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ABSTRACT

This article corrects a 60-year history of mis-application of the neoclassical theory of investment to interpret empirical work and guide policy analysis. Empirical estimates of firm-level user cost elasticities of investment or capital have in fact studied two distinct objects, an elasticity holding output fixed that identifies the elasticity of substitution between capital and labor (Eisner and Nadiri, 1968) and an elasticity holding the wage fixed that sheds light on the revenue elasticity of capital (Coen, 1969). A review finds a consensus range for the Coen short-run user cost elasticity that translates into a priori plausible values for the revenue elasticity of capital, offering validation to the neoclassical theory. The long-run general equilibrium capital elasticity to the user cost instead holds labor fixed and depends on both firm-level parameters. This synthesis offers a framework for future work and helps to explain why policy institutions have differing views on the effects of corporate taxes.

Gabriel Chodorow-Reich
Harvard University
Department of Economics
and NBER
chodorowreich@fas.harvard.edu

1 Introduction

The use of the tax code to promote investment has returned to the forefront of U.S. economic policy. The 2017 Tax Cut and Jobs Act (TCJA) reduced the corporate tax rate and allowed immediate expensing of equipment investment for five years, among other provisions. The 2025 One Big Beautiful Bill Act (OBBBA) made the immediate expensing permanent and allowed accelerated expensing of some structures. Such large changes in tax policy offer a rare opportunity to learn about firm behavior. Furthermore, their effect on aggregate investment, output, wages, and tax revenue is of direct concern to policy makers.

Studies of how tax incentives affect capital formation stretch back more than 60 years to the seminal analysis of the investment decision of a profit-maximizing firm in Jorgenson (1963) and Hall and Jorgenson (1967). Jorgenson (1963) derives the user cost of capital and shows that under a Cobb-Douglas production function the firm's "desired" or long-run capital stock satisfies $K = \alpha Y/R$, where K is the stock of capital, Y is the firm's revenue, R is the user cost, and α is the revenue elasticity of capital. Hall and Jorgenson (1967) measure R incorporating a detailed analysis of the corporate tax code and conduct counterfactual exercises for the effect of tax reforms on investment. In their framework, the long-run elasticity of capital and investment to the user cost is -1, which Council of Economic Advisers (2019, p. 59) refers to as the "neoclassical benchmark" and uses in its analysis of the TCJA.

A wave of empirical papers have estimated a user cost elasticity using variation from tax reforms and firm-level data. Hassett and Hubbard (2002, p.1321) summarize this evidence as finding "elasticities of investment to the user cost of capital between -0.5 and -1.0," a range seemingly consistent with the "neoclassical benchmark" and that has become enshrined in evaluations of the effect of tax policy on aggregate capital and output (Congressional Budget Office, 2018*b*; Council of Economic Advisers, 2019, 2025). Yet, despite this apparently happy confluence of theory and empirics, disagreement remains. A poll of economists taken in late 2023 found that 30% of respondents agreed with the statement that the corporate capital stock was substantially higher as a result of the TCJA, 33% disagreed, and 36% were uncertain (Clark Center for Global Markets, 2023). In their prospective analysis of TCJA, Barro and Furman (2018) used an elasticity of -1.6, outside of the Hassett and Hubbard (2002) range.

In fact, this disagreement reflects a 60-year history of mis-application of the neoclassical theory of investment to interpret empirical work and guide policy analysis. This history starts with the statement that the long-run elasticity of capital to the user cost in the neoclassical framework is -1. That error was first shown in a comment by Coen (1969). The reasoning is simple. Jorgenson (1963) and Hall and Jorgenson (1967) obtain $\partial \log K/\partial \log R = -1$ only by holding firm revenue Y fixed. But lower capital induced by an increase in the cost of capital will also reduce firm revenue, resulting in an "accelerator" feedback effect from lower revenue into even lower capital that the analysis of tax reforms in Hall and Jorgenson (1967) omits.

A second major source of confusion concerns the mapping from moments into parameters. Studies of the "user cost elasticity of investment" have implemented a diverse set of empirical specifications that connect to different structural objects. These include the magnitude of adjustment costs, the elasticity of investment inclusive of the feedback effect identified by Coen, and the elasticity of substitution between capital and labor. Comparisons of magnitudes across these specifications, while prevalent in literature reviews and review essays, make little sense. The literature does not even agree on what estimand underlies the Hassett and Hubbard (2002) range, with implications for the effect of taxes on investment that vary by a factor of five or more.

A third outstanding problem is the disconnect between empirical estimates of cross-firm, short-run investment responses and policy analysis interested in the aggregate long-run response of capital and investment. This disconnect involves both firm-level dynamics — a stable balanced growth path investment/capital ratio implies that investment and capital have the same long-run elasticity, but they differ in their short-run behavior — and the "missing intercept" problem of moving from firm-level responses to general equilibrium.

The objective of this article is to synthesize the accumulated evidence relating investment to tax incentives in a theory-consistent manner. This process involves laying out the theory, connecting empirical results to structural parameters, and using these parameters to relate the cross-sectional empirical evidence to the aggregate response. Section 2 reviews the neoclassical theory of investment of an individual firm i. This firm uses capital and labor to produce output, taking input prices as given. The steady state user cost $R_i = (\rho + \delta)P^K(1 - \Gamma_i)/(1 - \tau_i)$ consists of the before-tax discount rate and depreciation rate, $\rho + \delta$, multiplied by the price of capital P^K

and the "tax term" $(1-\Gamma_i)/(1-\tau_i)$, where τ_i is the firm's marginal tax rate and Γ_i the effective subsidy to the cost of new investment. The firm's first order condition for capital equates the marginal product of capital to the user cost.

A comparative static of the steady state capital first order condition with respect to the user cost gives a long-run elasticity. Much confusion stems from different choices of what to hold fixed in this comparative static. Jorgenson (1963) and Hall and Jorgenson (1967) hold output fixed with a Cobb-Douglas production function over capital and labor L. Eisner and Nadiri (1968) relax the Cobb-Douglas assumption and estimate a user cost elasticity conditional on output, which they show recovers the elasticity of substitution η between capital and labor (1 if Cobb-Douglas).

The user cost elasticity advocated by Coen (1969) instead holds fixed only variables exogenous to the firm, such as the wage, but includes the feedback from capital to revenue, which in turn increases the demand for capital. For the Cobb-Douglas case, $Y_{i,t} = K_{i,t}^{\alpha} L_{i,t}^{\beta}$, the long-run firm-level user cost elasticity is $-1/(1-\gamma) < -1$, where $\gamma = \alpha/(1-\beta)$. That is, the full neoclassical firm-level elasticity is not -1, but instead something (possibly much) larger in absolute value. The Coen (1969) elasticity generalizes with any constant elasticity of substitution (CES) over capital and labor, naturally with the elasticity of substitution also mattering. In short, the Eisner and Nadiri (1968) and Coen (1969) elasticities reflect different structural objects, despite both using the phrase "user cost elasticity".[1]

Bringing these results to the empirical literature requires relating the long-run formulas to short-run investment. With quadratic adjustment costs in capital, Summers (1981) obtains a third investment relationship, $I_{i,t}/K_{i,t} = \delta + (1/\phi)Q_{i,t}$, where $I_{i,t}$ denotes gross investment at date t, $Q_{i,t}$ is a transformation of Brainard-Tobin's q, and ϕ determines the scale of adjustment costs. However, this equation involves Q, not the user cost. Solving for the linearized transition path following a permanent tax change instead gives the useful result that the steady state Coen (1969) elasticity equals the short-run semi-elasticity of I/K scaled by the system's stable eigenvalue.

Section 3 revisits the empirical literature estimating firm or asset-level responses of investment

[1] This terminology overlap has often been overlooked. As one example among many, Chirinko, Fazzari and Meyer (1999) directly compare their estimated user cost elasticity from regressing investment on the user cost while controlling for firm revenue to a user cost elasticity they infer from a regression in Cummins, Hassett and Hubbard (1994) that does not control for firm revenue. Dwenger (2014); Council of Economic Advisers (2019); Congressional Budget Office (2018*b*); Council of Economic Advisers (2025); Hartley, Hassett and Rauh (2025) all repeat some version of this transgression.

to taxes. The literature has implemented a range of empirical specifications involving possibly minor seeming econometric choices in the dependent variable (I/K, $\log I$, or $\log K$), controls (non-tax component of Q or firm revenue) or timing (short or long-run). Yet, these specifications imply estimands that map into distinct parameters in the neoclassical theory. I provide a Lemma with the mapping for each empirical specification and group them into firm-level estimates of the Summers (1981) specification (Table 1), the Coen (1969) user cost elasticity allowing output to vary (Table 2), hybrid approaches (Table 3), the Eisner and Nadiri (1968) specification holding output fixed (Table 4), and asset-level specifications (Table 5). The Coen (1969) specifications translate into a short-run semi-elasticity of the investment-to-capital ratio to the user cost, $d\left[I_{i,t}/K_{i,t-1}\right]/d\log R_{i,t}$, in the range of -0.1 to below -1, with most values in the more compact range of -0.25 to -0.75. This range echoes the Hassett and Hubbard (2002) range of -0.5 to -1. However, my range concerns the semi-elasticity of investment to the user cost and contains adjustments to earlier estimates not made in Hassett and Hubbard (2002). Converting to an elasticity requires dividing by I/K, implying short-run investment elasticities of about -2 or below.[2]

Section 4 maps the empirical moments into parameter sets. The Summers (1981) and Eisner and Nadiri (1968) specifications map straightforwardly into the adjustment cost parameter and elasticity of substitution, respectively. The Coen (1969) specifications require more care, since even the transformed regression coefficients are functions of several underlying parameters. Figure 2 contains "trees" that start with different values of the short-run Coen (1969) semi-elasticity, "branch" for different values of the eigenvalue relating the short-to-long-run impulse response and for the elasticity of substitution between capital and labor, and end in "leaves" with the implied values of the revenue elasticity of capital α and adjustment cost parameter ϕ. For an empirical short-run semi-elasticity $d\left[I_{i,t}/K_{i,t-1}\right]/d\log R_{i,t}$ of -0.5, the implied long-run firm-level elasticity of capital to the user cost is around -3 or below. This result highlights the difference between the Coen (1969) elasticity and the elasticity of substitution between capital and labor and associated

[2]Indeed, the meaning of the Hassett and Hubbard (2002) range remains unsettled. Zwick and Mahon (2017, figure 4) compare the Hassett and Hubbard (2002) range to their coefficient from regressing I/K on the tax term $(1-\Gamma)/(1-\tau)$. Chodorow-Reich, Zidar and Zwick (2024) similarly plot coefficients from this specification and label values between -0.5 and -1 as the Hassett and Hubbard (2002) range. Council of Economic Advisers (2019, p.59) instead interprets the range as applying to the elasticity $d\log I/d\log R$. Since (starting from steady state) $d\left[I_{i,t}/K_{i,0}\right]/d\left[\left(1-\Gamma_i\right)/\left(1-\tau_i\right)\right] = \left(I_{i,0}/K_{i,0}\right)/\left[\left(1-\Gamma_i\right)/\left(1-\tau_i\right)\right] \times d\log I_{i,t}/d\log\left[\left(1-\Gamma_i\right)/\left(1-\tau_i\right)\right] = \delta_i/\left[\left(1-\Gamma_i\right)/\left(1-\tau_i\right)\right] \times d\log I_{i,t}/d\log R_i$, these interpretations of the Hassett and Hubbard (2002) range differ by the ratio of the depreciation rate to the tax term, or roughly a factor of 5 or more.

“neoclassical benchmark” of -1. The values of α range from 0.22 to 0.35, implying total returns to scale in the revenue function of 0.87 or above.

Section 5 relates the cross-firm studies to the effect of changes in tax incentives on aggregate capital, output, wages, and tax revenue. Individual firms face flat factor supply curves, taking the wage as fixed. In general equilibrium, factor supply curves may slope up. Under the simplifying assumptions that the economy consists of a large number of identical firms and aggregate labor is fixed, the general equilibrium user cost elasticity involves a third variant of the comparative static of the first order condition for capital, now holding labor rather than output (as in Eisner and Nadiri (1968)) or the wage (as in Coen (1969)) fixed. With Cobb-Douglas, the general equilibrium elasticity of capital to the user cost thus becomes $-1/(1-\mathcal{M}\alpha)$, where $\mathcal{M} \geq 1$ summarizes the component of decreasing returns in the revenue function due to downward-sloping demand. (For a monopolistically competitive firm it equals the markup.) I refer to this elasticity as the Barro and Furman (2018) elasticity, as these authors use it in their analysis of TCJA. Moving from the Coen (1969) elasticity identified from cross-firm responses to the Barro and Furman (2018) general equilibrium elasticity therefore requires relating α to γ. With CES, the formula again also involves the elasticity of substitution between capital and labor.

Figure 3 summarizes plausible values using trees, now starting from different values of the revenue elasticity of capital α and branching into values of the markup and elasticity of substitution. Under Cobb-Douglas, the long-run general equilibrium elasticity ranges from -1.3 to -1.7. Compared to the firm-level range of -3 or below, this range reflects substantial tightening and general equilibrium dampening due to the aggregate fixed factor of labor. The elasticity of -1.6 used in the actual Barro and Furman (2018) article falls inside it (they set $\alpha = 0.38$ implying $-1/(1-\alpha) = -1.6$). Thus, the general equilibrium neoclassical elasticity under Cobb Douglas also is not -1, but instead something again larger in absolute value.

A lower elasticity of substitution η reduces the general equilibrium elasticities of capital and output, as the fixed factor of labor generates faster decreasing returns to capital accumulation. Quantitatively, with constant returns to scale the Barro and Furman (2018) elasticity becomes $-\eta/(1-\mathcal{M}\alpha)$ and hence scales with the elasticity of substitution η. The values of α consistent with the short-run responses of investment imply returns to scale close enough to 1 that this formula roughly holds. Thus, both the Eisner and Nadiri (which identifies η) and Coen (which helps to identify α) firm-level specifications provide important inputs to the aggregate capital elasticity. The wage elasticity is much less sensitive to the value of η, as a lower elasticity of substitution implies less capital accumulation but faster wage growth per unit of capital. In the constant returns case the wage elasticity becomes $-\mathcal{M}\alpha/(1-\mathcal{M}\alpha)$ and is independent of η; it ranges from -0.3 to -0.7 for the parameter values consistent with the firm-level evidence.

These results characterize the general equilibrium elasticity of capital to the user cost, $d\log K/d\log R$. The elasticity of aggregate capital to the tax term may be smaller if the non-tax component of the user cost changes, for example because an upward-sloping savings schedule causes higher interest rates ($\rho\uparrow$) or limited capital goods supply increases the price of investment ($P^K\uparrow$). Letting ζ denote the inverse elasticity of the non-tax component to capital, Figure 4 shows that the overall elasticity of capital to the tax term is $(d\log K/d\log R)/(1-\zeta\times(d\log K/d\log R))$.

The final set of analytic results characterize the tax revenue response in the neoclassical framework. For changes to the corporate rate τ or generosity of expensing z, the change in revenue consists of a mechanical component holding the tax base fixed and a dynamic component as the base changes. Using values for the U.S. in 2024, the dynamic component offsets less than 10% of the mechanical decline when the tax rate falls and between 25% and 45% of the mechanical decline when expensing becomes more generous. Thus, in the relatively low-tax regime in the U.S. in 2024, the neoclassical framework cannot rationalize more than modest dynamic revenue offsets from further reductions in corporate taxation.

In summary, the neoclassical framework offers a guide both to interpreting the firm-level evidence and to connecting it to the aggregate effects of corporate tax changes. At the firm level, the Eisner and Nadiri output-fixed user cost elasticity identifies the elasticity of substitution between capital and labor η, while the Coen wage-fixed user cost elasticity additionally sheds light on the returns to scale of the firm and hence the revenue elasticity of capital α. A consensus range for the Coen specification of a short-run firm-level semi-elasticity of the investment-to-capital ratio to the user cost between -0.25 to -0.75 translates into *a priori* plausible values for the revenue elasticity of capital of between 0.22 and 0.35. The reasonableness of these values constitutes a victory for the neoclassical theory, especially in contrast to early estimates of adjustment costs implied by the Summers (1981) specification.

The long-run general equilibrium capital elasticity to the user cost instead holds labor fixed. The neoclassical elasticity is approximately $-\eta/(1-\mathcal{M}\alpha)$ and hence depends on the firm-level estimates of both the elasticity of substitution and the revenue elasticity. For $\alpha \subseteq (0.22, 0.34)$ and $\eta \subseteq (0.5, 1)$ it lies between -0.65 and -1.7. The wage elasticity is approximately $-\mathcal{M}\alpha/(1-\mathcal{M}\alpha)$ and lies between -0.3 and -0.7 for economy-wide changes in the user cost. The neoclassical framework cannot rationalize more than modest dynamic revenue offsets from further reductions in corporate taxation in the relatively low-tax regime in the U.S. in 2024.

The analysis concludes by contrasting the neoclassical framework with the practice of leading policy models. Council of Economic Advisers (2019, 2025) follow Hall and Jorgenson (1967) and apply an elasticity of -1 based on the Hassett and Hubbard (2002) range and an appeal to the neoclassical benchmark, but do not incorporate the additional feedback from output into capital. Congressional Research Service (2024) applies an elasticity of -0.6 based on the Eisner and Nadiri (1968) specification and likewise does not incorporate the additional feedback from output into capital. Congressional Budget Office (2018*a*) applies an elasticity of substitution of -0.7 to a user cost change holding output fixed, which it incorrectly justifies on the basis of a Coen (1969) firm-level regression rather than as an elasticity of substitution (Congressional Budget Office, 2018*b*). It then allows investment to respond to output, arriving at the long-run neoclassical formula via a two-step procedure.[3]

2 Theoretical Framework

This section derives key results starting from a firm-level production function. Section 2.1 provides the general setup and necessary conditions. Section 2.2 characterizes the steady state and defines the user cost and "tax term". Section 2.3 undergoes cross steady-state analysis with a Cobb-Douglas production function, including deriving the Hall and Jorgenson (1967) and Coen (1969) elasticities. Section 2.4 generalizes to a CES production function and derives the Eisner and Nadiri (1968) elasticity holding output fixed and the generalized Coen (1969) elasticity. Section 2.5 adds

[3]The critique of policy evaluation and counterfactuals echoes Lucas (1976), who also discussed Hall and Jorgenson (1967) in detail. My analysis differs from his however. Lucas (1976) focused on the interpretation of econometric evidence when tax policy is transitory. I instead assume throughout that firms expect current policy to continue and argue that even this simpler case has not properly integrated economic theory with econometric practice.

quadratic adjustment costs in capital and derives the Summers (1981) specification. Section 2.6 uses the quadratic adjustment cost environment to relate the cross steady state specifications to their short-run counterparts. For ease of reference, Table A.1 summarizes the notation.

Throughout this section, the analysis concerns a single firm that takes its input prices as given. This partial equilibrium perspective anticipates the firm, asset, or industry-level variation used in the studies reviewed in Section 3. Section 5 extends the results to general equilibrium and shows how the firm-level variation nonetheless disciplines aggregate elasticities.

2.1 General Setup

I set up the problem in continuous time. More substantively, I ignore uncertainty and assume zero inflation in order to arrive at the key relationships as neatly as possible.

Firm i produces revenue $Y_{i,t}$ using capital $K_{i,t}$ and labor $L_{i,t}$:

$$Y_{i,t} = A_{i,t} F\left(K_{i,t}, L_{i,t}\right), \tag{1}$$

where $A_{i,t}$ denotes exogenous Hicks-neutral technology and $F(.,.)$ is a twice differentiable globally concave function.[4] Equation (1) makes no assumption on the structure of demand, only that the function $F(.,.)$ maps capital and labor into revenue. The firm hires labor on the spot market each instant at a common wage W_t and discounts the future at rate ρ. The firm owns its capital stock, a fraction δ of which depreciates each period, giving the law of motion:

$$\dot{K}_{i,t} \equiv dK_{i,t}/dt = I_{i,t} - \delta K_{i,t}, \tag{2}$$

where $I_{i,t}$ denotes gross investment purchased at price P_t^K. Each dollar of investment also requires payment of $\Phi\left(I_{i,t}, K_{i,t}\right)$ dollars of installation costs.

The corporate tax code affects the firm in three ways. First, a firm can deduct from its taxable income a fraction of past investment expenditure. Specifically, for a dollar of investment at date t, the firm can deduct a fraction $B_{i,t}$ immediately and the remainder according to a specified schedule, where $c_{i,t+h|t}$ denotes the fraction of the remainder deductible at horizon h. The fraction $B_{i,t}$ is

[4] Allowing for multiple types of capital or labor would complicate some of the expressions below but not change the key results used to interpret empirical work. Likewise, with a constant intermediate inputs share of cost (i.e. Cobb-Douglas over intermediate inputs and factors of production), equation (1) may be reinterpreted as the function giving revenue net of intermediate inputs after concentrating these inputs out of the production function. With fixed costs, K and L in equation (1) should be interpreted as the variable factors of production.

referred to as "bonus depreciation" or "immediate expensing". Second, the firm may receive an additional investment tax credit $itc_{i,t}$ for investment at date t. The i subscripts in the depreciation schedule and investment tax credit reflect the possibility that they vary by asset type or industry. Third, the corporate income tax $\tau_{i,t}$ applies to revenue net of deductible costs. The i subscript in the tax rate reflects the possibility of credits, deductions, or net operating losses that make the marginal tax rate vary across firms.

The firm's problem simplifies under the maintained assumption that it expects the marginal tax rate to remain fixed from date t forward.[5] In that case, the present value of tax savings per dollar of investment at date t, denoted $\Gamma_{i,t}$, consists of the investment tax credit plus the tax rate multiplied by the present value of future depreciation allowances $z_{i,t}$:

$$\Gamma_{i,t} = itc_{i,t} + \tau_{i,t} z_{i,t}, \tag{3}$$

$$\text{where: } z_{i,t} = B_{i,t} + (1 - B_{i,t}) \int_0^\infty e^{-\rho h} c_{i,t+h|t} dh. \tag{4}$$

Furthermore, while accumulated allowances from past investment affect the value of the firm, they do not affect the choice of future investment or capital. The optimal choices of capital and labor therefore arise under the maximization problem:

$$\max_{\{L_{i,t+h}, K_{i,t+h}, I_{i,t+h}\}} \int_0^\infty e^{-\rho h} \left((1 - \tau_{i,t}) \left(A_{i,t} F\left(K_{i,t}, L_{i,t}\right) - W_t L_{i,t} - \Phi\left(I_{i,t}, K_{i,t}\right) \right) - (1 - \Gamma_{i,t}) P_t^K I_{i,t} \right) dh,$$

subject to the law-of-motion (2) and the initial conditions.

Let $\lambda_{i,t}$ denote the Lagrange multiplier associated with the constraint (2). Let $F_K\left(K_{i,t}, L_{i,t}\right) = \partial F\left(K_{i,t}, L_{i,t}\right) / \partial K_{i,t}$ and likewise for F_L, Φ_K, Φ_I. The necessary conditions are:

$$L_{i,t}: \qquad A_{i,t} F_L\left(K_{i,t}, L_{i,t}\right) = W_t, \tag{5a}$$

$$K_{i,t}: \qquad \frac{(1 - \tau_{i,t}) \left(A_{i,t} F_K\left(K_{i,t}, L_{i,t}\right) - \Phi_K\left(I_{i,t}, K_{i,t}\right) \right) - \delta \lambda_{i,t} + \dot{\lambda}_{i,t}}{\lambda_{i,t}} = \rho, \tag{5b}$$

$$I_{i,t}: \qquad (1 - \tau_{i,t}) \Phi_I\left(I_{i,t}, K_{i,t}\right) + (1 - \Gamma_{i,t}) P_t^K = \lambda_{i,t}. \tag{5c}$$

[5] Given initial capital $K_{i,t}$ and investment history $\{I_{i,t+h}\}_{h=-\infty}^0$, the firm chooses sequences $\{K_{i,t+h}, I_{i,t+h}, L_{i,t+h}\}_{h=0}^\infty$ to maximize $\int_0^\infty e^{-\rho h} CF_{i,t+h} dh$, where cash flows are $CF_{i,t} = (1 - \tau_{i,t}) \underbrace{\left(\left(Y_{i,t} - W_t L_{i,t} - \Phi\left(I_{i,t}, K_{i,t}\right) \right) \right)}_{\text{Earnings}} + \tau_{i,t} \underbrace{\left(B_{i,t} P_t^K I_{i,t} + \int_0^\infty (1 - B_{i,t-h}) c_{i,t|t-h} P_{t-h}^K I_{i,t-h} dh \right)}_{\text{Depreciation allowances}} + \underbrace{itc_{i,t} P_t^K I_{i,t}}_{\text{Tax credit}} - P_t^K I_{i,t}$. With constant τ, this problem gives the same choices as equations (5a) to (5c). In reality the availability of some credits, deductions, and net operating losses depend on firm choices, and hence even with fixed policy the marginal tax rate may not be constant or strictly exogenous to the firm. See Chodorow-Reich et al. (2025, Appendix C.5) for analysis of the determinants of firm-level heterogeneity in the change in $\tau_{i,t}$ after TCJA.

Equation (5a) states that the firm will hire labor up to the point where the marginal revenue product equals the wage. Equation (5b) equates the firm's discount rate to the internal rate of return on the shadow value of capital, equal to the after-tax marginal revenue product, less the increment to adjustment costs, and plus the change in the shadow value net of depreciation. Equation (5c) states that the shadow value of capital $\lambda_{i,t}$ must equal the cost of investment, given by the sum of the marginal adjustment cost and the market price net of tax incentives.

Equation (5b) solved forward yields:

$$\lambda_{i,t} = \int_t^{\infty} e^{-(\rho+\delta)h} \left(1-\tau_{i,t}\right) \left(A_{i,t+h} F_K \left(K_{i,t+h}, L_{i,t+h}\right) - \Phi_K \left(I_{i,t+h}, K_{i,t+h}\right)\right) dh. \tag{6}$$

Thus, the Lagrange multiplier $\lambda_{i,t}$ also equals the present discounted value of an additional unit of capital, where the composite discount rate $\rho+\delta$ accounts for both financial discounting and depreciation and the marginal value at horizon h consists of the after-tax marginal product of capital net of marginal adjustment costs.

2.2 Steady State

Variables without t subscripts denote steady state values. I assume no adjustment costs in steady state, $\Phi\left(I_i, K_i\right) = \Phi_I\left(I_i, K_i\right) = \Phi_K\left(I_i, K_i\right) = 0$. Equation (5c) then gives:

$$\lambda_i = \left(1-\Gamma_i\right) P^K. \tag{7}$$

Substitute this condition into equation (5b) and rearrange slightly:

$$A_i F_K\left(K_i, L_i\right) = (\rho+\delta) P^K \left(\frac{1-\Gamma_i}{1-\tau_i}\right) \equiv R_i. \tag{8}$$

Equation (8) equates the steady state marginal product of capital to the steady state user cost R_i.[6] The user cost has two terms. The component $(\rho+\delta) P^K$ comprises the opportunity cost and depreciation parts of the cost-of-capital, scaled by the market price. The second component is the "tax term" and will be a key object throughout the analysis. Denote it by $\Omega_{i,t}$:

$$\Omega_{i,t} \equiv \frac{1-\Gamma_{i,t}}{1-\tau_{i,t}}. \tag{9}$$

[6] Away from steady state, one can still define a user cost $R_{i,t}$. However, unlike the steady state R_i, this object contains components not directly measurable in the data such as the marginal increment to adjustment costs. This perspective takes the production function (1) as a technological constraint that holds at every date including outside of steady state. Chirinko (2008) instead views the production function itself as depending on whether the firm has its steady state quantity of capital.

The ratio $\Omega_{i,t}$ summarizes the taxation of capital, with a higher value indicating higher taxation.[7]

Using equation (8), the steady state choices of capital and labor also solve the static maximization problem:

$$\max_{K_i, L_i} A_i F(K_i, L_i) - W L_i - R_i K_i.$$

Key results that follow involve taking comparative statics of K_i with respect to R_i using equation (8) and holding fixed firm revenue Y_i (as in Hall and Jorgenson (1967); Eisner and Nadiri (1968)), the aggregate wage W (as in Coen (1969)), or, when moving to general equilibrium in Section 5, labor L_i.

2.3 Cobb-Douglas Across Steady States

The system (5a) to (5c) simplifies considerably with a Cobb-Douglas production function and across steady states. Thus, specialize equation (1) to:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{\beta}, \;\; \alpha > 0, \beta > 0, \alpha + \beta \leq 1. \tag{10}$$

Using $A_i F_K(K_i, L_i) = \alpha Y_i / K_i$ in equation (8) gives the Hall and Jorgenson (1967) relationship:

$$K_i = \frac{\alpha Y_i}{R_i}. \tag{11}$$

Intuitively, under Cobb-Douglas the factor shares of income are constant. With no adjustment costs, the price of capital is the user cost $(\rho + \delta) \Omega_i P^K$. Let lower case letters denote logs of upper case letters, $x \equiv \log X$ (where the lower case letter has an alternative meaning I write instead $\log X$, such as $\log I$ since i indexes individual firms). The Hall and Jorgenson (1967) user cost elasticity associated with equation (11) is:

$$\frac{\partial k_i}{\partial r_i}|_{Y_i = \bar{Y}} = -1. \tag{12}$$

[7] The first order conditions (5a) to (5c) do not incorporate deductibility of interest payments. Interest deductibility matters if the firm's debt choice depends on the capital stock. For example, with a fixed leverage ratio ψ such that the value of the flow interest deduction is $\tau_i \rho \psi K_i$, the user cost would also subtract $\tau_i / (1 - \tau_i) \rho \psi$. The combination of low tax rates, low interest rates, and modest leverage make interest deductibility less important to capital choice today than historically: Gravelle and Keightley (2025) obtain an average user cost for corporate equipment of 0.184 inclusive of interest deductibility and 0.189 without it (my calculations based on their model and parameters). Nonetheless, interest deductibility can still non-trivially affect the magnitude of the change in the user cost when τ changes, because τ directly affects the value of the interest deduction. It matters much less for changes in the user cost due to changes in z. See Goodman, Isen and Richmond (2025) for an empirical analysis that finds a small effect of interest deductibility on capital.

Notably, while this user cost elasticity holds Y_i fixed, in equation (11) Y_i is an endogenous object.

The Coen (1969) elasticity relates K_i to R_i holding fixed only variables exogenous to the firm. To that end, under Cobb-Douglas the first order condition for labor (5a) takes the form:

$$L_{i,t} = \frac{\beta Y_{i,t}}{W_t} = \frac{\beta A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{\beta}}{W_t} = \left(\frac{\beta A_{i,t} K_{i,t}^{\alpha}}{W_t}\right)^{\frac{1}{1-\beta}}. \tag{13}$$

Substitute equation (13) into the Cobb-Douglas production function (10) to find:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{\beta} = A_{i,t} K_{i,t}^{\alpha} \left(\frac{\beta A_{i,t} K_{i,t}^{\alpha}}{W_t}\right)^{\frac{\beta}{1-\beta}} = \mathcal{A}_{i,t} K_{i,t}^{\gamma}, \tag{14}$$

$$\text{where: } \mathcal{A}_{i,t} \equiv \left(\beta^{\beta} A_{i,t} W_t^{-\beta}\right)^{\frac{1}{1-\beta}}, \tag{15}$$

$$\gamma \equiv \frac{\alpha}{1-\beta} \subseteq [0,1]. \tag{16}$$

Importantly, $\mathcal{A}_i$ contains only variables exogenous to the firm. The Coen (1969) user cost elasticity follows from applying the chain rule using equation (14) to equation (11):

$$\frac{dk_i}{dr_i} = -1 + \frac{\partial y_i}{\partial k_i} \times \frac{dk_i}{dr_i} = -\frac{1}{1-\gamma} \leq -1. \tag{17}$$

This elasticity captures the full firm-level response of capital, taking the wage as exogenous. Since in the steady state $I_i = \delta K_i$, equation (17) also gives the long-run investment elasticity:

$$\frac{d \log I_i}{dr_i} = \frac{dk_i}{dr_i} = -\frac{1}{1-\gamma} \leq -1. \tag{18}$$

The feature that the Coen (1969) firm-level elasticity with Cobb Douglas is always weakly less than -1 clearly distinguishes it from the Hall and Jorgenson (1967) "neoclassical benchmark". The inequality is strict as long as $\alpha > 0$ and typically quite binding; for a plausible calibration of $\alpha = 0.25$ and $\beta = 0.65$, $\gamma = 5/7$ and $dk_i/dr_i = -3.5$.

It is instructive to arrive at equation (17) by instead starting from equation (11) and iterating in a cobweb adjustment process. Thus, suppose a firm starts with capital $K_{i,-1}$ and output $Y_{i,-1}$ satisfying equations (11) and (14), implying $K_{i,-1} = (\alpha \mathcal{A}_{i,-1}/R_i)^{1/(1-\gamma)}$. At date 0, the log user cost r_i changes by dr_i. To ease exposition, temporarily let $\mathcal{A}_{i,t} = 1 \; \forall t$. If the firm myopically treats equation (11) as a static updating rule and changes its capital stock only periodically at dates $t = 0, 1, 2, ...$, its capital and output will evolve over time as:

Horizon	$dk_{i,t}$	$dy_{i,t}$
0	$-dr_i$	$\gamma dk_{i,0}$
1	$-dr_i + dy_{i,0} = -dr_i\,(1+\gamma)$	$\gamma dk_{i,1}$
2	$-dr_i + dy_{i,1} = -dr_i\,(1+\gamma+\gamma^2)$	$\gamma dk_{i,2}$
$\vdots$		
$t>0$	$-dr_i \sum_{h=0}^{t} \gamma^h$	$\gamma dk_{i,t}$

The cobweb adjustment process sheds additional light on the Coen (1969) critique of Hall and Jorgenson (1967). Hall and Jorgenson analyzed tax reforms by applying only step 0 of the iterative process, yielding the elasticity -1. Coen correctly noted that iterating fully would yield the elasticity $-\lim_{t\to\infty}\sum_{h=0}^{t}\gamma^h = -1/(1-\gamma)$. Hall and Jorgenson (1969, p.369) replied that with constant returns to scale in the revenue function, $\alpha+\beta=1$, the parameter γ also equals 1 and the full firm-level elasticity becomes infinite, implying "serious shortcomings" of the Coen approach. While the statement that constant returns implies an infinite elasticity is true, an alternative perspective is that the coexistence of firms facing different user costs sheds light on the degree of returns to scale at the firm level. Furthermore, and as discussed in detail in Section 5, this issue disappears for aggregate analysis since aggregate labor is not infinitely elastic.

2.4 Constant Elasticity of Substitution (CES) Across Steady States

The Hall and Jorgenson (1967) and Coen (1969) elasticities generalize under more general CES production. Thus, now define the revenue function $F(K_{i,t}, L_{i,t})$ in equation (1) as:

$$F(K_{i,t}, L_{i,t}) = \left(\left(\frac{\alpha}{\alpha+\beta}\right) K_{i,t}^{1-1/\eta} + \left(\frac{\beta}{\alpha+\beta}\right) L_{i,t}^{1-1/\eta}\right)^{\frac{(\alpha+\beta)\eta}{\eta-1}}, \ \alpha>0, \beta>0, \eta>0, \alpha+\beta\le 1. \tag{19}$$

Here $\alpha+\beta$ still determines the returns to scale, while η is the elasticity of substitution between capital and labor. Using $\frac{(\alpha+\beta)\eta}{\eta-1} - 1 = \frac{(\alpha+\beta)\eta}{\eta-1}\left(1+\frac{1-\eta}{(\alpha+\beta)\eta}\right)$, the derivatives of this function are:

$$F_K(K_{i,t}, L_{i,t}) = \alpha F(K_{i,t}, L_{i,t})^{1+\frac{1-\eta}{(\alpha+\beta)\eta}} K_{i,t}^{-\frac{1}{\eta}}, \tag{20a}$$

$$F_L(K_{i,t}, L_{i,t}) = \beta F(K_{i,t}, L_{i,t})^{1+\frac{1-\eta}{(\alpha+\beta)\eta}} L_{i,t}^{-\frac{1}{\eta}}. \tag{20b}$$

Substitute equation (20a) into equation (8) relating the marginal product of capital to the user cost to obtain (see e.g. Eisner and Nadiri, 1968):

$$A_i \alpha \left(Y_i/A_i\right)^{1+\frac{1-\eta}{(\alpha+\beta)\eta}} K_i^{-\frac{1}{\eta}} = R_i. \tag{21}$$

The Eisner and Nadiri (1968) elasticity holding output fixed generalizes Hall and Jorgenson (1967) by applying to equation (21):

$$\frac{\partial k_i}{\partial r_i}|_{Y_i=\bar{Y}} = -\eta. \tag{22}$$

Next solve for the Coen (1969) specification holding only the wage fixed. Define $\alpha_i^* \equiv R_i K_i / Y_i$ and $\beta_i^* \equiv W_i L_i / Y_i$ as the steady state revenue shares of capital and labor, respectively. Take logs and implicitly differentiate equation (21) with respect to r_i and rearrange:

$$\frac{dk_i}{dr_i} = \left(\left(1 + \frac{1-\eta}{(\alpha+\beta)\,\eta}\right)\frac{\partial y_i}{\partial k_i} - \frac{1}{\eta}\right)^{-1}. \tag{23}$$

Substitute equation (20b) into equation (5a) with the wage held fixed, take logs and differentiate to obtain $\partial y_i / \partial k_i$ (see equation (A.2)), and substitute this expression into equation (23) to find the generalized Coen (1969) elasticity (see equation (A.3)):[8]

$$\frac{dk_i}{dr_i} = -\left(\frac{1-\beta_i^*\left(\eta + \frac{1-\eta}{\alpha+\beta}\right)}{1-\alpha-\beta}\right). \tag{24}$$

With $\eta = 1$, equation (24) simplifies to $-(1-\beta)/(1-\alpha-\beta) = -1/(1-\gamma)$ as in equation (17). Away from $\eta = 1$, the degree of returns to scale in the revenue function governs how far equation (24) differs from equation (17). As $\alpha + \beta \to 1$ (constant returns to scale), the Coen (1969) elasticity again diverges to negative infinity.

2.5 Adjustment Costs

Summers (1981) and Hayashi (1982) analyze quadratic adjustment costs in capital. The subsequent literature has emphasized more complicated adjustment costs including over investment rather than capital (Christiano, Eichenbaum and Evans, 2005) or fixed or irreversible costs of adjustment (Abel and Eberly, 1994, 1996; Caballero and Engel, 1999; Caballero, 1999; Cooper

[8] The derivation also uses the result that the sum of revenue shares equals the returns to scale of the revenue function, $\alpha_i^* + \beta_i^* = \alpha + \beta$ (see equation (A.1)).

and Haltiwanger, 2006; Winberry, 2021). Without disputing the importance of these features, the motivation for this article is that the empirical literature has often erred in relating moments to parameters even in the most "vanilla" setting.

The quadratic adjustment costs take the form:

$$\Phi\left(I_{i,t},K_{i,t}\right)=\frac{\phi}{2}\left(\frac{I_{i,t}}{K_{i,t}}-\delta\right)^{2}K_{i,t}, \tag{25}$$

$$\text{with: } \Phi_{I}\left(I_{i,t},K_{i,t}\right)=\phi\left(I_{i,t}/K_{i,t}-\delta\right),$$
$$\Phi_{K}\left(I_{i,t},K_{i,t}\right)=\Phi\left(I_{i,t},K_{i,t}\right)/K_{i,t}-\left(I_{i,t}/K_{i,t}\right)\Phi_{I}\left(I_{i,t},K_{i,t}\right).$$

Substituting the expression for $\Phi_I\left(I_{i,t},K_{i,t}\right)$ into the investment necessary condition (5c) yields the Summers (1981) equation:

$$\frac{I_{i,t}}{K_{i,t}}=\delta+\frac{1}{\phi}Q_{i,t}, \tag{26}$$

$$\text{where: } Q_{i,t}\equiv\frac{\lambda_{i,t}-P_{t}^{K}\left(1-\Gamma_{i,t}\right)}{1-\tau_{i,t}}=\frac{\lambda_{i,t}}{1-\tau_{i,t}}-P_{t}^{K}\Omega_{i,t}. \tag{27}$$

The recognition of the difference between the market value of an additional unit of capital, $\lambda_{i,t}$ (see equation (6)), and the tax-adjusted replacement cost, $P_t^K\left(1-\Gamma_{i,t}\right)$, as a determinant of investment dates to Brainard and Tobin (1968). The ratio of these values is referred to as Brainard-Tobin's q. The definition of Q in equation (27) is a transformation of this concept.

2.6 Coen (1969) in the Short-run

The linearized dynamic transition path links the steady state firm-level results to short-run empirical specifications. To that end, define the mapping from capital to revenue net of labor costs:

$$G_{i,t}\left(K_{i,t}\right)\equiv\max_{L_{i,t}}A_{i,t}F\left(K_{i,t},L_{i,t}\right)-W_{t}L_{i,t}, \tag{28}$$

where $G_{i,t}\left(K_{i,t}\right)$ has concentrated out labor using the first order condition (5a). Under Cobb-Douglas $G_{i,t}\left(K_{i,t}\right)=\left(1-\beta\right)\mathcal{A}_{i,t}K_{i,t}^{\gamma}$ (see equation (14)); more generally the i,t subscripts on the G function reflect the dependence on technology and the wage. Importantly, $G_{i,t}\left(K_{i,t}\right)$ inherits the concavity of equation (1), with in particular $G_{i,t}'\left(K_{i,t}\right)=A_{i,t}F_K\left(K_{i,t},L_{i,t}\right)$.[9]

[9]Using equation (5a) and defining $H_{i,t}\left(K_{i,t}\right)=L_{i,t}$ as the mapping from capital to labor, $G_{i,t}'\left(K_{i,t}\right)=A_{i,t}F_K\left(K_{i,t},L_{i,t}\right)+\left(A_{i,t}F_L\left(K_{i,t},L_{i,t}\right)-W_t\right)H_{i,t}'\left(K_{i,t}\right)=A_{i,t}F_K\left(K_{i,t},L_{i,t}\right)$, and $G_{i,t}''\left(K_{i,t}\right)=A_{i,t}\left(F_{KK}\left(K_{i,t},L_{i,t}\right)+F_{LL}\left(K_{i,t},L_{i,t}\right)H_{i,t}'\left(K_{i,t}\right)^2+2F_{KL}\left(K_{i,t},L_{i,t}\right)H_{i,t}'\left(K_{i,t}\right)\right)$.

Imposing quadratic adjustment costs, the system (5b) and (5c) takes the form:

$$\dot{\lambda}_{i,t} = (\rho + \delta)\,\lambda_{i,t} - (1 - \tau_{i,t})\left(G'_{i,t}\left(K_{i,t}\right) - \Phi_K\left(I_{i,t}, K_{i,t}\right)\right), \tag{29a}$$

$$\dot{K}_{i,t} = \frac{1}{\phi}\left(\frac{\lambda_{i,t} - (1 - \Gamma_{i,t})\,P^K_t}{1 - \tau_{i,t}}\right) K_{i,t}. \tag{29b}$$

Take a Taylor expansion in the neighborhood of the steady state (see Section A.2 for details):

$$\begin{pmatrix} \dot{\lambda}_{i,t} \\ \dot{K}_{i,t} \end{pmatrix} = \begin{pmatrix} \rho & -\left(1 - \tau_i\right) G''_i\left(K_i\right) \\ K_i/\left(\phi\left(1 - \tau_i\right)\right) & 0 \end{pmatrix} \begin{pmatrix} \lambda_{i,t} - \lambda_i \\ K_{i,t} - K_i \end{pmatrix}. \tag{30}$$

Consider a permanent tax change at date 0 that affects only firm i.[10] Using the approximation $k_{i,t} - k_i \approx (K_{i,t} - K_i)/K_i$, for an initial condition $k_{i,0}$, the solution is:

$$\lambda_{i,t} - \lambda_i = \phi\left(1 - \tau_i\right)\left(k_{i,0} - k_i\right)\chi e^{\chi_i t}, \tag{31a}$$

$$k_{i,t} - k_i = \left(k_{i,0} - k_i\right)e^{\chi_i t}, \tag{31b}$$

$$\text{where: } \chi_i \equiv \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{G''_i\left(K_i\right)K_i}{\phi}} < 0 \tag{32}$$

is the stable eigenvalue of the system.[11] Implicitly differentiate the capital first order condition $G'_{i,t}\left(K_{i,t}\right) = R_i$ with respect to R_i, giving $\frac{G''_i(K_i)K_i}{R} \times \frac{dk_i}{dr_i} = 1$, and substitute into equation (32):

$$\chi_i = \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}\right)^2 - \left(\frac{dk_i}{dr_i}\right)^{-1}\frac{R_i}{\phi}} < 0, \tag{33}$$

where dk_i/dr_i is the Coen (1969) elasticity in (17) or (24).

I can now state the key short-run results. The derivative of $k_{i,t}$ with respect to r_i in equation (31b) gives the elasticity of capital along the transition path:

$$\frac{dk_{i,t}}{dr_i} = \left(1 - e^{\chi_i t}\right)\frac{dk_i}{dr_i}. \tag{34}$$

The response of log net investment $\dot{k}_{i,t}$ is given by the time derivative of equation (31b):

$$\dot{k}_{i,t} = \frac{\dot{K}_{i,t}}{K_{i,t}} = \frac{I_{i,t}}{K_{i,t}} - \delta = \chi_i\left(k_{i,0} - k_i\right)e^{\chi_i t}. \tag{35}$$

[10] A reform affecting all firms would involve changes in aggregate variables such as the wage. These changes would appear as additional forcing terms in equations (31a) and (31b) and contribute to the regression intercept in the Coen (1969) specifications reviewed in Section 3, as discussed further in Section 5.

[11] Auerbach (1989) first derives the partial adjustment form of the transition path as in equation (31b). The formula (33) differs slightly from Auerbach's due to my assumption that adjustment costs are fully immediately depreciable from income. Auerbach (1989) and Auerbach and Hassett (1992) also allow for transitory tax policy changes, in which case k_i depends on both current and future tax policy.

The semi-elasticity of $I_{i,0}/K_{i,0}$ to the user cost after t periods is therefore:

$$\frac{dI_{i,t}/K_{i,t}}{dr_i} = -\chi_i e^{\chi_i t} \frac{dk_i}{dr_i}. \tag{36}$$

In particular, the short-run semi-elasticity is $-\chi_i dk_i/dr_i$. This neat relationship illustrates why my summary of Coen elasticities in Section 3 focuses on the short-run semi-elasticity of I/K rather than the short-run elasticity of $\log I$ or k. More generally, while the result that the eigenvalue scales the short-run response of the investment rate to the long-run elasticity of capital depends on the assumption of convex adjustment costs, one can always define χ as the ratio of the short-run semi-elasticity to the long-run elasticity, similar to Chodorow-Reich et al. (2025).[12]

The statement that a finite firm-level elasticity implies diminishing returns to scale in revenue does not carry over to the short-run elasticity. Take the Cobb-Douglas case for simplicity, in which $dk_i/dr_i = -1/(1-\gamma)$ (equation (17)). Apply L'Hopital's rule to $\chi_i/(1-\gamma)$ (see equation (36)):

$$\lim_{\gamma\to 1} \frac{dI_{i,0}/K_{i,0}}{dr_i} = \lim_{\gamma\to 1} \frac{\chi_i}{1-\gamma} = -\frac{R_i}{\rho\phi}. \tag{37}$$

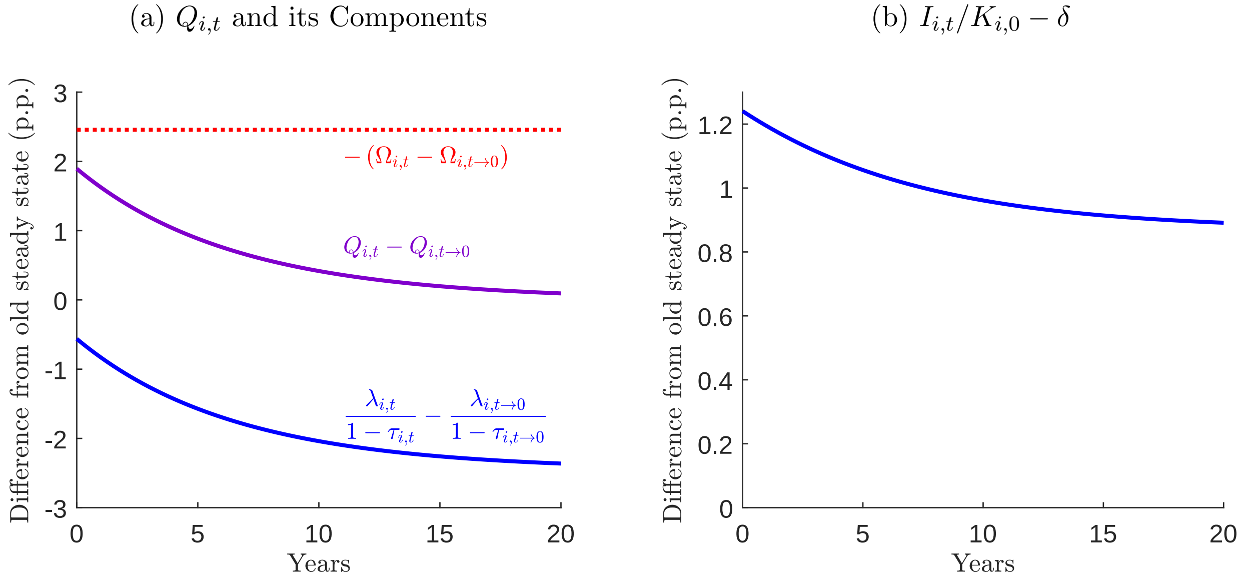
In the knife edge case of constant returns to scale, the short-run semi-elasticity contains no information about the long-run elasticity.

Figure 1 plots the transition path of the components of $Q_{i,t}$ and $I_{i,t}/K_{i,0}$ in response to a change in $\Omega_{i,t}$ due to more generous expensing. The top dotted line in Figure 1(a) shows the absolute value of the change in Ω, which in this example falls by 2.5p.p., roughly the difference between 0 and 100% expensing of equipment at the current marginal tax rate and depreciation schedule. The remainder of the calibration is illustrative but realistic: a Cobb-Douglas revenue function with $\alpha = 0.25, \beta = 0.65$, a depreciation rate $\delta = 0.1$, a discount rate $\rho = 0.06$, and adjustment cost parameter $\phi = 1.5$. The middle line shows that $Q_{i,t}$ jumps up at date 0, but by less than the reduction in $\Omega_{i,t}$. Arithmetically, $\lambda_{i,t}$ must then jump down, as shown in the bottom line. Figure 1(b) shows that investment jumps above the new long run steady state value and converges toward it from above.[13] The log change in $\Omega_{i,t}$ (and hence also r) is -2.4% and

[12]With more complicated adjustment costs, this ratio is not necessarily a closed form function of parameters. Section 4 will use the closed form expression in equation (33) to recover ϕ given other parameters. Otherwise, while I will refer to χ as the eigenvalue, for most purposes it should simply be understood as the ratio of the short run semi-elasticity to the long run elasticity.

[13]This qualitative pattern is not generic; with ϕ large enough, $\lambda_{i,t}$ can jump up and investment can converge to its long-run value from below.

Figure 1: Impulse Response of Q and I/K



Notes: In Panel (a), the dotted red line shows the absolute value of the change in the tax term Ω, the solid blue line shows the response of the market value of installed capital $\lambda/(1-\tau)$, and the solid purple line shows the response of Q. Panel (b) shows the percentage point increase in the investment-capital ratio I/K.

$\gamma = 5/7$, implying a long-run increase in investment of $2.4\%/(1-\gamma) = 8.4\%$ (see equation (17)), or a level change of 0.84p.p. of initial capital given $\delta = 0.1$. Furthermore, $\chi_i \approx -0.15$, giving $I_{i,0}/K_{i,0} - \delta = -\chi_i \times 8.4\% = 1.24\%$ of K_0 (see equation (36)), equal to the date 0 value of the impulse response shown in Figure 1(b).

3 A Reanalysis of Empirical Findings

This section reviews empirical estimates of the effect of taxes on capital and investment across firms or asset types. Section 3.1 considers the Summers (1981) specification that relates to the adjustment cost parameter ϕ. Section 3.2 reports variants of the Coen (1969) specification and relates empirical estimates to the long-run capital elasticity dk_i/dr_i and the short-run scalar given by the eigenvalue χ_i. Section 3.3 reviews hybrid specifications. Section 3.4 considers variants of the Eisner and Nadiri (1968) specification that relate to the elasticity of substitution η. Section 3.5 reports asset-level specifications. Section 4 further relates the Coen (1969) specifications to primitive parameters. As all of the regression specifications apply to individual units or asset types, they omit the effect of common changes to aggregate factor prices. Section 5 addresses this "missing intercept" problem and characterizes aggregate elasticities.

The set of papers builds on Zwick and Mahon (2017, table B.8).[14] I do not devote space to critiquing research designs. Most share the approach of using changes in tax law that have heterogeneous effects across industries or firms. Hence, they rely on the units most exposed to the tax changes not also being differentially exposed to other common shocks. My interpretation further assumes that tax changes specific to an individual firm or asset type do not cause firm or asset-specific changes in input prices, as such changes would contribute to the regression residuals.[15] I also do not critique the measurement of tax changes, but note here that papers differ in using statutory or effective tax rates and these differences can matter quantitatively.

For readers less interested in the details of individual studies, each sub-section concludes with a paragraph summarizing the evidence.

3.1 Estimates of the Summers (1981) Specification

A straightforward specification at short horizons is the Summers (1981) equation:

$$I_{i,t}/K_{i,t-1} = b_0^{I/K,Q} + b_1^{I/K,Q} Q_{i,t}, \tag{38}$$

where $Q_{i,t}$ is defined in equation (27). Lemma 1 formalizes the interpretation of $b_1^{I/K,Q}$ using equation (26):

Lemma 1 *In the environment of Section 2.5 with quadratic adjustment costs in capital, plim $b_1^{I/K,Q} = 1/\phi$, where ϕ scales the adjustment cost in equation (25).*

Table 1 reviews empirical estimates.

Summary. The implied values of ϕ of roughly 50 found in Cummins, Hassett and Hubbard (1994) and Desai and Goolsbee (2004) seem implausibly high; they suggest that a firm could pay more than $1 of adjustment costs per dollar of new capital.[16] The literature has interpreted the

[14] I restrict analysis to firm or asset-level studies relating investment to changes in corporate tax policy. Bond and Van Reenen (2007) review estimates of investment-related parameters using other sources of variation or structural methods. To keep the analysis manageable, with one exception I also restrict to results using U.S. data. Recent articles using non-U.S. data include Maffini, Xing and Devereux (2019); Guceri and Albinowski (2021); Harju, Koivisto and Matikka (2022); Link et al. (2024); Lerche (2025).

[15] See House and Shapiro (2008) for a joint analysis of investment and its price in response to bonus depreciation. Because their setup assumes a temporary tax change, I do not include it below, acknowledging here that whether firms perceived bonus depreciation in 2001 as transitory or permanent matters to analyses of that episode. In general, better measurement of firms' expectations of the persistence of tax policy would be an important step forward in refining the user cost changes during particular episodes and hence in comparing across studies.

[16] An additional failure of estimation of equation (38) concerns the absence of a theoretical rationale for a regression residual, suggesting the R^2 should be one (Romer, 2018).

Table 1: Estimates of the Summers (1981) Specification

Article	Table (col.)	Estimating equation	Data	Variation	b_1 (s.e.)	Implied ϕ
Cummins, Hassett and Hubbard (1994)	3(2)	$I_{i,t}/K_{i,t-1} = b_0 + b_1 Q_{i,t}$	Compustat annual 1953-1988	Firm-level with year and firm FE and control for cash flow (het. SEs)	$b_1 =$ 0.019 (0.001)	52.6
Cummins, Hassett and Hubbard (1996)	7	$I_{i,t}/K_{i,t-1} = b_0 + b_1 Q_{i,t}$	Global Vantage USA 1987	Firm-level first-differences IV using $\Omega_{i,t}$ and lagged $I_{i,t}/K_{i,t-1}$ and cash flow (het. SEs)	$b_1 =$ 0.650 (0.077)	1.5
Desai and Goolsbee (2004)	7(2)	$I_{i,t}/K_{i,t-1} = b_0 + b_1 Q_{i,t}$	Compustat annual 1962-2003	Four-digit SIC with year and firm FE (cluster by firm) and control for cash flow	$b_1 =$ 0.0231 (0.0011)	43.3

small coefficients shown in Table 1 (and earlier in Summers (1981) in the time series) as either a major failure of the neoclassical framework or as reflecting attenuation bias stemming from measurement error in the λ component of Q, including the difficulty of measuring the marginal rather than average value of a unit of capital. In support of the measurement error interpretation, Cummins, Hassett and Hubbard (1996) use tax variation as an excluded instrument and find a much larger regression coefficient and implied value of ϕ of 1.5.[17]

3.2 Estimates of the Coen (1969) Specification

Table 2 reviews articles that estimate variants of the equation:

$$I_{i,t}/K_{i,t-1} = b_0^{\text{Coen}I(0)/K,r} + b_1^{\text{Coen}I(0)/K,r} \log \Omega_{i,t} + e_{i,t}. \tag{39}$$

Equation (39) relates the investment-capital ratio, $I_{i,t}/K_{i,t-1}$, to the contemporaneous log of the tax term, $\log \Omega_{i,t}$. Alternatives, listed in equations (B.1a) to (B.1e), replace $I_{i,t}/K_{i,t-1}$ with $k_{i,t}$ or $\log I_{i,t}$ as the dependent variable, consider different horizons for the response of investment or capital, or replace the regressor $\log \Omega_{i,t}$ with the level of the tax term $\Omega_{i,t}$ or user cost $R_{i,t}$ or simply the present value of allowances $z_{i,t}$. The literature has estimated these specifications in first-differences, with firm fixed effects, or by first purging the left and right hand sides of date $t-1$ expectations.[18] In a common labor market, changes in the wage contribute to the intercept

[17]Technically, their excluded instrument is the variable $\tilde{Q}_{i,t}$ defined in equation (40). As discussed there, the identifying variation is simply the tax term $\Omega_{i,t}$.

[18]One can formally define the regression residual in equation (39) as a function of changes to idiosyncratic productivity or components of the user cost besides the tax term, such as capital goods prices. If b_1 is heterogeneous,

term b_0, such that b_1 gives the firm-level response holding the wage fixed. Lemma 2 provides a structural interpretation of $b_1^{\text{Coen}I(0)/K,r}$.

Lemma 2 *In the environment of Section 2.6 with quadratic adjustment costs in capital, equation (39) recovers plim* $b_1^{CoenI(0)/K,r} = -\chi_i dk_i/dr_i$, *where* χ_i *is the stable eigenvalue given in equation (33) and* dk_i/dr_i *is the long-run Coen elasticity given by equation (17) or (24).*

The close correspondence between the short-run semi-elasticity $b_1^{\text{Coen}I(0)/K,r}$ and the long-run elasticity dk_i/dr_i explains the appeal of summarizing the short-run evidence with this object. The alternative variants have similar structural interpretations after rescaling the regression coefficient, as shown in Lemmas B.1 to B.4. Thus, Table 2 reports estimates of variants of equation (39), my best attempt to standardize the regression coefficients by applying standard manipulations such as rescaling by the sample mean of the regressor to transform investment coefficients estimated h periods after a tax change into the semi-elasticity $b_1^{\text{Coen}I(h)/K,r}$ and capital coefficients into the elasticity $b_1^{\text{Coen}k(h),r}$, and the relationship to the long-run Coen (1969) elasticity dk_i/dr_i and the eigenvalue χ.

The setup in Cummins, Hassett and Hubbard (1994, table 5) illustrates some of the difficulties of interpretation and requires additional commentary. Cummins, Hassett and Hubbard (1994) is a landmark, comprehensive analysis of firm-level investment around major U.S. tax reforms and is among the most prominently-cited articles for policy analysis. They motivate their first main specification (equations (10)-(11) in their paper with results shown in their table 5) as a variant of the Summers (1981) specification in (38), but setting the regressor to $\tilde{Q}_{i,t} - \mathbb{E}_{i,t-1}\tilde{Q}_{i,t}$, where:

$$\tilde{Q}_{i,t} = \frac{\lambda_{i,t-2} - P^K_{i,t-2}\left(1 - \Gamma_{i,t}\right)}{1 - \tau_{i,t}}, \tag{40}$$

and $\mathbb{E}_{i,t-1}$ is an expectation operator implemented by projecting on lags of investment, cash flow, and a time trend. The purging of lagged information using $\mathbb{E}_{i,t-1}$ is innocuous (it isolates the shock). The definition of $\tilde{Q}_{i,t}$ as including $\lambda_{i,t-2}$, however, instead of $\lambda_{i,t}$ as in equation (27), changes the interpretation of their regression in a manner unrecognized at the time or since. Cummins, Hassett and Hubbard (1994, p.30) describe this substitution as intended to "focus on the tax surprise" given the possibility of measurement error in the λ component. Yet, this

the Lemmas also require that the heterogeneity be uncorrelated with the regressor.

Table 2: Estimates of the Coen (1969) Specification

Article	Table (col.)	Estimating equation	Data	Variation	b (s.e.)	Transformation
Cummins, Hassett and Hubbard (1994)	5	$I_{i,t}/K_{i,t-1} = b_0 + b_1\tilde{Q}_{i,t}$	Compustat annual	Firm-level projecting out expectations (het. SEs)		$b\times\Omega$ (Lemma B.3), lagged Ω from table 1 and figure 1
			1963		0.874 (0.23)	$\Omega = 1.35 : -b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 1.18$
			1973		0.47 (0.095)	$\Omega = 1.30 : -b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.61$
			1982		0.599 (0.10)	$\Omega = 1.11 : -b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.66$
			1987		0.613 (0.09)	$\Omega = 1.31 : -b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.80$
Cummins, Hassett and Hubbard (1994)	9	$I_{i,t}/K_{i,t-1} = b_0 + b_1 R_{i,t}$	Compustat annual equipment	Firm-level projecting out expectations (het. SEs)		$-b\times R$ (Lemma B.3), $R = 0.25$ (p. 43):
			1963		-0.605 (0.14)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.15$
			1973		-0.546 (0.14)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.14$
			1982		-0.757 (0.19)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.19$
			1987		-0.747 (0.23)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.19$
Cummins, Hassett and Hubbard (1996)	6	$I_{i,t}/K_{i,t-1} = b_0 + b_1\tilde{Q}_{i,t}$	Global Vantage USA 1987	Firm-level projecting out expectations (het. SEs)	0.603 (0.086)	$b\times\Omega$ (Lemma B.3), $\Omega = 1.21$ (Cummins, Hassett and Hubbard, 1994): $-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.73$
Zwick and Mahon (2017)		$\log I_{i,t} = b_0 + b_1 z_{i,t}$	SOI corporate sample	Four-digit NAICS with year and firm FE (cluster firm)		$b\times\delta\left(1-\tau z\right)/\tau$ (Lemma B.4), $z = 0.907, \delta = 0.1$ (table 2), $\tau = 0.35$ (statutory):
	3(1)		Pooled		3.69 (0.53)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.72$
	6(1)		Small firms		6.29 (1.21)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 1.23$
	6(2)		Large firms		3.22 (0.76)	$-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.63$
Curtis et al. (2021)	2(3)	$k_i = b_0 + b_1\left[\mathbb{I}z \leq P_{33}(z)\right]$	Census ASM, CM 2001-2011	Long-difference	0.1047 (0.0192)	$-\left(b/\Delta_{2001,2011}\Delta_{z\leq P_{33}(z),z>P_{33}(z)}\right)\times\left(1-\tau z\right)/\tau$ (Lemma B.4), $\Delta_{2001,2011}\Delta_{z\leq P_{33}(z),z>P_{33}(z)} = 0.034$, $z = 0.84$ (Appendix B), $\tau = 0.35$ (statutory): $b_1^{\text{Coen}\,k(10),r} = \left(1-e^{10\chi}\right)dk_i/dr_i = -6.21$
Chodorow-Reich et al. (2025)	3(3)	$\log I_{i,t} = b_0 + b_1\log\Omega_{i,t}$	SOI C-corporate sample, 2015-19	Firm-level in first-differences (het. SEs)	-4.27 (0.51)	$-b\times\delta$ (Lemma B.2), $\delta = 0.1$ (Zwick and Mahon, 2017): $-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.43$
Kennedy et al. (2026)	6(1)	$I_{i,t}/K_{i,t-1} = b_0 + b_1\log\Omega_{i,t}$	SOI corporate sample, 2013-2019	C-corp. indicator $\times$ post-2017 (cluster firm)	-0.424 (0.066)	None: $-b_1^{\text{Coen}\,I(0)/K,r} = \chi dk_i/dr_i = 0.42$

substitution also removes the endogenous response of $\lambda_{i,t}$ to tax changes. Furthermore, non-parametrically purging $\tilde{Q}_{i,t}$ of lagged information would remove the $\lambda_{i,t-2}$ term from the regressor altogether. Thus, their specification in fact is closest to equation (39) except with $-\Omega_{i,t}$ as the regressor. Their interpretation of the regression coefficient in terms of the magnitude of adjustment costs is therefore incorrect. Rather, focusing on the years with major tax reforms, their estimates map into values of $-b_1^{\text{Coen}I(0)/K,r} = \chi dk_i/dr_i$ ranging from 0.66-1.18. Cummins, Hassett and Hubbard (1996, table 6) implement the same specification and find a value of 0.73.

Cummins, Hassett and Hubbard (1994, table 9) replace $\log \Omega_{i,t}$ with the level of the user cost $R_{i,t}$, as in equation (B.1d). Using Lemma B.3, these estimates map into values of $-b_1^{\text{Coen}I(0)/K,r}$ ranging from 0.14 to 0.19 for major tax reform years, notably smaller than the values implied by their table 5. This difference is surprising, since the specifications should differ only in that the regressor in their table 9 is $-R/\Omega = -(\rho + \delta)$ multiplied by the regressor in their table 5. If $\rho + \delta$ were fixed across firms and ignoring the differences between their $\tilde{Q}$ and Ω, then the regression coefficients should also differ by a factor of $1/(\rho+\delta)$, or about five. Instead, Cummins, Hassett and Hubbard (1994) obtain coefficients of similar magnitude. One possible explanation is attenuation bias. Cummins, Hassett and Hubbard (1994) use firm-specific ρ based on bond yields and δ based on book capital; if these values have measurement error, the table 9 estimates will be attenuated.

Zwick and Mahon (2017) study the introduction of bonus depreciation in the early 2000s. Their specification regresses $\log I_{i,t}$ on $z_{i,t}$, in first differences. Applying Lemma B.4, the implied $-b_1^{\text{Coen}I(0)/K,r}$ pooling over all firms is 0.72. They also find important hetergeneity, with $-b_1^{\text{Coen}I(0)/K,r} = 1.23$ for smaller firms and 0.63 for larger firms.

Curtis et al. (2021) also study the introduction of bonus depreciation but look at outcomes after 10 years. Using Lemma B.4, their estimate for log capital translates into a value of $(1 - e^{10\chi})\, dk_i/dr_i = -6.21$. This number is not however directly comparable to the estimation in Zwick and Mahon (2017), because the Curtis et al. (2021) sample contains only manufacturing firms. For values of $\chi \subseteq (-0.18, -0.1)$ (see Section 4 for discussion of these values), the Curtis et al. (2021) coefficient implies $-b_1^{\text{Coen}I(0)/K,r} \subseteq (0.98, 1.34)$.

Chodorow-Reich et al. (2025) and Kennedy et al. (2026) study variation induced by the TCJA. Using firm-level variation for C corporations without international presence (to avoid complications from the TCJA's changes to international taxation), Chodorow-Reich et al. (2025) implement equation (39) but with $\log I_{i,t}$ as the dependent variable. Applying Lemma B.2, their regression coefficient translates into $-b_1^{\text{Coen}I(0)/K,r} = 0.43$. Kennedy et al. (2026) compare C and S corporations, as the corporate rate cut in TCJA applied only to the former. Their specification regressing $I_{i,t}/K_{i,t-1}$ on $\log \Omega_{i,t}$ (and firm fixed effects) gives $-b_1^{\text{Coen}I(0)/K,r} = 0.42$.

Summary. Comparing across the implied values of $plim b_1^{\text{Coen}I(0)/K,r} = -\chi dk_i/dr_i$ in Table 2 may appear to give the contours of the Hassett and Hubbard (2002) range of -0.5 to -1 for the short-run response. However, $plim b_1^{\text{Coen}I(0)/K,r}$ is the semi-elasticity of investment, not the elasticity. If one adopts the Council of Economic Advisers (2019) interpretation of the Hassett and Hubbard (2002) range as applying to the elasticity, then a proper comparison requires rescaling the values in Table 2 by I/K, yielding elasticities that are five to ten times larger in absolute value than the Hassett and Hubbard (2002) range. The difference does not reflect uniformly larger responses of investment to the more recent episodes of bonus depreciation and the TCJA than the earlier studies. Rather, my re-analysis contains a broader set of results and several adjustments to the estimates reviewed in Hassett and Hubbard (2002). The table also contains notable extremes. Only Cummins, Hassett and Hubbard (1994, table 9) implies values for $-b_1^{\text{Coen}I(0)/K,r}$ below 0.2. Only Cummins, Hassett and Hubbard (1994, table 5, 1963 and 1987), Curtis et al. (2021), and Zwick and Mahon (2017, small firms) find magnitudes greater than -0.75. A central tendency range of -0.25 to -0.75 therefore summarizes empirical evidence of the Coen (1969) cross-firm short-run semi-elasticity of investment to the user cost.

3.3 Estimates of Hybrid Specifications

Motivated by the possibility of measurement error in λ, Desai and Goolsbee (2004) and Edgerton (2010) split Q to allow different coefficients multiplying the $\lambda_{i,t}/(1-\tau_{i,t})$ and $\Omega_{i,t}$ components:[19]

$$I_{i,t}/K_{i,t-1} = b_0^{\text{hybrid}} + b_1^{\text{hybrid}} \frac{\lambda_{i,t}}{1-\tau_{i,t}} + b_2^{\text{hybrid}} \Omega_{i,t} + e_{i,t}. \tag{41}$$

This split is formally an over-identification test of the Summers (1981) specification, which requires $b_1 = -b_2$. If this condition fails, no clear structural interpretation of either coefficient exists:

[19] This exercise was first proposed by Caballero (1994) in his published comment on Cummins, Hassett and Hubbard (1994).

Table 3: Estimates of the Hybrid Specification

Article	Table (col.)	Estimating equation	Data	Variation	b_1 (s.e.)
Desai and Goolsbee (2004)	8(1)	$I_{i,t}/K_{i,t-1} = b_0 + b_1\lambda_{i,t} + b_2\Omega_{i,t}^{\text{equipment}} + b_3\frac{CF_{i,t-1}}{K_{i,t-1}}$	U.S. Compustat annual 1962-2003	Four-digit SIC with year and firm FE (cluster by firm); includes control for structures tax term	$b_1 =$ 0.0231 (0.0011), $b_2 = -0.8895$ (0.3173)
Edgerton (2010)	3(2)	$I_{i,t}/K_{i,t-1} = b_0 + b_1\lambda_{i,t} + b_2\Omega_{i,t}^{\text{equipment}}$	North America Compustat annual 1967-2005	Four-digit SIC with year and firm FE (cluster by firm); includes control for non-taxable status, its interaction with $\Omega_{i,t}$, and structures tax term	$b_1 =$ 0.038 (0.001), $b_2 = -0.846$ (0.323)

Lemma 3 *In the environment of Section 2.5 with quadratic adjustment costs in capital, plim* $b_1^{hybrid} = -$*plim* $b_2^{hybrid} = 1/\phi$*, where* ϕ *scales the adjustment cost in equation (25). If* $\lambda_{i,t}$ *is only noise, then plim* $b_1^{hybrid} = 0$ *and plim* $b_2^{hybrid} = (1/\Omega_i) \times$ *plim* $b_1^{CoenI(0)/k,r}$ *as in Lemma B.3. If* $\lambda_{i,t}$ *contains both signal and classical measurement error and* $Cov(\lambda_{i,t}, \Omega_{i,t}) < 0$*, then* $|\text{plim } b_2^{hybrid}| > |\text{plim } b_1^{I/K,Q}|$.

Summary. Desai and Goolsbee (2004) and Edgerton (2010) find much larger (in absolute value) coefficients multiplying the $\Omega_{i,t}$ component and near zero coefficients multiplying $\lambda_{i,t}/(1-\tau_{i,t})$. Desai and Goolsbee (2004) incorrectly interpret this result as the coefficient multiplying $\Omega_{i,t}$ providing a more accurate measure of adjustment costs.[20] While the interpretation of attenuation bias affecting b_1^{hybrid} may be correct, it does not follow that b_2^{hybrid} recovers the adjustment cost parameter. In the extreme case where measurement of $\lambda_{i,t}/(1-\tau_{i,t})$ results in only noise, the regression instead maps to the Coen (1969) elasticity. Applying this interpretation and $\Omega = 1.25$ (see Cummins, Hassett and Hubbard (1994, table 5) in Table 2) would imply $-b_1^{\text{Coen}I(0)/K,r} \approx 1$.

3.4 Estimates of the Eisner and Nadiri (1968) Specification

Table 4 reviews articles that estimate variants of the equation:

$$k_{i,t} = b_0^{\text{E-N}k,r} + b_1^{\text{E-N}k,r} r_{i,t} + b_2^{\text{E-N}k,r} y_{i,t} + e_{i,t}. \tag{42}$$

[20] Desai and Goolsbee (2004, p. 315): "the model predicts that these coefficients should be of the same magnitude as those on tax-adjusted q [λ in my notation], but of opposite sign. Here, instead, the coefficients on the equipment tax term are considerably larger. With measurement error in q, the coefficient on the tax term may provide a more realistic estimate of the true coefficient. Such a coefficient is considerably closer to 1 and, consequently, corresponds to more realistic estimates of adjustment costs."

Two key features distinguish estimation using equation (42) from the Coen (1969) specifications. First, the inclusion of firm log revenue $y_{i,t}$ as a second regressor. Second, an explicit focus on long-run outcomes by estimating in long differences.[21] As a result, it identifies the elasticity of substitution η. Using equation (21):

Lemma 4 *In the environment of Section 2.4 with CES production, equation (42) estimated across steady states recovers plim* $b_1^{E\text{-}Nk,r} = -\eta$ *and plim* $b_2^{E\text{-}Nk,r} = \eta + (1-\eta)/(\alpha+\beta)$.

Thus, taking the Cobb-Douglas case for simplicity, controlling for Y or not changes the "steady state user cost elasticity" from -1 to $-1/(1-\gamma)$.[22]

Chirinko, Fazzari and Meyer (2011) implement equation (42) by differencing across five-year averages of firm-level capital and tax-adjusted user cost. Treating the different five-year periods as distinct steady states, they find an elasticity of substitution of $\eta = 0.367$ and near constant (in fact, increasing) returns to scale in the revenue function of $\alpha + \beta = 1.135$.

Other Eisner and Nadiri (1968) specifications differ from equation (42) by replacing the dependent variable with either the log capital-output ratio $k_{i,t} - y_{i,t}$ or $I_{i,t}/K_{i,t-1}$, and by taking alternative approaches to allowing for non-instant adjustment of capital to its desired level (see equations (B.2a) to (B.2c)). Caballero, Engel and Haltiwanger (1995) estimate the cointegration model (B.2a) using annual data from the U.S. manufacturing sector. They motivate a steady state relationship between the capital-output ratio and user cost by starting from a Cobb-Douglas production function and an *ad hoc* coefficient scaling the user cost. Thus, instead of $k_i - y_i = \log\alpha - r_i$ (see equation (11)), they assume $k_i - y_i = \log\alpha - cr_i$ for a scalar c. One can instead rigorously derive this relationship by starting from CES with constant returns to scale, in which case the scalar c equals the elasticity of substitution η (see Lemma B.5). They find implied values of η

[21] At short horizons, equation (21) still holds for a suitably defined user cost $R_{i,t}$ that includes the marginal adjustment costs. In this case, the appropriate control is the output $y_{i,t}$ in the period in which the capital is used. Thus, with time-to-build that delays the effect of investment on output, the appropriate Eisner and Nadiri (1968) specification control is future output. Of course, this specification also requires the time-varying $r_{i,t}$ rather than the steady state r_i as the main regressor and capital rather than investment as the dependent variable, making clear the differences between Coen (1969) and Eisner and Nadiri (1968) specifications even at short horizons. In addition, there is no theoretical rationale for the regression residual in equation (42) that does not affect the interpretation of the coefficients. For example, variation in productivity $A_{i,t}$ across firms would also affect $y_{i,t}$, making the residual correlated with one of the regressors.

[22] In fact, in the knife-edge case where $\eta = 1$ exactly and with variation across firms only in the log user cost r_i, equation (42) suffers from a multicollinearity problem, since $y_i = \gamma/(1-\gamma)r_i$ plus a constant (see equations (11) and (14)).

Table 4: Estimates of the Eisner and Nadiri (1968) Specification

Article	Table (col.)	Estimating equation	Data	Variation	b (s.e.)	Transformation
Caballero, Engel and Haltiwanger (1995)	Fig. 4	$\Delta(k_{i,t} - y_{i,t}) = b_0 + b_1 r_{i,t} + b_2(k_{i,t-1} - y_{i,t-1}) + b_3 \Delta r_{i,t}$	LRD manufacturing 1972-1988	2-digit SIC industry, constrain b_1, b_2, b_3 by 2 digit sector	Not reported	b_1/b_2 (Lemma B.5): $\eta \subseteq [-0.01, -2]$ (mean=-1)
Chirinko, Fazzari and Meyer (1999)	4(orth. dev.)	$I_{i,t}/K_{i,t-1} = b_0 + \sum_{h=0}^{H} b_{1,h} \Delta r_{i,t-h} + \sum_{h=0}^{H} b_{2,h} \Delta y_{i,t-h}$	Compustat annual 1981-1991	2-digit SIC industry, Arellano and Bover (1995) orthogonal deviation estimator with $H = 2$ for b_1, $H = 4$ for b_2, control for cash flow	$\sum_h b_{1,h} = -0.26(0.046)$, $\sum_h b_{2,h} = 0.209(0.072)$	$\eta = -\sum_h b_{1,h}, \alpha + \beta = (1 + \sum_h b_{1,h}) / \sum_h (b_{1,h} + b_{2,h})$ (Lemma B.6): $\eta = 0.26, \alpha + \beta < 0$
Chirinko, Fazzari and Meyer (2011)	1(1)	$k_{i,t} = b_0 + b_1 r_{i,t} + b_2 y_{i,t}$	Compustat + DRI 1972-1991	Differences of averages over 1972-77, 1978-84, 1985-91, tax-adjusted user cost from DRI	$b_1 = -0.367(0.068)$, $b_2 = 0.925(0.038)$	$\eta = -b_1, \alpha + \beta = (1 + b_1)/(b_1 + b_2)$ (Lemma 4): $\eta = 0.367, \alpha + \beta = 1.135$
Dwenger (2014)	5(1)	$k_{i,t} = b_0 + b_1 r_{i,t-1} + b_2 y_{i,t-1} + b_3 k_{i,t-1}$	Hoppenstedt (German accounting data) 1987-2007	2-digit industry, control current and lagged changes in r and y	$b_1 = -0.606(0.107)$, $b_2 = 0.406(0.075)$, $b_3 = 0.352(0.067)$	$\eta = -b_1/(1-b_3), b_2/(1-b_3) = \eta + \frac{1-\eta}{\alpha+\beta}$ (Lemma B.7): $\eta = 0.935, \alpha + \beta < 0$

between 0 and 2 across 2 digit manufacturing industries, with a mean η of 1.

Chirinko, Fazzari and Meyer (2014) and Dwenger (2014) make alternative *ad hoc* assumptions about how investment or capital depends on lagged values of "desired capital", denoted by $K^*_{i,t}$ and defined as the solution to equation (21) taking $Y_{i,t}$ as given. Chirinko, Fazzari and Meyer (1999) regress $I_{i,t}/K_{i,t-1}$ on current and lagged changes in $r_{i,t}$ and $y_{i,t}$ (see equation (B.2b)). Under the assumption that the investment rate equals a weighted average of past changes in desired capital, this specification also identifies the elasticity of substitution (see Lemma B.6). Their preferred specification uses lagged values as instruments (Arellano and Bover, 1995). They find an elasticity of substitution $\eta = 0.26$ and negative implied returns to scale.[23]

Dwenger (2014) is the lone exception to this section's focus on results using U.S. data. The reasons are that this study has featured prominently in the U.S. policy debate (see e.g. Council of Economic Advisers, 2019; Congressional Research Service, 2024; Council of Economic Advisers, 2025) and that it offers a methodological critique of and alternative to the distributed lag assumption in Chirinko, Fazzari and Meyer (1999). Dwenger instead estimates an error correction model of log capital regressed on lags of the user cost, output, and capital, and shows that this specification identifies η if current capital is a function of lagged capital and current and lagged desired capital (see Lemma B.7). Using German data, she finds $\eta = 0.935$, but $\alpha + \beta < 0$.[24]

Curtis et al. (2021) also merit mention in a discussion of the elasticity of substitution. As described above, they estimate a 10 year firm-level response of log capital to bonus depreciation. They also estimate the 10 year response of log employment and find a response as high or higher than the capital response. Across steady states, equations (5a), (8), (20a) and (20b) give $dk_i - d\ell_i = -\eta\,(dr_i - dw)$. If dw doesn't vary across firms due to a common labor market and the 10 year response reflects the new steady state, then this result implies an elasticity of substitution $\eta \le 0$.

[23]Illustrating some of the difficulties in comparing across studies, Chirinko, Fazzari and Meyer (1999, pp. 69-71) directly compares their results to Cummins, Hassett and Hubbard (1994) without taking account of either the difference between the distributed lag specification in Chirinko, Fazzari and Meyer (1999) and the one-year specification in Cummins, Hassett and Hubbard (1994) or the importance of controlling for firm revenue. The same comparison is made in Chirinko (2008).

[24]Again illustrating the problematic cross-study comparisons prevalent in the literature, Dwenger (2014, pp. 162-163) includes a review of "the user cost elasticity" that discusses quantitative results in Eisner and Nadiri (1968), Auerbach and Hassett (1992), Cummins, Hassett and Hubbard (1994, 1996), Caballero, Engel and Haltiwanger (1995), Chirinko, Fazzari and Meyer (1999), Chirinko, Fazzari and Meyer (2011), and Hassett and Hubbard (2002) without accounting for the key specification differences across these studies in whether they control for output or not and the adjustment horizon. Council of Economic Advisers (2019, 2025) and Hartley, Hassett and Rauh (2025) likewise compare without adjustment the results in Cummins, Hassett and Hubbard (1994) and Dwenger (2014).

Summary. Estimates of the Eisner and Nadiri (1968) specification holding revenue fixed yield values for the elasticity of substitution between capital and labor stretching from 0.3 to 1. Several of these estimates also imply implausible returns to scale in the revenue function.

3.5 Estimates of Asset-Level Specifications

All of the studies reviewed so far use variation at the level of the firm or industry. An alternative approach uses variation and outcomes at the asset class level:

$$I_{j,t}/K_{j,t-1} = b_0^{\text{Asset } I/K,R} + b_1^{\text{Asset } I/K,R} R_{j,t} + e_{j,t}, \tag{43}$$

where the index j rather than i indicates asset rather than firm-level analysis.

The interpretation of $b_1^{\text{Asset } I/K,R}$ hinges on the degree to which asset types correspond to specific industries. Concretely, suppose there are capital types $K_1, K_2, \ldots$ and industry revenue functions $F^1(K_1, K_2, \ldots, L), F^2(K_1, K_2, \ldots, L), \ldots$, where $F^i()$ denotes the revenue function for industry i. If each industry uses only one type of capital, i.e. $F^i(K_1, K_2, \ldots, L) = F^i(K_i, L)$, then the variation in equation (43) mimics a Coen (1969) industry-level analysis and Lemma B.3 applies. If instead all industries share the same technology and use all types of capital, i.e. $F^i(K_1, K_2, \ldots, L) = F^{i'}(K_1, K_2, \ldots, L)\ \forall i, i'$, then equation (43) more closely resembles an Eisner and Nadiri (1968) specification but with substitution in the production process occurring across capital types rather than between capital and labor. (Here the regression intercept plays the role of controlling for total output.) However, because Lemma 4 relates long-run variation in log capital to the user cost, equation (43) does not afford easy interpretation in this case.

U.S. Bureau of Economic Analysis (2025*a*) provides asset-by-industry data for 74 industries and 94 asset types, which can help to distinguish these cases. For the median asset type, roughly half of economy-wide capital of that type concentrates in one industry. For the median industry, close to 40% of its capital is of one type. These concentration levels may support a Coen (1969) interpretation but also suggest prudence in comparing to firm-level analyses.

Table 5 reports results from two studies using asset-level variation. Auerbach and Hassett (1991) analyze investment patterns across asset classes after the 1986 reform and find assets with larger declines in the user cost had larger increases in investment.[25] Applying a Coen (1969) inter-

[25] Bosworth (1985) is the first article of which I am aware to examine the cross-sectional pattern of investment

Table 5: Estimates of Asset-Level Specifications

Article	Table (col.)	Estimating equation	Data	Variation	b (s.e.)	Transformation
Auerbach and Hassett (1991)	7(3)	$I_{j,t}/K_{j,t-1} = b_0 + b_1 R_{j,t}$	BEA equipment by asset type 1987	Asset level weighted by investment, projecting out expectations	-0.82 (0.26)	$-b \times R$ (Lemma B.3), $R = 0.226$ (Section B): $-b_1^{\text{Asset } I(0)/K,r} = \chi dk_i/dr_i = 0.15$
Hartley, Hassett and Rauh (2025)		$I_{j,t+h}/K_{j,t+h-1} = b_0 + b_1 R_{j,t}$	BEA Fixed Asset Tables	Asset-level in differences from 2016 (het. SEs)		$-b \times R$ (Lemma B.3), $R = 0.2058$ (Appendix table 2):
	1(2)		2018		-1.689 (0.890)	$-b_1^{\text{Asset } I(0)/K(0),r} = \chi dk_i/dr_i = 0.35$
	1(3)		2019		-1.057 (0.847)	$-b_1^{\text{Asset } I(1)/K(1),r} = \chi e^{\chi} dk_i/dr_i = 0.22$
	1(4)		2020		-1.115 (1.113)	$-b_1^{\text{Asset } I(2)/K(2),r} = \chi e^{2\chi} dk_i/dr_i = 0.23$
	1(5)		2021		-2.873 (1.166)	$-b_1^{\text{Asset } I(3)/K(3),r} = \chi e^{3\chi} dk_i/dr_i = 0.59$
	1(6)		2022		-3.053 (1.180)	$-b_1^{\text{Asset } I(4)/K(4),r} = \chi e^{4\chi} dk_i/dr_i = 0.63$
	1(7)		2023		-1.282 (1.154)	$-b_1^{\text{Asset } I(5)/K(5),r} = \chi e^{5\chi} dk_i/dr_i = 0.26$

pretation, their coefficient implies $-b_1^{\text{Coen}I(0)/K,r} = 0.15$, below the range established in Section 3.2.

Hartley, Hassett and Rauh (2025) use variation induced by the TCJA and report separate regressions for changes in the investment-capital ratio from 2016 to each year in 2018-2023. Applying a Coen (1969) interpretation, their 2018 coefficient translates into $-b_1^{\text{Coen}I(0)/K,r} = 0.35$. Dividing each horizon's rescaled coefficient by $e^{\chi h}$ and averaging across their coefficients for 2018-2023, for $\chi \subseteq (-0.18, -0.1)$ the translated values imply $-b_1^{\text{Coen}I(0)/K,r} \subseteq (0.51, 0.65)$, in the upper-middle part of the range shown in Table 2.[26]

Summary. Interpretation of asset-level specifications hinges on the degree to which asset types correspond to specific industries. The estimated semi-elasticities span much of the range of values of the Coen (1969) estimates reported in Table 2.

4 Implied Parameters

This section asks what light the empirical evidence summarized in Section 3 can shed on primitive parameters of the revenue function. For this purpose, consider a "representative" firm and drop the i index on parameters. I externally calibrate the labor share of revenue $\beta^* = 0.65$ and for simplicity set $\beta = \beta^*$. Where necessary, I also set the discount rate $\rho = 0.06$ and the user cost $R = 0.2$. While these values may have changed over time, I view the sampling uncertainty in the empirical studies as likely to swamp variation due to changes in these parameters and it facilitates comparisons to keep them fixed.[27] This leaves the following parameters: the adjustment cost scalar ϕ, the revenue elasticity of capital $\alpha = \alpha^*$, and the capital-labor substitution elasticity η.

Table 1 reviewed studies that estimate the adjustment cost parameter ϕ. Using tax variation

following a tax reform. It reported correlations of investment surprises and tax changes and concluded that non-tax forces dominated investment in the years after the 1981 and 1982 tax changes.

[26] Hartley, Hassett and Rauh (2025, abstract) conclude that their results are "larger than prior estimates of the responsiveness of investment with respect to the user cost of capital." Their conclusion differs from mine because they compare their asset-level regression coefficients to estimates of the elasticity of substitution between capital and labor based on the Eisner and Nadiri (1968) specification in Dwenger (2014) and Bitros and Nadiri (2017), as well as to only Cummins, Hassett and Hubbard (1994, table 9) in Table 2 (Hartley, Hassett and Rauh, 2025, p.13). They also compare their implied elasticity $d \log I_j / dr_j$ of -1.8 to -3.3 (obtained from multiplying their regression coefficients by the sample average of $R/(I/K)$) to a value of -0.7 reported in Congressional Budget Office (2018*b*). As discussed in Section 6, CBO applies that value to compute the elasticity holding output fixed and subsequently allows capital to adjust as output increases.

[27] For example, the corporate labor share in the U.S. has trended down from roughly 0.65 in the 1980s to 0.6 in the 2010s (Karabarbounis and Neiman, 2014), the cost of debt has declined (Gormsen and Huber, 2023), and the user cost has declined due to a shrinking tax term.

as an instrument, Cummins, Hassett and Hubbard (1996, table 7) found a value $\phi = 1.5$. The other studies found much larger values, but may suffer from attenuation bias.

Table 4 reviewed studies that estimate the capital-labor substitution elasticity η. Values ranged from 0.3 to above 1, with 0.9 being from the most recent study. By comparison, Chirinko (2008) reviews both tax and non-tax based approaches and argues for a value between 0.4 and 0.6 while Knoblach, Roessler and Zwerschke (2020) conduct a meta-analysis and argue for a value between 0.5 and 0.9. Congressional Budget Office (2018*b*) uses a value of 0.7, while Council of Economic Advisers (2019, 2025) use a value of 1.

Table 2 reviewed studies estimating variants of the Coen (1969) elasticity. A long-run Coen (1969) elasticity translates immediately into α, β, and η. A short-run specification also involves the eigenvalue χ given in equation (33) and hence the adjustment cost parameter ϕ. Using equations (33) and (36) and Lemma 2:

$$plimb_1^{\text{Coen}I(0)/K,r} = -\chi dk_i/dr_i = -\left(\frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}\right)^2 - \left(\frac{dk_i}{dr_i}\right)^{-1}\frac{R}{\phi}}\right) \times \left(\frac{dk_i}{dr_i}\right), \tag{44}$$

where dk_i/dr_i is given by equation (24).

Two approaches exist to infer primitive parameters from the short-run coefficients. One approach, pursued in Auerbach and Hassett (1992), assigns values of α, β, and η, which collectively characterize dk_i/dr_i, and backs out ϕ using equation (44). The alternative, pursued in Chodorow-Reich et al. (2025), sets χ directly, recovers the implied long-run coefficient and the parameter values α, β, and η, and backs out ϕ using the expression for χ in equation (33). The drawback of the second approach is that χ is not a deep structural parameter (in fact it depends on policy—see equation (33)). The advantages are: (i) calibration of χ can incorporate external information on adjustment costs and transition dynamics such as from moments of the firm investment distribution, (ii) the ratio of the short-to-long-run elasticity is a portable statistic that continues to apply under more complicated adjustment cost models than quadratic costs, and (iii) the transparency of setting χ directly allows readers to easily substitute their own preferred rescaling.

Figure 2 presents implications for α and ϕ under both approaches, taking as inputs various combinations of $b_1^{\text{Coen}I(0)/K,r}$, χ, and η. The figure contains "trees" based on representative values for the short-run semi-elasticity of the investment-capital ratio to the user cost in Table 2:

$b_1^{\text{Coen}I(0)/K,r} \in \{-0.25, -0.5, -0.75\}$. The Auerbach and Hassett (1992) "branches" assume a Cobb-Douglas revenue function with constant returns to scale, $\alpha + \beta = \eta = 1$, such that equation (37) applies and $\phi = -R/\left(\rho b_1^{\text{Coen}I(0)/K,r}\right)$. The Chodorow-Reich et al. (2025) branches consider $\chi \in \{-0.1, -0.14, -0.18\}$ and $\eta \in \{0.5, 1\}$. The value χ/δ has the interpretation of the ratio of the short-run to the long-run impulse response shown in Figure 1(b). In that figure's calibration $\chi \approx -0.15$, with an impact response of 1.24% and long-run response of 0.84% yielding a ratio of $-\chi/\delta = 1.5$. Thus, a value $\chi = -0.1$ implies a flat impulse response of investment, while $\chi = -0.18$ implies a more steeply declining impulse response function.[28] The values of $\eta \in \{0.5, 1\}$ span the Cobb-Douglas value of 1 and a lower elasticity of substitution as found in some of the studies in Table 4 and in the meta-analysis by Knoblach, Roessler and Zwerschke (2020).

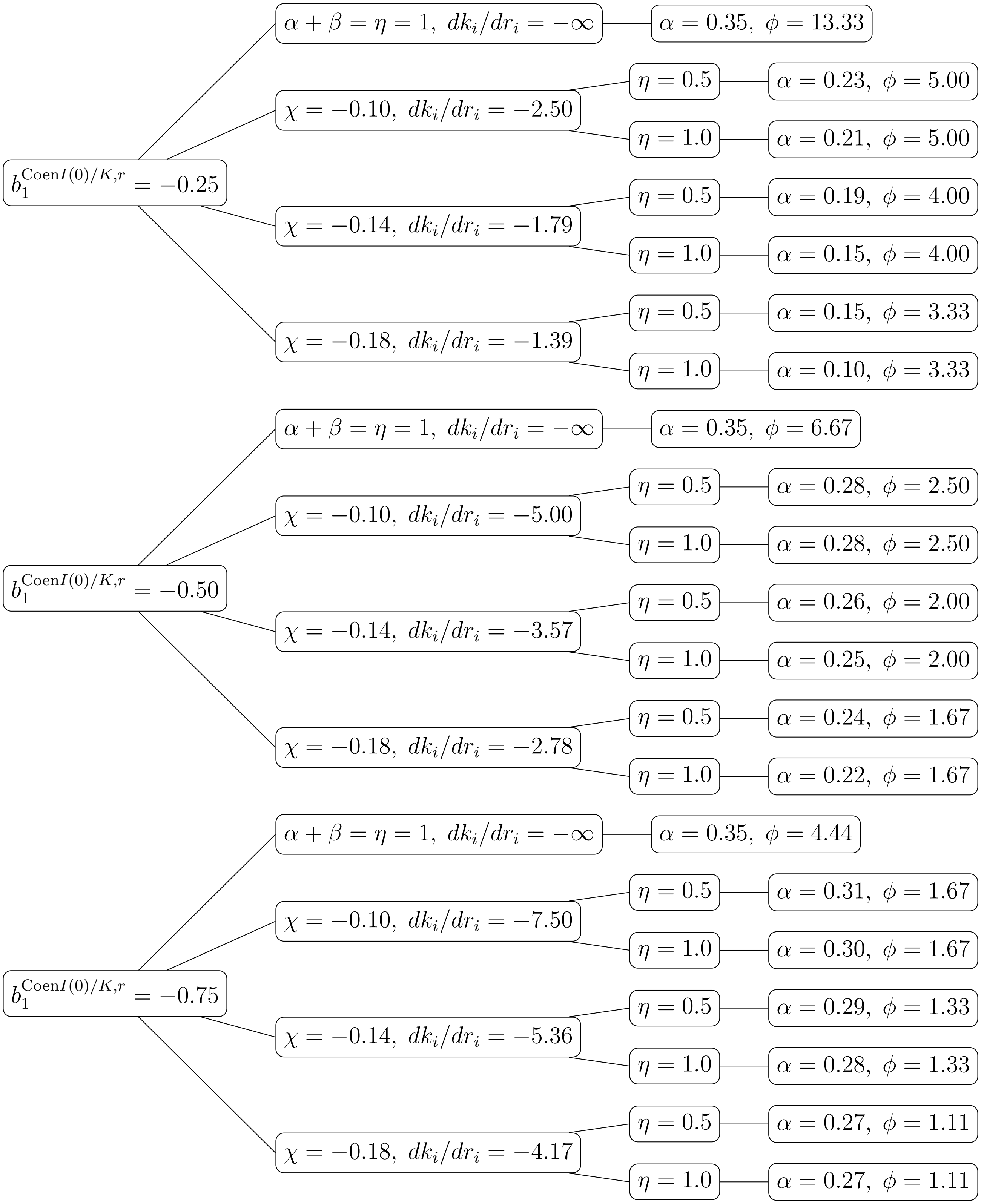
Figure 2 contains several lessons. To start, the long-run Coen (1969) firm-level elasticity of capital to the user cost, dk_i/dr_i, is always below -1 and sometimes substantially so. Equation (17) showed this inequality analytically in the Cobb-Douglas case, but it does not necessarily carry over to the CES case, where small enough η and α can make the long-run elasticity close to zero. For the mid-point short-run semi-elasticity of investment and the values of η considered, the long-run elasticity is around -3 or below. This conclusion highlights the difference between the long-run Coen (1969) firm-level elasticity, on the one hand, and the elasticity of substitution between capital and labor and associated "neoclassical benchmark" of -1 on the other. Notably, it does not depend on the assumption of quadratic adjustment costs, but only the existence of some plausibly-valued scalar relating the short and long-run elasticities.

Second, for $b_1^{\text{Coen}I(0)/K,r} \in \{-0.5, -0.75\}$, the implied values of α range from 0.22 to 0.35 and the values of ϕ vary between 1 and 7. The upper bounds come from the constant returns to scale specification; in the remainder, $\alpha \subseteq [0.22, 0.31]$ and $\phi \leq 2.5$. The range for α implies total returns to scale $\alpha + \beta$ above 0.87. The reasonableness of this range and the values for ϕ constitutes a victory for empirical assessment of the neoclassical model.

Third, the values of α vary only slightly with η within a χ branch of a tree. This result obtains because in these trees the total returns to scale are not too far from one (see equation (24)). The

[28] Chodorow-Reich et al. (2025) argue that $\chi = -0.14$ is a plausible value based on calibrations of firm-level adjustment costs such as in Winberry (2021). Their analysis focuses on the ratio of the short-to-long run elasticity $-\chi/\delta$. Using $\delta = 0.1$, this ratio is 1.4, consistent with their analysis of Winberry (2021).

Figure 2: Coen (1969) Specification Implied Parameters



Notes: Each "tree" starts from a value for the short-run semi-elasticity of I/K to the user cost r of $b_1^{\text{Coen}I(0)/K,r}$. The "branches" either set the total returns to scale $\alpha + \beta$ and elasticity of substitution between capital and labor η to 1, or values for the ratio of the short-run semi-elasticity to the long-run elasticity χ and the elasticity of substitution η. The "leaves" give the implied values for the revenue elasticity of capital α and the adjustment cost scalar ϕ.

fact that ϕ does not vary with η follows from equation (33), which shows that χ depends only on $\rho, R, dk_i/dr_i$, and ϕ.

5 From Cross-sectional Studies to Aggregate Investment

This section explores what the parameter values found in Section 4, based on firm or asset-level regressions, imply for the aggregate long-run elasticity of capital to the user cost as well as the effect on output and wages. The firm-level and aggregate elasticities differ because the individual firm faces flat factor supply curves, taking the wage as fixed, while in general equilibrium, factor supply curves may slope up.[29] Section 5.1 derives the general equilibrium Barro and Furman (2018) user cost elasticity and provides values consistent with the cross-sectional evidence. Section 5.2 shows that for changes to corporate taxes only, the aggregate user cost elasticity simply scales with the share of economic activity in the corporate sector. Section 5.3 extends the analysis to allow the interest rate or capital goods price to respond to higher investment demand. Section 5.4 gives formulas for the tax revenue response and shows that the dynamic component does not fully counteract the mechanical revenue loss in the current low tax regime of the U.S. Section 6 compares these results to the approaches in leading policy models.

5.1 The General Equilibrium User Cost Elasticity

Interest in the long-run aggregate elasticity dates to Hall and Jorgenson (1967), who as noted assumed an elasticity of capital of -1 and no effect on output. Their reply to the Coen (1969) critique anticipates some of the inconsistencies of subsequent approaches: "Another question which could be asked is the general equilibrium one: What were the overall effects of tax incentive policies, assuming no alternative compensatory policy? The answer waits for the development of a satisfactory model of general equilibrium in the United States economy" (Hall and Jorgenson, 1969, p. 394).

In fact, under some simplifying assumptions the Coen (1969) elasticity has an analytic general equilibrium counterpart. Thus, maintain the following assumptions in addition to the environment

[29] Following this logic, if firms have monopsony power and face upward-sloping labor supply curves, the gap between the cross-firm and general equilibrium user cost elasticities of capital is reduced. Kennedy et al. (2026) and Curtis et al. (2021) both test for and find no evidence that their treated firms increase wages for most workers relative to control firms, despite their increasing relative employment.

considered in Section 2:

1. Aggregate labor is fixed.

2. The economy consists of a large number of identical firms subject to the same tax schedule.

3. Firms face possibly downward-sloping demand curves with price elasticity of demand $\mathcal{M}/(\mathcal{M}-1) > 1$: $dy_{i,t} = -dp_{i,t}/(\mathcal{M}-1)$, where $P_{i,t}$ is the firm's output price (and recall that $Y_{i,t}$ is the firm's revenue).

Assumption 1 holds for example with balanced growth path preferences in which the income and substitution effects of a change in the wage on labor supply perfectly offset each other. Assumptions 1 and 2 together imply that each firm's labor input remains the same before and after a tax change. Assumption 2 also allows the dropping of i subscripts from parameters; I keep i subscripts on firm-level variables to distinguish them from aggregates without i subscripts. Totally differentiating the revenue function (1) across steady states and using the necessary conditions (5a) and (8) and Assumption 3 gives the firm's physical output across steady states:[30]

$$dy_i - dp_i = \mathcal{M}dy_i = \mathcal{M}\alpha dk_i + \mathcal{M}\beta d\ell_i + \mathcal{M}a_i. \tag{45}$$

I will sometimes refer to $\mathcal{M}$ as the markup, as it equals the equilibrium markup when firms have price-setting power.[31]

The property that aggregate labor remains fixed while the wage adjusts delivers the general equilibrium user cost elasticity as a comparative static of the first order condition for capital, now holding labor rather than output (as in Eisner and Nadiri (1968)) or the wage (as in Coen (1969)) fixed. Start with the Cobb Douglas case. Assumptions 1 to 3 together imply that the aggregate economy behaves as if there is a single representative firm with production function:

$$Y_t = A_t K_t^{\mathcal{M}\alpha} \bar{L}^{\mathcal{M}\beta},$$

where Y denotes real output. Thus, while the firm's revenue elasticity of capital holding the wage fixed is $\gamma \equiv \alpha/(1-\beta)$ (see equation (14)), the general equilibrium output elasticity holding labor

[30] That is: $dy_i = \frac{dY_i}{Y_i} = \frac{A_i F_K(K_i,L_i)K_i}{Y_i}\frac{dK_i}{K_i} + \frac{A_i F_L(K_i,L_i)L}{Y_i}\frac{dL_i}{L_i} + a_i = \alpha_i^* dk_i + \beta_i^* d\ell_i + a_i$. Setting $\alpha = \alpha^*$, $\beta = \beta^*$, and using $\mathcal{M}dy_i = dy_i - dp_i$ gives equation (45).

[31] Some of the studies reviewed in Section 3 exploit industry-level variation. In this case, the cross-sectional elasticities may still reflect downward-sloping demand even if no individual firm has price-setting power and charges a markup. Hence equations (46) to (49) still apply for an appropriately-calibrated demand elasticity.

fixed is $\mathcal{M}\alpha$.[32] The general equilibrium user cost elasticity follows from applying the chain rule using equation (45) to equation (11):

$$\frac{dk}{dr} = -1 + \frac{\partial y}{\partial k}|_{L \text{ fixed}} \times \frac{dk}{dr} = -\frac{1}{1-\mathcal{M}\alpha} \leq -1. \tag{46}$$

I refer to equation (46) as the Barro and Furman (2018) elasticity, as these authors use it (with $\mathcal{M} = 1$) to model the aggregate effects of the TCJA.[33] Moving from the Coen (1969) elasticity identified from cross-firm responses to the Barro and Furman (2018) general equilibrium elasticity therefore requires relating γ to $\mathcal{M}\alpha$.

Similar calculations give the Barro and Furman (2018) general equilibrium elasticity with CES production. Specifically, using the general equilibrium output elasticity in equation (45) in the implicit user cost elasticity (23) gives:

$$\frac{dk}{dr} = \left(\left(1 + \frac{1-\eta}{\mathcal{M}(\alpha+\beta)\eta}\right)\mathcal{M}\alpha - \frac{1}{\eta}\right)^{-1} = -\frac{\mathcal{M}(\alpha+\beta)\eta}{\mathcal{M}(\alpha+\beta) - (1-(1-\mathcal{M}\alpha-\mathcal{M}\beta)\eta)\mathcal{M}\alpha}. \tag{47}$$

The output and wage elasticities follow from equation (45) and the first order condition (5a) $A_i F_L(K_i, L_i) = W$ using equation (20b) and $d\ell = 0$:

$$\frac{dy}{dr} = \mathcal{M}\alpha \times \frac{dk}{dr}, \tag{48}$$

$$\frac{dw}{dr} = \left(1 + \frac{1-\eta}{\mathcal{M}(\alpha+\beta)\eta}\right) \times \frac{dy}{dr}. \tag{49}$$

Importantly, corporate taxes do not apply to output of the government, non-corporate, or household sectors. For changes to corporate taxes only, these formulas thus assume full segmentation of labor markets across these sectors (see assumption 1), as otherwise the corporate sector could pull in labor. Section 5.2 relaxes this assumption.

[32] In fact, with Cobb-Douglas one can easily incorporate any aggregate labor supply elasticity. Assume a labor supply equation $L_t = c_w W_t^{\xi}$ for a constant c_w and wage elasticity ξ. Substitute for W_t using the labor FOC (13) to find $L_{i,t} = c_w^{\frac{1}{1+(1-\beta)\xi}} \left(\beta A_{i,t} K_{i,t}^{\alpha}\right)^{\frac{\xi}{1+(1-\beta)\xi}}$. Using equation (45), the output elasticity of capital incorporating the positive labor supply response is $dy_i/dk_i = \mathcal{M}\alpha + \mathcal{M}\beta d\ell_i/dk_i = \frac{\mathcal{M}\alpha(1+\xi)}{1+(1-\beta)\xi}$, which exceeds $\mathcal{M}\alpha$ if $\xi > 0$. Using this expression in equation (46) in place of $\frac{\partial y}{\partial k}|_{L \text{ fixed}}$ gives the general equilibrium elasticity of capital to the user cost with aggregate labor supply elasticity ξ. Thus, a positive labor supply elasticity simply increases the effective output elasticity of capital.

[33] Barro and Furman (2018, with $\mathcal{M} = 1$) and Chodorow-Reich et al. (2025) both discuss versions of equation (46). The remaining results in this section are new, including the extension to CES with markups and non-constant returns to scale in the revenue function in equation (47), the extension to two sectors in Section 5.2, the analytic framework for crowd-out in the non-tax component of the user cost in Section 5.3, the revenue elasticity results in Section 5.4, and the quantitative explorations.

Using equations (47) to (49), Figure 3 gives values for the aggregate long-run elasticities of capital, output, and the wage consistent with the short-run cross-firm evidence. For concreteness, I consider values $\alpha \in \{0.22, 0.26, 0.3, 0.34\}$ (see Figure 2) and the equilibrium markup $\mathcal{M} \in \{1, 1.2\}$. I continue with a labor share $\beta = 0.65$ and the elasticity of substitution $\eta \in \{0.5, 1\}$.[34]

Several lessons again emerge. First, under Cobb Douglas, the long-run Barro and Furman (2018) elasticity is always below -1, ranging from -1.3 to -1.7. This range is notably tighter than the range of partial equilibrium long-run Coen (1969) elasticities, which varied between -1.1 and $-\infty$ in Figure 2 (or -1.1 and -7.5 away from the constant returns case). The reason is that labor is a fixed factor in general equilibrium, which dampens the overall response. The elasticity of -1.6 used in the actual Barro and Furman (2018) article falls inside this range, illustrating again the difference between a general equilibrium elasticity and the "neoclassical benchmark" of -1.

Second, the capital and output elasticities are smaller with $\eta = 0.5$, about half of their values with Cobb Douglas. Qualitatively, as the elasticity of substitution between capital and labor declines, the fixed factor of labor generates faster decreasing returns to capital accumulation. Quantitatively, with constant returns to scale, $\mathcal{M}\alpha + \mathcal{M}\beta = 1$, the Barro and Furman (2018) elasticity (47) simplifies to $-\eta/(1 - \mathcal{M}\alpha)$ and hence scales with η.[35] The empirical evidence from Table 2 requires values of α large enough that this approximation roughly holds.

Third, although capital and output respond less when η is smaller, the wage elasticity is quite similar. Qualitatively, while capital responds less when $\eta < 1$, the wage responds more per unit of capital. Quantitatively, with constant returns in the firm's revenue function the general equilibrium wage elasticity simplifies to $-\mathcal{M}\alpha/(1 - \mathcal{M}\alpha)$ and hence does not depend on η.

Fourth, the general equilibrium elasticities are increasing (in absolute value) in the markup $\mathcal{M}$. In partial equilibrium and fixing the returns to scale in the production function, a higher markup reduces the Coen (1969) elasticity because it lowers the returns to scale in the revenue

[34] As noted by Oberfield and Raval (2021), the firm-level and aggregate elasticities of substitution can differ due to cross-firm substitution. Assumption 2 rules out cross-firm substitution; in the more general case, the appropriate η would include such substitution.

[35] One can also arrive at this formula by following a cobweb adjustment process analogous to that in Section 2.3. With CES, the direct effect on capital holding output fixed is $dk_{i,0} = -\eta dr_i$. This additional capital increases output by $-\mathcal{M}\alpha\eta dr_i$ (see equation (45)). With constant returns to scale, additional output increases desired log capital one-for-one (see equation (21)), yielding $dk_{i,1} = -\eta dr_i (1 + \mathcal{M}\alpha)$. The additional capital further increases output and hence desired log capital by $\mathcal{M}\alpha dk_{i,1}$, giving $dk_{i,2} = -\eta dr_i \left(1 + \mathcal{M}\alpha + \mathcal{M}^2\alpha^2\right)$. Iterating until convergence, $dk_i = -\eta dr_i / (1 - \mathcal{M}\alpha)$.

Figure 3: General Equilibrium Elasticities

$\alpha = 0.22$	$\mathcal{M} = 1.00$	$\eta = 0.5$	$dk/dr = -0.65, dy/dr = -0.14, dw/dr = -0.31$
		$\eta = 1.0$	$dk/dr = -1.28, dy/dr = -0.28, dw/dr = -0.28$
	$\mathcal{M} = 1.20$	$\eta = 0.5$	$dk/dr = -0.67, dy/dr = -0.18, dw/dr = -0.35$
		$\eta = 1.0$	$dk/dr = -1.36, dy/dr = -0.36, dw/dr = -0.36$
$\alpha = 0.26$	$\mathcal{M} = 1.00$	$\eta = 0.5$	$dk/dr = -0.69, dy/dr = -0.18, dw/dr = -0.38$
		$\eta = 1.0$	$dk/dr = -1.35, dy/dr = -0.35, dw/dr = -0.35$
	$\mathcal{M} = 1.20$	$\eta = 0.5$	$dk/dr = -0.71, dy/dr = -0.22, dw/dr = -0.43$
		$\eta = 1.0$	$dk/dr = -1.45, dy/dr = -0.45, dw/dr = -0.45$
$\alpha = 0.30$	$\mathcal{M} = 1.00$	$\eta = 0.5$	$dk/dr = -0.72, dy/dr = -0.22, dw/dr = -0.44$
		$\eta = 1.0$	$dk/dr = -1.43, dy/dr = -0.43, dw/dr = -0.43$
	$\mathcal{M} = 1.20$	$\eta = 0.5$	$dk/dr = -0.76, dy/dr = -0.27, dw/dr = -0.51$
		$\eta = 1.0$	$dk/dr = -1.56, dy/dr = -0.56, dw/dr = -0.56$
$\alpha = 0.34$	$\mathcal{M} = 1.00$	$\eta = 0.5$	$dk/dr = -0.76, dy/dr = -0.26, dw/dr = -0.52$
		$\eta = 1.0$	$dk/dr = -1.52, dy/dr = -0.52, dw/dr = -0.52$
	$\mathcal{M} = 1.20$	$\eta = 0.5$	$dk/dr = -0.80, dy/dr = -0.33, dw/dr = -0.60$
		$\eta = 1.0$	$dk/dr = -1.69, dy/dr = -0.69, dw/dr = -0.69$

Notes: Each "tree" starts from a value for the revenue elasticity of capital α. The "branches" set the markup $\mathcal{M}$ and the elasticity of substitution between capital and labor η. The "leaves" give the implied values for the long-run general equilibrium user cost elasticity of capital, output, and the wage.

function. Fixing the Coen (1969) elasticity as estimated in the data, however, a larger markup implies a larger output elasticity, increasing the general equilibrium response.

5.2 The General Equilibrium User Cost Elasticity with 2 Sectors

In many countries including the U.S., businesses have a choice of legal form of organization. Changes to the corporate tax rate apply only to firms incorporated as C-corporations. These firms accounted for 43% of private sector employment in 2022 (U.S. Census Bureau, 2022), making it of interest to relax assumption 2. Thus, modify assumption 2 to:

4. The economy consists of a C-corporate sector and non C-corporate sector, each of which consists of a large number of identical firms. Firms exogenously choose their sector. The production function parameters α, β, and η and technology A are common across the sectors.

5. Workers are indifferent between the two sectors.

Assumption 4 ensures that starting from *ex ante* identical taxation, firms in the two sectors look identical.[36] Assumption 5 requires that the wage equalizes across sectors, with the labor market clearing condition $L_c + L_n = \bar{L}$ for corporate and non-corporate labor L_c and L_n, respectively.

Section A.3 characterizes the solution with two sectors. The presence of a non C-corporate sector without a tax change makes the responses of C-corporate capital and output larger, as the sector can pull in labor. This response is offset however by the decline in the non C-corporate sector.[37] Locally, the responses of aggregate capital, output, and the wage simply scale their counterparts in Figure 3 by the factor 0.43 of initial activity in the corporate sector.[38] For example, the elasticity of the wage to the C-corporate user cost is in the range of -0.1 to -0.2.[39]

[36] In fact, C-corporations have higher average capital per worker and productivity. The results in this section therefore serve as a benchmark without these differences as well as without an endogenous choice of corporate form.

[37] Kennedy et al. (2026, table 8) show exactly such a response following TCJA, with workers migrating from S corporations to C corporations.

[38] With *ex ante* differences, there would be additional gains or losses from the reallocation of activity. The scaling also only applies locally; in the limit with constant returns to scale, the C-corporate sector would immediately take over the whole economy after a tax cut and Figure 3 would apply with no scaling factor.

[39] This general equilibrium elasticity differs from the relative change in wages across the two sectors studied in Kennedy et al. (2026), which under Assumption 5 equals zero by assumption. Their market-level analysis implies general equilibrium elasticities slightly smaller than this range.

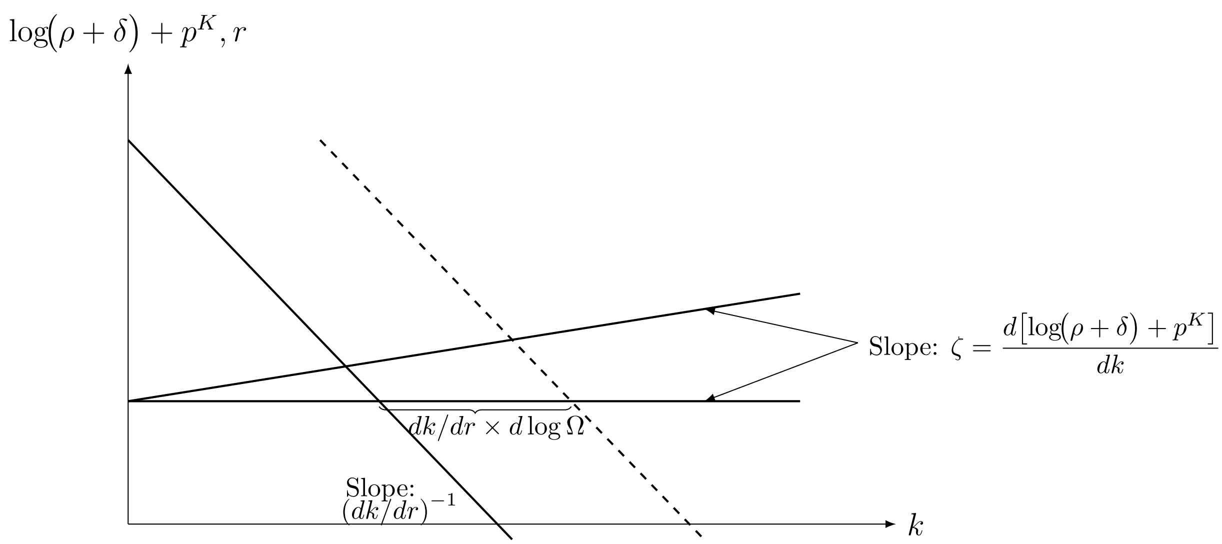
5.3 The General Equilibrium Tax-term Elasticity

The general equilibrium elasticities summarized in Figure 3 apply to changes in the user cost. As a result, they apply holding fixed the non-tax component $(\rho+\delta)P^K$ when the tax term $\Omega \equiv (1-\Gamma)/(1-\tau)$ changes. However, the non-tax component could change endogenously, for example due to crowd-out from an upward-sloping savings curve that makes ρ a function of K or an upward-sloping capital supply curve that makes P^K a function of K. Let $\zeta \equiv d\left[\log\left(\rho+\delta\right)+p^K\right]/dk$ denote the inverse supply elasticity.

Figure 4 summarizes the equilibrium inclusive of any changes to the non-tax component of the user cost. The vertical axis is the log of the non-tax component. The downward sloping lines have slope $(dk/dr)^{-1}$. A decline from Ω to $\Omega' < \Omega$ shifts the schedule to the right by the amount $dk/dr \times d\log\Omega$. With $\zeta = 0$, this shift gives the total effect of the tax change. With $\zeta > 0$, $dk/d\log\Omega$ is reduced. Algebraically, the equilibrium is the solution to the system of equations:

$$\begin{aligned} &\text{Capital demand:} && dk = (dk/dr)\, d\left[\log\left(\rho+\delta\right)+p^K+\log\Omega\right], \\ &\text{Capital supply:} && d\left[\log\left(\rho+\delta\right)+p^K\right] = \zeta dk, \\ &\text{Equilibrium:} && dk = \frac{dk/dr}{1-\zeta\left(dk/dr\right)} d\log\Omega. \end{aligned}$$

Figure 4: General Equilibrium Tax Elasticity



The total general equilibrium elasticity of capital to a tax change therefore combines dk/dr with a value for ζ. In an infinite horizon Ramsey economy or a small open economy facing a world interest rate, the interest rate ρ does not depend on investment and $\zeta = 0$. Moll, Rachel and Restrepo (2022) advocate a semi-elasticity of savings to the interest rate of 50, implying $\zeta = 1/(\rho+\delta) \times 1/50 = 0.125$ using $\rho+\delta = 0.16$ and $P^K = 1$. This value provides an upper bound for the response to corporate tax changes, since corporate capital is only about 40% of the total capital stock. Goolsbee (1998) finds a large response of capital goods prices to tax incentives. House and Shapiro (2008) and House, Mocanu and Shapiro (2022) dispute this conclusion, with House, Mocanu and Shapiro (2022) finding that the Goolsbee (1998) result disappears with revised data or in longer samples and arguing instead that $dp^K/dk \approx 0$.[40]

5.4 The General Equilibrium Elasticity of Tax Revenue

Steady state corporate tax payments are $\tau\left(Y - WL\right) - \Gamma I$.[41] Adding labor tax payments with a tax rate τ^L, setting $I = \delta K$, and using the labor first order condition (5a), total taxes T are:

$$T = \overbrace{\tau\left(A\left(F\left(K,L\right) - F_L\left(K,L\right)L\right) - \delta z K\right)}^{\text{Corporate taxes}\equiv CT} + \overbrace{\tau^L A F_L\left(K,L\right)L}^{\text{Labor taxes}\equiv LT}. \tag{50}$$

Let CT and LT denote corporate taxes and labor taxes, respectively. Start with changes in the corporate rate τ. It is convenient to express taxes paid as a share of baseline corporate revenue Y. For notational simplicity, let $\mathcal{M} = 1$. The semi-elasticities of corporate, labor, and total taxes to the corporate tax rate are (see equations (A.16) and (A.17)):

$$\frac{1}{Y} \times \frac{dCT}{d\log\tau} = \frac{CT}{Y} + c_{CT}\frac{dk}{d\log\tau}, \text{ where: } c_{CT} \equiv \tau\left(\alpha^*\left(1-\beta^*\right) - \frac{\partial\beta^*}{\partial k} - \frac{z\delta K}{Y}\right), \tag{51a}$$

$$\frac{1}{Y} \times \frac{dLT}{d\log\tau} = c_{LT}\frac{dk}{d\log\tau}, \text{ where: } c_{LT} \equiv \tau^L\left(\alpha^*\beta^* + \frac{\partial\beta^*}{\partial k}\right), \tag{51b}$$

$$\frac{1}{Y} \times \frac{dT}{d\log\tau} = \frac{CT}{Y} + \left(c_{CT} + c_{LT}\right)\frac{dk}{d\log\tau}, \tag{51c}$$

and where $\partial\beta^*/\partial k = \left(\frac{1-\eta}{(\alpha+\beta)\eta}\right)\alpha^*\beta^*$ denotes the semi-elasticity of the labor share of revenue to a change in capital, holding labor fixed (see equation (A.13)).

[40] Another possibility is $dp^k/dr > 0$, if the tax incentives concentrate in capital goods-producing firms that respond by increasing production. This possibility echoes the literature on investment-specific technical change (Greenwood, Hercowitz and Krusell, 1997).

[41] To streamline the analysis, this sub-section assumes uniform taxation of all business income, which with abuse of terminology I refer to as the corporate tax system. In the notation of Section 5.2, $\tau_c = \tau_n = \tau$ and $z_c = z_n = z$. It also ignores payout taxes (e.g. dividend taxes) and the transition path. See Chodorow-Reich et al. (2025) for a quantitative evaluation of TCJA incorporating these features.

The first term in equation (51c), CT/Y, gives the mechanical effect of a change in the tax rate holding the corporate and labor tax bases fixed. The coefficients c_{CT} and c_{LT} give the feedback from additional capital to corporate and labor taxes, respectively. The change in the corporate tax base when capital increases equals the change in output, $\alpha^* dk$, multiplied by the share of output accruing to owners of capital, $(1-\beta^*)$, less the change in the labor share when capital increases, $\partial\beta^*/\partial k \times dk$, less the additional depreciation deductions $z\delta dK$. The change in the labor tax base equals the change in output multiplied by the labor share, $\beta^*\alpha^* dk$, plus the change in the labor share. The coefficients c_{CT} and c_{LT} multiply the changes in the corporate and labor tax bases by their respective marginal tax rates. These coefficients therefore also apply to dynamic changes in capital induced by changes in z, while the mechanical effect in that case is instead the tax rate applied to steady state investment $-\tau I/Y$:

$$\frac{1}{Y} \times \frac{dT}{dz} = -\frac{\tau I}{Y} + (c_{CT} + c_{LT}) \frac{dk}{dz}. \tag{52}$$

In 2024, corporate taxes equaled roughly 4% of corporate value added, $CT/Y \approx 0.04$ (U.S. Bureau of Economic Analysis, 2025*b*). This value has remained remarkably stable since the 1980s, as a rising corporate profit share has offset a declining tax rate. Thus, the mechanical reduction in revenue from a 10% decline in the corporate tax rate is roughly 0.4% of corporate value added. For $\tau = 0.18$, $z = 0.85$, $\tau^L = 0.28$ (all approximately values for the average U.S. firm or worker in 2024), and values of α between 0.22 and 0.34 and $\eta \in \{0.5, 1\}$, the dynamic component $(c_{CT} + c_{LT})\, dk/d\log\tau$ offsets less than 10% of the mechanical decline in revenue (see Figure A.2). Alternatively, this range of parameter values implies a dynamic offset of between 25% and 45% of the mechanical revenue decline when z increases.

The result that the dynamic growth effects do not outweigh the mechanical revenue losses reflects the current corporate tax regime in the U.S. Higher values of τ or, especially, lower values of z could alter this conclusion. Several other caveats apply: these neoclassical formulas do not take account of international capital mobility and profit shifting, changes in corporate form in response to tax changes, or differences in the discounting of future depreciation allowances by firms and the government. They also do not incorporate higher interest payments if the interest rate increases (see Section 5.3), which may be non-trivial when the debt-GDP ratio is large.

6 Existing Policy Approaches

Major policy institutions routinely provide estimates of the effects of changes to corporate taxation on capital and output. The neoclassical framework offers a useful taxonomy to their approaches.

Council of Economic Advisers (2019, 2025) follow Hall and Jorgenson (1967). Council of Economic Advisers (2019, p. 68) describes their method:

> Calculations yield an estimated decline in the average user cost of capital of about 9 percent. Assuming a consensus user-cost elasticity of investment of -1.0, a 9 percent decline in the average user cost of capital would induce a 9 percent increase in the demand for capital. Following Jensen, Mathur, and Kallen (2017), it is then possible to use the Multifactor Productivity Tables from the Bureau of Labor Statistics in a growth accounting framework to increment the Congressional Budget Office's 10-year GDP growth projections by the additional contribution to output from a larger capital stock, assuming constant capital income shares, until we attain a new steady state.

This approach differs in three important ways from the analysis above. First, it follows Hall and Jorgenson (1967) in applying the "neoclassical benchmark" elasticity of -1.0 without allowing for feedback from higher output back into higher capital.[42] Second, the "consensus user-cost elasticity" of -1 differs from my preferred range and references estimates of both the Coen (1969) and the Eisner and Nadiri (1968) specifications.[43] Third, it does not explicitly account for the non C-corporate sector.

Congressional Research Service (2024, pp. 16-17) likewise omits feedback from higher output into higher capital. They apply an elasticity of 0.6 based on an average of Dwenger (2014) and a comparable study finding a smaller Eisner and Nadiri (1968) elasticity. Thus, Council of Economic Advisers (2019, 2025) and Congressional Research Service (2024) use smaller elasticities for aggregate capital than found in Figure 3 in part because they take only the first step of the cobweb adjustment process, as originally critiqued by Coen (1969).

The Congressional Budget Office (CBO) conventional score applies an aggregate Eisner and Nadiri (1968) investment elasticity of -0.7 to a user cost change holding output fixed (Congressional Budget Office, 2018*a*,*b*). The value of -0.7 comes from CBO's own analysis and is compared to

[42] The phrase "until we attain a new steady state" may seem to imply such feedback, but it does not. Jensen, Mathur and Kallen (2017) fully document the method. Starting from a baseline path of investment from 2017 forward, investment in each year is increased by 9% relative to the baseline path. This alternative path of investment implies an alternative path of capital using the perpetual inventory relationship $K_{t+1} = (1-\delta)K_t + I_t$. Multiplying the additional capital by the capital-income share gives the increase in GDP in each year.

[43] See the literature review in Council of Economic Advisers (2019, p. 58). In addition, Council of Economic Advisers (2019, p. 58) refers to Cummins, Hassett and Hubbard (1994, 1996) as finding "an estimated long-run elasticity of the capital stock of -0.67", despite these articles using short-run variation. See Table 2 and Figure 2 for my re-analysis of these studies.

the Hassett and Hubbard (2002) range of -0.5 to -1 (Congressional Budget Office, 2018*b*, p. 3), despite that range coming from studies mostly estimating the Coen (1969) specification across firms and at short horizons. CBO's dynamic score inputs the change in investment into a Solow-style macroeconomic model featuring feedback from output into investment as well as crowding out due to higher interest rates and inelastic labor (using an elasticity of labor supply of 0.2).[44]

7 Conclusion

Some of the most exciting current work in firm-level dynamics incorporates rich heterogeneity in production technology, product differentiation, financial frictions, and market size. How firms respond to changes in the cost of investment or taxation of profits can provide key moments to discipline these models (Koby and Wolf, 2020). The empirical literature has focused on tax reforms as a particularly good laboratory to measure these responses. Making these responses informative for modeling of the firm requires a clear understand of the moments estimated.

These episodes also inform policy makers considering further changes to corporate taxation. The interpretation of empirical work for this purpose requires properly relating the empirical moments to theory parameters. As often occurs in macroeconomics, such structural interpretation has particular import in settings such as tax policy where the infrequency of exogenous, major tax reforms makes analysis of micro data an important complement to time series analysis of the objects of interest to policy makers, such as the long-run effect on aggregate capital and wages.

This paper has attempted to re-connect the modern empirical literature studying firm-level responses to tax reforms to the neoclassical theory of the firm facing convex adjustment costs of capital. At the firm level, the Eisner and Nadiri output-fixed user cost elasticity identifies the elasticity of substitution between capital and labor η, while the Coen wage-fixed user cost elasticity additionally sheds light on the returns to scale of the firm and hence the revenue elasticity of capital α. Both parameters matter to the long-run general equilibrium user cost elasticity of capital. Throughout, I have suggested magnitudes consistent with the empirical evidence.

Recent work has enriched the neoclassical framework, including estimation of the extensive

[44]The CBO macroeconomic model combines a short run with demand-determined output with a long-run Solow-type growth model. In the short run, higher investment boosts demand and causes GDP to increase relative to potential. In the long run, higher output increases aggregate savings which then results in even higher capital, equivalent to the cobweb adjustment process described in Section 2.3.

margin of adjustment (Chen et al., 2023), substitution elasticities with multiple types of labor (Curtis et al., 2021), and extending the framework to include intangibles (Crouzet and Eberly, 2023) or a multinational setting (Chodorow-Reich et al., 2025). The paper has also largely avoided short-run issues including business cycle implications from changes in aggregate demand and transitory rather than permanent tax changes. Studies of these and other issues will benefit from drawing on proper interpretation of the extant body of empirical work on firm taxation.

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Online Appendix:

The Neoclassical Theory of Firm Investment and Taxes: A Reassessment

Gabriel Chodorow-Reich

Table A.1: Notation Summary

Symbol	Definition	Description	Eq.
$K_{i,t}$		Firm capital input	
$L_{i,t}$		Firm labor input	
$Y_{i,t}$	$A_{i,t}F\left(K_{i,t},L_{i,t}\right)$	Firm revenue	(1)
$I_{i,t}$	$\dot{K}_{i,t}+\delta K_{i,t}$	Gross investment	(2)
$z_{i,t}$		Present value of depreciation allowances	(4)
$\tau_{i,t}$		Corporate tax rate	
$\Gamma_{i,t}$	$itc_{i,t}+\tau_{i,t}z_{i,t}$	Effective subsidy to cost of new investment	(3)
ρ		Discount rate	
δ		Depreciation rate	
P_t^K		Common price of new investment	
W_t		Common wage	
$\lambda_{i,t}$		Marginal value of unit of capital	(6)
R_i	$\left(\rho+\delta\right)P^K\left(\frac{1-\Gamma_i}{1-\tau_i}\right)$	Steady state user cost	(8)
$\Omega_{i,t}$	$\frac{1-\Gamma_{i,t}}{1-\tau_{i,t}}$	Tax term	(9)
α		Cobb Douglas elasticity of revenue to capital	(10)
β		Cobb Douglas elasticity of revenue to labor	(10)
γ	$\alpha/(1-\beta)$	Cobb Douglas elasticity of revenue to capital allowing labor to adjust	(16)
η		Elasticity of substitution between capital and labor	(19)
α_i^*	R_iK_i/Y_i	Steady state capital share of revenue	
β_i^*	WL_i/Y_i	Steady state labor share of revenue	
ϕ		Scalar in quadratic adjustment costs	(25)
$Q_{i,t}$	$\frac{\lambda_{i,t}-P_t^K(1-\Gamma_{i,t})}{1-\tau_{i,t}}$	Transformation of Brainard-Tobin's q	(27)
$G_{i,t}\left(K_{i,t}\right)$	$\max_{L_{i,t}} A_{i,t}F\left(K_{i,t},L_{i,t}\right)-W_tL_{i,t}$	Revenue net of labor costs	(28)
χ_i	$\frac{\rho}{2}-\sqrt{\left(\frac{\rho}{2}\right)^2-\frac{G_i''(K_i)K_i}{\phi}}$	Stable eigenvalue	(32)
$b_1^{\text{Coen}k,r}$	dk_i/dr_i	Long-run firm-level Coen (1969) elasticity of capital to user cost	(B.1a)
$\mathcal{M}$		Equilibrium markup	
ζ	$d\left[\log\left(\rho+\delta\right)+p^K\right]/dk$	Inverse supply elasticity	

A Theory Appendix

This appendix provides additional results used in Sections 2 and 5. Table A.1 summarizes key notation used throughout the paper. In addition, lower case variables denote logs, variables or parameters with an i subscript pertain to the firm-level, and variables without an i pertain to the aggregate economy.

A.1 Additional CES Derivations

This appendix expands on results in Section 2.4. Using equations (20a) and (20b) and the necessary conditions equations (5a) and (8), the total cost share of revenue is:

$$\begin{aligned}\frac{R_i K_i + W_i L_i}{Y_i} &= \frac{F_K(K_i, L_i) K_i + F_L(K_i, L_i) L_i}{F(K_i, L_i)} = \frac{\alpha F(K_i, L_i)^{1+\frac{1-\eta}{(\alpha+\beta)\eta}} K_i^{1-\frac{1}{\eta}} + \beta F(K_i, L_i)^{1+\frac{1-\eta}{(\alpha+\beta)\eta}} L_i^{1-\frac{1}{\eta}}}{F(K_i, L_i)} \\ &= \alpha F(K_i, L_i)^{\frac{1-\eta}{(\alpha+\beta)\eta}} K_i^{1-\frac{1}{\eta}} + \beta F(K_i, L_i)^{\frac{1-\eta}{(\alpha+\beta)\eta}} L_i^{1-\frac{1}{\eta}} \\ &= \frac{\alpha K_i^{1-\frac{1}{\eta}} + \beta L_i^{1-\frac{1}{\eta}}}{\left(\frac{\alpha}{\alpha+\beta}\right) K_i^{1-1/\eta} + \left(\frac{\beta}{\alpha+\beta}\right) L_i^{1-1/\eta}} = \alpha + \beta. \end{aligned} \tag{A.1}$$

In the partial equilibrium Coen (1969) elasticity, the wage remains fixed. Then substituting equation (20b) into equation (5a), log differentiating, and setting $dw = 0$:

$$d\ell_i = \left(\eta + \frac{1-\eta}{(\alpha+\beta)}\right) dy_i = \left(\eta + \frac{1-\eta}{(\alpha+\beta)}\right)(\alpha_i^* dk_i + \beta_i^* d\ell_i) = \frac{\left(\eta + \frac{1-\eta}{(\alpha+\beta)}\right)\alpha_i^*}{1 - \left(\eta + \frac{1-\eta}{(\alpha+\beta)}\right)\beta_i^*} dk_i. \tag{A.2}$$

Substitute $\frac{\partial y_i}{\partial k_i} = \frac{\alpha_i^*}{1-\left(\eta + \frac{1-\eta}{\alpha+\beta}\right)\beta_i^*}$ from equation (A.2) into equation (23):

$$\begin{aligned}\frac{dk_i}{dr_i} &= \left(\left(\frac{\left(1 + \frac{1-\eta}{(\alpha+\beta)\eta}\right)\alpha_i^*}{1 - \left(\eta + \frac{1-\eta}{\alpha+\beta}\right)\beta_i^*}\right) - \frac{1}{\eta}\right)^{-1} = \left(\frac{1 - \left(\eta + \frac{1-\eta}{\alpha+\beta}\right)\beta_i^*}{\left(1 + \frac{1-\eta}{(\alpha+\beta)\eta}\right)\alpha_i^* - \frac{1}{\eta}\left(1 - \left(\eta + \frac{1-\eta}{\alpha+\beta}\right)\beta_i^*\right)}\right) \\ &= -\left(\frac{1 - \beta_i^*\left(\eta + \frac{1-\eta}{\alpha+\beta}\right)}{1-\alpha-\beta}\right). \end{aligned} \tag{A.3}$$

A.2 Linearized Transition Path

This appendix expands on results in Section 2.6. The Taylor expansion uses the result that $\Phi_K(I_{i,t}, K_{i,t}) = 0$ in the steady state and hence $d\Phi_K(I_{i,t}, K_{i,t}) = -\phi\delta d\dot{K}_{i,t}/K_{i,t} = -\delta/(1-\tau_i)(\lambda_{i,t} - \lambda_i)$. Restating equation (30) for convenience:

$$\begin{pmatrix} \dot{\lambda}_{i,t} \\ \dot{K}_{i,t} \end{pmatrix} = \begin{pmatrix} \rho & -(1-\tau_i) G_i''(K_i) \\ K_i/(\phi(1-\tau_i)) & 0 \end{pmatrix} \begin{pmatrix} \lambda_{i,t} - \lambda_i \\ K_{i,t} - K_i \end{pmatrix}$$

Denote by $(\chi_1, \mathbf{f}_1)$ and $(\chi_2, \mathbf{f}_2)$ the eigenvalue/eigenvector pairs of the coefficient matrix in equation (30), with:

$$\chi_1 = \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{G_i''(K_i) K_i}{\phi}} < 0, \chi_2 = \frac{\rho}{2} + \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{G_i''(K_i) K_i}{\phi}} > 0, \tag{A.4}$$

$$\mathbf{f}_1 = \begin{pmatrix} \chi_1 & K_i/((1-\tau_i)\phi) \end{pmatrix}', \mathbf{f}_2 = \begin{pmatrix} \chi_2 & K_i/((1-\tau_i)\phi) \end{pmatrix}'. \tag{A.5}$$

The general solution is:

$$\begin{pmatrix} \lambda_{i,t} - \lambda_i \\ K_{i,t} - K_i \end{pmatrix} = c_1 e^{\chi_1 t}\mathbf{f}_1 + c_2 e^{\chi_2 t}\mathbf{f}_2.$$

Immediately, $G'' < 0 \Rightarrow \chi_2 > \rho$, which requires $c_2 = 0$ or the transversality condition would be violated. Thus:

$$\begin{pmatrix} \lambda_{i,t} - \lambda_i \\ K_{i,t} - K_i \end{pmatrix} = c_1 e^{\chi_1 t} \begin{pmatrix} \chi_1 \\ K_i / \left(\left(1 - \tau_i\right) \phi \right) \end{pmatrix}. \tag{A.6}$$

The initial condition $K_{i,0}$ gives:

$$c_1 = \left(\frac{K_{i,0} - K_i}{K_i} \right) \left(1 - \tau_i\right) \phi. \tag{A.7}$$

Then defining $\chi_i \equiv \chi_1$ and using the approximation $k_{i,t} - k_i \approx (K_{i,t} - K_i)/K_i$ gives equations (32), (31a) and (31b) in the main text.

A.3 2 Sector General Equilibrium

This appendix derives the results described in Section 5.2.

Let $\omega_c \equiv L_c/\bar{L}$ denote the share of labor in the C-corporate sector and set $\mathcal{M} = 1$ for notational simplicity. Let $\xi \equiv 1 + \frac{1-\eta}{(\alpha+\beta)\eta}$. Applying the first order conditions (5a) and (8), the CES revenue function derivatives (20a) and (20b), the output elasticity formula (45), and the labor market clearing condition across steady states with changes in the user cost of dr_c and dr_n, respectively, gives a linear system of 7 equations in the 7 unknowns $dw, dk_i, d\ell_i, dy_i$:

$$dw = \xi dy_i - \frac{1}{\eta} d\ell_i, \;\; i \in \{c, n\}, \tag{A.8a}$$

$$dr_i = \xi dy_i - \frac{1}{\eta} dk_i, \;\; i \in \{c, n\}, \tag{A.8b}$$

$$dy_i = \alpha dk_i + \beta d\ell_i, \;\; i \in \{c, n\}, \tag{A.8c}$$

$$d\ell_c = -\left(\frac{1 - \omega_c}{\omega_c} \right) d\ell_n. \tag{A.8d}$$

In addition, $dx = \omega_c dx_c + (1 - \omega_c) dc_n$ for $x \in \{k, \ell, y\}$.

Using equation (A.8c),

$$d\ell_i = \frac{dy_i - \alpha dk_i}{\beta}.$$

Substitute into equation (A.8a):

$$dw = \xi dy_i - \frac{dy_i - \alpha dk_i}{\beta\eta},$$
$$dk_i = \frac{\beta\eta}{\alpha} \left(dw - \left(\xi - \frac{1}{\beta\eta} \right) dy_i \right).$$

Rewrite equation (A.8b) as $dy_i = \left(\frac{1}{\eta} dk_i + dr_i \right) / \xi$ and substitute:

$$\begin{aligned} dk_i &= \frac{\beta\eta}{\alpha} \left(dw - \left(\xi - \frac{1}{\beta\eta} \right) \frac{1}{\xi} \left(\frac{1}{\eta} dk_i + dr_i \right) \right) \\ &= \beta\eta \left(\frac{\eta\xi}{(\alpha + \beta)\eta\xi - 1} \right) \left(dw - \left(1 - \frac{1}{\beta\eta\xi} \right) dr_i \right). \end{aligned} \tag{A.9}$$

Next solve for labor as a function of prices using equations (A.8a) and (A.8b) to write $d\ell_i = -\eta\left(dw - dr_i\right) + dk_i$:

$$\begin{aligned} d\ell_i &= -\eta\left(dw - dr_i\right) + \beta\eta\left(\frac{\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)\left(dw - \left(1 - \frac{1}{\beta\eta\xi}\right)dr_i\right) \\ &= \eta\left(\frac{1-\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)dw + \eta\left(\frac{\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)dr_i. \end{aligned} \tag{A.10}$$

Using equation (A.10) for sector n and equation (A.8d):

$$d\ell_c = -\left(\frac{1-\omega_c}{\omega_c}\right)\left(\eta\left(\frac{1-\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)dw + \eta\left(\frac{\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)dr_n\right).$$

Equate with equation (A.10) for sector c and solve for w:

$$dw = -\left(\frac{\alpha\eta\xi}{1-\alpha\eta\xi}\right)\left(\omega_c dr_c + \left(1-\omega_c\right)dr_n\right). \tag{A.11}$$

Putting equations (A.9) to (A.11) and (A.8c) together gives the solution:

$$\frac{dw}{dr_c} = -\left(\frac{\alpha\eta\xi}{1-\alpha\eta\xi}\right)\omega_c, \tag{A.12a}$$

$$\frac{dk_n}{dr_c} = \beta\eta\left(\frac{\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)\frac{dw}{dr_c}, \tag{A.12b}$$

$$\frac{dk_c}{dr_c} = \frac{dk_n}{dr_c} - \eta\left(\frac{\beta\eta\xi - 1}{\left(\alpha+\beta\right)\eta\xi - 1}\right), \tag{A.12c}$$

$$\frac{d\ell_n}{dr_c} = \eta\left(\frac{1-\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right)\frac{dw}{dr_c}, \tag{A.12d}$$

$$\frac{d\ell_c}{dr_c} = \frac{d\ell_n}{dr_c} + \eta\left(\frac{\alpha\eta\xi}{\left(\alpha+\beta\right)\eta\xi - 1}\right), \tag{A.12e}$$

$$\frac{dy_i}{dr_c} = \alpha\frac{dk_i}{dr_c} + \beta\frac{d\ell_i}{dr_c}. \tag{A.12f}$$

Figure A.1 repeats Figure 3 in the economy with two sectors, starting from initial symmetry.

A.4 Tax Revenue

This appendix expands on results in Section 5.4. The derivation makes use of two preliminary calculations related to the CES production function. First, the partial (holding labor fixed) semi-elasticity of the labor share of revenue to capital is:

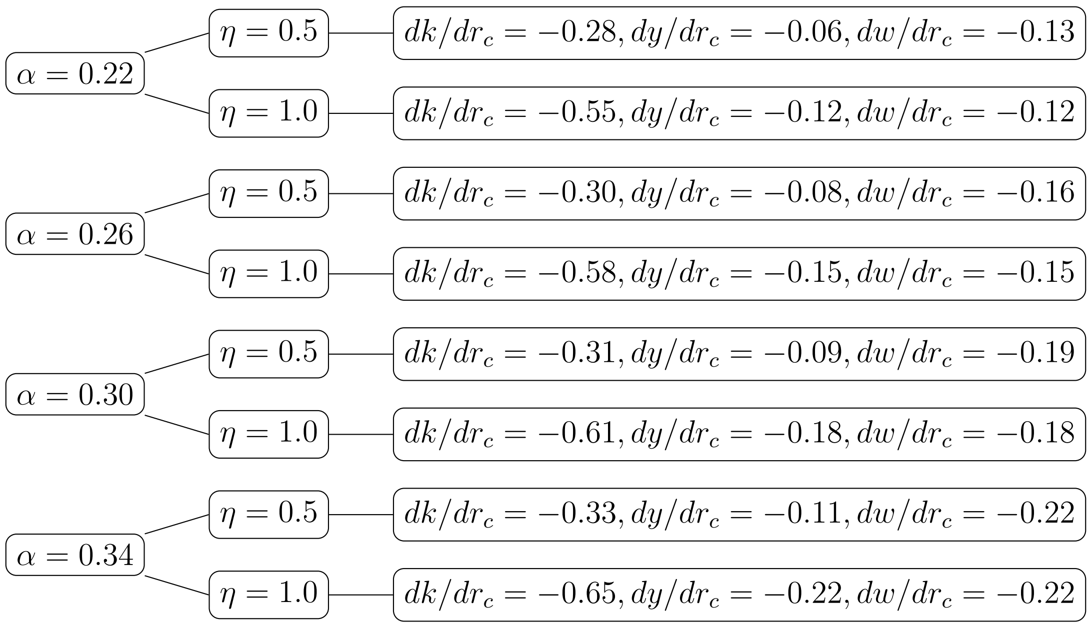
$$\frac{\partial\beta^*}{\partial k} = \left(\frac{1-\eta}{\left(\alpha+\beta\right)\eta}\right)\alpha^*\beta^*. \tag{A.13}$$

This result uses:

$$\beta^* = \frac{F_L\left(K_{i,t}, L_{i,t}\right)L_{i,t}}{F\left(K_{i,t}, L_{i,t}\right)} = \beta F\left(K_{i,t}, L_{i,t}\right)^{\frac{1-\eta}{(\alpha+\beta)\eta}} L_{i,t}^{1-1/\eta} = \frac{\beta}{\left(\frac{\alpha}{\alpha+\beta}\right)\left(K_{i,t}/L_{i,t}\right)^{1-1/\eta} + \left(\frac{\beta}{\alpha+\beta}\right)},$$

$$\alpha^* = \frac{F_K\left(K_{i,t}, L_{i,t}\right)K_{i,t}}{F\left(K_{i,t}, L_{i,t}\right)} = \alpha F\left(K_{i,t}, L_{i,t}\right)^{\frac{1-\eta}{(\alpha+\beta)\eta}} K_{i,t}^{1-1/\eta} = \frac{\alpha}{\left(\frac{\alpha}{\alpha+\beta}\right) + \left(\frac{\beta}{\alpha+\beta}\right)\left(K_{i,t}/L_{i,t}\right)^{1/\eta-1}}.$$

Figure A.1: General Equilibrium Elasticities with Two Sectors



Notes: Each "tree" starts from a value for the revenue elasticity of capital α. The "branches" set the elasticity of substitution between capital and labor η. The "leaves" give the implied values for the long-run general equilibrium user cost elasticity of corporate capital, corporate value added, and the wage. In all branches the markup is set to $\mathcal{M} = 1$ and the share of labor in the C-corporate sector is set to $\omega_c = 0.43$.

Second, the cross-partial of the CES production function is:

$$
\begin{aligned}
F_{LK}\left(K_{i,t}, L_{i,t}\right) &= \left(1 + \frac{1-\eta}{(\alpha+\beta)\,\eta}\right) \beta F\left(K_{i,t}, L_{i,t}\right)^{\frac{1-\eta}{(\alpha+\beta)\eta}} L_{i,t}^{-\frac{1}{\eta}} F_K\left(K_{i,t}, L_{i,t}\right) \\
&= \left(1 + \frac{1-\eta}{(\alpha+\beta)\,\eta}\right) \frac{F_K\left(K_{i,t}, L_{i,t}\right) F_L\left(K_{i,t}, L_{i,t}\right)}{F\left(K_{i,t}, L_{i,t}\right)}.
\end{aligned}
$$

Using the first order conditions (5a) and (8) and equation (A.13), in steady state this becomes:

$$
\frac{A_i F_{LK}\left(K_i, L_i\right) K_i L_i}{Y_i} = \left(1 + \frac{1-\eta}{(\alpha+\beta)\,\eta}\right) \alpha^* \beta^* = \alpha^* \beta^* + \frac{\partial \beta^*}{\partial k}. \tag{A.14}
$$

Change in τ Let $CTB = CT/\tau$ denote the corporate tax base:

$$
CTB = A\left(F\left(K, L\right) - F_L\left(K, L\right) L\right) - \delta z K.
$$

The change in corporate taxes in response to a 1% change in the corporate tax rate, expressed as a share of baseline total corporate value-added, is:

$$\begin{aligned}
\frac{1}{Y} \times \frac{dCT}{d\log\tau} &= \left(\frac{CT}{Y}\right)\frac{d\log CT}{d\log\tau} && \text{(A.15)}\\
&= \left(\frac{CT}{Y}\right)\left(1+\frac{d\log CTB}{d\log\tau}\right)\\
&= \left(\frac{CT}{Y}\right)\left(1+\frac{1}{CTB}\left(A\left(F_K\left(K,L\right)-F_{KL}\left(K,L\right)L\right)-\delta z\right)\frac{dK}{d\log\tau}\right)\\
&= \left(\frac{CT}{Y}\right)\left(1+\frac{1}{CTB}\left(A\left(F_K\left(K,L\right)K-F_{KL}\left(K,L\right)KL\right)-\delta zK\right)\frac{dk}{d\log\tau}\right)\\
&= \frac{CT}{Y}+\tau\left(\alpha^*\left(1-\beta^*\right)-\frac{\partial\beta^*}{\partial k}-\frac{zI}{Y}\right)\frac{dk}{d\log\tau}. && \text{(A.16)}
\end{aligned}$$

The change in labor taxes is:

$$\begin{aligned}
\frac{1}{Y} \times \frac{dLT}{d\log\tau} &= \frac{\tau^L A F_{KL}\left(K,L\right)L}{Y}\frac{dK}{d\log\tau}\\
&= \frac{\tau^L A F_{KL}\left(K,L\right)KL}{Y}\frac{dk}{d\log\tau}\\
&= \tau^L\left(\alpha^*\beta^*+\frac{\partial\beta^*}{\partial k}\right)\frac{dk}{d\log\tau}. && \text{(A.17)}
\end{aligned}$$

Change in z We have:

$$\begin{aligned}
\frac{1}{Y} \times \frac{dCT}{dz} &= \left(\frac{\tau}{Y}\right)\frac{d}{dz}\left[A\left(F\left(K,L\right)-F_L\left(K,L\right)L\right)-\delta zK\right]\\
&= \left(\frac{\tau}{Y}\right)\left(\left(A\left(F_K\left(K,L\right)K-F_{KL}\left(K,L\right)KL\right)-\delta zK\right)\frac{dk}{dz}-\delta K\right)\\
&= -\frac{\tau I}{Y}+\tau\left(\alpha^*\left(1-\beta^*\right)-\frac{\partial\beta^*}{\partial k}-\frac{zI}{Y}\right)\frac{dk}{dz}.
\end{aligned}$$

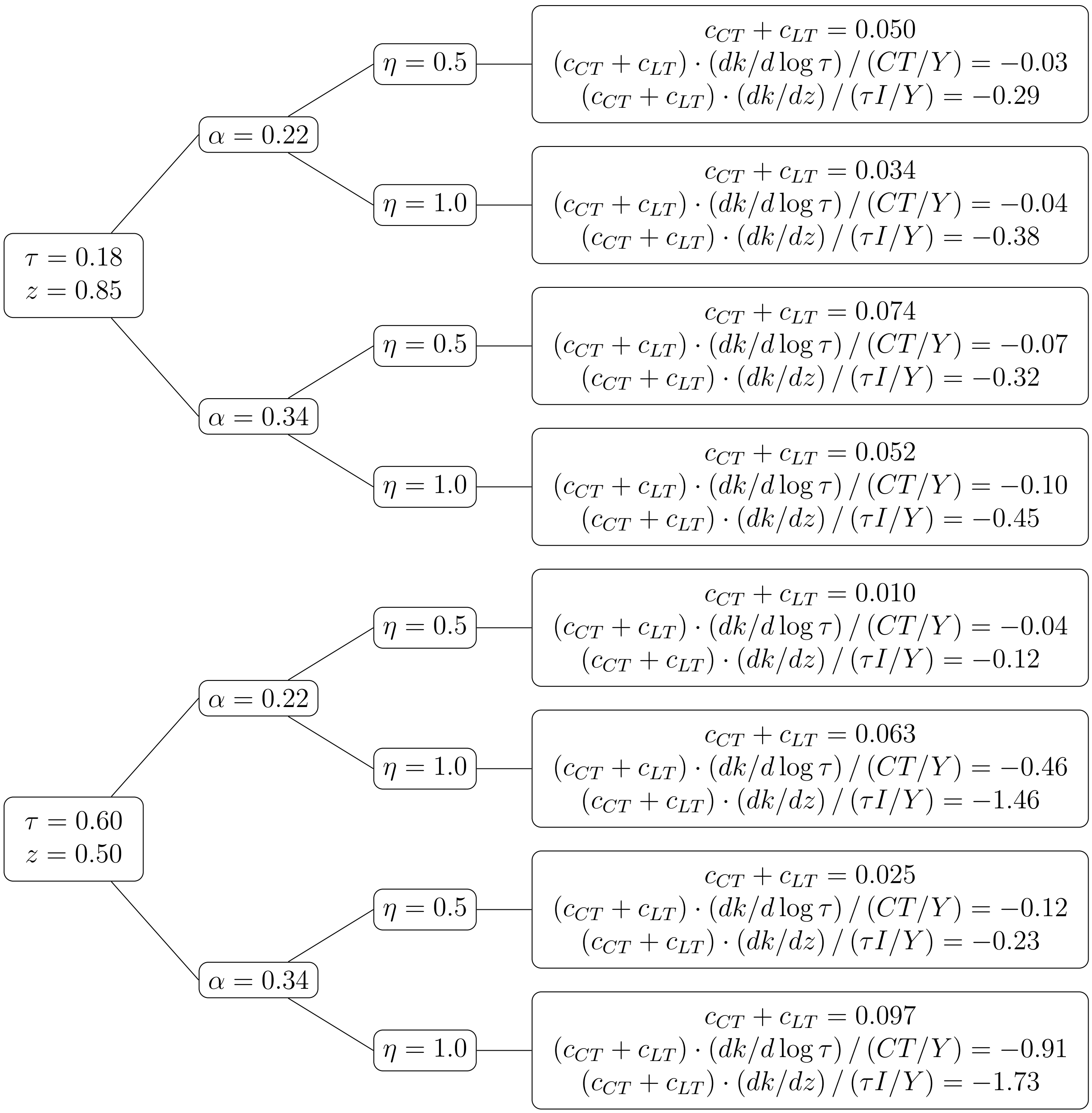
Thus, while the mechanical effect of a change in z differs from the mechanical effect of a change in $\log\tau$, the dynamic coefficient for corporate taxes is the same. The dynamic coefficient for labor taxes is likewise the same.

Figure A.2 shows values for the ratio of the dynamic to mechanical revenue change for different policy instruments and parameters. The top tree uses values of τ and z approximately equal to those in the U.S. in 2024. The bottom tree shows that with higher baseline tax rates, larger dynamic revenue effects are possible.

B Additional Notes on the Literature

This appendix contains additional notes on the papers listed in Tables 1 to 4.

Figure A.2: General Equilibrium Dynamic Tax Responses



Notes: Each "tree" starts from a value for the corporate rate τ and present value of expensing z. The "branches" set the capital output elasticity α and elasticity of substitution between capital and labor η. The "leaves" give the implied values for the long-run general equilibrium dynamic tax coefficient and the ratio of the dynamic to mechanical revenue change. All trees and branches use $\tau^L = 0.28$ and $\mathcal{M} = 1$.

B.1 Coen (1969) Specifications

The additional Coen (1969) estimating equations are:

$$k_{i,t} = b_0^{\text{Coen}k,r} + b_1^{\text{Coen}k,r} \log \Omega_{i,t} + e_{i,t}, \tag{B.1a}$$

$$\log I_{i,t} = b_0^{\text{Coen} \log I,r} + b_1^{\text{Coen} \log I,r} \log \Omega_{i,t} + e_{i,t}, \tag{B.1b}$$

$$I_{i,t}/K_{i,t-1} = b_0^{\text{Coen}I/K,\Omega} + b_1^{\text{Coen}I/K,\Omega} \Omega_{i,t} + e_{i,t}, \tag{B.1c}$$

$$I_{i,t}/K_{i,t-1} = b_0^{\text{Coen}I/K,R} + b_1^{\text{Coen}I/K,R} R_{i,t} + e_{i,t}, \tag{B.1d}$$

$$\log I_{i,t} = b_0^{\text{Coen} \log I,z} + b_1^{\text{Coen} \log I,z} z_{i,t} + e_{i,t}. \tag{B.1e}$$

Lemmas B.1 to B.4 provide the structural interpretations. Equation (B.1a) across steady states directly mirrors equations (17) and (24):

Lemma B.1 *In the environment of Section 2.3 with Cobb-Douglas production across steady states, plim $b_1^{Coenk,r} = dk_i/dr_i = -1/(1-\gamma)$, where $\gamma = \alpha/(1-\beta)$ as in equation (16). In the general CES case, plim $b_1^{Coenk,r} = dk_i/dr_i = -\left(\frac{1-\beta_i^*\left(\eta+\frac{1-\eta}{\alpha+\beta}\right)}{1-\alpha-\beta}\right)$.*

Lemma B.1 also applies to equation (B.1b) estimated across steady states, since $d\log I_i = dk_i$.

Let $k(h)$ and $I(h)$ denote log capital and investment h periods after a tax change; for compactness, continue to use the shorthand $k = k(\infty)$ when no explicit argument for h is given. Using equations (34) and (36):

Lemma B.2 *In the environment of Section 2.6 with quadratic adjustment costs in capital, equation (B.1a) estimated h periods after a tax reform recovers plim $b_1^{Coenk(h),r} = \left(1-e^{\chi_i h}\right) dk_i/dr_i$, where χ_i is the stable eigenvalue given in equation (33). Using $dI_{i,0}/K_{i,0} = \delta d\log I_{i,0}$, equation (B.1b) estimated immediately after a tax reform recovers plim $b_1^{Coen \log I(0),r} = (1/\delta)$ plim $b_1^{CoenI(0)/K,r} = -\left(\chi_i/\delta\right) dk_i/dr_i$.*

Equations (B.1c) to (B.1e) require additional steps since their regressors are $\Omega_{i,t}$, $R_{i,t}$, or $z_{i,t}$, respectively, rather than $\log \Omega_{i,t}$ as in equations (B.1a), (B.1b) and (39). Equation (B.1c) appears close to the Summers specification (38); both specifications have $I_{i,t}/K_{i,t-1}$ on the left hand side and they differ only in whether the regressor includes the component $\lambda_{i,t}/(1-\tau_{i,t})$ in $Q_{i,t}$. From the perspective of equation (38), this difference has an interpretation of omitted variables bias in equation (B.1c), since the tax change also causes a change in $\lambda_{i,t}$, as shown in Figure 1. The correct interpretation of equation (B.1c) therefore comes from equation (36) after rescaling the regression coefficient by $\Omega_{i,t}$, since equation (36) and Lemma B.2 relate $I_{i,t}/K_{i,t}$ to the log rather than the level of the tax term. A similar interpretation applies to equation (B.1d):

Lemma B.3 *In the environment of Section 2.6 with quadratic adjustment costs in capital, equation (B.1c) estimated immediately after a tax reform recovers plim $b_1^{CoenI(0)/K,\Omega} = (1/\Omega_i)$plim $b_1^{CoenI(0)/K,r} = -\left(\chi_i/\Omega_i\right) dk_i/dr_i$ and equation (B.1d) recovers plim $b_1^{CoenI(0)/K,R} = (1/R_i)$plim $b_1^{CoenI(0)/K,r} = -\left(\chi_i/R_i\right) dk_i/dr_i$. Equation (B.1d) estimated h periods after a tax reform recovers plim $b_1^{CoenI(h)/K(h),R} = -(\chi_i/R_i)e^{\chi_i h} dk_i/dr_i$.*

Finally:

Lemma B.4 *Assume the environment of Section 2.6 with quadratic adjustment costs in capital and tax policy changes to z only. Using $dz = \left(\frac{1-\Gamma}{\tau}\right)\frac{d\tau z}{1-\Gamma} = -\left(\frac{1-\Gamma}{\tau}\right) d\log\left(1-\Gamma\right)$, plim $b_1^{Coen \log I(0),z} = -\left(1/\delta\right)\left(\tau/\left(1-\Gamma\right)\right) b_1^{CoenI(0)/k,r}$; also using Lemma B.2, plim $b_1^{Coenk(h),z} = -\left(\tau/\left(1-\Gamma\right)\right)\left(1-e^{\chi_i h}\right) dk_i/dr_i$.*

B.2 Eisner and Nadiri (1968) Specifications

The additional Eisner and Nadiri (1968) estimating equations are:

$$\Delta\left(k_{i,t}-y_{i,t}\right)=b_0^{\text{E-NCRS}}+b_1^{\text{E-NCRS}}r_{i,t}+b_2^{\text{E-NCRS}}\left(k_{i,t-1}-y_{i,t-1}\right)+b_3^{\text{E-NCRS}}\Delta r_{i,t}+e_{i,t}, \tag{B.2a}$$

$$I_{i,t}/K_{i,t-1}=b_0^{\text{E-NDLM}}+\sum_{h=0}^{H}b_{1,h}^{\text{E-NDLM}}\Delta r_{i,t-h}+\sum_{h=0}^{H}b_{2,h}^{\text{E-NDLM}}\Delta y_{i,t-h}+e_{i,t}, \tag{B.2b}$$

$$\begin{aligned} k_{i,t}&=b_0^{\text{E-NECM}}+b_1^{\text{E-NECM}}r_{i,t-1}+b_2^{\text{E-NECM}}y_{i,t-1}+b_3^{\text{E-NECM}}k_{i,t-1}\\ &+\sum_{h=0}^{H-1}\left(b_{1,h}^{\text{E-NECM}}\Delta r_{i,t-h}+b_{2,h}^{\text{E-NECM}}\Delta y_{i,t-h}\right)+e_{i,t}. \end{aligned} \tag{B.2c}$$

The lemmas relating these equations to structural objects are given below in the context of the specific papers estimating them.

B.3 Individual Articles

Auerbach and Hassett (1991) They do not report summary statistics for the value of the user cost. For the non tax component, they use a discount rate $\rho = 0.04$ (p. 203). Using their table 1, I calculate an asset-weighted average depreciation rate for equipment of $\delta = 0.15$, where the asset weights come from assuming 1985 investment is equal to the replacement level of investment. For the tax component, Cummins, Hassett and Hubbard (1994, figure 1) implies a value $\Omega = 1.17$ for equipment in 1985. Together, this gives an average user cost for equipment $R = (\rho+\delta)\Omega = 0.226$. Note that this is close to the value of 0.25 reported in Cummins, Hassett and Hubbard (1994) for their full sample.

Cummins, Hassett and Hubbard (1994) I include their results for major tax reform years.

Zwick and Mahon (2017) I report their specification using $\log I$ because it is their preferred outcome and the only outcome for which they report results separately by firm size.

Curtis et al. (2021) This paper regresses the long-difference in equipment capital on an indicator for a firm's no-bonus z, denoted z_N, being in the bottom tercile of the distribution. The change in z given a value of bonus of B is $B + (1-B)\,z_N - z_N = B\,(1-z_N)$. Thus, the mean difference-in-difference change in z across the "treated" and "control" firms is $-B \times (\mathbb{E}_i[z_{i,N}|z_{i,N} \leq P_{33}(z_N)] - \mathbb{E}_i[z_{i,N}|z_{i,N} > P_{33}(z_N)])$. In the main text I use the difference-in-difference notation $\Delta_{2001,2011}\Delta_{z\leq P_{33}(z),z>P_{33}(z)}$ for this difference. Figure A.1 of their paper shows mean values of z_N in the "treated" and "control" groups of 0.795 and 0.863, respectively.[1] The value of bonus fluctuated during their sample period. I use $B = 50\%$ as the modal value, implying $\Delta_{2001,2011}\Delta_{z\leq P_{33}(z),z>P_{33}(z)} = 0.034$. Since these zs apply only to equipment, I report the equipment capital result in their table 2.

Chodorow-Reich et al. (2025) I report their results for domestic-only firms since these firms fit into the theory in Section 2.1.

Kennedy et al. (2026) Their featured regression estimates: $I_{i,t}/K_{i,t-1} = b_1 d\log(1-\tau) \times \mathbb{I}\text{year} \geq 2018 + \text{firm FE} + \text{industry} \times \text{size} \times yearFE$ in a panel of C and S corporations from 2013-2019. They treat $d\log(1-\tau)$ as an endogenous variable and use an indicator for being

[1]The precise values were provided via personal communication from the authors.

a C-corporation as the excluded instrument, since the corporate rate cut applied to C but not S corporations. One can relate this specification to a regression with $d\log\Omega$ as the regressor using $d\log\Omega = d\log(1-\Gamma) - d\log(1-\tau) = -\frac{\tau dz}{1-\Gamma} + \left(\frac{z}{\Omega}-1\right) d\log(1-\tau)$. Since the expensing provisions applied to both, dz is the same for both C and S corporations and its effect is essentially absorbed by the industry-size-year fixed effects. However, $d\log\Gamma$ depends on both dz and $d\tau$, requiring a re-scaling. Table 2 instead reports the coefficient from their regression with $d\log\Omega$ as the regressor, where they construct Ω taking account of credits, deductions, and NOLs.

Caballero, Engel and Haltiwanger (1995) Their Appendix B motivates their regression specification by starting from the Cobb-Douglas case with (ignoring productivity) $Y_{i,t} = K_{i,t}^{\gamma}$ (see equation (14)) and the associated first order condition for steady state capital $\gamma K_i^{\gamma-1} = R_{i,t}$, where $R_{i,t}$ here denotes the steady state user cost, which may vary over time. Subtract $k_{i,t}$ from both sides of the first order condition (in logs) to find:

$$k_i - k_{i,t} = -\frac{1}{1-\gamma} r_{i,t} - k_{i,t} = \frac{1}{1-\gamma}\left(y_{i,t} - k_{i,t} - r_{i,t}\right),$$

where the second equality uses the production function identity $y_{i,t} - k_{i,t} = (\gamma-1)k_{i,t}$. Caballero, Engel and Haltiwanger (1995) insert an *ad hoc* scalar multiplying $r_{i,t}$ and assume the left hand side is stationary to motivate a cointegrating relationship between the log capital-output ratio and the log user cost.

An alternative motivation without requiring the *ad hoc* scalar starts from CES. Using equation (21), take logs and rearrange to find:

$$k_{i,t} = \left(\eta + \frac{1-\eta}{(\alpha+\beta)}\right) y_{i,t} - \eta r_{i,t}.$$

Lemma B.5 summarizes:

Lemma B.5 *In the environment of Section 2.4 with CES production and constant returns to scale in the revenue function, $\alpha+\beta=1$, equation (B.2a) recovers $\eta = b_1^{E\text{-}NCRS}/b_2^{E\text{-}NCRS}$.*

Thus, a long-run cointegrating relationship between the log capital-output ratio and log user cost holds exactly only under constant returns to scale, $\alpha+\beta=1$, or the Cobb Douglas case $\eta=1$.

Chirinko, Fazzari and Meyer (1999) Let $K_{i,t}^*$ denote the firm's "desired capital" at date t, defined as the solution to equation (21) taking $Y_{i,t}$ as given. Instead of an explicit function for adjustment costs, Chirinko, Fazzari and Meyer (1999) assume that net investment is a function of lagged changes in log desired capital:

$$\frac{I_{i,t}}{K_{i,t-1}} = \delta + \sum_{h=0}^{H} \omega_h \Delta k_{i,t-h}^*,$$

where $\sum_{h=0}^{H} \omega_h = 1$. Substitute for $\Delta k_{i,t-h}^*$ using equation (21):

$$\frac{I_{i,t}}{K_{i,t-1}} = \delta + \sum_{h=0}^{H} \omega_h \left(-\eta \Delta r_{i,t-h} + \left(\eta + \frac{1-\eta}{\alpha+\beta}\right)\Delta y_{i,t-h}\right) + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ is a summation of past productivity changes. Summarizing:

Lemma B.6 *Assume $I_{i,t}/K_{i,t-1} = \delta + \sum_{h=0}^{H}\omega_h \Delta k_{i,t-h}^*$, where $\sum_{h=0}^{H}\omega_h = 1$. Then in the environment of Section 2.4 with CES production, equation (B.2b) recovers $\sum_{h=0}^{H} b_{1,h}^{E\text{-}NDLM} = -\eta$ and $\sum_{h=0}^{H} b_{2,h}^{E\text{-}NDLM} = \eta + \frac{1-\eta}{\alpha+\beta}$.*

Dwenger (2014) Dwenger assumes that current log capital is a function of lagged capital and current and lagged desired capital. After substituting using equation (21) as above:

$$k_{i,t} = \phi k_{i,t-1} + \sum_{h=0}^{H} \omega_h \left(-\eta r_{i,t-h} + \left(\eta + \frac{1-\eta}{\alpha+\beta} \right) y_{i,t-h} \right) + \epsilon_{i,t}.$$

Dwenger (2014) manipulates this expression as follows. In a steady state, we must have:

$$(1-\phi)\, k_i = k_i \sum_{h=0}^{H} \omega_h.$$

Using this fact, one can show that for any x:

$$\begin{aligned} \sum_{h=0}^{H} \omega_h x_{i,t-h} &= (1-\phi)\, x_{i,t-1} - \sum_{h=0}^{H-1} \mu_h \Delta x_{i,t-h}, \\ \text{where: } \mu_0 &= -\omega_0, \\ \mu_h &= \sum_{j=h+1}^{H} \omega_j \ \forall k > 0. \end{aligned}$$

Thus:

$$\begin{aligned} k_{i,t} = \phi k_{i,t-1} - (1-\phi)\, \eta r_{i,t-1} + (1-\phi) \left(\eta + \frac{1-\eta}{\alpha+\beta} \right) y_{i,t-1} \\ - \sum_{h=0}^{H-1} \mu_h \left(-\eta \Delta r_{i,t-h} + \left(\eta + \frac{1-\eta}{\alpha+\beta} \right) \Delta y_{i,t-h} \right) + \epsilon_{i,t}. \end{aligned}$$

Dwenger (2014) also includes a sample attrition term in her preferred specification. Summarizing:

Lemma B.7 *Assume $k_{i,t} = \phi k_{i,t-1} + \sum_{h=0}^{H} \omega_h k^*_{i,t-h}$, where $\sum_{h=0}^{H} \omega_h = 1-\phi$. Then in the environment of Section 2.4 with CES production, equation (B.2c) recovers $b_1^{E\text{-}NECM}/(1-b_3^{E\text{-}NECM}) = -\eta$ and $b_2^{E\text{-}NECM}/(1-b_3^{E\text{-}NECM}) = \eta + \frac{1-\eta}{\alpha+\beta}$.*