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ABSTRACT

This work develops a new framework to analyze the macroeconomic impact of trade distortions under global imbalances. Our New Keynesian Open Economy (NKOE) model incorporates trade and production networks with full input-output linkages, sectoral heterogeneity in price rigidities, and country heterogeneity in monetary policy. A key theoretical insight is that the dynamics of the inflation-output trade-off depend on the structure of global production networks and the degree of input complementarity; crucially, we introduce the NKOE Leontief inverse, which enables a more robust analysis of macroeconomic variables through the economy-wide propagation of tariff distortions. We show that international borrowing can dampen the inflationary impact of tariffs. The overall macroeconomic impact of tariffs depends on the endogenous monetary policy responses of both tariff-imposing and tariff-exposed countries, even in the absence of retaliation. The quantitative exercises based on data from 2025 tariffs imposed by the U.S., on Mexico, Canada, China, the Euro Area, and the ROW predict stagflation for all, with the largest increase in inflation in the U.S. and the biggest drop in output in Mexico. Exchange rate movements depend on the heterogeneous monetary policy responses and the nature of the shock—the dollar appreciates less or can even depreciate under tariff threats. Threats, even without implementation, are self-defeating as they lead to deflation and lower output.

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1 Introduction

We introduce a new framework to analyze the macroeconomic consequences of protectionist trade policies. Motivated by the goals of these policies to reduce the trade deficit and boost domestic manufacturing employment, we develop a global New Keynesian open-economy (NKOE) model that captures the complex interdependence of global trade, finance and production. Our framework incorporates realistic structural features—including full international input-output linkages, sector-specific nominal rigidities, and cross-country heterogeneity in monetary policy preferences—to provide a comprehensive assessment of both domestic and global effects of tariffs, when trade is unbalanced.

We address two central questions. First, how do tariffs affect key macroeconomic aggregates—such as output, consumption, the trade balance, inflation, and the exchange rate? Second, how do these effects vary in a global dynamic general equilibrium setting with international borrowing and production networks that span across countries and sectors?

To answer these broad questions, we aim to capture the key aspects of the current global trade system together with important domestic frictions. To this end, we introduce five primitives:

(i) Consumers in each country make choices prior to the imposition of tariffs, revealing their biases toward home and foreign goods. This is captured by the *consumption share matrix* $\mathbf{\Gamma}$; to relate to standard small open economy (SOE) and two-country settings, we use scalar counterparts for the home (H) and foreign (F) countries, denoted by γ_H and $\gamma_F = 1 - \gamma_H$.

(ii) Producers optimize their production by sourcing inputs globally. This is represented by the *input-output matrix* $\mathbf{\Omega}$, with scalar counterparts Ω_H and Ω_F capturing home and foreign input shares, respectively.

(iii) Goods from any country can be substituted—both in consumption and production—by goods within the same sector or across sectors. This is modeled through nested CES bundles. The *elasticities of substitution* (EoS) are given by the vector $\boldsymbol{\theta}$ (or scalar θ when a single elasticity is used).

(iv) Nominal rigidities may induce a sluggish adjustment of prices, captured by the frequency of price adjustment at the sectoral level, denoted by $\mathbf{\Lambda}$ (or scalar Λ in the simplified case).

(v) Central banks respond to price changes according to a *Taylor rule*, with response coefficients captured by the diagonal matrix $\mathbf{\Phi}$ (or scalar ϕ_π).

The effects of tariffs on prices and economic activity have been widely studied using some but not all of these features. For example, trade literature often assumes flexible prices, balanced trade with no international borrowing, and prefers static models given its long-run focus on productivity and welfare.[1] Open economy macro literature, on the other hand, relies mostly on SOE-NK models for a short-run analysis of transitory tariffs, ignoring intermediate inputs and supply chains.[2] We argue that, under both transitory and permanent tariffs, the dynamics of inflation-output trade-off, a key issue for the short-run approach, critically depends on the network structure and input complementarity—structural features that are only determined in the long-run.[3]

Let us start with a standard two-country (H and F) one-industry example to illustrate the intuition behind how the primitives shape the impact of tariffs. Suppose H places tariffs on F without retaliation. Under flexible prices, with low home bias (γ_H) for the tariff imposing country H, H is a relatively sizable buyer of F's goods. With terms of trade improvement in its favor, H's consumption can increase and its real exchange rate appreciates vis-a-vis F. We build on this standard case, in Section 4, adding production with endogenous labor supply and imported intermediate inputs. With endogenous labor supply, tariffs distort the labor-leisure choice and can either disincentivize or increase labor supply depending on income versus substitution effects under higher real wages. If production foreign bias Ω_H is high, where a large part of domestic production uses foreign intermediate inputs, then tariffs increase the cost of production and thereby act as a negative supply shock. These two forces (endogenous labor supply and intermediate inputs) dampen the original terms of trade gains and hence consumption might decline. Elasticity of substitution is important here: if both consumption goods and production inputs are highly substitutable within and across borders with a high θ, then tariffs can be expansionary, whereas if there is sufficient complementarity on the production side then tariffs maybe contractionary.

Next, in Section 5, we introduce nominal rigidity (Λ) and monetary policy (ϕ_π) to study the short-run effects of tariffs. As expected from standard New Keynesian theory, higher nominal rigidity (Λ) decreases inflation and increases the decline in output. While the terms

[1] Note that this literature treats tariffs as permanent and works with exact hat-algebra in two period models (see, for example, Costinot and Rodríguez-Clare, 2014).

[2] Early Keynesian literature studies the short-run impact of tariffs. See, for example, Mundell (1961), Eichengreen (1981), and Krugman (1982). This literature lacks the micro foundations of the modern-SOE-NK literature as in Galí and Monacelli (2005). Unfortunately, the early NK literature does not focus on tariffs but rather optimal exchange rate and monetary policies in SOEs. The paper by Barattieri et al. (2021) is an example who studied macro impact of tariffs in a SOE-NK model. In section 2, we overview the recent literature motivated by 2025 tariffs, focusing more on the normative aspects and discuss similarities and differences from our positive approach.

[3] The essential role of intermediate inputs and cross-border production chains in trade is well established (e.g., di Giovanni and Levchenko, 2010; Johnson, 2014).

of trade mechanism remains intact, if monetary policy targets inflation, higher home bias implies that consumption will get hit twice, once from higher domestic prices, next from higher interest rates via monetary policy.

The network setup granularizes these primitives and takes them to matrix scale with N countries and J industries. Instead of considering dependence on a single intermediate input, for example, we consider the full set of input-output linkages. Our model is summarized in a five-equation Global New Keynesian Representation: (i) the New Keynesian IS (NKIS) equation; (ii) the New Keynesian Phillips Curve (NKPC) for producer prices derived with Rotemberg costs; (iii) a definition of the consumption price vector, which deviates from producer prices due to exchange rate movements and tariff distortions; (iv) an Uncovered Interest Parity (UIP) condition that nests international arbitrage conditions; and (v) an equation of motion for external debt, which also incorporates the market-clearing condition. Together, these equations characterize the equilibrium and nest a broad class of NKOE models.

Our approach of writing down a N country-J sector model can be thought of as connecting two or more Rubbo (2023) economies under incomplete markets and trade imbalances. We do so by allowing representative households in countries to save in nominal local-currency, which are in net zero supply, and USD bonds, with which agents can save or dissave. We use portfolio adjustment costs to ensure that the steady-state level of debt is unique and this feature allows endogenous deviations from Uncovered Interest Parity (UIP). In the theoretical model, we linearize around a steady-state with non-zero debt that is consistent with the primitives γ and Ω. In the quantitative model, we discipline steady-state debt levels with real-life trade imbalances.

In Section 5, we solve the linearized model under three different policy rules: (1) monetary policy fixes the real rate and thereby stabilizes aggregate consumption, (2) monetary policy fixes the nominal demand (expenditure) and (3) monetary policy follows a Taylor rule. We find that network propagation is different under different policy regimes. Under a real rate rule that stabilizes consumption, tariffs lead to depreciation. Of the primitives of the foreign country, only γ_F enters the solution for home country's inflation. This is in marked contrast with the case when policy fixes nominal demand and this renders inflation in each sector and each country weakly positive. The intuition behind this result is that the policy choice when combined with Golosov and Lucas (2007) preferences fixes nominal wages and the nominal exchange rate. Then tariffs act as a cost-push shock and a cost-push shock in one part of the network propagates as a cost-push shock in all parts of the network. Finally, under a standard Taylor rule we find propagation is more flexible and inflation need not be strictly positive in all sectors and countries. The last case reveals the complexity of the non-linear interactions between the primitives when the policy primitive no longer fixes nominal demand or real demand and instead endogenously responds to inflation.

To understand and analytically decompose these interactions, we derive a key object with the method of undetermined coefficients—the NKOE Leontief inverse, which relates the tariff-related distortions on both consumption (demand) and production (supply) to the dynamics of inflation-output trade-off. Using this object, we decompose the general equilibrium response to the tariff shock into channels where demand distortions propagate to the economy through expectations, price stickiness, and monetary policy, and supply distortions propagate to the economy through the network. Intuitively, if a given sector is central to production—either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)—it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices—corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)—and is located in a country with relatively loose (or tight) monetary policy, the inflationary impact of a tariff on that sector will be amplified (or muted) by the network captured in the NKOE Leontieff inverse.

Having shown how input-output linkages affect macroeconomic aggregates in the context of tariff shocks, we explore when and why network granularity matters in Section 6. This involves two main answers. First involves aggregation of parameters under sectoral heterogeneity. As shown by Pasten et al. (2020) and Rubbo (2023) in a closed economy context, aggregating sectoral price stickiness parameters (Λ) across the economy yields a less precise picture of the aggregate Phillips curve in terms of its slope when the average is done with Λ terms and not price updating frequencies. We apply this result to our context and show that making the network coarser by collapsing sectors together and averaging the Λ terms across sectors can over-estimate the range of inflation outcomes that the central bank can achieve. That is, the inflation-output trade-off fears of inflation can be over-emphasized and fear of unemployment can be under-emphasized.

The second answer as to when and why network granularity matters has to do with international risk sharing. Production network models typically examine scenarios in which a sector-specific shocks propagate differently from aggregate shocks. This can occur because it is difficult for labor to switch sectors to serve as a substitute for inputs that are now more expensive or if a certain input is difficult to substitute. We find that this mechanism is sensitive to international risk sharing and the presence of a moving net foreign asset position between countries when the context is a *global* production network.

Many production network models are closed economy and many network models that do international trade restrict the net foreign asset position of countries to keep trade balanced, especially in quantitative simulations. Instead, ours is a setup with incomplete markets. In this setup, the representative household in each country makes a consumption and saving decision that equalizes the expected ratio of marginal utilities, taking into account differences in the relative price of each country's consumption basket. With this equalizing force in place, households choose their optimal labor supply. Depending on the substitutability of labor with intermediate inputs, labor in turn can smooth sectoral bottlenecks.

We find that, the structure of the network matters more and there are larger propagation effects in the absence of international risk sharing. We motivate this finding with an analytical solution that incorporates portfolio adjusment costs (ψ). At the financial autarky limit as $\psi \to \infty$, trade is restricted and sectoral rigidities and bottlenecks can lead to larger responses in prices and quantities. We test this finding with the quantitative model. The response by aggregate U.S. employment to tariffs being placed by the U.S. on different Chinese sectors differs more from sector to sector under financial autarky than under international risk sharing. Our findings suggest that DGE open economy models with production networks that attribute a central role to network structure—and that generate large differences between the effects of sectoral and aggregate shocks—may have these results critically depend on the absence of international borrowing.

For the quantitative exercise, we use the non-linear version of our model. The sectoral heterogeneity in price setting is disciplined by estimates from Nakamura and Steinsson (2008) and the steady state network using the OECD's Inter-Country Input–Output (ICIO) tables, imposing no *a priori* assumptions on whether a good is purely final or tradable. This modeling flexibility ensures that the quantitative results are not driven solely by the overall share of material inputs in marginal costs, as is often the case in conventional NKOE models. Instead, this relationship arises endogenously from the global I–O structure, which in turn allows our framework to nest other models as special cases.[4]

The first quantitative exercise involves road-testing the model with 2018 U.S. tariffs. Consistent with the existing empirical findings, the model predicts an inflation impact of 0.1 percentage points, which is in line with the estimate of Barbiero and Stein (2025). Our model also predicts a 4.5% appreciation of the U.S. dollar (USD) against the Chinese yuan, aligning with the observed 5.6% dollar appreciation against yuan between June 2018 and December 2018, where the broad dollar index appreciated over 7%. The predicted output loss of 0.3% is also consistent with Fajgelbaum et al. (2020), who estimate combined producer and consumer losses totaling 0.4% of U.S. GDP.

[4]For example, a model without intermediate inputs—where tariffs affect only demand—can be represented by collapsing the input–output matrix $\mathbf{\Omega}$. Likewise, a model with a single imported intermediate input and a final consumption good corresponds to a structure in which the columns of $\mathbf{\Omega}$ associated with final goods are zero vectors.

The next quantitative exercise predicts the impact of 2025 tariffs. Tariffs (implemented and announced to-be-implemented) are applied as near-permanent shocks, modeled as autoregressive processes with a persistence coefficient of 0.95. The model predicts inflation and falling output for all countries, with highest inflation for the U.S., a 0.5 percentage points rise, and the biggest drop of output for Mexico, 1.3 percent. A counterfactual exercise assuming symmetric retaliation predicts much larger effects, including output drops up to 1 percent for the U.S. Trade deficits improve only temporarily since tariffs do not change consumption-saving decisions permanently.

Last but not least, tariff threats are self-defeating. These are tariffs deployed with an announcement today, prompting trading partners to pledge retaliation in the future, but all tariffs are subsequently withdrawn. This "threat" shock highlights the role of the exchange rate as a forward-looking variable particularly transparent. In a perfect foresight setting, when tariffs are announced today and reversed tomorrow through a subsequent announcement, agents optimize based on the entire sequence of announcements. When the threat of permanent tariffs leads to anticipation of retaliation and a full trade war, the exchange rate immediately adjusts to front-load the anticipated change in consumption behavior. In this scenario, the U.S. NEER appreciates by 2.4% on impact—even in the absence of a contemporaneous change in monetary policy. Real GDP and consumption fall by 0.9% and 0.7%, respectively, almost as large as the case of symmetric retaliation, while inflation declines by 0.6 percentage points, resulting in deflation. These outcomes are driven primarily by the expectations channel: agents "price in" a future in which the U.S.—a net importer relative to the rest of the world—imports fewer foreign goods. Even before the mechanical price effects of actual tariffs materialize, anticipated trade distortions cause demand to contract, generating deflation on impact. Thus, while actual tariffs shift quantities by affecting supply chains and relative prices, reversed "tariff threats" generate limited quantity adjustments but sizable movements in prices, operating primarily through the expectations and demand channels.

Ultimately, our results imply that the inflationary impact of tariffs can be muted, while the effects on output and unemployment can be substantial in the presence of input–output linkages, country–sector heterogeneity in price stickiness, and open-economy channels. NKOE models that do not incorporate full global I–O linkages may systematically overestimate inflation and underestimate the real costs of tariffs, such as decline in employment.

2 Literature

Although we provide both flexible price and sticky price solutions, our preferred modeling approach is sticky prices to demonstrate the central role of monetary policy. In the flex-price setup, unless financial markets are incomplete, tariffs leads to real exchange rate appreciation offsetting tariffs fully by Lerner symmetry (Lerner, 1936).[5] However, there is a large empirical literature that shows the impact of tariffs is not fully offset by exchange rate movements. This literature also demonstrates that exchange rate pass-through to prices is much lower than tariff pass-through; the extent of tariff pass-thorough to border prices versus retail prices is subject of an extensive debate.[6] In addition, it is well-known in the two-country NKOE literature, (e.g., Obstfeld and Rogoff, 1995; Clarida et al., 2002), if exchange rate pass-through is less than full, domestic inflation in open economies can differ from CPI inflation. These two-country models also imply that domestic production and output are both affected from terms of trade and hence foreign activity becomes important for domestic prices both under sticky and flexible prices.

Our paper is also related to the normative literature on optimal tariff and networks. We explain below while drawing out key similarities and differences.

2.1 Optimal Monetary and Tariff Policies

The normative literature focuses on optimal tariffs that try to balance terms of trade gains with costs of tariff distortions. Most of this literature is composed of flexible price static models. The recent SOE-NK literature suggests that demand side considerations should be part of the thinking on optimal monetary policy and/or optimal tariffs in the short-run.

Although we do not study any normative implications, as we bring both demand and supply side of tariff distortions, similar to Ambrosino et al. (2024) who study fragmenta-

[5]See Erceg et al. (2018), Lindé and Pescatori (2019) and Costinot and Werning (2019) for modern treatments, laying out other conditions that under which the symmetry fails in different class of models. Jeanne and Son (2024) study exchange rate offset in a calibrated model and show that if trade elasticity is bigger than home bias, offset is less than 1.

[6]There is an active empirical debate on how much of the tariff is in the retail price faced by the consumer and how much of it impacts the marginal costs of both foreign and domestic firms? For example, for 2018 tariffs, Amiti et al. (2019), Fajgelbaum et al. (2020); Fajgelbaum and Khandelwal (2022), find complete pass-through of tariffs to consumer prices, whereas Cavallo et al. (2021) finds that the degree of pass-through from border to retailers and consumers is not complete. For categories like washing machines, the pass-through can be high (e.g., Flaaen et al., 2019). However, for more aggregated price indices that combine goods that are affected and unaffected by the tariffs, the pass-through is less clear-cut. Thus, retailers absorbing a significant share of the cost or raising their prices on goods that compete with the imports, or increasing the prices of goods not directly exposed are hard to separate. Inventory "front-running," moving supply chains away, or studying the early months with sticky prices can also blur the picture on aggregate price increases and inflationary impulse.

tion impacts, our work shares several features with this recent literature. For example, as in Monacelli (2025), we emphasize that the overall macro impact of tariffs depends on endogenous response of monetary policy. Yet, our work differs from Monacelli (2025) in that we show how other countries' monetary policy responses are also important in shaping the home country inflation-output trade-off, even without retaliation. The study by Bianchi and Coulibaly (2025) on the optimal monetary policy response to tariffs find similar results to Monacelli (2025), though through a different channel—a fiscal externality. In all these papers, tariffs work as a tax on consumption, similar to our demand side disturbance, while at the same time partially allocating demand from foreign to domestic goods. In all these papers, optimal policy stimulates the economy given the recession.

Adding intermediate input trade to the above SOE-NK models, Auclert et al. (2025) drive exact thresholds for recessionary and inflationary outcomes and argue that without taking the recession into account optimal tariff cannot be calculated. Temporary tariffs cause a recession whenever the import elasticity is below an openness-weighted (or home bias) average of the export elasticity and the intertemporal substitution elasticity. What we share with this paper is the possibility of a recession, but, in our case, this possibility arises implicitly due to the structure of the production network and involves additional features on top of trade elasticities and home bias.

Bergin and Corsetti (2023) find that optimal policy is contractionary. Whether that monetary policy stance involves tolerating an output contraction or expansion depends crucially on the value of the trade elasticity of substitution. Similarly, in our paper, monetary policy responses and trade elasticity interact together in shaping the macro impact of tariffs through the NKOE Leontieff inverse.

Finally, as in Auray et al. (2024b,a), if the steady state is inefficient and monetary policy is set by an inflation targeting rule, the optimal tariff is lower under sticky prices than under flexible prices. Similarly to our paper, they show that if labor supply and intermediate inputs are added, the tariff outcome depends critically on the monetary policy stance. They also show that if the country is a net debtor, it will set lower tariffs to reduce the payments on its net external debt. This resonates with our result that more debt smooths out the inflationary impact of tariffs.

2.2 Tariffs and Trade Deficits

Both Itskhoki and Mukhin (2025); Costinot and Werning (2025) share our long-run focus on trade imbalances. Similar to Auray et al. (2024a,b), Itskhoki and Mukhin (2025) highlight the importance of valuation effects for the determination of changes in steady state trade imbalances with tariffs in terms of nominal values. Costinot and Werning (2025) make the point that changes in real trade deficits with tariffs depend on the slope of Engel curves, which in turn depend on the extensive margin of trade. Both their paper and our paper share the view that for permanently changing real trade deficits in steady state, saving-investment decisions must change with tariffs.

2.3 Networks with and without Tariffs

Our work is related to the trade network literature connecting shocks to producers' marginal costs (see, for example, Caliendo and Parro, 2015), leading to amplification of cost-push inflation.[7] Other recent studies also integrate these production networks into global dynamic NK frameworks. One key related work is Cuba-Borda et al. (2025), which presents an empirical examination of trade distortions. While in Cuba-Borda et al. (2025) a numerical model is constructed to motivate empirical work, our approach provides analytical results that clarify the key mechanisms driving tariff-induced distortions. In another key paper, Ho et al. (2022) develop a global NK model with an analytical solution that relies on a real rate rule which fixes the path of consumption. In contrast, our model offers a richer analytical characterization of tariff transmission by emphasizing the critical role of endogenous networks and monetary policy in shaping macroeconomic outcomes. Unlike models that impose fixed relative and aggregate consumption paths, our framework allows both demand and policy to respond endogenously, capturing the dynamic interactions between tariffs, exchange rates, and monetary policy.[8]

As in the closed economy literature, relative price changes are important in understanding the behavior of aggregate inflation (e.g., Pasten et al., 2020, 2024; Rubbo, 2023, 2024; Afrouzi and Bhattarai, 2023). Similar to our work, Afrouzi et al. (2024) also implement network adjusted heterogeneity in price stickiness across sectors. This literature builds on the broader network research by Long and Plosser (1983); Acemoglu et al. (2012); Atalay (2017); Liu (2019); Baqaee and Farhi (2019, 2022); Baqaee (2018); Carvalho et al. (2021b); Carvalho and Tahbaz-Salehi (2019), among others. A recent focus of the literature, particularly in the context of open economies, has been to model the formation of firm-to-firm links. This

[7] All the network linkages are not necessary to create an inflationary impulse from tariffs. Even when we close the network linkages in our model (i.e., in a single sector economy), due to intermediate inputs and endogenous response of labor supply, we can generate factor price inflation. Werning et al. (2025), in a single sector SOE model with balanced trade and wage rigidity, also proposes the cost-push inflation as a result of distorted consumption and production decisions.

[8] Two-period closed and open economy network models are Baqaee and Farhi (2022), who incorporate both supply and demand shocks in a closed-economy context, di Giovanni et al. (2023) who extends this model to an open-economy setting, and Silva (2024) who explores the interaction between the CPI and production networks in small open economies.

strand of the literature, like in Chaney (2014) and Dhyne et al. (2021), takes discreteness seriously. In parallel to the study by Baqaee and Farhi (2024), our approach—rather than modeling the formation of links—implements a differentiable form of adjustment where links are determined by cost minimization. This means that we only handle the extensive margin via choke prices, but can have a general characterization of the equilibrium.

Roadmap. The remainder of this paper is organized as follows. Section 3 outlines our baseline New Keynesian open-economy model, detailing how we incorporate international production networks, nominal rigidity, and open-economy features. In Section 4, we solve for tariffs in the long run with flexible prices. In Section 5, we introduce nominal rigidities into the model to capture the dynamics in the short run. In particular, we solve the model under three different assumption. In Section 5.1, we use a real rate rule, in Section 5.2 we assume a fixed nominal demand scenario and Section 5.3 we solve the model under a Taylor rule. In Section 6 we focus on the question why the networks matter under our model. We introduce the data that we use in our quantitative exercises in Section 7 and present these results Section 8. Finally, Section 9 concludes.

3 Modeling Framework

We develop a multi-country New Keynesian model that incorporates nominal rigidities via Rotemberg costs, standard open-economy features such as portfolio adjustment costs and trade costs, and a production network.

Households optimize intertemporally, allocating consumption and labor supply while facing portfolio adjustment costs when holding foreign bonds. The production side follows a nested CES structure, with goods classified by sector and origin, and firms producing using labor and intermediate inputs. Prices are set in the producer's currency (PCP) and are subject to revenue-neutral tariffs. Monetary policy follows a Taylor rule, and exchange rates are fully endogenous in the model. Endogenous deviations from Uncovered Interest Parity (UIP) arise due to portfolio adjustment costs; as a country's real USD debt increases, the effective interest rate it pays also rises. The model provides a unified framework for analyzing macroeconomic dynamics in an interconnected global network economy.

3.1 Intertemporal problem.

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_n^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta}\right]$$

subject to

$$P_{n,t}C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}^{US}_{n,t}B^{US}_{n,t} + \mathcal{E}^{US}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t}) \leq$$
$$W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - (1+i_{n,t-1})B_{n,t-1} - \mathcal{E}^{US}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$

where $P_{n,t}$ is the price of the consumption bundle, $C_{n,t}$, at time t, β_n is the discount factor of country n, σ is the intertemporal elasticity of substitution, χ denotes labor disutility weight and η captures the elasticity of labor. $\mathcal{E}^{US}_{n,t}$ is the exchange rate between country n and the U.S. An increase in $\mathcal{E}^{US}_{n,t}$ implies a depreciation of the local currency relative to the U.S. dollar. $W_{n,t}$ is the wage in country n at time t, $L_{n,t}$ is the quantity of labor supplied in country n, $i_{n,t}$ is the nominal interest rate in local currency bond, $B_{n,t}$, and $i_{US,t}$ is the interest rate on the U.S. bond, $B^{US}_{n,t}$, where bonds are treated as liabilities. The term $\psi(B^{US}_{n,t}/P^{US}_{n,t})$ represents a stationarity-inducing portfolio adjustment cost that ensures a unique steady-state level of real debt (i.e., debt denominated in USD, deflated by the U.S. consumer price level). Taxes and transfers are denoted by $T_{n,t}$. In our model, tariffs are revenue-neutral; since there is a lump-sum rebate through $T_{n,t}$, tariff revenues and costs cancel out in the budget constraint.

Maximizing the household's lifetime utility subject to the present and future budget constraints yields the following standard first-order conditions (see Appendix B.1):

$$1 = \beta_n E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}}\right)^{-\sigma} \frac{P_{n,t}}{P_{n,t+1}}(1+i_{n,t})\right] \forall n \in N, \forall t \qquad \text{(Euler Equation)}, \tag{1}$$

$$\frac{1+i_{n,t}}{1+i^{US}_{n,t}} = E_t\left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}}\right] \frac{1}{1-\psi'(B^{US}_{n,t}/P^{US}_{n,t})} \qquad \text{(UIP) } n \in N-1. \tag{2}$$

The domestic bond is in net zero supply everywhere, and all countries save or dissave using U.S. bonds. In addition to the UIP condition, the arbitrage condition ensures that a country's bilateral exchange rates remain consistent with its exchange rates against the U.S. Finally, for completeness of notation, we define a country's exchange rate with itself.

$$\mathcal{E}_{n,m,t} = \frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{m,t}} \;\forall n \neq m \;\&\; m \neq US \; n,m \in N \tag{3}$$

$$\mathcal{E}_{n,n,t} = 1 \; \forall n \in N \tag{4}$$

We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

3.2 Intratemporal problem.

We now turn to the household's intratemporal problem. First, we introduce the consumption choices and labor supply decisions. Then, we turn to the production side. The first part of the intratemporal problem is the standard labor-consumption tradeoff that determines labor supply:

$$\frac{W_{n,t}}{P_{n,t}} = \chi L_{n,t}^{\eta} C_{n,t}^{\sigma} \; \forall n \in N, \forall t \tag{5}$$

Determining the intratemporal breakdown of consumption involves a nested CES structure. Outputs from different countries are first bundled into a country-sector consumption bundle, which is then aggregated into a country good:

$$C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i}^{\frac{1}{\theta_h^C}} C_{n,i,t}^{\frac{\theta_h^C - 1}{\theta_h^C}} \right]^{\frac{\theta_h^C}{\theta_h^C - 1}}, \tag{6}$$

where $C_{n,i,t}$ is country n's consumption of industry bundle i, and $\Gamma_{n,i}$ is the weight of bundle i. This bundle is then a combination of all goods of i procured by country n from countries $m \in N$ globally:

$$C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi}^{\frac{1}{\theta_{l,i}^C}} C_{n,i,mi,t}^{\frac{\theta_{l,i}^C - 1}{\theta_{l,i}^C}} \right]^{\frac{\theta_{l,i}^C}{\theta_{l,i}^C - 1}}, \tag{7}$$

where $\Gamma_{n,i,mi}$ is the weight of country m's good in this bundle (e.g., German automobiles –mi– in automobile bundle –i– for the U.S. consumers – n). We can then express the relevant price levels in line with the CES structure:

$$P_{n,t}^C = \left[\sum_{i \in J} \Gamma_{n,i} (P_{n,i,t}^C)^{1-\theta_h^C} \right]^{\frac{1}{1-\theta_h^C}}$$

$$P_{n,i,t}^C = \left[\sum_{m \in N} \Gamma_{n,i,mi} P_{n,mi,t}^{1-\theta_{l,i}^C} \right]^{\frac{1}{1-\theta_{l,i}^C}}$$

where $P^C_{n,i,t}$ is the local currency consumption price of the aggregated good basket i in country n at time t.

We assume that prices are set in the producer's currency and then converted to the consumer's currency using the exchange rate under the producer currency pricing (PCP) assumption:

$$P_{n,mi,t} = \mathcal{E}_{n,m,t}(1 + \tau_{n,mi,t})P_{mi,t} \tag{8}$$

where $\mathcal{E}_{n,m,t}$ is the bilateral exchange rate.

Remark 1. Given the prices that end users see and the aggregation of consumer prices, tariffs serve as a distortionary wedge, similar to a consumption tax or tax on labor income, in the labor-consumption tradeoff given by equation (5).

To complete the specification of demand on the household side, we need to define the relative demand conditions given the nested CES structure. Consumers choose:

$$C_{n,i,t} = \Gamma_{n,i} \left(\frac{P^C_{n,i,t}}{P_{n,t}} \right)^{-\theta^C_h} C_{n,t} \tag{9}$$

$$C_{n,mi,t} = \Gamma_{n,i,mi} \left(\frac{P_{n,mi,t}}{P^C_{n,i,t}} \right)^{-\theta^C_{l,i}} C_{n,i,t} \tag{10}$$

3.3 Production

Having defined the household's side, we now turn to the production side of the economy. Output in country n, sector i, at time t follows a CES production function:

$$Y_{ni,t} = A_{ni,t} \left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}} \right]^{\frac{\theta^P}{\theta^P-1}} \forall n \in N, \forall i \in J, \forall m \in N, \forall j \in J \tag{11}$$

All firms within a given country-sector combination are assumed to be identical, and each firm solves the following marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^p_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

As a firm faces this problem, it chooses labor and the quantities of the intermediate good specific to the producing industry in the given country. This intermediate good bundle is constructed as follows. Intermediate goods from different countries are first bundled into a country-industry-good bundle. This bundle and the relevant relative demand condition are defined below:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{\frac{1}{\theta_{l,j}^P}} X_{ni,mj,t}^{\frac{\theta_{l,j}^P - 1}{\theta_{l,j}^P}} \right]^{\frac{\theta_{l,j}^P}{\theta_{l,j}^P - 1}} \tag{12}$$

$$X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P_{ni,j,t}^p} \right)^{-\theta_{l,j}^P} X_{ni,j,t} \tag{13}$$

where $P_{ni,j,t}^p$ is the average price of j for producer sector i in country n, $\Omega_{ni,j,mj}$ captures the share of industry mj in bundle j for industry ni (e.g., Chinese steel –mj– in steel bundle –j– for the U.S. automobile industry – ni). The prices and intermediate inputs follow the same subscripts. Analogously, the intermediate bundle is constructed as follows:

$$\frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P_{ni,j,t}}{P_{ni,t}^p} \right)^{-\theta_h^P} \forall\ j \in J \tag{14}$$

$$X_{ni,t} = \left[\sum_{j \in J} \Omega_{ni,j}^{\frac{1}{\theta_h^P}} X_{ni,j,t}^{\frac{\theta_h^P - 1}{\theta_h^P}} \right]^{\frac{\theta_h^P}{\theta_h^P - 1}} \tag{15}$$

As we derive in detail in Appendix B.2, given the setup and definitions above, the firm's problem yields the following equilibrium conditions:

$$\frac{X_{ni,t}}{L_{ni,t}} = \frac{(1 - \alpha_{ni})}{\alpha_{ni}} \left(\frac{W_t}{P_{ni,t}^p} \right)^{\theta^P} \tag{16}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}} \left[\alpha_{ni} W_t^{1-\theta^P} + (1 - \alpha_{ni}) \left(\sum_j \Omega_{ni,j} P_{ni,j,t}^{1-\theta_h^P} \right)^{\frac{1-\theta^P}{1-\theta_h^P}} \right]^{\frac{1}{1-\theta^P}} \tag{17}$$

Within each country-sector, there is an infinite continuum of identical firms. A representative firm f in sector i of country n solves the following problem under the Rotemberg setup:

$$P_{ni,t}^f = \arg\max_{P_{ni,t}^f} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \left[Y_{ni,T}^f(P_{ni,T}^f) \left(P_{ni,T}^f - MC_{ni,T} \right) - \frac{\delta_{ni}}{2} \left(\frac{P_{ni,T}^f}{P_{ni,T-1}^f} - 1 \right)^2 Y_{ni,T} P_{ni,T} \right] \right]$$

where a bundler aggregates the sectoral output into a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}}\right)^{-\theta^R} Y_{ni,t}$. As we show in Appendix B.2.1, this problem yields the following equilibrium condition:

$$(\Pi_{ni,t} - 1)\,\Pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R}\right) + \beta_n \mathbb{E}_t\left[(\Pi_{ni,t+1} - 1)\,\Pi_{ni,t+1}\right] \tag{18}$$

Equation (18) constitutes a country- and sector-specific forward-looking New Keynesian Phillips Curve, expressed in terms of nominal marginal cost deflated by the sector's producer price. As $\delta_{ni} \to 0$, prices become more flexible, leading to $\Pi_{n,t} = 1$ and $\frac{MC_{ni,t}}{P_{ni,t}} = \frac{\theta^R-1}{\theta^R}$, which corresponds to the general pricing equation under monopolistic competition.

3.4 Balance of Payments and NIIP

We track the evolution of each country's net international investment position (NIIP) as follows:

$$\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}C_{n,mj,t}\right) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}X_{ni,mj,t}\right) + \mathcal{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$
$$+\, \mathcal{E}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t}) = \sum_{i\in\mathcal{J}}(P_{ni,t}Y_{ni,t}) + \mathcal{E}_{n,t}B^{US}_{n,t} \quad \forall n \in N-1 \tag{19}$$

where we account for the fact that tariffs are modeled as revenue-neutral by dividing relevant prices by $(1+\tau_{n,mi,t})$, since end-user prices reflect the impact of tariffs just as they do the impact of exchange rates. This will play an important role when we switch to vector notation in Section 3.6.

Given market-clearing conditions and budget constraints, one country's budget constraint is redundant as an equilibrium condition. Thus, we omit that of the first country, which corresponds to the U.S. in our model. However, we still need to ensure that the market for USD bonds is closed:

$$B^{US}_t = \sum_{m}^{N-1} B^{US}_{m,t} \tag{20}$$

3.5 Definitions, Market Clearing, Policy and Equilibrium

We assume that all goods markets clear. Goods can be used as final (consumption) goods and as intermediate inputs in all countries. Therefore, we write the goods market-clearing condition for country-sector ni at time t as:

$$Y_{ni,t} = \sum_{n \in \mathcal{N}} (C_{m,ni,t}) + \sum_{m \in \mathcal{N}} \sum_{j \in \mathcal{J}} (X_{mj,ni,t}), \tag{21}$$

where country m is the consuming country and n is the producing country.

To close the model, we need to specify the market-clearing condition for labor, define aggregate inflation, and specify policy. Policy in each country follows a standard Taylor rule.

$$L_{n,t} = \sum_{i \in J} L_{ni,t} \tag{22}$$

$$\Pi_{n,t} = \frac{P_{n,t}}{P_{n,t-1}} \quad \forall n \in N \tag{23}$$

$$1 + i_{n,t} = (\Pi_{n,t})^{\phi_\pi} e^{\hat{M}_{n,t}} \quad \forall n \in N \tag{24}$$

where $\hat{M}_{n,t}$ is a policy shock.

Definition 1. A non-linear competitive equilibrium for the model is a sequence of 11 endogenous variables $\{C_{nt}, C_{ni,t}, C_{n,mj,t}, X_{ni,mj,t}, X_{ni,j,t}, X_{ni,t}, Y_{ni,t}, L_{ni,t}, L_{n,t}, MC_{ni,t}, B^{US}_{n,t}\}_{t=0}^{\infty}$ and 11 prices $\{P_{nt}, P_{ni,t}, P^C_{ni,t}, P^p_{ni,t}, P^p_{ni,j,t}, P_{n,mi,t}, \Pi_{n,t}, \Pi_{ni,t}, \mathcal{E}_{n,t}, i_{n,t}, W_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\tau_t, A_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (1)-(24) hold for all countries and time periods.

3.6 Steady State with Trade Imbalances and Linearized Model

We linearize the 24 equations above and define an approximated equilibrium in order to use the method of undetermined coefficients and solve the model analytically.

Definition 2. A linearized competitive equilibrium for the model is a sequence of 11 endogenous variables $\{\hat{C}_{n,t}, \hat{C}_{ni,t}, \hat{C}_{n,mj,t}, \hat{X}_{ni,mj,t}, \hat{X}_{ni,j,t}, \hat{X}_{ni,t}, \hat{Y}_{ni,t}, \hat{L}_{ni,t}, \hat{L}_{n,t}, \widehat{MC}_{ni,t}, \hat{B}^{US}_{n,t}\}_{t=0}^{\infty}$ and 11 prices $\{\hat{P}_{nt}, \hat{P}_{ni,t}, \hat{P}^C_{ni,t}, \hat{P}^p_{ni,t}, \hat{P}^p_{ni,j,t}, \hat{P}_{n,mi,t}, \hat{\Pi}_{n,t}, \hat{\Pi}_{ni,t}, \hat{\mathcal{E}}_{n,t}, \hat{i}_{n,t}, \hat{W}_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\hat{\tau}_t, \hat{A}_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (C.4)-(C.27) hold for all countries and time periods.

It is common to linearize international open economy models around a steady state with net zero debt. We take a different approach and allow for asymmetry of the primitive parameters (i.e. home bias and imported intermediate input dependence) across countries, which implies a certain level of debt and net exports at the steady state that has to be consistent with these parameters. This level of steady state debt is then used to parametrize the portfolio adjustment costs that discourage deviations from steady-state levels of debt. In the quantitative section, we discipline the asymmetry of parameters and the steady-state level of debt using the ICIO Table. Further details on this and a scalar example can be found in Appendix D.

Solving the model analytically requires making some simplifying assumptions for our analytical work in Sections 5 and 4. The first simplifying assumption involves adopting elastic labor in the spirit of Golosov and Lucas (2007) preferences. That is we set $\chi = 1$ and the disutility of labor $\eta = 0$, making labor infinitely elastic, which simplifies the intratemporal labor-leisure chocie to: $\hat{W}_{n,t} - \hat{P}_{n,t} = \sigma \hat{C}_{n,t}$. This simplification allows us to focus on consumption in our five-equation Global New Keynesian Representation. In Section 5.2 we additionally assume $\sigma = 1$ and assume nominal demand is fixed, which more closely follows Golosov and Lucas (2007) preferences. While we do not track aggregate output in the Global New Keynesian Representation for brevity of the five-equation representation, one could add it with a sixth equation using market clearing conditions as we detail in Appendix D and we report aggregate output in our figures.

Second, while we assume that portfolio adjustment costs continue to ensure the uniqueness of the steady-state level of debt in the model, numerically we assume $\psi(B^{US}_{n,t}/P^{US}_{n,t}) \to 0$.[9] Real-world debt data exhibits high persistence, and when we calibrate portfolio adjustment costs accordingly, quantitative simulations show that the contribution of this term is small. Third, in the analytical model, for narrative ease, we assume that all shocks other than tariffs are set to zero (e.g., $A_{ni,t} = 0 \ \forall n, i, t$). Finally, we introduce generalized elasticity and weight terms that directly link the lowest-level bundles to the highest-level aggregates, such as:[10]

$$\hat{C}_{nt} = \sum_{m \in N} \sum_{i \in J} \Gamma_{n,mi} \hat{C}_{n,mi,t} = 0$$
$$\hat{C}_{n,mi,t} = -\theta^C_{l,i} \left(\hat{P}^p_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}^C_{ni,t} \right)$$

[9]Portfolio adjustment costs serve as our stationarity-inducing device. In our analytical work, when we forward forward-looking variables, we assume that portfolio adjustment costs, along with a sufficiently high ϕ_π, ensure that in the long run, all real variables return to steady-state levels in response to transitory shocks.

[10]To the first order, bundles presented in Sections 3.2 and 3.3 can be directly linked to the goods that form them. We can write these relations as:

$$\Gamma_{n,mi} = \Gamma_{n,i} \, \Gamma_{n,i,mi},$$
$$\Omega_{ni,mj} = (1 - \alpha_{ni}) \, \Omega_{ni,j} \, \Omega_{ni,j,mj},$$

3.6.1 Vector and Matrix Notation

Given the number of countries and industries involved, we can utilize the matrix form to write the equilibrium conditions. The most important aspect of the matrix notation pertains to deriving the New Keynesian Phillips Curve. To that end, let us consider the linearized producer price inflation equation:

$$\pi^p_{ni,t} = \frac{\theta^P_{l,i}}{\delta_{ni}} \left(\underbrace{\alpha_{ni}\hat{W}_t + \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}(\hat{P}^p_{mj,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mj,t})}_{\widehat{MC}_{ni,t}} - \hat{P}^p_{ni,t} \right) + \beta \mathbb{E}_t \pi^p_{ni,t+1} \tag{25}$$

This can be expressed in vector and matrix notation as follows:

$$\begin{aligned} \underbrace{\boldsymbol{\pi}^P_t}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ} \Bigg(\underbrace{\boldsymbol{\alpha}}_{NJ\times N} \underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} + \underbrace{(\boldsymbol{\Omega} - \boldsymbol{I})}_{NJ\times NJ} \underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times 1} + [\underbrace{\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\mathcal{E}}}_t}_{NJ\times NJ}] \underbrace{\mathbf{1}}_{NJ\times 1} \\ + [\underbrace{\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\tau}}_t}_{NJ\times NJ}] \underbrace{\mathbf{1}}_{NJ\times 1} \Bigg) + \boldsymbol{\beta}\mathbb{E}_t \underbrace{\boldsymbol{\pi}^P_{t+1}}_{NJ\times 1} \end{aligned} \tag{26}$$

where, with some slight abuse of notation, we define the tariff matrix as $\hat{\boldsymbol{\tau}}_{ni,mj,t} \equiv \tau_{n,mi,t}$, the exchange rate matrix as $\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t} = \hat{\mathcal{E}}_{n,m,t}$ and the diagonal matrix of discount rates as $\boldsymbol{\beta}_{nn} \equiv \beta_n$.

Thus, keeping in mind the labor-leisure tradeoff and using the fact that the price level at time t is the past price level plus inflation, we can express producer prices in levels as:

$$\hat{\boldsymbol{P}}^P_t = \underbrace{(\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega}))^{-1}}_{\tilde{\boldsymbol{\Psi}}} \left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda} \left(\boldsymbol{\alpha} \underbrace{\left(\hat{\boldsymbol{P}}^C_t + \sigma \hat{\boldsymbol{C}}_t \right)}_{\hat{\boldsymbol{W}}_t} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}^P_{t+1} \right]$$

where $\tilde{\boldsymbol{\Psi}}$ is a stickiness-adjusted Leontief Inverse.

We can also express the CPI using these matrices. For analytical tractability, we define the $NJ \times 1$ dimensional CPI vector $\boldsymbol{P}^C_t$ such that $\boldsymbol{P}^C_{mi,t} = P^C_{m,t}$. With this, we can write the CPI as:

$$\hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma} \cdot \hat{\boldsymbol{P}}^P_t + [\boldsymbol{\Gamma} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Gamma} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1},$$

where $\boldsymbol{\Gamma}$ is an $NJ \times NJ$ matrix such that $\boldsymbol{\Gamma}_{ni,mj} = \Gamma_{n,mj}$.

The matrix notation makes our expressions compact, generalizable, and useful for computational work. That said, in equilibrium definitions and macroeconomic interpretation, we additionally use vector notation. To that end, consider $\tilde{\boldsymbol{\mathcal{E}}}_t$ as the $N^2 \times 1$ vector of bilateral exchange rates that determine the entries of the matrix $\hat{\boldsymbol{\mathcal{E}}}_t$. Similarly, let $\tilde{\boldsymbol{\tau}}_t = \text{vec}(\boldsymbol{\tau}_t)$ be the vectorized form of $\boldsymbol{\tau}_t$, where $\tilde{\boldsymbol{\tau}}_t$ is an $N^2 J \times 1$ vector. In line with these vector representations, we also use $\mathbf{L}$ to denote loadings (i.e., how the subscript variable loads onto the superscript variable). These expressions compactly describe how vector variables load onto a given equation and serve as partial derivatives. Finally, in the linearized model we define $V_{n,t} = (1 + i^{US}_{n,t})B^{US}_{n,t}$ and linearize this variable. As we do so, we stack the balance of payments equations together with the market clearing condition for U.S. bonds as we detail below.

3.6.2 Global New Keynesian Representation

With the vector and matrix notation established, the full set of linearized equilibrium conditions in Appendix C can be written in vector form as an equilibrium that satisfies the Blanchard-Kahn stability conditions. We use this representation both for interpretation and to solve the model using the method of undetermined coefficients.[11] This five-equation representation is similar in spirit to the canonical three-equation New Keynesian model, if that model were extended to a context with N open economies, including input-output linkages.

Definition 3. A linearized equilibrium comprises vector sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{P}}^C_t, \tilde{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}^{\infty}_{t_0}$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}^{\infty}_{t_0}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (27)-(31) hold:

$$\textbf{NKIS+TR:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}(\hat{\boldsymbol{P}}^C_t - \hat{\boldsymbol{P}}^C_{t-1}) - \mathbb{E}_t(\hat{\boldsymbol{P}}^C_{t+1} - \hat{\boldsymbol{P}}^C_t) \tag{27}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_{\tau}\tilde{\boldsymbol{\tau}}_t \tag{28}$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}^P_t = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}^P_{\mathcal{E}}\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \tag{29}$$

$$\textbf{UIP+TR:} \quad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\tilde{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\tilde{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3(\hat{\boldsymbol{P}}^C_t - \hat{\boldsymbol{P}}^C_{t-1}) \tag{30}$$

$$\textbf{BoP:} \quad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\tilde{\boldsymbol{\mathcal{E}}}_t + \Xi_5\tilde{\boldsymbol{\tau}}_t \tag{31}$$

where "TR" denotes that the Taylor rule has been substituted in, and $\mathbf{L}$ notation represents loadings (i.e., how the subscript variable loads onto the superscript variable as a linear combination of the entries of the vector variable, as detailed above), which also serve as

[11] We depict prices in levels (e.g. $\hat{\boldsymbol{P}}^P_t$) rather than in first differences (e.g. $\boldsymbol{\pi}^P_t$) for two reasons in this representation. First, since prices appear both in levels and in first differences doing so allows us to write an equilibrium with 5 vector variables and 5 vector equations in a compact manner. Second, this representation is convenient for the algebra work we do with the method of undetermined coefficients.

partial derivatives. In the first and fourth of these equilibrium conditions, the Taylor rule is used to substitute out the nominal interest rate, where the diagonal matrix $\mathbf{\Phi}$ contains the Taylor rule's sensitivity to inflation in the respective countries. For example, in the two-country case, we have $\mathbf{\Phi} = \begin{bmatrix} \phi_\pi & 0 \\ 0 & \phi_\pi^* \end{bmatrix}$. That is, we have $\hat{\boldsymbol{i}}_t = \mathbf{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C)$ and the first $N-1$ rows of $\tilde{\mathbf{\Phi}}_3(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C)$ load the vector form of interest rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}_t^{US}$ for countries other than the first country in our system, the U.S.

The first of these equilibrium conditions is the Euler (New Keynesian IS, i.e., NKIS) equation, which is defined in terms of aggregate consumer prices. Intuitively, the impact of tariffs enters the demand side through how tariffs load onto consumer prices.

The second equation defines the consumer price index (CPI). As is typical in network models, the CPI and the producer price index (PPI) differ, with consumer prices being a weighted average of producer prices, exchange rates, and tariffs under our producer currency pricing assumption. Here, $\boldsymbol{L}_{\hat{\mathcal{E}}}^C$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\gamma_{HH})$, where $\gamma_{HH} \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}_\tau^C$ captures the share of goods exposed to tariffs. If one were to momentarily ignore the impact of tariffs on producer prices, their effect via consumer prices would be isomorphic to Euler equation shocks (e.g., discount rate shocks).

The third equation is the New Keynesian Phillips Curve for producer price inflation, defined in levels for convenience in the analytical solution. The impact of the input-output network is captured in the stickiness-adjusted Leontief inverse term $\tilde{\mathbf{\Psi}}$. This term multiplies the diagonal matrix of stickiness parameters, $\mathbf{\Lambda}$, and the matrix of nominal marginal costs. Additionally, $\tilde{\mathbf{\Psi}}$ multiplies both the vector of lagged producer prices, $\hat{\boldsymbol{P}}_{t-1}^P$, and the discounted expectation of future producer prices, $\beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P$. In this setup, the exchange rate loads onto nominal marginal costs via the dependence of producers on imported intermediate inputs, which is captured by $\mathbf{L}_{\hat{\mathcal{E}}}^P$. Similarly, tariffs have a direct impact, as they load onto the share of goods exposed to tariffs, captured by $\mathbf{L}_\tau^P$. If not for their additional impact on consumer prices, tariffs τ would be isomorphic to standard supply shocks in the New Keynesian context.

The fourth equation combines the UIP condition, exchange rate arbitrage conditions, and the definition of a country's exchange rate with itself (i.e., nesting linearized versions of equations (2), (3), and (4)). Here, the $\tilde{\mathbf{\Phi}}$ terms ensure that the ϕ_π terms for each country, along with the arbitrage conditions, are correctly loaded in each row.

The fifth equation combines market clearing for debt with the $N-1$ equations of motion for real debt, capturing the balance of payments as a function of prices, which reflect the terms of trade for each specific country-good variety, and the aggregate consumption vector.[12] This final equation describes how a country's net external position evolves in response to changes in good-specific terms of trade, as well as fluctuations in the interest rate and the balance sheet effect of debt via exchange rates. As such, it nests all the intratemporal relative demand conditions and pricing equations. Through this equation, debt responds to automatic debt dynamics and adjustments in exports following changes in the terms of trade.

This five-equation general representation can nest a broad class of open-economy New Keynesian models. For example, models with a bundle of intermediate inputs and a final good correspond to the case where $\mathbf{\Omega}$ involves $J = 2$, and one of the columns of $\mathbf{\Omega}$ is a column of zeros. This representation is general for N-country New Keynesian models (e.g., Clarida et al., 2002). However, by collapsing the number of countries to one and making the real rate exogenous, it reduces to a small open economy model reminiscent of Galí and Monacelli (2005).

4 Tariffs in the Long Run Under Flexible Prices

The impact of tariffs on our main variables of interest, exchange rate, inflation, output, output gap, trade balance and consumption, are complex and dependent on the primitive parameters. In this section, we start with the flexible-price version of the model to establish intuition. In order to do so, we will focus on a two-country setup ($N = 2$) with an arbitrary number of industries, J. As we detail in Appendix E, our Global New Keynesian Representation yields the following equilibrium under flexible prices:

Definition 4. A linearized equilibrium comprises vector sequences $\{\Delta\hat{\boldsymbol{C}}_t, \boldsymbol{\pi}_t^P, \boldsymbol{\pi}_t^C, \Delta\hat{\mathcal{E}}_t, \Delta\hat{\boldsymbol{V}}_t\}_{t_0}^{\infty}$ for a given sequence of $\{\Delta\hat{\tau}_t\}_{t_0}^{\infty}$ and an initial condition for $\Delta\hat{V}_0$ such that equations (32)-(36) hold:

$$\sigma\mathbb{E}_t\Delta\hat{\boldsymbol{C}}_{t+1} = \mathbf{\Phi}\boldsymbol{\pi}_t^C - \mathbb{E}_t\boldsymbol{\pi}_{t+1}^C \tag{32}$$

$$\boldsymbol{\pi}_t^C = \mathbf{\Gamma}\boldsymbol{\pi}_t^P + \tilde{\boldsymbol{L}}_{\mathcal{E}}^C\Delta\hat{\mathcal{E}}_t + \boldsymbol{L}_{\hat{\tau}}^C\Delta\hat{\tau}_t \tag{33}$$

$$\boldsymbol{\pi}_t^P = \mathbf{\Psi}\left(\boldsymbol{\alpha}\left(\boldsymbol{\pi}_t^C + \sigma\Delta\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\Delta\tilde{\mathcal{E}}_t + \boldsymbol{L}_{\hat{\tau}}^P\Delta\hat{\tau}_t\right) \tag{34}$$

$$\mathbb{E}_t\Delta\hat{\mathcal{E}}_{t+1} = \tilde{\mathbf{\Phi}}_3\boldsymbol{\pi}_t^C \tag{35}$$

$$\beta\Delta\hat{V}_t = \Delta\hat{V}_{t-1} + \mathbf{\Xi}_2\Delta\hat{\boldsymbol{C}}_t + \mathbf{\Xi}_3\boldsymbol{\pi}_t^P + \Xi_4\Delta\hat{\mathcal{E}}_t + \Xi_5\Delta\hat{\tau}_t \tag{36}$$

[12] The first $N - 1$ rows contain linearized versions of equation (19), while the last row captures the bond market clearing condition given by equation (20). In Appendix D, we derive this equation of motion.

In order to understand the long-run impact of tariffs under flexible prices we conside a permanent increase in tariffs, which implies that $\Delta\hat{\tau}_{t+j} = 0 \; \forall j > 0$. Using this, with the method of undetermined coefficients we find:[13]

Proposition 1. *The first period impact of a permanent increase in tariffs under flexible prices on the endogenous variables is as follows:*

$$\frac{\partial \Delta \hat{\mathcal{E}}_t}{\partial \Delta \hat{\tau}_t} = \Delta \hat{\mathcal{E}}_\tau = -\frac{\left(\Xi_2 + \sigma \Xi_3 \boldsymbol{\Psi} \boldsymbol{\alpha}\right) \left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{\alpha} \sigma\right)^{-1} \left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right) - \left(\Xi_3 \boldsymbol{\Psi} \boldsymbol{L}^P_{\hat{\tau}} + \Xi_5\right)}{\left(\Xi_2 + \sigma \Xi_3 \boldsymbol{\Psi} \boldsymbol{\alpha}\right) \left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{\alpha} \sigma\right)^{-1} \left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right) - \left(\Xi_3 \boldsymbol{\Psi} \boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)}$$

$$\underbrace{\frac{\partial \Delta \hat{\boldsymbol{C}}_t}{\partial \Delta \hat{\tau}_t}}_{N \times 1} = \Delta \hat{\boldsymbol{C}}_\tau = -\left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{\alpha} \sigma\right)^{-1} \left(\left(\boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right) \Delta \hat{\mathcal{E}}_\tau + \boldsymbol{\Gamma} \boldsymbol{\Psi} \boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right)$$

$$\underbrace{\frac{\partial \boldsymbol{\pi}^P_t}{\partial \Delta \hat{\tau}_t}}_{NJ \times 1} = \boldsymbol{\Psi} \left[\sigma \boldsymbol{\alpha} \Delta \hat{\boldsymbol{C}}_\tau + \boldsymbol{L}^P_{\mathcal{E}} \Delta \hat{\mathcal{E}}_\tau + \boldsymbol{L}^P_{\hat{\tau}}\right]$$

$$\underbrace{\frac{\partial \boldsymbol{\pi}^C_t}{\partial \Delta \hat{\tau}_t}}_{N \times 1} = \mathbf{0}, \quad \frac{\partial \Delta \hat{V}_t}{\partial \Delta \hat{\tau}_t} = 0$$

Proof. See Appendix E. □

Corollary 1. *Under flexible prices, a permanent shock has zero impact on consumer price inflation.*

This is because prices are flexible and the policy rule only targets inflation. As a result, in response to a permanent shock, the entire adjustment is done by variables other than inflation. Notably, producer price inflation is not zero as relative prices have to adjust. Similarly the exchange rate and consumption respond to tariffs.

Corollary 2. *Under flexible prices, a permanent shock does not change the net debt/asset position of either country denominated in the U.S. Dollar, which is the currency in which both countries save.*

This follows from the fact that $\frac{\partial \Delta \hat{V}_t}{\partial \Delta \hat{\tau}_t} = 0$. Under flexible prices, a permanent shock does not change the trade balance of either country expressed in U.S. Dollars. Note that the

[13] We verify these solutions with the quantitative model and ensure that the solution to the method of undetermined coefficients satisfies Blanchard-Kahn stability conditions. The first order approximation is around a given steady state, whereas a permanent shock will lead to the system settling at a different steady state. The first order solution based on an approximation around the initial steady state may not be valid when considering a permanent change that delivers the system to a new steady state. We confirm with our non-linear solution detailed in Section 8 that the first-order analytical solution here is numerically the same as the non-linear solution.

balance of payments can be summarized as follows from the perspective of the first country, US, whose local currency debt is used to facilitate global savings:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \beta^{-1}(1-\beta)\hat{NX}_t + \hat{i}_t$$

Since $\hat{i}_{n,t} = \phi_\pi \pi^C_{n,t}$ and since $\hat{V}_t = \pi^C_{n,t} = 0\ \forall n,t$, we necessarily have that the USD value of net exports do not change. That is $\hat{NX}_t = 0\ \forall t$. This is in line with the exact local neutrality result of Costinot and Werning (2025) and with the finding of Itskhoki and Mukhin (2025) that the long-run trade balance is determined by the financial position of a country.[14] Note that this does not rule out changes in quantities; trade balance in terms of quantities or expressed as a share of GDP can change, while the U.S. dollar value of net exports will remain constant. The intuition here is that in the presence of a permanent shock and flexible prices, the tariffs do not present an intertemporal tradeoff. In line with the permanent income hypothesis, the entire adjustment is done in quantities and prices, while debt is not utilized. As a corollary, then, the USD value of net exports does not change.

Remark 2. The impact on consumption is dependent on the response of the exchange rate to tariffs.

This follows from the first equation in Proposition 1, where we see that the impact of tariffs on consumption depend on $\Delta\hat{\mathcal{E}}_\tau$. If the the weighted sum of the entries for the first country in $\left(\left(\boldsymbol{\Gamma\Psi L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right)\Delta\hat{\mathcal{E}}_\tau + \boldsymbol{\Gamma\Psi L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right)$ were to be negative (i.e., appreciation and the terms of trade gain combined make it easier for home country to afford goods) and sufficiently large in magnitude then home country consumption could increase.

4.1 Scalar Example with One Industry ($N = 2$ & $J = 1$)

Let us now consider the scalar case for additional intuition. In order to do so, we set $J = 1$ and assume away self use by each industry. Then the matrices at hand will look as follows, when expressed in terms of the primitives:[15]

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \Omega_H \\ \Omega_F & 0 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 1-\Omega_H & 0 \\ 0 & 1-\Omega_F \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1-\gamma_H & \gamma_H \\ \gamma_F & 1-\gamma_F \end{bmatrix}$$

$$\boldsymbol{\Psi} = (\mathbf{I}-\boldsymbol{\Omega})^{-1}, \quad \tilde{\boldsymbol{L}}^C_{\mathcal{E}} = \begin{bmatrix} \gamma_H \\ -\gamma_F \end{bmatrix}, \quad \boldsymbol{L}^C_{\hat{\tau}} = \begin{bmatrix} \gamma_H L^C_{\hat{\tau}} \\ \gamma_F L^C_{\hat{\tau}} \end{bmatrix}, \quad \boldsymbol{L}^P_{\mathcal{E}} = \begin{bmatrix} \Omega_H \\ -\Omega_F \end{bmatrix}, \quad \boldsymbol{L}^P_{\hat{\tau}} = \begin{bmatrix} \Omega_H L^P_{\hat{\tau}} \\ \Omega_F L^P_{\hat{\tau}} \end{bmatrix}$$

[14] Itskhoki and Mukhin (2025) emphasize the gross position of the tariff-imposing country. While our modeling framework allows for countries to accumulate debt or assets in more than one currency, in our analytical and quantitative work we restrict countries to saving/dissaving in the dollar.

[15] To make the notation easier to follow in the scalar case we simplify subscripts such that $\gamma_{H,F}$ becomes γ_H and $\Omega_{H,F}$ becomes Ω_H.

where $L^C_{\hat{\tau}}$ and $L^P_{\hat{\tau}}$ are dummy variables that take on the value 0 or 1, indicating whether a given country imposes tariffs on the other one.

The first case to consider involves symmetry in parameters and symmetric retaliatory tariffs by both sides. Given symmetry we drop subscripts such that $\Omega_H = \Omega_F = \Omega$ and $\gamma_H = \gamma_F = \gamma$.

Corollary 3. *Under symmetric parameters and retaliation, the impact of tariffs on consumption and the exchange rate is:*

$$\frac{\partial \Delta C_{H,t}}{\partial \Delta \tau_t} = \frac{\partial \Delta C_{F,t}}{\partial \Delta \tau_t} = -\frac{1}{\sigma}\left[\gamma(1+\Delta\hat{\mathcal{E}}_\tau) + \frac{\Omega}{1-\Omega}\right] < 0$$

$$\frac{\partial \Delta \hat{\mathcal{E}}_t}{\partial \Delta \hat{\tau}_t} = \Delta\hat{\mathcal{E}}_\tau = 0$$

When parameters are symmetric and tariffs involve symmetric retaliation, the exchange rate response is zero. This, in turn, implies that a contraction in consumption by both countries is guaranteed. Import dependence both on the consumption side and production side sharpen this decline in consumption.

Next we consider the case where parameters are asymmetric across the two countries but there is no retaliaton; tariffs are only placed by H on F.

Corollary 4. *Under asymmetric parameters and no retaliation, the impact of tariffs on consumption is:*

$$\frac{\partial \hat{C}_{H,t}}{\partial \tau_t} = -\frac{(\Omega_H(1-\gamma_H)+\gamma_H)(\Delta\hat{\mathcal{E}}_\tau(1+\gamma_F-\gamma_H)+1-\gamma_H)}{\sigma(1-2\gamma_H)(1-\Omega_H)}$$

With the home bias assumption under which γ_H and γ_F are less than 1/2 and given boundary $\Omega < 1$ we can sign this expression. For tariffs to expand consumption a sufficiently large appreciation of the home country's currency is needed:

$$-\Delta\hat{\mathcal{E}}_\tau > 1 - \frac{\gamma_F}{1+\gamma_F-\gamma_H}$$

Two observations are noteworthy here. The first is that the rest of the world's parameters matter beyond picking export and import elasticities, when considering tariffs by the home country on the foreign country. This is in contrast with the small open economy approach. Secondly, the solution for the exchange rate turns into a complex object as soon as one leaves the case of symmetry combined with symmetric retaliation. In Appendix E, we show that the solution for the exchange rate is as follows under the symmetry assumption:

$$\Delta\hat{\mathcal{E}}_\tau = -\frac{\overbrace{((\theta+2\gamma)(1-2\gamma)+\Omega(1-2\gamma^2))\Omega}^{\text{Impact via } L^P_\tau>0} + \overbrace{(\theta(1-2\gamma)+2\gamma(1+\Omega(1+\theta))+\Omega^2\theta)\gamma}^{\text{Impact via } L^C_\tau>0}}{\underbrace{\Big[\Omega^2(1-2\gamma^2)+4\gamma^2+\Omega 6\gamma\Big]+2\theta\Big[\gamma(1-2\Omega)+\Omega(1++2\gamma^2+\Omega\gamma)\Big]}_{>0} - \underbrace{\Big[\theta 4\gamma^2+\Omega(1+4\gamma^2)\Big]}_{>0}}$$

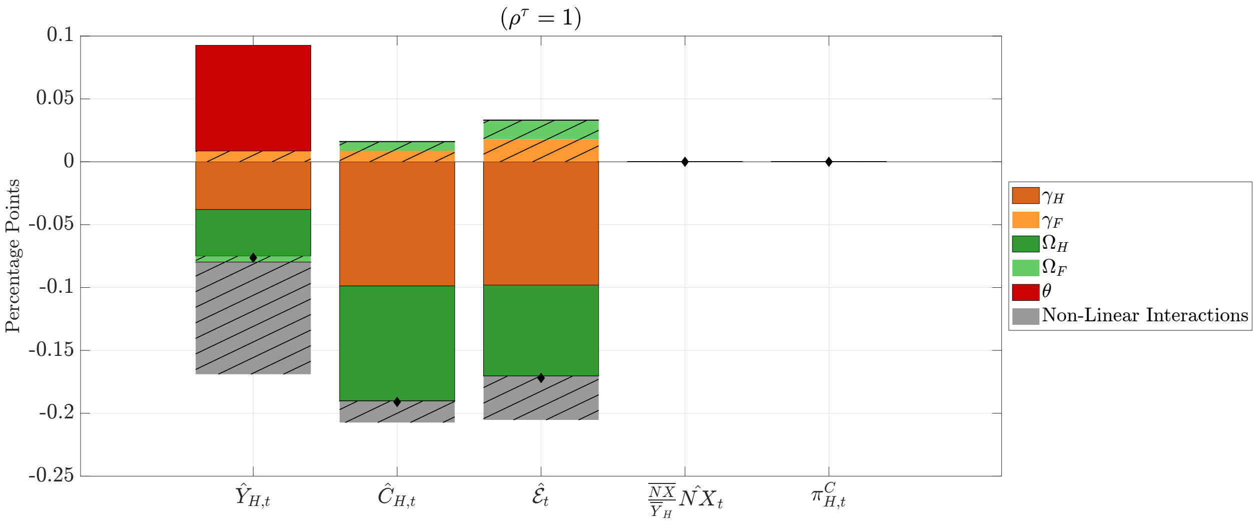
This implies that the impact of consumption and production tariffs have the same sign on the exchange rate and that the overall sign of the exchange rate is determined by the denominator. As we show in Appendix E, this implies that there is a range for the parameters θ, Ω and γ that result in depreciation. This particular result is dependent on the simplifying assumption of portfolio adjustment costs being set to zero. That is when the net external debt position of a country is allowed to follow a random walk, and when θ, Ω and γ are sufficiently low clearing the balance of payments can require depreciation. Assuming $\psi > 0$, however, rules out this range of outcomes.

As is evident in the expressions above, while the solution is linear in the state variables it is not necessarily linear in the parameters. Since the solved out terms can involve mathematically long expressions, below we visualize how the solution changes in response to changes in the primitives at hand: θ, Ω_H, Ω_F, γ_H and γ_F. That is we initialize these parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$ and look at changes in home country's macroeconomic variables of interest in the period of impact for a 10% tariff imposed by the home country on the foreign country, as one varies one parameter at a time. Each primitive's contribution comes from comparing the baseline results to the case when that primitives is set to 0.

Figure 1 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Specifically, to calculate contributions, we set each primitive of interest to 0 and recompute the outcome variables in that case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section A, we plot bivariate plots that show these impacts are monotonic and that is why, we interpret these as contributions.

In Figure 1, we see that consumption is declining in both γ_H and Ω_H, while they are increasing in the foreign country's parameters. The exchange rate appreciates in response to tariffs. This appreciation is stronger as one lowers the home bias in consumption and production for the home country. The intuition here is that as once increases Ω_H and γ_H, H becomes a larger buyer of goods produced by F and thus one has a larger change in the relative demand for F's goods, which in turn leads to a larger appreciation. This appreciation is not large enough to flip the sign of consumption into positive territory. Output is mostly responsive to the elasticity of substitution which allows both production and consumption to respond to prices in both countries.[16] Output is declining γ_H and γ_H, while it is not significantly responsive to foreign country parameters.

Figure 1. Contribution of Primitives to Macro Aggregates Under Flexible Prices



NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section A, we plot bivariate plots that show these impacts are monotonic and that is why, we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. This figure is consistent with our analytical work and simulations in Dynare.

5 Tariffs in the Short Run Under Sticky Prices

Having reviewed the impact of the first three of the five primitive factors, we now turn to the impact of the remaining two. That is, in this section, we add nominal rigidity in the form of sticky prices and policy. These additions change the impact of the first three primitives as well. To provide notational ease, in the $N = 2$ & $J = 1$ case the primitives we are adding

[16]Additionally, while output is solved out from the five-equation representation, we can compute it based on the solution of other variables. Thus, output as a variable of interest is included in Figure A.5.

correspond to the following matrices and scalar objects:

$$\mathbf{\Lambda} = \begin{bmatrix} \Lambda_H & 0 \\ 0 & \Lambda_F \end{bmatrix}, \mathbf{\Phi} = \begin{bmatrix} \phi_\pi^H & 0 \\ 0 & \phi_\pi^F \end{bmatrix} \tag{37}$$

To see these first, in Section 5.1 we start with the special case when there is a real rate rule that fixes consumption in all countries of interest. Next, in Section 5.2, we develop the case when policy fixes nominal demand, and the pressure from tariffs is shared equally by the the aggregate price level and aggregate consumption within each country. Finally in Section 5.3, we provide an analytical solution to our model with the standard Taylor Rule specified in Section 3.

In line with our two main research questions, our goal is twofold. First, we consider these cases to see what happens to macroeconomic aggregates in response to tariffs if monetary policy targets quantities (e.g. consumption in Section 5.2), or prices (e.g. inflation targeting in Section 3), or a mix of both (e.g., fixing nominal demand in Section 5.2). Second, we explore how network propagation changes under different policy regimes. To capture propagation, we develop New Keynesian Open Economy Leontief Inverse matrices for the last two cases, which allows us to go from scalar variables to the matrix scale, where the primitive parameters form non-linear interactions and cross-sectoral heterogeneity can amplify or mute impacts. Throughout this section our approach involves solving for inflation using the method of undetermined coefficients, and having solved prices we then analyze quantities as well. In the network setup, the NKPC equation contains both the lag and the expectation of sectoral prices. This leads to fixed point problems that are analytically hard to solve, even as numerical solutions are easy to obtain and verify. To arrive at analytical expressions throughout the section we make simplifying assumptions regarding policy, which allow us to solve parts of the model by forwarding one equation at a time.

We find that network propagation is different under different policy regimes. Under a real rate rule that stabilizes consumption, tariffs lead to depreciation. Of the primitives of the foreign country, only γ_F enters the solution for home country's inflation. This is in marked contrast with the case when policy fixes nominal demand; this renders inflation in each sector and each country weakly positive. The intuition behind this result is that the policy choice when combined with Golosov and Lucas (2007) preferences fixes nominal wages and the nominal exchange rate. Then tariffs act as a cost-push shock and a cost-push shock in one part of the network propagates as a cost-push shock in all parts of the network. Finally, under a standard Taylor rule we find propagation is more flexible and inflation need not be strictly positive in all sectors and countries.

5.1 Macroeconomic Outcomes Under a Real Rate Rule

Let us now assume that the policy rule in each country follows a real rate rule:

$$\hat{i}_{n,t} = \phi_\pi E_t \pi^C_{n,t+1}$$

where $\phi_\pi \to 1$. Having a constant real rate rule with a temporary shock, sets the path of consumption at zero ($\hat{\boldsymbol{C}}_t = \mathbf{0}$), which in turn implies a constant real exchange rate. This in turn implies that the exchange rate is $\hat{\mathcal{E}}_t = \hat{P}^C_{H,t} - \hat{P}^C_{F,t} = \underbrace{[1 - 1]}_{\mathbf{z}} \hat{\boldsymbol{P}}^C_t$.

Since we solve the model in vector notation in Appendix F, in this section, we focus on the case with $N = 2$ and $J = 1$ for intuition.

Proposition 2. *When $N = 2$ and $J = 1$, under a real rate rule in both countries that perfectly stabilizes consumption, the solution to the system following a tariff by the home country on the foreign country, which follows an AR(1) process of $\hat{\tau}_t = \rho^\tau \tau_t + \epsilon^\tau_t$ is as follows:*

$$\begin{aligned}
\hat{P}^P_{H,t} &= \hat{P}^P_{H,t-1} + [1-\beta\rho]^{-1}\Lambda_H \left(\frac{\gamma_H [1 - \gamma_F(1-\Omega_H)]}{1-\gamma_F-\gamma_H} L^C_\tau \; + \; \Omega_H \, L^P_\tau \right) \hat{\tau}_t \\
\hat{P}^P_{F,t} &= \hat{P}^P_{F,t-1} - [1-\beta\rho]^{-1}\Lambda_F \left(\frac{\gamma_H [(1-\Omega_F)\gamma_F + \Omega_F]}{1-\gamma_F-\gamma_H} L^C_\tau \right) \hat{\tau}_t \\
\hat{P}^C_{H,t} &= \hat{P}^P_{H,t} + \frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t \\
\hat{P}^C_{F,t} &= \hat{P}^P_{F,t} - \frac{\gamma_F\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t \\
\hat{\mathcal{E}}_t &= \hat{P}^P_{H,t} - \hat{P}^P_{F,t} + \frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t
\end{aligned}$$

Proof. See Appendix F. □

Corollary 5. *From the tariff-imposing home country's perspective inflation on impact will be:*

$$\frac{\partial \hat{P}^C_{H,t}}{\partial \hat{\tau}_t} = \left([1-\beta\rho]^{-1} \Lambda_H \cdot \frac{\gamma_H [1-\gamma_F(1-\Omega_H)]}{1-\gamma_F-\gamma_H} + \frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} \right) L^C_\tau + [1-\beta\rho]^{-1} \Lambda_H \cdot \Omega_H L^P_\tau$$

Remark 3. When $\gamma_H < \frac{1}{2}$,$\gamma_F < \frac{1}{2}$, the sign of $\frac{\partial \hat{P}^C_{H,t}}{\partial \hat{\tau}_t}$ is unambigously positive since by construction $\Omega_H < 1$ and $\Omega_F < 1$. This shows the stagflationary impact of tariffs, because the policy that stabilizes consumption is unambiguously inflationary in the tariff imposing country.

When expressed in terms of first differences:

$$\pi^C_{H,t} = \underbrace{[1-\beta\rho]^{-1}\Lambda_H \left(\frac{\gamma_H\left[1-\gamma_F(1-\Omega_H)\right]}{1-\gamma_F-\gamma_H}\,L^C_\tau \;+\; \Omega_H\,L^P_\tau\right)\hat{\tau}_t}_{\pi^P_{H,t}} + \frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \Delta\hat{\tau}_t$$

Remark 4. The term $[1-\beta\rho]^{-1}$ is increasing in ρ. At the limit as $\rho \to 1$, this term grows very large when β is close to 1. This indicates that there is a permanent and high inflationary cost to permanent tariffs that are executed under perfect consumption stabilization. This comes in addition to the one-time increase in inflation that comes from the $\Delta\hat{\tau}_t$ term.

Corollary 6. *The impact of tariffs on the exchange rate is depreciationary under a real rate rule.*

$$\frac{\partial\hat{\mathcal{E}}_t}{\partial\hat{\tau}_t} = \frac{\partial\hat{P}^C_{H,t}}{\partial\hat{\tau}_t} - \frac{\partial\hat{P}^C_{F,t}}{\partial\hat{\tau}_t} = \frac{\partial\hat{P}^P_{H,t}}{\partial\hat{\tau}_t} - \frac{\partial\hat{P}^P_{F,t}}{\partial\hat{\tau}_t} + \frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau > 0$$

This result hinges on the fact that tariffs reduce demand for foreign goods and increase demands for home goods. This creates inflation at home and deflation abroad. When the real exchange rate is fixed because both countries follow a real rate rule, as a result the nominal exchange rate which follows the difference in the two price indices will move in a positive direction.

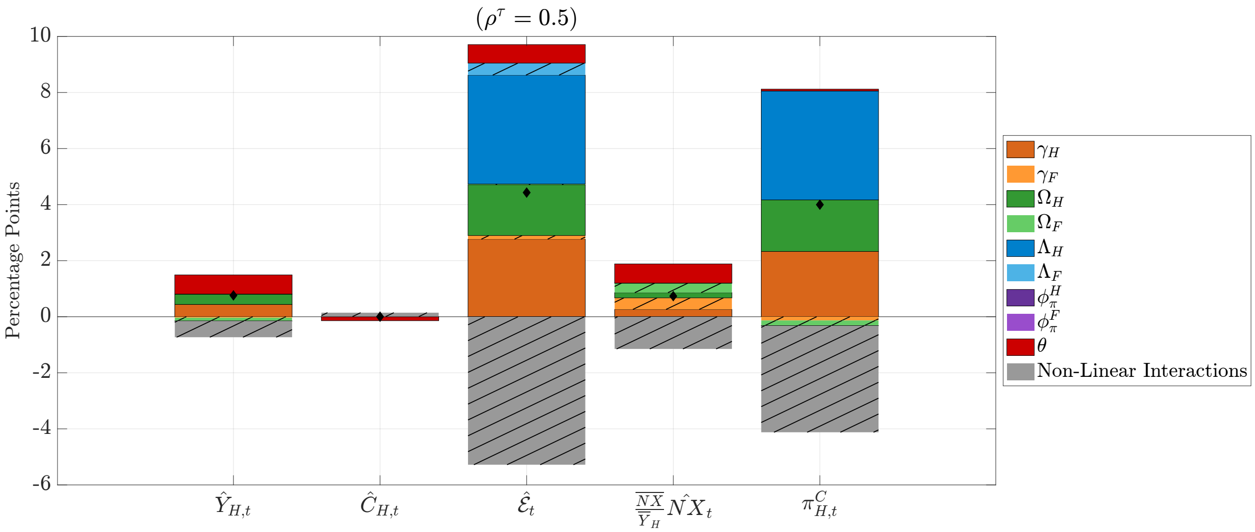
As in the previous section, using this analytical solution, we can visualize the contribution of primitives to macreconomic aggregates in Figure 2.[17] This version of the model shows that the impact of all five primitives can change in the short run once rigidity and policy is introduced. Now the exchange rate depreciates and the rate of depreciation is increasing (leading to further depreciation) in Ω_H and γ_H. The real rate rule fixes consumption and the real exchange rate so the foreign country's parameters matter less for inflation and consumption; however, they do matter for output, exchange rate and net exports. All the primitives provide positive impulse to the variables of interest, excluding non-linear interactions. Relying more on the foreign country on the consumption (γ_H) or prouction side (Ω_H) implies that the policy that stabilizes consumption involves stimulating demand in an inflationary and depreciationary manner to make up for lost consumption and production. We see the expenditure switching channel at play. At the cost of inflation and depreciation, the tariff-imposing home country can achieve an increase in output that stabilizes consumption, increases output and improves the trade balance.

Two primitives are added here relative to the earlier sections: stickiness and policy.

[17] Additionally we visualize the parameter sensitivity of the impact of tariffs in Figure A.6.

Figure 2 demonstrates that increasing price flexibility positively contributes to depreciation and inflation. The intuition at hand is that a higher Λ corresponds to a more vertical supply curve; as a corollary the depreciation and inflation that is necessary to achieve consumption stabilization in creases. The policy primitive at hand fixes consumption and thereby has a significant impact on other variables.[18]

Figure 2. Contribution of Primitives to Macro Aggregates Under Real Rate Rule



NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section A, we plot bivariate plots that show these impacts are monotonic and that is why, we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. The real rate rule $(\hat{i}_{n,t} = \phi_\pi E_t \pi^C_{n,t+1})$ represents a knife-edge case for determinacy. To ensure it holds, in this visualization we approach $\phi_\pi \to 1$ from the right and numerically remain above 1 by a small amount. This numerical departure from $\phi_\pi = 1$ is why $\hat{C}_{H,t}$ is not exactly zero and Ω_F is present in the decomposition for $\pi^C_{H,t}$ in this figure, whereas it does not appear in the analytical solution.

It is worth remarking on the impact on net exports. What explains the fact that the trade balance moves into positive territory, whereas in Section 4 remained at steady state levels?

[18] Since the primitive does not involve ϕ in this case, ϕ terms does not contribute to macro aggregates.

Remark 5. Two facts in the setup of the model specifications in Section 5 create an intertemporal tradeoff that differs from the setup in the earlier section. First, in this section we study transitory shocks. Second, whereas the earlier setup involved flexible prices, in this Section the model has nominal rigidity. The fact that tariff rates in place today will decline in the future and the fact that price changes will take time to work their way through the system combined lead households to smooth consumption across time. This allows the trade balance to deviate from steady-state levels for a period of time in response to a transitory tariff shock.

5.2 Macroeconomic Outcomes Under Fixed Nominal Demand

In this subsection, we replace the Taylor rule with the equation: $\hat{P}_t + \hat{C}_t = \hat{M}_t$ which fixes nominal domestic demand. Additionally we set $\sigma = 1$ and this simplification, we obtain $\hat{W}_{n,t} = \hat{M}_{n,t} = \hat{P}_{n,t} + \hat{C}_{n,t}$. This approach is similar to menu cost models such as Golosov and Lucas (2007); Caratelli and Halperin (2023) and can be microfounded using a cash-in-advance constraint. [19]

The economic interpretation is that with an exogenous $\hat{M}_{n,t}$, policy sets the overall aggregate domestic demand stance, similar to earlier generations of models such as Salter-Swan (Swan, 1963; Salter, 1959). In a closed-economy setting, the policy rule would be analogous to nominal GDP targeting. As the Directed Acyclic Graph (DAG) representation in Figure G.1 illustrates how wages and the nominal exchange rate can be solved when $\hat{M}$ is fixed. Using $\hat{M}$, we first solve the nominal exchange rate, then derive the price and inflation vectors, which in turn determine all quantities.

5.2.1 Analytical Solution for Arbitrary N and Arbitrary J

In Appendix G we show that, under the assumption that tariff shocks and policy shocks are one-time shocks and that portfolio adjustment costs are strictly positive but numerically small, fixing nominal demand yields (i) a purely monetary exchange-rate equation and (ii) a forward-looking NKPC that embeds the production network. That is, fixing nominal demand renders the exchange purely a function of the differing monetary stances of country pairs ($\hat{\mathcal{E}}_{n,m,t} = \hat{M}_{n,t} - \hat{M}_{m,t}$). Defining the stickiness-adjusted Leontief inverse for the producer price inflation equation as $\tilde{\boldsymbol{\Psi}}_\pi = [\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I})]^{-1}$, we obtain the global NKPC for producer

[19] Since money balances drop from the balance of payments due to the government's budget constraint, we do not explicitly model money in the utility function. This approach can also be microfounded by incorporating money in the utility function.

price inflation:[20]

$$\boldsymbol{\pi}_t^P = \underbrace{\tilde{\boldsymbol{\Psi}}_\pi \boldsymbol{\Lambda}}_{\substack{\text{Propagation under}\\ \text{stickiness}}} \left[\underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})}_{\substack{\text{Policy impact}\\ \text{via Wages and ER}}} \hat{\boldsymbol{M}}_t + \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\text{Tariff incidence}} - \underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{P}_{t-1}^P}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \underbrace{\beta \boldsymbol{\Lambda}^{-1}\mathbb{E}_t \boldsymbol{\pi}_{t+1}^P}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right] \tag{38}$$

The term $(\boldsymbol{I}-\boldsymbol{\Omega})$ in front of $\hat{\boldsymbol{M}}_t$ consists of the summation of two components: (i) $\boldsymbol{\alpha}$, which arises from the demand channel via an increase in wages, and (ii) $(\boldsymbol{I}-\boldsymbol{\alpha}-\boldsymbol{\Omega})$, which originates from the exchange rate channel that raises input prices.

Nominal domestic demand policy affects producer price inflation through two channels: first, via the demand channel, and second, via the exchange rate channel. Since the labor-leisure tradeoff simplifies to $\hat{W}_t - \hat{P}_t = \hat{C}_t$ under the given parametrization, and since nominal wages depend on $\hat{M}_t$, stimulative demand policy increases labor supply. Through the exchange rate channel, stimulating domestic demand beyond its steady-state level results in depreciation, which raises firms' marginal costs by increasing the price of imported intermediate inputs.

Applying the method of undetermined coefficients to (38) we arrive at Proposition 3.

Proposition 3. *With future shocks set to zero such that (i.e. $\tau_{t+j} = \hat{M}_{t+j} = \hat{M}^*_{t+j} = 0\ \forall j > 0$) the solution for producer price inflation is:*

$$\boldsymbol{\pi}_t^P = \tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega})\hat{\boldsymbol{M}}_t + \tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1} + (\tilde{\boldsymbol{\Psi}}^{NKOE} - \boldsymbol{I})\boldsymbol{P}_{t-1}^P \tag{39}$$

where $\tilde{\boldsymbol{\Psi}}^{NKOE}$ is the NKOE Leontief inverse in this context. It transforms the stickiness-adjusted Leontief inverse by diagonalizing it and solving a quadratic equation to determine the matrix in front of the lagged vector $\boldsymbol{P}_{t-1}^P$, denoted as $\mathbf{C}_3$. The NKOE Leontief inverse $\tilde{\boldsymbol{\Psi}}^{NKOE}$ is constructed by taking this solution $\boldsymbol{C}_3$ and multiplying it as follows: $\tilde{\boldsymbol{\Psi}}^{NKOE} = \boldsymbol{Q}\boldsymbol{C}_3\boldsymbol{Q}^{-1}$, where $\boldsymbol{Q}$ diagonalizes $\tilde{\boldsymbol{\Psi}}_\pi$. In this formulation, $\tilde{\boldsymbol{\Psi}}^{NKOE}$ also serves as the matrix that multiplies the lagged price vector when producer price inflation is expressed in levels.

Proof. See Appendix G. □

Corollary 7. *The impact of a one-time tariff on the producer price inflation vector under price stickiness is:*

$$\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t} = \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE}}_{\substack{\textit{NKOE Leontief}\\ \textit{inverse}}} \underbrace{\boldsymbol{\Lambda}}_{\textit{Stickiness}} \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\substack{\textit{Tariff}\\ \textit{incidence}}} \tag{40}$$

[20] As described in Appendix G, we construct an $NJ \times 1$ dimensional vector $\hat{\boldsymbol{M}}_t$ by stacking each country's nominal demand change such that $\hat{\boldsymbol{M}}_{ni} = \hat{M}_n$.

where $\tilde{\boldsymbol{\Omega}}^F$ *is a* $NJ \times 1$ *vector whose elements are the row sum of the foreign elements of* $\boldsymbol{\Omega}$*.*

We can compare this with the impact under flexible prices:

$$\frac{\partial \boldsymbol{\pi}_t^{P,flex}}{\partial \tau_t} = \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}}_{\boldsymbol{\Psi}=\text{Leontief inverse}} \quad \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\text{Tariff incidence}} \tag{41}$$

Two points are noteworthy here. First, since aggregate nominal demand—and consequently the exchange rate—is determined by policy, tariffs have no impact through the nominal exchange rate in this setup. However, the real exchange rate and the terms of trade do depend on tariffs. Second, compared to the flexible-price expression (41) under price stickiness, it is the propagation mechanism that changes.

Remark 6. Equation (40) captures the core intuition: in DGE, network propagation ($\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}$) can amplify or mute the impact of tariffs in a given sector beyond what is implied by the raw sectoral shares. These matrices in turn comprise the primitives, Ω and Λ.

With the solution for NKPC in place, we can write the solution for CPI inflation is as follows:

Corollary 8. *With future shocks set to zero such that (i.e.* $\tau_{t+j} = \hat{M}_{t+j} = \hat{M}^*_{t+j} = 0 \; \forall j > 0$*) the solution for producer price inflation is:*

$$\begin{aligned} \boldsymbol{\pi}_t^C = & \left(\boldsymbol{\Gamma} \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}}_{\substack{\text{NKPC} \\ \text{propagation}}} \quad \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{via Wages and} \\ \text{via ER for producers}}} + \underbrace{(\boldsymbol{I} - \boldsymbol{\Gamma})}_{\text{via ER for consumers}} \right) \hat{\boldsymbol{M}}_t \\ & + \left(\boldsymbol{\Gamma} \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}}_{\substack{\text{NKPC} \\ \text{propagation}}} \quad \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence} \\ \text{for Producers}}} + \underbrace{[\boldsymbol{\Gamma} \odot \hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence} \\ \text{for consumers}}} \right) \mathbf{1} \\ & + \underbrace{\boldsymbol{\Gamma} \left(\tilde{\boldsymbol{\Psi}}^{NKOE} - \boldsymbol{I} \right) \hat{\boldsymbol{P}}_{t-1}^P}_{\text{Impact of lagged prices}} \end{aligned} \tag{42}$$

Proof. See Appendix G ☐

As seen above in Equation (42), policy and tariffs affect consumer price inflation through two channels: first, via producer prices, and second, through the exchange rate and tariffs that convert a producer price into a consumer price. A helpful interpretation of the expression above is that the terms labeled "NKPC Propagation" illustrate how the production network propagates shocks in a forward-looking setup, whereas the other terms represent the first-order impacts. For example, when a τ_t% tariff is imposed, these terms capture what share of the consumption basket is affected, considering both its indirect effect through producers' input baskets and its direct effect on consumers' consumption baskets.

Proposition 4. *The impact of a one-time tariff* ($\tau_t \geq 0$) *on consumer price inflation is always weakly positive under fixed nominal demand. That is, let* $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t}$ *be an* $NJ \times 1$ *vector such that* $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$.

Proof. We can derive the necessary derivative from (42) as follows:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{L}_\tau^C + \boldsymbol{\Gamma} \tilde{\boldsymbol{\Psi}}^{NKOE} \boldsymbol{\Lambda} \boldsymbol{L}_\tau^P \tag{43}$$

In this context, $\boldsymbol{L}_\tau^P = \tilde{\boldsymbol{\Omega}}^F$ and $\boldsymbol{L}_\tau^C = \tilde{\boldsymbol{\Gamma}}^F$ correspond to the row sums of the foreign elements in intermediate inputs and final consumption, respectively. All matrices on the right-hand side of Equation (43) contain weakly positive entries. As a result, $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$.

This is the case because $\boldsymbol{\Lambda}$ has weakly positive entries by construction and $\tilde{\boldsymbol{\Psi}}^{NKOE}$ is a sign-preserving transformation of the stickiness-adjusted Leontief inverse, $\tilde{\boldsymbol{\Psi}}_\pi = [\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I})]^{-1}$ and matrices like the standard Leontief inverse will have weakly positive entries since one can express the matrix $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ as the following Neumann series:

$$(\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{k=0}^{\infty} \boldsymbol{\Omega}^k.$$

Each power $\boldsymbol{\Omega}^k$ has nonnegative entries, implying that $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ also has nonnegative entries. The term $\tilde{\boldsymbol{\Omega}}^F$ also retains nonnegative entries. Since $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ is an $NJ \times NJ$ matrix with nonnegative entries and $\tilde{\boldsymbol{\Omega}}^F$ is an $NJ \times 1$ vector with nonnegative entries, their product is an $NJ \times 1$ vector with nonnegative entries. Thus, every entry of $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t}$ is weakly positive.

□

Proposition 4 demonstrates that tariffs imposed by any country is inflationary for all countries in a setup where nominal demand is fixed. This is the case because with the nominal exchange rate and wages fixed by policy, the distortion from tariffs in one country propagates as an added increase in the cost of goods made in another country. The conclusion of this proposition also extends to producer prices; an increase that serves as a cost-push in one place translates to weakly increase prices in every country-industry combination.

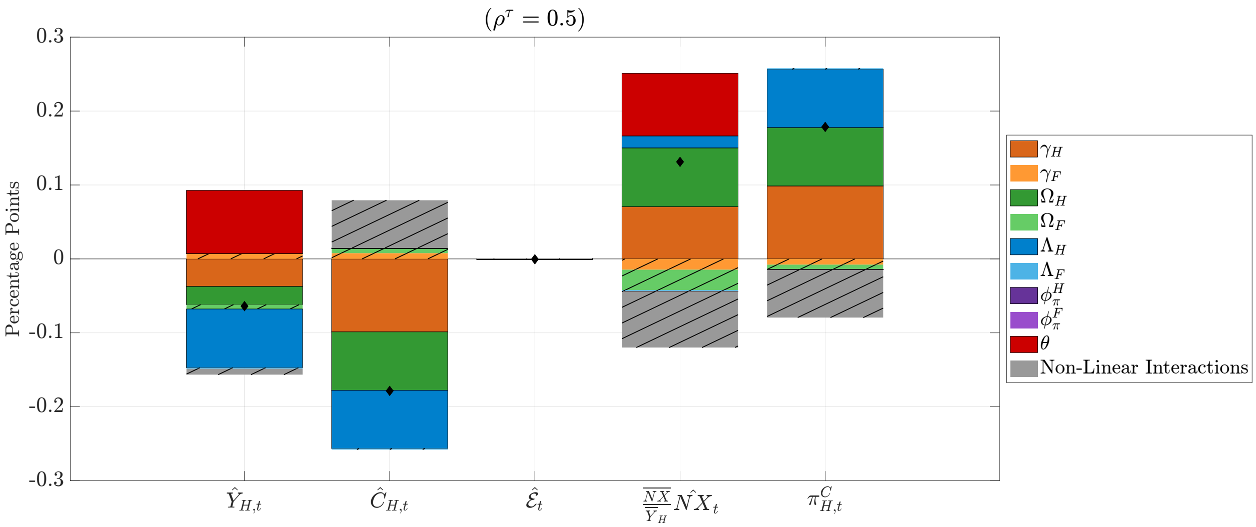
Corollary 9. *Under flexible prices (efficient allocation), impact of tariffs on consumer prices consists of the following direct effects through the consumption basket and producer's input* *basket:*

$$\frac{\partial \boldsymbol{\pi}_t^{C^{flex}}}{\partial \tau_t} = \boldsymbol{L}_\tau^C + \boldsymbol{\Gamma\Psi} \boldsymbol{L}_\tau^P \tag{44}$$

and the difference between Equation (43) *and Equation* (44) *yields the allocative efficiency term.*

5.2.2 Scalar Example with One Industry ($N = 2$ & $J = 1$)

Figure 3. Contribution of Primitives to Macro Aggregates Under Fixed Nominal Demand



Note: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section A, we plot bivariate plots that show these impacts are monotonic and that is why we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. This figure is consistent with our analytical work and simulations in Dynare.

In Figure 3, we plot the linear contribution of the primitives, in the $N = 2$ and $J = 1$ case. Above we have analytically solved for exchange rate and prices, showing that the former is fixed and the latter will be weakly positive. The figure below demonstrates that γ_H and Ω_H positively contribute to inflation and net exports, while they negatively contribute to output and consumption. Elasticity of substitution θ contributes positively to production and net exports; however, it does not to contribute to other variables meaningfully. Home country's price stickiness parameter Λ_H indicates that higher flexibility of prices contributes positively to inflation and negatively to output and consumption. Differing from the case in the real rate rule, the rest of the world's parameters work in the opposite direction.

When studying the impact of tariffs, monetary policy stance is fixed. That is $\hat{P}^C_{H,t} + \hat{C}_{H,t} = \hat{M}_t = 0$. This explains why and how $\hat{C}_{H,t}$ is the inverse image of $\hat{P}^C_{H,t}$. Once one solves prices, that then allows one to solve for consumption. With prices and consumption solved, production quantities (and net exports) adjust to make markets clear across the two countries. In that response a higher elasticity of substitution allows home country's production (and net exports) to respond more strongly.

As is evident in this example, fixed nominal demand offers a restrictive setting, where aggregate demand's response to tariffs and the exchange rate are fixed. That is tariffs have no impact on aggregate nominal demand or the exchange rate, and these effects are linearly separable with no interactions between them. In such a setup, tariffs are always expected to be inflationary. As we show with the DAG representation in Appendix G, this is a setup in which knowing tariffs and the nominal demand stance of policy allows one to determine all other variables in the system. While this is useful from a modeling point of view and is one of the reasons this approach is common in production network models, important two-way interactions in DGE are ruled out. To study these, we now turn to a setting where policy follows a Taylor Rule.

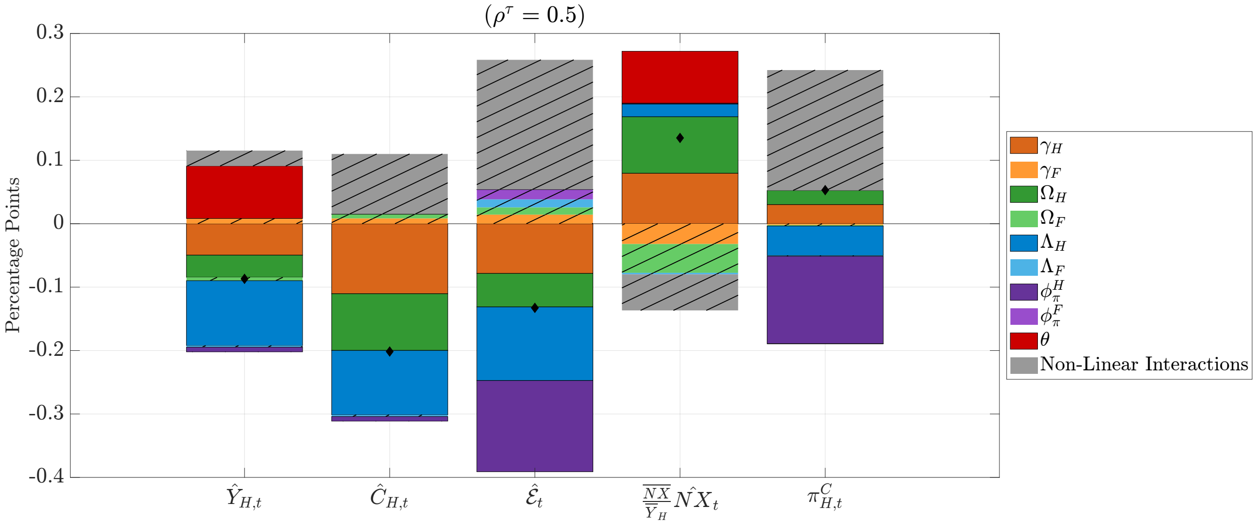
5.3 Macroeconomic Outcomes Under a Taylor Rule

In this section we consider the case, whereby the central bank follows a Taylor rule as in our baseline model in Section 3. Here we shall diverge from the flow of earlier sections, whereby the $N = 2$ & $J = 1$ example follows the analytical solution at matrix scale. Instead, we will start with the the $N = 2$ & $J = 1$ to motivate the analytical work.

5.3.1 Scalar Example with One Industry ($N = 2$ & $J = 1$)

In Figure 4, we plot the linear contribution of the primitives, in the $N = 2$ and $J = 1$ case. The figure below demonstrates that γ_H and Ω_H positively contribute to inflation and net exports, while they negatively contribute to output, consumption and exchange rate (i.e. creating appreciationary pressure). Elasticity of substitution θ contributes positively to production and net exports, while it does not to contribute to other variables meaningfully once again.

Figure 4. Contribution of Primitives to Macro Aggregates Under a Taylor Rule



Note: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section A, we plot bivariate plots that show these impacts are monotonic and that is why, we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. This figure is consistent with our analytical work and simulations in Dynare.

Policy and stickiness play a different role in Figure 4, which is a core motivation for our work in the section below. A higher central bank sensitivity to inflation (i.e. higher ϕ_π tends to put downward pressure on inflation and it creates appreciationary pressure for the exchange rate in response to tariffs. The latter follows from the UIP condition and the policy rule. In the figure, there are large residuals that come from non-linear interactions in the model. This a core motivation for our work below. The sign and the size of the non-linear interaction terms indicate that in dynamic general equilibrium, once one fully endogenizes monetary policy and makes it reactive to inflaton, as opposed to the earlier two cases where it targeted consumption and nominal demand, linear contributions can fail to provide sufficient intuition. A setup with input-output linkages, where all five variables of interest can move requires one to develop a solution that can decompose propagation into channels with matrices that incorporate the primitives.

5.3.2 Analytical Solution for $N = 2$ and Arbitrary J When $\phi_\pi \approx 1$

We now assume that $N = 2$ and that policy does not fix nominal domestic demand at an exogenously determined level and instead, a Taylor rule is followed, given by $\hat{i}_t = \phi_\pi \pi_t^C$ as specified in the baseline modeling framework. We do so as it allows us to solve for wages and the nominal exchange rate in the general expression in Equation (29) using a different approach, accounting for the endogenous impact of tariffs on demand and the exchange rate.

As derived in Appendix H, forwarding the Euler equation yields the following expression for consumption when shocks are transitory and ϕ_π is close to 1:

$$\hat{\boldsymbol{C}}_t = \overline{\boldsymbol{C}} - \frac{1}{\sigma}\boldsymbol{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) \tag{45}$$

For this expression to be numerically valid, we once again assume portfolio adjustment costs are strictly positive to ensure all variables return to the initial steady state while still being numerically small enough to be simplified away. When the steady-state level of debt is made globally stable through portfolio adjustment costs, the shock at hand is transitory, and as long as standard determinacy conditions are met (e.g., $\phi_\pi > 1$), it is guaranteed that $\lim_{t\to\infty} \hat{C}_t = \overline{C} = 0$ and Equation (45) serves as a valid numerical approximation that we verify with the quantitative model.[21]

Similarly, forwarding the UIP condition yields $\hat{\mathcal{E}}_t = \overline{E} + \phi_\pi \hat{P}_{H,t-1}^C - \phi_\pi^* \hat{P}_{F,t-1}^C$ where $\lim_{t\to\infty} \hat{\mathcal{E}}_t = \overline{E}$ as we show in Appendix H. By the same assumptions used above, we set $\overline{E} = 0$ and use the following expression to substitute out the exchange rate from the equilibrium conditions:[22]

$$\hat{\mathcal{E}}_t = \begin{bmatrix} 1 & -1 \end{bmatrix} \boldsymbol{\Phi} \hat{P}_{t-1}^C \tag{46}$$

Setting labor elasticity to $\gamma = 0$, as we did earlier in this section, the labor-leisure condition once again yields: $\hat{\boldsymbol{W}}_t = \hat{\mathbf{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t = (\mathbf{I} - \boldsymbol{\Phi})\hat{\mathbf{P}}_t^C - \boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C$. Plugging these into Equation (29), grouping terms, and rearranging, we obtain:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}_\phi \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left((\boldsymbol{L}_C^P + \mathbf{L}_{\hat{\mathcal{E}}}^P)\boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C + \left[\boldsymbol{L}_C^P(\mathbf{I} - \boldsymbol{\Phi})\boldsymbol{L}_\tau^C + \mathbf{L}_\tau^P\right]\tau_t \right) + \beta \mathbb{E}t \hat{\boldsymbol{P}}_{t+1}^P \right] \tag{47}$$

[21] The intuition behind this expression becomes clearer by considering the limit $\phi_\pi \to 1$. In this case, we obtain $\hat{\boldsymbol{C}}_t = -\boldsymbol{\pi}_t^C$, which indicates a downward-sloping aggregate demand curve once the Taylor Rule is substituted into the NKIS.

[22] We confirm the validity of the approximations here with the quantitative model in Dynare.

where $\tilde{\boldsymbol{\Psi}}_\phi = \left[\boldsymbol{I}(1+\beta) - \boldsymbol{\Lambda}\left[\boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{L}_C^P(\boldsymbol{I} - \boldsymbol{\Phi})\boldsymbol{\Gamma}\right]\right]^{-1}$ is now the *stickiness and policy-adjusted Leontief Inverse*.

Using (45) and (46) we can substitute out $\hat{\boldsymbol{C}}_t$ and $\hat{\boldsymbol{\mathcal{E}}}_t$ in the CPI equation in (28) and the equation of motion for debt in (36). Combining the resulting expressions with (47) we have a block that maps τ_t to $\hat{\mathbf{P}}_t^P, \hat{\mathbf{P}}_t^C, \hat{V}_t$. With that, once again using the method of undetermined coefficients, we can find an analytical solution. We confirm that our solution is numerically accurate, especially when ϕ_π is close to 1.[23] Additionally, in Appendix H.5.1 we show how our solution can collapse to the standard solution of the three-equation New Keynesian model when $N = 1$ and $J = 1$.

Proposition 5. *The impact of a one-time tariff on CPI inflation is*

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Gamma}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\left[\boldsymbol{L}_\tau^P + \Big(\boldsymbol{L}_C^P(\mathbf{I} - \boldsymbol{\Phi}) + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\Big)\boldsymbol{L}_\tau^C\right] + \boldsymbol{L}_\tau^C \tag{48}$$

where $\boldsymbol{L}_{\mathcal{E}}^C = \overline{\rho}(\boldsymbol{I} - \beta\overline{\rho}\tilde{\boldsymbol{L}}_{\mathcal{E}}^C)^{-1}$, *and* $\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}$ *is the stickiness- and policy-adjusted NKOE Leontief inverse. This expression endogenizes the demand and exchange rate response to the imposition of tariffs.*[24] *It transforms the stickiness- and policy-adjusted Leontief inverse* $\tilde{\boldsymbol{\Psi}}_\phi$ *by diagonalizing it and solving a quadratic equation to determine the matrix in front of the diagonalized lagged vector* $\breve{\boldsymbol{P}}_{t-1}^P$, *denoted as* $\mathbf{C}_1$. *The NKOE Leontief inverse* $\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}$ *is then constructed by taking this solution* $\boldsymbol{C}_1$ *and applying the transformation:*$\tilde{\boldsymbol{\Psi}}_\phi^{NKOE} = \boldsymbol{Q}\boldsymbol{C}_1\boldsymbol{Q}^{-1}$, *where* $\boldsymbol{Q}$ *diagonalizes* $\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}$. *In this formulation,* $\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}$ *also serves as the matrix multiplying the lagged price vector when producer price inflation is expressed in levels. The matrix* $\mathbf{Q}$ *represents the collection of eigenvectors that diagonalize the stickiness-adjusted Leontief inverse,* $\tilde{\boldsymbol{\Psi}}$.

Proof. See Appendix H. □

This analytical solution allows us to decompose the impact of tariffs into five indirect reallocation channels that extend beyond the direct impact of tariffs on CPI and PPI: (i) the contemporaneous demand channel inclusive of policy, (ii) the expected demand channel inclusive of policy, (iii) the expected exchange rate channel, (iv) price stickiness, and (v) the network channel. These channels correspond directly to the five primitives we highlight. As

[23]In our baseline comparison with both countries' parameter set to $\phi_\pi = \phi_\pi^* = 1.01$, our Dynare simulation finds U.S. inflation to be 0.8123%, while our linearized approximation matrices find this impact to be 0.8104%.

[24]The dimensions of the loadings are as follows: $\boldsymbol{L}_\tau^P$ is $NJ \times 1$, $\boldsymbol{L}_C^P$ is $NJ \times N$, $\boldsymbol{L}_{\mathcal{E}}^P$ is $NJ \times N$, $\boldsymbol{L}_{\mathcal{E}}^C$ is $N \times N$, $\boldsymbol{L}_\tau^C$ is $N \times 1$.

such, they can serve as model-based, ex-ante sufficient statistics.[25]

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \underbrace{\boldsymbol{\Gamma} \boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma} \boldsymbol{L}_C^P (\mathbf{I} - \boldsymbol{\Phi}) \boldsymbol{L}_\tau^C}_{\text{Demand channel}} + \underbrace{\beta \boldsymbol{\Gamma} \boldsymbol{L}_C^P \boldsymbol{\Phi} \boldsymbol{L}_\mathcal{E}^C \boldsymbol{L}_\tau^C}_{\text{Expected demand channel}} + \underbrace{\beta \boldsymbol{\Gamma} \boldsymbol{L}_\mathcal{E}^P \boldsymbol{\Phi} \boldsymbol{L}_\mathcal{E}^C \boldsymbol{L}_\tau^C}_{\text{Expected ER channel}} + \underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}}$$
$$+ \underbrace{\boldsymbol{\Gamma}(\tilde{\boldsymbol{\Psi}}_\phi^{NKOE} \boldsymbol{\Lambda} - \mathbf{I}) \mathbf{Z}}_{\text{Propagation}} \quad (49)$$

The propagation term captures the combined impact of the input–output structure, price stickiness, and policy. These components are difficult to analytically disentangle due to the definition of the stickiness- and policy-adjusted Leontief inverse prior to solving in the NKOE setting: $\tilde{\boldsymbol{\Psi}}_\phi = \left[\boldsymbol{I}(1+\beta) - \boldsymbol{\Lambda}\big[\boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{L}_C^P(\boldsymbol{I} - \boldsymbol{\Phi})\boldsymbol{\Gamma}\big] \right]^{-1}$. For this reason, we numerically decompose the propagation term into the contributions of $\boldsymbol{\Omega}$, $\boldsymbol{\Phi}$, and the remainder. Specifically, we set $\boldsymbol{\Omega} = \mathbf{0}$ and $\boldsymbol{\Phi} = \mathbf{I}$ one at a time, labeling these as the contributions of the network and policy to propagation, respectively. The remaining portion is attributed to price stickiness.

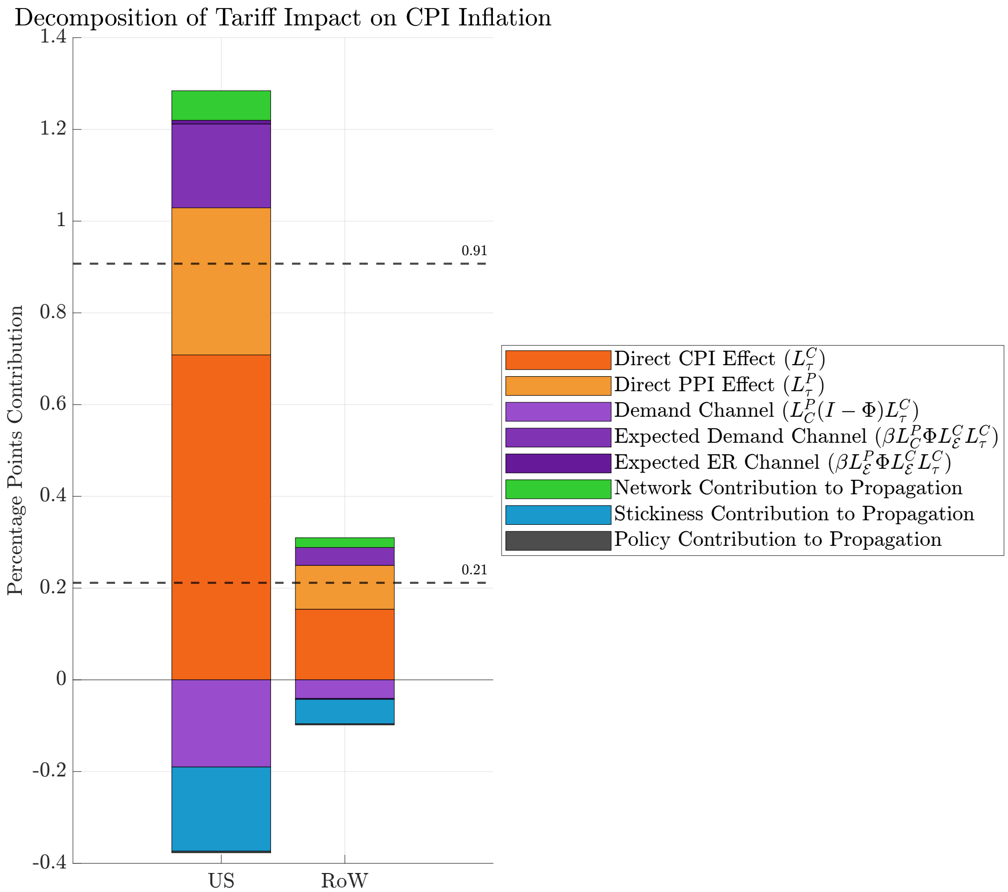
To illustrate how these channels operate and to build intuition around the model, let us consider an example based purely on the analytical solution above. Our objective here is not to conduct a full quantitative exercise—that is reserved for Section 8. Imagine dividing the world into two regions: the United States and the rest of the world. Suppose the United States imposes a 10% tariff on all goods and industries imported from the rest of the world for one period. In response, the rest of the world retaliates during the same period. Agents in both regions anticipate that these tariffs are transitory and will be lifted in the following period. We use the parameter values described in greater detail in Section 8 and Table 2, except where simplifications of the analytical model apply (e.g., $\sigma = 1$, $\eta = 0$). The impact of this theoretical tariff shock is illustrated in Figure 5 below.

When this transitory tariff shock occurs, the direct impact on CPI and PPI generates an inflationary impulse of approximately 1 percentage point in the tariff-imposing country. The magnitude of these direct affects is related to the trade openness of the United States. Beyond these direct effects, we also observe indirect effects. As expected, the contemporaneous demand channel carries a negative sign. Under policy, aggregate demand slopes downward in response to inflation. As this is a New Keynesian framework, this arises because the central bank raises real interest rates in response to rising headline inflation, thereby contracting demand. Consequently, when the tariff shock hits, agents choose to forego consumption today in favor of consuming tomorrow. Meanwhile, the expected demand channel generates

[25] Details are available in Appendix I.

an additional inflationary impulse as agents anticipate that the tariffs are one-time, transitory shocks and expect them to dissipate in the following period.

Figure 5. US Against the Rest: Decomposing Impact of Global Tariff War



Note: Here, we decompose CPI inflation in a two-country case, namely the U.S. and the rest of the world (RoW). We assume both regions impose an additional 10% tariff on each other. Using Equation 49, we break down the different contributing effects. The dashed line represents the total effect, showing an inflation increase of 0.91% in the U.S. and 0.21% in the rest of the world. In this theoretical example based on our analytical solution, we use annual price updating frequencies, whereas in the quantitative model we use quarterly frequencies.

What partially offsets the initial inflationary impulse of approximately 1.3 percentage points—bringing the overall effect down to 0.91 percentage points—is the combined influence of price stickiness and the contemporaneous demand channel. The primary impact of policy operates through contemporaneous demand, while policy's contribution to propagation is limited. In contrast, the input–output network generates positive inflationary pressure—a mechanism we explore in greater detail in Section 6.

Note that, first, the impact on the rest of the world follows the same directional pattern as in the United States but is smaller in magnitude. This is because the rest of the world is larger than the U.S., making the distortion a relatively smaller shock in the context of the global economy. Second, The transmission from the exchange is relatively muted due to transitory nature of the shock. The contemporaneous exchange rate response—abstracted from in this section due to the simplifying assumptions—exhibits larger movements when the shock is permanent. As indicated by equation (46), the exchange rate closely follows changes in the price level given our assumptions. Third, this figure underscores what the full model captures compared to standard trade or small open economy (SOE) models. In the absence of intertemporal optimization and forward-looking behavior, the contemporaneous demand channel—as well as the expected demand and expected exchange rate channels—would be absent. In the SOE case, loadings from the rest of the world would not be present. Finally, in models without network effects, the network channel would also be absent.

This analytical decomposition, additionally offers a way to see the impact of primitives as they feature in different matrices, thereby shedding light on the gray-colored non-linear interaction terms in Figure 4. The direct CPI effect and direct PPI effect respectively contain, γ and Ω. If tariffed goods are γ_H share of the consumption basket (or Ω_H of inputs to production) and a 10% tariff is imposed will have a γ (or $0.1 \cdot \Omega_H$) direct impact on CPI (or PPI). The indirect effects in the decomposition, similarly involve matrices that take include the same five primitives we highlight. The loadings sum through the primitives (e.g. $\boldsymbol{L}_C^P$, which contains labor shares that can be found by subtracting the sum of Ω terms from 1). The channels highlighted in (49) fully decomposes the impact of tariffs on inflation and thereby captures the non-linear interactions between the primitives that the linear contributions in Figure 4 do not capture.

Further intuition can be gained by comparing the solution in Equation (43) under fixed nominal demand to that in Equation (48) under a Taylor rule. In the former, the impact on demand and the exchange rate is linearly separable from tariffs. Thus, the two expressions differ in the following ways: (i) in how the NKOE Leontief inverse is reshaped by policy, and (ii) through the term $\left(\boldsymbol{L}_C^P(\mathbf{I} - \boldsymbol{\Phi}) + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\right)\boldsymbol{L}_\tau^C$. This term captures how tariffs impact contemporaneous demand, expected demand, and expected exchange rates. Part of this impact operates through lagged consumer prices, which enter contemporaneous inflation via the expected inflation term in the Phillips Curve, hence the presence of β in the expression. We can analyze this term further by separating it into its three components:

$$\underbrace{\boldsymbol{L}_C^P(\mathbf{I} - \boldsymbol{\Phi})}_{\substack{\text{Tariff Impact} \\ \text{via Demand}}} + \beta \underbrace{\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_\tau^C}_{\substack{\text{Tariff Impact} \\ \text{via Expected Demand}}} + \beta \underbrace{\boldsymbol{L}_{\mathcal{E}}^P\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_\tau^C}_{\substack{\text{Tariff Impact} \\ \text{via Expected ER}}}$$

The way a term loads onto $\hat{C}_t$ and $\hat{\mathcal{E}}_t$ is by first loading onto consumer prices. In this sense, $\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}$ captures how tariffs affect consumer prices, which in turn impact the exchange rate, thereby influencing producer prices. Similarly, $\boldsymbol{L}^P_{C}\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}$ captures how tariffs load onto consumer prices and, consequently, influence demand. As this process unfolds, these effects are mediated by policy, as captured by $\boldsymbol{\Phi}$.

Remark 7. The network structure, when combined with price stickiness and sectoral heterogeneity, can either amplify or dampen some entries, thereby shaping the overall sign and magnitude of inflation in the two countries. This differs from the setup with fixed nominal demand where all inflation entries were weakly positive.

6 When and Why Network Granularity Matters in Global DGE

Section 5 demonstrated that one needs at least $N = 2$ and $J = 1$ with input-output linkages (i.e. an $\boldsymbol{\Omega}$ matrix that is at least 2×2) to accurately capture feedback from the rest of the world as there are non-linear interactions between the primitives. That itself is a network. The question is what one gains by making that network more granular (i.e. increasing number of industries, J beyond one) and how does this answer change in global DGE with international risk sharing?

The first point to note is a caveat: in this section we will be working with the linearized model. To a first-order approximation, the aggregation of any CES bundle behaves similarly to a Cobb–Douglas function. In a more general non-linear setting, however, the structure of $\boldsymbol{\Omega}$ is important because a more granular depiction of the production network significantly affects outcomes, especially when shocks are large as elasticities matter more.

Beyond numerical precision, we are interested in when and why network granularity matters in global DGE. Our first answer involves aggregation under sectoral heterogeneity. Making an IO table less granular, necessarily involves summing entries. It makes a difference if an average of parameters (Λ) is applied to a sum of weights (Ω) across sectors first, as opposed to first multiplying the parameters and weights separately to later sum them as implied by the granular model (see equation (25)).

In Section 6.1, we show how the sectoral heterogeneity in the stickiness parameters interact with the input-output networks. In Section 6.2, we explore the role of net foreign assets and international risk sharing for network propagation.

6.1 Aggregation Under Sectoral Heterogeneity

Given that price stickiness and input–output linkages are key primitives that enter into propagation matrices, we now turn to the question of how the network channel outlined in Section 5.3 operates, how the structure of the $\mathbf{\Omega}$ matrix influences transmission and how this interacts with the price stickiness matrix $\mathbf{\Lambda}$. Why is the sign of the network channel in Figure I positive? To understand this and the broader impact of the network channel we begin by establishing several key insights that follow from the propositions above. The importance of network granularity for precision in the context of aggregation has been well-documented (Pasten et al., 2020; Rubbo, 2023) and in this section we apply this insight to our context.

Remark 8. In a first-order approximation setting, the regular Leontief inverse ($\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$) and the stickiness-adjusted Leontief inverse, multiplied by the stickiness matrix ($\tilde{\mathbf{\Psi}}_\pi \mathbf{\Lambda} = (\boldsymbol{I} - \mathbf{\Lambda}(\mathbf{\Omega} - \boldsymbol{I}))^{-1}\mathbf{\Lambda}$), will behave similarly, provided there is no heterogeneity in the stickiness parameter across sectors. The key difference between them lies in the magnitude of inflation.

Proof. Each of the two objects involve Neumann series. In the absence of sectoral heterogeneity, $\mathbf{\Lambda} = \lambda \boldsymbol{I}$. Then:

$$\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1} = \sum_{k=0}^{\infty} \mathbf{\Omega}^k$$

Similarly, for the stickiness-adjusted Leontief inverse:

$$\begin{aligned} \tilde{\mathbf{\Psi}}_\pi \mathbf{\Lambda} &= (\boldsymbol{I} - \mathbf{\Lambda}(\mathbf{\Omega} - \boldsymbol{I}))^{-1} \mathbf{\Lambda} \\ &= \frac{\Lambda}{1+\Lambda} \sum_{k=0}^{\infty} \left(\frac{\Lambda}{1+\Lambda} \right)^k \mathbf{\Omega}^k. \end{aligned}$$

As long as $\mathbf{\Omega}_{ij} \neq 0$ for some i, j, the relative importance—or centrality—of sectors remains unchanged in the absence of heterogeneity in price stickiness across sectors. However, the overall impact on inflation will be scaled by a constant factor. As established in Rubbo (2023), when a finer I-O matrix captures more goods within the $\mathbf{\Omega}$ matrix, the aggregate Phillips Curve becomes flatter. This occurs because, as the number of sectors increases, the individual input–output coefficients $\mathbf{\Omega}_{ij}$ decrease, reflecting a more granular production network. Since $\mathbf{\Omega}$ enters the Neumann series multiplicatively, and assuming $\mathbf{\Omega}_{ij} \in (0, 1)$, smaller $\mathbf{\Omega}_{ij}$ entries attenuate the aggregate impact of sectoral shocks. As a result, the aggregate Phillips Curve flattens: nominal rigidities become more diffuse across a fragmented network, reducing the responsiveness of inflation to shocks. Consequently, as prices respond less, quantities respond more. □

Remark 9. In a NKOE setting, as shown in Equation (48), the combination of cross-sectoral heterogeneity in the price stickiness term $\mathbf{\Lambda}$ and the stickiness- and policy-adjusted NKOE Leontief inverse, $\tilde{\mathbf{\Psi}}^{NKOE}$, can exert downward pressure on inflation. This occurs in part because $\tilde{\mathbf{\Psi}}^{NKOE}$ is not restricted to having weakly positive entries. When there is heterogeneity in price stickiness or policy preferences—either across sectors or across countries—$\tilde{\mathbf{\Psi}}^{NKOE}$ can amplify negative entries from other channels, further dampening the aggregate inflation response.

In our context, $\mathbf{\Lambda}$ captures price stickiness; an extreme case of flexible prices corresponds to a vertical supply curve with a fixed quantity. In this scenario, the inelasticity of supply can amplify the influence of a given sector or country. Suppose a particular country–sector combination constitutes only a small share of the home country's producer price basket. If its supply is inelastic, the NKOE impact of a tariff on this country–sector will be disproportionately large. This type of effect may be overlooked in models where all intermediate goods are bundled together under flexible pricing.

Relatedly, omitting a fully specified input–output structure can lead to an overestimation of inflation and an underestimation of the quantity response, including effects on unemployment. Suppose intermediate inputs are bundled with some degree of price stickiness (or even under flexible pricing), while, for notational simplicity, final goods production follows either flexible or staggered pricing. In such a case, inflation in final goods may be overstated, since the network channel can exert downward pressure on overall inflation.

Remark 10. Given interlinkages between sectors, heterogeneity in price stickiness parameters can compress the range of inflation outcomes that the central bank can achieve through endogenous rate hikes under a Taylor Rule, thereby reducing the effectiveness of monetary policy.

Rubbo (2023) notes that eliminating heterogeneity in stickiness parameters does not necessarily alter the slope of the aggregate Phillips Curve. This outcome depends on how the average price updating frequency is computed, how correlated labor, input and consumption shares are. While we do not report Phillips Curve slopes in our setting, the exercise we have in mind is similar in spirit.

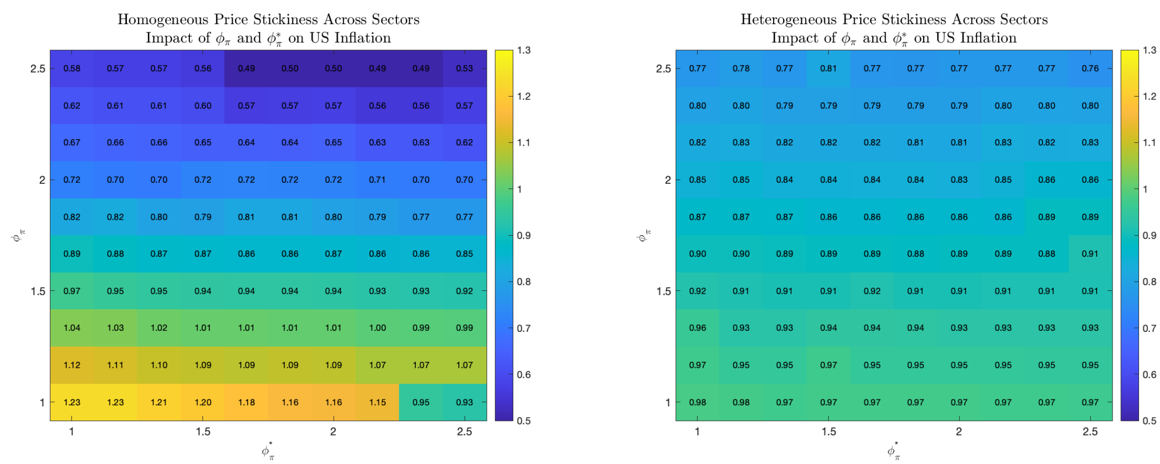
With the model analytically solved, what can we say about the NKOE impact on inflation from changing the weight on inflation in the Taylor Rule, ϕ_π? And how is this impact affected by heterogeneity in price updating frequencies? We explore these questions in Figure 6. These heatmaps are based on the analytical solution and reflect the same setup as in Figure 5, where the United States imposes 10% tariffs on the rest of the world, and the rest of the world retaliates.[26] The two axes in each heatmap vary the central banks' weights on

[26]Since our analytical solution involves approximations, we have verified the relative magnitudes and

inflation, ϕ_π and ϕ_π^*, in the two blocs. The heatmap color indicates the resulting inflation in the United States. The right-hand panel shows the case in which price stickiness parameters are heterogeneous across sectors, using the values from our full quantitative simulations based on Nakamura and Steinsson (2008).[27] The left-hand panel shows the case in which a single stickiness parameter is applied to all sectors. To match the overall magnitude across both panels, the stickiness parameter used in Figure 6a is set equal to the sales-weighted average of sectoral stickiness.[28] These figures suggest that, in our context and using the ICIO input–output table, cross-sectoral heterogeneity in price stickiness compresses the range of inflation outcomes that the central bank can achieve through endogenous rate hikes: from 0.49% to 1.23% in the homogeneous case, versus 0.76% to 0.98% in the heterogeneous case.

Figure 6. Impact of Heterogeneity: Price Stickiness vs. ϕ_π

(a) **(b)**



NOTE: Heatmaps show U.S. CPI inflation in a two-country setting (the United States and the rest of the world), where both regions impose a 10% tariff on each other. The horizontal and vertical axes vary the inflation response parameters ϕ_π and ϕ_π^* in the Taylor Rule for the U.S. and the rest of the world, respectively. The heatmaps reflect the resulting U.S. inflation as these policy parameters vary.

Two additional observations are worth noting. First, this result is specific to the input–output (I–O) table we use. Intuitively, and based on Equation (48), the slope of the Phillips Curve matters for how $\mathbf{\Phi}$ affects inflation, and this, in turn, depends on $\boldsymbol{L}_C^P$—which, in the context of our analytical solution, contains only labor shares.[29] Then it will matter

ranges of these estimates using Dynare.

[27] We conducted simulations using alternative stickiness parameterizations, including Monte Carlo simulations with randomly drawn vectors of sectoral stickiness. Across these exercises, we consistently find that heterogeneity in stickiness compresses the range of inflation outcomes attainable by varying ϕ_π.

[28] Specifically, we take the weighted average of the diagonal entries of $\mathbf{\Lambda}$.

[29] This is because labor is assumed to be elastic in the analytical solution, so there is no Frisch elasticity

if sectors with high vs. low labor shares get higher or lower stickiness parameters as Rubbo (2023) notes. Second, the simulations in Figure 6 suggest that, when tariffs are modeled as one-time transitory shocks, heterogeneity in ϕ_π across countries does not significantly affect inflation outcomes in the home country. However, in quantitative simulations using a multi-country setup, we find that the response of variables such as the exchange rate and inflation to near-permanent shocks does depend on cross-country heterogeneity in ϕ_π.

Remark 11. The matrix of price stickiness parameters $\mathbf{\Lambda}$ influences inflation in three different ways: (i) via the average level of price stickiness,[30] (ii) via cross-sectoral heterogeneity, whereby it will matter if a sector with high vs. low labor shares get higher or lower stickiness parameters, and (iii) via the interaction with $\mathbf{\Omega}$ inside the NKOE Leontief inverse.

Of the three ways in which $\mathbf{\Lambda}$ influences inflation, the first is present both in models without multiple sectors and without input-output linkages. The second could be present in a model that has multiple sectors without explicitly introducing input-output linkages. However, the third can only be present in models with input-output linkages.

This brings up our final point regarding the impact of $\mathbf{\Omega}$ on inflation; this impact is a nuanced one. On the one hand, having a finer or more granular network flattens the Phillips Curve and as such would mute the impact of shocks on inflation as outlined above. On the other hand, the very reliance of one sector on another introduces positive weights inside the marginal cost expression for each sector such that for a given network $\mathbf{\Omega}$ will have a positive impact on inflation. This second and positive impact is what makes the network contribution to propagation positive in Figure 5. Inside the stickiness and policy-adjusted Leontief Inverse, $\mathbf{\Omega}$ is multiplied by $\mathbf{\Lambda}$ before we arrive at the NKOE Leontief Inverse. This implies that the positive inflationary impulse from input-output linkages are highly dependent on the distribution of price stickiness parameters. If a given sector's reliance on an input from another sector is multiplied by a high (low) price stickiness parameter, the inflation (quantity) impact from a shock to that sector will be amplified. Put differently, using different price stickiness parameters can make the network contribution to propagation larger in Figure 5.

Intuitively, if a given sector is central to production whether because it is widely used in different industries (e.g., steel and aluminum) or its downstream linkages (e.g., semiconductor chips)—it will carry a high weight in the standard Leontief inverse. If this sector also exhibits highly flexible (rigid) prices indicating a vertical (horizontal) supply curve with fixed quantity

term in the slope of our New Keynesian Phillips Curve.

[30] As noted in Rubbo (2023) how this average is calculated matters. Averaging Calvo price updating frequencies first and then calculates a single price stickiness parameter yields a different result than calculating price stickiness parameters and then averaging them. We find it also matters whether the final scalar price stickiness parameter that is used is a weighted average or a simple average.

(highly elastic supply), the inflationary impact of a tariff on this sector will be amplified (muted) by $\tilde{\mathbf{\Psi}}_{\phi}^{NKOE}$. Since $\tilde{\mathbf{\Psi}}_{\phi}^{NKOE}$ also includes distribution of central banks' weights on inflation, whether the shocks hit countries with loose or tight monetary policy will be an additional amplification or deamplification channel.

6.2 The Role of Net Foreign Assets and International Risk Sharing for Network Propagation

Production network models typically examine scenarios in which a sector-specific shocks propagate differently from aggregate shocks. In such a setting with input-output linkages and sectoral heterogeneity, granular shocks can lead to large aggregate impacts if there are bottlenecks. Bottlenecks, in turn, occur because pressure in one part of the network cannot be alleviated. In the case where tariffs are placed on intermediate inputs, for example, if it is difficult for labor to switch sectors to serve as a substitute for inputs that are now more expensive or if a certain input is difficult to substitute (i.e. low θ), a small tariff can lead to large impacts.

We find that this result is sensitive to international risk sharing and the presence of a moving net foreign asset position between countries. Many production network models are closed economy and many network models that do international trade restrict the net foreign asset position of countries, especially in quantitative simulations. If one allows for countries to accumulate claims on each other in the form of financial assets, this leads to international risk sharing in expectation and this has the impact of smoothing price and quantity changes. International finance and international trade are two sides of the same coin; restricting one restricts the other. Beyond, this familiar insight, however, we find that it matters if the Euler equation holds for each country and countries' net foreign asset position is allowed to move.[31]

To understand how international risk sharing impacts the propagation of inflation in an international production network consider a version of our five-equation Global New Keynesian Representation with portfolio adjustment costs added back to the linearized model, where we have $N = 2$ countries and arbitrary J number of industries. As detailed in Appendix K, following Itskhoki and Mukhin (2021), we assume that the aggregate consumer price levels in both countries are fixed by policy: $\hat{P}^C_{H,t} = \hat{P}^C_{F,t} = 0$. Our model then acts similar to the baseline model of Itskhoki and Mukhin (2021) with input-output linkages and nominal rigidity. Our goal in this exercise is to use the real portfolio adjustment cost, ψ, to examine the effect of restricting financial flows between countries as it impacts network

[31] Allowing trade imbalances with transfer terms, for example, does not lead to international risk sharing.

propagation since at the limit, as $\psi \to \infty$, we have financial autarky.

Proposition 6. *Under the assumption that policy stabilizes the nominal price level and that the tariff shock is a one-time shock, portfolio adjusment costs can mute or amplify the propagation of inflation in the network:*

$$\frac{\partial \hat{\boldsymbol{P}}_t^P}{\partial \tau_t} = \left[(\tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda})^{-1} + \boldsymbol{\Theta}_1\right]^{-1}\left[\boldsymbol{\Theta}_2 - \left(\boldsymbol{L}_{\mathcal{E}}^P \frac{\partial \hat{V}_t}{\partial \hat{\tau}_t}\right)\psi\right]$$

Proof. See Appendix K □

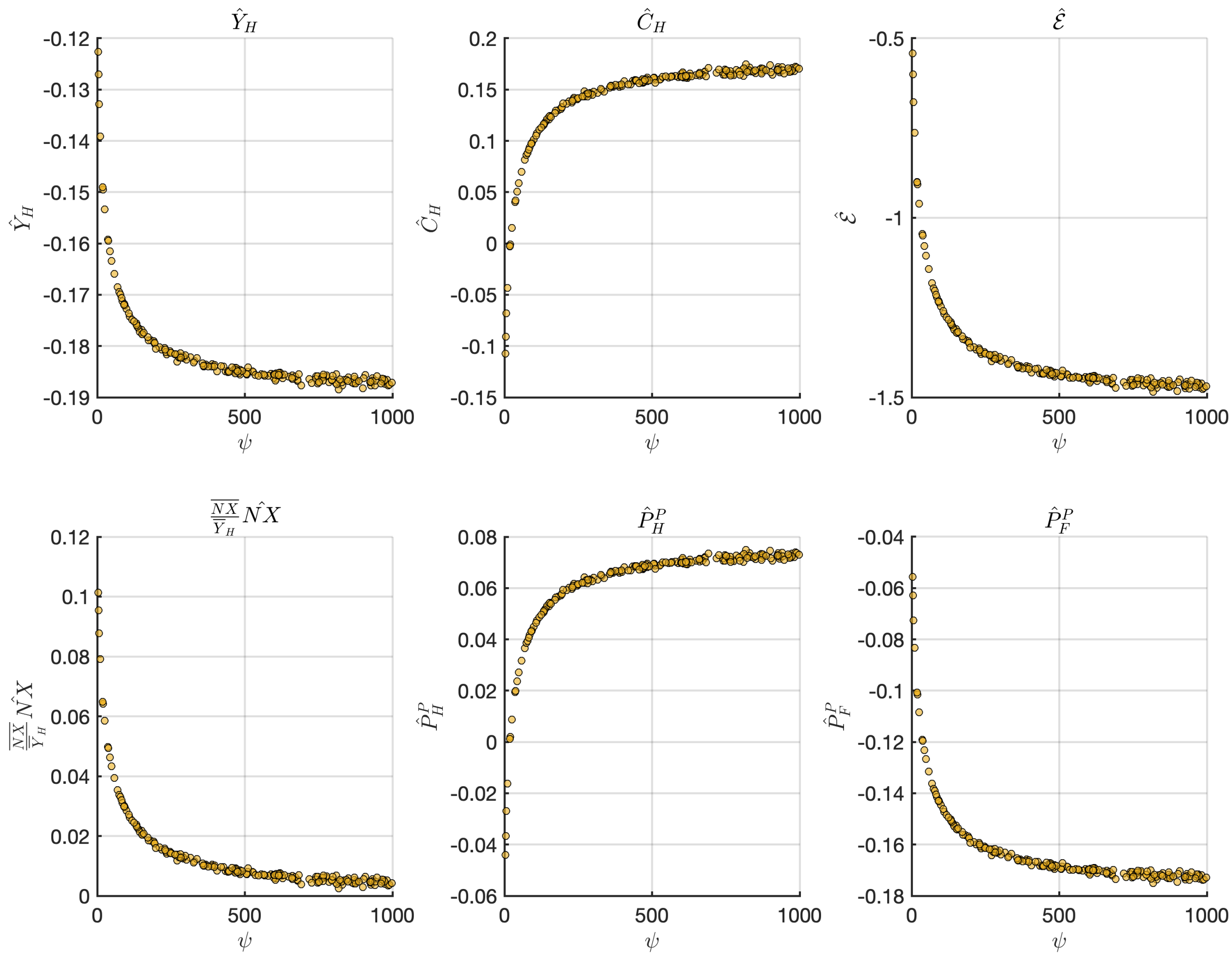
Portfolio adjustment costs, ψ, in this setup impacts the network propagation of a tariff shock in this setting (i.e. ψ is multiplied by the matrix inverse containing $(\tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda})^{-1}$). Since ψ impacts $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$ through small interactions, so we compute $\frac{\partial^2 \hat{\boldsymbol{P}}_t^P}{\partial \tau_t \, \partial \psi}$ numerically to sign it. The intuition is that the impact of tariffs on the net external debt position of the home country is negative and the first entry of $\boldsymbol{L}_{\mathcal{E}}^P$ is positive while its second entry is negative. For that reason, in an example with $N = 2$ and $J = 1$, we should expect the impact of tariffs on the home country's domestically produced good to be positive and that of the foreign country should be negative. Figure 7 confirms this intuition.

Our is a setup with incomplete markets. There is one nominal bond denominated in the U.S. dollar that all countries use to accumulate net claims or net debt. In this setup, the representative household in each country makes a consumption and saving decision that equalizes the expected ratio of marginal utilities, taking into account differences in the relative price of each country's consumption basket. With this equalizing force in place, households choose their optimal labor supply. Depending on the substitutability of labor with intermediate inputs, labor in turn can smooth sectoral bottlenecks.

In the absence of international risk sharing, then, one would expect to see larger network effects and to see the structure of the network matter more. For example, bottlenecks in one sector could lead to larger impacts and the impact of a shock on a given sector would be expected to differ from the impact of a shock on other sectors more significantly. To explore this, Figure 8 utilizes the quantitative model in Section 8 to depict the impact of unilateral tariffs by U.S. on different Chinese sectors. Each IRF represents the impact of a 100% tariff on a different Chinese sector. The two simulations only differ in that the subplot on the left assumes $\psi = 0.00001$, while the subplot on the right assumes $\psi = 1000$. The IRFs are scaled to treat each sector as though its share in the U.S. import basket is equal to the weight of the average Chinese sector. As predicted by theory, under financial autarky, the structure of the network matters more. The response by aggregate U.S. employment to tariffs being placed by the U.S. on different Chinese sectors differs more from sector to sector under financial autarky.

Figure 7. Numerical Second Derivative: $\frac{\partial^2 \hat{\boldsymbol{P}}_t^P}{\partial \tau_t \, \partial \psi}$

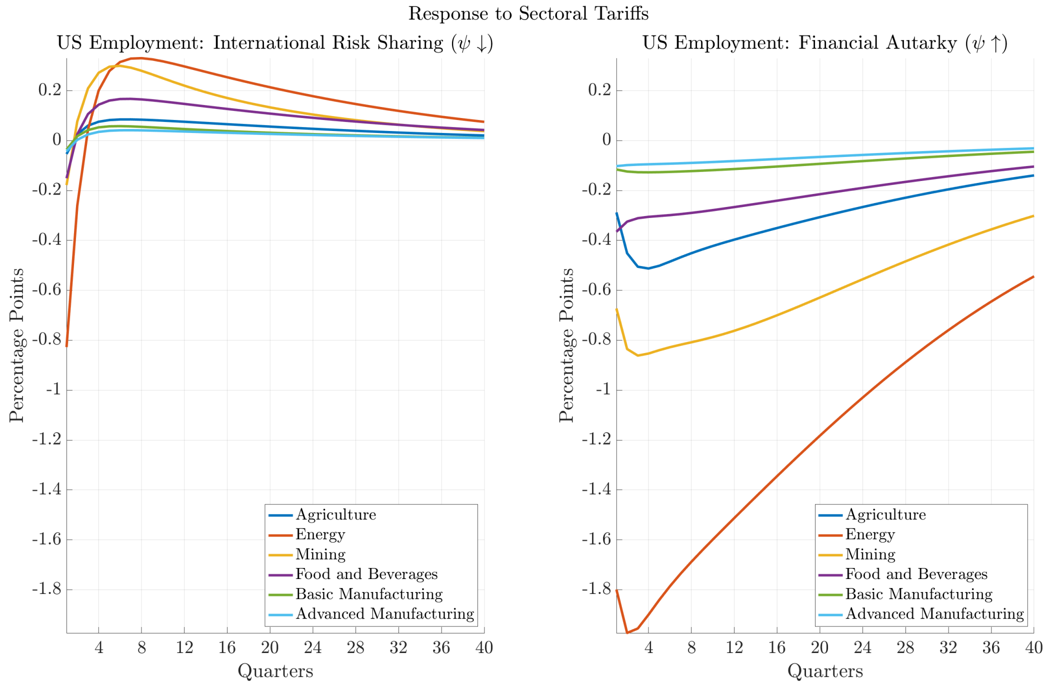
Effect of PAC (ψ) on Endogenous Variables



Note: Figure visualizes how the first period impact of endogenous variables of interest of changes as the primitive parameters are changed in the context of 10% tariffs being imposed by the home country. We initialize primitive parameters respectively at $\theta = 0.6$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0$ to match the analytical solution. Aggregate inflation is not plotted as it is fixed at 0 by policy in this setup.

We explore the financial autarky limit by taking $\psi \to \infty$, which serves as a reduced-form method to restrict both trade and financial flows in our model. Our findings suggest that DGE open economy models with production networks that attribute a central role to network structure—and that generate large differences between the effects of sectoral and aggregate shocks—may have these results critically depend on the absence of international borrowing.

Figure 8. Sectoral Shocks and International Risk Sharing



NOTE: Figure utilizes the quantitative model in Section 8 to depict the impact of unilateral tariffs by U.S. on different Chinese sectors. Each IRF represents the impact of a 100% tariff on a different Chinese sector. The two simulations only differ in that the subplot on the left assumes $\psi = 0.00001$, while the subplot on the right assumes $\psi = 1000$. The IRFs are scaled to treat each sector as though its share in the U.S. import basket is equal to the weight of the average Chinese sector.

7 Data and Calibration

7.1 Input - Output Network

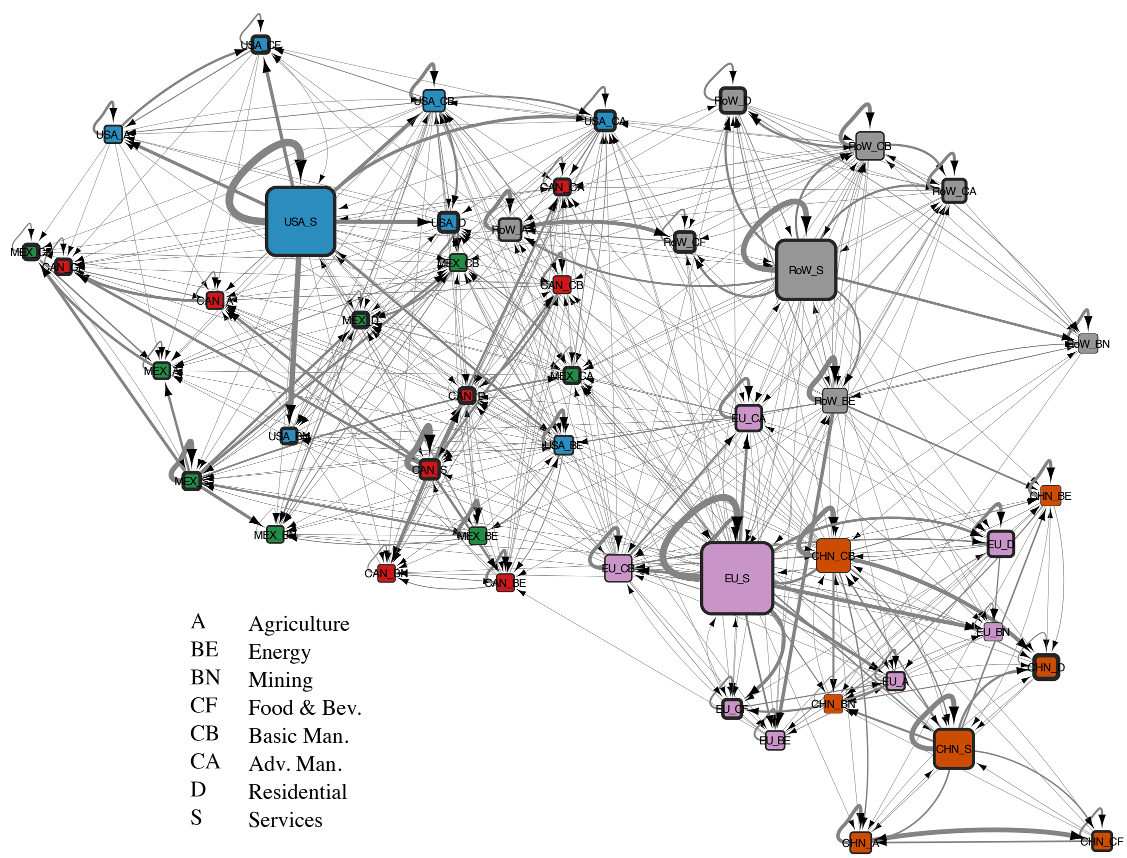
As the basis for consumption shares and intermediate input shares, we use the OECD Inter-Country Input–Output (ICIO) tables (OECD, 2020) for the year 2019.[32] We aggregate the ICIO data to align with the country and industry groupings used in our analysis. we include the United States, euro area, China, Canada, and Mexico—reflecting the countries most affected by the tariff announcements as of April 2025—along with an aggregate entity representing the Rest of the World (RoW). On the industry side, we aggregate sectors into eight broad categories: agriculture, energy, mining, food, basic manufacturing, advanced

[32]Although the latest available data at the time of writing was for 2020, we use 2019 data to avoid distortions arising from the COVID-19 pandemic.

manufacturing, residential services, and other services to match with sectoral rigidity data of Nakamura and Steinsson (2008) (see below).

We visualize the input-output network in Figure 9. The thickness of the edges in this network captures the input shares. The layout of the network was generated automatically using the edge-weighted spring embedded layout feature of Cytoscape. Global shocks could be carried over the links shown on this network. Strikingly, many Canadian and Mexican sectors are naturally grouped together with American industries. Chinese, sectors, in contrast, are not very well integrated. This might be due to the fact that many Chinese goods imported by the U.S. could be for the final consumption.

Figure 9. Visualizing the Input-Output Network



Note: Here, we show the inter-country inter-industry input-output network. The color of the node represents the country. Size of the node represents the total output. The thickness of the edges show the share of inputs of target node coming from the source node (we do not show the edges smaller than 1%). The thickness of the borders of nodes represents the share of final goods in the output of the sector. The layout was generated automatically using the edge-weighted spring embedded layout using the openly available Cytoscape software.

In Table 1, we show the basic stats for the U.S. industries. The U.S. economy heavily relies on services, with more than 75% GDP attributed to this sector. Most of the U.S. output is consumed domestically, with shares ranging from 80 to 99 %. The home share in consumption and intermediate inputs exhibit the lowest rates in manufacturing sectors. Interestingly, close to one third of consumer goods and intermediate inputs are sourced from foreign countries in advanced manufacturing. The energy sector's intermediate products are sourced at a higher level internationally. In Table A.2 of the Appendix, we provide a more detailed breakdown of the final and intermediate input shares at country-sector level.

Table 1. Sector Statistics for USA (%)

Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

NOTES: The values are calculated from OECD ICIO for year 2019 OECD (2020). Output Share is the share of the sector in total U.S. output. VA share is the share of the sector in total U.S. GDP. Consumption share is calculated as the sector's weight in the household expenditure. Output Home Share represents the share of the output of the sector sold domestically. Consumption Home Share captures the share of domestic production in consumption and Intermediate Home Share captures the share of intermediate goods supplied domestically.

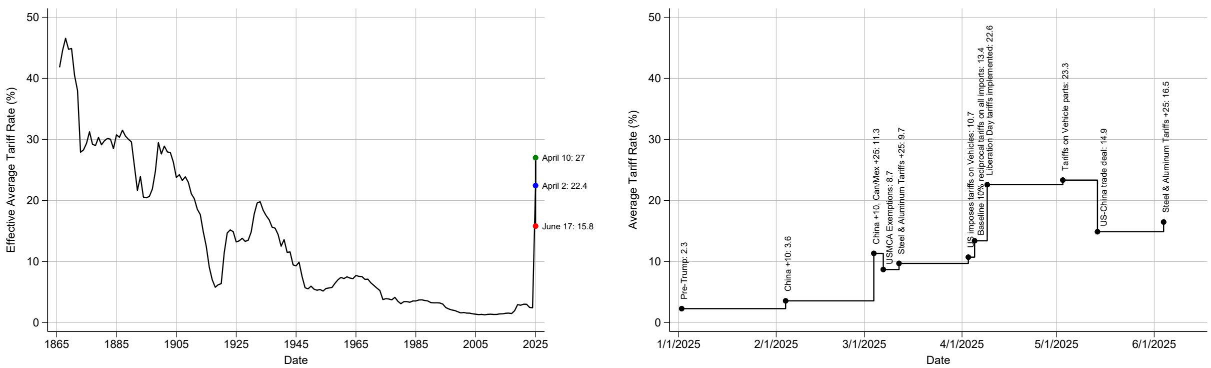
7.2 Tariffs

In the quantitative exercises that follow, we are motivated by the renewed interest among policymakers in using tariffs as a tool to manage external imbalances and exert geopolitical influence. This interest predates the second Trump presidency and reflects a broader global re-evaluation of trade policy not only for the standard terms of trade manipulation but also both for strategic and retaliatory purposes. In the quantitative section of our paper we solely focus on the tariffs announced in the early months of the second Trump administration.

As shown in Figure 10a, the tariffs proposed on April 2—referred to as "Liberation Day" by the administration—are projected to raise the effective U.S. tariff rate to 22.4%, the highest level in over a century. We obtain the country - sector levels tariffs from the WTO – IMF Tariff Tracker (WTO and IMF, 2025) at Harmonized System 6-digit level. We aggregate these tariff rates to ICIO sectoral level by weighing them with the imports of the countries, provided in the same dataset. Figure 10b shows the implemented tariff rates since January 1, 2025 until June 20, 2025. The "liberation day tariffs," were announced on April 2, 2025 but with most tariffs going into effect on April 9th. Between these two dates, there was also a steep escalation between the U.S. and China tariffs to each other, resulting in tariff rates exceeding 125% for Chinese goods in the U.S.

Figure 10. Effective Tariff Rates

(a) Historic and Estimated, (%) **(b)** Since January 1, 2025, (%)

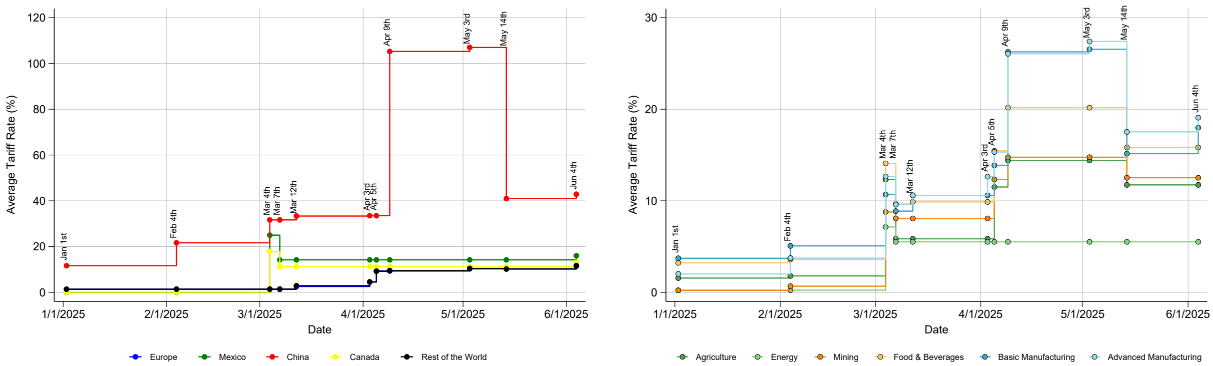


Note: (a) Effective tariff rate stands for customs duty revenue as a proportion of goods imports. Data from *Historical Statistics of the United States* Ea424-434, *Monthly Treasury Statement*, Bureau of Economic Analysis. Estimated effective tariff rates of for 2025 provided by Yale Budget Lab using the GTAP Model v7 (Corong et al., 2017). (b) Estimated effective tariff rates based on WTO - IMF Tariff Tracker (WTO and IMF, 2025). The dates here correspond to the actual implementation change of the tariffs. The data was accessed on June 20, 2025.

In Table A.1, we document the episodes of implemented tariff changes for the U.S. reported by the WTO-IMF Tariff Tracker (WTO and IMF, 2025). We summarize the tariff rates at the country and sector level in Figure 11. The largest swings are observed for China with escalating tariff announcements with a moratorium on May 14, 2025 (Figure 11a). At the sectoral level, the tariffs are the highest for basic and advanced manufacturing goods. Figure A.2 in the Appendix shows the size of country-sector-level tariffs implemented in 2025 until the time of our writing in panel (a). Panel (b) focuses on the "Liberation Day" tariffs. Figure A.2a shows that the highest tariff rates are applied to the Chinese goods. Among Chinese sectors, basic manufacturing (e.g., textiles), food and beverages, and agriculture have the highest values with tariffs ranging from 45-50%. For most other countries, the tariffs started from very low levels but increased around 10-20% for many goods.We will use the most recent data (June 4, 2025) levels for our quantitative analysis. In Table A.2 of the Appendix, we provide detailed breakdown of the tariff rates as of June 20, 2025 and maximum tariff rate observed between January 1, 2025 and June 20, 2025.

Figure 11. Effective Country and Sector Level Tariff Rates

(a) Country Level Tariffs (%) **(b)** Sector Level Tariffs (%)



NOTE: Estimated effective tariff rates at the (a) country level (b) sectoral level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) between January 1, 2025 and June 4, 2025 (last available data as the manuscript was prepared). Both country level and sectoral level tariff rates are calculated as the weighted average of the 6-digit tariff rates by using the latest available import values reported in the dataset as weights.

According to both the St Louis Fed[33] and the Tax Foundation[34], the 2018 tariffs affected \$376 billion of goods from China, which is around 1.66% of the 2018 U.S. GDP. As of June 2025, most of the tariffs enacted on the "Liberation Day" have been halted via an injunction by the U.S. Court of International Trade. Those not affected still represent \$500 billion worth of U.S. imports, or 1.68% of the 2024 U.S. GDP. If all of the "Liberation Day" tariffs were to come into effect again, they would represent \$2.3 trillion worth of U.S. imports, which is 7.7% of 2024 U.S. GDP.

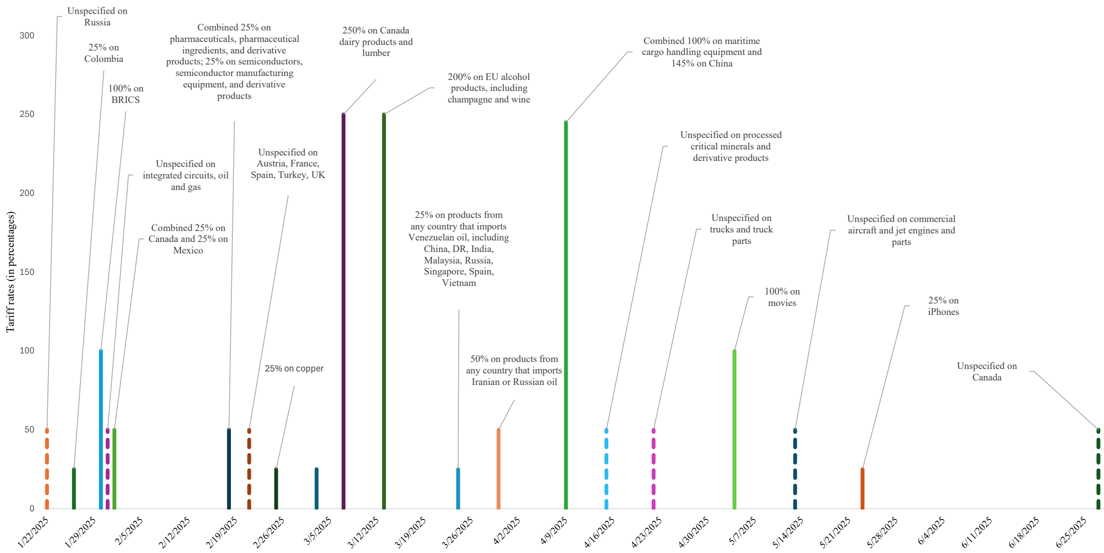
The tariff rates changed considerably with very frequent announcements, repeals, threats, deals, and various negotiations. In Figure 12 we show some of the tariff threats which includes not implemented and some announcements with the future implementation uncertain. In Appendix, we also show tariffs announcements and impementations by date. This also leads to a great deal of uncertainty surrounding which tariffs will be implemented at the end. That is why, we also model the in our quantitative section.

As validation, we also model the trade war between United States on China and other countries with tariffs imposed from February 2018 to September 2018. In this period, the U.S. implemented tariffs ranging from 10% to 25% to China, 10% tariff to aluminum, 25% to iron and steel, 30% to solar and 20 to 50 % tariffs to washers with some exceptions at country levels. In return, Canada, China, European Union, Mexico, Russia and Turkey retaliated with tariffs ranging from 5 to 20%. We obtained the detailed tariff data for this episode from

[33] https://www.stlouisfed.org/on-the-economy/2025/may/what-have-we-learned-us-tariff-increases-2018
[34] https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/

Fajgelbaum et al. (2020) and trade values to calculate the weighted tariff rates from USITC website.[35]

Figure 12. Tariff Threats - not implemented and future implementation uncertain



NOTE: Tariff threats between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far). The tariffs that classified as "threats" are those that –as of June 30th —had not been implemented and were unlikely to be implemented based on available information. These threats were identified by extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

7.3 Calibration Parameters

The calibration parameters are summarized in Table 2. The model employs sector-specific Calvo parameters based on the empirical estimates in Nakamura and Steinsson (2008), adjusted to a quarterly frequency. The production and intratemporal consumption structures are similar to those in Çakmaklı et al. (2025) and di Giovanni et al. (2023). On the produc-

[35] Exports: https://dataweb.usitc.gov/trade/search/TotExp/HTS, Imports: https://dataweb.usitc.gov/trade/search/GenImp/HTS.

tion side, firms combine labor and intermediate input bundles to produce goods. Based on Atalay (2017), we set the elasticity of substitution between labor and intermediates $\theta^P = 0.6$. Boehm et al. (2023) estimate short-run trade elasticities of approximately 0.76 and long-run elasticities around 2. For our tariff scenarios, we adopt the lower short-run elasticity of 0.76, which better captures the immediate effects that are more relevant for monetary policy. In contrast, USTR (2025) uses a higher value of 4 for the trade elasticity. Intermediate input bundles are composed of sectoral bundles, which are assumed to be complements. Following Boehm et al. (2019) and Baqaee and Farhi (2024), we set this elasticity in the range of $\theta_h^P = 0.001 - 0.2$. Each sectoral bundle consists of varieties sourced from different countries. In our baseline specification, we set the Armington elasticity across countries at the sectoral level to $\theta_{li}^P = 0.6$. On the intratemporal consumption side, we follow Baqaee and Farhi (2024) and assume Cobb–Douglas preferences across sectors, setting the sectoral elasticity to $\theta_h^C = 1$. For the aggregation of varieties within sectoral consumption bundles, we adopt the same approach as in the production structure.

Additionally, we incorporate monetary policy inertia by modifying the baseline Taylor rule. Specifically, Equation (24) is replaced with the following specification:

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho_m^n} (\Pi_{n,t})^{\phi_\pi^n} (Y_{n,t})^{\phi_y^n} e^{\hat{M}_{n,t}} \quad \forall n \in N$$

Here, ρ_m^n captures the degree of interest rate smoothing (or policy inertia), ϕ_π^n and ϕ_y^n are the inflation and output coefficients in the Taylor rule, and $\hat{M}_{n,t}$ denotes a monetary policy shock. This specification is applied to all countries $n \in N$ in the model.

For the United States, we set $\rho_m^{\text{US}} = 0.82$ and $\phi_\pi^{\text{US}} = 1.29$, based on the estimates provided by Carvalho et al. (2021a). Following Clarida et al. (2000), we use $\rho_m^n = 0.95$ and $\phi_\pi^{EA} = 1$ for the rest of the world and the euro area, respectively. For other countries in the rest of the world, we assume $\phi_\pi^n = 0.2$, except for Mexico, where we use a slightly higher value of $\phi_\pi^{\text{MX}} = 0.3$. These ϕ_π values are calibrated using a model-consistent interpretation of the long-run average of quarterly inflation rates. Specifically, following the logic in Clarida et al. (2000), we set $\phi_\pi^n = \frac{1-\rho_m^n}{\overline{\pi}_n^C}$, where $\overline{\pi}_n^C$ denotes the long-run average of quarterly CPI inflation in country n. Using quarterly data from 2002Q2 to 2024Q4 and setting $\rho_m^n = 0.95$, we calibrate the inflation response coefficients accordingly. This calibration captures the empirical observation that central banks in many countries outside the United States are less responsive to inflation fluctuations and are therefore less likely to adhere strictly to a Taylor rule.

Table 2. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal EoS	2	e.g., Itskhoki and Mukhin (2021)
η	Elasticity of Labor	1	e.g., Itskhoki and Mukhin (2021)
ψ	Reactivity of UIP to Debt	0.001 – 0.0001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82	Carvalho et al. (2021a)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	1.29	Carvalho et al. (2021a)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^P	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	0.6	Calibrated for consistency
θ_h^P	EoS among intermediate inputs	0.001 – 0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6	di Giovanni et al. (2023)
θ_{li}^P	Sector level input bundle EoS	0.6	di Giovanni et al. (2023)

NOTES: "EoS" is the elasticity of substitution.

8 Quantitative Results

We now return to the exact version of the model without any linearization. Tariffs follow an AR(1) process (i.e., $\tau_t = \rho^\tau \tau_{t-1} + \epsilon_t^\tau$) and we specify the value of ρ^τ below in each case. The quantitative model also incorporates a permanent real capital account wedge to treat the year 2018 as the steady state to which the economy eventually returns. This allows us to embed a realistic net foreign asset (NFA) position for all relevant country blocks. With these wedges, we can discipline the debt dynamics such as running permanent current account/trade deficit in the steady state requires accumulated returns from positive net foreign assets.

As the model is non-linear, we solve the model with Dynare under three alternative solution methods: first-order approximation, second-order approximation, and MIT shocks under perfect foresight. For small shocks, these methods yield nearly identical impulse response functions. However, our preferred solution approach employs MIT shocks under perfect foresight, given the presence of non-linearities in both the production and consumption structures, as well as the sizeable nature of the trade shocks we analyze. We experiment with both permanent (or near-permanent) tariff shocks—modeled as autoregressive processes with coefficients of 0.95 or higher—and transitory shocks, such as one-time tariff increases. While local solution methods (e.g., first-order approximation) are valid only in the neighborhood of the steady state, perfect foresight solutions are better suited for analyzing the effects of permanent shocks that drive the system further from its baseline. Accordingly, for scenarios involving persistent policy changes, the perfect foresight approach provides additional insights beyond what local approximations can offer.

8.1 Case 1: 2018's Trade War

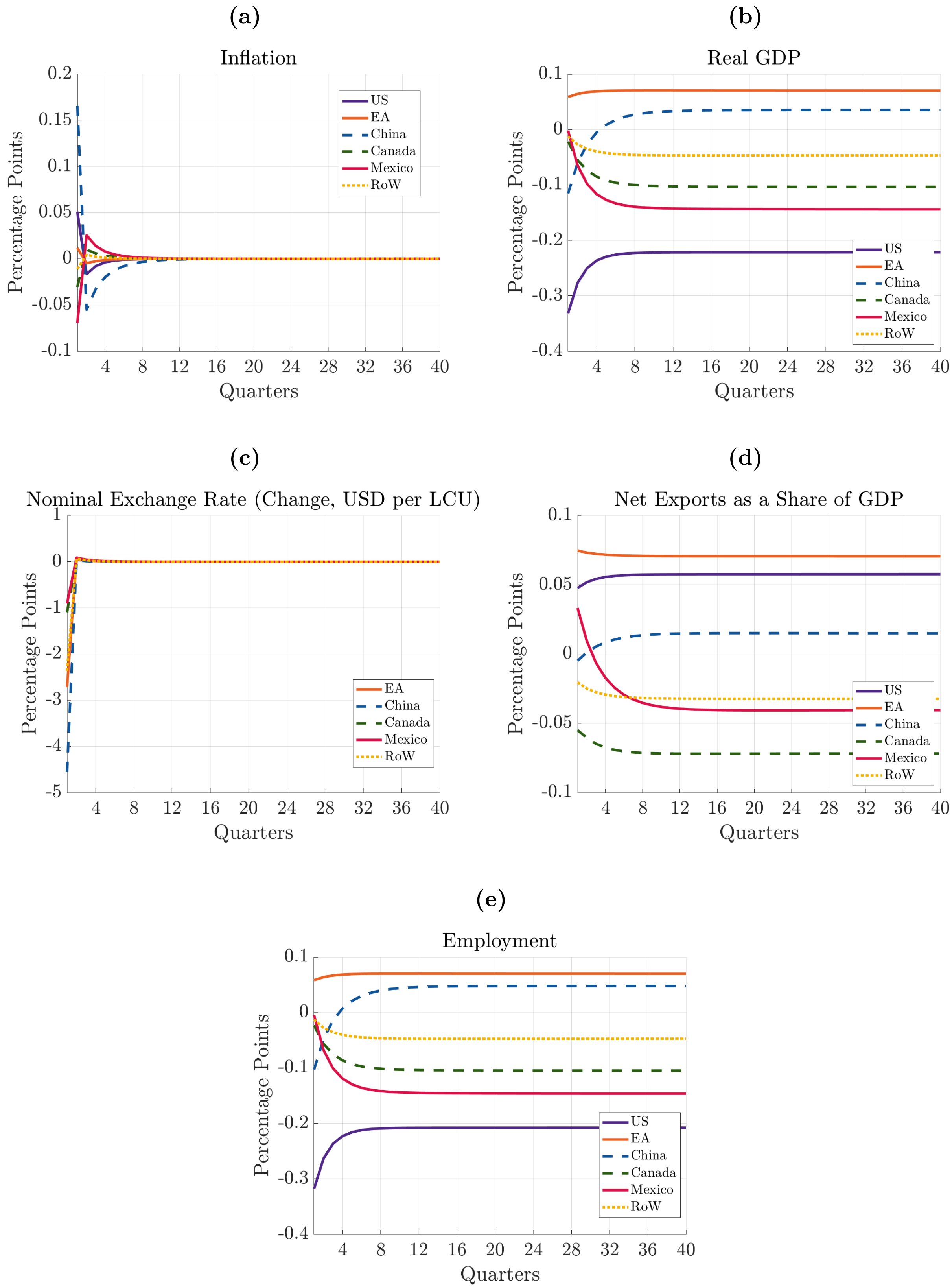
We begin by validating the model using the case of the tariffs imposed by the United States on China and other countries between February 2018 and September 2018 (See Section 7.2 for details of the data). We model this as a fully permanent shock with $\rho^\tau = 1$. We assume that the central banks involved did not place a weight on deviation from pre-tariff output (i.e. $\phi_y = 0$). As shown in Figure 13, the model predicts a 4.5% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan. This closely aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018.

The model estimates the impact of the 2018 tariffs on U.S. inflation to be 0.1 percentage points. This is in line with the magnitude of the static estimate of Barbiero and Stein (2025), who find that the tariff war may have contributed between 0.1 and 0.2 percentage points to U.S. PCE inflation using a static partial equilibrium model. Our estimate lies at the lower end of this range, which is consistent with the structure of our model—featuring nominal rigidities and network complementarities—tending to produce smaller inflationary effects and larger real responses when shocks are realized. On impact, real U.S. GDP declines by 0.33%. This magnitude is comparable to the findings of Fajgelbaum et al. (2020), who estimate that the tariffs resulted in producer and consumer losses totaling 0.4% of GDP. Notably, the model also captures changes in external balances: U.S. net exports increase by 0.1% of steady-state GDP, while China's net exports decline less by 0.01%. These are meaningful magnitudes as they are relative to steady-state GDP. For context, U.S. overall trade balance improved around 1 percentage points from 2018 to 2019.

China experiences a modest contraction in real GDP, with output declining by 0.12%, accompanied by much larger declines in consumption (0.6%) and real wages (0.9%) and the highest inflation among all countries. The renminbi depreciates more than 4% in nominal and in real terms. In contrast, the Rest of the World (RoW) experiences a negligible output loss (0.01%), with only minor movements in macroeconomic indicators.

Figure 13 illustrates the model's dynamics over a ten-year horizon. Recall that this is a permanent shock. As shown in Figure 13a, all regions experience an initial inflationary shock, followed by a deflationary adjustment. U.S. real GDP contracts on impact (Figure 13b) and remains approximately 0.5 percentage points below its pre-shock level in the long run. In contrast, China exhibits a gradual recovery. While the Rest of the World (RoW) has the minimal loss, Euro area experiences modest gains, benefiting from the opportunity to substitute for Chinese exports in the U.S. market. Interestingly, both Mexico and Canada also loses together with the U.S. given their tight production links to the U.S. Employment patterns, shown in Figure 13e, closely follow the path of real GDP.

Figure 13. Case 1: Impact of 2018's Trade War



NOTE: Simulated responses to the 2018 U.S. tariffs on China. Impulse responses are computed under with MIT shocks, with a near-permanent tariff shock ($\rho^{\tau} = 1$).

Table 3. On-Impact Response of Variables in Case 1: 2018's Trade War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.33%	0.06%	-0.12%	-0.02%	-0.00%	-0.01%
C_n	-0.08%	-0.08%	-0.54%	0.16%	0.31%	0.07%
π_n	0.05%	0.01%	0.17%	-0.03%	-0.07%	-0.01%
i_n	0.07%	0.01%	0.03%	-0.01%	-0.02%	-0.00%
$\Delta\mathcal{E}_n$	0.00%	-2.71%	-4.55%	-1.10%	-0.91%	-2.36%
ΔRER_n	0.00%	-2.75%	-4.44%	-1.18%	-1.03%	-2.42%
L_n	-0.32%	0.06%	-0.10%	-0.02%	-0.00%	-0.01%
$\frac{W_n}{P_n}$	-0.47%	-0.11%	-1.18%	0.30%	0.62%	0.13%
$\frac{NX_n}{NGDP_n^{ss}}$	0.05%	0.07%	-0.01%	-0.06%	0.03%	-0.02%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.00%	-0.02%	-0.56%	-0.01%	0.01%	-0.22%

NOTE: First-period impact of the U.S. tariffs in 2018. Effects are reported in deviation from the pre-tariff steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

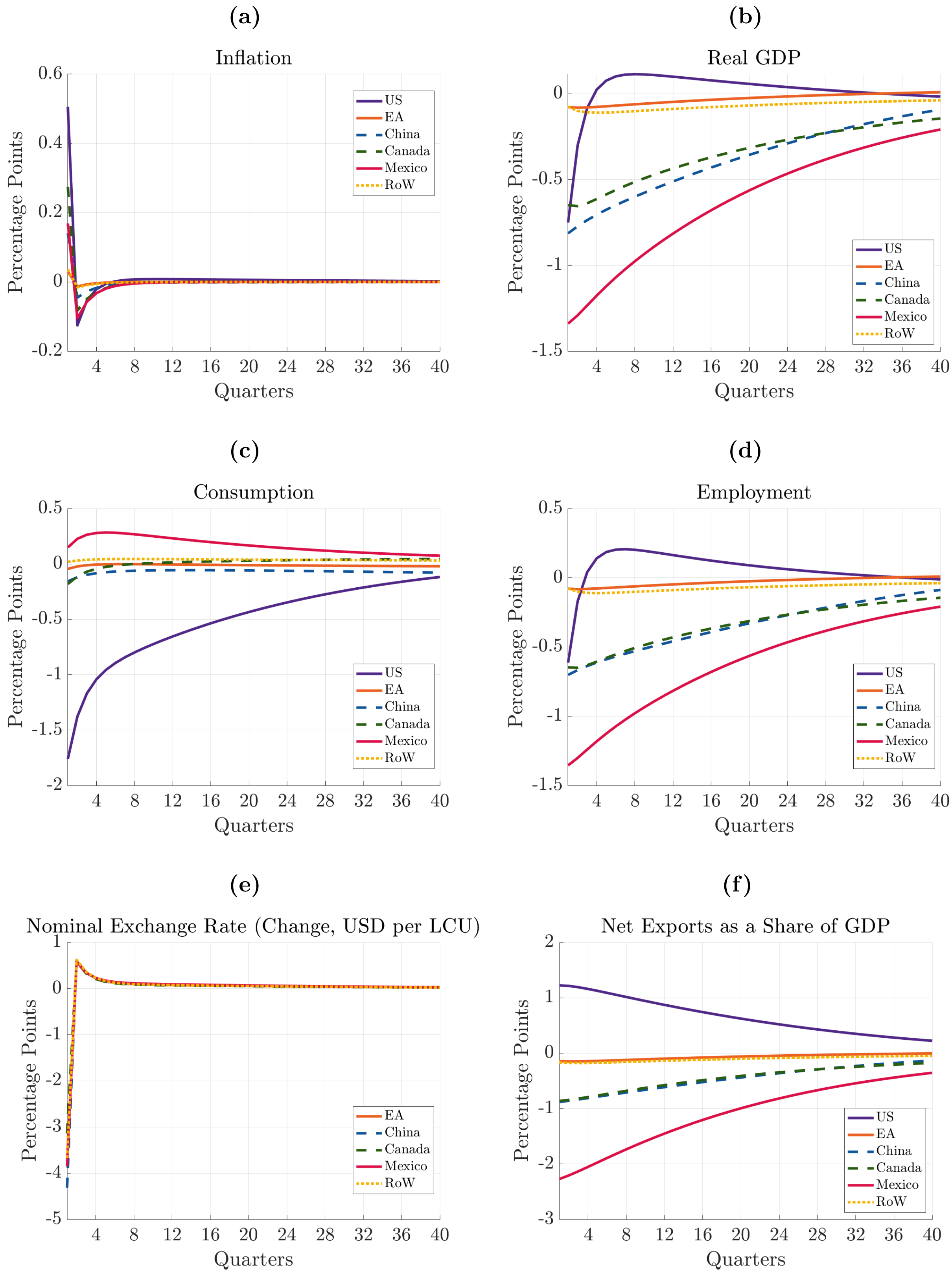
8.2 Case 2: 2025's Trade War

In 2025, the United States announced several rounds of tariffs targeting Mexico, Canada, Europe, China, and many other countries. We explained in detail the tariffs announced, implemented, changed and limited retaliation from others happened so far, at the time of this writing, in Section 7.2 . Even though we set the retaliation to zero, or use what happened in reality, we get the same results as shown under Case 2 since retaliation so far stays limited. We apply 10% to the Rest of the World (RoW). We set the tariff persistence parameter to $\rho^\tau = 0.95$.

As shown in Figure 14 and Table 4, the model predicts a contraction in U.S. real GDP, declining by almost 0.8% on impact. This is accompanied by almost 2.0% decrease in consumption, a 1.2% increase in net exports as a share of steady-state GDP (improvement in trade deficit), and a 4% decline in real wages. Inflation rises by 0.5 percentage points, prompting a 0.7 percentage point increase in the nominal interest rate. Additionally, the U.S. trade-weighted nominal effective exchange rate (NEER) appreciates by 4%.

The effects are most pronounced for Mexico and Canada. Mexico's real GDP contracts by 1.3%, while Canada's declines by 0.7%. Labor market impacts are also substantial, with employment falling by 1.4% in Mexico and 0.7% in Canada, same as in China. Net exports

Figure 14. Case 2: Impact of 2025 Tariffs (w/Limited Retaliation)



NOTE: Simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, Europe and the RoW. Impulse responses are computed with MIT shocks persistence of $\rho^{\tau} = 0.95$.

decline sharply, by 2.2% and 0.8% of steady-state GDP, respectively. Inflation rises by 0.2 percentage points in Mexico and 0.3 percentage points in Canada, less than U.S.

Table 4. On-Impact Response of Variables in Case 2: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.75%	-0.08%	-0.81%	-0.65%	-1.34%	-0.08%
C_n	-1.76%	-0.04%	-0.16%	-0.18%	0.15%	0.02%
π_n	0.51%	0.03%	0.14%	0.27%	0.17%	0.04%
i_n	0.66%	0.03%	0.03%	0.06%	0.05%	0.01%
$\Delta\mathcal{E}_n$	0.00%	-3.82%	-4.31%	-3.14%	-3.85%	-3.67%
ΔRER_n	0.00%	-4.28%	-4.66%	-3.36%	-4.17%	-4.12%
L_n	-0.61%	-0.08%	-0.70%	-0.65%	-1.36%	-0.08%
$\frac{W_n}{P_n}$	-4.08%	-0.17%	-1.01%	-1.01%	-1.06%	-0.04%
$\frac{NX_n}{NGDP_n^{ss}}$	1.24%	-0.15%	-0.88%	-0.83%	-2.19%	-0.16%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.16%	0.04%	-0.42%	-0.02%	0.07%	-0.35%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package. Tariff rates vary by country-sector; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

China experiences the same amount of contraction as the U.S., a decline of –0.8% in GDP and –0.7% in employment. Consumption decline is much more muted. Inflation increases modestly like Mexico by 0.1 percentage points. Notably, the renminbi depreciates by 4.3% against the U.S. dollar in nominal terms. The euro area (EA) experiences very small output effects, like ROW. Consumption decline is also very small (–0.04%). Inflation in the EA rises only by 0.03 percentage points.

As shown in Figure 14a, inflation declines across all regions after the initial period, with everyone except Euro Area (EA) experiencing mild deflation. In the medium to long run, only the U.S. a positive effect on real GDP (Figure 14b). This is driven by the high degree of substitution that drives employment and output higher through higher production, under a near-permanent but not fully permanent shock like the 2018 case. Consumption stays depressed though. This is in spite of the fact that, as shown in, Figure 14e, U.S. dollar initially appreciates relative to all other currencies on impact; thereafter there is a small depreciation in the second period, after which the changes in the exchange rate are minimal.

In terms of trade balances, Figure 14f shows that net exports improve only slightly for the US, while all other regions see some deterioration. Employment dynamics, depicted in Figure 14d, closely track real GDP patterns given the household's labor supply decision.

8.3 Case 3: All-Out Trade War

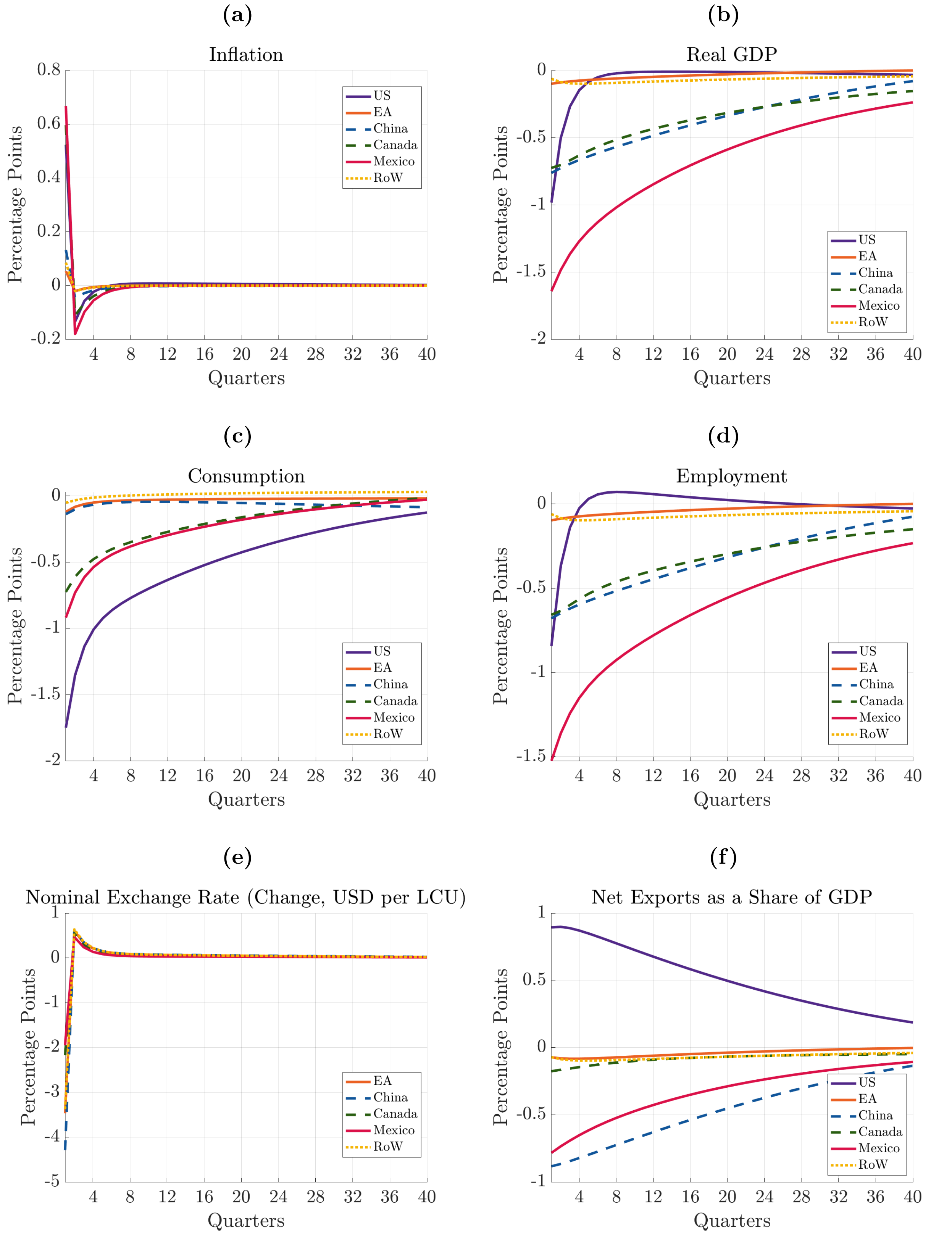
We now turn to a counterfactual quantitative exercise that mirrors the theoretical simulation presented earlier but rather uses 2025 tariffs implemented and proposed by the U.S. administration, where the countries retaliate in return with the exact same amounts—an all-out symmetric tariff war. In this case, the United States imposes tariffs on all major trade partners at the same rates as specified in Case 2. However, unlike the unilateral shock in Case 2, with some limited retaliation, trade partners retaliate by imposing symmetric tariffs on U.S. exports. The persistence of the tariff shock is set to $\rho^{\tau} = 0.95$, reflecting a near-permanent policy change.

China experiences a contraction in GDP, declining by 0.8%, while consumption drop is limited (–0.1%). The real exchange rate depreciates by 5%. Inflation rises by 0.5 percentage points, and employment declines marginally by 0.1%. Real wages decline is not as large as other countries, 1%. The euro area experiences a very moderate contraction. Real GDP declines by 0.1%, consumption falls by 0.1%, and real wages decrease by 0.3%. Inflation rises modestly by 0.1 percentage points. The euro depreciates by 3.4% against the U.S. dollar, partly reflecting the divergence in inflation and interest rate responses between the two regions. This exchange rate adjustment helps absorb a portion of the external shock, mitigating further declines in output. The Rest of the World experiences a mild contraction overall.

As illustrated in Figure 15, the model predicts a substantial contraction in U.S. real GDP, which declines by 1% on impact. Consumption falls by almost 2%, while net exports increase by 1% as a share of steady-state GDP. Inflation rises by 0.5 percentage points, prompting a corresponding increase in the nominal interest rate of 0.7 percentage points. Labor market effects are pronounced, with real wages falling by 4.3% and employment declining by 0.8%. The U.S. NEER appreciates by 2.6%.

The effects of the global tariff war extend across regions, though with heterogeneous intensity. Canada, Mexico, and China are again among the most adversely affected, but China is better off than the U.S. Real GDP contracts by 0.7% in Canada and by 1.7% in Mexico. Net exports decline by 0.7% of steady-state GDP in Mexico, but much less in Canada, while employment falls by 0.7% in Canada and 1.5% in Mexico. Inflation rises by 0.6 percentage points in Canada and 0.7 percentage points in Mexico. Real wages decline

Figure 15. Case 3: Impact of All-Out Tariff War



Note: All-out tariff war scenario in which trade partners retaliate symmetrically. Impulse responses are calculated with MIT shocks and with shock persistence is set to $\rho^{\tau} = 0.95$. Tariff rates same as Case 2.

by 3% and 2%, respectively, indicating substantial labor market strain.

Table 5. On-Impact Response of Variables in Case 3: All-Out Tariff War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.98%	-0.10%	-0.76%	-0.72%	-1.64%	-0.06%
C_n	-1.75%	-0.12%	-0.14%	-0.73%	-0.92%	-0.05%
π_n	0.52%	0.05%	0.13%	0.60%	0.67%	0.08%
i_n	0.68%	0.05%	0.03%	0.12%	0.20%	0.02%
$\Delta\mathcal{E}_n$	0.00%	-3.47%	-4.29%	-2.17%	-1.95%	-3.37%
ΔRER_n	0.00%	-3.92%	-4.66%	-2.10%	-1.81%	-3.80%
L_n	-0.84%	-0.10%	-0.68%	-0.66%	-1.52%	-0.06%
$\frac{W_n}{P_n}$	-4.28%	-0.34%	-0.95%	-2.09%	-3.33%	-0.16%
$\frac{NX_n}{NGDP_n^{ss}}$	0.92%	-0.08%	-0.89%	-0.16%	-0.73%	-0.07%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.09%	0.02%	-0.41%	-0.02%	0.03%	-0.35%

NOTE: First-period outcomes from a global tariff war scenario with full retaliation. Tariff magnitudes and persistence match Case 2. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

The dynamics of the model under a full trade war, depicted in Figure 15, resemble those in Figure 14, albeit with significant differences in magnitude. Notably, initial inflation is higher across all regions, while the exchange rate and net export effects are more muted. Interestingly, U.S. turns out to be the loser in this war as consumption stays depressed and now output and employment also do not recover as in the realistic case 2 with limited retaliation. This exercise underscores that retaliation entails significant costs, especially for the country imposing tariffs, in spite of the terms of trade gains.

As a robustness check, we also examine the implications of a higher Armington elasticity of 4, on all imported goods, final and intermediate, consistent with the assumption used by USTR (2025). As shown in Figure A.10 in Appendix, the magnitude of the quantity responses shown here in the counterfactual trade war case are all significantly attenuated under the high-elasticity scenario for the intermediate input substitution. This is expected given the important role of network complementarity in the model in amplification of the tariff shocks.

8.4 Case 4: Reversed Tariff Threats

As seen in Figure 12, there has been many tariff threats that are not implemented or uncertain to be implemented. In this section, we apply our model to the case of reversed tariff threats—scenarios in which a country announces future tariffs but subsequently reverses the decision before implementation. This case also incorporates retaliation: specifically, the United States announces in period 1 that tariffs will be imposed in period 2, prompting other countries to announce retaliatory measures for the same period. However, when period 2 arrives, it is announced that no tariffs will be levied by either side. This scenario not only mimics the reality of 2025 geopolitics but also it allows us to isolate the role of the expectations channel, while examining a real-world dynamic that has become increasingly common in the context of U.S. trade policy, where tariff threats are frequently issued and later postponed or rescinded.

Our approach is inspired by the *fake news* algorithm of Auclert et al. (2021), in which agents receive information about a future increase in income and optimize accordingly, only to later discover that the anticipated change does not materialize. While Auclert et al. (2021) employ this construct as a computational device for solving models in sequence space, we interpret and apply it literally to study the macroeconomic implications of trade policy reversals.

To analyze the effects of reversed tariff threats, we construct two impulse responses under perfect foresight. First, we simulate the all-out tariff war shock examined in Case 3, assuming it is both announced and implemented in the first period of the simulation. Second, we simulate the same shock—identical in magnitude—but announced to take effect in the second period, only to be withdrawn before implementation. The impulse response to the reversed tariff threat is then obtained by shifting the first (implemented) impulse response forward by one period and subtracting it from the second (announced-but-not-implemented) response. This approach isolates the effect of the anticipatory behavior triggered by the announcement, net of the effects of actual implementation. Importantly, we observe that from the second period onward, the quantity variables in both simulations converge and remain nearly identical. This reflects the fact that agents discount the future and adjust quantities in response to the announcement, but not to the same extent as they would if the shock were immediate and fully realized.

Figure 16 compares the impact on GDP, inflation, consumption, employment, and U.S. dollar appreciation against the Chinese yuan in Case 3 (Tariff Shock) to the reversed tariff threat scenario (we only plot single currency for the country subject to largest number of threats—China). Although tariffs are never actually implemented, real variables respond: real GDP and consumption decline by approximately 0.9 and 0.7 percentage points, respectively. Because prices are forward-looking, their responses are of greater magnitude. The near-permanent nature of the anticipated shock induces a pronounced increase in prices, as households and firms adjust their behavior in light of expected future income streams.

A future in which the United States demands fewer goods from China prompts an immediate appreciation of the USD, as agents incorporate these expectations into current pricing. In this scenario, the U.S. trade-weighted nominal effective exchange rate appreciates by 2.4% on impact. In contrast, quantity variables respond more gradually. Consumption declines as households begin smoothing in anticipation of a lower future consumption path. Although consumption begins adjusting toward the level consistent with an immediate tariff shock, it does not fall fully in the first period. When agents realize in the second period that the shock will not materialize, they reoptimize, resulting in a partial recovery. Output follows a similar pattern—declining on impact and gradually recovering thereafter.

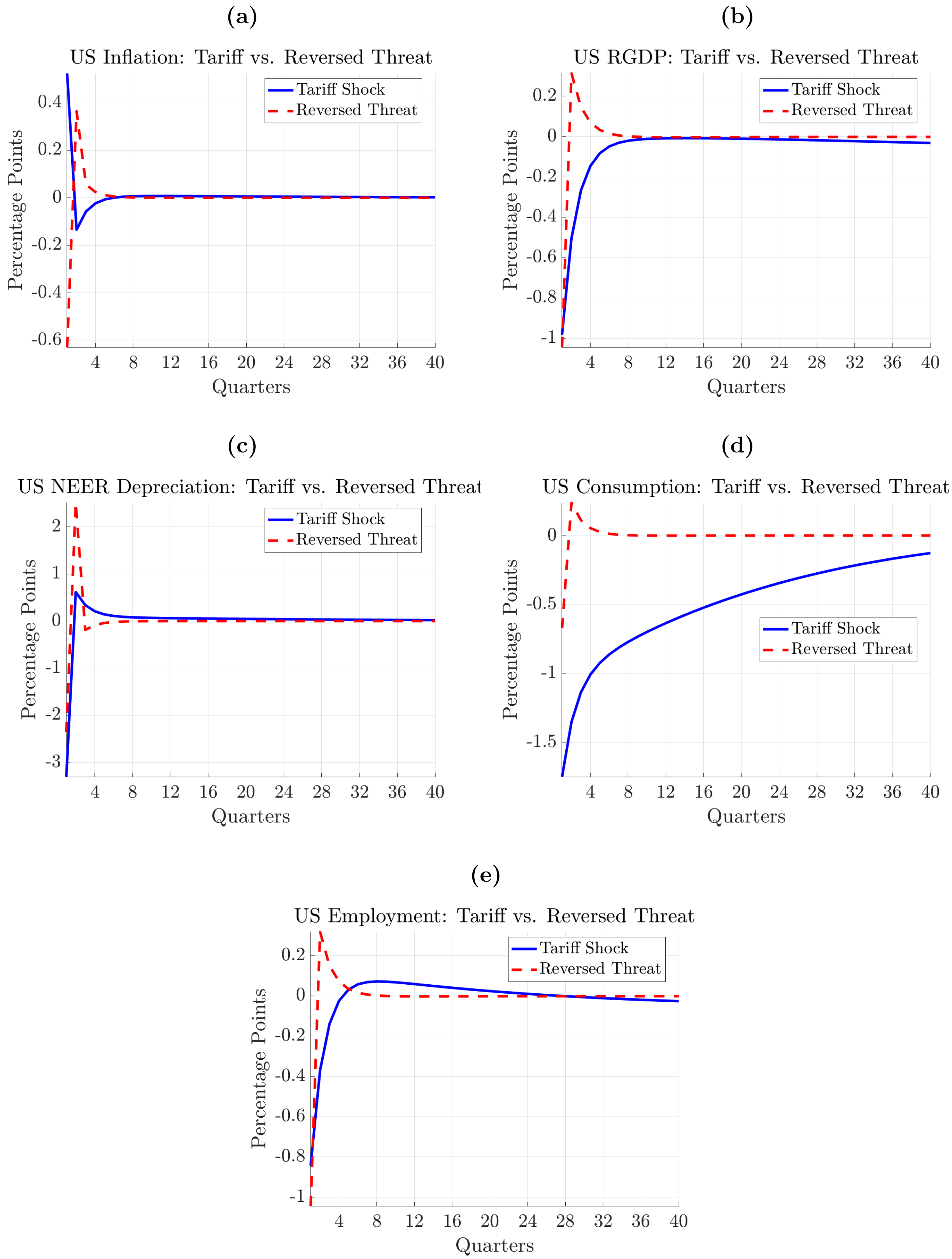
Overall, this exercise demonstrates that the expectations channel, emphasized in our theoretical analysis, plays a central role. Reversed tariff announcements operate similarly to domestic demand shocks, particularly when announcements are perceived as credible. Importantly, the macroeconomic distortion introduced through the expectations channel does not dissipate immediately with the reversal announcement. Variables exhibit persistence, and the economy does not return to steady state instantaneously.

It is notable that, once tariffs are reversed, the U.S. dollar depreciates: agents had previously priced in a future in which the U.S. would reduce demand for foreign goods, but upon receiving new information in the second period that this scenario would not materialize, the exchange rate response is reversed. Although expectations linked overshooting is interesting since this does not happen with regular tariffs. A more realistic interpretation of the observed and somewhat sustained U.S. dollar depreciation in response to tariffs requires accounting for a much larger uncertainty (VIX) shock and policy volatility more than our simple one period on-off tariff threat exercise, or other shocks such as fiscal uncertainty, that are outside the scope of our paper.

8.5 Discussion

Our analytical and quantitative analyses allow us to engage with several central questions. Under what conditions are tariffs appreciationary or depreciationary for the nominal exchange rate? Under what conditions are tariffs inflationary or deflationary? And under what conditions tariffs can be contractionary? We know these answers from the model but here in the light of the quantitative results that takes into account non-linearities, we provide further discussion.

Figure 16. Case 4: Impact of Reversed Tariff Threats



NOTE: Simulated response to reversed tariff announcements. Tariffs are announced in the first period but canceled in the second period.

8.5.1 Trade Deficits and the Dollar

In our quantitative framework, we find that tariffs can lead to an appreciation of the currency of the tariff-imposing country on impact. However, once retaliation is introduced, the exchange rate response becomes sensitive to the relative hawkishness of central banks. For instance, in a scenario where the U.S. imposes tariffs and the rest of the world responds, the U.S. dollar (USD) may depreciate on impact if the rest of the world has higher ϕ_π parameters—leading to greater interest rate differentials in favor of non-USD currencies.

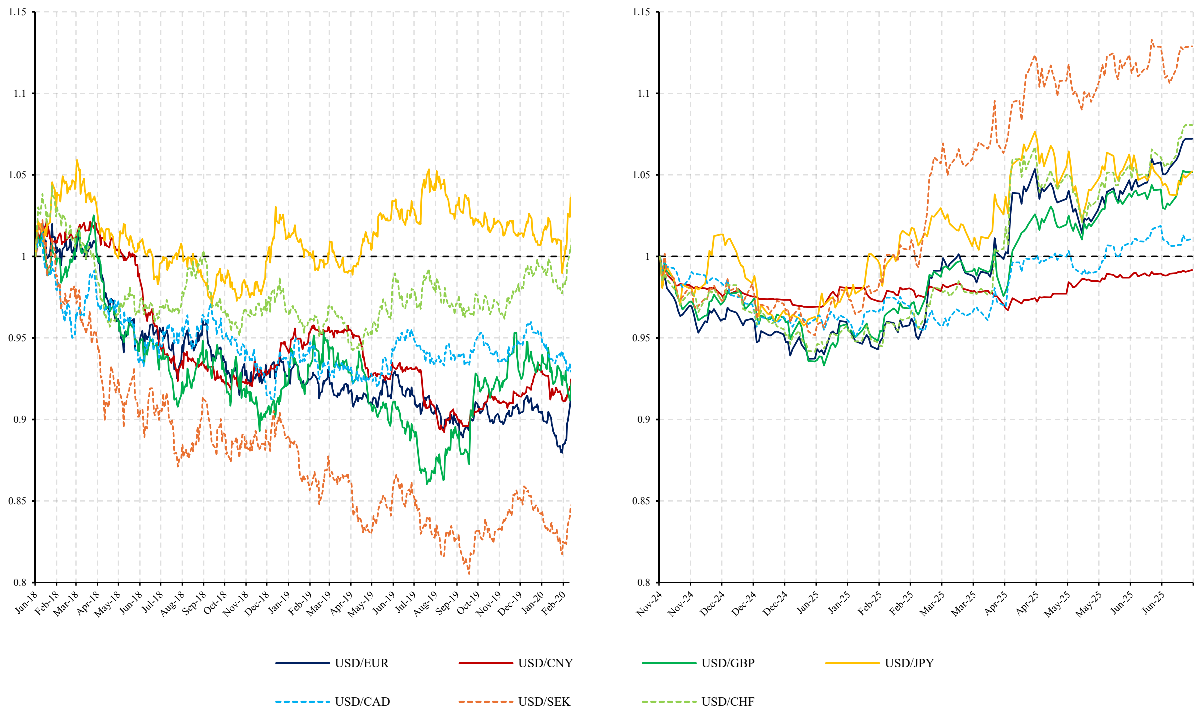
Other work, such as Jiang et al. (2025) and Itskhoki and Mukhin (2025), explain the observed depreciation of the dollar, since the beginning of April 2025, with the loss of safe heaven status or convenience yield, where the two are related as shown before (e.g., Kekre and Lenel, 2024). We agree that this is a likely possibility but it is also important to acknowledge that there was a huge uncertainty shock captured by the VIX rise in April 2025, which was as high as what was observed during the pandemic. Thus, another alternative for the observed dollar depreciation is that the impact of policy uncertainty embedded in tariff threats dwarfs the standard appreciationary effect of a regular tariff shock. In fact, the early appreciation followed by a depreciation of the dollar shown in Figure 16 as a result of tariff threats is similar to what is observed after inauguration in January 2018 and January 2025, as shown in Figure 17. Although the depreciation of the dollar, especially vis-a-vis classic safe haven currencies, is in excess of 10 percent at the time of this writing, it is still small viewed in a historical context as shown in Appendix Figure A.3. Interestingly, the initial appreciation of around 2% and then the depreciation of 4% plotted in this Appendix figure over very short horizons are both in the ballpark of what we get with our tariff threat calibration.[36]

On trade deficits: In our model, there is a change in trade deficit during transition but under transitory tariffs countries go back to where they started in terms of trade deficits. Even with permanent tariff shocks, as shown in our quantitative exercise, the proposed tariffs do not affect trade deficits decisively. This is because tariffs do not have an affect on consumption-saving patterns permanently, even they can be affected during transition. So, steady state trade imbalances will not change. A similar argument has put forth also by Obstfeld (2025).

[36] If we want to match the observed movements in dollar, we can do it by putting an uncertainty wedge in the UIP equation for idiosyncratic local risk factor linked to policy volatility as done in Kalemli-Özcan and Varela (2021).

Figure 17. USD Exchange Rates against to Major Currencies, following 2018 and 2025 Inaugurations

(a) January 2018 – February 2020 **(b)** November 2024 – June 2025



Note: USD vs. EUR, CNY, GBP, JPY, CAD, SEK, CHF normalized exchange rates during (a) 2018 Trade war episode between USA and China between January 2018 and February 2020 (before Covid-19 pandemic). (b) Since the election of President Trump for his second term (November 2024) until the latest available date (June 30, 2025). Data Source: Bloomberg.

8.5.2 Inflation-Output Trade-Off and Employment

Our analytical work and calibrations show that tariffs can be inflationary or deflationary for the country on which they are imposed, as they reduce demand for that country's goods, but also can raise them with trade diversion. A more subtle question is whether tariffs can be deflationary for the tariff-imposing country itself, such as the United States. Within our modeling framework, and barring extreme parameterizations, the direct effect of tariffs, which mechanically exerts upward pressure on prices, dominates the deflationary forces from other channels. If inflation were to turn negative, monetary policy would reverse direction and cut interest rates, thereby supporting prices. Consequently, in both our analytical solution and baseline simulations (Cases 1, 2, and 3), tariffs are inflationary for the imposing country and output declines in the short-run and also in the long-run with retaliation. The key exception is Case 4, in which tariffs threats lead to deflation due to expectations channel.

Overall tariffs can create a stagflationary outcome with increasing inflation and declining employment and output. The response of monetary policy is critical here. We assume the central bank target inflation, alternative formulations, such as central banks put equal weight on inflation and output can change the results.[37]

9 Conclusion

We develop a new global general equilibrium framework to study the macroeconomic impact of tariffs under global imbalances. Our NKOE model incorporates full global input–output linkages, heterogeneity in sectoral price rigidities, and endogenous monetary policy responses to tariffs across all countries involved in a potential trade war. We formulate the model around five primitives composed of structural parameters (consumption shares, production shares, elasticities of substitution), frictions (nominal rigidities), and endogenous monetary policy (Taylor rule).

The presence of nominal rigidities, production network structure, and input complementarities play a crucial role in shaping the inflation and output responses to tariffs, influencing Phillips Curve dynamics, inflation-output trade-off by introducing new wedges relative to the predictions of both flex-price trade and also NK-SOE models. Our quantitative results highlight the inflationary and contractionary effects of tariff shocks in an environment with forward-looking agents, where these effects are further amplified through the expectations channel. We decompose the general equilibrium response of key macroeconomic variables to trade shocks into channels—each of which maps directly onto structural components of the model. We demonstrate that the net impact of tariffs on domestic inflation and output critically depends on the endogenous monetary policy responses in both the tariff-imposing and tariff-exposed countries and international risk sharing.

Our work yields two key implications, relevant both for scholars and policy makers. First, models that omit a multi-sector structure may underestimate the impact of tariffs on real economic quantities—such as output and employment—while overestimating their effect on inflation, especially under the assumption of balanced trade. Second, tariff threats carry real macroeconomic consequences—even when they are subsequently reversed. When agents expect future price increases, they begin to smooth consumption downward in anticipation.

[37] In our network setup, we are aware, as Rubbo (2023) noted, that standard divine coincidence does not hold. Targeting only CPI inflation can, in theory, leave behind a permanent cost-push shock unless one targets the divine coincidence index. In our simulations, however, we find that central bank targeting of only CPI inflation does not result in a major discernible permanent inflationary impulse. This might be because in our model, the CPI shares that are based on the ICIO table happen to be close to the shares that one would assign based on the divine coincidence index proposed by Rubbo (2023).

Because the exchange rate is forward-looking, it appreciates immediately in response to these expectations, but then reverses itself and depreciates when threat turns empty. In this way, tariff threats function as contractionary demand shocks, even in the absence of actual tariff implementation. A deeper understanding of both production network structures and expectation-driven dynamics—such as those modeled here—can help central banks navigate a policy environment in which tariffs, retaliation, and related threats are becoming increasingly common. As Federal Reserve Chair Jerome Powell recently emphasized, navigating the current environment poses significant challenges. In April 2025, he cautioned, “We may find ourselves in the challenging scenario in which our dual-mandate goals are in tension.” Then, in June 2025, he further acknowledged the uncertainty surrounding trade policy transmission: “There aren’t historical experiences we can consult here. So it may turn out that the tariff pass-through is less or more than we think. We are perfectly open to the idea that the pass-through will be less than we think, and, if so, that will matter for our policy.” Our analysis can shed light on these pressing policy questions.

By theoretically unifying long- and short-run perspectives on the impact of trade barriers, our framework echoes foundational insights from classical economic literature, dating back to Hume (1758), which emphasized the price–specie flow mechanism. This mechanism illustrates how price levels adjust endogenously through trade flows, ultimately rendering trade restrictions self-defeating. Restrictions on exports and imports induce exchange rate movements that offset perceived gains. For countries imposing import restrictions, rising labor and input costs typically follow, forcing firms to reduce employment and scale back production—ultimately undermining domestic economic performance. This core insight traces back even further to Gervaise (1720), underscoring the long-standing understanding that trade barriers distort price signals, resource allocation and economic growth.

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APPENDIX

A Additional Results

Table A.1. U.S. Tariff events from the WTO-IMF Tariff Tracker.

Event Date	Average Tariff (%)	Event Label	Event Description
1/1/2025	2.3	Pre-Trump	The baseline tariff rates for U.S. imports from China have been updated to reflect actual tariff rates applied per tariff line, based on data from the U.S. Census for 2024. These were then compared with the Most Favored Nation (MFN) tariff rates for 2024 to identify pre-existing tariff hikes before the start of 2025. The resulting tariff rates were rounded to the nearest 0.5%. For other exporters, the baseline tariff rates are a combination of MFN and preferential rates for 2024.
2/4/2025	3.6	China +10	On February 4, 2025, the United States imposed an additional 10% tariff on all imports from China.
3/4/2025	11.3	China +10	On March 3, 2025, the United States further increased tariffs from 10% to 20% on all imports from China.
3/4/2025	11.3	Can/Mex +25	On March 4, 2025, the United States implemented additional 25% tariff on imports from Canada and Mexico. Energy resources from Canada will have a lower 10% tariff.
3/7/2025	8.7	USMCA Exemptions	Effective on 7 March 2025 the United States announced an exemption for all imports complying with the United States-Mexico-Canada Agreement (USMCA). Compliance rate has been estimated using 2023 imports notification submitted by the U.S. to WTO's IDB. Additionally, tariff on potash imports have been reduced from 25% to 10%.
3/12/2025	9.7	Steel & Alum. Tariffs +25	On March 12, 2025, the United States imposed additional duties on steel and aluminum imports. Specifically, a 25% tariff was applied to steel and aluminum imports, with the exception of Russian Federation, which faced a 200% tariff on aluminum.
4/3/2025	10.7	U.S. tariffs on Vehicles	Effective April 3, 2025, the United States imposed new tariffs on vehicle imports. Additional 25% tariff was applied to vehicles from all countries.
4/5/2025	13.4	Baseline 10% reciprocal tariffs	On April 05, 2025, the United States imposed a baseline additional 10% tariff on imports (there are exemptions) from all countries, except for Canada, Mexico, and countries subject to rates set forth in Column 2 of the HTSUS (Russian Federation, Cuba and Belarus, which is a WTO Observer).
4/9/2025	22.6	Liberation Day tariffs implemented	On April 9, 2025, the United States imposed additional tariffs of 34% on imports from China. On April 9, 2025, the United States increased the additional tariffs from 34% to 84% on imports from China. On April 10, 2025, the United States increased the additional tariffs from 84% to 125% on imports from China. The increased tariffs on imports from the other 55 countries with implementation date on April 9, 2025, were suspended effective April 10, 2025 for 90-days until July 9, 2025.
5/3/2025	23.3	Tariffs on Vehicle parts	Effective May 3, 2025, new tariffs were imposed on vehicle part imports. A 25% tariff was applied to vehicles' parts from all countries.
5/14/2025	14.9	U.S.-China trade deal	U.S. and China agreed to a trade deal that reduces 125% tariffs to 10%.
6/4/2025	16.5	Steel & Alum. Tariffs +25	U.S. doubles tariffs on foreign steel and aluminum imports to 50%. This applies to all trading partners except the UK.

NOTE: The tariff events are described by WTO - IMF Tariff Tracker (WTO and IMF, 2025). Note that this table only include the actual implemented tariffs but do not include the tariffs to be implemented until June 20, 2025.

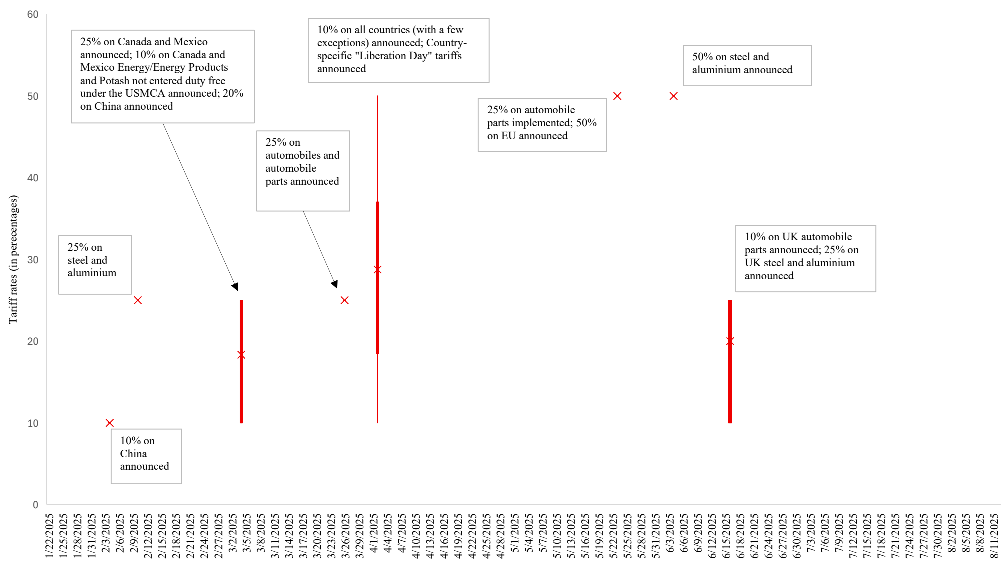
Table A.2. Sectoral Shares and Tariffs for the U.S.

Country	Industry	World Share (%)	U.S. Share (%)	U.S. Final Share (%)	U.S. Int Share (%)	U.S. Curr. Tariff (%)	U.S. Max. Tariff (%)	Ret. Curr. Tariff (%)	Ret. Max. Tariff (%)
USA	Agriculture	7.0	89.1	88.5	89.3	0	0	0	0
USA	Energy	15.0	79.0	89.4	75.0	0	0	0	0
USA	Mining	11.1	94.8	98.5	89.9	0	0	0	0
USA	Food & Bev.	13.1	91.3	91.2	91.7	0	0	0	0
USA	Basic Man.	11.2	77.4	66.0	82.5	0	0	0	0
USA	Adv. Man.	13.1	67.0	67.0	66.9	0	0	0	0
USA	Resid. Serv.	13.1	99.7	99.9	99.5	0	0	0	0
USA	Services	29.1	96.5	96.7	96.2	0	0	0	0
EUU	Agriculture	13.3	2.1	2.3	2.1	9.3	9.3	0	0
EUU	Energy	14.0	1.7	2.0	1.6	0	0	0	0
EUU	Mining	13.9	0.7	0.4	1.2	9.2	9.2	0	0
EUU	Food & Bev.	23.9	2.6	2.8	2.1	10.6	10.6	0	0
EUU	Basic Man.	23.2	7.6	12.4	5.5	6.0	6.0	0	0
EUU	Adv. Man.	29.2	8.8	8.7	9.0	14.9	14.9	0	0
EUU	Resid. Serv.	28.5	0.1	0.1	0.2	0	0	0	0
EUU	Services	30.7	1.5	1.4	1.7	0	0	0	0
CHN	Agriculture	31.7	0.4	0.4	0.4	42.8	156.3	39.7	138.4
CHN	Energy	17.6	0.1	0.1	0.1	31.9	31.9	33.6	145.1
CHN	Mining	21.4	0.1	0.1	0.1	28.8	78.4	26.7	140.7
CHN	Food & Bev.	24.1	0.8	0.8	0.8	39.1	147.6	20.8	130.5
CHN	Basic Man.	38.0	5.3	8.6	3.8	41.4	117.6	19.0	126.6
CHN	Adv. Man.	32.1	9.0	8.9	9.2	38.7	93.9	16.8	128.0
CHN	Resid. Serv.	27.2	0	0	0	0	0	0	0
CHN	Services	12.9	0.3	0.3	0.3	0	0	0	0
CAN	Agriculture	1.1	1.7	1.8	1.6	13.9	25.0	4.0	4.0
CAN	Energy	2.3	5.9	2.0	7.4	8.2	10.0	0	0
CAN	Mining	3.1	1.4	0.5	2.6	23.5	25.0	2.3	2.3
CAN	Food & Bev.	1.4	1.2	1.1	1.4	10.4	24.5	5.8	5.8
CAN	Basic Man.	1.2	2.3	1.5	2.7	21.9	25.0	9.0	9.0
CAN	Adv. Man.	1.1	2.3	2.3	2.2	15.4	25.0	6.7	6.7
CAN	Resid. Serv.	1.9	0.1	0.1	0.2	0	0	0	0
CAN	Services	2.1	0.4	0.3	0.5	0	0	0	0
MEX	Agriculture	1.0	1.7	1.8	1.6	6.2	25.0	0	0
MEX	Energy	1.4	1.5	0.5	1.8	14.9	25.0	0	0
MEX	Mining	1.7	0.1	0	0.2	20.3	25.0	0	0
MEX	Food & Bev.	2.0	0.8	0.9	0.8	18.1	25.0	0	0
MEX	Basic Man.	1.0	1.3	1.2	1.3	18.9	25.0	0	0
MEX	Adv. Man.	2.1	6.3	6.3	6.3	16.2	25.0	0	0
MEX	Resid. Serv.	1.1	0	0	0	0	0	0	0
MEX	Services	1.1	0.3	0.3	0.3	0	0	0	0
ROW	Agriculture	45.9	5.0	5.2	5.0	9.9	9.9	0	0
ROW	Energy	49.8	11.8	5.9	14.0	0	0	0	0
ROW	Mining	48.8	2.9	0.5	6.1	6.2	6.3	0	0
ROW	Food & Bev.	35.5	3.2	3.2	3.2	9.8	10.0	0	0
ROW	Basic Man.	25.5	6.1	10.3	4.2	10.6	10.6	0	0
ROW	Adv. Man.	22.4	6.6	6.7	6.4	12.7	12.7	0	0
ROW	Resid. Serv.	28.0	0	0	0.1	0	0	0	0
ROW	Services	24.2	1.0	1.0	1.0	0	0	0	0

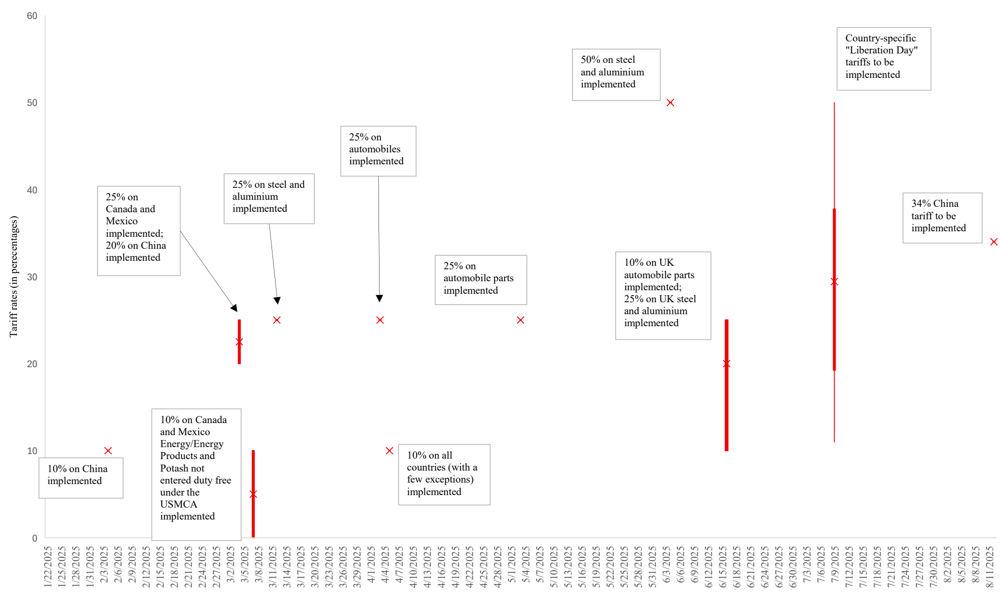
NOTE: Share data is obtained from OECD ICIO Tables (OECD, 2020) and tariff data is obtained from WTO - IMF Tariff Tracker database (WTO and IMF, 2025). World Share is the share of the industry in the world in that industry, U.S. Share is the share of the industry in both U.S. final goods and intermediate goods, U.S. Final Share is the share in the final good consumption in that industry, U.S. Int. Share is the intermediate use share in that industry, U.S. Curr. Tariff is the tariff as of June 30, 2025, U.S. Max Tariff is the maximum tariff observed since January 1, 2025. Ret. Curr. Tariff and Ret. Max. Tariff are the retaliatory levels from the countries to the U.S.

Figure A.1. Tariff Announcements and Implementations

(a) Tariff Announcements



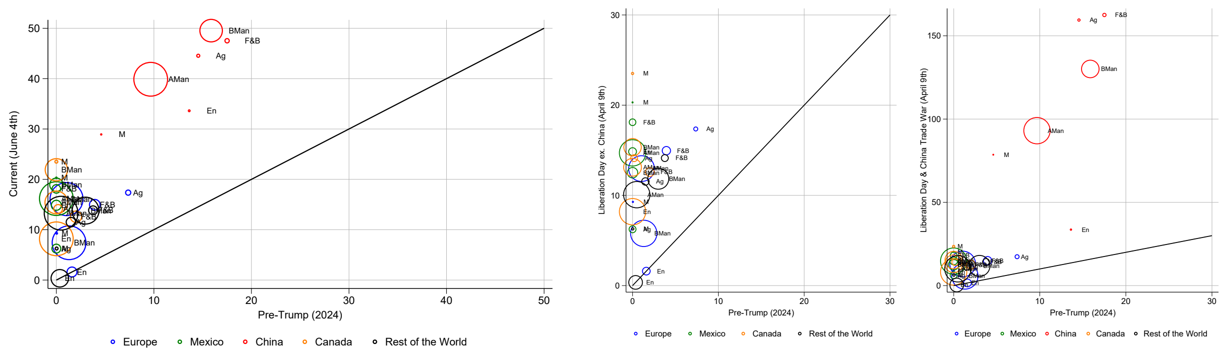
(b) Tariffs - Implemented (and to be Implemented)



Note: Tariff announcements and implementations between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (`https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates`). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (`https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/`), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (`https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far`). The tariffs that classified as "threats" are those that –as of June 30th —had not been implemented and were unlikely to be implemented based on available information. These threats were identified by extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

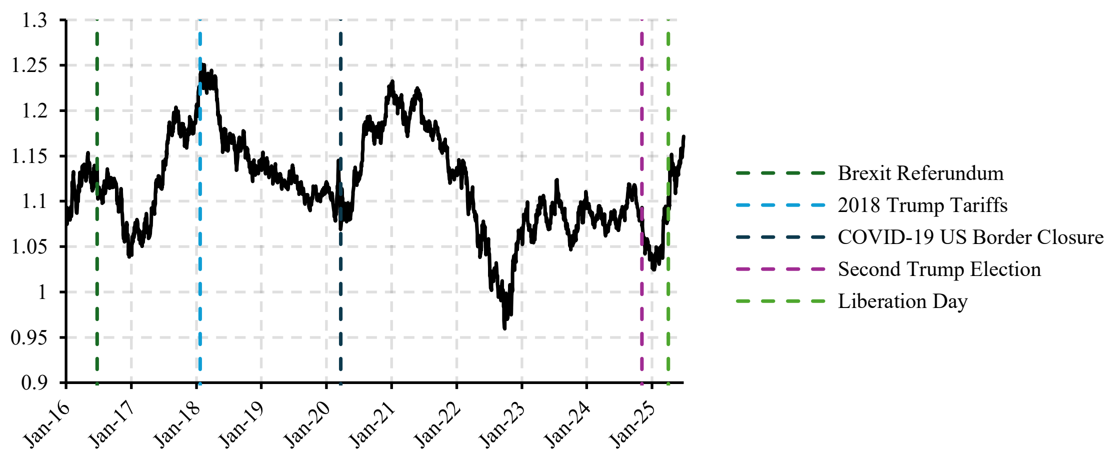
Figure A.2. Effective Country-Sector Level Tariff Rates

(a) As of June 4, 2025 (%) **(b)** As of the "Liberation Day", (%)



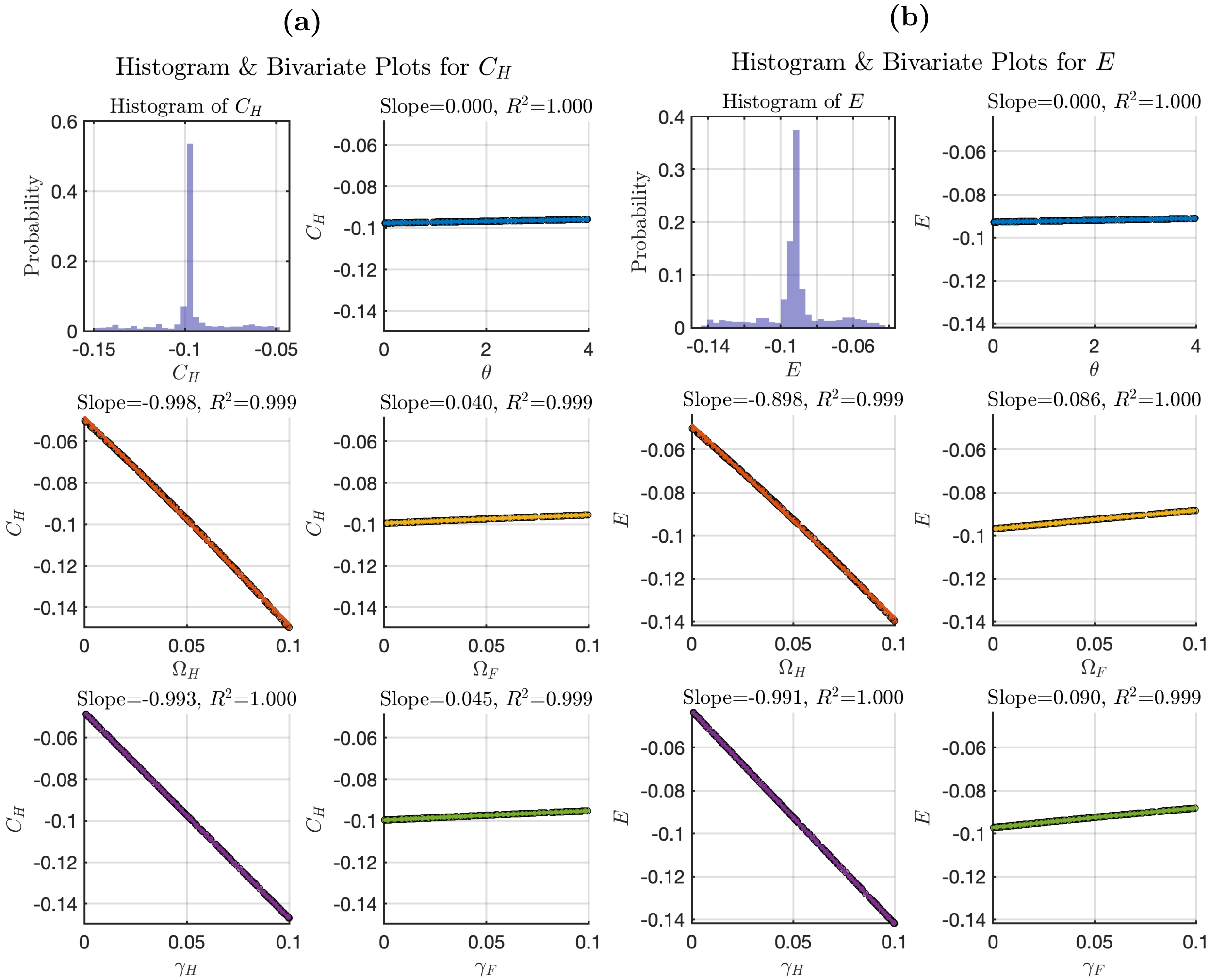
NOTE: (a) Estimated effective tariff rates at the country sector level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) as of the last available day (June 4, 2025) when we accessed the data on June 20, 2025. (b) Estimated effective tariff rates at the country sector level when the tariffs announced on the "liberation day" and extra tariffs on China went into effect. In the left panel, we remove the Chinese sectors. In the right panel, we show all country-sector combinations. Size of the bubbles corresponds to the U.S. imports from that country-sector pair for the last available data at WTO. The colors code for countries: Canada, China, euro area, Mexico and the Rest of the World. Sectoral Acronyms are Ag: Agriculture, En: Energy, M: Mining, F&B: Food & Beverages, BMan: Basic Maufacturing, AMan: Advanced Manufacturing.

Figure A.3. USD - Euro Exchange Rate 2016-2025



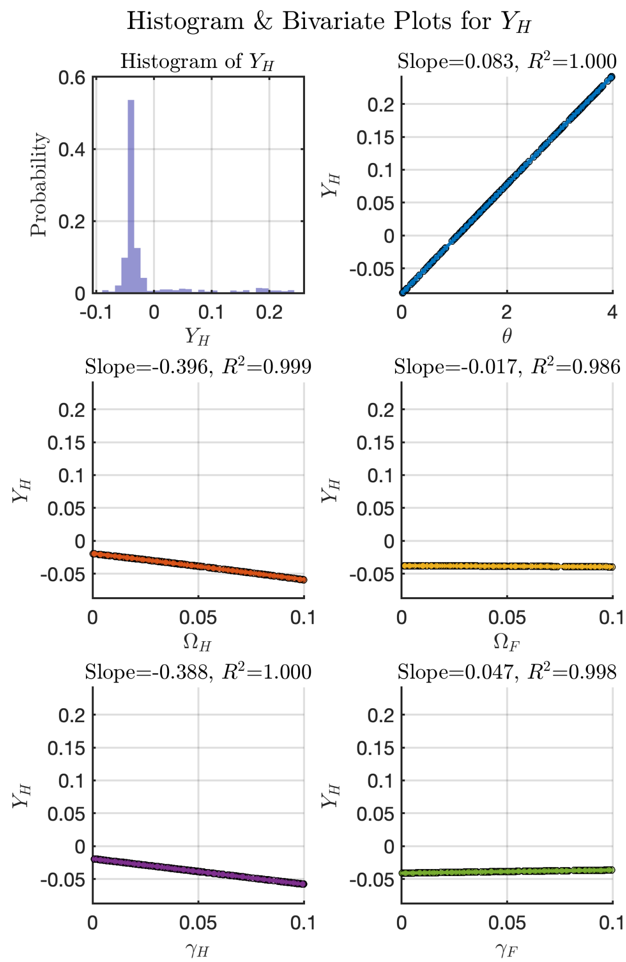
NOTE: USD Euro Exchange Rate from 2015 to 2025. The vertical lines indicate different events. Source: Bloomberg.

Figure A.4. Tariff Impact as a Function of Model Primitives Under Flexible Prices



NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Vertical axis variables are measured in percentage points. We see that consumption is declining in both γ_H and Ω_H, while they are increasing in the foreign country's parameters. The exchange rate appreciates in response to tariffs. This appreciation is stronger as one lowers the home bias in consumption and production for the home country. The intuition here is that as once increases Ω_H and γ_H, H becomes a larger buyer of goods produced by F and thus one has a larger change in the relative demand for F's goods, which in turn leads to a larger appreciation. This appreciation is not large enough to flip the sign of consumption into positive territory. Additionally, while output is solved out from the five-equation representation, we can compute it based on the solution of other variables. Thus, output as a variable of interest is included in Figure A.5. Output is mostly responsive to the elasticity of substitution which allows both production and consumption to respond to prices in both countries. Output is declining γ_H and γ_H, while it is not significantly responsive to foreign country parameters.

Figure A.5. Tariff Impact as a Function of Model Primitives Under Flexible Prices

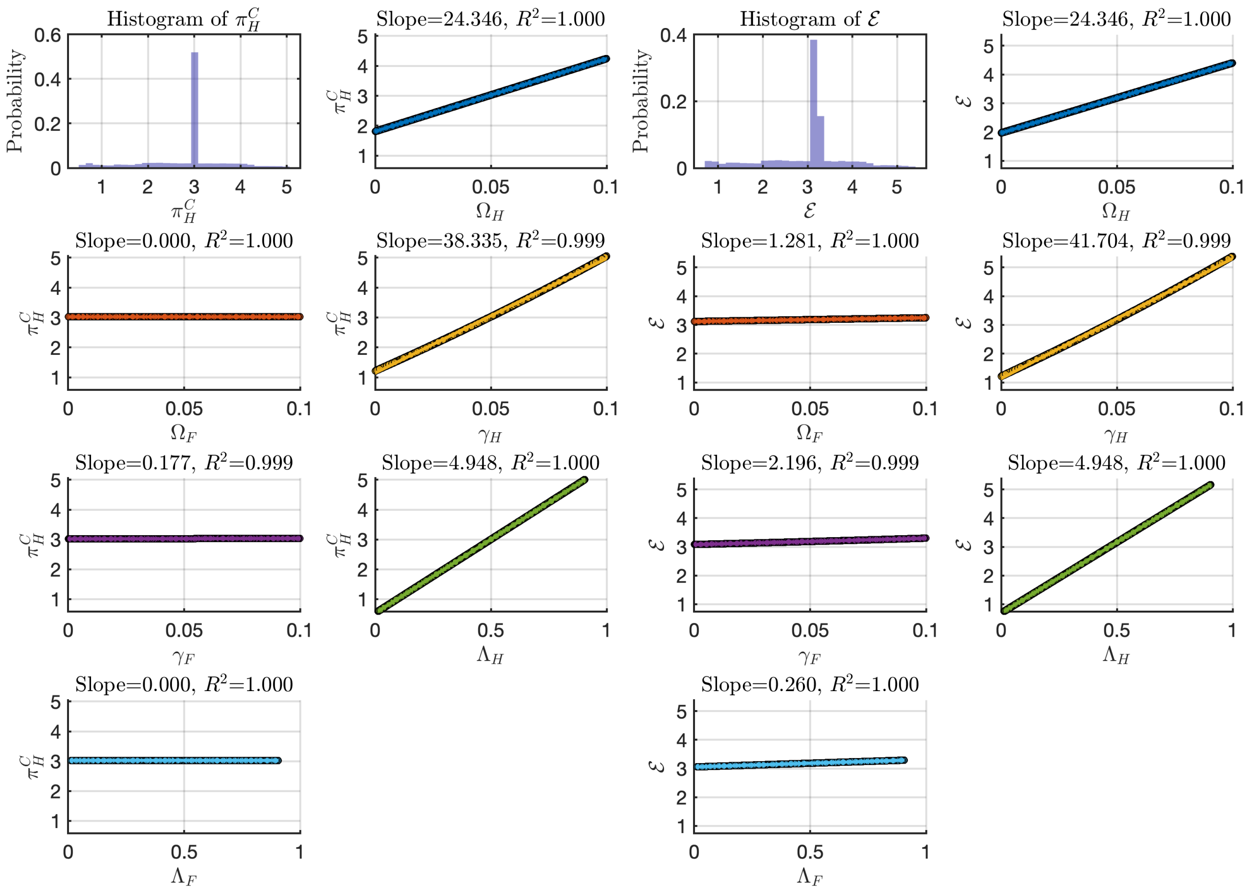


NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed one at a time. Vertical axis variables are measured in percentage points.

Figure A.6. Tariff Impact as a Function of Model Primitives Under Real Rate Rule

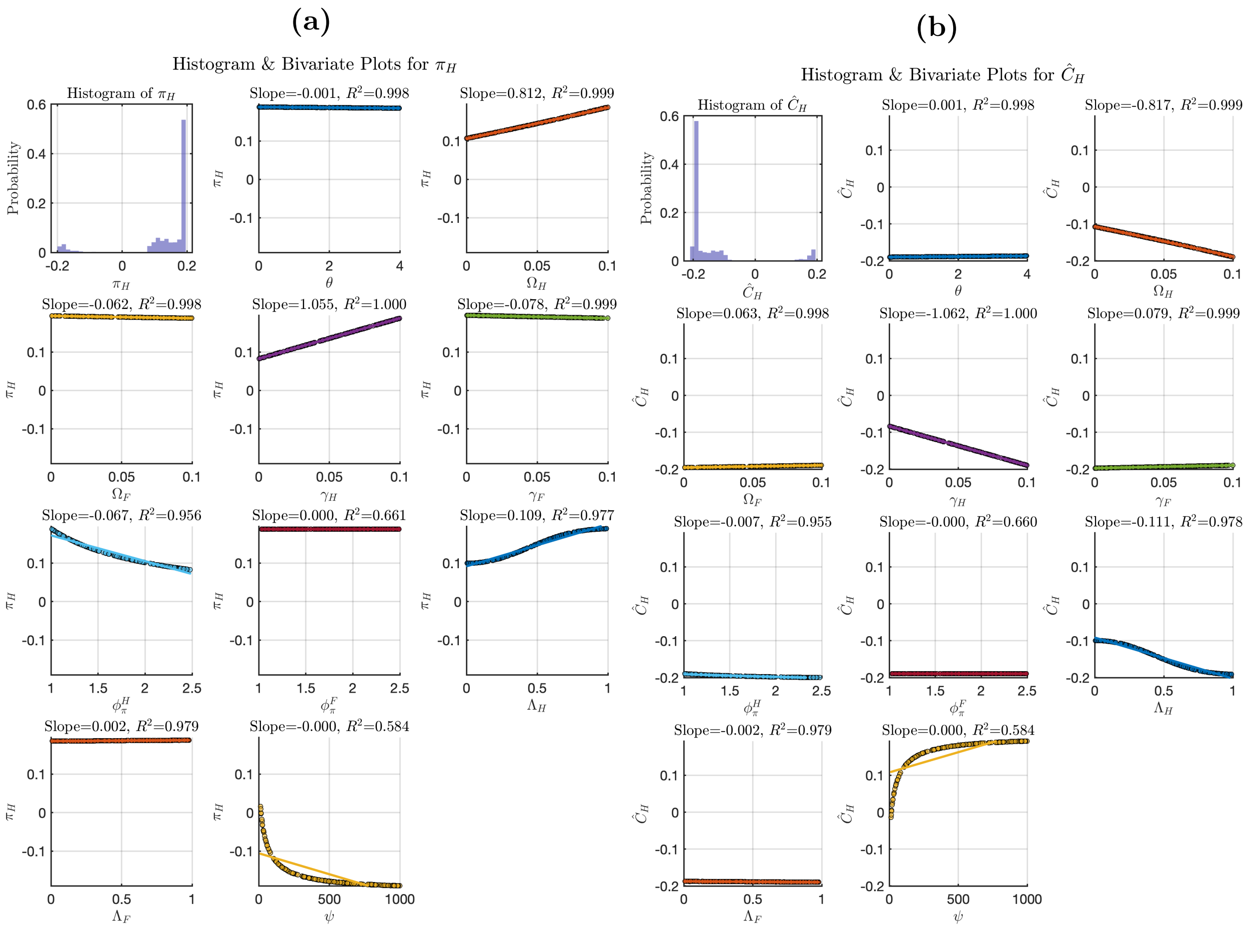
(a) Histogram & Bivariate Plots for π_H^C

(b) Histogram & Bivariate Plots for $\mathcal{E}$



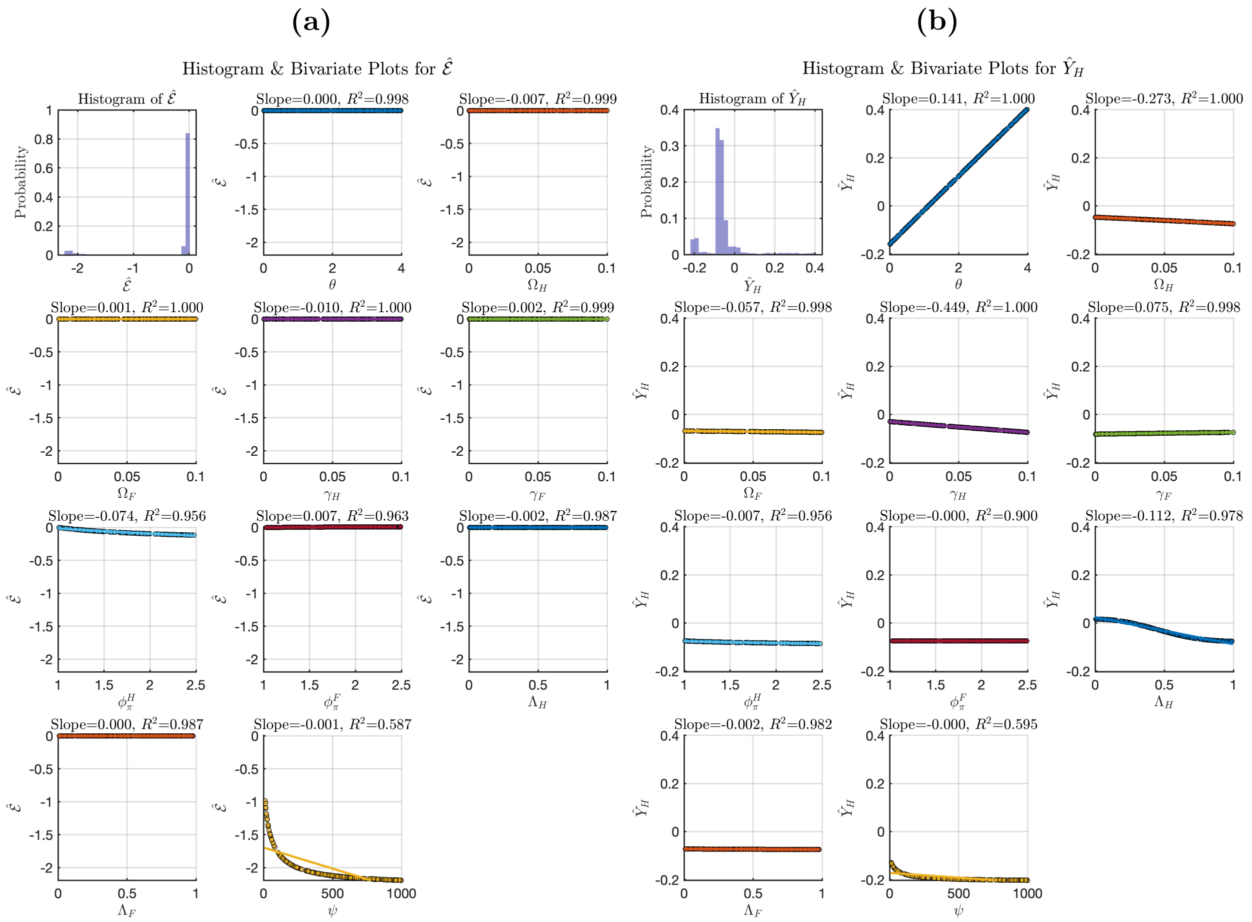
Note: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed one at a time. Vertical axis variables are measured in percentage points. Persistence of the shock is set to $\rho^\tau = 0.5$.

Figure A.7. Tariff Impact as a Function of Model Primitives Under Taylor Rule



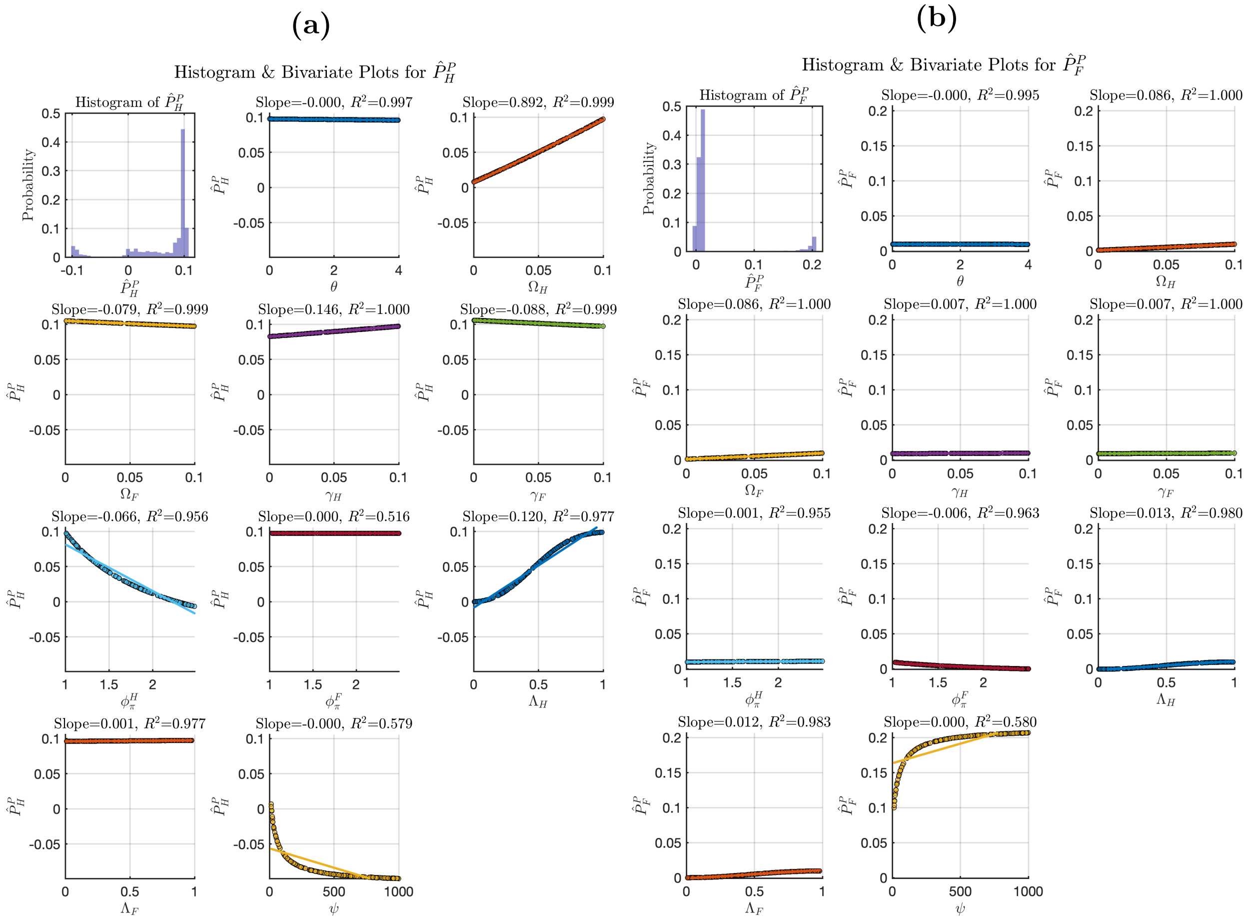
Note: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed one at a time. Vertical axis variables are measured in percentage points. Persistence of the shock is set to $\rho^\tau = 0$.

Figure A.8. Tariff Impact as a Function of Model Primitives Under Taylor Rule



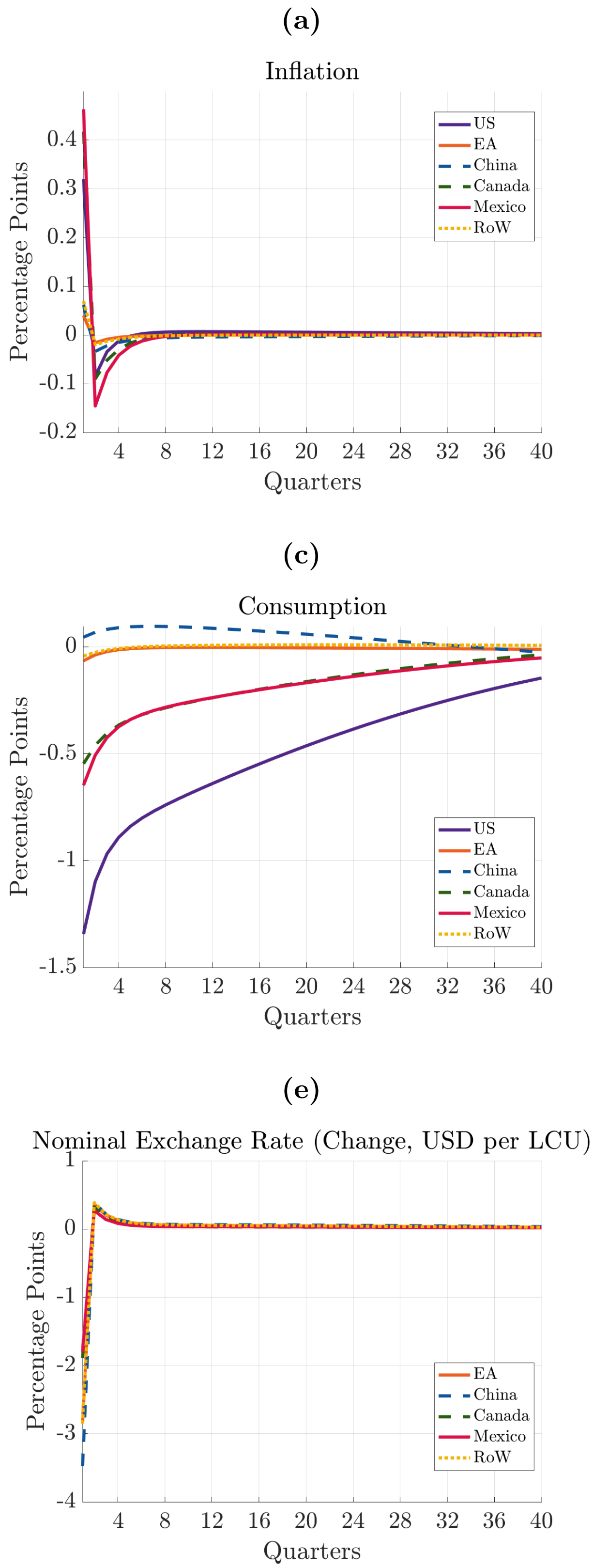
NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed one at a time. Vertical axis variables are measured in percentage points. Persistence of the shock is set to $\rho^{\tau} = 0$.

Figure A.9. Tariff Impact as a Function of Model Primitives Under Taylor Rule



NOTE: Figure visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed one at a time. Vertical axis variables are measured in percentage points. Persistence of the shock is set to $\rho^\tau = 0$.

Figure A.10. Case 3: Impact of All-Out Tariff War Under High Elasticity of Substitution



Note: All-out tariff war scenario in which trade partners retaliate symmetrically. Impulse responses are calculated with MIT shocks and with shock persistence is set to $\rho^{\tau} = 0.95$. Tariff rates same as Case 3; however, all CES elasticites are set to 4.

B Derivations

B.1 Household's Problem

The Lagrangian for the household's problem is:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right] \right.$$
$$+ \lambda_t \left[\sum_i (W_{n,t} L_{ni,t} + \Pi_{ni,t}) - (1+i_{n,t-1}) B_{n,t-1} - \mathcal{E}_{n,t}(1+i_{n,t-1}^{US}) B_{n,t-1}^{US} \right.$$
$$\left. \left. - P_{n,t} C_{n,t} - T_{ni,t} + B_{n,t} + \mathcal{E}_{n,t} B_{n,t}^{US} - \mathcal{E}_{n,t} P_{n,t}^{US} \psi(B_{n,t}^{US} / P_{n,t}^{US}) \right] \right\}.$$

Given $L_{n,t} = \sum_i L_{ni,t}$, the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_{n,t}} = \beta^t C_{n,t}^{-\sigma} - \lambda_t P_{n,t} = 0, \quad \forall t$$
$$\frac{\partial \mathcal{L}}{\partial L_{n,t}} = -\beta^t \chi L_{n,t}^{\eta} + \lambda_t W_{n,t} = 0, \quad \forall t$$
$$\frac{\partial \mathcal{L}}{\partial B_{n,t}} = \lambda_t - E_t \lambda_{t+1} (1+i_{n,t}) = 0, \quad \forall t$$
$$\frac{\partial \mathcal{L}}{\partial B_{n,t}^{US}} = \lambda_t \mathcal{E}_{n,t} - E_t \lambda_{t+1} \mathcal{E}_{n,t+1} (1+i_{n,t}^{US}) - \lambda_t \mathcal{E}_{n,t} \psi'(B_{n,t}^{US} / P_{n,t}^{US}) = 0, \quad \forall t.$$

Rearranging the first-order conditions, we derive the key equilibrium conditions.

Euler Equation

Rearranging the FOC for $B_{n,t}$:

$$\lambda_t = E_t \lambda_{t+1} (1+i_{n,t})$$

Substituting $\lambda_t = \frac{\beta^t C_{n,t}^{-\sigma}}{P_{n,t}}$ from the FOC for $C_{n,t}$:

$$1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P_{n,t}}{P_{n,t+1}} (1+i_{n,t}) \right].$$

Intratemporal Labor-Consumption Choice

Rearranging the FOC for $L_{n,t}$:

$$\chi L_{n,t}^{\eta} = \frac{\lambda_t W_{n,t}}{\beta^t}.$$

Substituting $\lambda_t = \frac{\beta^t C_{n,t}^{-\sigma}}{P_{n,t}}$ from the FOC for $C_{n,t}$:

$$\chi L_{n,t}^{\eta} = \frac{C_{n,t}^{-\sigma} W_{n,t}}{P_{n,t}}$$
$$\frac{W_{n,t}}{P_{n,t}} = \chi L_{n,t}^{\eta} C_{n,t}^{\sigma}$$

Uncovered Interest Parity (UIP) Condition with Portfolio Adjustment Costs

Rearranging the FOC for $B_{n,t}^{US}$:

$$\lambda_t \mathcal{E}_{n,t} = E_t \lambda_{t+1} \mathcal{E}_{n,t+1}(1 + i_{n,t}^{US}) + \lambda_t \mathcal{E}_{n,t} \psi'(B_{n,t}^{US}/P_{n,t}^{US}).$$

Dividing both sides by $\lambda_t \mathcal{E}_{n,t}$:

$$1 = E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} (1 + i_{n,t}^{US}) \right] + \psi'(B_{n,t}^{US}/P_{n,t}^{US}).$$

Using $\lambda_t = E_t \lambda_{t+1}(1 + i_{n,t})$:

$$1 = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \frac{1 + i_{n,t}^{US}}{1 + i_{n,t}} \right] + \psi'(B_{n,t}^{US}/P_{n,t}^{US})$$
$$\frac{1 + i_{n,t}}{1 + i_{n,t}^{US}} \left(1 - \psi'(B_{n,t}^{US}/P_{n,t}^{US})\right) = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right]$$
$$\frac{1 + i_{n,t}}{1 + i_{n,t}^{US}} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1 - \psi'(B_{n,t}^{US}/P_{n,t}^{US})}$$

B.2 Firm Problem

Output in country n sector i at firm f at time t each firm has some CRS production function:

$$Y_{ni,t} = A_{n,i} F_i(L_{ni,t}, \{X_{ni,j,t}\}_{i=1,j=1}^{i=J,j=J})$$

Intermediate goods from different countries are first bundled into a country-industry-good bundle:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{1/\theta_{l,j}^P} X_{ni,mj,t}^{\frac{\theta_{l,j}^P - 1}{\theta_{l,j}^P}} \right]^{\frac{\theta_{l,j}^P}{\theta_{l,j}^P - 1}} \tag{B.1}$$

Relative demand condition is appropriately defined as:

$$X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P_{ni,j,t}} \right)^{-\theta_{l,j}^P} X_{ni,j,t} \tag{B.2}$$

where $P_{ni,j,t}$ is the price of bundle j for producer sector i in country n and can be shown that:

$$P_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj} P_{n,mj,t}^{1-\theta_{l,j}^P} \right]^{\frac{1}{1-\theta_{l,j}^P}} .$$

The price of good j from country m in country n is given by:

$$P_{n,mj,t} = \tau_{n,mj,t} \mathcal{E}_{n,m,t} P_{mj,t}.$$

Intermediate bundle for sector i in country n is an aggregation of country-industry-good bundle:

$$X_{ni,t} = \left[\sum_{j \in J} \Omega_{ni,j}^{1/\theta_h^P} X_{ni,j,t}^{\frac{\theta_h^P - 1}{\theta_h^P}} \right]^{\frac{\theta_h^P}{\theta_h^P - 1}}$$

Relative demand condition is appropriately defined as:

$$X_{ni,j,t} = \Omega_{ni,j} \left(\frac{P_{ni,j,t}}{P_{ni,t}^P} \right)^{-\theta_h^P} X_{ni,t}$$

where $P_{ni,t}^P$ is the price index for intermediate bundle for producer sector i in country n with:

$$P_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj} P_{ni,mj,t}^{1-\theta_{l,j}^P} \right]^{\frac{1}{1-\theta_{l,j}^P}}$$

Note that:

$$P_{ni,t}^P X_{ni,t} = \sum_{j \in J} P_{ni,j,t} X_{ni,j,t} = \sum_{m \in N} \sum_{j \in J} P_{ni,mj,t} X_{ni,mj,t}.$$

We next define marginal cost; assuming all firms in a country-sector combination are identical:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^P_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

Production is CES:

$$Y_{ni,t} = A_{ni,t} \left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P - 1}{\theta^P}} + (1 - \alpha_{ni})^{1/\theta^P} X_{ni,t}^{\frac{\theta^P - 1}{\theta^P}} \right]^{\frac{\theta^P}{\theta^P - 1}} \forall n \in N, \forall i \in J.$$

This problem yields the following equilibrium conditions:

$$\begin{aligned}
\frac{X_{ni,t}}{L_{ni,t}} &= \left(\frac{(1 - \alpha_{ni}) W_t}{\alpha_{ni} P^P_{ni,t}} \right)^{\theta^P} \\
X_{ni,t} &= (1 - \alpha_{ni}) \left(\frac{P^P_{ni,t}}{MC_{ni,t}} \right)^{-\theta^P} Y_{ni,t} \\
MC_{ni,t} &= \frac{1}{A_{ni,t}} \left[\alpha_{ni} W_t^{1-\theta^P} + (1 - \alpha_{ni}) (P^P_{ni,j,t})^{1-\theta^P} \right]^{\frac{1}{1-\theta^P}}
\end{aligned}$$

Combining all equilibrium conditions, we can write:

$$\begin{aligned}
X_{ni,mj,t} &= \underbrace{(1 - \alpha_{ni}) \Omega_{ni,j,mj} \Omega_{ni,j}}_{=\Omega_{ni,mj}} \left(\frac{\tau_{n,mj,t} P_{mj,t}}{P_{ni,j,t}} \right)^{-\theta^P_{l,j}} \left(\frac{P_{ni,j,t}}{P^P_{ni,t}} \right)^{-\theta^P_h} \left(\frac{P^P_{ni,t}}{MC_{ni,t}} \right)^{-\theta^P} Y_{ni,t} \\
&= \Omega_{ni,mj} \left(\frac{P_{n,mj,t}}{P_{ni,j,t}} \right)^{-\theta^P_{l,j}} \left(\frac{P_{ni,j,t}}{P^P_{ni,t}} \right)^{-\theta^P_h} \left(\frac{P^P_{ni,t}}{MC_{ni,t}} \right)^{-\theta^P} Y_{ni,t}
\end{aligned}$$

B.2.1 Rotemberg Adjustment Costs

Within each country sector there is an infinite continuum of identical firms. Representative firm f in sector i of country n solves the following problem Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \left[Y^f_{ni,T}(P^f_{ni,T}) \left(P^f_{ni,T} - MC_{ni,T} \right) - \frac{\delta_{ni}}{2} \left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1 \right)^2 Y_{ni,T} P_{ni,T} \right] \right]$$

where a bundler puts together the sectoral output as a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}} \right)^{-\theta^R} Y_{ni,t}$. Bundler has log utility; it takes firm-level output and produces sectoral level output and net-zero bond-supply such that the nominal SDF will be $\text{SDF}_{t,T} = \beta^{T-t}\frac{Y_{ni,t}P_{ni,t}}{Y_{ni,T}P_{ni,T}}$. Plugging this in and writing the Lagrangian:

$$\mathcal{L} = \mathbb{E}_t\left[\sum_{T=t}^{\infty}\text{SDF}_{t,T}\left[\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta^R} Y_{ni,T}\left(P^f_{ni,T} - MC_{ni,T}\right) - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

$$\mathcal{L} = \sum_{T=t}^{\infty}\beta^{T-t}Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[\frac{1}{Y_{ni,T}P_{ni,T}}\left[(P^f_{ni,T})^{1-\theta^R}P_{ni,T}{}^{\theta^R}Y_{ni,T} - \left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta^R} Y_{ni,T}MC_{ni,T} - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

$$\mathcal{L} = \sum_{T=t}^{\infty}\beta^{T-t}Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[(P^f_{ni,T})^{1-\theta^R}(P_{ni,T})^{\theta^R-1} - \left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta^R}\frac{MC_{ni,T}}{P_{ni,T}} - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2\right]$$

Taking the FOC with respect to $P^f_{ni,T}$:

$$\begin{aligned}\frac{\partial Z_t}{\partial P^f_{ni,T}} =& \mathbb{E}_t\left[Y_{ni,t}P_{ni,t}\left[(1-\theta^R)(P^f_{ni,T})^{-\theta^R}(P_{ni,T})^{\theta^R-1} + \theta^R\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta^R-1}\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P^f_{n,T}}{P^f_{n,T-1}} - 1\right)\frac{1}{P^f_{n,T-1}}\right]\right]\\ &+ \beta Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[\delta_{ni}\left(\frac{P^f_{n,T+1}}{P^f_{n,T}} - 1\right)\frac{P^f_{n,T+1}}{(P^f_{n,T})^2}\right] = 0\end{aligned}$$

With $Y_{ni,t}P_{ni,t} \neq 0$ we can divide both sides by $Y_{ni,t}P_{ni,t}$. Additionally firms within an industry are symmetric so $P^f_{n,T} = P_{n,T}$.

$$\begin{aligned}&\mathbb{E}_t\left[(1-\theta^R)(P^f_{ni,T})^{-\theta^R}(P_{ni,T})^{\theta^R-1} + \theta^R\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta^R-1}\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P^f_{n,T}}{P^f_{n,T-1}} - 1\right)\frac{1}{P^f_{n,T-1}}\right]\\ &+ \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P^f_{n,T+1}}{P^f_{n,T}} - 1\right)\frac{P^f_{n,T+1}}{(P^f_{n,T})^2}\right] = 0\\ &\mathbb{E}_t\left[(1-\theta^R)P^{-1}_{ni,T} + \theta^R\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P_{n,T}}{P_{n,T-1}} - 1\right)\frac{1}{P_{n,T-1}}\right]\\ &+ \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P_{n,T+1}}{P_{n,T}} - 1\right)\frac{P_{n,T+1}}{(P_{n,T})^2}\right] = 0\end{aligned}$$

Since T is arbitrary, let us set $t = T$:

$$\left[(1-\theta^R)P^{-1}_{ni,t} + \theta^R\frac{MC_{ni,t}}{(P_{ni,t})^2} - \delta_{ni}\left(\frac{P_{n,t}}{P_{n,t-1}} - 1\right)\frac{1}{P_{n,t-1}}\right] + \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P_{n,t+1}}{P_{n,t}} - 1\right)\frac{P_{n,t+1}}{(P_{n,t})^2}\right] = 0$$

Defining gross inflation and multiplying both sides by $\frac{P_{n,t}}{\delta_{ni}}$ and rearranging:

$$\left(\Pi_{ni,t} - 1\right)\Pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R}\right) + \beta\mathbb{E}_t\left[\left(\Pi_{ni,t+1} - 1\right)\Pi_{ni,t+1}\right] \tag{B.3}$$

The FOCs for the MC minimization problem pins down demand for inputs (including labor), so jointly equations (16)-(18) constitute a forward-looking New Keynesian Phillips Curve. As $\delta_{ni} \to 0$ prices are more flexible and we have $\Pi_{n,t} = 1$ and $\frac{MC_{ni,t}}{P_{ni,t}} = \frac{\theta^R - 1}{\theta^R}$, which is the general pricing equation under monopolistic competition. For the zero inflation steady state where prices are all 1, the equation above can be rewritten as follows:

$$(\Pi_{ni,t} - 1)\,\Pi_{ni,t} = \frac{\theta^R - 1}{\delta_{ni}} \left(\frac{e^{\widehat{MC}_{ni,t}}}{e^{\hat{P}_{ni,t}}} - 1 \right) + \beta \mathbb{E}_t \left[(\Pi_{ni,t+1} - 1)\,\Pi_{ni,t+1} \right]$$

C Approximated Linear Equilibrium Conditions

Before simplifications are introduced, linearized equilibrium conditions are as follows:[38]

$$E_t \hat{C}_{n,t+1} - \hat{C}_{n,t} = \frac{1}{\sigma}\left(\hat{i}_t - E_t \pi_{n,t+1}\right) \tag{C.4}$$

$$\hat{i}_{n,t} - \hat{i}_{US,t} = E_t \hat{\mathcal{E}}_{n,t+1} - \hat{\mathcal{E}}_{n,t} + \hat{\psi} \tag{C.5}$$

$$\hat{\mathcal{E}}_{n,m,t} = \hat{\mathcal{E}}^{US}_{n,t} - \hat{\mathcal{E}}^{US}_{m,t} \tag{C.6}$$

$$\hat{\mathcal{E}}_{n,n,t} = 0 \tag{C.7}$$

$$\hat{W}_{n,t} - \hat{P}_{n,t} = \eta \hat{L}_{n,t} + \sigma \hat{C}_{n,t} \tag{C.8}$$

$$\hat{C}_{nt} = \sum_{j \in J} \Gamma_{n,j} \hat{C}_{n,j,t} \tag{C.9}$$

$$\hat{C}_{n,j,t} = \sum_{m \in N} \Gamma_{n,j,mj} \hat{C}_{n,mj,t} \tag{C.10}$$

$$\hat{P}_{n,mj,t} = \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,m,t} + \hat{P}_{mj,t} \tag{C.11}$$

$$\hat{C}_{n,j,t} = \hat{C}_{n,t} - \theta^C_h \left(\hat{P}_{n,j,t} - \hat{P}_{n,t} \right) \tag{C.12}$$

$$\hat{C}_{n,mj,t} = \hat{C}_{n,j,t} - \theta^C_{l,j} \left(\hat{P}_{n,mj,t} - \hat{P}_{n,j,t} \right) \tag{C.13}$$

$$\hat{X}_{ni,j,t} = \sum_{m \in N} \Omega_{ni,j,mj} \hat{X}_{ni,mj,t} \tag{C.14}$$

$$\hat{X}_{ni,mj,t} = \hat{X}_{ni,j,t} - \theta^P_{l,j} \left(\hat{P}_{n,mj,t} - \hat{P}_{ni,j,t} \right) \tag{C.15}$$

$$\hat{X}_{ni,t} = \sum_{j \in J} \Omega_{ni,j} \hat{X}_{ni,j,t} \tag{C.16}$$

$$\hat{X}_{ni,j,t} = \hat{X}_{ni,t} - \theta^P_h \left(\hat{P}_{ni,j,t} - \hat{P}^P_{ni,t} \right) \tag{C.17}$$

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \alpha_{ni} \hat{L}_{ni,t} + (1 - \alpha_{ni}) \hat{X}_{ni,t} \tag{C.18}$$

[38] Please note in this set of equilibrium conditions the highest layer of the intermediate input bundle is simplified away.

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_{n,t} + (1-\alpha_{ni})\hat{P}^P_{ni,t} \tag{C.19}$$

$$\hat{X}_{ni,t} - \hat{L}_{ni,t} = \theta^P \hat{W}_{n,t} - \theta^P \hat{P}^P_{ni,t} \tag{C.20}$$

$$\pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta \mathbb{E}_t \pi_{ni,t+1} \tag{C.21}$$

$$\bar{B}^{US}\hat{B}^{US}_t = \sum_m^{N-1} \bar{B}^{US}_m \hat{B}^{US}_{m,t} \tag{C.22}$$

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{n\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{C.23}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{C.24}$$

$$\pi_{n,t} = \hat{P}_{n,t} - \hat{P}_{n,t-1} \tag{C.25}$$

$$\hat{i}_{n,t} = \phi_\pi \pi_{n,t} + \hat{M}_{n,t} \tag{C.26}$$

and

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&+ \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) \\
=&\sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t})
\end{aligned} \tag{C.27}$$

D Relating the Balance of Payments to Prices

Let us define the total expenditure of country n in USD as $\chi_{n,t} = P_nC_n/\mathcal{E}_{n,t}$ and the output of industry in USD as $\lambda_{ni,t} = P_{ni,t}Y_{ni,t}/\mathcal{E}_{n,t}$. Let $\mathbf{\Sigma}^{\mathcal{N}}$ denote the $NJ \times N$ matrix, which sums up the industries to country level. Let $\boldsymbol{\chi}_t$ denote the N dimensional row-vector for country expenditures and $\boldsymbol{\lambda_t}$ denote the NJ dimensional row-vector for the outputs.

$\mathbf{\Omega}$ and $\mathbf{\Gamma}$ matrices, by definition, include the trade costs. We can define the versions of these matrices without the trade costs as:

$$\Omega^\tau_{ni,mj,t} = \frac{1}{\tau_{n,mj,t}}\frac{P_{n,mj,t}X_{ni,mj,t}}{P_{ni,t}Y_{ni,t}} \qquad \text{and} \qquad \Gamma^\tau_{n,mj,t} = \frac{1}{\tau_{n,mj,t}}\frac{P_{n,mj,t}C_{n,mj,t}}{P_{n,t}C_{n,t}}.$$

With these matrices at hand, we can write the total expenditure of the countries as:

$$\boldsymbol{\chi}_t = (\underbrace{\boldsymbol{\lambda_t}\,\mathrm{diag}[(\boldsymbol{I} - \mathbf{\Omega})\mathbf{1}]}_{\text{Wages \& Markups}} + \underbrace{\boldsymbol{\lambda_t}\,\mathrm{diag}[(\mathbf{\Omega} - \mathbf{\Omega}^\tau)\mathbf{1}]}_{\text{Tariff Revenue Intermediate Inputs}})\mathbf{\Sigma}^{\mathcal{N}} + \underbrace{\boldsymbol{\chi}_t\,\mathrm{diag}[(I - \mathbf{\Gamma}^\tau)\mathbf{1}]}_{\text{Tariff Revenue Consumption Inputs}}$$

$$
\begin{aligned}
&+ \underbrace{(1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{n,t}^{US}}_{\text{Debt Position}} \\
&= \boldsymbol{\lambda_t}\,\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}} + \boldsymbol{\chi}_t\,\mathrm{diag}[(I-\boldsymbol{\Gamma}^{\tau})\mathbf{1}] + (1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US} \\
\boldsymbol{\chi}_t\,\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}] &= \boldsymbol{\lambda_t}\,\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}} + (1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US} \\
\boldsymbol{\chi}_t &= \boldsymbol{\lambda_t}\,\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}}\,\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1} + [(1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US}]\,\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1}
\end{aligned}
$$

We can re-write market clearing conditions as:

$$
\begin{aligned}
P_{ni,t}Y_{ni,t} &= \sum_{n\in\mathcal{N}} P_{ni,t}C_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} P_{ni,t}X_{mj,ni,t} \\
&= \sum_{n\in\mathcal{N}} \frac{P_{ni,t}C_{m,ni,t}}{P_{m,t}C_{m,t}} P_{m,t}C_{m,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \frac{P_{ni,t}X_{mj,ni,t}}{P_{mj,t}Y_{mj,t}} \\
\lambda_{ni,t} &= \sum_{n\in\mathcal{N}} \Gamma^{\tau}_{m,ni,t}\chi_{m,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \Omega^{\tau}_{mj,ni,t}\lambda_{mj,t}
\end{aligned}
$$

In matrix notation:

$$
\begin{aligned}
\boldsymbol{\lambda}_t &= \boldsymbol{\chi}_t\,\boldsymbol{\Gamma}^{\tau} + \boldsymbol{\lambda}_t\,\boldsymbol{\Omega}^{\tau} \\
&= \boldsymbol{\chi}_t\,\boldsymbol{\Gamma}^{\tau}\,[\boldsymbol{I}-\boldsymbol{\Omega}^{\tau}]^{-1} \\
&= \left(\boldsymbol{\lambda_t}\,\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}}\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1} + [(1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US}]\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1}\right)\,\boldsymbol{\Gamma}^{\tau}\,[\boldsymbol{I}-\boldsymbol{\Omega}^{\tau}]^{-1}
\end{aligned}
$$

Therefore, we can write:

$$
\begin{aligned}
\left((1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US}\right)\,\underbrace{\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1}\,\boldsymbol{\Gamma}^{\tau}\,[\boldsymbol{I}-\boldsymbol{\Omega}^{\tau}]^{-1}}_{\equiv\boldsymbol{A}} &= \boldsymbol{\lambda}_t(\boldsymbol{I}-\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\,\boldsymbol{\Sigma}^{\mathcal{N}}\,\boldsymbol{A}) \\
(1+i_{t-1}^{US})\boldsymbol{B}_{t-1}^{US} - \boldsymbol{B}_{t}^{US} &= \boldsymbol{\lambda}_t\left(\boldsymbol{I}-\mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}}\,\boldsymbol{A}\right)\boldsymbol{A}^{\dagger}(\boldsymbol{A}\,\boldsymbol{A}^{\dagger})^{-1} \\
&= \boldsymbol{\lambda}_t\left(\boldsymbol{A}^{\dagger}(\boldsymbol{A}\,\boldsymbol{A}^{\dagger})^{-1} - \mathrm{diag}[(\boldsymbol{I}-\boldsymbol{\Omega}^{\tau})\mathbf{1}]\boldsymbol{\Sigma}^{\mathcal{N}}\right)
\end{aligned}
$$

Note that all the terms in the right hand side depends on the prices, wages and tariffs. Because of our nested CES production and consumption choices, changes in the elements of $\boldsymbol{\Omega}$, $\boldsymbol{\Omega}^{\tau}$, $\boldsymbol{\Gamma}$ and $\boldsymbol{\Gamma}^{\tau}$ are also functions of price changes and tariffs.

Plugging the last equation into the income equation:

$$
\boldsymbol{\chi}_t = \boldsymbol{\lambda}_t\boldsymbol{A}^{\dagger}(\boldsymbol{A}\,\boldsymbol{A}^{\dagger})^{-1}\,\mathrm{diag}[\boldsymbol{\Gamma}^{\tau}\,\mathbf{1}]^{-1}
$$

D.1 Deriving the Fifth Equation of the Global NK Representation

Here, we would like to show that the changes in BoP can be written as:

$$\beta\hat{V}^{US}_{n,t} = \Xi_1\hat{V}^{US}_{n,t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\mathcal{E}_t + \Xi_5\tau_t.$$

The expressions below are algebraically involved, but at the end we show that there is a way to write the BoP as such.

To start with, we can rewrite the BoP as follows:

$$\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t}+\hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t}+\hat{X}_{ni,mj,t})$$
$$+\bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t}+\hat{i}^{US}_{n,t-1}+\hat{B}^{US}_{n,t-1}\right) = \sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t}+\hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t}+\hat{B}^{US}_{n,t})$$
$$\bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t}+\hat{i}^{US}_{n,t-1}+\hat{B}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t}+\hat{B}^{US}_{n,t})$$

Redefining $\hat{V}_t$ as dollar-denominated debt inclusive of interest payments: $\hat{V}_t = B^{US}_{n,t}(1+i_t)$:

$$\bar{\mathcal{E}}_n\bar{V}^{US}_n\left(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \frac{\bar{\mathcal{E}}_n\bar{V}^{US}_n}{1+\bar{i}^{US}}(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t}-\hat{i}^{US}_t)$$

WLOG $\bar{\mathcal{E}}_n = 1$. Also noting $(1+\bar{i}^{US}_n) = \beta^{-1}$, and $\overline{NX} = (1-\beta)\bar{V}^{US}_n$

$$\begin{aligned}
\bar{V}^{US}_n\left(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t-1}\right) &= (1-\beta)\bar{V}^{US}_n\widehat{NX}_{n,t} + \beta\bar{V}^{US}_n(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t}-\hat{i}^{US}_t)\\
\left(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t-1}\right) &= (1-\beta)\widehat{NX}_{n,t} + \beta(\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t}-\hat{i}^{US}_t)\\
(1-\beta)\hat{\mathcal{E}}_{n,t}+\hat{V}^{US}_{n,t-1} &= (1-\beta)\widehat{NX}_{n,t} + \beta\hat{V}^{US}_{n,t} - \beta\hat{i}^{US}_t\\
\beta\hat{V}^{US}_{n,t} - \hat{V}^{US}_{n,t-1} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}^{US}_t
\end{aligned}$$

Using the market clearing condition and production function in vector notation, we can then express net exports as a function of prices. This yields the fifth equation in the five-equation representation.

$$\underbrace{\widehat{NX}_{n,t}}_{1\times1} = \underbrace{\boldsymbol{S}_1}_{1\times NJ}\left(\underbrace{\hat{\boldsymbol{Y}}^{ni}_t}_{NJ\times1} + \underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times1}\right) - \underbrace{\boldsymbol{S}_2}_{1\times N}\left(\underbrace{\hat{\boldsymbol{C}}_t}_{N\times1} + \underbrace{\hat{\boldsymbol{P}}^C_t}_{N\times1}\right) - \underbrace{\boldsymbol{S}_3}_{1\times NJNJ}\left(\underbrace{\hat{\boldsymbol{X}}_t}_{NJNJ\times1} + \underbrace{\hat{\boldsymbol{P}}^{nimj}_t}_{NJNJ\times1}\right) \quad \text{(D.1)}$$

where $\boldsymbol{S}$ denotes selector matrices. For example, $\boldsymbol{S}_1$ selects the country for whom net exports are calculated and additionally includes steady state ratios (e.g., $\frac{\overline{Y}^{ni}\overline{P}^{P}_{ni}}{\overline{NX}}$ if country is n and 0 if not).

Note vector of end user prices can be written as follows for firms and consumers:

$$\underbrace{\hat{\boldsymbol{P}}_t^X}_{NJNJ\times 1} = \left(\underbrace{\boldsymbol{S}_4}_{NJNJ\times NJ} \underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_5}_{NJNJ\times 1} \underbrace{\hat{\mathcal{E}}_t}_{1\times 1} + \underbrace{\boldsymbol{S}_6}_{NJNJ\times 1} \underbrace{\tau_t}_{1\times 1} \right) \tag{D.2}$$

$$\underbrace{\hat{\boldsymbol{P}}_t^{CX}}_{NNJ\times 1} = \left(\underbrace{\boldsymbol{S}_7}_{NNJ\times NJ} \underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_8}_{NNJ\times 1} \underbrace{\hat{\mathcal{E}}_t}_{1\times 1} + \underbrace{\boldsymbol{S}_9}_{NNJ\times 1} \underbrace{\tau_t}_{1\times 1} \right) \tag{D.3}$$

Market clearing conditions can be written as follows:

$$\underbrace{\hat{\boldsymbol{Y}}_t}_{NJ\times 1} = \underbrace{\boldsymbol{\Omega}^C}_{NJ\times NNJ} \underbrace{\hat{\boldsymbol{C}}^{CX}}_{NNJ\times 1} + \underbrace{\boldsymbol{\Omega}^X}_{NJ\times NJNJ} \underbrace{\hat{\boldsymbol{X}}}_{NJNJ\times 1} \tag{D.4}$$

Here, for simplification, we will shy away from the sectoral bundles for both production and consumption and we will assume that $\theta^P = \theta^P_h = \theta^P_{l,i}$ and $\theta^C = \theta^C_h = \theta^C_{l,i}$ for all i.[39] From the CES structure we have $\hat{C}_{n,mj,t} = \hat{C}_{n,t} + \theta^C(\hat{P}^C_t - \hat{P}_{n,mj,t})$. In matrix notation this will be:

$$\underbrace{\hat{\boldsymbol{C}}_t^{nmj}}_{NNJ\times 1} = \underbrace{\boldsymbol{S}_{10}}_{NNJ\times N} \underbrace{\hat{\boldsymbol{C}}_t}_{N\times 1} + \theta^C \left(\underbrace{\boldsymbol{S}_{11}}_{NNJ\times N} \underbrace{\hat{\boldsymbol{P}}_t^C}_{N\times 1} - \underbrace{\hat{\boldsymbol{P}}_t^{CX}}_{NNJ\times 1} \right), \tag{D.5}$$

where $\hat{\boldsymbol{P}}_t^{CX}$ is the vector of consumption prices. From the production function we have:

$$\underbrace{\hat{\boldsymbol{Y}}_t}_{NJ\times 1} = \underbrace{\boldsymbol{\alpha}}_{NJ\times NJ} \underbrace{\hat{\boldsymbol{L}}_t}_{NJ\times 1} + \underbrace{\boldsymbol{\Omega}}_{NJ\times NJNJ} \underbrace{\hat{\boldsymbol{X}}_t}_{NJNJ\times 1} \tag{D.6}$$

From the CES structure we have $\hat{X}_{ni,mj,t} = \hat{L}_{ni,t} + \theta^P(\hat{W}_{n,t} - \hat{P}_{ni,mj,t})$. In matrix notation this will be:

$$\underbrace{\hat{\boldsymbol{X}}_t}_{NJNJ\times 1} = \underbrace{\boldsymbol{S}_{12}}_{NJNJ\times NJ} \underbrace{\hat{\boldsymbol{L}}_t}_{NJ\times 1} + \theta^P \left(\underbrace{\boldsymbol{S}_{13}}_{NJNJ\times N} \underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} - \underbrace{\hat{\boldsymbol{P}}_t^X}_{NJNJ\times 1} \right), \tag{D.7}$$

where $\hat{\boldsymbol{P}}_t^X$ is the vector of input prices. Using (D.7), we can substitute out $\hat{\boldsymbol{X}}_t$ in (D.6). Then we shall solve for $\hat{\boldsymbol{L}}_t$ in that equation. Call this equation, (D.8), "labor-output mapping,"

[39] More general case follows the same logic depicted here, but the notation becomes heavily involved.

$$\hat{\boldsymbol{Y}}_t = \boldsymbol{\alpha}\,\hat{\boldsymbol{L}}_t \;+\; \boldsymbol{\Omega}\Big(\boldsymbol{S}_{12}\,\hat{\boldsymbol{L}}_t \;+\; \theta^P\big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\big]\Big)$$

$$\hat{\boldsymbol{Y}}_t = \Big(\boldsymbol{\alpha} \;+\; \boldsymbol{\Omega}\,\boldsymbol{S}_{12}\Big)\,\hat{\boldsymbol{L}}_t \;+\; \theta^P\,\boldsymbol{\Omega}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\Big)$$

Rearranging and solving for

$$\Big(\boldsymbol{\alpha} \;+\; \boldsymbol{\Omega}\,\boldsymbol{S}_{12}\Big)\,\hat{\boldsymbol{L}}_t = \hat{\boldsymbol{Y}}_t \;-\; \theta^P\,\boldsymbol{\Omega}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\Big),$$

$$\hat{\boldsymbol{L}}_t = \Big(\boldsymbol{\alpha} \;+\; \boldsymbol{\Omega}\,\boldsymbol{S}_{12}\Big)^{-1}\Big[\hat{\boldsymbol{Y}}_t \;-\; \theta^P\,\boldsymbol{\Omega}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\Big)\Big] \tag{D.8}$$

Then we use (D.7) to substitute out $\hat{\boldsymbol{X}}_t$ and use (D.5) to substitute out $\hat{\boldsymbol{C}}_t^{nmj}$ in (D.4). This equation will now have $\hat{\boldsymbol{L}}_t$ in it. Substituting that out using "labor-output mapping," we can then solve for $\hat{\boldsymbol{Y}}_t$ and call this "output-consumption mapping." First, substitute (D.5) and (D.7) into the market-clearing condition (D.4). This yields an expression in terms of $\hat{\boldsymbol{Y}}_t$ and $\hat{\boldsymbol{L}}_t^{ni}$:

$$\hat{\boldsymbol{Y}}_t = \boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t \;+\; \theta^C\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \;+\; \boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{12}\,\hat{\boldsymbol{L}}_t^{ni} \;+\; \theta^P\big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\big)\Big].$$

Next, use the labor–output mapping (equation (D.8)) to substitute out $\hat{\boldsymbol{L}}_t^{ni}$. Let

$$\boldsymbol{A} \;\equiv\; \boldsymbol{\alpha} \;+\; \boldsymbol{\Omega}\,\boldsymbol{S}_{12}.$$

Then

$$\hat{\boldsymbol{L}}_t^{ni} = \boldsymbol{A}^{-1}\Big[\hat{\boldsymbol{Y}}_t \;-\; \theta^P\,\boldsymbol{\Omega}\big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\big)\Big].$$

Substituting this into the above expression and collecting terms in $\hat{\boldsymbol{Y}}_t$ gives

$$\begin{aligned}\hat{\boldsymbol{Y}}_t = {}& \boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t \;+\; \theta^C\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \\ & +\; \boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{12}\,\boldsymbol{A}^{-1}\big(\hat{\boldsymbol{Y}}_t \;-\; \theta^P\,\boldsymbol{\Omega}\big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\big)\big) \;+\; \theta^P\big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\big)\Big].\end{aligned}$$

Rearranging to isolate $\hat{\boldsymbol{Y}}_t$ on the left-hand side and then inverting the resulting coefficient matrix gives us equation, (D.9), which is the *output–consumption mapping*:

$$\begin{aligned}\hat{\boldsymbol{Y}}_t = {}& \Big[\boldsymbol{I} \;-\; \boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\boldsymbol{A}^{-1}\Big]^{-1}\Big\{\boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t + \theta^C\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \;+\; \theta^P\,\boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big] \\ & -\; \theta^P\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\boldsymbol{A}^{-1}\,\boldsymbol{\Omega}\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big]\Big\}.\end{aligned} \tag{D.9}$$

We now return to (D.1). Let us substitute out $\hat{\boldsymbol{X}}_t$ in that equation using (D.7). Next we substitute $\hat{\boldsymbol{L}}_t^{ni}$ in the resulting expression using (D.8). Finally we substitute out $\hat{\boldsymbol{Y}}_t$ in the resulting expression using (D.9), ending up with an expression that expresses net exports as a function of the aggregate consumption vector and prices. Recalling that we defined

$$\boldsymbol{A} \equiv \boldsymbol{\alpha} + \boldsymbol{\Omega}\, \boldsymbol{S}_{12}, \quad \text{and} \quad \hat{\boldsymbol{L}}_t^{ni} = \boldsymbol{A}^{-1}\Big[\hat{\boldsymbol{Y}}_t - \theta^P\, \boldsymbol{\Omega}\big(\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\big)\Big].$$

From the *output–consumption mapping* (D.9), we have

$$\begin{aligned}\hat{\boldsymbol{Y}}_t = \Big[\boldsymbol{I} - \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\Big]^{-1}\Big\{&\boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\, \hat{\boldsymbol{C}}_t + \theta^C\big(\boldsymbol{S}_{11}\, \hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \\ &+ \theta^P\, \boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big] - \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big]\Big\}.\end{aligned}$$

Starting again from (D.1),

$$\widehat{NX}_{n,t} = \boldsymbol{S}_1\big(\hat{\boldsymbol{Y}}_t^{ni} + \hat{\boldsymbol{P}}_t^P\big) - \boldsymbol{S}_2\big(\hat{\boldsymbol{C}}_t + \hat{\boldsymbol{P}}_t^C\big) - \boldsymbol{S}_3\big(\hat{\boldsymbol{X}}_t + \hat{\boldsymbol{P}}_t^X\big),$$

we substitute (D.7) for $\hat{\boldsymbol{X}}_t$, then (D.8) for $\hat{\boldsymbol{L}}_t^{ni}$, and finally (D.9) for $\hat{\boldsymbol{Y}}_t$. Inserting each expression carefully and gathering terms gives:

$$\begin{aligned}\widehat{NX}_{n,t} = \boldsymbol{S}_1\Bigg(&\Big[\boldsymbol{I} - \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\Big]^{-1}\Big\{\boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\, \hat{\boldsymbol{C}}_t + \theta^C\big(\boldsymbol{S}_{11}\, \hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \\ &+ \theta^P\, \boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big] - \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big]\Big\} + \hat{\boldsymbol{P}}_t^P\Bigg) \\ - \boldsymbol{S}_2&\Big(\hat{\boldsymbol{C}}_t + \hat{\boldsymbol{P}}_t^C\Big) \\ - \boldsymbol{S}_3&\Bigg[\boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\Big(\Big[\boldsymbol{I} - \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\Big]^{-1}\Big\{\boldsymbol{\Omega}^C\Big[\boldsymbol{S}_{10}\, \hat{\boldsymbol{C}}_t + \theta^C\big(\boldsymbol{S}_{11}\, \hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmj}\big)\Big] \\ &+ \theta^P\, \boldsymbol{\Omega}^X\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big] - \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\Big[\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big]\Big\} \\ &- \theta^P\, \boldsymbol{\Omega}\Big(\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\big)\Big) + \theta^P\Big(\boldsymbol{S}_{13}\, \hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big) + \hat{\boldsymbol{P}}_t^{nimj}\Bigg].\end{aligned}$$

This final expression shows $\widehat{NX}_{n,t}$ as a function of the aggregate consumption vector, the wage vector, and the relevant price vectors. In our analytical solution we use $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t$, so we plug that in. Defining $\widetilde{A} = \big[\boldsymbol{I} - \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\big]^{-1}$ we can multiply terms out and rearrange:

$$\begin{aligned}
\widehat{NX}_{n,t} = \Big(& \boldsymbol{S}_1\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \boldsymbol{S}_{10} \;+\; \boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{13} \;-\; \boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \\
&- \boldsymbol{S}_2 \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \boldsymbol{S}_{10} \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{13} \\
&+ \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \;+\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \theta^P\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \;-\; \theta^P\, \boldsymbol{S}_3\, \boldsymbol{S}_{13}\Big)\, \hat{\boldsymbol{C}}_t \\
+ \Big(& \boldsymbol{S}_1\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \theta^C\, \boldsymbol{S}_{11} \;+\; \boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{13} \;-\; \boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \\
&- \boldsymbol{S}_2 \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \theta^C\, \boldsymbol{S}_{11} \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{13} \\
&+ \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \theta^P\, \boldsymbol{\Omega}\, \boldsymbol{S}_{13} \;-\; \theta^P\, \boldsymbol{S}_3\, \boldsymbol{S}_{13}\Big)\, \hat{\boldsymbol{P}}_t^C \\
+ \Big(& \boldsymbol{S}_1\Big)\, \hat{\boldsymbol{P}}_t^P \\
+ \Big(& -\boldsymbol{S}_1\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \theta^C \;+\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \boldsymbol{\Omega}^C\, \theta^C \;+\; \theta^C\, \boldsymbol{S}_3\Big)\, \hat{\boldsymbol{P}}_t^{CX} \\
+ \Big(& -\boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X \;+\; \boldsymbol{S}_1\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega} \\
&+ \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X \;-\; \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \widetilde{A}\, \theta^P\, \boldsymbol{\Omega}^X\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \boldsymbol{\Omega} \\
&- \boldsymbol{S}_3\, \boldsymbol{S}_{12}\, \boldsymbol{A}^{-1}\, \theta^P\, \boldsymbol{\Omega} \;+\; (\theta^P \;-\; 1)\, \boldsymbol{S}_3\Big)\, \hat{\boldsymbol{P}}_t^X.
\end{aligned}$$

In this expression, $\hat{\boldsymbol{P}}_t^X$ and $\hat{\boldsymbol{P}}_t^{CX}$ are also linear combinations of producer prices, exchange rate and tariffs, and given that the U.S. nominal interest rate is a function of U.S. price level. Thus, we can write:

$$\begin{aligned}
\beta \hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta \hat{i}_t^{US} \\
\beta \hat{V}_{n,t}^{US} &= \Xi_1 \hat{V}_{n,t-1}^{US} + \boldsymbol{\Xi}_2 \hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3 \hat{\boldsymbol{P}}_t^P + \Xi_4 \mathcal{E}_t + \Xi_5 \tau_t
\end{aligned}$$

where $\Xi_1 = 1$ in the case of the two-country model; aggregating this yields the fifth equation in the five-equation representation. We will not specify the elements of $\boldsymbol{\Xi}$ matrices explicitly and for our purposes here, it is enough to show that the balance of payments could be written as in the expression above.

From the expression above and from intuition, we can see that a higher elasticity of substitution makes the balance of payments more reactive to changes in prices. More broadly we see net exports react to the aggregate demand stance of countries and the terms of trade in each sector.

Stacking the final expression above for different countries n, alongside a market-clearing condition for U.S. bonds, yields the fifth equation in the five-equation Global New Keynesian

Representation.

D.2 N=2 J=1

Let us focus on the case where $N = 2$ and $J = 1$ under flexible prices. As opposed to the N-country setting, when $N > 1$, it is sufficient to track only one balance of payments equation, which in turn can be written from the perspective of the home country whose bonds are used by both countries to save and dissave.

Starting with the budget constraint and simplifying by setting domestic bonds $B_{n,t} = 0$ and portfolio adjustment cost $\psi(\cdot) = 0$:

$$\begin{aligned}
P_{n,t}C_{n,t} &= W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} + T_{n,t} + \mathcal{E}^{US}_{n,t}B^{US}_{n,t} - \mathcal{E}^{US}_{n,t}(1 + i^{US}_{n,t-1})B^{US}_{n,t-1} \\
P_{n,t}C_{n,t} + T_{n,t} - \mathcal{E}^{US}_{n,t}B^{US}_{n,t} &= W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - \mathcal{E}^{US}_{n,t}(1 + i^{US}_{n,t-1})B^{US}_{n,t-1} \\
NX_{n,t} &\equiv \sum_i \Pi_{ni,t} - W_{n,t}L_{n,t} - P_{n,t}C_{n,t} - T_{n,t} \\
\mathcal{E}^{US}_{n,t}(1 + i^{US}_{n,t-1})B^{US}_{n,t-1} &= NX_{n,t} + \mathcal{E}^{US}_{n,t}B^{US}_{n,t}
\end{aligned}$$

Redefining V_t as dollar-denominated debt inclusive of interest payments: $V_t = B^{US}_{n,t}(1 + i_t)$. Will drop superscript for ease of notation. Additionally note that we can write this in terms of home country, the U.S., for which $\mathcal{E}_t = 1\ \forall t$:

$$V_{t-1} = NX_t + \frac{V_t}{1 + i_t} \tag{D.10}$$

At steady state this equation will read:

$$\begin{aligned}
\overline{V} &= \overline{NX} + \beta\overline{V} \\
\overline{NX} &= (1 - \beta)\overline{V}
\end{aligned}$$

In light of this, and the fact that $1 + \bar{i} = \beta^{-1}$ we can rewrite (D.10):

$$\begin{aligned}
\overline{V}\hat{V}_{t-1} &= \overline{NX}\hat{NX}_t + \beta\overline{V}(\hat{V}_t - \hat{i}_t) \\
\overline{V}\hat{V}_{t-1} &= (1 - \beta)\overline{V}\hat{NX}_t + \beta\overline{V}(\hat{V}_t - \hat{i}_t) \\
\beta\hat{V}_t &= \hat{V}_{t-1} - (1 - \beta)\hat{NX}_t + \beta\hat{i}_t
\end{aligned}$$

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \frac{(1-\beta)}{\beta}\hat{NX}_t + \hat{i}_t$$

For the sake of simplicity, we will assume away home country's use of its own goods as intermediate input and label the home country H and foreign country F. Recalling in our subscript notation the fact that the first subscript is user and the second is producer, then the two market clearing conditions then are:

$$Y_{H,t} = C_{H,H,t} + C_{F,H,t} + X_{F,H,t} \tag{D.11}$$

where $C_{H,H,t}$ is home country's consumption of goods made in the home country, $C_{F,H,t}$ is foreign country's consumption of goods made in the home country, and $X_{F,H,t}$ is the foreign country's use of goods made in the home country as intermediate inputs. By symmetry we also have:

$$Y_{F,t} = C_{F,F,t} + C_{H,F,t} + X_{H,F,t} \tag{D.12}$$

Note that in a flexible price setting producer price equals marginal cost so we can write the following in light of the CES structure of production:

$$\frac{X_{F,H,t}}{Y_{F,t}} = \Omega^F \left(\frac{P^P_{H,t}(1+\tau^F_t)/\mathcal{E}_t}{MC_{F,t}} \right)^{-\theta^P}$$

$$X_{F,H,t} = \Omega^F \left(\frac{P^P_{H,t}(1+\tau^F_t)}{P^P_{F,t}\mathcal{E}_t} \right)^{-\theta^P} Y_{F,t} \tag{D.13}$$

where $P^P_{H,t}$ and $P^P_{F,t}$ are respectively the producer price of the good made in the home country and foreign country under producer currency pricing. By symmetry:

$$X_{H,F,t} = \Omega^H \left(\frac{P^P_{F,t}(1+\tau^H_t)\mathcal{E}_t}{P^P_{H,t}} \right)^{-\theta^P} Y_{H,t} \tag{D.14}$$

Next, we denote from home country's perspective and in home country's currency (i.e. in USD) net exports:

$$NX_t = P^P_{H,t}(C_{F,H,t} + X_{F,H,t}) - P^P_{F,t}\mathcal{E}_t(C_{H,F,t} + X_{H,F,t})$$

Using market clearing conditions in (D.11) and (D.12) let us substitute out intermediate inputs:

$$NX_t = P^P_{H,t}(Y_{H,t} - C_{H,H,t}) - P^P_{F,t}\mathcal{E}_t(Y_{F,t} - C_{F,F,t}) \tag{D.15}$$

Next we can note that under the CES structure consumption can be expressed as follows given the standard relative demand conditions:

$$\begin{aligned}
C_{H,H,t} &= (1-\xi_H)\left(\frac{P^P_{H,t}}{P_{H,t}}\right)^{-\theta^P} C_{H,t} \\
C_{H,F,t} &= \xi_H\left(\frac{P^P_{F,t}(1+\tau^H_t)\mathcal{E}_t}{P_{H,t}}\right)^{-\theta^P} C_{H,t} \\
C_{F,F,t} &= (1-\xi_F)\left(\frac{P^P_{F,t}}{P_{F,t}}\right)^{-\theta^P} C_{F,t} \\
C_{F,H,t} &= \xi_F\left(\frac{P^P_{H,t}(1+\tau^F_t)}{P_{F,t}\mathcal{E}_t}\right)^{-\theta^P} C_{F,t}
\end{aligned}$$

where $C_{H,t}$ and $P_{H,t}$ are aggregate consumption and CPI price index for the home country. Then (D.15) becomes:

$$\begin{aligned}
NX_t &= P^P_{H,t}(Y_{H,t} - C_{H,H,t}) - P^P_{F,t}\mathcal{E}_t(Y_{F,t} - C_{F,F,t}) \\
&= P^P_{H,t}Y_{H,t} - P^P_{H,t}(1-\xi_H)\left(\frac{P^P_{H,t}}{P_{H,t}}\right)^{-\theta^P} C_{H,t} - P^P_{F,t}\mathcal{E}_t Y_{F,t} + P^P_{F,t}\mathcal{E}_t(1-\xi_F)\left(\frac{P^P_{F,t}}{P_{F,t}}\right)^{-\theta^P} C_{F,t} \\
&= P^P_{H,t}Y_{H,t} - (1-\xi_H)(P^P_{H,t})^{1-\theta^P} P^{\theta^P}_{H,t}C_{H,t} - P^P_{F,t}\mathcal{E}_t Y_{F,t} + (1-\xi_F)(P^P_{F,t})^{1-\theta^P} P^{\theta^P}_{F,t}\mathcal{E}_t C_{F,t}
\end{aligned} \tag{D.16}$$

Note that steady state output in both countries can be normalized to 1. This then implies $\overline{C}_H = 1 - \Omega_H = \alpha_H$, where α_H is labor share and relatedly Ω_H is imported input share in country H. There are two ways to parametrize exports at the steady state. The first follows from the balance of payments equation above evaluated at the steady state, which yields $\overline{NX} = (1-\beta)\overline{V}$. The second involves evaluating (D.16) at the steady state, where prices are normalized to 1, as follows:

$$\begin{aligned}
\overline{NX}_t &= \overline{P}^P_{H,t}\overline{Y}_H - (1-\xi_H)\left(\overline{P}^P_H\right)^{1-\theta^P} \overline{P}^{\theta^P}_H \overline{C}_H - \overline{P}^P_F\overline{\mathcal{E}}\,\overline{Y}_F + (1-\xi_F)\left(\overline{P}^P_F\right)^{1-\theta^P} \overline{P}^{\theta^P}_F \overline{\mathcal{E}}_t \overline{C}_F \\
&= 1 - (1-\Omega_H)(1-\xi_H) - 1 + (1-\Omega_F)(1-\xi_F)
\end{aligned}$$

$$= -(1-\Omega_H)(1-\xi_H) + (1-\Omega_F)(1-\xi_F)$$

Then linearizing the net exports equation we have:

$$\begin{aligned}
\overline{NX}\hat{NX}_t &= \overline{Y}_H(\hat{P}^P_{H,t} + \hat{Y}_{H,t}) - \overline{C}_H\left[(1-\theta^P)\hat{P}^P_{H,t} + \theta^P\hat{P}^C_{H,t} + \hat{C}_{H,t}\right] \\
&\quad - \overline{Y}_F(\hat{P}^P_{F,t} + \hat{\mathcal{E}}_t + \hat{Y}_{F,t}) + \overline{C}_F\left[(1-\theta^P)\hat{P}^P_{F,t} + \theta^P\hat{P}^C_{F,t} + \hat{\mathcal{E}}_t + \hat{C}_{F,t}\right] \\
&= (\hat{P}^P_{H,t} + \hat{Y}_{H,t}) - (1-\Omega_H)\left[(1-\theta^P)\hat{P}^P_{H,t} + \theta^P\hat{P}^C_{H,t} + \hat{C}_{H,t}\right] \\
&\quad - (\hat{P}^P_{F,t} + \hat{\mathcal{E}}_t + \hat{Y}_{F,t}) + (1-\Omega_F)\left[(1-\theta^P)\hat{P}^P_{F,t} + \theta^P\hat{P}^C_{F,t} + \hat{\mathcal{E}}_t + \hat{C}_{F,t}\right] \\
&= \left[1-(1-\Omega_H)(1-\theta^P)\right]\hat{P}^P_{H,t} + \left[-1+(1-\Omega_F)(1-\theta^P)\right]\hat{P}^P_{F,t} \\
&\quad + (\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\Omega_F)(\hat{C}_{F,t} + \theta^P\hat{P}^C_{F,t}) - (1-\Omega_H)(\hat{C}_{H,t} + \theta^P\hat{P}^C_{H,t}) - \Omega_F\hat{\mathcal{E}}_t
\end{aligned}$$

With steady-state consumption normalized to 1, we can express steady-state values for variables like $C_{H,H,t}$ and $X_{F,H,t}$ in terms of home bias in consumption $(1-\xi_H)$ and imported input dependence Ω_H, which is transformed into $\Psi_H = \frac{1}{1-\Omega_H}$. Thus, when linearized we have the following equations:

$$\begin{aligned}
\hat{Y}_{H,t} &= (1-\Omega_H)(1-\xi_H)\hat{C}_{H,H,t} + (1-\Omega_F)\xi_F\hat{C}_{F,H,t} + \Omega_F\hat{X}_{F,H,t} \\
\hat{Y}_{F,t} &= (1-\Omega_F)(1-\xi_F)\hat{C}_{F,F,t} + (1-\Omega_H)\xi_H\hat{C}_{H,F,t} + \Omega_H\hat{X}_{H,F,t} \\
\hat{X}_{F,H,t} &= -\theta^P\left(\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t - \hat{P}^P_{F,t}\right) + \hat{Y}_{F,t} \\
\hat{X}_{H,F,t} &= -\theta^P\left(\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t - \hat{P}^P_{H,t}\right) + \hat{Y}_{H,t} \\
\hat{C}_{H,H,t} &= -\theta^P\left(\hat{P}^P_{H,t} - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{H,F,t} &= -\theta^P\left(\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{F,F,t} &= -\theta^P\left(\hat{P}^P_{F,t} - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{C}_{F,H,t} &= -\theta^P\left(\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{P}^C_{H,t} &= (1-\xi_H)\hat{P}^P_{H,t} + \xi_H(\hat{P}^P_{F,t} + \mathcal{E}_t + \hat{\tau}^H_t) \\
\hat{P}^C_{F,t} &= (1-\xi_F)\hat{P}^P_{F,t} + \xi_F(\hat{P}^P_{H,t} - \mathcal{E}_t + \hat{\tau}^F_t) \\
\overline{NX}\hat{NX}_t &= \left[1-(1-\Omega_H)(1-\theta^P)\right]\hat{P}^P_{H,t} + \left[-1+(1-\Omega_F)(1-\theta^P)\right]\hat{P}^P_{F,t} \\
&\quad + (\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\Omega_F)(\hat{C}_{F,t} + \theta^P\hat{P}^C_{F,t}) - (1-\Omega_H)(\hat{C}_{H,t} + \theta^P\hat{P}^C_{H,t}) - \Omega_F\hat{\mathcal{E}}_t
\end{aligned}$$

These equations can express net exports as a share of prices, which can then be plugged into the following balance of payments equation:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \frac{(1-\beta)}{\beta}\hat{NX}_t + \hat{i}_t$$

E Analytical Solution Under Flexible Prices

Our original 5-equation global NK representation was as follows. A linearized equilibrium comprises vector sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}_t^P, \hat{\boldsymbol{P}}_t^C, \tilde{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}_{t_0}^{\infty}$ for a given sequence of $\{\hat{\tau}_t\}_{t_0}^{\infty}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (E.1)-(E.5) hold:

$$\textbf{NKIS+TR:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) - \mathbb{E}_t(\hat{\boldsymbol{P}}_{t+1}^C - \hat{\boldsymbol{P}}_t^C) \tag{E.1}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \tilde{\boldsymbol{L}}_{\mathcal{E}}^C\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\hat{\tau}}^C\hat{\boldsymbol{\tau}}_t \tag{E.2}$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\hat{\tau}}^P\hat{\tau}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{E.3}$$

$$\textbf{UIP+TR:} \quad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\tilde{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\tilde{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) \tag{E.4}$$

$$\textbf{BoP:} \quad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}_t^P + \Xi_4\tilde{\boldsymbol{\mathcal{E}}}_t + \Xi_5\hat{\boldsymbol{\tau}}_t \tag{E.5}$$

$$\tag{E.6}$$

To study the long-run behavior of the exchange rate in a tractable way let us assume we are in the two-country case and prices are fully flexible. We will study the impact of a permanent tariff When prices are flexible entries of $\boldsymbol{\Lambda} \to \infty$ so we have

$$\boldsymbol{0} = \left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right) + (\boldsymbol{\Omega} - \boldsymbol{I})\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\hat{\tau}}^P\hat{\tau}_t\right)$$

$$(\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{P}}_t^P = \left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\hat{\tau}}^P\hat{\tau}_t\right)$$

$$\boldsymbol{\pi}_t^P = \boldsymbol{\Psi}\left(\boldsymbol{\alpha}\left(\boldsymbol{\pi}_t^C + \sigma\Delta\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\Delta\tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\hat{\tau}}^P\Delta\hat{\tau}_t\right)$$

where $\boldsymbol{\Psi}$ is the regular Leontief inverse (different from our NKOE Leontief Inverse, which is a short-run DGE object).

The following is the case with us as is the standard three-equation NK model: With the shock being permanent and the policy rule targeting only inflation, all the adjustment will take place via other variables (e.g. quantities and exchange rate), while inflation's deviation from steady state will be zero. We confirm this analytically and quantitatively with our model coded in Dynare.

Then in first differences a linearized equilibrium comprises vector sequences $\{\Delta\hat{\boldsymbol{C}}_t, \boldsymbol{\pi}_t^P, \boldsymbol{\pi}_t^C, \Delta\tilde{\boldsymbol{\mathcal{E}}}_t, \Delta\hat{\boldsymbol{V}}_t\}_{t_0}^{\infty}$ for a given sequence of $\{\Delta\hat{\tau}_t\}_{t_0}^{\infty}$ and an initial condition for $\boldsymbol{\Delta}\hat{\boldsymbol{V}}_0$ such that equations (E.7)-(E.11) hold:

$$\sigma \underbrace{\mathbb{E}_t\Delta\hat{\boldsymbol{C}}_{t+1}}_{N\times 1} = \underbrace{\boldsymbol{\Phi}}_{N\times N}\underbrace{\boldsymbol{\pi}_t^C}_{N\times 1} - \mathbb{E}_t\underbrace{\boldsymbol{\pi}_{t+1}^C}_{N\times 1} \tag{E.7}$$

$$\underbrace{\boldsymbol{\pi}_t^C}_{N\times 1} = \underbrace{\boldsymbol{\Gamma}}_{N\times NJ}\underbrace{\boldsymbol{\pi}_t^P}_{NJ\times 1} + \underbrace{\tilde{\boldsymbol{L}}_{\mathcal{E}}^C}_{N\times 1}\Delta\hat{\mathcal{E}}_t + \underbrace{\boldsymbol{L}_{\hat{\tau}}^C}_{N\times 1}\Delta\hat{\tau}_t \tag{E.8}$$

$$\mathbb{E}_t\Delta\hat{\mathcal{E}}_{t+1} = \underbrace{\tilde{\boldsymbol{\Phi}}_3}_{1\times N}\underbrace{\boldsymbol{\pi}_t^C}_{N\times 1} \tag{E.9}$$

$$\beta\Delta\hat{V}_t = \Delta\hat{V}_{t-1} + \underbrace{\boldsymbol{\Xi}_2}_{1\times N}\underbrace{\Delta\hat{\boldsymbol{C}}_t}_{N\times 1} + \underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\pi}_t^P}_{NJ\times 1} + \Xi_4\Delta\hat{\mathcal{E}}_t + \Xi_5\Delta\hat{\tau}_t \tag{E.10}$$

$$\underbrace{\boldsymbol{\pi}_t^P}_{NJ\times 1} = \underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\left(\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\left(\underbrace{\boldsymbol{\pi}_t^C}_{N\times 1} + \sigma\underbrace{\Delta\hat{\boldsymbol{C}}_t}_{N\times 1}\right) + \underbrace{\boldsymbol{L}_{\mathcal{E}}^P}_{NJ\times 1}\Delta\tilde{\mathcal{E}}_t + \underbrace{\boldsymbol{L}_{\hat{\tau}}^P}_{NJ\times 1}\Delta\hat{\tau}_t\right) \tag{E.11}$$

E.1 Method of Undetermined Coefficients

Let us postulate that

$$\begin{aligned}
\Delta\hat{\boldsymbol{C}}_t &= \underbrace{\boldsymbol{C}_1}_{N\times 1}\Delta\hat{V}_{t-1} + \underbrace{\boldsymbol{C}_2}_{N\times 1}\Delta\hat{\tau}_t \\
\boldsymbol{\pi}_t^C &= \underbrace{\boldsymbol{C}_3}_{N\times 1}\Delta\hat{V}_{t-1} + \underbrace{\boldsymbol{C}_4}_{N\times 1}\Delta\hat{\tau}_t \\
\boldsymbol{\pi}_t^P &= \underbrace{\boldsymbol{C}_5}_{NJ\times 1}\Delta\hat{V}_{t-1} + \underbrace{\boldsymbol{C}_6}_{NJ\times 1}\Delta\hat{\tau}_t \\
\Delta\hat{V}_t &= C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t \\
\Delta\hat{\mathcal{E}}_t &= C_9\Delta\hat{V}_{t-1} + C_{10}\Delta\hat{\tau}_t
\end{aligned}$$

Iterating one period forward and taking expectation at t. Keeping in mind the fact that a permanent shock means $\Delta\hat{\tau}_t$ is 0 for all periods after the initial period of impact (so in first differences it is a one-time shock).

$$\begin{aligned}
\mathbb{E}_t\Delta\hat{\mathbf{C}}_{t+1} &= \mathbf{C}_1\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right) \\
\mathbb{E}_t\boldsymbol{\pi}_{t+1}^C &= \mathbf{C}_3\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right)
\end{aligned}$$

$$\mathbb{E}_t\Delta\hat{\mathcal{E}}_{t+1} = C_9\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right)$$

Plugging these in

$$\sigma\left(\mathbf{C}_1\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right)\right) = \mathbf{\Phi}\left(\mathbf{C}_3\Delta\hat{V}_{t-1} + \mathbf{C}_4\Delta\hat{\tau}_t\right) - \left(\mathbf{C}_3\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right)\right)$$
$$\mathbf{C}_3\Delta\hat{V}_{t-1} + \mathbf{C}_4\Delta\hat{\tau}_t = \mathbf{\Gamma}\left(\mathbf{C}_5\Delta\hat{V}_{t-1} + \mathbf{C}_6\Delta\hat{\tau}_t\right) + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}(C_9\Delta\hat{V}_{t-1} + C_{10}\Delta\hat{\tau}_t) + \boldsymbol{L}^C_{\hat{\tau}}\Delta\hat{\tau}_t$$
$$C_9\left(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t\right) = \tilde{\mathbf{\Phi}}_3(\mathbf{C}_3\Delta\hat{V}_{t-1} + \mathbf{C}_4\Delta\hat{\tau}_t)$$
$$\beta(C_7\Delta\hat{V}_{t-1} + C_8\Delta\hat{\tau}_t) = \Delta\hat{V}_{t-1} + \mathbf{\Xi}_2(\mathbf{C}_1\Delta\hat{V}_{t-1} + \mathbf{C}_2\Delta\hat{\tau}_t) + \mathbf{\Xi}_3\boldsymbol{\pi}^P_t + \Xi_4\Delta\hat{\mathcal{E}}_t + \Xi_5\Delta\hat{\boldsymbol{\tau}}_t$$
$$\mathbf{C}_5\Delta\hat{V}_{t-1} + \mathbf{C}_6\Delta\hat{\tau}_t = \mathbf{\Psi}\Bigg(\boldsymbol{\alpha}\left((\mathbf{C}_3\Delta\hat{V}_{t-1} + \mathbf{C}_4\Delta\hat{\tau}_t) + \sigma(\mathbf{C}_1\Delta\hat{V}_{t-1} + \mathbf{C}_2\Delta\hat{\tau}_t)\right)$$
$$+ \boldsymbol{L}^P_{\mathcal{E}}(C_9\Delta\hat{V}_{t-1} + C_{10}\Delta\hat{\tau}_t) + \boldsymbol{L}^P_{\hat{\tau}}\Delta\hat{\tau}_t\Bigg)$$

That is we have:

$$\left(\sigma\mathbf{C}_1C_7 - \mathbf{\Phi}\mathbf{C}_3 + \mathbf{C}_3C_7\right)\Delta\hat{V}_{t-1} + \left(\sigma\mathbf{C}_1C_8 - \mathbf{\Phi}\mathbf{C}_4 + \mathbf{C}_3C_8\right)\Delta\hat{\boldsymbol{\tau}}_t = 0 \tag{1'}$$

$$\left(\mathbf{C}_3 - \mathbf{\Gamma}\mathbf{C}_5 - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}C_9\right)\Delta\hat{V}_{t-1} + \left(\mathbf{C}_4 - \mathbf{\Gamma}\mathbf{C}_6 - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}C_{10} - \mathbf{L}^C_{\hat{\tau}}\right)\Delta\hat{\boldsymbol{\tau}}_t = 0 \tag{2'}$$

$$\left(C_9C_7 - \tilde{\mathbf{\Phi}}_3\mathbf{C}_3\right)\Delta\hat{V}_{t-1} + \left(C_9C_8 - \tilde{\mathbf{\Phi}}_3\mathbf{C}_4\right)\Delta\hat{\boldsymbol{\tau}}_t = 0 \tag{3'}$$

$$\begin{aligned}&\left(\beta C_7 - 1 - \mathbf{\Xi}_2\mathbf{C}_1 - \mathbf{\Xi}_3\mathbf{C}_5 - \Xi_4C_9\right)\Delta\hat{V}_{t-1}\\ &\qquad + \left(\beta C_8 - \mathbf{\Xi}_2\mathbf{C}_2 - \mathbf{\Xi}_3\mathbf{C}_6 - \Xi_4C_{10} - \Xi_5\right)\Delta\hat{\boldsymbol{\tau}}_t = 0\end{aligned} \tag{4'}$$

$$\begin{aligned}&\left[\mathbf{C}_5 - \mathbf{\Psi}\boldsymbol{\alpha}(\mathbf{C}_3 + \sigma\mathbf{C}_1) - \mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}}C_9\right]\Delta\hat{V}_{t-1}\\ &+ \left[\mathbf{C}_6 - \mathbf{\Psi}\boldsymbol{\alpha}(\mathbf{C}_4 + \sigma\mathbf{C}_2) - \mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}}C_{10} - \mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}}\right]\Delta\hat{\tau}_t = \mathbf{0}\end{aligned} \tag{5'}$$

Resulting system of 10 equations:

$$\sigma\,\mathbf{C}_1C_7 - \mathbf{\Phi}\mathbf{C}_3 + \mathbf{C}_3C_7 = 0 \tag{E.12}$$

$$\mathbf{C}_3 - \mathbf{\Gamma}\mathbf{C}_5 - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}C_9 = 0 \tag{E.13}$$

$$C_9C_7 - \tilde{\mathbf{\Phi}}_3\mathbf{C}_3 = 0 \tag{E.14}$$

$$\beta C_7 - 1 - \mathbf{\Xi}_2\mathbf{C}_1 - \mathbf{\Xi}_3\mathbf{C}_5 - \Xi_4C_9 = 0 \tag{E.15}$$

$$\left[\mathbf{C}_5 - \mathbf{\Psi}\boldsymbol{\alpha}(\mathbf{C}_3 + \sigma\mathbf{C}_1) - \mathbf{\Psi}\boldsymbol{L}_{\mathcal{E}}^{P} C_9\right] = 0 \tag{E.16}$$

$$\sigma\,\mathbf{C}_1 C_8 - \mathbf{\Phi}\mathbf{C}_4 + \mathbf{C}_3 C_8 = 0 \tag{E.17}$$

$$\mathbf{C}_4 - \mathbf{\Gamma}\mathbf{C}_6 - \tilde{\boldsymbol{L}}_{\mathcal{E}}^{C} C_{10} - \mathbf{L}_{\hat{\tau}}^{C} = 0 \tag{E.18}$$

$$C_9 C_8 - \tilde{\mathbf{\Phi}}_3 \mathbf{C}_4 = 0 \tag{E.19}$$

$$\beta C_8 - \mathbf{\Xi}_2\mathbf{C}_2 - \mathbf{\Xi}_3\mathbf{C}_6 - \Xi_4 C_{10} - \Xi_5 = 0 \tag{E.20}$$

$$\left[\mathbf{C}_6 - \mathbf{\Psi}\boldsymbol{\alpha}(\mathbf{C}_4 + \sigma\mathbf{C}_2) - \mathbf{\Psi}\boldsymbol{L}_{\mathcal{E}}^{P} C_{10} - \mathbf{\Psi}\boldsymbol{L}_{\hat{\tau}}^{P}\right] = 0 \tag{E.21}$$

To solve the system with ten equations we begin as follows:

$$C_7 = \frac{1}{C_9}\tilde{\mathbf{\Phi}}_3\mathbf{C}_3$$
$$\left(\frac{\sigma}{C_9}\mathbf{C}_1\tilde{\mathbf{\Phi}}_3 - \mathbf{\Phi} + \frac{1}{C_9}\mathbf{C}_3\tilde{\mathbf{\Phi}}_3\right)\mathbf{C}_3 = 0$$

Keep in mind that $\tilde{\mathbf{\Phi}}_3 = \begin{bmatrix}1 & -1\end{bmatrix}\mathbf{\Phi} = \boldsymbol{Z}\mathbf{\Phi}$. Then:

$$\frac{1}{C_9}\left((\sigma\mathbf{C}_1 + \mathbf{C}_3)\boldsymbol{Z} - \boldsymbol{I}C_9\right)\mathbf{\Phi}\mathbf{C}_3 = 0$$

$$C_8 = \frac{1}{C_9}\tilde{\mathbf{\Phi}}_3\mathbf{C}_4$$
$$\left(\frac{\sigma}{C_9}\mathbf{C}_1\tilde{\mathbf{\Phi}}_3 - \mathbf{\Phi} + \frac{1}{C_9}\mathbf{C}_3\tilde{\mathbf{\Phi}}_3\right)\mathbf{C}_4 = 0$$
$$\mathbf{C}_4 = \mathbf{\Gamma}\mathbf{C}_6 + \tilde{\boldsymbol{L}}_{\mathcal{E}}^{C} C_{10} + \mathbf{L}_{\hat{\tau}}^{C}$$
$$\beta C_8 - \mathbf{\Xi}_2\mathbf{C}_2 - \mathbf{\Xi}_3\mathbf{C}_6 - \Xi_4 C_{10} - \Xi_5 = 0$$
$$\mathbf{C}_6 = \mathbf{\Psi}\left[\boldsymbol{\alpha}(\mathbf{C}_4 + \sigma\mathbf{C}_2) + \boldsymbol{L}_{\mathcal{E}}^{P} C_{10} + \boldsymbol{L}_{\hat{\tau}}^{P}\right]$$

Plugging in last equation into 3rd equation yields:

$$\mathbf{C}_4 = (\boldsymbol{I} - \mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha})^{-1}\left[\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\mathbf{C}_2 + \left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}_{\mathcal{E}}^{P} + \tilde{\boldsymbol{L}}_{\mathcal{E}}^{C}\right)C_{10} + \left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}_{\hat{\tau}}^{P} + \mathbf{L}_{\hat{\tau}}^{C}\right)\right]$$

Also have:

$$\beta\frac{1}{C_9}\tilde{\mathbf{\Phi}}_3\mathbf{C}_4 = \mathbf{\Xi}_2\mathbf{C}_2 + \mathbf{\Xi}_3\mathbf{C}_6 + \Xi_4 C_{10} + \Xi_5$$

When price equals marginal cost (in deviation from the steady state), $C_4 = 0$, so is C_3. We have confirmed this with the code. So now system is:

$$\mathbf{C}_4 = 0$$
$$C_8 = 0$$
$$\mathbf{C}_6 = \left[\boldsymbol{\Psi}\left[\boldsymbol{\alpha}(\sigma\mathbf{C}_2) + \boldsymbol{L}^P_{\mathcal{E}}C_{10} + \boldsymbol{L}^P_{\hat{\tau}}\right]\right]$$

That leaves two equations and two unknowns:

$$0 = \boldsymbol{\Gamma}\left[\boldsymbol{\Psi}\left[\boldsymbol{\alpha}(\sigma\mathbf{C}_2) + \boldsymbol{L}^P_{\mathcal{E}}C_{10} + \boldsymbol{L}^P_{\hat{\tau}}\right]\right] + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}C_{10} + \mathbf{L}^C_{\hat{\tau}}$$
$$0 - \boldsymbol{\Xi}_2\mathbf{C}_2 - \boldsymbol{\Xi}_3\left[\boldsymbol{\Psi}\left[\boldsymbol{\alpha}(\sigma\mathbf{C}_2) + \boldsymbol{L}^P_{\mathcal{E}}C_{10} + \boldsymbol{L}^P_{\hat{\tau}}\right]\right] - \Xi_4 C_{10} - \Xi_5 = 0$$

So we have:

$$0 = \underbrace{\boldsymbol{\Gamma}}_{N\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\sigma\underbrace{\mathbf{C}_2}_{N\times 1} + \underbrace{\boldsymbol{\Gamma}}_{N\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\mathcal{E}}}_{NJ\times 1}C_{10} + \underbrace{\boldsymbol{\Gamma}}_{N\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\hat{\tau}}}_{NJ\times 1} + \underbrace{\tilde{\boldsymbol{L}}^C_{\mathcal{E}}}_{N\times 1}C_{10} + \underbrace{\mathbf{L}^C_{\hat{\tau}}}_{N\times 1}$$

$$0 = -\underbrace{\boldsymbol{\Xi}_2}_{1\times N}\underbrace{\mathbf{C}_2}_{N\times 1} - \underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\sigma\underbrace{\mathbf{C}_2}_{N\times 1} - \underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\mathcal{E}}}_{NJ\times 1}C_{10} - \underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\hat{\tau}}}_{NJ\times 1} - \Xi_4 C_{10} - \Xi_5$$

$$0 = \underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{\alpha}\sigma}_{N\times N}\mathbf{C}_2 + \left(\underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\mathcal{E}}}_{N\times 1} + \underbrace{\tilde{\boldsymbol{L}}^C_{\mathcal{E}}}_{N\times 1}\right)C_{10} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\hat{\tau}}}_{N\times 1} + \underbrace{\mathbf{L}^C_{\hat{\tau}}}_{N\times 1}$$

$$0 = -\left(\underbrace{\boldsymbol{\Xi}_2}_{1\times N} + \sigma\underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\right)\mathbf{C}_2 - \left(\underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\mathcal{E}}}_{NJ\times 1} + \Xi_4\right)C_{10} - \left(\underbrace{\boldsymbol{\Xi}_3}_{1\times NJ}\underbrace{\boldsymbol{\Psi}}_{NJ\times NJ}\underbrace{\boldsymbol{L}^P_{\hat{\tau}}}_{NJ\times 1} + \Xi_5\right)$$

We will use the first equation to solve for C_2:

$$\mathbf{C}_2 = -\left(\underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{\alpha}\sigma}_{N\times N}\right)^{-1}\left(\left(\underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\mathcal{E}}}_{N\times 1} + \underbrace{\tilde{\boldsymbol{L}}^C_{\mathcal{E}}}_{N\times 1}\right)C_{10} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\hat{\tau}}}_{N\times 1} + \underbrace{\mathbf{L}^C_{\hat{\tau}}}_{N\times 1}\right)$$

First expand and group terms:

$$0 = \boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{\alpha}\sigma\mathbf{C}_2 + \left(\boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right)C_{10} + \boldsymbol{\Gamma}\boldsymbol{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}$$

$$0 = -\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\mathbf{C}_2 - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)C_{10} - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \Xi_5\right)$$

Solve the first equation for $\mathbf{C}_2$:

$$\mathbf{C}_2 = -\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right)C_{10} + \mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right)$$

Substitute this into the second equation:

$$0 = -\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(-\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right)C_{10} + \mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right)\right) - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)C_{10} - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}\right.$$

Expand:

$$0 = \left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right)C_{10} + \mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right) - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)C_{10} - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \Xi_5\right.$$

Group terms with C_{10} and constants:

$$0 = \left[\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right) - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)\right]C_{10} + \left[\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}\right.\right.$$

Thus solving for C_{10}:

$$C_{10} = -\frac{\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \mathbf{L}^C_{\hat{\tau}}\right) - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\hat{\tau}} + \Xi_5\right)}{\left(\mathbf{\Xi}_2 + \sigma\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{\alpha}\right)\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{\alpha}\sigma\right)^{-1}\left(\mathbf{\Gamma}\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\right) - \left(\mathbf{\Xi}_3\mathbf{\Psi}\boldsymbol{L}^P_{\mathcal{E}} + \Xi_4\right)}$$

E.2 N=2 J=1

Under flexible prices and permanent tariffs, with a standard Taylor Rule $\pi^C_{H,t} = \pi^C_{F,t} = 0$ and $\hat{V}_t = 0$. Then we have consumption taking on a new permanent value starting from the first period. With that we can solve analytically for the impact of tariffs in a number of different illustrative cases.

E.2.1 Symmetry and Retaliation

Under symmetry, flexible prices and retaliation, if both sides start raising tariffs:

$$\frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} = \frac{\partial \hat{P}^C_{H,t}}{\partial \hat{\tau}_t} = \frac{\partial \hat{P}^C_{F,t}}{\partial \hat{\tau}_t} = 0$$

$$\frac{\partial \hat{C}_{H,t}}{\partial \hat{\tau}_t} = \frac{\partial \hat{C}_{F,t}}{\partial \hat{\tau}_t} = \frac{1}{\sigma}\left[-\frac{\Omega}{1-\Omega} L^P_\tau - \gamma L^C_\tau\right]$$

$$\frac{\partial \hat{P}^P_{H,t}}{\partial \hat{\tau}_t} = \frac{\partial \hat{P}^P_{F,t}}{\partial \hat{\tau}_t} = -\gamma L^C_\tau$$

This case highlights the core intuition. Impact of tariffs is bigger when dependence on imports is high on the consumption and production side. Secondly, the notation allows us to separately see the impact of tariffs on the demand and supply side. While aggregate inflation has to be zero, the impact on producer prices is negative and this is exclusive from the loading of tariffs onto the consumption basket. As is expected under flexible prices, the direct impact is the entirety of the impact, so if there is a 10% tariff placed on all imports, which constitute 10% of the consumption basket, producer prices would decline by %1.

E.2.2 Symmetry and No Retaliation

Under symmetry and no retaliation the exchange rate's response is:

$$C_{10} = \frac{\partial \hat{\mathcal{E}}_t}{\partial \tau_t} = -\frac{\Xi_2(\gamma - \Omega - \Omega\gamma) + \Xi_3(\gamma - \Omega\gamma) + \Xi_5(-1 + \Omega + 2\gamma - 2\Omega\gamma)}{2\Xi_2(\gamma - \Omega - \Omega\gamma) + 2\Xi_3(\gamma - \Omega\gamma) + \Xi_4(-1 + \Omega + 2\gamma - 2\Omega\gamma)}$$

where

$$\Xi_2 = \Xi_{21} = -\frac{2\gamma + \Omega}{\Omega + 1}(1 - \Omega) < 0$$

$$\Xi_{22} = -\Xi_2$$

$$\Xi_3 = \Xi_{31} = \frac{\Omega^2 - 2\Omega\theta - 2\theta\gamma + 4\theta\gamma^2 + \Omega^2 + 4\Omega\theta\gamma - 4\Omega\theta\gamma^2 - 2\Omega^2\theta\gamma}{\Omega + 1}$$

$$= \frac{2\left(\Omega^2 + \theta(\Omega(-\Omega\gamma - 2\gamma^2 + 2\gamma - 1) + 2\gamma^2 - \gamma)\right)}{\Omega + 1}$$

$$\Xi_{32} = -\Xi_3$$

$$\Xi_4 = -\Xi_3$$

$$\Xi_5 = \frac{\Omega^2\gamma + \Omega + \gamma(1 - 2\gamma)}{\Omega + 1}\theta > 0$$

Equivalently we can write:

$$\hat{\mathcal{E}}_t = -\frac{\tau(\Omega\theta + 2\Omega\gamma - 4\Omega\gamma^2 - 2\Omega^2\gamma^2 + \Omega^2 - 2\Omega\theta\gamma)}{D} L^P_\tau - \frac{\tau(\theta\gamma + 2\Omega\gamma^2 - 2\theta\gamma^2 + 2\gamma^2 + 2\Omega\theta\gamma^2 + \Omega^2\theta\gamma)}{D} L^C_\tau$$

where $D = 2\Omega\theta - \Omega + 6\Omega\gamma + 2\theta\gamma - 4\Omega\gamma^2 - 2\Omega^2\gamma^2 - 4\theta\gamma^2 + \Omega^2 + 4\gamma^2 - 4\Omega\theta\gamma + 4\Omega\theta\gamma^2 + 2\Omega^2\theta\gamma$. Rearranging, we find that when $\gamma < 1/2$, the terms multiplying L^P_τ and L^C_τ will be positive.:

$$\hat{\mathcal{E}}_t = -\frac{1}{D}\left[\underbrace{(\theta(1-2\gamma) + \Omega(1-2\gamma^2) + 2\gamma(1-2\gamma))\Omega L^P_\tau}_{>0} + \underbrace{(\theta(1-2\gamma) + 2\Omega\gamma + 2\gamma + 2\Omega\theta\gamma + \Omega^2\theta)\gamma L^C_\tau}_{>0}\right]\hat{\tau}_t$$

Then the denominator will determine the sign of the exchange rate:

$$D = \underbrace{\left[\Omega^2(1-2\gamma^2) + 4\gamma^2 + \Omega 6\gamma\right] + \theta\left[2\Omega + 2\gamma(1-2\Omega) + 4\Omega\gamma^2 + 2\Omega^2\gamma\right]}_{>0} - \underbrace{\left[\theta 4\gamma^2 + \Omega(1+4\gamma^2)\right]}_{>0}$$

There will be appreciation if:

$$\left[\Omega^2(1-2\gamma^2) + 4\gamma^2 + \Omega 6\gamma\right] + \theta\left[2\Omega + 2\gamma(1-2\Omega) + 4\Omega\gamma^2 + 2\Omega^2\gamma\right] > \left[\theta 4\gamma^2 + \Omega(1+4\gamma^2)\right]$$

Let us consider some cases. First, evaluating this at $\theta \to 0$ we find:

$$\underbrace{\Omega^2(1-2\gamma^2) + 4\gamma^2(1-\Omega)}_{>0} + \Omega(6\gamma - 1) > 0$$

Then when $\theta \to 0$, a sufficient condition for appreciation is $\gamma > \frac{1}{6}$. Secondly, if for example we have $\theta \to 0$ and $\gamma \to 0$ then the expression above collapses to

$$\Omega^2 > \Omega$$

This is false since $0 \leq \Omega \leq 1$. That is when both θ and γ are low that can generate depreciation. If however, both, $\Omega \to 0$ and $\theta \to 0$ we have $4\gamma^2 > 0$, which holds true.

F Analytical Solution Under Real Rate Rule

Let us now set $N = 2$ for an arbitrary J and assume that the policy rule in each country follows a real rate rule:

$$\hat{i}_{n,t} = \phi_\pi E_t P^C_{n,t+1}$$

where $\phi_\pi \to 1$. Then the equilibrium conditions read as follows:

$$\begin{aligned}
&\sigma(\mathbb{E}_t \hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}(\hat{\boldsymbol{P}}^C_t - \hat{\boldsymbol{P}}^C_{t-1}) - \mathbb{E}_t(\hat{\boldsymbol{P}}^C_{t+1} - \hat{\boldsymbol{P}}^C_t) \\
&\hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t \\
&\hat{\boldsymbol{P}}^P_t = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^P_{\hat{\tau}}\hat{\tau}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \\
&\mathbb{E}_t\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t = \tilde{\boldsymbol{\Phi}}_3(\hat{\boldsymbol{P}}^C_t - \hat{\boldsymbol{P}}^C_{t-1}) \\
&\beta\hat{V}_t = \hat{V}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\hat{\mathcal{E}}_t + \Xi_5\hat{\tau}_t
\end{aligned}$$

Having a constant real rate rule with a temporary shock, sets the path of consumption at zero ($\hat{\boldsymbol{C}}_t = \mathbf{0}$), which in turn implies a constant real exchange rate. This in turn implies that the exchange rate is $\hat{\mathcal{E}}_t = \hat{P}^C_{H,t} - \hat{P}^C_{F,t} = \underbrace{[1 - 1]}_{\mathbf{Z}}\hat{\boldsymbol{P}}^C_t$.

In light of this rearranging CPI equation:

$$\begin{aligned}
\hat{\boldsymbol{P}}^C_t &= \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z}\hat{\boldsymbol{P}}^C_t + \boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t \\
(\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})\hat{\boldsymbol{P}}^C_t &= \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t \\
\hat{\boldsymbol{P}}^C_t = (\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})^{-1}\boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t &+ (\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})^{-1}\boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t
\end{aligned}$$

Plugging in the CPI equation and these into the NKPC equation yields:

$$\begin{aligned}
\hat{\boldsymbol{P}}^P_t &= \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\left(\boldsymbol{\alpha} + \boldsymbol{L}^P_{\mathcal{E}}\mathbf{Z}\right)\hat{\boldsymbol{P}}^C_t + \boldsymbol{L}^P_{\hat{\tau}}\hat{\tau}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \\
\hat{\boldsymbol{P}}^P_t &= \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\left(\boldsymbol{\alpha} + \boldsymbol{L}^P_{\mathcal{E}}\mathbf{Z}\right)\left((\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})^{-1}\boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + (\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})^{-1}\boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t\right) + \boldsymbol{L}^P_{\hat{\tau}}\hat{\tau}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \\
\hat{\boldsymbol{P}}^P_t &= \tilde{\boldsymbol{\Psi}}_{RR}\left[\hat{\boldsymbol{P}}^P_{t-1} + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1} + \underbrace{\boldsymbol{\Lambda}\left(\left(\boldsymbol{\alpha} + \boldsymbol{L}^P_{\mathcal{E}}\mathbf{Z}\right)(\mathbf{I} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\mathbf{Z})^{-1}\boldsymbol{L}^C_{\hat{\tau}} + \boldsymbol{L}^P_{\hat{\tau}}\right)}_{\mathbf{D}}\hat{\tau}_t\right]
\end{aligned}$$

As we show in Appendix J, a system of the following kind

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}_{RR}\left(\hat{\boldsymbol{P}}_{t-1}^P + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P + \mathbf{D}\tau_t\right)$$
$$\tau_t = \rho\tau_{t-1} + \epsilon_t$$

has the solution:

$$\hat{\boldsymbol{P}}_t^P = \left(\left[(\tilde{\boldsymbol{\Psi}}_{RR}^{-1} - \beta\sqrt{\tilde{\boldsymbol{\Psi}}_{\boldsymbol{RR}}^{\boldsymbol{-1}}}) - \rho\beta\mathbf{I}\right]^{-1}\mathbf{D}\right)\tau_t + \sqrt{\tilde{\boldsymbol{\Psi}}_{\boldsymbol{RR}}^{\boldsymbol{-1}}}\hat{\boldsymbol{P}}_{t-1}^P$$

where the square root operator is defined in Appendix J. This operator diagonalizes the Leontief Inverse, takes the square root of the diagonal entries and then pre and post multiplies with the diagonalizing matrix. With that we can also show the impact on inflation as follows:

$$\frac{\partial\hat{\boldsymbol{P}}_t^C}{\partial\tau_t} = (\mathbf{I} - \tilde{\boldsymbol{L}}_{\mathcal{E}}^C\mathbf{Z})^{-1}\left(\boldsymbol{\Gamma}\left(\left[(\tilde{\boldsymbol{\Psi}}_{RR}^{-1} - \beta\sqrt{\tilde{\boldsymbol{\Psi}}_{\boldsymbol{RR}}^{\boldsymbol{-1}}}) - \rho\beta\mathbf{I}\right]^{-1}\mathbf{D}\right) + \boldsymbol{L}_{\hat{\tau}}^C\right)\hat{\tau}_t$$

F.1 N=2 J=1

Under perfect consumption stabilization with a fixed real rate rule in the two countries we have the following system:

$$\begin{aligned}
\hat{P}_{H,t}^C &= (1-\gamma_H)\hat{P}_{H,t}^P + \gamma_H(\hat{P}_{F,t}^P + \hat{\mathcal{E}}_t + L_\tau^C\hat{\tau}_t)\\
\hat{P}_{F,t}^C &= (1-\gamma_F)\hat{P}_{F,t}^P + \gamma_F(\hat{P}_{H,t}^P - \hat{\mathcal{E}}_t)\\
\pi_{H,t}^P &= \Lambda_H\left(\alpha_H\hat{P}_{H,t}^C + \Omega_H\left(\hat{P}_{F,t}^P + \hat{\mathcal{E}}_t + L_\tau^P\hat{\tau}_t\right) - \hat{P}_{H,t}^P\right) + \beta\mathbb{E}_t\pi_{H,t+1}^P\\
\pi_{F,t}^P &= \Lambda_F\left(\alpha_F\hat{P}_{F,t}^C + \Omega_F\left(\hat{P}_{H,t}^P - \hat{\mathcal{E}}_t\right) - \hat{P}_{F,t}^P\right) + \beta\mathbb{E}_t\pi_{F,t+1}^P\\
\pi_{H,t}^P &= \hat{P}_{H,t}^P - \hat{P}_{H,t-1}^P\\
\pi_{F,t}^P &= \hat{P}_{F,t}^P - \hat{P}_{F,t-1}^P\\
\hat{\mathcal{E}}_t &= \hat{P}_{H,t}^C - \hat{P}_{F,t}^C
\end{aligned}$$

First we plug in the exchange rate into the first two equations:

$$\begin{aligned}
\hat{P}_{H,t}^C &= (1-\gamma_H)\hat{P}_{H,t}^P + \gamma_H\left(\hat{P}_{F,t}^P + (\hat{P}_{H,t}^C - \hat{P}_{F,t}^C) + L_\tau^C\hat{\tau}_t\right)\\
\hat{P}_{F,t}^C &= (1-\gamma_F)\hat{P}_{F,t}^P + \gamma_F\left(\hat{P}_{H,t}^P - (\hat{P}_{H,t}^C - \hat{P}_{F,t}^C)\right)
\end{aligned}$$

Solving this out:

$$\begin{aligned}
\hat{P}^C_{H,t} &= \hat{P}^P_{H,t} + \frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t \\
\hat{P}^C_{F,t} &= \hat{P}^P_{F,t} - \frac{\gamma_F\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t
\end{aligned}$$

Then the nominal exchange rate is:

$$\begin{aligned}
\hat{\mathcal{E}}_t &= \hat{P}^C_{H,t} - \hat{P}^C_{F,t} \\
&= \hat{P}^P_{H,t} - \hat{P}^P_{F,t} + \frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t
\end{aligned}$$

Now let us transform the NKPC equations into levels for the method of undetermined coefficients and also plug these in:

$$\begin{aligned}
\hat{P}^P_{H,t} - \hat{P}^P_{H,t-1} = \Lambda_H \Bigg(&(1-\Omega_H)\left(\hat{P}^P_{H,t} + \frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t\right) \\
&+ \Omega_H \left(\hat{P}^P_{F,t} + \left(\hat{P}^P_{H,t} - \hat{P}^P_{F,t} + \frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t\right) + L^P_\tau \hat{\tau}_t\right) - \hat{P}^P_{H,t}\Bigg) + \beta \mathbb{E}_t \hat{P}^P_{H,t+1} - \beta \hat{P}^P_{H,t} \\
\hat{P}^P_{F,t} - \hat{P}^P_{F,t-1} = \Lambda_F \Bigg(&(1-\Omega_F)\left(\hat{P}^P_{F,t} - \frac{\gamma_F\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t\right) \\
&+ \Omega_F \left(\hat{P}^P_{H,t} - \left(\hat{P}^P_{H,t} - \hat{P}^P_{F,t} + \frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau \hat{\tau}_t\right)\right) - \hat{P}^P_{F,t}\Bigg) + \beta \mathbb{E}_t \hat{P}^P_{F,t+1} - \beta \hat{P}^P_{F,t}
\end{aligned}$$

This yields:

$$\hat{P}^P_{H,t} = \frac{1}{1+\beta}\left[\Lambda_H \hat{\tau}_t \left[(1-\Omega_H)\frac{(1-\gamma_F)\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau + \Omega_H\left(\frac{\gamma_H}{1-\gamma_F-\gamma_H} L^C_\tau + L^P_\tau\right)\right] + \beta \mathbb{E}_t \hat{P}^P_{H,t+1} + \hat{P}^P_{H,t-1}\right]$$

Simplified:

$$\hat{P}^P_{H,t} = \frac{1}{1+\beta}\left[\Lambda_H \left(\frac{\gamma_H\left[1-\gamma_F(1-\Omega_H)\right]}{1-\gamma_F-\gamma_H} L^C_\tau + \Omega_H L^P_\tau\right)\hat{\tau}_t + \beta \mathbb{E}_t \hat{P}^P_{H,t+1} + \hat{P}^P_{H,t-1}\right]$$

This is equal to

$$\hat{P}^P_{H,t} = AD\hat{\tau}_t + A\beta \mathbb{E}_t \hat{P}^P_{H,t+1} + A\hat{P}^P_{H,t-1}$$

Setting up the system for the method of undetermined coefficients:

$$\hat{P}^P_{H,t} = C_1\hat{\tau}_t + C_2\hat{P}^P_{H,t-1}$$
$$E_t\hat{P}^P_{H,t+1} = C_1\rho\hat{\tau}_t + C_2\hat{P}^P_{H,t} = C_1\rho\hat{\tau}_t + C_2(C_1\hat{\tau}_t + C_2\hat{P}^P_{H,t-1}) = (\rho C_1 + C_2C_1)\hat{\tau}_t + C_2^2\hat{P}^P_{H,t-1}$$

Plugging these in:

$$C_1\hat{\tau}_t + C_2\hat{P}^P_{H,t-1} = AD\hat{\tau}_t + A\hat{P}^P_{H,t-1} + A\beta((\rho C_1 + C_2C_1)\hat{\tau}_t + C_2^2\hat{P}^P_{H,t-1})$$
$$[C_1 - AD - A\beta(\rho C_1 + C_1C_2)]\,\hat{\tau}_t + \left[C_2 - A - A\beta C_2^2\right]\hat{P}^P_{H,t-1} = 0$$

Then we have

$$\beta C_2^2 - A^{-1}C_2 + 1 = 0$$
$$\rightarrow C_2 = \frac{A^{-1} \pm \sqrt{(A^{-1})^2 - 4\beta}}{2\beta}$$

Since $A^{-1} = 1 + \beta$

$$\begin{aligned} C_2 &= \frac{1+\beta \pm \sqrt{(1+\beta)^2 - 4\beta}}{2\beta} \\ &= \frac{1+\beta \pm \sqrt{1+2\beta+\beta^2-4\beta}}{2\beta} \\ &= \frac{1+\beta \pm \sqrt{1-2\beta+\beta^2}}{2\beta} \\ &= \frac{1+\beta \pm (\beta-1)}{2\beta} \end{aligned}$$

That is $C_2 \in \left\{1, \frac{1}{\beta}\right\}$. We pick $C_2 = 1$ since that ensures system stability. So with $C_2 = 1$ then:

$$\begin{aligned} C_1 - AD - A\beta(\rho+1)C_1 &= 0 \\ [A^{-1} - \beta(\rho+1)]C_1 &= D \\ [1+\beta-\beta(\rho+1)]C_1 &= D \\ C_1 &= [1-\beta\rho]^{-1}D \end{aligned}$$

Since $D = \Lambda_H \left(\frac{\gamma_H[1-\gamma_F(1-\Omega_H)]}{1-\gamma_F-\gamma_H} L_\tau^C \; + \; \Omega_H L_\tau^P\right)$ we have:

$$\hat{P}^P_{H,t} = \hat{P}^P_{H,t-1} + [1-\beta\rho]^{-1}\Lambda_H \left(\frac{\gamma_H [1-\gamma_F(1-\Omega_H)]}{1-\gamma_F-\gamma_H} L_\tau^C \; + \; \Omega_H L_\tau^P\right)\hat{\tau}_t$$

Then the solution for the foreign price is:

$$\hat{P}^P_{F,t} = \underbrace{\frac{1}{1+\beta}}_{A}\left[\hat{P}^P_{F,t-1} + \beta\,\mathbb{E}_t\hat{P}^P_{F,t+1} - \underbrace{\Lambda_F \left(\frac{\gamma_H [(1-\Omega_F)\gamma_F + \Omega_F]}{1-\gamma_F-\gamma_H} L_\tau^C\right)}_{-D}\hat{\tau}_t\right]$$

Then the solution for the foreign price is:

$$\hat{P}^P_{F,t} = \hat{P}^P_{F,t-1} - [1-\beta\rho]^{-1}\Lambda_F \left(\frac{\gamma_H [(1-\Omega_F)\gamma_F + \Omega_F]}{1-\gamma_F-\gamma_H} L_\tau^C\right)\hat{\tau}_t$$

F.1.1 Small Open Economy Special Case with J=1

SOE assumption sets $\gamma_F = \Omega_F = 0$,:

$$\begin{aligned}
\hat{P}^P_{H,t} &= \hat{P}^P_{H,t-1} + [1-\beta\rho]^{-1}\Lambda_H \left(\frac{\gamma_H}{1-\gamma_H} L_\tau^C + \Omega_H L_\tau^P\right)\hat{\tau}_t \\
\hat{P}^C_{F,t} &= \hat{P}^P_{F,t} = \hat{P}^P_{F,t} = \hat{P}^P_{F,t-1} = 0 \\
\hat{\mathcal{E}}_t &= \hat{P}^C_{H,t} = \hat{P}^P_{H,t} + \frac{\gamma_H}{1-\gamma_H} L_\tau^C \hat{\tau}_t
\end{aligned}$$

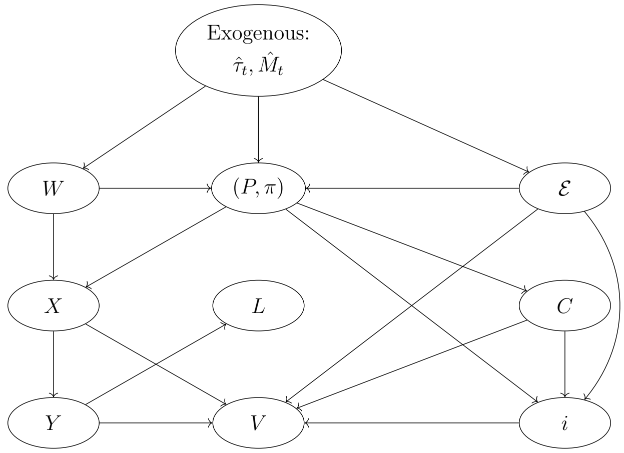
Or put differently:

$$\frac{\partial \hat{P}^C_{H,t}}{\partial \hat{\tau}_t} = \frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} = \left(\frac{\gamma_H}{1-\gamma_H}\left([1-\beta\rho]^{-1}\Lambda_H + 1\right)\right) L_\tau^C + [1-\beta\rho]^{-1}\Lambda_H \cdot \Omega_H L_\tau^P$$

G Analytical Solution with Fixed Nominal Demand

The nominal demand assumption allows us to break cyclical relationships in the system and as shown in the DAG representation below, one can solve for all endogenous quantities starting from $\hat{\tau}_t$ and $\hat{M}_t$.

Figure G.1. Directed Acyclic Graph (DAG) Representation of the Simplified Equilibrium



With the simplifying assumptions introduced for this section, the Backus Smith condition can be written and transformed as follows:

$$\begin{aligned} E_t \Delta \hat{Q}_{t+1} &= \sigma \left(E_t \Delta \hat{C}_{n,t+1} - E_t \Delta \hat{C}_{m,t+1} \right) \\ E_t \Delta \hat{\mathcal{E}}_{n,m,t+1} &= E_t \left(\Delta \hat{M}_{n,t+1} - \Delta \hat{M}_{m,t+1} \right) \\ \hat{\mathcal{E}}_{n,m,t} &= \overline{\mathcal{E}}_{n,m} + E_t \left[\sum_{j=0}^{\infty} -\Delta \hat{M}_{n,t+j+1} + \Delta \hat{M}_{m,t+j+1} \right] \end{aligned}$$

where $\hat{Q}_t$ is the real exchange rate. We consider transitory shocks. Additionally, we make the assumption that portfolio adjustment costs are strictly positive; however, numerically small that we omit them in our notation. The fact that PAC is strictly positive, implies that in response to the type of one-time shocks that we are interested it will be the case that $\overline{\mathcal{E}}_{n,m} = \lim_{t\to\infty} \hat{\mathcal{E}}_{n,m,t} \approx 0$. To that end, let us assume $M_{n,t+j} = M_{m,t+j} = 0 \; \forall \; j > 0$. Then, we have $\mathcal{E}_{n,m,t} = \hat{M}_{n,t} - \hat{M}_{m,t}$. That is, in the simplified version of the model with fixed nominal demand, nominal demand policy determines the path of nominal exchange rates. The intuitive interpretation of the expression above is that excessively stimulating demand (i.e., printing too much money) leads to depreciation, consistent with models of monetary exchange rate determination.

Having solved for the nominal exchange rate, we now turn to the Phillips Curve under fixed nominal demand. As detailed in Appendix G, in vector and matrix notation, we obtain:[40]

[40] As described in Appendix G, we construct an $NJ \times 1$ dimensional vector $\hat{\boldsymbol{M}}_t$ by stacking each country's nominal demand change such that $\hat{\boldsymbol{M}}_{ni} = \hat{M}_n$.

$$\boldsymbol{\pi}_t^P = \boldsymbol{\Lambda}\left((\boldsymbol{\Omega} - \boldsymbol{I})\hat{\boldsymbol{P}}_t^P + (\boldsymbol{I} - \boldsymbol{\Omega})\,\hat{\boldsymbol{M}}_{\boldsymbol{t}} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}\right) + \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1}^P \tag{G.1}$$

The term $(\boldsymbol{I} - \boldsymbol{\Omega})$ in front of $\hat{\boldsymbol{M}}_{\boldsymbol{t}}$ consists of the summation of two components: (i) $\boldsymbol{\alpha}$, which arises from the demand channel via an increase in wages, and (ii) $(\boldsymbol{I} - \boldsymbol{\alpha} - \boldsymbol{\Omega})$, which originates from the exchange rate channel that raises input prices.

An intuitive way to interpret (G.1) is to first examine the flexible price case, where marginal cost equals price:

$$\begin{aligned}\hat{\boldsymbol{\pi}}_t^P &= \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}}_{\text{Leontief Inverse}} \quad \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{Policy Impact}\\ \text{via Wages and ER}}} \quad \hat{\boldsymbol{M}}_t + (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\text{Tariff Incidence}} - \hat{\boldsymbol{P}}_{t-1}^P \\ &= \hat{\boldsymbol{M}}_t + (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} - \hat{\boldsymbol{P}}_{t-1}^P \end{aligned} \tag{G.2}$$

Equation (G.2) illustrates the impact on inflation under flexible prices. Nominal domestic demand policy affects producer price inflation through two channels: first, via the demand channel, and second, via the exchange rate channel. Since the labor-leisure tradeoff simplifies to $\hat{W}_t - \hat{P}_t = \hat{C}_t$ under the given parametrization, and since nominal wages depend on $\hat{M}_t$, stimulative demand policy increases labor supply. Through the exchange rate channel, stimulating domestic demand beyond its steady-state level results in depreciation, which raises firms' marginal costs by increasing the price of imported intermediate inputs.

Returning to the Rotemberg pricing case with the forward-looking NKPC in Equation (G.1), we simplify and define the stickiness-adjusted Leontief inverse for the producer price inflation equation as $\tilde{\boldsymbol{\Psi}}_\pi = [\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I})]^{-1}$, arriving at the global NKPC for producer price inflation:[41]

$$\boldsymbol{\pi}_t^P = \underbrace{\tilde{\boldsymbol{\Psi}}_\pi \boldsymbol{\Lambda}}_{\substack{\text{Propagation under}\\ \text{stickiness}}} \left[\underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})}_{\substack{\text{Policy impact}\\ \text{via Wages and ER}}} \hat{\boldsymbol{M}}_t + \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\text{Tariff incidence}} - \underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{P}_{t-1}^P}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \underbrace{\beta \boldsymbol{\Lambda}^{-1}\mathbb{E}_t \boldsymbol{\pi}_{t+1}^P}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right] \tag{G.3}$$

Applying the method of undetermined coefficients to (G.3) we arrive at Proposition 3.

Corollary 10. *The impact of a one-time tariff on the producer price inflation vector under price stickiness is:*

$$\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t} = \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE}}_{\substack{\textit{NKOE Leontief}\\ \textit{inverse}}} \underbrace{\boldsymbol{\Lambda}}_{\textit{Stickiness}} \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\substack{\textit{Tariff}\\ \textit{incidence}}}$$

where $\tilde{\boldsymbol{\Omega}}^F$ *is a* $NJ \times 1$ *vector whose elements are the row sum of the foreign elements of* $\boldsymbol{\Omega}$*.*

We can compare this with the impact under flexible prices:

$$\frac{\partial \boldsymbol{\pi}_t^{P,flex}}{\partial \tau_t} = \underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}}_{\boldsymbol{\Psi}=\text{Leontief inverse}} \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\text{Tariff incidence}} \tag{G.4}$$

Two points are noteworthy here. Firstly, since aggregate nominal demand—and consequently the exchange rate—is determined by policy, tariffs have no impact through the nominal exchange rate in this setup. However, the real exchange rate and the terms of trade do depend on tariffs. Secondly, the flexible-price expression captures a significant portion of the intuition. Under price stickiness, it is the propagation mechanism that changes, which is not surprising.

Proposition 7. *The impact of a one-time tariff* ($\tau_t \geq 0$) *on the producer price inflation is always weakly positive in the long run. That is let* $\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t}$ *be an* $NJ \times 1$ *vector, denoted as* $\boldsymbol{\pi}_\tau^P$*, such that* $\boldsymbol{\pi}_\tau^P \geq \mathbf{0}$*.*

Proof. Since the flexible-price equilibrium is the long run equilibrium, it would suffice to

[41]For intuition, in the closed-economy analogy, there is no exchange rate impact, and tariffs would act as a cost-push shock, with $\hat{\boldsymbol{M}}_t$ capturing NGDP.

$$\boldsymbol{\pi}_t^P = \underbrace{\tilde{\boldsymbol{\Psi}}_\pi \boldsymbol{\Lambda}}_{\substack{\text{Propagation under}\\ \text{stickiness}}} \left[\underbrace{\boldsymbol{\alpha}}_{\substack{\text{Policy impact}\\ \text{via Wages}}} \hat{Y}_t + \underbrace{\boldsymbol{\Omega}\boldsymbol{\mu}_t}_{\substack{\text{Impact of}\\ \text{cost-push shock}}} + \underbrace{\boldsymbol{\Omega}\boldsymbol{P}_{t-1}^P}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \underbrace{\beta \boldsymbol{\Lambda}^{-1}\mathbb{E}_t \boldsymbol{\pi}_{t+1}^P}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right]$$

work with (G.4). We can express the matrix $(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}$ as the following Neumann series:

$$(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}=\sum_{k=0}^{\infty}\boldsymbol{\Omega}^{k}.$$

Each power $\boldsymbol{\Omega}^k$ has nonnegative entries, implying that $(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}$ also has nonnegative entries. The term $\tilde{\boldsymbol{\Omega}}^F$ also retains nonnegative entries. Since $(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}$ is an $NJ \times NJ$ matrix with nonnegative entries and $\tilde{\boldsymbol{\Omega}}^F$ is an $NJ \times 1$ vector with nonnegative entries, their product is an $NJ \times 1$ vector with nonnegative entries. Thus, every entry of $\boldsymbol{\pi}_t^{P,flex}$ is weakly positive.

□

With the NKPC describing producer price inflation, we next define consumer price inflation as follows. Aggregate consumption price indices in all countries are a linear combination of granular consumption prices, which in turn depend on producer prices, the exchange rate, and τ_t. Then, [42]

$$\hat{\boldsymbol{P}}_t^C=\boldsymbol{\Gamma}\cdot\boldsymbol{P}_{n,t}^P+\underbrace{[\boldsymbol{\Gamma}\odot\boldsymbol{\mathcal{E}}_t]\mathbf{1}}_{\tilde{\boldsymbol{L}}_{\mathcal{E}}^C\hat{\boldsymbol{M}}_t}+\underbrace{[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\boldsymbol{L}_{\tau}^C\tilde{\boldsymbol{\tau}}_t} \tag{G.5}$$

where $\boldsymbol{\Gamma}$ captures the share of each good i from country m in country n's consumption basket. Applying Lemma 1 from the Appendix, we can express

$$[\boldsymbol{\Gamma}\odot\boldsymbol{\mathcal{E}}_t]\mathbf{1}=(\boldsymbol{I}-\boldsymbol{\Gamma})\hat{\boldsymbol{M}}_t=\tilde{\boldsymbol{L}}_{\mathcal{E}}^C\hat{\boldsymbol{M}}_t.$$

Then, consumer price inflation can be written as:

$$\boldsymbol{\pi}_t^C=\Delta\hat{\boldsymbol{P}}_t^C=\boldsymbol{\Gamma}\cdot\boldsymbol{\pi}_t^P+\tilde{\boldsymbol{L}}_{\mathcal{E}}^C\Delta\hat{\boldsymbol{M}}_t+\boldsymbol{L}_{\tau}^C\Delta\tilde{\boldsymbol{\tau}}_t \tag{G.6}$$

For simplicity, assuming lagged values are zero, i.e., $\hat{M}_{t-1}=\tau_{t-1}=0$ (meaning the shock occurs at $t=0$ and the economy was previously at steady state), and substituting the expression for producer price inflation from Proposition 3, we arrive at a solution for consumer price inflation. This solution maps lagged prices, policy, and tariffs to the consumer

[42] As described in Appendix G, we construct an $NJ \times NJ$ dimensional matrix $\boldsymbol{\Gamma}$ and an $NJ \times 1$ dimensional consumer price vector by stacking each country's consumer demand matrix and consumer price vector.

price inflation vector:

$$\boldsymbol{\pi}_t^C = \left(\underbrace{\boldsymbol{\Gamma}\,\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}}_{\substack{\text{NKPC}\\\text{propagation}}} \quad \underbrace{(\boldsymbol{I}-\boldsymbol{\Omega})}_{\substack{\text{via Wages and}\\\text{via ER for producers}}} + \underbrace{(\boldsymbol{I}-\boldsymbol{\Gamma})}_{\text{via ER for consumers}} \right) \hat{\boldsymbol{M}}_t$$
$$+ \left(\underbrace{\boldsymbol{\Gamma}\,\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}}_{\substack{\text{NKPC}\\\text{propagation}}} \quad \underbrace{[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence}\\\text{for Producers}}} + \underbrace{[\boldsymbol{\Gamma}\odot\hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence}\\\text{for consumers}}} \right) \mathbf{1}$$
$$+ \underbrace{\boldsymbol{\Gamma}\left(\tilde{\boldsymbol{\Psi}}^{NKOE} - \boldsymbol{I}\right)\hat{\boldsymbol{P}}_{t-1}^{P}}_{\text{Impact of lagged prices}} \tag{G.7}$$

As seen above in Equation (G.7), policy and tariffs affect consumer price inflation through two channels: first, via producer prices, and second, through the exchange rate and tariffs that convert a producer price into a consumer price. A helpful interpretation of the expression above is that the terms labeled "NKPC Propagation" illustrate how the production network propagates shocks in a forward-looking setup, whereas the other terms represent the first-order impacts. For example, when a $\tau_t\%$ tariff is imposed, these terms capture what share of the consumption basket is affected, considering both its indirect effect through producers' input baskets and its direct effect on consumers' consumption baskets.

Corollary 11. *Under flexible prices (efficient allocation), impact of tariffs on consumer prices consists of the following direct effects through the consumption basket and producer's input basket:*

$$\frac{\partial \boldsymbol{\pi}_t^{C^{flex}}}{\partial \tau_t} = \boldsymbol{L}_\tau^C + \boldsymbol{\Gamma\Psi}\boldsymbol{L}_\tau^P \tag{G.8}$$

and the difference between Equation (43) *and Equation* (44) *yields the allocative efficiency term.*

G.1 NKPC

Recalling producer inflation:

$$\pi_{ni,t}^p = \frac{\theta_l}{\delta_{ni}} \left(\alpha_{ni} \underbrace{\hat{W}_{n,t}}_{\hat{M}_{n,t}} + \sum_{m\in N}\sum_{j\in J} \Omega_{ni,mj}(\hat{P}_{mj,t}^p + \underbrace{\hat{\mathcal{E}}_{n,m,t}}_{\hat{M}_{n,t}-\hat{M}_{m,t}} + \tau_{n,mj,t}) - \hat{P}_{ni,t}^p \right) + \beta_n \mathbb{E}_t \pi_{ni,t+1}^p$$

Then in vector and matrix notation we have:

$$\underbrace{\boldsymbol{\pi}_t^P}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ}\Bigg(\underbrace{\boldsymbol{\alpha}}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{M}}_t}_{NJ\times 1} + \underbrace{(\boldsymbol{\Omega}-\boldsymbol{I})}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\underbrace{[\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\mathcal{E}}}_t]}_{NJ\times NJ}\underbrace{\mathbf{1}}_{NJ\times 1}}$$

$$+ \underbrace{[\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\tau}}_t]}_{NJ\times NJ}\underbrace{\mathbf{1}}_{NJ\times 1}\Bigg) + \underbrace{\boldsymbol{\beta}}_{NJ\times NJ}\mathbb{E}_t\underbrace{\boldsymbol{\pi}_{t+1}^P}_{NJ\times 1},$$

where α is the diagonal matrix whose non-zero elements are the labor-shares (i.e., α_{ni}) and $\hat{\boldsymbol{M}}_t$ is a $NJ \times 1$ vector such that $\hat{\boldsymbol{M}}_{ni,t} = \hat{M}_{n,t}$. We will use the following Lemma to simplify the equations.

Lemma 1. *Given $\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t} = \hat{\boldsymbol{M}}_{ni,t} - \hat{\boldsymbol{M}}_{mj,t}$, we can write:*

$$[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} = (\boldsymbol{I} - \boldsymbol{\alpha} - \boldsymbol{\Omega})\hat{\boldsymbol{M}}_t$$

The proof follows from calculating each element:

$$\sum_{mj}\Omega_{ni,mj}\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t} = \sum_{mj}\Omega_{ni,mj}(\hat{\boldsymbol{M}}_{ni,t} - \hat{\boldsymbol{M}}_{mj,t}) = \hat{\boldsymbol{M}}_{ni,t}\underbrace{\sum_{mj}\Omega_{ni,mj}}_{1-\alpha_{ni}} - \sum_{mj}\Omega_{ni,mj}\hat{\boldsymbol{M}}_{mj,t}.$$

Lemma 2. *We can write:*

$$[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} = \breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t,$$

where $\breve{\boldsymbol{\Omega}}$ is a $NJ \times (N \times NJ)$ block diagonal matrix with:

$$\breve{\boldsymbol{\Omega}}_{ni,l*(NJ-1)+mj} = \begin{cases} \boldsymbol{\Omega}_{ni,mj} & \textit{if} \quad l = n \\ 0 & \textit{otherwise} \end{cases}$$

and $\breve{\boldsymbol{\tau}}_t$ is a $(N \times NJ) \times 1$ vector whose elements are given by $\breve{\boldsymbol{\tau}}_{n*(NJ-1)+mj,t} = \hat{\tau}_{n,mj,t}$.

This equality can be seen easily by calculating the summations. Therefore, we can write the producer inflation as:

$$\boldsymbol{\pi}_t^P = \boldsymbol{\Lambda}\Bigg([\boldsymbol{\Omega} - \boldsymbol{I}]\hat{\boldsymbol{P}}_t^P + [\boldsymbol{I} - \boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t + \breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t\Bigg) + \boldsymbol{\beta}\mathbb{E}_t\boldsymbol{\pi}_{t+1}^P.$$

G.2 Method of Undetermined Coefficients

Rewriting (G.3) purely in terms of the price level as follows, we can solve it analytically:

$$\begin{aligned}
\boldsymbol{\pi}_t^P &= \boldsymbol{\Lambda}\left([\boldsymbol{\Omega}-\boldsymbol{I}]\hat{\boldsymbol{P}}_t^P + [\boldsymbol{I}-\Omega]\hat{\boldsymbol{M}}_{\boldsymbol{t}} + \breve{\Omega}\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{\beta}\mathbb{E}_t\boldsymbol{\pi}_{t+1}^P \\
(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P) &= \boldsymbol{\Lambda}\left([\boldsymbol{\Omega}-\boldsymbol{I}]\hat{\boldsymbol{P}}_t^P + [\boldsymbol{I}-\Omega]\hat{\boldsymbol{M}}_{\boldsymbol{t}} + \breve{\Omega}\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{\beta}\mathbb{E}_t\left(\hat{\boldsymbol{P}}_{t+1}^P - \hat{\boldsymbol{P}}_t^P\right) \\
(\boldsymbol{I}+\boldsymbol{\beta}-\boldsymbol{\Lambda}[\boldsymbol{\Omega}-\boldsymbol{I}])\,\hat{\boldsymbol{P}}_t^P &= \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P + \boldsymbol{\Lambda}\left([\boldsymbol{I}-\Omega]\hat{\boldsymbol{M}}_t + \breve{\Omega}\breve{\boldsymbol{\tau}}_t\right)
\end{aligned}$$

Then:

$$\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \underbrace{(\boldsymbol{I}+\boldsymbol{\beta}+\boldsymbol{\Lambda}-\boldsymbol{\Lambda}\boldsymbol{\Omega})^{-1}}_{\tilde{\boldsymbol{\Psi}}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P + \underbrace{\boldsymbol{\Lambda}[\boldsymbol{I}-\Omega]}_{\boldsymbol{A}}\hat{\boldsymbol{M}}_t + \underbrace{\boldsymbol{\Lambda}\breve{\Omega}}_{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t\right] \\
\boldsymbol{P}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{M}}_t + \tilde{\boldsymbol{\Psi}}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t + \tilde{\boldsymbol{\Psi}}\hat{\boldsymbol{P}}_{t-1}^P + \tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P)
\end{aligned}$$

We next do a manipulation to find a system where the matrix on the lagged vector is diagonal. To do so we diagonalize $\tilde{\boldsymbol{\Psi}}$. Defining:[43]

$$\begin{aligned}
\tilde{\boldsymbol{\Psi}} &= \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1} \\
\tilde{\boldsymbol{P}}_t^P &= \boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_t^P \\
\tilde{\boldsymbol{A}} &= \boldsymbol{Q}^{-1}\boldsymbol{A} \\
\tilde{\boldsymbol{B}} &= \boldsymbol{Q}^{-1}\boldsymbol{B} \\
\tilde{\boldsymbol{\beta}} &= \boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}
\end{aligned}$$

Multiplying both sides on the left by $\boldsymbol{Q}^{-1}$ we have:

$$\begin{aligned}
\boldsymbol{P}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{M}}_t + \tilde{\boldsymbol{\Psi}}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t + \tilde{\boldsymbol{\Psi}}\hat{\boldsymbol{P}}_{t-1}^P + \tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P) \\
\boldsymbol{Q}^{-1}\boldsymbol{P}_t^P &= \boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{A}\hat{\boldsymbol{M}}_t + \boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t + \boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}\boldsymbol{Q}^{-1}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P) \\
\tilde{\boldsymbol{P}}_t^P &= \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}}\hat{\boldsymbol{M}}_t + \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{P}}_{t-1}^P + \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}(\mathbb{E}_t\tilde{\boldsymbol{P}}_{t+1}^P)
\end{aligned}$$

Now we have the coefficient on the lag and forward price vector being diagonal, which will

[43] it is important to note that $\tilde{\boldsymbol{\Psi}}$ is almost diagonal to begin with. Hence, in an approximation sense this step might not be needed.

come in handy. We can next postulate:

$$\begin{aligned}\tilde{\boldsymbol{P}}_t^P &= \boldsymbol{C}_1\hat{\boldsymbol{M}}_t + \boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t + \boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P \\ \mathbb{E}\tilde{\boldsymbol{P}}_{t+1}^P &= \boldsymbol{C}_3\tilde{\boldsymbol{P}}_t^P = \boldsymbol{C}_3\boldsymbol{C}_1\hat{\boldsymbol{M}}_t + \boldsymbol{C}_3\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t + \boldsymbol{C}_3\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P\end{aligned}$$

Plugging these into the expression above:

$$\boldsymbol{C}_1\hat{\boldsymbol{M}}_t + \boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t + \boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P = \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}}\hat{\boldsymbol{M}}_t + \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{P}}_{t-1}^P + \tilde{\boldsymbol{\beta}}\breve{\boldsymbol{\Psi}}\left(\boldsymbol{C}_3\boldsymbol{C}_1\hat{\boldsymbol{M}}_t + \boldsymbol{C}_3\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t + \boldsymbol{C}_3\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P\right)$$
$$\left(\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_1\right)\hat{\boldsymbol{M}}_t + \left(\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}} - \tilde{\boldsymbol{\beta}}\breve{\boldsymbol{\Psi}}\boldsymbol{C}_3\boldsymbol{C}_2\right)\breve{\boldsymbol{\tau}}_t + \left(\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_3\right)\tilde{\boldsymbol{P}}_{t-1}^P = \boldsymbol{0}$$

We have a system of three matrix equations and three unknown matrices ($\boldsymbol{C}_1$,$\boldsymbol{C}_2$,$\boldsymbol{C}_3$):

$$\begin{aligned}\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_1 = \boldsymbol{0} &\to \boldsymbol{C}_1 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}} \\ \boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_2 = \boldsymbol{0} &\to \boldsymbol{C}_2 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}} \\ \boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_3 = \boldsymbol{0} &\to \boldsymbol{C}_3 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\end{aligned}$$

Hence, we can solve $\boldsymbol{C}_3$ and then plug it into other coefficients.

$$\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3^2 - \boldsymbol{C}_3 + \breve{\boldsymbol{\Psi}} = \boldsymbol{0}$$

If we assume all discount factors are the same for the countries, i.e., $\beta_n = \beta \quad \forall n$, $\tilde{\boldsymbol{\beta}}$ becomes $\beta\boldsymbol{I}$. Hence, $\boldsymbol{C}_3$ will be diagonal like $\breve{\boldsymbol{\Psi}}$, so we can solve for its diagonal elements explicitly. Since $\boldsymbol{C}_3$ and $\breve{\boldsymbol{\Psi}}$ are diagonal, let their ith diagonal elements be $C_{3,i}$ and $\breve{\Psi}_i$, respectively. The quadratic equation for each diagonal element is:

$$\beta\breve{\Psi}_i C_{3,i}^2 - C_{3,i} + \breve{\Psi}_i = 0$$

Solving for $C_{3,i}$:

$$C_{3,i} = \frac{1 \pm \sqrt{1 - 4\beta\breve{\Psi}_i^2}}{2\beta\breve{\Psi}_i}$$

Since $\boldsymbol{C}_3$ is diagonal, it is constructed as:

$$\boldsymbol{C}_3 = \mathrm{diag}(C_{3,1}, C_{3,2}, \ldots, C_{3,n})$$

where each $C_{3,i}$ is obtained from the quadratic solution above. Given stability requirements, we select the root that satisfies $|C_{3,i}| \leq 1$, ensuring the process does not diverge.[44]

In effect $\tilde{\boldsymbol{\Psi}}$ is already almost diagonal so $\tilde{\boldsymbol{\Psi}}$ is numerically very close to being the identity matrix. For that reason, going forward we will simplify away the tilde notation.

$$\begin{aligned}\boldsymbol{C}_1 &= \boldsymbol{C}_3\tilde{\boldsymbol{A}} = \boldsymbol{C}_3\boldsymbol{Q}^{-1}\boldsymbol{A}\\ \boldsymbol{C}_2 &= \boldsymbol{C}_3\tilde{\boldsymbol{\Lambda}} = \boldsymbol{C}_3\boldsymbol{Q}^{-1}\boldsymbol{\Lambda}\end{aligned}$$

Let us call the matrix that is constructed to transform $\boldsymbol{C}_3$ back to the industry coordinates with $\boldsymbol{\rho} = \boldsymbol{Q}\boldsymbol{C}_3\boldsymbol{Q}^{-1}$. Thus, substituting these into our expression for $\boldsymbol{P}_t^P$:

$$\boldsymbol{P}_t^P = \boldsymbol{\rho}(\boldsymbol{A}\hat{\boldsymbol{M}}_t + \boldsymbol{\Lambda}\breve{\boldsymbol{\tau}}_t + \boldsymbol{P}_{t-1}^P)$$

Substituting for $\boldsymbol{A}$, and $\boldsymbol{B}$ and subtracting $\boldsymbol{P}_{t-1}^P$ from both sides:

$$\begin{aligned}\boldsymbol{\pi}_t^P &= \boldsymbol{\rho}\boldsymbol{\Lambda}[\boldsymbol{I} - \boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t + \boldsymbol{\rho}\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t + (\boldsymbol{\rho} - \boldsymbol{I})\boldsymbol{P}_{t-1}^P\\ &= \boldsymbol{\rho}\boldsymbol{\Lambda}[\boldsymbol{I} - \boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t + \boldsymbol{\rho}\boldsymbol{\Lambda}[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\boldsymbol{1} + (\boldsymbol{\rho} - \boldsymbol{I})\boldsymbol{P}_{t-1}^P\end{aligned}$$

Similar to Blanchard-Kahn conditions we need the solution that ensures all the eigenvalues of $\boldsymbol{\rho}$ are inside the unit circle.

With this expression, we can quantify the effect of a tariff by country n to sector j in country m on producer prices globally as:

$$\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_{n,mi,t}} = \boldsymbol{\rho}\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}\boldsymbol{e}_{n,mi}$$

where $\boldsymbol{e}_{n,mi}$ is the basis vector whose $[(n-1) \times NJ + (m-1) \times J + i]^{\text{th}}$ entry is 1 and all other entries are 0.

Now let's assume that the countries increase their tariffs with the same amount $\hat{\tau}_{n,mi,t} = \hat{\tau}$ with $\hat{\tau}_{n,ni,t} = 0, \quad \forall n, ni, mi$, since there are no tariffs domestically. With these assumptions,

$$\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t = \tilde{\boldsymbol{\Omega}}^F\hat{\tau}$$

where $\tilde{\boldsymbol{\Omega}}^F$ is an $NJ \times 1$ dimensional vector that represent the foreign weight in the inputs, respectively. Hence, the impact of a one-time tariff on the producer price inflation vector

[44] We allow for the price *level* can have persistence in the long-run; hence the weak inequality.

under price stickiness is:

$$\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t} = \boldsymbol{\rho}\boldsymbol{\Lambda}\tilde{\boldsymbol{\Omega}}^F$$

where $\tilde{\boldsymbol{\Omega}}^F$ is a $NJ \times 1$ vector whose elements are the row sum of the foreign elements of $\boldsymbol{\Omega}$.

H Analytical Solution under $\phi_\pi \to 1$

H.1 Forwarding the Euler Equation

Plug in the Taylor Rule and assume $\sigma = 1$, we have:

$$\hat{C}_{n,t} = E_t\hat{C}_{nt+1} - (\phi_\pi^n \pi_{n,t} - E_t\pi_{n,t+1})$$

Forwarding this we can write today's consumption as the sum of future expected real rates, which in turn can be expressed in terms of inflation differentials, under the assumption that $\lim_{t\to\infty} \hat{C}_{n,t} = 0$:

$$\hat{C}_{n,t} = -E_t \sum_{j=0}^{\infty} [\phi_\pi^n \pi_{n,t+j} - \pi_{n,t+j+1}] = -\phi_\pi^n \pi_{n,t} + (1-\phi_\pi^n) E_t \sum_{j=1}^{\infty} \pi_{n,t+j}$$

Taking the limit of $\phi_\pi \to 1$:

$$\hat{C}_{n,t} = -\pi_{n,t} \tag{H.1}$$

Our simulations confirm that Equation (H.1) is identical to the standard Euler equation as $\phi_\pi \to 1$. The intuition is that as inflation rises, central bank will raise rates (and even if it only infinitesimally raises the real rate) that will reduce consumption. More broadly we are deriving an aggregate demand curve that is downward sloping in inflation and can be written as a contemporaneous equation.

This is similar in spirit to fixing nominal demand with $M_{n,t} = P_{n,t}C_{n,t}$; however, this allows for there to be fluctuation in both the nominal and real exchange rates. In general this setup makes it easier to see the feedback loop from prices to demand as opposed to approaches that fix consumption and make it almost exogenous.

In our analytical work instead of taking the limit to 1, we will assume $\phi_\pi \approx 1$ such that we write (H.1) as follows:

$$\hat{C}_{n,t} \approx -\phi_\pi \pi_{n,t}$$

Numerically this serves as an accurate approximation when $\phi_\pi \approx 1$ and when the shocks at hand are transitory.

H.2 Solving the Exchange Rate

Simplifying away the stationarity inducing device of portfolio adjustment costs, the UIP condition is:

$$\hat{i}_{n,t} - \hat{i}_{m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} - \hat{\mathcal{E}}_{n,m,t}$$

Rearranging:

$$\hat{\mathcal{E}}_{n,m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} - (\hat{i}_{n,t} - \hat{i}_{m,t})$$

Plugging in policy rule:

$$\hat{\mathcal{E}}_{n,m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} + \phi_\pi(\pi_{m,t} - \pi_{n,t})$$

Forwarding:

$$\hat{\mathcal{E}}_{n,m,t} = \overline{\mathcal{E}}_{n,m} + \ \phi_\pi E_t \left[\sum_{j=0}^{\infty} (\pi_{m,t+j} - \pi_{n,t+j}) \right]$$

where $\overline{\mathcal{E}}_{n,m} = \lim_{t\to\infty} \hat{\mathcal{E}}_{n,m,t}$ (i.e., we allow for the nominal exchange rate to settle at a permanently different level after shocks as opposed to requiring all nominal variables to return to steady state- real variables will do so).

Defining the real exchange rate between countries and its first difference:

$$\hat{Q}_{m,n,t} = \hat{P}_{m,t} + \hat{\mathcal{E}}_{n,m,t} - \hat{P}_{n,t} \tag{H.2}$$

$$\Delta\hat{Q}_{m,n,t} = \pi_{m,t} + \Delta\hat{\mathcal{E}}_{n,m,t} - \pi_{n,t} \tag{H.3}$$

Recalling the Backus Smith condition:

$$\sigma \left(E_t \Delta \hat{C}_{n,t+1} - E_t \Delta \hat{C}_{m,t+1} \right) = E_t \Delta \hat{Q}_{n,m,t+1}$$

Plugging in $\sigma = 1$, $\hat{C}_{n,t} = -\pi_{n,t}$ and $\hat{C}_{m,t} = -\pi_{m,t}$:

$$E_t \Delta \hat{Q}_{n,m,t+1} = \phi_\pi (\pi_{n,t} - E_t \pi_{n,t+1} - \pi_{m,t} + E_t \pi_{m,t+1}) \tag{H.4}$$

Rewriting (H.4):

$$\begin{aligned} E_t \hat{Q}_{n,m,t+1} - \hat{Q}_{n,m,t} &= \phi_\pi (\pi_{n,t} - E_t \pi_{n,t+1} - \pi_{m,t} + E_t \pi_{m,t+1}) \\ \hat{Q}_{n,m,t} &= E_t \hat{Q}_{n,m,t+1} + \phi_\pi (E_t \pi_{n,t+1} - \pi_{n,t}) - \phi_\pi (E_t \pi_{m,t+1} - \pi_{m,t}) \end{aligned}$$

Forwarding the previous equation yields:

$$\hat{Q}_{n,m,t} = \phi_\pi E_t \left[\sum_{j=0}^{\infty} (\pi_{n,t+j+1} - \pi_{n,t+j}) - (\pi_{m,t+j+1} - \pi_{m,t+j}) \right]$$

since under steady state stability long-run real variables will return to zero; that is $\lim_{t \to \infty} \hat{Q}_{n,m,t} = 0$. Everything other than initial inflation appears twice so it cancels out:

$$\hat{Q}_{n,m,t} = \phi_\pi (\pi_{m,t} - \pi_{n,t}) \tag{H.5}$$

Using the definition of the real exchange rate in (H.2):

$$\hat{Q}_{n,m,t} = \hat{P}_{m,t} + \hat{\mathcal{E}}_{m,n,t} - \hat{P}_{n,t} = \phi_\pi (\pi_{m,t} - \pi_{n,t}) \tag{H.6}$$

$$\hat{P}_{m,t} + \hat{\mathcal{E}}_{m,n,t} - \hat{P}_{n,t} = (\hat{P}_{m,t} - \hat{P}_{m,t-1}) - (\hat{P}_{n,t} - \hat{P}_{n,t-1}) \tag{H.7}$$

$$\hat{\mathcal{E}}_{m,n,t} = \hat{P}_{n,t-1} - \hat{P}_{m,t-1} \tag{H.8}$$

Equations (H.5) and (H.8) pin down the nominal and real exchange rates under the assumption that $\phi_\pi^n = \phi_\pi^m \to 1$. Similar to the approach above, in our analytical work instead of fully taking the limit to $\phi_\pi \to 1$, we assume $\phi_\pi \approx 1$.

H.3 Method of Undetermined Coefficients

Recall that:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left(\boldsymbol{\alpha} \left(\hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t \right) + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t] \mathbf{1} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t] \mathbf{1} \right) + \boldsymbol{\beta} \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$$

Note that $\hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t = \hat{\boldsymbol{P}}_t^C - \boldsymbol{\pi}_t = \hat{\boldsymbol{P}}_{t-1}^C$. Therefore we can write the equation of motion for the price indices as:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\boldsymbol{\alpha}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{\Lambda}[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + \boldsymbol{\Lambda}[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1} + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{H.9}$$

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\cdot\hat{\boldsymbol{P}}_t^P + [\boldsymbol{\Gamma}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Gamma}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1} \tag{H.10}$$

Using Lemmas 1 and 2, and using Equation H.8 above, we can write:

$$\begin{aligned}
[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} &= (\boldsymbol{I}-\boldsymbol{\alpha}-\boldsymbol{\Omega})\hat{\boldsymbol{P}}_{t-1}^C \\
[\boldsymbol{\Gamma}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} &= (\boldsymbol{I}-\boldsymbol{\Gamma})\hat{\boldsymbol{P}}_{t-1}^C \\
\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t &= \breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t \\
\boldsymbol{\Gamma}\odot\hat{\boldsymbol{\tau}}_t &= \breve{\boldsymbol{\Gamma}}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

Then we can write:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega})}_{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}}_{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{H.11}$$

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\cdot\hat{\boldsymbol{P}}_t^P + \underbrace{(\boldsymbol{I}-\boldsymbol{\Gamma})}_{\boldsymbol{D}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Gamma}}\breve{\boldsymbol{\tau}}_t \tag{H.12}$$

That is we have:

$$\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{P}_{t-1}^P + \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{P}}_{t-1}^C + \tilde{\boldsymbol{\Psi}}\boldsymbol{B}\tau_t + \tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\boldsymbol{P}_{t+1}^P) \\
\hat{\boldsymbol{P}}_t^C &= \boldsymbol{\Gamma}\boldsymbol{P}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Gamma}}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

We will now diagonalize $\tilde{\boldsymbol{\Psi}} = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$. We then define:

$$\begin{aligned}
\breve{\boldsymbol{P}}_t^P &= \boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_t^P \\
\breve{\boldsymbol{A}} &= \boldsymbol{Q}^{-1}\boldsymbol{A} \\
\breve{\boldsymbol{B}} &= \boldsymbol{Q}^{-1}\boldsymbol{B} \\
\breve{\boldsymbol{\beta}} &= \boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}
\end{aligned}$$

So now the system is

$$\breve{\boldsymbol{P}}_t^P = \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{P}}_{t-1}^P + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}(\mathbb{E}_t\breve{\boldsymbol{P}}_{t+1}^P)$$

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\boldsymbol{Q}\breve{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Gamma}}\breve{\boldsymbol{\tau}}_t$$

Let us now postulate:

$$\begin{aligned}
\breve{\boldsymbol{P}}_t^P &= \underbrace{\boldsymbol{C}_1}_{NJ\times NJ}\breve{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{C}_2}_{NJ\times NJ}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{C}_3}_{NJ\times N^2J}\breve{\boldsymbol{\tau}}_t \\
\hat{\boldsymbol{P}}_t^C &= \underbrace{\boldsymbol{C}_4}_{NJ\times NJ}\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{C}_5}_{NJ\times NJ}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{C}_6}_{NJ\times N^2J}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

Iterating the first equation forward and taking expectation at time t, under the assumption that the tariff is a one-time shock:

$$\begin{aligned}
E_t\breve{\boldsymbol{P}}_{t+1}^P &= \boldsymbol{C}_1\breve{\boldsymbol{P}}_t^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_t^C \\
&= \boldsymbol{C}_1\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{C}_2\left(\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t\right)
\end{aligned}$$

Plugging these into the two original equations:

$$\begin{aligned}
\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) &= \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{P}}_{t-1}^P + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t \\
&+ \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\Bigg(\boldsymbol{C}_1\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) \\
&+ \boldsymbol{C}_2\left(\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t\right)\Bigg) \\
\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t &= \boldsymbol{\Gamma}\boldsymbol{Q}\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Gamma}}\tau_t
\end{aligned}$$

Expanding and grouping terms:

$$\begin{aligned}
&\left(\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_4\boldsymbol{Q}\right)\breve{\boldsymbol{P}}_{t-1}^P + \left(\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_5\right)\hat{\boldsymbol{P}}_{t-1}^C \\
&+ \left(\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_6\right)\breve{\boldsymbol{\tau}}_t = 0
\end{aligned}$$

And:

$$\left(\boldsymbol{C}_4\boldsymbol{Q} - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_1\right)\breve{\boldsymbol{P}}_{t-1}^P + \left(\boldsymbol{C}_5 - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_2 - \boldsymbol{D}\right)\hat{\boldsymbol{P}}_{t-1}^C + \left(\boldsymbol{C}_6 - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Gamma}}\right)\breve{\boldsymbol{\tau}}_t = 0$$

This yields a system of 6 (matrix) equations and 6 unknowns:

$$\begin{aligned}
\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_4\boldsymbol{Q} = 0 \\
\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_5 = 0
\end{aligned}$$

$$
\begin{aligned}
&\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_6 = 0\\
&\boldsymbol{C}_4\boldsymbol{Q} - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_1 = 0\\
&\boldsymbol{C}_5 - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_2 - \boldsymbol{D} = 0\\
&\boldsymbol{C}_6 - \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Gamma}} = 0
\end{aligned}
$$

Dependent Blocks

$$
\begin{aligned}
\mathbf{C}_4 &= \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_1\boldsymbol{Q}^{-1},\\
\mathbf{C}_5 &= \boldsymbol{D} + \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_2,\\
\mathbf{C}_6 &= \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_3 + \breve{\boldsymbol{\Gamma}},
\end{aligned}
$$

Core Fixed-Point Equations

$$
\begin{aligned}
&\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_1 = 0\\
&\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_2 = 0\\
&\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Gamma}} = 0
\end{aligned}
$$

After multiplying on the left by $\breve{\boldsymbol{\Psi}}^{-}1$, the first equation can be rewritten as:

$$
\begin{aligned}
&\underbrace{\left(\breve{\boldsymbol{\Psi}}^{-1} - \breve{\boldsymbol{\beta}}\mathbf{C}_1 - \breve{\boldsymbol{\beta}}\mathbf{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}\right)}_{=\mathbf{C}_1^{-1}}\mathbf{C}_1 = \boldsymbol{I}\\
&\breve{\boldsymbol{\Psi}}\mathbf{C}_1^{-1} = \boldsymbol{I} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\mathbf{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\mathbf{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}
\end{aligned}
$$

Plugging this expression into the second and third equations gives us:

$$
\begin{aligned}
&\breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D} = \breve{\boldsymbol{\Psi}}\boldsymbol{C}_1^{-1}\boldsymbol{C}_2 \Rightarrow \boldsymbol{C}_2 = \boldsymbol{C}_1(\breve{\boldsymbol{A}} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D})\\
&\breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Gamma}} = \breve{\boldsymbol{\Psi}}\boldsymbol{C}_1^{-1}\boldsymbol{C}_3 \Rightarrow \boldsymbol{C}_3 = \boldsymbol{C}_1(\breve{\boldsymbol{B}} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Gamma}})
\end{aligned}
$$

Hence, $\boldsymbol{C}_3$ can be written as a function of $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. So we need to solve for these two matrices.

We can rewrite the first equation:

$$
\breve{\boldsymbol{\beta}}\mathbf{C}_1^2 - (\breve{\boldsymbol{\Psi}}^{-1} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Gamma}\boldsymbol{Q})\mathbf{C}_1 + \boldsymbol{I} = \mathbf{0}
$$

This expression, along with the expression for $\boldsymbol{C}_2$ can be numerically solved. Here we will make two simplifying assumptions to arrive at an analytical expression. First, we assume all discount factors are the same for the countries, i.e., $\beta_n = \beta \quad \forall n$, $\tilde{\boldsymbol{\beta}}$ becomes $\beta \boldsymbol{I}$. Second, we will ignore the term $\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Gamma}\boldsymbol{Q}$ since this term is relatively small number numerically. With these simplifying assumptuons, we can now solve for $\mathbf{C}_1$ with the quadratic formula. We wish to solve for the diagonal matrix $\mathbf{C}_1$ in

$$\beta\, \mathbf{C}_1^2 - \breve{\boldsymbol{\Psi}}^{-1}\mathbf{C}_1 + \mathbf{I} = \mathbf{0},$$

assuming

$$\mathbf{C}_1 = \operatorname{diag}(c_1, c_2, \ldots, c_n) \quad \text{and} \quad \breve{\boldsymbol{\Psi}} = \operatorname{diag}(\psi_1, \psi_2, \ldots, \psi_n).$$

For each i, the i-th diagonal element satisfies

$$\beta\, c_i^2 - \frac{1}{\psi_i} c_i + 1 = 0.$$

Dividing by β yields

$$c_i^2 - \frac{1}{\beta\psi_i}\, c_i + \frac{1}{\beta} = 0.$$

Applying the quadratic formula gives

$$c_i = \frac{\frac{1}{\psi_i} \pm \sqrt{\frac{1}{\psi_i^2} - 4\beta}}{2\beta}$$

With $\mathbf{C}_1$ is close to $\overline{\rho}\mathbf{I}$, where $\overline{\rho}$ is the average of the elements in the diagonal, we can now solve for $\mathbf{C}_2$

$$\mathbf{C}_2 = \overline{\rho}\breve{\boldsymbol{A}}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}$$

Finally, $\mathbf{C}_3$ is given by:

$$\boldsymbol{C}_3 = \overline{\rho}\breve{\boldsymbol{B}} + \beta\overline{\rho}\breve{\boldsymbol{A}}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}\breve{\boldsymbol{\Gamma}}$$

With these we can now return to $\boldsymbol{C}_6$, our object of interest which captures the impact of tariffs on consumer price inflation.

$$\begin{aligned} \mathbf{C}_6 &= \boldsymbol{\Gamma}\boldsymbol{Q}\mathbf{C}_3 + \breve{\boldsymbol{\Gamma}} \\ &= \boldsymbol{\Gamma}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}} + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}\breve{\boldsymbol{\Gamma}}\right) + \breve{\boldsymbol{\Gamma}} \end{aligned}$$

where we used $\boldsymbol{B} = \boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}$.

With this expression, we can quantify the effect of a tariff by country n to sector j in country m on producer prices globally as:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_{n,mi,t}} = \boldsymbol{\Gamma Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}} + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}\breve{\boldsymbol{\Gamma}}\right)\boldsymbol{e}_{n,mi} + \breve{\boldsymbol{\Gamma}}\boldsymbol{e}_{n,mi}$$

where $\boldsymbol{e}_{n,mi}$ is the basis vector whose $[(n-1)\times NJ + (m-1)\times J + i]^{\text{th}}$ entry is 1 and all other entries are 0.

If we assume that the countries increase their tariffs with the same amount $\hat{\tau}_{n,mi,t} = \hat{\tau}$ and $\hat{\tau}_{n,ni,t} = 0, \quad \forall n, ni, mi$. The second equation specifies that there are no tariffs domestically. With these assumptions,

$$\breve{\boldsymbol{\Gamma}}\breve{\boldsymbol{\tau}}_t = \tilde{\boldsymbol{\Gamma}}^F\hat{\tau}$$
$$\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t = \tilde{\boldsymbol{\Omega}}^F\hat{\tau}$$

where $\tilde{\boldsymbol{\Gamma}}^F$ and $\tilde{\boldsymbol{\Omega}}^F$ are $NJ \times 1$ dimensional vectors that represent the foreign weight in the final consumption and the inputs, respectively. Hence:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \boldsymbol{\Gamma Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\tilde{\boldsymbol{\Omega}}^F + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}\tilde{\boldsymbol{\Gamma}}^F\right) + \tilde{\boldsymbol{\Gamma}}^F$$

where $\boldsymbol{Q}$ comes from the diagonalization of the stickiness-adjusted Leontief inverse: $\tilde{\boldsymbol{\Psi}} = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$. Let us call this $\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1} = \boldsymbol{\Psi}^{NKOE}$, indicating that this is now the New Keynesian Open Economy Leontief inverse (taking the stickiness adjusted Leontief inverse to NKOE setting with expectations). Let us now define loadings:

$$\boldsymbol{A} = \boldsymbol{\Lambda}(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)$$
$$\boldsymbol{B} = \boldsymbol{\Lambda}\boldsymbol{L}_\tau^P$$
$$\overline{\rho}(\mathbf{I} - \beta\overline{\rho}\boldsymbol{D})^{-1} = \boldsymbol{L}_{\mathcal{E}}^C$$
$$\boldsymbol{F} = \boldsymbol{L}_\tau^C$$

Then:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\beta}\boldsymbol{\Psi}^{NKOE}\boldsymbol{\Lambda}(\boldsymbol{L}_\tau^P + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_\tau^C) + \boldsymbol{L}_\tau^C$$

H.4 Generalizing the Result: Two Country Case

If $\phi_\pi \to 1$ is not the case, in the general case only the loadings change. This is because $\hat{W} - \hat{P}_t^C = -\hat{P}_t^C + \phi_\pi \hat{P}_t^C$ and the exchange rate is more generally

$$\hat{E}_t = \overline{E} + \phi_\pi \hat{P}_{t-1}^C - \phi_\pi^* \hat{P}_{t-1}^{*C}$$

We know both from numerical simulations and similar models that the $\overline{E}$ will be a function of the real debt position. Since it is linearly separable and the quantitative impact is small when the elasticities of substitution are small (i.e. below 1 indicating goods are complements on the production side), we will momentarily ignore it in the following section.

That is in vector form, in the two-country case we have $\hat{\mathbf{W}}_t = \mathbf{\Phi}\hat{\boldsymbol{P}}_{t-1}^C$ and $\hat{\mathcal{E}}_t \approx \tilde{\mathbf{\Phi}}\hat{\boldsymbol{P}}_{t-1}^C$ where $\tilde{\mathbf{\Phi}} = \begin{bmatrix}1 & -1\end{bmatrix}\mathbf{\Phi}$. With $\begin{bmatrix}1 & -1\end{bmatrix}$ already defined within the loading, this means all that changes is:

$$\boldsymbol{A} = \mathbf{\Lambda}(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\mathbf{\Phi}$$

Then:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\beta}\mathbf{\Psi}^{NKOE}\mathbf{\Lambda}(\boldsymbol{L}_\tau^P + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\mathbf{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_\tau^C) + \boldsymbol{L}_\tau^C$$

H.4.1 Impact of Policy

$$\hat{\boldsymbol{P}}_t^P = \underbrace{(\boldsymbol{I}(1+\beta) + \mathbf{\Lambda}(\boldsymbol{I} - \mathbf{\Omega}))^{-1}}_{\tilde{\mathbf{\Psi}}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \mathbf{\Lambda}\left(\boldsymbol{\alpha}\underbrace{\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right)}_{\hat{\boldsymbol{W}}_t} + [\mathbf{\Omega}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\mathbf{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1}\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

where $\tilde{\mathbf{\Psi}}$ is a stickiness-adjusted Leontief Inverse. Let us plug in our approximation of the Euler equation:

$$\hat{\boldsymbol{C}}_t = -\mathbf{\Phi}(\boldsymbol{P}_t^C - \boldsymbol{P}_{t-1}^C)$$

which implies under $\sigma = 1$:

$$\hat{\boldsymbol{W}}_t = \hat{\mathbf{P}}_t^C + \hat{\boldsymbol{C}}_t = (\mathbf{I} - \mathbf{\Phi})\hat{\mathbf{P}}_t^C - \mathbf{\Phi}\boldsymbol{P}_{t-1}^C$$

We also have in vector form, in the two-country case $\hat{\mathbf{W}}_t = \mathbf{\Phi}\hat{\boldsymbol{P}}_{t-1}^C$. Plugging this into the NKPC:

$$(\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega}))\hat{\boldsymbol{P}}_t^P=\left[\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left((\mathbf{I}-\boldsymbol{\Phi})\hat{\mathbf{P}}_t^C-\boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C\right)+\mathbf{L}_{\hat{\mathcal{E}}}^P\hat{\mathcal{E}}_t+\mathbf{L}_\tau^P\tau_t\right)+\beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Next we substitute out consumer prices, using $\hat{\boldsymbol{P}}_t^C=\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C+\boldsymbol{L}_\tau^C\tau_t$ and the exchange rate given $\hat{\mathcal{E}}_t\approx\tilde{\boldsymbol{\Phi}}\hat{\boldsymbol{P}}_{t-1}^C$ where $\tilde{\boldsymbol{\Phi}}=\begin{bmatrix}1 & -1\end{bmatrix}\boldsymbol{\Phi}$. With $\begin{bmatrix}1 & -1\end{bmatrix}$ already defined within the loading:

$$(\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega}))\hat{\boldsymbol{P}}_t^P=$$
$$\left[\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{\Lambda}\left(\boldsymbol{L}_C^P\left((\mathbf{I}-\boldsymbol{\Phi})\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C+\boldsymbol{L}_\tau^C\tau_t\right)-\boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C\right)+\mathbf{L}_{\hat{\mathcal{E}}}^P\hat{\boldsymbol{P}}_{t-1}^C+\mathbf{L}_\tau^P\tau_t\right)+\beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Grouping terms and rearranging:

$$\hat{\boldsymbol{P}}_t^P=\underbrace{\left[\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}\big[\boldsymbol{I}-\boldsymbol{\Omega}+\boldsymbol{L}_C^P(\boldsymbol{\Phi}-\boldsymbol{I})\boldsymbol{\Gamma}\big]\right]^{-1}}_{\tilde{\boldsymbol{\Psi}}_\phi}$$
$$\left[\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{\Lambda}\left[\big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D}+\mathbf{L}_{\hat{\mathcal{E}}}^P-\boldsymbol{L}_C^P\big]\boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C+\big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C+\mathbf{L}_\tau^P\big]\tau_t\right]+\beta\mathbb{E}t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Going back to earlier solution we have:

$$\frac{\partial\hat{\boldsymbol{P}}_t^C}{\partial\hat{\tau}_t}=\boldsymbol{\Gamma}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\tilde{\boldsymbol{\Omega}}^F+\beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\tilde{\boldsymbol{\Gamma}}^F\right)+\tilde{\boldsymbol{\Gamma}}^F$$

Or alternatively:

$$\frac{\partial\hat{\boldsymbol{P}}_t^C}{\partial\tau_t}=\boldsymbol{\Gamma}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\boldsymbol{B}+\beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\boldsymbol{F}\right)+\boldsymbol{F}$$

Let us now define loadings (keeping in mind that the cross term $\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D}\approx 0$ due to home bias, so for narrative simplicity we'll omit it in the expression below):

$$\begin{aligned}\boldsymbol{A}&=\big[\boldsymbol{L}_C^P+\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D}+\mathbf{L}_{\hat{\mathcal{E}}}^P\big]\boldsymbol{\Phi}\rightarrow\boldsymbol{\Lambda}(\boldsymbol{L}_C^P+\boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi}\\ \boldsymbol{B}&=\big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C+\mathbf{L}_\tau^P\big]\\ \overline{\rho}(\mathbf{I}-\beta\overline{\rho}\boldsymbol{D})^{-1}&=\boldsymbol{L}_{\mathcal{E}}^C\end{aligned}$$

$$\boldsymbol{F} = \boldsymbol{L}_{\tau}^{C}$$

Then:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Gamma}\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE}\boldsymbol{\Lambda}\left[\boldsymbol{L}_{\tau}^{P} + \Big(\boldsymbol{L}_{C}^{P}(\mathbf{I}-\boldsymbol{\Phi}) + \beta(\boldsymbol{L}_{C}^{P} + \boldsymbol{L}_{\mathcal{E}}^{P})\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^{C}\Big)\boldsymbol{L}_{\tau}^{C}\right] + \boldsymbol{L}_{\tau}^{C}$$

H.5 Examples

H.5.1 Case 1: N=1,J=1, standard NK model

We can begin by comparing how the model and its solution to the three-equation canonical New Keynesian model recopied below. For simplicity, let us have demand shocks given by ϵ_t^d and supply (cost-push) shocks given by μ_t:

$$\begin{aligned}
\sigma(\mathbb{E}_t\hat{Y}_{t+1} - \hat{Y}_t) &= \hat{i}_t - \mathbb{E}_t\pi_{t+1} + \epsilon^d \\
\pi_t &= \kappa\hat{Y}_t + \mu_t + \beta\mathbb{E}_t\pi_{t+1}^P \\
\hat{i}_t &= \phi_\pi\pi_t
\end{aligned}$$

The standard solution in this model for π_t, when the shocks in question are one-time shocks, reads as follows:

$$\pi_t = \frac{\sigma}{\kappa\phi_\pi + \sigma}\mu_t + \frac{\kappa}{\kappa\phi_\pi + \sigma}\epsilon_t^d \tag{H.13}$$

We can reduce our model to the scalar case, by setting $N = 1$ and $J = 1$ to compare our solution to the standard one. Relative to the general case with N countries and J industries, the exchange rate drops out and τ_t on the production side is isomorphic to a cost-push shock. Additionally, lagged prices disappear. In a closed economy there would not be tariffs. However, to see the analogy and the intuition here we can treat $\epsilon_t^d = L_\tau^C\tau_t$ as a demand shock as a wedge between producer prices and consumer prices would be isomorphic to one (i.e. the loading in this analogy would be different as we show below). $\kappa = \Lambda L_C^P$ would be the slope of the NKPC and let $\mu_t = \Lambda L_\tau^P\tau_t$ be a cost-push shock. Written with the notation we developed, with the Taylor rule plugged in, and keeping $\sigma = 1, \psi = 0$ we would have:

$$\mathbb{E}_t\hat{Y}_{t+1} - \hat{Y}_t = \underbrace{\phi_\pi\pi_t}_{\hat{i}_t} - \mathbb{E}_t\pi_{t+1} + \underbrace{L_\tau^C\tau_t}_{\epsilon_t^d}$$

$$\pi_t = \underbrace{\Lambda L_C^P}_{\kappa} \hat{Y}_t + \underbrace{\Lambda L_\tau^P \tau_t}_{\mu_t} + \beta \mathbb{E}_t \pi_{t+1}^P$$

Plugging in the parameters into the standard solution in (H.13) we find:

$$\pi_t = \frac{\Lambda}{1 + \phi_\pi \Lambda L_C^P} \left[L_\tau^P + L_C^P L_\tau^C \right] \tau_t \tag{H.14}$$

After performing an adjustment for the fact that our model's solution was derived in a setup with lags, this would be the same as the solution in (48).

H.5.2 Case 2: N=2, J=1, no intermediate inputs

This set up is similar to the one solved by Monacelli (2025). Here, I-O matrix is a matrix of zeros, i.e., $\boldsymbol{\Omega} = \mathbf{0}$. Then:

$$(\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda})\hat{\boldsymbol{P}}_t^P = \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left((\mathbf{I} - \boldsymbol{\Phi}) \hat{\mathbf{P}}_t^C - \boldsymbol{\Phi} \boldsymbol{P}_{t-1}^C \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$$

Next we substitute out consumer prices, using

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{D} \hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C \tau_t$$

we arrive at:

$$(\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda})\hat{\boldsymbol{P}}_t^P = \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left((\mathbf{I} - \boldsymbol{\Phi}) \left(\boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{D} \hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C \tau_t \right) - \boldsymbol{\Phi} \boldsymbol{P}_{t-1}^C \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$$

Grouping terms and rearranging:

$$\hat{\boldsymbol{P}}_t^P = \underbrace{\left[\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda} \big[\mathbf{I} + (\boldsymbol{\Phi} - \boldsymbol{I}) \boldsymbol{\Gamma} \big] \right]^{-1}}_{\tilde{\boldsymbol{\Psi}}_{\boldsymbol{\phi}}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left[\big[(\mathbf{I} - \boldsymbol{\Phi})(\mathbf{I} - \boldsymbol{\Gamma}) \big] \boldsymbol{\Phi} \boldsymbol{P}_{t-1}^C + (\mathbf{I} - \boldsymbol{\Phi}) \boldsymbol{L}_\tau^C \tau_t + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right] \right]$$

Let's assume the matrices are defined as:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \qquad \boldsymbol{\Phi} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix}, \qquad \boldsymbol{\Gamma} = \begin{bmatrix} \xi_1 & (1-\xi_1) \\ (1-\xi_2) & \xi_2 \end{bmatrix}$$

with ξ_1 and ξ_2 capturing the domestic consumption bias of home and foreign, respectively. Then $\tilde{\Psi}_\phi$ is given by:

$$\tilde{\boldsymbol{\Psi}}_\phi = \Bigg[\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda}\big[\mathbf{I} + (\boldsymbol{\Phi} - \boldsymbol{I})\boldsymbol{\Gamma}\big] \Bigg]^{-1} = \frac{1}{\Delta} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

where

$$A = 1+\beta+\lambda_1(1+\xi_1(\phi_1-1)), \quad B = \lambda_1(1-\xi_1)(\phi_1-1)$$
$$C = \lambda_2(1-\xi_2)(\phi_2-1), \quad D = 1+\beta+\lambda_2(1+\xi_2(\phi_2-1))$$
$$\Delta = (1+\beta+\lambda_1(1+\xi_1(\phi_1-1)))(1+\beta+\lambda_2(1+\xi_2(\phi_2-1))) - \lambda_1\lambda_2(1-\xi_1)(\phi_1-1)(1-\xi_2)(\phi_2-1)$$

Let's assume symmetric countries with $\phi_1 = \phi_2 = \phi$, $\lambda_1 = \lambda_2 = \lambda$ and $\xi_1 = \xi_2 = \xi$. Then the expression simplifies to:

$$\tilde{\boldsymbol{\Psi}}_\phi = \frac{1}{\Delta} \begin{bmatrix} 1+\beta+\lambda(1+\xi(\phi-1)) & -\lambda(1-\xi)(\phi-1) \\ -\lambda(1-\xi)(\phi-1) & 1+\beta+\lambda(1+\xi(\phi-1)) \end{bmatrix}$$

where

$$\Delta = (1+\beta)^2 + 2(1+\beta)\lambda(1+\xi(\phi-1)) + 4\lambda^2\xi(\phi-1)$$

If we do the eigendecomposition of $\tilde{\boldsymbol{\Psi}}_\phi$ such that $\tilde{\boldsymbol{\Psi}}_\phi = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$, then:

$$\breve{\boldsymbol{\Psi}} = \begin{bmatrix} \tilde{\psi}_1 & 0 \\ 0 & \tilde{\psi}_2 \end{bmatrix}, \qquad \boldsymbol{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad \boldsymbol{Q}^{-1} = \boldsymbol{Q}^T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

where the eigenvalues $\tilde{\psi}_1$ and $\tilde{\psi}_2$ are given by:

$$\tilde{\psi}_1 = \frac{1+\beta+\lambda(1+\xi(\phi-1)) - \lambda(1-\xi)(\phi-1)}{\Delta},$$
$$\tilde{\psi}_2 = \frac{1+\beta+\lambda(1+\xi(\phi-1)) + \lambda(1-\xi)(\phi-1)}{\Delta}.$$

Now, we can solve for: $\beta\,\mathbf{C}_1^2 - \breve{\mathbf{\Psi}}^{-1}\mathbf{C}_1 + \mathbf{I} = \mathbf{0}$:

$$\mathbf{C}_1 = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}, \qquad c_1 = \frac{\frac{1}{\tilde{\psi}_1} \pm \sqrt{\left(\frac{1}{\tilde{\psi}_1}\right)^2 - 4\beta}}{2\beta}, \qquad c_2 = \frac{\frac{1}{\tilde{\psi}_2} \pm \sqrt{\left(\frac{1}{\tilde{\psi}_2}\right)^2 - 4\beta}}{2\beta}.$$

Then:

$$\tilde{\mathbf{\Psi}}_\phi^{NKOE} \approx \boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1} = \frac{1}{2}\begin{bmatrix} c_1 + c_2 & c_1 - c_2 \\ c_1 - c_2 & c_1 + c_2 \end{bmatrix}$$

Going back to earlier solution we have:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \beta\mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\overline{\rho}\mathbf{\Lambda}(\mathbf{I} - \beta\overline{\rho}(\boldsymbol{I} - \mathbf{\Gamma}))^{-1}\tilde{\mathbf{\Gamma}}^F\mathbf{1} + \tilde{\mathbf{\Gamma}}^F\mathbf{1}$$

where $\overline{\rho} = (c_1 + c_2)/2$ and $\tilde{\mathbf{\Gamma}}^F\mathbf{1} = [1-\xi, 1-\xi]^T$. Hence:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \beta(1-\xi)\mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\overline{\rho}\mathbf{\Lambda}(\mathbf{I} - \beta\overline{\rho}(\boldsymbol{I} - \mathbf{\Gamma}))^{-1}\mathbf{1} + (1-\xi)\mathbf{1},$$

where we resize $\mathbf{1}$ vector to $N \times 1$ dimensions.

I Decomposing the Impact on Inflation

Starting with Equation (48) we can write:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\left[\boldsymbol{L}_\tau^P + \left(\boldsymbol{L}_C^P(\mathbf{I} - \mathbf{\Phi}) + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_\mathcal{E}^P)\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\right)\boldsymbol{L}_\tau^C\right] + \boldsymbol{L}_\tau^C$$

Rearranging:

$$\begin{aligned}
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_\tau^P + \mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_C^P(\mathbf{I} - \mathbf{\Phi})\boldsymbol{L}_\tau^C \\
&+ \beta\mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_C^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\mathbf{\Gamma}\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_\mathcal{E}^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^C \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \mathbf{\Gamma}\Big(\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_\tau^P + \tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_C^P(\mathbf{I} - \mathbf{\Phi})\boldsymbol{L}_\tau^C \\
&+ \beta\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_C^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}\boldsymbol{L}_\mathcal{E}^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C\Big) + \boldsymbol{L}_\tau^C \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \mathbf{\Gamma}\left(\underbrace{\boldsymbol{L}_\tau^P + \boldsymbol{L}_C^P(\mathbf{I} - \mathbf{\Phi})\boldsymbol{L}_\tau^C + \beta\boldsymbol{L}_C^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\boldsymbol{L}_\mathcal{E}^P\mathbf{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C}_{\mathbf{Z}} + (\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda} - I)\mathbf{Z}\right)
\end{aligned}$$

$$
\begin{aligned}
&+ \boldsymbol{L}_{\tau}^{C} \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \boldsymbol{\Gamma} \boldsymbol{L}_{\tau}^{P} + \boldsymbol{\Gamma} \boldsymbol{L}_{C}^{P} (\mathbf{I} - \boldsymbol{\Phi}) \boldsymbol{L}_{\tau}^{C} + \beta \boldsymbol{\Gamma} \boldsymbol{L}_{C}^{P} \boldsymbol{\Phi} \boldsymbol{L}_{\mathcal{E}}^{C} \boldsymbol{L}_{\tau}^{C} + \beta \boldsymbol{\Gamma} \boldsymbol{L}_{\mathcal{E}}^{P} \boldsymbol{\Phi} \boldsymbol{L}_{\mathcal{E}}^{C} \boldsymbol{L}_{\tau}^{C} + \boldsymbol{L}_{\tau}^{C} \\
&+ \boldsymbol{\Gamma} (\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \boldsymbol{\Lambda} - I) \mathbf{Z}
\end{aligned}
$$

This is the desired decomposition:

$$
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \underbrace{\boldsymbol{\Gamma} \boldsymbol{L}_{\tau}^{P}}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma} \boldsymbol{L}_{C}^{P} (\mathbf{I} - \boldsymbol{\Phi}) \boldsymbol{L}_{\tau}^{C}}_{\text{Demand channel}} + \underbrace{\beta \boldsymbol{\Gamma} \boldsymbol{L}_{C}^{P} \boldsymbol{\Phi} \boldsymbol{L}_{\mathcal{E}}^{C} \boldsymbol{L}_{\tau}^{C}}_{\text{Expected demand channel}} + \underbrace{\beta \boldsymbol{\Gamma} \boldsymbol{L}_{\mathcal{E}}^{P} \boldsymbol{\Phi} \boldsymbol{L}_{\mathcal{E}}^{C} \boldsymbol{L}_{\tau}^{C}}_{\text{Expected ER channel}} + \underbrace{\boldsymbol{L}_{\tau}^{C}}_{\text{Direct CPI effect}}
+ \underbrace{\boldsymbol{\Gamma} (\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \boldsymbol{\Lambda} - \mathbf{I}) \mathbf{Z}}_{\text{Propagation}}
$$

J General Solution for Price Vector

Initial step:

$$
\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \boldsymbol{\Psi} \left(\mathbf{A} \hat{\boldsymbol{P}}_{t-1}^P + \mathbf{D} \hat{\boldsymbol{P}}_t^P + \mathbf{B} \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P + \mathbf{C}_t \tau_t \right) \\
\tau_t &= \rho \tau_{t-1} + \epsilon_t
\end{aligned}
$$

In a broad class of cases the whole system can be collapsed into a single endogenous vector $\hat{\mathbf{P}}_t^P$ which is a function of its own lag, expectation and an exogenous marginal cost shock variable that has an AR(1) process. Assume the system is:

$$
\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \boldsymbol{\Psi} \left(\hat{\boldsymbol{P}}_{t-1}^P + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P + \mathbf{D} \tau_t \right) \\
\tau_t &= \rho \tau_{t-1} + \epsilon_t
\end{aligned}
$$

Then we can hypothesize:

$$
\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \mathbf{C}_1 \tau_t + \mathbf{C}_2 \hat{\boldsymbol{P}}_{t-1}^P \\
E_t \hat{\boldsymbol{P}}_{t+1}^P &= \mathbf{C}_1 \rho^{\tau} \tau_t + \mathbf{C}_2 \hat{\boldsymbol{P}}_t^P = \mathbf{C}_1 \rho^{\tau} \tau_t + \mathbf{C}_2 (\mathbf{C}_1 \tau_t + \mathbf{C}_2 \hat{\boldsymbol{P}}_{t-1}^P) = (\rho \mathbf{C}_1 + \mathbf{C}_2 \mathbf{C}_1) \tau_t + \mathbf{C}_2 \mathbf{C}_2 \hat{\boldsymbol{P}}_{t-1}^P
\end{aligned}
$$

Method of undetermined coefficients system is:

$$
\begin{aligned}
\mathbf{C}_1 \tau_t + \mathbf{C}_2 \hat{\boldsymbol{P}}_{t-1}^P &= \boldsymbol{\Psi} \left(\hat{\boldsymbol{P}}_{t-1}^P + \beta ((\rho \mathbf{C}_1 + \mathbf{C}_2 \mathbf{C}_1) \tau_t + \mathbf{C}_2 \mathbf{C}_2 \hat{\boldsymbol{P}}_{t-1}^P) + \mathbf{D} \tau_t \right) \\
0 &= \left[\mathbf{C}_1 - \beta \boldsymbol{\Psi} (\rho \mathbf{I} + \mathbf{C}_2) \mathbf{C}_1 - \boldsymbol{\Psi} \mathbf{D} \right] \tau_t + \left[\mathbf{C}_2 - \boldsymbol{\Psi} - \beta \boldsymbol{\Psi} \mathbf{C}_2 \mathbf{C}_2 \right] \hat{\boldsymbol{P}}_{t-1}^P
\end{aligned}
$$

We have two equations and two unknowns:

$$
\begin{aligned}
[\mathbf{C}_1 - \beta\mathbf{\Psi}(\rho\mathbf{I} + \mathbf{C}_2)\mathbf{C}_1 - \mathbf{\Psi}\mathbf{D}] &= 0 \\
[\mathbf{C}_2 - \mathbf{\Psi} - \beta\mathbf{\Psi}\mathbf{C}_2\mathbf{C}_2] &= 0
\end{aligned}
$$

$\mathbf{C}_2$ is solved with the quadratic method we described.

$$
\begin{aligned}
\beta\mathbf{C}_2\mathbf{C}_2 - \mathbf{\Psi}^{-1}\mathbf{C}_2 + \mathbf{I} = 0 \\
\mathbf{\Psi}^{-1} - \beta\mathbf{C}_2 = \mathbf{C}_2^{-1} \\
\mathbf{\Psi}^{-1} = \beta\mathbf{C}_2 + \mathbf{C}_2^{-1}
\end{aligned}
$$

We will now diagonalize $\mathbf{\Psi} = \boldsymbol{Q}\breve{\mathbf{\Psi}}\boldsymbol{Q}^{-1}$. By definition: $\mathbf{\Psi}^{-1} = \boldsymbol{Q}\breve{\mathbf{\Psi}}^{-1}\boldsymbol{Q}^{-1}$. We then define:

$$
\breve{\boldsymbol{C}}_{\mathbf{2}} = \boldsymbol{Q}^{-1}\mathbf{C}_2\boldsymbol{Q}
$$

Hence:

$$
\breve{\mathbf{\Psi}}^{-1} = \beta\breve{\boldsymbol{C}}_{\mathbf{2}} + \breve{\boldsymbol{C}}_{\mathbf{2}}^{-1}
$$

Since $\breve{\mathbf{\Psi}}$ is diagonal and β is scalar, then there is a solution for $\breve{\boldsymbol{C}}_{\mathbf{2}}$ which is a diagonal. Let's denote the diagonal elements of $\breve{\mathbf{\Psi}}^{-1}$ with $\breve{\Psi}_i^{-1}$. Hence:

$$
\begin{aligned}
\breve{\Psi}_i^{-1} = \beta\breve{C}_{2i} + \breve{C}_{2i}^{-1}. \\
\beta\breve{C}_{2i}^2 - \breve{\Psi}_i^{-1}\breve{C}_{2i} + 1 = 0.
\end{aligned}
$$

The solutions are given by:

$$
\breve{C}_{2i} = \frac{\breve{\Psi}_i^{-1} \pm \sqrt{\breve{\Psi}_i^{-2} - 4\beta}}{2\beta}
$$

With $\mathbf{C}_2$ solved $\mathbf{C}_1$ is:

$$
\begin{aligned}
\left[\mathbf{\Psi}^{-1} - \beta(\rho\mathbf{I} + \mathbf{C}_2)\right]\mathbf{C}_1 = \mathbf{D} \\
\left[\mathbf{C}_2^{-1} - \beta\rho\mathbf{I}\right]\mathbf{C}_1 = \mathbf{D} \\
\mathbf{C}_1 = \left[\mathbf{C}_2^{-1} - \beta\rho\mathbf{I}\right]\mathbf{D}
\end{aligned}
$$

$\mathbf{D}$ will capture how tariffs load onto consumer and producer prices directly and indirectly.

K Analytical Solution with Portfolio Adjustment Costs

We start with the five-equation Global New Keynesian representation equilibrium conditions, which read as follows when we bring back portfolio adjustment costs:

$$
\begin{aligned}
&\sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \hat{\boldsymbol{i}}_t - \mathbb{E}_t(\hat{\boldsymbol{P}}^C_{t+1} - \hat{\boldsymbol{P}}^C_t)\\
&\hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t\\
&\hat{\boldsymbol{P}}^P_t = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^P_{\hat{\tau}}\hat{\tau}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right]\\
&\mathbb{E}_t\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t = \boldsymbol{Z}\hat{\boldsymbol{i}}_t + \psi\hat{V}_t\\
&\beta\hat{V}_t = \hat{V}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\hat{\mathcal{E}}_t + \Xi_5\hat{\tau}_t + \beta\boldsymbol{\Xi}_6\hat{\boldsymbol{i}}_t\\
&\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi}(\hat{\boldsymbol{P}}^C_t - \hat{\boldsymbol{P}}^C_{t-1})
\end{aligned}
$$

where $\boldsymbol{Z} = [1\ \ -1]$ and $\boldsymbol{\Xi}_6 = [1\ 0]$.

We now assume that the central bank's policy rule perfectly stabilizes the price level such that $\hat{\boldsymbol{P}}^C_t = \mathbf{0}$; this replaces the Taylor rule as the policy rule in the equation above. Secondly, let us momentarily shut down forward looking behavior by the firm to focus on network effects in conjunction with portfolio adjustment costs.[45] The equilibrium conditions now read as follows:

$$
\begin{aligned}
&\sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \hat{\boldsymbol{i}}_t\\
&\mathbf{0} = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\hat{\tau}}\hat{\tau}_t\\
&\hat{\boldsymbol{P}}^P_t = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}\hat{\boldsymbol{C}}_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^P_{\hat{\tau}}\hat{\tau}_t\right)\right]\\
&\mathbb{E}_t\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t = \boldsymbol{Z}\hat{\boldsymbol{i}}_t + \psi\hat{V}_t\\
&\beta\hat{V}_t = \hat{V}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\hat{\mathcal{E}}_t + \Xi_5\hat{\tau}_t + \beta\boldsymbol{\Xi}_6\hat{\boldsymbol{i}}_t
\end{aligned}
$$

K.1 Method of Undetermined Coefficients

Let us postulate that

$$
\begin{aligned}
\hat{\boldsymbol{C}}_t &= \underbrace{\boldsymbol{C}_1}_{N\times 1}\hat{V}_{t-1} + \underbrace{\boldsymbol{C}_2}_{N\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{C}_3}_{N\times 1}\hat{\tau}_t\\
\hat{\boldsymbol{P}}^P_t &= \underbrace{\boldsymbol{C}_4}_{NJ\times 1}\hat{V}_{t-1} + \underbrace{\boldsymbol{C}_5}_{NJ\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{C}_6}_{NJ\times 1}\hat{\tau}_t
\end{aligned}
$$

[45] Mathematically we can assume that the firm's β is different and we take the limit of that β to 0.

$$\hat{V}_t = C_7 \hat{V}_{t-1} + \underbrace{\boldsymbol{C}_8}_{1\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + C_9 \hat{\tau}_t$$

$$\hat{\mathcal{E}}_t = C_{10} \hat{V}_{t-1} + \underbrace{\boldsymbol{C}_{11}}_{1\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + C_{12} \hat{\tau}_t$$

$$\hat{\boldsymbol{i}}_t = \underbrace{\boldsymbol{C}_{13}}_{N\times 1} \hat{V}_{t-1} + \underbrace{\boldsymbol{C}_{14}}_{N\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{C}_{15}}_{N\times 1} \hat{\tau}_t$$

Suppose the shock is one-time:

$$\begin{aligned}
\mathbb{E}_t \hat{\boldsymbol{C}}_{t+1} &= \boldsymbol{C}_1 \hat{V}_t + \boldsymbol{C}_2 \hat{\boldsymbol{P}}^P_t \\
&= \boldsymbol{C}_1 \left(C_7 \hat{V}_{t-1} + \boldsymbol{C}_8 \hat{\boldsymbol{P}}^P_{t-1} + C_9 \hat{\tau}_t \right) + \boldsymbol{C}_2 \left(\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_6 \hat{\tau}_t \right) \\
&= \left(\boldsymbol{C}_1 C_7 + \boldsymbol{C}_2 \boldsymbol{C}_4 \right) \hat{V}_{t-1} + \left(\boldsymbol{C}_1 \boldsymbol{C}_8 + \boldsymbol{C}_2 \boldsymbol{C}_5 \right) \hat{\boldsymbol{P}}^P_{t-1} + \left(\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2 \boldsymbol{C}_6 \right) \hat{\tau}_t \\
\mathbb{E}_t \hat{\mathcal{E}}_{t+1} &= C_{10} \hat{V}_t + \boldsymbol{C}_{11} \hat{\boldsymbol{P}}^P_t \\
&= C_{10} \left(C_7 \hat{V}_{t-1} + \boldsymbol{C}_8 \hat{\boldsymbol{P}}^P_{t-1} + C_9 \hat{\tau}_t \right) + \boldsymbol{C}_{11} \left(\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_6 \hat{\tau}_t \right) \\
&= \left(C_{10} C_7 + \boldsymbol{C}_{11} \boldsymbol{C}_4 \right) \hat{V}_{t-1} + \left(C_{10} \boldsymbol{C}_8 + \boldsymbol{C}_{11} \boldsymbol{C}_5 \right) \hat{\boldsymbol{P}}^P_{t-1} + \left(C_{10} C_9 + \boldsymbol{C}_{11} \boldsymbol{C}_6 \right) \hat{\tau}_t
\end{aligned}$$

Plugging these into the first equation:

$$\begin{aligned}
& \sigma(\mathbb{E}_t \hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \hat{\boldsymbol{i}}_t \\
& \sigma(\boldsymbol{C}_1 \left(C_7 \hat{V}_{t-1} + \boldsymbol{C}_8 \hat{\boldsymbol{P}}^P_{t-1} + C_9 \hat{\tau}_t \right) + \boldsymbol{C}_2 \left(\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_6 \hat{\tau}_t \right)) \\
& \quad - \sigma(\boldsymbol{C}_1 \hat{V}_{t-1} + \boldsymbol{C}_2 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_3 \hat{\tau}_t) = \boldsymbol{C}_{13} \hat{V}_{t-1} + \boldsymbol{C}_{14} \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_{15} \hat{\tau}_t \\
\Rightarrow & \quad + \left[\sigma(\boldsymbol{C}_1 C_7 + \boldsymbol{C}_2 \boldsymbol{C}_4 - \boldsymbol{C}_1) - \boldsymbol{C}_{13} \right] \hat{V}_{t-1} \\
& \quad + \left[\sigma(\boldsymbol{C}_1 C_8 + \boldsymbol{C}_2 \boldsymbol{C}_5 - \boldsymbol{C}_2) - \boldsymbol{C}_{14} \right] \hat{\boldsymbol{P}}^P_{t-1} \\
& \quad + \left[\sigma(\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2 \boldsymbol{C}_6 - \boldsymbol{C}_3) - \boldsymbol{C}_{15} \right] \hat{\tau}_t = 0
\end{aligned}$$

Plugging these into the second equation:

$$\begin{aligned}
\boldsymbol{0} &= \boldsymbol{\Gamma} \hat{\boldsymbol{P}}^P_t + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\hat{\tau}} \hat{\tau}_t \\
&= \boldsymbol{\Gamma} \left(\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_6 \hat{\tau}_t \right) + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \left(C_{10} \hat{V}_{t-1} + \boldsymbol{C}_{11} \hat{\boldsymbol{P}}^P_{t-1} + C_{12} \hat{\tau}_t \right) + \boldsymbol{L}^C_{\hat{\tau}} \hat{\tau}_t \\
&= \left(\boldsymbol{\Gamma} \boldsymbol{C}_4 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} \right) \hat{V}_{t-1} + \left(\boldsymbol{\Gamma} \boldsymbol{C}_5 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{C}_{11} \right) \hat{\boldsymbol{P}}^P_{t-1} + \left(\boldsymbol{\Gamma} \boldsymbol{C}_6 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{12} + \boldsymbol{L}^C_{\hat{\tau}} \right) \hat{\tau}_t
\end{aligned}$$

Plugging these into the third equation:

$$
\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_6 \hat{\tau}_t \\
&= \tilde{\boldsymbol{\Psi}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \hat{\boldsymbol{C}}_t + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t + \boldsymbol{L}_{\hat{\tau}}^P \hat{\tau}_t \right) \right]
\end{aligned}
$$

Substitute the conjectured laws of motion:

$$
\begin{aligned}
\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_6 \hat{\tau}_t = \tilde{\boldsymbol{\Psi}} \hat{\boldsymbol{P}}_{t-1}^P + \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \Big[& \sigma \boldsymbol{\alpha} \left(\boldsymbol{C}_1 \hat{V}_{t-1} + \boldsymbol{C}_2 \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_3 \hat{\tau}_t \right) \\
& + \boldsymbol{L}_{\mathcal{E}}^P \left(C_{10} \hat{V}_{t-1} + \boldsymbol{C}_{11} \hat{\boldsymbol{P}}_{t-1}^P + C_{12} \hat{\tau}_t \right) + \boldsymbol{L}_{\hat{\tau}}^P \hat{\tau}_t \Big]
\end{aligned}
$$

Group by state variables:

$$
\begin{aligned}
\boldsymbol{C}_4 \hat{V}_{t-1} &+ \boldsymbol{C}_5 \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_6 \hat{\tau}_t \\
& - \tilde{\boldsymbol{\Psi}} \hat{\boldsymbol{P}}_{t-1}^P \\
& - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_1 + \boldsymbol{L}_{\mathcal{E}}^P C_{10} \right) \hat{V}_{t-1} \\
& - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_2 + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{C}_{11} \right) \hat{\boldsymbol{P}}_{t-1}^P \\
& - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_3 + \boldsymbol{L}_{\mathcal{E}}^P C_{12} + \boldsymbol{L}_{\hat{\tau}}^P \right) \hat{\tau}_t = 0
\end{aligned}
$$

Group final expression for third equation:

$$
\begin{aligned}
& \left[\boldsymbol{C}_4 - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_1 + \boldsymbol{L}_{\mathcal{E}}^P C_{10} \right) \right] \hat{V}_{t-1} \\
& + \left[\boldsymbol{C}_5 - \tilde{\boldsymbol{\Psi}} - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_2 + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{C}_{11} \right) \right] \hat{\boldsymbol{P}}_{t-1}^P \\
& + \left[\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_3 + \boldsymbol{L}_{\mathcal{E}}^P C_{12} + \boldsymbol{L}_{\hat{\tau}}^P \right) \right] \hat{\tau}_t = 0
\end{aligned}
$$

Plugging undetermined coefficients into the fourth equation:

$$
\begin{aligned}
& \left[\left(C_{10} C_7 + \boldsymbol{C}_{11} \boldsymbol{C}_4 \right) \hat{V}_{t-1} + \left(C_{10} \boldsymbol{C}_8 + \boldsymbol{C}_{11} \boldsymbol{C}_5 \right) \hat{\boldsymbol{P}}_{t-1}^P + \left(C_{10} C_9 + \boldsymbol{C}_{11} \boldsymbol{C}_6 \right) \hat{\tau}_t \right] \\
& \quad - \left[C_{10} \, \hat{V}_{t-1} + \boldsymbol{C}_{11} \, \hat{\boldsymbol{P}}_{t-1}^P + C_{12} \, \hat{\tau}_t \right] \\
& = \boldsymbol{Z} \left[\boldsymbol{C}_{13} \, \hat{V}_{t-1} + \boldsymbol{C}_{14} \, \hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_{15} \, \hat{\tau}_t \right] + \psi \left[C_7 \, \hat{V}_{t-1} + \boldsymbol{C}_8 \, \hat{\boldsymbol{P}}_{t-1}^P + C_9 \, \hat{\tau}_t \right].
\end{aligned}
$$

Now we plug in the undetermined coefficients into the fifth equation:

$$
\beta (C_7 \hat{V}_{t-1} + \boldsymbol{C}_8 \hat{\boldsymbol{P}}_{t-1}^P + C_9 \hat{\tau}_t) = \hat{V}_{t-1} + \Xi_5 \hat{\tau}_t
$$

$$
\begin{aligned}
&+ \boldsymbol{\Xi}_2 \left(\boldsymbol{C}_1 \hat{V}_{t-1} + \boldsymbol{C}_2 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_3 \hat{\tau}_t \right) \\
&+ \boldsymbol{\Xi}_3 \left(\boldsymbol{C}_4 \hat{V}_{t-1} + \boldsymbol{C}_5 \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_6 \hat{\tau}_t \right) \\
&+ \Xi_4 \left(C_{10} \hat{V}_{t-1} + \boldsymbol{C}_{11} \hat{\boldsymbol{P}}^P_{t-1} + C_{12} \hat{\tau}_t \right) \\
&+ \beta \boldsymbol{\Xi}_6 (\boldsymbol{C}_{13} \hat{V}_{t-1} + \boldsymbol{C}_{14} \hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{C}_{15} \hat{\tau}_t)
\end{aligned}
$$

Grouping terms:

$$
\begin{aligned}
0 =& \Big[(C_{10} C_7 + \boldsymbol{C}_{11} \boldsymbol{C}_4 - C_{10}) - \boldsymbol{Z} \boldsymbol{C}_{13} - \psi \, C_7 \Big] \hat{V}_{t-1} \\
&+ \Big[(C_{10} \boldsymbol{C}_8 + \boldsymbol{C}_{11} \boldsymbol{C}_5 - \boldsymbol{C}_{11}) - \boldsymbol{Z} \boldsymbol{C}_{14} - \psi \, \boldsymbol{C}_8 \Big] \hat{\boldsymbol{P}}^P_{t-1} \\
&+ \Big[(C_{10} C_9 + \boldsymbol{C}_{11} \boldsymbol{C}_6 - C_{12}) - \boldsymbol{Z} \boldsymbol{C}_{15} - \psi \, C_9 \Big] \hat{\tau}_t.
\end{aligned}
$$

Then we have:

$$
\begin{aligned}
&[\beta C_7 - 1 - \boldsymbol{\Xi}_2 \boldsymbol{C}_1 - \boldsymbol{\Xi}_3 \boldsymbol{C}_4 - \Xi_4 C_{10} - \beta \boldsymbol{\Xi}_6 \boldsymbol{C}_{13}] \, \hat{V}_{t-1} \\
&+ [\beta \boldsymbol{C}_8 - \boldsymbol{\Xi}_2 \boldsymbol{C}_2 - \boldsymbol{\Xi}_3 \boldsymbol{C}_5 - \Xi_4 \boldsymbol{C}_{11} - \beta \boldsymbol{\Xi}_6 \boldsymbol{C}_{14}] \, \hat{\boldsymbol{P}}^P_{t-1} \\
&+ [\beta C_9 - \Xi_5 - \boldsymbol{\Xi}_2 \boldsymbol{C}_3 - \boldsymbol{\Xi}_3 \boldsymbol{C}_6 - \Xi_4 C_{12} - \beta \boldsymbol{\Xi}_6 \boldsymbol{C}_{15}] \, \hat{\tau}_t = 0
\end{aligned}
$$

K.2 System of 15 Equations and 15 Unknowns

With the method of undetermined coefficients we have the following system

$$
\begin{aligned}
&[\sigma(\boldsymbol{C}_1 C_7 + \boldsymbol{C}_2 \boldsymbol{C}_4 - \boldsymbol{C}_1) - \boldsymbol{C}_{13}] = 0 \\
&[\sigma(\boldsymbol{C}_1 C_8 + \boldsymbol{C}_2 \boldsymbol{C}_5 - \boldsymbol{C}_2) - \boldsymbol{C}_{14}] = \boldsymbol{0} \\
&[\sigma(\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2 \boldsymbol{C}_6 - \boldsymbol{C}_3) - \boldsymbol{C}_{15}] = 0 \\
&\boldsymbol{\Gamma} \boldsymbol{C}_4 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} = 0 \\
&\boldsymbol{\Gamma} \boldsymbol{C}_5 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{C}_{11} = \boldsymbol{0} \\
&\boldsymbol{\Gamma} \boldsymbol{C}_6 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{12} + \boldsymbol{L}^C_{\hat{\tau}} = 0 \\
&\boldsymbol{C}_4 - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_1 + \boldsymbol{L}^P_{\mathcal{E}} C_{10} \right) = 0 \\
&\boldsymbol{C}_5 - \tilde{\boldsymbol{\Psi}} - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_2 + \boldsymbol{L}^P_{\mathcal{E}} \boldsymbol{C}_{11} \right) = \boldsymbol{0} \\
&\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda} \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_3 + \boldsymbol{L}^P_{\mathcal{E}} C_{12} + \boldsymbol{L}^P_{\hat{\tau}} \right) = 0 \\
&\Big[(C_{10} C_7 + \boldsymbol{C}_{11} \boldsymbol{C}_4 - C_{10}) - \boldsymbol{Z} \boldsymbol{C}_{13} - \psi \, C_7 \Big] = 0
\end{aligned}
$$

$$\left[(C_{10}\boldsymbol{C}_8 + \boldsymbol{C}_{11}\boldsymbol{C}_5 - \boldsymbol{C}_{11}) - \boldsymbol{Z}\boldsymbol{C}_{14} - \psi\, \boldsymbol{C}_8\right] = \boldsymbol{0}$$
$$\left[(C_{10}C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - C_{12}) - \boldsymbol{Z}\boldsymbol{C}_{15} - \psi\, C_9\right] = 0$$
$$[\beta C_7 - 1 - \boldsymbol{\Xi}_2\boldsymbol{C}_1 - \boldsymbol{\Xi}_3\boldsymbol{C}_4 - \Xi_4 C_{10} - \beta\boldsymbol{\Xi}_6\boldsymbol{C}_{13}] = 0$$
$$[\beta\boldsymbol{C}_8 - \boldsymbol{\Xi}_2\boldsymbol{C}_2 - \boldsymbol{\Xi}_3\boldsymbol{C}_5 - \Xi_4\boldsymbol{C}_{11} - \beta\boldsymbol{\Xi}_6\boldsymbol{C}_{14}] = \boldsymbol{0}$$
$$[\beta C_9 - \Xi_5 - \boldsymbol{\Xi}_2\boldsymbol{C}_3 - \boldsymbol{\Xi}_3\boldsymbol{C}_6 - \Xi_4 C_{12} - \beta\boldsymbol{\Xi}_6\boldsymbol{C}_{15}] = 0$$

We are interested in $\boldsymbol{C}_3$, $\boldsymbol{C}_6$, $\boldsymbol{C}_9$, $\boldsymbol{C}_{12}$ and $\boldsymbol{C}_{15}$. These appear in the following equations

$$[\sigma(\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2\boldsymbol{C}_6 - \boldsymbol{C}_3) - \boldsymbol{C}_{15}] = 0$$
$$\boldsymbol{\Gamma}\boldsymbol{C}_6 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{12} + \boldsymbol{L}^C_{\hat{\tau}} = 0$$
$$\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}\boldsymbol{C}_3 + \boldsymbol{L}^P_{\mathcal{E}} C_{12} + \boldsymbol{L}^P_{\hat{\tau}}\right) = 0$$
$$[C_{10}C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - C_{12} - \boldsymbol{Z}\boldsymbol{C}_{15} - \psi\, C_9] = 0$$
$$[\beta C_9 - \Xi_5 - \boldsymbol{\Xi}_2\boldsymbol{C}_3 - \boldsymbol{\Xi}_3\boldsymbol{C}_6 - \Xi_4 C_{12} - \beta\boldsymbol{\Xi}_6\boldsymbol{C}_{15}] = 0$$

K.2.1 C_3

First:

$$\boldsymbol{C}_3 = (\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2\boldsymbol{C}_6) - \sigma^{-1}\boldsymbol{C}_{15}$$

Plugging that in:

$$\boldsymbol{C}_6 = \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}((\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2\boldsymbol{C}_6) - \sigma^{-1}\boldsymbol{C}_{15}) + \boldsymbol{L}^P_{\mathcal{E}} C_{12} + \boldsymbol{L}^P_{\hat{\tau}}\right)$$

K.2.2 C_{12}

Then we plug in $C_{12} = (C_{10} - \psi)C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - \boldsymbol{Z}\boldsymbol{C}_{15}$

$$\boldsymbol{C}_6 = \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}((\boldsymbol{C}_1 C_9 + \boldsymbol{C}_2\boldsymbol{C}_6) - \sigma^{-1}\boldsymbol{C}_{15}) + \boldsymbol{L}^P_{\mathcal{E}}((C_{10} - \psi)C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - \boldsymbol{Z}\boldsymbol{C}_{15}) + \boldsymbol{L}^P_{\hat{\tau}}\right)$$

Multiplying out:

$$\begin{aligned}
&\boldsymbol{C}_6 = \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_1 C_9 + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_2\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{\alpha}\boldsymbol{C}_{15} \\
&\quad + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}} C_{10}C_9 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\psi C_9 + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{C}_{11}\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z}\boldsymbol{C}_{15} + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\hat{\tau}} \\
\Rightarrow\quad &\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_2\boldsymbol{C}_6 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{C}_{11}\boldsymbol{C}_6 \\
&\quad = \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_1 C_9 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{\alpha}\boldsymbol{C}_{15} + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}} C_{10}C_9 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\psi C_9 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z}\boldsymbol{C}_{15} + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\hat{\tau}}
\end{aligned}$$

Grouping terms we have three equations three unknowns:

$$\begin{aligned}
\left[\boldsymbol{I} - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_2 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{C}_{11}\right]\boldsymbol{C}_6 &= \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\hat{\tau}} + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}\boldsymbol{C}_1 + \boldsymbol{L}^P_{\mathcal{E}} C_{10} - \boldsymbol{L}^P_{\mathcal{E}}\psi\right) C_9 \\
&\quad + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(-\boldsymbol{\alpha} - \boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z}\right)\boldsymbol{C}_{15}
\end{aligned} \tag{K.1}$$

K.2.3 C_{15}

Let us turn to the CPI equation plugging into it $C_{12} = (C_{10} - \psi)C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - \boldsymbol{Z}\boldsymbol{C}_{15}$:

$$\begin{aligned}
&\boldsymbol{\Gamma}\boldsymbol{C}_6 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}((C_{10} - \psi)C_9 + \boldsymbol{C}_{11}\boldsymbol{C}_6 - \boldsymbol{Z}\boldsymbol{C}_{15}) + \boldsymbol{L}^C_{\hat{\tau}} = 0 \\
&\tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{Z}\boldsymbol{C}_{15} = \boldsymbol{\Gamma}\boldsymbol{C}_6 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10}C_9 - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\psi C_9 + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{C}_{11}\boldsymbol{C}_6 + \boldsymbol{L}^C_{\hat{\tau}} \\
&\tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{Z}\boldsymbol{C}_{15} = \left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{C}_{11}\right)\boldsymbol{C}_6 + \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\psi\right) C_9 + \boldsymbol{L}^C_{\hat{\tau}}
\end{aligned}$$

$\tilde{\boldsymbol{L}}^C_{\mathcal{E}}$ is $N \times 1$ while Z is $1 \times N$, so the matrix on the left is invertible. Then:

$$\boldsymbol{C}_{15} = (\tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{Z})^{-1}\left[\left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{C}_{11}\right)\boldsymbol{C}_6 + \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\psi\right) C_9 + \boldsymbol{L}^C_{\hat{\tau}}\right]$$

Plugging this back to (K.1)

$$\begin{aligned}
&\left[\boldsymbol{I} - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\sigma\boldsymbol{\alpha}\boldsymbol{C}_2 - \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{C}_{11}\right]\boldsymbol{C}_6 = \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\boldsymbol{L}^P_{\hat{\tau}} + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(\sigma\boldsymbol{\alpha}\boldsymbol{C}_1 + \boldsymbol{L}^P_{\mathcal{E}} C_{10} - \boldsymbol{L}^P_{\mathcal{E}}\psi\right) C_9 \\
&\quad + \tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\left(-\boldsymbol{\alpha} - \boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z}\right)((\tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{Z})^{-1}\left[\left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\boldsymbol{C}_{11}\right)\boldsymbol{C}_6 + \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}}\psi\right) C_9 + \boldsymbol{L}^C_{\hat{\tau}}\right])
\end{aligned} \tag{K.2}$$

K.2.4 C_9

We use the last of the 5 equations and findings above to express C_9 as a function of C_6:

$$\begin{aligned}
\beta C_9 &= \Xi_5 + \boldsymbol{\Xi}_2\boldsymbol{C}_1 C_9 + \boldsymbol{\Xi}_2\boldsymbol{C}_2\boldsymbol{C}_6 - \sigma^{-1}\boldsymbol{\Xi}_2\boldsymbol{C}_{15} + \boldsymbol{\Xi}_3\boldsymbol{C}_6 \\
&\quad + \Xi_4 C_{10}C_9 - \Xi_4\psi C_9 + \Xi_4\boldsymbol{C}_{11}\boldsymbol{C}_6 - \Xi_4\boldsymbol{Z}\boldsymbol{C}_{15} + \beta\boldsymbol{\Xi}_6\boldsymbol{C}_{15}
\end{aligned}$$

$$
\begin{aligned}
&= \boldsymbol{\Xi}_2 \boldsymbol{C}_1 C_9 + \Xi_4 C_{10} C_9 - \Xi_4 \psi C_9 \\
&\quad + \boldsymbol{\Xi}_2 \boldsymbol{C}_2 \boldsymbol{C}_6 + \boldsymbol{\Xi}_3 \boldsymbol{C}_6 + \Xi_4 \boldsymbol{C}_{11} \boldsymbol{C}_6 + \Xi_5 \\
&\quad + \left(-\sigma^{-1} \boldsymbol{\Xi}_2 - \Xi_4 \boldsymbol{Z} + \beta \boldsymbol{\Xi}_6\right) \boldsymbol{C}_{15} \\
&= \boldsymbol{\Xi}_2 \boldsymbol{C}_1 C_9 + \Xi_4 C_{10} C_9 - \Xi_4 \psi C_9 \\
&\quad + \boldsymbol{\Xi}_2 \boldsymbol{C}_2 \boldsymbol{C}_6 + \boldsymbol{\Xi}_3 \boldsymbol{C}_6 + \Xi_4 \boldsymbol{C}_{11} \boldsymbol{C}_6 + \Xi_5 \\
&\quad + \left(-\sigma^{-1} \boldsymbol{\Xi}_2 - \Xi_4 \boldsymbol{Z} + \beta \boldsymbol{\Xi}_6\right) ((\tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{Z})^{-1} \left[\left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{C}_{11}\right) \boldsymbol{C}_6 + \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \psi\right) C_9 + \boldsymbol{L}^C_{\hat{\tau}}\right])
\end{aligned}
$$

Solving for C_9:

$$
C_9 = \frac{\boldsymbol{\Xi}_2 \boldsymbol{C}_2 \boldsymbol{C}_6 + \boldsymbol{\Xi}_3 \boldsymbol{C}_6 + \Xi_4 \boldsymbol{C}_{11} \boldsymbol{C}_6 + \Xi_5 + \left(-\sigma^{-1} \boldsymbol{\Xi}_2 - \Xi_4 \boldsymbol{Z} + \beta \boldsymbol{\Xi}_6\right) \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{Z}\right)^{-1} \left[\left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{C}_{11}\right) \boldsymbol{C}_6 + \boldsymbol{L}^C_{\hat{\tau}}\right]}{\beta - \boldsymbol{\Xi}_2 \boldsymbol{C}_1 - \Xi_4 C_{10} + \Xi_4 \psi - \left(-\sigma^{-1} \boldsymbol{\Xi}_2 - \Xi_4 \boldsymbol{Z} + \beta \boldsymbol{\Xi}_6\right) \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{Z}\right)^{-1} \left(\tilde{\boldsymbol{L}}^C_{\mathcal{E}} C_{10} - \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \psi\right)}
$$

K.2.5 C_6

Now we plug in our findings above into (K.2)

$$\boldsymbol{A} := (-\sigma \boldsymbol{\alpha} \boldsymbol{C}_2 - \boldsymbol{L}^P_{\mathcal{E}} \boldsymbol{C}_{11}) \tag{K.3}$$

$$\boldsymbol{B} := \left(\sigma \boldsymbol{\alpha} \boldsymbol{C}_1 + \boldsymbol{L}^P_{\mathcal{E}} C_{10}\right), \tag{K.4}$$

$$\boldsymbol{D} := \left(-\boldsymbol{\alpha} - \boldsymbol{L}^P_{\mathcal{E}} \boldsymbol{Z}\right) (\tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{Z})^{-1} \left(\boldsymbol{\Gamma} + \tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{C}_{11}\right), \tag{K.5}$$

$$\boldsymbol{F} := (\boldsymbol{L}^P_{\hat{\tau}} + -\boldsymbol{\alpha} - \boldsymbol{L}^P_{\mathcal{E}} \boldsymbol{Z}\Big) (\tilde{\boldsymbol{L}}^C_{\mathcal{E}} \boldsymbol{Z})^{-1} \boldsymbol{L}^C_{\hat{\tau}}) \tag{K.6}$$

With these definitions (K.2) becomes

$$\frac{\partial \hat{\boldsymbol{P}}^P_t}{\partial \tau_t} = \boldsymbol{C}_6 = ((\tilde{\boldsymbol{\Psi}} \boldsymbol{\Lambda})^{-1} + \boldsymbol{A} - \boldsymbol{D})^{-1} \left[\boldsymbol{F} + (\boldsymbol{B} - \boldsymbol{L}^P_{\mathcal{E}} \psi) C_9\right] \tag{K.7}$$

Rewriting:

$$\boldsymbol{\Theta}_1 := \boldsymbol{A} - \boldsymbol{D}, \tag{K.8}$$

$$\boldsymbol{\Theta}_2 := \boldsymbol{F} + \boldsymbol{B} C_9. \tag{K.9}$$

$$\boxed{\frac{\partial \hat{\boldsymbol{P}}_t^P}{\partial \tau_t} = \left[(\tilde{\boldsymbol{\Psi}}\boldsymbol{\Lambda})^{-1} + \boldsymbol{\Theta}_1\right]^{-1}\Big[\boldsymbol{\Theta}_2 - \big(\boldsymbol{L}_{\mathcal{E}}^P \frac{\partial \hat{V}_t}{\partial \hat{\tau}_t}\big)\psi\Big]}$$

ψ impacts $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$ through small interactions, so we compute $\frac{\partial^2 \hat{\boldsymbol{P}}_t^P}{\partial \tau_t \, \partial \psi}$ numerically to sign it. The intuition is that the impact of tariffs on the net external debt position of the home country is negative and the first entry of $\boldsymbol{L}_{\mathcal{E}}^P$ is positive while its second entry is negative. For that reason we should expect the impact of tariffs on the home country's domestically produced good to be positive and that of the foreign counry should be negative.