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STRATEGIC DIGITIZATION IN CURRENCY AND PAYMENT COMPETITION

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ABSTRACT

We model the competition between digital forms of fiat money and private digital money (PDM). Countries strategically digitize their fiat money — upgrading existing or launching new payment systems (including CBDCs) — to enhance adoption and counter PDM competition. A pecking order emerges: less dominant currencies digitize earlier, reflecting a first-mover advantage; dominant currencies delay digitization until they face competition; the weakest currencies forgo digitization. Delayed digitization allows PDM to gain dominance, eventually weakening fiat money's role. We also highlight how geopolitical considerations, stablecoins, and interoperability between fiat and private digital money shape the digitization of money and monetary competition.

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As technology advances, economic transactions and interactions have become increasingly digital, with the value of digital payments globally being in the hundreds of trillions of U.S. dollars according to BIS data (BIS Statistics, 2023; Glowka, Kosse, and Szemere, 2023).[1] While bank-centric and government-led payment systems (e.g., ACH, SWIFT, or credit cards) have traditionally dominated digital payments, non-bank providers like PayPal and M-Pesa, along with BigTech firms such as Apple and Alibaba, have gained prominence in recent decades. These non-bank payment services challenge traditional payment systems, for instance, by offering faster, more efficient payments with broader functionalities. Digitization has also enabled Hayek's vision of private currency issuance and competition with fiat money (Hayek, 1976). Indeed, cryptocurrencies, stablecoins, and decentralized finance have shown the potential to challenge traditional monetary systems (Brunnermeier, James, and Landau, 2019; Adrian and Mancini-Griffoli, 2019). Such developments have prompted many countries to explore currency and payment system digitization, including upgrading existing or introducing new payment systems (e.g., Brazil's Pix or India's UPI), or launching Central Bank Digital Currencies (CBDCs, see, e.g., Auer, Cornelli, and Frost, 2023).

Against this backdrop, we examine the evolving competition among currencies and forms of money amid the rise of private digital money (PDM), the ongoing digitization of payments, and countries' efforts to digitize their fiat payment systems. Among many new insights and predictions, we find that countries with less dominant, yet well-adopted currencies (e.g., China) digitize their fiat money earlier, showing a first-mover advantage. In contrast, countries with dominant currencies (e.g., the U.S.) exhibit a second-mover advantage and delay digitization until their dominance is challenged through the rise of PDM or the digitization of other fiat currencies. Our analysis highlights how strategic considerations, the nature of currency competition — both among fiat currencies and between fiat and private digital money — as well as trends like the rise of U.S. dollar-backed stablecoins shape the evolution and digitization of money and payment.

In our model, national fiat monies (also referred to as "fiat currencies") and PDM compete for adoption in digital payments. Instead of modeling different forms of fiat money and PDM separately, we consider representative forms (i.e., monetary aggregates). Fiat money refers to digital means of payment and payment services directly tied to a country's banking system, central bank, or government. Fiat money includes bank deposits with their associated payment rails, as well as government-led payment systems and CBDCs once introduced. The digitization of fiat money may involve upgrading existing bank-centric or government-led payment systems, introducing new ones, or launching CBDCs, thereby enhancing the convenience of fiat money in digital payments.

[1] The digital economy, even without counting point-of-sale systems in offline stores or bank transfers for businesses not directly tied to digital platforms, is projected to contribute 25% of global GDP in 2025 (PwC Report, 2025).

Private digital money (PDM) describes digital means of payment and payment services that largely bypass traditional bank-centric and government-led payment systems. PDM includes (C1) cryptocurrencies and tokens (e.g., Ether), (C2) currencies introduced by digital platforms (studied in Brunnermeier and Payne, 2024) (e.g., Libra, had it succeeded), (C3) stablecoins (e.g., Tether), and (C4) certain non-bank payment systems and services (e.g., Alipay).

Two fiat currencies, A and B, issued by countries A and B, respectively, and a representative PDM C provide convenience utility. We micro-found convenience utility by modeling payments subject to random search/matching between users and sellers of services and a cash-in-advance constraint. In this micro-foundation, the payment convenience of a currency depends on (i) the probability of users encountering sellers who accept it (reflecting the currency's level of acceptance), (ii) the efficiency, speed, and cost of transactions involving the currency, and (iii) the bargaining power of users relative to sellers who accept it, which, as we argue, is influenced by the currency's privacy features. While our micro-foundation highlights the medium-of-exchange function of money, we acknowledge that convenience may also reflect its store-of-value and unit-of-account functions, complementing its role as a medium of exchange. Thus, although our analysis focuses on payment competition, it may also apply more broadly to monetary competition in other dimensions.

Our micro-foundation suggests that transaction fees charged by payment intermediaries (e.g., credit card fees), inefficiencies in bank-railed payments (e.g., slow settlement speeds), and the limited payment functionalities and usability (e.g., the inability to support blockchain, and some cross-border and digital platform payments) are key factors limiting the convenience of fiat money in digital payments. Some forms of PDM or the digitization of fiat money can address these frictions, for instance, by facilitating faster payments or expanding usability. Additionally, factors such as payment privacy (e.g., in cryptocurrencies), unique functionalities (e.g., smart contracting), reduced reliance on costly payment intermediaries, and integration with digital platforms (e.g., Alipay's integration with Alibaba) contribute to the convenience of PDM.

In general, PDM competes with fiat money by (i) facilitating transactions traditionally settled with bank deposits, thereby reducing reliance on bank-centric and government-led payment systems, and (ii) enabling new types of transactions that fiat money cannot support without digitization (e.g., blockchain transactions). We assume increasing competition from PDM and stipulate that its convenience grows over time at an endogenous rate that rises with PDM adoption. This reflects PDM's ability to compete more effectively on margin (i) or an increasing share of transactions that can only be settled using PDM, raising the importance of margin (ii). This dynamic may be driven by the growing importance of digital platforms, technological advancements, or the introduction of new forms of PDM with unique functionalities.

Countries undertake costly efforts to enhance the convenience of their fiat money through digitization. This increased convenience may stem from enhanced payment technologies, broader usability, stronger privacy features, or reduced transaction costs. We model fiat digitization as a one-time stochastic event that occurs with an intensity proportional to a country's efforts. Formally, we study a dynamic game in which two countries, acting as large strategic players, choose their digitization efforts to maximize a time average of their currencies' adoption for payment, net of digitization costs. Price-taking users act as non-strategic players, and allocate their endowment (representing demand for digital payments) among three currencies, considering their payment convenience. As PDM convenience grows over time, users gradually adopt it, reducing their adoption of fiat currencies and influencing countries' incentives to digitize their fiat currencies.

In our model, countries digitize fiat money to increase its adoption and relevance in digital payments. In line with this assumption, empirical evidence from Berg, Keil, Martini, and Puri (2024) suggests that a key motive behind launching CBDCs is to enhance payment autonomy, which may involve ensuring the adoption of fiat money in digital payments and reducing reliance on non-bank payment providers. Similarly, Brunnermeier et al. (2019) argue that countries may need to digitize their currencies to preserve the relevance and adoption of fiat money in digital payment, a plausible concern for many countries. More generally, the objective function for countries also reflects that the wide adoption of a country's currency as means of payment is a source of valuable geoeconomic power and autonomy (Clayton, Maggiori, and Schreger, 2023).

We focus on the relevant case of asymmetric currency competition and solve for a Markov equilibrium with two state variables: one capturing the competition from PDM and the other capturing the state of countries' digitization processes. Country A is considered "strong" relative to a "weaker" country B due to the higher initial convenience and adoption of its currency. For example, the dominant currency A could represent the U.S. dollar — which derives high convenience from its broad usability as means of payment — while B represents a less dominant currency, such as the Euro or Renminbi. Although not included in the baseline model, we discuss, in an extension, very weak currencies, including the ones from small open economies, whose adoption remains low despite digitization, leading to negligible or no digitization efforts.

While countries' total digitization efforts are initially strong, these efforts gradually diminish and may even cease altogether as PDM gains adoption over time. At the outset, the weaker country accounts for most of the digitization efforts, reflecting an endogenous first-mover advantage. The stronger country exhibits a second-mover advantage and undertakes significant efforts only after its currency's dominance is challenged by PDM or the weaker currency. These findings align with the observation that less dominant currencies, like the Euro and Renminbi, are among the first to be digitized via CBDCs, while the United States is not actively pursuing CBDCs.

In our model, countries have strong incentives to digitize fiat currency before PDM achieves widespread adoption. However, failing to act sufficiently early creates a vacuum in the digital payment space, which PDM fills. As PDM gains dominance due to the absence of digitized fiat money, countries' incentives to digitize fiat money diminish or may disappear entirely, delaying or preventing digitization. This may lead to an equilibrium where fiat currencies play a diminished role and PDM dominates digital payments. Thus, our findings suggest that the relevance of fiat money in digital payments over the long run depends on whether countries act early in fiat digitization.

Intuitively, a country's incentives to digitize its currency reflect both the potential increase in adoption and the persistence of this effect. For the less dominant currency, B, digitization generates larger and more persistent adoption gains, especially when competition from PDM is weak or when country A has not yet launched its CBDC — both of which are true at the beginning of the game. Thus, B has strong incentives to move early in digitizing its currency, creating a first-mover advantage. However, when B fails to digitize its currency early on, the increasing competition from PDM or the digitization of currency A diminishes this first-mover advantage and so reduces B's efforts over time. Specifically, the digitization of the dominant currency A serves as a strategic substitute for B's digitization efforts. Overall, B's incentives to digitize are high initially, exceeding those of A, but, absent early success, they gradually decline and eventually fall below A's.

In contrast, A's incentives to digitize are low when competition from PDM is either weak or strong. That is, A's digitization efforts initially rise and then fall as competition from PDM increases, resulting in an inverted U-shaped pattern. At first, A's adoption level is high due to its dominance, limiting the gains it can achieve through digitization. However, as competition from PDM intensifies or B digitizes its currency, A's adoption level decreases, which raises the gains from digitization. In particular, the digitization of currency B acts as a strategic complement and increases A's digitization efforts. Finally, when A's adoption level is low and digitization offers limited benefits due to PDM's dominance, A's incentives to digitize diminish again. Overall, country A has strong incentives to digitize its currency only when its dominance is challenged, and digitization enables it to restore dominance. Country A's incentives respond earlier to rising competition from PDM when its growth is expected to accelerate. This reflects a dynamic strategic motive: early digitization curtails PDM adoption and its growth, limiting future competition.

We find that while increased competition from PDM hampers fiat currency digitization, increased competition among fiat currencies accelerates it. These differential effects arise because the nature of increased competition influences the endogenous growth of PDM, which rises with PDM adoption. Increased competition from PDM — driven by higher adoption or expected growth — may initially boost countries' efforts to digitize, specifically by challenging the dominance of A and raising A's efforts. However, it gradually undermines these efforts as it accelerates PDM growth, ultimately allowing PDM to dominate and limiting the gains from digitization for both countries. In contrast, stronger fiat currencies curb PDM adoption and growth, incentivizing countries to sustain their digitization efforts. Although not explicitly modeled, we consider regulation as a factor that reduces the convenience and competitiveness of PDM. Our model predicts that if regulation (by one or multiple countries) does indeed reduce PDM convenience, it accelerates fiat currency digitization, in that regulation complements countries' digitization efforts.

We contrast the dynamics of currency digitization with the planner's solution, where digitization efforts are chosen to maximize overall welfare or countries independently maximize the welfare (convenience utility) generated by their currencies. Since, in the baseline, countries maximize a time average of their currency's adoption in the digital economy, net of digitization costs, they care about their currency's convenience only insofar as it leads to higher adoption (i.e., shifts users' investment toward their currency). However, they do not internalize that users' utility increases with the convenience of currencies, holding investment fixed. As a result, countries' baseline digitization efforts are inefficiently low compared to welfare-maximizing efforts, causing inefficiently late digitization of fiat currency. Additionally, welfare would be maximized if the stronger country exerted higher efforts and moved first, since its currency is more widely held. This analysis suggests that, because the competitive outcome is inefficient, collaboration among countries and coordination of their digitization efforts — such as the BIS mBridge project — helps to achieve efficient outcomes in currency and payment digitization.

In an extension, we consider (i) interoperability between fiat money and PDM and (ii) public-private collaborations in payment digitization. For instance, payment systems like Alipay process their own transactions but can also link to bank accounts, making them partially interoperable with bank payment rails. Regarding (ii), government-led digitization efforts often involve collaboration with the private sector, as exemplified by the Digital Euro Project, and also enhance interoperability. We demonstrate that such public-private collaborations lead to more persistent digitization efforts by countries, thereby advancing the digitization of fiat money. However, countries begin collaborating only after PDM achieves widespread adoption and collaboration becomes inevitable, whereas low PDM adoption prompts them to digitize their fiat currencies to compete. That is, as

PDM gains widespread adoption, public-private collaborations and enhanced interoperability are necessary to ensure fiat money's relevance in digital payments.

Our framework also applies to stablecoins (e.g., USDC), which are typically pegged to the U.S. dollar and partially backed by dollar-denominated assets. Extending our model, we capture the interdependence between fiat money and PDM by assuming that PDM is partially backed by currency A. In this setup, PDM adoption drives further adoption of currency A, reducing digitization efforts—both overall and for currency A—and ultimately delaying digitization. This variant highlights how dollar-backed stablecoins can increase the U.S. dollar's relevance in digital payments. It also suggests that the United States might benefit from pursuing crypto-friendly policies to encourage stablecoin growth while slowing its own digitization efforts, as a digitized U.S. dollar could displace stablecoins. Broadly, the private sector, through stablecoins, effectively creates a digital dollar, substituting for government-led dollar digitization initiatives.

We extend our model to incorporate exchange rates determined in a frictionless bond market and nominal interest rates that accrue to currency holders subject to imperfect passthrough, reflecting that bank deposits typically earn interest below the policy rate. In this variant, we show that uncovered interest parity (UIP) holds, and our key findings remain robust. Since the passthrough of nominal interest to users is imperfect and UIP holds, a higher interest rate raises the cost of holding a specific currency, which reduces its "effective" convenience and adoption. We find that a higher nominal interest rate, potentially reflecting higher inflation, or worse passthrough for the weaker currency, B, reduces competition among fiat currencies and countries' total digitization efforts. Conversely, a higher interest rate or worse passthrough for the dominant currency, A, increases total digitization efforts. We also show that our key findings hold when modeling the benefits of digitization solely as improved interest rate passthrough (Chiu, Davoodalhosseini, Jiang, and Zhu, 2023). In this scenario, countries exert greater digitization efforts when interest rates are high.

The extension incorporating interest rates enables us to account for "very weak" currencies, specifically those characterized by very high nominal interest rates and high inflation. In our model, the store-of-value and medium-of-exchange functions complement each other: very weak currencies perform poorly as stores of value, making their adoption for payment costly and leading to low adoption rates, even with digitization. The model predicts minimal or no digitization for such weak currencies, leading to a novel pecking order of currency digitization. In particular, less dominant but well-adopted currencies are digitized first, followed by more dominant currencies, while very weak currencies are digitized last or not at all.

We also show that countries, especially those with less dominant currencies, tend to intensify digitization efforts when they prioritize short-term objectives, exhibiting more myopic behavior. This is because they place less emphasis on how increasing competition from PDM erodes long-term digitization gains. Finally, while our baseline model reveals an endogenous first-mover advantage for weaker countries and a second-mover advantage for stronger ones, we introduce a variant where digitization costs decline after a competitor digitizes. This cost reduction, driven by learning or technological spillovers, incentivizes strategic delays in digitization.

Literature. Our paper analyzes the competition between private digital money (PDM) and fiat currency, contributing to the literature on digital currencies (Schilling and Uhlig, 2019; Cong, Li, and Wang, 2021; Sockin and Xiong, 2021, 2022; Biais, Bisiere, Bouvard, Casamatta, and Menkveld, 2023; Guennewig, 2024) and associated risks (Uhlig, 2022; Li and Mayer, 2021). It also connects to policy debates and the academic literature on central bank digital currencies (CBDCs, see Bech and Garratt, 2017; Duffie and Gleeson, 2021; Fernández-Villaverde, Schilling, and Uhlig, 2020; Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2021), with Bai, Cong, Luo, and Xie (2025) analyzing the initial adoption of e-CNY in China. Studies have examined CBDC interactions with the banking sector, including deposits and lending (Brunnermeier and Niepelt, 2019; Andolfatto, 2021; Keister and Sanches, 2023; Garratt and Zhu, 2021; Chiu et al., 2023; Niepelt, 2024). Whited, Wu, and Xiao (2022) structurally estimate CBDCs' impact on the banking system.

We also add to the broader field of digital payment studies (e.g., Cong, Easley, and Prasad, 2024). Sarkisyan (2023) analyzes the effects of Brazil's Pix on banking competition. Duarte, Frost, Gambacorta, Koo Wilkens, and Shin (2022); Kahn (2024) also examine the effects of the government-led launch of fast payment systems. Parlour, Rajan, and Zhu (2022) and Bian, Cong, and Ji (2023) examine payment competition and how payments interact with credit provision. We contribute to these studies on digital payments and CBDCs by analyzing the competition among fiat currencies (captured by monetary aggregates including both bank deposits and CBDCs) and PDMs. We focus on the endogenous digitization of money and dynamics of monetary competition, while abstracting from interactions between CBDCs and banks.

Benigno, Schilling, and Uhlig (2022) analyze currency competition between fiat currencies and a global cryptocurrency, demonstrating that the latter's adoption synchronizes monetary policy across countries. Our analysis differs by (i) modeling countries' incentives to digitize their currencies and (ii) capturing the dynamics of competition between fiat currencies and PDM. This article also contributes to the literature on the international monetary system, focusing on reserve and safe assets, their determinants, and competition among them (e.g., Farhi and Maggiori, 2018; Gopinath and Stein, 2021; He, Krishnamurthy, and Milbradt, 2019; Coppola, Krishnamurthy, and Xu, 2023).

Clayton, Dos Santos, Maggiori, and Schreger (2024a) examine the competition among currencies as stores of value. Although it could apply more broadly, our model emphasizes the medium-of-exchange function of money and the competition between various means of payment. More importantly, our theory adds to the literature on currency competition by capturing countries' strategic digitization efforts and rich dynamics arising from the competition.

Our study is related to the growing literature on geoeconomics and how global hegemons extract benefits (Clayton et al., 2023; Pflueger and Yared, 2024). In our model, a country's objective reflects the value of widespread adoption of its currency or payment system, particularly internationally—a source of geoeconomic power. Our findings suggest that while the United States initially has little incentive to pursue fiat digitization due to its status as a hegemon with a dominant currency, it responds by raising its digitization efforts once it faces competition and its dominance is challenged. In contrast, countries with widely adopted yet not dominant currencies, such as China, have the strongest incentives to digitize fiat money to expand (international) currency adoption and geoeconomic influence. This prediction aligns with China's efforts to digitize its currency, as seen in the launch of the e-CNY, and its broader initiatives to internationalize the renminbi (Clayton, Dos Santos, Maggiori, and Schreger, 2024b; Bahaj and Reis, 2024).

Finally, we set up and solve a dynamic game where fiat monies of varying strength (convenience) compete both among themselves and with PDM, with digitization as an innovation with endogenous effort and completed at a stochastic time (Aghion and Howitt, 1992).[2] From a modeling perspective, this paper is also linked to studies on real options (McDonald and Siegel, 1986; Dixit and Pindyck, 1994). While the literature has explored firms' real option exercises under (symmetric) competition (e.g., Fudenberg and Tirole, 1985; Grenadier, 1996, 2002; Kogan, 2001; Novy-Marx, 2007; Dai, Jiang, and Wang, 2022), the application to currency competition is new and important given how monetary economics traditionally abstracts from (i) endogenous investments that enhance currencies' monetary functions and (ii) aggregate time dynamics shaping currency competition.

1 A Dynamic Model of Currency Digitization and Competition

We present a dynamic model in which national fiat monies (also referred to as "fiat currencies") and private digital (non-bank) money — abbreviated as PDM — compete for adoption for mediating digital payments. Our analysis studies the competition between different forms of money, with an emphasis on the medium-of-exchange function of money. While the model primarily focuses

[2]The Markov equilibrium we study involves two state variables — one capturing dynamic PDM competition and the other capturing currency digitization status—and is characterized by a system of coupled differential equations.

on retail payment activities and their digitization (e.g., retail CBDC or fast payment system), it can also be applied to wholesale digital payments. Interpreted more broadly, despite its focus on payment, our theoretical model also applies to currency competition along other monetary functions, i.e., competition between stores of value and units of account. Before introducing the model, we provide a brief contextual description.

Digital Money. In our framework, a country's fiat money refers to digital means of payment and payment services directly tied to this country's banking system, central bank, or government. Fiat money includes (F1) bank deposits and the associated bank-centric payment rails and (F2) government-led payment systems (e.g., Brazil's Pix or India's UPI) and CBDCs, once introduced.[3] Note that government-led payment systems are often also bank-centric or linked to banks, as they facilitate payments using bank deposits (e.g., Pix, which is linked to bank accounts); also, CBDCs are considered government-led payment systems.[4] Rather than modeling these types of fiat money separately, we consider representative fiat money (i.e., a monetary aggregate) that encompasses (F1) and (F2). Thus, bank deposits and payment rails, CBDCs, and government-led payment systems collectively contribute to the digital payment convenience of representative fiat money. The digitization of fiat money may involve upgrading existing bank-centric or government-led payment systems, introducing new ones, or launching CBDCs. Either way, digitization enhances the overall payment convenience of representative fiat money. With some abuse, we refer to "fiat money" also as "fiat currency," and use these terms interchangeably.

Private digital money (in short, PDM) refers to digital means of payment and payment services that operate largely outside the traditional bank-centric and government-led payment systems. PDM includes (C1) cryptocurrencies and tokens (e.g., Ether and Bitcoin), (C2) digital currencies provided by digital platforms (as studied in Brunnermeier and Payne, 2024)) once launched (e.g., Libra, had it succeeded), (C3) stablecoins (e.g., USDC and Tether), and (C4) non-bank payment services and systems that largely bypass traditional fiat payment rails. Rather than modeling these forms of PDM separately, we consider a representative PDM — a monetary aggregate encompassing all categories (C1)–(C4) and representing technological disruptions to traditional payment systems

[3] Our notion of fiat money includes bank deposits, even though they are not direct liabilities of the central bank, as they are considered equivalent to public money for retail transactions (e.g., due to deposit insurance). While our analysis focuses on retail payments, fiat money could also encompass central bank reserves in the context of wholesale payments.

[4] Thus, the notions of bank-centric and government-led payment systems may overlap; we do not strictly differentiate between bank-centric and government-led payment systems. Further, note that CBDCs share many similarities with government-led payment systems layered on top of bank-centric payment systems, especially when implemented through banks. In many cases, such as the Digital Euro Project, CBDCs are effectively a form of government-led payment system and are designed to serve as a means of payment, rather than acting as a new store of value.

that come from the private sector. Some non-bank payment services are partially integrated with or interoperable with the bank-centric payment system, enabling them to process both transactions that use bank deposits and those that bypass banks. For instance, users can transact with Alipay by linking their bank account (to use bank deposits) or credit cards, or by using separate funds held in their Alipay e-wallet (Bian et al., 2023). Section 3.1 introduces a model variant that accounts for the interoperability between fiat money and PDM.

As detailed in our micro-foundation of the payment convenience in Appendix E, frictions limiting the convenience of digital fiat money and bank-centric payment rails include high transaction costs (e.g., fees charged by payment intermediaries like credit card companies), slow settlement speeds, outdated payment technology, and limited payment functionalities and usability (e.g., the inability to support blockchain or certain cross-border or digital platform transactions). PDM may address some of these limitations, offering payment convenience through unique functionalities, improved payment technology, an expanded scope of usability, or by reducing reliance on costly payment intermediaries. In general, PDM competes with fiat money in digital payments in two ways: (i) by facilitating transactions traditionally settled using bank deposits, thereby reducing reliance on bank-centric and government-led payment systems, and (ii) by enabling new types of transactions that fiat money cannot support without digitization (e.g., blockchain or certain digital platform transactions).

We assume that competition from PDM will intensify over time, reflecting its increasing ability to compete more effectively on margin (i) or by an increasing share of transactions that can only be settled using PDM, raising the importance of margin (ii). Digitizing fiat money can address these challenges by enhancing its competitiveness on margin (i) — such as through improved settlement efficiency/speed or lower transaction costs — or by broadening its usability to better compete on margin (ii) — for instance, digitized fiat money could better facilitate digital platform or cross-border payments, or transactions based on blockchains and smart contracts.

Users and Money. Time (indexed by t) is infinite. To introduce users and money, we set up the model "as if" time runs discretely with time increments $dt > 0$, i.e., $t = 0, dt, 2dt, 3dt, ...$ We take the continuous time limit $dt \to 0$ once we complete the model description. The economy is populated by one representative OLG user who takes prices as given and does not discount.[5] Cohort t is born at t with lifespan dt and exits at $t + dt$ when a new cohort is born. At birth, each cohort is

[5] We model overlapping generations ("OLG") users in a continuous time economy following the modeling approach of He and Krishnamurthy (2013). Biais et al. (2023) use OLG in discrete-time economy when modeling equilibrium Bitcoin pricing. In our OLG setting where users live for one instant, the assumption of no discounting is without loss of generality and one could easily introduce a discount rate without changing the model's outcomes.

endowed with one unit of the perishable generic consumption good, which serves as the numeraire that all prices are quoted in. Cohort t derives utility from consumption only at time $t+dt$ and thus would like to store its users' endowment (consumption good) from t to $t+dt$, yet the consumption good cannot be stored. Thus, money (explained below) facilitates transactions across different cohorts and time, thereby functioning as an inter-temporal medium of exchange and store of value. In addition, money delivers convenience utility, reflecting its function as intra-temporal medium of exchange. The total demand for money from users is fixed to one unit of the consumption good; it is supposed to capture the specific demand for digital money (as opposed to money demand in general). The assumption of fixed currency demand and thus demand for digital payment is for simplicity; one could relax this assumption without altering the key economic insights of the model.

Different Forms of Money. Two countries, A and B, have their representative fiat currencies A and B, respectively. Meanwhile, there is one representative PDM, C. Each currency $x \in \{A, B, C\}$ is in fixed unit supply and has an equilibrium value (equal to its adoption) P_t^x in consumption goods.[6] To consume at time $t+dt$, users in cohort t spend their consumption good endowment to buy money from the previous cohort (i.e., cohort $t-dt$) at time t. At time $t+dt$, cohort t users exchange money for the consumption good with cohort $t+dt$ users and so on.[7]

We denote by m_t^x cohort t's holdings of currency x in terms of the consumption good over its users' lifetime $[t, t+dt]$. As cohort t does not derive any utility from consuming early at time t and there are no other investment opportunities than money, cohort t users invest their entire endowment of one consumption good into money, which implies:

$$m_t^A + m_t^B + m_t^C = 1. \tag{1}$$

In our model, cohort t users are the only holders of currencies. Thus, market clearing implies $m_t^A = P_t^A, m_t^B = P_t^B$ and $m_t^C = P_t^C$, and $P_t^x = m_t^x$ is the endogenous level of adoption of currency x. In our baseline setup, the market clearing conditions (1) and $m_t^x = P_t^x$ uniquely pin down the exchange rate dynamics of different currencies. Section 3.3 presents a model variant where the exchange rates are determined according to arbitrageurs trading government bonds (which implies the uncovered interest parity); our key findings remain robust in this modified setting.

[6] One could extend the model by allowing money supply to vary over time. Because we aim to model currency competition in the digital economy, we abstract from the money demand that is not directly related to the digital economy. In Section 3.3, we provide a model variant with interest rates and exchange rates determined in international bond markets. In this model variant, the exact money supply does not play *any* role, since the market for currency clears due to reasons outside of the model.

[7] We assume that the first cohort born at $t = 0$ is simply endowed with the currency supply.

Money as a Medium of Exchange: Micro-Founded Convenience Utility. Our analysis focuses on the payment function of money. In addition to facilitating exchange across cohorts, the three currencies A, B, and C generate convenience utility, which is micro-founded in Appendix (E) (as discussed below) and captures the medium-of-exchange function of money related to digital payment. Specifically, the user's expected utility over $[t, t + dt]$ reads

$$U_t = \mathbb{E}_t[c_{t+dt}] + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt. \tag{2}$$

Here, c_{t+dt} denotes cohort t's consumption at time $t + dt$ and the remainder terms capture the convenience utility of money over $[t, t + dt]$ (which, unlike consumption, is of order dt). As in Cong et al. (2021) and Gryglewicz, Mayer, and Morellec (2021), convenience utility increases in m_t^x, i.e., the "real" money holdings in consumption goods, and increases with a convenience scale parameter Z_t^x for $x = A, B$ and Y_t for $x = C$, respectively. Further, it is characterized by a concave, smooth function $v(m_t^x)$ satisfying $v(m_t^x) \geq 0$, $v'(m_t^x) > 0$, $v''(m_t^x) < 0$, and $\lim_{m\to 0} v'(m) = +\infty$ — which implies imperfect currency substitutability and ensures that equilibrium money holdings satisfy $m_t^x \in (0, 1)$.[8] In what follows, we will take the CRRA functional form $v(m) = \frac{m^{1-\eta}}{1-\eta}$ for $\eta \in (0, 1)$ which satisfies these properties — our results go through under other functional forms too. In the baseline model, currencies do not pay interest and differ only in their convenience. Section 3.3 introduces interest rates.

Appendix E models payments subject to random search and matching with bargaining and a cash-in-advance constraint, thereby micro-founding the convenience utility in (2), specifically the quantities Z_t^x and Y_t. In this micro-foundation, over an instant $[t, t + dt]$, the user randomly encounters a seller of a service and holds money in advance to transact. At the beginning of the period $[t, t + dt]$ — before knowing whether a meeting will occur — the user chooses its currency holdings, considering the likelihood of meeting a seller who accepts the currency for payment, as well as the transaction costs and service prices involved. When such a meeting takes place, the user and the seller engage in bargaining over the service price. The seller then delivers the service in exchange for payment, and the user derives utility from the service.

In this micro-foundation, the medium-of-exchange and store-of-value functions of money complement each other. If a currency offers higher expected returns and serves as a better store of

[8] The the representative user's demand for digital money of sums over the demand from many individual users (across different locations) with potentially different demands for currencies A, B, and C. In particular, that the representative user buys currency x should be interpreted as some but not necessarily all users buying currency x. Hence, our modeling is consistent with some users (e.g., users within a certain country) having high needs for one currency in digital usage while others have low or no needs for that currency.

value, it becomes less costly for users to hold this currency "in advance" for payments, thereby reinforcing its role as a medium of exchange.[9] Likewise, as shown in Doepke and Schneider (2017) and Gopinath and Stein (2021), the unit-of-account function of money can be viewed as complementary to the medium-of-exchange and store-of-value functions.[10] Thus, while we link the convenience of money to its medium-of-exchange function, this convenience may also reflect other monetary functions. Thus, although our analysis focuses on payment competition, it could also apply more broadly to monetary competition across other dimensions.

Determinants of Convenience. As we show in Appendix E, particularly in equation (E.60), the convenience parameters Z_t^x and Y_t are determined by several factors: (i) the probability that a buyer encounters a seller who accepts the respective currency (reflecting the currency's level of acceptance), (ii) transaction costs — both monetary (e.g., fees charged by payment intermediaries) and utility costs (e.g., settlement delays) — and (iii) the user's bargaining power relative to sellers. We discuss each of these factors below and argue how they drive the convenience of fiat monies and PDM. Appendix E.5 presents a more detailed discussion.

First, (i) reflects a currency's payment technology (e.g., settlement speed), payment functionalities, and the scope of its payment applications and usability (i.e., the ability to handle specific types of payments, including digital platform or cross-border payments). Slow settlement speeds, as well as limited payment functionalities and usability (e.g, the inability to handle blockchain or digital platform transactions) reduce the "digital" convenience of fiat money, frictions that digitization can address. Note that (i) also captures the currency's overall level of acceptance and usage, which is subject to network effects. This implies that widely accepted currencies, such as the U.S. dollar, inherently provide high convenience; due to network effects, even minor improvements in payment technology or costs can amplify and significantly boost payment convenience.

Regarding (i), the digital payment convenience of representative PDM stems from technological factors such as settlement speed (e.g., fast payments using Alipay) and unique functionalities (e.g., smart contracting features in cryptocurrencies). Additionally, it arises from PDM's integration with digital platforms and ecosystems, where transactions often require PDM for settlement — for example, Alipay on the Alibaba platform or Ether on the Ethereum platform. Due to this

[9]Our micro-foundation of payment convenience relies on random search/matching subject to a "cash-in-advance constraint" (money-in-advance constraint) — under these assumptions, the store-of-value and medium-of-exchange functions of money arise as complements. In contrast, abstracting away from search but instead focusing on coordination, Goldstein, Yang, and Zeng (2023) establish a conflict between the store-of-value and medium-of-exchange functions of money.

[10]An example of the complementarity between medium-of-exchange and unit-of-account functions of money can be found in the U.S. dollar. The U.S. dollar is widely accepted as a means of payment because it is also widely adopted as a unit of account internationally, and vice versa.

integration, PDM offers unique payment convenience by enabling a wide range of digital platform transactions, including some that fiat money may not support. Moreover, adopting PDM for payments on a digital platform may provide benefits related to product bundling.[11]

Second, transaction costs (ii) charged by payment intermediaries, such as credit card fees or cross-border payment fees, and settlement delays, causing a utility cost of transacting, are likely key frictions that limit the convenience of fiat money. Fiat digitization can mitigate these frictions and transaction costs by reducing reliance on costly payment intermediaries, exerting competitive pressure on them (Duarte et al., 2022), and enabling faster payments. Certain features of cryptocurrencies and tokens (e.g., smart contracting or decentralization) and non-bank payment systems can also reduce dependence on costly payment intermediaries by bypassing traditional bank payment rails. These factors enhance the convenience of PDM relative to fiat money.

Third, regarding (iii), we argue that, following the findings in Garratt and Van Oordt (2021), enhanced payment privacy features can strengthen users' bargaining power relative to sellers.[12] As such, privacy features contribute to the convenience of cryptocurrencies and tokens. The digitization of fiat currency can also enhance privacy features, as, e.g., highlighted in Ahnert, Hoffmann, and Monet (2022) and Garratt, Yu, and Zhu (2022).

Currency Digitization and CBDC. We model currency digitization in a technology-neutral manner, recognizing that it can take various forms: It may involve an upgrade of bank-centric payment rails or the government-led introduction of new payment systems (e.g., Brazil's Pix), the launch of CBDCs, or other measures.[13] We interpret currency digitization as technological innovations that enhance the digital payment convenience of representative fiat currency. As discussed, fiat digitization can enable faster, more efficient transactions; broaden the scope of transactions (e.g., by enabling digital platform or cross-border payments) and payment functionalities; reduce reliance on costly intermediaries; and enhance privacy features. Digitization of currency $x = A, B$ is a one-time stochastic event at endogenous time T^x. When country $x = A, B$ digitizes its currency

[11] For example, using Alipay generates valuable data that enhances a user's access to loans offered on the Alibaba platform (Ouyang, 2021).

[12] In Garratt and Van Oordt (2021), firms use data collected through payments to price discriminate future consumers. Such price discrimination is akin to assuming a lower bargaining power that users have vis-a-vis sellers in our micro-foundation — this lower bargaining power translates into a lower currency convenience scale Z_t^x, Y_t. In contrast, enhanced privacy features strengthen bargaining power and increase a currency's convenience scale. See Appendix E for additional discussion.

[13] We model currency digitization and, specifically, CBDCs, as technology-neutral and are agnostic of the (technical) details on the design and implementation. See, e.g., Auer and Böhme (2020) and Duffie, Mathieson, and Pilav (2021) regarding the technical implementation of CBDCs, which is beyond the scope of the paper.

at time T^x, convenience scale Z_t^x increases:

$$Z_t^x = \begin{cases} Z_L^x & \text{for} \quad t < T^x \\ Z_H^x & \text{for} \quad t \geq T^x, \end{cases} \tag{3}$$

where $Z_H^x \geq Z_L^x > 0$. Z_t^x is public knowledge.

The digitization of fiat currency requires time, effort, investment, and incurs significant costs. To capture these features, we assume that the (stochastic) time T^x of currency digitization arrives according to an observable jump process $dJ_t^x \in \{0, 1\}$, with intensity $\frac{\mathbb{E}_t[dJ_t^x]}{dt} = e_t^x$. That is, the probability of successful digitization by country x (if it has not occurred yet) over $[t, t + dt]$ is $e_t^x dt$. Here, $e_t^x \geq 0$ is the endogenous, unobservable digitization effort/investment of country x, which entails a flow cost $g^x(e_t^x)$ for country x in consumption goods. Hence, we model the digitization process similar to an innovation project in Aghion and Howitt (1992) and related papers, which implies that currency digitization requires time: For instance, a constant effort level $e^x > 0$ would imply an expected time to successful digitization of $1/e^x$ for x. In what follows, we take the following linear-quadratic cost function: $g^x(e_t^x) = \phi e_t^x + \frac{\lambda (e_t^x)^2}{2}$ for parameters $\phi, \lambda \geq 0$; our results would go through under different (convex) cost functions too. We assume symmetry in costs across the two countries, while the benefits of digitization — captured by convenience scale parameters — differ across the two countries. The cost broadly captures the direct monetary expenses of digitization (e.g., implementation and development) as well as the indirect costs arising from disruptions to existing structures, such as the banking system (Whited et al., 2022).

Dominant versus Less Dominant Currencies. We refer to the country with the initially more convenient and more adopted currency as the "strong" country, and to the other country as the "weak" one. Without loss of generality, we set country A to be strong, in that $Z_L^A \geq Z_L^B$. One can think of the "dominant" currency A as the U.S. dollar, whose convenience also reflects its wide use internationally, while B is a relatively weaker, less dominant currency (e.g., Euro or RMB). We study the asymmetric competition between a dominant and a less dominant (but still widely used) fiat currency. Our baseline abstracts from "very weak" currencies, often of small, open economies, whose adoption remains low regardless of digitization, leading to negligible or no digitization efforts. See Section 3.3 for a study of very weak currencies.

Private Digital Money (PDM). As will become clear, Y_t is linked to PDM adoption and captures the competition from PDM that fiat money faces. Therefore, we may refer to high (low) Y_t as strong (weak) PDM competition. We model the growing competition from PDM by assuming that Y_t grows *endogenously* according to:

$$\frac{dY_t}{Y_t} = \mu m_t^C dt, \tag{4}$$

for $\mu > 0$.[14] As detailed in our micro-foundation of convenience utility in Appendix E, the gradual increase in PDM convenience may reflect technological advancements, the expansion of digital platforms and ecosystems (broadening PDM use cases), or the launch of new forms of PDM offering unique payment functionalities and convenience.

Importantly, according to (4), the growth rate of Y_t increases with PDM's adoption level m_t^C, so that higher PDM adoption in the present boosts PDM adoption in the future. In particular, convenience of and competition from PDM increase more rapidly when PDM adoption m_t^C is high and so fiat adoption $1 - m_t^C$ is low. This implies that PDM emerges primarily when fiat currencies provide limited digital payment convenience, thereby leaving a gap in the digital payment space that PDM fills by offering superior convenience, driving its adoption for payments.

Unlike the convenience of fiat money (Z_t^x), which follows a jump process, the convenience of PDM changes according to (4). This is because the digitization of fiat money—such as through the launch or upgrade of payment systems or the introduction of a CBDC—occurs infrequently and represents significant disruptions to existing structures and payment systems. In contrast, PDM encompasses various payment systems and digital currencies, each evolving over time, with some achieving breakthroughs. When aggregated, the convenience of PDM evolves more gradually than that of fiat money. See Appendix E.7 for further discussion.

We assume that the potential convenience of PDM is bounded, in that $Y_t \leq \overline{Y}$ for some exogenous constant $\overline{Y} > 0$. This assumption ensures that PDM convenience cannot fully outgrow the convenience of fiat currencies and so cannot gain full dominance; it is also helpful for solving the model, as it yields a well-defined boundary condition for the ODE system characterizing the Markov equilibrium; one can take $\overline{Y}$ arbitrarily large. Formally, the drift of dY_t vanishes as it reaches $\overline{Y}$ (i.e., $dY_t = 0$ if $Y_t = \overline{Y}$) while (4) holds for $Y_t < \overline{Y}$. We set $Y_0 > 0$. Both the assumptions of a time-increasing yet bounded Y_t and that Z_t^x jumps up only once are made for simplicity and tractability, but they could be relaxed.

In practice, the usability, convenience, and growth rate of PDM may depend on whether it is banned or regulated by governments; however, such regulation might not be feasible because

[14] Eq. (4) should capture the average, long-run growth of PDM, which may be interrupted by crashes or setbacks. Our results are robust to the specific growth path of Y_t, as long as Y_t grows over time on average. We could allow for occasional setbacks/crashes that arrive according to a Poisson process, or add a Brownian component or a negative drift component to the law of motion in (4) (which can also generate crashes and setbacks).

PDM operates outside the banking system. Although not modeled, regulation (by one or multiple countries) could be interpreted as a factor that reduces Y or μ, so Y and μ should be understood as net of the effects of regulation or a ban.[15] Section 2.4.1 studies comparative statics in μ, Y.

Objective Function and Optimization. At any time t, country $x = A, B$ chooses its effort (taking the effort of the other country as given) to maximize:

$$V_t^x = \max_{(e_s^x)_{s \geq t}} \mathbb{E}_t^x \left[\int_t^\infty e^{-\delta(s-t)} \left[\delta f_s^x - g(e_s^x) \right] ds \right], \tag{5}$$

where $\mathbb{E}_t^x[\cdot]$ denotes the time-t expectation from the perspective of country x (which is conditional on time-t public information and effort $(e_s^x)_{s \geq t}$). In (5), f_s^x is a flow payoff that may depend on state variables or currency adoption levels. In what follows, we take $f_t^x = P_t^x$; we also scale this flow payoff by δ in the objective, which has no bearing on our key findings. See Section 3.4 for an analysis of the effects of δ.

Thus, country x maximizes a time average of its currency's adoption and usage in digital payment, net of the costs of digitization.[16] We abstract from monetary policy, other governmental or central bank considerations, and broader macroeconomic factors. Instead, we stipulate that countries digitize their currencies to obtain a stake, influence, and control in the digital payment space by promoting the adoption and relevance of their currency. Our modeling of countries' objective functions aligns with recent empirical evidence from Berg et al. (2024), which suggests that a key benefit — and potential motive — for launching CBDCs is to enhance payment autonomy. This includes maintaining the relevance of fiat currency in digital payments and reducing reliance on foreign or non-bank payment providers. Similarly, Brunnermeier et al. (2019) argues that digitizing currencies may be necessary to preserve the adoption of fiat money (e.g., bank deposits) in the digital economy, a concern for many nations. Public fiat money anchors the monetary system, but this role depends on its widespread adoption. Ensuring the use of fiat money in digital payments (see, e.g., Ahnert, Assenmacher, Hoffmann, Leonello, Monnet, and Porcellacchia, 2022) is a key concern driving CBDC initiatives. Thus, countries' efforts to maximize currency adoption reflect their goal to preserve the relevance and anchoring role of fiat money.

Our modeling aligns with the growing literature on geoeconomics (Clayton et al., 2023; Clayton,

[15] PDM may also offer unique convenience, therefore, compete with fiat money in digital payment even when regulated or banned. As such, countries might need to react to PDM competition through digitization rather than bans and regulation.

[16] While it is natural to take the linear specification $f_t^x = P_t^x$, the results would remain similar under a monotonic transformation of P_t^x as flow payoff. Likewise, not scaling the flow payoff by δ, for instance, by setting $f_t^x = 1/\delta P_t^x$ would lead to similar results.

Maggiori, and Schreger, 2024; Pflueger and Yared, 2024), which examine the sources, value, and privileges of geoeconomic power and hegemony. Consistent with this literature, our framework captures countries' strategic pursuit of influence through the widespread adoption of their currencies in digital and international payments. A widely adopted currency and its digital payment system confer geoeconomic power, enabling a country to influence others and extract economic benefits. For instance, the United States derives significant power from the dollar-based payment system.

Taken together, our baseline objective function is consistent with the aforementioned intuitive arguments, studies, empirical observations, as well as policy debates. Section 2.5 contrasts the baseline to the planner solution as an alternative objective, where a planner maximizes overall welfare as we define next.

Welfare. Overall welfare derives from consumption utility and currency convenience net of the costs of digitization. We note that any cohort's consumption is fixed and determined by the exogenous endowment. Instead, the currency convenience and the costs of digitization are endogenous. When analyzing welfare, we solely focus on the endogenous part, i.e., on total convenience utility net of digitization costs (discounted over time), which reads

$$W_t = \mathbb{E}_t\left[\int_t^\infty e^{-\gamma(s-t)}\Big[\gamma\big(Z_s^A v(m_s^A) + Z_s^B v(m_s^B) + Y_s v(m_s^C)\big) - g^A(e_s^A) - g^B(e_s^B)\Big]ds\right], \tag{6}$$

where $\gamma > 0$ is the exogenous discount rate in the welfare function. In line with the stipulation of (5), we scale the convenience utility flow by γ, but this has no bearing on the model implications. Note that γ captures how much welfare weight is put on current versus future convenience utilities. Observe that because we assume $f_t^x = P_t^x$ in the objective (5), countries maximize a time average of their currencies' digital adoption instead of the convenience utility generated by their currencies. As such, equilibrium digitization efforts are generally not welfare-maximizing, as we also show in Section 2.5, where we characterize welfare-maximizing efforts.

Equilibrium Concept. We study a dynamic game with two large, strategic players, that is, countries A and B, and a non-strategic player, that is, the price-taking OLG user, which can equivalently be interpreted as a mass of atomistic players. The key difference between fiat money and PDM is that the countries, issuing fiat money, strategically act to increase their currency's convenience through digitization, whereas the convenience of PDM evolves endogenously according to the pre-determined law of motion (4). Importantly, due to the convex cost of effort, countries cannot launch CBDC immediately by setting $e_t^x = +\infty$, for instance, to react to a competitor's launch, as this would lead to infinite costs.

We solve for a Markov equilibrium of this dynamic game in the continuous time limit $dt \to 0$. Let $z \in \{0, A, B, AB\}$ denote which countries have digitized their currencies up to date. Specifically, $z = 0$ means that no country has digitized its currency, $z = x \in \{A, B\}$ means that only country x has digitized, and $z = AB$ means that both countries have digitized. We characterize a Markov equilibrium with state variables (Y, z), so that all equilibrium quantities can be expressed as functions of (Y, z). In a Markov equilibrium, at any time $t \geq 0$, cohort t users choose the holdings of currencies A, B, C to maximize the expected utility U_t from (2), given prices (P_t^A, P_t^B, P_t^C). The markets for all currencies clear, i.e., $m_t^x = P_t^x$ for $x = A, B, C$. And, both countries A and B choose their efforts according to (5), taking the effort of the other country as given, while Y_t evolves according to (4).

2 Model Solution and Analysis

2.1 Solving for the Markov Equilibrium

We define expected returns of currency x in terms of the consumption good as:

$$r_t^x = \frac{\mathbb{E}_t[dP_t^x]}{P_t^x dt}, \tag{7}$$

where $\mathbb{E}_t[\cdot]$ denotes the time-t expectation, which is conditional on all public information that is available at time t. Notice that r_t^x is the expected rate of appreciation of currency x in terms of consumption good. That is, if $r_t^x > 0$, currency x is expected to appreciate and, if $r_t^x < 0$, currency x is expected to depreciate relative to the consumption good. In the Markov equilibrium, r_t^x is endogenous and a function of (Y, z), i.e., $r_t^x = r^x(Y, z)$.

Next, we can write cohort t's consumption c_{t+dt} at $t + dt$ as:

$$c_{t+dt} = \sum_{x \in \{A,B,C\}} \frac{m_t^x P_{t+dt}^x}{P_t^x}. \tag{8}$$

Basically, cohort t's consumption consists of the proceeds from selling their nominal holdings of currency x, m_t^x / P_t^x, at price P_{t+dt}^x to cohort $t + dt$. We can write $P_{t+dt}^x = P_t^x + dP_t^x$ and, inserting this relation into (8), we obtain:

$$c_{t+dt} = \sum_{x \in \{A,B,C\}} m_t^x + \sum_{x \in \{A,B,C\}} \frac{m_t^x dP_t^x}{P_t^x}. \tag{9}$$

Because cohort t only derives utility from consuming at time $t + dt$, it is optimal to use the entire endowment of one unit of consumption good to purchase money at time t, so that $\sum_{x\in\{A,B,C\}} m_t^x = 1$ must hold for given prices (P_t^A, P_t^B, P_t^C) (see (1)). As a result, cohort t maximizes:

$$\max_{m_t^A, m_t^B, m_t^C \geq 0} U_t \quad \text{s.t.} \quad \sum_{x\in\{A,B,C\}} m_t^x = 1, \tag{10}$$

taking (P_t^A, P_t^B, P_t^C) as given. With (2), (9), and $\sum_{x\in\{A,B,C\}} m_t^x = 1$, the objective in (10) becomes:

$$U_t = 1 + \sum_{x\in\{A,B,C\}} m_t^x r_t^x dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt. \tag{11}$$

The first two terms represent cohort t's expected consumption at time $t + dt$. Observe that in equilibrium, consumption equals the fixed endowment of cohort $t + dt$, in that, as we show, $\sum_{x\in\{A,B,C\}} m_t^x r_t^x dt = 0$. The last three terms represent the convenience utility to holding currencies. Recall that a micro-foundation of (11) is provided in Appendix E where we model payments and link convenience to the medium-of-exchange function of money. In light of $\sum_{x\in\{A,B,C\}} m_t^x = 1$, it must hold at any optimum that:

$$\frac{\partial U_t}{\partial m_t^A} = \frac{\partial U_t}{\partial m_t^B} = \frac{\partial U_t}{\partial m_t^C}, \tag{12}$$

provided $m_t^x \in (0, 1)$. That is, in equilibrium, the user (which takes prices as given) is on the margin indifferent between substituting a unit of currency x towards another currency $-x$. This relationship implies the following equilibrium pricing equations:

$$Y_t v'(m_t^C) + r_t^C = Z_t^A v'(m_t^A) + r_t^A = Z_t^B v'(m_t^B) + r_t^B. \tag{13}$$

Condition (13) states that in equilibrium, the sum of the marginal convenience utility and expected appreciation must be equal across currencies. Due to $\lim_{m_t^x \to 0} v'(m_t^x) = \infty$, optimal currency holdings $m_t^x = P_t^x$ satisfy $m_t^x, P_t^x \in (0, 1)$ for $x = A, B, C$. In a Markov equilibrium with state variables (Y, z), we can write $m_t^x = P_t^x = m^x(Y, z) = P^x(Y, z)$ for $x = A, B, C$ as well as $r_t^x = r^x(Y, z)$, so that (13) will depend on (Y, z) only.

Next, we characterize countries' time-t value function from (5) as well as the optimal levels of efforts. By the dynamic programming principle, the governments' value function V_t^x from (5) satisfies the HJB equation (for $x = A, B$):

$$\delta V_t^x = \max_{e_t^x \geq 0} \left(\delta f_t^x - \frac{\lambda (e_t^x)^2}{2} - \phi e_t^x + \frac{\mathbb{E}_t^x[dV_t^x]}{dt} \right), \tag{14}$$

where f_t^x is the flow payoff that countries derive — set to $f_t^x = P_t^x$ in our baseline. Again, in a Markov equilibrium with state variables (Y, z), we can express V_t^x as a function of (Y, z) only, i.e., $V_t^x = V^x(Y, z)$ for $x = A, B$. As optimal effort e_t^x is determined according to the HJB equation (14), it depends on the government's value function $V_t^x = V^x(Y, z)$ and adoption $P_t^x = P^x(Y, z)$. Since V_t^x and P_t^x are functions of (Y, z) only, the optimal effort is a function of (Y, z) too, in that $e_t^x = e^x(Y, z)$. Indeed, as shown in Appendix A, one solves for the Markov equilibrium by conjecturing and then verifying that equilibrium quantities are functions of (Y, z) only.

As shown in Appendix A, effort satisfies for $x = A, B$ (correspondingly $-x = B, A$):

$$e^x(Y, 0) = \frac{\left[V^x(Y, x) - V^x(Y, 0) - \phi\right]^+}{\lambda} \quad \text{and} \quad e^x(Y, -x) = \frac{\left[V^x(Y, AB) - V^x(Y, -x) - \phi\right]^+}{\lambda}. \tag{15}$$

where $[\cdot]^+ = \max\{\cdot, 0\}$. In addition, $e^x(Y, AB) = e^x(Y, x) = 0$, i.e., efforts become zero after successful digitization. Because countries maximize a time average of their currency's adoption net of digitization costs, optimal digitization efforts reflect the potential gain in adoption, as well as the persistence of this effect — all of which are captured by $V^x(Y, x) - V^x(Y, 0)$ in state $z = 0$ and by $V^x(Y, AB) - V^x(Y, -x)$ in state $z = x$. In particular, when current adoption is low (high) in $z = 0$, then $V^x(Y, 0)$ tends to be low (high), boosting the incentives and, therefore, efforts to digitize. Also note that optimal digitization efforts can be zero (even in state $z = 0$), in which case countries abandon their initiatives to digitize currency.

Finally, welfare W_t can be written as $W_t = W(Y, z)$ satisfying the HJB equation

$$\gamma W_t = \sum_{x=A,B} (\gamma Z_t^x v(m_t^x) - g^x(e_t^x)) + \gamma Y_t v(m_t^C) + \frac{\mathbb{E}_t[dW_t]}{dt}, \tag{16}$$

given m_t^x and efforts e_t^A and e_t^B chosen by countries. Section 2.5 later characterizes the solution when efforts are chosen to maximize welfare. We summarize our findings:

Proposition 1. *In the Markov equilibrium with state variables (Y, z), the following holds:*

1. *Users invest their entire endowment in currencies, i.e., (1) holds. The markets for all currencies clear, so that $m_t^A = P_t^A$, $m_t^B = P_t^B$, $m_t^C = P_t^C$.*

2. *Optimal currency adoption levels m_t^x, P_t^x for $x = A, B, C$ satisfy $m_t^x, P_t^x \in (0, 1)$. The equi-*

librium pricing condition (13) holds, and government value functions V_t^A and V_t^B solve the HJB equation (14). Welfare W_t solves the HJB equation (16).

3. *For $x = A, B, C$ and $(Y_t, z_t) = (Y, z)$, currency adoption satisfies $P_t^x = m_t^x = P^x(Y, z) = m^x(Y, z)$, expected returns satisfy $r_t^x = r^x(Y, z)$, value functions satisfy $V_t^x = V^x(Y, z)$ for $x = A, B$, welfare satisfies $W_t = W(Y, z)$, and efforts satisfy $e_t^A = e^A(Y, z)$ and $e_t^B = e^B(Y, z)$ according to (15).*

4. *The Markov equilibrium is characterized by a system of coupled first order ODEs and non-linear equations, all of which are presented in Appendix A.5.*

Appendix A provides a detailed characterization of the model solution and the Markov equilibrium in terms of a system of coupled ODEs that describe the dynamics of the currency adoption $P^x(Y, x) = m^x(Y, x)$, governments' value functions $V^A(Y, z)$ and $V^B(Y, z)$, digitization efforts $e^A(Y, z)$ and $e^B(Y, z)$, and welfare $W(Y, z)$. Note that because our dynamic game features two state variables and asymmetric competition among currencies, there is no analytical solution and we do not provide formal existence and uniqueness arguments. The Markov equilibrium needs to be solved numerically.

2.2 Numerical Solution and Parameter Choice

Similar to Krishnamurthy and Vissing-Jorgensen (2012), Cong et al. (2021), or Gryglewicz et al. (2021), we pick for the convenience utility the functional form $v(m) = \frac{m^{1-\eta}}{1-\eta}$, where $\eta \in (0, 1)$ ensures $v(m) > 0$ for $m > 0$; we pick $\eta = 0.9$. We set $\delta = \gamma = 0.1$, and normalize $Z_L^A = 1$. Further, we choose $Z_L^B = 0.2$, i.e., currency B is initially less convenient than the dominant currency A. The dynamics of PDM competition are characterized through $Y_0 = 0.025$, $\overline{Y} = 5$, and $\mu = 0.2$.[17] Digitization improves the convenience of currency $x = A, B$ according to $Z_H^x = \Delta^{Fixed} + (1 + \Delta^{Prop})Z_L^x$ where we stipulate $\Delta^{Fixed} = \Delta^{Prop} = 1$. That is, currency digitization increases the convenience Z_t^x both by a fixed amount and proportionally relative to the base level Z_L^x. The flow cost of digitization satisfies $g^x(e^x) = \phi e^x + \frac{\lambda(e^x)^2}{2}$ for $\phi = 0.15$ and $\lambda = 1$. Importantly, the model's implications, which are qualitative in nature, are robust to the choice of these parameters, as we verify and the following analysis also highlights.[18]

[17] Since the drift of Y in (4) is always positive, the lower bound of the state space, Y_0, does not imply relevant boundary conditions for the ODE system, characterizing the equilibrium. Consequently, the value of Y_0 has no impact on the equilibrium values of model quantities in states $Y > Y_0$. We just picked "relatively low" Y_0 for illustrating the equilibrium over a large state space.

[18] Due to the lack of closely related quantitative studies and extensive historical data on CBDC issuance and currency digitization and, more generally, due to the forward-looking nature of our analysis, there is no straightforward way

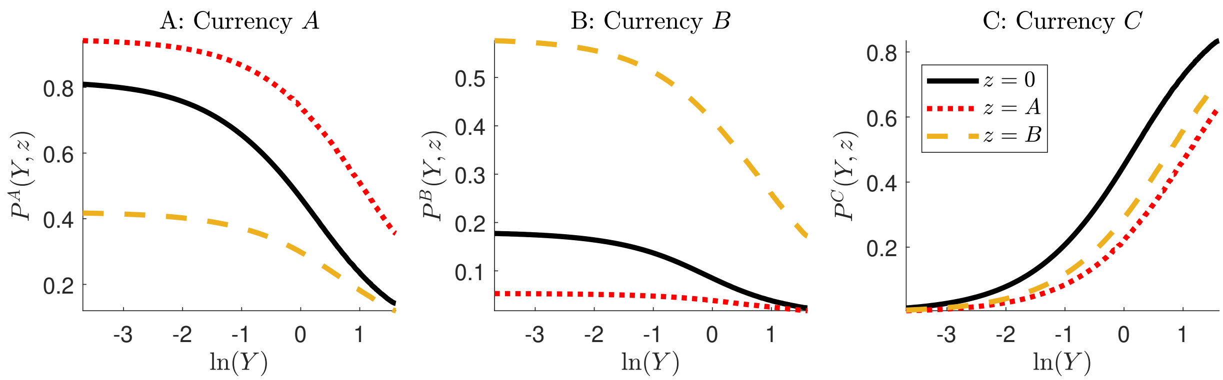

Figure 1: The Dynamics of Adoption. This figure plots the adoption levels of currencies A, B, and C in Panels A, B, and C, respectively against $\ln(Y)$ in states $z = 0, A, B$. The solid black line depicts $z = 0$, the dotted red line depicts state $z = A$, and the dashed yellow line depicts $z = B$. We use our baseline parameters from Section 2.2.

Figure 1 illustrates the dynamics of currency adoption (values) by plotting $P^x(Y, z) = m^x(Y, z)$ as a function of $\ln(Y)$ — which is a monotonic transformation of Y (we use it for the sake of illustration, simply because Y grows exponentially) — over the entire range $[\ln(Y_0), \ln(\overline{Y})]$ in states $z = 0, A, B$. Recall that Y and, therefore, $\ln(Y)$ increase over time. Naturally, the adoption of currencies A and B declines with PDM convenience Y, while PDM adoption increases in Y. As such, Y or, equivalently, $\ln(Y)$ quantifies PDM adoption and dominance, as well as the competition that fiat currencies face from PDM.

Panel C shows that digitization by country x, representing a shift from $z = 0$ to $z = x$, spurs the adoption of currency x but reduces the adoption of currency $-x$ and PDM. Since C's adoption is always higher in state $z = B$ than in state $z = A$, the digitization of the dominant currency A harms C relatively more. Interestingly, as can be seen on Panel A, digitization of the less dominant currency B has a relatively large, negative effect on the adoption of currency A when Y is small, but this effect diminishes for larger values of Y. Indeed, when Y is low, currency A's primary competitor is B, especially so after B is digitized. When Y is large, PDM competes away market share from A and B, limiting the effects of B's digitization and making C the main competitor of A. Panel B shows that currency B's adoption increases the most from digitization when Y is small, whereas, for A, digitization yields larger adoption gains for larger values of Y. As discussed next, these patterns shape countries' incentives to digitize their currencies or upgrade payment systems.

to rigorously calibrate the model and to make quantitative predictions. However, the model's outcomes, which are qualitative in nature, are robust across various parameter configurations.

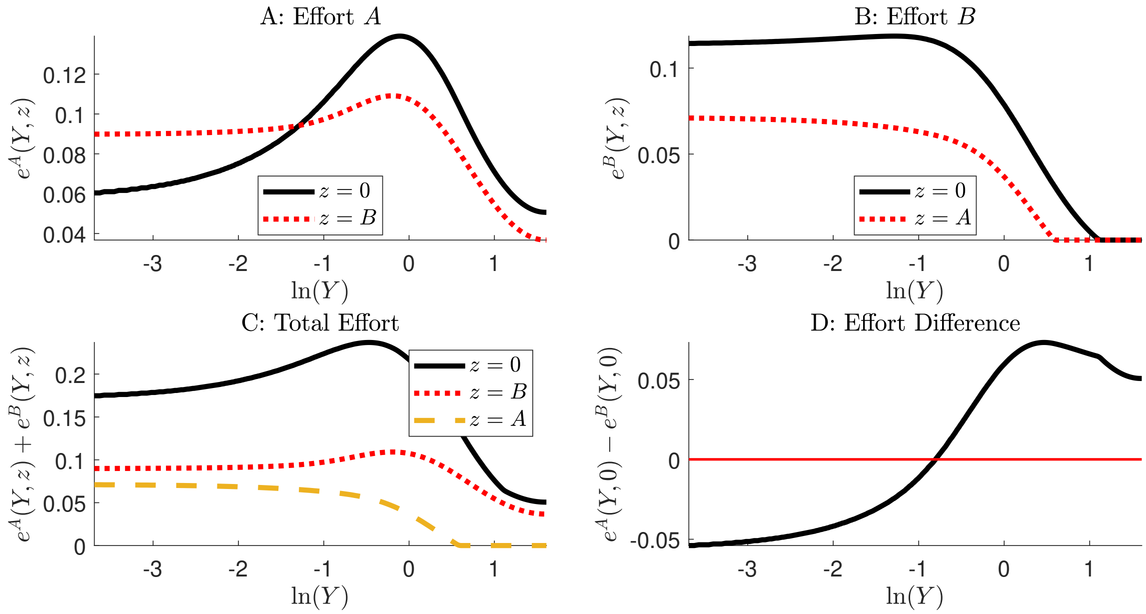

Figure 2: The Dynamics of Digitization Efforts. Panel A depicts A's effort as a function of $\ln(Y)$ in states $z = 0$ (solid black line) and $z = B$ (dotted red line). Panel B depicts B's effort as a function of $\ln(Y)$ in states $z = 0$ (solid black line) and $z = A$ (dotted red line). Panel C plots the sum of efforts in states $z = 0$ (solid black line), $z = B$ (dotted red line), and $z = A$ (dashed yellow line), while Panel D plots their difference in state $z = 0$. We use our baseline parameters from Section 2.2.

2.3 The Dynamics of Currency Digitization

We investigate the dynamics of currency digitization initiatives against the backdrop of growing competition from PDM. To this end, Figure 2 plots outcomes as a function of $\ln(Y)$, both in state $z = 0$ (solid black line) and in states $z = B, A$, respectively (dotted red line). According to (4), Y increases over time and PDM gradually gains adoption. Therefore, Figure 2 also depicts the time dynamics of currency digitization, highlighting how efforts change as fiat currencies face more competition from PDM over time.

2.3.1 Effort Dynamics and Strategic Interactions

To begin with, observe from Panels A and B of Figure 2 that in state $z = 0$, the efforts (i.e., digitization incentives) of the strong country A follow an inverted U shape in $\ln(Y)$, while the efforts of country B decrease in $\ln(Y)$. Panel A shows that digitization by B, i.e., a move from $z = 0$ to $z = B$, increases A's efforts for low Y, while decreasing them for high Y. In contrast, the digitization of A always reduces B's efforts, as shown in Panel B. Panel C shows that countries' total efforts in state $z = 0$, that is, the sum of individual efforts, tend to decrease in $\ln(Y)$. In addition, Panel D reveals that country B's incentives to digitize its currency are highest and exceed those of A at the beginning of the game, i.e., for low levels of Y when competition from PDM is weak. In contrast, later in the game, i.e., for higher levels of Y, A has stronger incentives than B.

To gain intuition, recall from (5) that countries maximize the time average of their currencies' adoption, net of the cost of digitization. As such, the incentives to digitize currency reflect the potential increase in adoption upon digitization, as well as the persistence of this effect. Digitization increases currency B's convenience and adoption in the future. This effect is relatively larger and more persistent when competition from PDM is weak (i.e., Y is small) and currency A has not been digitized yet (i.e., in state $z = 0$). At the same time, in state $z = 0$ and for low levels of Y, the current level of adoption of B is low compared to that of A, implying higher adoption gains upon digitization relative to the status quo. Therefore, B's digitization efforts are highest for low levels of Y in state $z = 0$ and exceed those of A. However, B's efforts taper off as Y increases and the gains of digitization diminish due to PDM's strength and wider adoption. Notably, under our baseline parameters, B sets $e^B(Y, 0) = 0$ and even stops the digitization process for large Y. Likewise, the digitization of currency A limits the adoption gains from digitizing currency B, reducing B's incentives in state $z = A$ relative to $z = 0$ for any Y (see Panel B).

In state $z = 0$, country A's digitization efforts are highest for intermediate levels of PDM competition and Y. For low levels of Y, currency A's adoption is high reflecting its dominance, which limits the additional adoption that A can gain upon digitization. However, as competition from PDM intensifies, A's adoption decreases and, therefore, the gains from digitization rise, increasing A's effort incentives. Finally, when Y becomes sufficiently large, A's current level of adoption is low, but digitization yields low benefits due to PDM's dominance, which again limits A's incentives to digitize. As a result, A's incentives to digitize first increase and then decrease in Y, resulting in the inverted U-shaped pattern. In sum, country A's incentives peak when PDM threatens dominance but is not yet entrenched. As discussed in the next Section, A's incentives to digitize also reflect the dynamic component that digitization hampers the further growth in PDM convenience Y. This effect generates a motive to respond early to competition from PDM, especially when this competition is expected to grow fast. More generally, country A has high incentives to digitize its currency, when its dominance is challenged by PDM or a competing fiat currency and digitization allows to reassert its dominance.

Taken together, country B, with the less dominant currency, enjoys an endogenous first-mover advantage in currency digitization, while country A with the dominant currency has a second-mover advantage. In particular, B possesses strong incentives to move first or early in digitizing its cur-

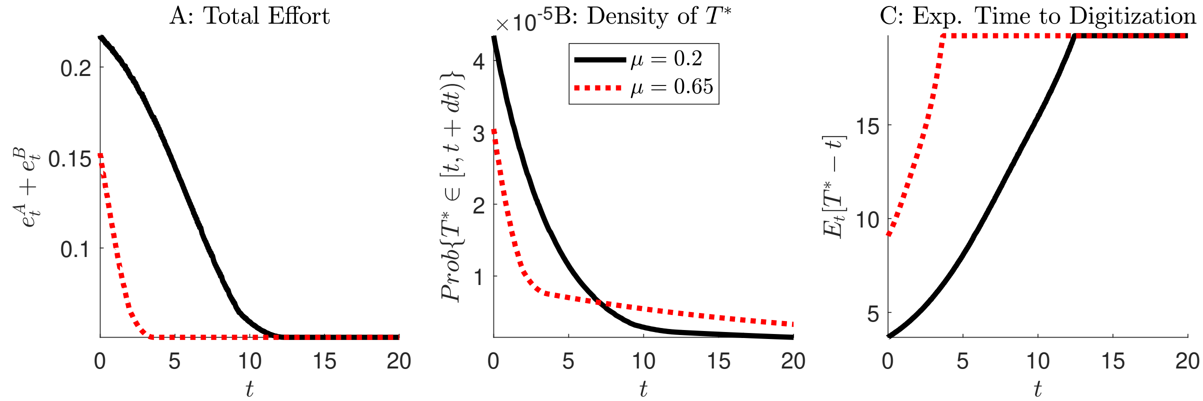

Figure 3: The Dynamics and Timing of Digitization. This Figure depicts time dynamics, conditional on remaining in state $z = 0$. Panel A plots countries total efforts, $e_t^A + e_t^B = e^A(Y_t, 0) + e^B(Y_t, 0)$ against calendar time t, conditional on remaining in state $z = 0$, for $\mu = 0.2$ (solid black line) and $\mu = 0.65$ (dotted red line). Panel B plots the probability density function of $T^* = \min\{T^A, T^B\}$, i.e., the first time of digitization, against t, for $\mu = 0.2$ (solid black line) and $\mu = 0.65$ (dotted red line). Panel C depicts the expected time to digitization, being in $z = 0$ and at time t. We use our baseline parameters from Section 2.2, but set $Y_0 = 1$.

rency. Indeed, Panel D shows that initially, i.e., for low levels of Y and $z = 0$, the digitization efforts of B exceed those of A. However, both the increasing competition from PDM and the digitization of currency A effectively remove this first-mover advantage, thereby reducing B's digitization incentives and effort. In contrast, A's incentives to digitize are initially low, but increase over time with the growing competition from PDM or with the digitization of currency B. When B is being digitized and Y is low, A's incentives to digitize increase, meaning that A has incentives to move second in currency digitization. Indeed, as B challenges A through digitization, A's adoption and dominance decline, boosting A's incentives to digitize its currency as well. This effect vanishes, however, when competition from PDM is strong, limiting gains from digitization.

Importantly, fiat currency digitization efforts can endogenously emerge as either strategic complements or substitutes. For country B, digitization by the dominant country A acts as a strategic substitute, consistently reducing B's incentives to digitize. This occurs because B's incentives are driven by a first-mover advantage, which vanishes once A digitizes. In contrast, for country A, digitization by B initially serves as a strategic complement, increasing A's incentives to digitize when Y is small, reflecting a second-mover advantage. However, when Y is large and PDM competition intensifies, the digitization of B becomes a strategic substitute, reducing A's efforts because the growing dominance of PDM erodes the second-mover advantage.

2.3.2 When Do Countries Digitize Fiat Currencies?

Although countries' digitization efforts are initially high (see Panel C of Figure 2), they tend to decrease over time and may even cease altogether if early success is not achieved and PDM gains widespread adoption. In the beginning, the weaker country accounts for most of the digitization efforts, reflecting an endogenous first-mover advantage, whereas the stronger country generally reacts later by increasing its effort, reflecting a second-mover advantage. This finding aligns with the observation that relatively less dominant currencies (such as the Renminbi) are being digitized first through CBDC, while the United States is delaying or halting CBDC development.

Figure 3 illustrates the dynamics of currency digitization over time t, starting from $Y_0 = 1$ for two different levels of μ.[19] Panel A shows that countries' total efforts $e_t^A + e_t^B$ in state $z = 0$ decline over time. This finding is consistent with the dynamics of CBDC initiatives around globe, many of which have been slowed down, stalled, or stopped despite initial enthusiasm.

Next, Panel B of Figure 3 plots the probability density function of $T^* = \min\{T^A, T^B\}$, representing the first time of currency digitization, over time in state $z = 0$. Panel C plots in state $z = 0$ the expected time to first digitization against time t, measuring the average time it takes for currency digitization to occur (being in state $z = 0$ at time t). Appendix D shows how to calculate the expected time to digitization and the probability density function of T^*. Observe that the density of T^* is unimodal and decreases in t, while the expected time to digitization increases in t.[20] Intuitively, Panel A illustrates the time-t conditional probability of digitization over $[t, t + dt)$, i.e., $Prob_t\{T^* \in [t, t + dt)\} = (e_t^A + e_t^B)dt$, while Panel B shows the unconditional probability $Prob_0\{T^* \in [t, t + dt)\}$. The solid black (dotted red) line depicts $\mu = 0.2$ ($\mu = 0.65$).[21]

Because overall efforts tend to decrease over time (Panel A), the model predicts relatively early digitization. If it indeed occurs early, digitization, especially by the dominant currency, hampers PDM adoption and its future growth. However, if digitization is not achieved early on, it is delayed significantly or may never occur, as digitization efforts decline over time. Consequently, the density features a large probability mass close to $t = 0$ (including a maximum at $t = 0$), and decreases with t. In addition, the time-t expected time to digitization in Panel C increases in t. Thus, initial failures or setbacks in the digitization process increase, rather than decrease, the expected time to

[19] For the sake of illustration, we pick higher Y_0 than in other figures to obtain more meaningful time dynamics, since the growth of Y is very slow for low levels of Y_0 (according to (4)) and the illustration of the results would require many periods of time.

[20] There is still a probability mass of about 10% that $T^* > 20$, which is not depicted here. For larger values of ϕ, countries' effort(s) may become zero and the digitization is stalled altogether. In this case, the distribution (density) features an atom of probability at $+\infty$, i.e., $Prob\{T^* = +\infty\} > 0$. Under these circumstances, digitization occurs either relatively early or never.

[21] We note the patterns are robust to changes in μ; we discuss the effects of μ in greater detail in the next Section.

successful digitization.

In particular, failure to digitize fiat money early creates a vacuum in the digital payment space, which PDM fills. As PDM adoption grows due to a lack of digitization, countries' digitization efforts diminish or may cease altogether, delaying or preventing digitization. This can lead to a long-run equilibrium where fiat currencies play a diminished role and PDM dominates digital payments. Therefore, our model suggests that the long-term relevance of fiat money in digital payments depends on whether countries act early to digitize it.

2.4 Does Currency Competition Spur Digitization?

We now analyze how the competition from PDM and among fiat currencies shape the dynamics of currency and payment digitization, the timing of CBDC issuance, and overall welfare. Our findings suggest that while competition from PDM slows digitization, competition among fiat currencies accelerates it. Notably, both forms of competition contribute to overall welfare by enhancing convenience utility over time. These differential effects reflect that the nature of increased competition has distinct effects on the endogenous growth of PDM, which accelerates as PDM adoption rises. Stronger fiat currencies curb PDM adoption and growth, encouraging countries to sustain their digitization efforts and ultimately boosting digitization. In contrast, increased competition from PDM — both in the short term (due to higher current adoption) and over time (due to faster growth in PDM convenience) — may initially boost digitization efforts but gradually undermines them, as it accelerates PDM growth and its path to dominance, ultimately limiting the benefits of digitization for both countries.

2.4.1 The Effects of Competition from Private Digital Money and Regulation

The level of Y quantifies the competition that fiat currencies face from PDM in the present, thereby affecting countries' incentives to digitize their currency. The law of motion (4) implies that higher PDM adoption today increases Y and PDM adoption in the future, adding a dynamic component to currency digitization considerations. When countries digitize their currencies, they reduce PDM adoption in the present, reducing growth of PDM convenience, competition, and adoption in the future. The incentives arising from this dynamic channel depend on the parameter μ, governing growth of PDM convenience and thus fiat currencies' (dynamic) competition from PDM.

As can be seen from Figure 2, an increase in competition from PDM through higher Y reduces B's digitization efforts, while having an ambiguous effect on A's incentives, which follow an inverted U-shaped pattern in $\ln(Y)$. Recall that overall, an increase in PDM competition through higher Y

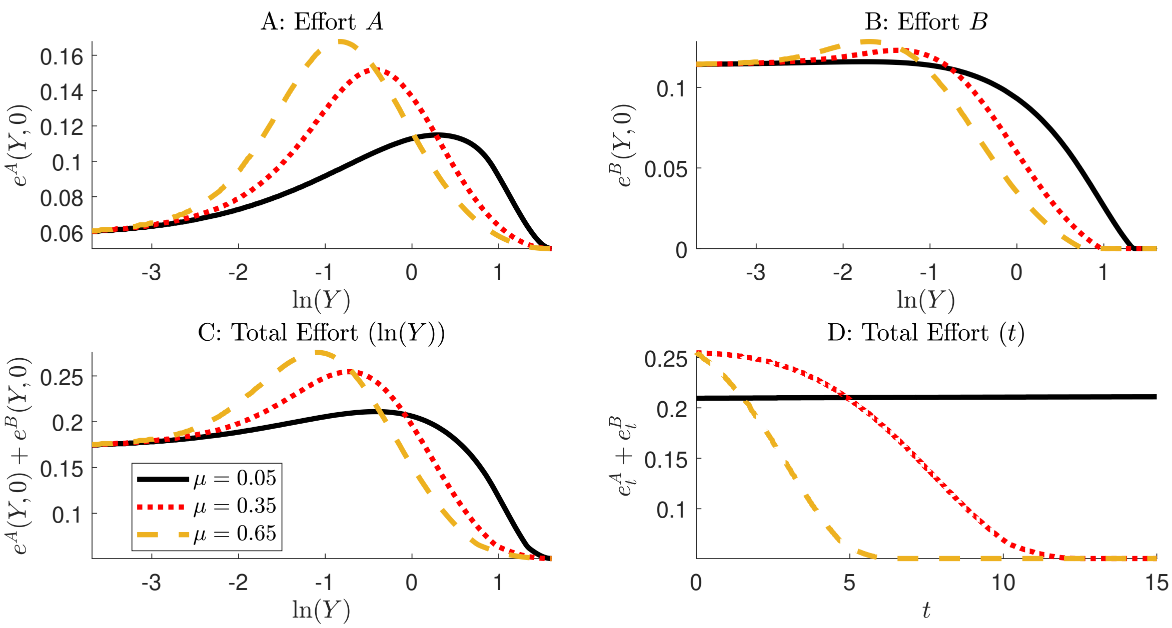

Figure 4: Effort Dynamics and PDM Competition. This Figure presents comparative statics in μ. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of μ. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. Panel D plots total efforts against time, t, being state $z = 0$. We use our baseline parameters from Section 2.2 (incluing $Z_L^A = 1$), but set $Y_0 = 0.5$ for Panel D.

reduces total digitization efforts. To analyze how the growing competition from PDM affects the *dynamics* of digitization, we perform comparative statics in μ. Figure 4 plots country A's efforts (Panel A), country B's efforts (Panel B), and their sum against $\ln(Y)$ in Panel C and over time in Panel D, for $\mu = 0.05$, $\mu = 0.35$, and $\mu = 0.65$. Panel A shows that as μ increases, A's digitization efforts increase for low Y, peaking at a lower level of Y.

When PDM competition grows faster and μ is larger, A responds more strongly and earlier to the rising competition from PDM, i.e., its incentives become more forward-looking. Indeed, when μ is larger, country A is incentivized to digitize its currency early to slow PDM growth and prepare for increased future competition. This effect reflects the endogenous nature of PDM competition, which grows at a rate that accelerates with PDM adoption. By digitizing its currency, country A curbs PDM growth, thereby mitigating future competition.

In contrast, the concern of slowing future growth of PDM adoption has a much smaller effect on B's incentives to digitize (see Panel B). Loosely speaking, B mostly cares about the additional adoption it can gain through digitization, as its effect on the dynamics of Y is relatively small. Panel C shows that when μ is larger, total digitization efforts respond more strongly and earlier to the rising competition from PDM, in that they are higher for low Y and peak earlier.

Panel D plots countries' total digitization efforts over time, starting from $Y_0 = 0.5$. When μ is larger, countries' total efforts are higher initially but decline more quickly, resulting in less persistence. That is, countries abandon the digitization of their currency more rapidly, when PDM competition grows faster. As discussed before, the model predicts that digitization occurs relatively early on in the game. However, if countries do not succeed early on, digitization is delayed significantly or never occurs. Notably, this pattern is more pronounced for larger values of μ, as shown in Figure 3. Panel B of Figure 3 shows that an increase in μ reduces the probability that digitization occurs early, while increasing the probability that it occurs later. Panel C of Figure 3 shows that an increase in μ delays digitization, i.e., raises the expected time to digitization for all t. Likewise, Panel C of Figure 3 shows that the expected time to digitization increases with Y. Both findings indicate that competition from PDM delays fiat currency digitization.

Finally, the convenience of PDM and its growth rate may reflect whether they are banned or regulated by governments. Although not explicitly modeled, we consider regulation as a factor that reduces the convenience and competitiveness of PDM. Our model predicts that if regulation (by one or multiple countries) does indeed reduce PDM convenience and adoption, it accelerates the digitization of fiat currencies, but hampers overall welfare by reducing payment convenience. Interpreted differently, the regulation of PDM and the digitization of fiat money complement each other in maintaining the relevance and adoption of their currency in digital payments.

2.4.2 Competition Among Fiat Currencies

To examine the effects of competition among fiat currencies, we perform comparative statics in their relative convenience. For this sake, we hold fixed Z_L^A at one (a normalization), and vary Z_L^B, whereby an increase in Z_L^B corresponds to increased competition among fiat currencies. To this end, Figure 5 plots country A's efforts (Panel A), country B's efforts (Panel B), and total efforts (Panel C) against $\ln(Y)$ and, in Panel D, total efforts against time t starting at $Y_0 = 0.5$, for $Z_L^B = 0.1$ (solid black line), $Z_L^B = 0.5$ (dotted red line), and $Z_L^B = 0.9$ (dashed yellow line).

Note that increased competition from currency B, i.e., an increase in Z_L^B, boosts A's digitization incentives for low levels of Y, while curbing them for higher levels of Y. Indeed, higher Z_L^B implies lower adoption of currency A, raising A's adoption gain upon digitization. At the same time, higher Z_L^B raises adoption for currency B, thereby reducing B's incentives for low levels of Y. When B's adoption is relatively low (because Z_L^B is low), country B has strong incentives to move first in currency competition, resulting in high efforts by B for low Y. In contrast, A's incentives to move early are diminished. In other words, more asymmetric competition among fiat currencies

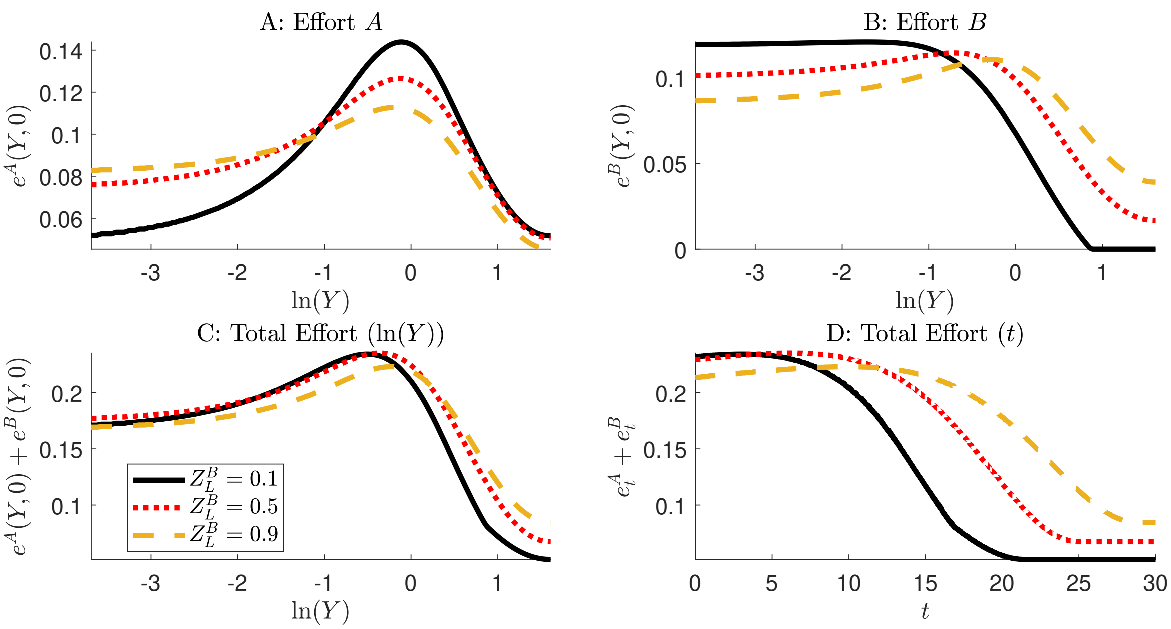

Figure 5: Effort Dynamics and Fiat Currency Competition. This Figure presents comparative statics in Z_L^B. Panels A and B plot country A's and B's efforts in state $z=0$ against $\ln(Y)$ for three different levels of Z_L^B. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. Panel D plots total efforts against time, t, being state $z=0$. We use our baseline parameters from Section 2.2 (including $Z_L^A=1$), but set $Y_0=0.5$ for Panel C.

implies a relatively larger first-mover advantage for B, and greater incentives to move second for A. Interestingly, as shown in Panel C, the level of competition among fiat currencies hardly affects the total digitization efforts for a *given level* of Y, i.e., the effects more or less cancel out.

Because currency B also competes with C for adoption, a higher level of Z_L^B reduces PDM adoption and, by (4), slows the growth of PDM competition. Recall that countries' digitization efforts tend to decrease over time and in particular reach low levels when PDM has gained widespread adoption (i.e., Y is large). Consequently, as shown in Panel D, higher Z_L^B implies that countries' digitization efforts are more persistent, which raises the likelihood that digitization occurs relatively early, rather than very late or never. In other words, increased competition among fiat currencies accelerates currency digitization and digital upgrades of traditional payment systems.

2.4.3 The Welfare Effects of Competition

We examine how currency competition affects the timing of digitization and welfare. Panel A of Figure 6 shows that the expected time to digitization increases in Y and in μ for given $\ln(Y)$, which, in line with previous findings, indicates that PDM competition delays the digitization of fiat currencies. Next, Panel C shows that the expected time to digitization in state Y decreases in Z_L^B, so fiat currency competition accelerates digitization. Regarding welfare, Panel B displays that welfare increases in Y and μ for given $\ln(Y)$, in that PDM competition benefits welfare. Likewise, Panel D shows that welfare increases in Z_L^B, i.e., increased fiat currency competition boosts welfare.

The intuition behind these findings is that because users benefit from increased convenience, any type of competition increases their welfare. However, while increased competition among fiat currencies accelerates digitization, increased competition from PDM delays or slows digitization. The reason is that increased competition from PDM (i.e., larger μ or Y) limits fiat currencies' adoption gain upon digitization, which dynamically reduces countries' efforts and delays digitization. Differently, increased competition among national currencies, i.e., higher Z_L^B, raises countries' digitization efforts and their persistence, which accelerates digitization.

2.5 Welfare Maximization and Planner Solution

In our baseline specification, countries maximize the time average of their currency's adoption in the digital economy, net of the costs of digitization. As a consequence, countries care about the convenience of their currencies only insofar as it leads to higher adoption (i.e., shifts users' investment toward their currency). However, countries do not internalize that user utility and, therefore, welfare increase in currencies' convenience, holding the investment decision fixed. That

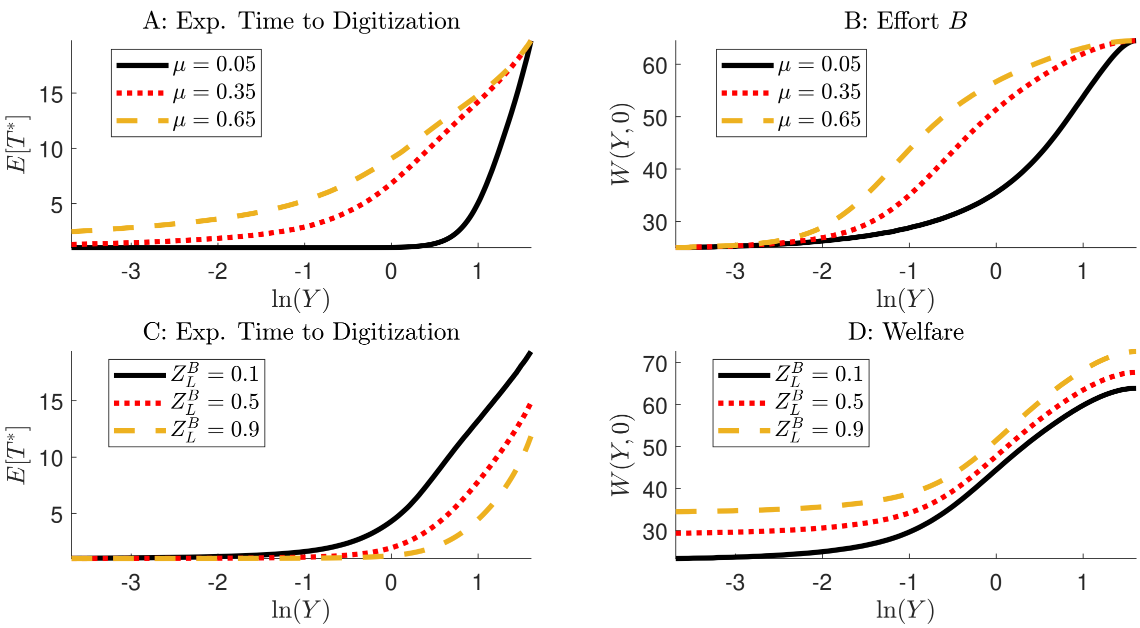

Figure 6: Time to Digitization and Welfare. This Figure plots the expected time to digitization (in Panels A and C) and welfare against $\ln(Y)$ (Panels B and D) in state $z = 0$. The upper Panels A and B depict comparative statics in μ, while the lower Panels C and D present comparative statics in Z_L^B. We use our baseline parameters from Section 2.2.

is, countries only focus on the users' investment on the margin, while they do not internalize that digitization improves convenience on infra-marginal investments. These effects harm total welfare and lead to "inefficiently low" digitization efforts, especially by the dominant currencies, as we illustrate below.

We compare the dynamics of countries' digitization efforts to two those that would obtain in two benchmarks. First, we consider that a planner chooses efforts to maximize the welfare from (6), subject to $m_t^x = P_t^x$ satisfying the pricing relationship (13). That is, the planner only decides on digitization efforts, but cannot control users' choice among currencies. Appendix B characterizes the planner solution in greater detail, specifically optimal efforts and the ODE system characterizing the Markov equilibrium. Second, we consider that countries $x = A, B$ separately maximize the welfare (convenience utility) generated by their own currencies x, net of digitization costs. Specifically, we set $f_t^x = Z_t^x v(m_t^x)$ for the flow utility in the objective (5). Further, set $\delta = \gamma$ (i.e., planner and countries discount at the same rate), so countries internalize the full welfare generated by their currencies. The solution is formally analogous to the baseline.

Figure 7 plots welfare-maximizing effort levels against $\ln(Y)$, both when countries maximize their currencies' welfare independently (solid black line) and the planner maximizes welfare (dotted

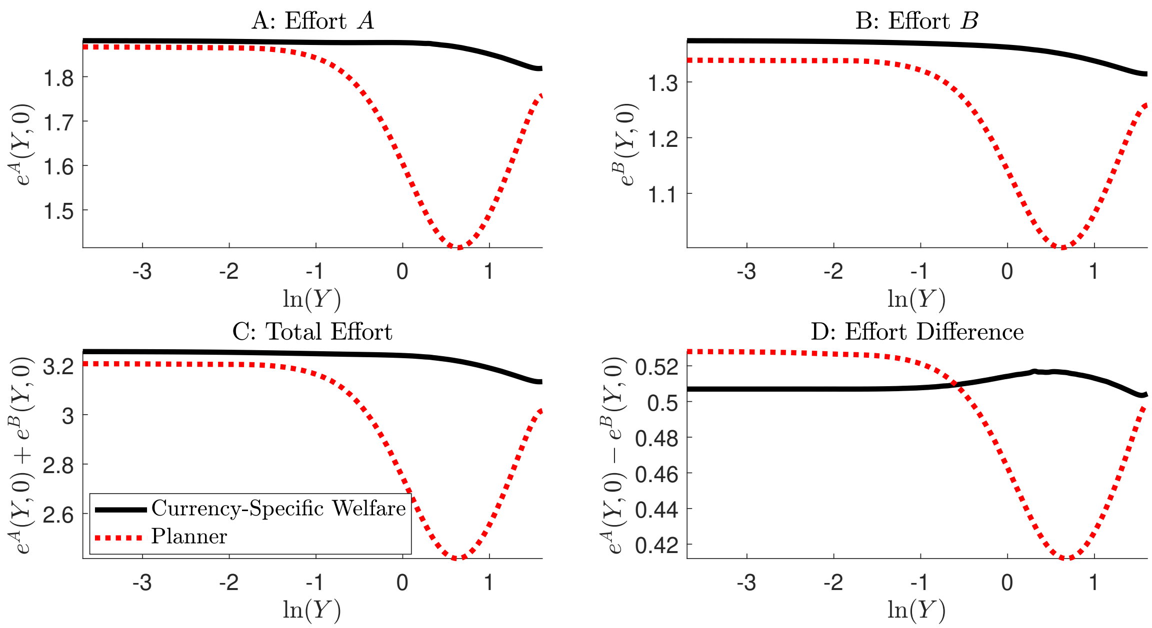

Figure 7: Welfare-Optimizing Efforts. This figure plots welfare-maximizing efforts in Panels A and B (for countries A and B, respectively) against $\ln(Y)$ in state $z = 0$, both under the planner solution (dotted red line) and currency-specific welfare maximization (solid black line). Panel C plots the sum of these efforts and Panel D their difference against $\ln(Y)$. We use our baseline parameters from Section 2.2.

red line). Panels A and B depict A's and B's welfare-maximizing effort levels in state $z = 0$, while Panel C plots their sum and Panel D their difference. Comparing effort levels from Figures 2 and 7, it is evident that the baseline effort levels lie below the welfare-maximizing levels of all Y in both benchmarks. We highlight countries' failure to fully internalize the convenience utility generated by digitization as an economic mechanism leading to this under-investment. However, we acknowledge that the magnitude of baseline effort levels (and whether they fall below welfare-maximizing levels) also depends on the specific functional forms we assumed (e.g., in the baseline).

Panels A, B, and C show that welfare-maximizing levels exhibit a U-shaped pattern in $\ln(Y)$ under the planner solution, i.e., they first increase and then decrease in Y. In contrast, efforts tend to decrease in Y, when countries maximize their currencies' welfare independently. This U-shaped pattern reflects that for intermediate levels of Y, users benefit from the growth of PDM convenience. The digitization of fiat currency in this region would slow down the growth of Y, which would harm welfare, curbing the planner's digitization efforts.

Interestingly, when countries maximize their currencies' welfare separately, their individual and joint efforts exceed those that would prevail under the planner solution. The intuition is that when maximizing their currencies' welfare only, countries do not internalize the reduction in the adoption of other currencies caused by their digitization. This effect leads them to over-invest in digitization, and causes efforts to decline in Y, rather than to follow a U-shaped pattern. In the baseline, with countries only focusing on the adoption of their own currency, country A's efforts follow an inverted U shape in Y, and country B's efforts decrease in Y.

Finally, as shown in Panel D, it is welfare maximizing to have country A exert higher efforts than country B and thus to move first in currency digitization, because currency A is held more widely and thus benefits more from digitization. In contrast, in the baseline, country A's incentives lie below those of B for low Y, showing a first-mover advantage for the weaker country B and a second-mover advantage for the strong country A. In other words, country B's first-mover advantage and country A's second-mover advantage are not welfare-maximizing.

3 Discussion and Model Extensions

Our baseline setting omits many realistic and relevant features of currency competition and digitization. We now present several model variants and extensions to demonstrate the flexibility and robustness of our theory.

3.1 Interoperability and Public-Private Collaborations

In practice, fiat money and PDM may be interconnected through (i) interoperability or (ii) public-private collaborations in payment digitization. Some private payment systems, for instance, can facilitate transactions that bypass traditional banks while also linking to bank accounts or credit cards, enabling bank deposit-based transactions. A notable example is Alipay, which allows users to transact using Alipay credit or wallet balances without having each transaction go through banks. At the same time, Alipay can connect to credit cards and bank accounts, making it interoperable with bank payment rails. Governments and central banks may also collaborate with private payment firms to digitize their currencies and bank-based payment systems. The benefits of currency digitization depend on the state of the payment technology underlying PDM. For example, Brazil's Pix system was developed by the Brazilian central bank in partnership with industry experts. Similarly, the Digital Euro Project is a collaboration between the ECB and European payment firms (Berg et al., 2024). Alipay and WeChat Pay also promote the usage of e-CNY (Xia, Gao, and Zhang, 2023; Bai et al., 2025). Such public-private collaborations can further enhance the interoperability between government-led or bank-centric payment rails and PDM (e.g., Duarte et al., 2022).

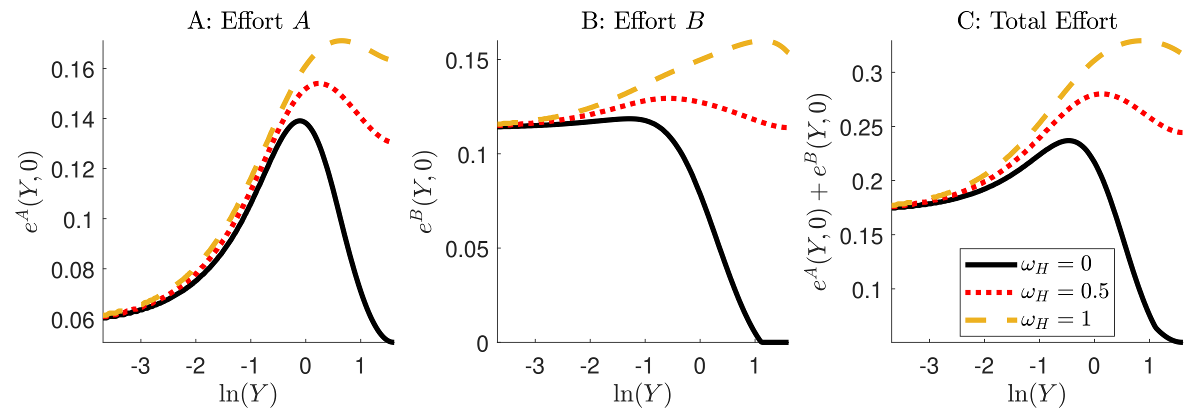

Figure 8: Public-Private Collaborations and Digitization Efforts. This Figure presents comparative statics in ω_H^x. Panels A and B plot country A's and B's efforts in state $z=0$ against $\ln(Y)$ for three different levels of $\omega_H^A=\omega_H^B$, where $\omega_L^A=\omega_L^B=0$. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We use our baseline parameters from Section 2.2.

To capture the interdependence of fiat money and PMD as well as to allow for public-private collaborations in currency digitization, we stipulate that payment convenience satisfies:

$$Z_t^x = \begin{cases} \zeta_L^x + \omega_L^x Y_t & \text{for} \quad t < T^x \\ \zeta_H^x + \omega_H^x Y_t & \text{for} \quad t \geq T^x, \end{cases} \tag{17}$$

where $\omega_L^x \leq \omega_H^x$. We define $Z_L^x = \zeta_L^x + \omega_L^x Y_t$ and $Z_H^x = \zeta_H^x + \omega_H^x Y$. Note that the convenience of fiat money, Z_t^x, increases with the convenience of PDM, Y_t, reflecting the partial interoperability and linkage of PDM payment technologies with bank-centric or government-led payment systems.

We next model public-private collaborations in fiat currency digitization by assuming $\omega_H^x > \omega_L^x$. Under this assumption: (i) the relative convenience gain from digitization, $Z_H^x - Z_L^x$, increases with Y; and (ii) digitization strengthens the link between the convenience of fiat money and that of PDM. These two features, (i) and (ii), are intrinsic to public-private digitization initiatives, where payment firms collaborate with central banks or the government to leverage private-sector payment technologies in digitizing fiat money or bank-centric payment systems. First, the gains from such collaborations are greater when private-sector payment technology is more advanced, as reflected by a higher Y. Second, digitizing fiat currency with private-sector technology typically involves linking these solutions to bank-centric or government-led payment systems, enhancing interoperability (as captured by $\omega_H^x > \omega_L^x$) and the convenience of fiat money.

3.1.1 Public-Private Collaborations and Fiat Digitization

We now investigate how currency digitization efforts change, when digitization is structured as a public-private collaboration. For this sake, we perform comparative statics in the parameter ω_H^x, assuming, for symmetry, $\omega_H^A = \omega_H^B$ as well as $\omega_L^A = \omega_L^B = 0$. Further, we adopt our baseline parameters and, specifically and analogously to the baseline, set $\zeta_L^A = 1, \zeta_L^B = 0.2$, as well as $\zeta_H^x = \Delta^{Fixed} + (1 + \Delta^{Prop})\zeta_L^x$; we stipulate $\Delta^{Fixed} = \Delta^{Prop} = 1$.

Figure 8 presents the results, with Panel A showing A's effort, Panel B showing B's effort, and Panel C showing total digitization effort. The figure illustrates that an increase in ω_H^x not only raises the overall level of digitization efforts but also alters their timing. Naturally, a higher ω_H^x — indicating more intense public-private collaboration — increases the gains from digitization, thereby boosting digitization efforts. More intriguingly, the model predicts that public-private collaborations in currency digitization are associated with backloaded and more persistent digitization efforts. Specifically, countries exert relatively low efforts when PDM adoption is limited, gradually increasing their efforts as PDM adoption and convenience grow.

As such, when currency digitization involves relatively little public-private collaboration, as in our baseline scenario, digitization efforts are initially high at low levels of PDM adoption but gradually taper off. In this case, countries might even discontinue digitization as PDM becomes widely adopted, with the model predicting relatively early digitization. In contrast, when currency digitization involves a high degree of collaboration, digitization efforts are higher at elevated levels of Y and, crucially, increase over time. This makes digitization efforts more persistent and increases the likelihood of eventual digitization. Nevertheless, several findings hold regardless of the level of collaboration, and the key patterns remain robust to this modification. For instance, in both scenarios, the relatively weaker currency has a first-mover advantage and stronger incentives to digitize early compared to the stronger currency.

3.1.2 When to compete and when to collaborate?

To analyze the choice between collaboration and competition in currency digitization, we now allow countries to choose the level of ω_H^x at starting state $Y_0 = Y$ — after that, the choice ω_H^x remains fixed at future dates for simplicity. Specifically, country x chooses in state $(Y, 0)$ the level of $\omega_H^x \in \{0, \omega_H\}$ against a linear cost:[22]

$$\max_{\omega_H^x \in \{0,\omega_H\}} V^x(Y,0) - \mathcal{C}\omega_H^x, \tag{18}$$

where $\omega_H \geq 0$ and $\mathcal{C} \geq 0$ are constants. The cost $\mathcal{C}$ may reflect payments required by private-sector participants, the coordination costs of involving multiple parties, or (in reduced form) the loss of payment autonomy when engaging the private sector. Countries $x = A, B$ choose ω_H^x simultaneously, taking the other country's choice as given. This choice induces a static game, for which we characterize the pure-strategy Nash equilibria at a given level of Y. Let $\mathcal{E}(Y) \subseteq \{(0,0), (0,\omega_H), (\omega_H, 0), (\omega_H, \omega_H)\}$ denote the set of pure-strategy Nash equilibria, represented as tuples of the form (ω_H^A, ω_H^B).

It is clear that for sufficiently low values of Y, the unique Nash equilibrium is to compete, i.e., $\omega_H^x = 0$ for $x = A, B$ and $(0,0) = \mathcal{E}(Y)$. Likewise, sufficiently high cost $\mathcal{C}$ preclude digitization and yield our baseline. Such high cost may capture that the private sector is unwilling to collaborate with government entities or collaboration is not feasible.[23] In contrast, when Y is sufficiently large (relative to $\mathcal{C}$), a Nash equilibrium features at least one country collaborating, e.g., $\omega_H^x = \omega_H$ for $x = A$ or $x = B$; that is, $(0,0) \notin \mathcal{E}(Y)$.

To illustrate this outcome, we consider a numerical example where $\mathcal{C} = 0.05$ and $\omega_H = 1$, and solve for the pure-strategy Nash equilibria. In this example, the pure-strategy Nash equilibrium exists and is unique. For low levels of Y (i.e., $Y \leq 0.703$), the unique equilibrium is where both countries set $\omega_H^x = 0$, meaning they do not collaborate in currency digitization and instead compete. For intermediate values of Y (i.e., $0.704 \leq Y \leq 1.042$), the unique equilibrium entails country A choosing $\omega_H^A = 0$ and country B choosing $\omega_H^B = \omega_H$, indicating that the weaker country collaborates while the stronger one does not. Finally, for high values of Y (i.e., $Y \geq 1.042$), the unique equilibrium features $\omega_H^x = \omega_H$, meaning both countries collaborate with the private sector to digitize fiat currency.

In summary, when PDM adoption is low, countries digitize their fiat currencies to *compete* with PDM and with each other, with country B exerting higher efforts due to a first-mover advantage. As PDM adoption increases, countries transition to *collaborating* with the private sector to digitize

[22] One could also allow countries to choose dynamically the value of ω_H^x at any point in time. However, this modeling would complicate the model analysis, while likely generating similar insights.

[23] For instance, many cryptocurrencies and blockchain-based payment systems were designed to establish a decentralized financial system free from government oversight, which allows for privacy in payments. This feature is crucial to the convenience and adoption of cryptocurrency as PDM, as discussed in Appendix E, where we link payment privacy to PDM convenience. Collaboration with government entities would undermine the core purpose and appeal of cryptocurrency, leading to reluctance among crypto practitioners to collaborate.

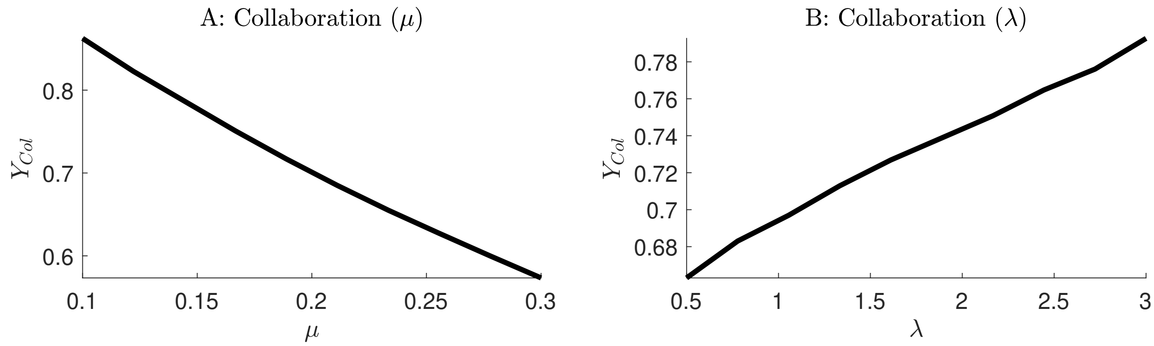

Figure 9: When to Collaborate or to Compete? This Figure presents comparative statics in Y_{Col} defined in (19). Panel A presents comparative statics with respect to μ and Panel B presents comparative statics with respect to λ. We use our baseline parameters from Section 2.2, and set $\omega_H^x = \omega_H = 1$, $\omega_L^x = 0$, and $\mathcal{C} = 0.05$.

their currencies. Thus, the paradigm in currency digitization shifts over time from competition to collaboration. Initially, countries focus on competing with PDM, but as PDM becomes widely adopted, collaboration becomes both inevitable and optimal.

Finally, we investigate in Figure 9 how countries' propensity to collaborate with the private sector in currency digitization depends on the growth of PDM convenience μ (see Panel A) and the cost of digitization λ (see Panel B). For this sake, we plot

$$Y_{Col} = \inf\{Y \in (0, \overline{Y}) : (0,0) \notin \mathcal{E}(Y) \ \wedge \ \mathcal{E}(Y) \neq \emptyset\}. \tag{19}$$

against μ in Panel A and against λ in Panel B. Observe that for $Y \geq Y_{Col}$, any pure-strategy Nash equilibrium involves at least one country engaging in public-private collaboration. Therefore, a lower value of Y_{Col} indicates a higher propensity for countries to engage in such collaborations.

Panel A shows that Y_{Col} decreases with μ. This implies that when the convenience or adoption of PDM is expected to grow faster, and PDM poses greater competition to fiat money over time, the gains and necessity for collaboration increase, leading to more public-private collaboration.

Panel B shows that Y_{Col} increases with countries' digitization cost, as captured by λ. This suggests that countries are more likely to engage in public-private collaborations when the cost of achieving digitization is low. The intuition is that when λ is high and digitization is more challenging, digitization efforts and the likelihood of digitization are low to begin with, making it suboptimal to incur the additional costs associated with public-private collaborations.

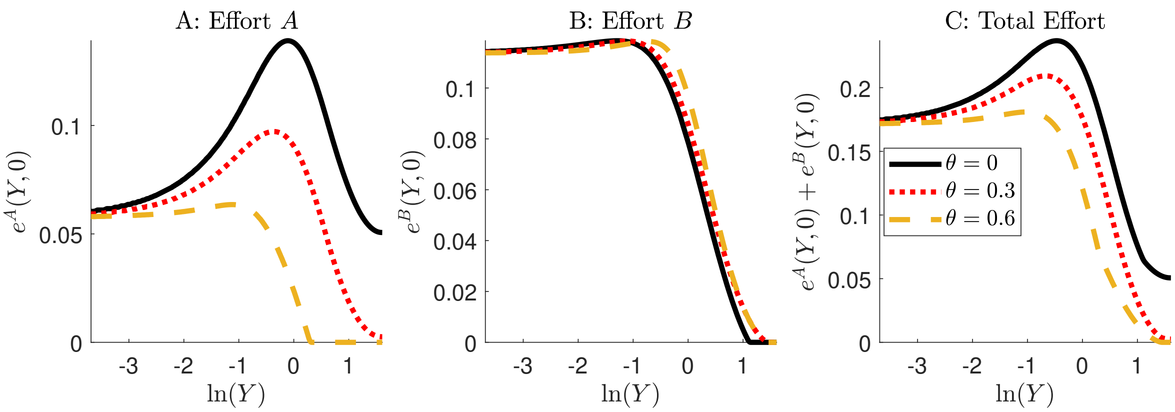

Figure 10: The Role of Dollar-Backed Stablecoins. This Figure presents comparative statics in θ. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of θ. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We use our baseline parameters from Section 2.2.

3.2 Stablecoins and Dollar-Backed Cryptocurrencies

In our model, the representative PDM encompasses the broader cryptocurrency market, including stablecoins —cryptocurrencies pegged to a reference unit. Many of the largest stablecoins (e.g., USDC or Tether) are pegged to the U.S. dollar and are (partially) backed by U.S. dollar reserve assets, such as deposits or cash equivalents.[24] To model the interdependence between the dominant fiat currency A and PDM C related to stablecoins, we extend our baseline by assuming that a fraction $\theta \in [0, 1)$ of the PDM adoption value (i.e., market capitalization) P_t^C is backed by currency A. An increase in θ could reflect regulatory reserve requirements on stablecoins, mandating that a greater portion be backed by U.S. dollar assets. Similarly, a higher θ could represent the growing significance of stablecoins — both within the cryptocurrency ecosystem and globally.

The introduction of the parameter θ changes the model as follows. At time t, the total reserves backing PDM are worth θP_t^C units of the consumption good, i.e., they consist of $\theta P_t^C / P_t^A$ units of currency A. The total value of currency A in goods becomes $P_t^A = m_t^A + \theta P_t^C$ while $m_t^B = P_t^B$ and $m_t^C = P_t^C$. The market clearing condition $\sum_{x'=A,B,C} m_t^{x'} = 1$ implies

$$P_t^A + P_t^B + P_t^C(1 - \theta) = 1. \tag{20}$$

All other elements remain unchanged.[25] The arguments in Appendix A already allow for $\theta > 0$,

[24]While stablecoins can also be backed by fiat currencies other than the U.S. dollar, this is rare in practice. The fraction of cryptocurrency market capitalization backed by non-dollar fiat currencies is negligible. Therefore, we focus on the case where currency C is backed exclusively by currency A, although our framework is flexible enough to accommodate backing with B as well.

[25]Because fraction θ of total PDM holdings $m_t^C = P_t^C$ (in terms of consumption good) are backed by currency A,

which nests the baseline upon setting $\theta = 0$.

Figure 10 illustrates how θ affects digitization incentives. As shown in Panel A, an increase in θ reduces A's digitization efforts. This occurs because, when $\theta > 0$, currency A gains additional adoption from its role as a reserve asset for PDM. More intuitively, the adoption of stablecoins pegged to currency A in digital payments effectively increases the adoption of currency A, thereby strengthening its role and importance in digital payments. As a result, country A partially benefits from the rise of PDM, which provides additional adoption for its currency. This effect reinforces the dominance of currency A and reduces its incentive to compete with PDM through digitization, thereby delaying the digitization of currency A. Panel B shows that changes in θ have little to no effect on B's incentives, leading to a decline in total digitization efforts as θ increases (see Panel C). In other words, higher θ reduces countries' efforts to digitize their currencies and delays digitization.

Our analysis highlights that the increasing adoption of dollar-backed stablecoins in digital payments enhances the relevance and influence of the U.S. dollar, thereby strengthening the geoeconomic power of the United States. This dynamic incentivizes the United States to adopt crypto-friendly policies and regulations to support the growth of stablecoins. Additionally, it reduces the incentive to digitize the U.S. dollar, as doing so could undermine stablecoin usage. Broadly interpreted, the private sector effectively creates a form of digital dollar through stablecoins, substituting for the U.S. government's efforts to digitize the dollar. In a way, U.S. crypto-friendly policies have offered a strategic substitute for CBDCs.

3.3 Interest Rates, Uncovered Interest Parity, and a Pecking Order of Adoption

In our baseline, the exchange rate dynamics (in terms of the consumption good and across currencies) are pinned down through market clearing (i.e., $m_t^x = P_t^x$) and the user's allocation of endowment across currencies. This modeling implicitly assumes that the adoption and usage of a currency in the digital economy is relevant enough to influence exchange rates.[26] We now consider that exchange rates are determined outside of the model in a frictionless bond market, and show

an additional capital gain of $\theta r_t^A dt$, which arises because part of the PDM value is invested in currency, accrues to PDM as a whole over $[t, t + dt)$. For simplicity and to enhance the comparability with the baseline, we assume that the capital gain from investing in reserves consisting of currency A is fully captured by the PDM developers (who are outside of the model), so that (13) still applies. If fraction $\hat{\omega}$ of this capital gain accrued to the PDM investors, equilibrium pricing conditions (13) would change to

$$Y_t v'(m_t^C) + r_t^C + \hat{\omega}\theta r_t^A = Z_t^A v'(m_t^A) + r_t^A = Z_t^B v'(m_t^B) + r_t^B,$$

which would lead to qualitatively similar outcomes. Moreover, if the dollar reserves are interest-bearing, there would be another capital gains term accruing to PDM.

[26]In fact, in the baseline, the extreme case obtains that exchange rates are solely pinned down through the currency adoption in the digital space, which is extreme.

that our key findings remain robust in this model variant (see Section 3.3.3). Consider $\theta = 0$, as in the baseline. We assume that the expected return (in consumption goods) $r_t^x + i^x$ from investing in currency $x = A, B$ through risk-free short-term bonds, which pay a nominal interest rate i^x, must equal an exogenous required rate of return ρ:[27]

$$\rho = r_t^x + i_t^x. \tag{21}$$

Using (21) for $x = A, B$, we obtain the uncovered interest parity, i.e., $r_t^A - r_t^B = i^B - i^A$. We assume that the (constant) interest rates i^x are exogenous, i.e., determined outside of the model. the user cannot invest in the newly introduced bonds and allocates its entire endowment across the three monies A, B, and C, as in the baseline.

We now make the following assumptions as to how users, holding currencies for their needs in the digital economy, can benefit from the currencies' interest rate. Specifically, we assume that only fraction $1 - \alpha^x$ of the interest rate is passed-through to the user, potentially reflecting imperfect deposit rate passthrough. Crucially, $\alpha^x = \alpha_t^x = \alpha^x(z)$ is a parameter that may change with digitization, for instance, because the launch of CBDC may improve the interest rate passthrough on deposits Chiu et al. (2023). Likewise, Sarkisyan (2023), who studies the introduction of Brazil's Pix payment system, shows how currency digitization can influence banking competition and deposit rates. As shown in Appendix C, the pricing equation (13) then changes to

$$Y_t v'(m_t^C) + r_t^C = Z_t^A v'(m_t^A) + \rho - \alpha_t^A i^A = Z_t^B v'(m_t^B) + r_t^B + \rho - \alpha_t^B i^B. \tag{22}$$

Equation (22) illustrates that when interest rate passthrough is not perfect and $\alpha^x > 0$, users effectively incur a cost of holding currency x, hampering this currency's effective convenience and adoption in the digital economy. That is, a higher interest rate i^x or a worse passthrough α^x renders

[27] For a micro-foundation, consider that each country $x = A, B$ has short-term government bonds with maturity dt outstanding. For simplicity, these bonds are risk-free and pay a nominal interest at rate i^x — which is fixed and exogenous. Let P_t^x the value of one unit of currency x in terms of the consumption good, and let $r_t^x = \mathbb{E}[dP_t^x]/(P_t^x dt)$ the expected rate of appreciation of currency x in terms of the consumption good, as in the baseline. One bond has a face value of one unit of currency x. Investing one unit of the consumption good in x's bond at time t, i.e., buying $1/P_t^x$ units of the bond, and holding this bond up to maturity at time $t + dt$ yields (undiscounted) payoff

$$-1 + \frac{1}{P_t^x} \left(i_t^x P_{t+dt}^x dt + P_{t+dt}^x \right) = r_t^x dt + i_t^x dt,$$

where we used that $P_{t+dt}^x = P_t^x + dP_t^x$ and ignored terms of order $(dt)^2$ or higher (which are negligible in the continuous time limit). Government bonds are bought by international bond investors with deep pockets, who can short-sell and buy bonds, are risk-neutral, and require a return at rate ρ in terms of the consumption good. Thus, for the bond market to clear, the payoff to buying bonds, $r_t^x dt + i_t^x dt$, must equal bond investors' required rate of return, ρdt, so that (21) must hold after canceling out dt.

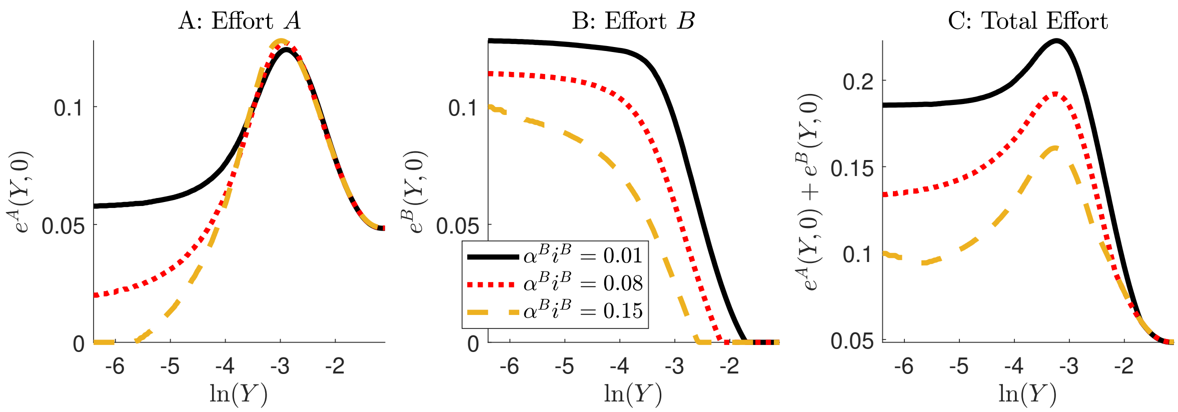

Figure 11: Interest Rates and Digitization. This Figure presents comparative statics in $\alpha^B i^B$. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of $\alpha^B i^B$. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We divide all baseline parameters from Section 2.2, which are related to currency convenience, by 15, leading to $Z_L^A = 1/15$, $Z_L^B = 0.2/15$, $\Delta^{Fixed} = 1/15$, $\overline{Y} = 5/15$, and $Y_0 = 0.1/15$ while all other parameters remain unchanged. We set $\alpha^A i^A = 0.01$, and $\rho = \theta = 0$.

currency x *effectively* less convenient, i.e., has similar effects as a decrease in Z_t^x.

All other model elements remain unchanged. Appendix C presents further solution details and characterizes the ODE system of the Markov equilibrium.

3.3.1 The Effects of Interest Rates and Passthrough

We apply this model variant to examine how changes in the interest earnings $\alpha^x i^x$ — which may stem from changes in interest passthrough $1 - \alpha^x$ or monetary policy (i.e., changes in the nominal rate i^x) — affect currency digitization. To this end, Figure 11 performs comparative statics in $\alpha^B i^B$ by plotting A's efforts (Panel A), B's efforts (Panel B), and total efforts (Panel C) against $\ln(Y)$, for three different levels of $\alpha^B i^B$, holding fixed $\alpha^A i^A$. To be able to better highlight the effects of interest rates and to ensure they have reasonable quantitative effects on currency digitization (without stipulating unrealistic interest rates), we divide all baseline parameters from Section 2.2, which are related to currency convenience, by 15. This leads to $Z_L^A = 1/15$, $Z_L^B = 0.2/15$, $\Delta^{Fixed} = 1/15$, $\overline{Y} = 5/15$, and $Y_0 = 0.1/15$ while all other parameters remain unchanged relative to the baseline.

Higher $\alpha^B i^B$ could capture increased inflation in country B, precipitating a rise in the nominal rate, or worsened interest passthrough, for instance, because B's banking or financial system is less digitized or competitive. Both features make currency B less convenient, which, all else equal, reduces the competition among fiat currencies. In line with the findings of Section 2.4.2, Figure 11 illustrates that such reduced fiat currency competition due to higher $\alpha^B i^B$ reduces both countries' digitization efforts and, hence, slows the digitization of fiat currencies. Notably, higher $\alpha^B i^B$ reduces

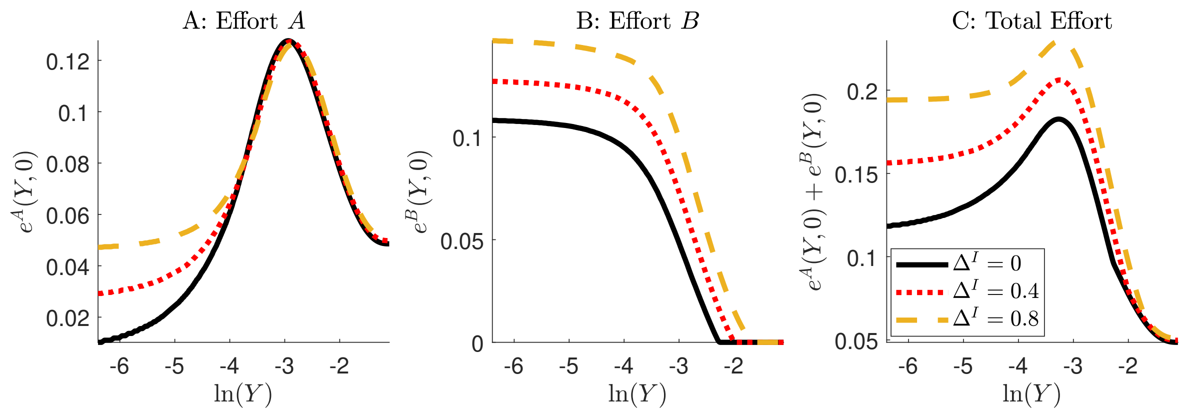

Figure 12: Interest Rate Passthrough and Digitization. This Figure presents comparative statics in Δ^I. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of Δ^I. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We divide all baseline parameters from Section 2.2, which are related to currency convenience, by 15, leading to $Z_L^A = 1/15$, $Z_L^B = 0.2/15$, $\Delta^{Fixed} = 1/15$, $\overline{Y} = 5/15$, and $Y_0 = 0.1/15$ while all other parameters remain unchanged. We set $i^A = 0.01$ and $i^B = 0.1$, and $\rho = \theta = 0$.

both countries' digitization efforts, and thus total efforts. For country B, higher $\alpha^B i^B$ raises the cost of holding currency B hampers adoption, both before and after digitization, thereby limiting the gains from digitization. For country A, higher $\alpha^B i^B$ implies weaker competition from B and thus weaker incentives to invest in currency digitization.

Appendix Figure F.3 presents comparative statics in $\alpha^A i^A$. Interestingly, we find that higher $\alpha^A i^A$, resulting in more competition among fiat currencies, has the opposite effect and increases total digitization efforts, thereby accelerating digitization.

Our model can also capture that CBDCs improve interest passthrough, for instance, via $\alpha^x(0) = \alpha^x(-x) > \alpha^x(x) = \alpha^x(AB)$, as micro-founded in Chiu et al. (2023). Naturally, when currency digitization improves the interest passthrough (while all else remains equal), countries have stronger incentives to digitize their currency, especially when interest rates are high. Figure 12 illustrates this outcome by plotting country A's efforts in Panel A, country B's efforts in Panel B, and total efforts in Panel C against $\ln(Y)$. It uses $\alpha^x(0) = 1$, while $\alpha^x(x) = \alpha^x(AB) = 1 - \Delta^I$. When Δ^I is larger, digitization improves the interest passthrough more significantly, thereby stimulating efforts by both countries. Since country B starts with a higher interest rate $i^B = 0.1$ (while $i^A = 0.01$), its incentives react more strongly to an increase in Δ^I. This outcome highlights that insofar currency digitization enhances the interest rate passthrough, countries with higher interest rates have stronger incentives to digitize their currencies.

3.3.2 Very Weak Currencies and Pecking Order

The model variant incorporating interest rates allows us to capture "very weak" currencies, characterized by excessively high nominal interest rates (e.g., due to hyperinflation). Examples include the Turkish Lira, with a nominal interest rate of about 50% and inflation around 70%, and the Argentine Peso, with an interest rate near 40% and inflation around 240%, as of October 2024. One can show that as $i^B \to \infty$ while $\alpha^B > 0$, both $P^B(Y, z) \to 0$ and $V^B(Y, z) \to 0$ for any z and Y. This implies $\lim_{i^B \to \infty} e^B(Y, z) = 0$. Loosely speaking, as $i^B \to \infty$, it becomes infinitely costly to adopt currency B (due to imperfect interest passthrough), causing the adoption level of B to approach zero in all states. The reason is that in our model and unlike in Goldstein et al. (2023), the store-of-value and medium-of-exchange functions complement each other: when a currency is a better store-of-value and thus offers higher returns, it becomes less costly to adopt it for payment, reinforcing the medium-of-exchange function. Very weak currencies perform poorly as stores of value, making their adoption for payment costly and, therefore, low, even with digitization.

In other words, countries with very weak currencies exhibit low adoption, regardless of digitization efforts, thereby limiting the incentives for digitization. In unreported results, we verify numerically that as i^B becomes sufficiently large, country B's efforts, $e^B(Y, z)$, indeed approach zero. Consequently, the model predicts minimal or no digitization for very weak currencies with high nominal interest rates, leading to a pecking order in fiat currency digitization. Specifically, less dominant but not overly weak currencies are digitized first, followed by more dominant currencies, while very weak currencies are digitized last, if at all.

3.3.3 Robustness Checks

Notably, our key findings from the baseline remain unchanged in this model variant and, consequently, are robust to incorporating interest rates and international finance elements. First, Appendix Figure F.1 replicates the key Figure 2 in this model variant (assuming $\alpha^A i^A = 0.01$ and $\alpha^B i^B = 0.03$ and otherwise using baseline parameters). Comparing Figures 2 and F.1, we note that our findings remain unchanged relative to the baseline, both qualitatively and quantitatively. In particular, note that the effort dynamics remain similar, with A's effort being inverted U-shaped and B's effort declining in $\ln(Y)$. In addition, B exerts higher efforts than A for low Y, showing a first-mover advantage, while A's efforts exceed those of B for high Y.

Second, Appendix Figure F.2 shows that one obtains similar effort patterns as in the baseline, when modeling the benefits of digitization *solely* as improved interest passthrough, that is, *without* stipulating any changes and differences in the convenience parameters Z_L^x, Z_H^x. In particular, Figure

F.2 sets $Z_L^A = Z_L^B = Z_H^A = Z_H^B = 1/15$, while $\alpha^A(0)i^A = 0.01$ and $\alpha^A(A)i^A = \alpha^A(AB)i^A = 0.001$ as well as $\alpha^B(0)i^B = 0.1$ and $\alpha^B(B)i^B = \alpha^B(AB)i^B = 0.01$.[28] Indeed, observe that A's effort is inverted U-shaped in $\ln(Y)$, while effort of B decreases in $\ln(Y)$. Total efforts decrease in $\ln(Y)$ too. Qualitatively, the effort dynamics resemble those of Figure 2.

3.4 Discounting and Myopia

Our baseline specification allows us to examine how government myopia affects the dynamics of currency digitization. We now show that myopia accelerates currency digitization. In particular, in the government objective (5), the parameter δ captures how present-focused a government is. Indeed, an increase in δ increases i the weight that the country puts on current adoption while reducing the weight of future adoption in its objective function.

Appendix Figure F.4 presents comparative statics in δ for different levels of $\ln(Y)$. Panel A shows that as δ increases, country A's efforts tend to decline for low values of $\ln(Y)$, while they increase for larger values of $\ln(Y)$. Notably, Panel B shows that country B's efforts increase strongly in δ for any level of $\ln(Y)$. Panel C illustrates that countries' total digitization efforts unambiguously increase in δ for any $\ln(Y)$. In particular, larger δ implies both higher and more persistent digitization efforts, which accelerates digitization and reduces T^* (not shown explicitly).

Thus, when countries are more myopic, placing greater emphasis on the current adoption of their currency rather than future adoption, they exert overall higher digitization efforts, which accelerates digitization. This occurs because they give less consideration to the fact that growing competition from PDM erodes the long-term adoption gains from digitization. Especially the weaker country B, which has strong incentives to move early in currency digitization, exerts higher digitization efforts, when δ is larger. The reason is that an increase in δ shifts the countries' focus toward the presence, thereby making the first-mover advantage from digitization in state $z = 0$ more appealing. This effect strengthens the first-mover advantage in currency digitization for relatively less dominant currencies.

3.5 Learning From Others and Second-Mover Advantage

We now allow the cost function $g(e_t^x) = g_z(e_t^x)$ to depend on the state z to capture that currency digitization by one country may generate technological spillovers, effectively reducing the cost of CBDC issuance for the other country. Specifically, we assume that in state $z = 0$, the cost of

[28]Following earlier practices, we again divide convenience-related parameters by 15 (relative to the baseline) to get meaningful effects from interest rates only. This is important because the benefits of digitization are solely modeled through interest rates and improvements in the passthrough.

launching CBDC (i.e., effort) follows $g_0(e_t^x) = \phi e_t^x + \frac{\lambda(e_t^x)^2}{2}$, while, in states $z = A, B$, the cost is reduced by fraction α, i.e., the cost becomes $g_z(e_t^x) = (1-\alpha)g_0(e_t^x)$. Formally, this model variant is a straightforward extension of the baseline and can be solved numerically.

Appendix Figure F.5 presents comparative statics in α. Panels A and B show that when the spillover effect, captured by α, is larger, both countries exert lower digitization efforts in state $z = 0$. Moreover, higher α lowers the cost of digitization when moving second, which increases digitization efforts in states $z = A$ and $z = B$, as shown in Panels C and D. In other words, countries strategically delay digitization to benefit from positive spillovers from another currency's digitization — they wait for the other country to act first, thereby postponing the initial time of digitization, T^*. However, as spillovers reduce the cost of digitization in states $z = A$ and $z = B$, the interval between individual currencies' digitization shortens. For sufficiently large $\alpha \approx 1$, the digitization of one currency would be followed almost immediately by that of other currencies.

Alternatively, spillovers could also affect the convenience of digitized fiat money and thus the benefits of digitization. To account for such effects, we now stipulate that

$$Z_H^x = Z_L^x(1 + \Delta^{Proportional}) + \Delta^{Fixed} + \alpha^C \mathbb{I}\{z = AB\}.$$

Thus, the convenience of currency x upon digitization increases by $\alpha^C \geq 0$ when the other currency has been digitized too, i.e., in state $z = AB$. Similar to Appendix Figure F.5, Appendix Figure F.6 presents comparative statics in α^C.

One would expect that higher α^C should boost countries' digitization efforts. This is indeed true for the less dominant currency, B. As shown in Panels B and D of Figure F.6, a higher α^C increases B's efforts across all levels of Y in states $z = 0$ and $z = A$. Interestingly, the effect of α^C on A's incentives can be either positive or negative. Specifically, when Y is low, A's main concern is competition from B. If A digitizes its currency in state $z = 0$, this action either accelerates B's digitization (in state $z = 0$) or enhances B's convenience (in state $z = B$). Both effects reduce A's incentives to digitize when Y is low. This pattern reverses when Y is high, and A's primary concern shifts to competition from PDM. In this case, spillovers from B's digitization strengthen A's incentives, as they make digitization a more effective tool for competing with C.

4 Conclusions

We develop a novel framework to study the competition among fiat currencies and private digital money (PDM) in digital payments, set against the backdrop of rising PDM adoption and countries' initiatives to digitize fiat money. We micro-found different currencies' digital payment convenience by modeling payments subject to random search and matching. To enhance the adoption of their currencies in digital payments or to counter growing competition from PDM, countries invest in increasing payment convenience through digitization. Fiat digitization includes launching central bank digital currencies (CBDCs), upgrading existing payment systems, or introducing government-led payment innovations. Our analysis reveals an endogenous pecking order: less dominant fiat currencies tend to digitize earlier, exhibiting a first-mover advantage, while more dominant currencies digitize later with greater effort, showing a second-mover advantage. The weakest currencies forgo digitization altogether.

Interestingly, total digitization efforts by countries are highest when PDM competition is weak but decline as PDM gains traction in digital transactions. A failure to digitize fiat currencies early creates a vacuum that PDM fills, potentially leading to a tipping point where PDM becomes dominant. As PDM gains market share, countries' incentives to digitize fiat money diminish, potentially delaying or halting digitization efforts. This dynamic can result in an equilibrium where fiat currencies play a diminished role, leaving PDM to dominate digital payments. Our findings suggest that the long-term relevance of fiat money in digital payments hinges on early action by countries to digitize their currencies. Finally, we explore model variants and extensions, shedding light on the roles of interoperability, public-private collaboration in payment digitization, and the influence of stablecoins in shaping digital currency competition and adoption.

To maintain tractability amidst complex economic trade-offs, we have abstracted from other realistic elements such as monetary policy implications, the impact of digitization on the banking system, and broader macroeconomic dynamics. Incorporating these features and extending the analysis would be an interesting direction for future research. Moreover, while our micro-foundation links convenience to the medium-of-exchange function of money, in practice, convenience may also reflect the store-of-value and unit-of-account functions of money, often complementing its role as a medium of exchange. Thus, although our analysis focuses on payment competition, our theory could also apply more broadly to currencies competing in multiple functionalities.

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Appendix

A Solving the Markov Equilibrium and Proof of Proposition 1

To avoid repetition, we provide the model solution for the generalized version presented in Section 3.2. In particular, relative to the baseline, we assume that fraction θ of the representative private digital money (in short, PDM) is backed by currency A, changing the market clearing condition for currency A from $m_t^A = P_t^A$ in the baseline to $m_t^A = P_t^A - \theta P_t^C$, i.e., $P_t^A = m_t^A + \theta P_t^C$. Clearly, the baseline is nested and obtains for $\theta = 0$. This is the only place where θ enters (i.e., all other parts remain as in the baseline from the main text), and the introduction of θ does not change the flow of argument. Part I of the argument (presented below) shows in greater detail how the introduction θ affects the market clearing conditions and related relationships. Again, Proposition 1 and the baseline solution can be obtained by simply setting $\theta = 0$ in what follows.

To begin with, we introduce the "CBDC state variable" or "digitization state variable:" $z_t = z = 0$ denotes that no country has digitized its currency by time t (i.e., prior to and including time t); $z_t = z = A$ ($z_t = z = B$) denotes that only country A (B) has digitized its currency by time t; and, $z_t = z = AB$ means that both countries have digitized their currencies by time t. We solve for a Markov equilibrium with state variables (Y, z) so that all equilibrium quantities can be expressed as functions of (Y, z), as we show.

In this Markov equilibrium, at any time $t \geq 0$, the following must hold. First, cohort t chooses the holdings of currencies A, B, C to maximize the expected utility U_t (with U_t from (2)), given prices (P_t^A, P_t^B, P_t^C) subject to $m_t^A + m_t^B + m_t^C = 1$ (i.e., it is optimal to invest the entire endowment since there is no consumption utility at birth at time t). Second, the markets for all currencies clear, that is,

$$m_t^A = P_t^A - \theta P_t^C, \; m_t^B = P_t^B, \; \text{ and } \; m_t^C = P_t^C.$$

Third, both countries A and B choose their efforts according to (5), taking the choice of the other country as given. Finally, Y_t evolves according to (4), while the dynamics of z_t are governed by countries' endogenous digitization efforts.

We solve for the equilibrium in several steps, i.e., the proof has several parts. Part I further characterizes and rewrites the market clearing conditions. Part II characterizes the user optimization. Part III characterizes currency values and adoption as functions of (Y, z). Part IV characterizes the government value function as a function (Y, z). Part V summarizes the systems of coupled ODEs and associated boundary conditions that describe the Markov equilibrium. Overall, to solve for the equilibrium, we conjecture and verify that all quantities can be expressed as functions of (Y, z). Finally, we solve this system of coupled ODEs and boundary conditions numerically for the Markov equilibrium.

We do not provide a formal uniqueness and existence proof for our equilibrium. The numerical solution suggests that the Markov equilibrium we derive exists and is unique among the class of Markov equilibria with states (Y, z).

In what follows, we denote by x the respective currency; unless otherwise mentioned, $x \in \{A, B, C\}$.

A.1 Part I — Market Clearing Conditions with $\theta > 0$

To begin with, recall the market clearing conditions, $m_t^B = P_t^B$ and $m_t^C = P_t^C$, for currencies B and C, respectively, while $P_t^A = m_t^A + \theta m_t^C$ Recall that fraction θ of PDM value P_t^C is backed by currency A reserves, where $\theta \in [0, 1)$ is an exogenous constant.

As a result, total reserves backing PDM are worth θP_t^C units of the consumption good. Thus, the reserves backing PDM consist of $\theta P_t^C / P_t^A$ units of currency A, leaving the circulating supply of currency A at $(1 - \theta P_t^C / P_t^A)$ units. For the market for currency A to clear, the user holds this circulating supply, i.e.,

$$m_t^A / P_t^A = 1 - \theta P_t^C / P_t^A$$

units of currency A. Therefore, the user's holdings of currency A in units of the consumption good is:

$$m_t^A = P_t^A - \theta P_t^C. \tag{A.1}$$

The condition (1), i.e., $m_t^A + m_t^B + m_t^C = 1$, then becomes:

$$P_t^A + P_t^B + P_t^C(1 - \theta) = 1 \quad \Longrightarrow \quad P_t^C = \frac{1 - P_t^A - P_t^B}{1 - \theta} \tag{A.2}$$

and, inserting P_t^C from (A.2) into (A.1), we obtain

$$m_t^A = P_t^A - \theta P_t^C = P_t^A - \frac{\theta(1 - P_t^A - P_t^B)}{1 - \theta} = \frac{P_t^A - \theta(1 - P_t^B)}{1 - \theta}, \tag{A.3}$$

which is the rewritten market clearing condition for currency A.

A.2 Part II — User Optimization

For digitization state $z_t = z$, we denote the set of possible next-period state transitions by $\mathcal{S}(z)$. That is, when $z_t = z$, then $z_{t+dt} = z'$ is possible for all $z' = \mathcal{S}(z)$ with $z' \neq z$; we explicitly exclude z itself from the set $\mathcal{S}(z)$. In particular, $\mathcal{S}(0) = \{A, B\}$, $\mathcal{S}(A) = \mathcal{S}(B) = \{AB\}$, and $\mathcal{S}(AB) = \emptyset$. Thus, when $z = 0$, the digitization state may transition toward A or B. When $z = A$ or $z = B$, the transition state may only transition toward AB — in state $z = AB$, no more state transitions are possible.

We postulate that equilibrium currency values (i.e., prices) $P_t^x = P^x(Y, z)$ for $(Y, z) = (Y_t, z_t)$ follow the law of motion:

$$\frac{dP_t^x}{P_t^x} = \mu^x(Y, z)dt + \sum_{z' \in \mathcal{S}(z)} \Delta^x(Y, z; z') dJ_t^{z,z'}, \tag{A.4}$$

where $\mu^x(Y, z)$ is the endogenous price drift in state $(Y_t, z_t) = (Y, z)$. In (A.4), $\Delta^x(Y, z; z')$ is the endogenous (percentage) value change of currency x if the digitization state changes from z to z'. The jump process $dJ_t^{z,z'} \in \{0, 1\}$ equals one if and only if the digitization state changes from z to z' at time t; otherwise, $dJ_t^{z,z'} = 0$. Note that the arrival rate $\mathbb{E}_t[dJ_t^{z,z'}]/dt$ is endogenous and depends on efforts and state (Y, z).

Recall the definition of expected currency returns in terms of the consumption good, that is,

$$r_t^x := \frac{\mathbb{E}_t[dP_t^x]}{P_t^x dt},$$

which will be a function of the state variables in the Markov equilibrium. We can then write cohort t's consumption c_{t+dt} at $t+dt$ as

$$c_{t+dt} = \sum_{x \in \{A,B,C\}} \frac{m_t^x P_{t+dt}^x}{P_t^x}. \tag{A.5}$$

Observe that $P_{t+dt}^x = P_t^x + dP_t^x$. Because the representative user invests its entire endowment one to buy currencies at time t, it follows that $\sum_{x \in \{A,B,C\}} m_t^x = 1$. We can therefore rewrite (A.5) as follows:

$$c_{t+dt} = 1 + \sum_{x \in \{A,B,C\}} \frac{m_t^x dP_t^x}{P_t^x}. \tag{A.6}$$

Now, note that the representative user maximizes her expected lifetime utility/payoff, i.e.,

$$\max_{m_t^x \geq 0} U_t \quad \text{s.t.} \quad \sum_{x \in \{A,B,C\}} m_t^x = 1, \tag{A.7}$$

taking prices P_t^x as given. Here, the expected lifetime utility/payoff U_t reads:

$$U_t = \mathbb{E}_t[c_{t+dt}] + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt,$$

so that

$$U_t = 1 + \sum_{x \in \{A,B,C\}} m_t^x r_t^x dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt. \tag{A.8}$$

Thus, in light of $\sum_{x \in \{A,B,C\}} m_t^x = 1$ and (A.8), the solution (m_t^A, m_t^B, m_t^C) to (A.7) satisfies

$$(m_t^A, m_t^B, m_t^C) = \arg\max_{m_t^x \geq 0} \Omega(m_t^A, m_t^B, m_t^C) \quad \text{s.t.} \quad \sum_{x \in \{A,B,C\}} m_t^x = 1,$$

with

$$\Omega(m_t^A, m_t^B, m_t^C) := \sum_{x \in \{A,B,C\}} m_t^x r_t^x + Z_t^A v(m_t^A) + Z_t^B v(m_t^B) + Y_t v(m_t^C).$$

Due to $\sum_{x \in \{A,B,C\}} m_t^x = 1$, it must hold in optimum that the user is indifferent between substituting a marginal unit of any currency for another one, i.e.,

$$\frac{\partial \Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^A} = \frac{\partial \Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^B} = \frac{\partial \Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^C}, \tag{A.9}$$

provided $m_t^x \in (0,1)$. Note that condition (A.9) becomes equivalent to (12) from the main text, as desired.

Taking the derivative in (A.9) and using the definition of $\Omega(m_t^A, m_t^B, m_t^C)$, we get:

$$Y_t v'(m_t^C) + r_t^C = Z_t^A v'\left(m_t^A\right) + r_t^A \quad \text{and} \quad Y_t v'(m_t^C) + r_t^C = Z_t^B v'(m_t^B) + r_t^B. \tag{A.10}$$

Inserting the market clearing condition $m_t^A = \frac{P_t^A - \theta(1-P_t^B)}{1-\theta}$ from (A.3), $m_t^B = P_t^B$, and $m_t^C = P_t^C$ into (A.10), we obtain

$$\begin{aligned} Y_t v'(P_t^C) + r_t^C &= Z_t^A v'\left(\frac{P_t^A - \theta(1-P_t^B)}{1-\theta}\right) + r_t^A \\ Y_t v'(P_t^C) + r_t^C &= Z_t^B v'(P_t^B) + r_t^B. \end{aligned} \tag{A.11}$$

Notice that (A.10) is equivalent to (13) upon setting $\theta = 0$. Because $\lim_{m_t^x \to 0} v'(m_t^x) = \infty$, any solution to (13) or (A.11) must satisfy $m_t^x, P_t^x \in (0,1)$.

A.3 Part III — Solving for Currency Values and Adoption, and Equilibrium Conditions

We now express the currency values P_t^x, adoption levels m_t^x, and currency returns r_t^x, as well as the countries'digitiation efforts e_t^x as functions of Y and state $z \in \{0, A, B, AB\}$, and we omit time subscripts unless necessary. In doing so, we also derive useful equilibrium relations and conditions that we invoke later on. Unless otherwise mentioned, we denote by x the respective currency, where $x \in \{A, B, C\}$.

We conjecture and verify that $P_t^x = P(Y_t, z_t)$, $m_t^x = m^x(Y_t, z_t)$, and $e_t^{x'} = e^{x'}(Y_t, z_t)$ for $x = A, B, C$ and $x' = A, B$, for functions $P^x(\cdot)$, $m^x(\cdot)$, and $e^{x'}(\cdot)$. It then follows that r_t^x is a function of (Y, z) too, in that $r_t^x = r^x(Y, z)$. Also write $dY = \mu^Y(Y, z)dt$ whereby the drift of dY reads according to (4):

$$\mu^Y(Y, z) = \begin{cases} \mu Y m^C(Y, z) & \text{if} \quad Y < \overline{Y} \\ 0 & \text{if} \quad Y = \overline{Y}. \end{cases} \tag{A.12}$$

Next, market clearing in equilibrium implies $P_t^x = P^x(Y, z) = m_t^x = m^x(Y, z)$ for $x \in \{A, B, C\}$, and, according to (A.3):

$$m_t^A = m^A(Y, z) = \frac{P^A(Y, z) - \theta(1 - P^B(Y, z))}{1 - \theta}.$$

Also, we get from (A.2):

$$P^A(Y, z) + P^B(Y, z) + P^C(Y, z)(1 - \theta) = 1 \quad \Longleftrightarrow \quad P^C(Y, z) = \frac{1 - P^A(Y, z) - P^B(Y, z)}{1 - \theta}. \tag{A.13}$$

Recall (A.4), and observe that:

$$\Delta^x(Y, z; z') = \frac{P^x(Y, z')}{P^x(Y, z)} - 1. \tag{A.14}$$

Thus, we obtain $\Delta^x(Y, z; z')P^x(Y, z) = P^x(Y, z') - P^x(Y, z)$.

Denote $(P^x)'(Y, z) = \frac{\partial}{\partial Y} P^x(Y, z)$. By Ito's Lemma, the drift of currency value x, that is,

$\mu^x(Y,z)$, becomes

$$\mu^x(Y,z) = \left(\frac{(P^x)'(Y,z)}{P^x(Y,z)}\right)\mu^Y(Y,z), \tag{A.15}$$

where $\mu^Y(Y,z)$ is the drift of dY from (A.12) (which vanishes for $Y = \overline{Y}$). Thus, for $Y = \overline{Y}$, the price drifts $\mu_t^x = \mu^x(Y,z)$ from (A.4) equals zero.

Also note that because $P_t^A + P_t^B + P_t^C(1-\theta) = 1$ — that is, $P^A(Y,z) + P^B(Y,z) + P^C(Y,z)(1-\theta) = 1$ from (A.13) — we have $dP_t^A + dP_t^B + dP_t^C(1-\theta) = 0$. This implies by means of (A.4)

$$\mu^A(Y,z)P^A(Y,z) + \mu^B(Y,z)P^B(Y,z) + \mu^C(Y,z)P^C(Y,z)(1-\theta) = 0 \tag{A.16}$$

as well as

$$\Delta^A(Y,z,z')P^A(Y,z) + \Delta^B(Y,z,z')P^B(Y,z) + \Delta^C(Y,z,z')P^C(Y,z)(1-\theta) = 0 \tag{A.17}$$

for all $z' \in \mathcal{S}(z)$, i.e., for all states z' that can be reached from state z.

In light of (A.16), (A.17), and $P_t^A + P_t^B + P_t^C(1-\theta) = 1$, it suffices to characterize the currency values and dynamics for currencies A and B, and the value and the dynamics for currency C follow as the residual, and can be backed out knowing $P^A(Y,z)$ and $P^B(Y,z)$ (and their dynamics).

Next, we can characterize expected returns r_t^x, and write $r_t^x = r^x(Y,z)$. We start by analyzing the arrival rates of the process $dJ^{z,z'}$ for $z, z' \in \{0, A, B, AB\}$. Note that the only possible transitions from state $z = 0$ are $z' = A, B$. The only possible transition from states $z = A, B$ is $z' = AB$. We can calculate the transition probabilities in these cases over a short period of time $[t, t+dt)$:

$$\begin{aligned} \mathbb{E}[dJ^{0,A}] &= e^A(Y,0)dt \quad \text{and} \quad \mathbb{E}[dJ^{0,B}] = e^B(Y,0)dt \\ \mathbb{E}[dJ^{A,AB}] &= e^B(Y,A)dt \quad \text{and} \quad \mathbb{E}[dJ^{B,AB}] = e^A(Y,B)dt. \end{aligned} \tag{A.18}$$

In all other cases, $dJ^{z,z'}$ equals zero with certainty, so that $dJ^{AB,z'} = 0$, $dJ^{0,AB} = 0$, $dJ^{A,B} = dJ^{B,A} = dJ^{A,0} = dJ^{B,0} = 0$. Likewise, we also obtain that $e^{x'}(Y,x) = e^{x'}(Y,AB) = 0$ for $x' = A, B$, i.e., there is no more effort after successful digitization.

Taking the expectation in (A.4) and using (A.14) and (A.18), we can calculate for $x = A, B, C$:

$$\begin{aligned} r^x(Y,0) &= \mu^x(Y,0) + e^A(Y,0)\left(\frac{P^x(Y,A)}{P^x(Y,0)} - 1\right) + \ e^B(Y,0)\left(\frac{P^x(Y,B)}{P^x(Y,0)} - 1\right), \\ r^x(Y,A) &= \mu^x(Y,A) + e^B(Y,A)\left(\frac{P^x(Y,AB)}{P^x(Y,A)} - 1\right), \\ r^x(Y,B) &= \mu^x(Y,B) + e^A(Y,B)\left(\frac{P^x(Y,AB)}{P^x(Y,B)} - 1\right), \\ r^x(Y,AB) &= \mu^x(Y,AB). \end{aligned} \tag{A.19}$$

Combining (A.16), (A.17), and (A.19) as well as (A.14), we also obtain

$$r^A(Y,z)P^A(Y,z) + r^B(Y,z)P^B(Y,z) + r^C(Y,z)P^C(Y,z)(1-\theta) = 0. \tag{A.20}$$

The equilibrium condition (A.11) yields for $x' = A, B$:

$$Yv'\big(P^C(Y,z)\big) + r^C(Y,z) = Z^{x'}(Y,z)v'\big(m^{x'}(Y,z)\big) + r^{x'}(Y,z), \tag{A.21}$$

where $Z^A(Y,z) = Z_L^A$ for $z = 0, B$ and $Z^A(Y,z) = Z_H^A$ for $z = A, AB$. Likewise, $Z^B(Y,z) = Z_L^B$ for $z = 0, A$ and $Z^B(Y,z) = Z_H^B$ for $z = B, AB$. Note that by (A.3), $m^A(Y,z) = \frac{P^A(Y,z)-\theta(1-P^B(Y,z))}{1-\theta}$, and $m^B(Y,z) = P^B(Y,z)$. It was also used that $m^C(Y,z) = P^C(Y,z)$.

As a result, under the assumption that optimal effort e_t^x is a function of (Y,z) (i.e., $e_t^x = e^x(Y,z)$), we have verified that the equilibrium pricing condition (A.11) depends only on state variables (Y,z). As such, currency values can be expressed in terms of (Y,z). The next Part IV shows that indeed, optimal effort e_t^x is a function of (Y,z).

A.4 Part IV: Solving Government Objective

We characterize the government/country value function as a function of (Y,z) for both countries $x = A, B$. In this part, x refers to a country and, unless otherwise mentioned, takes the values $x = A, B$.

At a given time t, the government $x = A, B$ chooses effort $(e_s^x)_{s\geq t}$ to maximize the objective function V_t^x as follows:

$$V_t^x = \max_{(e_s^x)_{s\geq t}} \mathbb{E}_t^x\left[\int_t^\infty e^{-\delta(s-t)}\left(\delta f_s^x - \frac{\lambda(e_s^x)^2}{2} - \phi e_s^x\right)ds\right], \tag{A.22}$$

where we set $f_s^x = P_s^x$ in the baseline.

By the dynamic programming principle, the government's value function solves the following HJB equation:

$$\delta V_t^x = \max_{e_t^x\geq 0}\left(\delta f_t^x - \frac{\lambda(e_t^x)^2}{2} - \phi e_t^x + \frac{\mathbb{E}_t^x[dV_t^x]}{dt}\right), \tag{A.23}$$

which is (14). Notice that the expectation $\mathbb{E}_t^x[dV_t^x]$ depends on the levels of (e^A, e^B) and is conditional on country x's time$-t$ information (which includes time-t public information and e^x); country x takes the effort of the other country $-x$ as given. Effort e_t^x is not observable for the user or the competing country, and countries cannot commit to effort levels. As such, the choice of effort e_t^x at any time t is privately optimal for x. Clearly, effort e_t^x is redundant after time T^x, i.e., after country x has digitized. As such, we set $e^x(Y,x) = e^x(Y,AB) = 0$ for $x = A, B$.

Likewise, by the dynamic programming principle and the integral expression (6), welfare satisfies

$$\gamma W_t = \sum_{x=A,B}\left[\gamma Z_t^x v(m_t^x) - g^x(e_t^x)\right] + \gamma Y_t v(m_t^x) + \frac{\mathbb{E}_t[dW_t]}{dt},$$

which coincides with (16) from the main text.

Next, we can express V_t^x and time-t welfare W_t as functions of (Y,z) only, i.e., $V_t^x = V^x(Y_t, z_t)$ and $W_t = W(Y,z)$. Further, we solve for efforts $e_t^x = e^x(Y,z)$ and derive eight first order ODEs that characterize the functions $V^x(Y,z)$ for $x = A, B$. To do so, we now consider all states $z = 0, A, B, AB$ separately. In what follows, x is either A or B. When $x = A$, then $-x = B$ and vice versa (i.e., when $x = B$, then $-x = A$). In what follows, we suppress the dependence of $\mathbb{E}_t^x$ on (x,t)

and simply write $\mathbb{E}$ for the expectation. Likewise, we suppress time subscripts, unless confusion arises. Last, to simplify notation, we define $(V^x)'(Y,z) := \frac{\partial V^x(Y,z)}{\partial Y}$, where $x = A, B$, as well as $W'(Y,z) = \frac{\partial W(Y,z)}{\partial Y}$.

In what follows, we consider Markovian flow payoff functions f_t^x, satisfying $f_t^x = f^x(Y,z)$. Again, this is the case in our baseline, where we assume that $f_t^x = P_t^x$, which implies in the Markov equilibrium that $f_t^x = P^x(Y,z) \equiv f^x(Y,z)$.

Finally, we note that below ordinary differential equations hold for $Y \in (0, \overline{Y})$, where, given $Y_0 > 0$, $Y = 0$ is never attained. At $Y = \overline{Y}$, the drift of dY vanishes, in which case the derived ODEs collapse to non-linear equations. That is to say, the equilibrium relations we derive next also apply for $Y = \overline{Y}$ (upon setting the drift of dY to zero).

A.4.1 State $z = AB$

In state $z = AB$, efforts are redundant, so clearly $e^x(Y, AB) = 0$. Using Ito's Lemma, we can calculate for $x = A, B$:

$$\frac{\mathbb{E}[dV^x(Y,AB)]}{dt} = (V^x)'(Y,AB)\mu^Y(Y,AB),$$

where $\mu^Y(Y,z)$ is the drift of dY from (A.12). Inserting these relations into (A.23), we obtain

$$\delta V^x(Y,AB) = \delta f^x(Y,AB) + (V^x)'(Y,AB)\mu^Y(Y,AB), \tag{A.24}$$

which are two first-order ODEs in Y for $x = A, B$, given $z = AB$.

Next, in state $z = AB$, we have $W_t = W(Y, AB)$. Since efforts are zero, welfare $W(Y, AB)$ solves

$$\gamma W(Y,AB) = \sum_{x=A,B} \gamma Z_H^x v\big(m^x(Y,AB)\big) + \gamma Y v\big(m^C(Y,AB)\big) + W'(Y,AB)\mu^Y(Y,AB), \tag{A.25}$$

which is a first-order ODE in Y.

A.4.2 State $z = x$

Consider state $z = x$ for $x = A$ or $x = B$. Recall that when $x = A$, then $-x = B$ and vice versa. Then, $e^x(Y,x) = 0$. Using Ito's Lemma for jump processes, we can calculate

$$\frac{\mathbb{E}[dV^x(Y,z)]}{dt} = (V^x)'(Y,x)\mu^Y(Y,x) + e^{-x}(Y,x)(V^x(Y,AB) - V^x(Y,x)), \tag{A.26}$$

and

$$\frac{\mathbb{E}[dV^{-x}(Y,z)]}{dt} = (V^{-x})'(Y,x)\mu^Y(Y,x) + e^{-x}(Y,x)(V^{-x}(Y,AB) - V^{-x}(Y,x)), \tag{A.27}$$

Inserting (A.27) into (A.23) for country $-x$, we obtain

$$\delta V^{-x}(Y,x) = \max_{e^{-x}(Y,x)\geq 0} \Big\{ \delta f^{-x}(Y,x) + (V^{-x})'(Y,x)\mu^Y(Y,x) \\ + e^{-x}(Y,x)\big[V^{-x}(Y,AB) - V^{-x}(Y,x)\big] - \frac{\lambda(e^{-x}(Y,x))^2}{2} - \phi e^{-x}(Y,x) \Big\}. \tag{A.28}$$

The optimization with respect to effort $e^{-x}(Y,x)$ yields (with some abuse of notation)

$$e^{-x}(Y,x) = \frac{\max\{0, V^{-x}(Y,AB) - V^{-x}(Y,x) - \phi\}}{\lambda} = \frac{\big[V^{-x}(Y,AB) - V^{-x}(Y,x) - \phi\big]^+}{\lambda}, \tag{A.29}$$

where $[\cdot]^+ = \max\{0,\cdot\}$ is the positive part of a real number.

Performing similar steps for country x (i.e., inserting (A.26) and $e^x(Y,x) = 0$ into (A.23) and rearranging), we have

$$\delta V^x(Y,x) = \delta f^x(Y,x) + (V^x)'(Y,x)\mu^Y(Y,x) + e^{-x}(Y,x)\big[V^x(Y,AB) - V^x(Y,x)\big]. \tag{A.30}$$

subject to optimal effort $e^{-x}(Y,x)$ from (A.29). Note that in state $z = x \in \{A,B\}$, the two ODEs (A.28) and (A.30), which characterize the value functions and optimal digitization efforts, are interconnected, i.e., coupled.

Finally, in state x, welfare $W(Y,x)$ solves the ODE

$$\gamma W(Y,x) = \gamma\big[Z_H^x v\big(m^x(Y,x)\big) + Z_L^{-x} v\big(m^{-x}(Y,x)\big) + Yv(m^C(Y,x))\big] + W'(Y,x)\mu^Y(Y,x) \\ + e^{-x}(Y,x)\big[W(Y,AB) - W(Y,x)\big] - \frac{\lambda(e^{-x}(Y,x))^2}{2} - \phi e^{-x}(Y,x), \tag{A.31}$$

subject to optimal effort $e^{-x}(Y,x)$ from (A.29).

A.4.3 State $z = -x$

The analysis of state $z = -x$ is analogous when we replace x by $-x$.

A.4.4 State $z = 0$

In state $z = 0$, we can calculate for $x = A, B$:

$$\mathbb{E}[dV^x(Y,z)] = (V^x)'(Y,0)\mu^Y(Y,0) \\ + e^x(Y,0)(V^x(Y,x) - V^x(Y,0)) + e^{-x}(Y,0)(V^x(Y,-x) - V^x(Y,0)). \tag{A.32}$$

We can now insert (A.32) into (A.23) and obtain (after omitting time subscripts) in state $(Y,0)$ for $x = A, B$:

$$\delta V^x(Y,0) = \max_{e^x(Y,0)\geq 0} \Big\{ \delta f^x(Y,0) - \frac{\lambda(e^x(Y,0))^2}{2} - \phi e^x(Y,0) + (V^x)'(Y,0)\mu^Y(Y,0) \\ + e^x(Y,0)\big[V^x(Y,x) - V^x(Y,0)\big] + e^{-x}(Y,0)\big[V^x(Y,-x) - V^x(Y,0)\big] \Big\}. \tag{A.33}$$

Country x takes the other country's effort $e^{-x}(Y,0)$ as given. The optimization with respect to effort $e^x(Y,0)$ in state $z=0$ yields

$$e^x(Y,0) = \frac{\left[V^x(Y,x) - V^x(Y,0) - \phi\right]^+}{\lambda} \tag{A.34}$$

for $x = A, B$. Analogously, one can solve for country $-x$'s effort. Welfare solves in state $z=0$

$$\begin{aligned}\gamma W(Y,0) = &\sum_{x=A,B} \gamma Z_L^x v\big(m^x(Y,0)\big) + \gamma Y v\big(m^C(Y,0)\big) + W'(Y,0)\mu^Y(Y,0) \\ &+ \sum_{x=A,B}\left[e^x(Y,0)\big[W(Y,x) - W(Y,0)\big] - \frac{\lambda(e^x(Y,0))^2}{2} - \phi e^x(Y,0)\right],\end{aligned} \tag{A.35}$$

with efforts $e^x(Y,0)$ satisfying (A.34).

A.5 Part V: System of ODEs and Non-Linear Equations

To get a better overview, we now explicitly gather the ODEs that characterize the Markov equilibrium by collecting and summarizing our findings from Parts I through IV. We separately consider the states $z=0$, $z=x\in\{A,B\}$, and $z=AB$, starting with state $z=AB$.

Next, recall that

$$\begin{aligned}m^A(Y,z) &= \frac{P^A(Y,z) - \theta(1-P^B(Y,z))}{1-\theta} \\ m^B(Y,z) &= P^B(Y,z) \\ m^C(Y,z) &= P^C(Y,z) = \frac{1-P^A(Y,z)-P^B(Y,z)}{1-\theta}.\end{aligned} \tag{A.36}$$

These relations will be used throughout for any $z \in \{0, A, B, AB\}$.

Also recall that in the baseline, the flow payoff function $f_t^x = f^x(Y,z)$ satisfies $f^x(Y,z) = P^x(Y,z)$.

A.5.1 State $z = AB$

In state $z=AB$, we combine (A.19), (A.15), and (A.20) (as well as (A.36)) to calculate

$$r^A(Y,AB) = \left(\frac{(P^A)'(Y,AB)}{P^A(Y,AB)}\right)\mu^Y(Y,AB) \tag{A.37}$$

$$r^B(Y,AB) = \left(\frac{(P^B)'(Y,AB)}{P^B(Y,AB)}\right)\mu^Y(Y,AB) \tag{A.38}$$

$$r^C(Y,AB) = -\left(\frac{r^A(Y,AB)P^A(Y,AB) + r^B(Y,AB)P^B(Y,AB)}{1-P^A(Y,AB)-P^B(Y,AB)}\right) \tag{A.39}$$

Then, (A.21) implies

$$\begin{aligned} Yv'\left(m^C(Y,AB)\right) + r^C(Y,AB) &= Z_H^A v'\left(m^A(Y,AB)\right) + r^A(Y,AB) \\ Yv'\left(m^C(Y,AB)\right) + r^C(Y,AB) &= Z_H^B v'\left(m^B(Y,AB)\right) + r^B(Y,AB). \end{aligned} \tag{A.40}$$

And, from (A.24), we know

$$\begin{aligned} \delta V^A(Y,AB) =& \delta f^A(Y,AB) + (V^A)'(Y,AB)\mu^Y(Y,AB), \\ \delta V^B(Y,AB) =& \delta f^B(Y,AB) + (V^B)'(Y,AB)\mu^Y(Y,AB). \end{aligned} \tag{A.41}$$

Further, (A.25) holds, yielding a system of five first-order ODEs.

At the boundary $Y = \overline{Y}$, the drift of dY vanishes (i.e., $\mu^Y(\overline{Y}, z) = 0$), and the solution, that is, $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB), W(\overline{Y}, AB))$ for $x = A, B$, is characterized by the following system of five equations (for $x = A, B$):

$$\begin{aligned} \overline{Y}v'\left(m^C(\overline{Y},AB)\right) &= Z_H^x v'\left(m^x(\overline{Y},AB)\right) \\ \delta V^x(\overline{Y},AB) &= \delta f^x(\overline{Y},AB), \\ \gamma W(\overline{Y},AB) &= \sum_{x=A,B} \gamma Z_H^x v\big(m^x(\overline{Y},AB)\big) + \gamma \overline{Y} v\big(m^C(\overline{Y},AB)\big), \end{aligned} \tag{A.42}$$

where $m^A(\overline{Y}, 0), m^B(\overline{Y}, 0), m^C(\overline{Y}, 0)$ satisfy (A.36). To solve for the Markov equilibrium in state $z = AB$, we first solve the system of non-linear equations (A.42) for the five unknowns $P^A(\overline{Y}, AB)$, $P^B(\overline{Y}, AB)$, $V^A(\overline{Y}, AB)$, $V^B(\overline{Y}, AB)$, and $W(\overline{Y}, AB)$ — there is no closed-form solution. The calculation of $m^C(\overline{Y}, AB) = P^C(\overline{Y}, AB)$ follows from (A.36).

Then, we solve the system of five coupled first order ODEs in (A.40), (A.41), as well as (A.25) subject to the boundary conditions/boundary values $\left(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB), W(\overline{Y}, AB)\right)_{x=A,B}$ — which then yields values $P^x(Y, AB)$ for $x = A, B$ as well as $P^C(Y, AB)$ via $P^C(Y, AB) = m^C(Y, AB) = \frac{1-P^A(Y,z)-P^B(Y,z)}{1-\theta}$. We follow this approach in the other states $z = A, B, 0$ too in order to solve the system of differential equations, starting at the upper boundary $\overline{Y}$.

A.5.2 State $z = x \in \{A, B\}$

In state $z = x$, we have $e^x(Y, x) = 0$ and $e^{-x}(Y, x) = \frac{\left[V^{-x}(Y,AB)-V^{-x}(Y,x)-\phi\right]^+}{\lambda}$. Then, we can combine (A.19), (A.15), and (A.20) to obtain

$$\begin{aligned} r^A(Y,x) &= \left(\frac{(P^A)'(Y,x)}{P^A(Y,x)}\right)\mu^Y(Y,x) + e^{-x}(Y,x)\left(\frac{P^A(Y,AB)}{P^A(Y,x)} - 1\right). \\ r^B(Y,x) &= \left(\frac{(P^B)'(Y,x)}{P^B(Y,x)}\right)\mu^Y(Y,x) + e^{-x}(Y,x)\left(\frac{P^B(Y,AB)}{P^B(Y,x)} - 1\right) \\ r^C(Y,x) &= -\left(\frac{r^A(Y,x)P^A(Y,x) + r^B(Y,x)P^B(Y,x)}{1 - P^A(Y,x) - P^B(Y,x)}\right) \end{aligned}$$

Then, (A.21) implies

$$Yv'\left(m^C(Y,x)\right) + r^C(Y,x) = Z^A(Y,z)v'\left(m^A(Y,x)\right) + r^A(Y,x)$$
$$Yv'\left(m^C(Y,x)\right) + r^C(Y,x) = Z^B(Y,x)v'\left(m^B(Y,x)\right) + r^B(Y,x), \tag{A.43}$$

where $Z^A(Y,z) = Z^A_L$ for $z = 0, B$ and $Z^A(Y,z) = Z^A_H$ for $z = A, AB$. Likewise, $Z^B(Y,z) = Z^B_L$ for $z = 0, A$ and $Z^B(Y,z) = Z^B_H$ for $z = B, AB$.

Further, we recall that $V^A(Y,x)$ and $V^B(Y,x)$ solve the ODEs (A.28) and (A.30), while $W(Y,x)$ solves the ODE (A.31). This yields a system of five interconnected first-order ODEs in state $x = AB$, which are linked to state $z = AB$ via the jump processes, facilitating stochastic state transitions from state $z = x$ to state $z = AB$.

To solve the model for the Markov equilibrium in state $z = x$, we need to solve the system of five coupled first order ODEs, which is characterized in (A.43), (A.28), (A.30), and (A.31), for $P^A(Y,x)$, $P^B(Y,x)$, $V^A(Y,x)$, $V^B(Y,x)$, and $W(Y,x)$. Given the solution, we then also obtain $P^C(Y,x) = m^C(Y,x) = \frac{1-P^A(Y,x)-P^B(Y,x)}{1-\theta}$.

At the boundary $Y = \overline{Y}$, the drift of dY vanishes so that the system characterized in (A.43), (A.28), (A.30), and (A.31) becomes a system of five non-linear equations, which can be solved for the four unknowns $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$, $V^A(\overline{Y},x)$, $V^B(\overline{Y},x)$, and $W(\overline{Y},x)$, given the values of $P^A(\overline{Y},AB)$, $P^B(\overline{Y},AB)$, $V^A(\overline{Y},AB)$, $V^B(\overline{Y},AB)$, and $W(\overline{Y},x)$. Given these boundary conditions at $\overline{Y}$, we can then solve the system of ODEs — characterized via (A.43), (A.28), (A.30), and (A.31) — on $(0,\overline{Y}]$ in state $z = x$.

A.5.3 State $z = 0$

In state $z = 0$, we have

$$e^A(Y,0) = \frac{\left[V^A(Y,A) - V^A(Y,0) - \phi\right]^+}{\lambda} \quad \text{and} \quad e^B(Y,0) = \frac{\left[V^B(Y,B) - V^B(Y,0) - \phi\right]^+}{\lambda}.$$

Then, we can combine (A.19), (A.15), and (A.20) to obtain

$$r^A(Y,0) = \left(\frac{(P^A)'(Y,0)}{P^A(Y,0)}\right)\mu^Y(Y,0) + \sum_{x=A,B} e^x(Y,0)\left(\frac{P^A(Y,x)}{P^A(Y,0)} - 1\right).$$
$$r^B(Y,0) = \left(\frac{(P^B)'(Y,0)}{P^B(Y,0)}\right)\mu^Y(Y,0) + \sum_{x=A,B} e^x(Y,0)\left(\frac{P^B(Y,x)}{P^B(Y,0)} - 1\right)$$
$$r^C(Y,0) = -\left(\frac{r^A(Y,0)P^A(Y,0) + r^B(Y,0)P^B(Y,0)}{1 - P^A(Y,0) - P^B(Y,0)}\right)$$

Then, (A.21) implies

$$Yv'\left(m^C(Y,0)\right) + r^C(Y,0) = Z^A_L v'\left(m^A(Y,0)\right) + r^A(Y,0)$$
$$Yv'\left(m^C(Y,0)\right) + r^C(Y,0) = Z^B_L v'\left(m^B(Y,0)\right) + r^B(Y,0) \tag{A.44}$$

Moreover, $V^A(Y,0)$ and $V^B(Y,0)$ solve the ODE system (A.33) and $W(Y,0)$ solves the ODE (A.35). To solve the model for the Markov equilibrium in state $z=0$, we need to solve this system of five interconnected first order ODEs, which is characterized in (A.44), (A.33), and (A.35), for $P^A(Y,0)$, $P^B(Y,0)$, $V^A(Y,0)$, $V^B(Y,0)$, and $W(Y,0)$. We then also obtain $P^C(Y,0) = m^C(Y,0) = \frac{1-P^A(Y,0)-P^B(Y,0)}{1-\theta}$.

At the boundary $Y=\overline{Y}$, the drift of dY vanishes so that the system characterized in (A.44), (A.33), and (A.35) becomes a system of five non-linear equations, which can be solved for the five unknowns $P^A(\overline{Y},0)$, $P^B(\overline{Y},0)$, $V^A(\overline{Y},0)$, $V^B(\overline{Y},0)$ and $W(\overline{Y},0)$, given the values of $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$, $V^A(\overline{Y},x)$, $V^B(\overline{Y},x)$, and $W(\overline{Y},x)$ for $x=A,B$.

A.6 Discussion: Numerical Solution Method

The numerical solution requires to solve the system of ODEs from Section A.5.

Because the currency values in states $z=A$ and $z=B$ depend on the currency values in state $z=AB$, one has to solve the model backward in terms of the state variable z, starting with state $z=AB$.

Having obtained $P^x(Y,AB)$, $V^x(Y,AB)$, and $W(Y,AB)$ for $Y\in(0,\overline{Y}]$, one can solve for $P^x(Y,A)$ and $P^x(Y,B)$, value functions $V^x(Y,A)$ and $V^x(Y,B)$ and efforts (determining the transition probabilities for z), and welfare $W(Y,A)$ and $W(Y,B)$. Here, $x=A,B$.

Having obtained $P^x(Y,A)$ and $P^x(Y,B)$ as well as $V^x(Y,A)$ and $V^x(Y,B)$, one can solve for currency values $P^x(Y,0)$ and value functions $V^x(Y,0)$, efforts, and welfare $W(Y,0)$.

In other words, the solution admits the hierarchy in terms of the state variable: (i) $z=AB$ (no more transitions possible), (ii) $z=A,B$ (only possible transition: $z'=AB$), and (iii) $z=0$ (possible transitions: $z'=A$ and $z'=B$). We solve the equilibrium system obeying to the order of hierarchy, (i), (ii), and (iii). The solution can be numerically obtained via a standard ODE solver, such ode15s in Matlab.

B Planner Solution

We assume $\theta=0$. In the planner solution, efforts are chosen according to the HJB equation:

$$\gamma W_t = \max_{e_t^A, e_t^B \geq 0} \left\{ \sum_{x=A,B} (\gamma Z_t^x v(m_t^x) - g^x(e_t^x)) + \gamma Y_t v(m_t^x) + \frac{\mathbb{E}_t[dW_t]}{dt} \right\}. \tag{B.45}$$

We solve for a Markov equilibrium, where we can express all equilibrium quantities as functions of (Y,z). In this Markov equilibrium, at any point in time t, cohort t chooses the holdings of currencies A,B,C to maximize the expected utility U_t (with U_t from (2)), given prices (P_t^A, P_t^B, P_t^C), yielding $m_t^A + m_t^B + m_t^C = 1$. The markets for all currencies clear, i.e., $m_t^A = P_t^A$, $m_t^B = P_t^B$, and $m_t^C = P_t^C$. And, (e_t^A, e_t^B) is chosen according to (B.45).

Analogous to the baseline, we characterize the relevant ODEs for $z=0,x,AB$ where $x=A,B$. The optimal choice of the planner's effort in state (Y,z) is denoted $e_*(Y,z)$. We will show that (for

$x = A, B$ and $-x = B, A$):

$$e_*^x(Y,0) = \frac{[W(Y,x) - W(Y,0) - \phi]^+}{\lambda} \quad \text{and} \quad e_*^x(Y,-x) = \frac{[W(Y,AB) - W(Y,-x) - \phi]^+}{\lambda}. \tag{B.46}$$

In addition, $e_*^x(Y,x) = e_*^x(Y,AB) = 0$. Throughout, we have for all $x = A, B, C$ that $m^x(Y,z) = P^x(Y,z)$ by means of market clearing.

State $z = AB$. Since there is no effort anymore in state $z = AB$, the solution and equilibrium coincide with the ones from the baseline.

State $z = x$. In state $z = x$, we have $e_*^x(Y,x) = 0$ and $e_*^{-x}(Y,x) = \frac{\left[W(Y,AB) - W(Y,x) - \phi\right]^+}{\lambda}$. The currency returns are characterized via

$$r^A(Y,x) = \left(\frac{(P^A)'(Y,x)}{P^A(Y,x)}\right)\mu^Y(Y,x) + e_*^{-x}(Y,x)\left(\frac{P^A(Y,AB)}{P^A(Y,x)} - 1\right).$$

$$r^B(Y,x) = \left(\frac{(P^B)'(Y,x)}{P^B(Y,x)}\right)\mu^Y(Y,x) + e_*^{-x}(Y,x)\left(\frac{P^B(Y,AB)}{P^B(Y,x)} - 1\right)$$

$$r^C(Y,x) = -\left(\frac{r^A(Y,x)P^A(Y,x) + r^B(Y,x)P^B(Y,x)}{1 - P^A(Y,x) - P^B(Y,x)}\right)$$

In addition, we have the pricing relationship (A.43), that is,

$$Yv'\left(m^C(Y,x)\right) + r^C(Y,x) = Z^A(Y,z)v'\left(m^A(Y,x)\right) + r^A(Y,x) = Z^B(Y,x)v'\left(m^B(Y,x)\right) + r^B(Y,x),$$

where $Z^A(Y,z) = Z_L^A$ for $z = 0, B$ and $Z^A(Y,z) = Z_H^A$ for $z = A, AB$. Likewise, $Z^B(Y,z) = Z_L^B$ for $z = 0, A$ and $Z^B(Y,z) = Z_H^B$ for $z = B, AB$.

Further, in state x, welfare $W(Y,x)$ in the planner solution solves the ODE

$$\begin{aligned} \gamma W(Y,x) = \max_{e_*^{-x}} \Big\{ &\gamma\big[Z_H^x v\big(m^x(Y,x)\big) + Z_L^{-x} v\big(m^{-x}(Y,x)\big) + Yv(m^C(Y,x))\big] + W'(Y,x)\mu^Y(Y,x) \\ &+ e_*^{-x}(Y,x)\big(W(Y,AB) - W(Y,x)\big) - \frac{\lambda(e_*^{-x}(Y,x))^2}{2} - \phi e_*^{-x}(Y,x)\Big\}. \end{aligned} \tag{B.47}$$

Taking the first-order condition with respect to e_*^{-x} yields

$$e_*^{-x}(Y,x) = \frac{[W(Y,AB) - W(Y,-x) - \phi]^+}{\lambda},$$

as desired.

To solve the model for the Markov equilibrium in state $z = x$, we need to solve the system of three coupled first order ODEs, which is characterized in (A.43) and (B.47), for $P^A(Y,x)$, $P^B(Y,x)$, and $W(Y,x)$. At the boundary $Y = \overline{Y}$, the drift of dY vanishes so that the system characterized in (A.43) and (B.47) becomes a system of three non-linear equations, which can be solved for the four unknowns $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$, and $W(\overline{Y},x)$, given the values of $P^A(\overline{Y},AB)$, $P^B(\overline{Y},AB)$, $V^A(\overline{Y},AB)$, $V^B(\overline{Y},AB)$, and $W(\overline{Y},x)$. Given these boundary conditions at $\overline{Y}$, we can then solve the system of ODEs.

State $z = 0$. In state $z = 0$, we can combine (A.19), (A.15), and (A.20) to obtain

$$r^A(Y,0) = \left(\frac{(P^A)'(Y,0)}{P^A(Y,0)}\right)\mu^Y(Y,0) + \sum_{x=A,B} e_*^x(Y,0)\left(\frac{P^A(Y,x)}{P^A(Y,0)} - 1\right).$$

$$r^B(Y,0) = \left(\frac{(P^B)'(Y,0)}{P^B(Y,0)}\right)\mu^Y(Y,0) + \sum_{x=A,B} e_*^x(Y,0)\left(\frac{P^B(Y,x)}{P^B(Y,0)} - 1\right)$$

$$r^C(Y,0) = -\left(\frac{r^A(Y,0)P^A(Y,0) + r^B(Y,0)P^B(Y,0)}{1 - P^A(Y,0) - P^B(Y,0)}\right),$$

where optima (equilibrium) efforts $e_*^A(Y,0)$ and $e_*^B(Y,0)$ are characterized below.

Then, pricing equation (A.44) applies in that

$$Yv'\left(m^C(Y,0)\right) + r^C(Y,0) = Z_L^A v'\left(m^A(Y,0)\right) + r^A(Y,0) = Z_L^B v'\left(m^B(Y,0)\right) + r^B(Y,0)$$

Welfare solves in state $z = 0$

$$\gamma W(Y,0) = \max_{e_*^A, e_*^B}\Bigg\{\sum_{x=A,B}\gamma Z_L^x v\big(m^x(Y,0)\big) + \gamma Y v\big(m^C(Y,0)\big) + W'(Y,0)\mu^Y(Y,0) \\ + \sum_{x=A,B}\left[e_*^x(Y,0)(V^x(Y,x) - V^x(Y,0)) - \frac{\lambda(e_*^x(Y,0))^2}{2} - \phi e_*^x(Y,0)\right]\Bigg\}. \tag{B.48}$$

Optimizing over e_*^x, we obtain

$$e_*^A(Y,0) = \frac{\left[W(Y,A) - W(Y,0) - \phi\right]^+}{\lambda} \quad \text{and} \quad e_*^B(Y,0) = \frac{\left[W(Y,B) - W(Y,0) - \phi\right]^+}{\lambda}.$$

To solve the model for the Markov equilibrium in state $z = 0$, we need to solve this system of three interconnected first order ODEs, which is characterized in (A.44) and (B.48), for $P^A(Y,0)$, $P^B(Y,0)$ and $W(Y,0)$.

At the boundary $Y = \overline{Y}$, the drift of dY vanishes so that the system characterized in (A.44) and (B.48) becomes a system of five non-linear equations, which can be solved for the five unknowns $P^A(\overline{Y},0)$, $P^B(\overline{Y},0)$, and $W(\overline{Y},0)$, given the values of $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$ and $W(\overline{Y},x)$ for $x = A, B$.

C Model Variant with Interest Rates and UIP

In the model variant of Section 3.3, we have $\theta = 0$ and cohort t's lifetime utility (i.e., the user's lifetime utility) becomes:

$$U_t = \mathbb{E}_t[c_{t+dt}] + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt + m_t^A(1-\alpha_t^A)i^A dt + m_t^B(1-\alpha_t^B)i^B dt. \tag{C.49}$$

where $\mathbb{E}_t[c_{t+dt}] = 1 + \sum_{x=A,B,C} m_t^x r_t^x dt$. The novel terms relative to (2) capture that the holding currency $x = A, B$ allows users to earn interest subject to imperfect passthrough, namely at rate $(1-\alpha_t^x)i^x$. When $\alpha_t^x = 0$, interest passthrough is perfect and currency holders earn interest at rate

i^x, while no interest is earned and passed on to currency-x holders when $\alpha_t^x = 1$.

It is assume that the uncovered interest parity (UIP) holds, in that:

$$\rho = r_t^x + i_t^x$$

for $x = A, B$. Using UIP, we obtain for $x = A, B$:

$$m_t^x(r_t^x + (1 - \alpha_t^x)i^x) = m_t^x(\rho - \alpha_t^x i^x).$$

As such, we can rewrite the user's expected utility as follows:

$$U_t = 1 + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt + m_t^A(\rho - \alpha_t^A i^A)dt + m_t^B(\rho - \alpha_t^B i^B)dt + m_t^C r_t^C dt$$

Note that $\theta = 0$ is assumed, so without loss we denote by $P_t^{x'} = m_t^{x'}$ the level of adoption of currency $x' = A, B, C$. Clearly,

$$P_t^A + P_t^B + P_t^C = m_t^A + m_t^B + m_t^C = 1$$

holds in equilibrium. The user cohort t chooses currency holdings to maximize its expected utility, taking prices (i.e., r_t^x) and interest rates as given.

The user takes prices and interest as given, and solves

$$\max_{m_t^x \geq 0} U_t \quad \text{s.t.} \quad m_t^A + m_t^B + m_t^C = 1.$$

After taking first-order conditions, the pricing equation becomes

$$Y_t v'(m_t^C) + r_t^C = Z_t^A v'(m_t^A) + \rho - \alpha_t^A i^A = Z_t^B v'(m_t^B) + \rho - \alpha_t^B i^B,$$

which is (22). The analogous pricing condition for the baseline model is presented in (13).

We now present the relevant equations and ODEs for states $z = 0, x, AB$ where $x \in \{A, B\}$ to solve for the Markov equilibrium. The arguments are analogous to those in the baseline, and we provide additional details only where needed. The only difference to the baseline lies in the pricing equation, which changes from (13) to (22).

C.1 State $z = AB$

In state $z = AB$, we have

$$r^C(Y, AB) = \frac{(P^C)'(Y, AB)}{P^C(Y, AB)} \mu^Y(Y, AB) = (P^C)'(Y, AB)\mu Y,$$

for $Y < \overline{Y}$ while $\mu^Y(\overline{Y}, AB) = 0$, where we used $\mu^Y(Y, AB) = Y\mu P^C(Y, AB)$, as well as $\theta = 0$ implying $m^C(Y, AB) = P^C(Y, AB)$. The pricing equation (22) yields (for $\alpha_t^x = \alpha^x(z)$ with $x = A, B$):

$$Z_H^A v'(m^A(Y, AB)) + \rho - \alpha^A(AB)i^A = Z_H^B v'(m^B(Y, AB)) + \rho - \alpha^B(AB)i^B.$$

We can eliminate $m^B(Y, AB) = 1 - m^A(Y, AB) - m^C(Y, AB)$. We can then solve above non-linear equation for $m^A(Y, AB)$ as a function of $m^C(Y, AB) = P^C(Y, AB)$. Next, we use (22) to obtain

$$Yv'(m^C(Y, AB)) + r^C(Y, AB) = Z_H^A v'(m^A(Y, AB)) + \rho - \alpha^A(AB)i^A. \tag{C.50}$$

Note that (C.50) represents a first-order ODE, which we can solve on $[0, \overline{Y}]$ subject to an appropriate boundary condition. This boundary condition is obtained by setting the drift of dY to zero at $Y = \overline{Y}$.

At $Y = \overline{Y}$, the drift of dY vanishes, so $r^C(\overline{Y}, AB) = 0$ and

$$\overline{Y}v'(m^C(\overline{Y}, AB)) = Z_H^A v'(m^A(\overline{Y}, AB)) + \rho - \alpha^A(AB)i^A,$$

which can be solved together with

$$Z_H^A v'(m^A(\overline{Y}, AB)) + \rho - \alpha^A(AB)i^A = Z_H^B v'(m^B(\overline{Y}, AB)) + \rho - \alpha^B(AB)i^B$$

for $m^C(\overline{Y}, AB), m^A(\overline{Y}, AB)$ and ultimately for $m^B(\overline{Y}, AB) = 1 - m^C(\overline{Y}, AB) - m^A(\overline{Y}, AB)$.

Finally, the value functions $(V^A(Y, AB), V^B(Y, AB))$ solve (A.24) and welfare solves (A.25), subject to appropriate boundary conditions obtained by setting the drift of dY to zero at $Y = \overline{Y}$.

C.2 State $z = x$

In state $z = x \in \{A, B\}$, we have

$$r^C(Y, x) = \frac{(P^C)'(Y, x)}{P^C(Y, x)}\mu^Y(Y, x) + e^{-x}(Y, x)\left(\frac{P^C(Y, AB)}{P^C(Y, x)} - 1\right),$$

whereby $e^{-x}(Y, x) = \frac{\left[V^{-x}(Y, AB) - V^{-x}(Y, x) - \phi\right]^+}{\lambda}$.

The pricing equation (22) yields (for $\alpha_t^x = \alpha^x(z)$):

$$Z^A(Y, x)v'(m^A(Y, x)) + \rho - \alpha^A(x)i^A = Z^B(Y, x)v'(m^B(Y, x)) + \rho - \alpha^B(x)i^B,$$

where $Z^A(Y, z) = Z_L^A$ for $z = 0, B$ and $Z^A(Y, z) = Z_H^A$ for $z = A, AB$. Likewise, $Z^B(Y, z) = Z_L^B$ for $z = 0, A$ and $Z^B(Y, z) = Z_H^B$ for $z = B, AB$.

We can eliminate $m^B(Y, x) = 1 - m^A(Y, x) - m^C(Y, x)$. We can then solve above non-linear equation for $m^A(Y, x)$ as a function of $m^C(Y, x) = P^C(Y, x)$.

Next, we use (22) to obtain

$$Yv'(m^C(Y, x)) + r^C(Y, x) = Z^A(Y, x)v'(m^A(Y, x)) + \rho - \alpha^A(x)i^A. \tag{C.51}$$

This is a first-order ODE, which we can solve on $[0, \overline{Y}]$ subject to an appropriate boundary condition obtained by setting the drift of dY to zero at $Y = \overline{Y}$.

Finally, the value functions $(V^A(Y, AB), V^B(Y, AB))$ solve (A.28) and (A.30), and welfare solves (A.31), subject to appropriate boundary conditions obtained by setting the drift of dY to zero at $Y = \overline{Y}$.

C.3 State $z = 0$

In state $z = 0$, we have for $x = A, B$ that

$$r^C(Y,0) = \frac{(P^C)'(Y,0)}{P^C(Y,0)}\mu^Y(Y,0) + \sum_{x=A,B} e^x(Y,0)\left(\frac{P^C(Y,x)}{P^C(Y,0)} - 1\right),$$

whereby

$$e^x(Y,0) = \frac{\left[V^x(Y,x) - V^x(Y,0) - \phi\right]^+}{\lambda}.$$

The pricing equation (22) yields (for $\alpha_t^x = \alpha^x(z)$):

$$Z_L^A v'(m^A(Y,0)) + \rho - \alpha^A(0)i^A = Z_L^B v'(m^B(Y,0)) + \rho - \alpha^B(0)i^B.$$

We can eliminate $m^B(Y,0) = 1 - m^A(Y,0) - m^C(Y,0)$. We can then solve above non-linear equation for $m^A(Y,0)$ as a function of $m^C(Y,0) = P^C(Y,0)$.

Next, we use (22) to obtain

$$Yv'(m^C(Y,0)) + r^C(Y,0) = Z_L^A v'(m^A(Y,0)) + \rho - \alpha^A(0)i^A, \tag{C.52}$$

which is a first-order ODE. This ODE can be solved on $[0, \overline{Y}]$ subject to an appropriate boundary condition. As before, this boundary condition is obtained by setting the drift of dY at $\overline{Y}$ to zero.

Finally, the value functions $(V^A(Y,AB), V^B(Y,AB))$ solve (A.33) and welfare solves (A.31), subject to appropriate boundary conditions obtained by setting the drift of dY to zero at $Y = \overline{Y}$.

D Calculation of Model Quantities

The first time of digitization reads $T^* = \min\{T^A, T^B\}$. Assume that $e^A(\overline{Y},0) + e^B(\overline{Y},0) > 0$ to guarantee that T^* is finite in expectation.

We calculate the expected time to first digitization $\mathcal{T}_t$ at time t for $Y_t = Y$ in state $z = 0$, which is defined as

$$\mathcal{T}_t = \mathbb{E}_t[T^* - t | z = 0] = \int_t^\infty e^{-\int_t^s (e_u^A + e_u^B)du} ds.$$

Let $\mathcal{T}(Y) = \mathcal{T}_t$ for $Y_t = Y$. Then, $\mathcal{T}(Y)$ solves the ODE on $(Y_0, \overline{Y})$:

$$(e^A(Y,0) + e^B(Y,0))\mathcal{T}(Y) = 1 + \mathcal{T}'(Y)\mu^Y(Y,0)$$

subject to the boundary condition

$$\mathcal{T}(\overline{Y}) = \frac{1}{e^A(\overline{Y},0) + e^B(\overline{Y},0)} < +\infty.$$

Conditional on remaining in state $z = 0$, there is a one-to-one mapping from $Y_t < \overline{Y}$ to t, so, having obtained $\mathcal{T}(Y)$, we can also calculate $\mathcal{T}_t = \mathcal{T}(Y_t)$ for $Y_t < \overline{Y}$. Conditional on remaining in state $z = 0$, defining $t' = \inf\{t \geq 0 : Y_t \geq \overline{Y}\}$. For times $t \geq t'$, $\mathcal{T}_t = \mathcal{T}(\overline{Y}) = \mathcal{T}_{t'}$, i.e., $\mathcal{T}_t$ remains constant after Y_t reaches $\overline{Y}$.

Next, the probability density function of T^* as a function of t equals

$$k_t = e^{-\int_0^t (e_u^A + e_u^B)du}(e_t^A + e_t^B),$$

where $e^{-\int_0^t (e_u^A + e_u^B)du} = Prob(\{T^* \geq t\}|z = 0)$ and $(e_t^A + e_t^B)dt = Prob(\{T^* \in [t, t + dt)\}|T^* \geq t\}$.

E Micro-Foundation of Convenience Utility

We now provide a micro-foundation of the money-in-the-utility approach, specifically, the formulation of expected utility in (2) entailing convenience utility. This micro-foundation of the convenience utility is based on a cash-in-advance constraint as well as random search/matching and bargaining between users (buyers) and sellers, highlighting the medium-of-exchange function of money. The micro-foundation shares some similarities with the new monetarist approach, as developed in Lagos and Wright (2005), but we make certain simplifying assumptions (as the micro-foundation is not the paper's key focus). We set up the micro-foundation in a rather general form: It nests the baseline specification in (2), but it could also accommodate more general forms of convenience utility. Importantly, the micro-foundation also sheds light on the factors that affect currency convenience and, specifically, determine the values of the convenience scale Z_t^x and Y_t in (2).

For the micro-foundation, consider cohort t of the representative user that is born with one unit of consumption good at time t. At the beginning of its lifetime, i.e., an instant $[t, t + dt]$, cohort t chooses its holdings of currency x (in terms of the consumption good), m_t^x at price P_t^x for $x = A, B, C$ — that is, cohort t holds m_t^x/P_t^x units of currency x. Over its lifetime, cohort t either utilizes its money holdings to pay a seller for services, yielding some utility from transacting with the seller (characterized below), or sells its money to cohort $t + dt$ of the user at price P_{t+dt}^x.

We characterize the transaction activity between sellers and users (buyers) over a short time period $[t, t + dt]$. Crucially, transactions between sellers and users occur before uncertainty about the digitization outcome over $[t, t + dt]$ is realized and thus before the next-period prices P_{t+dt}^x are realized. Users and sellers can also not write contracts contingent on next-period prices. Thus, when transacting, users and sellers form expectations about P_{t+dt}^x. All expectations (denoted shorthand $\mathbb{E} = \mathbb{E}_t$) are understood as conditional on time-t public information. Further, we will assume the CRRA function form for $v(m_t^x) = \frac{m^{1-\eta}}{1-\eta}$, although most of our results carry through under more general forms. Also, note that

$$\mathbb{E}[P_{t+dt}^x] = P_t^x + \mathbb{E}[dP_t^x] = P_t^x(1 + r_t^x dt),$$

which will be used repeatedly below to simplify the expressions. When laying out the micro-foundation, we can focus on a specific currency $x = A, B, C$ — the micro-foundation is analogous for all three currencies (for notational convenience, we write $Z_t^C := Y_t$ and similar).

E.1 Random Search/Matching and Transactions

For any $x = A, B, C$, there exists a mass of type-x sellers. For simplicity, any type-x seller only accepts currency x as payment for its services. There are no other types of sellers, i.e., any seller accepts precisely one type of currency x. Any type-x seller can produce an arbitrary number of service goods at a marginal (utility) cost normalized to one unit of the consumption good. Meetings and thus transactions between the user and sellers occur randomly over $[t, t+dt]$.

With probability $\hat{Z}_t^x dt$, the user meets a (single) type-x seller and transacts using its holdings of currency x. While the user and seller could theoretically pass on the opportunity, we show that in our formulation, a transaction always occurs when the opportunity arises. With probability $1 - \hat{Z}_t^x dt$, the user does not transact with a type-x seller. This scenario could capture that the user does not have transaction needs, is unable to locate a seller, or, in reduced form, that it meets a seller but the transaction does not go through on time (e.g., due to settlement failures or delays). Thus, one can think of $\hat{Z}_t^x$ capturing the currency's general level of acceptance by sellers, as well as the technology and transaction speed associated with currency x (more on this later).

Note that the probability of meeting two sellers of different types is of order $(dt)^2$ and, therefore, negligible in continuous time. We do not account for these events, when calculating payoffs.

E.2 Transaction Utility and Payoffs

Suppose now that the representative user meets a type-x seller and that a transaction occurs. In this transaction, the type-x seller is paid $\mathcal{M}$ units of currency x and delivers $\mathcal{S}$ units of service to the user. Here, $\mathcal{M}$ must adhere to the cash-in-advance constraint $\mathcal{M} \leq m_t^x / P_t^x$ — that is, at time t, the user acquires m_t^x / P_t^x units of currency x and cannot pay more than this amount to the seller.

The seller incurs a (utility) production cost $\mathcal{S}$ (i.e., the seller produces services at a marginal cost normalized to one) and a transaction cost proportional to the real (expected) time-$t+dt$ value of money $\hat{\kappa}_t^x \mathcal{M} \mathbb{E}[P_{t+dt}^x]$ for some $\hat{\kappa}_t^x \in [0,1)$. After being paid, the seller of the service sells its currency holdings to cohort $t+dt$ at price P_{t+dt}^x, yielding expected payoff $\mathcal{M}\mathbb{E}[P_{t+dt}^x]$ at the time the seller is paid by the user. Recall that transactions between users and sellers occur before next-period prices are realized.[29]

Thus, the seller's expected utility from the transaction — given $(\mathcal{M}, \mathcal{S})$ — reads

$$U_t^x(\mathcal{M}, \mathcal{S}) = -\mathcal{S} + \mathcal{M}(1 - \hat{\kappa}_t^x)\mathbb{E}[P_{t+dt}^x]. \tag{E.53}$$

The proportional transaction cost could reflect monetary costs associated with the payment network that the seller must pay to intermediaries (e.g., credit card companies or payment processors) or a utility cost (for instance, due to settlement and payment delays).

The transaction cost implies that for each unit of currency transferred between buyer and seller, the seller incurs $\hat{\kappa}_t^x$ in transaction fees; total transaction costs are then $\hat{\kappa}_t^x \mathcal{M}\mathbb{E}[P_{t+dt}^x]$. While the transaction cost enters the seller's utility, we note that the terms of trade $(\mathcal{M}, \mathcal{S})$ are endogenous and eventually determine whether the seller or buyer covers these transaction costs. Our model is flexible and allows for a split of these transaction costs.

The user derives utility from the transaction of

$$\hat{U}_t^x(\mathcal{M}, \mathcal{S}) = v(\mathcal{S}) + \mathcal{S} + \nu_t^x \hat{\kappa}_t^x \mathcal{M}\mathbb{E}[P_{t+dt}^x] \tag{E.54}$$

[29] We could equally assume that the transaction cost is based on the time-t value of money and price P_t^x. This would not affect the payoffs and outcomes in the end, since the difference induced by this alternative modeling is of order $(dt)^2$ and, therefore, negligible. This arises, because (i) the probability of a transaction occurring is of order dt and (ii) the difference between P_t^x and $\mathbb{E}[P_{t+dt}^x]$ is of order dt.

Here, the user's utility from service consumption reads $v(\mathcal{S}) + \mathcal{S}$ — that is, the production of $\mathcal{S}$ units of service (at marginal cost of one) generates a surplus of $v(\mathcal{S})$.

Next, the transaction costs $\hat{\kappa}_t^x \mathcal{M}\mathbb{E}[P_{t+dt}^x]$ could represent interchange fees in credit card transactions (when $x = A, B$ represents a fiat currency). In practice, such interchange fees are predominantly borne by sellers, while buyers even receive rebates (e.g., in form of credit card points or cash back). To capture such rewards, we assume that the user is rebated fraction ν_t^x of the transaction fees, where $\nu_t^x \leq 1$ is a parameter. The rebate is modeled as a positive utility payoff entering $\hat{U}_t^x(\mathcal{M}, \mathcal{S})$ above. By assuming $\nu_t^x < 0$, we could capture that, like the seller, the user incurs a transaction cost (in utility) too. We assume for simplicity, that $\nu_t^x \hat{\kappa}_t^x \mathcal{M}\mathbb{E}[P_{t+dt}^x]$ is in utility units and does not have to be paid from the cash balance (i.e., does not affect the cash-in-advance constraint) — this would be consistent with credit card rewards and points.

Note that $\nu_t^x = 1$ implies that transaction fees are a pure transfer from the seller to the buyer, occurring at time $t + dt$ after the transaction. In this case, transaction fees do not constitute a deadweight loss. However, our analysis will show that these fees can still distort transactions and limit the convenience of a currency, even when $\nu_t^x = 1$.

We observe that the user's transaction utility net of payment equals $\hat{U}_t^x(\mathcal{M}, \mathcal{S}) - \mathcal{M}\mathbb{E}[P_{t+dt}^x]$, where the payment to the seller can be regarded as an opportunity cost. That is, when the user transacts with a seller, it pays the seller and, therefore, "foregoes" the opportunity to sell its currency holdings to cohort $t + dt$, which would give payoff $\mathcal{M}\mathbb{E}[P_{t+dt}^x]$. The transaction surplus equals then

$$v(\mathcal{S}) - \mathcal{M}(1 - \nu_t^x)\hat{\kappa}_t^x \mathbb{E}[P_{t+dt}^x].$$

Without transaction costs, i.e., $\hat{\kappa}_t^x = 0$, surplus increases in the amount of service produced.

Finally, we make a parameter assumption to render tractability to our analysis and avoid tedious case distinctions. Specifically, we assume that

$$v'(1) \geq \frac{\hat{\kappa}_t^x}{1 - \hat{\kappa}_t^x}. \tag{E.55}$$

By strict concavity, note that $v'(\mathcal{S}) > v'(1)$ for $\mathcal{S} \in [0, 1)$. This condition ensures that in optimum, the user spends all its cash holdings to buy services, when given the opportunity. In other words, the cash-in-advance constraint $\mathcal{M} \leq m_t^x / P_t^x$ binds in optimum under this condition, thereby simplifying the analysis. Whenever $\mathcal{M} = m_t^x / P_t^x$, we have

$$\mathcal{M}\mathbb{E}[P_{t+dt}^x] = \frac{m_t^x}{P_t^x}\mathbb{E}[P_t^x + dP_t^x] = m_t^x(1 + r_t^x dt),$$

where we used $P_{t+dt}^x = P_t^x + dP_t^x$ and $\mathbb{E}[dP_t^x] = P_t^x r_t^x dt$.

E.3 Transaction Terms and Bargaining

When the user and a type-x seller meet, they bargain over $(\mathcal{M}, \mathcal{S})$, i.e., the terms of trade. We model the bargaining process (in reduced form) as follows. When a meeting occurs between the user and the type-x seller, then, with probability χ_t^x, the user has full bargaining power and makes a take-it-or-leave-it (TIOLI) offer to the seller. With probability $1 - \chi_t^x$, the seller has full bargaining power and makes a TIOLI offer to the user. In bargaining, the seller has an outside option of zero. The user, entering the bargaining with m_t^x/P_t^x units of currency x, has the outside option of not transacting and selling its currency holdings to cohort $t+dt$, delivering expected utility $\mathcal{M}\mathbb{E}[P_{t+dt}^x] = m_t^x(1+r_t^x dt)$.

The parameter χ_t^x can be interpreted as the user's bargaining power vis-a-vis sellers accepting currency x. We offer some interpretation of this parameter later on. We note that this modeling of bargaining is more tractable in our setting than the more commonly adopted Nash-bargaining, while allowing us to capture the relevant economic forces related to bargaining power. We adopt it for simplicity and tractability, but note that we could equally employ Nash bargaining, as, e.g., in money search models such as Lagos and Wright (2005).

We now distinguish two cases: (1) user has full bargaining power (which happens with probability χ_t^x), and (2) Seller has full bargaining power (which happens with probability $1-\chi_t^x$).

E.3.1 User has Full Bargaining Power

If the user has full bargaining power, the user makes a take-it-or-leave-it offer $(\mathcal{M},\mathcal{S})$ to the seller which, stipulates proposed (nominal) payment $\mathcal{M}$ and service delivery $\mathcal{S}$. When choosing the offer, the user maximizes its utility net of payment subject to the seller's participation constraint and the cash-in-advance constraint, in that:

$$\max_{(\mathcal{M},\mathcal{S})} \hat{U}_t^x(\mathcal{M},\mathcal{S}) - \mathcal{M}\mathbb{E}[P_{t+dt}^x] \ \text{ s.t. } \ U_t^x(\mathcal{M},\mathcal{S}) \geq 0 \text{ and } \mathcal{M} \leq \frac{m_t^x}{P_t^x},$$

where $\hat{U}_t^x(\mathcal{M},\mathcal{S})$ is from (E.54). It is optimal for the user extract full transaction surplus. Thus, in optimum, the seller just breaks even and earns its outside option of zero, so that $U_t^x(\mathcal{M},\mathcal{S})=0$. Therefore, by (E.53), we obtain $\mathcal{S}=\mathcal{M}(1-\hat{\kappa}_t^x)\mathbb{E}[P_{t+dt}^x]$. Under this relation, the user's net utility becomes (according to (E.54)):

$$\mathcal{K}(\mathcal{M}) := v(\mathcal{M}(1-\hat{\kappa}_t^x)\mathbb{E}[P_{t+dt}^x]) - \hat{\kappa}_t^x(1-\nu_t^x)\mathcal{M}\mathbb{E}[P_{t+dt}^x],$$

where we used that $\mathbb{E}[P_{t+dt}^x]/P_t^x = 1+r_t^x dt$. Calculate $\mathcal{K}'(\mathcal{M}) = v'(S)(1-\hat{\kappa}_t^x)\mathbb{E}[P_{t+dt}^x] - \hat{\kappa}_t^x(1-\nu_t^x)\mathbb{E}[P_{t+dt}^x]$. By condition (E.55), $\mathcal{K}'(\mathcal{M})>0$. Consequently, the user's payoff $\mathcal{K}(\mathcal{M})$ increases in $\mathcal{M}$ and thus is maximized (subject to cash-in-advance constraint) by setting $\mathcal{M}=m_t^x/P_t^x$. That is, the cash-in-advance constraint optimally binds.

As a result, the seller is paid $\mathcal{M}=m_t^x/P_t^x$ units of currency x. Solving $U_t^x(m_t^x/P_t^x,\mathcal{S})=0$ for $\mathcal{S}=\overline{\mathcal{S}}_t^x$, we get

$$\overline{\mathcal{S}}_t^x = m_t^x(1-\hat{\kappa}_t^x)(1+r_t^x dt). \tag{E.56}$$

The user's net utility then becomes $v(\overline{\mathcal{S}}_t^x) - \hat{\kappa}_t^x(1-\nu_t^x)m_t^x(1+r_t^x dt)$. The user's gross utility (excluding payment) is $v(\overline{\mathcal{S}}_t^x) + \overline{\mathcal{S}}_t^x + m_t^x\nu_t^x\hat{\kappa}_t^x(1+r_t^x dt)$.

E.3.2 Seller has Full Bargaining Power

If the seller has full bargaining power, the seller makes a take-it-or-leave-it offer $(\mathcal{M},\mathcal{S})$ to the user which, stipulates proposed payment $\mathcal{M}$ and service delivery $\mathcal{S}$. Formally, the seller maximizes its payoff subject to the user's participation constraint and the cash-in-advance constraint:

$$\max_{(\mathcal{M},\mathcal{S})} U_t^x(\mathcal{M},\mathcal{S}) \;\text{ s.t. }\; \hat{U}_t^x(\mathcal{M},\mathcal{S}) \geq \mathcal{M}\mathbb{E}[P_{t+dt}^x] \text{ and } \mathcal{M} \leq \frac{m_t^x}{P_t^x}.$$

In optimum, the user just breaks even, so $\hat{U}_t^x(\mathcal{M},\mathcal{S}) = \mathcal{M}\mathbb{E}[P_{t+dt}^x]$ — which implies by (E.54) that $\mathcal{S} + v(\mathcal{S}) = \mathcal{M}\mathbb{E}[P_{t+dt}^x](1-\nu_t^x\hat{\kappa}_t^x)$. We can solve for

$$\mathcal{M}\mathbb{E}[P_{t+dt}^x] = \frac{v(\mathcal{S})+\mathcal{S}}{1-\nu_t^x\hat{\kappa}_t^x}. \tag{E.57}$$

Utilizing the cash-in-advance constraint, i.e., $\mathcal{M} \leq m_t^x/P_t^x$, the condition (E.57) implies that

$$v(\mathcal{S}) + \mathcal{S} \leq m_t^x(1-\nu_t^x\hat{\kappa}_t^x)(1+r_t^x dt).$$

In particular, due to $m_t^x < 1$, $\mathcal{S} \leq 1$.

Using (E.57), we get $\mathcal{S} = -v(\mathcal{S}) + (1-\nu_t^x\hat{\kappa}_t^x)\mathcal{M}\mathbb{E}[P_{t+dt}^x]$. Using this relationship (in line 2), the seller's payoff becomes

$$\begin{aligned}
U_t^x(\mathcal{M},\mathcal{S}) &= -\mathcal{S} + \mathcal{M}(1-\hat{\kappa}_t^x)\mathbb{E}[P_{t+dt}^x] \\
&= v(\mathcal{S}) - \mathcal{M}\hat{\kappa}_t^x(1-\nu_t^x)\mathbb{E}[P_{t+dt}^x] \\
&= v(\mathcal{S}) - \frac{\hat{\kappa}_t^x(1-\nu_t^x)(v(\mathcal{S})+\mathcal{S})}{1-\nu_t^x\hat{\kappa}_t^x} \\
&= \frac{v(\mathcal{S})(1-\hat{\kappa}_t^x) - \hat{\kappa}_t^x(1-\nu_t^x)\mathcal{S}}{1-\nu_t^x\hat{\kappa}_t^x},
\end{aligned}$$

where we used (E.57) to transition from line 2 to 3. Provided $v'(\mathcal{S}) \geq \frac{\hat{\kappa}_t^x(1-\nu_t^x)}{1-\hat{\kappa}_t^x}$ for all $\mathcal{S} \in [0,1]$ — which holds by (E.55) — it follows that $U_t^x(\mathcal{M},\mathcal{S})$ increases in $\mathcal{S}$. Since $\mathcal{S}$ increases in $\mathcal{M}$ by (E.57), we have $\mathcal{M} = \frac{m_t^x}{P_t^x}$, i.e., the cash-in-advance constraint optimally binds.

The seller is paid m_t^x/P_t^x units of currency and, by (E.57), the seller produces $\mathcal{S} = \underline{\mathcal{S}}_t^x$ service units, satisfying

$$v(\underline{\mathcal{S}}_t^x) + \underline{\mathcal{S}}_t^x = m_t^x(1-\nu_t^x\hat{\kappa}_t^x)(1+r_t^x dt). \tag{E.58}$$

Clearly, $\overline{\mathcal{S}}_t^x > \underline{\mathcal{S}}_t^x$. The seller's payoff becomes $m_t^x(1-\hat{\kappa}_t^x)(1+r_t^x dt) - \underline{\mathcal{S}}_t^x > 0$.

E.4 Expected Utility

Next, the model can allow for currency x to pay a nominal interest at rate $\hat{i}_t^x$; this assumption allows us to encompass the formulation of Section 3.3. We assume that the user earns the interest payment on currency x of $\hat{i}_t^x m_t^x dt$ regardless of whether it spends its currency-x holdings in a transaction to pay the seller. Since the interest payment is of order dt and transactions with sellers occur with probabilities of order dt, this assumption is inconsequential (i.e., the probability of transacting times the interest payment is of order $(dt)^2$ and, thus, negligible).

Then, at time t, the user's expected utility becomes (ignoring terms of order $(dt)^2$ and higher):

$$\mathcal{U}_t := \sum_{x=A,B,C} \hat{Z}_t^x dt \Big\{ \chi_t^x \left[v(\overline{\mathcal{S}}_t^x) + \overline{\mathcal{S}}_t^x + m_t^x \nu_t^x \hat{\kappa}_t^x (1 + r_t^x dt) \right] + (1 - \chi_t^x) m_t^x (1 + r_t^x dt) \Big\}$$
$$+ \sum_{x=A,B,C} (1 - \hat{Z}_t^x dt) m_t^x (1 + r_t^x dt) + \sum_{x=A,B,C} m_t^x \hat{i}_t^x dt.$$

Using (E.56), we can write $v(\overline{S}_t^x) = v\big(m_t^x(1 - \hat{\kappa}_t^x)\big) + o(dt)$. We can insert this relationship into above expression for $\mathcal{U}_t$, ignore terms of order $(dt)^2$ or higher, and simplify to obtain:

$$\mathcal{U}_t = \sum_{x=A,B,C} \chi_t^x \hat{Z}_t^x \Big[v\big(m_t^x(1 - \hat{\kappa}_t^x)\big) - \hat{\kappa}_t^x (1 - \nu_t^x) m_t^x \Big] dt + \sum_{x=A,B,C} m_t^x (r_t^x + \hat{i}_t^x) dt + 1.$$

Employing CRRA utility, i.e., $v(m) = \frac{m^{1-\eta}}{1-\eta}$, we get

$$\mathcal{U}_t = 1 + \sum_{x=A,B,C} Z_t^x v(m_t^x) dt + \sum_{x=A,B,C} m_t^x (r_t^x + \hat{i}_t^x - \kappa_t^x) dt. \tag{E.59}$$

Here,

$$\kappa_t^x := \hat{Z}_t^x \chi_t^x \hat{\kappa}_t^x (1 - \nu_t^x)$$

and, most importantly,

$$Z_t^x := \hat{Z}_t^x \chi_t^x (1 - \hat{\kappa}_t^x)^{1-\eta}, \tag{E.60}$$

where we write for notational convenience $Z_t^C = Y_t$. We note that the convenience parameters Z_t^x reflect the (1) the probability that a buyer encounters a seller who accepts the respective currency (i.e., $\hat{Z}_t^x$), (2) transaction costs $\hat{\kappa}_t^x$ and (3) the user's bargaining power relative to sellers χ_t^x. Interestingly, even if transaction fees are fully rebated to the user, i.e., $\nu_t^x = 1$, the transaction fees $\hat{\kappa}_t^x$ still distort transactions away from the optimum and thus limit convenience.[30]

Note that the baseline obtains upon setting $\hat{i}_t^x = 0$ (currency does not earn interest) and either of (i) $\hat{\kappa}_t^x = 0$ or (ii) $\nu_t^x = 1$, bearing in mind that $Y_t = Z_t^C$. In particular, under these assumptions, $\mathcal{U}$ from (E.59) coincides with (2) and (11).

The stipulation of expected utility in the model variant with UIP and interest rates in Appendix C — specifically, (C.49) — is obtained upon setting $\hat{i}_t^x = (1 - \alpha_t^x) i^x$ for $x = A, B$ and $i^C = 0$. In addition, either of (i) $\hat{\kappa}_t^x = 0$ or (ii) $\nu_t^x = 1$ must hold too.

Finally, we note that in our micro-foundation, the medium-of-exchange and store-of-value functions of money complement each other. When a currency offers higher expected returns and serves as a better store of value, it becomes less costly for users to hold this currency "in advance" for payments, thereby reinforcing its role as a medium of exchange. In contrast to our micro-foundation based on random search and matching, Goldstein et al. (2023) abstract away from search, instead focusing on coordination. They establish a conflict between the store-of-value and medium-of-

[30] When the transaction cost is fully rebated and $\nu_t^x = 1$, the transaction cost is not a deadweight loss, but represents a (suboptimal) transfer between the user and the sellers. Without cash-in-advance constraint, one could offset this transfer through higher payments from the user to the sellers. However, because the cash-in-advance constraint binds, this is not possible. As a result, the transaction costs represent a distortion limiting payment convenience, even if $\nu_t^x = 1$.

exchange functions of money, which can lead to fragility.

Likewise, as shown in Doepke and Schneider (2017); Gopinath and Stein (2021), the unit-of-account function of money can be viewed as complementary to the medium-of-exchange and store-of-value functions. For instance, the U.S. dollar acts as store of value and widely accepted medium of exchange, as it is often used as a unit of account (e.g., for invoicing in international trade); at the same time, medium-of-exchange and store-of-value functions reinforce the U.S. dollar's role as international unit of account. In short, while we base the convenience of money on its medium-of-exchange function, the complementarity of the three functions suggests that this convenience may also reflect other monetary functions.

E.5 Determinants of Payment Convenience

The micro-foundation sheds light on the determinants of the convenience utility from (2) and, specifically, the parameters Z_t^x and $Y_t = Z_t^C$. Equation (E.60) shows that the convenience scale parameters Z_t^x and Y_t depend on (1) the probability that a buyer encounters a seller who accepts the respective currency (i.e., $\hat{Z}_t^x$), (2) transaction costs $\hat{\kappa}_t^x$ and (3) the user's bargaining power relative to sellers χ_t^x. We discuss each of these factors of in greater detail and what they represent in reality. We also argue how they drive the convenience of fiat monies and PDM.

1. **Probability of Meeting a Seller and Transacting.** The variable $\hat{Z}_t^x$ represents the probability of a successful transaction using currency x. This probability is influenced by the payment technology underlying currency x as well as the overall level of its acceptance or adoption.

   To better see why $\hat{Z}_t^x$ captures payment technology (e.g., in terms of settlement speed), suppose that the user meets a type-x seller with a probability normalized to $\pi_t^x dt$. Provided a meeting occurs, the seller and the user try to transact but the payment is successfully processed within $[t, t+dt]$ only with probability $\hat{z}_t^x dt$ — if the payment is not successful (for instance, because it fails or is not processed on time), the transaction is not successful, i.e., does not occur. Then, the probability that a transaction occurs can be written $\hat{Z}_t^x dt$, with $\hat{Z}_t^x = \pi_t^x \hat{z}_t^x$ being the product of a meeting rate and the probability that the payment is successful and settled fast enough.

   Naturally, a currency becomes more convenient when it is widely accepted and easy to transact with. The likelihood that a buyer meets a seller accepting x, that is, $\hat{Z}_t^x$ should reflect the number of sellers accepting or offering services in exchange for currency x. This mechanism is subject to network effects. As we sketch in Section E.6 — where we endogenize the number of services that can be bought with currency x — a higher number of services that can be bought with x makes currency x more convenient for users, drawing users to currency x, which, in turn, makes it more appealing for sellers to accept currency x for a wider range of services.

   Consequently, currencies like the U.S. dollar are convenient relative to other fiat currencies (i.e., have a high Z_t^x) because they are widely accepted and benefit from the large size of the U.S. economy, which facilitates numerous dollar-based transactions (including digital ones). Similarly, the convenience of PDM could arise from its integration with digital platforms and ecosystems, where transactions often require specific types of PDM for settlement. For example, Ether is convenient because it is widely used within the expansive Ethereum ecosystem, and Alipay is convenient due to its applicability across a broad range of services within Alibaba's ecosystem. This source of payment convenience becomes increasingly significant as the size of the digital platform on which the PDM is integrated or accepted grows. Specifically, the growing importance of digital platforms contributes to the rise in PDM convenience, modeled in (4).

Finally, we argue that slow settlement speeds, outdated payment technologies, and limited payment functionalities or usability (e.g., the inability to support blockchain transactions, certain digital platforms, or cross-border payments) constrain the digital payment convenience of fiat money. Certain forms of PDM may have an advantage over fiat money in these areas by offering faster transactions, unique payment functionalities, or a broader scope of payment services and usability. Fiat digitization, however, can address these issues by enabling faster payments and enhancing payment technologies and functionalities. Additionally, digitization may increase the convenience of fiat money by expanding its usability—both geographically (e.g., by supporting cross-border transactions) and across services (e.g., by supporting blockchain-based transactions).

2. **Transaction Costs.** The transaction cost parameter $\hat{\kappa}_t^x$ inversely affects convenience, i.e., Z_t^x decreases with $\hat{\kappa}_t^x$. The transaction cost can be of monetary nature (e.g., credit card interchange fees) or a utility cost (e.g., settlement delays, cost of processing transactions via PoS, or similar). Specifically, the transaction cost may capture that due to delays in payment settlement, either the seller receives payment late or the user receives the service late — both of which are costly.[31]

   We think that such transactions costs, both monetary (e.g., fees charged by costly payment intermediaries such as credit card companies) and utility costs (reflecting settlement delays), are important frictions limiting the convenience of fiat money in digital payment. Certain features of cryptocurrencies and tokens (e.g., smart contracting or decentralization in cryptocurrencies) and non-bank payment systems can reduce dependence on costly payment intermediaries by bypassing traditional bank payment rails, thereby reducing transaction costs. Together, these factors enhance the convenience of PDM relative to fiat money.[32]

   The digitization of fiat money can also mitigate these frictions by reducing reliance on costly payment intermediaries (such as credit card companies) and enabling faster, even instant, payments. For instance, the introduction of a fast, government-led payment system—such as

[31] To model this more formally, one could assume that upon a successful transaction between the user and a type-x seller, the payment is initiated at time $t + dt$, but only succeeds at time T^S arriving at a Poisson rate λ_t^x. Thus, the seller must wait on average $\mathbb{E}[T^S - t + dt] = 1/\lambda_t^x$ units of time before receiving the payment. Further, consider that the seller discounts at rate $\rho > 0$. Then, a payment worth one unit of consumption good at initiation at $t + dt$ is worth to the seller only $\frac{\lambda_t^x}{\rho+\lambda_t^x} < 1$. Abstracting from price movements between payment initiation and settlement, one could then $\hat{\kappa}_t^x \simeq \frac{\lambda_t^x}{\rho+\lambda_t^x}$. For instance, we could assume that an intermediary stands ready to hedge exchange rate movements at zero cost so that a payment has the same consumption good value at initiation and settlement. Otherwise, the seller values a payment of one (nominal) unit of currency at time $t + dt$ as $\frac{\lambda_t^x \mathbb{E}[P_{T^S}^x]}{\rho+\lambda_t^x} < 1$, which would lead to qualitatively similar outcomes.

[32] Cryptocurrencies and tokens promise decentralization and the ability to bypass costly payment intermediaries. In particular, their smart contracting features (see, e.g., Cong and He (2019)) enable intermediary-free transactions. These features, combined with their decentralized nature, can reduce transaction fees for certain transactions, contributing to the digital payment convenience of PDM.

Brazil's Pix or India's UPI—could increase payment speed and reduce reliance on credit card companies. In addition, a government-led payment system may exert competitive pressure on payment intermediaries, thus lowering their fees.

3. **Bargaining Power.** We argue that a larger user bargaining power vis-a-vis sellers (i.e., higher χ_t^x) could be linked to better privacy features inherent in currency x. For this sake, we draw on a large literature in industrial organization, which shows that sellers can leverage user data to price discriminate (see, e.g., Bergemann, Brooks, and Morris (2015); Brunnermeier, Lamba, and Segura-Rodriguez (2023)), effectively allowing them to charge higher prices for their services. More relevant in our context, Garratt and Van Oordt (2021) show how sellers could use data collected through payments with currency x to price discriminate future consumers that pay using currency x. Thus, according to their argument, enhanced privacy features imply less such price-discrimination and better prices for users, which we could capture by higher χ_t^x.[33]

   Therefore, strong privacy features (as seen in, e.g., some cryptocurrencies and tokens) contribute to the digital payment convenience of PDM. In contrast, digital fiat money (especially in the form of bank deposits) offers weaker privacy features absent digitization. However, privacy-enhancing currency digitization — for instance, through launching CBDC (as discussed in Garratt and Van Oordt (2021); Ahnert et al. (2022); Garratt et al. (2022)) — could improve privacy features in digital fiat money, thus increasing its convenience.

E.6 Endogenous Network Effects and Payment Adoption

Taking our micro-foundation one step further, we could endogenize the number of (homogeneous) services that the user can pay for with currency x. For instance, assume that there is one representative user and one representative seller only accepting currency x. There is a large mass of potential services that the seller may offer. Offering n_t^x services entails a quadratic cost $\frac{k_t^x (n_t^x)^2}{2} dt$ with cost parameter k_t^x. This cost parameter could capture the cost of using the payment system and accepting currency x as payment for certain services (e.g., the cost of setting up international payment or setting up PoS).[34]

The user buys at most one service from the seller. The probability of successfully transacting and buying an individual service is assumed to be $\hat{Z}_t^x dt$ with $\hat{Z}_t^x = \hat{z}_t^x n_t^x$ where $\hat{z}_t^x > 0$ is taken as given and n_t^x is the endogenous number of services. Hence, the number of successfully transacting increases in the number of services offered by the representative seller. While we set up this variant with one representative user and one representative seller, one can interpret it also as describing a mass of users and a mass of sellers (offering different services).

The heuristic timing over $[t, t + dt]$ is as follows. The type-x seller first chooses n_t^x against cost $\frac{k_t^x (n_t^x)^2}{2} dt$. Then, our aforementioned micro-foundation applies. Specifically, the seller is matched

[33] Likewise, Ahnert et al. (2022) show how enhanced payment privacy can benefit users through better "service prices" when banks bundle lending and payment services.

[34] The increasing cost reflects that, for some services, it is easy or natural for merchants to accept currency x as payment (e.g., a U.S.-based service catering to U.S. customers and paid in dollars). However, for other services, it is more difficult and costly to accept the currency—for instance, a Chinese seller incurs additional costs when selling goods in the U.S. for dollar payments. As the representative seller expands the range of services offered in currency x, they first exhaust the easier, low-cost options and then take on services that involve higher payment costs.

with the user with probability $\hat{Z}_t^x dt = \hat{z}_t^x n_t^x dt$ — in which case the transaction occurs. The seller cannot extract any surplus with probability χ_t^x. With probability $1 - \chi_t^x$, however, the seller has full bargaining power and makes a TIOLI offer to the user. The seller then is paid m_t^x / P_t^x units of currency x and delivers $\underline{S}_t^x$ units of service, with $\underline{S}_t^x$ characterized in (E.58); recall the seller incurs a proportional transaction cost, so its payoff then reads $\left[(1 - \hat{\kappa}_t^x) m_t^x (1 + r_t^x dt) - \underline{S}_t^x\right]$.

The seller's total expected payoff can be written as (ignoring terms of order $(dt)^2$ or higher):

$$\underbrace{\hat{z}_t^x n_t^x (1 - \chi_t^x)\left[m_t^x(1 - \hat{\kappa}_t^x) - \underline{S}_t^x\right] dt}_{\text{Expected Transaction Utility}} - \underbrace{\frac{k_t^x (n_t^x)^2}{2}}_{\text{Initial Cost}} dt,$$

where $\underline{S}_t^x$ solves (E.58). Observe that $\underline{S}_t^x$ is strictly smaller than m_t^x and increases in m_t^x.

Now, consider that the seller chooses n_t^x to maximize its payoff, taking m_t^x as given (which depends on n_t^x in equilibrium). Then, n_t^x satisfies

$$n_t^x = \frac{\hat{z}_t^x}{k_t^x}(1 - \chi_t^x)\left[m_t^x(1 - \hat{\kappa}_t^x) - \underline{S}_t^x\right].$$

This relation leads to network effects: Higher m_t^x implies higher n_t^x, while higher n_t^x leads to higher $\hat{Z}_t^x$ and Z_t^x, and thus higher m_t^x. One can then solve for equilibrium m_t^x and n_t^x by solving a fixed point problem (that possibly admits multiple solutions).

The analysis highlights the importance of network effects in determining currency convenience. It also shows that potentially small changes in $\hat{z}_t^x$ — e.g., due to technological improvements of the payment technology — can lead to rather large increases in currency convenience due to these network effects. Moreover, note that a decrease in k_t^x implies higher $\hat{Z}_t^x$ and Z_t^x. Thus, digitization that decreases the cost of adopting a currency for payment (e.g., reducing the seller's cost of accepting payments in a currency for a specific service) improves that currency's convenience. For instance, CBDC or instant payment systems reduce the cost for sellers of accepting digital payment for a specific service, thus enhancing the currency's convenience.

E.7 The Evolution of PDM Convenience

We discuss the assumption that the convenience of PDM evolves continuously according to (4), while the convenience of fiat money changes discontinuously, experiencing a jump following successful digitization at time T^x. This assumption reflects that the digitization of fiat money—whether through the launch of a new payment system, an upgrade to an existing system, or the introduction of a CBDC—occurs infrequently and represents significant changes or disruptions to the bank-railed and government-led payment systems. For this reason, we model the effects of fiat currency digitization as a jump rather than a gradual process. For simplicity, we consider only a one-time digitization, though the model could be extended to allow for multiple stages in the digitization process.

In contrast, PDM encompasses various forms of payment systems and digital currencies, each evolving over time, with some achieving significant breakthroughs. When aggregated, their convenience evolves more gradually over time compared to that of fiat money. To illustrate this argument more formally, consider that PDM encompasses a continuum of different payment services, indexed by $i \in [0,1]$ and with individual payment convenience Y_t^i. For any $i \in [0,1]$, the payment convenience evolves according to

$$\frac{dY_t^i}{Y_t^i} = \hat{\mu} dJ_t^i,$$

where $dJ_t^i \in \{0,1\}$ is a jump process with $\mathbb{E}[dJ_t^i] = \Lambda_t^i dt$ and $\hat{\mu} \geq 0$. Then, by the law of large numbers and under standard regularity conditions, we have that aggregate PDM convenience Y_t defined as

$$Y_t = \int_0^1 Y_t^i di,$$

evolves according to

$$\frac{dY_t}{Y_t} = \hat{\mu} \left[\int_0^1 \Lambda_t^i di \right] dt.$$

Assuming that $\int_0^1 (\Lambda_t^i) di \propto m_t^C$, the law of motion (4) follows. This assumption could potentially be micro-founded by considering that individual payment services evolve proportionally to their adoption level. For instance, let m_t^{Ci} the adoption of PDM service i and define $m_t^C \equiv \int_0^1 m_t^{Ci}$, and assume that $\Lambda_t^i \propto m_t^{Ci}$, which implies $\int_0^1 (\Lambda_t^i) di \propto m_t^C$.

F Robustness and Additional Figures

This Appendix presents additional (non-essential) figures, yielding additional results and underscoring the robustness of our findings. These figures are discussed and referenced in the main text.

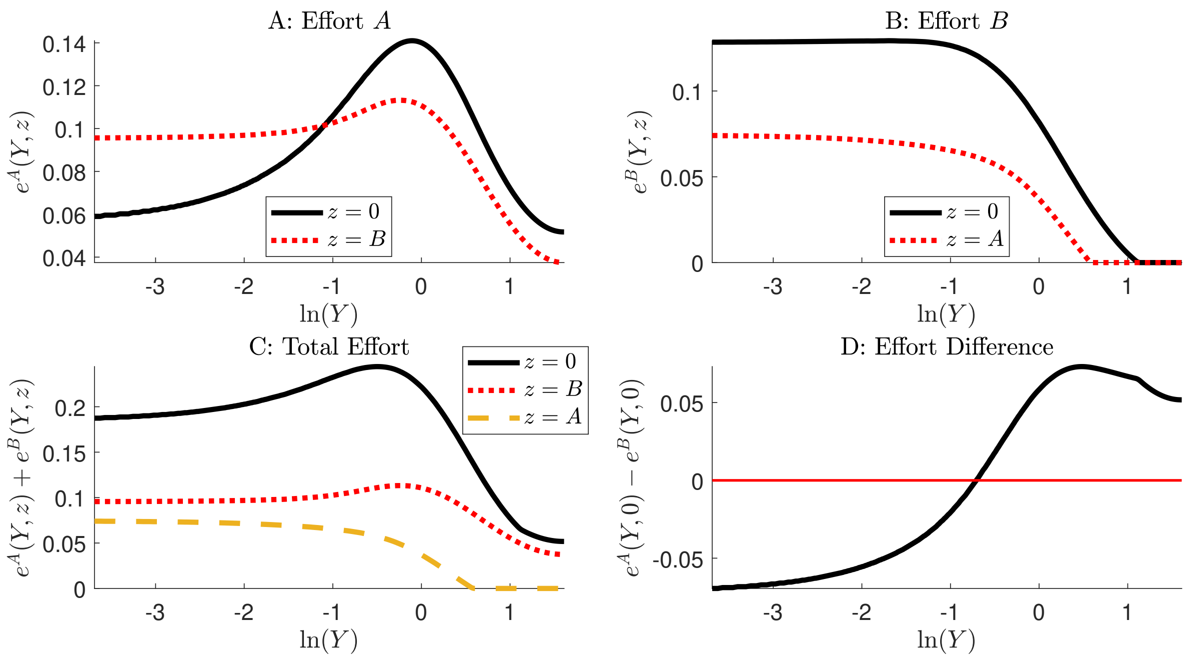

Figure F.1: Replication of Figure 2: The Dynamics of Digitization Efforts. Panel A depicts A's effort as a function of $\ln(Y)$ in states $z = 0$ (solid black line) and $z = B$ (dotted red line). Panel B depicts B's effort as a function of $\ln(Y)$ in states $z = 0$ (solid black line) and $z = A$ (dotted red line). Panel C plots the sum of countries' efforts against $\ln(Y)$ in states $z = 0$ (solid black line), $z = B$ (dotted red line), and $z = A$ (dashed yellow line). Panel D plots the effort difference against $\ln(Y)$ in state $z = 0$. We use our baseline parameters from Section 2.2, in addition to $\alpha^A i^A = 0.01$ and $\alpha^B i^B = 0.03$ (across all states z).

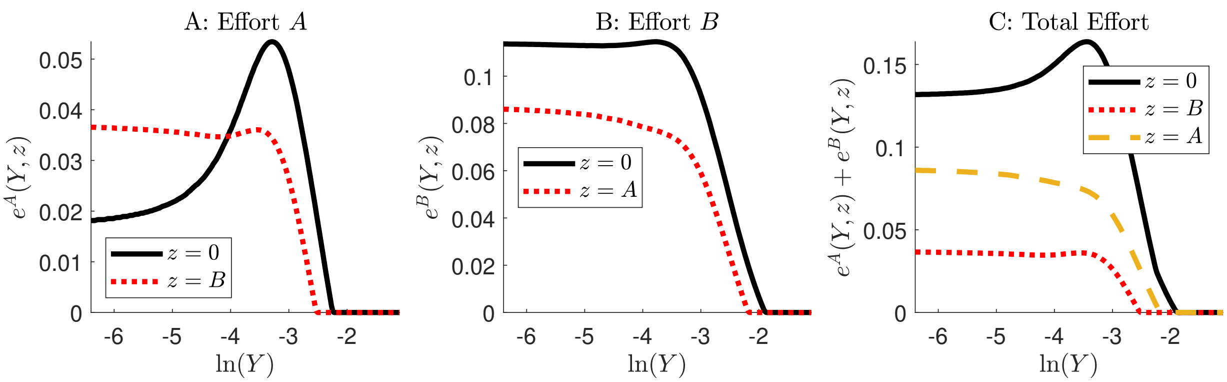

Figure F.2: Modeling the Benefits of Digitization through Improved Passhtrough. Panel A plots country A's against $\ln(Y)$ in state $z=0$ (solid black line) and state $z=B$ (dotted red line). Panel B plots country B's against $\ln(Y)$ in state $z=0$ (solid black line) and state $z=A$ (dotted red line). Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$, for $z=0$ (solid black line), $z=B$ (dotted red line), and $z=A$ (dashed yellow line). We set $i^A=0.01$, $i^B-0.1$, $\alpha^x(-x)=\alpha^x(0)=1$, and $\alpha^x(x)=\alpha^x(AB)=0.1$. And, $\rho=\theta=0$.

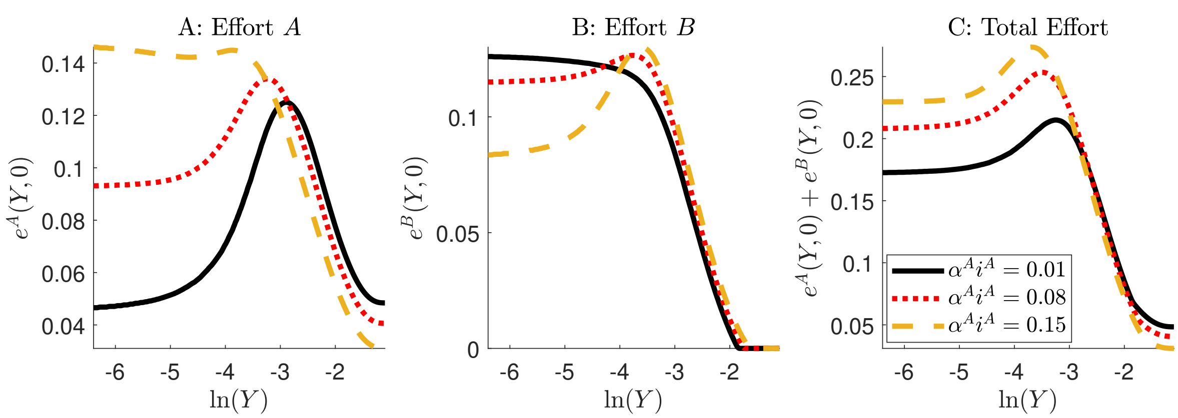

Figure F.3: Interest Rates and Digitization. This figure presents comparative statics in $\alpha^A i^A$. Panels A and B plot country A's and B's efforts in state $z=0$ against $\ln(Y)$ for three different levels of $\alpha^A i^A$. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We divide all baseline parameters from Section 2.2, which are related to currency convenience, by 15, leading to $Z_L^A=1/15$, $Z_L^B=0.2/15$, $\Delta^{Fixed}=1/15$, $\overline{Y}=5/15$, and $Y_0=0.1/15$ while all other parameters remain unchanged. We set $\alpha^B i^B=0.03$, and $\rho=\theta=0$.

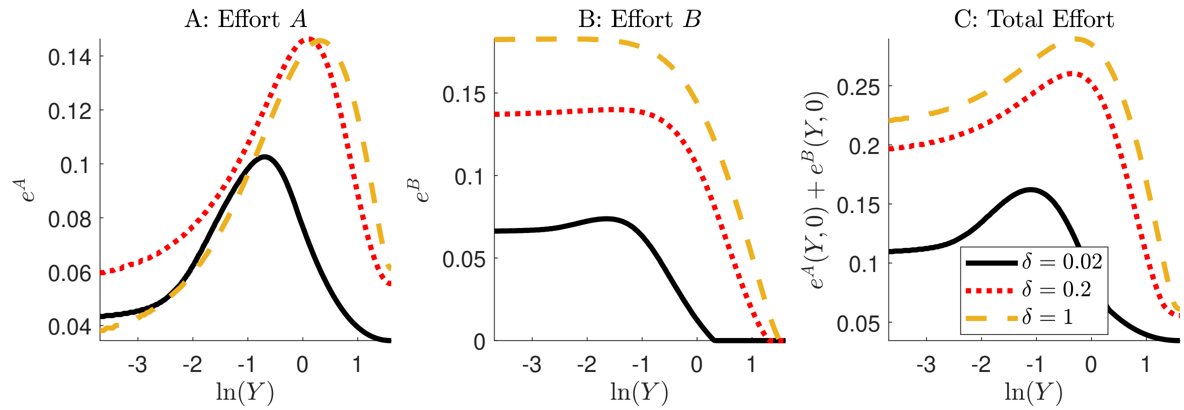

Figure F.4: Myopia and Effort. This figure presents comparative statics in δ. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of δ. Panel C plots total efforts, i.e., the sum of individual efforts, against $\ln(Y)$. We use our baseline parameters from Section 2.2, but set $Y_0 = 0.5$ for Panel C.

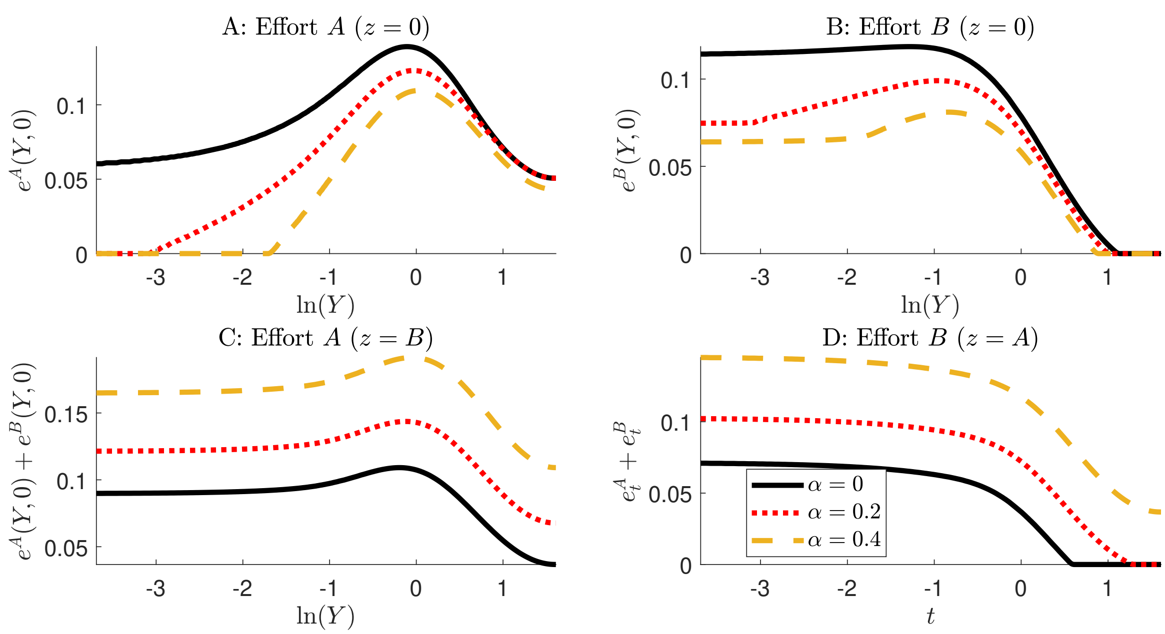

Figure F.5: Positive Spillovers in Digitization Costs. This figure presents comparative statics in α, starting from $\alpha = 0$. Panels A and B plot country A's and B's efforts in state $z = 0$ against $\ln(Y)$ for three different levels of α. Panel C plots A's effort in state $z = B$, while Panel D plots B's effort in state $z = A$.

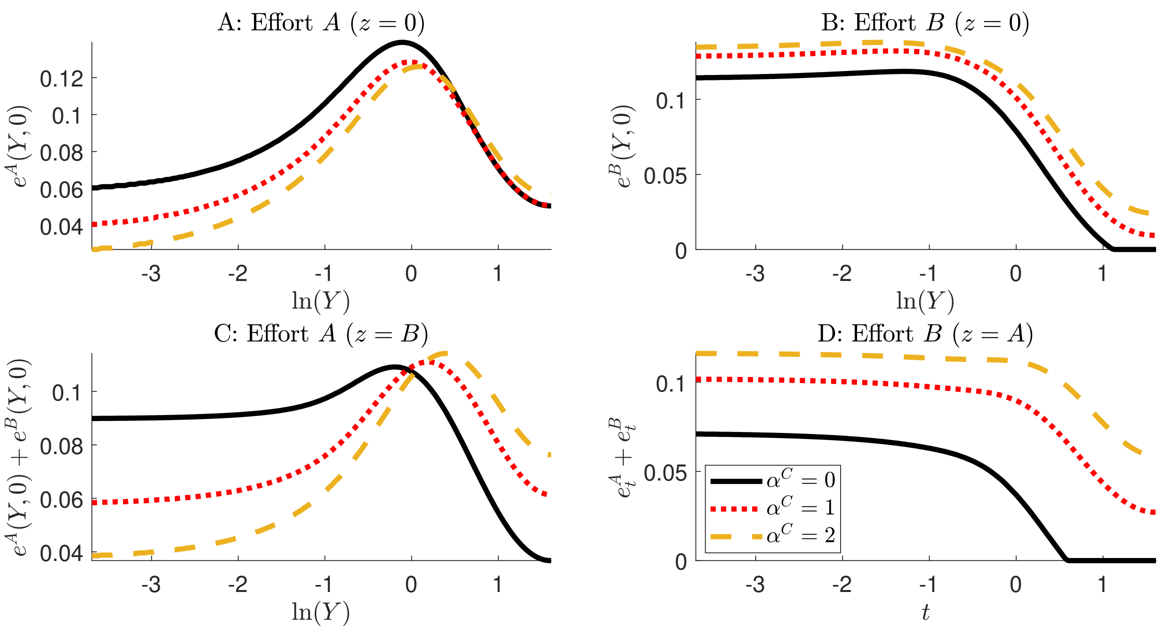

Figure F.6: Positive Spillovers in Digitization Benefits. This figure presents comparative statics in α^C, starting from $\alpha^C=0$. Panels A and B plot country A's and B's efforts in state $z=0$ against $\ln(Y)$ for three different levels of α^C. Panel C plots A's effort in state $z=B$, while Panel D plots B's effort in state $z=A$. We use our baseline parameters from Section 2.2.