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ABSTRACT

We develop a multi-country, multi-sector New Keynesian model with incomplete markets, input-output linkages, and heterogeneous sectoral price rigidities to study the macroeconomic effects of tariffs. Tariffs act simultaneously as demand and supply shocks. A risk-sharing wedge, driven by terms-of-trade effects and revalued net foreign assets, summarizes the wealth transfer in general equilibrium and determines whether the tariff-imposing country gains or loses. This wedge interacts with a propagation matrix encoding network structure, sectoral rigidity, and cross-country monetary policy heterogeneity, which governs how inherited real marginal-cost distortions feed into inflation and consumption. Through input-output linkages, transitory tariffs generate persistent distortions, unlike standard New Keynesian benchmarks. These distortions exceed what N-country monetary policy can offset, even under flexible exchange rates. Quantitatively, the 2025-2026 tariffs are stagflationary for the U.S. and yield inflation or deflation abroad depending on trade diversion and monetary policy heterogeneity. Tariff threats alone generate inflation shaped by retaliation expectations.

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1 Introduction

This paper studies the macroeconomic impact of trade distortions, theoretically and quantitatively, using a novel open-economy New Keynesian framework (NKOE). Understanding the aggregate consequences of trade distortions requires confronting three features that canonical models treat in isolation: global input-output (I-O) linkages, sectorally heterogeneous nominal rigidities, and incomplete financial markets. We build a multi-country, multi-sector NKOE model incorporating these features and ask: what is the macroeconomic impact of tariffs, and how does a global production network shape that impact?

Tariffs transmit to the macroeconomy through three channels. First, tariffs act as demand shocks. By raising the domestic price of foreign goods, they shift expenditure from imports toward domestically produced varieties. As relative demand shocks, they reallocate spending from taxed to untaxed varieties. With finite elasticity of substitution across varieties, taxing a subset of goods impacts the aggregate consumption-basket's price, which affects consumption.

Second, tariffs act as supply shocks. Modern production networks are global, interconnected and complex: firms rely on imported intermediate inputs that are complementary to each other, and taxes on those inputs flow directly into marginal costs and producer prices, affecting home and global production and output.

Third, tariffs generate wealth transfers. Under incomplete markets, tariff-induced relative price changes redistribute resources across countries in global general equilibrium. We summarize these wealth transfers through a risk-sharing wedge, the deviation of relative consumption across countries from the complete-markets Backus–Smith benchmark. In response to a tariff shock, the wedge opens as a one-time martingale, and hence the long-run effect is priced-in on impact reflecting the permanent change in wealth. The sign and magnitude of the wedge help determine consumption dynamics and are shaped by two opposing forces. The wedge is negative when favorable terms-of-trade movements and balance-sheet gains on net foreign assets produce a one-time wealth transfer toward home large enough to dominate intertemporal substitution. Then, even though tariffs make today a more expensive period to consume, the wealth gain raises home consumption on impact. The wedge is positive when terms of trade move against home and net foreign asset position deteriorates; both forces push foreign wealth up and home consumption down.

Whether the home country is a net winner or loser depends on three primitives: the global network structure, which establishes the dependence on foreign inputs and governs the terms-of-trade response; elasticities of substitution across consumption goods and production inputs, which can vary across country-sector and variety; and the persistence of the tariff shock, which governs the strength of intertemporal substitution relative to the wealth effect impacting the time path of consumption.

Our N-country, J-sector model, with Rotemberg pricing, portfolio adjustment costs, an empirically relevant mix of producer- and dominant-currency pricing, and a Taylor rule, features all the channels through which tariffs impact the macroeconomy. The linearized equilibrium is characterized by five vector equations: an IS curve, a Phillips curve for producer prices, a CPI definition, an Uncovered Interest Parity (UIP) condition, and a balance-of-payments (BoP) equation, nesting a broad class of NKOE models. We solve the model analytically in two blocks, an NK block and a BoP block, by extending the method of undetermined coefficients to matrix scale.

The production network matters for the macroeconomic impact of tariffs for two reasons. First, it impacts the terms of trade and the sign of the risk-sharing wedge. Without intermediate inputs, the wedge is negative for standard parameterizations, so tariffs favor the tariff-imposing country. Once intermediate inputs are included, home dependence on foreign inputs can move terms of trade against home. To see this, suppose home imports cars and also semiconductors used to produce chips, and imposes tariffs on both. This hurts home chip production while raising demand for home cars. Foreign car production is hurt, since foreign firms need home chips, making them relatively more scarce and valuable in spite of the higher demand for home cars, tilting terms of trade against home.

Second, the production network generates persistence in real marginal cost deviations, governed by the NKOE propagation matrix $\mathbf{\Psi}^{\mathrm{NKOE}}$. This matrix is the coefficient on lagged real marginal costs and determines how past cost distortions feed into current inflation and output. The persistence result requires more than one sector: with $J = 1$, there is one sectoral price distortion per country and $\mathbf{\Psi}^{\mathrm{NKOE}} = \mathbf{0}$, while with $J > 1$ there are NJ sectoral distortions and $\mathbf{\Psi}^{\mathrm{NKOE}} \neq \mathbf{0}$. Conditional on $J > 1$, stronger input–output linkages (e.g. a higher foreign input dependence) in many sectors makes inherited cost distortions unwind more slowly and thus raise inflation persistence. Final propagation depends on the number of sectors, sectoral price rigidity, and heterogeneity in cross-country monetary policy offsetting cost distortions differentially, while the persistence threshold remains $J > 1$ regardless of whether the exchange rate is treated as a separate national instrument. Country-level aggregate demand cannot span all sectoral distortions. Thus N-dimensional policy cannot offset NJ lagged states.

We apply the model to the 2025–2026 U.S. tariff episode.[1] Feeding the country–sector

[1] We first validate the model against the 2018 U.S.–China trade war, for which we predict a 1.1% nominal dollar appreciation against the yuan, a 0.05% decline in U.S. real GDP, and a 0.12% increase in the aggregate U.S. price level. These are broadly consistent with available estimates on the impact of 2018 tariffs: the dollar appreciated by about 1.1% around tariff announcements, the aggregate price level rose by 0.1–0.2%,

tariffs implemented as of March 2026 using applied border rates in each quarter, leads to stagflation in the U.S. with sizable international spillovers, including trade diversion from the U.S. towards Europe. This process is persistent up to 10 quarters (if monetary policy looks-through), leading to a cumulative rise of 0.35pp in the U.S. inflation. We also study reversed tariff threats, where the home country announces future tariffs that are withdrawn before implementation, mimicking events after the Liberation Day announcement. Even when the threat is never implemented, there are sizable macro effects through the expectations channel: U.S. inflation rises 0.34pp on impact and does not return to steady state for some time, despite the reversal being announced next period. Additional counterfactuals involve the Liberation Day tariffs implemented as announced, stronger production-side complementarities that make tariffs globally stagflationary, and cross-country monetary-policy heterogeneity, including exchange-rate stabilization in China and a Euro Area real-rate rule that lowers U.S. output by weakening Euro Area import demand. In general, we show that open-economy models without I-O linkages can overstate the inflation and understate the output decline, as they miss slow-moving propagation across countries, sectors, and time.

Our positive analysis carries normative implications. The persistence result implies that tariffs in a multi-sector economy generate distortions that take longer to unwind. We quantify this by comparing convergence across one- and multi-sector networks under the same tariff shock and show that with I-O linkages and more than one-sectors, real marginal cost, inflation, and output deviations remain farther from their terminal equilibrium for longer.

The paper proceeds as follows. Section 2 presents the baseline model. Section 3 analyzes the flexible-price solution and characterizes the risk-sharing wedge. Section 4 introduces nominal rigidities, derives the NKOE propagation matrix, and establishes why production networks generate persistence in real marginal cost deviations. Section 5 presents the quantitative analysis and policy counterfactuals. Section 6 concludes.

1.1 Relation and Contribution to the Literature

Our paper builds on three distinct literatures: NKOE, production networks, and trade. We extend the NKOE literature's canonical two-country models of Obstfeld and Rogoff (1995) and Clarida et al. (2002) to N countries and J sectors with endogenous current accounts, showing the importance of I–O linkages and N-country monetary policies for global imbalances. A large part of the NKOE literature focuses on SOEs, such as Barattieri et al. (2021), omitting intermediate input imports.[2] Recent work adds intermediate inputs but not

and aggregate real income fell by about 0.04% of GDP (Barbiero and Stein, 2025; Fajgelbaum et al., 2020).

[2]This literature, rooted in Galí and Monacelli (2005), focuses on optimal exchange-rate and monetary policies.

full I-O linkages that we show to have a central role for macro impact of tariffs (e.g., Auray et al., 2024; Ambrosino et al., 2024; Auclert et al., 2025).[3] A related small open economy (SOE) literature studies optimal monetary policy under tariff shocks such as Bergin and Corsetti (2023), Bianchi and Coulibaly (2025), Werning et al. (2025), and Monacelli (2025). Our contribution to this literature is showing how the presence of production networks can change benchmark results from the one-sector SOE baseline in global general equilibrium by impacting inflation-output trade-off and persistence of inflation.

The closed-economy NK-networks literature establishes that productivity shocks induce endogenous cost-push effects and welfare losses, that the Phillips curve is flatter with production networks (Rubbo, 2023; Pasten et al., 2020, 2024; Afrouzi et al., 2024), and that when sectoral prices outnumber aggregate policy instruments, monetary policy cannot close all sectoral gaps simultaneously (Guerrieri et al., 2021; La'O and Tahbaz-Salehi, 2022). We show analytically that the *persistence of real marginal cost deviations is larger with global I-O linkages*. Our eigenvalue-based persistence result complements the spectral analysis in Liu and Tsyvinski (2024): while the relevant object there is the I-O matrix, here it is the NKOE propagation matrix, which governs the slow unwinding of tariff-induced distortions. This result is also in the spirit of Afrouzi and Bhattarai (2023), which studies monetary and sectoral shocks in a continuous-time NK-production network closed economy. We differ by studying tariffs in a discrete-time open economy with global I-O linkages and incomplete risk sharing. Differing from closed-economy propagation, $\mathbf{\Psi}^{\text{NKOE}}$ contains the impact of the exchange rate: heterogeneous monetary policy maps into exchange-rate movements, and imported-input linkages feed them back into sectoral marginal costs. Cross-country monetary policy heterogeneity can increase inflation persistence relative to the closed-economy benchmark. The final impact on consumption depends on the wealth transfer between countries which is pinned down by the risk sharing wedge.

We build on the quantitative general-equilibrium trade models (e.g., Caliendo and Parro, 2015; Baqaee and Farhi, 2024) by adding dynamics, nominal rigidities, and incomplete markets. Our model's long-run flexible-price equilibrium nests a structure similar to these frameworks. The contribution of our framework is the short-to-medium-run transitional dynamics: nominal rigidities generate persistent deviations from the long-run allocation, and the speed of convergence depends on the network structure and monetary policy. We share the importance of incomplete markets with Itskhoki and Mukhin (2025), who study the long-run

[3] A few recent papers also incorporate cross-border production networks or multilateral trade linkages, including Qiu et al. (2025), Cuba-Borda et al. (2025), and Ho et al. (2022). Relative to these papers, our contribution is to deliver a closed-form analytical characterization of tariff transmission and inflation persistence.

impact of tariffs on trade balances and optimal tariff policies.[4] Static trade models with exogenous transfers may overstate the magnitude of the tariff-induced risk-sharing wedge, while complete-markets models set it to zero by construction. In our dynamic incomplete markets model, the risk sharing wedge shapes the consumption response and the inflation–output trade-off, and thus central to whether tariffs are expansionary on impact.[5]

2 Environment

We develop a multi-country, multi-sector New Keynesian model that incorporates input-output linkages, nominal rigidities via Rotemberg costs, and portfolio adjustment costs.

2.1 Households

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t},B_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right]$$

subject to:

$$P^C_{n,t} C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}^{US}_{n,t} B^{US}_{n,t} + \mathcal{E}^{US}_{n,t} P^{US}_t \psi(B^{US}_{n,t}/P^{US}_t) =$$
$$W_{n,t} L_{n,t} + \sum_i \mathcal{D}_{ni,t} - (1+i_{n,t-1}) B_{n,t-1} - \mathcal{E}^{US}_{n,t}(1+i^{US}_{t-1}) B^{US}_{n,t-1},$$

where $P^C_{n,t}$ is the price of the consumption bundle $C_{n,t}$, $\mathcal{E}^{US}_{n,t}$ the exchange rate against the U.S. (increase denotes local-currency depreciation), $W_{n,t}$ the wage, $L_{n,t}$ labor supply, and $i_{n,t}$ and i^{US}_t the nominal rates on the local-currency bond $B_{n,t}$ and the U.S. bond $B^{US}_{n,t}$ (net foreign liabilities). $\psi(B^{US}_{n,t}/P^{US}_t)$ is a portfolio adjustment cost inducing stationarity of real debt. $T_{n,t}$ denotes taxes and transfers, including lump-sum rebates of tariff revenue, and $\mathcal{D}_{ni,t}$ firm profits. β, σ, χ, η are the discount factor, intertemporal elasticity of substitution, labor disutility weight, and labor supply elasticity. The domestic bond is in zero net supply, so all countries save or dissave in U.S. bonds; bilateral exchange rates are pinned down by arbitrage against the dollar.

Maximizing the household's lifetime utility subject to the budget constraints yields:

[4]In the long run, Costinot and Werning (2025) show that trade deficits depends on the extensive margin. We have a similar result in our long-run flexible-price equilibrium.

[5]Our risk-sharing wedge is conceptually related to the Backus–Smith wedge in Aguiar et al. (2025), who show how exchange rate disconnect can open this wedge.

Euler Equation: $1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P^C_{n,t}}{P^C_{n,t+1}} (1 + i_{n,t}) \right] \forall n \in N$ and $\forall t$ and

UIP Condition: $\frac{1+i_{n,t}}{1+i^{US}_t} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1-\psi'(B^{US}_{n,t}/P^{US}_t)}$ $\forall n \in N - 1$ (excluding the U.S).

Finally, for completeness of notation, we define arbitrage conditions: $\mathcal{E}_{n,m,t} = \mathcal{E}^{US}_{n,t}/\mathcal{E}^{US}_{m,t}$ $\forall n, m \in N$, which also pins down a country's exchange rate with itself (with $\mathcal{E}_{n,n,t} = 1$). We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

We now turn to the household's intratemporal problem. The first part of the intratemporal problem is the standard labor-consumption trade-off that determines labor supply $W_{n,t}/P^C_{n,t} = \chi L^{\eta}_{n,t} C^{\sigma}_{n,t}$ where $W_{n,t}$ is the wage in country n at time t.

The intratemporal breakdown of consumption uses a nested CES structure: country varieties bundle into country-sector bundles, which then aggregate into the country consumption bundle. For example, U.S. consumption of automobiles is a CES aggregate of automobiles sourced from each country, which in turn enters U.S. aggregate consumption. Mathematically:

$$C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i}^{\frac{1}{\theta^C_h}} C_{n,i,t}^{\frac{\theta^C_h - 1}{\theta^C_h}} \right]^{\frac{\theta^C_h}{\theta^C_h - 1}} \quad \text{and} \quad C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi}^{\frac{1}{\theta^C_{l,i}}} C_{n,i,mi,t}^{\frac{\theta^C_{l,i} - 1}{\theta^C_{l,i}}} \right]^{\frac{\theta^C_{l,i}}{\theta^C_{l,i} - 1}}.$$

The index (n, i) denotes sector i bundle in country n, with consumption $C_{n,i,t}$ and weight $\Gamma_{n,i}$; θ^C_h governs substitution across sectors (e.g., automobiles vs. food). Each sectoral bundle aggregates country-sector varieties indexed (n, i, mi), with weights $\Gamma_{n,i,mi}$ (e.g., German automobiles in the U.S. automobile bundle) and within-sector elasticity $\theta^C_{l,i}$. Prices and quantities are indexed analogously. The optimality conditions are:

$$P^C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i} (P^C_{n,i,t})^{1-\theta^C_h} \right]^{\frac{1}{1-\theta^C_h}} \quad \text{and} \quad C_{n,i,t} = \Gamma_{n,i} \left(\frac{P^C_{n,i,t}}{P^C_{n,t}} \right)^{-\theta^C_h} C_{n,t},$$

where $P^C_{n,i,t}$ is the local-currency price of bundle i in country n.[6] Let $P_{mi,t}$ denote the producer price of industry i in country m (in m's currency). Buyers in country n face $P_{n,mi,t} = \mathcal{E}_{n,m,t}(1 + \tau_{n,mi,t}) P_{mi,t}$, applying the bilateral exchange rate and the tariff $\tau_{n,mi,t}$. The price index of $C_{n,i,t}$ is:

$$P^C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi} P_{n,mi,t}^{1-\theta^C_{l,i}} \right]^{\frac{1}{1-\theta^C_{l,i}}} \quad \text{and} \quad C_{n,mi,t} = \Gamma_{n,i,mi} \left(\frac{P_{n,mi,t}}{P^C_{n,i,t}} \right)^{-\theta^C_{l,i}} C_{n,i,t}.$$

[6]Superscript C denotes consumption-side price bundles.

2.2 Production

Output in country-sector ni at time t, $Y_{ni,t}$, follows a CES production function:

$$Y_{ni,t} = \left[\alpha_{ni}^{1/\theta^X} L_{ni,t}^{\frac{\theta^X-1}{\theta^X}} + (1-\alpha_{ni})^{1/\theta^X} (X_{ni,t})^{\frac{\theta^X-1}{\theta^X}}\right]^{\frac{\theta^X}{\theta^X-1}} \forall n \in N, \forall i \in J,$$

where θ^X governs the elasticity between labor and the intermediate bundle $X_{ni,t}$, and α_{ni} is the labor weight. Firms within a country-sector are identical and solve:

$$MC_{ni,t} = \min_{\{X_{ni,t}, L_{ni,t}\}} W_{n,t} L_{ni,t} + P_{ni,t}^X X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1,$$

where $P_{ni,t}^X$ is the price of the intermediate bundle (superscript X denotes production-side price indices, given by the relevant CES dual).

The intermediate bundle for ni aggregates sectoral bundles (ni, j, t) for $j \in \mathcal{J}$, each formed from sector-j varieties sourced across countries (e.g., U.S. automobile production uses steel from any $m \in N$). The bundle and relative demand condition are:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{\frac{1}{\theta_{l,j}^X}} X_{ni,mj,t}^{\frac{\theta_{l,j}^X-1}{\theta_{l,j}^X}}\right]^{\frac{\theta_{l,j}^X}{\theta_{l,j}^X-1}} \text{ and } X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P_{ni,j,t}^X}\right)^{-\theta_{l,j}^X} X_{ni,j,t},$$

where $P_{ni,j,t}^X$ and $X_{ni,j,t}$ are the price index and quantity of the bundle, and $\theta_{l,j}^X$ is the within-sector elasticity of substitution across varieties. The aggregate intermediate bundle for ni is:

$$X_{ni,t} = \left[\sum_{j \in J} \Omega_{ni,j}^{\frac{1}{\theta_h^X}} X_{ni,j,t}^{\frac{\theta_h^X-1}{\theta_h^X}}\right]^{\frac{\theta_h^X}{\theta_h^X-1}} \text{ and } \frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P_{ni,j,t}^X}{P_{ni,t}^X}\right)^{-\theta_h^X}, \forall j \in \mathcal{J}.$$

The firm's problem yields:

$$MC_{ni,t} = \left[\alpha_{ni} W_{n,t}^{1-\theta^X} + (1-\alpha_{ni}) \left(P_{ni,t}^X\right)^{1-\theta^X}\right]^{\frac{1}{1-\theta^X}} \text{ and } \quad \frac{X_{ni,t}}{L_{ni,t}} = \frac{(1-\alpha_{ni})}{\alpha_{ni}} \left(\frac{W_{n,t}}{P_{ni,t}^X}\right)^{\theta^X}.$$

Each country-sector contains a continuum of identical firms. Representative firm f in ni sets its price subject to Rotemberg adjustment costs:

$$P_{ni,t}^f = \arg\max_{P_{ni,t}^f} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \left[Y_{ni,T}^f (P_{ni,T}^f) \left(P_{ni,T}^f - MC_{ni,T}\right)\right.\right.$$

$$-\frac{(1-\vartheta_{ni})\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}}-1\right)^2 Y_{ni,T}P_{ni,T}-\frac{\vartheta_{ni}\delta_{ni}}{2}\left(\frac{P^f_{ni,T}/\mathcal{E}^{US}_{n,T}}{P^f_{ni,T-1}/\mathcal{E}^{US}_{n,T-1}}-1\right)^2 Y_{ni,T}P_{ni,T}\Bigg]\Bigg]$$

A CES bundler aggregates individual firms' output, yielding demand $Y^f_{ni,t}(P^f_{ni,t}) = \left(P^f_{ni,t}/P_{ni,t}\right)^{-\theta^R} Y_{ni,t}$. The parameters $(1-\vartheta_{ni})\delta_{ni}$ and $\vartheta_{ni}\delta_{ni}$ are the real costs of adjusting prices in producer and dominant (USD) currency, respectively, so $\vartheta_{ni} \to 0$ is PCP, $\vartheta_{ni} \to 1$ is DCP, and intermediate values yield a hybrid. In Section 5 we additionally incorporate pricing to market (PTM) by relabeling sector i in a destination-specific manner, generating sticky producer-importer prices (e.g., Japanese steel *intended for American end users*). We discipline ϑ_{ni} using export dollar-invoicing shares from the DCP literature, setting it to zero for domestically sold goods. The resulting framework combines PCP, DCP, and PTM in a manner consistent with empirical evidence.

This problem yields the following equilibrium condition:

$$\begin{aligned}(1-\vartheta_{ni})(\Pi_{ni,t}-1)\Pi_{ni,t}+\vartheta_{ni}\left(\frac{\Pi_{ni,t}}{D^{US}_{n,t}}-1\right)\frac{\Pi_{ni,t}}{D^{US}_{n,t}} &= \frac{\theta^R}{\delta_{ni}}\left[\frac{MC_{ni,t}}{P_{ni,t}}-\frac{\theta^R-1}{\theta^R}\right]\\ +\beta\,\mathbb{E}_t\left[(1-\vartheta_{ni})(\Pi_{ni,t+1}-1)\Pi_{ni,t+1}+\vartheta_{ni}\left(\frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}}-1\right)\frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}}\right] \end{aligned} \tag{1}$$

where $\Pi_{ni,t} = P_{ni,t}/P_{ni,t-1}$ is gross inflation and $D^{US}_{n,t} = \mathcal{E}^{US}_{n,t}/\mathcal{E}^{US}_{n,t-1}$ is gross depreciation against the USD. Equation (1) is a country- and sector-specific forward-looking New Keynesian Phillips Curve in nominal marginal cost deflated by the producer price. As $\delta_{ni} \to 0$, it collapses to the flexible-price allocation with constant markup: $\Pi_{ni,t} = 1$ and $MC_{ni,t}/P_{ni,t} = (\theta^R-1)/\theta^R$. Linearization is conducted around a subsidized steady state that removes the monopolistic distortion (subsidy notation suppressed); we abstract from exogenous markup shocks but track real marginal costs since tariffs induce endogenous markup variation through nominal rigidities.[7]

A natural question is whether stickiness applies to pre- or post-tariff (and pre- or post-exchange-rate) producer prices; our framework accommodates both. In the baseline, under both PCP ($\vartheta_{ni} = 0$) and DCP ($\vartheta_{ni} = 1$), tariffs are applied on top of the sticky producer price and pass through immediately. Since full instant pass-through to end users is not empirically realistic, Section 5 introduces domestic importing firms that intermediate all imports and face sticky retail prices, restoring the distribution margin absent from standard I-O tables (Horowitz and Planting, 2009).

[7] Markups also move because wages and intermediate-input prices need not adjust together.

2.3 Market Clearing, and Policy Equilibrium

The evolution of net debt is governed by the balance of payments. For country n, with tariff revenue rebated to households lump-sum:

$$\begin{aligned}&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}(P_{n,mj,t}C_{n,mj,t})+\sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}(P_{n,mj,t}X_{ni,mj,t})+\mathcal{E}_{n,t}(1+i_{t-1}^{US})B_{n,t-1}^{US}\\&+\mathcal{E}_{n,t}P_t^{US}\psi(B_{n,t}^{US}/P_t^{US})=\sum_{i\in\mathcal{J}}P_{ni,t}Y_{ni,t}+\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{\tau_{n,mj,t}}{1+\tau_{n,mj,t}}P_{n,mj,t}C_{n,mj,t}\right)\\&+\sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{\tau_{n,mj,t}}{1+\tau_{n,mj,t}}P_{n,mj,t}X_{ni,mj,t}\right)+\mathcal{E}_{n,t}B_{n,t}^{US}\quad\forall n\in N-1.\end{aligned}\tag{2}$$

All terms are in country n's domestic currency, and the second and third right-hand-side terms are tariff revenues. By Walras' Law, one country's budget constraint is redundant; we omit the U.S.

All markets clear: labor satisfies $L_{n,t}=\sum_{i\in J}L_{ni,t}$; USD bonds satisfy $B_t^{US}=\sum_{n=2}^{N}B_{n,t}^{US}$, where $B_t^{US}=B_{1,t}^{US}$ is U.S. aggregate bond supply; and goods, used both for final consumption and as intermediates, satisfy $Y_{ni,t}=\sum_{m\in\mathcal{C}}C_{m,ni,t}+\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}X_{mj,ni,t}$ for producing country n and consuming country m.

Monetary policy in each country follows a generalized rule with interest-rate smoothing and a flexible target basket:

$$P_{n,t}^T=\left(\prod_{m\in N}\prod_{j\in J}(P_{mj,t})^{\Upsilon_{n,mj}^P}\right)\left(\prod_{m\in N\setminus\{n\}}\mathcal{E}_{n,m,t}^{\Upsilon_{n,m}^{\mathcal{E}}}\right)\quad\text{and}\quad\Pi_{n,t}^T=\frac{P_{n,t}^T}{P_{n,t-1}^T}.$$

$\Upsilon_{n,mj}^P$ and $\Upsilon_{n,m}^{\mathcal{E}}$ are the weights on producer prices and bilateral exchange rates in country n's target basket, nesting CPI targeting, producer-price combinations, and exchange-rate targeting.[8] The Taylor rule with inertia is $1+i_{n,t}=(1+i_{n,t-1})^{\rho_m^n}\left(\Pi_{n,t}^T\right)^{\phi_\pi^n}$ for all $n\in N$, with $\rho_m^n\in[0,1)$ governing smoothing.

2.4 Linearization, Matrix Notation and Analytical Solution

We linearize the 25 equations above and apply the method of undetermined coefficients (MUC) to solve the model analytically. Hat variables denote percent deviations from the zero-tariff pre-shock steady state rather than the flexible-price allocation.[9] Departing from the standard zero-debt linearization, we allow primitive parameters (home bias, imported-input dependence) to be asymmetric across countries (e.g., Obstfeld and Rogoff, 1995), which

[8] It also nests PPI targeting and the divine-coincidence index of Rubbo (2023).

[9] Output is therefore measured relative to pre-tariff steady-state output rather than as a gap, since our interest is in the world with tariffs relative to a world without.

Table 1. Notation guide

	Scalar notation	Matrix/vector notation	Object
Key Primitives	γ_H, γ_F	$\mathbf{\Gamma}$ $(N \times NJ)$	consumption shares
	Ω_H, Ω_F	$\mathbf{\Omega}$ $(NJ \times NJ)$	input–output shares / production network
	θ^C, θ^X	$\boldsymbol{\theta}$	CES elasticity of substitution
	Λ_{ni}	$\mathbf{\Lambda}(NJ \times NJ)$	nominal rigidity parameters
	ϕ_π	$\mathbf{\Phi}$ $(N \times N)$	monetary-policy coefficients
Key Variables	$\hat{C}_{n,t}$	$\hat{\boldsymbol{C}}_t$ $(N \times 1)$	real consumption
	$\hat{P}_{ni,t}$	$\hat{\boldsymbol{P}}^P_t$ $(NJ \times 1)$	producer prices
	$\pi^P_{ni,t}$	$\boldsymbol{\pi}^P_t$ $(NJ \times 1)$	producer-price inflation
	$\hat{P}^C_{n,t}$	$\hat{\boldsymbol{P}}^C_t$ $(N \times 1)$	consumer prices
	$\mu_{ni,t}$	$\boldsymbol{\mu}_t$ $(NJ \times 1)$	real marginal cost
	$\hat{\mathcal{E}}_{n,m,t}$	$\hat{\boldsymbol{\mathcal{E}}}_t$ $(N^2 \times 1)$	nominal exchange rates
	$\hat{V}_{n,t}$	$\hat{\boldsymbol{V}}_t$ $(N \times 1)$	net external debt
	$\hat{\tau}_{n,mj,t}$	$\hat{\boldsymbol{\tau}}_t$ $(N^2 J \times 1)$	tariffs
Key Objects	$\hat{w}_t$	–	risk-sharing wedge
	–	$\mathbf{\Psi}$ $(NJ \times NJ)$	NKOE propagation matrix

Note: n, m index countries, i, j index industries, and boldface denotes stacked country or country-sector objects. Hat notation denotes deviation from steady state. Elasticity of substitution refers to CES bundles on the consumption and production side.

implies nonzero steady-state debt and net exports consistent with those parameters. The implied debt level parametrizes the portfolio adjustment costs that anchor deviations from steady state. The quantitative section disciplines parameter asymmetry and steady-state debt using the ICIO Table (Appendix B).

We make four simplifying assumptions for analytical tractability. First, following Golosov and Lucas (2007), we set $\chi = 1$ and $\eta = 0$ (infinitely elastic labor), reducing the intratemporal labor-leisure condition to $\hat{W}_{n,t} - \hat{P}^C_{n,t} = \sigma \hat{C}_{n,t}$ and letting us focus on consumption while tracking aggregate output separately. Second, we set $\psi(B^{US}_{n,t}/P^{US}_t) = 0$: portfolio adjustment costs serve as a stationarity device and are numerically small. Third, we assume producer-currency price rigidity, $\vartheta_{ni} = 0 \ \forall n \in N, i \in J$.

Fourth, policy targets only a producer-price basket, $\Upsilon^{\mathcal{E}}_{n,m} = 0 \ \forall n, m \in N$, with target weights given by consumption shares so that $\hat{\boldsymbol{\imath}}_t = \mathbf{\Phi} \mathbf{\Gamma} \boldsymbol{\pi}^P_t$.[10] Finally, we introduce generalized elasticities linking the lowest-level bundles directly to the highest-level aggregates on both the consumption and production sides.[11]

Given the N-country, J-industry structure, we adopt matrix notation. Table 1 collects the main primitives, variables, and objects. Consider the linearized producer-price inflation

[10] Results do not hinge on using $\mathbf{\Gamma}$; alternative producer-price mixes deliver similar conclusions.

[11] To first order, $\Gamma_{n,mi} = \Gamma_{n,i}\Gamma_{n,i,mi}$ and $\Omega_{ni,mj} = (1-\alpha_{ni})\Omega_{ni,j}\Omega_{ni,j,mj}$, so e.g. $\hat{C}_{n,t} = \sum_{m,i} \Gamma_{n,mi}\hat{C}_{n,mi,t}$ and $\hat{C}_{n,mi,t} = -\theta^C_{l,i}(\hat{P}_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}_{ni,t})$.

equation:

$$\pi^P_{ni,t} = \underbrace{\frac{\theta^R_{l,i}}{\delta_{ni}}}_{\Lambda_{ni}} \left(\underbrace{\alpha_{ni}\hat{W}_{n,t} + \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}(\hat{P}_{mj,t} + \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,mj,t})}_{\widehat{MC}_{ni,t}} - \hat{P}_{ni,t} \right) + \beta\mathbb{E}_t\pi^P_{ni,t+1} \quad (3)$$

which vectorizes as $\boldsymbol{\pi}^P_t = \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t + (\boldsymbol{\Omega} - \boldsymbol{I})\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_\tau\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\boldsymbol{\pi}^P_{t+1}$, with $\hat{\boldsymbol{\mathcal{E}}}_t$ the $N^2 \times 1$ vector of bilateral exchange rates and $\hat{\boldsymbol{\tau}}_t$ the $N^2 J \times 1$ vector of tariff rates. We use $\mathbf{L}$ to denote loadings, i.e., how a subscript variable loads onto a superscript variable as a linear combination of vector entries, serving as partial derivatives.[12]

2.4.1 Global New Keynesian Representation

The linearized equilibrium conditions in Appendix A can be written in vector form as a Blanchard-Kahn-stable equilibrium, used both for interpretation and for solving via MUC.[13] This five-equation representation extends the canonical three-equation New Keynesian model to N open economies with I-O linkages.

Definition 1. A linearized equilibrium comprises sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{P}}^C_t, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}^\infty_{t_0}$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}^\infty_{t_0}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (4)-(8) hold:

$$\textbf{NKIS:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1}) - \mathbb{E}_t(\hat{\boldsymbol{P}}^C_{t+1} - \hat{\boldsymbol{P}}^C_t) \quad (4)$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_\tau\hat{\boldsymbol{\tau}}_t \quad (5)$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}^P_t = \boldsymbol{\Psi}_\Lambda\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_\tau\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \quad (6)$$

$$\textbf{UIP:} \quad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\hat{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\hat{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1}) \quad (7)$$

$$\textbf{BoP:} \quad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t + \boldsymbol{\Xi}_6\hat{\boldsymbol{P}}^P_{t-1} \quad (8)$$

where $\boldsymbol{\Psi}_\Lambda = [(1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega})]^{-1}$ is a stickiness-adjusted Leontief inverse, and $\hat{\boldsymbol{V}}_t$ is the vectorized debt variable $V_{n,t} = (1 + i^{US}_t)B^{US}_{n,t}$. The NKIS, UIP, and BoP equations substitute out the nominal interest rate via the Taylor rule, $\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1})$, where the diagonal $\boldsymbol{\Phi}$ contains country-specific sensitivities; the first $N-1$ rows of $\tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1})$ load interest-rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}^{US}_t$ relative to the U.S. The BoP equation accordingly features $\hat{\boldsymbol{P}}^P_{t-1}$, because the interest rate is substituted out.

The first equation is the NKIS equation, with tariffs entering the demand side through their loadings on the consumer price index (CPI). The second equation defines CPI. Here $\boldsymbol{\Gamma}$

[12] E.g., $(\boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_{ni} = \sum_{m,j}\Omega_{ni,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^P_\tau\hat{\boldsymbol{\tau}}_t)_{ni} = \sum_{m,j}\Omega_{ni,mj}\hat{\tau}_{n,mj,t}$.

[13] Expressing prices in levels yields a compact five-variable system convenient for MUC algebra.

is an $N \times NJ$ matrix, containing consumption shares[14] and $\boldsymbol{L}^C_{\mathcal{E}}$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\gamma_H)$, where $\gamma_H \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}^C_{\tau}$ captures the share of goods exposed to tariffs.

The third equation is the NKPC for producer-price inflation, expressed in levels. The stickiness-adjusted Leontief inverse $\boldsymbol{\Psi}_\Lambda$ captures the I-O network and multiplies $\boldsymbol{\Lambda}$, nominal marginal costs, lagged prices $\hat{\boldsymbol{P}}^P_{t-1}$, and discounted expectations $\beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}$. Exchange rates and tariffs load onto marginal costs via imported-input dependence ($\boldsymbol{L}^P_{\mathcal{E}}$) and tariff-exposure shares ($\boldsymbol{L}^P_{\tau}$). The fourth equation combines UIP, exchange-rate arbitrage, and self-arbitrage; the $\tilde{\boldsymbol{\Phi}}$ terms load each country's ϕ_π alongside the arbitrage restrictions row by row.

The fifth equation combines bond-market clearing with the $N-1$ laws of motion for net debt, expressing the balance of payments as a function of prices (reflecting good-specific terms of trade) and aggregate consumption.[15] It nests all relevant intratemporal demand and pricing relationships through the $\boldsymbol{\Xi}$ coefficients (Appendix B), governing how net debt responds to terms-of-trade changes, balance-sheet effects, and interest payments.

This five-equation representation nests a broad class of open-economy New Keynesian models: intermediate-input models with a final good correspond to $J=2$ with one column of $\boldsymbol{\Omega}$ zeroed out, while collapsing to a single country with an exogenous real rate recovers the canonical SOE model of Galí and Monacelli (2005).

2.5 Wealth Transfers Summarized By the Risk-Sharing Wedge

Through Section 4, we focus on $N=2$, with H denoting the tariff-imposing home country and F the foreign country, so bilateral variables (including $\hat{\mathcal{E}}_t$ and, without loss of generality, $\hat{\tau}_t$) are scalars. Section 5 returns to the general case.

Under incomplete markets, tariffs generate a cross-country wealth transfer summarized by a risk-sharing wedge $\hat{w}_t$ measuring deviations from perfect risk sharing. This object is useful for two reasons. Substantively, it captures the wealth transfer to home (foreign) arising from terms-of-trade gains (losses) and balance-sheet effects on net debt, operating through both nominal and real channels (currency appreciation/depreciation and trade-deficit adjustments). Methodologically, tracking $\hat{w}_t$ as a state variable is more convenient in our MUC approach than tracking net debt $\hat{V}_t$.

Begin with the perfect risk-sharing benchmark. Under complete markets (e.g., in the presence of Arrow-Debreu securities), the linearized Backus-Smith condition holds: $\sigma\big(\hat{C}_{H,t}-$

[14] Similar to the production case, $(\boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^C_{\tau}\hat{\boldsymbol{\tau}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\tau}_{n,mj,t}$.

[15] The first $N-1$ rows linearize (2); the last row is bond-market clearing. See Appendix B.

$\hat{C}_{F,t}\big) = \hat{Q}_t$, where $\hat{Q}_t \equiv \hat{P}^C_{F,t} + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}$ is the real exchange rate.[16] Ex-ante full insurance implements the allocation via state-contingent transfers, so the planner reallocates resources toward states where consumption is more valuable: when $\hat{Q}_t$ is high (home basket relatively cheap), efficiency requires higher home consumption. Under incomplete markets, households trade only a single non-state-contingent (U.S.) bond. Combining the Euler equation with UIP yields the Backus-Smith condition *in expectation*, $\sigma(\mathbb{E}_t\Delta\hat{C}_{H,t+1} - \mathbb{E}_t\Delta\hat{C}_{F,t+1}) = \mathbb{E}_t\Delta\hat{Q}_{t+1}$. Defining $\hat{w}_t \equiv \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$ gives $\hat{w}_t = \mathbb{E}_t\hat{w}_{t+1}$: the wedge is a martingale.[17]

For a tariff shock revealed at $t = 0$ with persistence ρ_τ, $\hat{w}_t$ is a linear function of the initial shock: $\hat{w}_t = f(\hat{\tau}_0)\ \forall t$. Since the shock is unexpected and uninsurable, it acts as a wealth transfer when revealed. If terms-of-trade gains and balance-sheet effects favor home, $\hat{w}_t < 0$ under standard parameterizations of θ, so wealth flows to home, generating appreciationary pressure and pushing home consumption above the perfect-risk-sharing benchmark.[18]

2.6 Tariffs Transmission and Propagation Channels

Since $\hat{w}_t$ is a martingale, we split the five-equation representation into two blocks. In the New Keynesian block, we treat the risk-sharing wedge as a state variable alongside tariffs and substitute out the exchange rate using the definition of the risk-sharing wedge: $\hat{\mathcal{E}}_t = \hat{w}_t - \hat{P}^C_{F,t} + \hat{P}^C_{H,t} + \sigma\big(\hat{C}_{H,t} - \hat{C}_{F,t}\big)$. In the five-equation representation, the lagged price vector is a state variable because the I-O structure makes inflation depend on the level of producer prices. For interpretability, we instead track lagged sectoral real marginal costs $\boldsymbol{\mu}_{t-1}$ as a state variable in the New Keynesian block, since these capture deviations from the flexible-price benchmark, yielding:

$$\textbf{NKIS:} \qquad \sigma\mathbb{E}_t\Delta\hat{\boldsymbol{C}}_{t+1} = \underbrace{(\boldsymbol{I} - \boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\pi}^P_t - \boldsymbol{\Gamma}\,\mathbb{E}_t\boldsymbol{\pi}^P_{t+1} - \boldsymbol{L}^C_\tau\,\mathbb{E}_t\Delta\hat{\tau}_{t+1}}_{\hat{\boldsymbol{\imath}}_t - \mathbb{E}_t\boldsymbol{\pi}^C_{t+1}} \tag{9}$$

$$\textbf{NKPC:} \qquad \boldsymbol{\pi}^P_t = \boldsymbol{\Lambda} \underbrace{\boldsymbol{\mu}_t}_{\text{Real Marginal Cost (RMC)}} + \beta\,\mathbb{E}_t\boldsymbol{\pi}^P_{t+1} \tag{10}$$

$$\textbf{RMC} \qquad \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\mu}_1\,\boldsymbol{\pi}^P_t + \boldsymbol{\mu}_2\sigma\Delta\hat{\boldsymbol{C}}_t + \big(\boldsymbol{\mu}_2\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau\big)\,\Delta\hat{\tau}_t + \boldsymbol{\mu}_4\Delta\hat{w}_t \tag{11}$$

[16] This follows from equating state-contingent marginal utilities valued in a common currency, $U_C(C_{H,t})/P^C_{H,t} = \lambda\,U_C(C_{F,t})/(\mathcal{E}_t P^C_{F,t})$, and linearizing under CRRA.

[17] The martingale property relies on zero portfolio adjustment costs; nonzero but small costs yield near-martingale behavior without materially altering conclusions.

[18] Other shocks can move $\hat{w}_t$: Section E.3 adds a tariff-uncertainty shock that raises the UIP premium (Kalemli-Özcan et al., 2026), leading to dollar depreciation, $\hat{w}_t > 0$, and a wealth transfer to foreign.

where Δ notation indicates first difference, $\boldsymbol{Z} = \begin{bmatrix} 1 & -1 \end{bmatrix}$ differences home and foreign entries, $\boldsymbol{\mu}_1 \equiv \boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\Gamma} + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})m\boldsymbol{Z}\boldsymbol{\Gamma}$, capturing the impact of producer prices via input-output linkages, wages and the exchange rate, $\boldsymbol{\mu}_2 \equiv \Big(\boldsymbol{\alpha} + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})m\boldsymbol{Z}\Big)$, capturing the impact of consumption and tariffs on real wages and the exchange rate, and $\boldsymbol{\mu}_4 \equiv (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})m$ where $m \equiv \left(1 - \boldsymbol{Z}\boldsymbol{L}^C_{\mathcal{E}}\right)^{-1}$ capturing the impact of the risk-sharing wedge via the exchange rate's impact on real wages and producer prices.

As the New-Keynesian block above demonstrates: (i) Non-transitory tariffs induce a demand shock (e.g., a patience shock). Tariffs are a tax on the consumption of some goods; as such they are a *relative* demand shock. In the absence of perfect substitutability, however, this consumption tax has a direct effect on the aggregate consumption price index and acts similarly to an aggregate consumption tax. We denote this direct effect with $\boldsymbol{L}^C_{\tau}$. This shock makes consumption more expensive in some periods relative to others, thereby impacting both intertemporal substitution and the household's labor supply as other demand shocks do.[19] Because it distorts the household's labor supply, $\boldsymbol{L}^C_{\tau}$ also impacts real marginal cost, $\boldsymbol{\mu}_t$. (ii) Tariffs are a tax placed on intermediate inputs and hence they are a supply shock that makes it more expensive for the industries of the tariff-imposing country to produce. We denote this direct effect on the real marginal cost basket with $\boldsymbol{L}^P_{\tau}$. (iii) As noted in Section 2.5 tariffs constitute a wealth transfer between countries, summarized by the risk-sharing wedge. If the wealth transfer via the risk-sharing wedge favors the home country, then it can partially or more than offset the increase in $\boldsymbol{\mu}_t$ from tariffs via exchange rate appreciation.

Remark 1. The wealth transfer captured by the risk-sharing wedge impacts the inflation-output tradeoff.[20] When the wedge is negative (positive), and thus favors the tariff-imposing country (tariffed country), it can increase (decrease) output and decrease (increase) inflation, offsetting (amplifying) the negative supply shock impact of tariffs that is stagflationary.

After solving the New Keynesian block, treating $\hat{w}_t$ as a state variable, we solve the open-economy block, comprising the martingale equation $\hat{w}_t = \mathbb{E}_t\hat{w}_{t+1}$ (which replaces the UIP), the balance of payments equation, and the necessary variable definitions, using the coefficients from the first block to pin down $\hat{w}_t$ and $\hat{V}_t$. There is a unique level of $\hat{w}_t$ that satisfies the martingale condition and ensures that $\hat{V}_t$ is not on an explosive path.[21] This two-block approach – solving in terms of the risk-sharing wedge and then determining the wedge from the remaining equilibrium conditions – is a methodological contribution to production network models with incomplete markets and to general open-economy models.

[19] When tariffs are permanent, instead of intertemporal substitution, there is a one-time and permanent change in all variables. Here, we focus on the case when $0 \leq \rho_\tau < 1$.

[20] Even though our model is written to track consumption as the aggregate quantity, our argument here extends to the inflation output tradeoff.

[21] We confirm numerically that solving in two blocks yields the same result as solving the full model.

3 Tariffs Under Flexible Prices

We first study the flexible-price two-country, one-good economy ($N = 2$, $J = 1$), then extend to multiple sectors to isolate the network channel. With $J = 1$, we consider unilateral tariffs (so foreign entries of $\boldsymbol{L}_\tau^C$ and $\boldsymbol{L}_\tau^P$ vanish) and rule out self-use. The relevant matrices are:[22]

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \Omega_H \\ \Omega_F & 0 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 1-\Omega_H & 0 \\ 0 & 1-\Omega_F \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1-\gamma_H & \gamma_H \\ \gamma_F & 1-\gamma_F \end{bmatrix}, \quad \boldsymbol{\Psi} = (\mathbf{I}-\boldsymbol{\Omega})^{-1},$$
$$\boldsymbol{L}_\mathcal{E}^C = \begin{bmatrix} \gamma_H \\ -\gamma_F \end{bmatrix}, \quad \boldsymbol{L}_\tau^C = \begin{bmatrix} \gamma_H L_\tau^C \\ 0 \end{bmatrix}, \quad \boldsymbol{L}_\mathcal{E}^P = \begin{bmatrix} \Omega_H \\ -\Omega_F \end{bmatrix}, \quad \boldsymbol{L}_\tau^P = \begin{bmatrix} \Omega_H L_\tau^P \\ 0 \end{bmatrix}$$

L_τ^C and L_τ^P indicate whether one country tariffs the other; H, F denote home and foreign. Under symmetry we set $\Omega_H = \Omega_F = \Omega$ and $\gamma_H = \gamma_F = \gamma$, with $0 \le \gamma < \frac{1}{2}$ and $0 \le \Omega < 1$.

With this notation, consider the one-good economy under flexible prices and assume monetary policy stabilizes aggregate prices rather than following a Taylor rule.[23] Setting $\sigma = 1$, the equilibrium implied by the New Keynesian block (i.e., $\Lambda_{ni} \to \infty$ for all $n \in N, i \in J$), is such that for exogenous $\{\hat{\tau}_t, \hat{w}_t\}_{t=0}^{\infty}$ the following equations hold $\forall t$:

1. Euler equations hold: $(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t}$ and $(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t}$.
2. Basket prices satisfy $0 = (1-\gamma)\hat{P}_{H,t}^P + \gamma(\hat{\mathcal{E}}_t + \hat{P}_{F,t}^P + \hat{\tau}_t)$ and $0 = (1-\gamma)\hat{P}_{F,t}^P + \gamma(\hat{P}_{H,t}^P - \hat{\mathcal{E}}_t)$.
3. Prices equal marginal cost: $\hat{P}_{H,t}^P = (1-\Omega)\hat{C}_{H,t} + \Omega(\hat{P}_{F,t}^P + \hat{\tau}_t + \hat{\mathcal{E}}_t)$ and $\hat{P}_{F,t}^P = (1-\Omega)\hat{C}_{F,t} + \Omega(\hat{P}_{H,t}^P - \hat{\mathcal{E}}_t)$.
4. Risk sharing is imperfect: $\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t$.

These equilibrium conditions link the risk-sharing wedge with the terms of trade. Defining the terms of trade $\hat{s}_t \equiv \hat{\mathcal{E}}_t + \hat{P}_{F,t}^P - \hat{P}_{H,t}^P$, the intratemporal conditions imply

$$\hat{w}_t = \frac{1+\Omega}{1-\Omega}\hat{s}_t + \frac{\Omega}{1-\Omega} L_\tau^P \hat{\tau}_t,$$

so the wedge embeds the terms-of-trade gains emphasized in optimal-tariff analysis. Without intermediates, elastic labor implies $\hat{w}_t = \hat{s}_t$; with intermediates, tariffs on inputs can raise the wedge and depreciate the home currency. Although the wedge grows more complex once these assumptions are relaxed, the central mechanism is clear: terms-of-trade movements favoring the tariff-imposing country transfer wealth to it, allowing consumption above the

[22] In the scalar case, $\gamma_{H,F} \to \gamma_H$ and $\Omega_{H,F} \to \Omega_H$.
[23] Under flexible prices this leaves real allocations unchanged and simplifies notation.

perfect-risk-sharing benchmark. If home is highly dependent on foreign intermediates, this can reverse, transferring wealth abroad.

Proposition 1. *Under symmetry and flexible prices, solving the model yields*

$$\hat{C}_{H,t} = -\underbrace{\frac{\Omega(1-\gamma)+\gamma}{1+\Omega}}_{>0}\left(\frac{1}{1-\Omega}\hat{\tau}_t + \hat{w}_t\right) \tag{12}$$

$$\hat{\mathcal{E}}_t = -\underbrace{\frac{(\Omega(1-\gamma)+\gamma)}{1+\Omega}}_{>0}\hat{\tau}_t + \underbrace{\frac{(1-\Omega)(1-2\gamma)}{1+\Omega}}_{>0}\hat{w}_t \tag{13}$$

Proposition 1 follows from MUC algebra; Appendix C contains the derivation. Two immediate corollaries follow.

Corollary 1. *Under perfect risk sharing ($\hat{w}_t = 0$), a unilateral home import tariff lowers home consumption and appreciates the home currency ($\frac{\partial \hat{C}_{H,t}}{\partial \hat{\tau}_t} < 0, \frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} < 0$).*

Tariffs shift demand toward domestic goods. With home bias, the home consumption basket becomes more expensive and the home currency appreciates in real terms. Under perfect risk sharing, Arrow-Debreu transfers reallocate resources toward states in which home consumption is relatively cheap; because the tariff makes home consumption relatively expensive on impact, contemporaneous home consumption falls. The decline in the tariff-imposing country's consumption in frameworks such as Caliendo et al. (2025) reflects this mechanism.

Corollary 2. *For tariffs to raise home consumption, tariffs must induce a deviation from perfect risk sharing (i.e., $\hat{w}_t \neq 0$), thereby producing a wealth transfer towards home in states when consumption is expensive.*

Under imperfect risk sharing, the fall in consumption generated by intertemporal substitution (consumption is expensive today with tariffs) can be offset by the permanent wealth effect embodied in the martingale wedge. In equilibrium, consumption rises if the wedge is sufficiently negative. Consumption falls with a sufficiently positive wedge that also implies depreciation and a wealth transfer to the rest of the world, strengthening foreign consumption and appreciating the foreign currency. Under incomplete markets, the sign and magnitude of $\hat{w}_t$ can therefore change the sign and magnitude of consumption and exchange rate. The tariff-induced wedge that opens on impact and remains constant thereafter is ($\forall\, k \geq 0$):

$$\hat{w}_{t+k} = \underbrace{\left[\frac{1-\beta}{1-\beta\rho_\tau}\right]}_{>0} \frac{\overbrace{\mathcal{A}\left(\gamma(1-\Omega) + \frac{2\Omega}{1-\Omega}\right) + \theta\left[\Omega + \gamma(1-\gamma)(1-\Omega)^2\right]}^{>0}}{\underbrace{\mathcal{A}(1-2\gamma)(1-\Omega)}_{>0} - \underbrace{2\theta\left[\Omega + \gamma(1-\gamma)(1-\Omega)^2\right]}_{>0}} \tag{14}$$

where $\mathcal{A} \equiv \gamma + (1-\gamma)\Omega > 0$. Appendix C provides details for Equation (14). In this expression, the persistence-discount prefactor and numerator are strictly positive, so the denominator determines the sign. The threshold is:

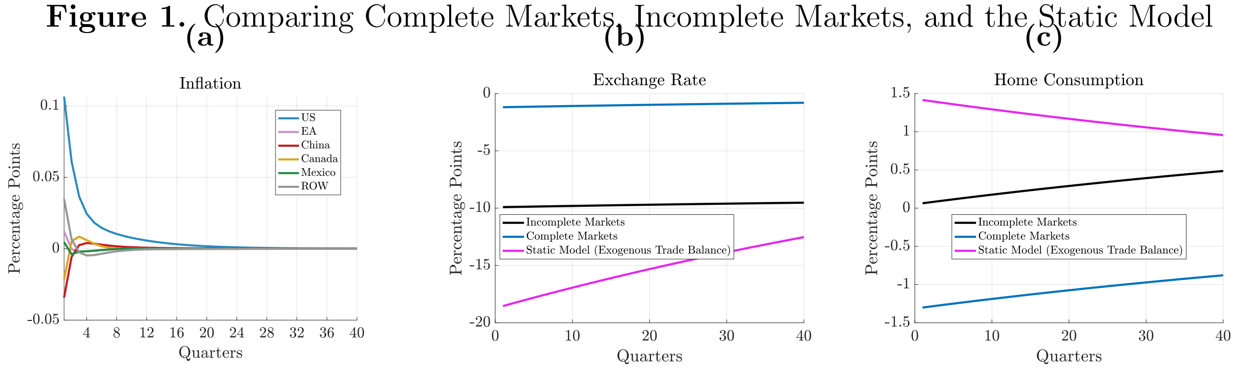
$$\theta^{\text{crit}} \equiv \tfrac{\mathcal{A}(1-2\gamma)(1-\Omega)}{2[\Omega+\gamma(1-\gamma)(1-\Omega)^2]}, \text{ where } \hat{w}_t > 0 \text{ for } \theta < \theta^{\text{crit}} \text{ and } \hat{w}_t < 0 \text{ for } \theta > \theta^{\text{crit}}.$$

Greater substitutability makes it more likely that terms-of-trade gains favor the home country, transferring wealth to home, increasing consumption at home and appreciating the home currency (a sufficiently negative wedge). The magnitude of the wedge is also sensitive to tariff persistence (ρ_τ) as persistent tariffs lead to larger wealth effects.

Remark 2. Greater tariff persistence enlarges the risk-sharing wedge in absolute value: terms-of-trade gains then last longer and imply a larger wealth transfer. While primitives such as $\mathbf{\Gamma}$, $\mathbf{\Omega}$, and θ can reverse the sign of $\hat{w}_t$, persistence can also make the wedge large enough to flip the signs of the consumption and exchange-rate responses in (12) and (13).

To highlight the importance of imperfect risk sharing, we compare incomplete markets with two standard alternatives: complete markets and exogenous transfers, as in static trade models that pin down the trade balance. Figure 1 reports the wedge in panel (a), the exchange rate in panel (b), and home consumption in panel (c) after a 10% home tariff with $\rho_\tau = 0.99$, $\gamma = 0.05$, and $\Omega = 0.1$. Under complete markets (blue), the wedge is zero. Under incomplete markets (black), it is permanently negative. Under exogenous transfers (pink), it is also negative but larger in absolute value, as eliminating financial assets creates a greater departure from perfect risk sharing. In the static model, the wedge declines in absolute value over time, reflecting the fact that it is not a martingale: there, $\hat{w}_t$ depends only on contemporaneous tariffs and therefore reverts to zero as tariffs decline. The exchange rate appreciates in all cases, with negligible appreciation under complete markets and a large one in the static transfer case. The home-consumption response is positive in both the static transfer and incomplete-markets cases, but negative under complete markets. With financial assets, consumption depends on the wedge (as in (12)): when $\hat{w}_t = 0$, perfect risk sharing implies a negative consumption response, whereas a sufficiently negative wedge lets the intertemporal wealth effect dominate substitution and raise consumption, as in panel (c). Consistent with our theory, the large absolute value of $\hat{w}_t$ is partly driven by the high tariff persistence ($\rho_\tau = 0.99$).

Figure 1. Comparing Complete Markets, Incomplete Markets, and the Static Model



NOTE: Panel (a) plots the risk-sharing wedge, panel (b) the exchange rate, and panel (c) home consumption after a 10% home tariff on the rest of the world with persistence $\rho_\tau = 0.99$. We set $\gamma = 0.05$ and $\Omega = 0.1$. Black denotes incomplete markets, blue complete markets, and pink the static exogenous-transfer specification that fixes the trade balance at its steady-state level. Under complete markets the wedge is zero; under incomplete markets and the static specification it is negative, so consumption and exchange-rate responses differ across cases.

Figure 1 therefore shows that neither complete markets nor static exogenous transfers constitute innocuous benchmarks. Complete markets impose $\hat{w}_t = 0$, shutting down tariff-induced wealth transfers and thereby making the home-consumption response negative by construction. Static models move in the opposite direction and can overstate the wedge since the trade balance is unable to adjust endogenously, which can generate an unrealistically large appreciation; in this calibration, the appreciation response is nearly twice the size of the tariff. Because $\hat{w}_t$ enters both the consumption response and the inflation–output tradeoff, these two benchmarks can change whether tariffs are expansionary on impact. Some form of incomplete markets—at minimum, a single non-state-contingent nominal bond—is therefore essential to understanding the macroeconomic impact of tariffs.

3.1 N Countries and J Sectors: Why the Network Matters?

We next examine how production networks affect tariff transmission under flexible prices. When intermediate inputs are absent ($\Omega = 0$), Equation (14) reduces to ($\forall\ k \geq 0$):

$$\hat{w}_{t+k} = \left[\frac{1-\beta}{1-\beta\rho_\tau}\right]\ \frac{\gamma+\theta(1-\gamma)}{(1-2\gamma)-2\theta(1-\gamma)}.$$

Thus, without intermediate inputs, $\theta > \frac{(1-2\gamma)}{2(1-\gamma)}$ is sufficient for the risk-sharing wedge to be negative, favoring the tariff-imposing home country. This threshold is satisfied under most calibrations in the literature, as θ often exceeds 0.5. Once $\Omega \neq 0$, however, the sign of the wedge can vary over a wider range.

To illustrate, let us add a second foreign good, denoted T. It is upstream, does not enter final consumption, has fixed output, and is used by the home country as an intermediate input (its share denoted by $\Omega_{H,T}$). Then the risk-sharing wedge coefficient with respect to a tariff innovation is ($\forall\ k \geq 0$):

$$\hat{w}_{t+k} = -\frac{1-\beta}{1-\beta\rho_\tau}\, \frac{(\Omega + \Omega_{H,T})\Big[\Omega\big(\Omega_{H,T}y(2-\theta^X) - \beta\big) + \alpha_F(A_2 - \beta)\Big] - A_1\, A_3}{A_1\Big(1 - y\mathcal{D}\,\alpha_H\Big) - \alpha_H\Big[\Omega\,\Omega_{H,T}y(2-\theta^X) + \alpha_F A_2\Big]},$$

where $\mathcal{D} \equiv \frac{1}{1+\Omega}$, $A_1 \equiv \Omega\Omega_{H,T} + \Omega\alpha_F + \Omega\alpha_H + \Omega_{H,T}\alpha_F + \alpha_F\alpha_H > 0$, $A_2 \equiv y\Big(1 + \Omega_{H,T} + \mathcal{D}\big[-\alpha_H - \theta^X(2\Omega + \Omega_{H,T})\big]\Big)$, and $A_3 \equiv -y^2\mathcal{D}\,\theta^X L^P_\tau\big(\Omega\alpha_F + \Omega_{H,T}\big) < 0$.

When θ is low, that is inputs are highly complementary, the sign of $\hat{w}_{t+k}$ can reverse with Ω_{HT}. Tariffs on the upstream good make the home good a scarce input to foreign production as foreign production also uses it as an input, the resulting relative-supply and -demand shifts can make foreign goods more valuable, turning the wedge positive and favoring the foreign country.

4 Tariffs Under Sticky Prices

We now reintroduce nominal rigidities and monetary policy and thereby turn to the analytical solution for our model under sticky prices. In the $N = 2$ and $J = 1$ case, the primitives we add are the following matrices: $\boldsymbol{\Lambda} = \begin{bmatrix} \Lambda_H & 0 \\ 0 & \Lambda_F \end{bmatrix}$, $\boldsymbol{\Phi} = \begin{bmatrix} \phi^H_\pi & 0 \\ 0 & \phi^F_\pi \end{bmatrix}$. With that, we return to the system of equations in (9)-(11) which can be solved with the MUC as follows:

$$\begin{aligned}
\Delta\hat{\boldsymbol{C}}_t &= \boldsymbol{c}_\mu\boldsymbol{\mu}_{t-1} + \boldsymbol{c}_w\Delta\hat{w}_t + \boldsymbol{c}_\tau\hat{\tau}_t + \boldsymbol{c}_{\tau,-1}\hat{\tau}_{t-1},\\
\boldsymbol{\pi}^P_t &= \boldsymbol{p}_\mu\boldsymbol{\mu}_{t-1} + \boldsymbol{p}_w\Delta\hat{w}_t + \boldsymbol{p}_\tau\hat{\tau}_t + \boldsymbol{p}_{\tau,-1}\hat{\tau}_{t-1},\\
\boldsymbol{\mu}_t &= \boldsymbol{\Psi}^{\text{NKOE}}\boldsymbol{\mu}_{t-1} + \boldsymbol{\mu}_w\Delta\hat{w}_t + \boldsymbol{\mu}_\tau\hat{\tau}_t + \boldsymbol{\mu}_{\tau,-1}\hat{\tau}_{t-1},
\end{aligned}$$

Our first step is to introduce the NKOE propagation matrix, $\boldsymbol{\Psi}^{\text{NKOE}}$, which is the coefficient matrix in front of the lagged real marginal cost vector in the solution for the real marginal cost vector. As such, it governs the equation of motion for real marginal costs, in deviation from the pre-tariff steady state, across time. For a candidate propagation matrix

$\boldsymbol{X}$ and tariff persistence ρ, define

$$\mathcal{K}(\rho, \boldsymbol{X}) \equiv (1-\beta\rho)\Big[\left((1-\rho)\boldsymbol{I}-\boldsymbol{X}\right)(\boldsymbol{I}-\beta\boldsymbol{X})-(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{X}\Big]+\Big[(\boldsymbol{\alpha}+\boldsymbol{L}_{\mathcal{E}}^{P}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}-\rho(\boldsymbol{I}-\boldsymbol{\Omega})\Big]\boldsymbol{\Lambda}$$

Proposition 2. *The NKOE propagation matrix, $\boldsymbol{\Psi}^{\text{NKOE}}$, is the stable matrix that solves $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})\, \boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}$.*

Proposition 2 follows from MUC algebra (see Appendix D). Intuitively, for a candidate propagation matrix $\boldsymbol{X}$, three forces act on a cost distortion. First, $((1-\rho)\boldsymbol{I}-\boldsymbol{X})(\boldsymbol{I}-\beta\boldsymbol{X})$ captures forward-looking price adjustment and discounting. Second, $-(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{X}$ captures cost inheritance through sticky intermediate inputs: today's distortion reappears tomorrow as an input cost. Third, $\big[(\boldsymbol{\alpha}+\boldsymbol{L}_{\mathcal{E}}^{P}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}-\rho(\boldsymbol{I}-\boldsymbol{\Omega})\big]\boldsymbol{\Lambda}$ captures the stabilizing role of monetary policy and the exchange rate, alongside the continued loading of a persistent tariff onto marginal cost under price stickiness. In equilibrium, $\boldsymbol{\Psi}^{\text{NKOE}}$ is the stable propagation matrix balancing these forces.

The NKOE propagation matrix links tariff-related distortions on consumption and production to the dynamics of the real marginal cost vector. A sector central to production, whether through broad use (e.g., steel, aluminum) or downstream importance (e.g., semiconductors), carries significant weight in the standard Leontief inverse. If it also has highly flexible (or rigid) prices, i.e., a vertical (or horizontal) supply curve, the tariff's inflationary impact is amplified (or muted) through the network. Analogously to how the stickiness-adjusted Leontief inverse reweights sectors via $\boldsymbol{\Lambda}$, the NKOE propagation matrix also reweights sectors by the monetary stance of their country (e.g., through consumption and the exchange rate).

Next, we solve for the risk-sharing wedge in Proposition 3 (see Appendix D.9). As noted in Section 2.6, the wedge is the unique solution satisfying the martingale condition ($\hat{w}_t = \mathbb{E}_t\hat{w}_{t+1}$) implied by UIP and ensuring debt $\hat{V}_t$ is stable. It is the wealth transfer, relative to perfect risk sharing, required for goods and asset markets to clear at a given price sequence, including the exchange rate. This depends on how net debt responds to the terms of trade, balance sheet effects, and interest payments, which is why the balance of payments coefficients $\boldsymbol{\Xi}$ enter the solution.

Proposition 3. *The risk-sharing wedge in response to a transitory increase in tariffs ($0 < \rho < 1$) under sticky prices is as follows ($\forall k \geq 0$):*

$$\frac{\partial \hat{w}_{t+k}}{\partial \hat{\tau}_t} = w_\tau \equiv \frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{c}_\tau + \boldsymbol{c}_{\tau,-1}) + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\tau - (\tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_\mu + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\mu)(\boldsymbol{\Psi}^{\text{NKOE}} - \boldsymbol{I})^{-1}\boldsymbol{\mu}_\tau}{(1-\rho)\Big[(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_\mu + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\mu)(\boldsymbol{\Psi}^{\text{NKOE}} - \boldsymbol{I})^{-1}\boldsymbol{\mu}_w - \tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_w - \tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_w - \tilde{\Xi}_4\Big]}.$$

Proposition 3 shows that the wealth-transfer channel depends on the same propagation matrix $\boldsymbol{\Psi}^{\text{NKOE}}$ through $(\boldsymbol{\Psi}^{\text{NKOE}} - \boldsymbol{I})^{-1}$, tying the wedge directly to the dynamics governing real marginal cost distortions. With these two objects in hand, Proposition 4 solves for consumption and consumer price inflation, capturing two intuitions. First, the effect on consumption and the exchange rate depends on the tariff's impact on the risk-sharing wedge, w_τ: as in the flexible-price case, a sufficiently large negative wedge can generate simultaneous appreciation and a consumption increase. Second, the NKOE propagation matrix governs dynamics.

Proposition 4. *The first period impact of a transitory increase in tariffs* $(0 < \rho < 1)$ *under sticky prices on the endogenous variables is as follows:*

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \hat{\tau}_t} = \underbrace{\left[(\boldsymbol{I} + \boldsymbol{\mathcal{Z}})\boldsymbol{\Gamma}\Big((1-\rho)\boldsymbol{\Lambda}\,\mathcal{K}(\rho, \boldsymbol{\Psi}^{\text{NKOE}})^{-1}\Big) + \boldsymbol{\mathcal{Z}}\sigma \boldsymbol{R}_\tau - \boldsymbol{\Gamma}\right]\boldsymbol{L}_\tau^P}_{\textit{NKOE propagation}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{L}_\tau^P}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^P}$$
$$+ \underbrace{\boldsymbol{L}_\tau^C}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^C} + \underbrace{\Big[(1-\sigma)\boldsymbol{\mathcal{Z}}\Big]\boldsymbol{L}_\tau^C}_{\textit{Demand Propagation}} + \underbrace{\Big[\boldsymbol{\Gamma}\boldsymbol{p}_w + \boldsymbol{\mathcal{Z}}\big(\sigma \boldsymbol{c}_w + \boldsymbol{\Gamma}\boldsymbol{p}_w\big) + \boldsymbol{L}_{\mathcal{E}}^C\, m\Big] w_\tau}_{\textit{Contribution of Wealth Transfer}}$$

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \underbrace{\boldsymbol{R}_\tau \boldsymbol{L}_\tau^P}_{\textit{NKOE propagation}} - \underbrace{\boldsymbol{L}_\tau^C}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^C} + \underbrace{\boldsymbol{c}_w\, w_\tau}_{\textit{Contribution of Wealth Transfer}}$$

where $\boldsymbol{R}_\tau \equiv \boldsymbol{\mu}_2^\ell \Big\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\mathcal{K}(\rho, \boldsymbol{\Psi}^{\text{NKOE}})^{-1} - \boldsymbol{I}\Big\}$, $m \equiv \big(1 - \boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\big)^{-1}$, $\boldsymbol{\mathcal{Z}} \equiv \boldsymbol{L}_{\mathcal{E}}^C\, m\, \boldsymbol{Z}$, *and* $\boldsymbol{\mu}_2^\ell \equiv (\boldsymbol{\mu}_2^\top \boldsymbol{\mu}_2)^{-1}\boldsymbol{\mu}_2^\top$. *Moreover,* $\boldsymbol{c}_w$ *and* $\boldsymbol{p}_w$ *are the coefficient matrices on* $\Delta\hat{w}_t$ *in the sticky-price solutions for* $\Delta\hat{\boldsymbol{C}}_t$ *and* $\boldsymbol{\pi}_t^P$*, respectively.*

Proposition 4 follows from MUC algebra (see Appendix D). Written this way, $\boldsymbol{\Psi}^{\text{NKOE}}$ enters twice: through the equilibrium propagation condition $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})\boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}$, and through $\mathcal{K}(\rho, \boldsymbol{\Psi}^{\text{NKOE}})^{-1}$ in the impact responses. The same object that governs the persistence of tariff-induced cost distortions thus maps a persistent tariff into current inflation and consumption. Proposition 4 admits a transparent decomposition: the direct demand effect, $\boldsymbol{L}_\tau^C$, is the immediate rise in consumer prices of imported goods; the direct supply effect, $\boldsymbol{L}_\tau^P$, is the immediate rise in imported input costs; propagation collects the general-equilibrium feedback through sticky prices, production linkages, and monetary policy; and the risk-sharing wedge captures wealth-transfer effects.

To illustrate, consider a simple analytical example (the full quantitative exercise is reserved for Section 5). Divide the world into the United States and the rest of the world (RoW), and suppose the United States imposes a 10% tariff on all RoW imports with persistence $\rho_\tau = 0.99$.[24] Figure 2 reports the on-impact decomposition from Proposition 4. The rise in U.S. inflation is driven primarily by the two direct channels: $\boldsymbol{L}_\tau^C$ raises CPI because the tariff directly makes imported consumption more expensive, and $\boldsymbol{\Gamma}\boldsymbol{L}_\tau^P$ raises CPI because imported intermediates become costlier and pass through to consumer prices. The demand-propagation term is zero since $\sigma = 1$. The NKOE propagation term is negative: the monetary reaction to higher producer prices is strong enough to dampen inflation on impact. Intuitively, the tariff shifts wealth toward the imposing country, inducing an appreciation that lowers the domestic-currency price of imports and offsets part of the initial impulse, hence the negative wealth-transfer term for the U.S. For the RoW, the channel reverses: wealth flows out and inflation rises.

On the consumption side, the direct demand effect is negative: tariffs make the impact period relatively expensive, so households substitute toward cheaper periods. Imported-input tariffs affect consumption through the NKOE propagation term, which is negative, reflecting the contractionary mix of the supply shock, network spillovers, and monetary tightening. The wealth transfer toward the United States raises U.S. consumption, but not enough to overturn these negative effects. For the RoW, both wealth transfer and NKOE propagation reduce consumption.

4.1 N Countries and J Sectors: Why the Global Network Matters under Sticky Prices?

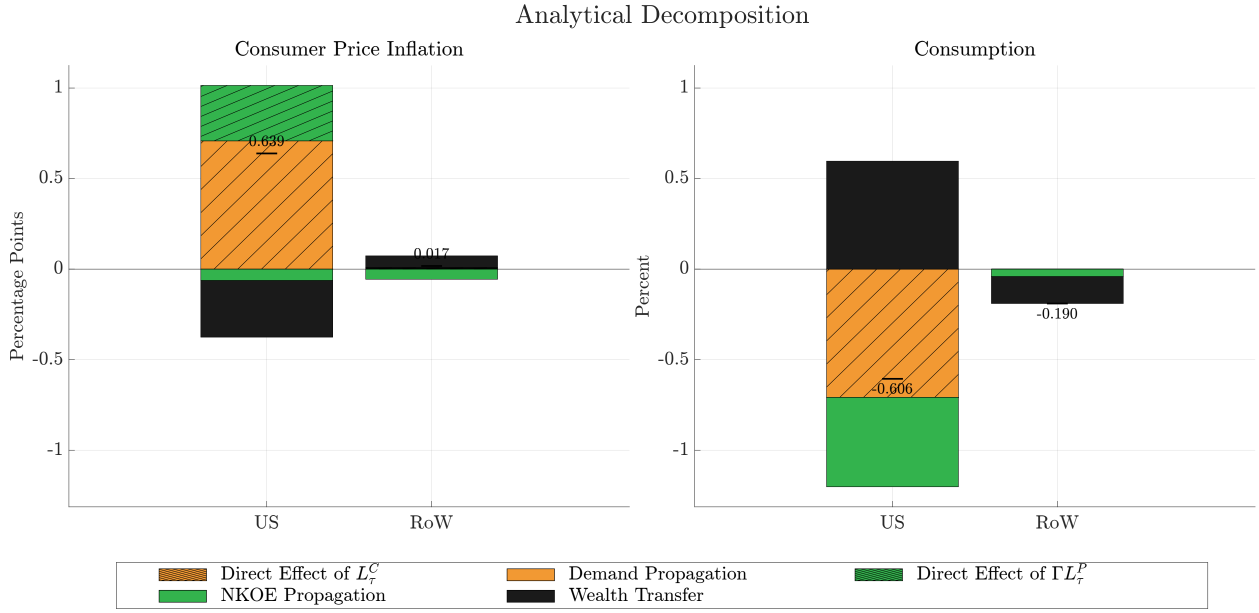
In network models, multiple sectors ($J > 1$) affect aggregate dynamics through three channels. First, under parameter heterogeneity, aggregation and multiplication do not commute: aggregating parameters before multiplying generally differs from multiplying at the sectoral level and then aggregating. Pasten et al. (2020) and Rubbo (2023) establish this result in closed economy settings; in our notation, it is reflected in Equation (3). One implication is that sectoral granularity can flatten the aggregate Phillips curve. Second, sectoral shocks interacting with sector-specific Phillips curves generate residual cost-push disturbances, so shocks to different sectors produce distinct aggregate responses. Third, and most important here, production networks make lagged deviations in real marginal costs (equivalently, lagged prices) relevant for inflation and consumption dynamics.

Proposition 5. *When $J = 1$, $\boldsymbol{\Psi}^{NKOE} = \mathbf{0}$, whereas when $J > 1$, $\boldsymbol{\Psi}^{NKOE} \neq \mathbf{0}$.*

Proof. The zero-persistence branch (i.e., $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$) requires aggregate-demand adjustments to eliminate all sectoral real marginal cost deviations on impact. In Appendix D.3,

[24] We use the parameter values in Section 5 and Table 2, except where the analytical model imposes simplifications (e.g., portfolio adjustment costs and η are set to zero, and $\sigma = 1$).

Figure 2. Theoretical Example: Tariffs Without Retaliation



NOTE: Figure 2 reports the on-impact decomposition of CPI inflation (left panel) and consumption (right panel) in a two-country analytical example with the U.S. and the rest of the world (RoW). The U.S. imposes a 10% tariff on the RoW with persistence $\rho_\tau = 0.99$. Using Proposition 4, the total response is decomposed into direct demand, demand propagation, direct supply, supply propagation, and wealth-transfer components. The black marker denotes the total effect.

this becomes $\boldsymbol{\mu}_2 \boldsymbol{c}_\mu = -\boldsymbol{I}_{NJ}$. When $J = 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{N \times N}$ is square and invertible, yielding the unique solution $\boldsymbol{c}_\mu = -\boldsymbol{\mu}_2^{-1}$ and hence $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$; under Blanchard-Kahn determinacy, this is the unique equilibrium. When $J > 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{NJ \times N}$ is tall and admits no right inverse: there are NJ sectoral distortions but only N aggregate-demand adjustments. The branch $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$ is thus impossible, so $\boldsymbol{\Psi}^{\text{NKOE}} \neq \mathbf{0}$. Appendix D.3 provides the rank argument in detail. □

Proposition 5 highlights the importance of multiple sectors in the open economy. When $J = 1$, a single sectoral price distortion per country means real marginal costs do not propagate. When $J > 1$, each country's monetary policy targets only a weighted average of prices, not the full vector of sectoral relative prices, so sectoral distortions survive aggregation and become inherited cost-push states. Hence $\boldsymbol{\mu}_t$ is a genuine state vector and the inflation–output trade-off becomes persistent. Input–output linkages then operate on this state; conditional on $J > 1$, they strengthen persistence by feeding today's sectoral cost distortions into tomorrow's marginal costs through intermediate-input use.

To understand the implications of Proposition 5, note that the solution to our model has a VAR(1) representation with the state vector defined above. Thus, the reduced-form solution can be written as $\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}\,\epsilon_t$, where $\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \epsilon_t$, $\boldsymbol{x}_t \equiv \begin{bmatrix} \boldsymbol{\mu}_t & \Delta \hat{\boldsymbol{C}}_t & \boldsymbol{\pi}_t^P & \Delta \hat{w}_t & \hat{\tau}_t \end{bmatrix}^\top$

and $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{\mu}_w w_\tau + \boldsymbol{\mu}_\tau & \boldsymbol{c}_w w_\tau + \boldsymbol{c}_\tau & \boldsymbol{p}_w w_\tau + \boldsymbol{p}_\tau & w_\tau & 1 \end{bmatrix}^\top$. Here $w_\tau \equiv \frac{\partial \Delta \hat{w}_t}{\partial \epsilon_t}$, so that $\frac{\partial \Delta \hat{w}_{t+k}}{\partial \epsilon_t} = 0$ for all $k > 0$, while the level of the wedge satisfies $\frac{\partial \hat{w}_{t+k}}{\partial \epsilon_t} = w_\tau$ for all $k \geq 0$. With the VAR(1) representation, we can derive analytical impulse response functions. For example, producer price inflation, k periods after tariffs with persistence ρ_τ are imposed can be written as $\frac{\partial \boldsymbol{\pi}^P_{t+k}}{\partial \epsilon_t} = \boldsymbol{S}_\pi \boldsymbol{A}^k \boldsymbol{B}$ where $\boldsymbol{S}_\pi \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$.

Remark 3. It follows from Proposition 5 that the VAR transition matrix $\boldsymbol{A}$ looks different when the number of sectors is $J = 1$ versus $J > 1$. When $J = 1$, Appendix D implies $\boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{p}_\mu = \boldsymbol{p}_w = \boldsymbol{\mu}_w = \mathbf{0}$, while $\boldsymbol{c}_\mu = -\boldsymbol{\mu}_2^{-1}$, $\boldsymbol{c}_{\tau,-1} = \boldsymbol{\mu}_2^{-1}\boldsymbol{\mu}_3$, $\boldsymbol{p}_{\tau,-1} = \mathbf{0}$, and $\boldsymbol{\mu}_{\tau,-1} = \mathbf{0}$. Thus

$$\boldsymbol{A}_{J=1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \rho_\tau \boldsymbol{\mu}_\tau \\ -\boldsymbol{\mu}_2^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\mu}_2^{-1}\boldsymbol{\mu}_3 + \rho_\tau \boldsymbol{c}_\tau \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \rho_\tau \boldsymbol{p}_\tau \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & \rho_\tau \end{bmatrix} \text{ and}$$

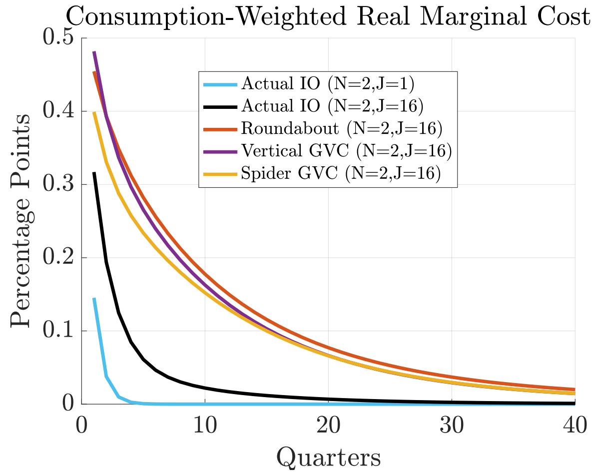
$$\boldsymbol{A}_{J>1} = \begin{bmatrix} \boldsymbol{\Psi}^{\text{NKOE}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \rho_\tau \boldsymbol{\mu}_\tau \\ \boldsymbol{c}_\mu & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{c}_{\tau,-1} + \rho_\tau \boldsymbol{c}_\tau \\ \boldsymbol{p}_\mu & \mathbf{0} & \mathbf{0} & \mathbf{0} & \rho_\tau \boldsymbol{p}_\tau \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & \rho_\tau \end{bmatrix}.$$

Thus, J changes how endogenous lagged variables propagate shocks. When $J = 1$, $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$ and lagged real marginal cost deviations have no effect on contemporaneous inflation and consumption. When $J > 1$, $\boldsymbol{\Psi}^{\text{NKOE}} \neq \mathbf{0}$ and the lagged real marginal cost vector generates persistence in the inflation–consumption (similarly, inflation–output) trade-off. This persistence point has a core implication for stabilization policy: with more granular global networks ($J > 1$), tariff-induced real marginal cost deviations take longer to clear.

Figure 3 illustrates this mechanism for a 10% tariff imposed by the U.S. on the rest of the world under passive monetary policy, implemented as $1+\hat{i}_{n,t} = \left(1 + \hat{i}_{n,t-1}\right) + \phi^n_\pi \pi_{n,t} \quad \forall n \in N$ with $\phi^n_\pi \to 0$.[25] Each line reports consumption-weighted real marginal cost in the tariff-imposing country. Across scenarios, aggregate home bias and aggregate intermediate-input dependence are held fixed, so all networks collapse to the same 2×2 matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\Omega}$; only granular between-sector flows differ. The light blue line is the $J = 1$ case; the remaining lines use $J = 16$. The figure demonstrates that relative to the one-sector benchmark, $J = 16$

[25]Taking the limit preserves determinacy.

Figure 3. Network Granularity and Persistence of Real Marginal Cost Deviations



NOTE: The figure plots consumption-weighted real marginal cost in the tariff-imposing country across network structures when a 10% unilateral tariff is imposed with persistence $\rho_\tau = 0.99$. All granular networks aggregate to the same 2×2 matrices $\mathbf{\Gamma}$ and $\mathbf{\Omega}$. Relative to $J = 1$, economies with $J > 1$ exhibit slower decay, and persistence varies across granular networks.

cases see higher persistence of real marginal cost deviations across time and different network setups (e.g. roundabout and vertical and spider global value chains) produce different persistence.

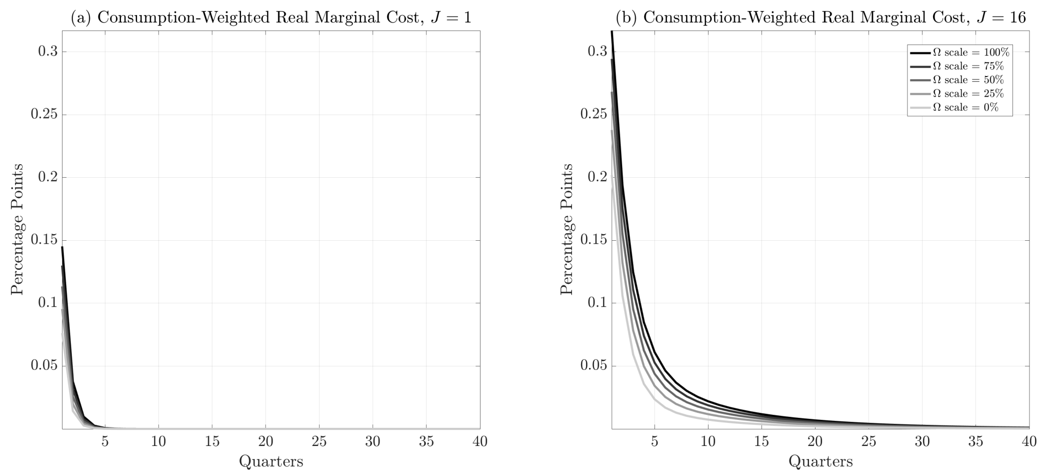
The effect is not purely dimensional. Proposition 5 identifies that, once $J > 1$, inherited real marginal cost becomes a propagating state even without input–output linkages. A separate question is what $\mathbf{\Omega}$ adds once propagation is on. Since $\mathbf{\Psi}^{\text{NKOE}}$ is the coefficient on lagged real marginal cost in the solution, a larger eigenvalue governing the slowest decay implies slower unwinding of inherited marginal-cost distortions. Holding J, $\mathbf{\Gamma}$, and $\mathbf{\Lambda}$ fixed, a larger $\mathbf{\Omega}$ raises the share of costs inherited through intermediate inputs. Proposition 6 gives sufficient conditions under which this increases persistence: $J > 1$ makes real marginal cost deviations persistent, and $\mathbf{\Omega}$ governs the strength of that persistence.

Proposition 6. *Input–output linkages intensify persistence conditional on $J > 1$. Fix J, $\mathbf{\Gamma}$, and $\mathbf{\Lambda}$, and compare economies along the admissible scaling path $\mathbf{\Omega}(s) = s\,\bar{\mathbf{\Omega}}, \quad s \in [\underline{s}, \bar{s}]$. Let $\mathbf{\Psi}(s) \equiv \mathbf{\Psi}^{NKOE}(s)$ denote the stable branch in $\boldsymbol{\mu}_t = \mathbf{\Psi}(s)\boldsymbol{\mu}_{t-1} + \cdots$. Consider the passive-policy limit ($\phi_\pi^n \to 0\ \forall n$). Suppose exactly N eigenvalues of $\mathbf{\Psi}(s)$ are zero, while the remaining $NJ - N$ eigenvalues are nonzero and stable.*[26] *Let $\lambda_*(s) \in (0,1)$ be the real, simple nonzero eigenvalue that governs the slowest decay: $|\lambda_*(s)| = \max\left\{|\lambda| : \lambda \in \sigma(\mathbf{\Psi}(s)),\ \lambda \neq 0\right\}$. Under the sufficient sign condition in Appendix D.7, $\lambda_*'(s) > 0$. Hence scaling up intermediate-input use raises the largest real non-zero eigenvalue of $\mathbf{\Psi}^{NKOE}$, which governs*

[26] We verify numerically that these assumptions hold.

the slowest decay. Equivalently, conditional on $J > 1$, stronger input–output linkages make tariff-induced real marginal-cost deviations unwind more slowly.

Figure 4. Input-Output Linkages, Network Granularity and Persistence of Real Marginal Cost Deviations



NOTE: The figure plots consumption-weighted real marginal cost in the tariff-imposing country when a 10% unilateral tariff is imposed with persistence $\rho_\tau = 0.99$. Panel (a) sets $J = 1$ and panel (b) sets $J = 16$. Within each panel, darker lines correspond to stronger input–output linkages, and the lightest line sets $\boldsymbol{\Omega} = \mathbf{0}$. The common y-axis shows that changing $\boldsymbol{\Omega}$ has little effect on persistence under $J = 1$ but a much larger effect under $J = 16$.

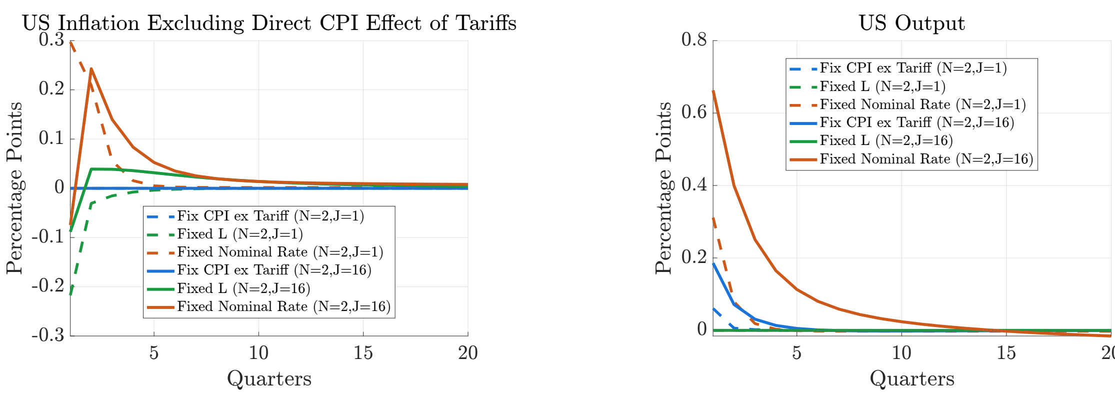
When $J > 1$, $\boldsymbol{\Psi}^{\text{NKOE}}$ solves $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})\,\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$, and, in the passive-policy limit along the scaling path ($\boldsymbol{\Omega}(s) = s\bar{\boldsymbol{\Omega}}$), $\boldsymbol{\Omega}$ enters only through $\boldsymbol{M}(s) \equiv (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\boldsymbol{\Omega}})\boldsymbol{\Lambda} = (1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda} - s\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}$. Under the conditions presented in Appendix D.7, a real simple nonzero eigenvalue $\lambda(s) \in (0, 1)$ of $\boldsymbol{\Psi}^{\text{NKOE}}(s)$ is linked to an eigenvalue $\zeta(s) = \beta\lambda(s) + \lambda(s)^{-1}$ of $\boldsymbol{M}(s)$. Scaling up s lowers this effective damping term and the stable-root relation then implies $\lambda'(s) > 0$. Applying this to the eigenvalue that governs the slowest decay gives Proposition 6: stronger input–output linkages make inherited real marginal-cost deviations unwind more slowly. See Appendix D.7 for the proof.

Figure 4 illustrates Proposition 6. Within each panel, darker lines correspond to stronger input–output linkages, lighter lines scale $\boldsymbol{\Omega}$ toward zero. Two margins stand out. First, moving from $J = 1$ to $J = 16$ substantially raises both the level and persistence of consumption-weighted real marginal cost deviations. Second, once $J > 1$, reducing $\boldsymbol{\Omega}$ compresses the real marginal cost response, and the effect is much larger when $J = 16$: in the $J = 1$ economy, shrinking $\boldsymbol{\Omega}$ mainly lowers the impact response while subsequent paths remain tightly clustered, whereas at $J = 16$ it lowers both the peak and the tail, showing that intermediate-input linkages materially slow the unwinding of tariff-induced cost distortions. Persistence is strongest when both forces operate jointly: $J > 1$ creates the endogenous propagation channel and $\mathbf{\Omega} \neq \mathbf{0}$ amplifies it.

Since the persistence of marginal cost deviations depends on network structure, inflation and output deviations can also be more persistent, as shown in Figure 5. In our context, if output is measured relative to the initial steady state rather than as a gap, it need not converge to zero. Inflation typically returns to zero, however, under policies stabilizing an aggregate quantity (e.g., consumption at steady state) adjustment may instead produce persistent inflation. We therefore define persistence as the number of periods required for variables to converge to their terminal values. Without portfolio adjustment costs, different policies generally yield different terminal steady states, so raw deviations from the initial steady state are not comparable across IRFs. Since our focus is *stabilization dynamics* rather than long-run incidence, the relevant comparison is the distance from each simulation's post-tariff equilibrium.

Figure 5. Network Granularity and Persistence of the Inflation-Output Trade-off

(a) **(b)**



NOTE: A 10% unilateral tariff is imposed with persistence $\rho_\tau = 0.99$. Each series is reported relative to its case-specific terminal steady state, since different simulations converge to different long-run equilibria. Under each policy regime, convergence is slower when $J > 1$: the multi-sector economy remains farther from its terminal equilibrium for longer than the $J = 1$ economy.

For this reason, Figure 5 reports variables as deviations from their case-specific *terminal* steady states under three regimes: (i) passive policy, with $\rho_m^n = 1$ and $\phi_\pi^n \to 0$; (ii) perfect stabilization of aggregate CPI excluding tariffs at its pre-tariff level; and (iii) stabilization of aggregate employment. In all three cases, greater network granularity slows convergence: under $J > 1$, inherited cost distortions unwind more slowly, leaving the economy farther from its terminal allocation than under $J = 1$.

The theoretical results above build on and broaden the closed-economy network literature. Our model additionally allows us to address a distinct open-economy question: relative to treating the world as one closed economy, how does allowing $N > 1$ countries—each with its own nominal price level and monetary-policy rule, and linked through exchange rates—affect the persistence of real marginal-cost deviations?

Remark 4. In an open economy ($N > 1$), the threshold for systematic persistence of marginal cost deviations (i.e., $\mathbf{\Psi}^{\text{NKOE}} \neq \mathbf{0}$) remains $J > 1$.

Persistence arises from the mismatch between aggregate-demand dimension (N) and the dimension of rigidities generating lagged endogenous variables (NJ). Persistence therefore requires sectoral nominal rigidities; purely national rigidities, such as sticky wages alone, are insufficient. Although the open economy adds the exchange rate as a choice variable, country-level aggregate demand still cannot span all sectoral distortions, leaving the one-sector threshold unchanged.

The open economy instead changes the propagation matrix, not the threshold. In $\mathcal{K}(0, \mathbf{\Psi}^{\text{NKOE}})$, policy enters through the closed-economy labor-supply channel, $\boldsymbol{\alpha}\mathbf{\Phi}\mathbf{\Gamma}\mathbf{\Lambda}$, and the policy-heterogeneity channel, $\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z}\mathbf{\Phi}\mathbf{\Gamma}\mathbf{\Lambda}$. The latter operates only if $J > 1$, so inherited sectoral cost states exist; sectors use foreign intermediates, $\boldsymbol{L}^P_{\mathcal{E}} \neq \mathbf{0}$; and policy rules create a heterogeneous cross-country interest-rate response to those states, $\boldsymbol{Z}\mathbf{\Phi}\mathbf{\Gamma}\mathbf{\Lambda} \neq \mathbf{0}$. This last condition can reflect different response coefficients, target baskets, or both.

To isolate the policy-heterogeneity channel, impose identical target baskets and let $\delta = \phi^H_\pi - \phi^F_\pi$ denote the Home-Foreign difference in policy responsiveness. Appendix D.8 shows that perturbing δ around zero can change the eigenvalue governing the slowest decay relative to the corresponding closed-economy block. The channel is UIP: different policy responses generate interest-rate differentials that move exchange rates and hence imported-input costs. After a Home tariff raises Home producer-price inflation, a more aggressive Home response appreciates the Home currency and compresses imported-input costs on impact, muting inflation today. Under Appendix D.8 conditions, if the inherited pressure remains Home-inflationary and exchange-rate-sensitive exposure is concentrated in Home sectors using Foreign intermediates, the expected unwinding of this appreciation raises imported-input costs tomorrow, increasing persistence relative to the closed-economy benchmark.

5 Quantitative Analysis

This section quantifies the macroeconomic effects of tariffs and the role of production networks in their transmission. Following the 2025–2026 tariff chronology, we first study reversed tariff threats: agents observe the Liberation Day tariff path and anticipate retaliation, but tariffs are not implemented. This isolates expectations: even one-period announcements reversed before implementation generate distortions that dissipate gradually through global networks. We then input U.S. tariff increases as of March 2026 and find stagflationary effects for the United States. Model and policy counterfactuals follow; additional pre-March-2026 exercises appear in the Appendix.

5.1 Data

We use the OECD Inter-Country Input-Output (ICIO) tables (Yamano et al., 2023) for 2022, aggregated to five countries (United States, Euro Area, China, Canada, Mexico) plus a rest-of-the-world block, and eight broad sectors: agriculture, energy, mining, food, basic manufacturing, advanced manufacturing, residential services, and other services, matched to the sectoral rigidity estimates of Nakamura and Steinsson (2008). As shown in Appendix F, services account for over 70% of U.S. GDP with most output consumed domestically, yet nearly one third of advanced-manufacturing inputs are imported, underscoring the importance of imported intermediates.

The empirical literature finds that tariff pass-through to consumer prices substantially exceeds exchange-rate pass-through, though estimates vary (e.g., Amiti et al., 2019; Fajgelbaum et al., 2020; Fajgelbaum and Khandelwal, 2022; Cavallo et al., 2021; Flaaen et al., 2019, 2025). This highlights the role of importers and the retail sector. Standard input-output tables net out retail and wholesale margins, attributing flows to originating sectors.[27] For tariff incidence and realistic passthrough, however, it is important to include importers and distributors explicitly. We therefore introduce a domestic importing sector that intermediates all imports, pays tariffs at the border, and faces sticky domestic prices, restoring the distribution margin absent from I-O tables.[28] We also introduce destination-specific pricing, indexing sector i by destination m so prices are sticky at the producer-importer pair level, and calibrate ϑ_{ni} to dollar-invoicing shares, setting it to zero for domestically sold goods.

Figure 6 shows the evolution of U.S. tariff rates from January 2025 to March 2026. We obtain country–sector tariffs from the WTO–IMF Tariff Tracker[29] at the HS 6-digit level and aggregate to ICIO sectors using 2024 bilateral import weights. Figure 6a shows that U.S. tariff rates reached 22.7% on May 3, 2025; as of March 2026, the effective rate implied by implemented measures is 10.8%. Figure 6b illustrates country-sector heterogeneity: rates reached as high as 160% on Chinese basic and advanced manufacturing goods.

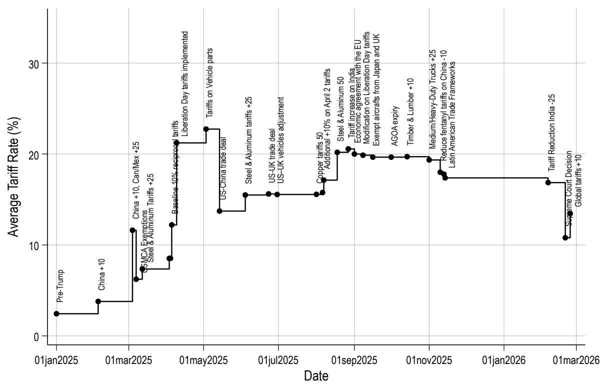
[27] See the U.S. Bureau of Economic Analysis's Concepts and Methods of the U.S. Input-Output Accounts for margin adjustments (Horowitz and Planting, 2009).

[28] Each original sector has a paired importing sector: in a given country, domestic steel producers and steel importers are separate sectors sharing the same stickiness parameter.

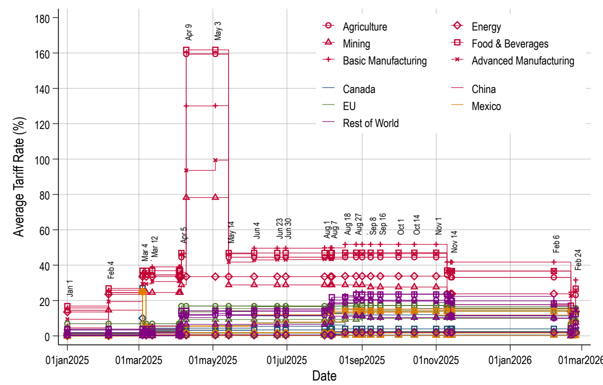
[29] `https://ttd.wto.org/en/analysis/tariff-actions`, last accessed March 31, 2026.

Figure 6. U.S. Effective Tariff Rates Since January 1, 2025

(a) Effective Tariff Rate (%)



(b) Country-Sector Level Tariffs (%)



Note: Panel (a) reports effective U.S. tariff rates weighted by 2024 trade volumes, and panel (b) reports country–sector-level effective U.S. tariff rates constructed from the WTO–IMF Tariff Tracker. Rates are expressed in percent.

5.2 Calibration

Calibration parameters are summarized in Table 2. Sector-specific Calvo parameters follow Nakamura and Steinsson (2008), adjusted to quarterly frequency.[30] Following Atalay (2017), the elasticity between labor and the intermediate bundle is $\theta^X = 0.6$. For the elasticity across intermediate inputs, we follow Boehm et al. (2019) and Baqaee and Farhi (2024), setting $\theta^X_h = 0.2$. On the consumption side, we assume Cobb–Douglas preferences across sectors ($\theta^C_h = 1$), fixing sectoral expenditure shares. Boehm et al. (2023) estimate short-run trade elasticities of about 0.76 and long-run elasticities around 2; we conduct sensitivity analyses with θ^C_{li} and θ^X_{li} in [0.6, 2]. The baseline Armington elasticity across country varieties within a sector is $\theta^C_{li} = \theta^X_{li} = 1.5$, so goods are substitutes at the lowest level of aggregation.

For the Taylor rule, we assume: $1+i_{n,t} = (1+i_{n,t-1})^{\rho^n_m} (\Pi_{n,t})^{\phi^n_\pi} \quad \forall n \in N$. For the United States, we set $\rho^{\text{US}}_m = 1$ and take the limit $\phi^{\text{US}}_\pi \to 0$ to capture the Federal Reserve *looking through* tariff-related increases in CPI. For all other countries we set $\rho^n_m = 0.95$ following Clarida et al. (2000), with $\phi^{EA}_\pi = 1$ for the Euro Area, $\phi^{\text{MX}}_\pi = 0.3$ for Mexico, and $\phi^n_\pi = 0.2$ for the remaining blocks. These coefficients are calibrated using $\phi^n_\pi = (1-\rho^n_m)/\overline{\pi}^C_n$, where $\overline{\pi}^C_n$ is the long-run average of quarterly CPI inflation in country n, computed from 2002Q2 to 2024Q4 data.

Finally, we make two adjustments for realism. First, the model incorporates a perma-

[30]We map Calvo updating frequencies to Rotemberg adjusting costs. Additionally, we conduct robustness exercises with the updating frequencies of Pasten et al. (2024). For less granular variants (e.g., $J = 1$), we use a weighted average of the diagonal elements of $\mathbf{\Lambda}$ rather than average updating frequencies.

nent real capital account wedge in each country so that 2018 data can serve as the steady state. The United States maintains persistent trade deficits alongside negative net foreign assets, which is inconsistent with standard steady-state algebra; the wedges reconcile this by allowing trade deficits and net debt to coexist in steady state. These wedges can be interpreted as persistent cross-country differences in patience or, equivalently, as an exogenous spread between interest paid on assets and liabilities. Second, while the analytical results assume that tariffs follow an AR(1) process, the quantitative exercises impose a tariff increase that remains in place for 100 periods, or 25 years. Over the horizon of interest, this closely approximates $\rho_\tau = 0.999$ but avoids the mechanical gradual decay implied by an AR(1). This specification better captures the empirically relevant path of tariffs, which are typically piecewise constant until policy changes. We solve the nonlinear model in Dynare under perfect foresight with MIT shocks.[31]

Table 2. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal elasticity of substitution (EoS)	2	Standard
η	Elasticity of Labor	1	Standard
ψ	Portfolio adjustment cost	0.001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82-1	Carvalho et al. (2021)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	0-1.29	Carvalho et al. (2021)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^X	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	1	di Giovanni et al. (2023)
θ_h^X	EoS among intermediate inputs	0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6-2	di Giovanni et al. (2023)
θ_{li}^X	Sector level input bundle EoS	0.6-2	di Giovanni et al. (2023)

As a validation exercise, we study the 2018 trade war episode, focusing on U.S. tariffs imposed on China and other trading partners between February and September 2018. In this period, the U.S. implemented tariffs ranging from 10% to 25% on China, a 10% tariff on aluminum, 25% on iron and steel, 30% on solar panels, and 20–50% tariffs on washers, with some country-level exceptions. In return, Canada, China, the European Union, Mexico, Russia, and Turkey retaliated with tariffs ranging from 5% to 20%. We obtained the detailed tariff data for this episode from Fajgelbaum et al. (2020) and trade values from the USITC website to compute weighted tariff rates.[32] Appendix E.2 reports the corresponding impulse responses and the announcement-based USD/CNY calculation. For this episode, the model

[31] For computational efficiency, the counterfactual IRFs are computed using a first-order approximation. Appendix E.1 shows that, for the range of shocks considered, the model's responses are consistent with the linear approximation.

[32] Exports: https://dataweb.usitc.gov/trade/search/TotExp/HTS; imports: https://dataweb.usitc.gov/trade/search/GenImp/HTS.

predicts a 1.1% nominal appreciation of the U.S. dollar against the Chinese yuan, a 0.05% decline in U.S. real GDP, and a 0.12% increase in the aggregate U.S. price level over 40 quarters. These predictions are broadly consistent with the available evidence on the 2018 tariffs: the dollar appreciated by about 1.1% around the main tariff announcements (for details see Appendix E.2), the aggregate price level rose by 0.1–0.2%, and aggregate real income fell by about 0.04% of GDP (Barbiero and Stein, 2025; Fajgelbaum et al., 2020).

5.3 Reversed Tariff Threats

Motivated by unimplemented tariffs observed in practice, we first simulate a reversed tariff threat: in period 1, the United States announces tariffs for period 2, and agents expect symmetric retaliation by other countries for the same period. When period 2 arrives, no tariffs are levied by either side, isolating the macroeconomic effects of credible announcements operating purely through expectations.

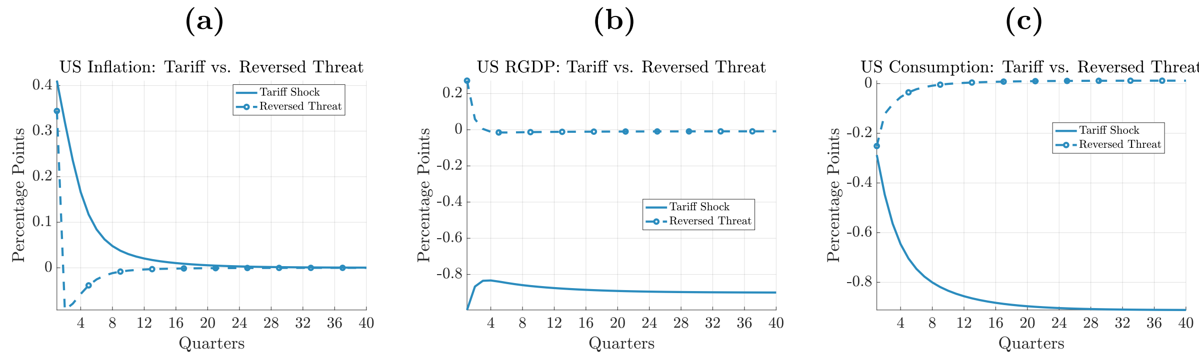
We construct two impulse responses under perfect foresight. The first simulates the actual tariff shock with retaliation, announced and implemented in period 1. The second simulates the same shock announced to take effect in period 2, only to be withdrawn before implementation. The reversed-threat impulse response is obtained by shifting the first response forward one period and subtracting it from the second, isolating the pure expectations effect. This approach is inspired by the *fake news* algorithm of Auclert et al. (2021), who use reversals of shocks as a computational device for sequence-space solutions; we apply it to study macroeconomic implications of trade policy reversals.

Figure 7 compares the reversed-threat scenario with actual tariffs under retaliation, using rates announced on Liberation Day. On impact, U.S. inflation rises by 0.34 percentage points, consumption falls by 0.25%, real GDP rises by 0.27%, and the NEER depreciates by 2.66%. Once the reversal is revealed in period 2, the NEER appreciates immediately given its forward-looking nature, while inflation, consumption, and output take 4-12 quarters to return to steady-state levels.

The USD depreciation in this case is driven by expected retaliation. Because the Liberation Day tariffs are large and the U.S. is smaller than the rest of the world, symmetric retaliation is sufficient to make the risk-sharing wedge positive, $\hat{w}_t > 0$, favoring the rest of the world, which, as shown in Section 3, lowers U.S. consumption through the intertemporal wealth effect. Once the shock is announced, $\hat{w}_t$ jumps immediately and the exchange rate adjusts even though tariffs take effect only next period. Agents lower consumption before tariffs are implemented, so on-impact effect of the reversed threat is comparable to the actual tariff. With a forward-looking NKPC, inflation is also similar across the two scenarios. Real

GDP rises modestly in the reversed-threat scenario, as depreciation improves U.S. export competitiveness before the supply-side effects of tariffs are realized. When tariffs are implemented, however, real GDP falls despite the depreciation induced by retaliation. The exercise shows that expectations alone generate sizable macroeconomic effects and that the resulting distortions dissipate only gradually, even when tariffs are ultimately not implemented.

Figure 7. Impact of Reversed Tariff Threats



NOTE: Figure 7 reports simulated responses to reversed tariff announcements. The dashed line denotes the reversed-threat experiment: tariffs are announced in period 1, expected to trigger retaliation in period 2, and canceled before implementation. The solid line denotes the implemented-tariff experiment, in which the same tariff shock is imposed from period 1 onward. Liberation Day announcement tariff rates are used in this exercise.

5.4 Implemented Tariffs

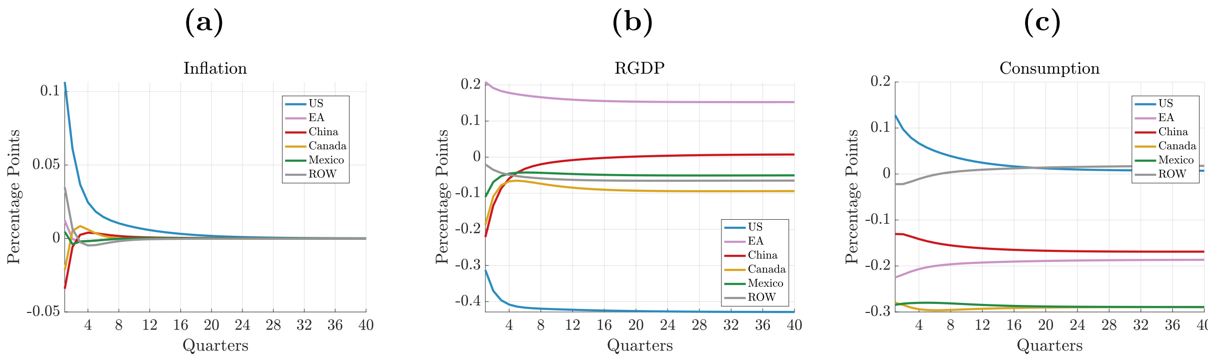
Next, we feed in tariffs implemented as of March 2026. As shown in Figure 8, implemented tariffs reduce U.S. real GDP by 0.31% on impact. Consumption rises by 0.13%, and inflation increases by 0.11 percentage points. The trade-weighted U.S. NEER appreciates by 4.86%.[33]

Effects are heterogeneous across trading partners. The Euro Area experiences an output expansion: real GDP rises by 0.21%, while consumption falls by 0.22%. Inflation rises slightly, by 0.01 percentage points. China contracts: real GDP falls by 0.22%, consumption by 0.13%, and inflation falls by 0.03 percentage points. Canada also contracts, with real GDP falling by 0.19%, consumption by 0.28%, and inflation declines by 0.02 percentage points. Mexico's real GDP falls by 0.11% and consumption by 0.28%. The rest of the world contracts only mildly, with real GDP falling by 0.02%, consumption by 0.02%, and inflation rises by 0.03 percentage points. Bilateral exchange-rate movements imply broad U.S. dollar

[33] The dollar depreciated after the Liberation Day announcements. Matching this depreciation requires a force that raises the risk-sharing wedge, $\hat{w}_t$. In Appendices E.3 and E.4, we model this force as an exogenous increase in the UIP premium, motivated by the contemporaneous rise in UIP deviations documented by Kalemli-Özcan et al. (2026). These exercises show how tariffs can coincide with dollar depreciation and can be microfounded by mechanisms such as tariff-induced uncertainty.

appreciation: the dollar appreciates by 5.16% against the Euro Area, 5.94% against China, 3.50% against Canada, 4.19% against Mexico, and 4.49% against the rest of the world.

Figure 8. Implemented Tariffs



NOTE: Figure 8 visualizes simulated responses to U.S. tariffs implemented as of March 2026, targeting China, Canada, Mexico, Europe, and the RoW. Impulse responses are computed with MIT shocks.

These impulse responses reflect the three transmission channels in Section 2.6. By shifting U.S. relative demand from foreign to domestic goods, tariffs act as a negative demand shock for the rest of the world. They also operate as a global supply shock transmitted via the production network. Together, these forces lower real GDP in all countries except the Euro Area, which benefits from trade diversion. Tariffs additionally generate a wealth transfer, summarized by the risk-sharing wedge $\hat{w}_t$. In contrast to the reversed tariff-threat scenario above, $\hat{w}_t$ favors the U.S. because implemented tariffs were not met by symmetric retaliation and the U.S. experiences terms-of-trade gains. As a result, the dollar appreciates and with the intertemporal wealth effect, U.S. consumption rises slightly in the short run. This effect mostly dissipates after 16 quarters, as importers pass tariffs through to final consumers. On impact, inflation rises in all countries except Canada and China. For the other countries, tariffs generate an inflationary impulse both through their supply-shock component and through dollar pricing, since the dollar appreciates. For Canada and China, the negative demand shock dominates these inflationary forces.

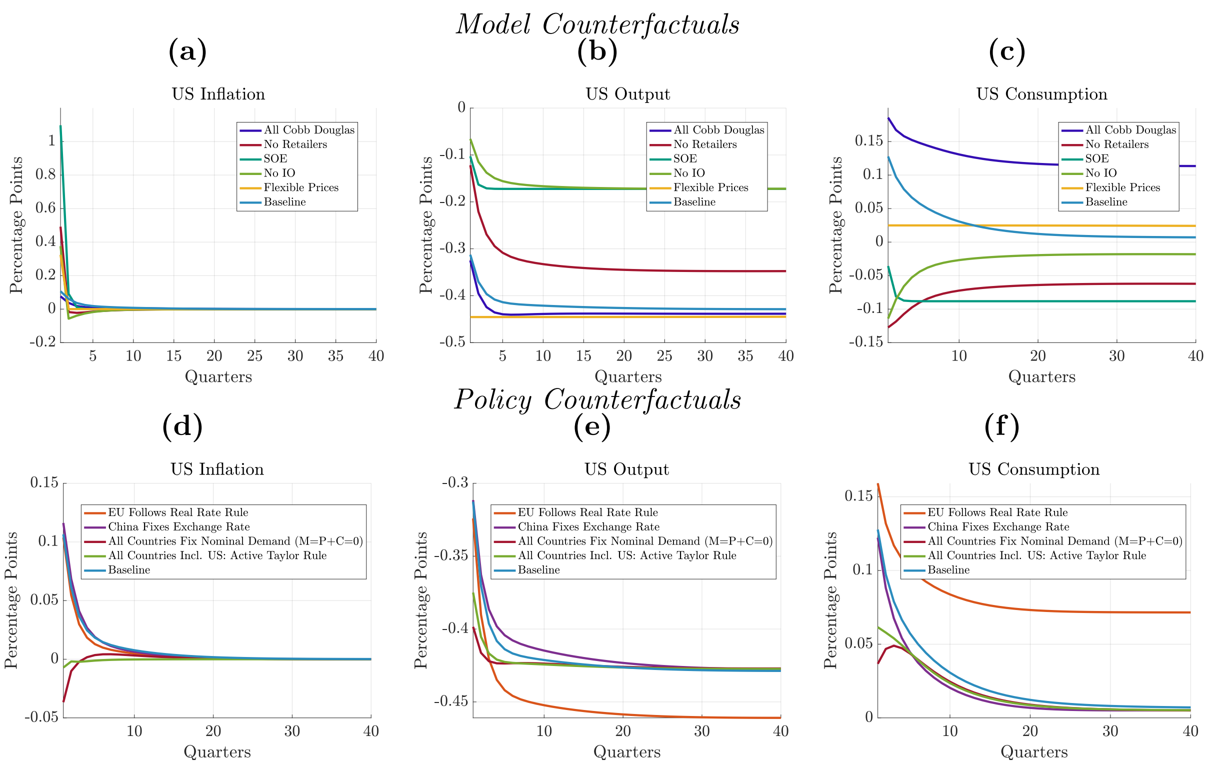
5.5 Model and Policy Counterfactuals

We next assess the sensitivity of the baseline results to alternative model features and policy specifications.[34] The top row of Figure 9 reports the model counterfactuals. "Baseline" is the scenario in Section 5.4 for the U.S., shown in Figure 8 . "All Cobb–Douglas" sets all

[34]Additional counterfactuals that explore the impact of DCP, more extreme complementarities and a widening UIP premium are presented in Appendix E.

elasticities to $\theta = 1$, so expenditure shares are fixed. "No Retailers" removes the domestic importing sector. "SOE" makes the rest of the world arbitrarily large, fixes foreign variables at steady state, sets $\mathbf{\Omega} = \mathbf{0}$, and removes DCP. "No IO" sets $\mathbf{\Omega} = \mathbf{0}$; retailers are also absent when $\mathbf{\Omega} = \mathbf{0}$. "Flexible Prices" replaces the NKPC with $MC_{ni,t} = P_{ni,t}$ in every sector. For inflation, the I-O matrix and domestic importers are quantitatively important. Removing domestic importers, as in "SOE," "No IO," and "No Retailers," implies full and more immediate passthrough to consumer prices. For output, removing the I-O matrix, as in "SOE" and "No IO," eliminates the supply-shock component of tariffs and raises output relative to the baseline. Thus, a model without I-O linkages would overstate inflation and understate the output loss. For consumption, reducing substitutability at the lowest level of the CES bundle, as in "All Cobb–Douglas," strengthens terms-of-trade gains. Scenarios such as "SOE" generate smaller terms-of-trade gains and lower consumption relative to the baseline. Notably, scenarios with more immediate passthrough, such as "No Retailers" and "No IO," generate a negative consumption response.

Figure 9. Model and Policy Counterfactuals



NOTE: Top row: model counterfactuals. Bottom row: policy counterfactuals. Panels plot U.S. inflation, real GDP, and consumption relative to the baseline tariffs implemented as of March 2026 from Section 5.4. Each line changes one model feature or one policy rule at a time, holding the remaining environment fixed.

Figure 9, bottom row, reports policy counterfactuals relative to the same baseline, varying the policy rule. "Fixed Nominal Demand" imposes $\hat{M}_{n,t} = \hat{P}^C_{n,t} + \hat{C}_{n,t} = 0$, fixing nominal spending at its steady-state level. "China Fixes Exchange Rate" stabilizes China's bilateral exchange rate. "EU Follows Real Rate Rule" has the Euro Area stabilize consumption with a real-rate rule. "All Countries Incl. US: Active Taylor Rule" makes the Fed target CPI with positive weight (setting $\rho_m^{\text{US}} = 0.82$ and $\phi_\pi^{\text{US}} = 1.29$ following Carvalho et al., 2021), as other countries do in the baseline, instead of looking through tariff-induced CPI movements. Inflation differs most under "Fixed Nominal Demand" and "All Countries Incl. US: Active Taylor Rule." In both cases, policy accommodates less of the tariff-induced increase in consumer prices, reducing inflation at the cost of lower output. U.S. output is also persistently lower under "EU Follows Real Rate Rule," where a weaker euro reduces Euro Area imports from the U.S. Consumption dynamics are broadly similar across specifications, except that the Euro Area real-rate rule generates a more persistent U.S. consumption increase through a stronger dollar.

6 Conclusion

This paper develops a multi-country, multi-sector New Keynesian open-economy framework to study the macroeconomic impact of trade distortions. Our central message is that the open-economy and network dimensions are not separable refinements of canonical NKOE models: they interact in ways that alter both the sign and persistence of tariffs' macro effects, and they do so through two novel analytical objects absent from canonical frameworks.

The first object is the risk-sharing wedge. Under incomplete markets, tariffs open a martingale wedge summarizing the wealth transfer across countries in global general equilibrium. Its sign, shaped jointly by the network structure (through terms of trade) and by shock persistence (through intertemporal substitution), determines whether the tariff-imposing country gains or loses. Neither complete-markets nor static exogenous-transfer closures are innocuous: the former shuts down the wealth-transfer channel by construction, the latter overstates it. A tractable incomplete-markets structure is thus an essential ingredient of tariff analysis.

The second object is the NKOE propagation matrix. With multiple sectors linked through input-output networks, tariffs generate real marginal cost distortions that become inherited states: the dimensionality of sectoral rigidities exceeds that of country-level aggregate demand, so monetary policy cannot span all sectoral gaps, even when the exchange rate is a separate national instrument. Transitory tariff shocks thus leave persistent distortions, which intermediate-input linkages amplify. This is a distinctly open-economy network result: exchange-rate adjustment does not eliminate propagation, and the one-sector NKOE benchmark, in which inflationary impulses are exhausted on impact, understates both the inflation–output trade-off and the stabilization burden on monetary policy.

Applied to the 2025–2026 episode, the framework delivers three quantitative lessons. Implemented U.S. tariffs are stagflationary, with sizable and heterogeneous international spillovers. Open-economy models without input-output linkages overstate the inflationary impact and understate the output decline, missing slow-moving propagation across sectors, countries, and time. And reversed tariff threats generate persistent macroeconomic effects through the expectations channel.

Our finding that sectoral distortions exceed what N-country monetary policy can offset invites normative analysis of international policy coordination in environments where the production network, rather than aggregate shocks, generates the residual instability. We leave this to future work.

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Supplemental Appendix

A Approximated Linear Equilibrium Conditions

The linearized equilibrium conditions used to arrive at the five-equation Global New Keynesian representation are:

$$E_t\hat{C}_{n,t+1} - \hat{C}_{n,t} = \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{n,t+1}\right) \tag{A.1}$$

$$\hat{i}_{n,t} - \hat{i}_{US,t} = E_t\hat{\mathcal{E}}_{n,t+1} - \hat{\mathcal{E}}_{n,t} + \hat{\psi} \tag{A.2}$$

$$\hat{\mathcal{E}}_{n,m,t} = \hat{\mathcal{E}}^{US}_{n,t} - \hat{\mathcal{E}}^{US}_{m,t} \tag{A.3}$$

$$\hat{\mathcal{E}}_{n,n,t} = 0 \tag{A.4}$$

$$\hat{W}_{n,t} - \hat{P}^C_{n,t} = \eta\hat{L}_{n,t} + \sigma\hat{C}_{n,t} \tag{A.5}$$

$$\hat{C}_{n,t} = \sum_{j\in J}\Gamma_{n,j}\hat{C}_{n,j,t} \tag{A.6}$$

$$\hat{C}_{n,j,t} = \sum_{m\in N}\Gamma_{n,j,mj}\hat{C}_{n,mj,t} \tag{A.7}$$

$$\hat{P}_{n,mj,t} = \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,m,t} + \hat{P}_{mj,t} \tag{A.8}$$

$$\hat{C}_{n,j,t} = \hat{C}_{n,t} - \theta^C_h\left(\hat{P}^C_{n,j,t} - \hat{P}^C_{n,t}\right) \tag{A.9}$$

$$\hat{C}_{n,mj,t} = \hat{C}_{n,j,t} - \theta^C_{l,j}\left(\hat{P}^C_{n,mj,t} - \hat{P}^C_{n,j,t}\right) \tag{A.10}$$

$$\hat{X}_{ni,j,t} = \sum_{m\in N}\Omega_{ni,j,mj}\hat{X}_{ni,mj,t} \tag{A.11}$$

$$\hat{X}_{ni,mj,t} = \hat{X}_{ni,j,t} - \theta^X_{l,j}\left(\hat{P}_{n,mj,t} - \hat{P}^X_{ni,j,t}\right) \tag{A.12}$$

$$\hat{X}_{ni,t} = \sum_{j\in J}\Omega_{ni,j}\hat{X}_{ni,j,t} \tag{A.13}$$

$$\hat{X}_{ni,j,t} = \hat{X}_{ni,t} - \theta^X_h\left(\hat{P}^X_{ni,j,t} - \hat{P}^X_{ni,t}\right) \tag{A.14}$$

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t} \tag{A.15}$$

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_{n,t} + (1-\alpha_{ni})\hat{P}^X_{ni,t} \tag{A.16}$$

$$\hat{X}_{ni,t} - \hat{L}_{ni,t} = \theta^X\hat{W}_{n,t} - \theta^X\hat{P}^X_{ni,t} \tag{A.17}$$

$$\pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta\mathbb{E}_t\pi_{ni,t+1} \tag{A.18}$$

$$\bar{B}^{US}\hat{B}^{US}_t = \sum_{m}^{N-1}\bar{B}^{US}_m\hat{B}^{US}_{m,t} \tag{A.19}$$

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{m\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{A.20}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{A.21}$$

$$\pi_{n,t} = \hat{P}^C_{n,t} - \hat{P}^C_{n,t-1} \tag{A.22}$$

$$\hat{i}_{n,t} = \phi_\pi \pi_{n,t} + \hat{M}_{n,t}. \tag{A.23}$$

The linearized external budget constraint is

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&\qquad + \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) \\
&= \sum_{i\in\mathcal{J}} \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}),
\end{aligned} \tag{A.24}$$

where bars denote steady-state values.

B Relating the Balance of Payments to Prices

Define gross dollar debt by $V^{US}_{n,t} \equiv (1+i^{US}_t)B^{US}_{n,t}$. Using $\bar{\mathcal{E}}_n = 1$, $1+\bar{i}^{US} = \beta^{-1}$, and $\overline{NX}_n = (1-\beta)\bar{V}^{US}_n$, the linearized balance-of-payments condition implies

$$\beta\hat{V}^{US}_{n,t} - \hat{V}^{US}_{n,t-1} = (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}^{US}_t. \tag{B.1}$$

Stacking across countries and replacing one debt equation with U.S.-bond market clearing yields $\beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t$. To arrive at this expression, we express net exports in terms of $(\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{\tau}}_t)$. Their baseline decomposition is

$$\underbrace{\overline{\boldsymbol{NX}}_n}_{N\times N}\underbrace{\widehat{\boldsymbol{NX}}_{n,t}}_{N\times 1} = \underbrace{\overline{\boldsymbol{Y}}^{N,NJ}}_{N\times NJ}\left[(\hat{\boldsymbol{P}}^P_t + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^{C,\tau}_t + \hat{\boldsymbol{C}}_t\right) - \boldsymbol{\Omega}\left(\hat{\boldsymbol{P}}^{X,\tau}_{ni,t} + \hat{\boldsymbol{X}}_{ni,t}\right)\right]. \tag{B.2}$$

When an object that is inherently lower-dimensional is mapped to a higher dimension, we replicate rows or columns to reach the final object that has the right dimensions. For example, steady-state outputs are $NJ\times 1$, but the new object is $N\times NJ$: we transpose the vector and repeat it N times to arrive at $\overline{\boldsymbol{Y}}^{N,NJ}$.

B.1 Market-Clearing Condition

Linearized goods-market clearing is

$$\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\hat{\boldsymbol{C}}_t^{nmj} + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\hat{\boldsymbol{X}}_t^{nimj},$$

where $\boldsymbol{\Gamma}^M$ is $NJ \times N^2J$ and $\boldsymbol{\Omega}^M$ is an $NJ \times N^2J^2$ matrix, obtained by shifting $\boldsymbol{\Gamma}$ and $\boldsymbol{\Omega}$ so that multiplication loads onto the correct entries. Relative-demand conditions imply[35]

$$\begin{aligned}\hat{\boldsymbol{C}}_t^{nmj} &= \boldsymbol{S}_1\hat{\boldsymbol{C}}_t + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmj}\right),\\ \hat{\boldsymbol{X}}_t^{nimj} &= \boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^X\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^P - \hat{\boldsymbol{P}}_{ni,mj,t}^P\right),\end{aligned}$$

where the $\boldsymbol{S}$ matrices are selector matrices that pick the appropriate producer price, bilateral exchange rate, and tariff entries, and

$$\hat{\boldsymbol{P}}_t^{nmj} = \boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{S}_5\hat{\boldsymbol{\tau}}_t, \qquad \hat{\boldsymbol{P}}_{ni,mj,t}^P = \boldsymbol{S}_6\hat{\boldsymbol{P}}_t^{nmj}.$$

Using the selector identities

$$\overline{\boldsymbol{C}}_n\,\boldsymbol{\Gamma}^M\boldsymbol{S}_1 = \boldsymbol{\Gamma}^\top\,\overline{\boldsymbol{C}}, \qquad \boldsymbol{\Omega}^M\boldsymbol{S}_2 = \boldsymbol{\Omega}^\top, \qquad \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3 = \boldsymbol{\Omega}^\top,$$

and

$$\boldsymbol{L}_{\mathcal{E}}^P \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4, \qquad \boldsymbol{L}_\tau^P \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5,$$

we obtain

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = {} & \overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t}\\ & + \theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\left(\boldsymbol{T}_P^C\hat{\boldsymbol{P}}_t^P + \boldsymbol{T}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{T}_\tau^C\hat{\boldsymbol{\tau}}_t\right)\\ & + \theta^X\left(\left[\boldsymbol{\Omega}^\top\boldsymbol{\Omega} - \boldsymbol{\Omega}^\top\right]\hat{\boldsymbol{P}}_t^P + \left[\boldsymbol{\Omega}^\top\boldsymbol{L}_{\mathcal{E}}^P - \boldsymbol{L}_{\mathcal{E}}^P\right]\hat{\boldsymbol{\mathcal{E}}}_t + \left[\boldsymbol{\Omega}^\top\boldsymbol{L}_\tau^P - \boldsymbol{L}_\tau^P\right]\hat{\boldsymbol{\tau}}_t\right),\end{aligned} \tag{B.3}$$

where

$$\boldsymbol{T}_P^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3, \quad \boldsymbol{T}_{\mathcal{E}}^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4,$$

$$\boldsymbol{T}_\tau^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_\tau^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5.$$

[35] We abstract from higher- and lower-level θ varieties; the results become more complicated with additional elasticities but the intuition is unchanged. Mathematically, we take $\theta_h^X = \theta_{l,j}^X = \theta^X \quad \forall j \in \mathcal{J}$ and $\theta_h^C = \theta_{l,j}^C = \theta^C \quad \forall j \in \mathcal{J}$.

B.2 Substituting out $\hat{X}_{ni,t}$

From the production bundle and production function,

$$\hat{X}_{ni,t} = \hat{Y}_{ni,t} + \theta^X \alpha_{ni}(\hat{W}_t - \hat{P}^X_{ni,t}),$$

so in vector form

$$\hat{\boldsymbol{X}}_{ni,t} = \hat{\boldsymbol{Y}}_{ni,t} + \theta^X(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}^P_{ni,t}).$$

Here, $\boldsymbol{\alpha}$ is $NJ \times N$ matrix with entries $\boldsymbol{\alpha}_{nj,n} = \alpha_{nj}$ and all other entries 0. $\tilde{\alpha}$, on the other hand, is the diagonal matrix whose diagonal entries are α_{nj}. Substituting this into (B.3), and then using

$$\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t, \qquad \hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_\tau\hat{\boldsymbol{\tau}}_t,$$

$$\hat{\boldsymbol{P}}^X_{ni,t} = \boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_\tau\hat{\boldsymbol{\tau}}_t, \qquad \boldsymbol{\Psi}_T \equiv (\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1},$$

gives

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = \boldsymbol{\Psi}_T\Big[&\Big(\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\sigma\Big)\hat{\boldsymbol{C}}_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_P + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_P\Big)\hat{\boldsymbol{P}}^P_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_{\mathcal{E}} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^X_{\mathcal{E}}\Big)\hat{\boldsymbol{\mathcal{E}}}_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_\tau + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_\tau\Big)\hat{\boldsymbol{\tau}}_t\Big],\end{aligned} \tag{B.4}$$

with

$$\boldsymbol{T}^P_P \equiv \boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega},$$

$$\boldsymbol{T}^X_{\mathcal{E}} \equiv \boldsymbol{L}^P_{\mathcal{E}} - (\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}^P_{\mathcal{E}} + \boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\mathcal{E}}, \qquad \boldsymbol{T}^P_\tau \equiv \boldsymbol{L}^P_\tau - (\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}^P_\tau + \boldsymbol{\alpha}\boldsymbol{L}^C_\tau - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_\tau.$$

Substituting (B.4) into (B.2) gives

$$\overline{\boldsymbol{NX}}_n\widehat{\boldsymbol{NX}}_{n,t} = \Xi_P\,\hat{\boldsymbol{P}}^P_t + \Xi_{\mathcal{E}}\,\hat{\boldsymbol{\mathcal{E}}}_t + \Xi_\tau\,\hat{\boldsymbol{\tau}}_t + \Xi_C\,\hat{\boldsymbol{C}}_t, \tag{B.5}$$

where $\boldsymbol{\Psi}_\Delta \equiv (\boldsymbol{I} - \boldsymbol{\Omega})(\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1} = (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Psi}^\top$ and

$$\Xi_P = \overline{\boldsymbol{Y}}^{NNJ}\Big\{\boldsymbol{\Psi}_\Delta\Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_P + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_P\Big) - \theta^X\boldsymbol{\Omega}\,(\boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}) + \Big[\boldsymbol{I} - \boldsymbol{\alpha}\boldsymbol{\Gamma} - \boldsymbol{\Omega}^2\Big]\Big\},$$

$$\boldsymbol{\Xi}_{\mathcal{E}} = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\boldsymbol{\Psi}_{\Delta}\Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_{\mathcal{E}} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^X_{\mathcal{E}}\Big) - \theta^X\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\mathcal{E}}\right) - \Big[\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{\Omega}\boldsymbol{L}^P_{\mathcal{E}}\Big]\Bigg\},$$

$$\boldsymbol{\Xi}_{\tau} = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\boldsymbol{\Psi}_{\Delta}\Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_{\tau} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_{\tau}\Big) - \theta^X\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}^C_{\tau} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\tau}\right)\Bigg\},$$

$$\boldsymbol{\Xi}_{C} = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\boldsymbol{\Psi}_{\Delta}\Big(\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}} + \theta^X\sigma\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\Big) - \Big[\boldsymbol{I} + \theta^X\sigma\boldsymbol{\Omega}\Big]\boldsymbol{\alpha}\Bigg\}. \tag{B.6}$$

Stacking the country-level equations together with U.S.-bond market clearing gives the fifth equation in the five-equation representation. The $\boldsymbol{\Xi}_1$-$\boldsymbol{\Xi}_5$ coefficients in the fifth equation of the Global New Keynesian Representation, thus capture, how the balance of payments react to goods-specific terms of trade and to the balance sheet effect since $\overline{NX}_n = (1-\beta)\bar{V}_n^{US}$. $\boldsymbol{\Xi}_3$ and $\boldsymbol{\Xi}_6$ reflect how lagged prices load onto the balance of payments equation via $\beta\hat{i}_t^{US}$.

For illustration, in the $N = 2$ and $J = 1$ flexible-price version of our model, the fifth equation of the model—balance of payments, can be expressed in the following way with *symmetric* parameters:[36]

$$\begin{aligned}\beta\hat{V}_t = \hat{V}_{t-1} &+ \underbrace{\frac{\mathcal{A}}{1+\Omega}}_{\Xi_2}\left(\hat{C}_{H,t} - \hat{C}_{F,t}\right) + \underbrace{\frac{\mathcal{A}(\theta-1)}{1-\Omega}}_{\Xi_3}\left(\hat{p}_{H,t} - \hat{p}_{F,t}\right) \\ &+ \underbrace{\frac{\mathcal{A}(1+\Omega-2\theta)}{(1-\Omega)(1+\Omega)}}_{\Xi_4}\hat{\mathcal{E}}_t + \underbrace{\left(-\frac{\mathcal{A}\theta}{(1-\Omega)(1+\Omega)}\right)}_{\Xi_5}\hat{\tau}_t\end{aligned} \tag{B.7}$$

where $\mathcal{A} \equiv \gamma + (1-\gamma)\Omega > 0$.

This expression demonstrates that elasticity of substitution, θ, determines the sign of Ξ_4, which in turn determines whether the Marshall-Lerner condition holds and a depreciation improves the trade balance. For intuition let us consider $\theta \to 0$: when goods are impossible to substitute, an exchange rate depreciation means imports (exports) become more expensive in domestic currency (less valuable in foreign currency), while export revenues in domestic currency (the import bill in foreign currency) remain the same. Via this mechanism depreciation can worsen the trade balance and increase the net debt of the home country, which corresponds to the case when $\Xi_4 > 0$ as $\theta \to 0$. A rise in the domestic-currency value of the import bill, with little offsetting quantity adjustment (decline in import quantity), can worsen the trade balance. Put differently, if a tariff mechanically pushes the external

[36]Under symmetry, we have symmetric coefficients inside the $\boldsymbol{\Xi}$ vectors. To demonstrate this with an example consider $\boldsymbol{\Xi}_2$, which is an $N \times 1$ vector. Under symmetry, we have $\boldsymbol{\Xi}_2 = [\Xi_{21}\Xi_{22}]'$, where $\Xi_{21} = -\Xi_{22} = \Xi_2$.

balance toward surplus, equilibrium may require a depreciation to offset that surplus.

C Appendix for Section 3

C.1 Symmetry Case

Under incomplete markets,

$$\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t.$$

Combining the Euler equations with UIP gives the Backus–Smith martingale condition

$$E_t \hat{w}_{t+1} = \hat{w}_t.$$

Under symmetry, $y_H = y_F = (1-\Omega^2)^{-1}$ and

$$\begin{aligned}
\mathcal{A} &\equiv \gamma + (1-\gamma)\Omega, \\
\Xi_2 &= \frac{\mathcal{A}}{1+\Omega}, \\
\Xi_3 &= \frac{1}{1-\Omega}\left[\mathcal{A}(\theta^C - 1) + \frac{2\Omega}{1+\Omega}(\theta^X - \theta^C)\right], \\
\Xi_4 &= -(1-\gamma) - \frac{\theta^X - 1}{1-\Omega} - \frac{2\gamma\theta^C - \theta^X}{1+\Omega}, \\
\Xi_5 &= -\frac{L_\tau^C\,\gamma\,\theta^C}{1+\Omega} - \frac{L_\tau^P\,\Omega\,\theta^X}{1-\Omega^2}.
\end{aligned}$$

Solving the full open-economy block by MUC yields

$$\hat{w}_t = \frac{\beta - 1}{(\beta\rho_\tau - 1)\,\mathcal{D}}\mathcal{N}\,\varepsilon_t^\tau + \frac{(\beta-1)\rho_\tau}{(\beta\rho_\tau - 1)\,\mathcal{D}}\mathcal{N}\,\tau_{t-1} + \frac{(\beta-1)(1+\Omega)}{\mathcal{D}}\hat{V}_{H,t-1}, \tag{C.8}$$

where

$$\mathcal{N} = L_\tau^C\gamma(1+\Omega)(\Xi_2+\Xi_3+\Xi_4) - L_\tau^P\Omega\Big[2\gamma(\Xi_2+\Xi_3+\Xi_4) - (\Xi_2+\Xi_4)\Big] - (1+\Omega)\Xi_5,$$

$$\mathcal{D} = 2\Omega\gamma(\Xi_2+\Xi_3+\Xi_4) - 2\Omega\Xi_2 - \Omega\Xi_4 - 2\gamma(\Xi_2+\Xi_3+\Xi_4) + \Xi_4.$$

Hence, for every $k \geq 0$,

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = \frac{1-\beta}{1-\beta\rho_\tau}\frac{\mathcal{N}}{\mathcal{D}}. \tag{C.9}$$

Therefore

$$\Xi_2 + \Xi_3 + \Xi_4 = -\frac{\mathcal{A}(\theta^C - 1)}{1+\Omega},$$

and the wedge simplifies to $\forall k \geq 0$:

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = \frac{1-\beta}{1-\beta\rho_\tau} \times \frac{(1+\Omega)\, L_\tau^C\, \gamma\big(\theta^C - \mathcal{A}(\theta^C - 1)\big) + L_\tau^P\, \Omega\big[\frac{2\mathcal{A}}{1-\Omega} + \theta^X - 2\gamma\mathcal{A} - 2\gamma(1-\gamma)(1-\Omega)\theta^C\big]}{\mathcal{A}(1-2\gamma)(1-\Omega) - 2\Omega\,\theta^X - 2\gamma(1-\gamma)(1-\Omega)^2\theta^C}$$

Setting $L_\tau^P = L_\tau^C = 1$ and $\theta^X = \theta^C = \theta$ gives (14) in the text.

Conditioning on a given constant wedge, the reduced static block implies

$$\hat{C}_{H,t} = -\frac{1}{1+\Omega}\Bigg\{\Big[L_\tau^C\, \gamma(1+\Omega) + L_\tau^P\, \frac{\Omega\big(1-\gamma(1-\Omega)\big)}{1-\Omega}\Big]\hat{\tau}_t + \big(\Omega(1-\gamma)+\gamma\big)\hat{w}_t\Bigg\}, \tag{C.10}$$

$$\hat{C}_{F,t} = \frac{1}{1+\Omega}\Bigg\{ - L_\tau^P\, \frac{\Omega\big(\Omega(1-\gamma)+\gamma\big)}{1-\Omega}\,\hat{\tau}_t + \big(\Omega(1-\gamma)+\gamma\big)\hat{w}_t\Bigg\}, \tag{C.11}$$

$$\hat{\mathcal{E}}_t = \frac{1}{1+\Omega}\Bigg\{ - \Big[L_\tau^C\, \gamma(1+\Omega) + L_\tau^P\, \Omega(1-2\gamma)\Big]\hat{\tau}_t + (1-\Omega)(1-2\gamma)\hat{w}_t\Bigg\}. \tag{C.12}$$

Setting $L_\tau^P = L_\tau^C = 1$ gives the expressions reported in Section 3.

C.2 With a Second Foreign Good

We add a second foreign good whose output is fixed and which enters production but not final consumption.

C.2.1 Scalar MUC

Notation. Here $C_1, \ldots, C_{18}$ are the scalar MUC coefficients from conjecturing linear rules in the two states $(\hat{V}_{H,t-1}, \hat{\tau}_t)$. Coefficients are numbered in pairs: the first coefficient multiplies $\hat{V}_{H,t-1}$, and the second multiplies $\hat{\tau}_t$. For the wedge calculation only the following decision rules matter:[37]

$$\hat{C}_{H,t} = C_1\hat{V}_{H,t-1} + C_2\hat{\tau}_t, \qquad \hat{C}_{F,t} = C_3\hat{V}_{H,t-1} + C_4\hat{\tau}_t,$$

$$\hat{\mathcal{E}}_t = C_{11}\hat{V}_{H,t-1} + C_{12}\hat{\tau}_t, \qquad \hat{V}_{H,t} = C_{17}\hat{V}_{H,t-1} + C_{18}\hat{\tau}_t.$$

Since

$$\hat{w}_t = \hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}),$$

[37] The missing intermediate coefficients are the analogous price and interest-rate coefficients, solved jointly in the scalar MUC system but not needed explicitly for the wedge expression.

the martingale property implies

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^{\tau}} = C_{12} - (C_2 - C_4) = -\frac{\big[(C_1 - C_3) - C_{11}\big]\, C_{18}}{1-\rho_\tau}, \qquad \forall\ k \geq 0.$$

Solving for the coefficients yields

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^{\tau}} = -\frac{1-\beta}{1-\beta\rho_\tau}\, \frac{(\Omega + \Omega_{H,T})\Big[\Omega\big(\Omega_{H,T}\, y(2-\theta^X) - \beta\big) + \alpha_F(A_2 - \beta)\Big] - A_1\, A_3}{A_1\big(1 - y\mathcal{D}\,\alpha_H\big) - \alpha_H\Big[\Omega\,\Omega_{H,T}\, y(2-\theta^X) + \alpha_F A_2\Big]},$$

where

$$\mathcal{D} \equiv \frac{1}{1+\Omega}, \qquad A_1 \equiv \Omega\Omega_{H,T} + \Omega\alpha_F + \Omega\alpha_H + \Omega_{H,T}\alpha_F + \alpha_F\alpha_H > 0,$$

$$A_2 \equiv y\Big(1 + \Omega_{H,T} + \mathcal{D}\big[-\alpha_H - \theta^X(2\Omega + \Omega_{H,T})\big]\Big), \quad A_3 \equiv -y^2 \mathcal{D}\,\theta^X L_\tau^P\big(\Omega\alpha_F + \Omega_{H,T}\big) < 0.$$

D Appendix for Section 4

Notation bridge to Section 4. This appendix solves the sticky-price block first in terms of the MUC coefficients $\{\boldsymbol{C}_1, \ldots, \boldsymbol{C}_{12}\}$ as it is easier to track numbered subscripts in algebra work. For compactness within this appendix, we redefine $\boldsymbol{\mu}_2$ to denote the full coefficient on $\Delta\hat{\boldsymbol{C}}_t$ in the real-marginal-cost equation: $\boldsymbol{\mu}_2 \equiv \sigma\Big(\boldsymbol{\alpha} + (\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)m\boldsymbol{Z}\Big)$, $\quad m \equiv \big(1 - \boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\big)^{-1}$. The coefficient on $\Delta\hat{\tau}_t$ is denoted by $\boldsymbol{\mu}_3 \equiv \boldsymbol{\alpha}\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^P + (\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)m\boldsymbol{Z}\boldsymbol{L}_\tau^C = \sigma^{-1}\boldsymbol{\mu}_2\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^P$. We label

$$\boldsymbol{C}_9 \equiv \boldsymbol{\Psi}^{\text{NKOE}}, \qquad \boldsymbol{C}_5 \equiv \boldsymbol{p}_\mu, \qquad \boldsymbol{C}_1 \equiv \boldsymbol{c}_\mu, \qquad \boldsymbol{C}_6 \equiv \boldsymbol{p}_w, \qquad \boldsymbol{C}_2 \equiv \boldsymbol{c}_w,$$
$$\boldsymbol{C}_7 \equiv \boldsymbol{p}_\tau, \qquad \boldsymbol{C}_3 \equiv \boldsymbol{c}_\tau, \qquad \boldsymbol{C}_{10} \equiv \boldsymbol{\mu}_w, \qquad \boldsymbol{C}_{11} \equiv \boldsymbol{\mu}_\tau,$$

with

$$\boldsymbol{C}_4 \equiv \boldsymbol{c}_{\tau,-1}, \qquad \boldsymbol{C}_8 \equiv \boldsymbol{p}_{\tau,-1}, \qquad \boldsymbol{C}_{12} \equiv \boldsymbol{\mu}_{\tau,-1}.$$

The matrix $\mathcal{K}(\rho, \boldsymbol{C}_9)$ is therefore the same object as $\mathcal{K}(\rho, \boldsymbol{\Psi}^{\text{NKOE}})$ in Section 4. In the decomposition formulas below we abbreviate it as $\boldsymbol{H}_\tau \equiv \mathcal{K}(\rho, \boldsymbol{\Psi}^{\text{NKOE}})$. With this bridge, these formulas map directly into Propositions 2, 3, and 4.

D.1 Method of Undetermined Coefficients- New Keynesian Block

Postulate

$$\begin{aligned}
\Delta\hat{\boldsymbol{C}}_t &= \boldsymbol{C}_1\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_2\Delta\hat{w}_t + \boldsymbol{C}_3\hat{\tau}_t + \boldsymbol{C}_4\hat{\tau}_{t-1},\\
\boldsymbol{\pi}_t^P &= \boldsymbol{C}_5\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_6\Delta\hat{w}_t + \boldsymbol{C}_7\hat{\tau}_t + \boldsymbol{C}_8\hat{\tau}_{t-1},\\
\boldsymbol{\mu}_t &= \boldsymbol{C}_9\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_{10}\Delta\hat{w}_t + \boldsymbol{C}_{11}\hat{\tau}_t + \boldsymbol{C}_{12}\hat{\tau}_{t-1}.
\end{aligned}$$

Using $E_t\hat{\tau}_{t+1} = \rho\hat{\tau}_t$ and $E_t\Delta\hat{w}_{t+1} = 0$, coefficient matching delivers the nonlinear system below.

D.2 System of 12 Equations and 12 Unknowns

Let $\boldsymbol{A}_2 \equiv (\boldsymbol{I} - \boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{Z})\boldsymbol{\Phi\Gamma}$. The coefficient restrictions are

$$\begin{aligned}
0 &= \sigma\boldsymbol{C}_1\boldsymbol{C}_9 - \boldsymbol{A}_2\boldsymbol{C}_5 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_9, \qquad & 0 &= \boldsymbol{C}_5 - \boldsymbol{\Lambda}\boldsymbol{C}_9 - \beta\boldsymbol{C}_5\boldsymbol{C}_9,\\
0 &= \boldsymbol{C}_9 - \boldsymbol{I} - \boldsymbol{\mu}_1\boldsymbol{C}_5 - \boldsymbol{\mu}_2\boldsymbol{C}_1, & 0 &= \sigma\boldsymbol{C}_1\boldsymbol{C}_{10} - \boldsymbol{A}_2\boldsymbol{C}_6 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_{10},\\
0 &= \boldsymbol{C}_6 - \boldsymbol{\Lambda}\boldsymbol{C}_{10} - \beta\boldsymbol{C}_5\boldsymbol{C}_{10}, & 0 &= \boldsymbol{C}_{10} - \boldsymbol{\mu}_1\boldsymbol{C}_6 - \boldsymbol{\mu}_2\boldsymbol{C}_2 - \boldsymbol{\mu}_4,\\
0 &= \sigma(\boldsymbol{C}_1\boldsymbol{C}_{11} + \rho\boldsymbol{C}_3 + \boldsymbol{C}_4) - \boldsymbol{A}_2\boldsymbol{C}_7 + \boldsymbol{\Gamma}(\boldsymbol{C}_5\boldsymbol{C}_{11} + & & \rho\,\boldsymbol{C}_7 + \boldsymbol{C}_8) - (1-\rho)\boldsymbol{L}_\tau^C,\\
0 &= \boldsymbol{C}_7 - \boldsymbol{\Lambda}\boldsymbol{C}_{11} - \beta(\boldsymbol{C}_5\boldsymbol{C}_{11} + \rho\boldsymbol{C}_7 + \boldsymbol{C}_8), & 0 &= \boldsymbol{C}_{11} - \boldsymbol{\mu}_1\boldsymbol{C}_7 - \boldsymbol{\mu}_2\boldsymbol{C}_3 - \boldsymbol{\mu}_3,\\
0 &= \sigma\boldsymbol{C}_1\boldsymbol{C}_{12} - \boldsymbol{A}_2\boldsymbol{C}_8 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_{12}, & 0 &= \boldsymbol{C}_8 - \boldsymbol{\Lambda}\boldsymbol{C}_{12} - \beta\boldsymbol{C}_5\boldsymbol{C}_{12},\\
0 &= \boldsymbol{C}_{12} - \boldsymbol{\mu}_1\boldsymbol{C}_8 - \boldsymbol{\mu}_2\boldsymbol{C}_4 + \boldsymbol{\mu}_3.
\end{aligned}$$

D.3 Defining Branches

For the NKOE propagation matrix it is enough to isolate the $\boldsymbol{\mu}_{t-1}$-block:

$$\sigma\boldsymbol{C}_1\boldsymbol{C}_9 = \boldsymbol{A}_2\boldsymbol{C}_5 - \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_9, \tag{D.13}$$

$$\boldsymbol{C}_5 = \boldsymbol{\Lambda}\boldsymbol{C}_9 + \beta\boldsymbol{C}_5\boldsymbol{C}_9, \tag{D.14}$$

$$\boldsymbol{C}_9 = \boldsymbol{I} + \boldsymbol{\mu}_1\boldsymbol{C}_5 + \boldsymbol{\mu}_2\boldsymbol{C}_1. \tag{D.15}$$

From (D.14),

$$\boldsymbol{C}_5 = \boldsymbol{\Lambda}\boldsymbol{C}_9(\boldsymbol{I} - \beta\boldsymbol{C}_9)^{-1}.$$

Substituting into (D.13)–(D.15), and using

$$\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 \sigma^{-1} \boldsymbol{\Gamma} = \boldsymbol{\Omega} - \boldsymbol{I}, \qquad \boldsymbol{\mu}_2 \sigma^{-1} \boldsymbol{A}_2 = (\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}) \boldsymbol{\Phi} \boldsymbol{\Gamma},$$

gives

$$\Big[(\boldsymbol{C}_9 - \boldsymbol{I})(\beta \boldsymbol{C}_9 - \boldsymbol{I}) - (\boldsymbol{I} - \boldsymbol{\Omega}) \boldsymbol{\Lambda} \boldsymbol{C}_9 + (\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}) \boldsymbol{\Phi} \boldsymbol{\Gamma} \boldsymbol{\Lambda} \Big] \boldsymbol{C}_9 = \boldsymbol{0}. \tag{D.16}$$

This is $\mathcal{K}(0, \boldsymbol{C}_9)\boldsymbol{C}_9 = \boldsymbol{0}$ in the notation of the main text.

D.4 Branch 1: $\boldsymbol{C}_9 = \boldsymbol{0}$

If $\boldsymbol{C}_9 = \boldsymbol{0}$, then $\boldsymbol{C}_5 = \boldsymbol{0}$ and (D.15) reduces to

$$\boldsymbol{\mu}_2 \boldsymbol{C}_1 = -\boldsymbol{I}_{NJ}. \tag{D.17}$$

Hence the zero-propagation candidate exists only if $\boldsymbol{\mu}_2$ admits a right inverse. Since $\boldsymbol{\mu}_2 \in \mathbb{R}^{NJ \times N}$, for $J > 1$

$$\operatorname{rank}(\boldsymbol{\mu}_2 \boldsymbol{C}_1) \le \operatorname{rank}(\boldsymbol{\mu}_2) \le N < NJ,$$

so (D.17) is impossible. Therefore,

$$\boxed{J > 1 \quad \Longrightarrow \quad \boldsymbol{\Psi}^{\text{NKOE}} \neq \boldsymbol{0}.}$$

When $J = 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{N \times N}$. Under the maintained regularity condition that $\boldsymbol{\mu}_2$ is nonsingular,

$$\boldsymbol{C}_1 = -\boldsymbol{\mu}_2^{-1}, \qquad \boldsymbol{C}_5 = \boldsymbol{0}, \qquad \boldsymbol{C}_9 = \boldsymbol{0}.$$

This is a stable solution, since all eigenvalues of $\boldsymbol{C}_9$ are zero. Because the linearized model is Blanchard–Kahn determinate, the stable equilibrium is unique. Hence

$$\boxed{J = 1 \quad \Longrightarrow \quad \boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}.}$$

There may still be other formal roots of $\mathcal{K}(0, \boldsymbol{X})\boldsymbol{X} = \boldsymbol{0}$, but under determinacy they are not admissible equilibrium solutions.

D.5 Solving the rest of the system:

For $J > 1$, equilibrium must lie on the propagating branch $\boldsymbol{C}_9 = \boldsymbol{\Psi}^{\text{NKOE}} \neq 0$.

D.5.1 Branch 2: $\boldsymbol{C}_9 \neq \boldsymbol{0}$

We begin by guessing and verifying $\boldsymbol{C}_{12} = \boldsymbol{C}_8 = \boldsymbol{0}$. Then, assuming $\boldsymbol{\mu}_2$ has left inverse $\boldsymbol{\mu}_2^\ell$ and $\boldsymbol{\Lambda}$ is nonsingular, coefficient matching yields

$$\boldsymbol{C}_4 = \boldsymbol{\mu}_2^\ell \boldsymbol{\mu}_3, \qquad \boldsymbol{C}_5 = \boldsymbol{\Lambda} \boldsymbol{C}_9 (\boldsymbol{I} - \beta \boldsymbol{C}_9)^{-1},$$

$$\mathcal{K}(\rho, \boldsymbol{C}_9) \equiv (1-\beta\rho)\Big[((1-\rho)\boldsymbol{I} - \boldsymbol{C}_9)(\boldsymbol{I} - \beta\boldsymbol{C}_9) - (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{C}_9\Big] + \Big[(\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma} - \rho(\boldsymbol{I} - \boldsymbol{\Omega})\Big]\boldsymbol{\Lambda},$$

$$\boldsymbol{C}_7 = (1-\rho)\boldsymbol{\Lambda}\,\mathcal{K}(\rho, \boldsymbol{C}_9)^{-1} \boldsymbol{L}_\tau^P, \qquad \boldsymbol{C}_{11} = (1-\beta\rho)(\boldsymbol{I} - \beta\boldsymbol{C}_9)\boldsymbol{\Lambda}^{-1}\boldsymbol{C}_7,$$

$$\boldsymbol{C}_3 = \boldsymbol{\mu}_2^\ell \Big\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I} - \beta\boldsymbol{C}_9) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\mathcal{K}(\rho, \boldsymbol{C}_9)^{-1}\boldsymbol{L}_\tau^P - \boldsymbol{\mu}_3\Big\},$$

$$\boldsymbol{C}_1 = \boldsymbol{\mu}_2^\ell \Big(\boldsymbol{C}_9 - \boldsymbol{I} - \boldsymbol{\mu}_1 \boldsymbol{\Lambda} \boldsymbol{C}_9 (\boldsymbol{I} - \beta\boldsymbol{C}_9)^{-1}\Big).$$

For the $\Delta\hat{w}_t$-block define

$$\boldsymbol{H}_w(\boldsymbol{C}_9) \equiv \Big[\sigma\boldsymbol{\mu}_2^\ell\big((\boldsymbol{C}_9 - \boldsymbol{I})(\boldsymbol{I} - \beta\boldsymbol{C}_9) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{C}_9\big) - \boldsymbol{A}_2\boldsymbol{\Lambda} + \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{C}_9\Big](\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}.$$

Then

$$\boldsymbol{C}_2 = -(\boldsymbol{H}_w(\boldsymbol{C}_9)\boldsymbol{\mu}_2)^{-1}\boldsymbol{H}_w(\boldsymbol{C}_9)\boldsymbol{\mu}_4,$$

$$\boldsymbol{C}_{10} = (\boldsymbol{I} - \beta\boldsymbol{C}_9)(\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}\big(\boldsymbol{\mu}_2\boldsymbol{C}_2 + \boldsymbol{\mu}_4\big), \quad \boldsymbol{C}_6 = \boldsymbol{\Lambda}(\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}\big(\boldsymbol{\mu}_2\boldsymbol{C}_2 + \boldsymbol{\mu}_4\big).$$

These are the coefficient formulas used in Propositions 2 and 4 once $\boldsymbol{C}_9 = \boldsymbol{\Psi}^{\text{NKOE}}$.

D.6 Decomposing the Impact on Inflation and Consumption

Since all endogenous variables are linear functions of $\hat{\tau}_t$ and $\Delta\hat{w}_t$, on impact we have:

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \boldsymbol{c}_\tau + \boldsymbol{c}_w w_\tau, \qquad \frac{\partial \boldsymbol{\pi}_t^P}{\partial \hat{\tau}_t} = \boldsymbol{p}_\tau + \boldsymbol{p}_w w_\tau, \qquad w_\tau \equiv \frac{\partial \Delta\hat{w}_t}{\partial \hat{\tau}_t}.$$

Let $\boldsymbol{\mu}_2^\ell \equiv (\boldsymbol{\mu}_2^\top \boldsymbol{\mu}_2)^{-1}\boldsymbol{\mu}_2^\top$. Then

$$\boldsymbol{p}_\mu = \boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})^{-1}, \quad \boldsymbol{c}_\mu = \boldsymbol{\mu}_2^\ell\Big[\boldsymbol{\Psi}^{\text{NKOE}} - \boldsymbol{I} - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})^{-1}\Big],$$

$$\boldsymbol{c}_{\tau,-1} = \boldsymbol{\mu}_2^\ell\boldsymbol{\mu}_3, \qquad \boldsymbol{p}_{\tau,-1} = \boldsymbol{0}, \qquad \boldsymbol{\mu}_{\tau,-1} = \boldsymbol{0}.$$

Define

$$\boldsymbol{H}_\tau \equiv (1-\beta\rho)\big((1-\rho)\boldsymbol{I} - \boldsymbol{\Psi}^{\text{NKOE}}\big)(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})$$

$$+ (1-\beta\rho)(\boldsymbol{\Omega}-\boldsymbol{I})\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}} + \Big(\rho(\boldsymbol{\Omega}-\boldsymbol{I}) + (\boldsymbol{\alpha} + \boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}\Big)\boldsymbol{\Lambda},$$

and

$$\begin{aligned}\boldsymbol{H}_w \equiv \Big[&\sigma\boldsymbol{\mu}_2^{\ell}\big((\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{I})(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}\big)\\ &- \boldsymbol{A}_2\boldsymbol{\Lambda} + \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}\Big](\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}.\end{aligned}$$

Then

$$\boldsymbol{p}_\tau = (1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1}\boldsymbol{L}^P_\tau, \quad \boldsymbol{c}_\tau = \boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1}\boldsymbol{L}^P_\tau - \boldsymbol{\mu}_3\right\},$$

$$\boldsymbol{c}_w = -(\boldsymbol{H}_w\boldsymbol{\mu}_2)^{-1}\boldsymbol{H}_w\boldsymbol{\mu}_4, \qquad \boldsymbol{p}_w = \boldsymbol{\Lambda}(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}(\boldsymbol{\mu}_2\boldsymbol{c}_w + \boldsymbol{\mu}_4),$$

$$\boldsymbol{\mu}_w = (\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}})\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_w, \qquad \boldsymbol{\mu}_\tau = (1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}})\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_\tau.$$

D.6.1 Inflation

Let

$$\boldsymbol{\mathcal{Z}} \equiv \boldsymbol{L}^C_{\mathcal{E}}\, m\, \boldsymbol{Z}, \qquad \boldsymbol{R}_\tau \equiv \boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\right\},$$

so that $\boldsymbol{c}_\tau = -\boldsymbol{L}^C_\tau + \boldsymbol{R}_\tau\boldsymbol{L}^P_\tau$. Substituting the exchange-rate solution into CPI inflation and grouping terms yields

$$\begin{aligned}\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = \underbrace{\boldsymbol{\Gamma}\boldsymbol{L}^P_\tau}_{\text{Direct Effect on inputs}} &+ \underbrace{\Big[\boldsymbol{\Gamma}\Big((1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\Big) + \boldsymbol{\mathcal{Z}}\Big(\boldsymbol{\Gamma}(1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1} + \sigma\boldsymbol{R}_\tau\Big)\Big]\boldsymbol{L}^P_\tau}_{\text{NKOE propagation}}\\ &+ \underbrace{\Big[\boldsymbol{I} + (1-\sigma)\boldsymbol{\mathcal{Z}}\Big]\boldsymbol{L}^C_\tau}_{\substack{\text{Contribution of tariffs}\\ \text{on consumption}}} + \underbrace{\Big[\boldsymbol{\Gamma}\boldsymbol{p}_w + \boldsymbol{\mathcal{Z}}\big(\sigma\boldsymbol{c}_w + \boldsymbol{\Gamma}\boldsymbol{p}_w\big) + \boldsymbol{L}^C_{\mathcal{E}}\, m\Big]w_\tau}_{\substack{\text{Contribution of the}\\ \text{risk-sharing wedge}}}.\end{aligned} \tag{D.18}$$

D.6.2 Consumption

Using the impact solution and $\boldsymbol{\mu}_3 = \boldsymbol{\mu}_2\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau$,

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \underbrace{-\boldsymbol{L}^C_\tau}_{L^C_\tau \text{ block}} + \underbrace{\boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\right\}\boldsymbol{L}^P_\tau}_{L^P_\tau \text{ block}} + \underbrace{\boldsymbol{c}_w\, w_\tau}_{\hat{w} \text{ block}}. \tag{D.19}$$

D.7 Why input–output linkages further increase persistence

Proposition 5 is the extensive-margin result: when $J > 1$, real marginal-cost deviations can become inherited states. This subsection studies the intensive margin: conditional on that propagation channel being active, how does scaling up input–output linkages affect the speed at which inherited marginal-cost distortions decay?

Throughout this subsection, we take the passive-policy limit $\boldsymbol{\Phi} \to \mathbf{0}$ and compare networks along the scalar path $\boldsymbol{\Omega}(s) = s\,\bar{\boldsymbol{\Omega}},\ s \in [\underline{s}, \bar{s}]$, where the interval is chosen so that input shares remain admissible.

We use the following terminology. A root of $\boldsymbol{\Psi}(s)$ means an eigenvalue of $\boldsymbol{\Psi}(s)$. A *propagated root* is a nonzero root of $\boldsymbol{\Psi}(s)$. If $\boldsymbol{\Psi}(s)\boldsymbol{v} = \lambda \boldsymbol{v},\ \lambda \neq 0$, then a marginal-cost distortion proportional to $\boldsymbol{v}$ survives one period with decay factor λ. Zero roots instead correspond to directions eliminated in one step. Thus persistence is governed by the nonzero roots. An *eigenvalue branch* is a differentiable function such that $\lambda(s)$ is an eigenvalue of the relevant matrix for every s in the interval. When the eigenvalue is simple, this branch is locally well defined and differentiable.

Proposition 7 (Scalar scaling of input–output linkages)**.** *Let $\boldsymbol{\Psi}(s) \equiv \boldsymbol{\Psi}^{NKOE}(s)$ denote the stable solution to $\mathcal{K}(0, \boldsymbol{\Psi}(s))\boldsymbol{\Psi}(s) = \mathbf{0}$ under $\boldsymbol{\Omega}(s) = s\bar{\boldsymbol{\Omega}}$. Define $\boldsymbol{M}(s) \equiv (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\boldsymbol{\Omega}})\boldsymbol{\Lambda}$. Let $\lambda(s) \in (0,1)$ be a real simple propagated-root branch of $\boldsymbol{\Psi}(s)$. Define $\zeta(s) \equiv \beta\lambda(s) + \lambda(s)^{-1}$. Then $\zeta(s)$ is an eigenvalue of $\boldsymbol{M}(s)$. Assume that $\zeta(s)$ is a simple eigenvalue branch of $\boldsymbol{M}(s)$. Let $\boldsymbol{x}(s)$ and $\boldsymbol{y}(s)$ denote right and left eigenvectors of $\boldsymbol{M}(s)$ associated with $\zeta(s)$, normalized by $\boldsymbol{y}(s)^\top \boldsymbol{x}(s) = 1$.*

If $\boldsymbol{y}(s)^\top \bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s) > 0$, then

$$\lambda'(s) = \frac{\lambda(s)^2}{1 - \beta\lambda(s)^2}\,\boldsymbol{y}(s)^\top \bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s) > 0.$$

Therefore scaling up input–output linkages raises the decay factor of this propagated marginal-cost mode.

Proof. Under $\boldsymbol{\Phi} = \mathbf{0}$,

$$\mathcal{K}(0, \boldsymbol{X}) = (\boldsymbol{I} - \boldsymbol{X})(\boldsymbol{I} - \beta\boldsymbol{X}) - (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{X}.$$

Along the path $\boldsymbol{\Omega}(s) = s\bar{\boldsymbol{\Omega}}$, this becomes $\mathcal{K}(0, \boldsymbol{X}) = \boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{X} + \beta\boldsymbol{X}^2$. Hence the equilibrium propagation matrix satisfies

$$\left[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\right]\boldsymbol{\Psi}(s) = \mathbf{0}. \tag{D.20}$$

Take a propagated root $\lambda(s) \neq 0$ of $\boldsymbol{\Psi}(s)$, with right eigenvector $\boldsymbol{u}(s)$:

$$\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s).$$

This vector is a self-replicating pattern of real marginal-cost distortions. After one period, the same pattern remains, scaled by $\lambda(s)$. Thus a larger $\lambda(s) \in (0, 1)$ means slower unwinding.

Apply (D.20) to $\boldsymbol{u}(s)$: $\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \mathbf{0}$. Using $\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s)$, this becomes

$$\lambda(s)\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{u}(s) = \mathbf{0}.$$

Because $\lambda(s) \neq 0$, we may divide by this scalar. This is the key step: we do not invert or cancel the full matrix $\boldsymbol{\Psi}(s)$. We only divide by the nonzero scalar $\lambda(s)$. Hence

$$\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{u}(s) = \mathbf{0}.$$

Using again $\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s)$ and $\boldsymbol{\Psi}(s)^2\boldsymbol{u}(s) = \lambda(s)^2\boldsymbol{u}(s)$, we obtain $\big[\boldsymbol{I} - \lambda(s)\boldsymbol{M}(s) + \beta\lambda(s)^2\boldsymbol{I}\big]\boldsymbol{u}(s) = \mathbf{0}$. Rearranging,

$$\boldsymbol{M}(s)\boldsymbol{u}(s) = \big(\beta\lambda(s) + \lambda(s)^{-1}\big)\,\boldsymbol{u}(s).$$

Thus $\zeta(s) \equiv \beta\lambda(s) + \lambda(s)^{-1}$ is an eigenvalue of $\boldsymbol{M}(s)$. Economically, $\zeta(s)$ is the effective damping term associated with the marginal-cost pattern $\boldsymbol{u}(s)$. A lower $\zeta(s)$ means weaker pullback toward steady state.

We now compute how $\zeta(s)$ moves with s. Let $\boldsymbol{x}(s)$ and $\boldsymbol{y}(s)$ be right and left eigenvectors of $\boldsymbol{M}(s)$ associated with $\zeta(s)$:

$$\boldsymbol{M}(s)\boldsymbol{x}(s) = \zeta(s)\boldsymbol{x}(s), \qquad \boldsymbol{y}(s)^\top\boldsymbol{M}(s) = \zeta(s)\boldsymbol{y}(s)^\top,$$

with normalization $\boldsymbol{y}(s)^\top\boldsymbol{x}(s) = 1$. To arrive at the standard simple-eigenvalue perturbation formula ($\zeta'(s) = \boldsymbol{y}(s)^\top\boldsymbol{M}'(s)\boldsymbol{x}(s)$), we begin by differentiating

$$\boldsymbol{M}(s)\boldsymbol{x}(s) = \zeta(s)\boldsymbol{x}(s)$$

with respect to s:

$$\boldsymbol{M}'(s)\boldsymbol{x}(s) + \boldsymbol{M}(s)\boldsymbol{x}'(s) = \zeta'(s)\boldsymbol{x}(s) + \zeta(s)\boldsymbol{x}'(s).$$

Premultiply by $\boldsymbol{y}(s)^\top$:

$$\boldsymbol{y}^\top \boldsymbol{M}'\boldsymbol{x} + \boldsymbol{y}^\top \boldsymbol{M}\boldsymbol{x}' = \zeta'\boldsymbol{y}^\top\boldsymbol{x} + \zeta\boldsymbol{y}^\top\boldsymbol{x}'.$$

Since $\boldsymbol{y}^\top\boldsymbol{M} = \zeta\boldsymbol{y}^\top$ and $\boldsymbol{y}^\top\boldsymbol{x} = 1$, the two terms involving $\boldsymbol{x}'$ cancel, leaving

$$\zeta'(s) = \boldsymbol{y}(s)^\top\boldsymbol{M}'(s)\boldsymbol{x}(s).$$

Because $\boldsymbol{M}(s) = (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\boldsymbol{\Omega}})\boldsymbol{\Lambda}$, we have $\boldsymbol{M}'(s) = -\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}$. Therefore

$$\zeta'(s) = -\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s).$$

Thus, if $\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s) > 0$, then increasing s lowers the effective damping term $\zeta(s)$.

Finally, differentiating the scalar relation $\zeta(s) = \beta\lambda(s) + \lambda(s)^{-1}$ yields:

$$\zeta'(s) = \left(\beta - \lambda(s)^{-2}\right)\lambda'(s) = -\frac{1-\beta\lambda(s)^2}{\lambda(s)^2}\lambda'(s).$$

Solving for $\lambda'(s)$,

$$\lambda'(s) = -\frac{\lambda(s)^2}{1-\beta\lambda(s)^2}\zeta'(s).$$

Substituting the expression for $\zeta'(s)$,

$$\lambda'(s) = \frac{\lambda(s)^2}{1-\beta\lambda(s)^2}\,\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s).$$

Since $0 < \beta < 1$ and $0 < \lambda(s) < 1$,

$$\frac{\lambda(s)^2}{1-\beta\lambda(s)^2} > 0.$$

Hence the sign of $\lambda'(s)$ is the sign of $\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s)$. Under the maintained sign condition, $\lambda'(s) > 0$. Thus scaling up intermediate-input use raises the fraction of this marginal-cost pattern that survives into the next period, which is precisely greater persistence. □

Applying Proposition 7 to a unique real simple dominant propagated-root branch $\lambda_*(s) \in (0,1)$ satisfying $\lambda_*(s) > \max\left\{|\lambda| : \lambda \in \sigma(\boldsymbol{\Psi}(s)),\ \lambda \neq 0,\ \lambda \neq \lambda_*(s)\right\}$ for every $s \in [\underline{s}, \bar{s}]$, with associated $\zeta_*(s) = \beta\lambda_*(s) + \lambda_*(s)^{-1}$ a simple eigenvalue branch of $\boldsymbol{M}(s)$, proves Proposition 6.

D.8 Open-economy monetary-policy heterogeneity and persistence

This appendix demonstrates how monetary policy heterogeneity can change the persistence of real marginal cost deviations.

Proposition 8. *Consider $N = 2$ and $J > 1$. Suppose countries have identical target baskets. Let $\delta = \phi_\pi^H - \phi_\pi^F$ summarize policy heterogeneity; if $\delta = 0$, the policy-heterogeneity channel, $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Lambda}$, is eliminated. Let $\boldsymbol{\Psi}^{\text{CE}}(\delta)$ and $\boldsymbol{\Psi}^{\text{OE}}(\delta)$ solve the corresponding closed- and open-economy kernel conditions, with the open-economy kernel adding $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}$. Let $\lambda_*^{\text{CE}}(\delta)$ and $\lambda_*^{\text{OE}}(\delta)$ be the local continuations of the largest real nonzero eigenvalue of the common $\delta = 0$ propagation matrix, and suppose this common eigenvalue $\lambda_* \in (0, 1)$ is simple. Let $\boldsymbol{M} \equiv (1 + \beta)\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}$, and let $\boldsymbol{A}_* \equiv \beta\lambda_*^2 \boldsymbol{I} - \lambda_* \boldsymbol{M} + \boldsymbol{I} + \phi_\pi \boldsymbol{\alpha}\boldsymbol{\Gamma}\boldsymbol{\Lambda}$, and let $\boldsymbol{v}_*$ and $\boldsymbol{u}_*$ satisfy $\boldsymbol{A}_*\boldsymbol{v}_* = \boldsymbol{0}$ and $\boldsymbol{u}_*^\top \boldsymbol{A}_* = \boldsymbol{0}^\top$. Then*

$$\partial_\delta[\lambda_*^{\text{OE}}(\delta) - \lambda_*^{\text{CE}}(\delta)]\big|_{\delta=0} = \frac{\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_*}{\boldsymbol{u}_*^\top (\boldsymbol{M} - 2\beta\lambda_* \boldsymbol{I})\boldsymbol{v}_*}.$$

Hence, if $\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_* \neq 0$, monetary-policy heterogeneity changes persistence; for small $\delta > 0$, it raises the largest real nonzero eigenvalue relative to the closed-economy policy block when the displayed ratio is positive, and lowers it when negative.

Proof. Fix $\boldsymbol{M} \equiv (1 + \beta)\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}$, $\boldsymbol{e}_H = (1, 0)^\top$, $\boldsymbol{e}_F = (0, 1)^\top$, $\boldsymbol{Z} = \boldsymbol{e}_H^\top - \boldsymbol{e}_F^\top$, and $\boldsymbol{\Phi}(\delta) = \phi_\pi \boldsymbol{I}_2 + \delta \boldsymbol{e}_H \boldsymbol{e}_H^\top = \left[\begin{smallmatrix} \phi_\pi+\delta & 0 \\ 0 & \phi_\pi \end{smallmatrix}\right]$. Identical target baskets imply $\boldsymbol{e}_H^\top \boldsymbol{\Gamma} = \boldsymbol{e}_F^\top \boldsymbol{\Gamma}$, so

$$\boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = (\boldsymbol{e}_H^\top - \boldsymbol{e}_F^\top)(\phi_\pi \boldsymbol{I}_2 + \delta \boldsymbol{e}_H \boldsymbol{e}_H^\top)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = \delta \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda},$$

and therefore $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = \delta \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}$, which is zero at $\delta = 0$.

Define

$$\boldsymbol{F}^{\text{CE}}(\lambda, \delta) \equiv \beta\lambda^2 \boldsymbol{I} - \lambda \boldsymbol{M} + \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}, \qquad \boldsymbol{F}^{\text{OE}}(\lambda, \delta) \equiv \boldsymbol{F}^{\text{CE}}(\lambda, \delta) + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}.$$

At $\delta = 0$, the two pencils coincide, and $\boldsymbol{F}^{\text{CE}}(\lambda_*, 0) = \boldsymbol{F}^{\text{OE}}(\lambda_*, 0) = \boldsymbol{A}_*$. For $s \in \{\text{CE}, \text{OE}\}$, the kernel condition for the local branch gives $\boldsymbol{F}^s(\lambda_*^s(\delta), \delta)\boldsymbol{v}^s(\delta) = \boldsymbol{0}$. Differentiating at $\delta = 0$ and premultiplying by $\boldsymbol{u}_*^\top$ gives, with the denominator nonzero by simplicity,

$$\left.\frac{d\lambda_*^s(\delta)}{d\delta}\right|_{\delta=0} = \frac{\boldsymbol{u}_*^\top \, \partial_\delta \boldsymbol{F}^s(\lambda, \delta)\big|_{(\lambda,\delta)=(\lambda_*,0)} \, \boldsymbol{v}_*}{\boldsymbol{u}_*^\top (\boldsymbol{M} - 2\beta\lambda_* \boldsymbol{I})\boldsymbol{v}_*}.$$

The required partial derivatives are

$$\partial_\delta \boldsymbol{F}^{\text{CE}}(\lambda, \delta)\big|_{(\lambda,\delta)=(\lambda_*,0)} = \boldsymbol{\alpha}\boldsymbol{e}_H \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}, \qquad \partial_\delta \boldsymbol{F}^{\text{OE}}(\lambda, \delta)\big|_{(\lambda,\delta)=(\lambda_*,0)} = \boldsymbol{\alpha}\boldsymbol{e}_H \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}.$$

Subtracting the closed-economy derivative from the open-economy derivative yields

$$\frac{d}{d\delta}\left[\lambda_*^{\mathrm{OE}}(\delta)-\lambda_*^{\mathrm{CE}}(\delta)\right]\bigg|_{\delta=0}=\frac{\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_*}{\boldsymbol{u}_*^\top(\boldsymbol{M}-2\beta\lambda_*\boldsymbol{I})\boldsymbol{v}_*}.$$

If the numerator is nonzero, the open-economy policy heterogeneity term changes the local response of the propagated eigenvalue to monetary-policy heterogeneity. Writing the displayed ratio as $\mathcal{R}$, $\lambda_*^{\mathrm{OE}}(\delta)-\lambda_*^{\mathrm{CE}}(\delta)=\delta\mathcal{R}+o(\delta)$. Since $\lambda_*\in(0,1)$ is the simple largest real nonzero eigenvalue at $\delta=0$, continuity of the local branch implies that, for sufficiently small δ, this branch remains the relevant largest real nonzero eigenvalue. Therefore, for small $\delta>0$, the policy heterogeneity term raises the largest real nonzero eigenvalue relative to the closed-economy policy block when $\mathcal{R}>0$, and lowers it when $\mathcal{R}<0$. This proves Proposition 8. □

D.9 Solving for the Risk-Sharing Wedge- Open Economy Block

Using

$$\hat{\mathcal{E}}_t=m\left[\boldsymbol{Z}\left(\sigma\hat{\boldsymbol{C}}_t+\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_\tau^C\hat{\tau}_t\right)+\hat{w}_t\right],\qquad m\equiv\left(1-\boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\right)^{-1},$$

the balance-of-payments block can be written as

$$\beta\hat{V}_t=\hat{V}_{t-1}+\widehat{NX}_t+\boldsymbol{\Xi}_6\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\pi}_t^P,$$

$$\widehat{NX}_t=\widehat{NX}_{t-1}+\tilde{\boldsymbol{\Xi}}_2\Delta\hat{\boldsymbol{C}}_t+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{\pi}_t^P+\tilde{\Xi}_4\Delta\hat{w}_t+\tilde{\Xi}_5(\hat{\tau}_t-\hat{\tau}_{t-1}),$$

where

$$\tilde{\boldsymbol{\Xi}}_2\equiv\boldsymbol{\Xi}_2+m\sigma\Xi_4\boldsymbol{Z},\qquad\tilde{\boldsymbol{\Xi}}_3\equiv\boldsymbol{\Xi}_3+m\Xi_4\boldsymbol{Z}\boldsymbol{\Gamma},\qquad\tilde{\Xi}_4\equiv m\Xi_4,\qquad\tilde{\Xi}_5\equiv\Xi_5+m\Xi_4\boldsymbol{Z}\boldsymbol{L}_\tau^C.$$

Combining this block with the NK solution above yields the wedge coefficient.

D.9.1 Solving for the Wedge

Let $\Delta\hat{w}_t=C_{26}\hat{\tau}_t$ on impact. In the nondegenerate martingale branch with $(\boldsymbol{C}_9-\boldsymbol{I})$ invertible,

$$C_{26}=\frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{C}_3+\boldsymbol{C}_4)+\tilde{\boldsymbol{\Xi}}_3(\boldsymbol{C}_7+\boldsymbol{C}_8)-\left(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\right)(\boldsymbol{C}_9-\boldsymbol{I})^{-1}(\boldsymbol{C}_{11}+\boldsymbol{C}_{12})}{(1-\rho)\left[\left(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\right)(\boldsymbol{C}_9-\boldsymbol{I})^{-1}\boldsymbol{C}_{10}-\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_2-\tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_6-\tilde{\Xi}_4\right]}.\qquad\text{(D.21)}$$

Under the Branch-2 restriction $\boldsymbol{C}_8 = \boldsymbol{C}_{12} = \boldsymbol{0}$, this becomes ($\forall\ k \geq 0$)

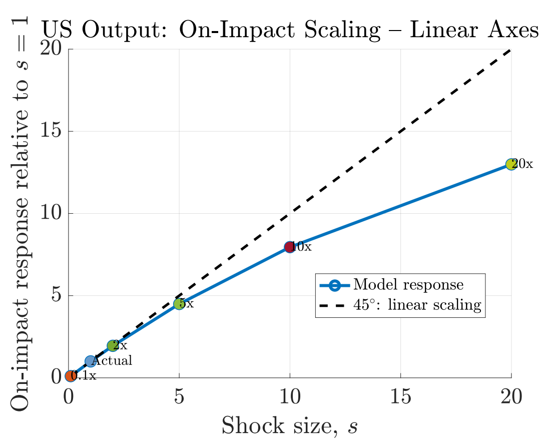
$$\frac{\partial \hat{w}_{t+k}}{\partial \hat{\tau}_t} = C_{26} = \frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{C}_3 + \boldsymbol{C}_4) + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_7 - \big(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\big)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}\boldsymbol{C}_{11}}{(1-\rho)\Big[\big(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\big)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}\boldsymbol{C}_{10} - \tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_2 - \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_6 - \tilde{\Xi}_4\Big]} \tag{D.22}$$

E Additional Quantitative Results

E.1 Non-Linearity of the Model

The model is nonlinear away from the local approximation. In the presence of tariffs, these nonlinearities may be quantitatively relevant for impulse responses. This is because the import margin is bounded: as tariffs become prohibitive, imports converge to zero, and further marginal increases in tariffs have vanishing effects through this margin. Hence the global policy functions for endogenous variables need not scale proportionally with the size of the tariff shock. Figure E.1 evaluates whether this concern is relevant in the neighborhood of implemented tariffs. The figure shows that, over this range, the on-impact responses are approximately homogeneous of degree one in the tariff shock. In particular, scaling the tariff shock scales the on-impact response of endogenous variables, such as output, nearly one-for-one. Detectable departures from this proportionality arise only when tariffs are scaled to roughly five times their implemented level.

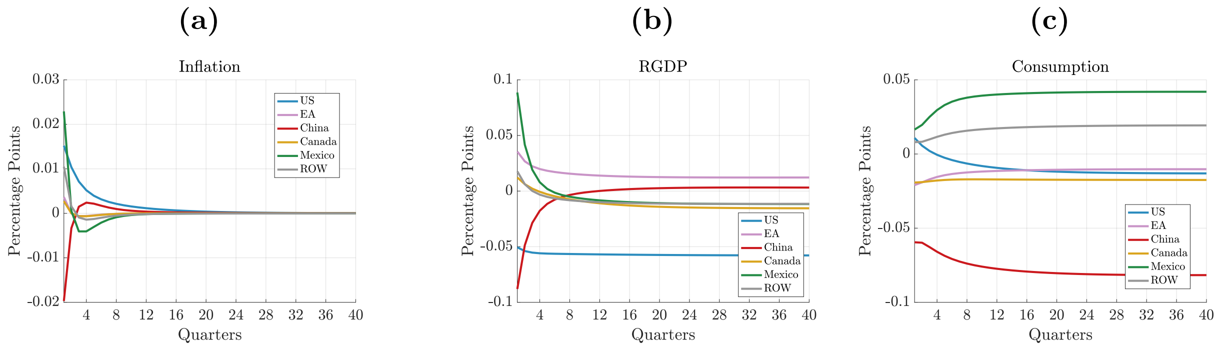
Figure E.1. Scaling Tariff Shocks and Non-Linearity of the Model



E.2 Validation: 2018 Tariffs

Figure E.2 reports the model-implied impulse responses for the 2018 tariff episode. On impact, U.S. real GDP falls by 0.05%, inflation rises by 0.015 percentage points, and consumption is essentially unchanged. With Table E.1 compares the model's exchange-rate response with an announcement-window statistic for the three main U.S.–China escalation dates. Following a high-frequency event-window approach, we sum the announcement-day changes in $\hat{\mathcal{E}}^{US}_{China,t}$, where $\mathcal{E}^{US}_{China,t}$ is yuan per dollar and t denotes the announcement day. Since an increase in $\hat{\mathcal{E}}^{US}_{China,t}$ denotes a yuan depreciation, the cumulative response implies a 1.126% dollar appreciation against the yuan. The model implies a 1.1% nominal dollar appreciation, a 0.05pp decline in U.S. real GDP, and, integrating the simulated inflation response over 40 quarters, a 0.12pp increase in the aggregate U.S. price level. These moments are close to the announcement-window exchange-rate response and to existing estimates that the 2018 tariffs raised the U.S. price level by 0.1–0.2pp and reduced aggregate real income by about 0.04% of GDP (Barbiero and Stein, 2025; Fajgelbaum et al., 2020).

Figure E.2. Impact of 2018 Tariffs



NOTE: Simulated responses to the announced 2018 U.S. tariff package under MIT shocks.

Table E.1. Major 2018 U.S.–China Tariff Announcements and Dollar Response

Date	Event	Implied $\hat{\tau}_t$	$\Delta\hat{\mathcal{E}}^{US}_{China,t}$
2018-03-22	New 25% China tariff threat on \$50–60bn	+2.47 to +2.97 pp	0.218%
2018-06-18	New 10% tariff threat on \$200bn	+3.96 pp	0.747%
2018-08-01	Raise proposed \$200bn List 3 tariff from 10% to 25%	+5.94 pp	0.162%
Sum	–	+12.37 to +12.87 pp	1.126%

NOTES: The implied tariff shock on announcement day t is $\hat{\tau}_t = 100 \times V_t \Delta\tau_t / M^{US \leftarrow China}_{2017}$, where V_t denotes the announced value of Chinese imports subject to the tariff change, $\Delta\tau_t$ denotes the announced change in the statutory tariff rate, and $M^{US \leftarrow China}_{2017} = 505.1651$ billion dollars denotes total U.S. goods imports from China in 2017. The exchange-rate response, $\Delta\hat{\mathcal{E}}^{US}_{China,t}$, is the announcement-day change in the yuan-per-dollar exchange rate relative to the previous day's end-of-day rate. Positive values indicate a U.S. dollar appreciation against the yuan.

E.3 Additional Model Counterfactuals

In this section we consider additional model counterfactuals. To do so we first introduce exogenous increases in the UIP premium. This is motivated by the real-life depreciation observed in the aftermath of Liberation Day.

In the baseline model, tariffs generate dollar appreciation in real and nominal terms for reasonable parameters, because $\hat{w}_t$ is sufficiently negative. The observed post–Liberation Day depreciation therefore requires an additional force that renders the wedge sufficiently positive. We capture that force with a reduced-form UIP premium κ_t, motivated by the empirically observed rise in UIP deviations in this period (Kalemli-Özcan et al., 2026). This modeling choice is consistent with several underlying mechanisms, including tariff-induced uncertainty or a decline in the dollar convenience yield.[38] Accordingly, the UIP condition becomes $\frac{1+i_{n,t}}{1+i_t^{US}} = E_t\left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}}\right]\frac{1-\kappa_t}{1-\psi'(B_{n,t}^{US}/P_t^{US})}$.

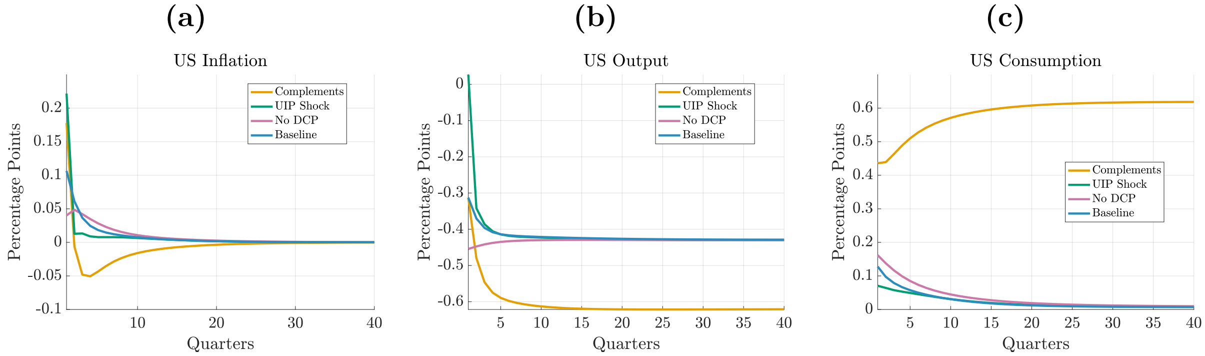
We next consider additional model counterfactuals. Figure E.3 reports U.S. responses under the baseline and three alternative specifications. "Baseline" is the scenario in Section 5.4. "Complements" sets the lowest-level production input elasticity to $\theta_{li}^X = 0.6$, relative to 1.5 in the baseline. "UIP Shock" adds a one-time disturbance to the UIP equation, following Kalemli-Özcan et al. (2026). "No DCP" sets $\vartheta_{ni} = 0$ for all n, i, reducing pricing to PCP. The largest deviations occur under "Complements": output falls more and remains persistently lower, while consumption rises because stronger complementarities amplify terms-of-trade gains. The "UIP Shock" dampens the consumption gain and raises impact inflation; its effects dissipate quickly, with output initially higher as dollar depreciation raises foreign demand for U.S. exports. DCP plays a limited quantitative role. "No DCP" lowers inflation and raises consumption modestly relative to the baseline, with little effect on output after the first five quarters.

E.3.1 Baseline with Complementarities

Figure E.4 reports IRFs for the "Complements" parametrization, which strengthens production-side complementarities by setting $\theta_{li}^X = 0.6$. On impact, U.S. real GDP falls by 0.31%, inflation rises by 0.18 percentage points, consumption rises by 0.44%, and the trade-weighted U.S. NEER appreciates by 10.64%. Spillovers are heterogeneous: real GDP expands by 0.33% in the euro area but falls by 0.22%, 0.27%, 0.35%, and 0.12% in China, Canada,

[38] We view this reduced-form UIP premium as an empirically-disciplined shorthand: even in a richer asset-market environment that endogenizes the premium, tariff-induced depreciation would still operate at least in part through a UIP wedge to counteract appreciationary forces. This is because the UIP premium interacts with the risk-sharing wedge from Section 3, which in this setup becomes $\hat{w}_t + \kappa_t = \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$. Even when tariffs generate $\hat{w}_t < 0$, a sufficiently large κ_t can overcome the appreciationary pressure, yielding depreciation and lower relative consumption.

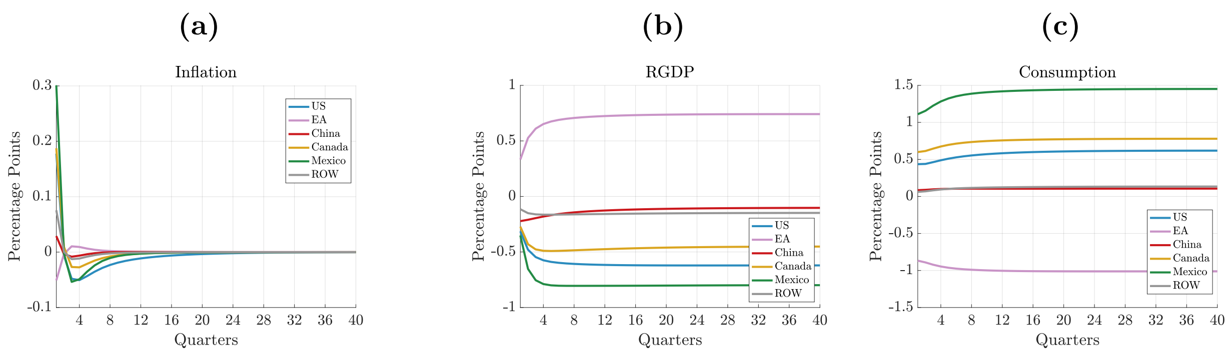
Figure E.3. Additional Model Counterfactuals



NOTE: Panels plot U.S. inflation, real GDP, and consumption under alternative model counterfactuals, relative to the baseline tariffs implemented as of March 2026 from Section 5.4. Each line changes one model feature at a time, holding the remaining environment fixed.

Mexico, and the rest of the world, respectively. Stronger production-side complementarity renders tariffs a globally stagflationary shock, except in the Euro Area, which benefits from trade diversion.

Figure E.4. Baseline under Complementarities



NOTE: Simulated responses to the announced 2025 U.S. tariff package under MIT shocks. The exercise sets $\theta_{li}^X = 0.6$, relative to 1.5 in the baseline.

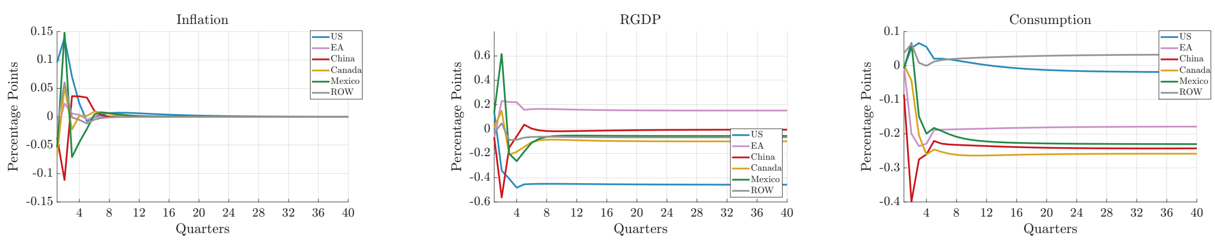
E.4 Gradual Implementation and UIP Premium

Here, we feed in tariffs as they were introduced gradually over 2025-2026, alongside exogenous increases in the UIP premium for realism (see Appendix E.3). Kalemli-Özcan et al. (2026) find that the UIP premium rose by 2.98 percentage points in 1Q2025 relative to 4Q2024 and remained elevated at 1.62 and 0.52 percentage points over the next two quarters.[39] We accordingly set $\kappa_0 = 0.0298$, 0.0162, and 0.0052 for tariff changes in the first, second, and third quarters.

Figure E.5 reports the gradual-implementation exercise with UIP premium shocks. On impact, the trade-weighted U.S. NEER depreciates by 1.86%, inflation rises by 0.10 percentage points, U.S. real GDP rises by 0.14% as the weaker dollar boosts U.S. export competitiveness, and consumption is essentially unchanged. As subsequent tariff changes arrive, U.S. output turns negative, reaching 0.48% below steady state in quarter 4 and remaining 0.46% below by quarter 40. U.S. inflation is front-loaded, peaking at 0.14 percentage points in quarter 2, while consumption rises temporarily by 0.07% in quarter 3 before drifting slightly below steady state. Effects are heterogeneous across trading partners. Mexico's inflation rises by 0.15 percentage points in quarter 2 and China's falls by 0.11 percentage points before reverting. The euro area expands after the first quarter, with real GDP rising by 0.23% in quarter 2, while consumption falls by 0.24% at its trough. China experiences the largest contraction, with real GDP and consumption falling by 0.56% and 0.40% in quarter 2. Mexico expands initially (real GDP rises 0.62% in quarter 2) but contracts as later tariff changes arrive, falling 0.26% below steady state in quarter 4. Canada and the RoW post smaller but persistent output losses, ending 0.10% and 0.07% below steady state.

Figure E.5. Gradual Implementation of Tariffs With Widening UIP Premium

(a) **(b)** **(c)**



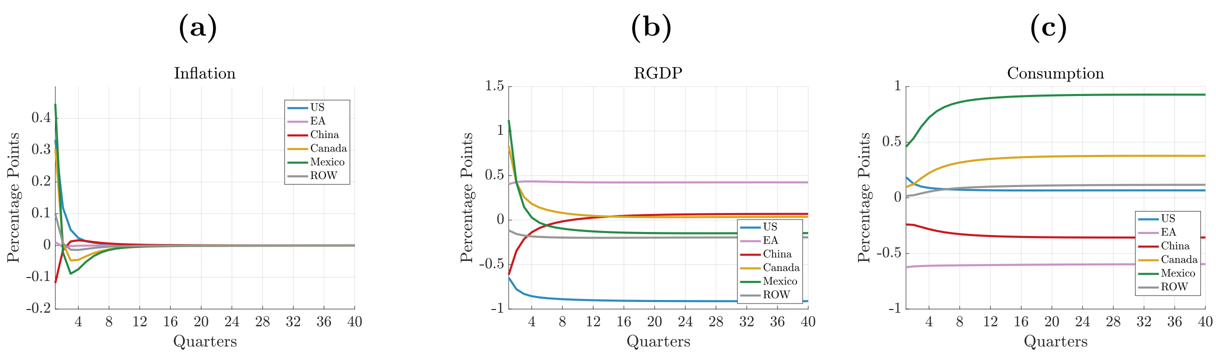
Note: Figure E.5 visualizes simulated responses to the 2025-2026 U.S. tariff packages, targeting China, Canada, Mexico, Europe and the RoW. Impulse responses are computed with MIT shocks. Instead of a single MIT shock, the figure overlays impulse responses to successive tariff changes, treating each as a new tariff level, and adds a one-time UIP premium shock at the start of each episode that matches empirical data from Kalemli-Özcan et al. (2026).

[39] As in Kalemli-Özcan et al. (2026), we shift quarters by 20 days so that 1Q2025 begins on Inauguration Day.

E.5 Counterfactual: Announced Liberation Day Tariffs Without Retaliation

Figure E.6 simulates the Liberation Day tariff package announced on April 2, 2025. On impact, U.S. real GDP falls by 0.65%, inflation rises by 0.33 percentage points, consumption rises by 0.18%, and the trade-weighted NEER appreciates by 11.31%. Spillovers are heterogeneous: China contracts, Mexico and Canada expand, and the Euro Area and rest of the world move modestly. Because announced tariffs exceeded those implemented, responses are larger than compared to the baseline.

Figure E.6. Impact of Liberation Day Tariffs



Note: Simulated responses to the announced 2025 U.S. tariff package under MIT shocks.

F Additional Tables and Figures

Table F.1. Descriptive Statistics for the U.S. (%)

Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

Notes: 'Output Share' is the share of the sector in total U.S. output. 'VA Share' is the share of the sector in total U.S. GDP. 'Consumption Share' is calculated as the sector's weight in the household expenditure. 'Output Home Share' represents the share of the output of the sector sold domestically. 'Consumption Home Share' captures the share of domestic production in consumption and 'Intermediate Home Share' captures the share of intermediate goods supplied domestically.