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PROPAGATION THROUGH PRODUCTION NETWORKS

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ABSTRACT

Skewness in macroeconomic time series may arise exogenously if shocks are asymmetrically distributed, or endogenously, as shocks propagate through production networks. These two possibilities are often considered in isolation by previous theoretical work. To evaluate the relative importance of these channels, we nest all possible sources of skewness in an empirical model where output has a network, a common, and an idiosyncratic component. In this model, skewness can arise not only from the three components, but also from coskewness due to their higher order covariation. An analysis of output growth in 43 U.S. sectors shows that coskewness, especially between the network and the common component, is a key source of asymmetry in the data and constitutes a connectivity channel not previously explored in the literature. To help interpret our results, we construct and estimate a micro-founded multi-sector general equilibrium model and show that it can generate skewness and coskewness consistent with the data.

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1 Introduction

Aggregate skewness in economic models can arise because of skewed common shocks that directly affect all sectors in the economy. It can also arise if a large shock hits a sector that happens to be granular.¹ Several studies in the literature develop models with sector-specific production technologies and heterogeneous elasticities of substitution to highlight the complex ways in which sectors are interconnected.² In these models, skewed outcomes arise from the asymmetric propagation of symmetric negative and positive shocks through the production network, especially when inputs are not easily substitutable. Thus, different sources of aggregate skewness can co-exist: while exogenous common shocks generate global connectivity, production networks endogenously generate local connectivity. Each of these channels can individually or jointly contribute to the variability and skewness of aggregate outcomes.

This paper develops a framework that encompasses both global and local connectivity. The framework is a network-augmented factor model that is a first-order approximation to the various multi-sector network models considered in the literature, but embodies a network propagation effect, common shocks, and sector-level shocks. Expressing sector-level output growth as the sum of three components allows us to quantify the network multiplier in the presence of common shocks. It also allows us to assess the relative importance of the components in aggregate variance and aggregate skewness. We highlight how common and sectoral shocks interact with the production structure to generate aggregate outcomes, and draw attention to features in the data that production network-based structural models of business cycles should match.

To assess the relative importance of the sources of skewness, we focus on Fisher’s coefficient of skewness,

$$\kappa_3 = \frac{\mu_3}{\sigma^3} = \frac{E[(X - \mu)^3]}{\sigma^3},$$

where μ_k is the k -th central moment of the random variable X and $\sigma^2 = \mu_2$ is the variance.³ Our point of departure is that if X is the sum of components, the skewness in X is not the sum of the skewness of the components. For example, if $X_1 \sim (\mu_1, \sigma_1^2)$ and $X_2 \sim (\mu_2, \sigma_2^2)$, then the coefficient

¹For skewness due to disasters, see Rietz (1988), Barro (2006), and Nakamura, Steinsson, Barro, and Ursua (2013). For skewed sector-specific shocks to granular sectors, see Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012).

²Examples include Atalay (2017), Baqaee and Farhi (2019), and Miranda-Pinto and Young (2022).

³Note that time-series tests for skewness need to account for weak dependence in the data. See Bai and Ng (2001), Bai and Ng (2005), and Bontemps and Meddahi (2005).

of skewness for $X = X_1 + X_2$ is

$$\begin{aligned}\kappa_3 &= E\left(\frac{(X_1 - \mu_1) + (X_2 - \mu_2)}{\sigma}\right)^3 \\ &= \frac{\sigma_1^3}{\sigma^3}\kappa_1 + \frac{\sigma_2^3}{\sigma^3}\kappa_2 + \frac{3\sigma_1^2\sigma_2}{\sigma^3}E(X_1 - \mu_1)^2(X_2 - \mu_2) + \frac{3\sigma_2^2\sigma_1}{\sigma^3}E(X_2 - \mu_2)^2(X_1 - \mu_1).\end{aligned}$$

Note that the skewness of X is not simply the (variance-weighted) sum of skewness of X_1 and X_2 , but includes two terms coming from the higher order covariability between X_1 and X_2 . In fact, X can be skewed even if X_1 and X_2 are symmetric, provided that the last two terms are non-zero. These two terms, known as coskewness, will not be zero unless X_1 and X_2 are independent. The decomposition is akin to an ANOVA analysis for variance, but involves more interaction terms because we analyze the third moment.

As an illustration of the quantitative importance of the coskewness terms that are the focus of this paper, consider the decomposition of the skewness of nominal GDP growth. The data is quarterly from 1950Q1 to 2024Q4 and, by construction, is the sum of real GDP growth and the growth rate of the implicit price deflator.⁴ The Fisher’s coefficient of skewness of nominal GDP growth is -0.99 , and, hence, this variable is negatively skewed as documented in previous literature. The terms of the decomposition above are -0.75 , 0.16 , -0.26 , and -0.14 , respectively. The first and second terms are, respectively, the contributions from the skewness of real GDP growth and the growth rate of the deflator, and the latter two terms are the coskewness terms. Note that the contribution of the coskewness terms is of the same order of magnitude as that of the first two terms. Their net effect is to overcome the positive skewness of inflation (as measured by the growth rate of the deflator) and increase the negative skewness of nominal GDP growth so that it is actually *larger* in absolute value than that of its real counterpart.

Our problem is more complicated because we have three components—a network (local), a common (global), and idiosyncratic—the dynamics are possibly involved, and the relation must be consistent with structural constraints based on the observed input-output matrix. Despite these complications, we show that the network-augmented factor model provides a framework to segregate the network propagation effect from that of the asymmetric shocks. In this sense, our paper extends previous literature (e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017)) that shows that network effects and asymmetric shocks jointly deliver skewed aggregate series. But by decomposing their relative contributions, we shed light on the channel of coskewness that has not previously been investigated.

We analyze data on sector-level output growth at the 3-digit level of the North American

⁴The raw data were taken from FRED at the Federal Reserve Bank of St. Louis. Real GDP is computed as the ratio of nominal GDP and the implicit price deflator, both of which are seasonally adjusted.

Industry Classification System (NAICS), and use data at the 2-digit level as a robustness check. Although the Bureau of Economic Analysis (BEA) constructs input-output (I-O) tables for the sample period 1957-2019, its accounting-based connectivity matrix may not necessarily reflect all the spillovers in the data that our econometric model seeks to explain. We capture the possible discrepancies using a matrix with a few parameters to be estimated together with the common and idiosyncratic components using the Generalized Method of Moments (GMM). Based on these estimates, an empirical inverse Leontief matrix identifies a few sectors—notably Trade, Finance and Insurance, Information Technology, and Services—to play a dominant role in the production network.

We then decompose aggregate skewness using both a factor-only model and our full network-augmented model. In the absence of network connections, the common component accounts for most of the skewness. However, once the network is introduced, the contribution of the common component declines significantly, with the majority of skewness attributed to the network and its interaction with other components. The network contributes to skewness directly—through production linkages—and indirectly—through coskewness with the common component—. In our baseline estimation, which allows for more flexible spillovers, the direct network effect rises to over 47% of total skewness, with the coskewness terms involving network and common components emerging as the dominant source of skewness, accounting for 50% of total skewness. These results support our central argument that higher-order co-variation, captured through network-based coskewness, is essential to understanding how production networks shape aggregate skewness. In terms of the central question of this paper, which is identifying possible sources of observed skewness and measuring their respective contribution, our results show that the production network accounts for a substantial part of aggregate skewness, in large part through network-based coskewness.

We also construct and estimate a micro-founded model with structural counterparts for the components of the reduced-form model and show that it can generate skewness and coskewness consistent with the U.S. data. This structural model helps us interpret the results from the network-augmented factor model and provides additional insights about the source of skewness in the data. As an additional contribution to the literature, we report method of moments estimates of the elasticity of substitution between material inputs and between consumption goods, respectively 0.83 and 0.75. The estimates are statistically different from 0 and 1 and imply gross complementarity. This result is important because some of the most interesting results in network analysis are based on the assumption of gross complementarity.

Harvey and Siddique (2000) appears to be one of the first works to consider conditional coskew-

ness as a feature in the cross-section of returns.⁵ This paper has generated significant interest in coskewness as a priced factor. For a recent review, see Harvey and Siddique (2023). Atalay, Drautzburg, and Wang (2018) compute the average coskewness of sectoral shocks implied by a calibrated multi-sector model to understand tail risk in aggregate output. We also analyze sectoral data, but our coskewness arises from higher-order covariation across the three components of aggregate output growth implied by our network-augmented factor model, not coskewness at the sectoral level. Furthermore, our focus is not on tail risk, but on the importance of skewness induced by exogenous shocks vis-à-vis the production network as an endogenous propagating mechanism. Our analysis was inspired by Dew-Becker and Vedolin (2021), which analyzes the time-series skewness implied by a multisector model with a rich production network structure but only idiosyncratic shocks exist. We additionally allow for global connectivity via common shocks, complementing the local connectivity enabled by the production network. Our analysis is also related to Dew-Becker and Vedolin (2025), which suggests conceptually new sources of skewness as being empirically plausible. In contrast, we suggest coskewness as a channel for skewness based on an empirical analysis of the data. It is encouraging that both analyses find that sectoral connections are a key driver of aggregate skewness.

The paper is organized as follows. Section 2 presents the data and econometric framework. Section 3 discusses the estimation of the network-augmented factor model and reports the reduced-form estimates. It also presents the results from the skewness decomposition, highlighting the relative importance of the network, common, and sector-specific components in explaining aggregate skewness. Section 4 constructs a multi-sector general equilibrium model with a production network, reports its structural estimates, and uses counterfactual experiments and decompositions to shed additional light on the sources of skewness observed in the U.S. data. Section 5 concludes. Appendix A contains additional information on the data. Online appendices B through D contain additional tables, figures, estimation details, and results using the 2-digit level of the NAICS.

2 Data and Econometric Framework

From early work by Neftci (1984), De Long and Summers (1988), Hamilton (1989) and Sichel (1993) to the more recent work by Salgado, Guvenen, and Bloom (2019), evidence of aggregate skewness is well documented. To get a glimpse of skewness in macroeconomic aggregates, we examine the output gap, the unemployment rate minus its natural rate (i.e., the unemployment

⁵The covariation between an asset i and the square of market returns is used as a measure of the skewness of asset i to the skewness of a well-diversified portfolio. Stocks with negative coskewness have higher downside risk and demand a higher risk premium, while stocks with high idiosyncratic volatility may have positive coskewness, thus low returns.

gap), and real output growth for 1960Q1 to 2019Q4 in the December 2024 vintage produced by the Congressional Budget Office (CBO). We also examine the data for 1960M1 to 2019M12 in the December 2024 vintage of FRED-MD. As explained in McCracken and Ng (2016), this panel of monthly macroeconomic variables is known to have a factor structure, meaning that it is well represented by the factor model $X = \mathcal{F}\Lambda + \varepsilon$. We estimate three factors by principal components, denoted respectively by $\hat{\mathcal{F}}_1$, $\hat{\mathcal{F}}_2$, and $\hat{\mathcal{F}}_3$, and find that these estimates explain about 50% of the variation in the data and load heavily on real activity, interest rate, and price variables, respectively. Table 1 shows that whether we consider quarterly or monthly data, these macro aggregates appear non-normal over the sample from 1960 to 2019, displaying negative skewness and/or excess kurtosis. Large negative values of $\hat{\mathcal{F}}_1$ are recorded in 1980M5, 1974M12, and 2008M12, which correspond to NBER recession dates. Even excluding the pandemic observations, both measures of skewness show stronger evidence of asymmetry in the 1984-2019 sample than in the pre-1984 sample.

It is also possible to define aggregate skewness from the bottom up by analyzing the GDP-by-industry database produced by the BEA. Since this is the primary data used in the rest of the paper, we describe it in more detail now. First, note that there have been changes to the definition of sectors over the years: At the 3-digit level, which is our base case, the number of sectors increased from 43 for the sample 1947-1962 to 61 for the sample 1963-1996 and further increased to 66 for 1997-2022. To strike a balance between the span of the data (T) and the number of sectors (N), we use the 1947 classification to aggregate data in subsequent years (i.e., 1947-2022) into 43 sectors. The sectors and their respective NAICS code are listed in Table A1 in Appendix A. Dropping the observations before the Korean War and after the pandemic leaves us with data for $N = 43$ sectors for the sample 1957-2019 with $T = 63$. Real gross sectoral output per capita is obtained by dividing total industry output (i.e., nominal gross output) from the Make Table of the Input-Output (I-O) accounts by the annual GDP deflator (from NIPA Table 1.1.9) and the estimate of the U.S. population produced by the BEA. The former series was obtained from the BEA website, and the latter two series were obtained from FRED at the Federal Reserve Bank of St. Louis. The growth rate of real output in sector i at time t , $y_{i,t}$, is computed as 100 times the log difference of real gross output per capita.

We assume that the $T \times N$ panel of data on real sectoral output growth rates Y has a factor representation, i.e., $Y = F\Lambda + e$. We estimate three factors and the corresponding idiosyncratic errors from the data. Summary statistics are reported in the Table B1. The three factors explain about 60% of the variation in the data. Most of the sectoral output growth rates and $\hat{e}$ series are not very persistent, as only around 30% have a first-order autocorrelation coefficient exceeding 0.3. The mean and median of the sectoral estimates of σ_i^y and $\sigma_i^{\hat{e}}$, for $i = 1, 2, \dots, 43$, are stable over the

sub-samples. The value of the first factor $\hat{F}_1$ is most negative in 2009, 1982, and 1975, which are years of economic downturns. Although the average (across sectors) of the skewness $\kappa_3^{\hat{e}}$ is close to zero, the average of the skewness κ_3^y is negative. Figure 1 plots the sample coefficient of skewness for each sector along with the sectoral weight. Though there is substantial heterogeneity, 33 of the 43 series are negatively skewed. Utilities, Apparel, and Air Transport (corresponding to Sectors 6, 21, 29) have the most negative κ_3^y while Farming, and Food and Tobacco Products (corresponding to sectors 1 and 19, respectively) are the most positively skewed.

A sector-based measure of aggregate output growth can be obtained by taking a weighted sum of sectoral output growth,

$$Y_t = \omega_t' \mathbf{y}_t$$

where $\mathbf{y}_t$ is the $N \times 1$ vector of sectoral output growth rates, y_{it} . The simplest approach is to use an equal weight of $1/N$. An alternative is to use the share of sector i 's output in total output at time t , $\omega_{it} = z_{it} / \sum_{i=1}^N z_{it}$. However, this measure can be sensitive to time variations. A more stable measure of weights is $\omega_{it} = \bar{\omega}_i = (1/T) \sum_{t=1}^T \omega_{it}$ for all t . The coefficients of skewness corresponding to the three weighting schemes are -1.257, -1.608, and -1.680. Regardless of the choice of weights ω_t , the aggregate output defined as above also exhibits negative skewness, consistent with the measures of skewness found for aggregate data reported earlier.

We construct a W_B matrix of input shares from the Use tables constructed by the BEA. The BEA records total purchases of inputs to industry production, as well as the commodities consumed by final users, in billions of current dollars at producer prices. We compute the average shares from the BEA data over the sample 1957-2019 and define a normalized matrix W_B satisfying $\sum_j W_{B,ij} = 1$. (See Appendix A for additional details.) The matrix W_B shows intuitive interactions between sectors. For instance, 66.5% of material expenditures by petroleum products goes to goods produced by oil and gas extraction, and 44.3% of material expenditures by food services goes to goods produced by the food and beverage sector. An important feature of the table at this level of disaggregation is the large values along the main diagonal, meaning that a substantial proportion of material expenditures by sector i goes to goods produced in the same sector. However, we show below that even if the I-O table is relatively sparse and features substantially larger diagonal entries than off-diagonal entries, it still implies strong connectivity across sectors.

2.1 A Network-augmented Factor Model

Foerster, Sarte, and Watson (2011) (hereafter, FSW) shows that the representation

$$(I_N - \ddot{\Phi}_1 L) \mathbf{y}_t = \ddot{\Pi}_0 \ddot{\epsilon}_t, \tag{1}$$

where I_N is an $N \times N$ identity matrix, L is the lag operator, and $\ddot{\Phi}_1$, and $\ddot{\Pi}_0$ are conformable matrices, encompasses the first-order approximate solution of many production network models previously considered in the literature. In this VAR specification, the contemporaneous effects occur through the shocks. Let α denote the diagonal matrix with sectoral intermediate input shares. If $\ddot{\Phi}_1 = \alpha W_B$ and $\ddot{\Pi}_0 = I$, (1) is the model in Long and Plosser (1983); if $\ddot{\Phi}_1 = 0$ and $\ddot{\Pi}_0 = (I - \alpha W_B)^{-1}$, it is the model in Carvalho (2010); and if $\ddot{\Pi}_0 = (I - \alpha W_B)^{-1}$ and $\ddot{\Phi}_1$ is a known diagonal matrix, it is the model in Hovarth (1998). FSW further assumes that $\ddot{e}_t = \ddot{\Lambda} \ddot{F}_t + \ddot{e}_t$ has a factor structure. In particular, $\ddot{\Lambda}$ is a $N \times r$ matrix of factor loadings corresponding to the $r \times 1$ vector of common factors $\ddot{F}_t$, and $\ddot{e}_t$ is a $N \times 1$ vector of idiosyncratic errors.

2.2 Decomposition of Y_t into Three Components

The focus of FSW is the relative importance of common and sector-specific shocks to output variance after controlling for the network effects.⁶ Our focus is the relative contribution of common shocks, sectoral shocks, and the production network to skewness. For our purpose, it is more convenient to consider the model

$$\mathbf{y}_t = \Phi_0 \mathbf{y}_t + \Phi_1 \mathbf{y}_{t-1} + \epsilon_t, \quad (2)$$

where Φ_0 and Φ_1 are $N \times N$ matrices. With $\epsilon_t = \Lambda F_t + e_t$, the baseline model is

$$\mathbf{y}_t = \Phi_0 \mathbf{y}_t + \Phi_1 \mathbf{y}_{t-1} + \Lambda F_t + e_t.$$

The inverse Leontief matrix is

$$\mathcal{L} = (I_N - \Phi_0)^{-1}.$$

The reduced form $\mathbf{y}_t = \mathcal{L}(\Phi_1 \mathbf{y}_{t-1} + \Lambda F_t + e_t)$ implies

$$\mathbf{y}_t = \underbrace{\mathcal{L}\Phi_1 \mathbf{y}_{t-1}}_{\text{network+dynamics}} + \underbrace{\mathcal{L}\Lambda F_t}_{\text{network+common}} + \underbrace{\mathcal{L}e_t}_{\text{network+idiosyncratic}}.$$

Thus, network effects are confounded with the common and idiosyncratic components in the data.

To gain more insights about the role of the production network, consider the static model and let $\mathcal{L}_{ij}$ be the (i, j) -th entry of $\mathcal{L} = (I_N - \Phi_0)^{-1}$. Then

$$\begin{aligned} \mathbf{y}_t &= \mathcal{L}\Lambda F_t + \mathcal{L}e_t \\ \begin{pmatrix} y_{1t} \\ y_{2t} \\ \dots \\ y_{Nt} \end{pmatrix} &= \begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} & \dots & \mathcal{L}_{1N} \\ \mathcal{L}_{21} & \mathcal{L}_{22} & \dots & \mathcal{L}_{2N} \\ \vdots & & & \\ \mathcal{L}_{N1} & \mathcal{L}_{N2} & \dots & \mathcal{L}_{NN} \end{pmatrix} \begin{pmatrix} \lambda_{11} \\ \lambda_{21} \\ \vdots \\ \lambda_{N1} \end{pmatrix} F_t + \begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} & \dots & \mathcal{L}_{1N} \\ \mathcal{L}_{21} & \mathcal{L}_{22} & \dots & \mathcal{L}_{2N} \\ \vdots & & & \\ \mathcal{L}_{N1} & \mathcal{L}_{N2} & \dots & \mathcal{L}_{NN} \end{pmatrix} \begin{pmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{Nt} \end{pmatrix} \end{aligned}$$

⁶Specifically, Foerster, Sarte, and Watson (2011) estimate the common component $\ddot{C}_t = \ddot{\Lambda} \ddot{F}_t$ after filtering out the network effects using a structural model i.e., by obtaining the model matrices $\ddot{\Phi}_1$, $\ddot{\Pi}_0$, and $\ddot{\Pi}_1$ to be functions of W_B and structural parameters.

For any $i = 1, \dots, N$

$$y_{it} = \sum_{j=1}^N \mathcal{L}_{ij} \lambda_{j1} F_t + \mathcal{L}_{ij} e_{it}.$$

We see that the output of sector i depends on the direct response of sector i to a change in F_t . But it also includes the indirect response of all sectors $j \neq i$ to the common factor F_t and the sector-specific shock e_{it} , both propagated through the network. The goal is to separate the total effect into direct and network effects, along the lines of Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2025), while incorporating lags and the common component.⁷ We do so by adding and subtracting terms:

$$\begin{aligned} \mathbf{y}_t &= \underbrace{(\mathcal{L} - I_N) \Lambda F_t + (\mathcal{L} - I_N) e_t}_{\text{current network}} + \underbrace{\mathcal{L} \Phi_1 \mathbf{y}_{t-1}}_{\text{dynamic network}} + \underbrace{\Lambda F_t}_{\text{common}} + \underbrace{e_t}_{\text{idiosyncratic}} \\ &= \underbrace{(\mathcal{L} - I_N) \Lambda F_t + (\mathcal{L} - I_N) e_t + \mathcal{L} \Phi_1 \mathbf{y}_{t-1}}_{\text{network}} + \underbrace{\Lambda F_t}_{\text{common}} + \underbrace{e_t}_{\text{idiosyncratic}}. \end{aligned}$$

Aggregate output growth is $Y_t = \omega' \mathbf{y}_t$ where ω' is $1 \times N$. A decomposition of aggregate output growth, $Y_t = \omega' \mathbf{y}_t$, is

$$\begin{aligned} Y_t &= \underbrace{\omega' (\mathcal{L} - I_N) \Lambda F_t + \omega' (\mathcal{L} - I_N) e_t + \omega' \mathcal{L} \Phi_1 \mathbf{y}_{t-1}}_{\text{network}} + \underbrace{\omega' \Lambda F_t}_{\text{common}} + \underbrace{\omega' e_t}_{\text{idiosyncratic}}. \quad (3) \\ &= \mathcal{N}_t + \mathcal{C}_t + \mathcal{E}_t. \end{aligned}$$

Thus, aggregate output growth has three components: local connectivity through the network, global connectivity through the common component, and idiosyncratic shocks. The spillover effects due to network connectivity are captured by $\mathcal{L} - I_N = \Phi_0 + \Phi_0^2 + \dots$. We refer to $(\mathcal{L} - I_N)$ as the Network Multiplier Matrix because it governs how shocks propagate through the network. Specifically, each (i, j) element of this matrix shows the multiplier effect of a shock in sector j on the output growth of sector i . We also define the Sectoral Network Multiplier, $\omega'(\mathcal{L} - I_N)$. This $N \times 1$ vector, calculated as a weighted sum along the rows for each column in the network multiplier matrix, quantifies the effect of a shock originating in any given sector on aggregate output growth.

The decomposition of the level of y_t leads to the following decomposition of variance of Y_t ,

$$\text{var}(Y_t) = \text{var}(\mathcal{N}_t) + \text{var}(\mathcal{C}_t) + \text{var}(\mathcal{E}_t) + 2\text{cov}(\mathcal{N}_t, \mathcal{C}_t) + 2\text{cov}(\mathcal{N}_t, \mathcal{E}_t) + 2\text{cov}(\mathcal{C}_t, \mathcal{E}_t).$$

A decomposition of skewness is more involved. As noted above, if a variable is the sum of two components, the skewness of the sum is not the sum of the skewness of the two components, but

⁷Acemoglu, Akcigit, and Kerr (2016) decomposes the impact of sectoral productivity (supply-side) shocks on the output of sector i into its own effect and the network effect. Ozdagli and Weber (2025) follows the same technique to decompose the impact of aggregate monetary policy shocks on sectoral asset returns into direct effect and network effect.

also depends on the higher order covariation of the components. Given that Y_t has three primitive components, a decomposition of the coefficient of skewness is

$$\begin{aligned}
\kappa_3^Y &= \frac{1}{T} \sum_{t=1}^T \left(\frac{Y_{it} - \mu_i}{\sigma_i} \right)^3 \\
&= s_{\mathcal{N}}^3 \kappa_3^{\mathcal{N}} + s_{\mathcal{C}}^3 \kappa_3^{\mathcal{C}} + s_{\mathcal{E}}^3 \kappa_3^{\mathcal{E}} \\
&\quad + 3s_{\mathcal{N}}^2 s_{\mathcal{C}} \text{CoS}(\mathcal{N}, \mathcal{N}, \mathcal{C}) + 3s_{\mathcal{C}}^2 s_{\mathcal{N}} \text{CoS}(\mathcal{N}, \mathcal{C}, \mathcal{C}) \\
&\quad + 3s_{\mathcal{N}}^2 s_{\mathcal{E}} \text{CoS}(\mathcal{N}, \mathcal{N}, \mathcal{E}) + 3s_{\mathcal{E}}^2 s_{\mathcal{N}} \text{CoS}(\mathcal{N}, \mathcal{E}, \mathcal{E}) \\
&\quad + 3s_{\mathcal{C}}^2 s_{\mathcal{E}} \text{CoS}(\mathcal{C}, \mathcal{C}, \mathcal{E}) + 3s_{\mathcal{E}}^2 s_{\mathcal{C}} \text{CoS}(\mathcal{C}, \mathcal{E}, \mathcal{E}) \\
&\quad + 6s_{\mathcal{N}} s_{\mathcal{C}} s_{\mathcal{E}} \text{CoS}(\mathcal{N}, \mathcal{C}, \mathcal{E}),
\end{aligned}$$

where $s_H = \sigma_H / \sigma_Y$, and $H, J = \mathcal{N}, \mathcal{C}$, or $\mathcal{E}$,

$$\text{CoS}(H, H, J) = \frac{E((H - \mu_H)^2(J - \mu_J))}{\sigma^2 H \sigma_J}.$$

A system of k components has $k!(k-2)!/2$ coskewness terms. Since $k = 3$ in our case, six possible sources can contribute to the skewness of Y . The decomposition takes σ_i^2 (i.e., the variance of Y_i) constant but allows for other types of time-varying second moments to contribute to skewness. For example, $\text{CoS}(\mathcal{N}, \mathcal{C}, \mathcal{C})$, is the covariance between the volatility (squared variations of the common component $\mathcal{C}$) and the network component.

2.3 From Theory to Empirics

The network-augmented factor model for the $N \times 1$ vector $\mathbf{y}_t = (\mathbf{y}_{1t}, \dots, \mathbf{y}_{Nt})'$ is

$$\mathbf{y}_t = \Phi_0 \mathbf{y}_t + \Phi_1 \mathbf{y}_{t-1} + \Lambda \mathbf{F}_t + e_t. \quad (4)$$

The estimation of (4) is straightforward in an ideal setting if we can take W_B to be Φ_0 . However, the BEA reports W_B as a matrix that links the level of sectoral output, not the sectoral growth that we analyze. To accommodate other discrepancies between Φ_0 and W_B that will be further discussed below, we let

$$\Phi_0 = A_0 \odot W_B,$$

where $\odot$ denotes element-by-element multiplication, and A_0 is an $N \times N$ matrix to be determined by the data. In this setting, the inverse Leontief matrix is

$$\mathcal{L} = (I_N - A_0 \odot W_B)^{-1}.$$

If W_B coincides with Φ_0 , $A_0 = 1_N 1_N'$. The homogeneous adjustment obtains as a special case with $A_0 = 1_N 1_N' a_0$.

To illustrate the importance of a_0 , Table 2 reports the decomposition of variance and skewness as a_0 varies. The top panel shows that the variance due to the network component increases monotonically with a_0 , while the covariance between $\mathcal{C}$ and $\mathcal{N}$ increases for values of a^0 up to about 0.3 and then decreases. The bottom panel shows that the direct contribution of the network component $\mathcal{N}$ to skewness increases sharply with a_0 at the expense of the common component $\mathcal{C}$. However, $\mathcal{C}$ still contributes to skewness, albeit indirectly through its higher-order covariation with $\mathcal{N}$. For many values of a_0 , the two coskewness terms $\mathcal{N}\mathcal{N}\mathcal{C}$ and $\mathcal{N}\mathcal{C}\mathcal{C}$ contribute more to skewness than $\mathcal{N}$, with a combined contribution to skewness that is maximized at values of a_0 between 0.3 and 0.4. Though useful to highlight that the relative importance of the sources of skewness depends on a_0 via the empirical inverse Leontief matrix in a non-linear way, a scalar adjustment parameter is restrictive.

Our more general specification encompasses various adjustments to W_B used in the network literature. In production network models with Cobb-Douglas technology and no capital, such as Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), the adjustment matrix A_0 simplifies to a constant (less than one) representing the share of intermediate inputs in the production process. In extensions of these models that incorporate dynamics in intermediate inputs, as in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), A_0 includes a time-discount factor as well. With heterogeneous sectors, the adjustment matrix becomes sector-specific, with each row containing the input shares corresponding to that sector (see Bigio and La'O (2020), von Lehn and Winberry (2022), and di Giovanni and Hale (2022)). Incorporating capital dynamics into production network models further modifies A_0 , making it dependent on the structure of the capital goods network as well (see Foerster, Sarte, and Watson (2011), Atalay (2017), and von Lehn and Winberry (2022)).

In models that adopt constant elasticity of substitution (CES) in production, such as those in Atalay (2017) and Baqaee and Farhi (2019), the adjustment matrix A_0 also depends on the sector-specific elasticity of substitution among intermediate inputs. Together, these examples highlight a range of model-based reasons why the BEA-provided input-output matrix is not an exact measure of its theoretical counterpart. We allow the data to determine the appropriate adjustment, thereby capturing a range of potential factors that could influence the empirical inverse Leontief matrix.

Since W_B is provided by the BEA, it remains to estimate A_0 and other parameters in the model. However, $A_0 = [a_{0,ij}]$ has N^2 parameters and unrestricted estimation is not feasible. The equation for sector i for the model defined by (4) is

$$y_{it} = \sum_j a_{0,ij} W_{B,ij} y_{jt} + \sum_j a_{1,ij} W_{B,ij} y_{jt-1} + \lambda_i \mathbf{F}_t + e_{it}.$$

We consider two models. The static version of Model I takes $a_{0,ij} = a_0$ for all (i, j) , while the static

version of Model II assumes $a_{0,ij} = a_{0,i}$ for all j . With some abuse of terminology, we will say that A_0 is heterogeneous when $a_{0,ij}$ varies across i . Model I has no variation in the rows and columns of A_0 . The rows of A_0 in Model II differ, but each row's column entries are the same. The dynamic versions of the two models make analogous assumptions on $a_{1,ij}$. We use the static version of Model II (i.e., without y_{t-1}) as our base model. The number of parameters in A_0 to be estimated is N in the static version of Model II, and only one for Model I.

3 Implementation

The static versions of Models I and II resemble a spatial panel model with interactive fixed effects. In the spatial econometrics literature, a scalar a_0 is used to capture the spillovers in the data. While spatial analysis estimates the spillover parameter taking Φ_0 as given, network analysis typically takes the estimation of the connectivity matrix as the goal. de Paula, Rasul, and Souza (2024) use exogenous covariates to identify the parameters, and Lewbel, Qu, and Tan (2023) recover the structural parameters from the reduced form coefficients. In the psychometrics literature, the Φ_0 matrix is estimated from the sample covariance structure, which can be high-dimensional, requiring regularization to prevent overfitting (see Epskamp, Rhemtulla, and Borsboom (2017)). In between the extremes of completely latent and completely observed Φ_0 is the case that treats Φ_0 as imperfectly observed. For example, Lam and Souza (2016, 2019) assume that $\Phi_0 = B_0 + \beta_0 W_B$ and only estimate B_0 when there is no information about W_B . Our use of A_0 is in this spirit of allowing the data matrix W_B to deviate from the model matrix Φ_0 .

Spatial autoregressive models are often estimated by maximum likelihood or by instrumental variables. Maximum likelihood is efficient but requires computation of the determinant of a $N \times N$ matrix with unknown parameters, which can be computationally costly when N is large. Less appealing in our context is that we want to learn about the possible asymmetry of the errors, so we refrain from specifying parametric distributions. Estimation by GMM-IV is also possible if there are covariates, but they are absent in our analysis. So we only consider estimation via GMM using the moment conditions that we explain in detail below.

Let the parameters of the model be collected into a vector $\beta = (a_0, a_1, \Lambda, \mathbf{F})$, and let β^* denote their true value. We will estimate β based on the covariance structure of the idiosyncratic errors. The following assumption is used:

Assumption A

- 1) $E[e_{it}^* e_{jt}^*] = E[e_{it}(\beta^*) e_{jt}(\beta^*)] = 0$ for $i \neq t$;
- 2) W_B satisfies $\|W_B\|_\infty < \delta_{W_B} < \infty$ and is such that for all i , $\sum_{j=1}^N W_{B,ij} = 1$;

3) (i) $\max(\|\Phi_0\|_1, \|\Phi_0\|_\infty) \leq \delta_0 < 1$, (ii) $\max(\|\Phi_1\|_1, \|\Phi_1\|_\infty) \leq \delta_1 < 1$, and (iii) $\delta_C = \delta_1/(1-\delta_0) < 1$.

Assumption A.2 is satisfied by construction. To ensure stationarity, A.3(i) controls contemporaneous dependence and A.3(ii) controls persistence. Bounding the maximum row sum $\|\Phi_0\|$ and column sum of $\|\Phi_0\|_1$ ensure that the adjusted Leontief $\mathcal{L} = (I_N - \Phi_0)^{-1}$ exists. Given that W_B is known, the assumptions on Φ_0, Φ_1 are implicit restrictions on A_0 and A_1 . Together with A.3(ii) and (iii), a stable reduced form $\mathbf{y}_t = \mathcal{L}\Phi_1\mathbf{y}_{t-1} + \mathcal{L}\epsilon_t$ is obtained.

Assumption A.1 is key to identification. Specifically, it requires that conditional on the common factors and network structure, the idiosyncratic errors e_{it}^* and e_{jt}^* are cross-sectionally uncorrelated when evaluated at the true β^* . Consider the static model without common factors and dynamics where $\beta = (a_0, \Lambda, \mathbf{F})$, $a_0 = (a_{0,1}, \dots, a_{0,N})'$. Then $y_i = a_0 W_{B,i} \mathbf{y} + \mathbf{e}_i$ for some a_0 not necessarily equal the true value a^* and $e_{it} = e_{it}^* - \sum_{k=1}^N (a_{0,ik} - a_{0,ik}^*) W_{B,ik} y_{kt}$. Under the maintained assumption that $\text{cov}(e_i^*, e_j^*) = 0$, the pairwise covariance of the errors $\text{cov}(e_i, e_j) = E[(y_i - a_0 W_{B,i} \mathbf{y})(y_j - a_0 W_{B,j} \mathbf{y})]$ satisfies

$$\text{cov}(e_i, e_j) = (a_0 - a_0^*) \left[(a_0 - a_0^*) E(W_{B,i} \mathbf{y} W_{B,j} \mathbf{y}) - E(W_{B,i} \mathbf{y} e_j + W_{B,j} \mathbf{y} e_i) \right].$$

The above can be zero only if $a_0 = a_0^*$. A similar idea is used in Liu, Liu, Li, and Lin (2023) to estimate homogeneous spatial models.⁸

The dynamic model is a bit more involved but conceptually similar. Now $\beta = (a_0, a_1, \Lambda, \mathbf{F})$. At the true β^* , the structural error corresponding to (4) is

$$e_{it}^* = e_{it}(\beta^*) = y_{it} - \sum_j a_{0,ij}^* W_{B,ij} y_{jt} - \sum_j a_{1,ij}^* W_{B,ij} y_{j,t-1} - \lambda_i^* \mathbf{F}_t^*.$$

The dynamic model satisfies the moment conditions

$$\mathbb{E}[g_1(\beta^*)] := \{ \mathbb{E}[e_{it}^* e_{jt}^*] = 0 \text{ for all } i \neq j \}, \quad (5a)$$

$$\mathbb{E}[g_2(\beta^*)] := \{ \mathbb{E}[(W_B \mathbf{y}_{t-1})_i e_{it}^*] = 0 \text{ for all } i \}. \quad (5b)$$

The $N(N-1)/2$ conditions in (5a) are the same as in the static model and follow from the assumption of no cross-sectional correlation in the sectoral shocks. The conditions in (5b) pertain to the orthogonality between the error and the regressor $(W_B \mathbf{y}_{t-1})_i$. Stacking up the conditions in (5a) and (5b) into $g(\beta^*) = (g_1(\beta^*)', g_2(\beta^*)')'$, we have $\mathbb{E}[g(\beta^*)] = 0$. For a given β not necessarily equal β^* , the moment conditions will not hold. With $\beta = (a_0, a_1, \Lambda, \mathbf{F})$, the estimation problem is

⁸More precisely, Liu, Liu, Li, and Lin (2023) consider the model $y_i = \rho W_i \mathbf{Y} + \mathbf{X}_i' \beta + \epsilon_i$ and show that $E[(Y_i - \rho W_i \mathbf{Y} - \mathbf{X}_i' \beta)(\mathbf{Y}_j - \rho W_j \mathbf{Y} - \mathbf{X}_j' \beta)] = 0$ only if ρ and β are at their true values. Their objective function is $Q(\rho, \beta) = \frac{1}{N(N-1)} \sum_{i \neq j} \epsilon_i(\rho, \beta) \epsilon_j(\rho, \beta)^2$.

to find $\hat{\beta}$ that best fits the data. Let $\bar{g}(\beta)$ be the vector of sample orthogonality conditions.. The GMM estimate of β is

$$\hat{\beta} = \operatorname{argmin}_{\beta} J(\beta),$$

where $J(\beta) = g(\beta)'Wg(\beta)$.

In the empirical work, we estimate β by coordinate descent. Let $\beta^{(k)}$ be the vector of estimates that emerges from the k -th iteration. The next iteration consists of the following steps:

- a. take $\Lambda^{(k+1)}$ $F^{(k+1)}$ to be the first principal component of $\mathbf{Y} - \mathbf{Y}(A_0^{(k)} \odot W_B) - \mathbf{Y}_{-1}(A_1^{(k)} \odot W_B)$;
- b. $a_0^{(k+1)} = \operatorname{argmin}_{a_0} J_T(a_0, a_1^{(k)}, F^{(k+1)}, \Lambda^{(k+1)})$;
- c. $a_1^{(k+1)} = \operatorname{argmin}_{a_1} J_T(a_0^{(k+1)}, a_1, F^{(k+1)}, \Lambda^{(k+1)})$.

This iterative estimation yields the network coefficients $\hat{a}_0$ and $\hat{a}_1$, along with estimates of the common component, $(\hat{\Lambda}, \hat{F})$, and sector-specific shocks, $\hat{\epsilon}_t$. We compute bootstrapped standard errors for all the empirical estimates.⁹

3.1 Estimation Results

The GMM estimates of the $\{\hat{a}_0\}$ determine the relative importance of each component—network, common, and sector-specific—in explaining sector-level output growth. A higher value of $\hat{a}_{0i}$ suggests that the network component plays a significant role in explaining $\mathbf{y}_i$, output growth in sector i . In contrast, if $\hat{a}_{0i}$ is close to zero, the common and sector-specific components are more important in driving output growth in that sector. As shown in Figure 2 (black dots), most of these estimates are significantly different from one another. The sectors with the largest $\hat{a}_{0i}$ are Oil and Gas, Apparel-Leather, and Finance and Insurance, while the values of $\hat{a}_{0i}$ are smallest for Education, Warehousing, and Retail trade. It is noteworthy that the sectors with the largest estimates of $\hat{a}_{0i}$ are not those with the highest output shares. This heterogeneity in the estimates plays a critical role in the skewness decomposition.

Before proceeding further, we verify the properties of the estimator using a simulation exercise. We use a calibrated Monte Carlo experiment to assess the performance of the GMM estimator based on the covariance structure. Specifically, for each $i = 1, \dots, N$, we generate data $y_{i,t}$ of different length T using estimates from Model II as ‘true values’, and use the simulated data to estimate the model. This exercise is repeated 1000 times. The estimates are reasonably accurate even with $T = 63$ as in our empirical application. Figure 2 plots the mean of $\hat{a}_0$ (blue stars) in 1000 replications, along with the one-standard-error confidence band. We see that the ‘true’ values

⁹We provide a detailed description of the step-by-step estimation routine in the online Appendix C.

are often close to the mean estimates, and that the mean estimates are within one standard error of the ‘true’ values in 42 of the 43 cases.

We now use the estimates to compute quantities that characterize the importance of the components across sectors and over time.

3.1.1 The Empirical Network Multiplier

As discussed in Section 2.2, the inverse Leontief matrix $\mathcal{L}$ identifies the sectors that are most influential in the propagation of shocks. In contrast, the Network Multiplier Matrix $\mathcal{L} - I_N$ isolates each sector’s impact by removing the direct effect. We compute the empirical Network Multiplier Matrix $\hat{\mathcal{L}} - I_N$ where $\hat{\mathcal{L}} = (I_N - \hat{A}_0 \odot W_B)^{-1}$. Each element (i, j) in the multiplier matrix represents the importance of shocks to sector j on sector i . We present the heatmap of the multiplier matrix in Figure 3 with blue blocks indicating lower importance and white blocks indicating greater importance. This figure shows that the multiplier matrix has large diagonal elements for some sectors, indicating that the shocks that matter most for network propagation originate in those sectors. The figure also shows that the multiplier matrix is relatively sparse, with just a few sectors dominating network propagation. For instance, shocks originating in Finance and Insurance, Trade, Information Technology, and Services exert the strongest multipliers on other sectors.

3.1.2 The Empirical Sectoral Multiplier

The sectoral network multipliers are given by the $N \times 1$ vector $\omega'(\mathcal{L} - I_N)$, where ω' represents the value-added shares of each sector. The i -th element in the vector quantifies how shocks to sector i affect aggregate output growth. The empirical sectoral network multipliers for the 43 sectors in our analysis are obtained by replacing $\mathcal{L}$ with $\hat{\mathcal{L}}$.¹⁰ Importantly, the Finance and Insurance sector has the highest sectoral network multiplier: a 1% shock in this sector leads to roughly a 0.39% change in aggregate output growth solely through network propagation. Transportation, Information Technology, Real Estate, Professional Services, and Wholesale Trade also exhibit relatively large network multipliers.

Comparing the Finance and Insurance sector with the Retail Trade sector provides insight into how sectoral network multipliers can vary significantly, even between sectors with similar value-added shares. Both sectors contribute approximately 6.4% to total value added, implying that the direct effect of a sectoral shock on aggregate output growth is roughly the same for each. However, their sectoral network multipliers differ markedly: 0.39 for Finance and Insurance versus 0.038 for Retail Trade. This means that the network-propagation effect of a shock in the Finance and

¹⁰See Figure B3 in online Appendix B.

Insurance sector is more than 10 times larger, highlighting how differences in sectoral connections within the production network can lead to starkly different aggregate implications. Summing all sectoral network multipliers yields approximately 2.53, indicating that a 1% shock across all sectors leads to a 2.53% change in aggregate output growth through network propagation.

3.1.3 Cyclicalities of $\mathcal{N}_t, \mathcal{C}_t, \mathcal{E}_t$:

To understand how different sources of fluctuations contribute to aggregate outcomes across the business cycle, we take as a starting point that

$$Y_t = \mathcal{N}_t + \mathcal{C}_t + \mathcal{E}_t.$$

Figure 4 presents the time series plot of aggregate growth (Y_t) along with the network ($\mathcal{N}_t$) and common ($\mathcal{C}_t$) components. During NBER recessions, the network component accounts for most of the negative aggregate growth. However, during normal times, both the network component and the common component contribute significantly to aggregate growth. To further understand the cyclical properties of the components, we classify each year as LOW or HIGH based on whether aggregate GDP growth lies in the bottom or top quartile of its empirical distribution. The relative importance of each component in states LOW and HIGH is computed as

$$\text{RL}(\mathcal{I}_t) = \frac{\sum_{t \in \text{LOW}} \mathcal{I}_t}{\sum_{t \in \text{LOW}} Y_t}, \quad \text{RH}(\mathcal{I}_t) = \frac{\sum_{t \in \text{HIGH}} \mathcal{I}_t}{\sum_{t \in \text{HIGH}} Y_t},$$

where $\mathcal{I} = \mathcal{N}, \mathcal{C}, \mathcal{E}$. In LOW states, the network component, $\text{RL}(\mathcal{N}_t)$, accounts for roughly 90 percent of aggregate growth compared to 65 percent in the HIGH states. By contrast, the common component becomes relatively more important in HIGH states, with $\text{RH}(\mathcal{C}_t)$ accounting for about 33 percent of aggregate growth compared to only 4 percent in LOW states. The idiosyncratic component is not particularly important in accounting for aggregate growth. These patterns indicate that adverse sectoral shocks and network-driven fluctuations typically dominate severe contractions, while strong expansions reflect favorable aggregate conditions combined with network propagation. As a robustness check, we consider other definitions of LOW and HIGH states.¹¹ In all the formulations, the network component is relatively more important in LOW states, and the common component is relatively more important in HIGH states.

3.2 Variance and Skewness Decomposition

Summarizing heterogeneity in Model II is more challenging as we estimate A_0 and possibly also A_1 . We focus on the variance and skewness implied by each model. The top panel of Table 3

¹¹We present these robustness results for different definitions of LOW and HIGH states in Table B2 of online Appendix B.

reports the share of variance attributed to the components; across all models considered, the total variance to be explained is always 10.088. In the ‘factor-only’ model (Column F), where the network component $\mathcal{N}$ is absent, the variability in the data can only be attributed to the common component $\mathcal{C}$ or to the idiosyncratic error $\mathcal{E}$, and they are orthogonal by construction. Absent the production network, the data find that $\mathcal{C}$ contributes significantly more to variance than $\mathcal{E}$.¹² However, once the network component is allowed, the importance of $\mathcal{C}$ falls.¹³ The precise importance of $\mathcal{N}$ depends on heterogeneity allowed in A_0 . When A_0 is homogeneous (Model I), the importance shifts from $\mathcal{C}$ to the covariance between $\mathcal{C}$ and $\mathcal{N}$. The more heterogeneous A_0 is (Models II and II(d)), the more important the network and covariance terms are. In our base case of Model II, the indirect contribution of $\mathcal{N}$ through covariances is as important as its direct contribution.

The lower panel of Table 3 shows the contribution of the components to total skewness, which for this data is -1.687. Absent the network component, the skewed common component directly contributes an amount of -1.363, with the remaining skewness coming from coskewness between $\mathcal{E}$ and $\mathcal{C}$. The direct contribution of estimated sectoral shocks, $\mathcal{E}$, to the aggregate skewness is almost zero. Further investigation reveals that sectoral shocks do not contribute to aggregate skewness because there are as many sectors with positively skewed shocks as with negatively skewed shocks.¹⁴

Introducing the network component drastically changes the decomposition, especially when A_0 is heterogeneous. As we allow more heterogeneity going from Model I to II(d), the skewness due directly to $\mathcal{N}$ and indirectly via the coskewness terms that involve $\mathcal{N}$, notably $\mathcal{N}\mathcal{N}\mathcal{C}$ and $\mathcal{N}\mathcal{C}\mathcal{C}$, increases at the expense of $\mathcal{C}$. Evaluated at our base case, Model II estimates, skewness from $\mathcal{N}\mathcal{N}\mathcal{C}$ and $\mathcal{N}\mathcal{C}\mathcal{C}$ (-0.626 and -0.182, respectively) are as important as the direct contribution of $\mathcal{N}$, the latter being -0.789. Thus, while the common and sectoral shocks do not directly contribute to skewness in a significant way, their indirect contribution is substantial. The most robust contribution is $\mathcal{N}\mathcal{N}\mathcal{C}$, accounting for one-quarter of total skewness.

As a robustness check, we also estimate our baseline Model II using a broader 2-digit NAICS classification comprising 14 sectors. Comparing the 14-sector results with those obtained using 43 sectors reveals that the coskewness components ($\mathcal{N}\mathcal{N}\mathcal{C}$ and $\mathcal{N}\mathcal{C}\mathcal{C}$) jointly account for more than 67% of skewness in the 14-sector classification, compared to about 48% in the 43-sector classification.¹⁵ These results further emphasize that the coskewness terms are more important, even though the estimates of A_0 for the 14-sector model exhibit less heterogeneity than those for the 43-sector data.

¹²Foerster, Sarte, and Watson (2011) also find that the common component explains almost all of the variation in industrial production when the production network is absent.

¹³Foerster, Sarte, and Watson (2011) and Atalay (2017) report a similar result whereby the share of variance explained by the common component decreases drastically when one accounts for network connections.

¹⁴See the Figure B4 in the online Appendix B, which plots the skewness of estimated sectoral shocks.

¹⁵Refer to Table D2 in online Appendix D.

We also investigate how these coskewness terms vary across individual sectors. To this end, we replicate the skewness decomposition at the sectoral level, focusing on how network-linked coskewness terms ($\mathcal{NNC}$, $\mathcal{NCC}$, $\mathcal{NNE}$, $\mathcal{NEE}$) contribute to each sector’s skewness.¹⁶ This sector-level analysis reveals that the relative importance of coskewness terms varies across the 43 sectors. The coskewness terms account for a substantial portion of overall skewness in nearly every sector. This finding further reinforces our argument that coskewness, capturing higher-order covariation, is crucial for understanding the network’s contribution to overall skewness.

4 A Structural Model

This section constructs and estimates a multi-sector general equilibrium model with a production network.¹⁷ The model is representative of the literature on production networks, but relaxes the assumption that shocks are normally distributed to allow instead for skewed shocks drawn from a generalized extreme value (GEV) distribution. This modeling choice is motivated by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), who find that for microeconomic shocks to translate into macroeconomic tail risk, the former need to exhibit some minimal degree of tail risk relative to the normal distribution. However, our model differs from Acemoglu et al. in a number of ways: 1) households suffer disutility from working, 2) the materials aggregator is constant elasticity of substitution (e.g., as in Baqaee and Farhi (2019)) rather than Cobb-Douglas, and 3) firms are subject to a skewed aggregate shock in addition to a sector-specific shock. The latter feature is important in this project because one of our goals is to examine the relative contribution of aggregate and sectoral shocks to the skewness of output growth.

In contrast to the literature, where network models are usually calibrated, we pursue the fully-fledged estimation of the model by the generalized method of moments (GMM). We complement the reduced-form (network-augmented factor) model in Section 2 with an estimated structural model to interpret the reduced-form results, shed additional light on the sources of skewness in the data, and obtain estimates of key parameters, such as the elasticities of substitution. Estimates are consistent with reduced-form results because we match the variance shares and components of skewness under our benchmark Model II in the GMM procedure.

¹⁶Refer to Figure B5 in online Appendix B.

¹⁷Early multi-sector models with an input-output structure include Long and Plosser (1983), Hovarth (1998), and Hovarth (2000) in real business cycles, Bouakez, Cardia, and Ruge-Murcia (2009) in monetary economics, as well as Foerster, Sarte, and Watson (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). Some of the subsequent literature is surveyed in Carvalho and Tahbaz-Salehi (2019) and cited elsewhere in this paper.

4.1 Producers

A period in the model is one year. In every period, the representative firm in sector $i \in N$ produces the output quantity $q_{i,t}$ using the technology¹⁸

$$q_{i,t} = z_{i,t}(z_t h_{i,t})^\eta (m_{i,t})^{1-\eta}, \quad (6)$$

where $z_{i,t}$ and z_t are productivity shocks, $h_{i,t}$ is labor input, $m_{i,t}$ is materials, and $\eta \in (0, 1)$ is a parameter. The shock $z_{i,t}$ is idiosyncratic and affects only firms in sector i , while the shock z_t is aggregate and affects all firms in all sectors. Labor is purchased from households in a frictionless market at the wage $w_{i,t}$ per period. Labor is completely mobile across sectors, so wages will be the same in all sectors in equilibrium. Materials are purchased from all sectors and combined according to

$$m_{i,t} = \left(\sum_{j=1}^N (\omega_{ji})^{1/\psi} (m_{j,i,t})^{(\psi-1)/\psi} \right)^{\psi/(\psi-1)}, \quad (7)$$

where $m_{j,i,t}$ is the quantity of good purchased from sector j , $\omega_{ji} \geq 0$ are weights that add up to 1, and $\psi > 0$ is a parameter that determines the elasticity of substitution between goods from different sectors. As it is well known, (7) includes as a special case the Leontief function when $\psi \rightarrow 0$ and the Cobb-Douglas function when $\psi = 1$. Baqaee and Farhi (2019) show that relaxing the Cobb-Douglas assumption and adopting a nonlinear solution method allows expenditure shares to react endogenously to shocks and give rise to nonlinearities that depend on the production structure. For example, if material inputs are gross complements (that is, $\psi < 1$), aggregate output is a concave function of sectoral productivity and, consequently, is negatively skewed. This implication of the model is studied by Dew-Becker and Vedolin (2021) using U.S. data and by Miranda-Pinto, Silva, and Young (2023) using cross-country data.

The firm maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Lambda_{\tau,t} d_{i,t}, \quad (8)$$

where $\Lambda_{\tau,t}$ is the household's marginal rate of substitution of consumption between periods τ and t , and $d_{i,t}$ is profits. Profits are total revenue minus total costs,

$$d_{i,t} = p_{i,t} \left(c_{i,t} + \sum_{j=1}^N m_{i,j,t} \right) - \left(w_{i,t} h_{i,t} + \sum_{j=1}^N p_{j,t} m_{j,i,t} \right). \quad (9)$$

Revenue is the sale of goods for final and intermediate consumption (respectively denoted by $c_{i,t}$ and $m_{i,j,t}$) at the unit price $p_{i,t}$. Costs are the wage bill and total expenditures on materials inputs.

¹⁸Since the firm is representative, we abstract from keeping separate indices for firms within the sector and, with a slight abuse of notation, equate a sector with its representative firm.

Since final and intermediate goods are physically the same good, and the firm is unable to price-discriminate across customers, all output $y_{i,t}$ is sold at the same price. As usual, the optimal choice of inputs (i.e., labor and materials) equates their marginal product to their cost.

4.2 Shocks

The sector-specific shock follows the process,

$$\ln z_{i,t} = \rho \ln z_{i,t-1} + \zeta_{i,t}, \quad (10)$$

where $\rho \in (-1, 1)$ is the autocorrelation coefficient and $\zeta_{i,t}$ is an independent and identically distributed (i.i.d.) innovation drawn from a generalized extreme value (GEV) distribution with mean zero. As in Eusepi, Hobijn, and Tambalotti (2011), innovations are drawn from a common distribution, but the draws are independent across sectors. The aggregate shock follows the process

$$\ln(z_t/z_{t-1}) = \mu + \varrho \ln(z_{t-1}/z_{t-2}) + \epsilon_t, \quad (11)$$

where μ is the mean gross rate of economic growth, $\varrho \in (-1, 1)$ is the autocorrelation coefficient, and ϵ_t is an i.i.d. innovation drawn from a GEV distribution with mean zero. Because there is long-run growth in this economy, and the model is rendered stationary by rescaling all variables by z_{t-1} .

The GEV distribution is due to Jenkinson (1955) and represents in a unified and flexible way the Type I, Type II, and Type III extreme value distributions, also known respectively as the Gumbel, Fréchet, and Weibull distributions. The GEV distribution is characterized by three parameters—a location, a scale, and a shape parameter—and it allows positive and negative skewness. The Gumbel, Fréchet, and Weibull distributions are the special cases of the GEV distribution when the shape parameter is zero, larger than zero, or smaller than zero, respectively. The probability density function (PDF) of the GEV distribution is

$$f(x) = (1/\kappa)g(x)^{\vartheta+1} \exp(-g(x)), \quad (12)$$

where $g(x) = ((1 + (x - \iota)\vartheta/\kappa))^{-1/\vartheta}$ when $\vartheta \neq 0$ and $g(x) = \exp(-(x - \iota)/\kappa)$ when $\vartheta = 0$, ι is the location parameter, κ is the scale parameter, and ϑ is the shape parameter (see Jenkinson (1955)). Coles (2001) and Embrechts, Klüppelberg, and Mikosch (2011) present applications of the GEV distribution to insurance and finance, and Kim and Ruge-Murcia (2019) study the implications of extreme GEV shocks for optimal monetary policy.

4.3 Households

Households are identical, and their total number is normalized to be one. The representative household maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} (\ln(c_t) - \gamma h_t), \quad (13)$$

where E_τ is expectation conditional on information available at time τ , $\beta \in (0, 1)$ is the subjective discount factor, c_t is final consumption, and h_t is total hours worked.¹⁹ Final consumption is a bundle of goods produced in all sectors,

$$c_t = \left(\sum_{i=1}^N (\xi_i)^{1/\chi} (c_{i,t})^{(\chi-1)/\chi} \right)^{\chi/(\chi-1)}, \quad (14)$$

where c_t^i is consumption of good i , $\xi^i \geq 0$ are weights that add up to 1, and $\chi > 0$ is a parameter that determines the elasticity of substitution between goods from different sectors. The case where $\xi^i = 0$ describes the situation where good i is not consumed directly by households. Notice, however, that the good may still be consumed indirectly by households if i is an input in the production of another good j for which $\xi^j > 0$ (think, for instance, of the lithium used in the production of electric car batteries). Total hours worked is the sum of the hours supplied to each sector.

The only financial asset in this economy is a one-period bond. The bond is real, non-contingent, and risk-less, meaning that it pays with certainty one unit of consumption at maturity regardless of the state of nature. The household's budget constraint is

$$\sum_{i=1}^N p_{i,t} c_{i,t} + b_t = \sum_{i=1}^N w_{i,t} h_{i,t} + (1 + r_{t-1}) b_{t-1}, \quad (15)$$

where b_t is the number of bonds purchased at time t and r_{t-1} is the real interest rate. The first-order conditions that characterize utility maximization include an Euler equation that equates the marginal cost (in terms of utility) of acquiring one additional bond and the discounted expected marginal benefit of keeping the bond till next period, and isoelastic demand functions for each good i .

In equilibrium, bond holdings are zero (because agents are identical), wages are the same in all sectors (because labor is completely mobile across sectors), and profits are zero (because firms are perfectly competitive). After aggregation,

$$c_t = \sum_{i=1}^N v_{i,t}, \quad (16)$$

¹⁹The assumption of logarithmic consumption is based on preliminary research where we estimated a general constant relative risk aversion (CRRA) specification and found the CRRA parameter to be statistically close to 1. The assumption of linear disutility of labor is based on the indivisible labor model in Hansen (1985).

where $v_{i,t}$ is value added in sector i (i.e., gross output minus the cost of materials inputs). That is, aggregate consumption equals aggregate output, measured in terms of value added as in the National Income and Product Accounts.

The model is solved using a first-order perturbation with the rational-expectation solution of the linearized system found using the approach in Klein (2000). The solution delivers linear decision rules on the state variables, and it is, therefore, consistent with the linear reduced-form network-augmented factor model in Section 2.²⁰

4.4 Estimation of Structural Model

We estimate the multi-sector by the simulated method of moments (SMM). SMM was originally developed by McFadden (1989) for the estimation of discrete response models. Lee and Ingram (1991) and Duffie and Singleton (1993) extend SMM to time-series models and show that it is consistent and asymptotically normal under general regularity conditions. In this project, the moments used to estimate the model are the variance shares and the components of skewness of real sectoral output computed under our benchmark Model II (see Table 3). The estimator minimizes the weighted distance between the moments computed from the U.S. data and from artificial data simulated from the model. The weighting matrix is the identity matrix. Note that the definition of real sectoral output from the model used to construct the moments is exactly the same as that based on the I-O data, that is $q_{i,t}p_{i,t}$, where $q_{i,t}$ is the quantity of output and $p_{i,t}$ is the real price of good i (see the discussion in Section 2). The total number of moments is 17. The length of the simulated series is equal to the actual data, that is, 63 observations. In order to limit the effect of starting values on the results, the simulated sample contains 100 additional “training” observations that are discarded when computing the moments.

The estimated parameters are the autoregressive coefficients of the aggregate and sectoral productivity shocks, the parameters of their GEV innovations, and the parameters χ and ψ that determine the elasticity of substitution between goods from different sectors in consumption production, respectively. The total number of estimated parameters is 8, and the number of degrees of freedom is 9. Monte-Carlo results reported in Ruge-Murcia (2007) indicate that asymptotic standard errors are not always a good approximation to the true variability of SMM estimates in small samples. For this reason, we compute standard errors using a parametric k -step bootstrap

²⁰Baqae and Farhi (2019) and Ruge-Murcia (2024) argue that nonlinear solution methods can also deliver skewed aggregate series. We acknowledge that these non-linear effects may be quantitatively important and have a bearing on the accuracy of our solution. Unfortunately, when we attempted to estimate a second-order perturbation, this proved unfeasible due to the large size of our model, which has 45 state variables and 135 choice variables. Thus, the most constructive way to interpret our analysis is as providing a linear benchmark, both in terms of the reduced-form and structural modeling, to be extended to the nonlinear case in future work.

with 99 replications. This bootstrap was proposed by Davidson and MacKinnon (1999) and is attractive because it reduces the computational costs of the standard bootstrap by performing a fixed number of quasi-Newton steps for each bootstrap sample. Andrews (2002) examines the higher-order properties of the k -step bootstrap for extremum estimators and finds that it delivers higher-order improvements over procedures based on first-order asymptotics. We use five steps of the Levenberg–Marquardt (LM) algorithm, which is a robust Gauss–Newton algorithm designed to solve non-linear least squares problems like ours. Preliminary experimentation showed basically no gain from employing more than five steps.

We use additional information to fix some parameters to economically plausible values as follows. The discount rate (β) is set to 0.99, which is the mean of the inverse real interest rate during the sample period (about 1% per year). The gross rate of aggregate productivity growth in the steady state (μ) is set to 1.012. This value implies a net growth rate of 1.2% per year, which is the average annual growth rate of total factor productivity (TFP) in the series constructed by John Fernald (see Fernald (2012)). The elasticity parameter (η) is set to 0.3 based on KLEM data from the Bureau of Economic Analysis (BEA) that record payments to all factors of production in all sectors. The weight of the disutility of work in the utility function (γ) is normalized to 1.²¹ The production weights ω_{ji} were computed using the Use tables of the Input-Output (I-O) accounts for the period 1957 to 2019. The consumption weights (ξ_i) for the same 43 sectors used above were computed using data from the column “Personal Consumption Expenditures” in the final-user part of the Use tables for the same period.

4.5 SMM Estimates

SMM estimates are reported in Table 4. Both the aggregate and sectoral shocks are persistent with relatively high and statistically significant autoregressive coefficients. The scale parameters of the innovations are not precisely estimated, but both shape parameters are negative and significant. That is, the null hypothesis that the shape parameter is zero (i.e., that innovations follow a Gumbel distribution) can be rejected at the 5% level in favor of the alternative that the parameter is less than zero. Hence, we conclude that aggregate and sectoral productivity innovations are characterized by Weibull distributions. The medians of the aggregate and sectoral productivity innovations are 0.045 and 0.130, respectively, but in both cases the distributions feature long left tails, meaning that very large negative productivity innovations can occur with a small but non-negligible probability.

The elasticity of substitution between consumption goods is $\chi = 0.735$. This estimate is statistically different from 0, which corresponds to the Leontief function, and from 1, which corresponds

²¹This normalization is inconsequential because this weight only scales the number of hours worked in steady state and does not affect the dynamics of the model.

to the Cobb-Douglas function, at the 5% level. This estimate is roughly in line with the value of 0.85 reported by Herrendorf, Rogerson, and Valentinyi (2013) based on a three-sector model and somewhat higher than the value of 0.445 reported by Ruge-Murcia (2024) based on asset prices from ten sectors. However, all these estimates are in agreement in that they are less than 1, indicating that consumption goods are complements. The elasticity of substitution between materials is $\psi = 0.828$ and statistically different from 0 at the 5% level and from 1 at the 10% level. This estimate is comparable to the value of 0.728 reported by Ruge-Murcia (2024), but higher than the estimates reported by Atalay (2017), which range between 0.07 and 0.13, and the values used in calibrations by Baqaee and Farhi (2019) and Dew-Becker and Vedolin (2021), who employ 0.001 and 0.1, respectively.

Column 1 of Table 6 lists the moments targeted by the GMM estimation procedure—namely, the variance shares and the components of skewness of real sectoral output computed under the reduced-form Model II (see Table 4)—. Column 2 reports the moments predicted by the structural model based on the GMM estimates in Table 5. Note that the correlation between both sets of moments is high (0.978) and statistically significant, and the R^2 is 0.952, meaning that the model explains about 95% of the variance of the targeted moments. Regarding untargeted moments, we examined the 903 cross-correlations between sectoral output growth and found that, although the model predicts larger cross-correlations than those observed in the data due to the strong effect of the network, both sets of moments are positively related: their correlation is 0.13 and significant.

4.6 Identification

Since the elasticities of substitution between consumption goods (χ) and materials (ψ) are key parameters of the model, it is important to understand the features of the data that permit their identification. To that end, we pursue the following strategy. Holding all other parameters constant, compute the variance and skewness of sectoral output growth predicted by the model in a grid around the SMM estimate of χ and ψ (separately). Then plot the difference between the predicted moments and their value at the SMM estimate. Suppose that a given moment does not change along the grid, then it follows that this moment does not contribute—at least locally—to the identification of the parameter. Suppose instead that a given moment varies considerably along the grid, then it follows that this moment is informative about the parameter and therefore helpful for its identification.

Figure 5 plots the results of this exercise for χ (top row) and ψ (bottom row). Each panel contains 43 lines, corresponding to the 43 sectors in our sample. However, we highlight the two that appear to vary the most along the grid. This figure leads to the following two conclusions. First,

the moments that contribute the most to the identification of the elasticity of substitution between consumption goods are the skewness of entertainment and apparel, and the standard deviations of furniture and apparel. Second, the moments that contribute the most to the identification of the elasticity of substitution between materials are the skewness of primary metal and computers, and the standard deviations of mining and computers.

4.7 Sources of Skewness

We use our estimated model to shed additional light on the sources of skewness observed in the data. This analysis complements that based on the reduced-form factor model reported above. Because the model is linear, the skewness of sectoral output growth arises solely from the skewness of structural shocks, but it is amplified by the network structure as in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). However, in contrast to their model, ours features an aggregate shock, so the question arises as to whether the skewness in the data is due to the amplification of aggregate shocks, sectoral shocks, or both. To answer this question, we perform counterfactual experiments. The first experiment assumes that aggregate productivity shocks are normally distributed while sectoral productivity shocks follow the estimated GEV distribution (and, hence, are negatively skewed). Instead, the second experiment assumes that aggregate productivity shocks follow the estimated GEV distribution while sectoral productivity shocks are normally distributed. In both cases, the I-O table is computed from the U.S. data.

Results are reported in Columns 3 and 4 of Table 5 and show that both versions of the model can account for the second moments of the data, but only the version with skewed sectoral shocks can generate substantial skewness, and do so via the network component. Put differently, skewed sectoral shocks propagated through the network are needed to generate the aggregate skewness observed in the data, as conjectured in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). In contrast, skewed aggregate shocks propagated through the network induce very limited skewness in the data.

Our work is complementary to Dew-Becker and Vedolin (2025), who construct a model where aggregate output is a concave function of shocks to sectoral productivity (as in Baqaee and Farhi (2019)) and labor wedges. This nonlinearity endogenously generates negative skewness as shocks propagate through the production network, provided that material inputs are gross complements (as shown to be the case by our empirical estimates). Dew-Becker and Vedolin (2025) provides a closed-form solution to the model and shows that it matches certain time-series and cross-sectional moments of U.S. data. Instead, we work in a linear framework, but allow for aggregate shocks and asymmetric shock distributions, and evaluate the relative contribution of the network to the skewness of aggregate output, directly and through co-skewness terms.

4.8 Quantities vs Real Prices

The value of using a structural model is illustrated in this section, where we draw attention to an issue neglected in the literature: sectoral real output data from the BLS is, by construction, the product of quantity and a relative price. An important feature of these data is that nominal gross output in the I-O table is measured as the product of the quantity of sectoral output times its price. Thus, a measure of real industry output – obtained by dividing nominal output by a price index – will unavoidably include both the output quantity and a relative price component. When the price index is the GDP deflator, this relative price is the real price of the sectoral good. Lack of data prevents us from segregating skewness into price and quantity, but we can deploy our model to decompose the skewness of real sectoral output growth into the parts arising from quantities and from real prices separately. As documented above in this paper, coskewness can be an important contributor to overall skewness, and hence, we write the skewness of $y_{n,t} = \Delta \ln(q_{n,t}) + \Delta \ln(p_{n,t})$ as

$$\kappa_3^y = s_q^3 \kappa_3^q + s_p^3 \kappa_3^p + 3s_q^2 s_p \text{CoS}(q, q, p) + 3s_p^2 s_q \text{CoS}(p, p, q).$$

Figure B8 in online Appendix B plots the model-based estimates of the skewness of sectoral output, κ_3^y , and its components κ_3^q , κ_3^p , $\text{CoS}(q, q, p)$, and $\text{CoS}(p, p, q)$. This figure shows that the skewness of the real price and the coskewness $\text{CoS}(q, q, p)$ are positive, while the skewness of quantity and the coskewness $\text{CoS}(p, p, q)$ are negative. The conclusion of this decomposition is that the overall negative skewness of the sectoral output as constructed from the I-O accounts is driven by the latter two terms.

5 Conclusion

This paper offers new insights into the structural mechanisms that drive aggregate skewness and the role of production networks in shaping macroeconomic fluctuations. We employ both a network-augmented factor model and a micro-founded structural model to examine the relative contribution of common shocks, sector-level shocks, and network effects to the skewness of output growth. Results indicate that while the common and network components exhibit skewness, the dominant sources of asymmetry are the coskewness terms, especially the higher-order covariation between the network and the common component. We show that even if the I-O table is relatively sparse and features substantially larger diagonal entries than off-diagonal entries, it still implies strong connectivity across sectors.

We also show that a fully specified structural model can generate skewness and coskewness consistent with the U.S. data, with asymmetric sectoral shocks playing a key role in their propagation

through the network. The model allows us to decompose the contribution of quantities and real prices to the skewness of real output growth and to document that their coskewness is an important contributor to overall skewness. Finally, this paper provides structural estimates of the elasticity of substitution between material inputs and between consumption goods, which may be used by future researchers in the calibration of structural models. We estimate the elasticity of substitution between intermediate inputs to be 0.834—significantly higher than the values typically assumed in the production network literature, which generally range from 0.001 to 0.13. A higher elasticity implies that sectors have greater flexibility in adjusting input combinations in response to shocks, potentially moderating the extent of shock amplification through the network.

Table 1: Skewness in the Aggregate Data: 1960-2019

	κ_3			κ_4		
	60-19	60-84	84-19	60-19	60-84	84-19
$y - y^*$	-0.099	-0.145	-0.456	3.221	2.679	2.479
$u - u^*$	-0.790	-0.507	-1.250	3.588	2.919	3.923
Δy	-0.289	-0.287	-1.307	4.770	3.113	7.598
$\hat{F}_1$	-1.238	-1.229	-1.739	6.750	5.219	9.718
$\hat{F}_2$	0.435	-1.022	0.565	9.735	16.197	7.571
$\hat{F}_3$	-0.333	-0.747	0.623	4.711	3.870	3.927

Notes: κ_3 and κ_4 respectively denote skewness and excess kurtosis. The quarterly data for output gap ($y - y^*$), unemployment minus the natural rate ($u - u^*$), and real output growth Δy were taken from the December 2024 vintage of the Congressional Budget Office database. The three factors $\hat{F}$ were estimated using monthly data from the December 2024 vintage of FRED-MD.

Table 2: Decomposition with $\Phi_0 = a_0 W_B$

	a_0								
	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.750
Total Variance = 10.088									
$\mathcal{N}$	0.048	0.065	0.091	0.157	0.240	0.343	0.463	0.601	0.844
$\mathcal{C}$	0.599	0.542	0.475	0.353	0.250	0.165	0.099	0.051	0.011
$\mathcal{E}$	0.020	0.020	0.020	0.019	0.018	0.017	0.016	0.014	0.015
$2 \mathcal{N}\mathcal{C}$	0.325	0.364	0.404	0.456	0.470	0.447	0.385	0.287	0.026
$2 \mathcal{N}\mathcal{E}$	0.007	0.009	0.011	0.015	0.021	0.029	0.037	0.046	0.104
$2 \mathcal{C}\mathcal{E}$	0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000
Total Skewness = -1.687									
$\mathcal{N}$	-0.016	-0.027	-0.046	-0.107	-0.208	-0.358	-0.567	-0.845	-1.413
$\mathcal{C}$	-0.749	-0.640	-0.520	-0.327	-0.189	-0.097	-0.041	-0.013	-0.000
$\mathcal{E}$	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	-0.001
$\mathcal{N}\mathcal{N}\mathcal{C}$	-0.172	-0.230	-0.307	-0.464	-0.599	-0.685	-0.692	-0.597	-0.114
$\mathcal{N}\mathcal{C}\mathcal{C}$	-0.620	-0.663	-0.691	-0.673	-0.581	-0.444	-0.290	-0.150	-0.008
$\mathcal{N}\mathcal{N}\mathcal{E}$	-0.005	-0.007	-0.011	-0.018	-0.028	-0.039	-0.050	-0.055	-0.137
$\mathcal{N}\mathcal{E}\mathcal{E}$	-0.003	-0.003	-0.004	-0.005	-0.006	-0.007	-0.007	-0.007	-0.019
$\mathcal{C}\mathcal{C}\mathcal{E}$	-0.076	-0.066	-0.055	-0.037	-0.024	-0.014	-0.007	-0.003	-0.002
$\mathcal{C}\mathcal{E}\mathcal{E}$	-0.007	-0.007	-0.006	-0.005	-0.004	-0.003	-0.002	-0.001	0.002
$\mathcal{N}\mathcal{C}\mathcal{E}$	-0.040	-0.044	-0.048	-0.050	-0.048	-0.042	-0.031	-0.017	0.006

Notes: The table reports decomposition of variance and skewness when the inverse Leontief matrix is $(I_N - a_0 W_B)^{-1}$ for different values of the scalar a_0 . Here, W_B is the normalized input-output matrix produced by the BEA. Output is expressed as the sum of three components: network ($\mathcal{N}$), common ($\mathcal{C}$), and idiosyncratic ($\mathcal{E}$). The top panel reports the proportion of the variance explained by $\mathcal{N}, \mathcal{C}, \mathcal{E}$ and the three covariances. The bottom panel reports the skewness due to the three components and the seven coskewness terms.

Table 3: Decomposition of Variance (κ_2) and Skewness (κ_3)

Model	F	I	II	II(d)	F	I	II	IID
κ_2	Share of Variance 10.088				S.E			
$\mathcal{N}$	-	0.149	0.570	0.683	-	0.072	0.070	0.077
$\mathcal{C}$	0.954	0.470	0.066	0.144	0.038	0.044	0.014	0.029
$\mathcal{E}$	-	0.026	0.011	0.013	-	0.006	0.007	0.007
$2\mathcal{NC}$	-	0.514	0.324	0.156	-	0.056	0.076	0.086
$2\mathcal{NE}$	0.046	-0.047	0.029	0.004	0.038	0.037	0.047	0.038
$2\mathcal{CE}$	0.000	-0.112	0.000	0.000	0.000	0.014	0.000	0.000
κ_3	Contribution to $\kappa_3 = -1.687$				S.E.			
$\mathcal{N}$	-	-0.099	-0.789	-0.916	-	0.132	0.297	0.285
$\mathcal{C}$	-1.363	-0.507	-0.019	0.018	0.140	0.040	0.005	0.004
$\mathcal{E}$	0.004	0.002	0.000	0.000	0.014	0.001	0.001	0.001
$\mathcal{N}\mathcal{N}\mathcal{C}$	-	-0.510	-0.626	-0.601	-	0.142	0.150	0.172
$\mathcal{N}\mathcal{C}\mathcal{C}$	-	-0.879	-0.182	-0.162	-	0.112	0.040	0.045
$\mathcal{N}\mathcal{N}\mathcal{E}$	-	0.044	-0.049	-0.034	-	0.042	0.071	0.065
$\mathcal{N}\mathcal{E}\mathcal{E}$	-	-0.010	-0.003	0.003	-	0.008	0.012	0.010
$\mathcal{C}\mathcal{C}\mathcal{E}$	-0.303	0.134	-0.002	0.014	0.181	0.018	0.002	0.006
$\mathcal{C}\mathcal{E}\mathcal{E}$	-0.025	-0.016	-0.001	0.001	0.042	0.004	0.002	0.002
$\mathcal{N}\mathcal{C}\mathcal{E}$	-	0.155	-0.016	-0.010	-	0.047	0.020	0.024

Notes: Output is expressed as the sum of three components: network ($\mathcal{N}$), common ($\mathcal{C}$), and idiosyncratic ($\mathcal{E}$). The top panel reports the proportion of the variance explained by $\mathcal{N}, \mathcal{C}, \mathcal{E}$ and the three covariances. The bottom panel reports the skewness due to the three components and the seven coskewness terms. Model F is a factor-only model with no network. Model I is the case when a_0 is a scalar. Static Model II lets a_0 be a $N \times 1$ vector. Model II(d) includes a lag-dependent variable.

Table 4: SMM Estimates

Parameter	Estimate	b.s.e
Aggregate shock		
AR coefficient	0.898	0.028
Scale parameter	0.065	0.067
Shape parameter	-1.827	0.014
Sectoral shock		
AR coefficient	0.691	0.031
Scale parameter	0.252	0.051
Shape parameter	-1.583	0.080
Elasticity of substitution		
Consumption	0.735	0.071
Production	0.828	0.049

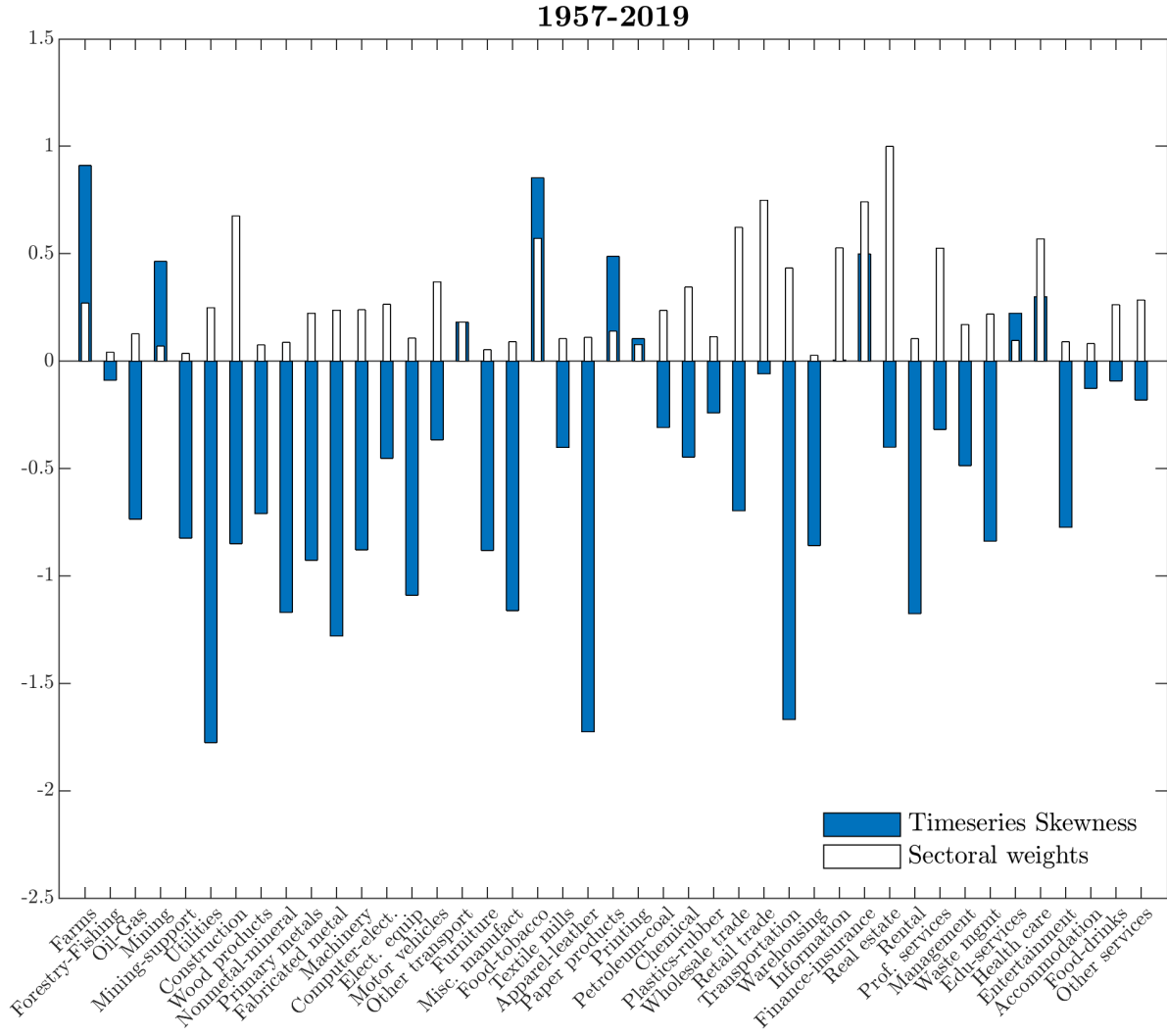
Notes: The table reports SMM estimates of the structural parameters of the multi-sector model and bootstrapped standard errors (b.s.e.) computed using a parametric k -step bootstrap with 99 replications. The scale and shape parameters are those of the respective GEV distributions and jointly determine the standard deviation and skewness of the shocks (see Section 4.2).

Table 5: Decompositions

	Reduced-Form Model (II)	Structural Model	Counter-factuals	
			1	2
Variance Shares				
$\mathcal{N}$	0.570	0.601	0.606	0.715
$\mathcal{C}$	0.066	0.051	0.049	0.024
$\mathcal{E}$	0.011	0.000	0.001	0.001
$2\ \mathcal{NC}$	0.324	0.346	0.340	0.259
$2\ \mathcal{NE}$	0.029	0.002	0.003	0.002
$2\ \mathcal{CE}$	0.000	0.000	0.000	0.000
Skewness				
Total	-1.687	-1.658	-0.425	-0.017
$\mathcal{N}$	-0.789	-0.763	-0.225	-0.027
$\mathcal{C}$	-0.019	-0.021	-0.003	-0.000
$\mathcal{E}$	0.000	-0.000	-0.000	0.000
$\mathcal{NNC}$	-0.626	-0.691	-0.164	-0.016
$\mathcal{NCC}$	-0.182	-0.209	-0.040	-0.003
$\mathcal{NNE}$	-0.049	0.016	0.005	-0.021
$\mathcal{NEE}$	-0.003	-0.000	-0.000	0.001
$\mathcal{CEE}$	-0.002	-0.001	0.001	0.001
$\mathcal{CEE}$	-0.001	-0.000	-0.000	0.000
$\mathcal{NCE}$	-0.211	0.009	0.004	0.007

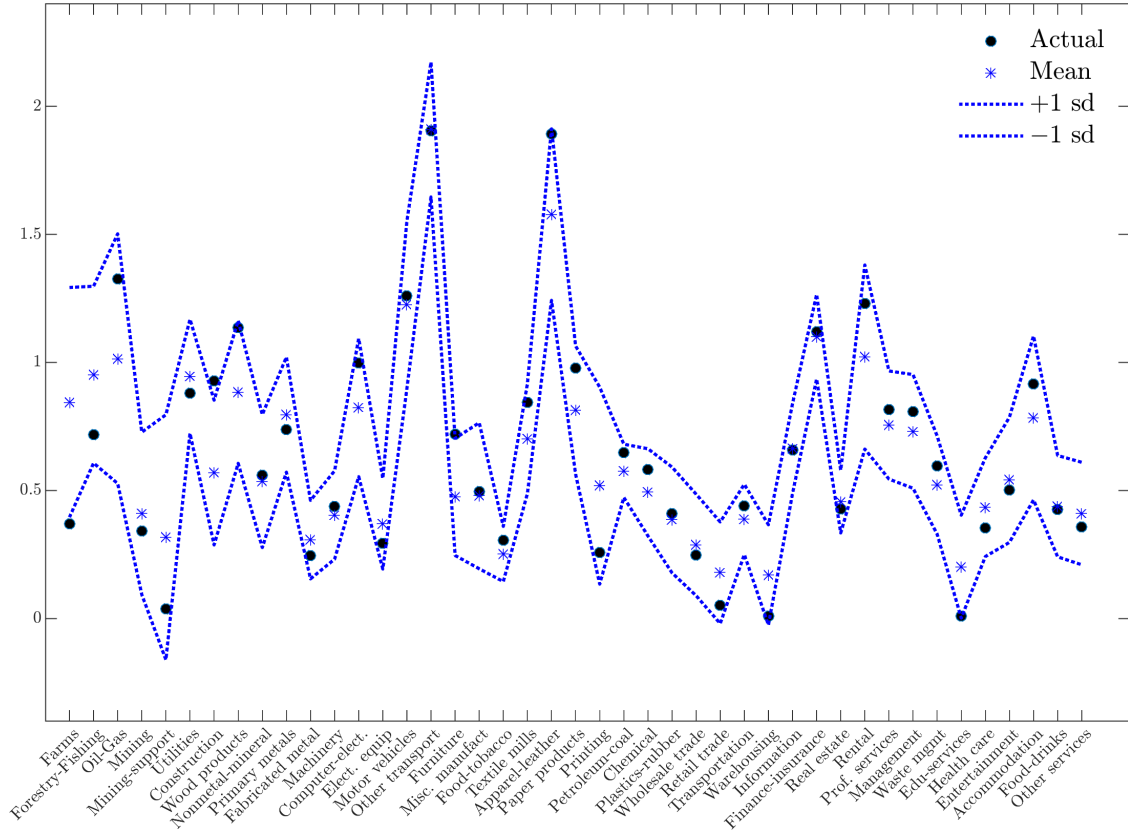
Notes: The table reports the contributions to the variance and skewness of aggregate output from each component. Counter-factual 1 refers to the experiment where aggregate productivity shocks are normally distributed and sectoral productivity shocks follow a GEV distribution. Counter-factual 2 refers to the experiment where aggregate productivity shocks follow a GEV distribution and sectoral productivity shocks are normally distributed.

Figure 1: Skewness of Output Growth and Value-Added Shares by Sector



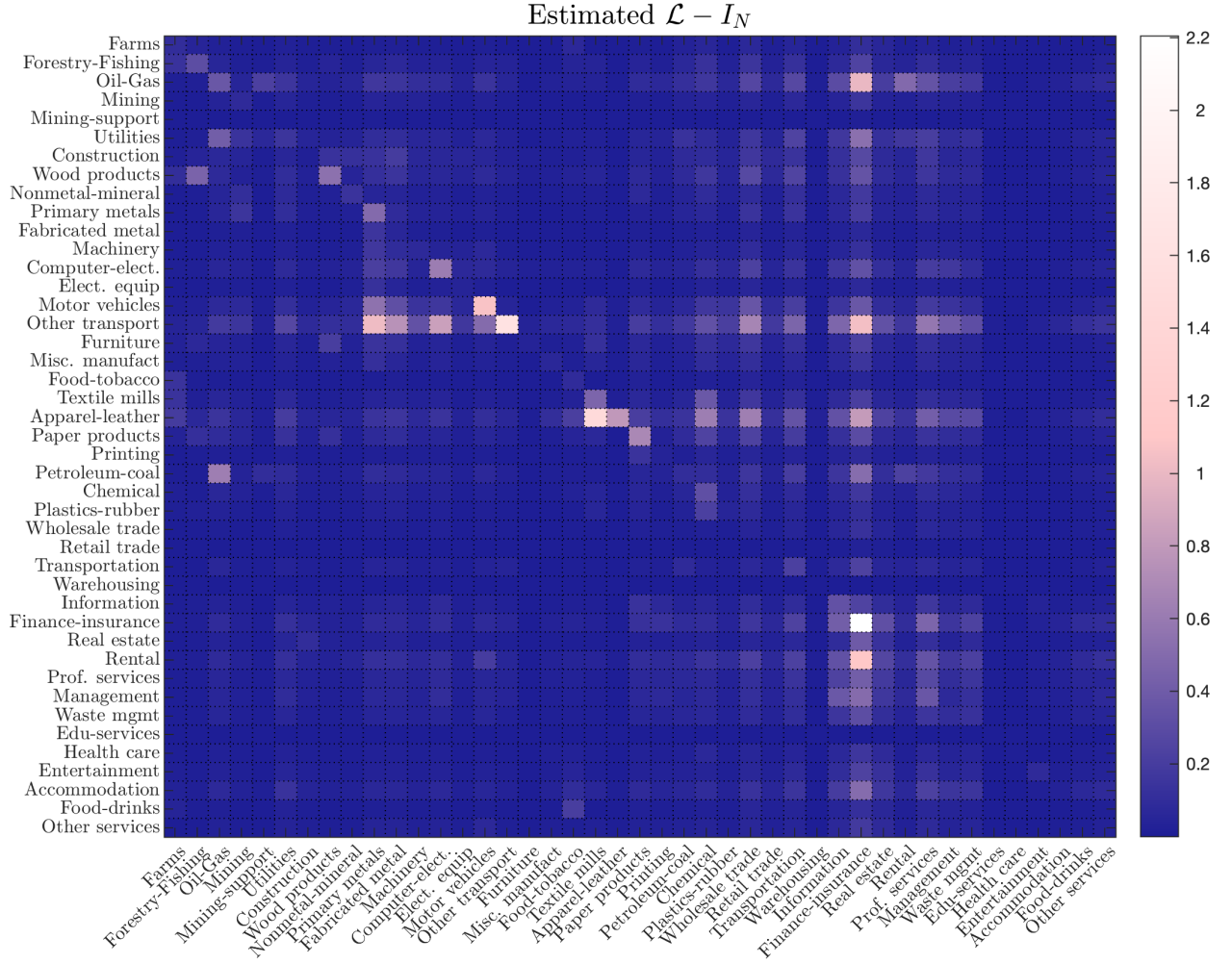
Notes: Blue bars are the coefficient of skewness for each of the 43 sectors over the sample 1957-2019. White bars are the corresponding (value-based) sectoral weights.

Figure 2: GMM Estimates of A_0 and Monte Carlo Results



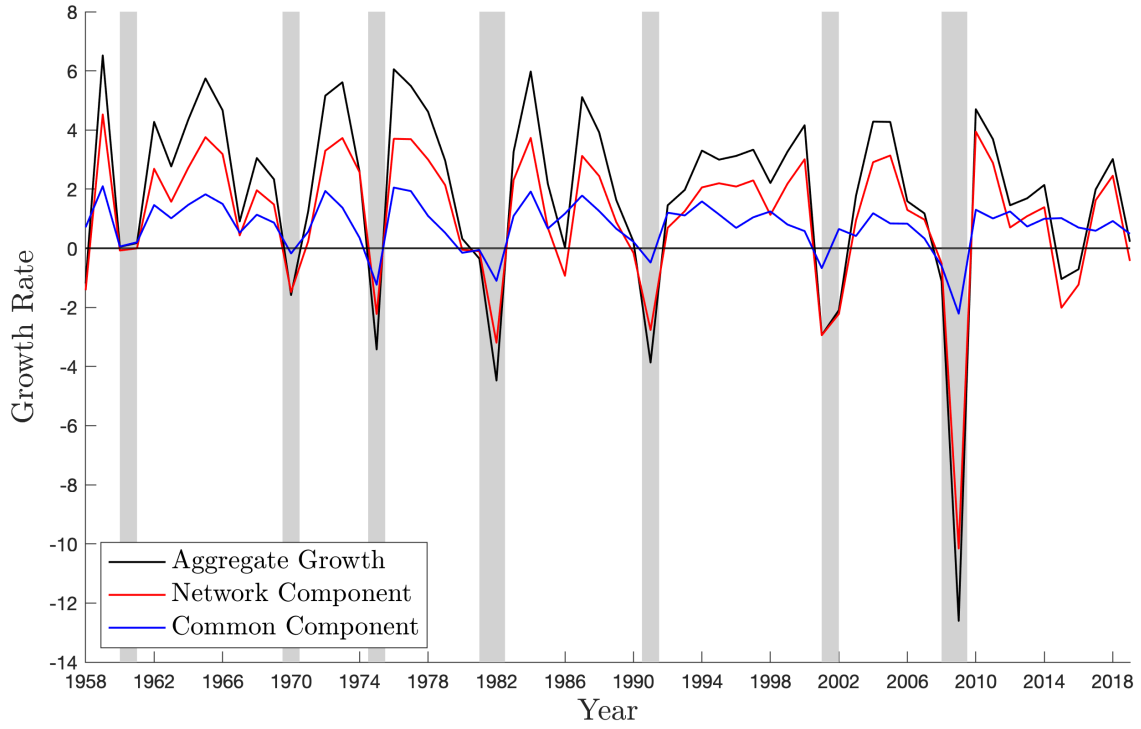
Notes: The calibrated Monte Carlo is based on parameters (in red) and the common component estimated from Model II with $T = 63, N = 43$. Each replication draws idiosyncratic errors with the same covariance structure as the residuals in Model II. The mean and ± 1 standard deviation of the estimates in 1000 replications are in blue.

Figure 3: Heatmap of Estimated Network Multiplier Matrix $\{\mathcal{L} - I_N\}$



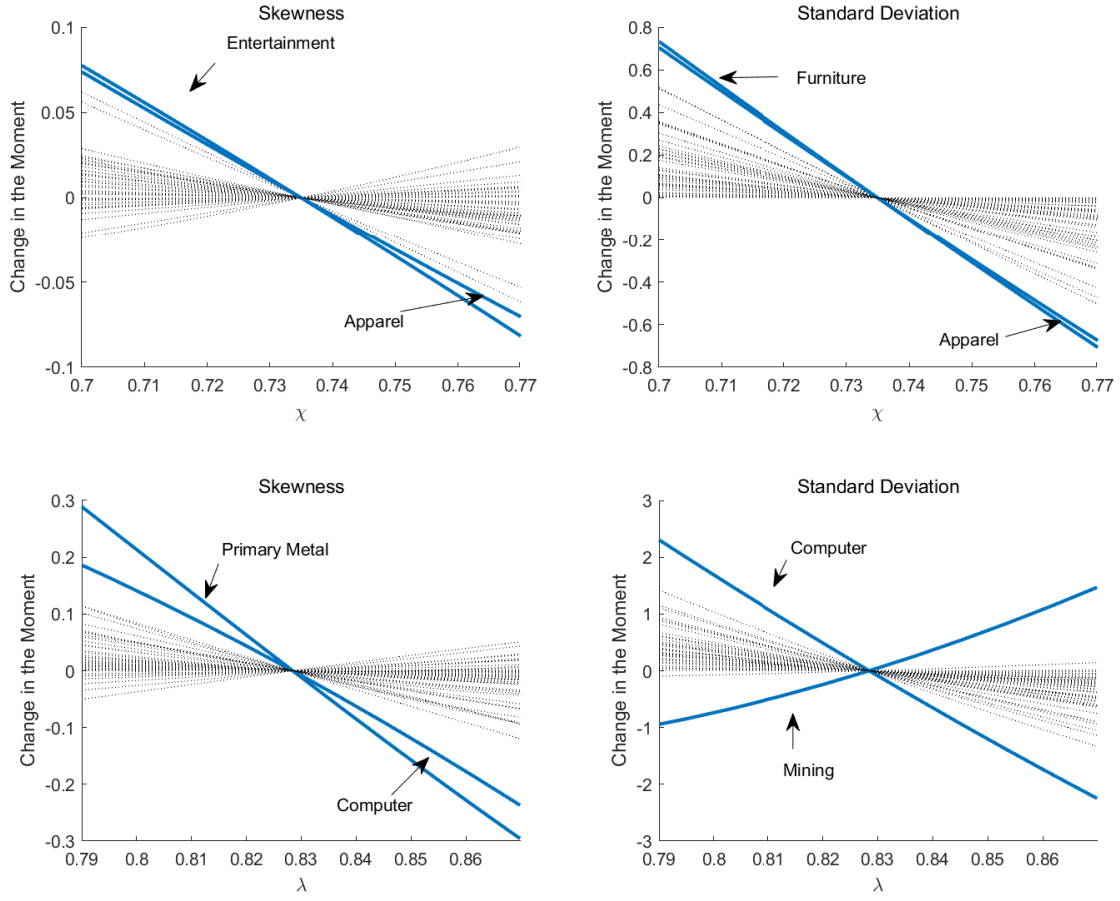
Notes: The figure displays a heatmap of the estimated value of the inverse Leontief ($\mathcal{L}$) minus the identity matrix (I_N) for 43 sectors. Each cell (i, j) represents the network multiplier of sector j 's shocks on sector i 's output growth.

Figure 4: Cyclicality of Network and Common Components



Notes: The black line denotes the aggregate growth (Y_t) with red and blue lines showing the contribution of network ($\mathcal{N}_t$) and common ($\mathcal{C}_t$) components, respectively. The shaded region represents the NBER recession periods.

Figure 5: Identification



Notes: Sectoral moments that identify the elasticity of substitution across consumption goods and materials.

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A Data Appendix

We construct a panel of U.S. production and input-output data using annual tables from the Bureau of Economic Analysis (BEA). Our primary source is the BEA historical input-output archive, which provides Make and Use tables for the period 1947-2019. Using this data, we construct sector-level measures of gross output, value added, intermediate input use, and consumption expenditures. To express variables in real terms, we use price deflators and population series from the BEA National Income and Product Accounts (NIPA). In particular, we use the GDP deflator (Table 1.1.9) and annual population measures consistent with BEA definitions.

The BEA input-output tables vary in their level of sectoral disaggregation over time: 43 sectors for 1947-1962, 61 sectors for 1963-1996, and 66 sectors for 1997-2019. To construct a consistent long-run panel, we harmonize all years to a common 43-sector classification based on the earliest period. This required us to construct concordances across classification systems using NAICS-based mappings. The list of 43 sectors used in the empirical analysis is given in Table A1. In addition, we aggregate the data to a 14-sector classification to facilitate a robustness check. All mappings are implemented using a unified concordance applied uniformly across time.

We construct the input-output table W_B using the BEA Make and Use tables following Pastén, Schoenle, and Weber (2024). Let make_{ij} denote the value of commodity j produced by industry i , and use_{ij} the value of commodity i used by industry j . We first compute production shares,

$$\text{share}_{ij} = \frac{\text{make}_{ij}}{\sum_{k=1}^N \text{make}_{kj}}$$

and then define revenue-weighted flows,

$$\text{revshare}_{ij} = \text{share}_{ij} \times \text{use}_{ij}.$$

Finally, input-output coefficients are given by

$$W_{Bij} = \frac{\text{revshare}_{ij}}{\sum_{k=1}^N \text{use}_{kj}}$$

This yields an $N \times N$ matrix capturing the network structure of intermediate input linkages across industries. For each year, we obtain the input-output matrix, gross output and value added by sector, consumption expenditures and consumption shares by sector, price deflators, and population series. All variables are available at both the 43-sector and 14-sector levels of aggregation.

Table A1: List of 43 NAICS Sectors

No.	Code	Name
1	111	Farms
2	113	Forestry, fishing, and related activities
3	211	Oil and gas extraction
4	212	Mining, except oil and gas
5	213	Support activities for mining
6	22	Utilities
7	23	Construction
8	321	Wood products
9	327	Nonmetallic mineral products
10	331	Primary metals
11	332	Fabricated metal products
12	333	Machinery
13	334	Computer and electronic products
14	335	Electrical equipment, appliances, and components
15	3361	Motor vehicles, bodies and trailers, and parts
16	3364	Other transportation equipment
17	337	Furniture and related products
18	339	Miscellaneous manufacturing
19	311	Food and beverage and tobacco products
20	313	Textile mills and textile product mills
21	315	Apparel and leather and allied products
22	322	Paper products
23	323	Printing and related support activities
24	324	Petroleum and coal products
25	325	Chemical products
26	326	Plastics and rubber products
27	42	Wholesale trade
28	44	Retail trade
29	48	Transportation
30	493	Warehousing and storage
31	51	Information
32	52	Finance and insurance
33	531	Real estate
34	532	Rental and leasing services and lessors of intangible assets
35	54	Professional, scientific, and technical services
36	55	Management of companies and enterprises
37	56	Administrative and waste management services
38	61	Educational services
39	62	Health care and social assistance
40	71	Arts, entertainment, and recreation
41	721	Accommodation
42	722	Food services and drinking places
43	81	Other services, except government

Notes: The table lists the 43-sector classification used in our empirical analysis. The column ‘Code’ is the NAICS sector code, and the column ‘Name’ is the NAICS sector name.

B Additional Tables and Figures

Table B1: Skewness in 43 Sectors: 1957-2019

	1957-2019	Pre 1984	Post 1984
R_C^2	0.560	0.628	0.611
$\text{AR}_{\hat{e}} > 0.3$	0.279	0.279	0.209
$\text{AR}_y > 0.3$	0.302	0.233	0.349
$\text{avg } \sigma_{\hat{e}}$	4.141	3.364	3.923
$\text{avg } \sigma_y$	6.698	6.019	6.805
$\text{med } \sigma_{\hat{e}}$	3.512	2.749	2.960
$\text{med } \sigma_y$	6.109	5.773	5.471
$\kappa_3(\hat{F}_1)$	-1.457	-0.479	-1.965
$\text{avg}(\kappa_3^{\hat{e}})$	0.128	0.089	0.148
$\text{avg}(\kappa_3^y)$	-0.464	-0.073	-0.587

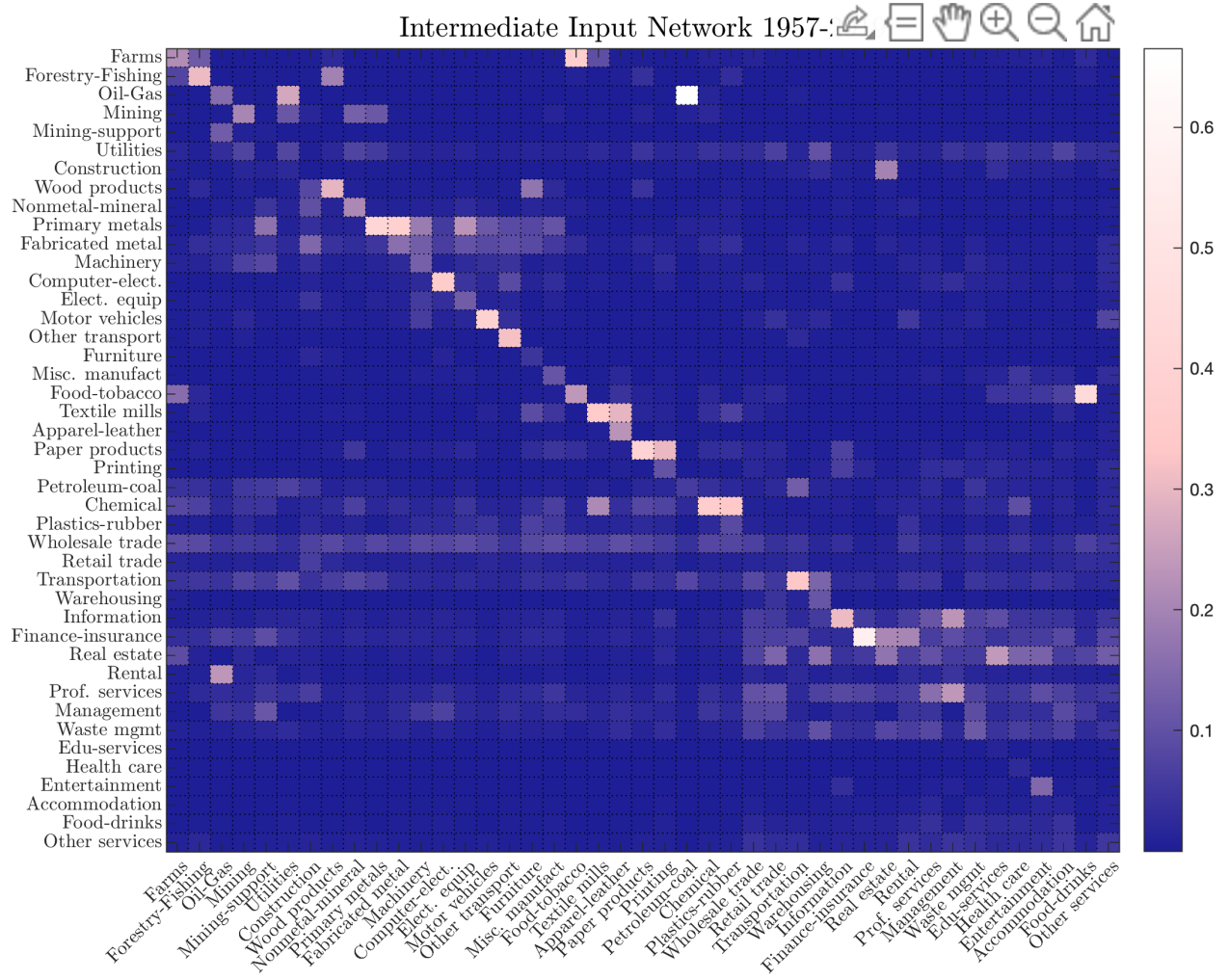
Notes: The table reports statistics for 43 sectors in the 3-digit BEA database; R_C^2 is the average R^2 in a regression of each sector's output on its common component $\hat{C}_i = \hat{F}\hat{\Lambda}'_i$; $\text{AR}_{\hat{e}}$ is the fraction of estimated idiosyncratic errors with first-order autocorrelation exceeding 0.3; $\text{avg } \sigma_{\hat{e}}$ and $\text{med } \sigma_{\hat{e}}$ are the mean and median standard deviation of $\hat{e}$ in the 43 sectors; $\kappa_3(\hat{F}_1)$ is the Fisher coefficient of skewness for the first estimated factor, $\hat{F}_1$; and $\text{avg}(\kappa_{ee})$ is the coefficient of skewness of $\hat{e}_i$ averaged over 43 sectors. AR_y , $\text{avg } \sigma_y$, $\text{med } \sigma_y$, and $\text{avg}(\kappa_3^y)$ are similarly defined for sectoral output growth y .

Table B2: Relative Importance of Components for Aggregate Growth

	Network		Common		Idiosyncratic	
	LOW	HIGH	LOW	HIGH	LOW	HIGH
	$RL_{\mathcal{N}}$	$RH_{\mathcal{N}}$	$RL_{\mathcal{C}}$	$RH_{\mathcal{C}}$	$RL_{\mathcal{E}}$	$RH_{\mathcal{E}}$
LOW $\equiv$ Bottom 25 percentile & HIGH $\equiv$ Top 25 percentile	0.908	0.655	0.044	0.333	0.048	0.012
LOW $\equiv$ Bottom 17 percentile & HIGH $\equiv$ Top 17 percentile	0.861	0.665	0.097	0.315	0.042	0.019
LOW $\equiv$ Bottom 10 percentile & HIGH $\equiv$ Top 10 percentile	0.799	0.653	0.172	0.316	0.029	0.030
LOW $\equiv$ Negative Agg. Growth & HIGH $\equiv$ Positive Agg. Growth	0.855	0.647	0.098	0.342	0.047	0.011

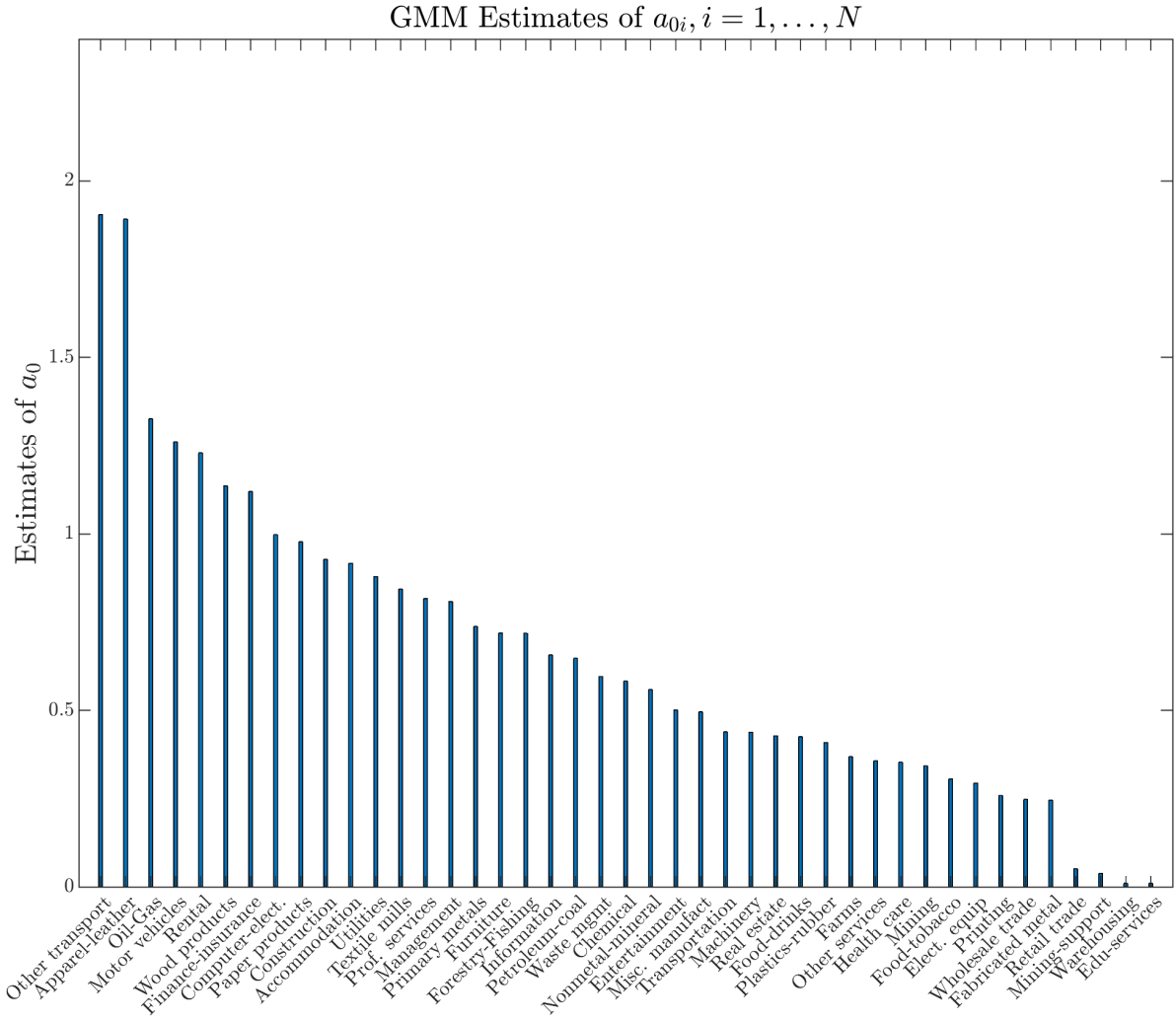
Notes: This table represents the importance of network and common components in LOW and HIGH states. Each year is classified into LOW or HIGH based on whether aggregate GDP growth lies in the bottom or top x percentile (given in the first column) of its empirical distribution. For each component in $\mathcal{I}_t \in \{\mathcal{N}, \mathcal{C}, \mathcal{E}\}$, the relative importance measures are given by $RL(\mathcal{I}_t) = \frac{\sum_{t \in \text{LOW}} \mathcal{I}_t}{\sum_{t \in \text{LOW}} Y_t}$ and $RH(\mathcal{I}_t) = \frac{\sum_{t \in \text{HIGH}} \mathcal{I}_t}{\sum_{t \in \text{HIGH}} Y_t}$.

Figure B1: Heatmap of Normalized Input-Output Matrix W_B



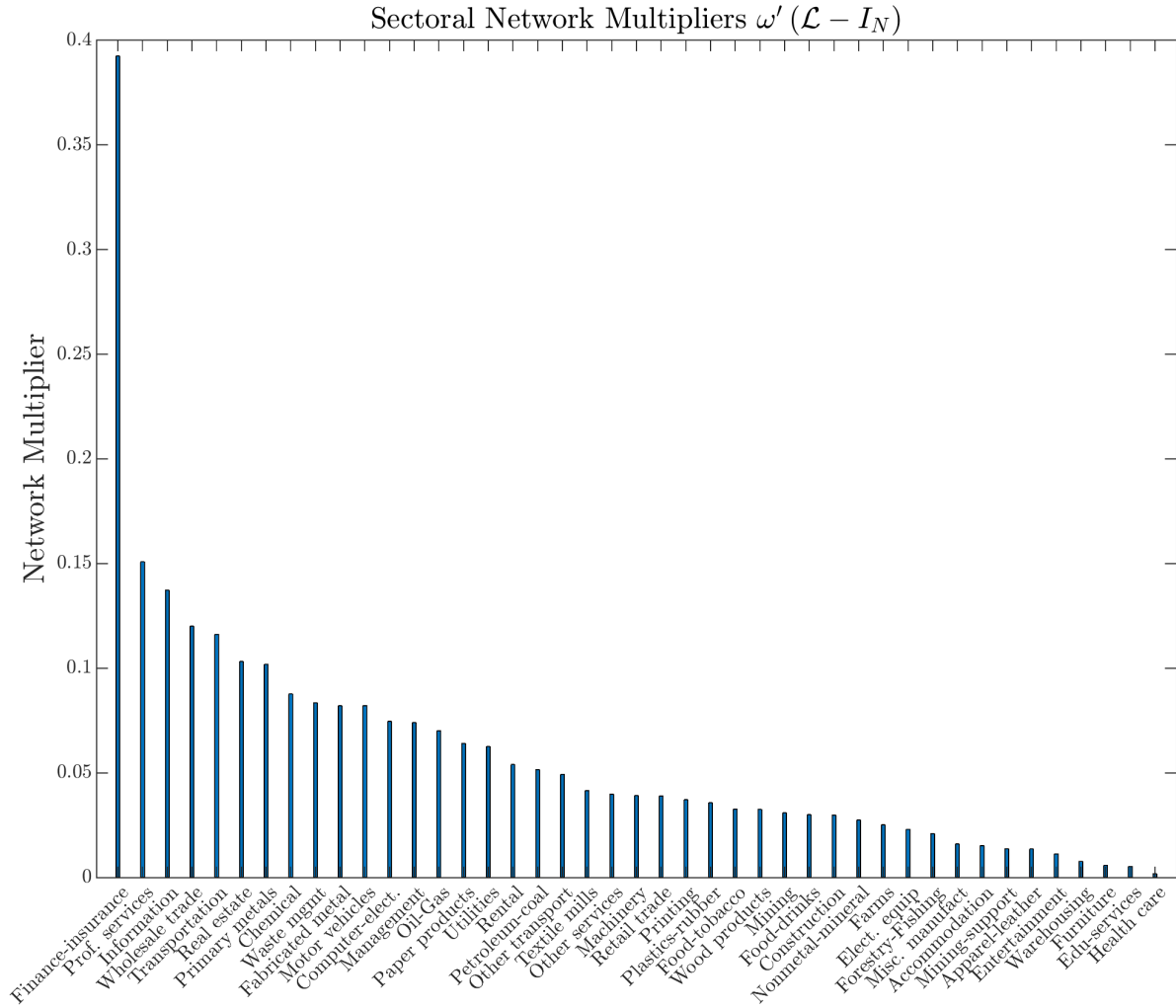
Notes: The figure displays a heatmap of the intermediate input-output network for 43 sectors. Each cell (i, j) represents the share of sector j 's total intermediate inputs expenditure sourced from sector i .

Figure B2: Baseline Estimates of A_0



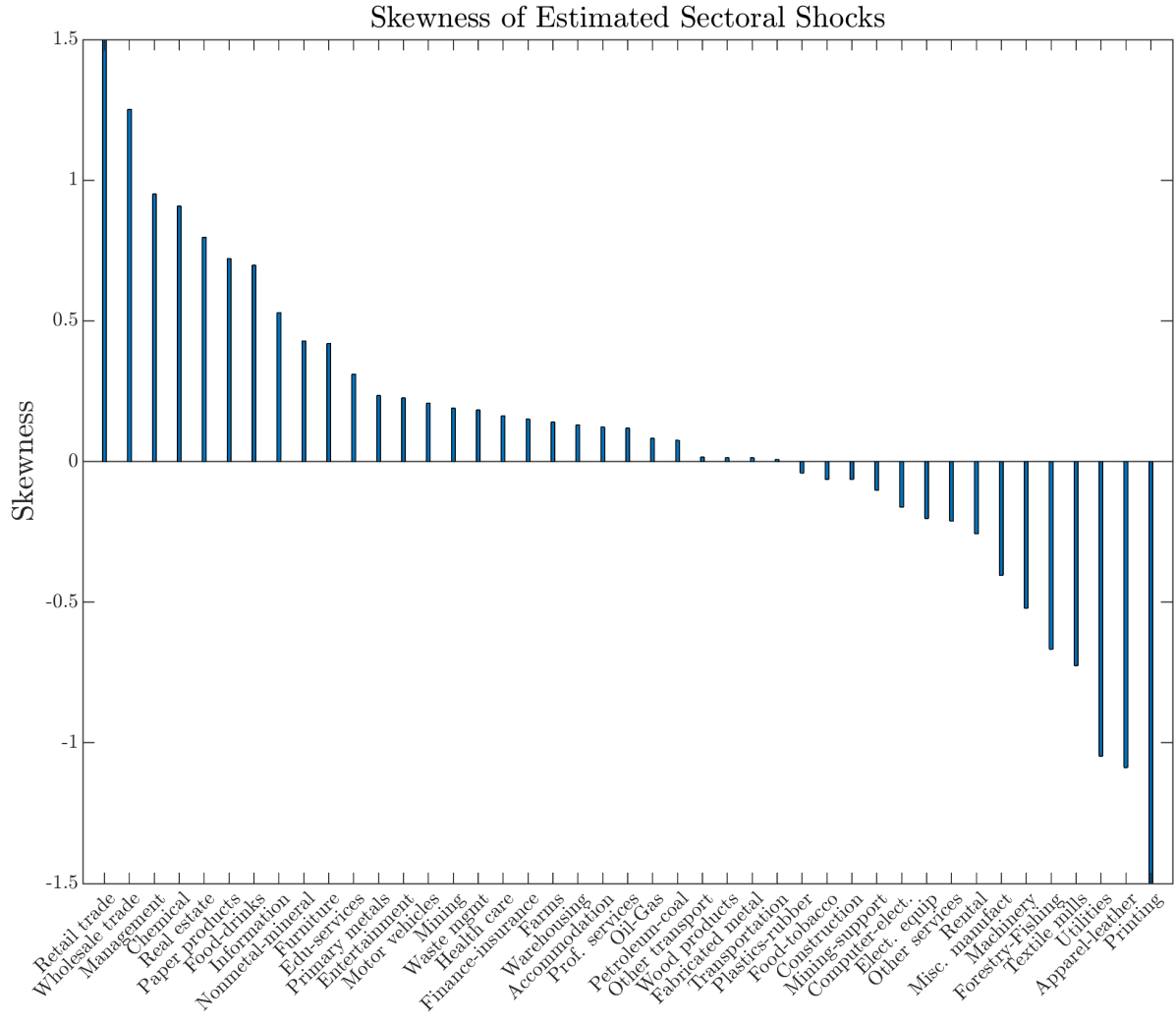
Notes: The bars denote the point estimates of the vector a_0 based on the GMM estimation of Model II. The sectors are ordered by their estimated coefficients to display the significant heterogeneity among them.

Figure B3: Relative Importance of Sectors for Network Propagation



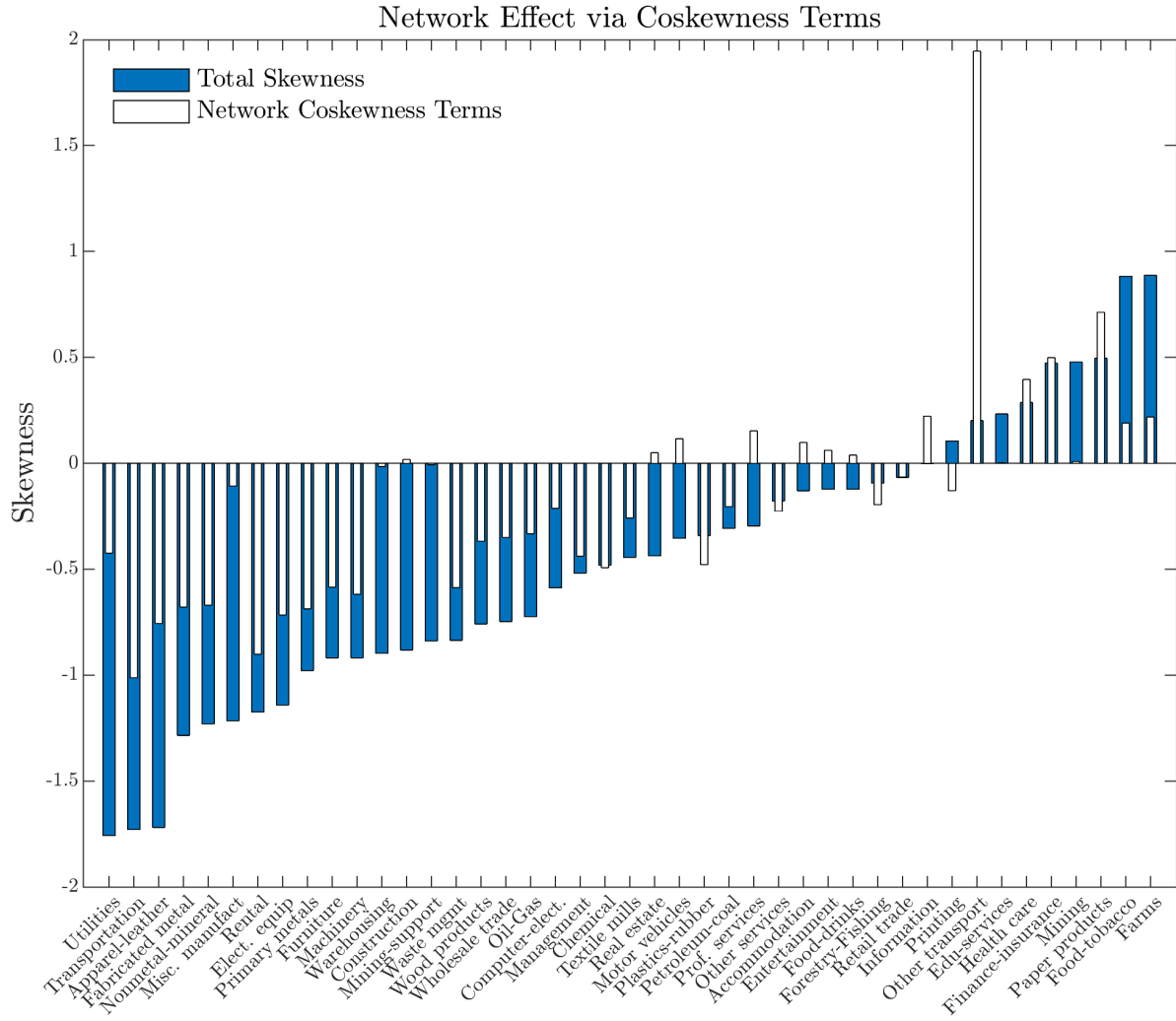
Notes: The bars denote estimated values of network multiplier effect $\omega'(\mathcal{L} - I_N)$ for 43 sectors from our baseline model. The values represent the importance of each sector to aggregate output growth via the intermediate-input network.

Figure B4: Skewness of Estimated Sectoral Shocks $\{\hat{e}_{it}\}$



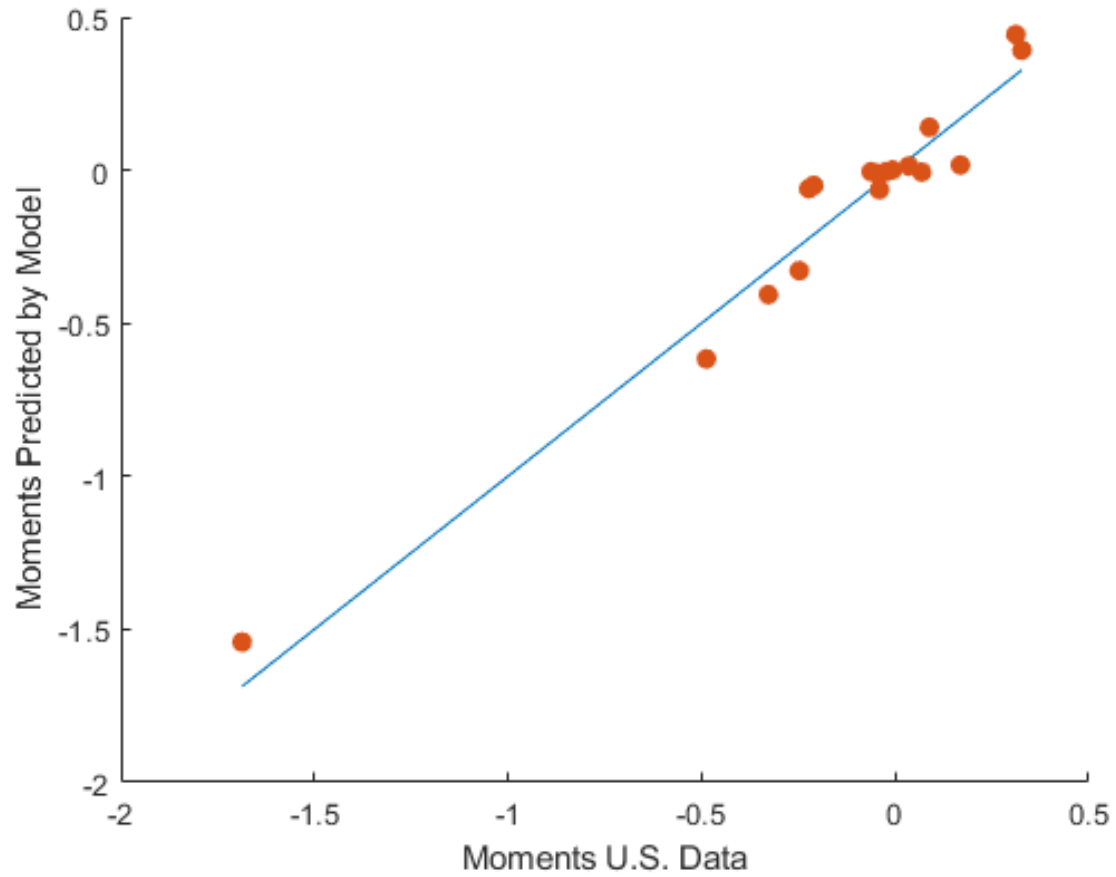
Notes: The bars denote skewness of the estimated sectoral shocks $\{\hat{e}_{it}\}$ for 43 sectors from our baseline model.

Figure B5: Relative Importance of Coskewness Terms for Network Propagation



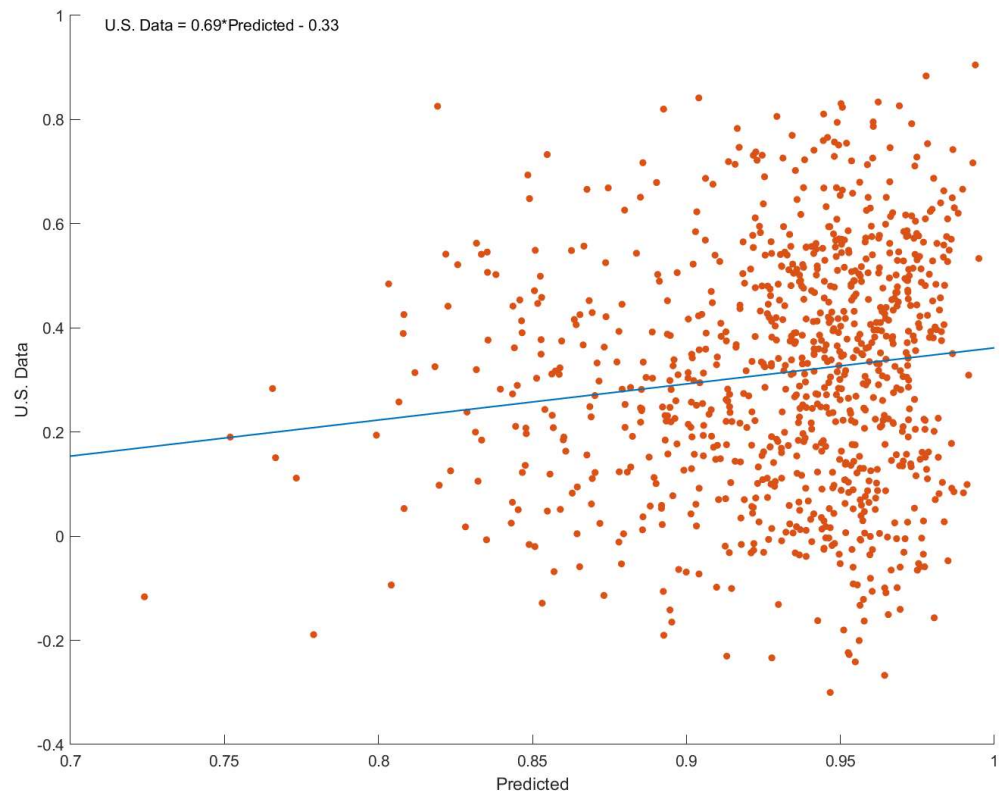
Notes: The blue bars denote the total skewness in each of the 43 sectors for our sample period from 1957 to 2019. The white bars denote the skewness explained by the intermediate input network through the four coskewness terms containing network: $\mathcal{NNC}$, $\mathcal{NCC}$, $\mathcal{NNE}$, and $\mathcal{NEE}$.

Figure B6: Model Fit



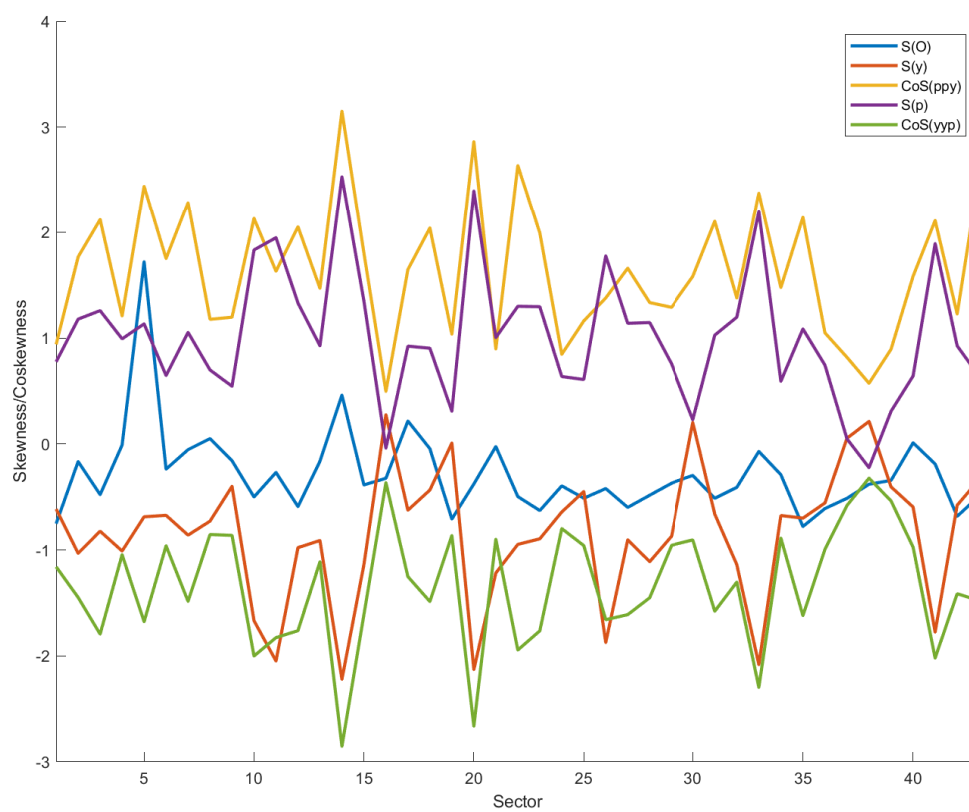
Notes: Comparison of the moments estimated using U.S. sectoral data and those predicted by the model based on the estimates reported in Table 6

Figure B7: Actual and Predicted Cross-correlations



Notes: Comparison of sectoral output growth cross-correlations estimated using U.S. sectoral data and those predicted by the model based on the estimates reported in Table 6

Figure B8: Decomposition of Skewness in Sectoral Output Growth



Notes: Contribution to the skewness of sectoral real output growth, $S(O)$, from quantity ($S(y)$), real price ($S(p)$), and the two coskewness terms.

C Estimation Details

The estimator for Model-II is implemented as a PC–GMM fixed-point iteration:

1. PC step (factor extraction): Given network parameters, form a network-filtered series and estimate a low-dimensional factor structure by PCA.
2. GMM step: Holding the estimated common component fixed (via the fitted common component and intercepts), update the network parameters by minimizing a GMM criterion.
3. Repeat until the network parameters converge.

This is an iterative procedure rather than a single convex optimization. The GMM objective is generally *non-convex* in ϕ , and, in the dynamic case, in (ϕ_0, ϕ_1) . The PC step depends on ϕ , and the GMM step depends on the PC step. Self-connectivity ($W_{ii} \neq 0$) and strong network propagation can lead to multiple basins of convergence. Consequently, different initial values may converge quickly but to different local fixed points.

Reduce dependence on initial values: The common component enters the residual as

$$y_t - \Phi y_t - \theta \hat{C}_t - \alpha,$$

where $\hat{C}_t$ is the estimated common component from the factor step. Empirically, allowing a free θ can introduce a weakly identified scaling direction that trades off with factor/loadings scaling in the PC step, increasing multimodality and sensitivity. Therefore, in the estimation, we set $\theta \equiv \mathbf{1}$ and remove θ from the numerical optimization. The GMM step then updates only the network parameters: ϕ in the static model, (ϕ_0, ϕ_1) in the dynamic model.

Standardize inputs in the PCA step: In the PC step we compute PCA from a time-demeaned matrix (as before), but optionally also standardize each series (z-scoring over time) so that high-variance sectors do not dominate factor extraction. Concretely, letting X denote the $T \times N$ matrix used for PCA, we compute

$$X_{t\cdot} \leftarrow X_{t\cdot} - \bar{X}, \quad X_{\cdot i} \leftarrow X_{\cdot i} / \hat{\sigma}_i,$$

where $\hat{\sigma}_i$ is the sample standard deviation of the demeaned i th series. Importantly, only the PCA uses standardized inputs. Subsequent time-series regressions estimating the intercepts and loadings are still run on the original, unstandardized series, so the fitted common component remains in the original units.

Global Multi-Start Scheme: The main mechanism for reducing dependence on initialized values is a deterministic global multi-start scheme:

1. Construct a *deterministic* collection of starting values for the network parameters. In our implementation, this includes a fixed baseline start (e.g., $\phi = 0.5$) plus a mixture of jittered and uniform random starts generated with a fixed seed.
2. For each start, run the full PC–GMM fixed-point iteration until convergence.
3. For each converged solution, compute a *comparable* final objective value by recomputing the factor step at the converged parameters and evaluating the criterion at that point.
4. Choose the solution that attains the smallest final objective among the explored starts.

This changes the numerical procedure from converging to the basin containing initial values to searching multiple basins and selecting the best by the criterion.

C.1 Note on the Dynamic Model

We use the same stabilization choices and impose moment conditions that include both sectoral shocks covariance restrictions and dynamic orthogonality restriction given by

$$\mathbb{E}[(W y_{t-1})_i e_{it}] = 0.$$

Even with these modifications, if the population criterion has multiple minima with very similar values (or identification is weak), two distinct parameter vectors can both appear “good.” The approach above provides: (i) reduced dependence on provided initial values, (ii) deterministic behavior under a fixed seed, and (iii) selection of the best solution found among the explored starts.

C.2 Additional Details

The estimation also relies on the following assumptions.

- The input-output matrix W_B is observed and fixed, while the sector-specific propagation coefficients collected in ϕ_0 and ϕ_1 are unknown.
- Aggregate comovement not captured by the network is well approximated by a low-dimensional factor structure.
- The contemporaneous propagation matrix satisfies $\rho(\Phi_0) < 1$, where ρ is the spectral radius of Φ_0 . This condition guarantees that $I - \Phi_0$ is nonsingular, so the inverse Leontief matrix exists and can be written as $(I - \Phi_0)^{-1} = \sum_{m=0}^{\infty} \Phi_0^m$.
- For dynamic stability, we additionally assume $\rho((I - \Phi_0)^{-1}\Phi_1) < 1$.

D Empirical Analysis: NAICS 2-Digit Sectors

Our primary dataset comes from the U.S. Bureau of Economic Analysis (BEA) GDP-by-industry database, which provides annual sector-level output data from 1947 to 2019. We used the 43-sector NAICS 3-digit classification for our empirical analysis in the main paper. Table A1 in the main paper lists the NAICS codes and names of the 43 sectors used in our analysis. Alternatively, we could use a sectoral classification at the NAICS 2-digit level, which yields 14 broad sectors. Table D1 lists the sector codes and sector names of these 14 sectors.

Table D1: NAICS 14 Sectors

No.	Code	Name
1	11	Agriculture, forestry, fishing, and hunting
2	21	Mining
3	22	Utilities
4	23	Construction
5	31	Manufacturing
6	42	Wholesale trade
7	44	Retail trade
8	48	Transportation and warehousing
9	51	Information
10	52	Finance, insurance, real estate, rental, and leasing
11	54	Professional and business services
12	6	Educational services, health care, and social assistance
13	7	Arts, entertainment, recreation, accommodation, and food services
14	81	Other services, except government

We begin by constructing all relevant data for the 14-sector classification using the procedures outlined in the main paper. To illustrate how the 14-sector framework differs from the 43-sector classification, we compare the structure of their respective intermediate input networks. A quick look at Figure D1 alongside Figure B1 in the main paper shows that the two networks follow distinctly different patterns. At the 14-sector level, diagonal dominance is less pronounced, and the network is more heavily concentrated among a few major supplier sectors. In particular, Manufacturing, Information, Finance, and Real Estate together provide a significant portion of the intermediate inputs consumed by the remaining sectors. Such a high concentration level may amplify propagation effects within the network von Lehn and Winberry (2022).

To investigate the significance of coskewness terms, as in the main paper, we use GMM to

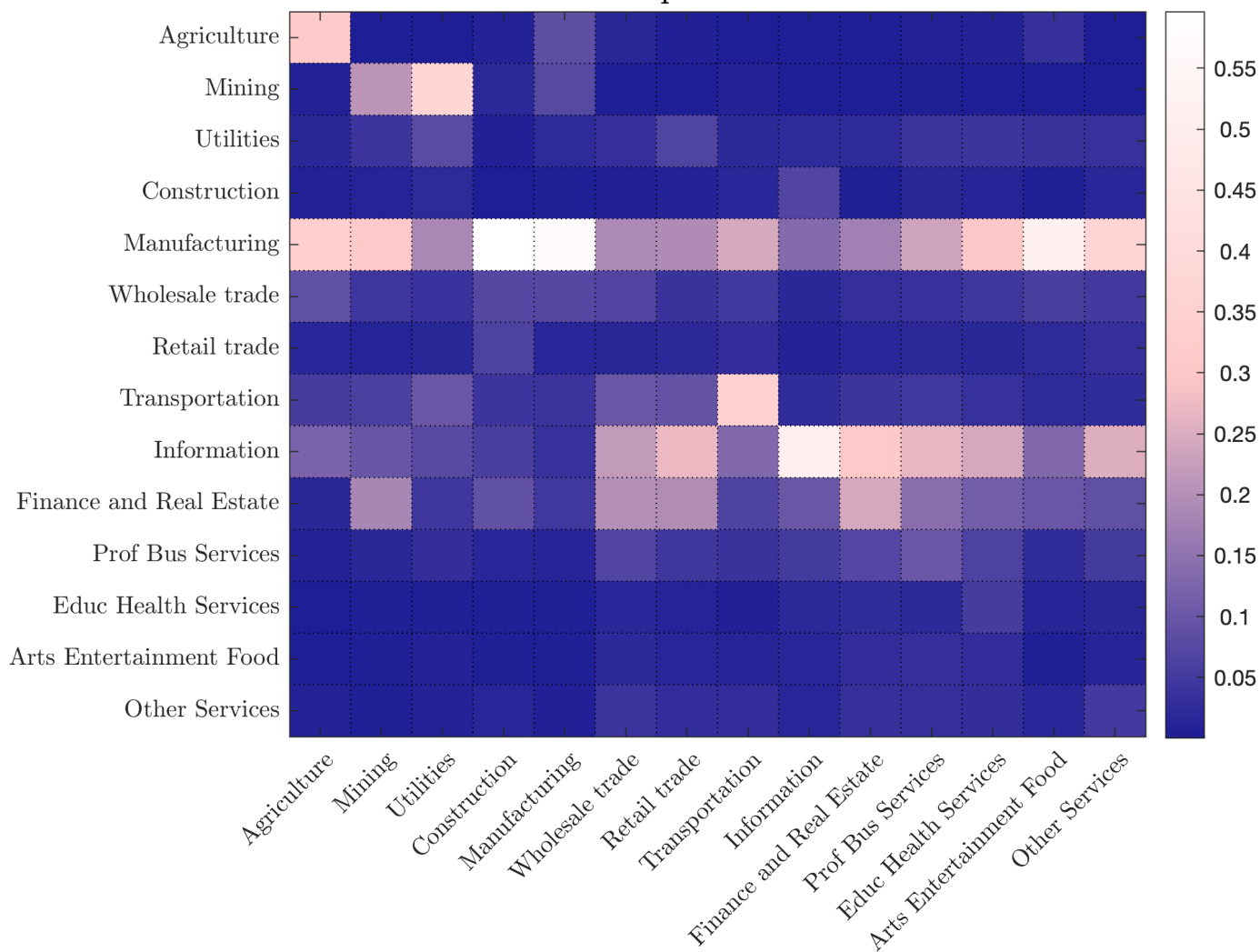
estimate our baseline model (the static version of Model II). Figure D2 shows the estimated values of A_0 for the 14 sectors and each sector's value-added share. Comparing these estimates with those in Figure B2 (based on the 43-sector classification) reveals that the 14-sector estimates exhibit less heterogeneity. As before, there is no evident relationship between each sector's estimates of A_0 and its value-added share. In this 14-sector setting, the estimates range from 0.35 to 1.32, with a mean of 0.61 and a standard deviation of only 0.27, lower than the 0.44 observed in the 43-sector case. This reduced variability in the estimates of A_0 places greater emphasis on the network's direct effects.

We next turn to the decomposition of variance and skewness to illustrate that, despite the network term's direct contribution being much lower than in the 43-sector case, higher-order comovement terms, coskewness, become more important for explaining aggregate variance and skewness. Table D2 presents the decomposition results. The first two columns, labeled $F(14)$ and $F(43)$, compare factor-only estimates under the respective classifications. Notably, the common component ($\mathcal{C}$), which captures network effects in the absence of network, plays a more prominent role in the 14-sector case and accounts for nearly all of the aggregate skewness. Meanwhile, our baseline model confirms that the covariance terms ($\mathcal{NC}$ and $\mathcal{NE}$) become more significant, accounting for about 48% of the variance (compared to 35% under the 43-sector classification).

The third and fourth columns of Table D2 emphasize the importance of coskewness terms in explaining aggregate skewness. Specifically, the coskewness components ($\mathcal{NNC}$ and $\mathcal{NNE}$) jointly account for more than 67% of skewness in the 14-sector classification, compared to about 48% in the 43-sector classification. Another noteworthy finding is the substantial increase in the direct contribution of the common ($\mathcal{C}$) component to aggregate skewness, which rises from 1% in the 43-sector case to nearly 8% in the 14-sector case. One possible explanation for this shift is that the lower heterogeneity in the A_0 estimates across the 14 sectors of the framework increases the relative importance of common factors and assigns greater weight to the direct effects.

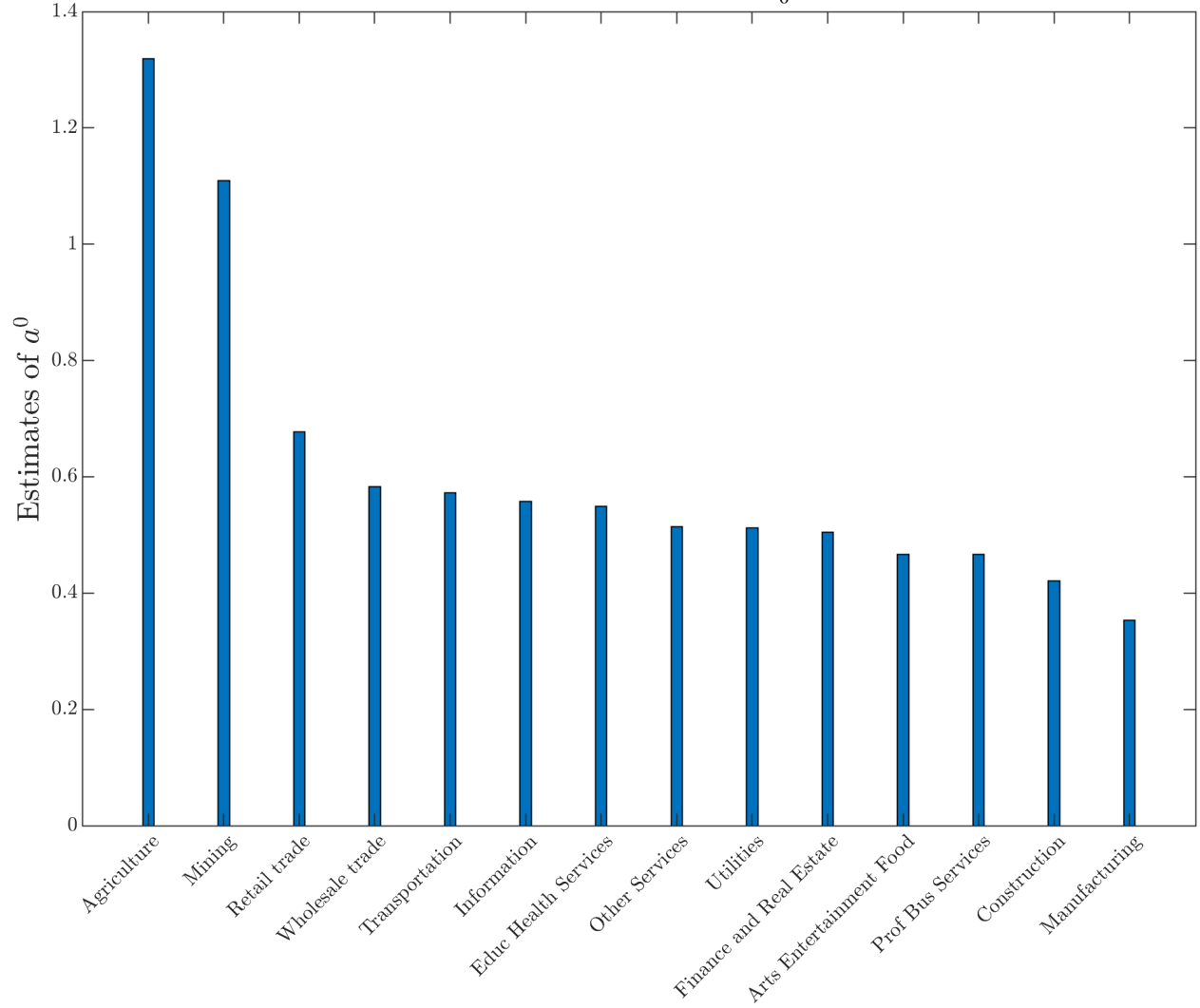
In conclusion, the coskewness terms remain pivotal for explaining aggregate skewness, revealing how sectoral shocks compound through the production network. Even in a more aggregated classification, where individual heterogeneity is reduced, these higher-order interactions continue to shape macroeconomic outcomes. The persistent influence of coskewness highlights the complex ways in which sectors are interconnected, emphasizing the importance of considering both direct and higher-order effects in understanding how shocks propagate across the economy.

Figure D1: Heatmap of W for 14 Sectors
Intermediate Input Network 1957-2019



Notes: The figure displays a heatmap of the 14-sector intermediate input network, averaged over the full 1957–2019 sample. Each cell (i, j) represents the share of sector j 's total intermediate inputs sourced from sector i .

Figure D2: GMM Estimates of A_0 for 14 Sectors
GMM Estimates of A_0



Notes: The figure presents the estimates of A_0 for the baseline Model II using GMM. The bars correspond to these estimates.

Table D2: Decomposition of Variance and Skewness: 14 sectors

Model	F(14)	F(43)	GMM(14)	GMM(43)	F(14)	F(43)	GMM(14)	GMM(43)
κ_2	Share of Variance 10.270				S.E			
$\mathcal{N}$	-	-	0.295	0.570	-	-	0.106	0.070
$\mathcal{C}$	0.965	0.954	0.191	0.066	0.036	0.038	0.044	0.014
$\mathcal{E}$	-	-	0.030	0.011	-	-	0.018	0.007
$2\mathcal{N}\mathcal{C}$	-	-	0.425	0.324	-	-	0.097	0.076
$2\mathcal{N}\mathcal{E}$	0.035	0.046	0.060	0.029	0.036	0.038	0.066	0.047
$2\mathcal{C}\mathcal{E}$	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
κ_3	Contribution to $\kappa_3 = -1.746$				S.E.			
$\mathcal{N}$	-	-	-0.291	-0.789	-	-	0.230	0.297
$\mathcal{C}$	-1.724	-1.363	-0.134	-0.019	0.186	0.140	0.033	0.005
$\mathcal{E}$	-0.004	0.004	0.004	0.000	0.013	0.014	0.002	0.001
$\mathcal{N}\mathcal{N}\mathcal{C}$	-	-	-0.665	-0.626	-	-	0.215	0.150
$\mathcal{N}\mathcal{C}\mathcal{C}$	-	-	-0.512	-0.182	-	-	0.125	0.040
$\mathcal{N}\mathcal{N}\mathcal{E}$	-	-	-0.043	-0.049	-	-	0.066	0.071
$\mathcal{N}\mathcal{E}\mathcal{E}$	-	-	0.012	-0.003	-	-	0.016	0.012
$\mathcal{C}\mathcal{C}\mathcal{E}$	0.011	-0.303	-0.034	-0.002	0.083	0.181	0.010	0.002
$\mathcal{C}\mathcal{E}\mathcal{E}$	-0.030	-0.025	0.001	-0.001	0.042	0.042	0.005	0.002
$\mathcal{N}\mathcal{C}\mathcal{E}$	-	-	-0.085	-0.016	-	-	0.038	0.020

Notes: This table presents the estimation results for the 14-sector classification, alongside a comparison to the 43-sector decomposition findings reported in the main paper. Output Y is expressed as the sum of three components: network ($\mathcal{N}$), common ($\mathcal{C}$), and idiosyncratic ($\mathcal{E}$). The top panel reports the fraction of variance explained by $\mathcal{N}, \mathcal{C}, \mathcal{E}$ and the three covariances. The bottom panel reports the magnitude of skewness due to the three components and seven coskewness terms. Columns F(14) and F(43) show the factor-only estimations (i.e., without a network component) for the 14-sector and 43-sector classifications, respectively. Columns GMM(14) and GMM(43) present the decomposition results derived from the baseline model (the static version of Model II).