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ABSTRACT

Theory suggests that in the face of fire sale externalities, banks have incentives to overinvest in order to issue excessive money-like deposit liabilities. The existence of a private market for insurance such as contingent capital can eliminate the overinvestment incentives, leading to efficient outcomes. However, it does not eliminate fire sales. A central bank that can infuse liquidity cheaply may be motivated to intervene in the face of fire sales. If so, it can crowd out the private market and, if liquidity intervention is not priced at higher-than-breakeven rates, induce overinvestment. We examine various forms of public intervention to identify the least distortionary ones. Our analysis helps understand the historical prevalence of private insurance in the era preceding central banks and deposit insurance, their subsequent disappearance, as well as the continuing incidence of banking crises and speculative excesses.

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1 Introduction

Over the past century, there has been a significant increase in the scale and scope of central bank interventions in response to banking crises, alongside the introduction of public insurance schemes such as deposit insurance (Bordo and Siklos, 2018; Metrick and Schmelzing, 2021). In particular, since World War II, central bank policy responses to financial crises have become "close to systematic" (Ferguson et al., 2023). Puzzlingly, despite the presence of ex-ante public backstops and large-scale public liquidity provision ex post, significant banking crises continue to occur across the world, including in advanced economies with comprehensive regulatory frameworks (for example, the widespread recent banking stress around the failure of Silicon Valley Bank in 2023). Why?

At the heart of banking crises lies the fundamental nature of liquidity transformation: banks borrow short to lend or invest long in illiquid assets. Occasionally, accidents happen, and banks fall short of liquidity, forcing them to liquidate assets in a fire sale. Contingent liquidity or capital can be helpful under such circumstances in reducing costly fire sales. While central bank interventions have become a familiar response, a fundamental question arises: what form should that contingent liquidity take? Should it be in the form of private insurance arrangements, e.g. private sector promises to provide transfers in exchange for a pre-committed fee (callable or contingent equity)? Or should it be public liquidity support in the form of central bank bailouts or loans at a high rate (à la Bagehot, 1873) after the need arises? Or should it be pre-positioned liquidity arrangements with the central bank (see Tuckman, 2012; King, 2016; Nelson, 2023; Hanson et al., 2024), and if so, what fee should the central bank charge? In this paper, we analyze all these mechanisms and how they influence each other.

In Stein (2012), the influential and tractable model we will base our analysis on, a banking sector raises funds from households by issuing a mix of money-like short-term liabilities and long-term bonds, and investing the funds in long-term projects (the banking sector in his model is clubbed together with the project managers for simplicity). Money-like liabilities are cheaper funding than bond funding because holders are willing to pay a liquidity premium for them, so long as they stay risk-free. However, at a future date, the projects may be hit by an adverse shock in some state of the world, in which case holders of short-term liabilities will come for their funds. Each (symmetric) bank will have to sell projects in the market to raise funds to repay short-term liability holders, possibly at fire-sale prices. Importantly, the amount of money-like liabilities the bank can issue initially can be constrained by the need to have enough saleable assets at that future date to pay off all the money-like liabilities – a kind of "collateral constraint" on initial money issuance.

The buyers of fire-sold assets in Stein's model are private (in his language, "patient") investors with a limited endowment of funds – and the fire-sale discount is because they will have to buy "fire-sold" projects instead of investing in their own opportunities so there is an opportunity cost of purchases.

In the Stein model, the bank may overinvest in projects in order to alleviate the collateral constraint on financing with cheap money-like liabilities. The bank does not internalize the fact that while its own collateral constraint is getting loosened, the additional investment and fire sales increase the size of the fire sale discount for all, tightening other banks' collateral constraint. Stein (2012) then examines supervisory and monetary actions that could limit the bank's incentive to overinvest in projects and overissue money.

In this paper, we start with a different question. First, is there a private fix to the problem? Can banks arrange insurance privately to relax the collateral constraint? Second, what can the central bank do to augment private sector efforts, and does it help or hurt? In particular, how does the public provision of (contingent) liquidity affect the private provision of (contingent) capital or liquidity?

We show that there is indeed a simple private solution to the potential problem of over-investment when money creation is collateral constrained: allow the bank to arrange for committed fund inflows from the private investor conditional on an adverse shock hitting, for which the bank will pay in future normal states. Such an arrangement ensures the collateral constraint never binds because a bank that wants to issue more money-like deposits simply buys more contingent capital insurance. Outcomes are once again socially optimal. Of course, the banking system may issue more money-like liabilities than in the Stein model where such contingent capital is not available. Indeed, under some conditions, it may even fund itself entirely with money-like liabilities, investing in both bonds and projects. In other words, if the premium that investors are willing to pay to hold money-like bank liabilities is high, the bank may take on more liquidity risk than strictly required by project financing in order to fully exhaust the benefit of issuing money-like assets. Nevertheless, the contingent capital inflows offset the quantum of fire sales sufficiently to ensure that the collateral constraint is not binding and private outcomes are socially optimal.

Such privately arranged contingent funding is not merely a theoretical curiosity. Banks in the past had unlimited liability in a number of countries (Hickson and Turner, 2003; Kenny and Ogren, 2021) and double liability in the United States till the Great Depression (Macey and Miller, 1992; Bodenhorn, 2015; Aldunate et al., 2019). Another version of greater liability was where bank shareholders paid in capital less than the par value of shares and were liable for the remainder on call (see, for example, the discussion in White (1995) on free banking in Scotland before 1844). All these are situations where the bank had a call on the capital/wealth of shareholders, and the call would come in distressed times. More recently, contingent convertible bonds (see Flannery, 2005; Kashyap, Rajan, and Stein, 2008; French et al., 2010; Flannery, 2014) have been proposed to reduce bank debt and enhance equity in distressed times, potentially allowing banks to raise funding. Vallee (2019) and Avdjiev et al. (2020) present empirical evidence that such contingent convertible capital securities can reduce bank fragility. The historical evidence, which we will discuss in detail later, also suggests such instruments helped bolster bank stability.

Of course, one can make assumptions that make contingent funding contractually difficult, thus recovering the spirit of overinvestment due to fire-sale externalities. We do believe, however, that such contracting difficulties should not be overstated. The absence of private contingent funding in recent times may have more to do with the scale and scope of central bank interventions than the contractual difficulties associated with such instruments. Indeed, we show that this is theoretically the case regardless of whether there are such contractual difficulties or not.

Let us elaborate. What happens if we add an interventionist central bank that can infuse liquidity more cheaply than the private sector into the mix (a possibility that became more realistic as countries left the gold standard and final settlement became possible in central bank issued fiat money)? The welfare implications turn on three issues. First, can the central bank commit to a policy of specific intervention, or does it react according to the needs of the moment? Second, does the central bank charge the needy bank for its intervention, and under what circumstances does it recover the appropriate charge? Third, is there a cost to the central bank of intervening, and what form does it take?

While we examine a variety of cases, in the introduction we will focus on two cases for illustrative purposes, assuming all players can game out the reactions of others so that we can restrict the analysis to subgame perfect equilibria. The first case is a "bailout" central bank, which seeks only to reduce fire sales ex post by infusing funds into distressed banks, without seeking to recover the funds later from the banks.[1] This is certainly the case if the central bank has no additional powers of obtaining repayment than the private sector. Given that a bank knows fire sales will be reduced ex post through central bank intervention, it has the incentive to increase up front financing through money-like liabilities – in the model, this exactly offsets central bank intervention. Furthermore, if the extent of the bailout is proportional to the investment (or bank size), the bank has the incentive to increase investment since it does not bear bail out costs. The net effect is that realized fire-sale costs do not change since banks take on more illiquidity risks up front to offset central

[1]Even if the central bank announces a penalty rate ex ante, it might be time-inconsistent. See, for example, Acharya and Yorulmazer (2007), Diamond and Rajan (2012), and Farhi and Tirole (2012).

bank intervention. And because banks do not pay the cost of central bank intervention, they also overinvest.

Importantly, the anticipation of central bank intervention could crowd out private insurance arrangements such as contingent capital support. In other words, the market for private contingent capital may disappear endogenously as a bailout central bank induces real investments (the asset-side of bank activity) to become the way to support money issuance in preference to arranging contingent capital (the liability-side of bank activity). The crowding-out of private insurance via the liability side is evidenced by the disappearance of additional or even unlimited shareholder liability, as well as the secular decline in banks' capital ratios in the past 140 years (see, e.g., Alessandri and Haldane, 2009).

In the second case, the central bank pre-commits to lend conditional on stress (think of this as resembling proposals for banks to pre-position assets with the central bank that they can borrow against in times of liquidity stress). By charging a premium for the liquidity support, the central bank can reduce the bank's incentive to overinvest. However, shielded from market forces and subject to political pressures, it is all to easy for the central bank to charge the wrong price for support to market participants. For instance, we argue the actuarially fair price (the price at which the central bank breaks even on the support) is too low a price to charge for pre-committed liquidity; the right "Bagehot" price is one that reflects the private costs of fire sales even though central bank intervention potentially alleviates fire sales.[2] Yet at this price the central bank will not help boost asset prices relative to what the private market would set, and would end up making money in a systematic way. Therefore, the actuarially fair price may anchor estimates of what is politically possible. Overall, we conclude that increasing anticipated interventions by central banks in stress situations without their adequately charging for public liquidity infusions can help explain the continuing incidence of banking stresses.

Worse still is if there is central bank moral hazard – that the central banker's personal costs of intervention may be quite different from societal costs – for instance, the central banker may worry about the cost of bank failures (or a perceived bailout) to their career, while society may pay a larger price in terms of enhanced moral hazard or higher taxes.[3] The augmented Stein model allows us to examine all such possibilities and their effects on welfare.

[2]There is some difference in opinion on the intent of Bagehot (1873)'s dictum for the central bank to lend freely but at a high price. Bernanke (2008) argues that the high price was meant to ration the central bank's holdings of scarce specie, and not to prevent moral hazard. So the dictum would not apply where the central bank controls an elastic currency. Our focus on moral hazard and investment distortions restores the need for the Bagehot dictum, even in a world of fiat money. Also see Acharya, Santos, and Yorulmazer (2010) on designing deposit insurance premium that can discourage systemic risk or bank-herding incentives.

[3]See, for example, Alesina and Tabellini (1990), Boot and Thakor (1993).

In the Stein (2012) model, the returns to taking on liquidity risk are capped by a constant money-financing premium. We extend the model in yet another direction by allowing the returns to taking liquidity risk to increase without bound. Specifically, in addition to real investments, we examine the effects of central bank intervention on speculative financial investments that are ultimately funded by banks (for example, via prime-brokerage services). One example is the financing of the Treasury cash-futures basis trade (undertaken by hedge funds) that has raised significant concerns with regards to financial stability. Although it is not socially beneficial in the model, banks engage in such lending because the profits (carry or prime brokerage fees) earned are privately attractive and increase with leverage. We assume that in the crisis state, speculation is subject to margin calls, that is calls on bank liquidity, which add to depositor demands. A bailout central bank not only continues to distort the mix of money creation via real investments versus private liquidity insurance, but also over-intervenes ex post. The availability of easy public liquidity induces greater financial speculation and amplifies liquidity demand during crises, resulting in a larger welfare loss compared to its effects in the baseline model.

More generally, cheap prospective liquidity can increase bank incentives to take advantage of that cheap liquidity. Asset-side actions of overinvestment and financial speculation can be even more distortionary than liability-side actions such as issuing demand deposits, though the two are interrelated. Of course, if central bank intervention is structured right, it can improve matters in states with fire sales. However, it is a tall order to design and price such intervention without distorting bank behavior significantly and without crowding out private markets for insurance.

1.1 Related Literature

As elaborated above, our analysis builds on the model of Stein (2012), which itself draws on a large strand of literature on fire-sale externality.[4] Closer to our analytical results, Davila and Korinek (2017) show in a model with price-dependent financial constraints that inefficient outcomes arise when insurance markets are incomplete, and therefore advocate for regulators to "improve insurance of financially constrained agents (e.g., by promoting contingent forms of financing) and stabilizing the value of assets used as collateral." Our analysis clarifies that such contingent financing need not be provided only by a lender of last resort or a public regulator, but also possibly by private agents. In fact, we demonstrate

[4]For an incomplete yet extensive list, see Shleifer and Vishny (1992, 1997), Kiyotaki and Moore (1997), Gromb and Vayanos (2002), Morris and Shin (2004), Allen and Gale (2005), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009), Stein (2009), Geanakoplos (2010), Caballero and Simsek (2013), Moore (2013), Bocola and Lorenzoni (2023).

that the private provision of such state-contingent insurance claims can get progressively eliminated – a form of endogenous incompleteness – due to moral hazard once a lender of last resort comes in, and that this could be welfare-reducing when public insurance is not appropriately priced.[5]

From a theoretical point of view, our first result that with state-contingent claims, collateral externality may not lead to inefficiencies in a frictionless world has also been established under different settings in Krishnamurthy (2003) and Asriyan (2020). What is novel in our setting, apart from the specific application, is that with frictions in private insurance markets (as shown in Appendix D), there is over-provision of such state-contingent claims in the private equilibrium, whereas earlier models typically feature an underprovision of private insurance when insurers face collateral constraints (Krishnamurthy, 2003) or face information and trading frictions that lead to illiquidity (Asriyan, 2020). This difference arises from the fact that in our model, banks are keen to pocket the "money premium" on deposit liabilities, which can also be manufactured by purchase of contingent capital in the form of insurance, but banks do not internalize the externality that makes fire-sale worse for everyone else, which leads to excessive money creation through *both* overinvestment in assets and over-purchase of insurance.

Finally, some of the discussion of the root causes of the Global Financial Crisis emphasizes the global savings glut (Bernanke, 2005) that led to an increase in demand for short-term money-like securities (e.g., Gorton and Ordonez, 2014) and an expansion of the financial sector's collateralizable assets to meet this demand. Our analysis clarifies that money can be privately manufactured by both asset- and liability-side adjustments. The public provision of liquidity can distort their optimal mix away from liability-side adjustments by crowding out the (perfect or imperfect) private market for contingent capital and leading to overinvestment in assets. In our telling, undue and unchecked leveraging was as responsible for the GFC as was overinvestment in hard assets.

2 Setup

We build on Stein (2012)'s framework. There are three periods: $t = 0, 1, 2$. There are three types of agents: households, banks, and private investors (PI). Traditional en-

[5] Jeanne and Korinek (2020) examine in detail the optimal ex-ante versus ex-post central bank policies in a fire-sale model of banking, while Davila and Walther (2022) examine optimal second-best corrective regulations in more general settings with uniform regulation and convex costs. Hachem and Kuncl (2025) explore how regulations could help increase banks' "haircuts" on withdrawals during banking crises. This parallels the private provision of contingent capital in our model. However, depositor haircuts might precipitate runs as depositors anticipate haircuts, so implementation might be difficult (see Diamond and Rajan (2012)).

trepreneurs are clubbed with banks to form a composite entity, which both initiates projects and raises finances for them.

2.1 Households

At time 0, the economy has an initial endowment Y. Private investors have initial endowments of W (assumed to be fixed).[6] Households start with disposable wealth $Y - W$ (and also are the ultimate owners of the incomes generated by private investors and banks). Households choose between immediate consumption C_0, investment in liquid money-like bank-issued money-like deposits (M), henceforth also called money, or risky bank-issued bonds (B). Households derive additional utility from holding money because it is effectively made riskless. Money pays a gross rate of R^M upon withdrawal at either time 1 or time 2. Risky bonds cannot be redeemed at time 1 and pay a gross rate R^B at time 2. At time 2, households consume C_2 out of the returns from money and bonds, as well as any income private investors or the banks generate.

The households' utility is given by

$$U = C_0 + \beta \mathbb{E}(C_2) + \gamma M. \tag{1}$$

Given linear preferences, the expected gross return on bonds is $R^B = \frac{1}{\beta}$ and the gross return on money is $R^M = \frac{1}{\beta+\gamma}$, where γ is the convenience yield on money. The money-bond spread is thus assumed to be fixed. Households consume and invest in bank assets.

2.2 Banks

There is a continuum of homogeneous banks with mass 1. The bank raises financing from households by issuing money or bonds, and can invest in real projects. The timing of events is detailed in Figure 1.

On the asset side, the representative bank invests I in real projects at time 0 and gets proceeds y at time 2.[7] At $t = 1$, uncertainty is revealed. With probability p, the economy is in a good state, and the output at $t = 2$ is $f(I) > I$, where f is increasing, concave, and twice differentiable. With probability $1 - p$, the economy is in a crisis state, where the time 2 expected output is $\lambda I \leq I$ and there is a positive probability $1 - q$ that the output is zero.[8]

[6] Under our baseline parameterization, endogenizing the household's choice of W, the amount of wealth allocated to the PIs, does not change any of the qualitative model results and has negligible effects on the numerical findings. For simplicity of exposition, we therefore treat W as fixed.

[7] In the rest of the text, we will often refer to the representative bank or the banking sector simply as "the bank."

[8] In our setup, the crisis probability $1-p$ is held fixed throughout for simplicity, including in the presence of central bank interventions introduced in Section 4. Section 6.3 presents empirical evidence broadly consistent

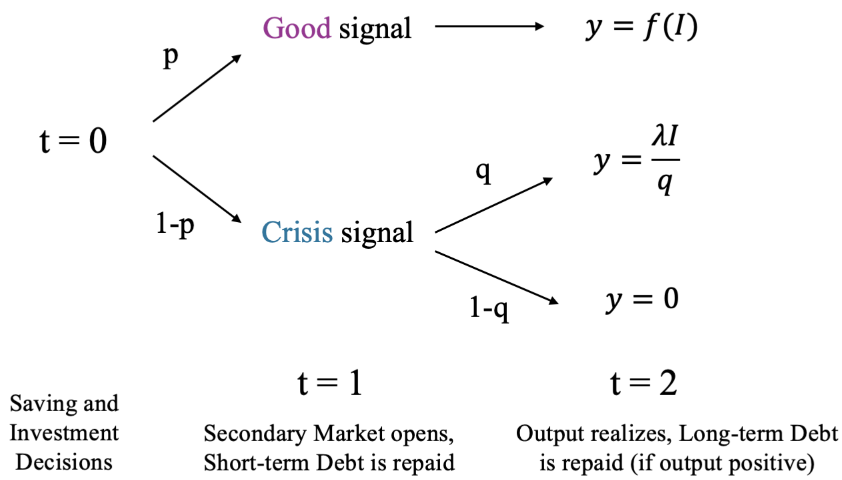

Figure 1: Timing of Events

On the liability side, each bank raises mI in money-like deposits, which can be withdrawn by the households at $t = 1$ or $t = 2$ and pays a gross rate R^M upon withdrawal. Each bank's total money liability is $M = mIR^M$. Worrying their deposits will turn risky, on seeing the crisis state at date 1, depositors will run.[9] To meet depositors' withdrawals amounting to mIR^M the representative bank sells a fraction $\Delta \in [0, 1]$ of its real assets to raise $\Delta k\lambda I$, where $k \in (0, 1)$ is an endogenous "fire-sale discounted" price, which the bank takes as given. As a result, in the crisis at $t = 1$, depositors are always repaid.

The bank finances real investment I partly with money liability mI. If $m \leq 1$, the bank finances the rest of the investment by issuing $(1-m)I$ in illiquid bonds that pays a gross rate of R^B at $t = 2$. If $m > 1$, the bank issues more money-like deposits than it needs to invest in projects. For now, we assume it puts the additional $(m-1)I$ raised in an illiquid storage technology (bonds issued, say, by other corporate entities) that also pays a gross rate of R^B at $t = 2$. In this case, the bank creates liquid money from illiquid financial investments. Later, we will allow it to make other financial investments.

2.3 Private Investors and Contingent Capital

In the Stein framework, the bank cannot make credible repayment promises in the crisis state – hence it cannot raise financing. This allows it two options to raise funds to repay depositors: sell assets or receive pre-contracted or contingent capital inflows (pre-committed lines of credit will not work because they require repayment). There is a continuum of

with this assumption, as crises remained frequent despite the global expansion of public interventions.

[9] To fix ideas, think of depositors as extremely risk averse corporate treasurers who will move their deposits as soon as there is any hint of risk for fear that they will not be able to meet future payroll or debt payments.

homogeneous private investors (PI) with mass 1 and wealth W. PIs absorb bank fire sales during crisis periods, make additional real investments at time 1, and new to the Stein (2012) framework, provide contingent capital commitments (equivalently, insurance) in the date-1 crisis state.

2.3.1 Contingent Capital

Specifically, at time 0, the bank can privately contract for the PI to pay an amount $E = \psi I$ as private contingent capital (equivalently, insurance payouts) to the bank if at $t = 1$, the economy is in the crisis state. Naturally, $E \leq W$. In exchange, whenever the economy reaches the good state, the bank pays the PI $r^E E$ at date 2. This arrangement resembles callable or partially paid-up bank equity capital, with the bank able to call up full payment in case of need. It could also represent equity with unlimited liability. It resembles today's contingent convertible capital bonds (Flannery, 2005; Kashyap, Rajan, and Stein, 2008), where the bond conversion in crisis times frees up additional borrowing capacity or adds to bank capital.

To make money riskless, in the crisis state the bank's promised payment to depositors $M = mIR^M$ must be covered by either fire sale of assets or by the private insurance $E = \psi I$ from private investors:

$$\Delta k\lambda I + \psi I = mIR^M \implies \Delta k\lambda I = M - E.$$

With private insurance, only $(M - E)$ of bank assets have to be fire sold. If $\Delta = 1$ (all assets are fire sold), the bank reaches the upper bound on private money creation, which is

$$M^{max} = k\lambda I + E \implies m^{max} = \frac{k\lambda + \psi}{R^M}.$$

Contingent capital thus enhances the maximum amount of money-like deposits that can be issued at date 0 by E $(= \psi I)$.

2.3.2 Late arrival projects

At $t = 1$, the PI can also invest K in a project that arrives at $t = 1$ to receive output $g(K)$ at $t = 2$, where $g(\cdot)$ is increasing, concave, twice differentiable, and satisfies Inada conditions at 0. In the good state, the PI receives $r^E E$ in insurance premium and can invest her entire W in late arrival projects. In the crisis state, the PI pays E to the bank, spends $(M - E)$ in fire sales, and invests the rest of its endowment, $W - E - (M - E) = (W - M)$, in late arrival projects.

3 Baseline Model

We now show that when there are no frictions in contracting private contingent capital, we get the socially optimal outcomes in Stein (2012). We will explore frictions in Appendix D.

3.1 Bank's Problem

The representative bank's problem can be written as follows:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}}$$
$$- p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} + (1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} - (1-p) \underbrace{z[mIR^M - \psi I]}_{\text{loss on fire sale}} \tag{2}$$

s.t.

$$m \leq m^{max} = \frac{k\lambda + \psi}{R^M}$$

where $z = \frac{1-k}{k}$ is the lost return on every dollar of fire sale proceeds. The Lagrangian is

$$\mathcal{L} = \text{objective} - \eta \left(m - \frac{k\lambda + \psi}{R^M} \right)$$

with Lagrange multiplier $\eta \geq 0$.

The bank's first order conditions (FOCs) then are as follows:

with respect to (w.r.t.) bank's fraction of real investment financed by deposits, m:

$$I[(R^B - R^M) - (1-p)zR^M] = \eta, \tag{3}$$

w.r.t. bank's fraction of real investment covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1-p)(1+z), \tag{4}$$

and w.r.t. bank's real investment, I:

$$pf'(I) + (1-p)\lambda - R^B = -\left[m(R^B - R^M) - (1-p)z\left(mR^M - \psi\right)\right] + pr^E\psi - (1-p)\psi$$
$$= -\frac{\eta}{I}\left[m - \frac{\psi}{R^M}\right]. \tag{5}$$

The first FOC states that the shadow cost of relaxing the money creation constraint, η, must equal the net marginal benefit of money creation, which is the money premium net of the expected fire sale loss. The second FOC equalizes the bank's marginal cost of arranging private contingent capital (the expected contingent capital insurance premium charged) with the marginal benefit of private contingent capital, including the expected avoided fire sale costs and the shadow benefit of money creation. The third FOC states that when the money creation constraint is not binding, the marginal benefit from investments is equal to the marginal financing cost R^B. When the constraint is binding, the bank can issue more uninsured money by increasing physical assets I to relax the money creation constraint, which creates an additional benefit of investment.[10]

3.2 Private Investor's Problem

The representative private investor's problem is

$$\max_{M,E} p\left[g(W)+r^E E\right]+(1-p)\left[g(W-M)+\frac{1}{k}(M-E)\right].$$

PI's FOC w.r.t. M, the PI's funds used for fire-sale purchases in the crisis state is:

$$g'(W-M)=\frac{1}{k} \tag{6}$$

which equalizes the marginal benefit of investing in the g technology with the marginal benefit of obtaining the return on buying fire-sold assets, $\frac{1}{k}$, in the crisis state at $t=1$.

PI's FOC w.r.t. private liquidity commitment E:

$$pr^E=(1-p)\frac{1}{k} \tag{7}$$

which equalizes the time-0 marginal expected premium on providing private contingent capital with the corresponding expected marginal cost (forgone return of $\frac{1}{k}$ in the bad state). This is the key new additional condition to Stein's framework.

3.3 Private Equilibrium

We first define a private equilibrium of this model.

Definition 1. An equilibrium is a set of prices and allocations such that taking the prices k, r^E as given, both bank's and PI's FOCs are satisfied, and markets clear. In this case, market clearing implies that $E=\psi I$ and $M=mIR^M$, so that the bank's choices of m,ψ,I are consistent with the PI's choices of M and E.

[10] Note that uninsured money in levels is $(M-E)$ and is $(m-\frac{\psi}{R^M})$ in time 0 terms per dollar of investment (by dividing $R^M I$).

To solve for the private equilibrium, we first note that from agents' FOC w.r.t. private insurance, (4) and (7),

$$(1-p)\frac{1}{k} = pr^E = \frac{\eta}{IR^M} + (1-p)(1+z).$$

By definition, $1+z = \frac{1}{k}$, which implies we must have $\eta = 0$: the constraint on money creation is not binding in an interior solution. In the baseline model, with $\eta = 0$, the fire-sale discount k is pinned down by the bank's FOC w.r.t. m (3):

$$\frac{R^B - R^M}{R^M} = (1-p)z = (1-p)(\frac{1-k}{k}), \tag{8}$$

and investment is pinned down by the bank's FOC w.r.t. I (5):

$$pf'(I) + (1-p)\lambda - R^B = 0. \tag{9}$$

The total amount of money liability created is pinned down by the PI's FOC w.r.t. M (6):

$$g'(W-M) = \frac{1}{k},$$

and the total amount of private insurance E is indeterminate as long as it satisfies

$$M - k\lambda I \leq E \leq M, \tag{10}$$

where the first inequality requires that the constraint on money creation is not binding and the second inequality is a natural limit on the amount of private insurance.

Note that in the private equilibrium, there is no solution where the constraint on money creation is binding. Suppose $\eta > 0$, then either the bank or the PI would not be on the respective FOC and E would be chosen to be as large as possible, which implies $E = M$. But in that case, the constraint on money creation would not be binding, which yields a contradiction. Therefore, we always have $\eta = 0$ in the baseline model. We now show that the private solution is socially optimal.

3.4 Social Planner's Problem

Consider next a benevolent social planner who seeks to maximize the households' utility (1) by choosing the private outcomes of M, E, and I. We can express the planner's problem as follows (detailed steps in Appendix A):

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) + (1-p)[g(W-M) + M]$$

such that the money creation constraint is met:

$$m \leq m^{max} = \frac{k\lambda + \psi}{R^M},$$

and the planner understands that the private investors' date-1 allocations require that the marginal returns available on late-arriving investment equal returns on assets bought in the fire sale so that

$$g'(W - M) = \frac{1}{k} \implies k = \frac{1}{g'(W - mIR^M)}.$$

The Lagrangian for the planner's problem is

$$\mathcal{L}^{\mathcal{P}} = \text{planner's objective} - \eta^P \left(m - \frac{k\lambda + \psi}{R^M} \right)$$

with Lagrange multiplier $\eta^P > 0$. Thus, the planner's FOC w.r.t. the fraction of investment financed by deposits, m, is

$$I[(R^B - R^M) - (1-p)zR^M] = \eta^P \left[1 - \frac{g''(W - M)}{(g'(W - M))^2} \lambda I \right] = \eta^P \left(1 + \Omega(M, I) \right) \qquad (11)$$

where the concavity of g implies that $g'' < 0$, and $\Omega(M, I) = -\frac{g''(W-M)}{(g'(W-M))^2}\lambda I > 0$ is a measure of the fire-sale externality associated with increasing the issuance of money, the extent of which depends on the curvature of g. This externality is internalized by the planner in her choice of m if the money-issuance constraint is binding.

The planner's FOC w.r.t. ψ, the fraction of I covered by insurance, is

$$0 = \frac{\eta^P}{IR^M}. \qquad (12)$$

Because there is no social cost of private insurance, the planner can choose as much ψ as necessary to ensure the money constraint is not binding (i.e., $\eta^P = 0$).

Finally, the planner's FOC w.r.t. investment I is

$$\begin{aligned} pf'(I) + (1-p)\lambda - R^B + m(R^B - R^M) + (1-p)\left[-g'(W - M) + 1\right] mR^M \\ = -\frac{\eta^P \lambda}{R^M} \left(\frac{g''(W - M)}{(g'(W - M))^2} mR^M \right) \end{aligned}$$

which can be rewritten as

$$pf'(I) + (1-p)\lambda - R^B = -m\frac{\eta^P}{I}. \qquad (13)$$

After plugging $\eta^P = 0$ into the planner's FOCs and comparing with (8) and (9), it becomes clear that in the baseline model, the planner's choice coincides with the private outcomes.

Proposition 1. *The private equilibrium outcome in the baseline model with private contingent capital is efficient.*

Let the variables with a $*$ superscript denote the private equilibrium outcome and variables with a P superscript denote the outcomes from solving the planner's problem. Then,

1. The constraint on money creation never binds: $\eta^* = \eta^P = 0$.

2. The private choice of investment and money creation is socially optimal: $I^* = I^P$ and $M^* = M^P$.

3. The amount of private insurance E^* is indeterminate, as long as

$$\max(0, M - k\lambda I) := E^{min} \leq E^* \leq M,$$

and the same condition applies to E^P.

Consider two features about this efficient private equilibrium. First, the constraint on money creation never binds. In a frictionless market, the contingent capital insurance premium r^E equalizes the PI's marginal cost of forgone fire-sale return $\frac{1}{k}$. From the bank's perspective, this equals the bank's marginal benefit of getting insurance, which includes both the marginal savings from avoiding fire sales $\frac{1-k}{k}$ and the shadow value of relaxing the money constraint. Since in a frictionless market, the marginal savings from avoiding fire sales equals the PI's marginal cost of forgone fire-sale return, the shadow value of relaxing the money constraint is zero. Put differently, in equilibrium the insurance premium adjusts so that the shadow value of relaxing the constraint is driven to 0 and the constraint does not bind. Second, in equilibrium, for the marginal dollar in money liability M that is due at time 1, the bank is completely indifferent between paying depositors through fire sales at a cost of $\frac{1-k}{k}$ in the crisis state, or purchasing private insurance at a cost of r^E in the good state. This makes $E \in [E^{min}, M]$ indeterminate.

Since there are no frictions in providing contingent capital, there are no limits to money financing when accompanied by commensurate contingent capital, so there is no need to increase investment beyond the socially optimal in order to increase money financing. In Stein's model, by contrast, if the money premium $R^B - R^M$ is high, the bank may still want to finance with money but it may not have the ability to sell enough assets to make money riskless in the crisis state ($k\lambda I$ is too low). The binding money issuance constraint then gives the bank additional incentives to invest. Pecuniary externalities then matter because the bank does not take into account its higher investment in depressing fire sale values for other banks. Hence in his model, banks overinvest if the money premium is high.

From an economic standpoint, Proposition 1 clarifies that the market failure that leads to inefficiency in Stein (2012) is not the fire-sale externality per se, but the missing market for contingent capital provision. This market could be endogenously missing anticipating policy interventions (as we will show in Section 4) or due to frictions in committing contingent capital (which we will discuss in Section 4.2.6 and Appendix D).[11]

3.5 Numerical Illustration

Throughout the paper, unless otherwise stated, we illustrate the results with a numerical example with the following parameters and functional specifications: $p = 0.95$, $\lambda = 1$, $W = 140$, $R^B = 1.08$, R^M varies between 1 and 1.075, $f(I) = a\log(I) + I$ where $a = 3.5$, $g(K) = \theta\log(K)$ where $\theta = 140$.

In Figure 2, we plot the equilibrium levels of money liability M, investment I, and the minimum level of private insurance E^{min} for various levels of R^M (that is, various levels of the money-bond spread). In the "low spread" region where R^M is high, the planner's allocations are attainable without any private insurance. However, in the "high spread" region where R^M is low and money issuance attractive, private contingent capital is procured such that the level of investment and the amount of money created (which increases as the bond-money spread increases) are always at socially optimal level. In Stein's model by contrast, the money creation constraint binds in the high-spread region and pecuniary externalities operating through the pledgeability constraint would lead to inefficiently high levels of I and M in equilibrium.

3.6 Contingent Capital Over History

Historical parallels to pre-arranged injections of contingent capital into the banking sector from the private sector abound.

3.6.1 The Scottish Free Banking Era and Unlimited Liability

The Scottish free banking system from 1716-1845 enjoyed "remarkable monetary stability" with no central bank and very few legal restrictions (White, 1995), leading to "considerable agreement that lightly regulated banking was a success in Scotland" (Briones and Rockoff, 2005). While some argue that part of this success in the 17th Century was attributed to the presence of large privileged banks acting as quasi-central banks (Cowen and Kroszner, 1989), after 1810 the three chartered banks with limited liability were no larger

[11]Note that the indeterminacy of E arises only in the frictionless model. Introducing any friction in contracting private contingent capital would lead to a unique level of E in the constrained region, as shown in Appendix D. Any additional costs in the insurance market caused by such frictions would lead private agents to choose the minimum feasible level of E and banks would liquidate all available I in the fire-sale market. This motivates our focus in the graphical analysis on $E^{min} = M - k\lambda I$. Conceptually, the frictionless benchmark can be viewed as a limiting case as the costs associated with contracting frictions approach 0.

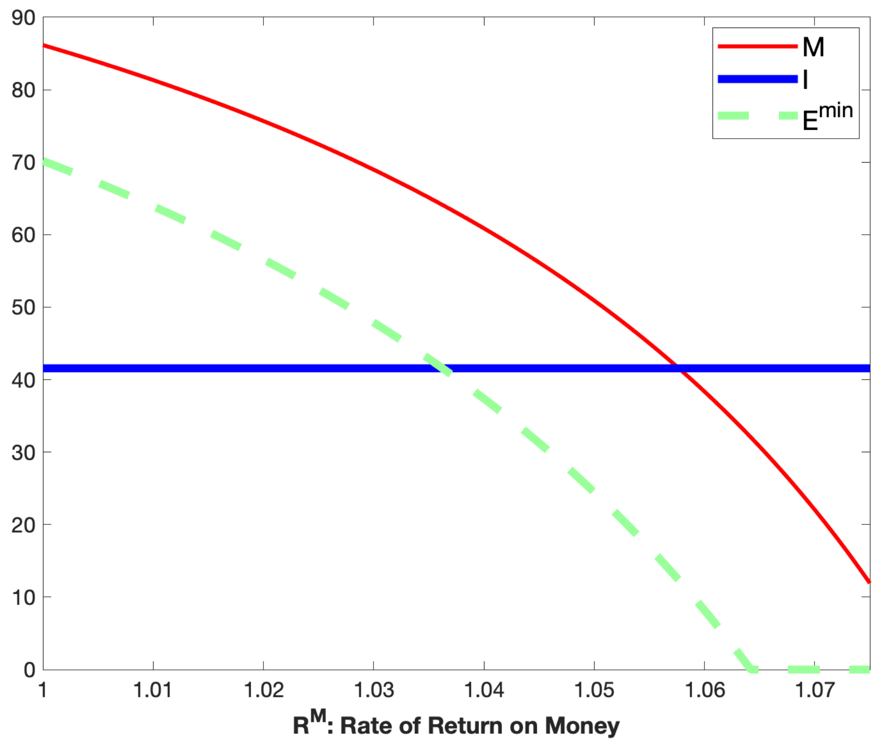

Figure 2: Equilibrium Outcomes in the Baseline Model, for different levels of return on money R^M

than the other non-chartered banks (with unlimited liability) and played no supervisory roles, leading to a period of a competitive and well-capitalized banking sector where "none [of the banks] were disproportionately large, all but a few were extensively branched" (White 1995, 2014).[12]

By giving the right of note issue only to the Bank of England, which became a lender of last resort to Scottish banks, the Peel Acts of 1844 and 1845 effectively ended the Scottish free banking era. However, many unchartered Scottish banks chose to retain unlimited liability in the 1860s-70s (without difficulty in raising capital on a large scale), even after the Companies Act of 1862 allowed them to choose a limited liability structure (White, 1995). While Bagehot was concerned that unlimited liability banks would eventually be owned by impecunious individuals with "few acres and few shillings" making the banks' liabilities effectively limited, Hickson and Turner (2003) examine the archives of an Irish joint stock

[12] Cowen and Kroszner (1989) also argue that the Bank of England may have acted as an implicit lender of last resort. White (1995) responds that "in a few cases, the BoE provided loans to Scottish banks, but in other cases (most importantly, during the crises of 1825-6 and 1836-7) it refused to lend." In fact the BoE explicitly rejected the idea that it had LOLR obligations. Thus, for our purposes, the Scottish free banking system can be viewed as our baseline model with a weak-to-nonexistent LOLR.

bank and conclude that the governing body successfully ensured that ownership remained in the hands of the largest and wealthiest shareholders.[13] White (1995) also presents evidence in Scotland that "all failed banks having more than nine partners were able to pay their liabilities to the public in full", and that the loss to the Scottish banking public during the entire free banking era were estimated as of 1841 to be merely half of the public losses on bank liabilities in London in 1840 alone. Interpreted through the lens of our model, it seems that the contingent capital (E) put down by equityholders was large enough to cover the losses in the crisis states.

Unfortunately, the failure in 1878 of the City of Glasgow Bank, a bank with unlimited liability that suffered losses more than six times its capital, rendered a large number of shareholders insolvent. This generated widespread public concern and led directly to the 1879 Companies Act that paved the way for the adoption of limited liability for all shareholders in all of Britain (Goodhart and Postel-Vinay, 2024).

Finally, Sweden also had a relatively free banking system from 1830 to 1903 with unlimited liability banks, despite the presence of a central bank (Lakomaa, 2007). Kenny and Ogren (2021) find that unlimited liability Swedish banks took on greater leverage (given the greater implicit equity) and generated higher dividends, return on equity, and maintained strong governance control relative to limited liability banks.

3.6.2 Contingent and Double Liability in European and US Banks

While unlimited liability became less common for British banks after the 1879 Companies Act, until early twentieth century many banks operated under contingent liability rules, where obligated shareholders were responsible for a portion of a bank's debts after insolvency. This effectively corresponds to contingent capital in our baseline model, where these obligated shareholders must provide additional capital in times of crisis in exchange for some return during good times. Grossman and Imai (2013) use data on British banks from 1878 to 1912 and find that banks with more contingent liability took less risk. They conclude that contingent capital could protect taxpayers by reducing their share of bank resolution costs and altering the risk-shifting incentives of banks by alleviating the moral hazard problem.

A similar contingent liability structure for banks, under the label of double liability, also prevailed in the United States roughly between the Civil War and the Great Depression.[14] Macey and Miller (1992) argue that double liability was "remarkably effective," with

[13] Unlimited liability banks were also established in Ireland from mid-1820s until the 1879 Act.

[14] It is important to note that double liability did not amount to providing insurance at the time of runs, but passing on the losses from bank liquidation or closure to shareholders ex post and up to an agreed liability. However, to the extent that double liability expands the bank's ability to borrow in crisis times because of the perceived additional implicit equity, it corresponds to our model of pre-arranged contingent

clear advantages over deposit insurance, which creates moral hazard and induces risk-taking. During that period, American banks subject to stricter liability rules (for equityholders and managers) had less equity and asset volatility, held less risky assets, and were less likely to become distressed during the Great Depression (Esty, 1998; Aldunate et al., 2019; Koudijs, Salisbury, and Sran, 2021). Banks with double liability operated with lower (book) capital ratios and therefore higher leverage, which suggests that bank creditors viewed the contingent liability as a credible guarantee or off-balance-sheet capital (Macey and Miller, 1992; Bodenhorn, 2015). These findings are consistent with our baseline model that suggests the purchase of contingent capital / private insurance can also increase the amount of money-like securities created by a bank.

Macey and Miller (1992) also find that double liability rules were effectively enforced through insolvency assessments with substantial recovery rates for depositors and other creditors. Despite its success, the wave of bank failures between 1929 and 1933 led to a widespread public perception that double liability had failed as a regulatory system, as damages were often imposed on innocent shareholders with no inside connections to the banks and who had purchased the shares during the economic boom of 1923-1929. With the Federal Deposit Insurance Company (FDIC) established in 1933, government deposit insurance at the time came to be perceived as a more effective remedy for the banking crises and double liability had been effectively extinguished from the US banking system by 1953, consistent with the crowding-out of private contingent capital by public backstops in our model. Below, we elaborate further on this development.

4 Central Bank Interventions

In the optimal private solution, at date 1 when the crisis state is realized the private investor diverts funds from investment in later arriving projects toward purchasing assets in the fire sale market. Seeing this, a socially-minded central bank may be tempted to intervene to infuse liquidity to enhance late investment (the fire sale is simply a transfer). Therefore, we add a central bank that can inject funds $L = \phi I$ (so that ϕ becomes the proportion of total I that is covered by central bank) into the bank to meet depositor withdrawals.[15] This reduces fire sales, thus allowing private investors to invest more. Specifically, the PI puts up E in contracted contingent capital but only spends $M - E - L$ on fire sales, and therefore now has $W - M + L$ to invest in the g technology.

liquidity or capital.

[15] Instead of banks perceiving the central bank intervention L to be in proportion with the size of real investments I, Appendix E illustrates the core results of crowding out persist under the setup where the LOLR support is in proportion to the size of M, the level of money liability created.

We assume that central bank funding is not entirely socially costless: it has a deadweight cost $C(L)$ of raising taxes to provide such bailout funding, which can be thought of as a distortionary fiscal or inflation cost. We assume $C(\cdot)$ is (weakly) increasing, convex, and twice differentiable. This cost is borne by households (so it also enters the planner's and the central bank's objectives) but is unrecognized by the representative bank and PI.

Regardless of whether the central bank chooses ϕ upfront or conditional on the crisis state, each bank will take ϕ as given, recognizing that L is affected by its investment, I. This form of central bank intervention incentivizes the bank to create money via overinvestment, because it now recognizes that public insurance also scales up as it increases investment I. The key then is how the central bank can offset this incentive by structuring intervention and the charge levied for intervention appropriately.

Before we go there, we first consider a benevolent planner that can dictate the levels of M, E, I and L, recognizing the social cost of intervention $C(L)$. Then, we consider different ways the central bank can intervene. We compare the welfare obtained in these different situations at the end.

4.1 Planner's Problem with Public Liquidity Provision

The planner's problem with contingent public liquidity provision now becomes

$$\max_{m,\psi,\phi,I} \quad pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) \\ + (1-p)\left[g(W - M + L) + M\right] - (1-p)C(L)$$

such that

$$m \leq m^{max} = \frac{k\lambda + \psi + \phi}{R^M}$$

with Lagrange multiplier on the constraint $\eta^P \geq 0$ and the planner knows

$$g'(W - M + L) = \frac{1}{k} \implies k = \frac{1}{g'(W - mIR^M + \phi I)}. \tag{14}$$

From the planner's FOC w.r.t. ψ, we still have that

$$0 = \frac{\eta^P}{IR^M}. \tag{15}$$

Substituting into the FOC w.r.t. m gives

$$I\left[(R^B - R^M) - (1-p)zR^M\right] = \eta^P\left[1 + \Omega(M, I)\right] = 0,$$

which implies the socially optimal fire-sale discount k^P must satisfy

$$\frac{R^B - R^M}{R^M} = (1-p)z = (1-p)(\frac{1}{k} - 1). \quad (16)$$

With $g'(W - M + L) = \frac{1}{k^P}$ and with $\eta^P = 0$, the planner's FOC w.r.t. ϕ can be written as

$$(1-p)g'(W - M + L) = (1-p)C'(L). \quad (17)$$

Equations (14) and (17) pin down the socially optimal level of central bank funding L^P and money creation M^P. As before, the socially optimal I^Psatisfies

$$pf'(I) + (1-p)\lambda - R^B = \frac{\eta^P}{I}\left(\frac{\phi}{R^M} - m\right) = 0.$$

We can see that k^P and I^P remain the same as in the planner's problem in Section 2.4, but M^P increases by central bank infusion L^P. Cheap central bank liquidity allows for more socially optimal money creation – one way in which the scales are tilted toward central bank intervention in the model.

We now explore a number of different ways the central bank can infuse L, assuming that M, E, I are chosen fully anticipating central bank actions. First, we examine what happens if it intervenes in the crisis state, after seeing fire sales (ex post lending). Second, we explore the possibility that the bank pre-contracts at date 0 for the central bank's infusion. This is ex ante commitment, along the lines of several current proposals (see King (2016) and Hanson et al. (2024)). In either case, the central bank can achieve the social optimal, but only if it charges the bank an appropriate rate. We will see why that is difficult.

4.2 Central Bank as Ex-post Lender of Last Resort (LOLR)

Suppose a central bank can lend $L = \phi I$ to the representative bank at date 1, provided it can recover a fixed multiple τ of the LOLR funding from the bank at $t = 2$. It will return the repaid amounts to households, where τ is the gross interest rate the central bank is statutorily required to charge, and $\tau - 1$ is the net return to households from the forced loan. Note that the private sector cannot finance the bank directly since it cannot trust the bank will repay – the only way for the bank to raise assistance from the private sector is by selling assets or by obtaining contingent capital. By allowing for τ=0, we nest the case where the central bank has no additional powers of recovery than the private sector, and central bank intervention is a pure bailout. When τ>0, the central bank has greater powers of recovery (for example, through bank taxation) than the private sector as in Holmstrom and Tirole (1998), but because it needs to recover in the crisis state, it is limited by the bank's net worth then.

Since the central bank can only intervene ex post when the crisis state realizes at $t = 1$, its objective is

$$\max_{\phi}\ g(W - M + \phi I) - C(\phi I)$$

with FOC w.r.t. ϕ:

$$g'(W - M + L) = C'(L) \tag{18}$$

where the LHS is the expected benefit of funding provision (increasing the investment in the g technology), and the RHS is the cost of providing such funding. This is the same FOC as the planner's FOC w.r.t. ϕ. [16] At time 0, the bank must ensure it meets the date-1 solvency constraint so that it has enough residual assets left to pay the central bank. This implies:

$$\lambda I - (1+z)(M - E - L) \geq \tau L \implies m \leq \frac{k\lambda + \psi + (1-\tau k)\phi}{R^M}, \tag{19}$$

which, for any τ >0, is tighter than the constraint faced by the bank in the planner's problem: $m \leq \frac{k\lambda+\psi+\phi}{R^M}$. Intuitively, the fact that the bank is required to pay the central bank for liquidity in the risky state rather than in the good state (which would be the case with pre-committed liquidity as we will see) constrains the space for money creation further.

4.2.1 Bank's Problem

Taking ϕ as given, the representative bank's problem is

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{moneybond spread}} - p \underbrace{r^E \psi I}_{\text{cont cap premium}} + (1-p) \underbrace{\psi I}_{\text{cont cap payout}}$$
$$+ (1-p) \underbrace{\phi I}_{\text{LOLR payout}} - (1-p) \underbrace{z[mIR^M - \psi I - \phi I]}_{\text{fire sale}} - (1-p) \underbrace{\tau \phi I}_{\text{payment to LOLR}}$$

such that

$$m \leq m^{max} = \frac{k\lambda + \psi + (1-\tau k)\phi}{R^M}$$

with the Lagrange multiplier on the solvency constraint $\eta \geq 0$.

Note that the bank's objective also changes, as in expectation it now has to pay $\tau\phi I$ back at $t = 2$ on the LOLR funding if there is a crisis. The bank's FOCs w.r.t. money creation

[16] The central bank that acts ex post at $t = 1$ after the crisis state is realized is indifferent between any level of tax rate τ to charge at $t = 2$, though that tax rate affects the incentives of the bank. The representative bank optimally chooses to always accept the LOLR funding in full as long as $\tau \leq 1 + z$, in which case the bank gains from utilizing public liquidity at date 1 instead of resorting to the fire sale. It is also natural to assume $\tau \leq 1 + z$, as otherwise banks would not accept central bank support.

m and private insurance ψ remain the same as (3) and (4), but in equilibrium with $\eta = 0$, the bank's FOC w.r.t. I now becomes

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau)\phi \tag{20}$$

with the additional $(1-p)(1+z-\tau)\phi$ term reflecting the bank's incentive to make additional investment due to the moral hazard created by central bank liquidity support that is underpriced to the bank for any $\tau < 1+z$.

Meanwhile, the PI takes L as given (correctly perceiving the central bank's choice of ϕ and the bank's choice of I), and the equilibrium fire-sale discount k is reached only after the value of aggregate M increases by the amount of aggregate public liquidity L (relative to the private equilibrium without a central bank).

4.2.2 PI's Problem

Taking L as given, the representative private investor's problem is

$$\max_{M,E}\ p\left[g(W) + r^E E\right] + (1-p)\left[g(W-M+L) + \frac{1}{k}(M-E-L)\right].$$

PI's FOCs w.r.t. M, the PI's funds used for fire-sale purchases and private insurance in the crisis state, is:

$$g'(W-M+L) = \frac{1}{k} \tag{21}$$

which equalizes the marginal benefit of investing in the g technology with the forgone cost of fire sales $\frac{1}{k}$ in the crisis state at $t = 1$.

PI's FOC w.r.t. private liquidity commitment E remains the same:

$$pr^E = (1-p)\frac{1}{k}. \tag{22}$$

4.2.3 Private Equilibrium with Central Bank Intervention as LOLR

With the usual definitions of equilibrium, it is straightforward to show

Proposition 2. *[Overinvestment and Underprovision of Private Insurance under an LOLR Central Bank].*

Let the superscript LOLR denote equilibrium outcomes with a LOLR central bank charging a rate $\tau \geq 0$ at $t = 2$ conditional on the crisis state. Let superscript P denote the corresponding planner's choices, where the planner is also subject to the bank solvency constraint in the crisis state.

1. The level of money creation and LOLR funding is socially optimal. That is, $M^{LOLR} = M^P$ and $L^{LOLR} = L^P$.

2. However, as long as $\phi^{LOLR} > 0$ and $\tau < 1 + z$ (and therefore $L^{LOLR} > 0$, so there is some LOLR funding), the private equilibrium has an inefficiently high level of investment. That is, $I^{LOLR} > I^P$.

3. As a result, the minimum level of private insurance, $E^{min,LOLR} = \max(M^{LOLR} - k^{LOLR}\lambda I^{LOLR} - (1 - \tau k)L^{LOLR}, 0)$, is lower than the socially optimal level if the planner is subject to the same solvency constraint (and the planner chooses a non-zero level of E^{min}).

4. There is less overinvestment and greater provision of private liquidity if the central bank charges more for public liquidity. That is, $I^{LOLR}$is decreasing in τ and $E^{min,LOLR}$ is increasing in τ.

Proof. See Appendix A.4. □

The prospect of liquidity provision by the central bank liquidity can cause banks to overinvest if the intervention is not priced right, as banks recognize that they can scale up, supported by underpriced liquidity from the LOLR. This also leads to underprovision of private insurance, which is effectively crowded out by LOLR funds.

Furthermore, suppose we can specify that the social cost of intervention to be a simple increasing and convex function of the amount of public liquidity provided: $C(L) = \frac{1}{2}cL^2$ so that $C'(L) = cL$. The parameter c reflects the marginal cost of public liquidity provision. We can then show that under some technical assumptions on the curvature of f that the private insurance market will be endogenously missing if this perceived cost of intervention is sufficiently low.

Proposition 3. *[Endogenously Missing Market for Private Insurance]*

Assume that $C(L) = \frac{1}{2}cL^2$ and $f'(I)I$ is (weakly) decreasing in I (equivalently, $-\frac{f''(I)I}{f'(I)} \geq 1$). If the LOLR central bank's tax rate on public liquidity, $\tau \geq 0$, is low enough, then there exists a cutoff $c \geq 0$ such that for all $c \in (0, \bar{c}]$, $E^{min,LOLR} = 0$.

Proof. See Appendix A.5.[17] □

4.2.4 Discussion of Moral Hazard

Note that overinvestment due to moral hazard arises precisely because each individual bank perceives that the LOLR central bank covers a fraction ϕ of all its investments. If we instead assume that the representative bank simply takes overall intervention L as given

[17]For $\tau = 0$, we can show a clean proof of the statement. For our specifications on f and g for the numerical results, we can also show an explicit upper bound on τ under which this statement will always hold. Otherwise, for the more general LOLR, there could be an implicitly defined upper bound on τ, but it is difficult to derive sufficient conditions where a smaller c always leads to $E^{min} = 0$.

and this LOLR funding does not vary with its individual choice of I, then there is no moral hazard and overinvestment. Which then is more appropriate? The larger the money-funded investment, the higher the required private investor intervention in the fire sale market if the central bank does not intervene, and the higher the cost of foregone late projects. So it is quite plausible that the ex-post LOLR, motivated by the desire to support real activity, will intervene more if the need arises.

Note also that the size of the eventual fire sale discount is determined by technology. So central bank intervention does nothing to reduce it. But once its intervention is anticipated and the bank issues more money claims, the fire sale discount would be much deeper if the central bank did not intervene. This is classic moral hazard.

The central bank can itself be subject to moral hazard that we will mention but not investigate in detail: The central bank's own perceived cost of intervention may be $\beta^P C(L)$ with $\beta^P \leq 1$ – the central bank perceives a deadweight cost lower than the true cost because it is either covering up past errors in supervision, or it is fearful of the political reaction if activity falls considerably. This will increase anticipated central bank intervention even further, crowding out private intervention, but also introduce additional costs as the central bank's intervention is excessive even by ex post considerations.

Finally, the fee the central bank charges for providing LOLR funds is important in determining the extent of bank moral hazard. Of course, if the central bank has no additional powers of recovery than the private sector, given it has not contracted support ex ante it must charge an ex-post tax rate of $\tau = 0$. We call this the *bail-out central bank*, which we explore below. Alternatively, assuming that the central bank has greater powers of recovery than the private sector, in Appendix B, we consider an actuarially fair LOLR (AF-LOLR) – one that will charge a break-even rate $\tau = 1$ at time 2. Finally, and theoretically, the LOLR could eliminate moral hazard by charging $\tau = 1 + z$. Of course, this argument was anticipated by Bagehot (1873), who enjoined the central bank to lend freely into a crisis to solvent entities but at a high rate. Our theoretical analysis indicates precisely what that high rate ought to be.

4.2.5 Bailout Central Bank

We consider the special case of $\tau = 0$, where the central bank exercises no additional powers of recovery than the private sector. The old constraint on money creation applies. We plot the equilibrium outcomes with such a bailout central bank versus the planner's allocations using the same parameters as in Section 3. We also specify that the social cost of intervention is $C(L) = \frac{1}{2}cL^2$ where $c = 0.02$ to get a sufficiently high level of bailout funding.

The numerical plots in Figure 3 support Proposition 2. Although the total level of money

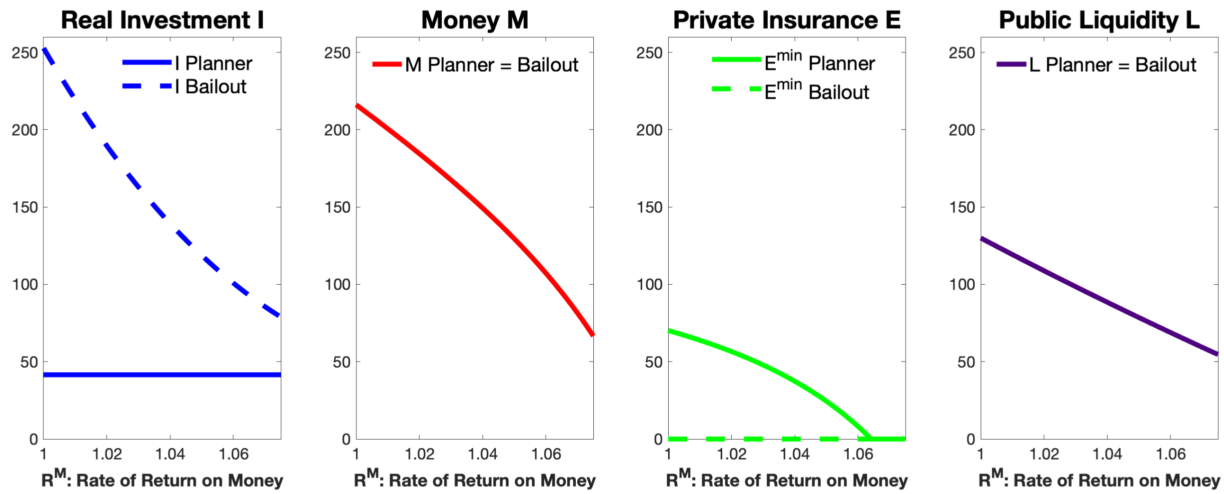

Figure 3: Equilibrium Outcomes with a Bailout Central bank, for different levels of return on money R^M

created M is efficient in the private case, it is now supported in equilibrium by an excessive level of investment and an inefficiently low level of private insurance than what is socially optimal. Moreover, under our baseline parameters, there is an endogenous "missing market" for insurance due to the presence of a bailout central bank. In Appendix B, we show that the same qualitative results prevail when $\tau = 1$ (AF-LOLR), though quantitatively there is less distortion resulting from moral hazard.

We also fix a level of money-bond spread by setting $R^M = 1.05$ and examine the effects of varying the cost of bailout funds c in Figure 4. A low cost of c maps to large levels of overinvestment and crowding out of private insurance, as well as high levels of bailout funding L (which in itself is not inefficient without the moral hazard problem). Hence, the lower the cost of bailout funds, the more severe the distortion due to moral hazard. As predicted by Proposition 3, with a sufficiently low $c < \bar{c}$ (the region to the left of the vertical dotted line in the plots of I and E), private insurance becomes endogenously missing (completely crowded out by public insurance L).

4.2.6 Frictional Insurance

In Appendix D, we relax the assumption of a frictionless insurance market by requiring the PI to escrow the amount pledged for private insurance until the risks of the crisis state dissipate entirely. This introduces an additional cost of providing insurance, over and above the forgone late investment. For the bank to wish to buy insurance, there must therefore be an additional marginal benefit to insurance than just preventing fire sales. In the model this comes from alleviating a binding money creation constraint. As a result, the model

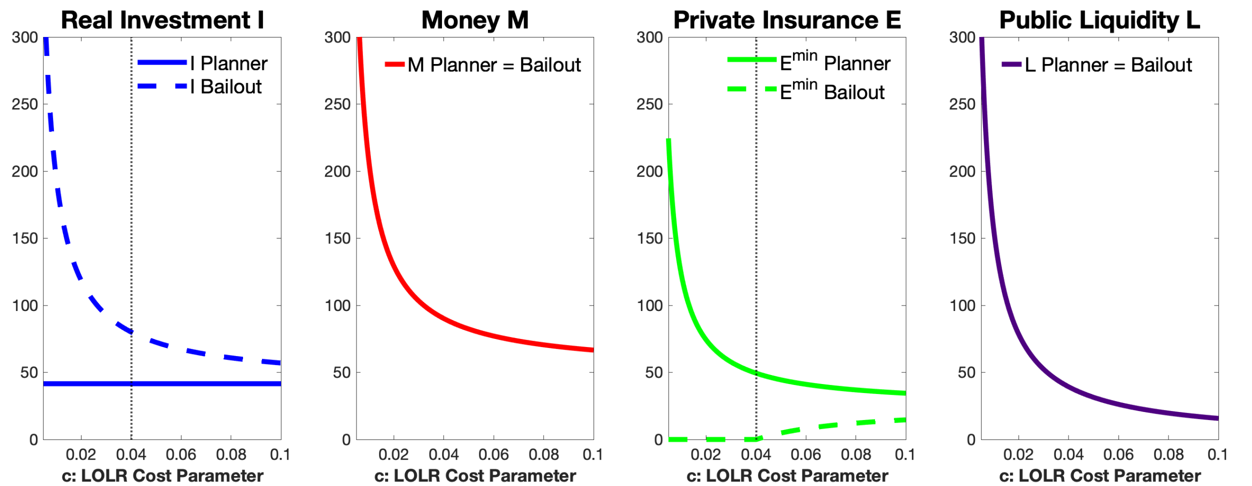

Figure 4: Equilibrium Outcomes with a Bailout Central bank at $R^M = 1.05$, for different levels of bailout cost c

reverts to that of Stein (2012) in that there is overinvestment and over-issuance of money due to fire-sale externality.[18] In addition, there is also over-provision of private insurance, as private agents enter into excessive insurance contracts to support the over-creation of money. Once we consider public liquidity provision, however, our earlier insights from the model with perfect private insurance hold, with public support crowding out the contingent capital.

4.3 Provision of Pre-committed Liquidity by Central Bank

An ex post central bank had limited ability to intervene. Now suppose we add to the baseline model a central bank that contracts ex ante (at $t = 0$) to provide a pre-committed level of liquidity to the bank in the crisis state at date 1 in return for payment in the good state. This could also be thought of as a form of deposit insurance. [19]

To the extent that banks purchase optimal support ex ante, even an interventionist central bank will not add additional liquidity ex post. Furthermore, since payment takes place in the good state, the central bank has fewer limits (such as the additional date-1 solvency constraint (19)) on the fee it can charge. So now in the bad state with probability $1 - p$, the CB provides $L = \phi I$ to the bank to alleviate the fire sale, but in the good state with

[18]Note that if there were frictional benefits to the bank of purchasing insurance (for instance in preventing sunspot runs) at the same time as there were frictional costs to the private insurer, we could get back to marginal benefits and costs being broadly equal, without the necessity for the collateral constraint to bind. In that case, the private solution would still be optimal.

[19]When the central bank charges a break-even rate, one can also interpret this as the CB charging p healthy banks to provide the deposit insurance to the $1 - p$ stressed banks, or just that if the time 0 to time 2 episode is repeated many times, the CB breaks even on average.

probability p, the central bank taxes at a rate τ to collect $\tau L = \tau\phi I$ from the bank. The central bank solves the time-0 planner's problem, but it can only choose ϕ and must respect the privately determined I, M, and E).

4.3.1 Actuarially Fair Price for Pre-committed Liquidity

Relative to the bailout central bank, the bank's objective now has an additional term that reflects the tax in the good state for using pre-committed liquidity:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} - p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} - p \underbrace{\tau\phi I}_{\text{CB tax}}$$
$$+ (1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} + (1-p) \underbrace{\phi I}_{\text{CB liquidity}} - (1-p) \underbrace{z[mIR^M - \psi I - \phi I]}_{\text{loss due to fire sale}}$$

Once again, the severity of the moral hazard problem depends on τ and to what extent the expected charge helps reduce moral hazard in the bank's FOC w.r.t. investments I. One politically plausible level for τ is the actuarially fair premium, $\tau = \frac{1-p}{p}$, that allows it to break even on loaned funds.

We solve for both central bank actions and bank responses (details of derivations in Appendix A.6), with the added complication that the central bank sets the level of intervention knowing it will affect private incentives, so it takes all private FOCs as given. We use the following functional specifications for our analytical results (the same ones as used for plotting): $f(I) = a\log(I) + I \implies f'(I) = \frac{a}{I} + 1$,$g(K) = \theta\log(K) \implies g'(K) = \frac{\theta}{K}$,$C(L) = \frac{1}{2}cL^2 \implies C'(L) = cL$.

We present the analytical and numerical results for this ex-ante central bank below.

Proposition 4. *[Pre-committed Liquidity at Actuarially Fair Price (AF-PL)]*

Let superscript $AFPL$ denote the equilibrium outcome with a central bank that provides pre-committed liquidity at an actuarially fair price. Under our current functional specifications on $f(\cdot)$, $g(\cdot)$, and $C(\cdot)$,

1. The level of money creation and central bank lending is lower than the socially optimal level (and the ex-post bailout level): $M^{AFPL} < M^P = M^{bailout}$, $L^{AFPL} < L^P = L^{bailout}$.

2. As in the case with a bailout central bank, there is overinvestment and underprovision of capital insurance: $I^{AFPL} > I^P$, $E^{min,AFPL} < E^{\min,P}$ (if $E^{\min,P} > 0$).

3. Compared to the bailout bank, moral hazard is less severe: $I^{AFPL} < I^{bailout}$, $E^{min,AFPL} > E^{min,bailout}$.

Proof. Appendix A1.7. □

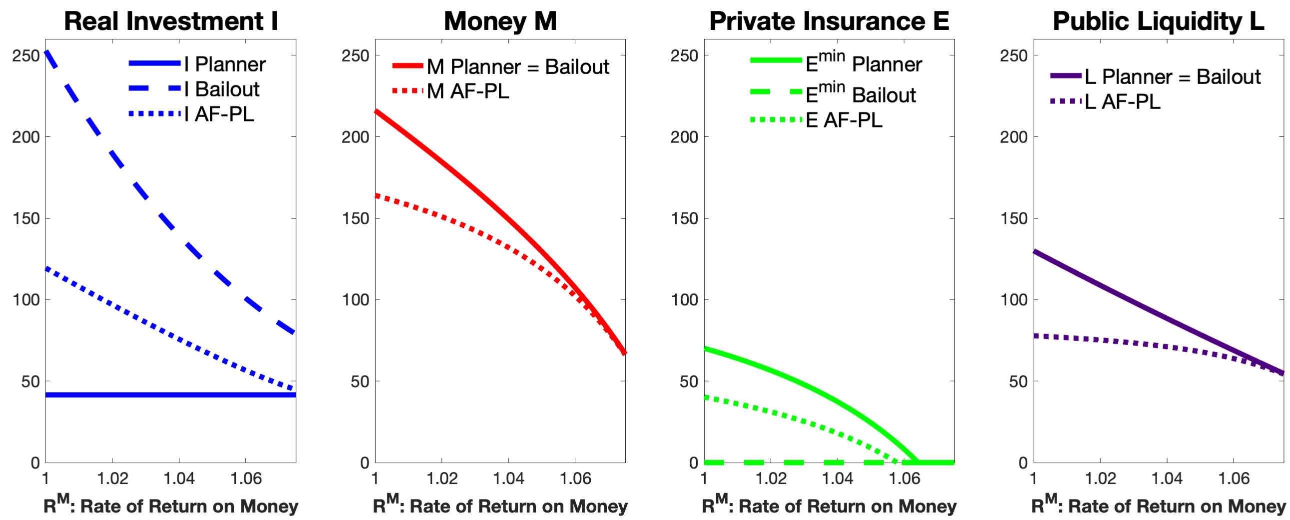

Figure 5: Equilibrium Outcomes for Actuarially Fair Pre-committed Liquidity, for different levels of return on money R^M

The dotted line in Figure 5 plots the pre-committed liquidity outcome along with the no central bank case and the bailout case. Note that in the model with an ex post bailout central bank, M and L are at the same level as the planner's allocations, while I is too high and E^{min} is too low. In contrast, as seen also from Proposition 4, the AF-PL central bank is acting in a constrained, second-best manner: by lowering L and M below the planner's allocations to alleviate moral hazard and push I and E closer to the efficient levels. So while there is still overinvestment and underprovision of private insurance, it is to a lesser extent than the ex-post case. However, only charging the break-even price for its pre-committed liquidity is not enough to restore efficient outcomes. As we discuss next, a higher price for L is needed to implement the planner's choice.

4.3.2 Appropriate Pigouvian Price

Now suppose instead of the actuarially fair tax rate for deposit insurance, the central bank charges $\tau = \frac{1-p}{p}(1+z)$ in the good state, the bank's objective now becomes the same as the objective (2) in the baseline without a central bank. The bank's choice of I now becomes socially optimal and moral hazard is entirely eliminated. The central bank's FOC now becomes the planner's condition $g'(W - M + L) = C'(L)$, because the private choice of investment I no longer depends on ϕ as there is no moral hazard. As a result, the central bank that provides pre-committed liquidity and charges $\tau = \frac{1-p}{p}(1+z)$ in the good state, which is above and beyond the actuarially fair premium, is able to achieve the efficient allocations.

The problem, of course, is that it is hard to explain to the public why the central bank should effectively charge the private sector rate (even more so when the private market has been crowded out) while its average cost of producing liquidity is far lower. A deposit insurance fund that continuously makes excess returns from the premia charged (which is what optimality requires) soon finds itself pressured by banks to reduce premia. The problem is even more acute if the central bank intervenes ex post to boost asset prices from their fire sale values. If it buys at depressed market prices, sellers are quick to point out that the central bank is no better than private sector bottom-fishers who are taking advantage of their vulnerability – even though market prices are invariably boosted by any central bank intervention. As our model suggests, the fire sale price k is the price the central bank should pay ex post to restore incentives, but political pressures will push it to pay the actuarially fair price of 1.[20] Unfortunately, publicly-provided insurance is invariably underpriced because of the difficulty of explaining incentive effects to the public (and the political class), something banks take full political advantage of.

4.3.3 Other Ways of Structuring Pre-committed Capital – Pre-positioned Collateral

There are other ways pre-committed intervention can be structured. If the central bank has the ability to recover more than the private sector in the crisis state, then instead of a capital infusion, the intervention can be a loan, perhaps against assets the private sector would not lend to without massive haircuts in a time of crisis. The pre-positioning of collateral that the central bank can lend against has been suggested by a number of authors (see, for example, King, 2016; Hanson et al., 2024). The repayment of the loan plus interest plus the "moral hazard" premium can be spread across good and crisis state based on the bank's ability to pay in those states. All that matters for date 0 incentives is that the expected payment be as in the previous sub section.

4.4 Welfare Comparison

Figure 6 plots the social welfare for the benchmark without a central bank, the three types of central bank interventions (bailout, LOLR that breaks even, and actuarially fair pre-committed liquidity), and the socially optimal case where the planner can choose L (which we showed is the case of pre-committed liquidity by central bank at the Pigovian price).

[20] Consistent with this, during the 2023 regional banking crisis in the United States, the Fed set up a facility to lend to banks at the full nominal face value of bonds posted as collateral rather than against their depressed market value.

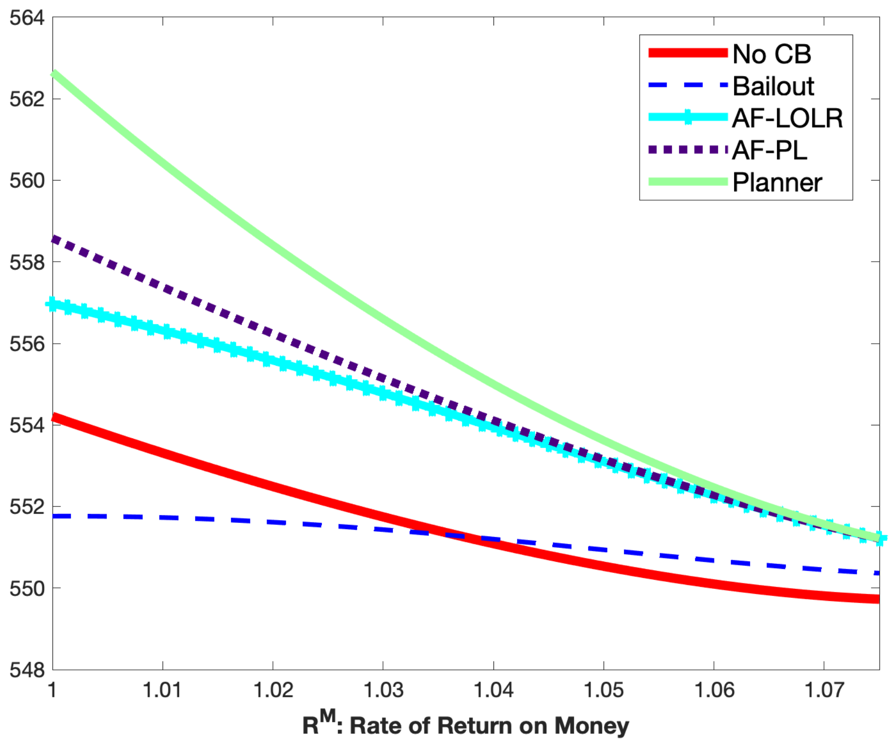

Figure 6: Welfare under Different Types of Interventions, varying the return on money R^M

When money-bond spreads are high (left part of the plot), moral hazard is extreme and adding a bailout central bank leads to lower welfare compared to the private equilibrium without a central bank. In the low spread region where the bank's incentive for excessive money creation is not too strong, adding any central bank leads to higher welfare, as central bank funding improves the outcome in the crisis state. The LOLR and ex-ante central bank that provides pre-committed liquidity also lead to higher welfare than the bailout central bank or the private outcome without a central bank, but are still socially suboptimal, especially in the region where the money-bond spread is large. Intuitively, the moral hazard from overinvestment and under-insurance is always stronger when the gains from money creation are large, and thus the support from underpriced liquidity particularly distortionary.

4.4.1 Welfare Effects of Varying c

Now for a fixed level of money spread, Figure 7 plots the welfare for different levels of c, where the dotted vertical line indicates the level below which private insurance becomes completely crowded out by the central bank. It is clear that a low level of c would not only crowd out private insurance but also severely reduce welfare.

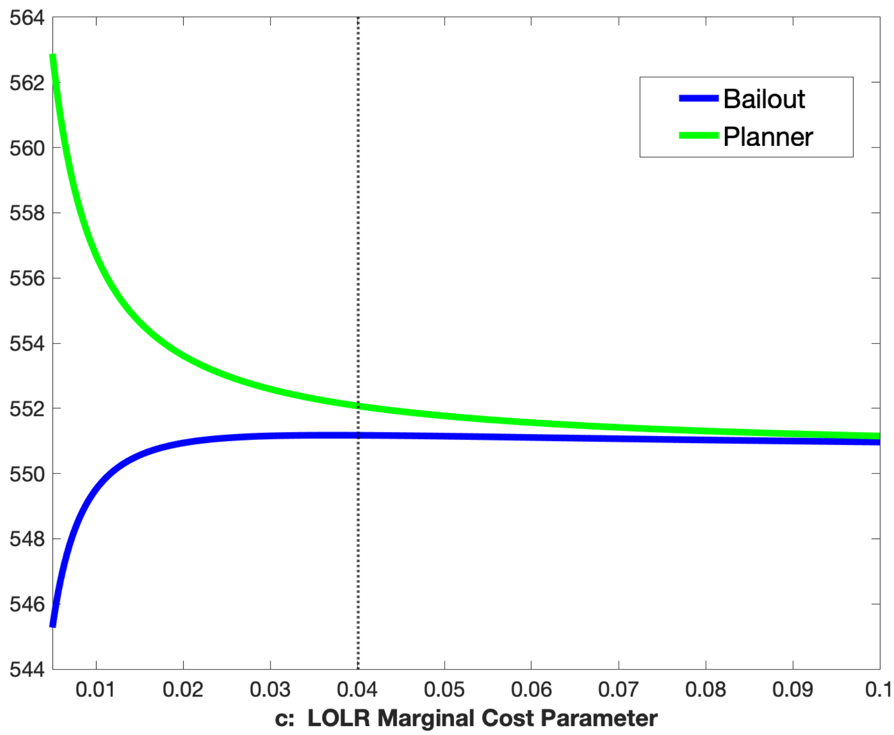

Figure 7: Welfare under Different Types of Interventions at $R^M = 1.05$, varying the central bank funding cost c

5 Financial Speculation

Thus far, we have assumed central bank liquidity support enhances real overinvestment because it reduces the effective cost of financing with cheap money. In the Stein model, the money premium is fixed, so it caps the return to money financing. What if investments with greater liquidity dependence have higher returns without cap? Interestingly, this may give the bank a more potent way to benefit from public liquidity intervention, in the process undermining the central bank's desire to enhance ex post real investment in the crisis state. Put differently, banks have creative ways of utilizing underpriced liquidity, some much more distortionary than others.

5.1 Speculation Technology

To fix ideas, we will make the case with an alternative financial speculation the bank can lend to, which has some implications for today's world. We therefore assume that in addition to the real technology $f(\cdot)$ — which can be financed by money (a fraction $m_1 \in [0, 1]$) or bonds (a fraction $1 - m_1$) — banks can also invest αI in a leveraged speculative financial technology as an alternative to the simple storage technology that pays R^B. For each dollar invested in speculation, the bank can choose leverage l on an existing, constant net spread s to earn $1 + ls$ in expectation. In the crisis state, this "search for yield" speculation leads to a margin call $v(l) > 0$, where $v(\cdot)$ is increasing and convex.

On the financing side of the αI invested in speculation, a fraction $m_2 \in [0, 1]$ is raised via money and the rest is raised via bonds. We impose an additional constraint that $m_2 \geq \bar{m}$, i.e., at least $\bar{m}$ fraction of αI must be money financed. We can attribute this to the bank's duration-hedging motives or regulations (this constraint is not essential but will ensure money financing is not crowded out). The liquidity call in the crisis state from this investment in speculation becomes $[m_2 + v(l)]\alpha IR^M$. Moreover, all else equal, margin call leads to additional fire sales.

The total amount of money issued in this economy is $M = m_1 IR^M + m_2\alpha IR^M = mIR^M$, but the total amount of liquidity demand at time 1 is $(1 + \theta)M = m_1 IR^M + m_2\alpha IR^M + v(l)\alpha IR^M$ with $\theta = \frac{v(l)\alpha}{m}$, where the last term reflects the additional liquidity demand from margin calls.

We focus on the case where in equilibrium, the speculative technology is lucrative enough so that it pays a net return above R^B (and thus banks do not invest in the simple storage technology). To be precise, $1 + ls - v(l)(1 - p)zR^M > R^B$ so that the gross return $1 + ls$, minus additional expected fire sale costs due to margin calls, $v(l)(1-p)zR^M$, is greater than R^B.

We model the gains from speculation $1+ls$ and any costs associated with the margin calls $v(l)$ as transfers, so the planner's problem remains the same from Section 4. For exposition, we add the speculation technology to both the benchmark model and the model with a bailout central bank.

5.2 Bank's Problem

The bank now chooses $m_1 \in [0, 1]$ (fraction of real investment financed by money), $m_2 \in [0, 1]$ (fraction of financial speculation financed by money), α (size of financial speculation relative to real investment I), leverage l, along with private insurance as a fraction of real investment ψ and real investment I to solve the following problem:

$$\max_{m_1\in[0,1],m_2\in[0,1],\alpha,l,\psi,I} \underbrace{pf(I)+(1-p)\lambda I - m_1 IR^M - (1-m_1)IR^B}_{\text{real investment}}$$
$$+\underbrace{\alpha I\cdot(1+ls) - m_2\alpha IR^M - (1-m_2)\alpha IR^B}_{\text{speculation}}$$
$$-p\underbrace{r^E\psi I}_{\text{Cont Cap premium}} + (1-p)\underbrace{\psi I}_{\text{Cont Cap payout}} + (1-p)\underbrace{\phi I}_{\text{LOLR payout}}$$
$$-(1-p)\underbrace{z[m_1 IR^M + [m_2+v(l)]\alpha IR^M - \psi I - \phi I]}_{\text{fire sale costs}}$$

s.t.

$$m_1+\alpha[m_2+v(l)] = m \leq \frac{k\lambda+\psi+\phi}{R^M}$$

$$m_2 \geq \bar{m}$$

5.3 Private Investor's Problem

Taking L and k as given, the PI's problem is essentially unchanged from the baseline, except that the total liquidity provided by the PI to the system is now $(1+\theta)M-L$:

$$\max_{(1+\theta)M,E} p\left[g(W)+r^E E\right]+(1-p)\left[g(W-(1+\theta)M+L)+\frac{1}{k}((1+\theta)M-E-L)\right].$$

The FOCs of the bank and the PI are shown and discussed in detail in Appendix C.1.

5.4 Private Equilibrium

When $1+ls-v(l)(1-p)zR^M > R^B$ (financial speculation earns a higher net return than the storage technology), the bank only invests in the real technology f and the speculation technology. In equilibrium, to finance speculation, the bank would like to use as much bond financing as possible (to pocket the difference between the net return on speculation and bond financing) and finance the rest using money at a point where the marginal cost of financing via money, which is effectively R^M plus the expected fire-sale cost $(1-p)zR^M$, is equalized with the net return on speculation.

As a result, banks are willing to accept a higher fire-sale cost that equals the difference between net speculation returns and R^M, whereas in the baseline model, the expected fire-sale cost equals the money-bond spread R^B-R^M. Therefore, in this model with lucrative speculative investments, real investments f are financed by bonds at cost R^B (because bond financing is cheaper than the all-in cost of money financing given higher fire sale costs). In short, we always have $m_1 = 0$, $m_2 = \bar{m}$, so money is created only to finance financial speculation. With more demand for liquidity through margin calls and a lower fire-sale price (though not necessarily more money created), the central bank also intervenes more in the presence of speculation. These results are formalized in the proposition below and fully derived in Appendix C.3 and C.4.

Proposition 5. *[Equilibrium in the Model with Speculation]*

Suppose $1 + ls - v(l)(1-p)zR^M > R^B$.

1. The bank finances real investments using bonds only ($m_1 = 0$*) and the bank is on the constraint (*$m_2 = \bar{m}$*)*

2. Let k^S *and* k^P *be the fire-sale price in the crisis state in the economy with speculation and under the planner's choice, respectively. Then* $k^S < k^P$.

3. With a bailout central bank, there is overinvestment in the real technology and over-intervention by the central bank: $I^S > I^P$, $L^S > L^P$.

4. With or without a central bank, there is higher private and total demand for liquidity in the crisis state in the economy with speculation than under the planner's choice: $(1 + \frac{v(l)}{\bar{m}})M^S - L^S > M^P - L^P$ *and* $(1 + \frac{v(l)}{\bar{m}})M^S > M^P$.

Proof. See Appendix C.4. □

In the baseline model with money financing, the fixed money premium pinned down the fire sale cost. Here, the variable return on the speculative investment increases the equilibrium fire sale cost above the money premium, and were it not for the constraint requiring minimum money financing, would crowd out the use of money financing entirely. Put differently, the speculative investment, even though it entails margin calls, is a better use of scarce liquidity (because of the higher returns) than money financing with attendant runs. Liquidity migrates to the highest net-of-liquidity-demand return activity, and this need not be money-financed real investment.

5.5 Numerical Results

We use the same parameters as the baseline setup. We choose $s = 0.01$ as the size of the primitive net "spread" that is magnified by the leveraged speculation trade. For the margin call function, we use $v(l) = 0.002l^2 + 0.001l$. We set the minimum level of money used to fund speculation to be $\bar{m} = 0.5$ (similar results prevail if we use $\bar{m} = 1$). These parameter

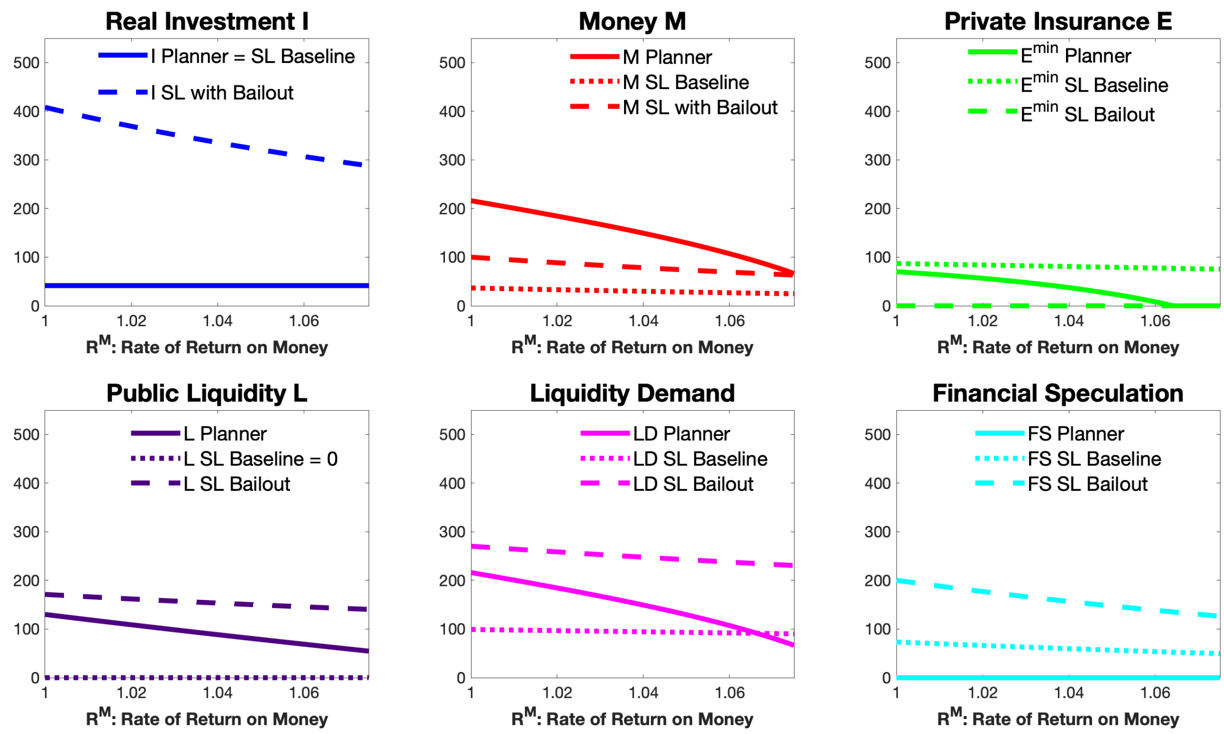

Figure 8: Equilibrium Outcomes with and without Speculative Lending (SL) Opportunities

choices give reasonable levels of speculation returns (net return of 0.1 to 0.13) and leverage (l is around 20-26; the size of the margin call, $v(l)$, is around 0.8 to 1.3).

Figure 8 plots the equilibrium outcomes of the model with speculation and no central bank, with speculation and with a bailout central bank, along with the efficient outcomes from Section 4.1. With speculation added to the baseline model, there is no distortion on the real investment I, though speculation leads to more private liquidity demand $M - L$ in the baseline model (since $L = 0$ in the baseline with no central banks) that is financed by a higher-than-socially-optimal level of private insurance E.

Once a bailout central bank is added to the model with speculation, the baseline model's results of overinvestment in the real technology and underprovision of liquidity insurance continues to prevail (with an endogenously missing private insurance market). There is also a larger level of speculation (as shown on the right panel) after adding a bailout central bank, and due to a more depressed fire-sale price with financial speculation, the level of central bank intervention is also inefficiently high. Furthermore, while the total amount of liquidity demand (dashed red line at the right panel, which includes both M and margin calls) is larger than that in the baseline setup (M Planner), fewer money-like deposits are issued because they compete with margin calls for liquidity in the crisis state.

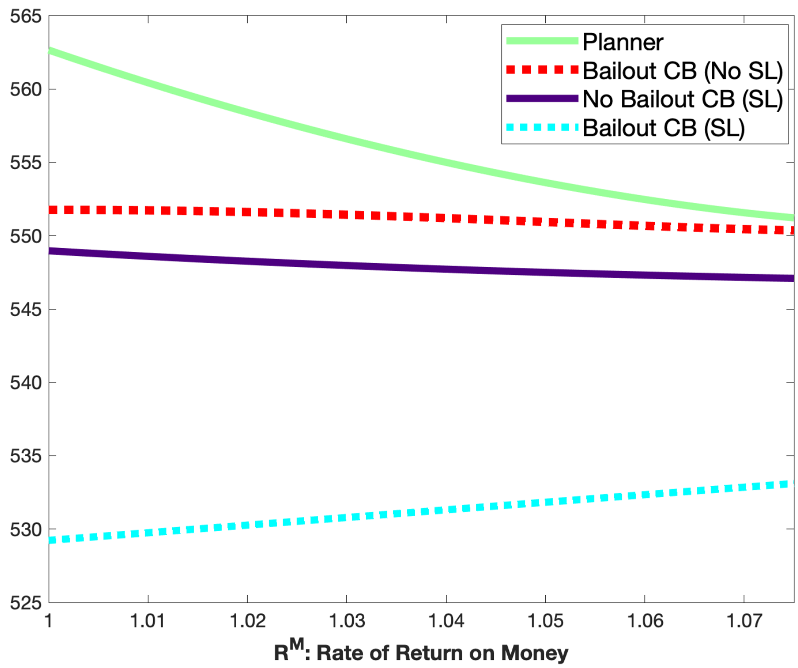

Figure 9: Welfare with vs without Speculative Lending Opportunities

5.5.1 Welfare

Figure 9 presents the welfare levels under four scenarios: the planner with contingent public liquidity provision, the benchmark model with a bailout central bank, the speculation model without a bailout central bank, and the speculation model with a bailout central bank.

Conceptually, one can view the presence of a bailout central bank and speculative opportunities as distortionary frictions that reduce welfare. However, the welfare loss with a bailout central bank in an economy with speculation is not merely the sum of the welfare losses from the baseline model with each friction individually — there is an additional loss from the interactions of the two distortions. As shown in Appendix C5, much of the interaction effect comes from the distortion in bank's FOC w.r.t. real investments I is increasing in the amount of fire-sale support from public intervention. Because speculation leads to larger fire sales, the overinvestment incentives from the bailout are exacerbated. Another novel distortion with speculation is that while it creates more liquidity demand at time 1, it creates fewer deposits M, which reduces welfare because of the loss of the household's convenience yield on money.

6 Lender of Last Resort and Financial Speculation

We have argued that banks can create safe, money-like liabilities by pre-arranging injections of contingent capital from the private sector. Such contingent capital provision is however crowded out by the provision of committed public liquidity if the latter is not suitably priced. Furthermore, the possibility of central bank intervention, perhaps intended to enhance real investment, can channel bank investments into financial speculation, warranting greater intervention. Historical evidence presented below provides support for the interaction between these channels.

6.1 The Emergence of Public Backstops

Early discussions of the lender of last resort are Thornton (1802) and Bagehot (1873), who emphasized that a LOLR should protect the aggregate money stock rather than individual institutions, lend only to sound institutions at penalty rates with good collateral, and preannounce these conditions in advance of crisis (Humphrey, 1989).[21] There is an extensive literature on the subject, with proponents favoring the LOLR's ability to overcome information asymmetries that lead to insolvency for otherwise sound banks as well as the LOLR's capacity to provide liquidity during episodes of systemic crises (Freixas et al., 2002). Meanwhile, the moral hazard costs of such a public backstop, potential losses to taxpayers,[22] and other potential costs of implementing liquidity provision that reduce a central bank's lending capacity (suggested in Goodhart (1999) and captured by the $C(\cdot)$ function in our model), have also been also well-recognized by policymakers and academics.

Even as proponents argue that deposit insurance could help reduce and alleviate bank runs (a view consistent with the sunspot bank run model of Diamond and Dybvig (1983)), there is also evidence that deposit insurance could lead to excessive risk-taking via moral hazard which increases systemic risk and causes banking crises (Wagster, 2007; Calomiris, 2010; Calomiris and Jaremski, 2019). The latter is captured by our analysis of the crowding-out effects of pre-committed public liquidity, e.g., a deposit insurance scheme or the formation of a central bank with commitment power to act as LOLR.

Irrespective of the normative questions on public backstops, interventions during banking crises expanded significantly in size and scale across the world in the 20th Century (Metrick

[21] Both the preannouncement corresponds more closely to pre-committed liquidity in our analysis, though it is unclear whether central banks have this level of commitment power ex ante, which is why we also analyze the bailout and ex-post LOLR central bank in Section 4.

[22] The bailout central bank in our analysis corresponds to one that completely neglects repayment to taxpayers which leads to severe moral hazard, whereas the actuarially fair LOLR repays taxpayers but moral hazard remains without charging an additional penalty rate.

and Schmelzing, 2021). Examining 17 major economies over the past four centuries, Ferguson et al. (2023) also show that since World War II, central bank policy responses to financial crisis have become "close to systematic," making crises the dominant driver of large central bank balance sheet expansions during the period. In contrast, wars were the primary cause of major central bank balance sheet growth in the centuries prior to World War II.

Overall, private arrangements of contingent capital appear to have been replaced during the 20th century by public provision of liquidity after episodes of large shocks where the equityholders became unable to inject sufficient capital to make the bank solvent again . However, in our model, while public liquidity provision is welfare-improving in a frictionless world, the effectiveness of a central bank that bails banks out, acts as a lender of last resort, or provides pre-committed liquidity, depends on the extent of moral hazard it induces in the banking sector, the political or economic costs of bailouts, as well as the myopia of the central bank. The historical and empirical evidence on whether different forms of public liquidity intervention are welfare-improving (in terms of severity and probability of crises) is unsurprisingly inconclusive and this remains a topic of debate.

6.2 Bank Liability under the Presence of a Public Backstop

Empirically, in addition to the gradual disappearance of private contingent capital in any form of contingent liability after the widespread adoption of public backstops in the 20th century, there has also been a secular decrease in bank equity as a fraction of assets, or equivalently an increase in banks' leverage, until capital regulations worldwide raised banks' capital ratio following the Global Financial Crisis (GFC) of 2009. For instance, Kaufman (1992) and Barth and Miller (2018) document that a secular decline in U.S. bank's book equity ratios (book capital as a fraction of total book assets), from above 50% in 1840 to below 10% in the late 1940s. Similarly, Alessandri and Haldane (2009) show the same downward trend in bank equity ratios in the UK from over 15% in 1880 to below 5% in 1960. This prolonged trend of increasing bank leverage, which started in mid or late-1800s in both the US and UK, is consistent with an increased willingness of banks to expand their assets and create more money-like securities to pocket the "money-bond spread" after a lender of last resort gradually became established and public bailouts of banks became the dominant expectation.

6.3 Continuing Recurrence of Severe Banking Crises

Despite central bank efforts, significant banking crises continue to occur across the world, including in advanced economies. Reinhart and Rogoff (2013) show that, especially since the early 1970s, the incidence of banking crises globally has been at least as frequent, if not more frequent, than in the pre-Great Depression era. In a cross-country study of 14 countries spanning 1870–2008, Schularick and Taylor (2012) also show that while monetary policy responses to financial crises became more aggressive after 1945, the output costs of crises have remained substantial. As an example, in the United States alone, banking and financial crises involving financial stability concerns since 1980 include the Savings and Loans (S&L) crisis in the 1980s, the bailout of Continental Illinois National Bank and Trust Company in 1984, the failure of Long-Term Capital Management in 1998, the Great Financial Crisis of 2007–2009, and the 2023 U.S. regional bank crisis (Bouis et al., 2025).

6.4 Modern Evidence: Financial Speculation

Although we abstract from the distinction between banks that fund speculation activities and the speculators themselves, our setup of financial speculation that magnifies a small primitive spread via leverage and leads to margin call costs in a crisis state mirrors the structure of real-world carry trades, including the Treasury cash-futures basis trade (also known as "the basis trade"), where hedge funds take a leveraged long position in cash US Treasuries while simultaneously holding a short position in Treasury futures. When there are temporary demand and supply imbalances in Treasury and its futures markets, these leveraged bets, often at 4060 times and even higher (Hauser, 2020), allows hedge funds to profit from the convergence between the futures price and bond price.[23]

Many studies have documented that the US Treasuries market dislocation in March 2020 during the Covid-19 Crisis were largely driven by fire sales from hedge funds that engaged in the basis trade (see, for example, Duffie, 2020; Barth and Kahn, 2021). In particular, the unwinding of leveraged futures positions by these funds were driven by a surge in Treasury futures margins (Schrimpf, Shin, and Sushko, 2020), leading to significant illiquidity in segments of the Treasuries market (see, for example, He, Nagel, and Song, 2022).[24] The Federal Reserve's purchase of $1 trillion in Treasuries subsequently helped restore normal functioning of the market (Vissing-Jorgensen, 2021). Kashyap et al. (2025) advocate for a more targeted policy response, where the central bank acts as a counterparty

[23] According to the Composite Indicator of Systemic Stress for the Euro Area, several other recent episodes of financial speculation have also raised financial stability concerns due to fire sales triggered by margin calls. Notable examples include the €100M trading losses from Einar Aas borne by Nasdaq and its clearing house members in September 2018, the collapse of Archegos Capital Management in April 2021, and the liability-driven investment (LDI) crisis involving UK pension funds in September 2022. We thank Francesco Mazzaferro for this list of examples.

[24] Schnabel (2020) also finds that with total variation margins posted by investment funds in the Euro area rose more than fivefold during the peak of the crisis, resulting in a large outflow in MMFs (money-market mutual funds) as investment funds facing margin calls sold their MMF holdings.

to hedge funds' unwinding trades. They argue that this approach would be more effective than broad-based liquidity provision in the Treasury market. While helping mitigate the associated moral hazard concerns, it also embeds duration neutrality for the central bank.

In our extension in Section 5, where margin calls take place when the system is liquidity stressed and financial speculation is funded by short-term money-like liabilities (as is common in practice), our model illustrates a mechanism through which an LOLR over-intervenes and the expectation of broad-based public liquidity provision can amplify financial speculation and reduce welfare.[25]

7 Conclusion

The incentive for banks to overissue money-like liabilities is an old concern, rendered more salient by recent theoretical modeling. Our work, however, suggests there are straightforward private remedies. The problem with public remedies is the political difficulty of charging an adequate fee to ensure the right private incentives. Public intervention thus is invariably underpriced and distortionary.

Our model could also find applications in international finance, where countries (substituting for banks in our model) might have the temptation to issue too much short term debt. Fee-based contingent credit lines from multilateral organizations like the IMF could reduce risk-taking, and resemble the contingent capital insurance in our model. Of course, the IMF's willingness to bail out countries without imposing sufficiently stiff conditionality could resemble the bailout in our model, though ordinary. The consequence would be excessive investment and risk taking.[26]

Ours is not a dynamic or a repeated game model. Arguably, though, while some aspects of central bank intervention can be rationally forecast, others become more likely as the central bank establishes a history of intervention. Put differently, expectations of future intervention grow with intervention. There is mounting evidence that central banks have become much more willing to intervene in the post World War II era (see, for example, Schularick and Taylor, 2012), perhaps because democracies have become more sensitive to public pain. This has been accompanied by the increased prevalence of public deposit insurance. While central bank intervention seems to have prevented banking systems from imploding or economies

[25]While unspecified in our model, the demand for speculation may also be closely connected to the supply of reserve assets. First, bank lending to speculators could be expanded if they have low return reserves on their balance sheet. Second, it could be that with more reserve assets leading to a higher level of demand deposits funding, central bank liquidity interventions are more likely to occur as it is more politically palatable for the LOLR to be saving the depositors' money rather than bailing out a small number of speculators.

[26]We thank Agustin Carstens for suggesting this.

from entering into outright deflation, cross-country studies such as Schularick and Taylor (2012) find that real output losses in the economic system are no lower and investment post-crisis no higher – perhaps because the banking system leverages and takes risk in anticipation. Indeed, Schularick and Taylor (2012) find the frequency of crises after World War II is higher than before. In a related vein, Ferguson et al. (2023) find that central bank liquidity support during financial crises has indeed reduced the severity of crises, but also raises the probability of future boom-bust episodes. Finally, as the size of the financial sector relative to the central bank increases, and as the fiscal situation of developed countries deteriorates, it is worth asking whether the capacity of the central bank to intervene could eventually be limited, even if it controls an elastic currency. In that case, could private sector risk taking in anticipation of intervention prove particularly costly.

What about regulation and supervision to reduce the gaming of intervention? As the continuing problems in the US financial system (for example, see the Barr report on the bank failures in March 2023) suggest, and as Barth, Caprio, and Levine (2005) argue, regulation and supervision is no panacea. All this suggests that finding the right mix of private contracting and public support will be an ongoing topic of debate and research.

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Appendix

A Key Steps and Proofs in Sections 3-4

A.1 Bank's FOC wrt I in the Baseline Model in Section 3.1

$$
\begin{aligned}
pf'(I) + (1-p)\lambda - R^B &= -\left[m(R^B - R^m) - (1-p)z[mR^M - \psi]\right] + pr^E\psi - (1-p)\psi \\
&= -\underbrace{m\left[(R^B - R^m) - (1-p)zR^M\right]}_{=\frac{\eta}{I}m} + \underbrace{\psi[pr^E - (1-p)(1+z)]}_{=\frac{\eta}{R^M I}\psi} \\
&= \frac{\eta}{I}\left[\frac{\psi}{R^m} - m\right],
\end{aligned}
$$

where we use the bank's FOCs (3) and (4) on the second line to arrive at the simplied expression (5).

A.2 Rewriting the Social Planner's Objective in Section 3.4

The social planner maximizes the utility of the representative household, who is endowed with Y units of output at time 0 and has utility function

$$U = C_0 + \beta\mathbb{E}[C_2] + \gamma M,$$

where $C_0 = Y - I - W$ and

$$C_2 = \begin{cases} f(I) + g(W) & \text{with prob } p \\ \lambda I + g(W - M) + M & \text{with prob } 1-p \end{cases}.$$

Therefore,

$$U = Y - I - W + \beta\left[pf(I) + pg(W) + (1-p)\lambda I + (1-p)g(W-M) + (1-p)M\right] + \gamma M.$$

Dropping constants Y and W, and dividing the expression by β (equivalent to multiplying by $R^B = \frac{1}{\beta}$) yields

$$U = -R^B I + pf(I) + pg(W) + (1-p)\lambda I + (1-p)g(W-M) + (1-p)M + \frac{\gamma}{\beta}M,$$

where $\dfrac{\beta+\gamma}{\beta} - 1 = \dfrac{R^B - R^M}{R^M}$. So we can rearrange and get

$$U = pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) + (1-p)[g(W-M) + M],$$

which arrives at the planner's objective in Section 3.4.

A.3 Planner's FOC wrt I in the Baseline Model in Section 3.4

$$\begin{aligned}
&pf'(I) + (1-p)\lambda - R^B + m(R^B - R^m) + \\
&+(1-p)\underbrace{\Big[-g'(W-M) + 1\Big]}_{=-\frac{1}{k}+1=-z} mR^M = -\frac{\eta^P\lambda}{R^M}\left(\frac{g''(W-M)}{(g'(W-M))^2} mR^M\right) \\
\Longrightarrow\ & pf'(I) + (1-p)\lambda - R^B + \underbrace{\Big[m(R^B - R^m) - (1-p)zmR^M\Big]}_{=m\frac{\eta^P}{I}\left[1-(\frac{g''(W-M)}{(g'(W-M))^2})\lambda I\right]} = -\frac{\eta^P\lambda}{R^M}\left(\frac{g''(W-M)}{(g'(W-M))^2} mR^M\right).
\end{aligned}$$

This can be further re-written as

$$pf'(I) + (1-p)\lambda - R^B = \eta^P\left(-\frac{g''(W-M)}{(g'(W-M))^2}\lambda m - \frac{m}{I} + \frac{g''(W-M)}{(g'(W-M))^2}\lambda m\right),$$

which leads to the expression in (13):

$$pf'(I) + (1-p)\lambda - R^B = -m\frac{\eta^p}{I}.$$

A.4 Proof of Proposition 2

Part 1. From comparing (8) with (16), note that the expressions for $(1-p)z$ are the same, so the private fire-sale price k is also socially optimal. Then by comparing from (18) and (21) with (14) and (17), we get $M^{LOLR} = M^P$, $L^{LOLR} = L^P$ as they are pinned down by the same FOCs.

Part 2-3. From the bank's FOC w.r.t. I,

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau)\phi,$$

whereas the RHS is 0 in the planner's case. The expressions for z are the same in both cases. Therefore, as long as $\phi > 0$ (some level of LOLR funding) and given that $f''(I) < 0$, we have that $I^{LOLR} > I^P$. It also follows that $E^{min} = M - k\lambda I - (1-k\tau)L$ is lower in the private case than what is socially optimal (assunig the social planner is subject to the solvency constraint under the same τ at $t = 2$ in the crisis state), because $M^{LOLR} - L^{LOLR} = M^P - L^P$ but $I^{LOLR} > I^P$.

Part 4. Consider two equilibria with LOLR policies that charge different levels of τ, with $\tau^H > \tau^L$. As shown in Part 1, the levels of M, L, k are at the planner's level regardless of τ. However, consider investment levels I^H, I^L, minimum private insurance $E^{min,H}$, $E^{min,L}$, and fraction of investment covered by public funding ϕ^H and ϕ^L under the equilibrium with an LOLR central bank that charges τ^H and τ^L, respectively. Then, suppose also that $\tau^H \leq 1 + z$.[27]

$$pf'(I^H) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau^H)\phi^H,$$
$$pf'(I^L) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau^L)\phi^L.$$

First, it cannot be the case that $I^H = I^L$. If so, because $L^H = L^L$, we would have $\phi^H = \phi^L$, but the RHS is different for the two conditions (because $\tau^H > \tau^L$), which would imply $I^H \neq I^L$, arriving at a contradiction.

Second, suppose for a contradiction that $I^H > I^L$. Then because $L^H = L^L$, it must be that $\phi^H < \phi^L$. Then we must have

$$\begin{aligned}
pf'(I^H) + (1-p)\lambda - R^B &= -(1-p)(1+z-\tau^H)\phi^H \\
&> -(1-p)(1+z-\tau^H)\phi^L \\
&> -(1-p)(1+z-\tau^L)\phi^{bailout} \\
&= pf'(I^L) + (1-p)\lambda - R^B.
\end{aligned}$$

This necessarily implies $f'(I^H) > f'(I^L) \implies I^H < I^L$, a contradiction.

Thus, it must be that $I^H < I^L$, and therefore $E^{min,H} = M - k\lambda I^H - (1-k\tau^H)L > M - k\lambda I^L - (1-k\tau^L)L = E^{min,L}$ (where M, L, k are the same for both equilibria).

A.5 Proof of Proposition 3

Part 1 (for $\tau = 0$): First, note that by definition, $E^{min,LOLR} = \max(M^{LOLR} - k\lambda I^{LOLR} - (1-\tau k)L^{LOLR}, 0) = 0$ if

$$M^{LOLR} - k\lambda I^{LOLR} - (1-\tau k)L^{LOLR} = M^{LOLR} - L^{LOLR} - k\lambda I^{LOLR} + \tau k L^{LOLR} \leq 0$$

which can be rewritten as

[27] If the central bank charges beyond the fire-sale savings, banks would simply choose not to receive central bank fundings.

$$k\lambda I^{LOLR} \geq M^{LOLR} - L^{LOLR} + \tau k L^{LOLR}$$

From the central bank's FOC w.r.t. L (18) and PI's FOC w.r.t. M (21), we have that

$$\frac{1}{k} = g'(W - M^{LOLR} + L^{LOLR}) = C'(L^{LOLR}) = cL^{LOLR} \implies L^{LOLR} = \frac{1}{ck^{LOLR}},$$

and

$$M^{LOLR} - L^{LOLR} = W - (g')^{-1}(\frac{1}{k})$$

where k is a constant that only depends on R^B, R^M, p (as can be seen from bank's FOC w.r.t. m (3) with $\eta = 0$, which remains to be the case with an LOLR central bank added). Since g is monotone increasing and strictly concave, $g' > 0$ and its inverse exists. We have that $M^{LOLR} - L^{LOLR} > 0$ (under the assumption that $g'(W) \geq 1 > \frac{1}{k}$ for any k).

As a result, we get $E^{min,LOLR} = 0$ if and only if

$$k\lambda I^{LOLR} \geq W - (g')^{-1}(\frac{1}{k}) + \frac{\tau}{c} \iff I^{LOLR} \geq \frac{W - (g')^{-1}(\frac{1}{k}) + \frac{\tau}{c}}{k\lambda}$$

So with a bailout central bank where $\tau = 0$, it suffices to show that I is monotone decreasing in c without any bounds, so that the LHS is increasing in c and the RHS is a constant.

From the bank's FOC w.r.t. I, if we multiply I on both sides, we get

$$pf'(I)I + (1-p)\lambda I - R^B I = -(1-p)(1+z-\tau)\phi I.$$

With the functional specification on $C(L)$, we always have that $C'(L) = cL = \dfrac{1}{ck} \implies L = \phi I = \dfrac{1}{ck}$. Therefore, we can rearrange and write

$$(R^B - (1-p)\lambda)I - pf'(I)I = (1-p)(1+z-\tau)\frac{1}{ck}.$$

Note that the LHS is increasing in I and is not bounded above, because we assume that $R^B > 1$ and $\lambda \leq 1$ which implies $(R^B - (1-p)\lambda)I$ is strictly increasing in I, and that $f'(I)I$ is (weakly) decreasing in I. Therefore, it is clear that I is a decreasing function of c, and $I \to \infty$ as $c \to 0$ (I grows without bounds as c decreases, which can be easily shown via contradiction).

Part 2 (general case): With $\tau > 0$, there is no guarantee on the existence of a positive $\bar{c}$ and it depends on the specific functional form f as well as the level of τ. For example, using our working assumptions on f and g, we would need $I \geq \dfrac{W - \theta k}{k\lambda} + \frac{\tau}{\lambda} L$ where

$$I = \frac{a + (1-p)z\frac{1}{ck}}{\frac{1}{p}[R^B - (1-p)\lambda - p]}, \qquad L = \frac{1}{ck}.$$

Thus, to make sure we get $E^{min} = 0$ with a small enough c, from equation (), we would need

$$\frac{(1-p)z}{\frac{1}{p}(R^B - (1-p)\lambda - p)} > \frac{\tau}{\lambda}$$

as a sufficient condition, which effectively imposes an upper bound on τ. In a more general setting, it might be possible to get the (possibly implicitly defined) derivative of I w.r.t. $\frac{1}{c}$ from

$$(R^B - (1-p)\lambda)I - pf'(I)I = (1-p)(1+z-\tau)\frac{1}{ck},$$

so we can simply impose that $I'(\frac{1}{c}) > \frac{\tau}{\lambda}$ which would guarantee $E^{min} = 0$ for a small enough c. This is again an upper bound on τ as long as I is increasing in $\frac{1}{c}$.

A.6 Derivation of the Ex-ante Central Bank's FOC in Section 4.4

The central bank's problem can be written in a way that makes fully transparent how ϕ affects private choices:

$$\max_{\phi} pf(I(\phi)) + (1-p)\lambda I(\phi) - R^B I(\phi) + m(\phi)I(\phi)(R^B - R^M)$$
$$+(1-p)\left[g\left(W - m(\phi)I(\phi)R^M + \phi I(\phi)\right) + m(\phi)I(\phi)R^M\right] - (1-p)C(\phi I(\phi)).$$

After some steps (detailed derivations below), the central bank's FOC w.r.t. ϕ can be written as

$$g'(W - M + L) = C'(L) + AL(C'(L) - 1), \tag{23}$$

where $A = \dfrac{(1-p)z}{ap} > 0$ is a constant that is increasing in the bank's marginal benefit from capital infusion (savings from fire sale), and decreasing in the productivity of time-0 investment. The LHS is the marginal social benefit from central bank funding in the crisis state and the RHS is the social cost of central bank funding in the crisis state, now scaled up by an additional term that reflects the severity of moral hazard (from the derivations, we show that it incorporates how a change in ϕ affects I). Note that this FOC takes the same form as the planner's and the bailout / LOLR central bank's, with the additional term $AL(C'(L) - 1)$ which reflects the effects of moral hazard.

A.6.1 Detailed Derivations

The central bank understands the following private decision rules:

1. Bank's FOC w.r.t. m pins down k:

$$(1-p)z = \frac{R^B - R^M}{R^M}.$$

Thus k is invariant to the central bank's actions as the constraint will never be binding in equilibrium.

2. Bank FOC w.r.t. I pins down I:

$$\begin{aligned} pf'(I) + (1-p)\lambda - R^B &= -(1-p)z\phi \\ \implies p[\frac{a}{I}+1] &= R^B - (1-p)\lambda - (1-p)z\phi \\ \implies \frac{a}{I} &= \frac{1}{p}\left[R^B - (1-p)\lambda - (1-p)z\phi\right] - 1 \\ \implies I &= ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-1}, \end{aligned}$$

where we can clearly observe that I is increasing in ϕ. So with moral hazard, we have $I'(\phi) > 0$, but if there is no moral hazard, $I'(\phi) = 0$. We can also formally derive the expression for $I'(\phi)$:

$$\begin{aligned} I'(\phi) &= -ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-2}(-(1-p)z) \\ &= ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-2}(1-p)z \\ &= I^2\frac{(1-p)z}{ap} \\ &=: AI^2, \end{aligned}$$

where $A = \frac{(1-p)z}{ap} > 0$ is a constant that depends entirely on parameters of the model. It is increasing in the bank's marginal benefit of insurance (savings from fire sale), and decreasing in the productivity of time-0 investment. It only shows up (so that $I'(\phi) \neq 0$) with the moral hazard term present in $-(1-p)z\phi$, otherwise, $I'(\phi) = 0$.

3. PI's FOC w.r.t. M pins down M:

$$g'(W - M + L) = \frac{1}{k} \implies \frac{\theta}{W - M + L} = \frac{1}{k} \implies \theta k = W - M + L.$$

We can think of $m(\phi)$ and $I(\phi)$ as separate functions of ϕ as the LOLR's choice affects the bank's financing and investment decisions, and write

$$\begin{aligned}\theta k &= W - m(\phi)R^M I(\phi) + \phi I(\phi) \\ \implies m(\phi)R^M I(\phi) &= W - \theta k + \phi I(\phi) \\ m(\phi) &= \frac{1}{R^M}\left[\frac{W-\theta k}{I(\phi)} + \phi\right],\end{aligned}$$

so that

$$\begin{aligned}m'(\phi) &= \frac{1}{R^M}\left[1 - \frac{W-\theta k}{I^2} I'(\phi)\right] \\ &= \frac{1}{R^M}\left[1 - A(W-\theta k)\right] \\ &= \frac{1}{R^M}\left[1 - A(M-L)\right].\end{aligned}$$

So without moral hazard, $m'(\phi) = \frac{1}{R^M} > 0$, but with moral hazard, we can no longer sign $m'(\phi)$ in general but $m'(\phi) < 0$ when $M - L$ (money that is not publicly insured) is large (which is the case with a large W).

For the central bank that charges the acturially fair τ =1 to break even, its problem can be written in a way that makes clear how ϕ affects private choices:

$$\begin{aligned}\max_{\phi}\ U(\phi) =& \{pf(I(\phi)) + (1-p)\lambda I(\phi) - R^B I(\phi)\} + m(\phi)I(\phi)(R^B - R^M) \\ & (1-p)\left[g\left(W - m(\phi)I(\phi)R^M + \phi I(\phi)\right) + m(\phi)I(\phi)R^M\right] - (1-p)C(\phi I(\phi)).\end{aligned}$$

The central bank's FOC w.r.t. ϕ is

$$\begin{aligned}0 =& \left[pf'(I) + (1-p)\lambda - R^B\right] I'(\phi) + (R^B - R^M)\left[m'(\phi)I + mI'(\phi)\right] \\ & + (1-p)g'(W - M + L)\left\{-\left[m'(\phi)I + mI'(\phi)\right]R^M + \phi I'(\phi) + I\right\} \\ & + (1-p)\left[m'(\phi)I + mI'(\phi)\right]R^M - (1-p)C'(L)\left[\phi I'(\phi) + I(\phi)\right].\end{aligned}$$

Substituting $pf'(I) + (1-p)\lambda - R^B = -(1-p)z\phi$, as well as $I'(\phi) = AI^2$ and dividing across by I gives

$$\begin{aligned}0 =& \left[-(1-p)z\phi\right]AI + \frac{R^B - R^M}{R^M}\left[m'(\phi) + mAI\right]R^M \\ & + (1-p)g'(W - M + L)\left\{-\left[m'(\phi) + mAI\right]R^M + \phi AI + 1\right\} \\ & + (1-p)\left[m'(\phi) + mAI\right]R^M - (1-p)C'(L)\left[\phi AI + 1\right].\end{aligned}$$

Now substitute that $L = \phi I$, $M = mIR^M$ everywhere to obtain

$$
\begin{aligned}
(1-p)C'(L)\left[AL+1\right] = & -(1-p)zAL + \frac{R^B - R^M}{R^M}\left[m'(\phi)R^M + AM\right] \\
& + (1-p)g'(W-M+L)\left\{-\left[m'(\phi)R^M + AM\right] + AL + 1\right\} \\
& + (1-p)\left[m'(\phi)R^M + AM\right].
\end{aligned}
$$

Recognizing that $m'(\phi)R^M = 1 - A(M-L) \implies m'(\phi)R^M + AM = 1 + AL$:

$$
\begin{aligned}
(1-p)C'(L)(1+AL) = & -(1-p)zAL + \frac{R^B - R^M}{R^M}(1+AL) \\
& + (1-p)g'(W-M+L)\left[-(1+AL) + AL + 1\right] + (1-p)(1+AL).
\end{aligned}
$$

Substituting $\frac{R^B - R^M}{R^M} = (1-p)z$, we can simplify as follows:

$$
-(1-p)zAL + (1-p)z(1+AL) + (1-p)(1+AL) = (1-p)C'(L)(1+AL).
$$

Using $g'(W-M+L) = \dfrac{1}{k}$, this can be written as Equation (23):

$$
(1-p)(1+z) = (1-p)\left[C'(L) + AL(C'(L)-1)\right].
$$

A.7 Proof of Proposition 4

Part 1. Consider equation (23):

$$
\frac{1}{k} = g'(W-M+L) = C'(L) + AL(C'(L)-1).
$$

and denote the RHS as $s(L) = C'(L) + AL(C'(L)-1) = cL + AL(cL-1)$.

First, note that the socially optimal L^P satisfies

$$
\frac{1}{k} = 1 + z = C'(L^P) = cL^P \implies L^P = \frac{1}{ck}.
$$

Second, note that $RHS(L)$ is increasing for any $L \geq L^P$ because $cL^P - 1 = \frac{1}{k} - 1 = 1 + z - 1 = z > 0$, so that all parts of $s(\cdot)$ are increasing in L. The same observation can be made by looking at $s'(L) = c + 2AcL - A > 0$ for any $L \geq \frac{1}{ck}$.

Third, suppose for a contradiction that the ex-ante central bank chooses the level of pre-commited liquidity to be some $L^{DI} \geq L^P$. Then because s is increasing for any $L \geq L^P$,

$$
s(L^{DI}) \geq s(L^P) = cL^P + AL^P(cL^P - 1) > cL^P = \frac{1}{k}\ ,
$$

because $AL^P(cL^P-1) > 0$ with $L^P = \frac{1}{ck}$. This implies

$$
C'(L^{DI}) + AL^{DI}(C'(L^{DI}) - 1) = s(L^{DI}) > \frac{1}{k}\ .
$$

which implies L^{DI} does not satisfy the CB's FOC (23). This is a contradiction, as the CB's problem must have an interior solution since $g(\cdot)$ is concave and $C(\cdot)$ is convex.

Therefore, we must have that $L^{DI} < L^P = L^{bailout}$. Because $(M - L)$ is pinned down by the same FOC $g'(W - M + L) = \frac{1}{k}$ and k is the same across all baseline cases, we also have $M^{DI} < M^P = M^{bailout}$.

Part 2. From the bank's FOC w.r.t. I,

$$pf'(I^{DI}) + (1-p)\lambda - R^B = -(1-p)z\phi^{DI} < 0.$$

whereas the planner's choice satisfies

$$pf'(I^P) + (1-p)\lambda - R^B = 0,$$

with the RHS being 0 in the planner's case. The expressions for $(1-p)z$ are the same in all cases. Therefore, as long as $\phi^{DI} > 0$ (some level of bailout funding, which is always the case in our specification), we have that $I^{DI} > I^P$. It also follows that $E^{min} = M - k\lambda I - L$ is lower with deposit insurance than what is socially optimal, because $M^{DI} - L^{DI} = M^P - L^P$ but $I^{LOLR} > I^P$.

Part 3. Suppose, for a contradiction, $I^{DI} \geq I^{bailout}$. Then because $L^{DI} < L^{bailout}$, it must be that $\phi^{DI} < \phi^{bailout}$. From the bank's FOC w.r.t. I:

$$pf'(I^{DI}) + (1-p)\lambda - R^B = -(1-p)z\phi^{DI},$$

and

$$pf'(I^{bailout}) + (1-p)\lambda - R^B = -(1-p)(1+z)\phi^{bailout}.$$

To have $I^{DI} \geq I^{bailout}$, we must have

$$-(1-p)z\phi^{DI} < -(1-p)(1+z)\phi^{bailout},$$

but

$$\phi^{DI} < \phi^{bailout} \implies -(1-p)z\phi^{DI} > -(1-p)z\phi^{bailout} > -(1-p)(1+z)\phi^{bailout},$$

which yields a contradiction.

Thus, it must be that $I^{DI} < I^{bailout}$, and therefore $E^{min,DI} > E^{min,bailout}$.

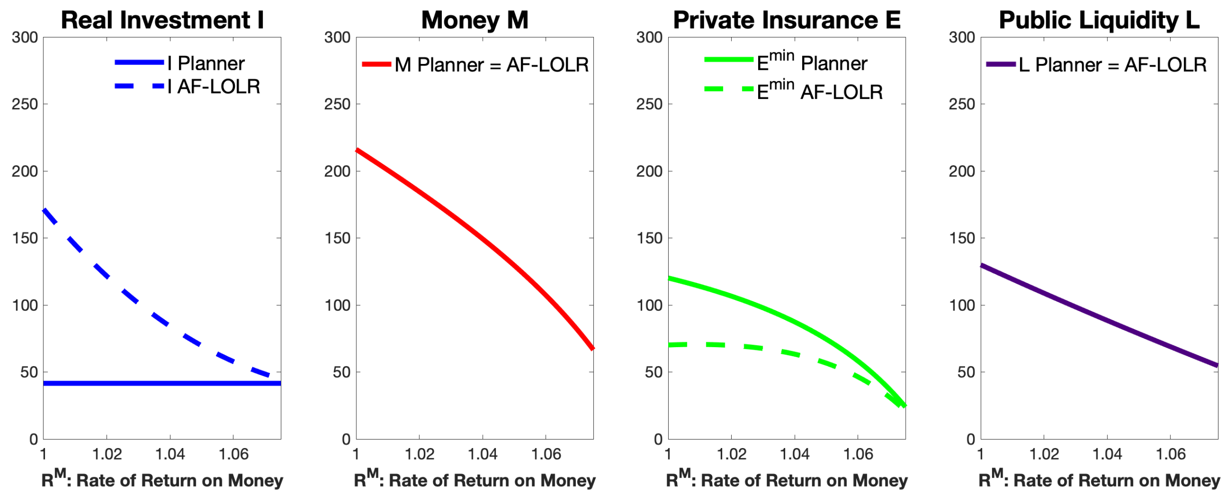

Figure 10: Equilibrium Outcomes with an Actuarially Fair LOLR central bank, for different levels of return on money R^M

B Acturially Fair LOLR

B.1 Actuarially Fair Ex-post LOLR

Consider here an LOLR central bank that is perceived to charge a break-even rate $\tau = 1$ at time 2 conditional on the crisis state. As shown in Figure 10, such an ex-post actuarially-fair LOLR (AF-LOLR) alleviates moral hazard relative to the bailout central bank in Figure 3, as there is less severe overinvestment and underprovision of private insurance, but it still cannot attain the planner's solution.[28] To counteract the moral hazard effect of LOLR, the LOLR has to charge a higher than actuarially fair rate on public liquidity provision. This may not be feasible if the high rate is not politically feasible (banks claim the central bank is gouging them) or if it is not collectible because the central bank has limited additional powers of recovery than the private sector.

[28]To ensure comparability, the planner's solutions plotted with the AF-LOLR also respects the solvency constraint for the repayment of taxes by the banks.

C Model with Speculative Technology

C.1 Bank's Problem

$$\max_{m_1\in[0,1],m_2\in[0,1],\alpha,l,\psi,I} \underbrace{pf(I)+(1-p)\lambda I - m_1 IR^M-(1-m_1)IR^B}_{\text{real investment}}$$
$$+\underbrace{\alpha I\cdot(1+ls)-m_2\alpha IR^M-(1-m_2)\alpha IR^B}_{\text{speculation}}$$
$$-p\underbrace{r^E\psi I}_{\text{insurance premium}}+(1-p)\underbrace{\psi I}_{\text{insurance payout}}+(1-p)\underbrace{\phi I}_{\text{LOLR payout}}$$
$$-(1-p)\underbrace{z[m_1IR^M+[m_2+v(l)]\alpha IR^M-\psi I-\phi I]}_{\text{fire sale}}$$

s.t.

$$m_1+\alpha[m_2+v(l)]=m\leq\frac{k\lambda+\psi+\phi}{R^M}$$

$$m_2\geq\bar{m}$$

where $z=\dfrac{1-k}{k}$ is the net return on fire sale assets. The Lagrangian is

$$\mathcal{L}=\text{objective}-\eta\left(m_1+\alpha[m_2+v(l)]-\frac{k\lambda+\psi+\phi}{R^M}\right)+\eta_2(m_2-\bar{m})$$

with Lagrange multipliers $\eta,\eta_2\geq 0$.

The bank's first order conditions (FOCs) then are as follows:

w.r.t. bank's fraction of real investment financed by deposits, $m_1\in[0,1]$:

$$I[(R^B-R^M)-(1-p)zR^M]=\eta,$$

where if LHS<RHS, we run into the corner solution that $m_1=0$ and the real investments are entirely bond financed.

w.r.t. the bank's choice of leverage on the speculative trade, l:

$$s-v'(l)(1-p)zR^M=\frac{\eta}{I}v'(l),$$

so for each dollar invested, the marginal benefit of increasing leverage, s, is equal to the additional fire sale costs due to the margin calls and increased shadow costs (if the constraint is binding) caused by the increase in leverage.

w.r.t. α, bank's speculative technology as a fraction of real investment:

$$I[(1 + lS - m_2 R^M - (1 - m_2)R^B - (1-p)z(m_2 + v(l))R^M] = \eta(m_2 + v(l)),$$

which can be written as

$$1 + lS - R^B + m_2 R^B - m_2 R^M - m_2(1-p)zR^M - v(l)(1-p)zR^M = \frac{\eta}{I}(m_2 + v(l))$$

w.r.t. the fraction of speculative technology financed by money, m_2:

$$I[(\alpha R^B - \alpha R^M) - (1-p)z\alpha R^M] + \eta_2 = \eta\alpha,$$

w.r.t. bank's fraction of real investment covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1-p)(1+z),$$

and w.r.t. bank's real investment, I:

$$\begin{aligned} &\underbrace{pf'(I) + (1-p)\lambda - m_1 R^M - (1-m_1)R^B}_{\text{real investment}} \\ &+ \underbrace{\alpha \cdot (1 + ls) - m_2 \alpha R^M - (1-m_2)\alpha R^B}_{\text{speculation}} \\ -p \underbrace{r^E\psi}_{\text{insurance premium}} &+(1-p) \underbrace{\psi}_{\text{insurance payout}} +(1-p) \underbrace{\phi}_{\text{LOLR payout}} \\ &-(1-p) \underbrace{z[m_1 R^M + [m_2 + v(l)]\alpha R^M - \psi - \phi]}_{\text{fire sale}} = 0 \end{aligned}$$

which implies

$$\begin{aligned} &pf'(I) + (1-p)\lambda - R^B \\ = &- \left[m_1(R^B - R^M) + \alpha[1 + ls - m_2 R^M - (1-m_2)\alpha R^B] - (1-p)zR^M\left[(m_1 + (m_2 + v(l))\alpha)\right]\right] \\ &+ pr^E\psi - (1-p)\psi - (1-p)z\psi - (1-p)(1+z)\phi. \end{aligned}$$

At the bank's optimal solution, $m_1 = 0$, substituting the early FOCs into the FOC w.r.t. I, we recover

$$pf'(I) + (1-p)\lambda - R^B = -\frac{\eta}{I}\left[\alpha(m_2 + v(l)) - \frac{\psi}{R^M}\right] - (1-p)(1+z)\phi.$$

C.2 Private Investor's Problem

Taking L and k as given, the PI's problem is essentially unchanged from the baseline, except that the total liquidity provided by the PI to the system is now $(1+\theta)M - L$[29]:

$$\max_{(1+\theta)M,E} p\left[g(W) + r^E E\right] + (1-p)\left[g(W-(1+\theta)M+L) + \frac{1}{k}((1+\theta)M - E - L)\right].$$

PI's FOC w.r.t. $(1+\theta)M$, the PI's funds used for fire-sale purchases and private insurance in the crisis state is:

$$g'(W-(1+\theta)M+L) = \frac{1}{k}$$

which equalizes the marginal benefit of investing in the g technology with the forgone cost of fire sales $\frac{1}{k}$ in the crisis state at $t=1$.

PI's FOC w.r.t. private liquidity commitment E:

$$pr^E = (1-p)\frac{1}{k}.$$

C.3 Private Equilibrium

To solve for the private equilibrium, we first note that from agents' FOC w.r.t. private insurance,

$$(1-p)\frac{1}{k} = pr^E = \frac{\eta}{IR^M} + (1-p)(1+z).$$

By definition, $1+z = \frac{1}{k}$, which implies we must have $\eta = 0$: the constraint on money creation is not binding in an interior solution.

For a given level of fire-sale price k, the levels of investment and central bank's intervention are pinned down by the bank's FOC w.r.t. I and the central bank's FOC w.r.t. ϕ (in the baseline model with out a central bank, $\phi = 0$):

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z)\phi. \tag{24}$$

$$\frac{1}{k} = C'(L) = C'(\phi I) \tag{25}$$

In the baseline model, with $\eta = 0$, given a level of k and leverage l, the bank's FOC w.r.t. α and m_1 are fairly similar:

$$1 + ls - R^B + m_2 R^B - m_2 R^M - m_2(1-p)zR^M - v(l)(1-p)zR^M = \frac{\eta}{I}(m_2 + v(l)) = 0,$$

[29]With $m_1 = 0$ and $m_2 = \bar{m}$ in equilibrium, $m = \bar{m}\alpha$ and $\theta = \frac{v(l)}{\bar{m}}$

$$[(R^B - R^M) - (1-p)zR^M] = \frac{\eta}{I} = 0,$$

which implies

$$\frac{1}{m_2}(1 + ls - R^B) + R^B - R^M = [1 + \frac{v(l)}{m_2}](1-p)zR^M,$$

$$(R^B - R^M) = (1-p)zR^M.$$

Note that as long as $(1 + ls - R^B) > v(l)(1-p)zR^M$, it must be that the first FOC holds but LHS<RHS for the second FOC. Therefore, $m_1 = 0$ and $m = m_2\alpha$: money is only created to finance speculation, while the real investment is entirely financed by illiquid bonds.

Moreover, when $m_1 = 0$, from the bank's FOC w.r.t. m_2:

$$I[(\alpha R^B - \alpha R^M) - (1-p)z\alpha R^M] + \eta_2 = \eta\alpha = 0,$$

it must be that $\eta_2 = -\alpha[(R^B - R^M) - (1-p)zR^M] > 0$ which implies $m_2 = \bar{m}$ if $\alpha > 0$. That is, when where is speculation ($\alpha > 0$), the funding constraint on the speculative technology is binding, as the banks creates as little money as possible to be able to earn more than R^B on the asset side and finance at the cost of R^B.

To pin down k and l, note that the bank's FOC w.r.t. l implies that

$$s - v'(l)(1-p)zR^M = \frac{\eta}{I}v'(l) = 0 \implies \frac{s}{v'(l)} = (1-p)zR^M. \tag{26}$$

Substituting into the bank's FOC w.r.t. α at $m_2 = \bar{m}$ yields:

$$\frac{1}{\bar{m}}(1 + ls - R^B) + R^B - R^M = [1 + \frac{v(l)}{\bar{m}}](1-p)zR^M = [1 + \frac{v(l)}{\bar{m}}]\frac{s}{v'(l)}. \tag{27}$$

Moreover, the PI's FOC w.r.t. M pins down $M = \bar{m}\alpha IR^M$ (since $m_1 = 0, m_2 = \bar{m}$) which effectively pins down α:

$$g'(W - (1 + \frac{v(l)}{\bar{m}})M + L) = \frac{1}{k}, \tag{28}$$

Therefore, assuming $(1 + ls - R^B) > v(l)(1-p)zR^M$ which in equilibrium amounts to $(1 + ls - R^B) > v(l)\frac{S}{v'(l)}$, we will always have $m_1 = 0$ and $m_2 = \bar{m}$. We have that l is pinned down by (27), k is pinned down by (26), L is pinned down by (25), M is pinned down by (28), and I is pinned down by (24).

Lastly, the total amount of private insurance E is indeterminate as long as it satisfies

$$(1 + \frac{v(l)}{\bar{m}})M - k\lambda I - L \leq E \leq (1 + \frac{v(l)}{\bar{m}})M - L, \tag{29}$$

where the first inequality requires that the constraint on money creation is not binding and the second inequality is a natural limit on the amount of private insurance.

C.4 Proof of Proposition 5

Suppose the primitive parameters are such that in equilibrium in the economy with speculation, $1 + ls - v(l)(1-p)zR^M > R^B$.

1. $m_1 = 0$ and $m_2 = \bar{m}$ are shown in Section C.3.

2. To see that $k^S < k^P$, note that from Section C.3, $(R^B - R^M) < (1-p)z^S R^M$ but in the baseline model $(R^B - R^M) = (1-p)z^P R^M$. Therefore, $z^S > z^P$ and therefore $k^S < k^P$ as $z = \frac{1}{k} - 1$.

3. With a bailout central bank, since we always have $\frac{1}{k} = C'(L)$, a lower level of k implies a higher level of L, as $C(\cdot)$ is increasing and convex so $C'(\cdot)$ is increasing in L. Moreover, in both the benchmark and this extension, when a bail-out central bank is added, I is pinned down by:

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z)\phi,$$

   Note that we must have a larger I under the speculation extension. For a contradiction, suppose that I were smaller in the extended model with speculators, then because L is now higher, it must be that ϕ is higher, which from this FOC implies I must be higher, yielding a contradiction.

4. In the model with speculation, we have that

$$g'(W - (1 + \frac{v(l)}{\bar{m}})M^S + L^S) = \frac{1}{k^S}$$

   where $L = 0$ maps to the case without a central bank. In contrast, in the baseline model, $g'(W - M^P + L^P) = \frac{1}{k^P} < \frac{1}{k^S}$, therefore since $g'(\cdot)$ is decreasing, we must have that the total private liquidity demand at time 1 in the crisis state is higher with speculation: $(1 + \frac{v(l)}{\bar{m}})M^S - L^S > M^P - L^P$. Since $L^S > L^P$, it is also the case that the total liquidity demand is also higher: $(1 + \frac{v(l)}{\bar{m}})M^S > M^P$. The amount of money liability created, however, could be lower than the efficient level, as $(\frac{v(l)}{\bar{m}})M$ is used to satisfy margin calls.

C.5 Welfare Decomposition

Social welfare can be decomposed into four parts (ignoring constants):

$$\underbrace{pf(I) + (1-p)\lambda I - R^B}_{\text{real investment}} + \underbrace{mI(R^B - R^M)}_{\text{Money Premium}}$$
$$+ \underbrace{pg(W) + (1-p)\left[g(W - M + L) + M\right]}_{\text{PI Profit}} \underbrace{-(1-p)C(L)}_{\text{LOLR Cost}}$$

With speculation, the social welfare for the allocations are computed as

$$\underbrace{pf(I) + (1-p)\lambda I - R^B}_{\text{real investment}} + \underbrace{mI(R^B - R^M)}_{\text{Money Premium}}$$
$$+ \underbrace{pg(W) + (1-p)\left[g(W - (1+\theta)M + L) + M\right]}_{\text{PI Profit}} \underbrace{-(1-p)C(L)}_{\text{LOLR Cost}}$$

where $[m_2 + v(l)]\alpha I R^M := (1+\theta)M$ represents the total liquidity demand at time 1, to meet both money liabilities and margin calls.

In the baseline model, the welfare loss only comes from real investment (overinvestment and therefore underprovision of private insurance to create an optimal amount of money), as the level of money and LOLR intervention is efficient relative to the planner that can choose L. With speculation added and without a central bank, there is no distortion on time 0 investment, though the lack of L leads to too little money created relative to the planner's choice. Moreover, with speculation and margin calls, less "good money" is created so the society earns less from the premium on demandable money liabilities. After adding a central bank to the speculation model, we can see that

1. Moral hazard becomes more severe than without speculation so there is too much time 0 real investment I.

2. Money premium is extremely low because of speculation (slightly counterbalanced by the central bank),

3. There is too much central bank intervention L, more than the case without speculation.

4. While there is too much $g(W - (1+\theta)M + L) + (1+\theta)M$, their relative proportion is not at the level of what's socially optimal (there is too much fire sale and too little g as k is smaller). Moreover, relative to what's socially optimal , while there is also too much liquidity need $(1+\theta)M$ created that is set aside from g, only M earns the money convenience yield and the rest does not. All of this can be seen from

$$g'(W - (1+\theta)M + L) = \frac{1}{k} = C'(L)$$

whereas the socially optimal solution satisfies

$$g'(W - M^{social} + L^{social}) = \frac{1}{k^{social}} = C'(L^{social})$$

With $k^{social} > k$, we have that too much CB intervention $L > L^{social}$, too much liquidity demand $(1+\theta)M > M^{social}$ but with a large margin call θ, we have that $M < M^{social}$ in our numerical illustrations.

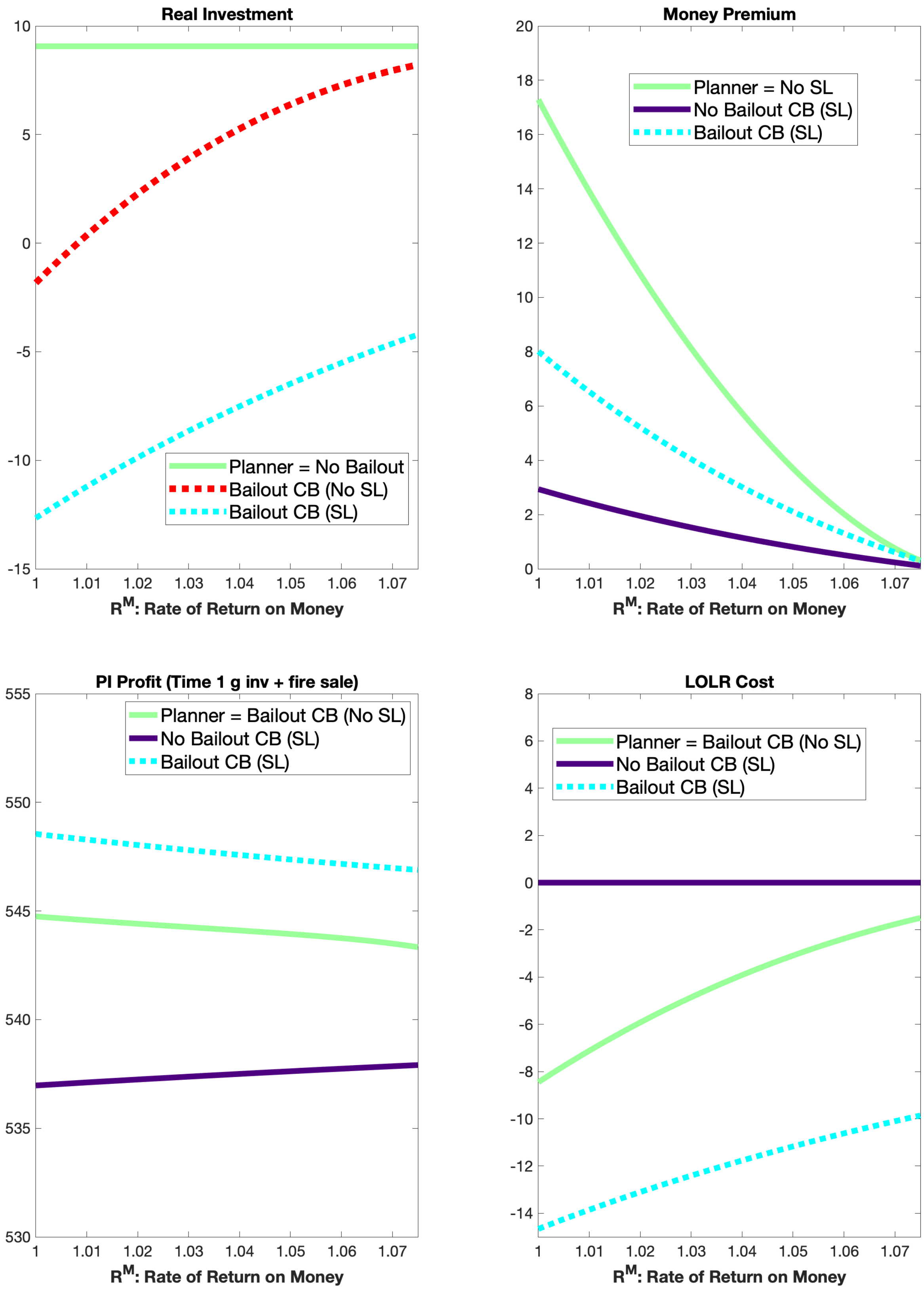

Figure 11: Welfare Decomposition

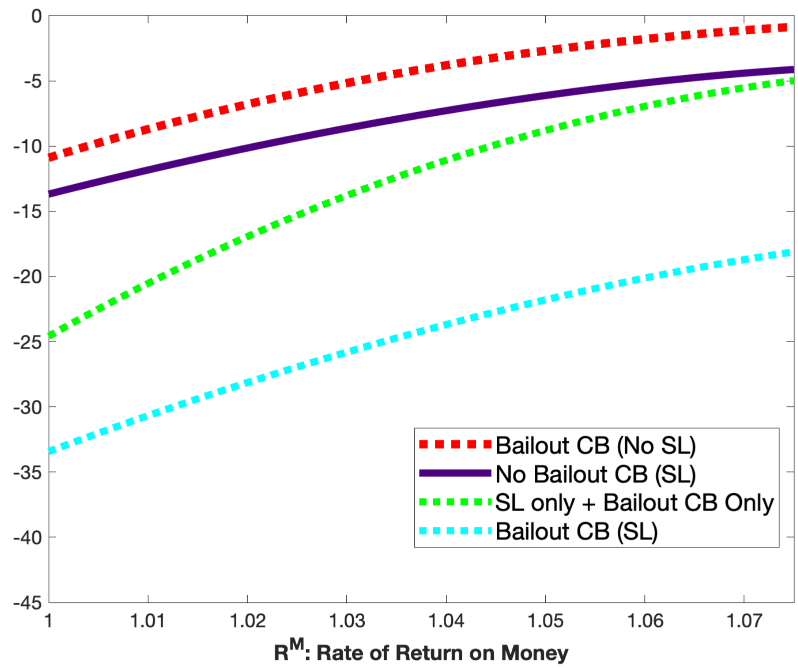

Figure 12: Welfare Loss Relative to Planner's Choice

D Model with Frictions in the Insurance Market

D.1 Baseline Model with Limited Commitment

In this section, we consider the same setup as the baseline model, except that because their commitment cannot be trusted, the PI must put E, the funds they promise for contingent capital / private insurance, into a liquid technology in advance and hold it till any possible contingency passes. This requirement can be seen as solving an unmodeled agency problem where insurers can pocket the premium and walk away. Conceptually, one can think of $t = 0$ to $t = 2$ as a fixed period with continuous time, and the liquidity shock at $t = 1$ could arise at any time between the start and the end, so the funds committed to insurance have to be permanently available as liquid collateral. Under this interpretation, the "good" state is the case where the economy reaches $t = 2$ without experiencing any liquidity shock.

Therefore, the PI cannot invest E into the g technology even in the good state. In the good state at time 2, the PI gets $h(E)$ from the storage technology and the insurance premium $r^E E$ from the bank. We assume that h is increasing and weakly concave.[30]. In the crisis state, the PI's capital payment E is sent to the bank. A very conservative base case for the liquid storage technology, which we consider in the numerical illustration below, is simply setting aside the entire E in a cash account so that $h(E) = E$.

We show below that with limited commitment in the private insurance technology, the model setup reverts to that of Stein (2012) in that there is overinvestment and over-issuance of money. However, there is also over-provision of private insurance in the private equilibrium. Nevertheless, once we consider public insurance, our earlier insights from the model with perfect private insurance continue to prevail.

D.1.1 Private Equilibrium

The bank's problem and FOCs remain the same as in Section 2.1. However, the private investor's problem now becomes

$$\max_{M,E}\ p\left[g(W-E)+h(E)+r^E E\right]+(1-p)\left[g(W-M)+\frac{1}{k}(M-E)\right]$$

PI's FOC w.r.t. M, the PI's funds used for fire-sale purchases and private insurance in the crisis state, is:

$$g'(W-M)=\frac{1}{k}. \tag{30}$$

PI's FOC w.r.t. private liquidity commitment E is:

[30] Intuitively, the diminishing marginal returns from the liquid storage technology corresponds to increasing marginal costs due to limited commitment

$$pr^E = p\left[g'(W-E) - h'(E)\right] + (1-p)\frac{1}{k}, \tag{31}$$

which equalizes the time-0 marginal benefit of providing private insurance (premium r^E in the good state) with the corresponding marginal cost (forgone return of $\frac{1}{k}$ in the bad state and putting the funds in the liquid technology which loses the opportunity to invest in g in the good state).

From the bank's FOC w.r.t. private insurance, (4), we can see that when $E > 0$ (interior solution), the insurance market equalizes the bank's marginal benefit of receiving insurance with the PI's marginal cost of providing insurance:

$$\frac{\eta}{R^M I} + (1-p)\frac{1}{k} = pr^E = p\left[g'(W-E) - h'(E)\right] + (1-p)\frac{1}{k}.$$

which implies the bank's shadow cost of money creation equals the PI's costs incurred due to limited commitment.

The equilibrium definition and market clearing conditions remain the same as in the baseline model.

D.1.2 Planner's Solution

Next, consider the benevolent planner who also faces the limited commitment problem when allocating the level of E. The planner's objective becomes

$$\begin{aligned} U =& pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) \\ &+ p\left[g(W-E) + h(E)\right] + (1-p)\left[g(W-M) + M\right]. \end{aligned}$$

As a result, the planner's FOC w.r.t. ψ takes the form

$$p\left[g'(W-E) - h'(E)\right] = \frac{\eta^P}{IR^M}, \tag{32}$$

whereas the planner's FOC w.r.t. m and I still have the same expressions as (11) and (13).

D.1.3 Private outcomes are no longer socially optimal

Proposition 6. *With limited commitment in the private insurance market, Stein (2012)'s baseline results are restored.*

Let I^*, M^*, E^* denote the private outcomes and I^P, M^*, E^* denote the planner's choices in the model with limited commitment without a central bank.

1. In the low spread region where the constraint on money creation does not bind ($\eta = \eta^P = 0$), the private outcome is socially optimal. That is, $I^* = I^P$, $M^* = M^P$, $E^* = E^P$.

2. In the high spread region where the constraint on money creation is binding ($\eta, \eta^P > 0$), there is not only overinvestment and over-issuance of money, but also over-provision of private insurance. That is, $I^* > I^P$, $M^* > M^P$, $E^* > E^P$. Fire sale is also suboptimally severe ($k^P > k^*$).

Proof. Appendix D.3. □

Using the same parameters as before and assuming that the liquidity technology is a cash technology ($h(E) = E$, $h'(E) = 1$), the numerical results from Figure 13 confirm the findings from Proposition 6, where the variables with "Social" subscripts denote the allocations from a planner that does not have access to public liquidity L. Notably, there is also over-provision of capital insurance both in aggregate ($E > E^P$) and in terms of the fraction of bank size I insured ($\psi > \psi^P$) where $\psi = \frac{E}{I}$. Importantly, this result will be reversed after adding a bailout central bank.

Remark. As in Stein (2012), in the constrained region, the bank's shadow value of money creation is too high relative to the socially optimal level, because the bank fails to internalize the externality that makes the fire-sale worse for everyone else. Because the bank's shadow value equals the PI's limited commitment costs, there is too much private insurance provided because private agents over-perceive its benefit when the constraint is binding. Over-provision of private insurance arises because of the fire sale externality that is present with the bank's money creation constraint. If there were no fire sale externality and over-creation of money (that is, $\eta = \eta^P$), then there would be no over-provision of private insurance.

D.2 Adding a Bailout Central Bank

As an illustration of the effects of public insurance in case of frictional private insurance, we add a bailout central bank to the model in Section 5.1, with the same modifications as were made in Section 4.2 assuming $\tau = 0$. Given the difficulty in obtaining clean analytical results with limited insurance commitment, the analysis in this subsection is mainly based on numerical plots using the same specifications as before. Figure 14 compares the level of equilibrium allocations under a bailout central bank relative to the planner's allocations. There is substantial overinvestment due to moral hazard engendered by the presence of a bailout central bank, which leads to over-creation of money (in the high spread region) despite underprovision of private insurance (an endogenously missing market where $E = 0$). We emphasize here that there is underprovision of private insurance in the presence of a bailout central bank, which reverses the over-provision result from Section 5.1 both in the presence of limited commitment in the private insurance market. Under the same mechanism

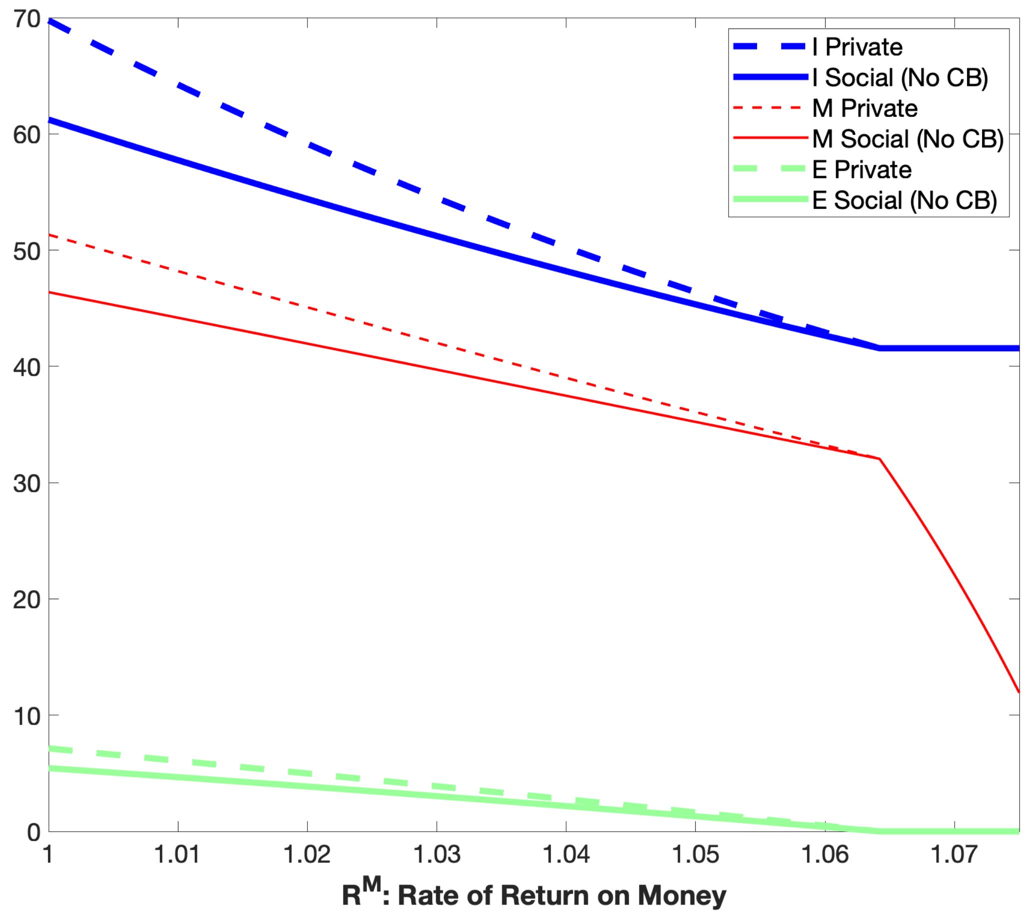

Figure 13: Equilibrium Outcomes in the Model with Limited Commitment, for different levels of return on money R^M

that is more cleanly illustrated in Section 4 with a frictionless insurance market, the presence of LOLR crowds out private insurance.

Under this parameterization, the level of bailout funds, L, is very close to L from the planner's solution. This can be understood by looking at two opposing forces that affect $L = \phi I$. On the one hand, due to moral hazard, there is substantial overinvestment that raises L in the private equilibrium relative to the planner's allocation. On the other hand, the bailout central bank only acts ex post, so it does not recognize that ex ante, a higher level of ϕ, the fraction of investment covered by public liquidity, also relaxes the money creation constraint of the bank in the high spread region where the constraint is binding. As a result, it under-supplies ϕ relative to the planner's choice. Under our baseline parameters, the two opposing forces happen to almost cancel each other out.[31] Regardless, as shown in Figure

[31] This seems to hold only under when we use $p = 0.95$. It is not a general result in the model with limited

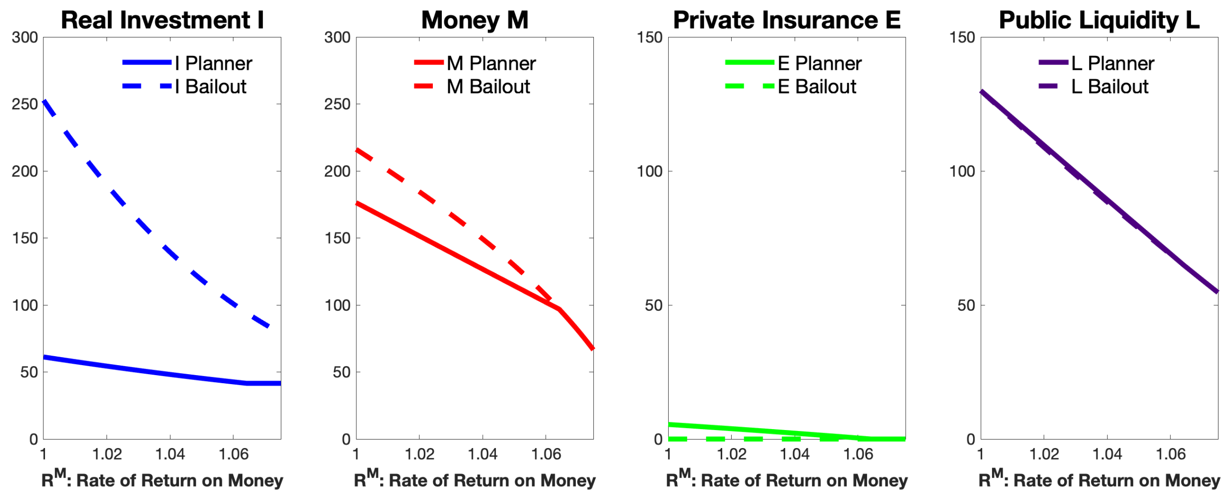

Figure 14: Equilibrium Outcomes with Limited Commitment in the Private Insurance Market and a Bailout Central Bank, for different levels of return on money R^M

15, in the model with limited commitment, adding a bailout central bank still crowds out private insurance which remains potentially welfare-reducing.

Finally, the welfare results from adding the actuarially fair ex-post LOLR or pre-committed liquidity to the model with limited commitment remain qualitatively similar as the results from the baseline model. The core mechanism in Section 4 that the provision of public liquidity crowds out private liquidity arrangements and leads to overinvestment unless it is judiciously priced remains significant even in the presence of a binding constraint on money creation.

D.3 Proof of Proposition 7

Proof for E:

In the low-spread region with $\eta = \eta^P = 0$, the private FOC coincides with the planner's FOC. If $\eta > 0$, note that the private FOCs w.r.t. private insurance can be combined and written as

$$p(g'(W - E^*) - h'(E^*)) = \frac{\eta^*}{I^* R^M}.$$

Similarly, the socially optimal E^P must satisfy

$$p\left[g'(W - E^P) - h'(E^P)\right] = \frac{\eta^P}{I^P R^M}.$$

Suppose, for a contradiction, that $E^* \leq E^P$, then because the LHS is increasing in E, we

commitment that L is also very close to the planner's level.

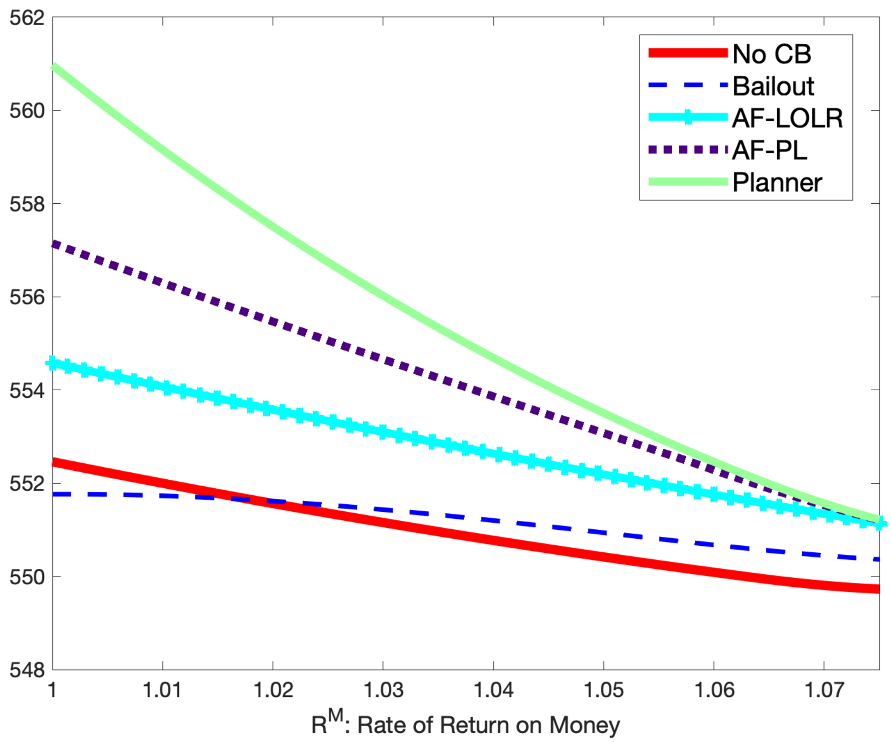

Figure 15: Welfare with Limited Commitment of Private Insurance

must have

$$\frac{\eta^P}{I^P} \geq \frac{\eta^*}{I^*}.$$

Note that the private FOC w.r.t. m gives

$$\left[(R^B - R^M) - (1-p)z^* R^M\right] = \frac{\eta^*}{I^*},$$

whereas the planner's FOC w.r.t. m gives

$$\left[(R^B - R^M) - (1-p)z^P R^M\right] = \frac{\eta^P}{I^P}[1 + \Omega(M^P, I^P)] > \frac{\eta^P}{I^P} \geq \frac{\eta^*}{I^*}.$$

So it must be that $z^P < z^*$ which implies $k^P > k^*$. Then we have

$$g'(W - M^*) = \frac{1}{k^*} > \frac{1}{k^P} = g'(W - M^P) \implies M^* > M^P.$$

The planner's FOC w.r.t. I is

$$pf'(I^P) + (1-p)\lambda - R^B = -\frac{\eta^P}{I^P}\left(m^P - \frac{\psi^P}{R^M}\right)$$

whereas the bank's FOC w.r.t I can be written as

$$pf'(I^*) + (1-p)\lambda - R^B = -\frac{\eta^*}{I^*}\left[m^* - \frac{\psi^*}{R^m}\right]$$

Note that in the constrained region, we must have that $m^P - \frac{\psi^P}{R^M}, m^* - \frac{\psi^*}{R^m} > 0$. There are two cases, both leading to a contradiction.

Case 1: $I^P \geq I^*$, then in the constrained region $k^P\lambda I^P + E^P = M^P < M^* = k^*\lambda I^* + E^*$contradicts with $k^P > k^*$, $E^P \geq E^*$and $I^P \geq I^*$.

Case 2: $I^P < I^*$. Then from the two FOCs w.r.t. I, the following must hold:

$$m^P - \frac{\psi^P}{R^M} < m^* - \frac{\psi^*}{R^m} \implies \frac{k^P\lambda}{R^M} < \frac{k\lambda}{R^M} \implies k^P < k,$$

which yields a contradiction with $k^P > k$.

Therefore, we must have $E^* > E^P$ and hence $\frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$.

Proof for M: Suppose $M^P \geq M^*$. Then we must have

$$M^* \leq M^P \implies \frac{1}{k^*} = g'(W - M^*) \leq g'(W - M^P) = \frac{1}{k^P} \implies k^P \leq k^*.$$

We have already established $E^P < E^*$,so in the binding region, we must have that

$$k^P\lambda I^P = M^P - E^P \geq M^* - E^* = k^*\lambda I^* \implies k^P\lambda I^P \geq k^*\lambda I^*.$$

Since $k^P < k^*$,this necessarily implies that $I^P > I^*$.

The planner's FOC w.r.t. I is

$$pf'(I^P) + (1-p)\lambda - R^B = -\frac{\eta^P}{I^P}\left(m^P - \frac{\psi^P}{R^M}\right),$$

whereas the bank's FOC w.r.t I can be written as

$$pf'(I^*) + (1-p)\lambda - R^B = -\frac{\eta^*}{I^*}\left[m^* - \frac{\psi^*}{R^m}\right].$$

Since $I^P > I^*$and $\frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$, we must have $m^P - \frac{\psi^P}{R^M} > m^* - \frac{\psi^*}{R^m} \implies \frac{k^P\lambda}{R^M} > \frac{k^*\lambda}{R^M} \implies k^P > k^*$, yielding a contradiction with $k^P < k^*$ from earlier.

Therefore, we must have $M^* > M^P$ and $k^P > k$.

Proof for I: We have already established that $E^P < E^* \implies \frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$,$M^P < M^* \implies k^P > k^* \implies m^P - \frac{\psi^P}{R^M} > m^* - \frac{\psi^*}{R^m}$ in the binding region.

Private and social FOCs w.r.t. I then imply that

$$pf'(I^P)+(1-p)\lambda-R^B=-\frac{\eta^P}{I^P}\left(m^P-\frac{\psi^P}{R^M}\right)>-\frac{\eta^*}{I^*}\left[m^*-\frac{\psi^*}{R^m}\right]=pf'(I^*)+(1-p)\lambda-R^B,$$

which implies $I^P > I^*$ as required.

E Private Insurance and Public Liquidity Support as a Fraction of M

Now suppose both private and public insurance are on the amount of money created (payments to depositors), M, rather than on the banks' level of assets. The contingent capital purchased by banks from PIs are in proportion with M. We show in this appendix that overinvestment continues to prevail when a central bank is added to the model.

To make money riskless, in the crisis state, the bank's promised payment to depositors $M = mIR^M$ must be covered by either fire sale of assets or by the private insurance $E = \psi M$ from private investors:

$$\Delta k\lambda I+\psi M=M \implies \Delta k\lambda I=(1-\psi)M=(1-\psi)mIR^M.$$

With private insurance, only $(M-E)$ of bank assets have to be fire sold. If Δ =1 (all assets are fire sold), the bank reaches the upper bound on private money creation, which is

$$M^{max}=\frac{k\lambda I}{1-\psi} \implies m^{max}=\frac{1}{1-\psi}\frac{k\lambda}{R^M}.$$

We assume that the central bank's liquidity injection takes the form $L = \phi M$, so that ϕ becomes the proportion of total M that is insured / covered by central bank. In this case, the money creation constraint simplifies to

$$m\leq\frac{1}{1-\psi-\phi}\frac{k\lambda}{R^M}.$$

The central bank then chooses ϕ, but each bank takes ϕ as given yet recognizes that L is affected by its money creation, M, which can possibly lead to a moral hazard problem if the incentives of money creation M through real investment I are not appropriately corrected. The presence of this form of central bank intervention allows the bank to create money via overinvestment, because they now recognize that public insurance also scales up as they increase their investment I which in turn increases M.

E.1 Private Investor's Problem

Taking L as given, the representative private investor's problem is

$$\max_{M,E} \; p\left[g(W) + r^E E\right] + (1-p)\left[g(W - M + L) + \frac{1}{k}(M - E - L)\right].$$

PI's FOCs w.r.t. M, the PI's funds used for fire-sale purchases and private insurance in the crisis state, is:

$$g'(W - M + L) = \frac{1}{k} \tag{33}$$

which equalizes the marginal benefit of investing in the g technology with the forgone cost of fire sales $\frac{1}{k}$ in the crisis state at $t = 1$.

PI's FOC w.r.t. private liquidity commitment E remains the same:

$$pr^E = (1-p)\frac{1}{k}. \tag{34}$$

The PI's FOCs are largely unchanged.

E.2 Bank's Problem

With an LOLR, the additional break-even constraint can be expressed as

$$\lambda I - (1+z)(M - E - L) - \tau L \geq 0 \implies (1 - \psi - \phi(1 - \tau k))m \leq \frac{k\lambda}{R^M}$$

which, for any τ >0, is tighter than the previous constraint faced by a planner: $[1 - \psi - \phi(1 - \tau k)]m \leq \frac{k\lambda}{R^M}$.

Taking ϕ as given, the representative bank's problem is

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} - p\underbrace{r^E\psi M}_{\text{insurance premium}}$$
$$+ (1-p)\underbrace{\psi M}_{\text{insurance payout}} + (1-p)\underbrace{\phi M}_{\text{LOLR payout}} - (1-p)\underbrace{z[mIR^M - \psi M - \phi M]}_{\text{fire sale}} - (1-p)\underbrace{\tau\phi M}_{\text{payment to LOLR}}$$

such that

$$[1 - \psi - \phi(1 - \tau k)]m \leq [1 - \psi - \phi(1 - \tau k)]m^{max} = \frac{k\lambda}{R^M}$$

with Lagrange multiplier on the break-even constraint $\eta \geq 0$.

Note that the bank's objective also changes, as in expectation it now has to pay back at $t = 2$ $\tau\phi M$ on the LOLR funding if there is a crisis. But as long as $\tau < 1 + z$, the bank will receive the benefit of avoiding fire-sale costs, so there is moral hazard.

with respect to (w.r.t.) bank's fraction of real investment financed by deposits, m:

$$I[(R^B - R^M) - (1-p)zR^M - pr^E\psi R^M + (1-p)(1+z)\psi R^M + (1-p)(1-\tau+z)\phi R^M] = \eta, \quad (35)$$

w.r.t. bank's fraction of real investment covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1-p)(1+z), \quad (36)$$

w.r.t. I (substituting in $\eta = 0$ as the constraint still does not bind):

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau)\phi m R^M \quad (37)$$

with the additional $(1-p)(1+z-\tau)\phi m R^M$ term reflecting that the bank are incentivized to make additional investment due to the moral hazard created by the presence of public funding that is not sufficiently costly to the bank for any $\tau < 1+z$. It is important to note that overinvestment due to moral hazard, the key feature of our benchmark model, continues to arise as the perception of an LOLR intervention that is proportional to M, which is in turn proportional to I.[32]

[32] The first FOC shows that there is further incentive to over-create money, as z would be higher and thus k would be lower, which implies M would be higher in this extension, all else equal. Our benchmark clarifies that even holding $M - L$ constant (both money and intervention at the efficient levels), we can see how the mix of money creation via I versus E becomes distorted due to moral hazard.