NBER WORKING PAPER SERIES

EFFICIENCY COSTS OF INCOMPLETE MARKETS

David R. Baqaee
Ariel Burstein

Working Paper 34233
http://www.nber.org/papers/w34233

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2025

We thank Joao Guerreiro, Oleg Itskhoki, Gianluca Violante, and Pierre-Olivier Weill for their comments. We thank Lautaro Adamovsky for exceptional research assistance. We acknowledge financial support from NSF grant No. 2343691. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Efficiency Costs of Incomplete Markets
David R. Baqaee and Ariel Burstein
NBER Working Paper No. 34233
September 2025
JEL No. E0, F0, F1, F30

ABSTRACT

This paper quantifies misallocation caused by limited risk-sharing and imperfect consumption-smoothing. We measure these losses in terms of how much of society's resources would be left over if financial markets were complete and each consumer were compensated to maintain their status-quo welfare. Using exact formulas and approximate sufficient statistics, we analyze standard incomplete-market environments—ranging from closed-economy Bewley-Aiyagari models to multi-country settings with input-output linkages. We find that incomplete insurance against household-level idiosyncratic risk is very costly—about 20% of aggregate consumption—based on both structural models and sufficient-statistics computed using household consumption panel data. By contrast, the cost of imperfect international financial markets (abstracting from within-country heterogeneity) is roughly 5%, driven by the inclusion of fast-growing economies such as China and India. Unexploited risk-sharing opportunities among countries at similar levels of development, on the other hand, are fairly limited (less than 1%).

David R. Baqaee
University of California, Los Angeles
Department of Economics
and CEPR
and also NBER
baqaee@econ.ucla.edu

Ariel Burstein
University of California, Los Angeles
Department of Economics
and NBER
arielb@econ.ucla.edu

1 Introduction

The inability of households to perfectly share risks across states of nature or to smooth consumption over time is a form of resource misallocation. This paper quantifies this misallocation by asking the following counterfactual question: if financial markets were complete and every household were compensated so that no one was worse off than under the status quo, how much of the available resources would be left over? We characterize this measure of misallocation in a range of workhorse models with incomplete financial markets — both in closed-economy (domestic) and open-economy (international) settings. We provide exact formulas for these welfare losses and show how to approximate them using sufficient statistics derived from observed consumption allocations.

The outline of the paper is as follows. In Section 2, we provide a general definition of misallocation using the aggregate consumption-equivalent defined by Baqaee and Burstein (2025b). We measure misallocation by how much the complete-markets consumption possibility set can shrink while keeping every agent at least indifferent to their status-quo allocation? The resulting contraction factor is a measure of the waste due to market incompleteness. Our measure is a multi-agent extension of the consumption-equivalent variation defined by Lucas (1987), and builds on ideas from Hicks (1939), Kaldor (1939), Allais (1979), and Debreu (1951). Its key property is that it quantifies the distance from the efficient frontier without making interpersonal utility comparisons. Our measure easily accommodates heterogeneous preferences, is invariant to monotone transformations of utility, and does not conflate inequality with inefficiency. We use a result from Baqaee and Burstein (2025b) to compute misallocation as the solution to a fictitious utility-maximization problem.

In Section 3, we set up a simple closed economy model with idiosyncratic risk. We show that misallocation is equal to the certainty-equivalent of the aggregate consumption process divided by the sum of certainty-equivalents of each households's consumption process. In this special case, our measure is similar to the efficiency measure proposed by Benabou (2002) (though ours is a result rather than a definition).

We provide a second-order approximation of misallocation in terms of deadweight-loss (Harberger) triangles. This sufficient statistic formula can be computed using only consumption panel data and a value for the elasticity of intertemporal substitution. In particular, we do not need to know the nature of households' financial market imperfections, income processes, or ownership of assets.

We apply these formulas to an off-the-shelf calibration of a Bewley (1972) economy. We find that imperfect insurance against idiosyncratic risk is equivalent to a loss of around

20 percent of aggregate consumption. That is, if markets were complete, we could make every agent indifferent to the status-quo and have 20% of the aggregate consumption good left over in every date and state. This loss is roughly three orders of magnitude larger than Lucas's (1987) estimate of the cost of aggregate volatility in a representative-agent economy.[1] Furthermore, although the economy is far from the efficient frontier, our second-order approximation is extremely accurate. The efficiency gains from completing financial markets are roughly half of as large as the distance to first-best according to the popular behind the veil-of-ignorance social welfare function. This is because the veil-of-ignorance social welfare function takes into account inequality as well as inefficiency.

In Section 4, we extend the simple model to allow for labor-leisure choice and for aggregate capital accumulation. Since our measure of misallocation is defined in very general terms, we can apply the same definition with these additional ingredients. In this case, the consumption possibility set also includes leisure and takes into account the fact that the aggregate capital stock is endogenous. With these ingredients, our measure of inefficiency also takes into account inefficiencies caused by excessive capital accumulation, due to the precautionary motive, and distortions in labor-leisure decisions. We also show that with labor-leisure choice, our measure differs from the measure of aggregate efficiency proposed by Benabou (2002), which has some counterintuitive properties. For example, we show that among points on the Pareto frontier, the Benabou (2002) measure may strictly prefer points with more inequality. By continuity, this also means that it may prefer Pareto-inefficient allocations to Pareto-efficient ones.

In Section 5, we introduce international trade. We study a setup with multiple countries, industries, and international input-output linkages. We extend our second-order approximation to allow for these ingredients. Our approximate sufficient-statistics formula depends only on observed nominal consumption expenditures and real-exchange rates by country over time, the static input-output network at one point in time, and elasticities of substitution in consumption and production. In particular, applying the formula does not require modelling financial market imperfections, ownership of assets, the exogenous productivity processes of each country, or separating consumption fluctuations due to wedges from those due to productivity shocks. We show, using Monte Carlo examples, that our second-order approximation performs well even for large shocks.

[1] Following Lucas' estimates for aggregate consumption, a series of papers have estimated the gains from eliminating volatility in settings with heterogeneous agents. See, for example, Imrohoroğlu (1989), Atkeson and Phelan (1994), Storesletten et al. (2001), and Constantinides (2025). To aggregate welfare gains across agents these papers rely either on a social welfare function (e.g. "average utility" or veil-of-ignorance) or have ex-ante symmetry (so that welfare gains are the same for all agents). Our paper complements this literature, since we do not use either a social welfare function nor assume ex-ante symmetry.

In Section 6 we take our sufficient statistics deadweight-loss triangle formulas to microeconomic (household-level) and macroeconomic (country-level) data. In Section 6.1, we study misallocation from incomplete risk-sharing in household consumption panel data from the United States (the Panel Study of Income Dynamics, PSID, database). We find that misallocation losses are similar to those in an off-the-shelf calibration of the Bewley model — roughly 20% of aggregate consumption in every period and state. Losses are greater, the lower is the elasticity of intertemporal substitution and the interest rate.

In Section 6.2, we study misallocation from imperfect consumption-smoothing and risk-sharing using international macroeconomic data. For this second exercise, we assume each country has a representative agent, and quantify losses from the absence of complete financial markets between countries. Our model has 32 countries, 54 industries in each country, and input-output linkages. We calibrate elasticities of substitution and apply our formula to a sample from 1970 to 2019. We find misallocation losses roughly in the 5% range. That is, if financial markets were complete and every household was compensated to be indifferent to the initial allocation, then there would be 5% of every consumption good left in every date and state. This result crucially depends on the inclusion of fast-growing countries. Hence, the gains are primarily due to unexploited gains from intertemporal trade between countries. If we exclude fast-growing countries, like China and India, then misallocation losses drop to around 1%. We consider sensitivity analysis and show, once again, that losses are higher if the intertemporal elasticity of substitution is lower. We also show that losses are larger if the Armington trade elasticity is higher. Intuitively, there are more unexploited opportunities to share risk and smooth consumption, conditional on the data, if the foreign and domestic goods are more substitutable.

These results underscore the potentially large welfare gains from more complete risk sharing, especially across individual households in domestic settings or between emerging and developed economies. Risk-sharing opportunities among countries at similar levels of development, on the other hand, are fairly limited.

Relation to companion papers. Although this paper is self-contained, it has two companions. Baqaee and Burstein (2025b) provides a general framework for studying aggregate efficiency with heterogeneous agents. Many of the results in this paper are applications or extensions of the general approach in that paper. In the other companion paper, Baqaee and Burstein (2025a), we apply the same framework to study changes in aggregate efficiency in spatial economies with discrete choice and heterogeneous consumer tastes.

Related literature. This paper is related to the literature that analyzes the efficiency implications of financial market incompleteness. There are two main branches of this literature. The first branch is concerned with domestic risk-sharing of idiosyncratic household-level risks in closed-economy settings. The second branch analyzes efficiency of risk-sharing in an international context with nontraded goods. Our paper provides a unified approach for both branches of the literature. We discuss these two branches in sequence.

The first branch derives from Bewley (1972) and its extensions, including Imrohoroğlu (1989), Huggett (1993), and Aiyagari (1994). To evaluate aggregate welfare in this class of models, there are two common approaches. The first is to use a social welfare function, typically by appealing to behind-the-veil of ignorance logic of Harsanyi (1955).[2] It is understood that social welfare functions, including the behind-the-veil one, embed some distributional judgement and require interpersonal comparisons. If preferences are risk-averse, the behind-the-veil measure is averse to inequality. The second approach, following Benabou (2002) and then Floden (2001), aims to separate Pareto-efficiency considerations from redistributional ones. These measures evaluate the value of an allocation using the sum of individual consumption certainty-equivalents. Both the veil-of-ignorance approach and the Benabou (2002) approach rely on the assumption that households have the same preferences. This means they are not applicable to international settings where households in different countries consume different goods.

In the closed-economy setting, our measure agrees with the one proposed by Benabou (2002) if we abstract from labor-leisure choice. However, even abstracting from this, our paper complements Benabou (2002) and the literature that followed it by providing a second-order approximation of the efficiency losses from imperfect risk-sharing. Our approximation formula, which is a Harberger (1964) triangles formula, requires only estimates of the elasticity of intertemporal substitution, the risk-free interest rate, and second moments of the household consumption allocations. This allows us to quantify misallocation with weaker structural assumptions.

Our focus is on the distance of the allocation from the Pareto frontier. This means that, when financial markets are completed, we allow for lump-sum transfers between households to ensure everyone is compensated. Some papers in this literature, including Benabou (2002), consider constrained efficiency and second best policies (with imperfect redistribution). Although our framework can be applied to study such questions, we do not pursue them in this paper. (See Baqaee and Burstein, 2025b for examples).[3]

[2]Some examples include Heathcote et al. (2008), Conesa et al. (2009), Dávila et al. (2012), Krueger et al. (2016), and Boar and Midrigan (2022).

[3]Namely, our Proposition 1 applies in situations where lump-sum transfers are not available. Extending our approach to second best scenarios means that we would apply Proposition 1 to a restricted consump-

The second branch of the risk-sharing literature focuses on the international dimension of the problem — taking the fact that some goods are non-tradeable into account. Some examples include Van Wincoop (1994), Gourinchas and Jeanne (2006), Fitzgerald (2012), Heathcote and Perri (2014), Fitzgerald (2024), Corsetti et al. (2024), and Aguiar et al. (2025). The approach to quantifying inefficiency, or aggregate welfare, in this branch of the literature is more eclectic. Some papers assume ex-ante symmetric countries, so that the loss from restricting trade in financial assets is also symmetric. Some papers eschew aggregate comparisons and report country-by-country welfare changes only. Lastly, some papers use Bergson (1954)-Samuelson (1956) social welfare functions, typically a so-called utilitarian function. Of course, there are also papers that analyze the efficiency properties of the decentralized equilibrium, without quantifying inefficiency per se, for example Cole and Obstfeld (1991) and Backus and Smith (1993).

Our paper also contributes to this literature. Given the way we measure aggregate efficiency, we do not need to impose that countries be ex-ante symmetrical or to use a social welfare function. Accordingly, our measure relaxes the unrealistic assumption of symmetry, while avoiding interpersonal utility comparisons and remaining neutral on distributional issues. On a methodological front, we derive sufficient statistics formulas that can be applied to the data without requiring knowledge of the productivity shocks that hit the economy.

Our paper is also related to recent work that provides approximate decompositions of changes in aggregate welfare using social welfare functions, for example Bhandari et al. (2021) and Dávila and Schaab (2022, 2023). Our paper is different because the measure of aggregate efficiency we use is distinct from the measures used in these papers in two ways. First, we do not specify a social welfare function. Second we define aggregate efficiency exactly and not as part of an approximate decomposition of aggregate welfare. This means that our measure can be integrated, allowing us to study the effect of large changes and, that generically, it does not coincide with what is referred to as efficiency in these papers.

Of course, our paper is also related to a different literature that studies the efficiency consequences of misallocation, following Harberger (1954), and more recently, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). From a methodological and concep-

tion possibility set accounting for limited redistribution. This is related to Farhi and Werning (2012), who quantify the aggregate welfare gains from capital taxation in an incomplete market model with private information, using the resources saved when implementing the inverse Euler equation while holding labor decisions and utilities unchanged. Another approach to evaluate second-best policies in incomplete market environments is to look for robust Pareto improvements, which weakly relax all constraints, as in Aguiar et al. (2024).

tual point of view, our paper is very closely related to this literature, though we study a different type of misallocation. Whereas this literature typically emphasizes static cross-sectional misallocation in production, we focus on dynamic stochastic misallocation in consumption. Notwithstanding this difference, our methodological approach is similar. We analyze the distance to the Pareto-efficient frontier, we use reduced-form wedges to capture the frictions in the decentralized equilibrium, and we repurpose the triangles formulas developed by Baqaee and Farhi (2020) to study a very different class of problems.[4]

2 Misallocation Due to Market Incompleteness

Consider an economy populated by agents indexed by h. Each agent h has non-satiated ordinal preferences $\succeq_h$ over bundles of commodities $\boldsymbol{c}_h$, which are time and state-contingent. The consumption stream $\boldsymbol{c}_h$ contains everything the agent has preferences over — so, in a model with labor-leisure choice, it would include leisure. Preferences are represented by utility functions $u_h(\boldsymbol{c}_h)$. A *consumption allocation* is a matrix $\boldsymbol{c} = \{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_H\}$ of consumption bundles for each agent. Denote the consumption allocation in the equilibrium with incomplete markets by $\boldsymbol{c}^0$. We refer to $\boldsymbol{c}^0$ as the *status-quo* allocation. Define $\mathcal{C}$ to be a set of feasible consumption allocations. For most of the paper, we focus on the case where $\mathcal{C}$ is the set of Pareto-efficient consumption allocations given complete markets. Although in principle, and in some examples, $\mathcal{C}$ can be a second-best feasible set (given other distortions).

We define misallocation as the distance to the frontier measured using the aggregate consumption-equivalent variation of Baqaee and Burstein (2025b).

Definition 1. *Misallocation* relative to the frontier of $\mathcal{C}$ is measured by the maximum contraction of $\mathcal{C}$ such that every agent can be kept at least indifferent to the status-quo allocation. Formally,

$$A(\boldsymbol{c}^0, \mathcal{C}) \equiv \max \left\{ \phi \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \phi^{-1}\mathcal{C} \text{ and } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\}. \quad (1)$$

The cardinal value of A is interpretable. For concreteness, say, $A = 1.01$. This means that it is possible to make everyone at least as well off as in the status-quo and discard 1% of every good (more precisely, $1 - 1/A$% of every good). Therefore, A is a measure

[4]Some papers, such as Buera et al. (2011), Midrigan and Xu (2014), Moll (2014), and Bigio and La'O (2016), study misallocation from financial frictions on firms. It would be interesting, but beyond the scope of this paper, to combine those frictions with the ones on households that we focus on. Proposition 1 can be applied to do this.

of the economic waste in the equilibrium allocation. Importantly, in calculating A, we do not need to take a stance on which agents would or should receive these extra resources if one were to complete markets. That is, this aggregate efficiency measure is silent on redistributions.

To calculate $A(\boldsymbol{c}^0, \mathcal{C})$, it is useful to define the consumption-equivalent variation at the individual level.

Definition 2. The *consumption-equivalent variation* for agent h, denoted by $\tilde{u}_h(\boldsymbol{c}_h)$, is the solution to

$$u_h\left(\frac{\boldsymbol{c}_h}{\tilde{u}_h}\right) = u_h(\boldsymbol{c}_h^0).$$

In words, $\tilde{u}_h(\boldsymbol{c}_h)$ is the amount the consumption stream $\boldsymbol{c}_h$ has to be scaled to make it exactly as desirable to agent h as their status-quo allocation. By construction, $\tilde{u}(\boldsymbol{c}_h)$ is homogeneous of degree one.

We use Theorem 1 from Baqaee and Burstein (2025b) to characterize A as the solution to a representative-agent planning problem.

Proposition 1 (Calculating misallocation via a planning problem)**.** *Misallocation relative to $\mathcal{C}$ is given by*

$$A(\boldsymbol{c}^0, \mathcal{C}) = \max_{\boldsymbol{c} \in \mathcal{C}} \left[\min\left\{\tilde{u}_1(\boldsymbol{c}_1), \cdots, \tilde{u}_H(\boldsymbol{c}_H)\right\}\right]. \tag{2}$$

This proposition converts the problem of calculating misallocation into one of maximizing utility for a fictional agent. This fictional agent has Leontief preferences over the consumption-equivalent variations of the households relative to status-quo.

The function $\min_{h \in H}\{\tilde{u}_h(\boldsymbol{c}_h)\}$ in Proposition 1 is not a Rawlsian social welfare function. First, this function depends on the minimum certainty-equivalent variation relative to the status-quo, whereas a Rawlsian social welfare function depends on the minimum level of utility. Second, whereas the allocation that maximizes a social welfare function is the optimal allocation, the allocation that maximizes (2) has no such interpretation. Rather, it is simply an analytical device for measuring misallocation A. Indeed, we do not ever define *the* optimal allocation on the consumption possibility set $\mathcal{C}$.

We use Proposition 1 throughout the paper to study misallocation due to incompleteness of financial markets in closed and open economies.

3 Baseline Closed Economy

We begin with a simple economy where all agents consume the same consumption good every period. This nests one-sector models like Bewley (1972) and Huggett (1993), but it also accommodates multi-sector versions of these models with input-output linkages, as long as every household's static consumption aggregator is the same and factors in each period are inelastically supplied (i.e. no physical capital accumulation).

We begin this section by setting up the environment. We then provide exact and approximation characterizations of misallocation due to incomplete consumption-smoothing. In the baseline, we abstract from labor-leisure choice, capital accumulation, production inefficiencies, and non-traded goods. We extend our results to allow for these additional ingredients in subsequent sections.

3.1 Environment

Each household, indexed by h, has intertemporal preferences over state-contingent consumption streams $\boldsymbol{c}_h$ represented by the utility function

$$u(\boldsymbol{c}_h) = \frac{1}{1 - 1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}. \tag{3}$$

Here, $c_{ht}(s)$ denotes consumption of household h at time t in state s (which may be a homothetic bundle of many goods), the discount factor is $\beta < 1$, and $\eta > 0$ is the elasticity of intertemporal substitution (EIS). The probability of state s is denoted by $\pi(s)$, where each state s indexes a sample path of shocks.

In every period, t, of every state s, production takes place. We assume that production in each period is statically efficient.[5] Since we abstract from capital accumulation, endogenous labor choice, and all households value the same consumption good, we do not need to specify the exact nature of the production structure. (There could be many producers, heterogeneous goods, input-output linkages, and arbitrary production functions and returns to scale, with any pattern of technological shocks). We can simply denote by $y_t(s)$ the aggregate quantity of the consumption good in period t and state s, knowing that $y_t(s)$ remains unchanged across points at the Pareto frontier.

The resource constraint for the consumption good is therefore

$$\sum_h c_{ht}(s) = y_t(s).$$

Denote the consumption possibility set of the economy to be the set of feasible consump-

[5]Production is neoclassical and all producers set price equal to marginal cost. This implies that, holding fixed consumption allocations in every other period and state, and focusing only on a single period and state, it is not possible to make one agent better off without making someone worse off.

tion allocations:

$$\mathcal{C} = \left\{ \boldsymbol{c} : \sum_h c_{ht}(s) \leq y_t(s), \text{ for every } t \text{ and } s \right\}.$$

3.2 Exact Characterization

First define the certainty-equivalent variation of a consumption process $\boldsymbol{c}_h$ to be the function $CE(\boldsymbol{c}_h)$ that solves

$$u(\mathbf{c}_h) = u(\mathbf{1}\, CE).$$

This can be rewritten as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{ht}^{1-\frac{1}{\eta}} = (1-\beta)^{-1} CE^{1-\frac{1}{\eta}},$$

where $\mathbb{E}_0$ denotes the time-zero expectation. In words, $CE(\boldsymbol{c}_h)$ is the amount of consumption the agent needs in every date and state to be indifferent to the consumption stream $\boldsymbol{c}_h$. Proposition 1 gives a simple characterization of misallocation in terms of certainty-equivalents.

Proposition 2 (Misallocation in economies with common consumption good)**.** *For the baseline closed-economy model, misallocation is*

$$A = \frac{CE(\sum_h \boldsymbol{c}_h^0)}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{4}$$

In words, A is the certainty-equivalent of the aggregate consumption process relative to the sum of the consumption equivalents of each households' consumption process.

To derive Proposition 2, observe that consumption allocations on the Pareto frontier must satisfy $c_{ht}(s) = \alpha_h y_t(s)$ for some household-specific $\alpha_h \geq 0$ with $\sum_h \alpha_h = 1$. Substituting this into (2) in Proposition 1, and manipulating yields

$$A = \max_{\boldsymbol{c} \in \mathcal{C}} \min_h \left\{ \tilde{u}_h(\boldsymbol{c}_h) \right\} = \max_{\substack{\boldsymbol{\alpha} \in \mathbb{R}^H \\ \alpha_h \geq 0,\, \sum_h \alpha_h = 1}} \min_h \left\{ \alpha_h\, \tilde{u}_h(\mathbf{y}) \right\}.$$

We know that the utility-maximizing problem above must satisfy $\alpha_h \tilde{u}_h(\boldsymbol{y}) = \alpha_{h'} \tilde{u}_{h'}(\boldsymbol{y})$. Given the functional form of preferences, the consumption-equivalent variation function is

$$\tilde{u}_h(\boldsymbol{c}_h) = \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}} = \frac{CE(\boldsymbol{c}_h)}{CE(\boldsymbol{c}_h^0)}.$$

Combining these equations and rearranging yields

$$\alpha_h = \frac{CE(\boldsymbol{c}_h^0)}{\sum_{h'} CE(\boldsymbol{c}_{h'}^0)}.$$

Substituting this into $A = \alpha_h \tilde{u}_h(\boldsymbol{y})$ and using $\sum_h \boldsymbol{c}_h^0 = \boldsymbol{y}$, yields (4).

Equation (4) is similar to the definition of aggregate efficiency provided by Benabou (2002), except here it is a result — given our general notion of misallocation — rather than a definition. The fact that we have a general definition of misallocation makes it straightforward to extend (4) to more complex environments with labor-leisure, capital accumulation, production distortions, and differences in consumption baskets across agents.

Contrast to the veil-of-ignorance. We contrast A to a popular measure of aggregate welfare in the literature: the veil-of-ignorance social welfare function. Since all households in the baseline model have common preferences, this measure is unambiguous to define.[6] Whereas our measure is designed to ignore inequality, a primary motivation for the veil-of-ignorance measure is to capture inequality-aversion using risk-preferences. Define the veil-of-ignorance certainty-equivalent of a consumption allocation $\boldsymbol{c}$ by:

$$u(\mathbf{1}CE^{VOI}) = \sum_{h \in H} \frac{1}{H} u(\boldsymbol{c}_h).$$

In words, CE^{VOI} is the certainty-equivalent of a population-weighted lottery of the consumption allocation of each agent. We can calculate the ratio of the value of first-best, according to CE^{VOI}, relative to the value of the status-quo allocation:

$$A^{VOI} = \frac{\max_{\boldsymbol{c} \in \mathcal{C}} CE^{VOI}(\boldsymbol{c})}{CE^{VOI}(\boldsymbol{c}^0)} = \frac{CE(\sum_h \boldsymbol{c}_h^0)}{\left(\sum_h \left(CE(\boldsymbol{c}_h^0)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}, \tag{5}$$

where the numerator uses the fact that the first-best allocation with this social welfare function would split aggregate consumption uniformly across all agents. (See Appendix A for a derivation). Comparing (4) to (5), we see that the veil-of-ignorance measure, A^{VOI}, uses risk-preferences to discipline inequality-aversion, whereas A does not feature

[6] See Eden (2020) for a detailed discussion of the veil-of-ignorance approach to quantifying social welfare, and how it must be adapted in the presence of heterogeneous preferences.

this effect.[7,8]

The simplest way to see the difference between A and A^{VOI} is to consider a Pareto efficient, but unequal, consumption allocation. Suppose that in the status-quo, each household consumes a constant fraction, α_h, of aggregate consumption. In this case, $CE(\boldsymbol{c}_h) = \alpha_h CE(\boldsymbol{y})$ and, using Proposition 2, it follows that misallocation is zero. However, as there is inequality in the status-quo (α_h varies across households), $A^{VOI} > 1$ (unless $\eta = \infty$ and households are risk-neutral).

3.3 Approximate Characterization

We now provide a second-order approximation of misallocation. This second-order approximation serves two purposes. First, it provides useful intuition about how parameters affect misallocation. Second, and more importantly, it identifies some approximate sufficient statistics that can be taken to the data without assuming complete knowledge of the entire distribution of consumption allocations and productivity shifters.

To derive this approximation, we introduce the concept of an equilibrium with wedges. We decentralize the status-quo allocation using household-specific state- and date-contingent consumption taxes. Denote the *wedge*, which is an implicit tax, on the consumption of household h at time t in state s by $\mu_{ht}(s)$.[9] The intertemporal budget constraint for household h, in the decentralization with wedges, is

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)p_t(s)c_{ht}(s) \leq I_h,$$

where $q_t(s)$ is the price of an Arrow security, $p_t(s)$ is the price of the consumption good h

[7] In other words, the veil-of-ignorance measures sets the Atkinson (1970) parameter for inequality-aversion equal to the coefficient of relative risk aversion.

[8] In this example, the veil of ignorance can also be thought of as a utilitarian social welfare function (sum of utilities) with a particular cardinalization of the utility function. However, with other cardinalizations of the same preferences, the sum of utilities will have different implications. This is because "the" utilitarian welfare function is not well-defined as it depends on how each utility function is cardinalized. For example, if instead of using the functional-form of u, defined above, we cardinalize the same preferences using the monotone nonlinear transformation $(u(\boldsymbol{c}_h)(1-1/\eta))^{\frac{\eta}{\eta-1}} = \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{ht}^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$, then the utilitarian social welfare function would have zero inequality aversion. We note that our measure of misallocation, A, is invariant to monotone transformations of utility.

[9] Because we assume that production is statically efficient, there are no production wedges. Propositions 1 through 4 still apply if there are production wedges that are held constant when completing markets. This is because households insuring each other would have no effect on the production of the aggregate consumption good in any date or state. With production wedges (due to, e.g., firms' borrowing constraints, market power, sticky prices, etc.), there is a different notion of misallocation where we complete markets and remove production wedges. Proposition 1 applies for this notion of misallocation, but Proposition 2 and Proposition 4 would need to be modified.

in state s at time t not including the wedge, and I_h is initial wealth.

We now define general equilibrium with wedges.

Definition 3 (Equilibrium with Wedges)**.** A general equilibrium with wedges is the collection of prices and quantities such that: (1) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and wealth as given; (2) every producer chooses inputs to minimize costs, taking input prices as given, and sets its price equal to marginal cost; (3) resource constraints are satisfied.

Denote household h's share of consumption in state s and date t in the status-quo allocation by

$$\omega_{ht}(s) = \frac{c^0_{ht}(s)}{\sum_{h'} c^0_{h't}(s)}.$$

The following proposition shows that the status-quo allocation, $\boldsymbol{c}^0$, can be decentralized using some pattern of household-state-date-specific consumption wedges.

Proposition 3 (Decentralization with Wedges)**.** *Consider some equilibrium status-quo allocation $\boldsymbol{c}^0$. Then, setting*

$$\log \mu_{ht}(s) = -\frac{1}{\eta} \left[\log \omega_{ht}(s) - \log \omega_{h0}(s)\right] \tag{6}$$

implies that $\mathbf{c}^0$ is a general equilibrium with those wedges. This equilibrium is supported by some lump-sum transfers across households.

We do not need to specify the lump-sum transfers explicitly. For our purposes, all that matters is that there exist an equilibrium with the wedges in (16), with appropriate transfers, that can support $\boldsymbol{c}^0$ as an equilibrium allocation.[10]

Denote deviations of aggregate output from some constant values by $\Delta \log y$. That is, for time t and state s, define

$$\Delta \log y_t(s) = \log \frac{y_t(s)}{\overline{y}},$$

where $\overline{y}$ is some constant (over time and states) level of output. Since production is invariant to changes in wedges $\Delta \log y$ is also invariant to changes in wedges.

The following proposition approximates misallocation losses in terms of Harberger deadweight loss triangles.

[10]The wedges in (16) are not the only ones that can decentralize $\boldsymbol{c}^0$. For example, if wedges $\mu_{ht}(s)$ are all raised by the same proportion for a given household h in every t and s, this can still decentralize $\boldsymbol{c}^0$, but the lump-sum transfers that support the status-quo allocations would change. That is, Proposition 3 is a particular normalization of wedges that can decentralize the status-quo.

Proposition 4 (Approximate Misallocation for Baseline)**.** *Consider the special case where there is a common consumption good in every period and state. Misallocation is approximately equal to*

$$\log A(\boldsymbol{c}^0, \mathcal{C}) \approx \frac{1}{2}\frac{1}{\eta}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\sum_h \omega_{h0}\left(\log\omega_{ht}(s) - \log\omega_{h0}\right)\left(\log\omega_{ht}(s) - \log\bar{\omega}_h\right)\right],$$

where r is the risk-free rate in the first period and $\log\bar{\omega}_h = r\mathbb{E}_0\left[\sum_{t=0}^{\infty}(1+r)^{-(t+1)}\log\omega_{ht}(s)|h\right]$ *is the conditional expected discounted consumption share of household h over states of nature s. The approximation error is order* $\log\mu^3$ *and* $\log\mu^2\Delta\log y$*.*

Proposition 4 is an application of Proposition 8 from Baqaee and Burstein (2025b) to this environment. The intuition for the expression above is exactly the same as the traditional deadweight-loss (Harberger) triangle logic. The height of the triangle is measured by the wedge, $\log\mu_{ht}(s) = -\frac{1}{\eta}[\log\omega_{ht}(s) - \log\omega_{h0}]$ from (6). The smaller is the EIS, the larger is the implied wedge necessary to reach the same distorted allocation. The base of the triangle is the gap between household h's share of consumption in state s and time t, $\omega_{ht}(s)$, and its expected share of consumption in net-present value terms $\bar{\omega}_h$. The area of the triangle divides the product of the base and the height by two. We then sum up deadweight loss triangles over households using net-present-value expenditure weights.

It is important to stress that in our context there is no single "first-best" allocation. That is, there is no implication that *the* first-best allocation is the one that sets each consumption share equal to its expected net present value in the status-quo. In the absence of wedges, there are many different allocations that are all Pareto dominant to the status-quo, and our measure takes no stance on which one is socially desirable. Instead, this allocation is used to form the Harberger triangles since it is the one chosen by the fictional agent in Proposition 1.

We apply these results in a calibrated version of the Bewley (1972) model.

3.4 Quantitative Example

We measure misallocation from market incompleteness using off-the-shelf calibration of Bewley (1972). We use this quantitative example to show how the costs of market incompleteness change as a function of parameters like idiosyncratic risk, borrowing constraints, and public debt. We also use this example to test the performance of our second-order approximation and its finite sample properties.

Model. There is a unit mass of households, indexed by $h \in [0,1]$, with preferences as in (3) subject to a per-period budget constraint

$$c_{ht} + a_{ht+1} = (1-\tau)z_{ht} + (1+r_t)a_{ht},$$

where a_{ht} is the quantity of a risk free bond held by h, z_{ht} is risky labor productivity, and τ is the tax rate. Each household faces a borrowing constraint

$$a \geq -\underline{a}.$$

Productivity evolves according to

$$\log z_{ht} = \rho \log z_{ht-1} + \sigma \epsilon_{ht},$$

where ϵ_{ht} is an idiosyncratic Gaussian disturbance. The government has issued B risk free bonds and runs a balanced budget every period using labor income taxes, so that

$$rB = \tau.$$

Market clearing condition for goods and bonds is

$$\int_0^1 c_{ht} dh = \int_0^1 z_{ht} dh = y = 1, \qquad \text{and} \qquad \int_0^1 a_{ht} dh = B.$$

The consumption possibility set is

$$\mathcal{C} = \left\{ \boldsymbol{c} : \int c_{ht}(s) dh \leq 1, \text{ for every } t \text{ and } s \right\}.$$

Parameterization. We use a quarterly calibration. We set quarterly persistence of log income to be $\rho = 0.975$ with standard deviation 0.16 to match estimates of the quarterly persistence and the cross-sectional standard deviation of the persistent component of log income in the United States.[11] We set the borrowing limit to be -5, so households can borrow at most 5 times their quarterly income. We set the annual risk-free $r = 5\%$ and the EIS $\eta = 0.5$. Finally, we set $B = 5.6$ — so that total bonds outstanding relative to quarterly output is 560% (or 140% of annual GDP).

[11] See Rognlie (2024). We target a cross-sectional standard deviation of log income equal to 0.7, which means that the standard deviation to the innovations must be $\sigma = 0.7 \times \sqrt{1 - 0.975^2}$.

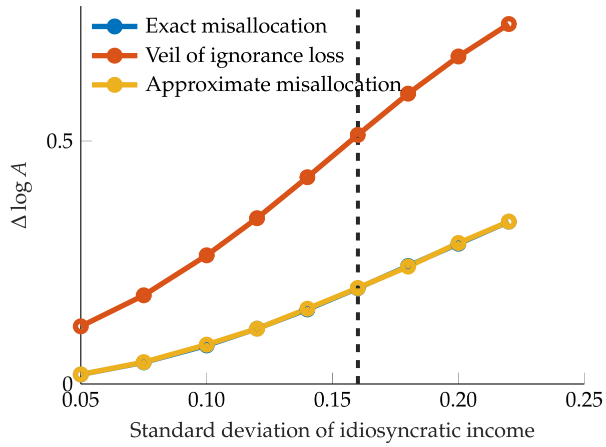


Figure 1: Losses as a function of idiosyncratic income risk. Dashed line is benchmark. Exact and approximate are visually identical.

Results. Figure 1 plots the extent of misallocation, calculated using Proposition 2, and compares it to the second-order approximation from Proposition 4, using the steady-state invariant distribution for the status-quo allocation. The approximation performs well and, as expected, becomes exact as $\sigma \to 0$. The benchmark values are indicated by the dashed black line, where misallocation is $\log A \approx 0.20$. This means that if agents perfectly insure each other and everyone is kept indifferent to their status-quo allocation, then there is 20 log points (or $1 - \exp(-0.20) \approx 18\%$ percentage points) of output left over to be split across agents as desired.

We also compare the status-quo allocation to the first-best allocation under the veil-of-ignorance criteria, using equation (5). The veil-of-ignorance measure, which penalizes inequality across agents analogously to uncertainty for each agent, assigns more than double the losses to the status-quo. That is, behind the veil, households would be prepared to give up 51 log points of aggregate consumption if they could equalize consumption across dates, states, and the cross-sectional population.

Figure 2 plots the quality of the second-order approximation against the exact misallocation losses as the sample length used in the approximation increases. The second-order approximation stabilizes after about 100 quarters (25 years), but suffers from some small sample bias when the number of quarters is significantly shorter than that. With a truncated sample, the second-order approximation underestimates the extent of misallocation because the Harberger triangles in the first few periods are, by construction, equal to zero.[12]

Figure 3 plots misallocation, relative to the status-quo in the invariant distribution,

[12]The second-order approximation is much less sensitive to the number of households in the sample.

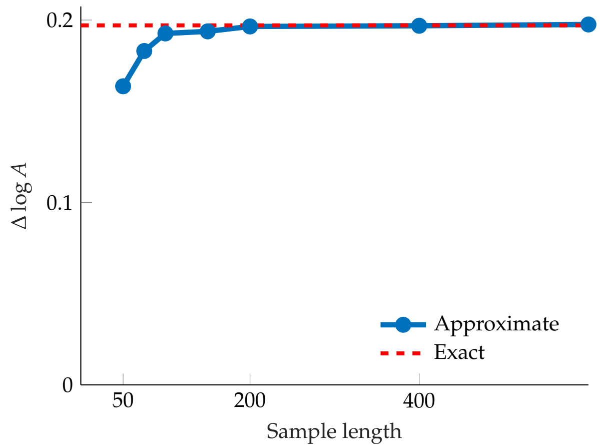
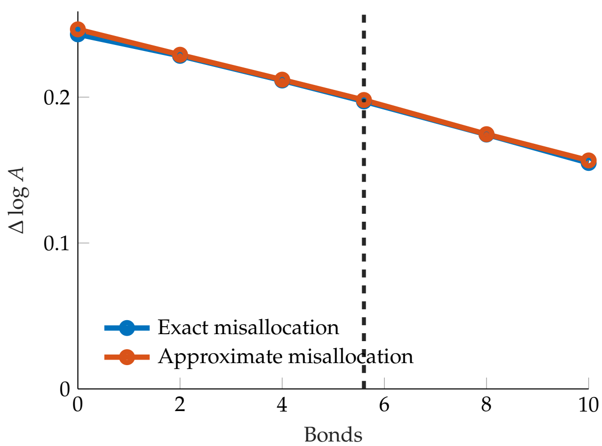
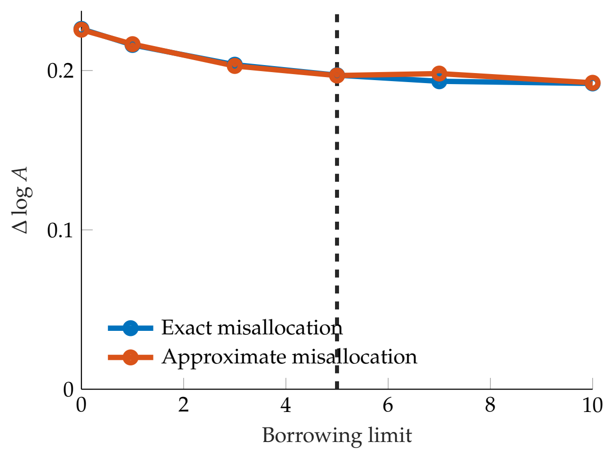


Figure 2: Quality of approximation for benchmark calibration



(a) Bond supply



(b) Borrowing limit

Figure 3: Misallocation as a function of parameters (dashed line is benchmark).

as a function of the aggregate supply of bonds and the borrowing limit. In both cases, misallocation falls mildly as the borrowing limit is increased and as the supply of bonds rises. The approximation continues to perform well. It is important to note that, as we change parameters, the invariant distribution changes — hence, in plotting these curves, we are not holding fixed the status-quo allocation. So, for example, if $\log A$ is 0.20 in the baseline, but $\log A = 0.15$ when aggregate bond supply is doubled, this means that the distance to the efficient frontier from the corresponding status-quo is 0.20 and 0.15, but this does not imply that misallocation falls by $0.20 - 0.15 \approx 0.05$ when bond supply is doubled. To answer this question (how much misallocation changes when we double bond supply), using Definition 1, we would have to specify the distributive tools available to society, which would give rise to a consumption possibility frontier $\mathcal{C}(B)$, hold fixed the status-quo allocation at $B = 5.6$, and solve the problem in Proposition 1.

4 Extensions with Leisure and Capital

In this section, we quantify misallocation allowing for endogenous labor-leisure choice and capital accumulation.

4.1 Extension with Labor-Leisure Choice

Suppose households have preferences over consumption goods and leisure:

$$u(\boldsymbol{c}_h, \boldsymbol{l}_h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(c_{ht}(s), l_{ht}(s)).$$

Each household has a unit endowment of time, which they devote either to leisure, $l_{ht}(s)$, or to work, $1 - l_{ht}(s)$. For simplicity, assume production converts labor into consumption good linearly. Hence, the resource constraint for consumption at date t in state s is

$$\sum_h c_{ht}(s) = y_t(s) = \sum_h z_{ht}(s) \left(1 - l_{ht}(s)\right), \tag{7}$$

where $z_{ht}(s)$ is the idiosyncratic productivity of household h in date t and state s. In the baseline model, leisure $l_{ht}(s)$ is normalized to be zero.

The consumption possibility set $\mathcal{C}(z)$ now consists of all consumption and leisure processes that are consistent with the resource constraint above. Elements of $\mathcal{C}(z)$ are consumption and leisure processes for each household, where each agent's leisure process must be in the unit interval, $l_{ht}(s) \in [0, 1]$, and consumption processes must satisfy (7).

We use the same definition of misallocation as in Definition 1. That is, by how much can we shrink the Pareto efficient consumption possibility set and still keep every agent at least indifferent to the status-quo. The scalar A measures how much of every good, including leisure, in every state and date is left over after every agent has been made indifferent. Equivalently, in this model, this is a measure of the amount of the time endowment that is being wasted.[13]

We modify the definition of consumption-equivalents, in Definition 2, to allow for leisure. Namely, define $\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h)$ implicity by the equation

$$u(\boldsymbol{c}_h^0, \boldsymbol{l}_h^0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v\left(\frac{c_{ht}(s)}{\tilde{u}}, \frac{l_{ht}(s)}{\tilde{u}}\right). \tag{8}$$

[13] Reducing leisure here does not mean we increase work — a proportional reduction in consumption and leisure is equivalent to reducing every household's time endowment by some fraction.

With this definition of $\tilde{u}_h$, Proposition 1 applies and can be used to calculate misallocation without change.

We provide some examples of $\tilde{u}_h$ for popular functional forms below.

Example 1 (Homothetic preferences). Suppose the intratemporal utility function, v, is

$$v(c_h, l_h) = \frac{1}{1 - 1/\eta} \left[c_h^{\gamma} l_h^{1-\gamma} \right]^{1-\frac{1}{\eta}}. \tag{9}$$

Then $\tilde{u}_h$ is

$$\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h) = \left[\frac{u(\boldsymbol{c}_h, \boldsymbol{l}_h)}{u(\boldsymbol{c}_h^0, \boldsymbol{l}_h^0)} \right]^{\frac{1}{1-1/\eta}}.$$

Example 2 (MaCurdy preferences). A popular class of intratemporal preferences, due to MaCurdy (1981), is

$$v(c_h, l_h) = \frac{1}{1 - 1/\eta} c_h^{1-\frac{1}{\eta}} + \phi_0 l_h^{1-\frac{1}{\phi}}. \tag{10}$$

In this case, $\tilde{u}(\boldsymbol{c}_h, \boldsymbol{l}_h)$ is implicitly defined by the equation

$$\tilde{u}_h = \left(\frac{1}{u_h^0} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_{ht}(s)^{1-\frac{1}{\eta}}}{1 - 1/\eta} + \tilde{u}_h^{\frac{1}{\phi} - \frac{1}{\eta}} \phi_0 l_{ht}(s)^{1-\frac{1}{\phi}} \right) \right] \right)^{\frac{\eta}{\eta-1}}.$$

Comparison with Benabou (2002) measure of aggregate efficiency. As mentioned earlier, our definition of efficiency is different to a frequently used alternative in the literature, like Benabou (2002), Floden (2001) and Boar and Midrigan (2022). To compare two allocations, the literature following Benabou (2002) defines the consumption-equivalent of a consumption process $\boldsymbol{c}_h$ and a leisure process $\boldsymbol{l}_h$ to be the function $CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$ that solves

$$u(\mathbf{1}CE, \mathbf{1}\bar{l}) = u(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

where $\bar{l}$ is some fixed level of leisure (e.g. average leisure). The efficiency of an allocation is defined by $\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$.

One way to see the difference between A and this measure is to note that this measure assigns different values to different allocations on the Pareto frontier. Suppose that preferences take the standard functional form in Example 1. In this case, this measure of efficiency can be written as

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \left(u(\boldsymbol{c}_h, \boldsymbol{l}_h) \right)^{\frac{\eta}{(\eta-1)\gamma}},$$

where the constant depends on β, η, γ, and $\bar{l}$. Consider the simple case where labor productivity is equal to one in every date and state for every agent. Then we can show that for every allocation $(\boldsymbol{c}, \boldsymbol{l})$ on the Pareto frontier, we can write

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \alpha_h^{\frac{1}{\gamma}}, \tag{11}$$

for some numbers on the unit simplex, $\{\boldsymbol{\alpha} \geq 0 : \sum_h \alpha_h = 1\}$. (See Appendix A for a derivation). In this expression, $\boldsymbol{\alpha}$ are coordinates of the Pareto frontier — i.e. they can be interpreted like Pareto weights — the higher is α_h for household h, the higher is the utility of that agent. Clearly, unless $\gamma = 1$, and there is no labor-leisure choice, (11) assigns different values to different points on the Pareto-frontier. Surprisingly, this measure assigns (weakly) higher values to more unequal Pareto weights since $\gamma \leq 1$. Indeed, by continuity, this shows that there are Pareto-inefficient allocations that receive a higher value according to this measure than alternative Pareto efficient allocations with less inequality. Therefore, once we have labor-leisure choice, this measure is not neutral with respect to pure redistributions (and indeed, prefers inequality). In contrast, our measure detects zero misallocation (i.e. $A = 1$) for any status-quo on the Pareto-efficient frontier.

Quantitative example with labor-leisure choice. Assume preferences take the MaCurdy (1981) form in (10). We calibrate the EIS and the Frisch elasticity of labor supply to equal $\eta = \phi = 0.5$, and set ϕ_0 so that leisure is, on average, equal to 40% of the time endowment. We re-calibrate the discount factor and the standard deviation of the productivity shocks to hit the same interest rate $r = 0.05$ and the cross-sectional variance of consumption as in the baseline calibration above. The remaining parameters the same as in the calibration in Section 3.4.

The losses are shown in Figure 4 as a function of idiosyncratic risk σ calculated using Proposition 1. Even as σ goes to zero, the losses are non-zero since there is a tax on labor. However, this effect is small because the baseline tax rate is small. Misallocation for the baseline parameters is 21 log points, which is very similar to the baseline model. Furthermore, the second-order approximation in Proposition 4 continues to perform very well, even though it is derived for a model without labor-leisure choice.

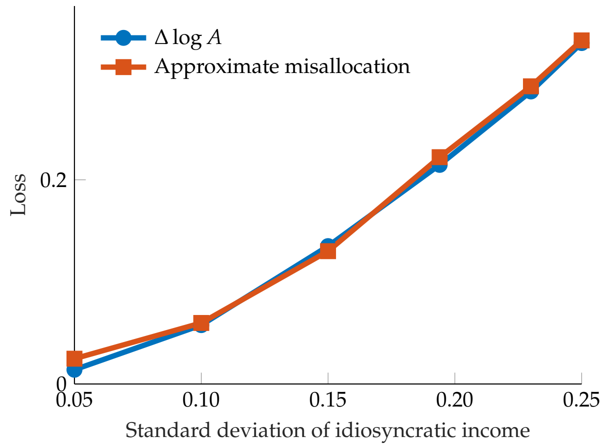


Figure 4: Losses with labor-leisure choice as a function of idiosyncratic income risk. Dashed line is σ in original calibration.

4.2 Extension with Capital Accumulation

We now consider an extension with capital accumulation along the lines of Aiyagari (1994). For simplicity, we assume labor is inelastically supplied. Aggregate output in each period and state is

$$y_t(s) = z_t(s)k_t(s)^{\alpha}, \tag{12}$$

where we imposed the requirement that the aggregate endowment of labor is equal to one. Capital accumulation satisfies

$$k_{t+1}(s) = (1-\delta)k_t(s) + x_t(s), \tag{13}$$

where $x_t(s)$ is investment. Denote the initial capital stock by k_0. The aggregate resource constraint for output is

$$y_t(s) = c_t(s) + x_t(s) = \sum_{h \in H} c_{ht}(s) + x_t(s). \tag{14}$$

Proposition 1 continues to hold with capital accumulation. Specifically, this means

$$A = \max_{\boldsymbol{c}} \left(\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} \right)^{\frac{\eta}{\eta-1}},$$

subject to (12), (13), (14), and $\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} = \frac{u(\boldsymbol{c}_{h'})}{u(\boldsymbol{c}_{h'}^0)}$ for every $h' \in H$ and some initial capital stock k_0. This implies the following.

Proposition 5 (Misallocation with capital accumulation). *Let $c_t^*(s)$ be the optimal (aggregate) consumption choice in period t and state s of a representative agent in the neoclassical growth model with initial capital stock k_0.*[14] *Then*

$$A(\boldsymbol{c}^0, \mathcal{C}) = \frac{CE(\{c_t^*(s)\}_{t,s})}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{15}$$

That is, A is equal to the ratio of the certainty-equivalent of the aggregate consumption process from a neoclassical growth model given the initial aggregate capital stock k_0 relative to the sum of the certainty-equivalent of each agent in the status-quo. Compared to Proposition 2, this means calculating misallocation now has one more step: solving for the transition dynamics in a standard neoclassical growth model given initial capital stock k_0.

Quantitative example with capital accumulation. Below, we describe the equilibrium that determines the status-quo. Household preferences are the same as before, but the per-period budget constraint is now

$$c_{ht} + x_{ht} = z_{ht} + R_t k_{ht},$$

where x_{ht} is investment by household h and R_t is the rental price of capital. Each household faces a borrowing constraint, so $k_{ht} \geq 0$. The labor income process is the same as before. Each household's capital stock follows

$$k_{ht+1} = (1-\delta)k_{ht} + x_{ht}$$

The aggregate resource constraints are as in (12)-(14). Aggregate output is produced by a perfectly competitive representative firm that hires labor and capital on competitive spot markets. The rental rate of capital clears the capital market: $\int_0^1 k_{ht} dh = k_t$.

We calibrate capital's share of GDP to be $\alpha = 0.35$, the depreciation rate δ to match a capital to (quarterly) output ratio of 14, and the discount factor β to match a steady-state annual interest rate of $r = 0.05$. We set the standard deviation of idiosyncratic income risk, σ, to match the same cross-sectional variance of consumption as in the baseline model without capital.

---

[14] In particular, c_t^* in state s solves $(c_t^*)^{-\frac{1}{\eta}} = \beta \mathbb{E}_t \left[(\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta) \left(c_{t+1}^*\right)^{-\frac{1}{\eta}} \right]$ and $c_t^* = z_t k_t^\alpha + (1-\delta)k_t - k_{t+1}$, given some initial k_0 and the standard transversality condition. The vector $\{c_t^*(s)\}_{t,s}$ is the stream of aggregate consumption across states and dates.

We calculate distance to the frontier, $\Delta \log A$, using Proposition 5. The results are plotted in Figure 5 as a function of idiosyncratic risk. Misallocation at the benchmark values is 19 log points, similar to the calibration of the baseline model without capital.

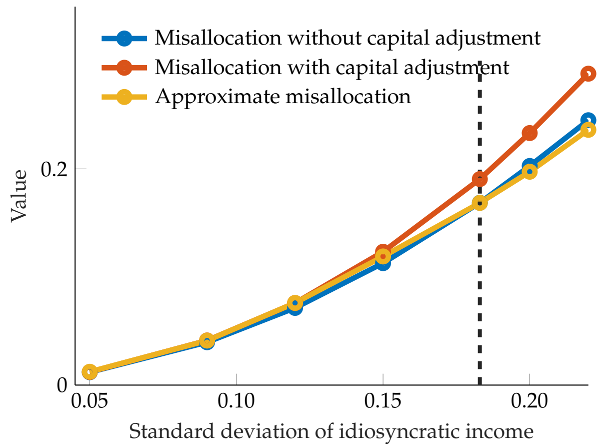


Figure 5: Losses as a function of idiosyncratic income risk. Dashed line is original calibration.

As mentioned above, due to the precautionary motive, households overinvest in capital relative to first-best allocations. To quantify the importance of overinvestment for misallocation, we compare the benchmark distance to the frontier with one where we impose that the capital stock remains constant. That is, we use the consumption possibility set that keeps the aggregate capital stock constant and equal to its status-quo value in Definition 1. This means we can use we use equation (4) rather than (15). As expected, the distance to the frontier holding the capital stock constant is smaller than the distance to the frontier allowing the capital stock to adjust — and the gap grows as idiosyncratic risk, and the strength of the precautionary motive, rise. At the benchmark values, allowing for the aggregate capital stock to adjust raises the distance from the efficient frontier from 17 log points (holding capital fixed) to 19 log points. For completeness, Figure 5 also reports the second-order approximation in Proposition 4, which continues to perform very well as an approximation to the case where the aggregate capital stock is held constant.

5 Incompleteness of International Markets

In this section, we quantify misallocation allowing for international trade in goods and assets between countries. To do so, we augment the baseline model of Section 3 with multiple household types with different consumption baskets (i.e. home bias).

5.1 Environment with Heterogeneous Consumption Baskets

To calculate misallocation from financial market incompleteness, we must define the Pareto frontier. In the baseline closed economy model in Section 3, the description of the Pareto frontier is very simple because all agents consume the same good and the aggregate quantity of the final good is unaffected by financial market incompleteness. When we relax this assumption, defining the Pareto frontier requires spelling out the production structure in more detail.

Preferences. Household h have the same intertemporal preferences as before:

$$u_h(\boldsymbol{c}_h) = \frac{1}{1-1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}.$$

Here, $c_{ht}(s)$ denotes consumption of household h at time t in state s. The crucial difference, relative to the baseline model in Section 3, is that each household h can consume a potentially different consumption good. This allows us to incorporate non-traded goods into the model (e.g. households in Germany consume a different bundle than households in China).

Technologies. In every period, t, of every state s, there is a set F of primary factor endowments and N of goods. The factors are inelastically supplied and owned by households, and used by producers in the same period (i.e. labor from t cannot be used by producers in $t+1$). Producer $i \in N$ has a CES production function that uses intermediate inputs and primary factor endowments with elasticity of substitution θ_i. Hence, the production function of i is

$$y_{it}(s) = z_{it}(s) \left(\sum_{j \in N} \alpha_{ij} \left(y_{ijt}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{f \in F} \alpha_{if} \left(l_{ift}(s) \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where z_i is a Hicks-neutral productivity shifter, and $y_{ijt}(s)$ and $l_{ift}(s)$ are intermediate input j and factor input f. The scalars α_{ij} and α_{if} are share parameters that affect expenditures shares across inputs for each i.

Note that this structure is general enough to accommodate any pattern of nested-CES producers. This model also accommodates any Armington-style model of trade, where productivity shifters, $z_{it}(s)$, for specialized intermediaries of imports and exports represent iceberg costs of trade. Without loss of generality, we treat the consumption bundle of each household $c_{ht}(s)$ as-if it is produced by one of the goods producers and order the consumption goods first among the commodities in N.[15]

Resource constraints. The resource constraints of the economy are as follows: consumption of good h equals its production,

$$y_{ht}(s) = c_{ht}(s), \qquad (h \in H)$$

use of intermediate input i equals its production,

$$\sum_{j \in N} y_{jit}(s) = y_{it}(s), \qquad (i \in N, i \notin H)$$

use of factor f equals its endowment

$$\sum_{j \in N} l_{jft}(s) = z_{ft}(s), \qquad (f \in F).$$

Given these technologies and resource constraints, denote the dynamic consumption possibility set of the economy by $\mathcal{C}(\boldsymbol{z})$, where $\boldsymbol{z}$ is the vector of all state-contingent technology processes. That is, each element of $\mathcal{C}(\boldsymbol{z})$ is a vector of state-contingent consumptions streams for every household. By the second welfare theorem, $\mathcal{C}(\boldsymbol{z})$ is the set of perfectly competitive equilibria, with complete markets, given unrestricted lump-sum transfers. The model in Section 3 is the special case of this model where all households consume the same good.[16]

5.2 Approximate Characterization

Proposition 1 applies to this economy, but computing misallocation this way requires fully specifying the Pareto frontier, including, for example, the productivity processes. Instead, we provide a (second-order) approximation for misallocation that does not require as much information to implement. We do this by using wedges to decentralize the status-quo allocation, as in Section 3.

[15]For more details, see the discussion of the "standard-form" representation of nested-CES economies in Baqaee and Farhi (2019).

[16]We also assume throughout that no country is in autarky. Or equivalently, if some country is in autarky, then it is excluded from the analysis. This is because if a country is in autarky, then that country is unaffected by incompleteness of financial markets since there is no way to transfer resources to that agent or insure them against fluctuations. Hence, by definition $A = 1$ if we include a country in autarky in the analysis.

Denote the *wedge*, which is an implicit tax, on the consumption of household h at time t in state s by $\mu_{ht}(s)$. The intertemporal budget constraint for household h, in the decentralization with wedges, is

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)p_{ht}(s)c_{ht}(s) \leq I_h,$$

where $q_t(s)$ is the price of an Arrow security, $p_{ht}(s)$ is the price of the consumption good h in state s at time t not including the wedge, and I_h is initial wealth (including factor endowments and revenues from consumption tax wedges).

We now extend the definition of general equilibrium with wedges. Since our focus is on misallocation from incomplete markets for households, we abstract from other possible distortions and assume that firms set prices equal to marginal cost.

Definition 4 (Equilibrium with Wedges)**.** A general equilibrium with wedges is the collection of prices and quantities such that: (1) the price of each good i equals its marginal cost of production; (2) each producer takes prices as given and chooses quantities to maximize profits; (3) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and income as given; (4) household h earns income from primary factors and tax revenues; (5) all resource constraints are satisfied.

The following proposition extends Proposition 3 to a setting with heterogeneous preferences. It shows that any feasible consumption allocation that is the equilibrium of a model with incomplete financial markets (and no other distortions) can be decentralized using only household-state-date-specific consumption taxes.[17]

Proposition 6 (Decentralization with Wedges)**.** *Consider the status-quo consumption allocation $\boldsymbol{c}^0$. Assume that for each period t and state s, the consumption vector $\{c^0_{ht}(s)\}_{h\in H}$ is statically efficient. Then, for any $\bar{h} \in H$, setting*

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \frac{\omega_{ht}(s)/\omega_{h0}}{\omega_{\bar{h}t}(s)/\omega_{\bar{h}0}}\right] + \frac{1-\eta}{\eta}\left[\log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}}\right] \tag{16}$$

[17] The wedges in Proposition 6 are distinct from the wedges in Berger et al. (2023). They consider preference shifters that, in a representative agent economy, replicate the path of aggregate outcomes (e.g. aggregate consumption, hours, etc.) from a heterogeneous agent New Keynesian model. They show that deviations from perfect risk-sharing map onto discount factor shocks in the representative agent model. They then consider the reduction in output volatility in the absence of these as-if discount factor shocks. In contrast, the wedges in Proposition 6 replicate a microeconomic, rather than just aggregate, allocation in a heterogeneous agent general equilibrium with wedges. We use these wedges to construct a deadweight loss triangle formula for incomplete market models.

implies that $\boldsymbol{c}^0$ is a general equilibrium with those wedges, where $\omega_{ht}(s)$ is the status-quo expenditures of consumer h in period t and state s as a share of total household spending in that period and state, and $p_{ht}(s)$ is the status-quo consumption price index for this household.[18] *The equilibrium allocation is supported by some lump-sum transfers across households.*

By construction, for some household $\bar{h}$, the wedge $\mu_{\bar{h}t}(s)$ equals 1 for every t and s. However, this choice of $\bar{h}$ has no bearing any of the results, since only relative wedges matter for equilibrium allocations (see proofs in the appendix). Moreover, we do not need to specify the lump-sum transfers explicitly. For our purposes, all that matters is that there exist an equilibrium with the wedges in (16), and appropriate transfers, that can support $\boldsymbol{c}^0$ as an equilibrium allocation.

If allocations are dynamically efficient, so that wedges are all equal to one, then according to (16), households whose consumption prices grow relatively more quickly (i.e. real exchange appreciation) experience relatively faster growth in consumption expenditures if $\eta < 1$. Setting wedges equal to one and rearranging yields the Backus and Smith (1993) condition for efficient risk-sharing. For this reason, we refer to $\log \mu_{ht}(s)$ as *Backus-Smith* wedges.

(2) If $\eta = 1$, then once again, the efficient allocation features constant consumption expenditure shares over time, even though households consume different goods. This is related to the observation by Cole and Obstfeld (1991) that an economy with $\eta = 1$ and constant expenditure shares in equilibrium delivers efficient risk-sharing even if there is financial autarky.

Given these wedges, we can now generalize Proposition 4 to this environment. To do so, denote the deviations of productivity shifters from some constant values by $\Delta \log z$. That is, for producer i at time t in state s,

$$\Delta \log z_{it}(s) = \log \frac{z_{it}(s)}{\bar{z}_i},$$

where $\bar{z}_i$ is some constant (over time and states) level of productivity for producer i.

The following proposition approximates misallocation losses in terms of Harberger deadweight loss triangles.

Proposition 7 (Approximate Misallocation for International Model)**.** *Misallocation compar-*

[18]More precisely, $\omega_{ht}(s) = (\sum_{i \in N} p_{it}(s) c^0_{iht}(s)) / (\sum_{h' \in H} \sum_{i' \in N} p_{i't}(s) c^0_{i'h't}(s))$ where i indexes different goods, with status-quo price $p_{it}(s)$, and $c^0_{iht}(s)$ is the consumption of good i by household h in date t and state s in the status-quo.

ing the status-quo allocation $\boldsymbol{c}^0$ to the Pareto-frontier $\mathcal{C}(\boldsymbol{z})$ is approximately

$$\log A \approx \frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\sum_{h\in H}\omega_h \log \mu_{ht}(s)\sum_{h'\in H}\mathcal{M}_{hh'}[\log \mu_{h't}(s)-\log \bar{\mu}_h]\right],$$

where $\log \mu_{ht}(s)$ is given by Proposition 6, $\log \bar{\mu}_h = \mathbb{E}_0[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\log \mu_{ht}(s)|h]$ is the expected discounted consumption wedge for household h, ω_h is the expenditure share of household h at any date or state, r is the risk-free rate at any date or state, and the scalar $\mathcal{M}_{hh'}$ depends only on the static input-output matrix and elasticities of substitution (including η). The approximation error is order $\log \boldsymbol{\mu}^3$ and $\log \boldsymbol{\mu}^2 \Delta \log \boldsymbol{z}$.

The explicit formula for the $H \times H$ matrix $\mathcal{M}$ in terms of the input-output table and elasticities of substitution is given in the appendix. The key is that the matrix $\mathcal{M}$ does not depend on either the date or the state.

Proposition 7 is a sufficient statistics formula: misallocation can be approximated conditional on knowledge of the (static) input-output table at some date, elasticities of substitution (include η), the risk-free rate r at one point in time, and wedges, $\log \mu_{ht}(s)$, which are recoverable from Proposition 6. Importantly, one does not need know the process driving productivity shocks $\Delta \log \boldsymbol{z}$ or the income process (which also depend on asset portfolios and returns, etc.) for each country.

To build some intuition for Proposition 7, consider the following simple example.

Example 3 (Symmetric country example). Consider two symmetric countries, $h \in \{1, 2\}$, and suppose that each country produces one good using a linear technology from the local factor endowment (i.e. there is one industry in each country and no intermediate inputs). Let α denote the import share in both countries in the first date in status-quo and let θ denote the elasticity of substitution between domestic and foreign goods.

Apply (16) and set log wedges for country 2 to zero (i.e. $\bar{h} = 2$). Then, apply Proposition 7 and rearrange to get

$$\log A \approx \frac{1}{2}\left[\frac{\alpha(1-\alpha)}{\left(\frac{1}{\eta}-\frac{1}{\theta}\right)4\alpha\left(1-\alpha\right)+\frac{1}{\theta}}\right]\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\mathbb{E}_0\left[\log \mu_{1t}(s)\left(\log \mu_{1t}(s)-\log \bar{\mu}_1\right)\right].$$

This expression illustrates several important lessons. First, holding the Backus-Smith wedges $\boldsymbol{\mu}$ constant, misallocation goes to zero if either the import share of consumption, α, approaches zero or one. Intuitively, if the two countries are consuming completely unrelated goods, then there is no insurance possible between them. Second, as the Armington elasticity, θ, rises to infinity misallocation rises because there is more scope for international risk sharing when foreign and domestic goods are more substitutable.

Finally, holding the Backus-Smith wedges constant, misallocation tends to zero as the EIS, η, tends to zero. This is because consumption choices do not respond to wedges when the EIS is close to zero. However, the Backus-Smith wedges in (16) are themselves functions of η (given data on consumption expenditures and real exchange rates). In particular, these wedges explode as η goes to zero. Intuitively, since consumption choices are insensitive to wedges when η is low, we require very large consumption wedges to justify deviations from perfect risk-sharing. This second effect always dominates (because it is order $1/\eta^2$) and so that misallocation is larger for lower values of η holding fixed the data. These three lessons are all borne out in our empirical application in Section 6.2.

5.3 Quantitative Example

We provide a quantitative example to assess the accuracy of the second-order approximation in Proposition 7. We simulate an Armington model of trade with 15 heterogeneous countries and homogeneous consumers within each country. Each country produces a single good, the elasticity of substitution between domestic and foreign goods is 3, and the EIS is 0.5. We randomize the country sizes and the input-output matrices. We draw productivity shocks and Backus-Smith wedges from a lognormal distribution.

We vary the standard deviation of the productivity and wedge process in Figure 6 and plot the exact gains from completing financial markets, computed using Proposition 1, and the approximate gains, computed using Proposition 7. To estimate the time-zero expectations in Proposition 7, we simulate the model for 100 periods and treat the observed realizations as one sample path from the distribution generating the data. Since we only have one realization of the sample path, we estimate the expectation using this single observation. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to an average of the observations in the last five years of the data.

Even with very large shocks, the second-order approximation performs very well. Notably, to compute the second-order approximation, we do not need to know the stochastic process driving either the wedges or the productivity shocks.

6 Empirical Applications

We provide two empirical applications. The first application applies the closed-economy result in Proposition 4 to US household consumption panel data from the Panel Study

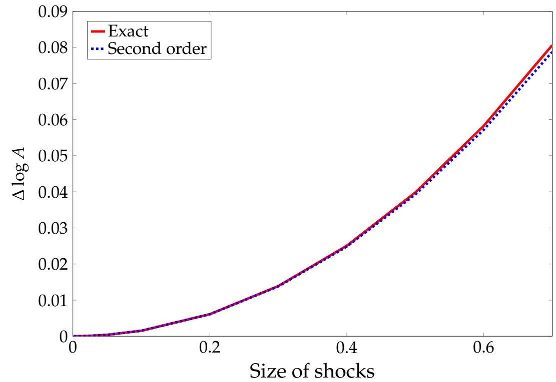


Figure 6: Performance of second-order approximation in Proposition 7. The x-axis is the standard deviation of log productivity and Backus-Smith wedge shocks.

of Income Dynamics (PSID). The second application applies the open-economy result in Proposition 7 to international macroeconomic data. Our first application measures misallocation due to incompleteness of financial markets among households in the PSID, whereas the second application measures misallocation due to incompleteness of international financial markets, where we abstract from micro-level heterogeneity and treat each country as a different representative agent.

Since we use our sufficient statistics approximations, we can calculate misallocation using only the data on expenditures and prices as well as estimates of elasticities of substitution — we do not specify the details of financial market imperfections or the determinants of the Pareto frontier like the technology/income processes.

6.1 Misallocation from Domestic Financial Market Incompleteness

In this application, we quantify misallocation from the lack of complete domestic insurance markets in the US. We study how much consumption (in every date and state) is left over if domestic insurance markets work perfectly and every household is kept indifferent relative to their status-quo allocation. The larger is this number, the greater is the extent of misallocation from incomplete risk-sharing in the status-quo. To avoid confusion, keep in mind that the status-quo allocation is the entire state- and date-contingent equilibrium consumption process for each household starting in the first period, rather than consumption in the first period.

Approach. We use Proposition 4 to examine the extent of misallocation. In particular, we assume the data arise from an economy that meets the assumptions laid out in Section

3 — that is, every household has the same static consumption aggregator, and production is efficient in a static sense, but consumption allocations may be Pareto inefficient over time or across different states of nature. In particular, this means that we abstract from labor-leisure choice at the individual level, and capital accumulation at the aggregate level. Nevertheless, our quantitative results in Section 4 give some assurance that abstracting from these two margins is quantitatively innocuous for $\log A$ and that our approximation formula continues to perform well even if these ingredients are added to the model.

Description of data. We use the PSID, which is a longitudinal panel survey of American households. We use a balanced panel of households from 1999 to 2021 with $2,096$ households. We use household consumption expenditures across six consumption categories collected once every two years. These categories are food (at home and away), child care, healthcare, education, transportation, and housing. We leave out other expenditure categories (like clothing and electronics) which are not collected in every wave. Housing expenditures do not measure owner-occupied rental value for home owners, so we use the methodology in Baqaee et al. (2024) to impute owner-occupied housing costs.[19]

Mapping data to terms in Proposition 4. To apply Proposition 4, we set the EIS $\eta = 0.5$ and the risk-free rate $r = 0.05$. We use household h's share of total consumption expenditures in 1999 to calibrate ω_h.[20] We perform sensitivity analysis with respect to these choices when we present our results.

To estimate the expected discounted consumption share of household h, $\log \bar{\omega}_h = r\mathbb{E}_0 \left[\sum_{t=0}^{\infty} (1+r)^{-(t+1)} \log \omega_{ht}(s') | h \right]$, we run a regression of household h's consumption share in period t on its date-zero consumption share (the first year of the sample, 1999), and a vector of household-level covariates.[21]

$$\log \omega_{ht} = \gamma_t \log(\omega_{h0}) + \boldsymbol{\psi_t X_{h0}} + \epsilon_{ht}. \tag{17}$$

The estimated regression equation is the best linear predictor of household h's consumption share at t conditional on observables at date zero (1999). We use this linear predictor

[19] Briefly, we regress rent on observables for non-owners, and then use the estimates to predict rents for home-owners.

[20] We experimented with using contemporaneous shares ω_{ht} every period instead of freezing them in 1999 and the results are very similar.

[21] The covariates are household wealth without home equity, state of residence, household size, home ownership status (0 or 1), household head's age, race, ethnicity, and college degree status, business assets of the head and spouse, household head's labor income, and spouse's labor income.

in place of the conditional expectation in the formulas (i.e. in place of the best nonlinear predictor). Hence, to calculate $\log \bar{\omega}_h = r\mathbb{E}_0 \left[\sum_{t=0}^{\infty}(1+r)^{-(t+1)} \log \omega_{ht}(s')|h\right]$ we predict household h's consumption share at each horizon t and sum them up discounted using r. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to the last observed value. As shown in Figure 2, this imputation does well in small sample settings.

Results. Figure 7 plots estimated misallocation losses in the PSID as a function of the annual interest rate. Our benchmark interest rate of 5% implies that misallocation is 21 log points (roughly 20%). That is, the gains from eliminating idiosyncratic consumption volatility are around 20% of consumption in every period and state. We can contrast this with the gains from eliminating aggregate volatility for a representative agent, which Lucas (1987) famously estimated to be three orders of magnitude smaller (0.05%).

The estimated gains in the PSID are in the same ballpark as those from the calibrated Bewley (1972) model in Section 3.4. Estimated misallocation decreases as the risk-free rate, or degree of impatience, rises. This is because deadweight loss triangles in the future are more heavily discounted. In the limit, as $r \to \infty$, households are infinitely impatient, there is no possibility to share risk, and misallocation is zero.

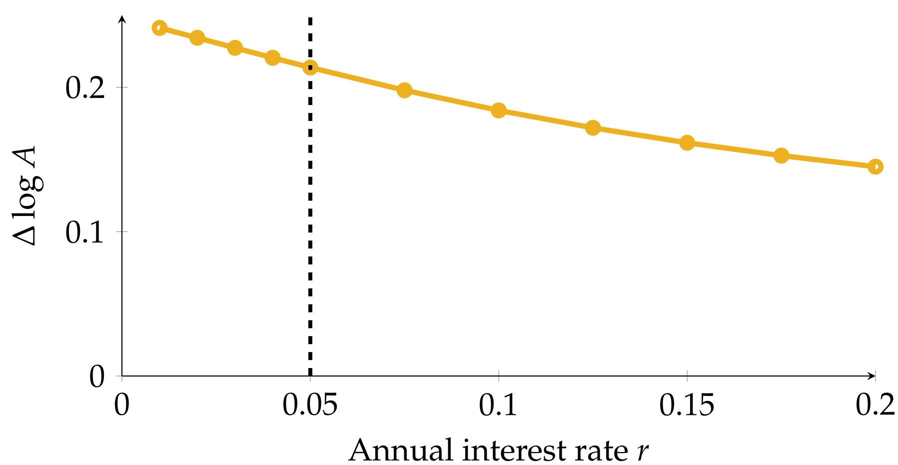


Figure 7: Estimated misallocation in PSID. Dashed line is benchmark.

Table 1 shows how estimates of misallocation change as we drop covariates from the regression in (17). As the regression becomes less informative, estimated misallocation rises. Intuitively, this is because with a less informative regression, we attribute some systematic differences between households to lack of risk-sharing rather than to differences in observable household characteristics. In other words, once the regression model stops controlling for certain variables, previously explained variation in consumption shares now appears as evidence of incomplete insurance, artificially inflating the measured misallocation. This upward bias in estimated misallocation can be non-negligible: including no additional controls besides initial consumption, causes the estimate to rise from 0.21 to 0.27.

Eliminated variable	**Estimated misallocation**
None (Baseline)	0.214
Spouse labor income	0.223
Household head labor income	0.223
Business assets (household & spouse)	0.231
Household head college degree	0.239
Household head race and ethnicity	0.244
Household head age	0.249
Renter status	0.252
Household size	0.258
State of residence	0.269
Wealth	0.266

Table 1: Estimated misallocation under the baseline calibration with annual interest rate $r = 0.05$. The top row includes all covariates; each subsequent row eliminates one additional covariate (e.g., the third row excludes both spouse and head labor income).

6.2 Misallocation from International Financial Market Incompleteness

In our final application, we quantify misallocation from the lack of complete international financial markets relative to the status-quo. More precisely, we calculate how much of every consumption good (in the world) is left over if financial markets are completed and agents in every country are kept indifferent relative to the status-quo. The larger is this number, the greater is the extent of misallocation from incomplete risk-sharing. Importantly, in this application abstract from within-country heterogeneity and assume that each country has a representative agent.

To study the extent of misallocation, we use Proposition 7, which allows for heterogeneity in household consumption bundles. We assume the data is generated by an economy satisfying the assumptions in Section 5 — allocations are efficient from a static perspective, but potentially inefficient over time and states of nature. The advantage of using the second-order approximation in Proposition 7, versus writing a fully-specified structural model and applying the exact result in Proposition 1, is that the informational requirements are much weaker. Specifically, we can apply Proposition 7 without taking a stance on the stochastic process driving either the wedges or the productivity shifters. For example, there may be productivity changes that we did not explicitly model, like changes in iceberg costs at the industry-country pair level, and they would not alter the validity of the second approximation.

Calibration of model. We specialize the technologies introduced in Section 5 as follows. There are 32 countries (households), 54 industries in each country, and one primary factor endowment per country (i.e. labor equipped by capital).[22] The static preferences of household h are an h-specific Cobb-Douglas aggregator across different industries. Consumption by h from industry i is an (h, i)-specific Armington CES aggregator over different origin countries, with elasticity of substitution θ_T.

The production function of industry i in country h is a Cobb-Douglas aggregator of the local primary factor and an (h, i)-specific bundle of intermediate inputs from other industries. This intermediate bundle is also an (h, i)-specific Cobb-Douglas aggregator. The industry-j input used by producers in country h is an (h, j)-specific Armington aggregator with elasticity θ_T across different origin countries.[23]

We calibrate the expenditure shares using the 2014 release of the world input-output database (Timmer et al., 2015).[24] That is, we calibrate the consumption share of each country ω_h using that country's share of total consumption, investment, and government expenditures. We calibrate the input-output matrix required for $A_{hh'}$ using the transaction flows in 2014. We calibrate each industry-country's expenditures on intermediate inputs from other industries and value-added from the WIOD.

We measure Backus-Smith wedges using the formula in Proposition 6, which expresses them as a function of consumption expenditure shares, real exchange rates, and the elasticity of intertemporal substitution (EIS).

[22]We drop the activities of private households as employers industry and the activities of extraterritorial organizations and bodies industry from the sample. The list of countries is Australia, Austria, Belgium, Brazil, Canada, Switzerland, China, Cyprus, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Hungary, Indonesia, India, Ireland, Italy, Japan, South Korea, Luxembourg, Mexico, Malta, Netherlands, Norway, Poland, Portugal, Sweden, Turkey, and the US.

[23]This means we assume the same country-composition of the intermediate input bundle by industry. For example, mining & quarrying and the manufacture of basic metals in Australia, have the same expenditure shares on rubber and plastic products from China relative to India. On other hand, the expenditure share of the rubber and plastic industry summed across all origins by the mining & quarrying industry in Australia can differ from that by the manufacture of basic metals industry.

[24]To calibrate Proposition 7, we take advantage of the fact that, since allocations are statically efficient by assumption, the observed relative prices of goods within each period and state, not including the consumption wedges, in the decentralized equilibrium with wedges are equal to the relative marginal costs of production. Hence, if, in the data, relative prices within periods and states reflect marginal costs, we can calibrate the expenditures shares in the model directly to those in the data.

We apply this formula at annual frequency from 1970 to 2019 using nominal consumption data and CPI-based real exchange rates from the Global Macro Database from Müller et al. (2025). This means that we treat the equilibrium starting in 1970 as part of the (date- and state-contingent) status-quo allocation. Hence, we treat the observed path of wedges as one realization (i.e. a sample path) of the wedges from the decentralized equilibrium with wedges in status-quo. The log wedges are nonzero if changes in log relative consumption and real exchange rates between countries do not comove perfectly. The correlation between annual changes in real exchange rates and real consumption between the US and each country is 0.17 for the median country, whereas perfect risk sharing implies that this correlation should be -1. We quantify the extent of misallocation that results from these wedges, abstracting from other possible distortions in the economy.

To compute the terms in Proposition 7, we assume an annual risk-free rate $r = 0.05$, an EIS $\eta = 0.5$, and a trade elasticity $\theta_T = 2$. We vary these parameters in sensitivity analyses.

To estimate the time-zero expectations in Proposition 7, we treat the wedges from 1970 to 2019 as one sample path from the distribution generating the data. Since we only have one realization of the sample path, we estimate the expectation using this single observation. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to an average of the observations in the last five years of the data (2015 to 2019).[25] This mirrors the procedure we used in our Monte Carlo simulation in Section 5.3.

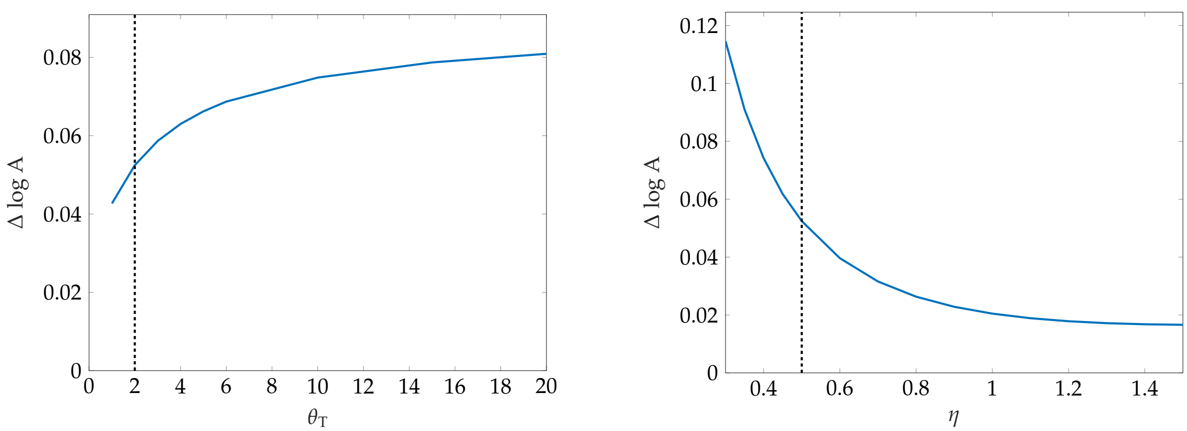
Results. Misallocation in our baseline calibration is 5.2% — that is, with complete insurance markets, every country can be made indifferent to the status-quo allocation with 5.2% of every good left over. Recall that the status-quo allocation is the date- and state-contingent consumption processes in the observed equilibrium. The extent of misallocation depends strongly on whether countries with rapid growth rates are included in the sample. For example, if we exclude just China, then the extent of misallocation falls to 1.9% instead. If we drop China, India, Korea, and Indonesia as well, misallocation falls to only 1.0%. After this, the results are quite stable to dropping more countries. This shows that if we include large countries with very different growth rates in the sample, then the extent of misallocation from lack of international financial markets becomes larger. In the rest of this section, we report results including all 32 countries.

We experimented with varying the start date, for example, if we start in 1980 instead of 1970, then misallocation is slightly larger at 6.3%. If we start in 1993, then we can increase

[25] Results are very similar if we set unobserved terms equal to the last observed year of these terms.

the number of countries in the sample by including 10 additional countries that belonged to the Eastern Bloc. This raises misallocation to 6.6%.

We also vary the WIOD release year we calibrate to. If we use an earlier release date, say 2006 instead of 2014, then misallocation is smaller, around 3.6% instead. This is because the world economy is less open in 2006 compared to 2014, so there is less scope for international risk-sharing, as discussed in Example 3.



(a) Armington elasticity

(b) EIS

Figure 8: Misallocation varying parameters. Dashed line is benchmark value.

Figure 8 shows how our estimates of misallocation change as a function of the Armington trade elasticity and the EIS. As expected from Example 3, misallocation is larger the higher is the Armington elasticity, since more substitutablity between domestic and foreign varieties facilitate more risk sharing; and misallocation is larger the lower is the EIS, since observed fluctuations in consumption are most costly for lower values of the EIS. Our estimates for misallocation are fairly insensitive to the Armington elasticity for the range the literature considers empirically plausible (e.g. from 1 to 5). However, our estimates are sensitive to lower values of the EIS. For example, if the EIS is 0.25, misallocation is around 10% — and these losses go to infinity as η approaches zero. We do not present graphs for how estimated misallocation varies as a function of the elasticity of substitution between industries or between intermediates and value-added. Estimated losses are slightly increasing in these elasticities.

7 Conclusion

We quantify misallocation due to households' inability to perfectly share risks across states of nature or to smooth consumption over time. We find that misallocation costs due to market incompleteness are substantial within countries, especially if the elasticity of substitution across time and states of nature is low. Misallocation losses from imperfect consumption-smooth across countries, assuming a representative agent in each country, are much smaller. This is particularly true among developed economies, who do not have very different growth rates, and if trade elasticities are relatively low, so that imports are poor substitutes for domestic goods. A promising area for future research is to extend our characterizations to study misallocation relative to the constrained efficient Pareto frontier, accounting for the presence of other distortions and imperfections of policy.

References

Aguiar, M., M. Amador, and C. Arellano (2024). Micro risks and (robust) pareto-improving policies. *American Economic Review 114*(11), 3669–3713.

Aguiar, M., O. Itskhoki, and D. Mukhin (2025). How good is international risk sharing? Technical report.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics 109*(3), 659–684.

Allais, M. (1979). La théorie générale des surplus.

Atkeson, A. and C. Phelan (1994). Reconsidering the costs of business cycles with incomplete markets. *NBER macroeconomics annual 9*, 187–207.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of economic theory 2*(3), 244–263.

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*(3-4), 297–316.

Baqaee, D. and A. Burstein (2025a). Aggregate efficiency with discrete choice. Technical report.

Baqaee, D. and A. Burstein (2025b). Aggregate efficiency with heterogeneous agents. Technical report.

Baqaee, D., A. Burstein, and Y. Koike-Mori (2024). Sufficient statistics for measuring forward-looking welfare. Technical report, National Bureau of Economic Research.

Baqaee, D. R. and E. Farhi (2019). The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem. *Econometrica 87*(4), 1155–1203.

Baqaee, D. R. and E. Farhi (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica 70*(2), 481–517.

Berger, D., L. Bocola, and A. Dovis (2023). Imperfect risk sharing and the business cycle. *The Quarterly Journal of Economics 138*(3), 1765–1815.

Bergson, A. (1954). On the concept of social welfare. *The Quarterly Journal of Economics 68*(2), 233–252.

Bewley, T. F. (1972). Existence of equilibria in economies with infinitely many commodities. *Journal of economic theory 4*(3), 514–540.

Bhandari, A., D. Evans, M. Golosov, and T. Sargent (2021). Efficiency, insurance, and redistribution effects of government policies. Technical report, Working paper.

Bigio, S. and J. La'O (2016). Financial frictions in production networks. Technical report.

Boar, C. and V. Midrigan (2022). Efficient redistribution. *Journal of Monetary Economics 131*, 78–91.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011, August). Finance and Development: A Tale of Two Sectors. *American Economic Review 101*(5), 1964–2002.

Cole, H. L. and M. Obstfeld (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of monetary economics 28*(1), 3–24.

Conesa, J. C., S. Kitao, and D. Krueger (2009). Taxing capital? not a bad idea after all! *American Economic Review 99*(1), 25–48.

Constantinides, G. M. (2025). Welfare costs of idiosyncratic and aggregate consumption shocks. *The Review of Asset Pricing Studies 15*(2), 103–120.

Corsetti, G., A. Lipińska, and G. Lombardo (2024). International risk sharing and wealth allocation with higher order cumulants.

Dávila, E. and A. Schaab (2022). Welfare assessments with heterogeneous individuals. Technical report, National Bureau of Economic Research.

Dávila, E. and A. Schaab (2023). Welfare accounting. Technical report, National Bureau of Economic Research.

Dávila, J., J. Hong, P. Krusell, and J. R. Rull (2012). Constrained efficiency in the one-sector neoclassical growth model with idiosyncratic uninsurable shocks. *Econometrica 80*, 2431–2467.

Debreu, G. (1951). The coefficient of resource utilization. *Econometrica: Journal of the Econometric Society*, 273–292.

Eden, M. (2020). Welfare analysis with heterogeneous risk preferences. *Journal of Political Economy 128*(12), 4574–4613.

Farhi, E. and I. Werning (2012). Capital taxation: Quantitative explorations of the inverse euler equation. *Journal of Political Economy 120*(3), 398–445.

Fitzgerald, D. (2012). Trade costs, asset market frictions, and risk sharing. *American Economic Review 102*(6), 2700–2733.

Fitzgerald, D. (2024). 3-d gains from trade. Technical report.

Floden, M. (2001). The effectiveness of government debt and transfers as insurance. *Journal of Monetary Economics 48*(1), 81–108.

Gourinchas, P.-O. and O. Jeanne (2006). The elusive gains from international financial integration. *The Review of Economic Studies 73*(3), 715–741.

Harberger, A. C. (1954). Monopoly and resource allocation. In *American Economic Association, Papers and Proceedings*, Volume 44, pp. 77–87.

Harberger, A. C. (1964). The measurement of waste. *The American Economic Review 54*(3), 58–76.

Harsanyi, J. C. (1955). Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of political economy 63*(4), 309–321.

Heathcote, J. and F. Perri (2014). Assessing international efficiency. In *Handbook of International Economics*, Volume 4, pp. 523–584. Elsevier.

Heathcote, J., K. Storesletten, and G. L. Violante (2008). Insurance and opportunities: A welfare analysis of labor market risk. *Journal of Monetary Economics 55*(3), 501–525.

Hicks, J. R. (1939). The foundations of welfare economics. *The economic journal 49*(196), 696–712.

Hsieh, C.-T. and P. J. Klenow (2009). Misallocation and manufacturing TFP in China and India. *The quarterly journal of economics 124*(4), 1403–1448.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control 17*(5-6), 953–969.

Imrohoroğlu, A. (1989). Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political economy 97*(6), 1364–1383.

Kaldor, N. (1939). Welfare propositions of economics and interpersonal comparisons of utility. *The economic journal 49*(195), 549–552.

Krueger, D., K. Mitman, and F. Perri (2016). On the distribution of the welfare losses of large recessions. Technical report, National Bureau of Economic Research.

Lucas, R. E. (1987). *Models of business cycles*, Volume 26. Basil Blackwell Oxford.

MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of political Economy 89*(6), 1059–1085.

Midrigan, V. and D. Y. Xu (2014, February). Finance and Misallocation: Evidence from Plant-Level Data. *American Economic Review 104*(2), 422–458.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review 104*(10), 3186–3221.

Müller, K., C. Xu, M. Lehbib, and Z. Chen (2025). The global macro database: A new international macroeconomic dataset. *Available at SSRN 5121271*.

Restuccia, D. and R. Rogerson (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics 11*(4), 707–720.

Rognlie, M. (2024). Econ411-3 lecture 3: Computing the steady state of the sim model. `https://github.com/mrognlie/econ411-3/blob/main/econ411_3_lecture3.ipynb`. GitHub repository, accessed 2025-08-26.

Samuelson, P. A. (1956). Social indifference curves. *The quarterly journal of economics 70*(1), 1–22.

Storesletten, K., C. I. Telmer, and A. Yaron (2001). The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk. *European Economic Review 45*(7), 1311–1339.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. De Vries (2015). An illustrated user guide to the world input–output database: the case of global automotive production. *Review of International Economics 23*(3), 575–605.

Van Wincoop, E. (1994). Welfare gains from international risksharing. *Journal of Monetary Economics 34*(2), 175–200.

Appendix A Proofs

Proof of Proposition 1. See Theorem 1 from Baqaee and Burstein (2025b) □

Proof of Proposition 2. See text. □

Proof of Proposition 3. A decentralized equilibrium with wedges satisfies

$$\max_{c_{ht}(s)} \frac{1}{1-\frac{1}{\eta}} \sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}$$

subject to

$$\sum_s \sum_t q_t(s) \mu_{ht}(s) c_{ht}(s) \leq I_h$$

and

$$\sum_h c_{ht}(s) = y_t(s).$$

Without loss of generality, we set the price of the consumption good to one in every period and state, $p_t(s) = 1$ (since only $p_t(s)q_t(s)$ is determined). Assume aggregate income is the numeraire. The first order conditions, resource constraints, budget constraint, and numeraire choice define equilibria:

$$\beta^t \pi(s) c_{ht}^{-\frac{1}{\eta}}(s) = \lambda_h q_t(s) \mu_{ht}(s).$$

$$\sum_h c_{ht}(s) = y_t(s).$$

$$\sum_s \sum_t q_t(s) \mu_{ht}(s) c_{ht}(s) = I_h$$

$$\sum_h I_h = 1.$$

We need to show that if

$$\mu_{ht}(s) = \left[\frac{\omega_{ht}^0(s)}{\omega_{h0}^0} \right]^{-\frac{1}{\eta}},$$

where $\omega_{ht}^0(s)$ are expenditure shares in the status-quo, then the following allocation is an equilibrium:

$$c_{ht}(s) = c_{ht}^0(s) = \omega_{ht}^0(s) y_t(s).$$

This requires showing that there is a set of ϕ_h, $q_t(s)$, and I_h such that all the equilibrium conditions are satisfied. Substituting in the wedges and the allocation into the first order condition yields the following restriction on ϕ_h and $q_t(s)$:

$$\phi_h q_t(s) = \beta^t \pi(s) y_t(s)^{-\frac{1}{\eta}} \left[\omega_{h0}^0\right]^{-\frac{1}{\eta}}.$$

Hence, dividing this equation for h' by H for some fixed household H gives

$$\phi_{h'} = \phi_H \left(\frac{\omega_{H0}^0}{\omega_{h'0}^0}\right)^{\frac{1}{\eta}}.$$

The resource constraint is satisfied automatically, since:

$$\sum_h c_{ht}(s) = \sum_h \omega_{ht}^0(s) y_t(s) = y_t(s).$$

Finally, substituting the FOC into the budget constraint yields

$$\frac{\sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s)}{\phi_h} = I_h.$$

Finally, the numeraire condition requires that

$$\sum_h I_h = 1.$$

Substituting the previous expression for I_h into this equation and rewriting every $\phi_{h'}$ in terms of ϕ_H for some h yields:

$$\sum_{h'} I_{h'} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{h'}} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_H \left(\frac{\omega_{H0}^0}{\omega_{h'0}^0}\right)^{\frac{1}{\eta}}} = 1.$$

Hence, we require that

$$\phi_H = \sum_h \left(\frac{\omega_{H0}^0}{\omega_{h0}^0}\right)^{-\frac{1}{\eta}} \sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s).$$

Since we can construct a collection of ϕ_h, $q_t(s)$, and I_h such that all equilibrium conditions are satisfied, with $c_{ht}(s) = \omega_{ht}^0(s) y_t(s)$ and $\mu_{ht}(s)$ given by (6), the proof is completed. $\square$

Define the fictitious compensated agent as follows.

Definition 5. The *compensated agent* is an agent whose preferences are represented by

$$U(\boldsymbol{c}) = \min_h \{\tilde{u}_h(\boldsymbol{c}_h)\},$$

where $\tilde{u}_h(\boldsymbol{c}_h) = \left[u_h(\boldsymbol{c}_h)/u_h(\boldsymbol{c}_h^0)\right]^{\frac{\eta}{\eta-1}}$.

Define the compensated equilibrium as follows.

Definition 6 (Compensated Equilibrium)**.** A *compensated equilibrium* is the general equilibrium of an economy with the same technologies, resource constraints, and wedges as the original economy but where there is a representative agent with preferences as in Definition 5. For any equilibrium variable $X(t)$, denote the same variable in the compensated equilibrium by $X^{\text{comp}}(t)$.

Theorem 2 from Baqaee and Burstein (2025b) implies that aggregate efficiency can be calculated via the utility of the compensated agent in the compensated equilibrium.

Proof of Proposition 4. We begin by showing the following lemma, drawn from Baqaee and Burstein (2025b).

Lemma 1. *At the status-quo, prices and quantities in the compensated equilibrium with wedges in Equation* (6) *coincide with those in the decentralized equilibrium with the same wedges.*

Proof of Lemma. A general proof can be found in Baqaee and Burstein (2025b). Here, we provide a self-contained derivation. We use this lemma both when households consume the same consumption good (as in the closed economy baseline) and when each household consumes a different bundle of goods (as in the international version of the model). Hence, in the proof of this lemma, we allow for consumption prices $p_{ht}(s)$ to vary by h, to allow for the possibility that households consume different consumption goods. The representative household maximizes

$$\min_h \{\tilde{u}_h\left(\boldsymbol{c}_h\right)\}$$

with

$$\tilde{u}_h\left(\boldsymbol{c}_h\right) = \left[\frac{\sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}}{\sum \beta^t \pi(s) c_{ht}^0(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}},$$

subject to a single budget constraint

$$\sum_h \sum_{s,t} q_t\left(s\right) p_{ht}(s) \mu_{ht}\left(s\right) c_{ht}\left(s\right) \leq I.$$

The solution to this problem can be found by two-step budgeting. The representative agent distributes income across h, and for each h maximizes $\tilde{u}_h(\boldsymbol{c}_h)$ subject to

$$\sum_{s,t} q_t(s)\, p_{ht}(s)\mu_{ht}(s)\, c_{ht}(s) = I_h.$$

The choice of $\{I_h\}_h$ in an interior equilibrium must be such that

$$\tilde{u}_h(\boldsymbol{c}_h) = \tilde{u}_{h'}(\boldsymbol{c}_h).$$

Setting $I_h = I_h^0$ for all h, where I_h^0 is the income level of household h in the status-quo of the decentralized equilibrium with wedges gives the status-quo allocation $\boldsymbol{c}^{\mathbf{0}}$, which satisfies $\tilde{u}_h\left(\boldsymbol{c}_h^0\right) = \tilde{u}_{h'}\left(\boldsymbol{c}_h^0\right) = 1$. □

Consider some exogenous parameter indexed by σ. For each value of σ, there is an endogenous set of wedges $\boldsymbol{\mu}(\sigma)$ that rationalize status-quo allocations, and for some value of this parameter, normalized to be zero, the wedges are all equal to one: $\boldsymbol{\mu}(0) = \mathbf{1}$. Two examples are the standard deviation of idiosyncratic risk and the inverse EIS.

From Proposition 8 in Baqaee and Burstein (2025b), we know that, to a second-order approximation in σ, misallocation is given by

$$\begin{aligned}\log A &\approx -\frac{1}{2}\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\frac{d\log c_{ht}^{\text{comp}}(s)}{d\log\boldsymbol{\mu}}\cdot\frac{d\log\boldsymbol{\mu}}{d\sigma}\right]\frac{d\log\mu_{ht}(s)}{d\sigma}\Delta\sigma^2 \\ &= -\frac{1}{2}\sum_h \sum_{s,t} q_t(s)c_{ht}(s)d\log c_{ht}^{\text{comp}}(s)d\log\mu_{ht}(s), \qquad (18)\end{aligned}$$

where we use the short-hand $d\log c_{ht}^{\text{comp}}(s)$ to mean $\frac{d\log c_{ht}^{\text{comp}}(s)}{d\log\boldsymbol{\mu}}\cdot\frac{d\log\boldsymbol{\mu}}{d\sigma}$ and $d\log\mu_{ht}(s)$ to mean $d\log\boldsymbol{\mu}/d\sigma$ in the compensated equilibrium evaluated at $\sigma = 0$. In this equation, we have used the convention that, in the compensated equilibrium with wedges, aggregate wealth is equal to one (this is our choice of numeraire).

Throughout, we use the fact that because of Lemma 1, we can use expenditures in the decentralized economy in place of expenditures in the compensated equilibrium with the same wedges that rationalize that allocation.

To calculate $d\log c_{ht}^{\text{comp}}(s)$, consider the first order condition of the representative agent in the compensated equilibrium:

$$\left[\frac{\sum_{t'}\beta^{t'}\pi(s)c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t'}\beta^{t'}\pi(s)c_{ht'}^{0}(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}}\frac{\beta^t\pi(s)c_{ht}^{\text{comp}}(s)^{-\frac{1}{\eta}}}{\sum_{t'}\beta^{t'}\pi(s)c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} = \tilde{\phi}_h q_t^{\text{comp}}(s)\,\mu_{ht}(s)$$

where $\tilde{\phi}_h$ is the Lagrange multiplier associated to h's budget constraint. Defining

$$\phi_h = \left[\tilde{\phi}_h \left(\sum_{t'} \beta^{t'} \pi(s) c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}\right)\right]^{-1}, \tag{19}$$

the first-order condition above can be written as

$$\tilde{u}_h \left(\boldsymbol{c}_h^{\text{comp}}\right) \beta^t \pi(s) c_{ht}^{\text{comp}}(s)^{-\frac{1}{\eta}} = \phi_h^{-1} q_t^{\text{comp}}(s)\, \mu_{ht}(s)$$

Taking ratios between h and H,

$$\frac{c_{ht}^{\text{comp}}(s)}{c_{Ht}^{\text{comp}}(s)} = \left(\frac{\mu_{ht}(s)\, \phi_h^{-1}}{\mu_{Ht}(s)\, \phi_H^{-1}}\right)^{-\eta} \left(\frac{\tilde{u}_h}{\tilde{u}_H}\right)^{\eta}. \tag{20}$$

Log differentiating and using the fact that the compensated representative agent keeps $\tilde{u}_h/\tilde{u}_H$ constant, we have

$$d\log c_{ht}^{\text{comp}}(s) - d\log c_{Ht}^{\text{comp}}(s) = -\eta \left(d\log \frac{\mu_{ht}(s)}{\phi_h} - d\log \frac{\mu_{Ht}(s)}{\phi_H}\right), \tag{21}$$

The condition $d\log \tilde{u}_h = d\log \tilde{u}_H$ can be written as

$$\sum_{t,s} \frac{\beta^t \pi(s) c_{ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t',s'} \beta^{t'} \pi(s') c_{ht'}^{\text{comp}}(s')^{1-\frac{1}{\eta}}} d\log c_{ht}^{\text{comp}}(s) = \sum_{t,s} \frac{\beta^t \pi(s) c_{Ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t',s'} \beta^{t'} \pi(s') c_{Ht'}^{\text{comp}}(s')^{1-\frac{1}{\eta}}} d\log c_{Ht}^{\text{comp}}(s). \tag{22}$$

Throughout the rest of the proof, we use the fact that for any compensated equilibrium value X^{comp} we have

$$X_{ht}^{comp}(s)(\boldsymbol{z}, \boldsymbol{\mu}) \approx X_{ht}^{comp}(s)(\bar{\boldsymbol{z}}, \mathbf{1}) + \frac{dX_{ht}^{comp}}{d\log z} \Delta \log z + \frac{dX_{ht}^{comp}}{d\log \boldsymbol{\mu}} \cdot \Delta \log \boldsymbol{\mu}.$$

Hence, to evaluate any coefficients in the second-order approximation, we can use $X_{ht}^{comp}(s)(\bar{\boldsymbol{z}}, \mathbf{1})$ or $X_{ht}^{comp}(s)(\boldsymbol{z}, \boldsymbol{\mu})$ because the difference between them is first order, which when multiplied by the other terms in the Taylor expansion, will result in terms that are order three or higher.

Hence, we can evaluate (22) at $\sigma = 0$, and without aggregate productivity shocks ($z_t = \bar{z}$), where we know that $c_{ht}^{\text{comp}}(s) = c_h^{comp}$ for all h, giving us

$$\sum_{t,s} \beta^t \pi(s) d\log c_{ht}^{\text{comp}}(s) = \sum_{t,s} \beta^t \pi(s) d\log c_{Ht}^{\text{comp}}(s).$$

Substituting (21) into this, gives

$$\sum_{t,s} \beta^t \pi(s) \left(d \log \frac{\mu_{ht}(s)}{\phi_h} - d \log \frac{\mu_{Ht}(s)}{\phi_H} \right) = 0.$$

so

$$d \log \phi_h - d \log \phi_H = \frac{\sum_{t,s} \pi(s) \beta^t \left(d \log \mu_{ht}(s) - d \log \mu_{Ht}(s) \right)}{\sum_{t''} \beta^{t'}}.$$

Plugging back into (21),

$$\begin{aligned} d \log c_{ht}^{\text{comp}}(s) - d \log c_{Ht}^{\text{comp}}(s) = & -\eta \left(d \log \mu_{ht}(s) - d \log \mu_{Ht}(s) \right) \\ & + \eta \frac{\sum_{t',s'} \pi(s') \beta^{t'} \left(d \log \mu_{ht'}(s') - d \log \mu_{Ht'}(s') \right)}{\sum_{t'} \beta^t}. \end{aligned} \tag{23}$$

Differentiating the resource constraint, at the status-quo, with respect to σ gives

$$\sum_{h'} c_{h't}^0(s) \, d \log c_{ht}^{\text{comp}}(s) = \sum_{h'} c_{h't}^0(s) \left[\frac{d \log c_{ht}^{\text{comp}}(s)}{d \log \boldsymbol{\mu}} \cdot d \log \boldsymbol{\mu} \right] = 0,$$

where the second equation is a definition. Substituting (23) into the expression above and rearranging yields:

$$\begin{aligned} d \log c_{ht}^{\text{comp}}(s) = -\eta \Bigg(& d \log \mu_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s') \, d \log \mu_{ht}(s)}{\sum_{t',s'} \beta^{t'} \pi(s')} \\ & - \sum_{h'} \omega_{h't}^0(s) \left(d \log \mu_{h't}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s') \, d \log \mu_{h't}(s)}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \Bigg). \end{aligned}$$

We also have

$$\frac{d}{d\sigma} \log \mu_{ht}(s) = -\frac{1}{\eta} \left[\frac{d \log \omega_{ht}^0(s)}{d\sigma} - \frac{d \log \omega_{h0}^0}{d\sigma} \right] \equiv -\frac{1}{\eta} \left[d \log \omega_{ht}^0(s) - d \log \omega_{h0}^0 \right],$$

where the last equality is a notational convention. We can now substitute these back into our Harberger triangle formula, (18):

$$
\begin{aligned}
\Delta \log A \approx\ & \frac{1}{2}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\eta\left(d\log\mu_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\mu_{ht'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right.\right. \\
& \left.\left. -\sum_{h'}\omega^{0}_{h't}(s)\left(d\log\mu_{h't}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\mu_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right)\right)\right] d\log\mu_{ht}(s) \\
=\ & -\frac{1}{2}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\left(d\log\omega^{0}_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{ht'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right.\right. \\
& \left.\left. -\sum_{h'}\omega^{0}_{h't}(s)\left(d\log\omega^{0}_{h't}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right)\right)\right] d\log\mu_{ht}(s) \\
=\ & \frac{1}{2}\frac{1}{\eta}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[d\log\omega^{0}_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{ht'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right] \\
& \times\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right] \\
& -\frac{1}{2}\frac{1}{\eta}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega^{0}_{h't}(s)\left(d\log\omega^{0}_{h't}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right)\right] \\
& \times\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right] \\
=\ & \frac{1}{2}\frac{1}{\eta}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[d\log\omega^{0}_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{ht'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right]\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right] \\
& +\frac{1}{2}\frac{1}{\eta}\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega^{0}_{h't}(s)\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right]\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right],
\end{aligned}
$$

where the final line uses the fact that $\sum_{h'}\omega^{0}_{h't}(s)\,d\log\omega^{0}_{h't}(s)=0$. We now focus on the last line of the expression above, and show that it is equal to zero to a second-order approximation. To do so, introduce a new symbol:

$$
x_{h'} \equiv \frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}.
$$

Using this symbol, we show that the following term is zero to a second-order:

$$
\begin{aligned}
\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega^{0}_{h't}(s)\frac{\sum_{t',s'}\beta^{t'}\pi(s')\,d\log\omega^{0}_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi(s')}\right]\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right] = \\
\sum_{h}\sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega^{0}_{h't}(s)\,x_{h'}\right]\left[d\log\omega^{0}_{ht}(s)-d\log\omega^{0}_{h0}\right]. \quad (24)
\end{aligned}
$$

Define

$$\bar{\omega}_h^0 = \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) \omega_{h't'}^0(s')}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}.$$

Note that

$$\omega_{ht}^0(s) = \bar{\omega}_h + \frac{d\omega_{ht}(s)}{d\sigma} d\sigma.$$

Hence, we can substitute this into (24) to get

$$\begin{aligned}
&\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \omega_{h't}^0\left(s\right) d\log\bar{\omega}_{h'}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \left[\bar{\omega}_{h'} + \frac{d\omega_{h't}(s)}{d\sigma} d\sigma\right] x_{h'}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right].
\end{aligned}$$

Dropping higher order terms gives

$$\begin{aligned}
&\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \omega_{h't}^0\left(s\right) d\log\bar{\omega}_{h'}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} x_{h'}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right].
\end{aligned}$$

Next substitute $x_{h'}$ back in to get

$$\begin{aligned}
&\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} x_{h'}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) \left[d\log\omega_{h't'}^0(s')\right]}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta) \sum_{t',s'} \beta^{t'} \pi\left(s'\right) \sum_{h'} \bar{\omega}_{h'} \left[d\log\omega_{h't'}^0(s')\right]\right] \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta) \sum_{t',s'} \beta^{t'} \pi\left(s'\right) \sum_{h'} \left[\omega_{h't}^0(s) - \frac{d\omega_{h't}(s)}{d\sigma} d\sigma\right] \left[d\log\omega_{h't'}^0(s')\right]\right] \\
&\times \left[d\log\omega_{ht}^0(s) - d\log\omega_{h0}^0\right].
\end{aligned}$$

Again drop higher order terms to get

$$\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} x_{h'}\right] \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right]$$
$$\approx \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta)\sum_{t',s'} \beta^{t'} \pi\left(s'\right) \sum_{h'} \omega^0_{h't}(s) d\log\omega^0_{h't'}(s')\right] \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right]$$
$$= 0,$$

since $\sum_{h'} \omega^0_{h't}(\mathrm{s})\mathrm{dlog}\,\omega^0_{h't'}(\mathrm{s'}) = 0$. This allows us to write

$$\Delta\log A \approx \frac{1}{2\eta} \sum_{h,s,t} q_t(s) c_{ht}(s) \left(d\log\omega^0_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) d\log\omega^0_{ht}(s)}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right) \left[d\log\omega^0_{ht'}(s') - d\log\omega^0_{h0}\right].$$

Next, we use the fact that, at $\sigma = 0$ and $z_t = \bar{z}$, we have $q_t(s) = (1+r)^{-t} / \sum(1+r)^{-t'}\bar{y}$ and $c_{ht}(s) = \omega_{h0}\bar{y}$. Note that the denominator in the Arrow security price is needed to ensure aggregate wealth is equal to one (our choice of numeraire).

So, we can again, evaluate the coefficients at this point (since differences are higher order):

$$\Delta\log A \approx$$
$$\frac{1}{2}\frac{1}{\eta} \sum_h \sum_{s,t} \frac{(1+r)^{-t}}{\sum_{t'}(1+r)^{-t'}\bar{y}} \omega_{h0}\bar{y} \left(d\log\omega^0_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) d\log\omega^0_{hst'}(s')}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right) \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right]$$
$$= \frac{1}{2}\frac{1}{\eta} \sum_h \sum_{s,t} \frac{(1+r)^{-t}}{\sum_{t'}(1+r)^{-t'}} \omega_{h0} \left(d\log\omega^0_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) d\log\omega^0_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right) \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right]$$
$$= \frac{1}{2}\frac{1}{\eta} \sum_h \sum_{s,t} \frac{r}{1+r} (1+r)^{-t} \omega_{h0} \left(d\log\omega^0_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) d\log\omega^0_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right) \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right]$$
$$= \frac{1}{2}\frac{1}{\eta} \sum_h \sum_{s,t} \frac{r}{(1+r)^{t+1}} \omega_{h0} \left(d\log\omega^0_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi\left(s'\right) d\log\omega^0_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi\left(s'\right)}\right) \left[d\log\omega^0_{ht}(s) - d\log\omega^0_{h0}\right].$$

Finally, we use the fact that

$$d\log\omega^0_{ht}(s) \approx \log\omega^0_{ht}(s) - \log\omega_h,$$

where $\log\omega_h$ is household $h's$ consumption share when $\sigma = 0$ (i.e. the point of approximation). Hence, if we substitute this into the Harberger formula and cancel, we get

$$
\begin{aligned}
\Delta \log A \approx & \frac{1}{2}\frac{1}{\eta}\sum_h \sum_{s,t} \frac{r}{(1+r)^{t+1}} \omega_{h0} \left[\left(\left[\log \omega_{ht}^{0}(s) - \log \omega_h \right] - \frac{\sum_{t',s'} \beta^{t'} \pi(s') \left[\log \omega_{ht}^{0}(s) - \log \omega_h \right]}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right] \\
& \times \left[\left(\log \omega_{ht}^{0}(s) - \log \omega_h \right) - \left(\log \omega_{h0}^{0}(s) - \log \omega_h \right) \right] \\
= & \frac{1}{2}\frac{1}{\eta}\sum_h \sum_{s,t} \frac{r}{(1+r)^{t+1}} \omega_{h0} \left[\left(\log \omega_{ht}^{0}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s') \left[\log \omega_{ht}^{0}(s) \right]}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right] \\
& \times \left[\log \omega_{ht}^{0}(s) - \log \omega_{h0}^{0}(s) \right].
\end{aligned}
$$

□

Proof of Proposition 5. In this proof, we use $C_t(s)$ and $\boldsymbol{C}$ to denote the aggregate consumption process, instead of lower case $c_t(s)$ and $\boldsymbol{c}$, so as to avoid confusion with consumption allocations, $\boldsymbol{c} = \{\boldsymbol{c}_h\}$. Observe that consumption allocations on the Pareto frontier must satisfy $c_{ht}(s) = \lambda_h C_t(s)$ for some household-specific λ_h with $\sum_h \lambda_h = 1$. Define $\Gamma(k_0)$ to the set of feasible aggregate consumption paths given initial capital stock k_0. Then substituting this into (2), and manipulating yields

$$
\begin{aligned}
A &= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \tilde{u}_h(\boldsymbol{c}_h) : \{\boldsymbol{c}_h\} \in \mathcal{C} \right\}, \\
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \lambda_h \tilde{u}_h(\boldsymbol{C}) : \boldsymbol{C} \in \Gamma(k_0) \right\},
\end{aligned}
$$

using the functional form for $\tilde{u}_h(\boldsymbol{c}_h) = \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}{}^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^{0}(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}}$, we have

$$
\begin{aligned}
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \lambda_h \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^{0}(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}, \\
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^{0}(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}, \\
&= \max_{\boldsymbol{\lambda}} \min_h \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^{0}(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \right\} \max_{\boldsymbol{C}} \left\{ \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}.
\end{aligned}
$$

Let $\boldsymbol{C}^*$ be the maximizing choice of aggregate consumption in the maximization problem above. Then, we know that $\lambda_h \tilde{u}_h(\boldsymbol{C}^*) = \lambda_{h'} \tilde{u}_{h'}(\boldsymbol{C}^*)$. Furthermore, we know that the solution to

$$\max_{\boldsymbol{\lambda}} \min_{h} \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \right\}$$

implies that for some fixed h and every h'

$$\lambda_h \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}} = \lambda_{h'},$$

as well as

$$\sum_{h'} \lambda_{h'} = 1.$$

Combining these equations yields

$$\lambda_h = \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}},$$

for every h. Hence,

$$\begin{aligned} A &= \lambda_h \tilde{u}_h(\boldsymbol{C}^*) \\ &= \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}} \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t^*(s)^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}} \\ &= \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t^*(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}. \end{aligned}$$

Substituting this into (4) and using the definition of certainty equivalent yields the desired result. □

Proof of Proposition 6. Recall from Definition 4 that a general equilibrium with wedges is a collection of prices and quantities such that: (1) the price of each good i equals its marginal cost of production; (2) each producer takes prices as given and chooses quantities to maximize profits; (3) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and income as given; (4) household h earns income from primary factors and tax revenues; (5) all resource constraints are satisfied.

We show that given the wedges in (16), the status-quo allocation, and status-quo static relative prices constitute a decentralized equilibrium with wedges. We do this by showing that there is some distribution of household wealth and Arrow security prices (to be determined), such that the status-quo allocations and prices maximize each household's problem, cause prices to equal marginal cost for each good, and statisfy all resource constraints.

By construction, the static relative prices satisfy the firms' first-order conditions and set prices equal to marginal costs. Furthermore, all resource constraints are satisfied by construction. Hence, we only need to check that the candidate equilibrium can satisfy the households' problem. Consider the households' problem:

$$\max_{c_{ht}(s)} \frac{1}{1-\frac{1}{\eta}} \sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}$$

subject to

$$\sum_s \sum_t q_t(s) p_{ht}(s) \mu_{ht}(s) c_{ht}(s) \leq I_h,$$

where I_h is the household's wealth. We assume that aggregate wealth is the numeraire, so that

$$\sum_h I_h = 1.$$

Equation (16) implies that the wedges are given by:

$$\mu_{ht}(s) = \left[\frac{p^0_{ht}(s) / p^0_{h0}}{p^0_{\bar{h}t}(s) / p^0_{\bar{h}0}}\right]^{\frac{1-\eta}{\eta}} \left[\frac{\omega^0_{ht}(s) / \omega^0_{h0}}{\omega^0_{\bar{h}t}(s) / \omega^0_{\bar{h}0}}\right]^{-\frac{1}{\eta}}, \tag{25}$$

where $\omega^0_{ht}(s)$ and $p^0_{ht}(s)$ are expenditures and goods' prices in the status-quo. In this equation, we explicitly distinguish between prices in the decentralized equilibrium with these wedges, $p_{ht}(s)$ and the prices in the primitive economy in status-quo $p^0_{ht}(s)$. This abuse of notation is harmless, since, as we show in this proof, $p^0_{ht}(s)$ is consistent with equilibrium in the decentralized economy with wedges.

We show that the status-quo allocation and static relative prices are decentralized equilibria by showing that all remaining equilibrium conditions, namely first-order conditions for utility maximization, household budget constraints, and the numeraire condition, can all be satisfied given these wedges, relative static prices, and quantities. To that end, in the decentralized equilibrium with wedges that we construct, the consumption allocation can be expressed as

$$c_{ht}(s) = c_{ht}^0(s) = \frac{\omega_{ht}^0(s)\, E_t^0(s)}{p_{ht}^0(s)},$$

where

$$E_t^0(s) = \sum_h p_{ht}^0(s)\, c_{ht}^0(s).$$

The first-order condition for $c_{ht}(s)$, given Lagrange multiplier ϕ_h and Arrow security prices $q_t(s)$ is

$$\beta^t \pi(s) c_{ht}^{-\frac{1}{\eta}}(s) = \phi_h q_t(s) p_{ht}(s) \mu_{ht}(s).$$

Substituting (25) into this first-order condition yields the following restriction on Lagrange multipliers ϕ_h and Arrow prices $q_t(s)$:

$$\beta^t \pi(s) p_{ht}^{\frac{1}{\eta}}(s) E_t^{-\frac{1}{\eta}}(s) \omega_{ht}^{-\frac{1}{\eta}}(s) = \phi_h q_t(s) p_{ht}(s) \left[\frac{p_{ht}^0(s)/p_{h0}^0}{p_{\bar{h}t}^0(s)/p_{\bar{h}0}^0}\right]^{\frac{1-\eta}{\eta}} \left[\frac{\omega_{ht}^0(s)/\omega_{h0}^0}{\omega_{\bar{h}t}^0(s)/\omega_{\bar{h}0}^0}\right]^{-\frac{1}{\eta}}. \quad (26)$$

We conjecture that $p_{ht}^0(s) = p_{ht}(s)$ is consistent with equilibrium. Then, dividing this equation for h' by $\bar{h}$ for some fixed household $\bar{h}$ allows us to express ϕ_h as a function of $\phi_{\bar{h}}$:

$$\phi_{h'} = \phi_{\bar{h}} \left(\frac{\omega_{\bar{h}0}^0}{\omega_{h'0}^0}\right)^{\frac{1}{\eta}} \left(\frac{p_{h0}^0}{p_{\bar{h}0}^0}\right)^{\frac{1-\eta}{\eta}}. \quad (27)$$

Substituting the FOC into the budget constraint pins down household wealth as a function of Lagrange multipliers ϕ_h:

$$\frac{\sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s)}{\phi_h} = I_h. \quad (28)$$

The numeraire condition pins down $\phi_{\bar{h}}$:

$$\sum_{h'} I_{h'} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{h'}} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{\bar{h}} \left(\frac{\omega_{\bar{h}0}^0}{\omega_{h'0}^0}\right)^{\frac{1}{\eta}} \left(\frac{p_{h0}^0}{p_{\bar{h}0}^0}\right)^{\frac{1-\eta}{\eta}}} = 1$$

Hence, we require that

$$\phi_{\bar{h}} = \sum_{h} \left(\frac{\omega_{\bar{h}0}^{0}}{\omega_{h0}^{0}} \right)^{-\frac{1}{\eta}} \left(\frac{p_{h0}^{0}}{p_{\bar{h}0}^{0}} \right)^{\frac{\eta-1}{\eta}} \sum_{s} \sum_{t} \beta^{t} \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s). \tag{29}$$

Hence, (29) pins down $\phi_{\bar{h}}$, (27) pins down ϕ_h for every other h, (28) pins down I_h, and (26), for any h, pins down the Arrow security prices $q_t(s)$ in the decentralized equilibrium with wedges that supports the status-quo allocation. Since expenditures in the decentralized equilibrium with wedges coincides with expenditures in the status-quo, and production technologies are all the same, and all firms set prices equal to marginal cost, static relative prices in the status-quo are consistent with equilibrium, which confirms the conjecture above.

□

Proof of Proposition 7. We follow the same steps as in the proof of Proposition 4, but this time allowing for differences in consumption baskets across households. First, index allocations by an exogenous scalar parameter σ. We assume that allocations are a smooth function of σ, which means that $\boldsymbol{\mu}(\sigma)$, defined by (16) is also smooth. We assume that $\boldsymbol{\mu}(0) = \mathbf{1}$ and $\boldsymbol{z}_t(\sigma) = \bar{\boldsymbol{z}}$, so that at $\sigma = 0$, wedges are all equal to one and productivities are constant over time and state. For example, the parameter σ could index the standard deviation of h-level productivities.

As before, following Proposition 8 from Baqaee and Burstein (2025b), we express misallocation to a second-order approximation in the parameter σ as

$$\log A = -\frac{1}{2} \sum_{h} \sum_{s,t} q_t(s) p_{ht}(s) c_{ht}(s) d \log c_{ht}^{\text{comp}}(s) d \log \mu_{ht}(s). \tag{30}$$

Here, $d \log c_{ht}^{\text{comp}}(s)$ is a short-hand to mean $\frac{d \log c_{ht}^{\text{comp}}(s)}{d \log \boldsymbol{\mu}} \cdot \frac{d \log \boldsymbol{\mu}}{d\sigma}$ and $d \log \mu_{ht}(s)$ to mean $d \log \boldsymbol{\mu}/d\sigma$ in the compensated equilibrium evaluated at $\sigma = 0$, with $\boldsymbol{\mu}(0) = \mathbf{1}$. As before, we take aggregate wealth to be the numeraire.

The first-order conditions of the compensated agent imply

$$\frac{c_{ht}^{\text{comp}}(s)}{c_{\bar{h}t}^{\text{comp}}(s)} = \left(\frac{p_{ht}(s) \mu_{ht}(s) \phi_{h}^{-1}}{p_{\bar{h}t}(s) \mu_{\bar{h}t}(s) \phi_{\bar{h}}^{-1}} \right)^{-\eta} \left(\frac{\tilde{u}_h}{\tilde{u}_{\bar{h}}} \right)^{\eta}. \tag{31}$$

The derivation is very similar to that of equation (20) but allows for difference in households' consumption baskets.

We now solve for $d \log c_{ht}^{\text{comp}}(s)$ up to a first-order approximation. Log differentiate

(31) and use the fact that the representative agent keeps $\tilde{u}_h / \tilde{u}_{\bar{h}}$ unchanged to obtain

$$d\log c_{ht}^{\text{comp}}(s) - d\log c_{\bar{h}t}^{\text{comp}}(s) = -\eta d\log \frac{p_{ht}^{\text{comp}}(s)}{p_{\bar{h}t}^{\text{comp}}(s)} - \eta\left(d\log\frac{\mu_{ht}(s)}{\phi_h} - d\log\frac{\mu_{\bar{h}t}(s)}{\phi_{\bar{h}}}\right), \tag{32}$$

where ϕ_h is household h's Lagrange multiplied defined in (19). This equation pins down final demand in the compensated economy as a function of relative prices in the compensated economy, wedges, and Lagrange multipliers.

To pin down these down, we must introduce some notation from the production side of the model. Define the within-period (static) $(H+N+F)\times(H+N+F)$ input-output matrix:

$$\Omega = \left[\begin{array}{ccc|ccc|ccc}
0 & \cdots & 0 & b_{11} & \cdots & b_{1N} & 0 & \cdots & 0 \\
\vdots & \cdots & \vdots & & \cdots & & & \cdots & \\
0 & \cdots & 0 & b_{H1} & \cdots & b_{HN} & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & \Omega_{11} & \cdots & \Omega_{1N} & \Omega_{1N+1} & \cdots & \Omega_{1N+F} \\
\vdots & \cdots & \vdots & & \ddots & & & & \\
0 & \cdots & 0 & \Omega_{N1} & & \Omega_{NN} & \Omega_{NN+1} & \cdots & \Omega_{NN+F} \\
\hline
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{array}\right]$$

The first H rows correspond to the households consumption baskets. The next N rows correspond to the expenditure shares of each producer on every other producer and factor. The last F rows correspond to the expenditure shares of the primary factors. Every row of Ω adds up to one or zero.

The Leontief inverse matrix is the $(H+N+F)\times(H+N+F)$ matrix defined as

$$\Psi \equiv (I-\Omega)^{-1}.$$

Let ω_h denote the share of expenditures of household h in total expenditures in a given period. That is,

$$\omega_{ht}(s) = \frac{p_{ht}(s)\, c_{ht}(s)}{\sum p_{h't}(s)\, c_{h't}(s)} = \frac{p_{ht}(s)\, c_{ht}(s)}{E_t(s)},$$

where $E_t(s)$ denotes total final expenditures in period t and date s (not including wedge revenues).

The within-period Domar weights are denoted by λ, where $\lambda_{it}(s)$ are the sales of i in period t and state s relative to total final expenditures, $E_t(s)$, in that date and state[26]

$$\lambda_{it}(s) = \frac{p_{it}(s) y_{it}(s)}{E_t(s)} \mathbf{1}[i \in N] + \frac{w_{it}(s) l_{it}(s)}{E_t(s)} \mathbf{1}[i \in F] + \frac{p_{it}(s) c_{it}(s)}{E_t(s)} \mathbf{1}[i \in H].$$

Market clearing identities imply that:

$$\lambda_t'(s) = \omega_t'(s) + \lambda_t'(s) \Omega_t(s) = \omega_t(s)' \Psi_t(s).$$

The following equations hold to a first-order in the compensated equilibrium. Goods' prices in a given period and state are given by the Leontief-inverse weighted changes in factor prices in that period and state:

$$d \log p_t^{\text{comp}}(s) = \sum_f \Psi_{(:,f)} d \log \lambda_{ft}^{\text{comp}}(s) + d \log E_t^{\text{comp}}(s), \tag{33}$$

where $\lambda_{ft}(s)$ is the sales of factor f in period t and state s relative to total final expenditures $E_t^{\text{comp}}(s) = \sum_h \mu_{ht}(s) p_{ht}^{\text{comp}}(s) c_{ht}^{\text{comp}}(s)$ and $\Psi_{(:,f)}$ is fth column of Ψ corresponding to factor f. We use the fact that changes in the wage of factor f in period t and state s is, in logs, the same as the change in expenditures on that factor (since factor quantity is held fixed in the variation). This equation is standard and follows from Shephard's lemma, factor market clearing in the compensated equilibrium, and the fact that $\sum_f \Psi_{(:,f)}$ is a vector of all ones.

Next, we note changes in the input-output matrix in period t and state s depend on changes in relative prices, which implies:

$$d\Omega_t^{\text{comp}}(s) = \Theta d \log \lambda_t^{\text{comp}}(s), \tag{34}$$

where Θ is some matrix involving cross-price elasticities and the Leontief inverse (we provide the specific formula for Θ below).

Finally, differentiating the identity that $\left(\lambda_t^{\text{comp}}(s)\right)' = (\omega_t^{\text{comp}}(s))' \Psi_t^{\text{comp}}(s)$ gives

$$\left(d\lambda_t^{\text{comp}}(s)\right)' = (\omega^{\text{comp}})' \Psi d\Omega_t^{\text{comp}}(s) \Psi + \left(d\omega_t^{\text{comp}}(s)\right)' \Psi \tag{35}$$

where we use the fact that at the point of approximation, $\Psi_t^{\text{comp}}(s) = \Psi_t(s)$ and where

[26]Within-period Domar weights are sales in a period divided by total consumption expenditures in that period. We refer to these as within-period Domar weights to contrast them with Arrow-Debreu Domar weights, which are net present value sales divided by net present value of total consumption using Arrow securities.

$\omega_{ht}^{\text{comp}}(s)$ denotes household h's share of total final spending in that period and state. Finally, by definition,

$$d\log c_t^{\text{comp}}(s) = d\log\omega_t^{\text{comp}}(s) - \left[d\log p_t^{\text{comp}}(s) - d\log E_t^{\text{comp}}(s)\right]. \tag{36}$$

Inspecting the above linearized system (32), (33), (34), (35), and (36), we can express $d\log c_{ht}^{\text{comp}}(s)$ as a function of $\{d\log\mu_{ht}(s) - d\log\phi_h\}$:

$$d\log c_{ht}^{\text{comp}}(s) = \mathcal{M}_{(h,:)}\left(d\log\boldsymbol{\mu}_t(s) - d\log\boldsymbol{\phi}\right),$$

where $d\log\boldsymbol{\phi} = \{d\log\phi_h\}_{h=1}^H$, $d\log\boldsymbol{\mu}_t = \{d\log\mu_{ht}\}_{h=1}^H$, and $M_{(h,:)}$ are vectors for each h with coefficients that depend on parameters and shares in the allocations without shocks (see below for the explicit functional form).

We now solve for $d\log\phi_h$. The condition $d\log\tilde{u}_h = d\log\tilde{u}_{\bar{h}}$ can be expressed as

$$\frac{\sum_{s,t}\pi(s)\beta^t c_{ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{s,t'}\pi(s)\beta^{t'} c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} d\log c_{ht}^{\text{comp}}(s) = \frac{\sum_{s,t}\pi(s)\beta^t c_{\bar{h}t}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t'}\pi(s)\beta^{t} c_{\bar{h}t'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} d\log c_{\bar{h}t}^{\text{comp}}(s)$$

We evaluate this expression at the point with no shocks, $\sigma = 0$, where we know that $c_{ht}^{\text{comp}}(s) = c_h$ for every h, giving us

$$\sum_{t,s}\beta^t\pi(s)d\log c_{ht}^{\text{comp}}(s) = \sum_{t,s}\beta^t\pi(s)d\log c_{\bar{h}t}^{\text{comp}}(s).$$

Substituting

$$\sum_{s,t}\pi(s)\beta^t\sum_{h'}\left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right]d\log\mu_{h't}(s) = \sum_{s,t}\pi(s)\beta^t\sum_{h'}\left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right]d\log\phi_{h'},$$

or,

$$\sum_{h'}\left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right]\frac{\sum_{s,t}\pi(s)\beta^t d\log\mu_{h't}(s)}{\sum_{s,t}\pi(s)\beta^t} = \sum_{h'}\left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right]d\log\phi_{h'}.$$

Therefore, a solution is to set

$$d\log\phi_{h'} = \frac{\sum_{s,t}\pi(s)\beta^t d\log\mu_{h't}(s)}{\sum_{s,t}\pi(s)\beta^t}.$$

Substitute this solution into (32) to get

$$d\log c_{ht}^{\text{comp}}(s) - d\log c_{\bar{h}t}^{\text{comp}}(s) = -\eta\left(d\log p_{ht}^{\text{comp}}(s) - d\log p_{\bar{h}t}^{\text{comp}}(s)\right) \tag{37}$$
$$-\eta\left(d\log\mu_{ht}(s) - \frac{\sum_{s',t'}\pi(s')\beta^{t'}d\log\mu_{ht'}(s')}{\sum\pi(s')\beta^{t'}}\right.$$
$$\left.-d\log\mu_{\bar{h}t}(s) + \frac{\sum_{s',t'}\pi(s')\beta^{t'}d\log\mu_{\bar{h}t'}(s')}{\sum\pi(s')\beta^{t'}}\right).$$

Combining this with (33), (34), (35), and (36) results in a linear system of equations that pins down $d\log c_{ht}^{\text{comp}}(s)$ in the compensated equilibrium as a function of Ψ, ω, cross-price elasticities in production and consumption (which determine Θ), and the forcing terms

$$d\log\mu_{ht}(s) - \frac{\sum_{t',s'}\pi(s')\beta^{t'}d\log\mu_{ht'}(s')}{\sum\pi(s')\beta^{t'}},$$

all of which can be recovered from the decentralized equilibrium at the status-quo. This results in the matrix of interest $\mathcal{M}$, mentioned in the statement of the proposition. We provide an explicit formula for $\mathcal{M}$ in terms of price elasticities and expenditure shares below. But first, we show that the choice of $\bar{h}$ does not affect the results of the proposition.

Showing that the choice of $\bar{h}$ is irrelevant. Here, we also note that the choice of $\bar{h}$ does not affect the approximation formula in (30). First, note that, by inspection, $\sum_{h'}\mathcal{M}_{h,h'} = 0$, since the forcing terms only show up in difference in (37). Hence, adding the same constant to every forcing does not alter $d\log \boldsymbol{c}_t^{\text{comp}}(s)$, which means that $\sum_{h'}\mathcal{M}_{h,h'} = 0$.

We now show that changing $\bar{h}$ has no effect on (30) by using this fact. Start by rearraing:

$$\begin{aligned}
\log A &\approx \sum_h\sum_{s,t} q_t(s)p_{ht}(s)c_{ht}(s)\,d\log c_{ht}^{\text{comp}}(s)\,d\log\mu_{ht}(s)\\
&= \sum_{s,t} q_t(s)E_t(s)\sum_h \omega_{ht}(s)\,d\log c_{ht}^{\text{comp}}(s)\,d\log\mu_{ht}(s)\\
&= \sum_{s,t} q_t(s)E_t(s)\sum_h \omega_{ht}(s)\sum_{h'}\mathcal{M}_{h,h'}\left[d\log\mu_{h't}(s) - d\log\bar{\mu}_{h'}\right]d\log\mu_{ht}(s).
\end{aligned}$$

We show that this is also equal to

$$\log A^{\text{alt}} = \sum_{s,t} q_t(s)E_t(s)\sum_h \omega_{ht}(s)\sum_{h'}\mathcal{M}_{h,h'}\left[d\log\mu_{h't}(s) - d\log\bar{\mu}_{h'} + y_t(s)\right]\left[d\log\mu_{ht}(s) + x_t(s)\right],$$

for any $x_t(s)$ and $y_t(s)$, which is what changing $\bar{h}$ does. To see, consider

$$
\begin{aligned}
\log A^{\text{alt}} &= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} + y_t(s) \right] \left[d\log \mu_{ht}(s) + x_t(s) \right] \\
&= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} \right] \left[d\log \mu_{ht}(s) \right] \\
&\quad + \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s)\, y_t(s)\, d\log \mu_{ht}(s) \underbrace{\sum_{h'} \mathcal{M}_{h,h'}}_{=0} \\
&\quad + \sum_{s,t} q_t(s) E_t(s)\, x_t(s) \underbrace{\sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} \right]}_{=0} \\
&\quad + \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s)\, y_t(s)\, x_t(s) \underbrace{\sum_{h'} \mathcal{M}_{h,h'}}_{=0} \\
&= \log A.
\end{aligned}
$$

The first underbrace follows from the fact that $\sum_{h'} \mathcal{M}_{h,h'} = 0$, the second underbrace follows from the Envelope theorem, which implies that production is statically efficient, so that aggregate real consumption in each date and state does not respond to wedges to a first order, and the last underbrace follows from $\sum_{h'} \mathcal{M}_{h,h'} = 0$.

Explicit Formula for $\mathcal{M}$. To make this more explicit, we now provide the explicit expression for $\mathcal{M}_{hh'}$. For any three positive vectors a, b, and c, define the covariance by

$$Cov_a(b,c) = \frac{\sum_i a_i b_i c_i}{\sum_{i'} a_{i'}} - \frac{\sum_i a_i b_i}{\sum_{i'} a_{i'}} \frac{\sum_i a_i c_i}{\sum_{i'} a_{i'}}.$$

Define the $F \times F$ matrix B with element (f, f) given by

$$B_{f,f'} = (1-\eta)\, Cov_{\omega'}(\Psi_{(:,f)}, \Psi_{(:,f')}) + \sum_{f' \in F} \sum_{j \in H \cup N} \lambda_j (1-\theta_j) Cov_{\Omega_{(j,:)}}(\Psi_{(:,f)}, \Psi_{(:,f')}),$$

where $\Omega_{(j,:)}$ is the jth row of Ω. Define the $F \times H$ matrix D with element (f, h) given by

$$D_{f,h} = Cov_{\omega'}(e_h, \Psi_{(:,f)})$$

where e_h is the h-th basis vector column vector (with hth element equal to 1). Define the $H \times H$ matrix.

$$F = -\eta \Psi_{HF} \left(diag(\lambda_F) - B \right)^{-1} D.$$

where Ψ_{HF} is the $H \times F$ block of Ψ corresponding to households' direct and indirect exposure to each factor and λ_F is the $F \times 1$ vector of static factor shares (the last F elements of λ). Finally, define the $H \times H$ matrix,

$$\mathcal{M} = -\eta\,(F + I) - (1-\eta)\mathbf{1}\omega' F + \mathbf{1}\eta\omega'.$$

The element (h, h') of $\mathcal{M}$ is $\mathcal{M}_{hh'}$ in Proposition 7. Note that $\mathcal{M}$ depends on the input-output matrix Ω, expenditure shares ω, and elasticities of substitution in production and consumption, θ_j for $j \in H \cup N$, and the EIS η.

Substituting our expression for $d \log c_t^{\text{comp}}(s)$ into (30) yields the desired expression. The last step is to recognize that at $\sigma = 0$, we have

$$q_t(s) p_{ht}(s) c_{ht}(s) = \frac{(1+r)^{-t} p_h c_h}{\sum_{h',t',s'} (1+r)^{-t'} p_{h'} c_{h'}} = \frac{(1+r)^{-t}}{\sum_{t'} (1+r)^{-t'}} \frac{p_h c_h}{\sum_{h'} p_{h'} c_{h'}}.$$

Hence, we can replace

$$q_t(s) p_{ht}(s) c_{ht}(s) = \frac{(1+r(0))^{-t}}{\sum_{t'} (1+r(0))^{-t'}} \omega_h(0),$$

where $\omega_h(0)$ and $r(0)$ make explicit the dependence of ω_h and r on $\sigma = 0$. We can replace these with the expenditure shares, $\omega_{h0}(\sigma)$, and risk free rate in the first period, $r(\sigma)$, both in the decentralized equilibrium, since any differences will be third order. □

Derivation of equation (5). Consider first the numerator. The maximization problem

$$\max_{\boldsymbol{c} \in \mathcal{C}} CE^{VOI}(\boldsymbol{c})$$

splits aggregate consumption uniformly across all agents:

$$\boldsymbol{c}_h^* = \frac{\sum_{h'} \boldsymbol{c}_{h'}^0}{H}.$$

The certainty-equivalent of a population-weighted lottery of $\{\boldsymbol{c}_h^*\}$ is

$$CE^{VOI}\left(\{\boldsymbol{c}_h^*\}\right) = \left((1-\beta) \left(1 - \tfrac{1}{\eta}\right) \sum_{h \in H} \frac{1}{H} u(\boldsymbol{c}_h^*) \right)^{\frac{\eta}{\eta-1}} = CE\left(\frac{\sum_{h'} \boldsymbol{c}_{h'}^0}{H} \right),$$

where we used the fact that

$$CE(\boldsymbol{c}_h) = \left((1-\beta)\left(1-\tfrac{1}{\eta}\right)u(\boldsymbol{c}_h)\right)^{\frac{\eta}{\eta-1}}.$$

Consider now the denominator:

$$CE^{VOI}(\{\boldsymbol{c}_h^0\}) = \left((1-\beta)\left(1-\tfrac{1}{\eta}\right)\sum_{h\in H}\frac{1}{H}u(\boldsymbol{c}_h^0)\right)^{\frac{\eta}{\eta-1}} = \left(\sum_{h\in H}\frac{CE(\boldsymbol{c}_h^0)^{\frac{\eta-1}{\eta}}}{H}\right)^{\frac{\eta}{\eta-1}}.$$

Taking the ratio of the numerator and denominator yields equation (5).

Derivation of equation (11) Suppose that preferences take the standard functional form in Example 1. In this case, $CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$ is the value of CE that solves

$$\frac{1}{1-\beta}\frac{1}{1-1/\eta}\left(CE^{\gamma}\bar{l}^{1-\gamma}\right)^{1-\frac{1}{\eta}} = u_h(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

so

$$CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \underbrace{\frac{\left[(1-\beta)(1-1/\eta)\right]^{\frac{\eta}{\gamma(\eta-1)}}}{\bar{l}^{\frac{1-\gamma}{\gamma}}}}_{\text{constant}} \times \left(u_h(\boldsymbol{c}_h, \boldsymbol{l}_h)\right)^{\frac{\eta}{(\eta-1)\gamma}},$$

and

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \left(u(\boldsymbol{c}_h, \boldsymbol{l}_h)\right)^{\frac{\eta}{(\eta-1)\gamma}}.$$

We now calculate $\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*)$ for any allocation $\{\boldsymbol{c}_h^*, \boldsymbol{l}_h^*\}$ in the Pareto efficient frontier. Under the assumptions of this example,

$$c_{ht}^*(s) = c_h^*, \quad l_{ht}^*(s) = l_h^*, \quad \text{with } \frac{\partial v(c_h^*, l_h^*)}{\partial c_h} = \frac{\partial v(c_h^*, l_h^*)}{\partial l_h},$$

or

$$l_h^* = \frac{1-\gamma}{\gamma}c_h^*.$$

Hence,

$$u_h(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \frac{1}{1-1/\eta}\frac{1}{1-\beta}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\gamma)(\eta-1)}{\eta}}(c_h^*)^{\frac{\eta-1}{\eta}},$$

and

$$\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \frac{[(1-\beta)(1-1/\eta)]^{\frac{\eta}{\gamma(\eta-1)}}}{\bar{l}^{\frac{1-\gamma}{\gamma}}} \times \sum_h (u(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*))^{\frac{\eta}{(\eta-1)\gamma}} = \underbrace{\left(\frac{1-\gamma}{\gamma}\frac{1}{\bar{l}}\right)^{\frac{1-\gamma}{\gamma}}}_{\text{constant}} \times \sum_h (c_h^*)^{\frac{1}{\gamma}}.$$

Any allocation $(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*)$ in the Pareto frontier satisfies

$$\sum_h c_h^* = y^* = \sum_h \left(1 - \frac{1-\gamma}{\gamma} c_h^*\right),$$

so

$$\sum_h c_h^* = y^* = \gamma \sum_h 1.$$

Setting $c_h^* = \alpha_h \gamma \sum_h 1$, with $\sum \alpha_h = 1$, we obtain

$$\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \underbrace{\left(\frac{1-\gamma}{\bar{l}}\right)^{\frac{1-\gamma}{\gamma}} \gamma}_{\text{constant}} \times \sum_h (\alpha_h)^{\frac{1}{\gamma}},$$

which corresponds to equation (11). This measure assigns (weakly) higher values to more unequal weights $\{\alpha_h\}$ since $\gamma \leq 1$.