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Jihene Arfaoui
Harald Uhlig

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ABSTRACT

Inspired by the Silicon Valley Bank run and building on Diamond- Dybvig (1993), we develop a model in which asset price fluctuations can trigger bank runs. Liquidation amounts to selling assets at their market price. Depositors can buy and hold the assets after paying an idiosyncratic cost. We characterize the equilibria. We introduce a withdrawal pressure function to distinguish between fundamental runs, driven by market price declines, and self-enforcing runs triggered by depositor panic. Deposit insurance can prevent self-enforcing runs but incurs losses during fundamental runs. Regulatory measures ensuring price resilience reduce run risks, but at the expense of depositor welfare.

Jihene Arfaoui
Goethe University Frankfurt
s4734159@stud.uni-frankfurt.de

Harald Uhlig
University of Chicago
and CEPR
and also NBER
huhlig@uchicago.edu

1 Introduction

The collapse of Silicon Valley Bank in early 2023 might have been viewed as likely ex post, but it is fair to consider it surprising ex ante. Rather than investing in opaque financial instruments or engaging in high-wire financial engineering, the Silicon Valley Bank followed a conventional maturity transformation model: it accepted deposits and invested in long-term U.S. Treasury bonds. However, when interest rates rose and therefore the market price for long-term bonds fell early in 2023, the SVB was bankrupt, when using mark-to-market accounting. If a sizeable share of depositors had held their deposits to maturity, the SVB would have still been profitable. However, many of these depositors were large, not FDIC-insured, and closely followed financial markets and social media; a bank run ensued.

In this paper, we aim to examine these events through the lens of a bank run model in the tradition of Diamond and Dybvig (1983), with one key difference: the asset can be traded in period 1. Depositors can withdraw early and purchase and hold the asset themselves after paying an idiosyncratic transaction cost, rather than relying on the bank deposit payout down the road. This setup allows patient depositors to exit the bank if expected returns in the market exceed the bank's promise. As a result, the risk of a run no longer stems only from early consumption needs but also from shifts in market valuations.

We begin by deriving a benchmark deposit contract that satisfies the incentive-compatibility condition, which requires that depositors earn at least as much by staying with the bank as by withdrawing and reinvesting in the market. This condition prevents premature withdrawals based only on gains from outside options. We show how asset price fluctuations can trigger bank runs. We identify three types. Fundamental runs occur when a drop in asset value below a critical threshold leaves the bank unable to meet future obliga-

tions, forcing withdrawals. Complete runs occur when negative expectations cause all depositors to withdraw their funds. Partial runs happen when only a fraction of depositors withdraw.

To examine these phenomena, we define a withdrawal pressure function, which maps asset prices and expectations into equilibrium withdrawal decisions. We introduce a critical withdrawal cost that leads depositors with lower outside investment costs to run when they expect others to do the same. We define a run threshold that is the minimal fraction of withdrawing depositors needed to make the run self-enforcing. We analyse the limits of standard policy tools. Deposit insurance reduces panic-driven runs by reassuring depositors, but it fails to prevent fundamental runs when asset prices fall enough to make early withdrawal optimal. In those cases, the deposit insurer absorbs the loss. Tighter regulation of deposit contracts can reduce run risks, but it also reduces depositor welfare and the ability to smooth consumption.

This note draws on the extensive literature following Diamond and Dybvig (1983). Allen and Gale (2009) provide an excellent survey and introduction. Gorton and Winton (2002) provide a more general overview of the literature on financial intermediation. Wallace (1988), McCulloch et al (1998), Selgin (1989, 2020), and McCulloch (2022) examine the framework in depth and discuss criticisms and alternative arrangements.

Our paper is motivated by the 2023 banking stress and the role of asset price declines. This is not a new phenomenon, of course. Rolnick and Weber (1984) provide historical empirical evidence on how asset price fluctuations contributed to past banking crises. Jamilov et al (2024) examine the macroeconomic consequences of systemic bank runs over two centuries.[1] Calomiris and Haber (2014) have argued that banks are "fragile by design", with Dowd (1992) providing a literature review of banking instability. Re-

[1]For a model of a systemic bank run, see Uhlig (2010).

garding recent events, excellent assessments can be found in Acharya et al. (2023), Jiang et al. (2023), the Basel Committee (2023), Angeloni et al. (2024), the "tracing in real time" in Cipriani et al (2024) and the account in Kelly and Rose (2025). DeMarzo et al. (2024) examine the interest rate risk on bank franchise values. Chen et al. (2024) discuss the importance of hold-to-maturity accounting rules. Blickle et al. (2024) argue that depositor flightiness has increased and that panic runs triggered by policy hikes become amplified with larger Fed balance sheets. Kim et al. (2024) argue that reciprocal deposits allowed banks to stem outflows in the 2023 banking crisis, but that the trade-offs need careful examination. It should be clear from these extensive descriptions and investigations, that a lot more went on than the main issue raised here, i.e., the possibility for depositors to invest in the assets themselves or to find deposits elsewhere offering that market return, but that this issue was central. We therefore deliberately chose to narrowly focus on it in our analysis.

We aim to provide a theoretical perspective, as do several recent contributions. Drechsler et al. (2023) point out the tension between hedging interest rate risk and liquidity risk exposure. They describe a dynamic game over time, where withdrawals by insured depositors are modelled following Drechsler et al. (2017), and they stress the role of uninsured depositors in triggering runs. Our analysis complements theirs by focusing on a few key features and necessarily omitting others. Like us, Amador and Bianchi (2024) stress the interplay between defaults driven by fundamentals, self-fulfilling runs, and asset prices. Their analysis differs in many details from ours, focusing on the externalities of bank leverage decisions and minimum capital requirements. Our analysis aligns with the perspective of Blickle et al. (2024), who suggest that depositors weigh the benefits of holding deposits against their outside investment options.

Our paper addresses issues related to deposit insurance and bank regu-

lation, but others have investigated them in greater depth. Like us, Dávila and Goldstein (2023) build on Diamond and Dybvig (1983) and stress the tension between fundamental insolvency and panic-driven runs. They examine optimal deposit insurance in considerably more detail. Likewise, Schilling (2023) builds on the global-games version in Goldstein and Pauzner (2005) of Diamond and Dybvig (1983), examining the challenges to an inefficient regulator in stepping in too early or too late in the case of a run. In light of these excellent in-depth analyses, we encourage the reader to turn there and read them in light of the analysis here.

2 The Model

As in Diamond-Dybvig (1983) and following the exposition in Allen and Gale (2007), we suppose that there are three periods $t = 0, 1, 2$ and that there is a continuum of depositors $i \in [0, 1]$, who have one unit of a good in $t = 0$, and either must consume in period $t = 1$ with probability $\lambda \in (0, 1)$ ("impatient") or may postpone consumption to period $t = 2$ with probability $1 - \lambda$ ("patient). We suppose their utility from consumption is given by a strictly increasing, strictly concave and twice continuously differentiable utility function $u(c)$ with relative risk aversion exceeding unity everywhere, $-u''(c)c/u'(c) > 1$ for all c. For numerical calculations, we use the CRRA utility function $u(c) = (c^{1-\eta} - 1)/(1 - \eta)$ with $\eta > 1$. We assume that there is a long-term asset that trades at a price of p in period $t = 1$ and pays off $R > 1$ units of the consumption good in period $t = 2$ for every unit invested in period $t = 1$. The benchmark price is $p = 1$. There are banks offering demand deposit contracts to agents which allow them to withdraw d in period $t = 1$. Competition between banks leads to a demand deposit contract with $c_1 = d$ maximizing expected ex ante utility

$$U = \lambda u(c_1) + (1 - \lambda)u(c_2) \tag{1}$$

subject to constraints, avoiding a run (more on that below) and subject to the budget constraints

$$\lambda c_1 = px \tag{2}$$

$$(1-\lambda)c_2 = R(1-x) \tag{3}$$

where $x \in (0,1)$ is the share of the long-term asset sold in $t=1$, provided that no one runs. We relax the latter restriction in the appendix A. Bank contract competition implies that the remaining agents receive the remaining asset payout pro rata when withdrawing in period $t=2$. Thus, in case of a bank run, i.e., if a share $\mu > \lambda$ of agents withdraws d in period 1, the remaining agents will receive $c_2 = c_{2,s}(\mu, p)$, where

$$\mu d = px(\mu, p) \tag{4}$$

$$(1-\mu)c_{2,s}(\mu,p) = R(1-x(\mu,p)) \tag{5}$$

provided that $\mu d \leq p$, and where asset sales $x(\mu, p)$ are needed to meet the period-1 withdrawals. The result is

$$c_{2,s}(\mu,p) = R\frac{1-\frac{\mu d}{p}}{1-\mu} \tag{6}$$

Note that $c_{2,s}(\mu,p)$ also depends on d. We avoid noting this dependency for ease of notation, but will return to that feature in section 6 and in appendix A. If $\mu d > p$, the withdrawing agents are paid pro rata in period $t=1$, $c_1 = p/\mu$, and $c_{2,s}(\mu,p) = 0$. At time $t=0$, and unlike Diamond and Dybvig (1983), we assume that p is stochastic. More precisely, we assume that agents assume $p=1$ "for sure" in $t=0$ and are then surprised when it drops to some other value $p<1$ instead in $t=1$.

In contrast to Diamond-Dybvig (1983), we assume that the asset is tradable in period $t=1$ and $p>0$ is its market price: "salvage value" there is "market price" here. We assume that agents can purchase the assets themselves and hold them to maturity. We assume that agent i has to pay a

transaction cost and loses resources γ_i when doing so.[2] That is, if a fraction $\mu \geq \lambda$ of agents withdraw in period $t = 1$, the asset market price is p and the bank can meet all withdrawals, $\mu d \leq p$, a patient agent i will compare the payoff $c_{2,s}(\mu, p)$ from staying in the deposit contract with the payoff $c_{2,w}(\gamma_i, p)$ achieved by withdrawing d in period $t = 1$ and investing $d - \gamma_i$ by purchasing the asset at price p. That payoff is given by

$$c_{2,w}(\gamma_i, p) = \frac{R}{p}(d - \gamma_i). \tag{7}$$

If $\mu d > p$, the patient agent compares $c_{2,s}(\mu, p) = 0$ to $\frac{R}{p}((p/\mu) - \gamma_i)$. We assume that γ_i has a bounded support in $(0, \infty)$. We denote the lower bound of the support with $\underline{\gamma} > 0$ and the upper bound with $\bar{\gamma} < \infty$. We assume that γ_i is drawn iid across agents from some distribution G on that interval.

In order to measure financial fragility, we introduce some additional terminology. For every $\mu \in [\lambda, 1]$ and every $p \in (0, 1]$, define the critical investment cost $\gamma_c(\mu, p)$ as that value, so that $c_{2,s}(\mu, p) = c_{2,w}(\gamma_i, p)$ for a patient agent with $\gamma_i = \gamma_c(\mu, p)$. In other words, a patient agent i with investment cost $\gamma_i = \gamma_c(\mu, p)$ is just indifferent between staying in the contract or withdrawing and investing, at $t = 1$, provided that a fraction μ of agents withdraws overall and that the market price is p. Thus, patient agents with $\gamma < \gamma_c(\mu, p)$ would strictly prefer to withdraw in period $t = 1$ and impatient agents do so anyway. The total fraction of agents that weakly prefer to withdraw is

$$m(\mu, p) = \lambda + (1 - \lambda)G(\gamma_c(\mu, p)), \tag{8}$$

provided a fraction of agents μ actually does so. We call $m(\mu, p)$ the **withdrawal pressure** at withdrawal fraction μ and price p.

[2]We assume that these transaction costs are lost to the economy and not paid to other agents. Our stylized assumption is intended to capture the range of possibilities available to withdrawing depositors in practice, including, for example, purchasing money market shares, which now offer higher interest rates between periods 1 and 2, with the fund investing in these assets.

If $m(\mu, p) > \mu$, then there are more than a fraction μ of agents willing to withdraw, when a fraction of agents μ are known to do so, reinforcing the withdrawals. If $m(\mu, p) < \mu$, then not all withdrawing agents are actually willing to do so.

An equilibrium for a price p is a value $\mu^e \in [\lambda, 1]$, so that $m(\mu^e, p) = \mu^e$. For a given price $p \in (0, 1]$, we define μ as a **run threshold**, if $\mu = \lambda$ or $m(\mu, p) = \mu$ and if, for every $\epsilon > 0$, there is a $\mu < \tilde{\mu} < \mu + \epsilon$, so that $m(\tilde{\mu}, p) > \tilde{\mu}$. In other words, beyond a run threshold, a run is self-enforcing: more agents than $\tilde{\mu}$ just beyond that threshold are willing to run, if a fraction $\tilde{\mu}$ is withdrawing. We define **financial fragility** $\mu^f(p)$ as the lowest run threshold μ, if the price p prevails. We call $\lambda < \mu < 1$ a **partial bank run**, if $m(\mu, p) = \mu$ and if for some $\epsilon > 0$ and all $\mu < \tilde{\mu} < \mu + \epsilon$, we have $m(\tilde{\mu}, p) < \tilde{\mu}$. In other words, at a partial bank run, the fraction of agents running is willing to do so, but locally, not more agents would be. A complete bank run $\mu = 1$ requires $m(1) = 1$. We seek to characterize the set of run thresholds, financial fragility, the set of partial bank runs, and the set of equilibria all as a function of p.

3 The benchmark deposit contract

We restrict deposit contracts so that no patient agent wishes to withdraw and buy the asset herself, when the asset trades at the benchmark price $p = 1$ and no run occurs.[3] That is, we restrict the search of deposit contracts maximizing expected utility (1) and allowing withdrawal of d in $t = 1$ to the **benchmark deposit contract** d that satisfies the incentive compatibility condition

$$R(d - \underline{\gamma}) \leq c_2 \tag{9}$$

[3]We relax that assumption in appendix A.

where $c_2 = c_{2,s}(\lambda, 1)$ is given in equation (6), and where we recall that $c_{2,s}(\lambda, 1)$ also depends on d.

To calculate the solution, let (c_1^*, c_2^*) denote the social planning solution of maximizing expected utility (1) subject to the budget constraints (2) and (3), but without imposing (9). As in Diamond and Dybvig (1983), the first-order condition for an interior solution is

$$u'(c_1^*) = Ru'(c_2^*) \tag{10}$$

Since $-u''(c)c/u'(c) > 1$ for all c, one can show that $c_1^* > 1$, see e.g. Allen and Gale (2007). With CRRA preferences, the solution is

$$c_1^* = \frac{R}{(1-\lambda)R^{\frac{1}{\eta}} + \lambda R} \text{ and } c_2^* = R^{1/\eta} c_1^* \tag{11}$$

There are now two cases.

1. Suppose that (9) is satisfied at $d = c_1^*$ and $c_2 = c_2^*$. The benchmark deposit contract is $d = c_1^*$, since then $c_{2,s}(\lambda, 1) = c_2^*$.

2. Suppose that $R(c_1^* - \underline{\gamma}) > c_2^*$. In that case, the withdrawal d needs to be lower than c_1^* to avoid that a positive fraction of patient agents withdraws and purchases the asset instead. As we seek to maximize expected utility (1), it is easy to verify that d should then be chosen as high as possible, subject to (9), i.e., solving for d by imposing equality in (9). Noting the dependency of $c_{2,s}(\lambda, 1)$ on d, the solution is

   $$d = 1 + \underline{\gamma}(1-\lambda) \tag{12}$$

   resulting in $c_2 = R(1 - \lambda\underline{\gamma})$.

We summarize the case distinction as

$$d = \min\{c_1^*, 1 + \underline{\gamma}(1-\lambda)\} \tag{13}$$

We note that $d > 1$, since $c_1^* > 1$ and since $\underline{\gamma} > 0$. The resulting second period consumption is $c_{2,s}(\lambda, p)$, if all patient agents stay, that is, if $\mu = \lambda$.

If (9) binds and if therefore $d = 1 + \underline{\gamma}(1 - \lambda)$ in (13), then a nonzero fraction of agents will wish to withdraw in period $t = 1$ for any $p < 1$. It follows that $\mu^f(p) = \lambda$ for any $p < 1$ in that case. Banking is then "fragile by design" as argued by Calomiris and Haber (2014), though for different reasons than in their book.

4 Financial Fragility

Assume that banks have offered the benchmark deposit contract (12). We now investigate the relationship between the market price p, the decision to run and the resulting equilibria. We seek to characterize the set of run thresholds, financial fragility, and the set of partial bank runs, all as a function of p in the range $0 < p \leq 1$, which we impose from here on out. We start by noting some monotonicity and continuity properties.

Lemma 1 *1. The second-period payout $c_{2,s}(\mu, p)$ for stayers is decreasing in μ and increasing in p, and a continuous function of both arguments.*

2. The critical investment cost $\gamma_c(\mu, p)$ is increasing in μ and decreasing in p, and is a continuous function of both arguments.

3. The withdrawal pressure $m(\mu, p)$ is increasing in μ and decreasing in p. It is a continuous function of both arguments, provided that $\gamma_c(\mu, p)$ is not a mass point of G.

Proof:

1. Direct per (6), since $d > 1$ and $p \leq 1$.

2. *Solving* $c_{2,s}(\mu, p) = c_{2,w}(\gamma_c(\mu, p), p)$ *for* $\gamma_c(\mu, p)$ *yields*

$$\gamma_c(\mu, p) = d - \frac{p}{R} c_{2,s}(\mu, p) = \frac{d - p}{1 - \mu}. \tag{14}$$

The result follows from the first part and by inspection.

3. *This follows from the second part and by inspecting (8) from the properties of distribution functions.*

•

We now restrict our attention to the more interesting situations, satisfying the following two assumptions and imposing them from here on out.

Assumption A. 1 *G is a continuous function, that is, it does not have mass points.*

This assumption ensures that $m(\mu, p)$ is a continuous function, according to the third part of lemma 1.

Assumption A. 2 $d - \bar{\gamma} > 0$

This assumption ensures that even the worst investment costs will yield a strictly positive payoff in period $t = 2$, as long as the bank can meet all withdrawal requests, $\mu d \leq p$.[4] The assumption is satisfied if $\bar{\gamma} \leq 1$.

We note some properties of the withdrawal pressure function $m(\mu, p)$.

Proposition 1 *1. For every p, there is a lowest* $\mu^*(p) < 1$*, so that* $\mu < m(\mu, p)$ *for all* $\mu^*(p) < \mu < 1$.

2. The threshold $\mu^*(p)$ *is increasing in p.*

[4] If $\mu d > p$, then some patient agents may prefer to receive nothing in $t = 2$ rather than invest their pro-rata payout in period $t = 1$, but the bank run is complete anyhow.

3. Financial fragility $\mu^f(p)$ is increasing in p and satisfies $\mu^f(p) \leq \mu^*(p)$.

$\mu^*(p)$ is the highest run threshold, triggering a complete bank run: the only equilibrium for $\mu > \mu^*(p)$ is $\mu = 1$. Part 1 of the proposition implies that a run threshold exists. The infimum μ^f of all these run thresholds is itself a run threshold and the financial fragility index.

Proof:

1. *We have $c_{2,s}(\mu, p) = 0$ whenever $\mu \geq p/d$. Since $d > 1$, $p/d < 1$. By assumption 2, all agents would then want to withdraw in period $t = 1$, i.e., $m(\mu, p) = 1$ for all $\mu > p/d$. The existence of $\mu^*(p)$ now follows from the continuity of $m(\mu, p) - \mu$ as a function of μ, guaranteed by assumption 1 and the last part of lemma 1.*

2. *Consider two prices $p_1 < p_2$. With the third part of lemma 1 and all $\mu \geq \mu^*(p)$, we have $m(\mu, p_1) \geq m(\mu, p_2) > \mu$ and therefore $\mu > \mu^*(p)$. It follows that $\mu^*(p)$ is increasing in p.*

3. *The monotonicity follows as in the second part. Note that $\mu^*(p)$ is a run threshold. Since $\mu^f(p)$ is the lowest run threshold, it follows that $\mu^f(p) \leq \mu^*(p)$.*

•

A bank run is **fundamental** for price p, if $m(\lambda, p) > \lambda$. In that case, λ cannot be an equilibrium: some patient agents are always withdrawing in $t = 1$. A fundamental bank run obtains, iff $c_{2,w}(\underline{\gamma}, p) > c_{2,s}(\lambda, p)$, i.e. iff

$$p < p^n = d - (1 - \lambda)\underline{\gamma}, \tag{15}$$

defining p^n as the threshold price for a fundamental bank run. Note that $p^n = 1$, if d is calculated per (12). Thus, if $R(c_1^* - \underline{\gamma}) > c_2^*$, we always have a fundamental bank run whenever $p < 1$.

5 Numerical Illustration

In this section, we illustrate the model's results through a numerical example. We assume that agents have constant relative risk aversion (CRRA) preferences. We set the parameters to $\eta = 2$, $\lambda = 0.2$, and $R = 2$. Therefore, $c_1^* = 1.306$ and $c_2^* = 1.847$. For the investment cost distribution $G(\gamma)$, we consider two alternative specifications. In the first case, G is uniform in the interval $[\underline{\gamma}, \bar{\gamma}]$, with $\underline{\gamma} = 0.1$ and $\bar{\gamma} = 1$, so that

$$G(\gamma) = \frac{\gamma - \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}}, \quad \gamma \in [\underline{\gamma}, \bar{\gamma}].$$

In the second case, G is piecewise uniform, placing equal mass of $1/3$ on each of the intervals $[0.1, 0.2], [0.3, .0.4]$ and $[0.8, 1]$, with a uniform distribution on these intervals, i.e. with density equal to $G'(\gamma) = 10/3$ on $[0.1, 0.2]$ and $[0.3, .0.4]$, and density equal to $G'(\gamma) = 5/3$ on $[0.8, 1]$. With that, the constraint (9) binds for both G, and thus $d = 1 + \bar{\gamma}(1 - \lambda) = 1.08$ per equation (12). This implies that some positive mass of patient agents will wish to withdraw and that $\mu^f(p) = \lambda$ for all $p < 1$. Figure 1 presents the results under the uniform specification. The left panel shows that the withdrawal pressure function $m(\mu, p)$ increases in μ and decreases in p, in line with lemma 1. For the line in the middle, i.e. for $p = 0.9$, there are two intersection points. The intersection point A is a fundamental bank run $\mu^n(p)$: for any withdrawal $\mu \in [\lambda, \mu^n(p)]$ "conjectured" by patient agents, $\mu^n(p)$ emerges as the equilibrium. There also is the threshold $\mu^*(p)$ at point B, beyond which only the complete bank run $\mu = 1$ can be an equilibrium. The right panel shows both types of run thresholds, as a function of p. The dark area denotes the area, where a conjectured μ will lead to a run, with the first point vertically above such a μ denoting the lowest equilibrium consistent with such a conjecture. The parabola separates the two thresholds for $p > 0.858$, while only a complete bank run is an equilibrium $p < 0.858$.

This behavior confirms Proposition 1(ii), which states that $\mu^*(p)$ is increasing in p, and Proposition 1(iii), which shows that in continuous G, the financial fragility threshold $\mu^f(p)$ coincides with the run threshold $\mu^*(p)$. The fragility response to prices is smooth and monotonic, meaning there is a single point at which the run becomes self-sustaining.

In contrast, Figure 2 shows the results under the piecewise distribution. We have shown the function $m(\mu, p)$ only for $p = 0.95$. There are now increasing as well as flat portions. Point A is a neccessary bank run: even starting at $\mu = \lambda$, at least that many patient agents will withdraw. A μ just above B will trigger a run at least all the way to point C. Finally, point D is the run threshold $\mu^*(p)$, beyond which only the complete bank run can emerge. The right panel of Figure 2 shows the bank run regions. For $p > 0.974$, the figure looks similar to the right region of the right panel of Figure 1, i.e., there are the fundamental bank runs at the bottom and the complete-run threshold μ^* at the top. For $p < 0.974$, a new intermediate region emerges, corresponding to the region between points B and C in the left panel of Figure 2. Below $p = 0.919$, points A and B disappear, and two bank run regions remain, similar again to the region on the right in the right panel of Figure 1. Finally, below $p = 0.866$, $\mu^* = \lambda$, and only complete bank runs can be an equilibrium.

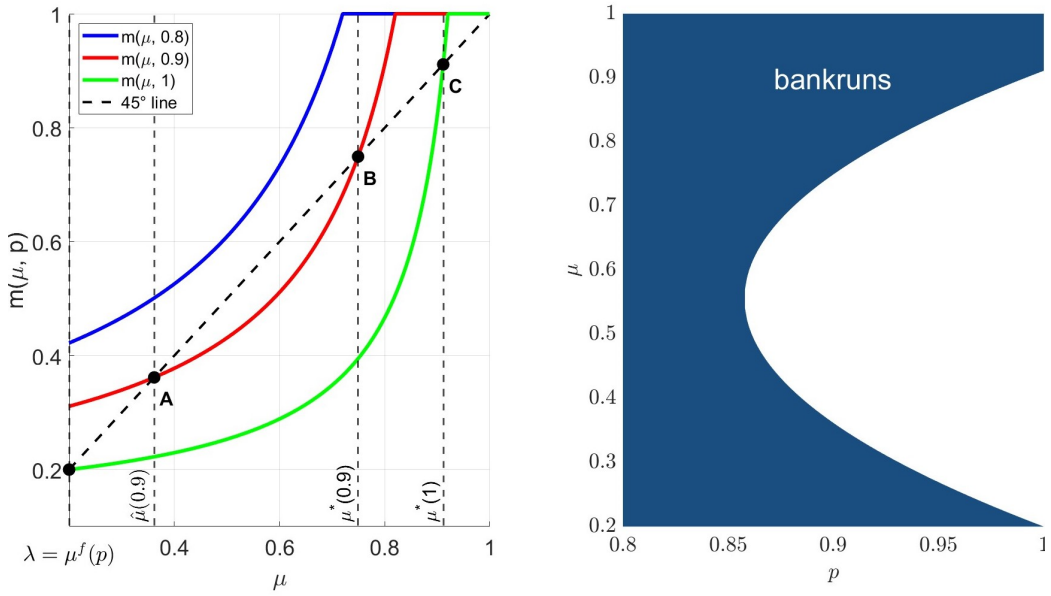


Figure 1: Withdrawal pressure and financial fragility (Linear G)

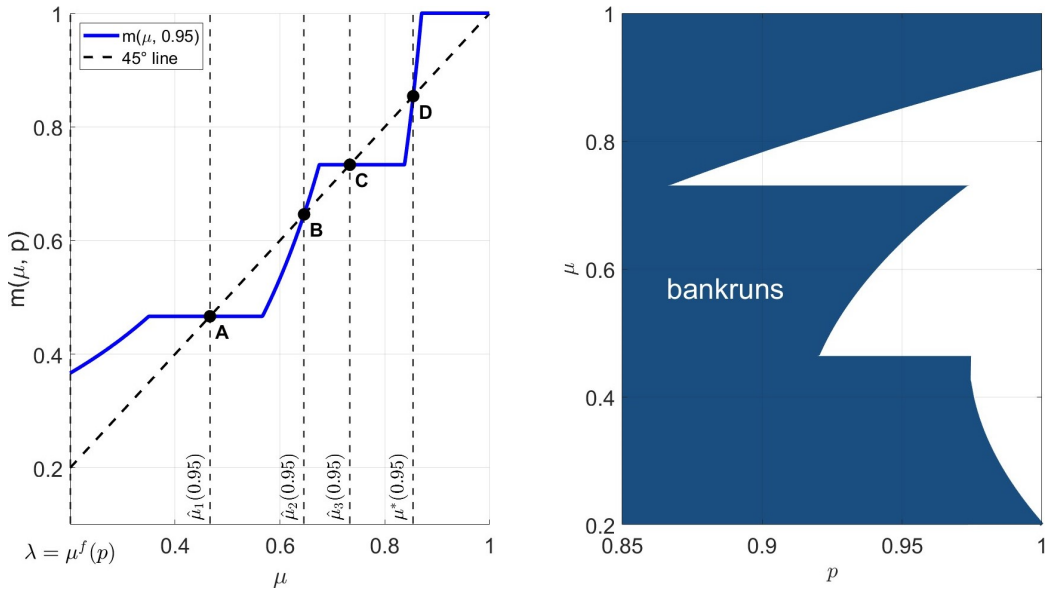


Figure 2: Withdrawal pressure and financial fragility (Piecewise G)

6 Deposit Insurance and Regulation

Deposit insurance and regulations are typically implemented to prevent or mitigate bank runs. Consider a coordination failure where fundamentals remain sound, but depositors withdraw simply because they expect others to do so. In this case, as long as financial fragility exceeds the share of impatient depositors, $\mu^f(p) > \lambda$, depositors have no incentive to run, if they expect $\mu < \mu^f(p)$ will do so. Deposit insurance is designed to keep the share of withdrawing depositors low by assuring patient depositors a sufficiently large payout $c_2(\bar{\mu}, p)$ in $t = 2$ for some $\bar{\mu} < \mu^f(p)$. The bank remains solvent, and the insurance provider does not incur losses, since the asset price, p, is still high enough to cover all obligations.[5] By contrast, if the asset price p drops below a critical threshold p^n defined by equation (15) and $\mu^f(p) = \lambda$, the run becomes fundamentally driven rather than panic-induced. In this situation, the bank's assets become insufficient to cover its liabilities, making a run fundamental regardless of depositor expectations. At this point, deposit insurance can no longer prevent the run without absorbing part of the losses. At best, a loss-free deposit insurance can assure that the bank run μ is no higher than the lowest partial bank run.[6]

An alternative policy is to require banks to offer deposit contracts that are resilient to asset price drops. Similar to condition (9), the regulation would impose that the choice $d = d(p^r)$ for the bank deposit contract must satisfy the constraint

$$\frac{R}{p^r}(d - \underline{\gamma}) \leq c_{2,s}(\lambda, p^r; d), \tag{16}$$

[5]Note that the deposit insurance, as formulated here, does not guarantee the $t = 2$ payout of the original deposit contract $c_{2,s}(\lambda, 1)$ for $p = 1$. Such an unconditional deposit insurance will make losses whenever $p < 1$.

[6] "Loss-free" is meant to apply to the on-equilibrium path only. If there is, say, a complete bank run, the deposit insurance will make losses.

where $p^r < 1$ is a *robustness price* set by the regulators, up to which p may fall without triggering a run, and where $c_{2,s}(\lambda, p; d)$ is given by the right hand side of equation (6), but now explicitly noting the dependency on d. If the constraint (16) is already satisfied by the social planner solution $d = c_1^*$ and $c_{2,s}(\lambda, 1) = c_2^*$, then the regulation does not impose a cost, but is also unnecessary, since the original deposit contract is already resilient enough. However, if the constraint (16) binds, banks have to reduce the promised early consumption to

$$d = d(p^r) = p^r + \underline{\gamma}(1 - \lambda) \tag{17}$$

in generalization of (12), while increasing the resulting pro-rata period-2 payout to $c_{2,s}(\mu, p; d(p^r))$ compared to the original $c_{2,s}(\mu, p) = c_{2,s}(\mu, p; d(1))$. This results in a drop in utility $u(d(p^r), c_{2,s}(\lambda, 1; d(p^r)))$, even if the price is $p = 1$ and there is no run, $\mu = \lambda$. The contract becomes safer, but also provides less insurance and consumption smoothing. To quantify the welfare cost of such regulation, we calculate the welfare loss in consumption-equivalent terms. Let $\Phi(p^r)$ be the factor that scales the utility given the original, unregulated deposit contract, captured by a regulator imposing $p^r = 1$, equal the utility under the regulated contract:

$$\begin{aligned} &\lambda\, u\big(d(p^r)\big) + \big(1-\lambda\big)\, u\big(c_{2,s}(\lambda, 1; d(p^r))\big) \\ &\quad = \quad \lambda\, u\big(\Phi(p^r)\, d(1)\big) + \big(1-\lambda\big)\, u\big(\Phi(p^r)\, c_{2,s}(\lambda, 1; d(1))\big) \end{aligned}$$

Since the regulated contract is less efficient, $\Phi(p^r)$ is an increasing function of p^r with $\Phi(1) = 1$. This reflects the drop in overall utility having to follow the regulation. We define the percentage welfare loss in consumption-equivalent terms as $L(p^r) = 100 \cdot (1 - \Phi(p^r))$.

In figure 3, we calculate and plot the utility and welfare losses for different robustness levels p^r, using the numerical parameterization of section 5. As p^r is set further below $p = 1$, the figure shows how more resilient deposit

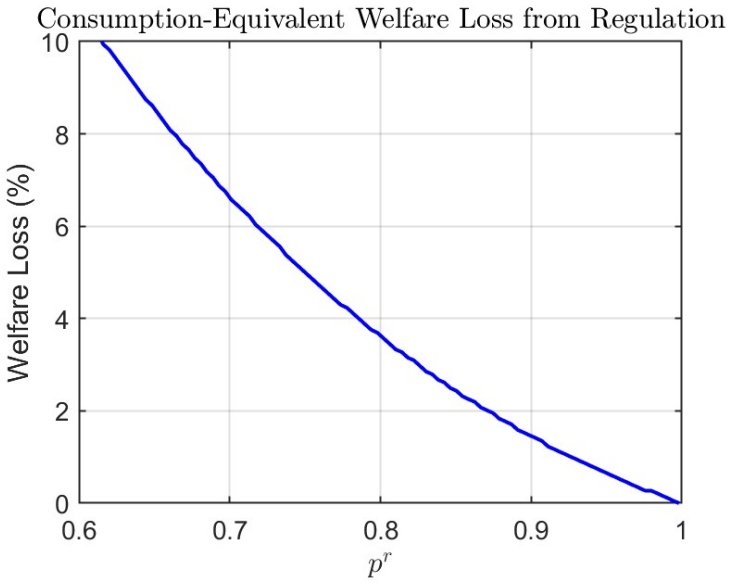


Figure 3: Consumption Equivalent Welfare Loss from Regulation

contracts cause greater losses in welfare, as they lower efficiency during normal times. This highlights a key regulatory trade-off between resilience and efficiency.

7 Discussion

We argue that SVB's collapse represents a fundamental bank run triggered by a sharp decline in asset prices. In early 2023, the rapid increase in interest rates led to a decline in the market price of long-term bonds. SVB had heavily invested in these bonds after expanding its balance sheet with large, uninsured deposits from tech startups and venture capital firms. As interest rates increased, the market value of SVB's bond holdings declined, creating substantial unrealized losses under mark-to-market accounting. Depositors, seeing the deterioration of the bank's balance sheet and fearing insolvency, withdrew more than $40 billion in a single day. This mass withdrawal trig-

gered the run, as documented in the Geneva report by Angeloni et al (2024).

In our model, this corresponds to a fundamental bank run. When the market price of the asset, p, falls below the critical threshold p^n, early withdrawal becomes the rational choice for all depositors. Once this threshold is crossed, the withdrawal pressure function increases sharply, leading to $m(\lambda, p) > \lambda$. Anticipating further declines in asset value, depositors rush to withdraw, reinforcing the run dynamics. It may well be that the only equilibrium was a complete run with $\mu^e = 1$, as shown in the left-most line of the left panel of Figure 1.

Other U.S. banks were close to failure or indeed failed, but not just there. In Switzerland, Credit Suisse collapsed. The fall in asset prices was likely a contributing factor for all of them. Aside from the interest-rate setting role of monetary policy, these episodes also illustrate the decisive role of regulatory timing. Deposit insurance, liquidity support, and emergency backstops are only effective if introduced before withdrawals exceed critical thresholds. As shown by Schilling (2023), the effectiveness of interventions depends critically on when they are implemented, as acting too late may make even well-designed policies ineffective.

8 Conclusion

In our model, we show that bank runs can be triggered both by asset price fluctuations and depositor expectations, explicitly distinguishing between two types of bank run: those driven by fundamentals and those driven by depositor panic. The collapse of Silicon Valley Bank illustrates a fundamental bank run, as rising interest rates reduced asset values below a critical threshold, making withdrawals rational and inevitable. Many other banks were close to failure or indeed failed at that time. These events emphasize the importance of timely interventions and robust bank contracts. Deposit

insurance effectively prevents self-enforcing runs by reassuring depositors, but becomes costly for preventing runs and less effective during fundamental runs when banks' asset values fundamentally deteriorate. Regulatory requirements for robust deposit contracts ensure stability during asset price declines but come at the expense of reduced depositor welfare and consumption smoothing. Our analysis highlights the key tradeoffs.

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APPENDIX

A Deposit contracts with some early withdrawals by patient agents

In this section, we relax condition (9) and allow d to take any value, recognizing that there may be a share of patient agents for which (9) is violated, even when $p = 1$. Therefore, agents with sufficiently low investment costs γ_i will also withdraw their demand deposit early in period $t = 1$ and invest in the long-term asset themselves, even in the absence of an asset price decline. We aim to find the optimal deposit contract, taking into account these endogenous withdrawals $\mu > \lambda$ at $p = 1$. We assume that the withdrawals can be kept to the minimal or necessary amounts, perhaps with the aid of deposit insurance.

Substituting second-period consumption (6) for agents who remain in the contract into the critical investment cost per (14) at $p = 1$, we obtain the total share of agents willing to withdraw as

$$m(\mu, 1) = \lambda + (1 - \lambda)G(\gamma_c(\mu, 1)) = \lambda + (1 - \lambda)G\left(\frac{d-1}{1-\mu}\right)$$

This defines the fixed point $\mu(d)$ as the lowest value μ satisfying

$$\mu = \lambda + (1 - \lambda)G\left(\frac{d-1}{1-\mu}\right) \tag{18}$$

Fixing the benchmark price $p = 1$, but recognizing the endogeneity of μ and its dependence on d, define

$$\gamma_c(d) = \frac{d-1}{1-\mu(d)} \text{ and } c_{2,s}(d) = R\frac{1- \mu(d)d}{1-\mu(d)}$$

with the help of equations (14) and (6). In bank competition, the deposit contract offering withdrawal d in $t = 1$ maximizes the expected utility of all

participants. Expected utility is

$$\begin{aligned} U(d) &= \lambda u(d) + (1-\lambda) \int_{\gamma \leq \gamma_c(d)} c_{2,w}(\gamma, 1) dG(\gamma) \\ &\quad + (1-\lambda) u(c_{2,s}(d)) \left(1 - G\left(\gamma_c(d)\right)\right) \end{aligned} \tag{19}$$

The planner's problem is to choose the optimal $d^* = d$ to maximize $U(d)$.

If G has a density, one can calculate the first-order conditions and seek to characterize the solution, see the online appendix. For the numerical optimization it is easier, though, to try out a range of values of d over a grid, solving the fixed-point equation (18) for the lowest value of the withdrawal share $\mu(d)$ and computing the resulting expected utility $U(d)$ for each d. As in section 5, we use a CRRA utility function with $\eta = 2$ and set $\lambda = 0.2$ as well as $R = 2$. We assume that the investment cost distribution $G(\gamma)$ follows a uniform distribution on $[\underline{\gamma}, \bar{\gamma}]$ with $\underline{\gamma} = 0.1$ and $\bar{\gamma} = 1$, as in the first case examined in section 5. Similar to the calculations there, solving the (18) amounts to solving a quadratic equation in μ, as long as $\gamma_c \in [\underline{\gamma}, \bar{\gamma}]$: $\mu(d)$ can neither fall below λ nor exceed unity. The lower value (provided it is real-valued) is the desired $\mu(d)$,

$$\mu(d) = \max\left\{\lambda\,,\, \min\left\{1\,,\, 1 - \frac{1}{2}\cdot\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma} - \sqrt{\frac{1}{4}\left(\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}\right)^2 - \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}(d-1)}\ \right\}\right\}$$

It is real-valued only as long as $d \leq \bar{d}$, where $\bar{d}$ is obtained by setting the determinant under the square root to zero and solving for d,

$$\bar{d} = 1 + \frac{1}{4}\cdot\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}^2 = 1.2222,$$

given our parameterization. For $d > \bar{d}$, one gets a complete bank run. The situation is similar to the situation depicted in the right panel of figure 1, except with d on the horizontal axis and the plot flipped from left to right. A complete bank run cannot be the optimal deposit contract. One can therefore

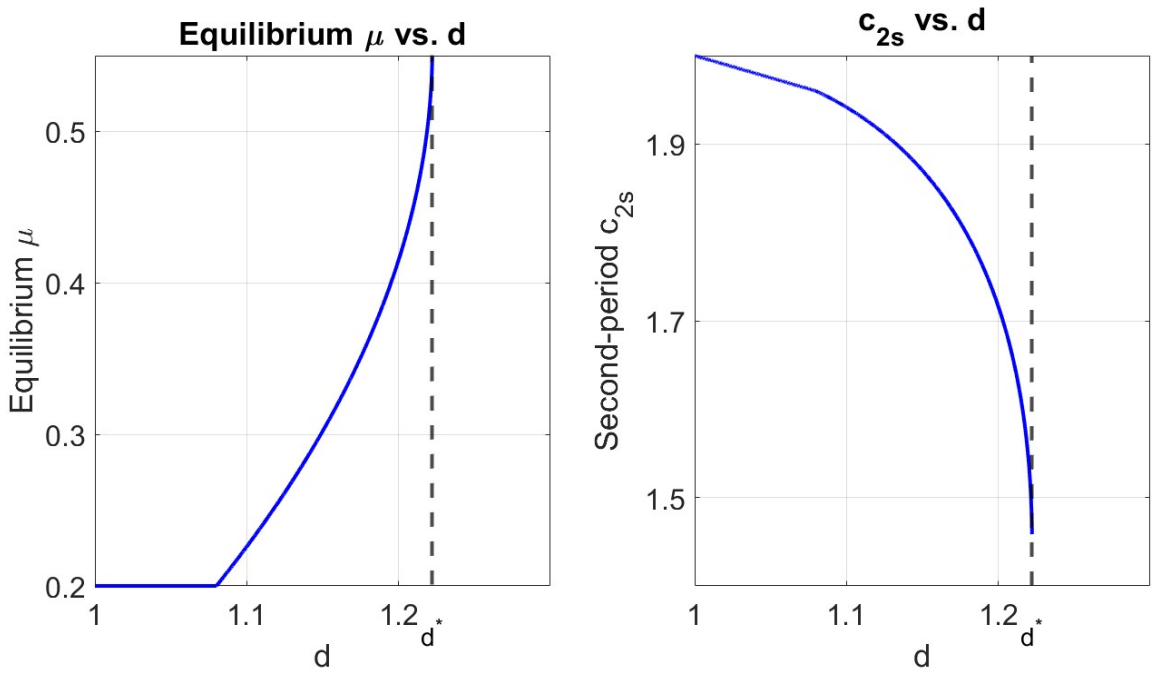


Figure 4: Key outcomes as a function of the deposit rate d.

restrict the search for the optimal d to the interval $d \in [1, \bar{d}]$. We use a grid of 5000 evenly spaced points on this interval, including the end points. It turns out that $\bar{d}$ is also the numerically optimal contract, i.e. $d^* = \bar{d}$, see figure 4. While optimal, this is now an extremely fragile deposit contract. Any price below one will lead to a complete bank run as opposed to the fundamental bank runs in figure 1. This highlights once more the tension between efficiency and robustness to price changes, that we also emphasize in section 6 and the conclusions.

The numerical solution yields an optimal deposit rate of $d^* = 1.2222$, which maximizes expected utility. In this contract, the equilibrium share of agents withdrawing early rises to $\mu(d^*) = 0.5536$, exceeding the baseline impatient share of $\lambda = 0.2$ due to the endogenous withdrawal of patient agents with low investment costs, up to the critical cutoff $\gamma_c^* = 0.4978$. Second-

period consumption for stayers is $c^*_{2,s} = 1.4488$, reflecting the trade-off between early payouts and long-term investment.

From figure 4 we can see how key outcomes change with the deposit rate d. In the left panel, we have the share $\mu(d)$ of agents who withdraw early. At low d, only impatient agents withdraw. As d increases, some patient agents also withdraw to invest directly, raising μ above the baseline $\lambda = 0.2$. In the right panel, we notice that consumption in the second period for those who stay in the contract, $c_{2,s}$, decreases as d increases. This decline reflects the reduced funds left for long-term investment when more is paid out early.

With this analysis, we demonstrate that even when the price is at the benchmark level, $p = 1$, and there are no shocks, some patient agents whose investment costs fall below the critical investment threshold choose to withdraw their deposits in period 1 and invest directly in the long-term asset. For this reason, it is vital to design a deposit contract that accounts for this behavior, recognizing that a share of patient agents withdraw early because they benefit more from investing on their own than from waiting until $t = 2$ for the payoff. In this appendix, we have examined choices for d, that would lead some patient agents to do so. Formally, this situation qualifies as a fundamental bank run, as illustrated in figure 1. However, it is essential to emphasize that this type of run is entirely benign, as it does not stem from panic or coordination failure, but rather from individually rational behavior given the investment opportunities and taken into account in the deposit contract design. The heightened fragility should, however, give pause before implementing such optimal deposit contracts at the edge of the abyss.

ONLINE APPENDIX

Numerical Example: Calculations

In this section of the Online appendix, we provide analytical derivations that support the numerical illustrations presented in Section 5. We explicitly solve for the equilibrium withdrawal behavior under three distinct assumptions about the distribution , which describes agents' transaction costs associated with investing in the long-term asset.

We begin with the expression for the cutoff agent type :

$$\gamma_c = \frac{d-p}{1-\mu} \tag{20}$$

Next, we define a parameter to clearly establish the lower bound on the early withdrawal payoff :

$$\underline{d} = 1 + \underline{\gamma}(1-\lambda) \tag{21}$$

Using this definition, we rewrite the cutoff agent type as follows:

$$\gamma_c = \frac{\underline{\gamma}(1-\lambda)}{1-\mu} + \frac{1-p}{1-\mu} \tag{22}$$

We then calculate the total fraction of agents who withdraw early, based on this cutoff type. The withdrawal function is given by:

$$m(\mu, p) = \lambda + (1-\lambda) \cdot G\left(\gamma_c(\mu, p)\right) \tag{23}$$

Example 1: G Uniform on $[\underline{\gamma}, 1]$

We assume that transaction costs follow a uniform distribution on the interval $[\underline{\gamma}, 1]$. The withdrawal function $m(\mu, p)$ depends on the position of the cutoff type γ_c. Specifically:

$$\begin{cases} \gamma_c < \underline{\gamma} & \Rightarrow m(\mu, p) = \lambda \\ \underline{\gamma} \leq \gamma_c \leq 1 & \Rightarrow m(\mu, p) = \lambda + (1-\lambda) \cdot \frac{\gamma_c - \underline{\gamma}}{1-\underline{\gamma}} \\ \gamma_c > 1 & \Rightarrow m(\mu, p) = 1 \end{cases} \tag{24}$$

Solving for Bank-Run Points: $\mu = m(\mu, p)$

To find fixed points, we assume that the cutoff lies within the support, $\underline{\gamma} \leq \gamma_c \leq 1$. Substituting the expression for γ_c, we write:

$$\mu = \lambda + \left(\frac{1-\lambda}{1-\underline{\gamma}}\right)\left(\frac{d-p}{1-\mu} - \underline{\gamma}\right) \tag{25}$$

We simplify this equation by introducing two parameters:

$$\kappa = \lambda - \left(\frac{1-\lambda}{1-\underline{\gamma}}\right) \cdot \underline{\gamma}, \quad \nu = \frac{1-\lambda}{1-\underline{\gamma}} \tag{26}$$

This allows us to rewrite the equation as:

$$\mu = \kappa + \nu \cdot \frac{d-p}{1-\mu} \tag{27}$$

We then multiply both sides by $(1-\mu)$ and rearrange terms to obtain the quadratic form:

$$0 = (\mu - \kappa)(\mu - 1) + \nu(d-p) \tag{28}$$

Expanding this expression gives:

$$\mu^2 - \alpha\mu + \beta(p) = 0 \tag{29}$$

where:

$$\alpha = \kappa + 1, \quad \beta(p) = \kappa + \nu(d-p) \tag{30}$$

We solve this quadratic and obtain the general solution:

$$\mu_{1,2} = \frac{\alpha}{2} \pm \sqrt{\delta(p)}, \quad \text{where } \mu^*(p) \text{ is the larger root, and } \mu_2(p) \text{ is the smaller one} \tag{31}$$

(A) We find two real solutions if the discriminant is non-negative:

$$\delta(p) = \frac{\alpha^2}{4} - \beta(p) \geq 0 \tag{32}$$

(B) If $\delta(p) < 0$, or equivalently:

$$p < p^* = \left(\frac{\alpha^2}{4} + \kappa + \nu d\right)/\nu \tag{33}$$

then no interior solution exists and we conclude that the only feasible equilibrium is a full bank run:

$$\mu^* = \lambda \tag{34}$$

Example 2

We now analyze a case where transaction cost types follow a bimodal distribution. In particular, we assume that G is uniform on two disjoint intervals:

$$[\underline{\gamma}, \gamma_a] \quad \text{and} \quad [\gamma_b, 1] \tag{35}$$

Each segment carries half of the total probability mass.

Defining the Withdrawal Function

The share of agents who withdraw depends on the value of the cutoff γ_c. We summarize the behavior of the withdrawal function below:

$$\begin{cases} \gamma_c < \underline{\gamma} & \Rightarrow m(\mu, p) = \lambda \\ \underline{\gamma} \leq \gamma_c \leq \gamma_a & \Rightarrow m(\mu, p) = \lambda + \frac{1-\lambda}{2} \cdot \frac{\gamma_c - \underline{\gamma}}{\gamma_a - \underline{\gamma}} \\ \gamma_a \leq \gamma_c \leq \gamma_b & \Rightarrow m(\mu, p) = \frac{1+\lambda}{2} \\ \gamma_b \leq \gamma_c \leq 1 & \Rightarrow m(\mu, p) = \frac{1+\lambda}{2} + \frac{1-\lambda}{2} \cdot \frac{\gamma_c - \gamma_b}{1-\gamma_b} \\ \gamma_c > 1 & \Rightarrow m(\mu, p) = 1 \end{cases} \tag{36}$$

(A) Increasing Region: $\mu \in \left[\lambda, \frac{1}{2}(1+\lambda)\right]$

We now focus on the first increasing segment of the withdrawal function, where the marginal cost types fall within the lower support of the bimodal distribution, i.e., $\gamma_c \in [\underline{\gamma}, \gamma_a]$. In this region, the withdrawal function is given by:

$$m(\mu, p) = \lambda + \frac{1-\lambda}{2} \cdot \frac{\gamma_c - \underline{\gamma}}{\gamma_a - \underline{\gamma}} \tag{37}$$

This expression captures how the share of withdrawing agents increases linearly with γ_c.

To determine equilibrium points, we impose the fixed-point condition $\mu = m(\mu, p)$. Substituting the expression for the cutoff threshold,

$$\gamma_c = \frac{d-p}{1-\mu} \tag{38}$$

into the withdrawal function yields:

$$\mu = \lambda + \frac{1-\lambda}{2} \cdot \left(\frac{1}{\gamma_a - \underline{\gamma}}\right)\left(\frac{d-p}{1-\mu} - \underline{\gamma}\right) \tag{39}$$

To simplify this expression, we introduce two auxiliary parameters:

$$\kappa = \lambda - \frac{1-\lambda}{2} \cdot \frac{\underline{\gamma}}{\gamma_a - \underline{\gamma}}, \quad \nu = \frac{1-\lambda}{2(\gamma_a - \underline{\gamma})} \tag{40}$$

This allows us to rewrite the fixed-point equation in a more compact form:

$$\mu = \kappa + \nu \cdot \frac{d-p}{1-\mu} \tag{41}$$

Multiplying both sides by $1-\mu$ to eliminate the denominator leads to a quadratic equation:

$$\mu(1-\mu) = \kappa(1-\mu) + \nu(d-p) \Rightarrow \mu^2 - \alpha\mu + \beta(p) = 0 \tag{42}$$

where:

$$\alpha = \kappa + 1, \quad \beta(p) = \kappa + \nu(d-p) \tag{43}$$

This quadratic equation governs the equilibrium values of μ. The number of real solutions depends on the discriminant:

$$\delta(p) = \frac{\alpha^2}{4} - \beta(p) \tag{44}$$

We obtain two distinct real solutions when $\delta(p) \geq 0$, which occurs for:

$$p < p_1^* = \left(\frac{\alpha^2}{4} + \kappa + \nu d \right) / \nu \tag{45}$$

In that case, the equilibrium values of μ are given by:

$$\mu_{1,2} = \frac{\alpha}{2} \pm \sqrt{\delta(p)} \tag{46}$$

We must then verify whether these roots fall within the admissible range $\left[\lambda, \frac{1}{2}(1+\lambda)\right]$. If they do, they represent feasible equilibrium levels of early withdrawals. If $\delta(p) < 0$, however, the quadratic equation has no real roots, and the model predicts that no partial-run equilibrium exists in this region. Instead, the only consistent outcome is a full bank run: $\mu = 1$.

(B) Constant Region: $\mu = \frac{1}{2}(1+\lambda)$

We now analyze the flat region of the withdrawal function, where the fraction of agents choosing to withdraw remains unchanged as long as the cutoff γ_c lies in the interval $[\gamma_a, \gamma_b]$. In this zone, the value of the withdrawal function is fixed at:

$$m(\mu, p) = \frac{1+\lambda}{2} \tag{47}$$

This reflects the fact that exactly half of the agents from the "high-cost" group (uniformly distributed over $[\gamma_b, 1]$) are not yet incentivized to withdraw. Consequently, the withdrawal mass remains constant at this value.

We aim to determine under which conditions this value of μ satisfies the fixed-point condition:

$$\mu = m(\mu, p) \tag{48}$$

Specifically, we test whether $\mu = \frac{1}{2}(1+\lambda)$ can be a valid solution. For this to hold, the corresponding cutoff γ_c must fall within the constant region, i.e.,

$$\gamma_a \leq \gamma_c \leq \gamma_b \tag{49}$$

Substituting the expression for the cutoff threshold, $\gamma_c = \frac{d-p}{1-\mu}$, into the condition above yields:

$$\gamma_a \leq \frac{d-p}{1-\frac{1}{2}(1+\lambda)} \leq \gamma_b \tag{50}$$

Simplifying the denominator:

$$1 - \frac{1}{2}(1+\lambda) = \frac{1-\lambda}{2} \tag{51}$$

we rewrite the inequality as:

$$\gamma_a \leq \frac{2(d-p)}{1-\lambda} \leq \gamma_b \tag{52}$$

Solving for p, we obtain the interval:

$$p \in \left[d - \frac{1}{2}(1-\lambda)\gamma_b,\ d - \frac{1}{2}(1-\lambda)\gamma_a \right] \tag{53}$$

This means that for any price p within this interval, the system admits a fixed point at the intermediate value $\mu = \frac{1}{2}(1+\lambda)$, corresponding to a "partial run" equilibrium. Importantly, in this region, small variations in p do not change the share of agents withdrawing, since γ_c remains inside the gap between the two modes of the transaction cost distribution. This behavior introduces a plateau in the withdrawal response and reflects a zone of coordination inertia, where agents' expectations are self-fulfilling and resistant to minor fluctuations in incentives.

(C) Increasing Region: $\mu \in \left(\frac{1}{2}(1+\lambda),\ 1\right)$

We now analyze the second increasing region of the withdrawal function, corresponding to the upper segment of the bimodal distribution. In this

region, the cutoff type γ_c lies in the interval $[\gamma_b, 1]$, meaning that an increasing share of the "high-cost" agents begins to withdraw as incentives improve. The withdrawal function in this region is given by:

$$m(\mu, p) = \frac{1+\lambda}{2} + \frac{1-\lambda}{2} \cdot \frac{\gamma_c - \gamma_b}{1-\gamma_b} \tag{54}$$

To find equilibrium points, we again impose the fixed-point condition $\mu = m(\mu, p)$, and substitute for the cutoff type using:

$$\gamma_c = \frac{d-p}{1-\mu} \tag{55}$$

This gives the equation:

$$\mu = \frac{1+\lambda}{2} + \frac{1-\lambda}{2} \cdot \left(\frac{1}{1-\gamma_b}\right)\left(\frac{d-p}{1-\mu} - \gamma_b\right) \tag{56}$$

To simplify, we define:

$$\kappa = \frac{1+\lambda}{2} - \frac{1-\lambda}{2} \cdot \frac{\gamma_b}{1-\gamma_b}, \quad \nu = \frac{1-\lambda}{2(1-\gamma_b)} \tag{57}$$

This allows us to rewrite the fixed-point equation as:

$$\mu = \kappa + \nu \cdot \frac{d-p}{1-\mu} \tag{58}$$

Multiplying both sides by $1-\mu$ and rearranging leads to the quadratic equation:

$$\mu^2 - \alpha\mu + \beta(p) = 0 \tag{59}$$

where:

$$\alpha = \kappa + 1, \quad \beta(p) = \kappa + \nu(d-p) \tag{60}$$

This equation characterizes equilibrium values of μ in the upper region of the withdrawal function. As before, the nature of the solution depends on the discriminant:

$$\delta(p) = \frac{\alpha^2}{4} - \beta(p) \tag{61}$$

There are two real solutions if $\delta(p) \geq 0$, which corresponds to the condition:

$$p < p_B^* = \left(\frac{\alpha^2}{4} + \kappa + \nu d \right) / \nu \tag{62}$$

If this condition is satisfied, the equilibrium values are:

$$\mu_{1,2} = \frac{\alpha}{2} \pm \sqrt{\delta(p)} \tag{63}$$

We then check whether these solutions fall in the admissible region:

$$\mu_{1,2} \in \left[\tfrac{1}{2}(1+\lambda),\ 1 \right] \tag{64}$$

If both roots lie in this range, they represent feasible equilibrium levels of early withdrawal in the upper region. Among them, the upper root $\mu^*(p)$ is typically interpreted as the stable equilibrium when multiple equilibria exist.

If the discriminant is negative, i.e., $\delta(p) < 0$, then the model predicts no interior fixed point in this region. In that case, all agents are induced to withdraw and a full-scale bank run occurs: $\mu = 1$.

Example 3: Trimodal Distribution

We now study a case where transaction cost types follow a trimodal distribution. Specifically, we assume that G is uniform over three disjoint intervals:

$$[\underline{\gamma}, \gamma_1], \quad [\gamma_2, \gamma_3], \quad [\gamma_4, \bar{\gamma}] \tag{65}$$

Each interval carries one-third of the total probability mass. This setup captures a population where agents cluster around three distinct transaction cost levels.

Defining the Withdrawal Function

We define the share of agents who withdraw, $m(\mu, p)$, based on the cutoff type $\gamma_c = \frac{d-p}{1-\mu}$. The function depends on where this cutoff falls relative to the three intervals:

$$
\begin{cases}
\gamma_c < \underline{\gamma} & \Rightarrow m(\mu, p) = \lambda \\
(A) \quad \underline{\gamma} \leq \gamma_c \leq \gamma_1 & \Rightarrow m(\mu, p) = \lambda + \frac{1-\lambda}{3} \cdot \frac{\gamma_c - \underline{\gamma}}{\gamma_1 - \underline{\gamma}} \\
(B) \quad \gamma_1 \leq \gamma_c \leq \gamma_2 & \Rightarrow m(\mu, p) = \frac{1+2\lambda}{3} \\
(C) \quad \gamma_2 \leq \gamma_c \leq \gamma_3 & \Rightarrow m(\mu, p) = \frac{1+2\lambda}{3} + \frac{1-\lambda}{3} \cdot \frac{\gamma_c - \gamma_2}{\gamma_3 - \gamma_2} \\
(D) \quad \gamma_3 \leq \gamma_c \leq \gamma_4 & \Rightarrow m(\mu, p) = \frac{2+\lambda}{3} \\
(E) \quad \gamma_4 \leq \gamma_c \leq \bar{\gamma} & \Rightarrow m(\mu, p) = \frac{2+\lambda}{3} + \frac{1-\lambda}{3} \cdot \frac{\gamma_c - \gamma_4}{\bar{\gamma} - \gamma_4} \\
\bar{\gamma} \leq \gamma_c & \Rightarrow m(\mu, p) = 1
\end{cases}
\tag{66}
$$

This function has five distinct regions. It increases in segments (A), (C), and (E), and remains flat in (B) and (D). The shape reflects how agents from each group start withdrawing once the price is low enough relative to their cost of waiting.

Cases (A), (C), and (E): Increasing Segments

In regions (A), (C), and (E), the withdrawal share increases with γ_c. To find the equilibrium value of μ, we solve the fixed-point condition:

$$\mu = m(\mu, p) \tag{67}$$

Using the structure of each segment, we write this condition as:

$$\mu = \kappa + \nu \cdot \frac{d - p}{1 - \mu} \tag{68}$$

We summarize the constants κ and ν for each increasing region:

Region	κ	ν
(A)	$\lambda - \frac{1-\lambda}{3} \cdot \frac{\underline{\gamma}}{\gamma_1 - \underline{\gamma}}$	$\frac{1-\lambda}{3(\gamma_1 - \underline{\gamma})}$
(C)	$\frac{1+2\lambda}{3} - \frac{1-\lambda}{3} \cdot \frac{\gamma_2}{\gamma_3 - \gamma_2}$	$\frac{1-\lambda}{3(\gamma_3 - \gamma_2)}$
(E)	$\frac{2+\lambda}{3} - \frac{1-\lambda}{3} \cdot \frac{\gamma_4}{\bar{\gamma} - \gamma_4}$	$\frac{1-\lambda}{3(\bar{\gamma} - \gamma_4)}$

(69)

Each pair (κ, ν) leads to a quadratic equation for μ. We rewrite the fixed-point condition as:

$$\mu^2 - \alpha\mu + \beta(p) = 0 \tag{70}$$

with:

$$\alpha = \kappa + 1, \quad \beta(p) = \kappa + \nu(d - p) \tag{71}$$

To find valid solutions, we compute the discriminant:

$$\delta(p) = \frac{\alpha^2}{4} - \beta(p) \tag{72}$$

We find two real solutions for μ when $\delta(p) \geq 0$, which happens if:

$$p < p^* = \frac{\alpha^2}{4} + \kappa + \nu d \Big/ \nu \tag{73}$$

The solutions are:

$$\mu_{1,2} = \frac{\alpha}{2} \pm \sqrt{\delta(p)} \tag{74}$$

We then check whether these roots fall within the relevant interval for each segment:

(A): $\mu_{1,2} \in \left[\lambda,\ \frac{1+2\lambda}{3}\right]$; (C): $\mu_{1,2} \in \left[\frac{1+2\lambda}{3},\ \frac{2+\lambda}{3}\right]$; (E): $\mu_{1,2} \in \left[\frac{2+\lambda}{3},\ 1\right]$.

Only roots inside these intervals represent valid equilibrium withdrawal shares. If $\delta(p) < 0$, then no fixed point exists in the region. In that case, the model predicts a jump to a higher withdrawal level or a full bank run.

Cases (B) and (D): Flat Segments

In regions (B) and (D), the withdrawal function $m(\mu, p)$ is constant because the cutoff type γ_c lies in a gap between two populated intervals of the trimodal distribution. No agents are located in these intervals, so small changes in γ_c do not alter the share of agents who withdraw.

To test for equilibrium, we check whether the fixed-point condition $\mu = m(\mu, p)$ holds for the constant values of μ that define these flat segments. In region (B), we set:

$$\mu = \mu_B = \frac{1 + 2\lambda}{3} \tag{75}$$

In region (D), we use:

$$\mu = \mu_D = \frac{2 + \lambda}{3} \tag{76}$$

We substitute each expression into the cutoff formula $\gamma_c = \frac{d-p}{1-\mu}$, and solve for the values of p that keep γ_c inside the corresponding flat region.

In region (B), we require $\gamma_c \in [\gamma_1, \gamma_2]$. Substituting μ_B, we obtain:

$$\gamma_1 \leq \frac{d-p}{1-\mu_B} \leq \gamma_2 \quad \Leftrightarrow \quad p \in \left[d - \frac{2+\lambda}{3}\gamma_2,\ d - \frac{2+\lambda}{3}\gamma_1 \right] \tag{77}$$

In region (D), we require $\gamma_c \in [\gamma_3, \gamma_4]$. Substituting μ_D, we find:

$$\gamma_3 \leq \frac{d-p}{1-\mu_D} \leq \gamma_4 \quad \Leftrightarrow \quad p \in \left[d - \frac{1+2\lambda}{3}\gamma_4,\ d - \frac{1+2\lambda}{3}\gamma_3 \right] \tag{78}$$

Appendix A: Calculations

First-Order Conditions

Generally, if G has a density $g(\gamma) = G'(\gamma)$, one can then calculate the first-order condition (FOC) with respect to d for maximizing $U(d)$ as

$$\begin{aligned} 0 = U'(d) = \lambda u'(d) + (1-\lambda) \big[& u'(c_{2,s}(d)) c'_{2,s}(d) \left(1 - G\left(\gamma_c(d)\right)\right) \\ & + \gamma'_c(d) \left(c_{2,w}(\gamma_c(d), 1) g(\gamma_c(d)) - u(c_{2,s}(d)) g\left(\gamma_c(d)\right)\right)] \end{aligned} \tag{79}$$

The relevant derivatives are

$$\begin{aligned} u'(c_{2,s}(d)) &= -\frac{R}{1-\mu(d)} \left(\frac{d-1}{1-\mu(d)} \mu'(d) + \mu(d) \right) \\ \gamma'_c(d) &= \frac{1}{1-\mu(d)} \left(\frac{d-1}{1-\mu(d)} \mu'(d) - 1 \right) \end{aligned}$$

where $\mu'(d)$ follows from (18) and the implicit function theorem or, more directly, by plugging in $\mu(d)$ for μ, taking the derivative with respect to d as

$$\mu'(d) = g(\gamma_c(d)) \gamma'_c(d)$$

and solving for $\mu'(d)$ as

$$\mu'(d) = \frac{(1-\mu(d)) g(\gamma_c(d))}{(1-\mu(d))^2 - g(\gamma_c(d))(d-1)}$$

Equilibrium with Uniform Distribution over $[\underline{\gamma}, \overline{\gamma}]$

In this section of the online appendix, we provide analytical derivations that support the numerical illustrations presented in Section A. We analyze the fixed-point condition for early withdrawals under the assumption that transaction costs are uniformly distributed over the interval $[\underline{\gamma}, \overline{\gamma}]$. This case offers a tractable benchmark for understanding equilibrium selection in the model.

The cutoff type γ_c, which separates early and late consumers, satisfies:

$$\gamma_c = \frac{d-1}{1-\mu} \tag{80}$$

Since agents withdraw if their type is below the cutoff, the share of agents who withdraw equals the cumulative distribution evaluated at γ_c. The fixed-point condition becomes:

$$\mu = \lambda + (1-\lambda)G\left(\frac{d-1}{1-\mu}\right) \tag{81}$$

Under the uniform distribution, the cumulative function is:

$$G(\gamma) = \frac{\gamma - \underline{\gamma}}{\overline{\gamma} - \underline{\gamma}} \tag{82}$$

Substituting into the fixed-point equation gives:

$$\mu = \lambda + (1-\lambda)\frac{1}{\overline{\gamma} - \underline{\gamma}}\left(\frac{d-1}{1-\mu} - \underline{\gamma}\right) \tag{83}$$

To simplify, define $\nu = 1 - \mu$. We then rewrite the equation as:

$$1 - \nu = \lambda + (1-\lambda)\frac{1}{\overline{\gamma} - \underline{\gamma}}\left(\frac{d-1}{\nu} - \underline{\gamma}\right) \tag{84}$$

Multiplying through by $(\overline{\gamma} - \underline{\gamma})$ gives:

$$(\overline{\gamma} - \underline{\gamma})(1-\nu) = (1-\lambda)\frac{d-1}{\nu} + \lambda(\overline{\gamma} - \underline{\gamma}) - (1-\lambda)\underline{\gamma} \tag{85}$$

Rearranging:

$$(1-\lambda)(\overline{\gamma} - \underline{\gamma}) - \nu(\overline{\gamma} - \underline{\gamma}) = (1-\lambda)\frac{d-1}{\nu} - (1-\lambda)\underline{\gamma} \tag{86}$$

or

$$(1-\lambda)\overline{\gamma} - \nu(\overline{\gamma} - \underline{\gamma}) = (1-\lambda)\frac{d-1}{\nu} \tag{87}$$

Multiplying both sides by $\nu/(\overline{\gamma} - \underline{\gamma})$ yields:

$$\frac{1-\lambda}{\overline{\gamma} - \underline{\gamma}}\overline{\gamma}\nu - \nu^2 = \frac{1-\lambda}{\overline{\gamma} - \underline{\gamma}}(d-1) \tag{88}$$

This is a quadratic equation in ν. Bringing all terms to one side:

$$\nu^2 - \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}\nu + \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}(d-1) = 0 \tag{89}$$

We solve using the quadratic formula:

$$\nu_{1,2} = \frac{1}{2}\cdot\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma} \pm \sqrt{\frac{1}{4}\left(\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}\right)^2 - \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}(d-1)} \tag{90}$$

Returning to $\mu = 1-\nu$, we get:

$$\mu_{1,2} = 1 - \frac{1}{2}\cdot\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma} \pm \sqrt{\frac{1}{4}\left(\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}\right)^2 - \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}(d-1)} \tag{91}$$

We obtain real-valued solutions whenever the discriminant is non-negative:

$$\delta(d) = \frac{1}{4}\left(\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}\right)^2 - \frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}(d-1) \geq 0 \tag{92}$$

Solving for the threshold where $\delta(d) = 0$, we find the maximum value of d that allows an interior solution:

$$\overline{d} = 1 + \frac{1}{4}\cdot\frac{1-\lambda}{\overline{\gamma}-\underline{\gamma}}\overline{\gamma}^2 \tag{93}$$

Equilibrium with interior withdrawal behavior exists if and only if $d \leq \overline{d}$. If $d > \overline{d}$, the quadratic has no real roots and the only solution is $\mu = 0$, implying a full bank run. Further, the quadratic solution is only valid for $\gamma_c \geq \underline{\gamma}$. Since $\mu = \lambda$ at that value, this implies that the quadratic solution is only valid for $d \geq \underline{d}$, where $\underline{d} = 1 + (1-\lambda)\underline{\gamma}$. We therefore restrict attention to values

$$d \in [\underline{d}, \overline{d}]$$

in order to use the solution to the quadratic equation (91). Outside, we have $\mu(d) \equiv \lambda$ for all $d \leq \underline{d}$ and $\mu(d) \equiv 1$ for all $d > \overline{d}$. For any such d, we can solve the fixed-point equation directly or use numerical methods to compute the equilibrium share of early withdrawals.