present exercise, on the ground not only that capital is already much more mobile within the European Community than are currencies substitutable, but that the U.K. proposal to promote currency substitutability as an alternative to a common currency is made within a context of a consensus regarding the Community's commitment to moving toward completely unrestricted capital mobility as part of Stage 1 of economic and monetary union.

[6] It can be shown that concavity of the function U implies the existence of a unique solution, at least for all values of $(R^1_t/I^1_t, ..., R^n_t/I^n_t)$ in a neighborhood of (1, ..., 1), i.e., if the spread between the return on bonds and on money is small enough for all currencies, by a direct extension of the argument in the single currency case. If the various types of goods and leisure are all sufficiently substitutable, there will also exist a unique solution globally.

[7] Note that it should be understood here, as in the case of (11) and (11'), that either the case described by (20) or that described by (20') must occur, for a lower interest rate on bonds than on money would imply that households would wish to hold an arbitrarily large quantity of the currency in question, financed by selling short a similarly large quantity of bonds.

[8] Measures of the volume of private transactions do not appear as arguments of these money demand functions because in the present model, with no supply shocks allowed for, the complete allocation of resources is a determinate function of the interest rate spreads and government purchases. In a stochastic model with shocks to the household's disutility for supply of goods, these preference parameters would also be arguments of the money demand functions. One could then substitute for them the vector of supplies or consumptions, in order to obtain a money demand function of a more standard form.

[9] See the references in Thomas and Wickens for other examples of the two contrasting approaches.

[10] On the other hand, this derivation does not necessarily provide support for the view that the appearance of foreign interest rates in a money demand equation is in itself proof of the importance of currency substitution. For it must be recalled that the equations derived here are equally applicable to a model in which each country's products must be purchased with that country's currency, and the n "types" of goods represent goods produced in different countries. In that case the effect of foreign interest rate spreads upon the demand for a particular currency results from substitution away from purchase of goods produced in countries whose currencies are particularly costly to hold due to a large interest rate spread, and not from any change in the currencies used for a particular type of transaction.

[11] This form makes it clear to what extent the difference equations obtained are a natural generalization of the equation obtained in Woodford (1988) for the single-currency case.

[12] Concavity of U implies that $G(m) > F(m)$ for all $0 < m < \bar{n}$.

[13] In this case there will also exist among the possible rational expectations equilibria what are sometimes called "sunspot equilibria", in which permanent exchange rate and price level shifts occur in response to random events that convey no information about any changes in the "fundamental" determinants of price levels and exchange rates.

[14] An even broader set of possibilities arises if we consider stochastic equilibria (i.e., "sunspot equilibria") as well as deterministic ("perfect foresight") solutions. For example, stochastic equilibria can occur in which the ratio between real balances of one currency and those of the other follows a random walk, visiting

arbitarily small and arbitrarily large positive values infinitely often, while the total level of real balances of both kinds remains bounded both above and away from zero. See, e.g., Chiappori and Guesnerie (1989).

[15]Hence the use of their own currencies by governments can play a crucial role in rendering determinate the equilibrium value of currencies, even if the share of government purchases is relatively small. See Giovannini (1989) for a related discussion.

[16]It might seem odd that the only account here given of how "competition" between currencies is supposed to work is presented in the context of a floating exchange rate system, when the UK Treasury paper explicitly envisions "competing currencies" within a system of fixed rates. But the existing theoretical justifications of the supposed advantages of "competing currencies" (e.g., Girton and Roper, or Salin, ed., 1984) are in fact in the context of a floating exchange rate world. Indeed, there would seem to be no possibility for competitive monetary policies in the case of a fixed exchange rate system, if this is interpreted as an asymmetrical peg of the kind discussed in section 4 below. The case of a "common currency" with independent central banks discussed by Casella and Feinstein (1989) cannot be interpreted as a fixed exchange rate system of the usual sort, in which only central banks are obligated to exchange the various currencies at par, but rather represents a world in which private sellers are required to exchange the various currencies at par (e.g., because they are physically indistinguishable). In any event, in the latter case, currency substitution has the consequence that noncooperative, seignorage-maximizing central banks would choose a higher rate of money growth, rather than a lower one, than would prevail in the case of less substitutability of currencies or in the case of cooperative behavior; so that such a regime cannot be what the authors of the UK Treasury view have in mind either.

[17]This objective is not the only possible one, but provides a simple explanation for why governments might wish to avoid policies that involve inflations so great that the use of their currency is severely reduced.

[18]Here we make an assumption with little obvious justification that avoids some very serious difficulties with the argument for "competing currencies". For without an arbitrary selection as to which equilibrium is expected to prevail in the case of any given pair of monetary policy choices, there is no well-defined game being played by the two countries. Furthermore, not all possible selection rules imply existence of the kind of "competitive" pressures usually assumed to result from currency substitution. For example, in the case of very high substitutability between currencies, all perfect foresight equilibria in which at least one currency is used forever, except those in which the ratio of the level of real balances of country 2's currency to real balances of country 1's currency is initially and forever extremely low, involve real balances of country 1's currency that asymptotically approach zero (while those of country 2's currency approach a positive constant), if I^1/μ^1 is even slightly *larger* than I^2/μ^2. And if (34) holds, a continuum of such equilibria exist. (In the case of perfect substitutability, this is true of all equilibria in which positive levels of real balances exist for both currencies, as shown by Kareken and Wallace.) This is sometimes taken to indicate that countries with higher rates of money growth (or lower rates of interest on their currencies) are rewarded with greater eventual use of their currencies, rather than the reverse. The argument is somewhat doubtful, because the initial levels of real balances are not given as an initial condition for the dynamics, but are instead determined by expectations about the future. And the fact that "most" initial levels of real balances correspond to one asymptotic behavior rather than another does not imply that that long run outcome is the one that should occur; for example, reasonable "learning" dynamics may converge to exactly the equilibrium that all but one perfect foresight paths diverge from (Grandmont (1985), Marcet and Sargent (1987)). But the grounds for a belief that a pressure toward lower rates of money growth should exist are also far from certain.

[19]This is the essential point of Girton and Roper's assumption of a convertible currency.

[20]This is an obvious analog for the two-country case of the kind of first-order condition obtained by authors such as Friedman (1971), Auernheimer (1974), and Calvo (1978).

[21]Note that there is a determinate optimal value only for the ratio, not for I^i or μ^i separately; similarly, in the Nash equilibrium of the policy choice game, only the ratio is determined for each country. The additional restriction that I^i is determined technologically to equal 1 will not affect the character of the game or the results obtained below.

[22]The analysis given here can be rigorously justified if utility is linear in the various types of consumption, so that each household is simply forecasting the expected rate of change of prices in terms of the various currencies.

[23] I assume that (10) holds at all times in the analysis of this section, for we are only interested in local stability of the learning dynamics near a stationary equilibrium in which both currencies are used, and so both currencies will be used along all paths sufficiently near that equilibrium.

[24] It will be observed that for any given values of the other parameters, (48) is satisfied for small enough positive λ. Hence the instability could be avoided, in the event of even a very great increase in the substitutability of currencies, if for some reason λ declined sufficiently at the same time, i.e., if expectations began to be adjusted sufficiently slowly. But it is hard to see why this should occur.

[25] The asymptotic behavior of the temporary equilibrium dynamics in the unstable case cannot be determined from a purely local analysis of the kind presented here. A plausible outcome would appear to be the eventual complete displacement of one of the currencies, but it is not clear that perpetual disequilibrium fluctuations in expectations can be ruled out.

[26] Note that if the interest rate on money is fixed institutionally (e.g., if it is necessarily zero), then the only aspect of world monetary policy that remains to be exogenously specified is a single variable per period chosen by the unconstrained country.

[27] Derivation of a determinate relationship of that sort would be possible here even without the assumption of a world representative consumer, because of the fixed exchange rates. Recall the derivation at the end of section 1.

[28] See Giovannini (1989) for discussion of this.

[29] On the desirability of a system of fixed exchange rates within the European Community, see Giavazzi and Giovannini (1989).

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