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FINANCIAL CONDITIONS TARGETING

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ABSTRACT

We present evidence that noisy financial flows influence financial conditions and macroeconomic activity. How should monetary policy respond to this noise? We develop a model where it is optimal for the central bank to target and (partially) stabilize financial conditions beyond their direct effect on output and inflation gaps, even though stable financial conditions are not a social objective per se. In our model, noise affects both financial conditions and macroeconomic activity, and arbitrageurs are reluctant to trade against noise due to aggregate return volatility. Our main result shows that Financial Conditions Index (FCI) targeting—announcing a (soft and temporary) FCI target and setting the policy rate in the near future to maintain the actual FCI close to the target—reduces the FCI volatility and stabilizes the output gap. This improvement occurs because a more predictable FCI enables arbitrageurs to trade more aggressively against noise shocks, thereby "recruiting" them to insulate FCI from financial noise. FCI targeting is similar to providing forward guidance about the FCI, and in our framework it is strictly superior to providing forward guidance about the policy interest rate. Finally, we extend recent policy counterfactual methods to incorporate our model's endogenous risk reduction mechanism and apply it to U.S. data. We estimate that FCI targeting could have reduced the variance of the output gap, inflation, and interest rates by 36%, 2%, and 6%, respectively, and decreased the conditional variance of the FCI by 55%. When compared with interest rate forward guidance, it would have reduced output gap variance by 21%. We also show that a significant share of the gains from FCI targeting can be attained by an augmented version of a Taylor rule that gives a large weight to a simplified financial conditions target.

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1. Introduction

"Financial conditions have tightened significantly in recent months... We remain attentive to these developments because persistent changes in financial conditions can have implications for the path of monetary policy." (Chair Jerome Powell, Economic Club of New York Luncheon, October 19, 2023)

Monetary policy has been transitioning from a narrow emphasis on short-term interest rates to a significantly wider focus on financial conditions—a summary measure of aggregate asset prices such as stocks, bonds, real estate, and exchange rates. This shift acknowledges the large role played by the price of risky assets in driving aggregate demand. In fact, Financial Conditions Indices (FCI), which aggregate asset classes based on their impact on aggregate spending, identify risky asset prices, especially stock prices, as their main driver in the U.S. and most major economies (see, e.g., Hatzius et al. (2017)). It is also well-documented in the finance literature that these types of risky asset prices fluctuate without meaningful changes in underlying fundamentals (see, e.g., Campbell (2014)). These fluctuations partly reflect *noise shocks*—changes in asset demand or supply that are orthogonal to fundamentals—which affect asset prices because sophisticated investors face constraints or risks that limit their ability to trade against noise (see De Long et al. (1990); Gabaix and Koijen (2021)). Consistent with this mechanism, we estimate (identified) vector-autoregression (VAR) models that show noisy financial flows can explain up to 55% of the variance of financial conditions and between 20% and 50% of the variance of output gaps in the U.S. (see Section 2). How should monetary policy react to this financial noise?

Bernanke and Gertler (2000, 2001) address this question within a New Keynesian model with asset bubbles. They argue that, in this context, central banks should not focus on asset prices directly but instead on stabilizing the inflation and output gaps that arise from asset price fluctuations. In this paper, we take this analysis one step further and propose a model in which it is optimal for the central bank to target and (partially) stabilize financial conditions *beyond* their direct impact on output and inflation gaps. Furthermore, we demonstrate that in our model financial conditions targeting is strictly superior to the traditional interest rate forward guidance. Lastly, extending recent policy counterfactual methods, we find that financial conditions targeting would have significantly reduced the volatility of the output gap and financial conditions in the U.S. over recent decades.

Our model builds on the "risk-centric" New Keynesian framework developed in Caballero and Simsek (2023). The distinctive feature of this framework is that monetary policy transmits to macroeconomic activity through financial conditions. Specifically, aggregate demand is influenced by the aggregate asset price (the FCI in our model), reflecting a consumption wealth effect (as a proxy for broader mechanisms linking financial conditions to aggregate demand). Monetary policy tries to steer the aggregate asset price, by adjusting the policy interest rate, to influence aggregate demand and close the output gap.

There are two key differences from Caballero and Simsek (2023). First, the aggregate asset price is not only influenced by the policy rate and standard financial forces but also by (financial) "noise" shocks. Specifically, households delegate their portfolio decisions to managers of three types: noise traders, inelastic funds, and risk-averse arbitrageurs. Noise traders create random market flows that must be absorbed by other investors. Inelastic funds cannot absorb these flows, since they passively invest according to the average optimal portfolio benchmark. Risk-averse arbitrageurs can absorb the noisy flows, but their limited capacity leaves substantial room for noise to impact aggregate asset prices. Second, monetary policy reacts to aggregate noise shocks only with a delay, preventing it from fully managing financial conditions and aggregate demand.

Our main result demonstrates that an expanded monetary policy framework, where the central bank announces a (soft and temporary) Financial Conditions Index (FCI) target and sets the policy interest rate in the near-future to keep the actual FCI close to the target, is welfare improving. The reason is that FCI targeting "recruits" arbitrageurs for monetary policy. That is, it enables arbitrageurs to absorb more of the noise flows in real time, without the reaction lags of monetary policy.

At the core of this result is an endogenous volatility feedback loop: noise has a greater impact on aggregate asset prices when return volatility is higher. This happens because higher return volatility makes arbitrageurs more reluctant to trade against noise. The larger price impact of noise leads to an endogenous increase in return volatility, which further amplifies the price impact of noise, and so on. This noise-driven volatility in aggregate asset prices affects macroeconomic activity and leads to "excessive" fluctuations in the output gap.

In this context, FCI targeting operates in two ways: First, it centers the target FCI at the expected level needed to close future output gaps, as in Caballero and Simsek (2023). Second, and central to our main result, it dampens the response of monetary policy to future macroeconomic data, reducing the return volatility associated to these responses. This lower return volatility is the key mechanism by which the policy recruits the arbitrageurs: with lower volatility, the arbitrageurs trade more aggressively to counter noisy financial flows and the market becomes more elastic with respect to these flows. This counterforce reverses the volatility feedback loop and thereby reduces the impact of noise on the FCI and output. Surprisingly, this reduction in the price impact of noise also implies that, in many instances, the policy objective gain can be achieved with lower policy rate volatility.

The downside of the volatility reduction mechanism is that it makes monetary policy less responsive to macroeconomic shocks, allowing these shocks to have a greater impact on the output gap. However, this trade-off is worthwhile. We show that starting from a perfect-flexibility (discretionary) benchmark, implementing some degree of FCI targeting is always optimal. This is because the reduced flexibility with respect to macroeconomic shocks results in only a second-order loss, while the significant reduction in the impact of noise generates first-order gains in stabilizing the output gap.

FCI targeting is akin to providing *forward guidance* about the future path of the FCI, assuming that this type of guidance entails some degree of commitment. We show that FCI forward guidance is strictly superior to guidance on the policy interest rate. This is because FCI forward guidance naturally allows the policy to react to future noise shocks, whereas interest rate forward guidance reduces the flexibility of the central bank to react to these shocks. In other words, the optimal policy reduces data-dependency with respect to macroeconomic shocks while increasing data-dependency with respect to noise shocks. FCI targeting achieves both goals, while interest rate forward guidance only achieves the former.

Beyond the specific mechanisms we emphasize, FCI targeting could stabilize asset prices for a variety of additional reasons. For instance, as emphasized by Jeanne and Rose (2002), noise itself tends to decline when a stabilizing policy framework is in place, creating a virtuous cycle of reduced volatility. This stabilization is not limited to noise traders but extends to belief-driven fluctuations in asset prices, such as the anticipation of transformative events like an AI revolution. In an extension of our model, we demonstrate that when markets anticipate the Fed's commitment to maintaining stable financial conditions, belief shocks —whether justified or not—lead to smaller asset price fluctuations, thereby mitigating their broader impact on macroeconomic stability.

In the final part of the paper we conduct an empirical evaluation of FCI targeting by building upon the counterfactual policy evaluation methods described in McKay and Wolf (2023b) and Caravello et al. (2024). The key idea is to combine estimated impulse responses to monetary policy *shocks* to approximate the effects of counterfactual monetary policy *rules*. Substituting shocks for rules in this way identifies the correct counterfactual as long as the model is linear and monetary policy operates through current or expected policy interest rates. However, in our model, this approach encounters a crucial complication: FCI targeting stabilizes the economy by reducing risk, and this risk reduction is a non-linear element. We adapt the methodology to account for this non-linearity and estimate counterfactuals for policies that reduce FCI volatility, along the lines of the FCI targeting rule in our theoretical model.

Our main empirical results examine a scenario in which the Fed minimizes a weighted average of output gaps, inflation gaps, interest rate changes, and the FCI deviations from a preannounced (optimized) FCI target. We show that this policy framework significantly reduces the impact of noise shocks on financial conditions, the output gap, and inflation compared to historical data. This stabilization is achieved with only a slight increase in frontloading but reduced volatility of the policy interest rate, as the framework encourages arbitrageurs to absorb more of the noise. With arbitrageurs playing a larger stabilizing role, the policy interest rate reacts less to these shocks. Beyond noise shocks, targeting financial conditions substantially reduces the volatility of macroeconomic and financial variables. Compared to historical data, the variance of the output gap, inflation, and interest rates decreases by 36%, 2%, and 6%, respectively, while the conditional variance of financial conditions decreases by 55%. These reductions outperform standard alternatives, such as a flexible dual mandate framework or interest rate forward guidance. A significant portion of these gains can be achieved with an augmented version of a Taylor rule that gives a large weight to a simplified financial conditions target.

Literature review. Our paper connects two main literatures: one in macroeconomics and one in finance. On the macroeconomics side, our paper is part of an emerging literature on New Keynesian models with risk and asset prices (e.g., Caballero and Farhi (2018); Caballero and Simsek (2020, 2021, 2023, forthcoming); Pflueger et al. (2020); Kekre and Lenel (2022); Kekre et al. (2023); Beaudry et al. (2024); Adrian and Duarte (2018); Adrian et al. (2020)). Our main new ingredient is the presence of financial noise, which interferes with the monetary policy transmission. Our main result demonstrates the benefits of FCI targeting in such an environment.

On the finance side, our paper is related to a large literature that emphasizes asset price fluctuations driven by noise and limits to arbitrage (see Black (1986); Shleifer and Summers (1990); De Long et al. (1990) for early contributions). Noise is a catch-all term for *nonfundamental* demand or supply by some market participants that might emerge from a variety of sources such as behavioral biases, institutional frictions, and segmented markets (see Gromb and Vayanos (2010)). Limits to arbitrage refers to the constraints faced by sophisticated investors in trading against noise (see Shleifer and Vishny (1997)). The literature has applied these ingredients to explain asset price fluctuations in many markets, including *aggregate* assets that affect financial conditions such as treasury bonds (Greenwood and Vayanos (2014); Vayanos and Vila (2021)), exchange rates (Gabaix and Maggiori (2015); Gourinchas et al. (2022); Greenwood et al. (2023)), and the aggregate stock market (Gabaix and Koijen (2021)). The VAR evidence that we present in Section 2 confirms these findings and indicates that noisy aggregate flows drive not only financial conditions but also *macroeconomic activity.* Our main contribution to this literature is to embed noise and limits-to-arbitrage into a macroeconomic model and show that these ingredients create a natural rationale for FCI targeting. In our model, FCI targeting works because it reduces the aggregate return volatility and recruits sophisticated investors to trade against noise.

Our model shares similarities with Jeanne and Rose (2002); Itskhoki and Mukhin (2021), who demonstrate that a monetary policy regime stabilizing the exchange rate can reduce exchange rate volatility without significantly changing other macroeconomic variables, offering an explanation of the Mussa puzzle. As mentioned earlier, Jeanne and Rose (2002) show that exchange rate stabilization deters the entry of noise traders creating a virtuous cycle of reduced volatility. Similarly, Itskhoki and Mukhin (2021) show that exchange rate stabilization enables sophisticated investors to trade against noise more aggressively. While our mechanism resembles that of Itskhoki and Mukhin (2021), the *macroeconomic* implications differ. In our model, noise-driven fluctuations in financial conditions have significant effects on macroeconomic activity, as we confirm in Section 2, whereas in their model, exchange rate fluctuations have minimal impact on aggregate activity. This large macroeconomic impact in our framework justifies the FCI-targeting policy.

Our paper is related to Woodford (2003), who shows that adding an interest-rate smoothing term to central bank objectives might be desirable, even though interest rate smoothing *per se* is not a social objective. In similar vein, we show that adding an FCI targeting to central bank objectives might be desirable, but the mechanism and the source of welfare gains are different. Interest rate smoothing affects the private sector's *expectations of future interest rates,* which in turn enables the central bank to shift the long-term interest rate through moderate changes in the short-term rate. In contrast, FCI targeting affects the private sector's expectations of *aggregate asset price volatility*, which encourages the arbitrageurs to trade against noise and stabilizes financial conditions.

Our paper connects with the large literature on forward guidance about the path of policy interest rates (see, e.g., Campbell et al. (2012); Woodford (2013); Svensson (2014); Bassetto (2019)). The recent literature emphasizes the role of forward guidance as a commitment device that might be especially useful when the policy rate is constrained by the effective lower bound (e.g., Eggertsson and Woodford (2003)). Our model shows that forward guidance about the FCI, viewed as a soft commitment to an FCI target, can stabilize financial conditions and output gaps.[1]

Our paper also belongs to a literature that empirically identifies the macroeconomic effects of financial shocks (Gilchrist et al., 2009; Gilchrist and Zakrajšek, 2012), and the transmission of monetary policy via financial markets (Gertler and Karadi, 2015; Caldara and Herbst, 2019).[2] While the previous literature focuses on the effects of shocks to credit spreads, our financial noise captures the price impact of equity flows. We also relate to several papers (Hatzius et al., 2017; Hatzius and Stehn, 2018; Ajello et al., 2023a) that show that: (i) innovations in various financial conditions indices are strongly correlated with output growth, (ii) equity is the main driver of financial conditions indices for the United States. Our identification strategy isolates plausibly exogenous variation in flows to equity, which allows for a causal interpretation of our estimates. Overall, our results highlight the significant role that noise shocks in the stock market play in macroeconomic fluctuations, which, although related, are distinct from other financial shocks identified in the literature.

Finally, our paper is part of a recent literature on semi-structural policy counterfactuals (Hebden and Winkler, 2021; Barnichon and Mesters, 2023; Beraja, 2023; McKay and Wolf, 2023b; Caravello et al., 2024). We contribute to this literature by showing how, within the class of models we consider, a simple departure from a purely linear setting is sufficient to account for the effects of endogenous changes in the level of risk in the counterfactuals. We use this approach to evaluate the efficacy of FCI targeting to stabilize macroeconomic and financial fluctuations.

The rest of the paper is organized as follows. Section 2 present facts on the macroeconomic

[1] Caballero and Simsek (2022) show that when central banks and markets disagree, forward guidance (about interest rates) can be beneficial by communicating the central bank's beliefs to the market and preventing misinterpretations. While we do not model disagreements in this paper, we conjecture that this communication channel would complement the commitment channel that we emphasize. Specifically, FCI forward guidance would help to communicate the central bank's beliefs to the market, which would reduce the policy risk premium (Caballero and Simsek (2023)) and further enable sophisticated investors to absorb noise.

[2] For a more structural approach, see (i.a.) Del Negro et al. (2013); Christiano et al. (2014) .

effects of financial noise that motivates our theoretical analysis. Section 3 describes the model, characterizes the equilibrium with discretionary policy, and demonstrates the destabilizing effects of noise. Section 4 presents our main theoretical results, which demonstrate that FCI targeting can reduce financial volatility and improve macroeconomic stability. This section also compares FCI targeting with interest rate targeting and discusses the robustness of FCI targeting in various model extensions, including an inflation-output trade-off and the presence of shocks to arbitrageurs' beliefs about future productivity. Section 5 extends recent methodology on counterfactual policy analysis to account for the endogenous volatility feedback loop in our model and uses it to empirically support the main implications of our model. Section 6 provides final remarks. The theory appendix A contains the derivations and various model extensions. The data appendix B presents the details of the empirical analysis and additional results.

2. The macroeconomic impact of financial noise

In this section, we explore the influence of stock market noise on financial conditions and macroeconomic activity. We focus on the stock market because it is the primary driver of FCI fluctuations in both the U.S. and other major economies. Our findings reveal that the effects of financial noise shocks are similar to those of classic demand shocks. These noise shocks account for a significant portion of the forecast variance of the FCI, contributing up to 55% at the initial impact. Moreover, they have a substantial effect on the variance of output gaps, peaking at a contribution of up to 50% to the forecast variance over a two-year horizon.

2.1. Data and methodology

We measure financial conditions with the Financial Conditions Impulse on Growth index (FCI-G) constructed by Ajello et al. (2023a). This index applies the macroeconomic models used by the Fed to predict the GDP growth over the next year implied by the recent changes (up to three years ago) in seven different financial variables—the Dow Jones stock price index, the Zillow house price index, the broad dollar index, the federal funds rate, the 10-year Treasury yield, the 30-year fixed mortgage rate, and the triple-B corporate bond yield. The index has the sign convention of interest rates and is scaled so that a reading of 1 implies that financial conditions will decrease next-year's GDP growth by 1 percentage point. Figure 1 illustrates the index along with the contribution of each financial variable over 1990Q1-2024Q2. The stock market is the key driver of the index followed by the exchange rate. The housing market is an important driver during the buildup of the GFC and the COVID-19 shock recovery.

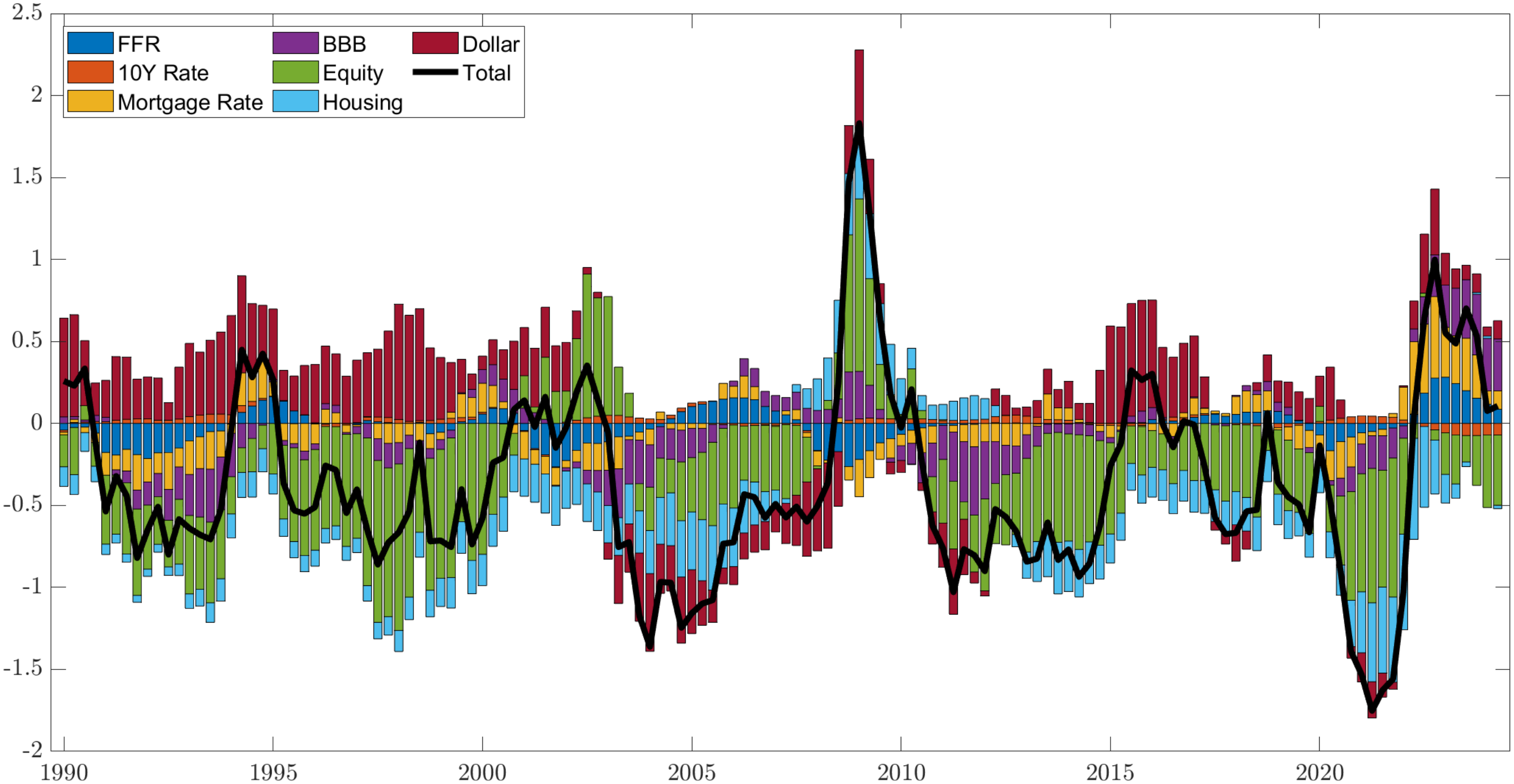

Figure 1: The FCI-G index (with a three-year lookback) and its drivers over 1990Q1-2024Q2. Positive values imply a decrease in GDP growth in the next year. Source: Ajello et al. (2023a)

We investigate the relationship between financial conditions and macroeconomic activity using a vector autoregression (VAR) that includes the FCI-G. Appendix B.1 contains a detailed discussion of our data construction. Our main sample is 1990Q1:2019Q4. We start in 1990 since the FCI-G index starts in this year and we stop before the Covid period to avoid outliers.[3] The baseline variables are the FCI-G, real potential GDP (estimated by the CBO), the output gap, real investment, real consumption, annualized PCE inflation, the Excess Bond Premium of Gilchrist and Zakrajšek (2012), and the 3-month nominal interest rate. Aside from the FCI-G, the rest of the variables are standard in monetary and financial VAR specifications (Gilchrist and Zakrajšek, 2012; Gertler and Karadi, 2015). We use this baseline set of variables to estimate the impulse responses in Section 2.2. We include two lags in the VAR as suggested by the BIC; and linearly detrend potential GDP, investment, and consumption. All other variables are included in levels.

Our variance decomposition analysis in Section 2.3 partly relies on the assumption of *invertibility*, which posits that structural shocks can be recovered from the VAR residuals. To make this assumption more plausible, we follow Caravello et al. (2024), and include three more variables in the VAR when computing variance decompositions, all in logs: (i) hours per worker, (ii) labor share, (iii) (detrended) labor productivity.

We proxy for the financial noise shock using a Granular IV (Gabaix and Koijen, 2024). In particular, we construct the proxy exactly as in Gabaix and Koijen (2021) using the Flow of Funds data. Appendix B.1.2 reviews the details of the construction. The gist of the idea is as follows: using flow-of-funds data, we can measure the proportional changes in equity held by different sectors at different points in time, Δq_{it}. Since these flows are endogenous, we residualize them using fixed effects, sector-specific trends, macro observables, and principal components, to obtain a residual $\Delta \tilde{q}_{it}$ of idiosyncratic flow shocks for each sector. This residual can be interpreted as sector-specific financial noise shocks. Finally, we do an equity-share-weighted-average of these residuals to construct the financial flow series Z_t^{μ} as:

$$Z_t^{\mu} = \sum_{i=1}^{I} S_{i,t-1} \Delta \tilde{q}_{it} \tag{1}$$

where $S_{i,t-1}$ is the fraction of total equity held by sector i at time $t-1$. Since households are large in the flow of funds data, in practice this measure is mostly driven by residual changes in households' equity holdings. Gabaix and Koijen (2021) argue that this is an appropriate measure of net flows into equities. For this procedure to be valid, $\Delta \tilde{q}_{it}$ must be uncorrelated with other aggregate shocks. In the Appendix, we show that our results are robust to various residualization methods.[4]

[3]Appendix B.2.3 shows what happens if the Covid period is included. In a nuthsell, the estimated IRFs are unchanged and the variance contribution to output gaps goes down given that the Covid shock is quite large and the noise shock plays a limited role in the post-Covid subsample.

[4]Note that the residualization step is key to tackle the issue that "for every buyer there is a seller." Absent residualization, prices always adjust such that the flows sum up to zero. The residualization eliminates the price

Given the shock and observables, we assume that the data generating process can be characterized (in demeaned terms) as:

$$Y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} \tag{2}$$

$$\tilde{Z}_t^\mu = \alpha \varepsilon_t^\mu + v_t, \tag{3}$$

$$Y_t = \sum_{\ell=1}^{L} A_\ell Y_{t-\ell} + u_t, \tag{4}$$

where Y_t is the vector of macro variables of interest (already demeaned and detrended), $\tilde{Z}_t^\mu = Z_t^\mu - L\left[Z_t^\mu | \{\tilde{Z}_\tau^\mu, Y_\tau\}_{\tau<t}\right]$ is the proxy shock after residualizing it with respect to lags of itself and other macro variables (where $L(x|y)$ denotes the linear projection of variable x onto variables y), ε_t^μ is the structural shock of interest, v_t is measurement error and u_t is the vector of Wold innovations. We assume that ε_t^μ, v_t are white noise, and independent of each other.

Equation (2) is the structural vector moving average representation of the data: Y_t depends on a set of structural shocks, ε_t, that propagate according to Θ_ℓ. Equations (3) and (4) are measurement equations: (3) relates the measured proxy Z_t^μ to the structural shock of interest ε_t^μ, whereas (4) is a reduced-form Wold representation of the data. Within this framework, different assumptions can be used to recover impulse responses and variance decompositions from the data. We explain the details in the following subsections.

2.2. Causal effects of noise shocks

First, we use the constructed proxy Z_t^μ to estimate the effect of noise shocks on macro and financial variables. In order to do so, we use a VAR that includes the baseline set of macro variables. Following Plagborg-Møller and Wolf (2021), we add the proxy to the VAR and use a recursive identification scheme, where the proxy is ordered first. That is, we consider the augmented vector $X_t = [Z_t^\mu, Y_t']'$, run a VAR on X_t, and use recursive identification. The red line in Figure 2 depicts the shock we use to obtain the impulse response. As can be seen, the series is relatively well mixed over different quarters. There is a large, negative shock around the GFC, but there are also other shocks of comparable magnitude in other points in the sample, as in the early 1990s or the mid 2000s.

First Stage. In order to evaluate instrument relevance, we perform an F-test for the relevance of the financial noise shock in explaining movements in the residuals of the FCI equation. The conventional F-statistic is 19.96, whereas the heteroskedasticity-robust F equals 13.75. Given that both values are above the conventional level of 10, we proceed using standard inference.

adjustment required for this identity to hold. See Section 4.1 of Gabaix and Koijen (2021) for details.

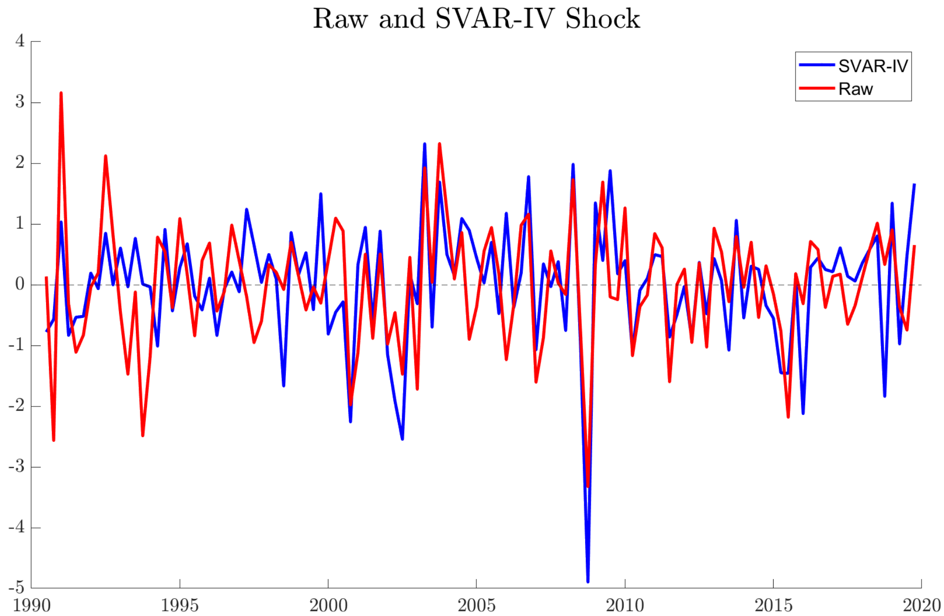

Figure 2: Red: raw shock, $\tilde{Z}_t^{\mu}$ as in (3). Blue: shock identified using SVAR-IV as in (5).

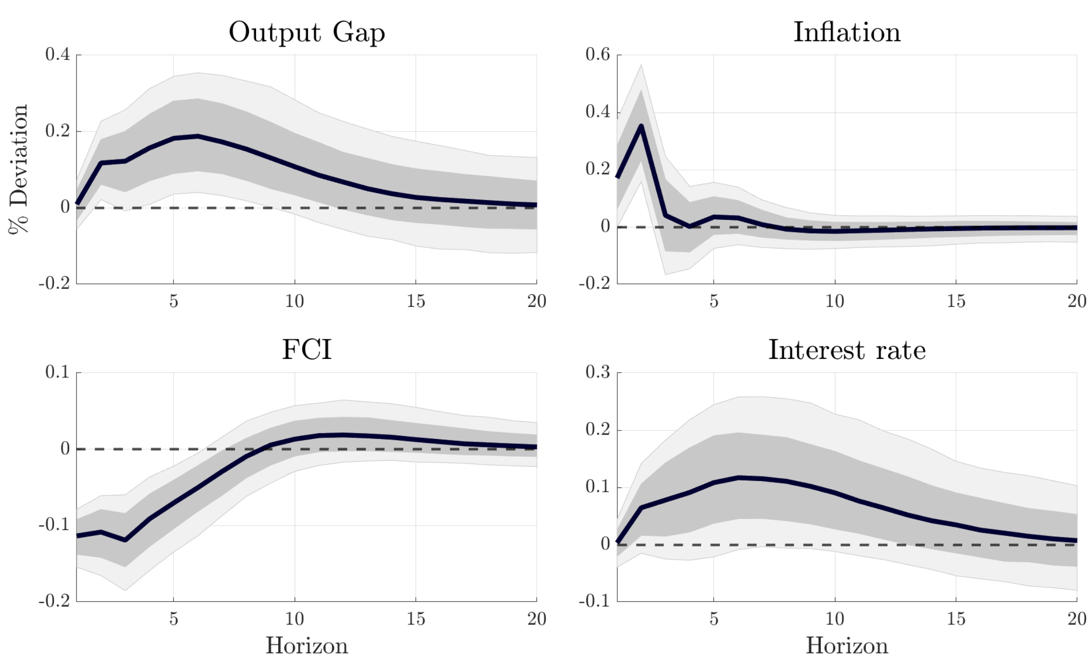

Figure 3: Impulse response to a financial noise shock. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

Impulse Responses. Figure 3 depicts the impulse-response of several macroeconomic outcomes of interests to an expansionary noise shock, i.e., an exogenous inflow into equity. The shock lowers the FCI index on impact, which implies looser financial conditions. This generates a positive output gap and inflation in the first few quarters. There is some positive response of the interest rate, but it is insufficient to fully stabilize the shock. Overall, the effect of the financial noise shock is that of a textbook demand shock, that is only imperfectly stabilized by monetary policy.[5]

2.3. Importance of noise shocks for macroeconomic fluctuations

In this subsection, we estimate the extent to which output fluctuations in this sample period are driven by the financial noise shocks. This is the key magnitude that determines the potential volatility reductions from adopting FCI targeting, since the policy works by endogenously reducing the impact of noise shocks.

Forecast Variance Ratios. Given its relevance, we present results under several alternative assumptions to estimate the contribution of the shock to forecast variance. The object of interest is the Forecast Variance Ratio (FVR). Following Plagborg-Møller and Wolf (2022), we define the FVR for variable i at horizon h as

$$FVR_{i,h} = 1 - \frac{\text{Var}\left(Y_{i,t+h} | \{Y_\tau\}_{\tau<t}, \{\varepsilon^\mu_\tau\}_{t\leq\tau<\infty}\right)}{\text{Var}\left(Y_{i,t+h} | \{Y_\tau\}_{\tau<t}\right)}.$$

This measures by how much does the forecast error (for variable i at horizon h) would be reduced if we knew with certainty the realization of the shock at all future dates.

Within the DGPs of the form (3)-(4), there are several alternative assumptions regarding the relation between $\tilde{Z}^\mu_t, \varepsilon^\mu_t$ and u_t, that yield different identified Forecast Variance Ratios.

First, the most common assumption is *invertibility*. Under this assumption, the structural shock satisfies

$$\varepsilon_{\mu,t} = q' u_t. \tag{5}$$

that is, there exists a linear combination of the (contemporaneous) Wold residuals that spans the shock of interest. Given that we have a proxy for this shock, we can use a SVAR-IV procedure (Mertens and Ravn, 2013) to identify q and, therefore, $\varepsilon_{\mu,t}$. Intuitively, in this case we are assuming that the true structural shock can be recovered from the Wold residuals. But since we can estimate the pattern of comovements that the structural shock generates using $\tilde{Z}^\mu_t$, we can back out the correct structural shocks from knowledge of the residuals and the proxy. The blue line in Figure 2 depicts the identified shock under this assumption. The potential problem of

[5]Appendix B.2 shows that the estimated responses are robust to i) controlling for additional principal components when constructing the shock; ii) controlling for changes in consumer sentiment when constructing the shock; iii) using an SVAR-IV procedure. Also, in Appendix B.2.2 we show that the shock does not predict TFP, which suggests that it is not a proxy for news about future fundamentals.

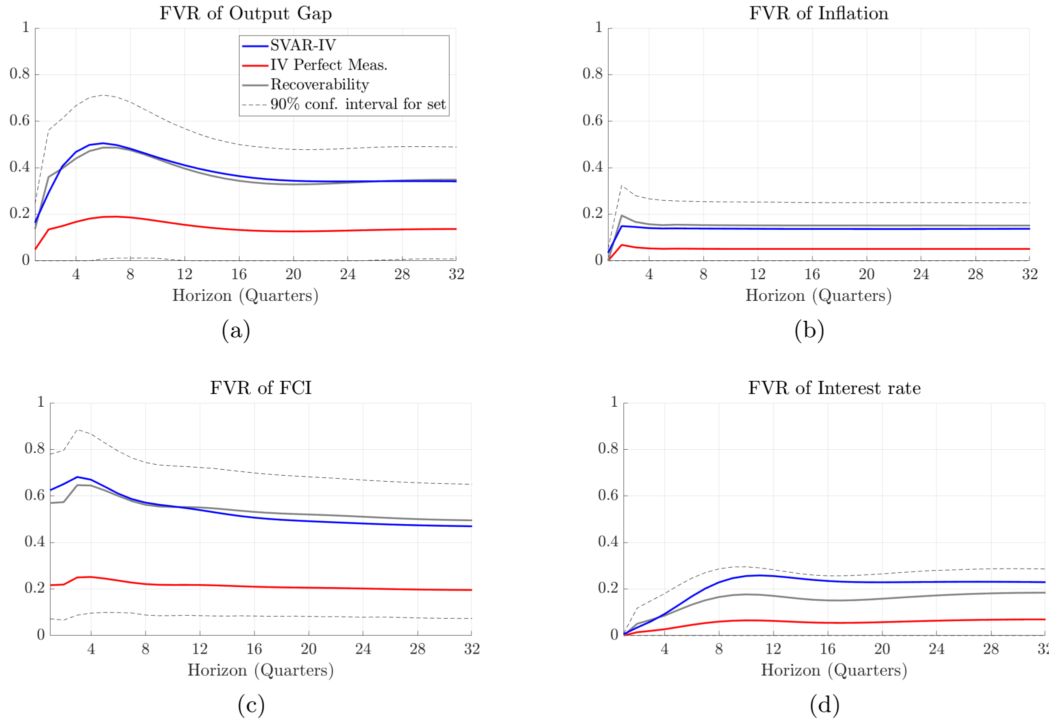

Figure 4: Identified Forecast Variance Ratios of the noise shock. Blue: SVAR-IV, assuming invertibility. Red: lower bound, assumes perfect measurement of the shocks. Grey: recoverability-based FVR. VAR includes the full set of macro outcomes (baseline + labor market variables). Dashed lines are 90% confidence intervals of the identified set, (Plagborg-Møller and Wolf, 2022) computed via bootstrap with 1000 repetitions.

this strategy is that, if the invertibility assumption is violated, it could lead to an overestimation of the identified shock's variance contribution.[6]

A second assumption that gives point identification of the FVR is to rule out measurement error, i.e., $v_t = 0$ for all t. In contrast to the previous case, if this assumption is violated, we would underestimate the shock's contribution: If $\tilde{Z}_t^{\mu}$ appears to have low correlation with Y_t, we would attribute that to the shock being unimportant instead of being caused by measurement error. In fact , this assumption provides a lower bound for the true FVR of the shock (Plagborg-Møller and Wolf, 2022). The red line in Figure 2 corresponds to the shock identified under this non-measurement error assumption. Of course, in practice, a prevalent type of measurement error is that we simply do not fully observe the shock. That is, the Gabaix and Koijen (2021) shock may be only one of many financial noise shocks. Thus, one interpretation under this assumption is that we are capturing the importance of the directly measured noise, whereas under the previous assumption we aim to capture the full importance of the noise shocks by imposing additional structure.

[6]See, e.g., Plagborg-Møller and Wolf (2022) for an illustration of this bias in the context of a model with monetary and oil shocks.

Finally, a third assumption that allows identification of the FVR is *recoverability* (Plagborg-Møller and Wolf, 2022; Forni et al., 2023). Under this assumption, the structural shock satisfies:

$$\varepsilon_{\mu,t} = q'\left(L^{-1}\right) u_t, \tag{6}$$

where $q(L^{-1})$ is now a lead polynomial. Thus, $\varepsilon_{\mu,t}$ can be recovered from the data, but we may need future values of u_t to do so. This assumption is less stringent than invertibility, and it provides an upper bound on the FVR (Plagborg-Møller and Wolf, 2022). However, even in this case, we are still assuming that we can properly recover the shock based on observables.

Figure 4 shows the FVRs identified under each of these three assumptions. As we can see, the standard invertibility-based SVAR-IV produces variance ratios that are almost indistinguishable to the recoverability-based estimates. Under any of these two assumptions, the noise shock explains up to 50% of the forecast error in output gap at a 2 year horizon. The share of unconditional output gap variance explained by the shock is around 35%.[7] Furthermore, the shock explains up to 55% of the contemporaneous variation in FCI. The lower bound, obtained under the perfect measurement assumption, shows lower contributions, but still a sizeable share of output gap's forecast variance at a 2 year horizon (20%) is driven by the shock, as well as a non-trivial portion of the unconditional output gap volatility (15%). The shock also explains a large share of FCI fluctuations. Overall, even at the lower bound, the evidence indicates that the shock is a significant driver of FCI and output gap fluctuations.[8]

Historical decomposition. In order to provide an interpretation of what kind of historical episodes are driven by the shock, we perform the following exercise: we take the estimated VAR, and feed in only the identified noise shock, setting all other innovations to zero. More specifically, we create alternative time series $\check{Y}_t$ using:

$$\check{Y}_t = \sum_{\ell=1}^{L} \hat{A}_\ell \check{Y}_{t-\ell} + \hat{p}\hat{\varepsilon}_t^{\mu}, \quad \check{Y}_j = 0 \text{ for } j < 0. \tag{7}$$

Where $\{\hat{A}_\ell\}$ are the estimated VAR coefficients, $\hat{p}$ is the vector of estimated contemporaneous effect of the structural shock on each VAR equation, and $\hat{\varepsilon}_t^{\mu}$ is the estimated time series for the shock. We do this for two identifying assumptions: (i) SVAR-IV, (ii) no measurement error. We omit recoverability due to similarity with SVAR-IV. We emphasize that this lacks a direct counterfactual interpretation, but it is a useful accounting device to see when the shock is important in the sample.

Figure 5 shows the results of this exercise. The dashed line is the raw data, and the solid lines

[7] In the Figure, we can proxy this contribution by looking at the FVRs at the longest horizon, since there is essentially no predictability left at that horizon.

[8] We also note that, for all sets of assumptions, the shock explains a modest amount of inflation fluctuations (up to 15% under at the upper bound, around 5% at the lower bound), and also a modest amount of interest rate fluctuations.

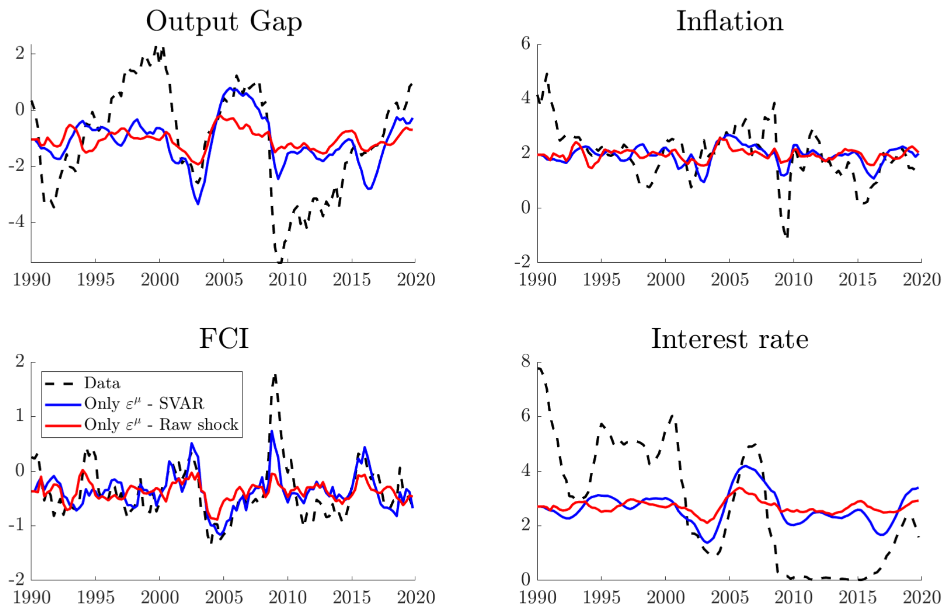

Figure 5: Alternative time series constructed by setting to zero all shocks other than the identified noise shock, following (7). Red: raw shock, $\tilde{Z}_t^\mu$ as in (3). Blue: shock identified using SVAR-IV as in (5). Note: inflation is reported in year-on-year terms.

show the alternative time series $\check{Y}_t$ based on the SVAR-IV identified shock (in blue), and the shock identified under the perfect measurement assumption (in red). The alternative series based on the SVAR-IV assumption track the data on FCI quite well for the 1999-2008 period. Before and after, there is some relation but the gap between the data and the alternative series is wider. Something similar happens with the output gap: the blue line tracks the data very well during the 1999-2008 period, just before the GFC. During the GFC, the noise shock explains some of the drop, but it is far from explaining the full depth and slow recovery from the recession. Similar patterns are observed with interest rates and inflation. Overall, the SVAR-identified shock explains most of the FCI and output fluctuations in the 1999-2008 period, less so before and after. This is consistent with the narrative that attributes macro fluctuations preceding the GFC to exuberance in financial markets, associated with positive noise shocks. When the GFC happened, this is partially amplified by negative noise shocks, but other factors (such as rising risk premium and the binding ZLB) also must have played a role, since financial noise shocks alone cannot explain the data after the GFC. Focusing on the shock identified under the perfect measurement assumption, the overall patterns point to the same direction, but magnitudes are smaller. From our earlier discussion, this may reflect measurement error, in particular the fact that the Gabaix and Koijen (2021) shock is only a subset of all financial noise shocks.

3. A macroeconomic model with financial noise

In this section, we present a risk-centric macroeconomic model with the following key features: (i) financial noise can drive the aggregate asset price (FCI) away from the central bank's desired level due to policy reaction lags; (ii) sophisticated investors (arbitrageurs) trade against this noise; and (iii) the arbitrageurs face uncertainty about the future level of FCI. In this setup, we demonstrate that noise shocks affect both financial conditions and macroeconomic activity, consistent with our motivating evidence. In the subsequent section, we will use this model to investigate the effects of an explicit FCI targeting policy.

3.1. Environment

We relegate the details of the environment to Appendix A.1. Here, we summarize the real side of the economy and describe in more detail the financial market side.

Real economy. On the supply side, (the log of) potential output follows the process:

$$y_t^* = y_{t-1}^* + \varepsilon_{z,t}, \text{ where } \sigma_z^2 \equiv var\left(\varepsilon_{z,t}\right) \tag{8}$$

and $\varepsilon_{z,t}$ denotes an i.i.d. *aggregate supply shock*. Due to nominal price stickiness, (the log of) output, y_t, is determined by aggregate demand and can depart from potential output. For the baseline model, we assume firms' prices are fully sticky. Our main results are robust to allowing for partially flexible prices and a trade-off between inflation and output stabilization, as we show in Appendix A.5 and discuss in Section 4.6.2.

On the demand side, there are two types of households: hand-to-mouth agents and (asset holding) households. Hand-to-mouth agents do not play an important role beyond decoupling the labor supply decisions from household consumption behavior. They supply all of the labor and spend all of their income. Since their spending is driven by output, which is endogenous, they create a Keynesian multiplier effect but they do not drive the aggregate demand.

Aggregate demand is driven by (asset holding) households. These households own the aggregate risky asset (the market portfolio): a claim on firms' share of output (αY_t). They have expected log utility and make portfolio allocation and consumption-savings decisions. They delegate the portfolio decision to the portfolio managers that we describe later. Their consumption rule is centered around the optimal rule with log utility, but it can deviate by an amount denoted by δ_t, which we refer to as an *aggregate demand shifter*. This is a modeling device to capture various factors that affect aggregate spending, e.g., a consumer sentiment shock, a fiscal policy shock, or a discount rate shock.

The upshot of these assumptions is the *output-asset price* relation

$$y_t = m + p_t + \delta_t, \tag{9}$$

where p_t denotes (the log of) the price of the market portfolio and m is a derived parameter that combines households' marginal propensity to consume and the multiplier. All else equal, higher aggregate asset prices raise spending and output through a consumption wealth effect. In practice, a higher price for aggregate assets such as stocks, bonds, and real estate raises spending for various reasons including wealth effects. Thus, we view p_t as the model counterpart to an FCI, and Eq. (9) as capturing the broader set of channels that links spending to asset prices.[9]

Naturally, a higher aggregate demand shifter δ_t also induces higher spending and output. We assume the aggregate demand shifter follows an AR(1) process

$$\delta_t = \varphi_\delta \delta_{t-1} + \varepsilon_{\delta,t}, \quad \text{where } \sigma_\delta^2 \equiv var\left(\varepsilon_{\delta,t}\right) \tag{10}$$

and $\varepsilon_{\delta,t}$ is an i.i.d. *aggregate demand shock*, which is independent from supply shocks.

Remark 1 (Policy transmits via FCI)**.** *Observe that the short-term interest rate does not enter into the output-asset price relation as a separate variable: it affects output only through its impact on the price of the market portfolio. This feature is driven by our assumptions (such as log utility) but it is supported by empirical evidence once we interpret* p_t *as the model counterpart to an FCI. In Appendix B.2, we use our empirical estimates from Section 2 to perform counterfactuals that show that broad financial conditions are a critical pathway for the transmission of monetary policy. In fact, we find that a monetary policy shock that significantly alters short-term interest rates* without *affecting financial conditions does not meaningfully impact the output gap or inflation.*

Financial markets. Households make a portfolio choice between two assets: the market portfolio and the risk-free asset (normalized to have zero net supply). The (log) return on the risk-free asset $r_t^f = \log R_t^f$ is set by the central bank as we describe later. The (log) return on the market portfolio, $r_{t+1} = \log R_{t+1}$, is approximately given by (see the appendix)

$$r_{t+1} = \rho - (1-\beta)\, m + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t,$$

where $\rho \equiv -\log \beta$ is the (log of) households' discount rate. After substituting the output asset price relation (9) in it, we can write the return as

$$r_{t+1} = \rho + p_{t+1} - p_t + (1-\beta)\, \delta_{t+1}. \tag{11}$$

That is, the aggregate return is driven by the (log) price change and the aggregate demand shock (through its impact on cash flows).

Households delegate their portfolio choice to managers.[10] In each period, a fraction η of

[9] We caution that p_t is stated in price, rather than the usual rates convention of FCIs. Thus, it has the opposite sign-convention of the standard FCIs, where an increase in the index typically means tightening (see Section 2).

[10] Managers do not consume themselves; they simply make a portfolio choice for households. For simplicity, each

these managers are "noise traders" and their portfolio weight is given by $\omega_t^N = 1 + \frac{1}{\eta}\mu_t$. That is, they deviate from the optimal portfolio benchmark by an amount given by $\frac{1}{\eta}\mu_t$. We refer to μ_t as the aggregate noise—the total flow that needs to be absorbed by other investors (see Remark 2 for a discussion). We assume the aggregate noise μ_t follows an AR(1) process

$$\mu_t = \varphi_\mu \mu_{t-1} + \varepsilon_{\mu,t}, \qquad \text{where } \sigma_\mu^2 \equiv var\left(\varepsilon_{\mu,t}\right) \tag{12}$$

and $\varepsilon_{\mu,t}$ is an i.i.d. *financial noise shock*, which is independent from supply and demand shocks.

Among the remaining managers, a mass $1-\eta-\alpha$ represent "inelastic funds" and their portfolio weight is given by $\omega_t^I = 1$. That is, they simply invest according to the average optimal portfolio benchmark. Finally, a mass α of managers are "arbitrageurs" (or elastic funds) who choose their portfolio weights optimally as we describe below. Combining the managers' positions, we obtain the market clearing condition

$$\alpha\omega_t^A + \eta\left(1 + \frac{\mu_t}{\eta}\right) + (1-\eta-\alpha) = 1 \Longrightarrow \omega_t^A = 1 - \frac{\mu_t}{\alpha}. \tag{13}$$

In equilibrium, arbitrageurs must adjust their portfolio weight ω_t^A to absorb the aggregate noise.

The arbitrageurs choose their portfolio weight to maximize expected log assets-under-management, after observing the risk-free rate and the current noise μ_t:[11]

$$\max_{\omega_t^A} E_t\left[\log\left(\alpha W_t\left(R_t^f + \omega_t\left(R_{t+1} - R_t^f\right)\right)\right)\right].$$

In equilibrium, this implies a standard optimality condition $E_t\left[M_{t+1}\left(R_{t+1} - R_t^f\right)\right] = 0$, where $M_{t+1} = \frac{1}{R_t^f + \omega_t^A\left(R_{t+1} - R_t^f\right)}$. Assuming the market and the portfolio returns are log-normally distributed, we obtain the approximate optimality condition:

$$\omega_t^A \sigma_{t,r_{t+1}} = \frac{E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f}{\sigma_{t,r_{t+1}}}. \tag{14}$$

The arbitrageurs' demand for risk is equal to their (perceived) equilibrium Sharpe ratio. Combining this with (13), we derive the *financial market equilibrium condition:*

$$E_t\left[r_{t+1}\right] = r_t^f + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2 - \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}\mu_t. \tag{15}$$

Substituting (11) into this condition, we further obtain a present discounted value relation that

household invests with a random sample of managers. This ensures there is no portfolio and wealth heterogeneity across households, even though there is heterogeneity across managers.

[11]The equilibrium price fully reveals the noise, so this assumption is without loss of generality.

describes the equilibrium aggregate asset price:

$$p_t = \rho + E_t\left[p_{t+1}\right] + (1-\beta)\, E_t\left[\delta_{t+1}\right] - \left(r_t^f + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2\right) + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}\mu_t. \tag{16}$$

Eqs. (16) shows that, all else equal, the effect of noise on the aggregate asset price *increases with return variance.* As market volatility rises, arbitrageurs become more reluctant to counteract the noise traders' flows and require a greater change in the price (and expected return) to absorb these flows. Moreover, the impact of noise is amplified when the mass of arbitrageurs α is smaller, leading to more inelastic aggregate asset demand.

Remark 2 (Interpreting noise shocks)**.** *In practice, noisy flows emerge from various sources. Retail investors may engage in sentiment-driven trading based on excessive optimism or pessimism about assets. Institutional investors often face client inflows and outflows that require them to expand or reduce positions regardless of market conditions. Portfolio rebalancing by both retail and institutional investors can generate mechanical trading needs unrelated to asset fundamentals. While we remain agnostic about the specific sources of noisy flows, the key feature of our model is that these flows are relatively insensitive to return variance compared to arbitrageurs' portfolios (in the model, we make the stark assumption that noisy flows are completely insensitive to variance, though this is stronger than necessary for our results). This assumption is supported by extensive research on limits to arbitrage showing that return variance and risk management are primary constraints on arbitrage activities but play a smaller role in noise trader behavior (see, e.g., Shleifer and Vishny (1997)).*

3.2. Equilibrium with discretionary monetary policy

We next introduce standard *discretionary* monetary policy and characterize the resulting equilibrium. The central bank sets the nominal interest rate i_t^f. Since nominal prices are sticky (until Section 4.6.2), this is the same as the real interest rate, $r_t^f = i_t^f$, so we assume the central bank sets r_t^f. The central bank focuses on closing the output gap $\tilde{y}_t \equiv y_t - y_t^*$.

3.2.1. Benchmark without policy reaction lags

As a benchmark, we start with the (unrealistic) first-best scenario in which the central bank *can* condition its interest rate choice on the realized noise μ_t. In this case, the central bank can set $\tilde{y}_t = 0$ in all periods and states. Using (9) , (11) and (16), along with the shock processes in (8) and (10), the equilibrium is given by

$$\begin{aligned}
p_t &= p_t^* \equiv y_t^* - m - \delta_t, \\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1 - \beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\mu_t, \\
r_{t+1} &= \rho + \delta_t - \beta\delta_{t+1} + \varepsilon_{z,t+1}, \\
\text{where } \sigma^2 &\equiv var_t\left(r_{t+1}\right) = \sigma_z^2 + \beta^2\sigma_\delta^2.
\end{aligned} \tag{17}$$

The first line defines p_t^* (pstar), which is the aggregate asset price that ensures output is equal to its potential. The second line describes the interest rate the central bank sets to achieve pstar (rstar). Note that noise affects the interest rate but it does not affect the aggregate asset price or output. The central bank fully adjusts the interest rate in response to noise to prevent noise-driven fluctuations in output. The third line describes the return conditional on $p_t = p_t^*$ (and $y_t = y_t^*$) at all times. The last line shows that the conditional return variance depends on supply and demand variance (macro-induced variance) but it *does not* depend on noise variance.

3.2.2. Equilibrium with policy reaction lags

Set against this benchmark, our key assumption is that the central bank chooses r_t^f *before* observing the current-period noise μ_t. Therefore, the central bank cannot condition its decision on the current noise shock $\varepsilon_{\mu,t}$. In practice, financial markets are noisy even over short horizons and central banks adjust their policy with some lags (both the inter-meeting lags as well as the reaction lags). This creates a large scope for noise to affect asset prices beyond the central banks' intentions.

For simplicity only, we assume that the central bank still conditions its decision on the macroeconomic shocks $\varepsilon_{\delta,t}, \varepsilon_{z,t}$. We show in Section 4.6.1 that our main results extend to an environment in which the central bank sets r_t^f before observing all current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$.

Formally, we assume the central bank sets the risk-free interest rate (without commitment) to solve:

$$G_t = \min_{r_t^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \tilde{y}_{t+h}^2 \right]. \tag{18}$$

The central bank minimizes the expected discounted sum of quadratic log output gaps (henceforth, output-gap loss) under its information set. We use the notation $\underline{E}_t[\cdot]$ to denote expectations in period t *before* the realization of the noise shock $\varepsilon_{\mu,t}$ (but after the realization of macroeconomic shocks $\varepsilon_{\delta,t}, \varepsilon_{z,t}$).

The first-best equilibrium described in (17) is no longer feasible. Our main result in this section characterizes the (second-best) equilibrium with discretionary policy.

Proposition 1 (Equilibrium with Discretionary Policy)**.** *Suppose the planner sets policy according to* (18) *and the parameters satisfy* $\alpha^2 \geq 4\sigma_\mu^2\left(\sigma_z^2 + \beta^2\sigma_\delta^2\right)$. *Then, there is a (locally stable)* *equilibrium in which the asset price, output, and the interest rate are given by*

$$p_t = p_t^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \qquad \textit{where } p_t^* \equiv y_t^* - m - \delta_t, \tag{19}$$

$$y_t = y_t^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \tag{20}$$

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}. \tag{21}$$

The return is given by

$$r_{t+1} = \rho + \delta_t + \varepsilon_{z,t+1} - \beta\delta_{t+1} + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right), \tag{22}$$

and its variance $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smallest positive solution to the following fixed point problem:*

$$\sigma^2 = \sigma_{macro}^2 + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, \qquad \textit{where } \sigma_{macro}^2 = \sigma_z^2 + \beta^2\sigma_\delta^2. \tag{23}$$

Greater noise variance σ_μ^2 *increases the total return variance* σ^2*, and the output-gap loss* $G_t = \frac{\left(\frac{\sigma^2}{\alpha}\right)^2 \sigma_\mu^2}{1-\beta}$.

We relegate the proof of this Proposition to Appendix A.2 and discuss here the intuition for the equilibrium. Eq. (19) shows that, unlike in the benchmark case, the surprise component of noise $\varepsilon_{\mu,t}$ affects the aggregate asset price (cf. (17)). Eq. (20) shows that the noise-driven fluctuations in the aggregate asset price affect output through the output-asset price relation. Eq. (21) shows that the central bank adjusts the interest rate to insulate output from the predictable component of noise $E_{t-1}\left[\mu_t\right] = \varphi_\mu \mu_{t-1}$.

Eqs. $(22-23)$ characterize the equilibrium return and its variance. Note that the total return variance is greater than macro-induced variance because noise shocks are not fully stabilized by monetary policy. Crucially, the noise variance is *endogenous* and increasing with total return variance (see (16)): a greater variance allows noise shocks to have a greater impact, which then leads to even greater variance, and so on. Eq. (23) formalizes these feedbacks and shows that the equilibrium variance corresponds to the solution to a fixed point problem. This problem is a quadratic that has two positive solutions (under appropriate parametric restrictions). We focus on the smaller solution, as this solution is locally stable, whereas the larger solution is locally unstable.[12]

The last part of the result shows that greater noise variance raises the total variance. Moreover, this channel is amplified by the above reinforcement feedbacks. Importantly, by increasing

[12]The larger solution is locally unstable in the sense that a small increase (resp. decrease) in volatility would further increase (resp. descrease) the price impact, which would further increase (resp. decrease) the variance, and so on. In contrast, the smaller solution is robust to small fluctuations in volatility.

asset price volatility, greater noise variance also increases the output-gap loss. That is, financial noise *worsens the macroeconomic performance of monetary policy.*

Quantifying the impact of noise. How large is the potential impact of noise on the return and output variance? For a simple calibration, observe that the price impact of a unit change in asset demand (as a fraction of supply) is given by

$$\mathcal{I} \equiv \frac{dp_t}{d\varepsilon_{\mu,t}} = \frac{\sigma^2}{\alpha}.$$

Recent empirical analyses find that this price impact is large. For instance, Gabaix and Koijen (2021) suggest that for the stock market it could be as large as 5. To be conservative, suppose we set the fraction of elastic funds α to target a price impact equal to one, $\mathcal{I} = \frac{\sigma^2}{\alpha} = 1$. Combining this with (23), we obtain

$$\alpha = \sigma_{macro}^2 + \sigma_\mu^2 = \sigma^2. \tag{24}$$

With the appropriate choice of α, there is a "candidate" solution in which the price impact is equal to one and the total variance is the sum of the macro-induced variance and noise variance. We verify that this corresponds to an actual solution as long as the noise variance is not too large, $\sigma_\mu^2 \le \sigma_{macro}^2$.[13] In this calibration, the variance of noise affects the return variance additively.

This potentially large effect of noise on market volatility is consistent with the finance literature emphasizing asset price fluctuations driven by noise and limits to arbitrage (see De Long et al. (1990)). Proposition 1 demonstrates that this type of noise destabilizes not only financial markets but also the broader macroeconomy when it affects the aggregate asset price. These findings highlight the need for an alternative policy framework in which the central bank aims to mitigate the impact of such noise.

4. FCI targeting

In this section, we show that a framework where the central bank sets a (soft) FCI target for the upcoming period and strives to maintain the FCI near this target, in addition to focusing on its conventional objectives, enhances the central bank's ability to achieve its standard macroeconomic goals. Compared to the standard discretionary policy, this approach results in greater FCI stability and recruits the arbitrageurs to absorb aggregate noise, thereby lessening its impact on economic activity. We also illustrate that FCI targeting dominates committing to future interest rates; in essence, "FCI-based forward guidance" outperforms traditional in-

[13] In the appendix, we show that a candidate corresponds to the stable solution to (23) only if it satisfies $2\sigma_\mu^2\sigma^2 \le \alpha^2$. Together with (24), this implies $\sigma_\mu^2 \le \sigma_{macro}^2$. As long as noise variance is not too large (relative to macro-news variance), the candidate solution with $\mathcal{I} = \frac{\sigma^2}{\alpha} = 1$ corresponds to a locally stable equilibrium. If $\sigma_\mu^2 > \sigma_{macro}^2$, then noise is so large that when its unit-price impact is equal to one, it induces destabilizing dynamics. Specifically, there is no locally stable equilibrium in which $\sigma_\mu^2 > \sigma_{macro}^2$ and $\mathcal{I} = \frac{\sigma^2}{\alpha} = 1$.

terest rate forward guidance. Moreover, despite relying solely on interest rate adjustments as its instrument, FCI targeting may lower interest rate volatility. Finally, our analysis shows FCI targeting does *not* require the central bank to identify in real time whether asset prices are driven by noise or fundamentals (see Remark 3); and in fact, allowing asset prices to fluctuate due to arbitrageurs' information about future cash flows (that is not available to the central bank) further increases the stabilization benefits from FCI targeting (see Section 4.6.3).

4.1. Equilibrium with FCI targeting

Formally, suppose the central bank solves the following *modified* problem:

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(p_{t+h} - \overline{p}_{t+h} \right)^2 \right] \right], \tag{25}$$

where $\overline{p}_{t+h}$ denotes an FCI target announced by the central bank in the previous period, $t+h-1$ (the initial target $\overline{p}_0$ is given). That is, in addition to minimizing the output gaps as usual, the central bank penalizes the deviations of the aggregate asset price from its pre-announced target. The parameter $\psi \geq 0$ captures the strength of the FCI targeting objective relative to the central bank's usual objectives. The standard model is a special case with $\psi = 0$.

While we change the central bank's *operational* objective function, it is important to note that the *true* objective function is unchanged and given by the output-gap loss in (18). That is, merely stabilizing asset prices does not improve welfare or the policy performance. Our goal is to analyze whether adopting an operational FCI targeting framework can improve the true policy performance. Our next result characterizes the equilibrium with $\psi \geq 0$.

Proposition 2 (Equilibrium with FCI Targeting)**.** *Suppose the planner follows the FCI targeting policy in* (25) *with $\psi \geq 0$, the parameters satisfy $\alpha^2 \geq 4\sigma_\mu^2 \left(\sigma_z^2 + \beta^2 \sigma_\delta^2 \right)$ (and $\beta > 1 - \beta$), and the initial target satisfies $\overline{p}_0 = E_{-1}[p_0^*]$. Then, there is a (stable) equilibrium in which the planner announces the expected "pstar" for the next period as its target*

$$\overline{p}_{t+1} = \underline{E}_t \left[p_{t+1}^* \right] \text{ where } p_{t+1}^* = y_{t+1}^* - m - \delta_{t+1}. \tag{26}$$

The equilibrium asset price, output, and interest rate are

$$\begin{aligned}
p_t &= \underline{E}_{t-1}[p_t^*] + \frac{1}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t} \right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, && (27) \\
y_t &= y_t^* + \frac{\psi}{1+\psi} \left(\varepsilon_{\delta,t} - \varepsilon_{z,t} \right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, && (28) \\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta \varphi_\delta \right) \delta_t + \frac{\sigma^2}{\alpha} \varphi_\mu \mu_{t-1} + \frac{\psi}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t} \right). && (29)
\end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t\left[r_{t+1}\right] + \frac{1}{1+\psi}\varepsilon_{z,t+1} - \left(\frac{1}{1+\psi} - (1-\beta)\right)\varepsilon_{\delta,t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \tag{30}$$

where the expected return $E_t\left[r_{t+1}\right]$ *is given by (A.36). The return variance* $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smaller positive solution to the following fixed point problem*

$$\begin{aligned} \sigma^2 &= \sigma^2_{macro}\left(\psi\right) + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_{\mu}, \\ \text{where } \sigma^2_{macro}\left(\psi\right) &= \sigma^2_z\left(\frac{1}{1+\psi}\right)^2 + \sigma^2_{\delta}\left(\frac{1}{1+\psi} - (1-\beta)\right)^2. \end{aligned} \tag{31}$$

Let $\overline{\psi} = \arg\min_{\psi\geq 0}\sigma^2_{macro}\left(\psi\right)$. *Over the range* $\psi \in [0, \overline{\psi})$, *increasing* ψ *strictly reduces* σ^2 *as well as* $\sigma^2_{macro}\left(\psi\right)$ *and* $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_{\mu}$. *That is, stronger FCI targeting reduces the return variance and* ***both*** *of its components.*

We provide the equilibrium's intuition here and relegate the proof to Appendix A.2. Eq. (26) says that the central bank optimally announces its expected "pstar" as its target for the next period. Given this target, the central bank's optimal (interest rate) policy implies

$$\underline{E}_t\left[p_t\right] = \frac{1}{1+\psi}p^*_t + \frac{\psi}{1+\psi}\underline{E}_{t-1}\left[p^*_t\right]. \tag{32}$$

That is, the central bank's expected asset price for the current period (i.e., before observing the noise shock) is a weighted average of the current "pstar" and the last period's expected "pstar", which it had announced as a target. This implies Eq. (27), which says the asset price reflects the *surprises* in "pstar" but only *partially*: A positive supply shock $\varepsilon_{z,t} > 0$ raises the asset price but less than in the case with discretionary policy ($\psi = 0$); a positive demand shock $\varepsilon_{\delta,t} > 0$ decreases the asset price but less than with discretionary policy. This in turn implies Eq. (28), which says that the slow adjustment of asset prices to macroeconomic shocks affects output. A positive supply shock $\varepsilon_{z,t} > 0$ has a smaller effect on output than with discretionary policy, because the policy does not allow asset prices (and demand) to adjust to the higher supply immediately. Conversely, a positive demand shock $\varepsilon_{\delta,t} > 0$ has some effect on output, because the policy does not undo the effect of demand fully. Eq. (29) characterizes the policy interest rate that induces these outcomes. We discuss this policy rate response later in Section 4.4.

Eqs. $(30-31)$ describe the equilibrium return and its conditional variance. The total return variance is still determined as the solution to a fixed point problem. The difference is that the macro-induced variance is now endogenous to the degree of FCI targeting and typically lower than with discretionary policy. In particular, supply shocks always have a smaller impact on the return. Demand shocks also have a smaller impact as long as the FCI targeting is not too strong.[14] Consequently, there is a large range $\left[0, \overline{\psi}\right]$ (where $\overline{\psi} > \frac{\beta}{1-\beta}$) over which FCI targeting reduces the macro-induced variance $\sigma^2_{macro}(\psi)$. Importantly, over the same range, FCI targeting also reduces the total return variance σ^2. In fact, the reduction in total variance is greater than the reduction in macro-induced variance, because a lower variance enables the arbitrageurs to trade against noise more aggressively, reducing the noise-induced variance $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_\mu$.

In summary, FCI targeting (typically) reduces the return volatility and the impact of noise on asset prices and output. Since the policy keeps the asset price close to the announced target, the arbitrageurs become more willing to absorb noise shocks. This reverses the adverse return volatility feedback loop generated by noise shocks.

Remark 3 (Informational requirements to implement FCI targeting)**.** *Proposition 2 shows that implementing the FCI targeting rule requires* only *forecasts of potential output and aggregate demand—variables that central banks already routinely forecast. The central bank does* not *need to identify whether asset price movements are driven by fundamentals or noise in real time, nor does it need to estimate the fundamental value of stocks or other assets. This is because the optimal target $\overline{p}_{t+1} = \underline{E}_t\left[p^*_{t+1}\right]$ depends only on the expected potential output y^*_{t+1} and the expected aggregate demand δ_{t+1}. When the aggregate asset price (FCI) deviates from this target due to noise or other factors, the policy response helps stabilize the economy regardless of the source of the deviation. In Section 4.6.3, we allow arbitrageurs to have additional information about the future potential output that is not available to the central bank when it announces its optimal target. We demonstrate that financial conditions targeting is not only desirable in this extended setting but also provides additional stabilization benefits.*

4.2. Macro-stabilization effects of FCI targeting

Proposition 2 demonstrates that a central bank adopting an FCI targeting policy mitigates market volatility. However, the central bank in our model is not concerned with market volatility *per se*. The question then arises: does FCI targeting aids the central bank in fulfilling its standard macro-stabilization goals? Our main result in this section confirms this: *some FCI targeting always improves macroeconomic stabilization.*

We evaluate the policy performance with the output-gap loss function G_t defined in (18). This function might depend on the current realizations of supply and demand shocks $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. To obtain a welfare measure that averages across different shocks, we consider the expected

[14] The coefficient, $\frac{1}{1+\psi} - (1-\beta)$, implies that when $\psi < \frac{\beta}{1-\beta}$ a positive demand shock decreases the return, although less than with discretionary policy. When $\psi > \frac{\beta}{1-\beta}$, a positive demand shock increases the return since its impact on output (cash flows) dominates its dampened effect on the aggregate asset price.

output-gap loss given by:[15]

$$G^e(\psi) = E[G_t(\psi)] = E\left[\sum_{h=0}^{\infty} \beta^h \left(y_{t+h}(\psi) - y^*_{t+h}\right)^2\right]. \tag{33}$$

Here, $E[\cdot] = E[\underline{E}_t[\cdot]]$ denotes the unconditional distribution over all shocks. To evaluate this expectation, observe that Eq. (28) implies the output gap is given by:

$$\tilde{y}_t = (\varepsilon_{\delta,t} - \varepsilon_{z,t}) \frac{\psi}{1+\psi} + \varepsilon_{\mu,t} \frac{\sigma^2}{\alpha}. \tag{34}$$

Using this expression, we calculate and decompose the expected output gap-loss as follows

$$\begin{aligned} G^e(\psi) &= G^e_{macro}(\psi) + G^e_{noise}(\psi), \qquad (35) \\ \text{where } G^e_{macro}(\psi) &= \frac{\left(\sigma_z^2 + \sigma_\delta^2\right)\left(\frac{\psi}{1+\psi}\right)^2}{1-\beta} \quad \text{and} \quad G^e_{noise}(\psi) = \frac{\sigma_\mu^2 \left(\frac{\sigma^2}{\alpha}\right)^2}{1-\beta}. \end{aligned}$$

$G^e_{macro}(\psi)$ and $G^e_{noise}(\psi)$ are the contributions of macroeconomic shocks and noise shocks to the expected output-gap loss, respectively. Our next result describes how FCI targeting affects $G^e(\psi)$ and its components.

Proposition 3 (Macrostabilization Effects of FCI Targeting)**.** *Consider the equilibrium in Proposition 2. Then, a small degree of FCI targeting reduces the expected output-gap loss*

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} < 0, \text{ with } \frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0 \text{ and } \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} < 0.$$

Therefore, $\psi^* = \arg\min_{\psi \geq 0} G^e(\psi) > 0$*; i.e., the output-gap loss minimizing policy features FCI targeting.*

For intuition, observe from Eqs. $(34-35)$ that FCI targeting has competing effects on output gaps. On the one hand, the policy creates new sources of output gaps as it does not fully allow output to adjust to supply surprises and it allows demand surprises to influence output (the terms $(\varepsilon_{\delta,t} - \varepsilon_{z,t}) \frac{\psi}{1+\psi}$). On the other hand, the policy reduces return variance σ^2, and this mitigates the asset price and output impact of noise surprises (the term $\varepsilon_{\mu,t}\frac{\sigma^2}{\alpha}$). However, the noise-reduction force always dominates for sufficiently low levels of ψ because $\frac{dG_{macro}(\psi)}{d\psi}|_{\psi=0} = 0$: starting from the baseline discretionary policy, allowing macroeconomic surprises to affect the output gap induces only a second-order increase in the output-gap loss. In contrast, Proposition 2 implies that $\frac{d\sigma^2}{d\psi}|_{\psi=0} < 0$ and thus $\frac{dG_{noise}(\psi)}{d\psi}|_{\psi=0} < 0$: increasing ψ

[15]Our main result qualitatively also holds with the current gap G_t; that is, some degree of FCI targeting improves welfare for *any* given realization of shocks $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. However, the magnitude of welfare gains and the optimal degree of FCI targeting depends on $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. The expected value $G^e(\psi)$ ensures that we evalute the welfare gains and the optimal FCI targeting by averaging across a variety of shocks.

induces a first-order reduction on return variance caused by noise $\sigma_\mu^2\left(\frac{\sigma^2}{\alpha}\right)^2$, and this "excess" variance affects output gaps and asset prices. Therefore, adopting an FCI targeting policy with positive ψ reduces the output-gap loss.

Although the central bank cannot directly counteract the price fluctuations caused by market noise, FCI targeting allows it to indirectly alleviate these effects by recruiting rational investors (the market) to absorb more of the noise. Consequently, FCI targeting reduces the impact of noise on the output gap.

4.3. Numerical illustration of FCI targeting

For a numerical illustration of Propositions 2 and 3, consider the calibration we introduced in Section 3.2.2 (see (24))

$$\begin{aligned} \sigma_\mu^2 &= \sigma_{macro}^2(0), \\ \sigma_\delta^2 &= \sigma_z^2 = \frac{\sigma_{macro}^2(0)}{1+\beta^2}, \\ \alpha &= \sigma_\mu^2 + \sigma_{macro}^2(0) = \sigma^2, \\ \text{with } \sigma_{macro}^2(0) &= (0.01)^2 \text{ and } \beta = 0.99. \end{aligned} \tag{36}$$

We equalize the noise variance to the macro-induced variance, and set the variances of demand and supply to be identical. We set α to target a price impact coefficient of one, $\mathcal{I} = \sigma^2/\alpha = 1$. We consider a quarterly calibration and set the discount rate to 1%. Finally, we set the macro-induced standard deviation to 1% to match (roughly) the standard deviation of quarterly output growth in the data.[16]

The left panel of Figure 6 illustrates the impact of FCI targeting on return variance and its components (see (31)). Stronger FCI targeting reduces the return variance as well as both of its components. The reduction is substantial: at the optimum level of targeting, $\psi = \psi^*$ (illustrated by the vertical line), the total variance decreases by approximately two-thirds. Notably, the variance due to noise diminishes by more than ninety percent. In essence, optimal FCI targeting nearly eradicates the noise-induced variance, which significantly lowers the total return variance.

The right panel of Figure 6 shows how FCI targeting affects the output-gap loss and its components (see (35)). Starting from the discretionary policy, FCI targeting substantially reduces the noise-component and total output-gap loss, while having a second-order effect on the macro-induced output-gap loss. As FCI targeting intensity rises, it continues to reduce noise-induced losses but begins to increase macro-induced losses more rapidly. The optimal level of FCI targeting, $\psi^* \simeq 0.4$, corresponds to a central bank targeting an asset price that roughly assigns a one-third weight on its pre-announced target and two-thirds to the current "pstar" (see

[16]In this calibration, the level of $\sigma_{macro}^2(0)$ does not change the optimal level of FCI targeting since all other variances scale with $\sigma_{macro}^2(0)$.

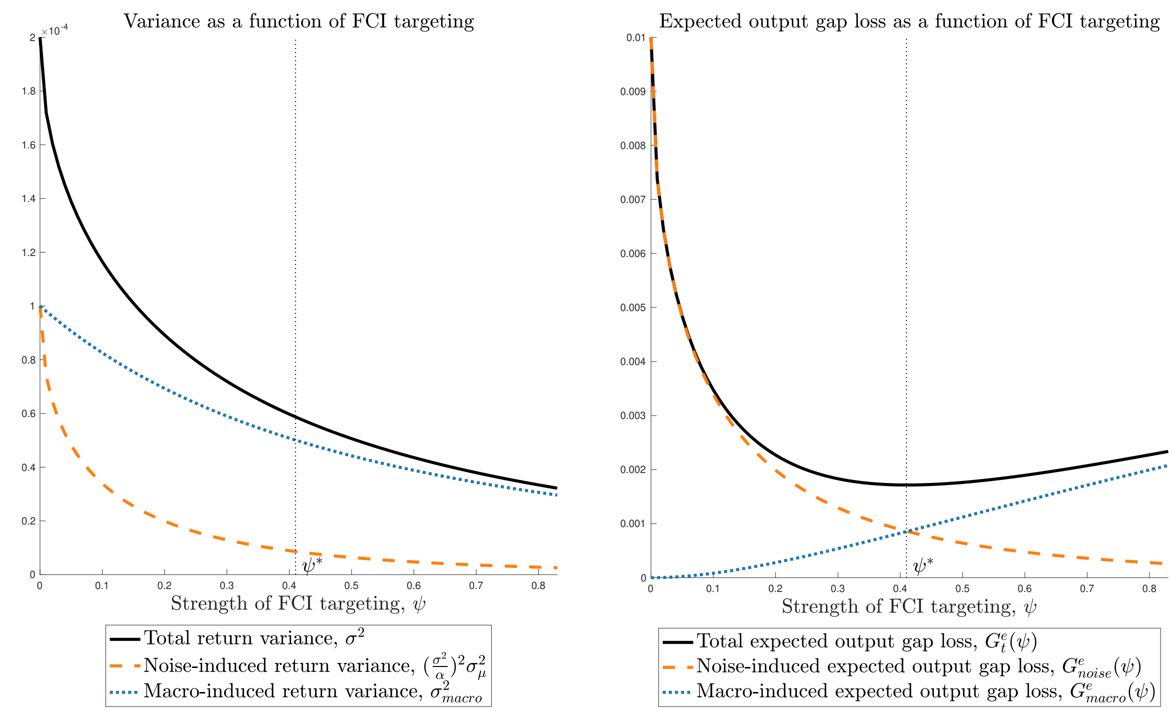

Figure 6: The left panel shows the effect of FCI targeting ψ on return variance (solid black line) and its components induced by noise shocks (dashed orange line) and by macroeconomic shocks (dotted blue line). The right panel shows the effect on the expected output gap loss (solid black line) and its components induced by noise shocks (dashed orange line) and by macroeconomic shocks (dotted blue line). The vertical lines illustrate the gap-minimizing level of FCI targeting. We use the parameters in (36).

(32)). Although this represents a relatively mild form of FCI targeting, it effectively eliminates nearly all noise-driven loss, as depicted in the left panel.

4.4. FCI targeting and interest rate volatility

One concern with FCI targeting is that it might require large movements in the policy interest rate to keep financial conditions close to the target. However, our model reveals that FCI targeting has competing effects on interest rate volatility and can, in fact, reduce it—even though reducing rate volatility is not an explicit policy goal.

In order to analyze the effects on interest rate volatility, we write Eq. (29) as

$$r_t^f = \underline{E}_{t-1}\left[r_t^f\right] + \frac{\psi}{1+\psi}\varepsilon_{z,t} + \left[1 - \beta\varphi_\delta - \frac{\psi}{1+\psi}\right]\varepsilon_{\delta,t} + \frac{\sigma^2}{\alpha}\varphi_\mu\varepsilon_{\mu,t-1}, \tag{37}$$

where $\underline{E}_{t-1}\left[r_t^f\right]$ is the expected interest rate in the previous period $t-1$. The remaining terms reflect the interest rate surprises induced by supply shocks, demand shocks, and financial noise shocks, respectively. FCI targeting increases the policy rate's responsiveness to supply shocks. It may also increase the sensitivity to demand shocks (although in the opposite direction)—this happens when φ_δ is high and ψ is not too low. In scenarios of persistent demand shocks, asset prices react in anticipation of future policy rate changes in response to the demand shift. Consequently, the central bank might need to adjust the current policy rate in the opposite direction to counteract these asset price movements. Conversely, FCI targeting diminishes the policy rate's sensitivity to financial noise shocks by reducing the return variance σ^2 and the noise's impact on asset prices. The overall effect hinges on the balance between this decreased sensitivity to financial noise and the generally increased sensitivity to macroeconomic shocks.

For a quantitative exploration, consider the parameters in (36) along with

$$\varphi_\delta = \varphi_\mu = 0.95. \tag{38}$$

We set the persistence of demand and noise shocks to match (roughly) the quarterly autocorrelation of the policy interest rate observed in the data. Figure 7 depicts the impact of FCI targeting on the conditional interest rate variance, and its macro-induced and noise-induced components. Stronger FCI targeting increases the macro-induced rate variance but significantly reduces the noise-induced rate variance. The reduction in the noise-induced variance is notably more substantial. As a result, FCI targeting overall *lowers* the total interest rate variance.

Why is the effect of FCI targeting on noise-induced interest rate variance more pronounced? In this calibration, under a discretionary policy ($\psi = 0$), financial noise shocks are the primary contributors to interest rate volatility, despite macroeconomic shocks and noise shocks contributing equally to the return variance (cf. the left panel of Figure 6). Thus, as FCI targeting diminishes the price impact of noise, it also lessens the need for substantial rate adjustments.

The broader point is that, as arbitrageurs absorb the majority of the noise, the burden on

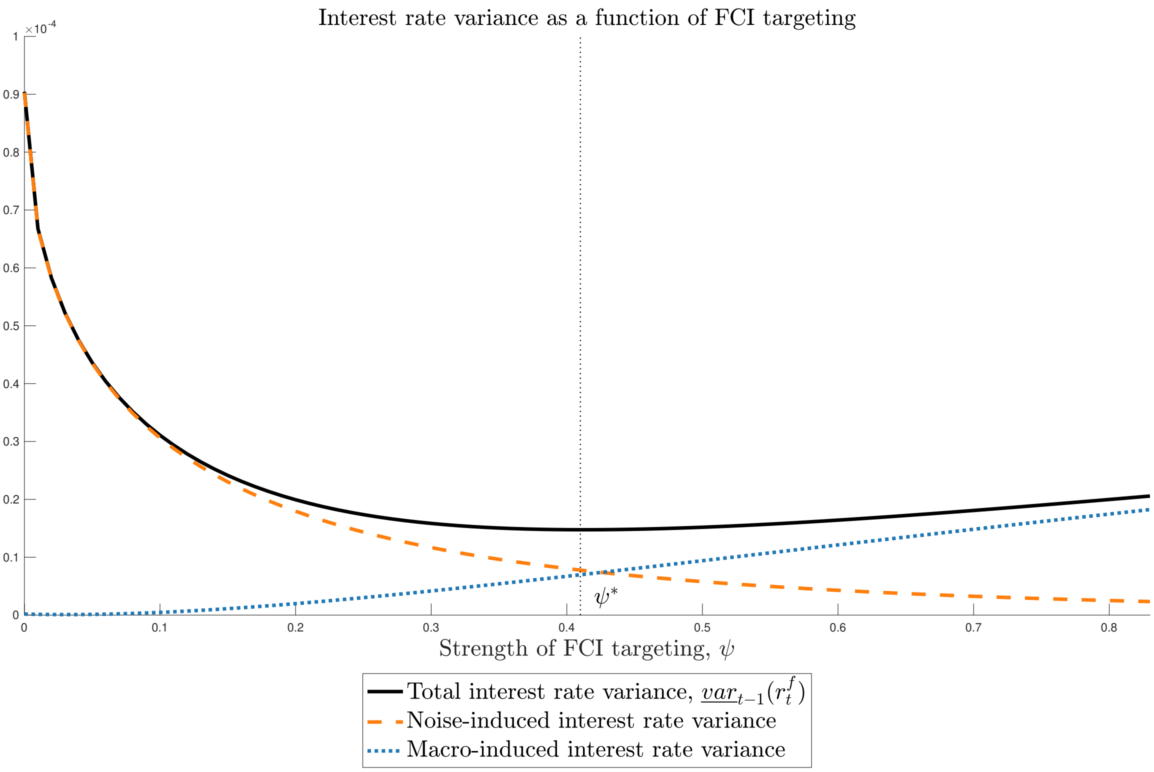

Figure 7: This figure shows the effect of FCI targeting ψ on the conditional interest rate variance (solid black line) and its components driven by noise shocks (dashed orange line) and by macroeconomic shocks (doted blue line). The line illustrates the gap-minimizing level of FCI targeting. We use the parameters in (36) and (38).

the central bank to react to noise shocks is reduced. This can result in greater stability of the policy interest rate, especially if noise shocks are the primary driver of interest rate volatility.

4.5. FCI targeting vs interest rate forward guidance

In our model, FCI targeting functions similarly to issuing *forward guidance* about the future trajectory of the FCI, assuming that this type of guidance implies a soft degree of commitment. This similarity raises the question of whether (more conventional) *interest rate forward guidance*, interpreted as a soft commitment to a future interest rate, could yield similar advantages. We explore this question in Appendix A.3, where we analyze a policy framework in which the central bank targets the future interest rate rather than the future FCI. Specifically, suppose the central bank solves the following modified problem:

$$G_t^{r^f\text{-target}} = \min_{r_t^f, \overline{r}_{t+1}^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(r_{t+h}^f - \overline{r}_{t+h}^f \right)^2 \right] \right]. \tag{39}$$

In each period, the central bank sets the policy rate r_t^f and announces a target interest rate for the subsequent period $\overline{r}_{t+1}^f$. This problem leads to a similar equilibrium as in Proposition 2, with the key distinction that the central bank does not fully respond to *recent noise shocks* (in addition to the current noise shock). Consequently, this strategy leads to a less effective policy performance compared to a similar FCI targeting policy.

The solution is particularly tractable for the special case in which there are no supply shocks, $\varepsilon_{z,t} = 0$, and demand shocks are transitory, $\varphi_\delta = 0$. However, the insights apply more generally. For this special case, Proposition 5 in the appendix shows that the equilibrium interest rate is

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\varphi_\mu^2 \mu_{t-1} + \frac{1}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} \right).$$

Compared to FCI targeting, the central bank underreacts to not only demand shocks but also to the persistent component of recent noise shocks $\varphi_\mu \varepsilon_{\mu,t-1}$ (cf. (28)). As a result, this predictable noise also generates volatility in asset prices and output. Moreover, current noise shocks have a greater effect on asset prices and output because financial markets anticipate that the future interest rate will underreact to these shocks. Specifically, the equilibrium asset price is

$$p_t = y_t^* - m - \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi} \varphi_\mu \right) \varepsilon_{\mu,t} \right),$$

and output is described by a similar expression, detailed in the appendix.

Comparing these expressions with those in Proposition 2, it becomes evident that interest rate targeting results in larger output gaps and achieves a smaller reduction in return volatility than FCI targeting. In fact, interest rate targeting might even *increase* return volatility by amplifying the price impact of noise shocks for a given σ^2. Therefore, the interest rate targeting policy is strictly dominated by a comparable FCI targeting policy in this environment.

Intuitively, interest rate targeting reduces the flexibility of the central bank to control *the aggregate asset price (FCI)*. Given that output is driven by the aggregate asset price rather than the policy interest rate, this loss of control results in a larger output-gap loss.

4.6. Robustness of FCI targeting

Our baseline model is stylized. In this section, we demonstrate that the logic behind the benefits of FCI targeting survives in richer economic environments. These extensions also serve as a transition to the empirical counterfactual analysis presented in the next section.

4.6.1. FCI targeting with policy lags to all current shocks

In Appendix A.4, we explore the implications of FCI targeting when the central bank sets policy before observing *all* current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$ (as opposed to only $\varepsilon_{\mu,t}$). The analysis mirrors that of our baseline model but with the difference that the policy reacts to macroeconomic shocks with a delay. Nevertheless, since markets are forward-looking and anticipate *future* policy responses to shocks, FCI targeting still reduces return volatility and improves macroeconomic stability as in the baseline model.

The analysis is particularly tractable when there are no supply shocks, $\varepsilon_{z,t} = 0$, although the insights apply more generally. For this special case, Proposition 6 in the appendix demonstrates

$$p_t = y_t^* - m - \varphi_\delta^2 \delta_{t-2} - \frac{1}{1+\psi} \varphi_\delta \varepsilon_{\delta,t-1} - \left(\frac{1}{1+\psi} - (1-\beta) \right) \varphi_\delta \varepsilon_{\delta,t} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}.$$

Recall that FCI targeting in the baseline model operates via reducing data-dependency with respect to macroeconomic shocks. The same logic applies here. Under a discretionary policy ($\psi = 0$), a positive demand shock $\varepsilon_{\delta,t} > 0$ still lowers the asset price because $\left(\frac{1}{1+\psi} - (1-\beta) \right) \varphi_\delta = \beta \varphi_\delta > 0$, although its effect is less pronounced compared to the baseline model (cf. (27)). While the policy does not immediately react to the demand shock, it will respond in the subsequent period, and financial markets price this anticipated response. With FCI targeting ($\psi > 0$), the future policy response is dampened, and this reduces the price impact of demand shocks. This reduces the equilibrium volatility σ^2 as in the baseline model.

4.6.2. FCI targeting with inflation and output trade-off

In Appendix A.5, we investigate the effects of FCI targeting when prices are partially flexible and the central bank faces a trade-off between stabilizing inflation and output. We find that our main results continue to hold in this more realistic setting.

We endogenize inflation via the standard New Keynesian Phillips Curve (NKPC)

$$\pi_t = \kappa \tilde{y}_t + \beta E_t \left[\pi_{t+1} \right] + u_t, \tag{40}$$

where π_t denotes (the log of) nominal price inflation and u_t denotes cost-push shocks that follow:

$$u_t = \varphi_u u_{t-1} + \varepsilon_{u,t}.$$

We also adjust the central bank's *true* objective function to incorporate the costs of inflation gaps. In particular, with discretion, the central bank targets the real interest rate r_t^f to solve:[17]

$$\min_{r_t^f} G_t = \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left(\tilde{y}_{t+h}^2 + \zeta \pi_{t+h}^2 \right) \right], \tag{41}$$

where ζ denotes the relative welfare weight for the inflation gaps (we normalize the inflation target to zero). The rest of the model is the same as in Sections 3 and 4.

In the appendix, we show that the equilibrium with discretion satisfies (see $(A.60)$)

$$\begin{aligned}
p_t &= p_t^o + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \quad \text{where } p_t^o = y_t^* - m - \delta_t - Y_u u_t, \\
\pi_t &= \Pi_u u_t + \kappa \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
y_t &= y_t^* - Y_u u_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.
\end{aligned}$$

The parameters $\Pi_u, Y_u > 0$ are derived coefficients [see $(A.59)$] and p_t^o is the central bank's optimal asset price target absent noise. In equilibrium, cost-push shocks result in positive inflation gaps and negative output gaps, and they create a new source of asset price volatility. Importantly, noise shocks remain an important driver of output and (now) inflation gaps.

We then consider an FCI targeting framework in which the central bank minimizes

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} E_{t-1} \left[\sum_{h=0}^{\infty} \beta^h \left[\tilde{y}_{t+h}^2 + \zeta \pi_{t+h}^2 + \psi \left(1 + \kappa^2 \zeta\right) \left(p_{t+h} - \overline{p}_{t+h}\right)^2 \right] \right],$$

where $\left(1 + \kappa^2 \zeta\right)$ is a normalizing term. Proposition 7 in the appendix shows that in equilibrium:

$$\begin{aligned}
p_t &= \underline{E}_{t-1}\left[p_t^o\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
y_t &= y_t^* - Y_u u_t + \frac{\psi}{1+\psi} Y_u \varepsilon_{u,t} + \frac{\psi}{1+\psi}\left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
\pi_t &= \Pi_u u_t + \frac{\psi}{1+\psi}\kappa Y_u \varepsilon_{u,t} + \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}.
\end{aligned}$$

FCI targeting mitigates the aggregate asset price reaction to cost-push shocks $\varepsilon_{u,t}$ as well as to supply and demand shocks. Therefore, FCI targeting still reduces the return volatility σ^2 and

[17] We assume the central bank sets the nominal interest rate i_t^f and show that (under appropriate assumptions) the central bank can still target the real interest rate r_t^f. Along the equilibrium path, the central bank implements a particular r_t^f by setting i_t^f after accounting for expected inflation and the inflation risk premium.

the impact of noise shocks $\varepsilon_{\mu,t}$.[18]

Finally, Proposition 8 in the appendix shows that, as in the baseline model, some degree of FCI targeting always improves the central bank's *true* objective function in (41). Intuitively, while cost-push shocks induce nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by these shocks. Therefore, small deviations from this policy generate only second-order losses, while still inducing first-order gains via the same noise-reduction mechanism as in our baseline model (see Proposition 3).

4.6.3. FCI targeting with time-varying arbitrageur beliefs

In Appendix A.6, we investigate the effects of FCI targeting when asset prices fluctuate not only because of noisy flows by a group of agents, but also due to changes in arbitrageurs' beliefs. We find that our main results continue to hold in this more realistic setting. Moreover, in our model, beliefs-driven asset price fluctuations create an additional mechanism by which FCI targeting stabilizes the output gap.

We capture belief shocks by allowing arbitrageurs to receive a signal about future productivity, $n_{z,t} = \varepsilon_{z,t+1} + e_{zt}$. Their posterior expectation for future productivity is then given by $E_t\left[y^*_{t+1}\right] = y^*_t + b_t$. Here, b_t denotes *the belief shock*, which is proprtional to $n_{z,t}$ and has an ex-ante distribution $b_t \sim N\left(0, \sigma_b^2\right)$. When the news is good and $b_t > 0$, arbitrageurs expect productivity and cash-flows to grow faster than usual, and vice versa when the news is bad and $b_t < 0$. We remain agnostic about whether these beliefs are correct or biased—this distinction does not affect the equilibrium. We assume the central bank shares the same belief as arbitrageus and it sets policy before observing current-period belief shock b_t as well as the current-period noisy flow shock $\varepsilon_{\mu,t}$.

In the appendix, we show that the equilibrium with discretion satisfies (see Eqs. $(A.87 - A.89)$)

$$\begin{aligned}
p_t &= \underline{E}_{t-1}\left[p^*_t\right] + \varepsilon_{z,t} - \varepsilon_{\delta,t} + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
y_t &= y^*_t + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
r_{t+1} &= E^A_t\left[r_{t+1}\right] + \left(\varepsilon_{z,t+1} - b_t\right) - \beta\varepsilon_{\delta,t+1} + b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}.
\end{aligned}$$

Compared to the baseline model, the main difference is that the asset price and output are also influenced by the arbitrageurs' belief shocks b_t. When arbitrageurs become more optimistic about the future supply, the aggregate asset price increases and this increases the *current* output—because the policy cannot immediately react to the shock. Therefore, belief shocks create another source of asset price and output gap fluctuations. The impact of the belief shock

[18]Note also that FCI targeting implies that cost-push shocks have a larger impact on inflation gaps and a smaller effect on output gaps.

does not depend on its variance, because, unlike noisy flow shocks, these shocks do not induce trade. However, these belief-driven fluctuations influence the return process, increasing the return variance σ^2 both directly and indirectly by amplifying the price impact of noisy flows.

We then show that FCI targeting changes the equilibrium as follows (see Proposition 9)

$$\begin{aligned} p_t &= \underline{E}_{t-1}\left[p_t^*\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\ y_t &= y_t^* + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \end{aligned} \tag{42}$$

where the variance σ^2 is obtained by solving a fixed-point problem and is decreasing in ψ. As before, FCI targeting mitigates the effect of supply and demand shocks on the asset price (and raises their impact on output). Importantly, FCI targeting also *mitigates* the effect of belief shocks on both asset prices and output. Under FCI targeting, the central bank commits to limit the immediate response of *future* asset price to *future*, $p_{t+1} = \underline{E}_t\left[p_t^*\right] + \frac{1}{1+\psi}\varepsilon_{z,t+1} + \ldots$. The anticipation of this effect reduces the impact of belief shocks. It follows that FCI targeting reduces the macro-induced variance by reducing the asset price impact of belief shocks as well as supply and demand shocks. This, in turn, reduces the noise-induced variance as in the main model.

Eq. (42) shows that in this setup FCI targeting ($\psi > 0$) is beneficial for two distinct reasons. As before, FCI targeting reduces variance σ^2 and mitigates the output gaps induced by noise shocks. In addition, FCI targeting also mitigates the output gaps induced by belief shocks. FCI targeting is still costly because it allows supply and demand shocks to induce some output gaps, but as before these costs are second order. It follows that FCI targeting is robust to allowing for belief-driven asset price fluctuations. In fact, our analysis reveals that belief-driven fluctuations create a new channel by which FCI targeting might be beneficial.

5. Policy Counterfactuals for the U.S.

In this section, we conduct an empirical evaluation of a counterfactual scenario in which U.S. monetary policy had adopted an FCI-targeting strategy over the past few decades. Our findings reveal that FCI targeting would have delivered substantial improvements over historical outcomes, particularly by stabilizing the output gap, inflation (less so), and financial market volatility. Furthermore, FCI targeting outperforms both a dual mandate-based optimal rule and an interest rate-based forward guidance strategy, achieving lower volatility across macroeconomic and financial indicators. We also demonstrate the specific benefits of FCI targeting from the end of the Dot-com to the pre-GFC economic cycle, which is characterized by large noise shocks. Finally, we show how to approximate the optimal policy with simpler Taylor-style rules and simple FCI-targets. We show that an important part of the stabilization gains are obtained under this approximate, simple version of FCI targeting.

5.1. Methodology

We adapt the methodology described in McKay and Wolf (2023b) and Caravello et al. (2024) to our problem. These papers combine estimated VARs with estimated impulse responses to *policy shocks* to approximate counterfactual *policy rules.* This approach generates accurate counterfactuals as long as the model is linear and monetary policy operates through current or expected policy interest rates—that is, if the source of changes in the interest rate path, whether from shocks or policy rules, is inconsequential. However, directly applying this methodology in our context presents a key challenge: our mechanism operates by reducing risk, and this risk reduction introduces a non-linear component. In the following discussion, we explain the necessary extensions to address this non-linearity.

Set-up and objects of interest. Our baseline set up is similar to Caravello et al. (2024). In particular, we observe data from a data generating process (DGP) of the form:

$$Y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} = \sum_{\ell=0}^{\infty} \Theta_{\mu,\ell} \varepsilon_{\mu,t-\ell} + \sum_{\ell=0}^{\infty} \Theta_{-\mu,\ell} \varepsilon_{-\mu,t-\ell}. \tag{43}$$

That is, a linear SVMA(∞), where Y_t is again a vector of macroeconomic aggregates, the shock vector ε_t is distributed as

$$\varepsilon_t \sim N(0, I), \tag{44}$$

and the $n_y \times n_\varepsilon$-dimensional matrices Θ_ℓ denote the impulse response of the vector of observables Y_t at horizon ℓ to a date-t vector of shocks ε_t. We partition the shock vector as $\varepsilon_t = (\varepsilon_{\mu,t}, \varepsilon'_{-\mu,t})'$ where $\varepsilon_{\mu,t}$ is the financial noise shock and $\varepsilon_{-\mu,t}$ stands for the rest of the structural macroeconomic shocks. Analogously, we partition the full impulse response matrices $\Theta_\ell = (\Theta_{\mu,\ell}, \Theta_{-\mu,\ell})$, where $\Theta_{\mu,\ell}$ is a $n_y \times 1$ column vector that collects the impulse response to the financial noise shock, and $\Theta_{-\mu,\ell}$ is a $n_y \times (n_\varepsilon - 1)$ matrix that collects the response to the rest of the shocks. Next, define also the Wold representation of (43) as:

$$Y_t = \sum_{\ell=0}^{\infty} \Psi_\ell u_{t-\ell}, \tag{45}$$

where u_t are orthogonalized Wold innovations, $u_t = P\varepsilon_t$ for some orthogonal matrix P, and $\Psi_\ell = \Theta_\ell P'$.

We assume that the impulse responses Θ_ℓ can be obtained as the solution to a linear system of dynamic equations:

$$\begin{aligned} \mathcal{F}_w \boldsymbol{w} + \mathcal{F}_x \boldsymbol{x} + \mathcal{F}_z \boldsymbol{z} + \mathcal{F}_\mu(\sigma_r^2 \varepsilon_{\mu,0}) &= \mathbf{0}, & (46)\\ \mathcal{H}_w \boldsymbol{w} + \mathcal{H}_x \boldsymbol{x} + \mathcal{H}_z \boldsymbol{z} + \mathcal{H}_\varepsilon \varepsilon_0 &= \mathbf{0}, & (47)\\ \mathcal{A}_x \boldsymbol{x} + \mathcal{A}_z \boldsymbol{z} + \mathcal{A}_v v_0 &= \mathbf{0}. & (48) \end{aligned}$$

Here, $\boldsymbol{x} = (x_0, x_1, \dots)$ denotes the infinite sequence of variable x (analogously for $\boldsymbol{w}, \boldsymbol{z}$). As in McKay and Wolf (2023b), $\boldsymbol{x}$ collects all private sector variables, $\boldsymbol{z}$ is the path of the policy instrument (the policy interest rate), and $\boldsymbol{w}$ collects variables that are unobserved to the econometrician. $\Theta_{-\mu,\ell}$ includes the impulse response to ε_0 (macroeconomic shocks) and v_0 (policy shocks), and $\Theta_{\mu,\ell}$ collects the impulse responses to $\varepsilon_{\mu,0}$ (financial noise shocks).

As explained by Caravello et al. (2024), Eqs. $(46-48)$ embed common macroeconomic models that satisfy two key restrictions: (i) structural equations are linear, (ii) policy operates through current and expected policy interest rates—i.e., it is inconsequential if these policy rates change because of rule-based policy response captured by $\mathcal{A}_x\boldsymbol{x}$, or a policy shock captured by $\mathcal{A}_v v_0$.

Our main departure from the previous literature is the addition of equation (46), which represents the Financial Block of the model. The key restriction embedded in (46) is that the (endogenous) conditional variance of returns, σ_r^2, only affects the transmission of the financial noise shock, and it does so proportionally. In particular, we consider models in which the $\mathcal{F}, \mathcal{H}$ or $\mathcal{A}$ matrices do not depend on σ_r^2. This condition is satisfied in our model. This is because (i) the noise shock influences the remaining equilibrium variables through the aggregate asset price, (ii) the noise shock affects the equilibrium asset price in proportion to the conditional variance σ_r^2 (see Eq. (16)), and (iii) the model is conditionally homoskedastic, so the conditional variance of returns is constant. Although admittedly stringent, these assumptions allow us to depart from full linearity to study how an FCI targeting policy can reduce risk.

In this setup, our goal is to obtain three objects:

1. the counterfactual impulse responses for the noise shock, i.e., how would the economy react to the shock if policy had been different?
2. counterfactual second moments, i.e., what would have been the variance of the variables if the policy had been different?
3. the counterfactual historical evolution between two dates t_1 and t_2, i.e., what would have been the realized path of variables in between those dates had the policy been different?

To this end, we consider alternative policy rules, parameterized by $\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z, \tilde{\mathcal{A}}_v, \tilde{\mathcal{A}}_\varepsilon$, such that

$$\tilde{\mathcal{A}}_x\boldsymbol{x} + \tilde{\mathcal{A}}_z\boldsymbol{z} + \tilde{\mathcal{A}}_v v_0 + \tilde{\mathcal{A}}_\varepsilon \varepsilon_0 = \mathbf{0}. \tag{49}$$

In equilibrium, the counterfactual rule induces different impulse response matrices $\tilde{\Theta}_\ell$. Our first object of interest is the counterfactual impulse response corresponding to the noise shock. Our second object of interest is counterfactual second moments given by,

$$\tilde{\Gamma}_Y(\ell) = \sum_{m=0}^{\infty} \tilde{\Theta}_m \tilde{\Theta}'_{m+\ell}, \tag{50}$$

where $\tilde{\Gamma}_Y(\ell)$ is the counterfactual autocovariance function of vector Y_t. We can compute the counterfactual historical evolution as:

$$\tilde{Y}_t = \sum_{\ell=0}^{t-t_1} \tilde{\Theta}_\ell \varepsilon_{t-\ell} + \tilde{Y}_t^1, \quad \forall t \in [t_1, t_1+1, \dots, t_2], \tag{51}$$

where $\tilde{Y}_t^1 = E_{t_1-1}[\tilde{Y}_t]$ is an initial conditions term.

Accounting for endogenous risk. If we followed Caravello et al. (2024) directly, coupling (49) with (46) and (47) would yield (a rotation of) counterfactual impulse responses $\tilde{\Theta}_\ell$, which can then be used to mechanically construct the counterfactual second moments using (50), and the counterfactual impulse response as a by product. However, in the present setting, this would yield an incorrect counterfactual, since this would not take into account the endogenous reaction of σ_r^2. In order to account for the endogenous reduction in risk, we use the following proposition.

Proposition 4. *Suppose that the SVMA(∞) process* (43) *is invertible; i.e., that*

$$\varepsilon_t \in span(\{Y_\tau\}_{-\infty<\tau\leq t}). \tag{52}$$

Then knowledge of: (i) the Wold representation of Y_t (i.e., the history of innovations $\{u_{t-\ell}\}_{\ell=0}^{\infty}$ together with $\Psi(L)$); (ii) policy causal effects Θ_ν; and iii) and identified time series for the financial noise shock, $\{\varepsilon_{\mu,t}\}$ suffices to construct the counterfactuals of interest— $\tilde{\Theta}_\ell$, $\tilde{\Gamma}_Y(\ell)$, and $\tilde{Y}_t$.

Appendix C.1 contains the proof. The essence of the proof begins by implementing the procedure described in Caravello et al. (2024), followed by rescaling the IRF of the financial noise shock by $\tilde{\sigma}_r^2/\sigma_r^2$ (where $\tilde{\sigma}_r^2$ is the counterfactual conditional variance, obtained via solving a quadratic analogous to that in the model section). This rescaling accounts for the endogenous variance reduction, and allows us to construct the counterfactuals of interest.

Implementation. We use the same data as in Section 2, with the augmented set of variables that includes labor market series. We use CBO output gap as our measure of output gap, and PCE inflation as our measure of inflation, the Financial Conditions Index built by Ajello et al. (2023a) as a proxy for p_t, and the 3 month interest rate as a the policy rate. All variables are demeaned to capture deviations from steady state. For our measured noise shock, we use the shock identified under SVAR-IV as the baseline.

We employ a fully semi-structural approach, using directly measurable impulse responses to approximate the counterfactual policy, as detailed in McKay and Wolf (2023b); Caravello et al. (2024). Although this is an approximation, Caravello et al. (2024) show in their applications for counterfactual second moments and counterfactual historical evolution, the approximation obtained with only one shock is quite good.

We obtain monetary policy impulse responses using the shocks provided by Romer and Romer (2004) and Aruoba and Drechsel (2022). We use a VAR with the baseline set of macro variables described in Section 2, but for the extended sample 1973Q1:2019Q4 in order to exploit a longer time series for monetary policy shocks. We include both shocks in the VAR, and use a recursive identification scheme as suggested in Plagborg-Møller and Wolf (2021) and implemented in McKay and Wolf (2023b). In particular, the Aruoba and Drechsel (2022) shock is ordered first, then output gap, potential output, investment, consumption, inflation, then the Romer and Romer (2004) shock, and then the rest of the variables. Appendix B.2.4 depicts the estimated impulse responses to the variables of interest.

We take the Wold innovations and identified noise shock as given, but account for estimation uncertainty in the monetary IRFs. Specifically, we estimate the confidence bands for the IRFs using a parametric bootstrap method. Subsequently, for each bootstrap sample of the IRFs, we construct the relevant counterfactual. We then report the distribution of these counterfactual outcomes as a means to assess the significance of estimation uncertainty. This is analogous to the procedure outlined in McKay and Wolf (2023b) or Caravello et al. (2024).

5.2. Evaluation of FCI Targeting

5.2.1. Description of the policy rules

We consider a central bank that minimizes a loss of the form:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{y}_t^2 + \lambda_{\Delta i}(i_t - i_{t-1})^2 + \psi(FCI_t - \overline{FCI}_t))^2\right]. \tag{53}$$

The benchmark policy rule has $\psi = 0$. This kind of loss is considered, for example, in the optimal control exercises reported by the Fed staff to FOMC ahead of their interest rate decision (Federal Reserve Tealbook, 2016). As in previous sections, the planner takes conditional volatility as given when choosing the interest rate. The main departure from the theoretical model is the inclusion of an interest rate smoothing term, $\lambda_{\Delta i}(i_t - i_{t-1})^2$, as in Woodford (2003). We refer to policies that arise from minimizing (53) as "Flexible Dual Mandate" (FDM). We choose the degree of smoothing $\lambda_{\Delta i}$ to match the interest rate variance observed in the data.[19] We compare this benchmark to the case with ψ^*, i.e.,the value of ψ that minimizes the (true) loss *omitting* the FCI term.

Construction of the optimal target under timing constraints. To align our analysis with the theoretical model, we introduce a timing constraint: policy responses are determined with a one-period lag to the shock. When constructing counterfactual impulse responses, we implement the timing restriction by assuming that, at time $t = 0$, the planner targets i_0 at the average rate, and then from $t = 1$ onwards it sets its policy optimally in order to minimize a

[19]We find $\hat{\lambda}_{\Delta i} = 2.5965$.

quadratic loss as in McKay and Wolf (2023a). We also need to obtain the optimal FCI target. This is achieved by solving for the policies that minimize (53) as a function of the target. We then select this target to minimize (53) subject to the timing constraints.

In order to see how this works in more detail, consider the policy response to a non-policy shock $\boldsymbol{\varepsilon} = (\varepsilon, 0, 0, \cdots)'$, which is the building block of all counterfactuals presented in this section. Denote the sequences of output gap, inflation, nominal interest rate and FCI by $\tilde{\boldsymbol{y}}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{f}$ respectively. Then the optimal FCI target in response to a shock, $\bar{\boldsymbol{f}}$, is obtained by solving:

$$
\begin{aligned}
&\min_{\nu, \bar{\boldsymbol{f}}} \quad \tilde{\boldsymbol{y}}'\tilde{W}_{\tilde{y}}\tilde{\boldsymbol{y}} + \boldsymbol{\pi}'\tilde{W}_{\pi}\boldsymbol{\pi} + \lambda_{\Delta i}\boldsymbol{i}'\tilde{W}_i\boldsymbol{i} + \psi(\boldsymbol{f} - \bar{\boldsymbol{f}})'\tilde{W}_f(\boldsymbol{f} - \bar{\boldsymbol{f}}) \\
&s.t. \quad \boldsymbol{x}_i = \Theta_{x_i,v}\nu + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}, \quad \text{for } \boldsymbol{x}_i = \tilde{\boldsymbol{y}}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{f} \qquad \text{(Implementation)} \\
&R_f\bar{\boldsymbol{f}} = 0 \qquad \text{(Announcement Timing)}
\end{aligned}
$$

Several additional pieces of notation must be clarified. First, $\tilde{W}_{\tilde{y}}, \tilde{W}_{\pi}, \tilde{W}_i, \tilde{W}_f$ are matrices that encode discounting and timing restrictions. For example, under the assumption of a one period lag and discount factor β, $\tilde{W}_{\tilde{y}}$ is a diagonal matrix with $\text{diag}(\tilde{W}_{\tilde{y}}) = (0, \beta, \beta^2, \cdots)'$, where the first zero comes from the one period lag. Second, the constraints $\boldsymbol{x}_i = \Theta_{x_i,v}\nu + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}$ encode how the planner can affect outcomes via choosing policy paths. In particular, the $\Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}$ term is the path of the variable under the baseline rule, i.e., in the IRF estimated in sample. On top of this, the policymaker can commit to different policy paths by choosing ν appropriately.[20] Finally, the constraint $R_f\bar{\boldsymbol{f}} = 0$ encodes that the FCI path is announced one period in advance. Since the planner reacts to shocks with a one period lag, for a shock that hits at $t = 0$, any changes in $F\bar{C}I_t$ are announced at $t = 1$ and take effect in $t = 2$. Thus, R_f encodes that the first two elements of $\bar{f}$ must be zero. Appendix C.2 expands on the details.

5.2.2. Results

Impulse Responses. Figure 8 shows the counterfactual impulse response to a noise shock under FCI targeting. As a preliminary step, the beige line shows the outcome if we did not account for endogenous risk reduction, essentially applying McKay and Wolf (2023b)'s methodology directly. In this scenario, monetary policy stabilizes the noise shock more effectively than observed in the historical data, with interest rates raised more aggressively and the noise shock having a smaller effect on the FCI, output gap, and inflation. Our main result is the blue line, which shows the accurate counterfactual that accounts for the endogenous reduction in FCI volatility under the FCI targeting policy. This scenario demonstrates even greater stabilization, with noise shocks having a further reduced impact on the FCI, output gap, and inflation. Observe also that interest rates are raised *less* than in the first scenario and only slightly more than in the data. In fact, future interest rates actually increase even *less* than in the data. This

[20] Since we have only a subset of policy shocks in our empirical analysis (i.e only two as opposed to one for each time horizon), then ν is a two-dimensional vector, and the interpretation is that of a constrained optimal policy in which the planner can only alter baseline policy in the directions spanned by the two monetary policy shocks.

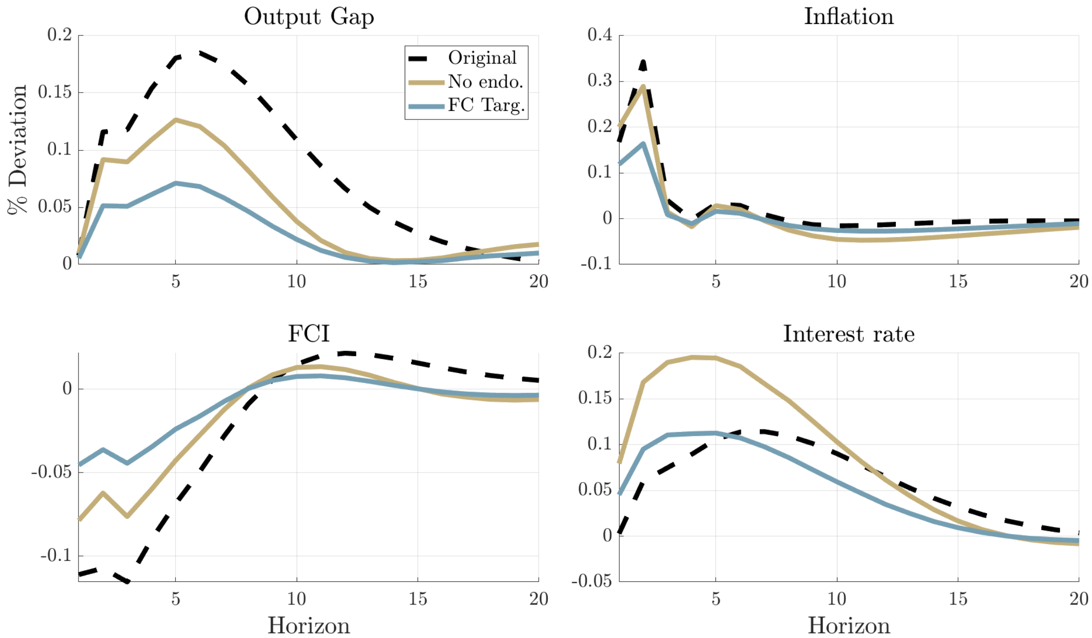

Figure 8: Counterfactual Impulse Responses for the noise shock under FCI Targeting. Beige: not accounting for endogenous risk reduction. Blue: accounting for endogenous risk reduction.

enhanced stabilization with a muted interest rate response occurs because the FCI targeting policy effectively "recruits" arbitrageurs to help the Fed by trading more aggressively against noise. As arbitrageurs take on a larger stabilizing role, the policy interest rate needs to react less to noise shocks. This dynamic is explained in detail in subsection 4.4.

Counterfactual historical evolution. To enhance understanding of how FCI targeting operates in practice, we demonstrate how FCI targeting would have impacted the realized paths of the output gap, inflation, FCI, and interest rates in the period leading up to the Global Financial Crisis. We choose this period because our historical decomposition in Section 2.3 shows that the financial noise shock was a significant driver of the 2001 recession and the main driver of the later expansion. We consider the period 2000Q1-2007Q4, starting at the peak of the expansion preceding the 2001 recession and continuing until the start of the GFC. We use the version of FCI with the interest rate smoothing term discussed before, with the same values of $\lambda_{\Delta i}$ and ψ^*.

Figure 9 shows the results. First, the initial part of the recession appears unavoidable. However, thanks to FCI targeting, the recession is less deep, with the output gap plateauing between 2001Q4 and 2003Q2 instead of falling. At the start of the 2001 recession, the planner targets looser financial conditions in order to stabilize the drop in output gap. Around this period, FCI targeting makes financial conditions less restrictive than in the data. Interestingly, this is *not* due to additional interest rates cuts in that period; if anything, interest rates are

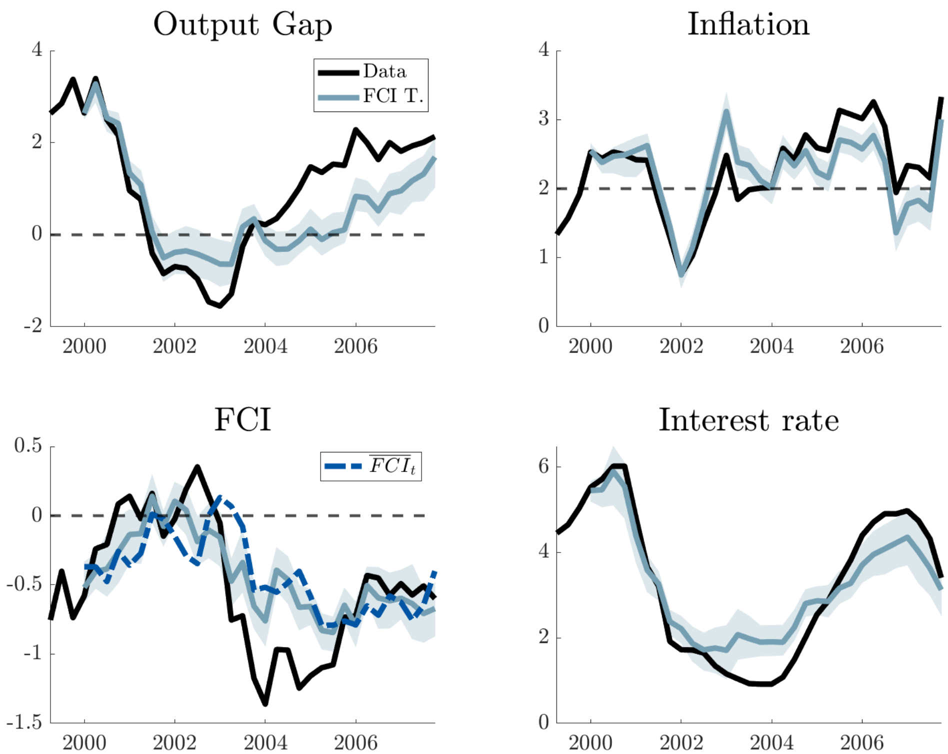

Figure 9: Counterfactual Historical Evolution for 2000Q1-2007Q4. Black: data. For output gap, this is demeaned CBO output gap. Inflation is in year-on-year terms. The rest of the variables are in levels. Blue: FCI targeting, i.e minimize 53 with $\psi = \psi^*$. Solid: median. Shaded area: 16 and 84 confidence bands. Dashed line in the FCI panel indicates the target $\overline{FCI}_t$.

higher than in the data starting on 2001Q4. Thus, we can attribute these looser financial conditions to the positive effects of FCI targeting via encouraging the arbitrageurs.

Turning to the expansionary phase of the cycle stating in 2004Q1, the data shows that financial conditions became quite loose, a positive output gap opened up, and this was accompanied by above-target inflation. If policy had followed FCI targeting, looser financial conditions would have been counteracted by policy, both via interest rate hikes and announcements of tighter FCI targets. This would have generated tighter financial conditions, which would have helped to achieve lower output gaps, and consequently, inflation closer to the 2% target. Overall, the adoption of FCI targeting would have meaningfully smoothed both phases of the cycle.

Counterfactual macro and financial volatility. In order to get a sense of how much macro and financial volatility would have been reduced had policy followed FCI targeting, Figure 10 displays the counterfactual standard deviation for FDM (i.e., minimize (53) with $\psi = 0$) in red, and contrasts these with FCI targeting, shown in blue. Under FCI targeting, the volatility of *all* macroeconomic variables is reduced. Relative to the data, these reductions are substantial: the

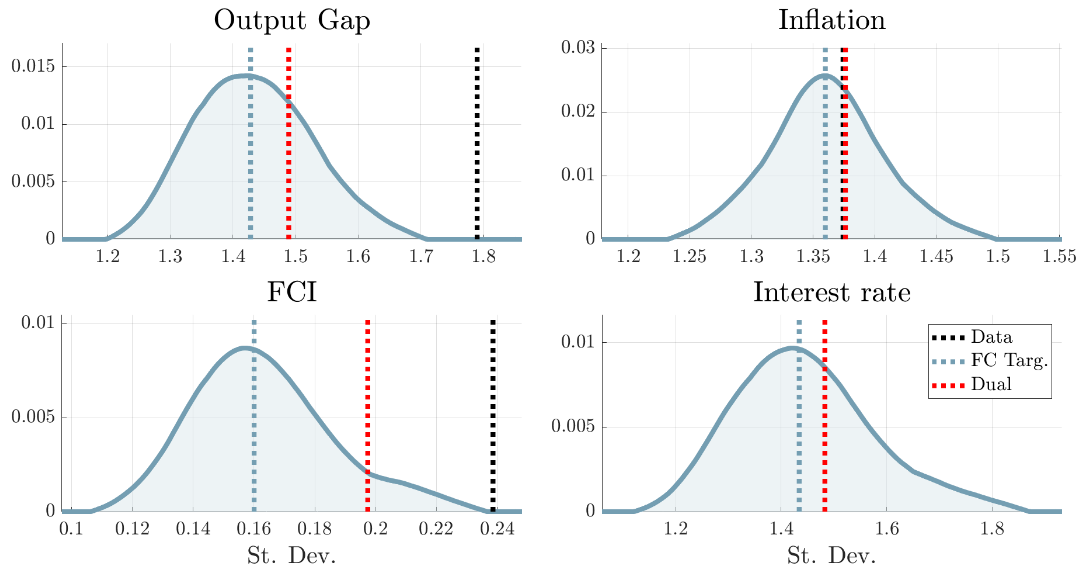

Figure 10: Counterfactual Standard Deviations. For Output Gap, Inflation and interest rates, this is the unconditional standard deviation, for FCI this is the conditional SD. Black dashed: data. Dashed lines the median. Red: Flexible Dual Mandate, i.e minimize 53 with $\psi = 0$. Blue: FCI targeting, i.e minimize 53 with $\psi = \psi^*$. Beige: FCI targeting, i.e minimize 53 with $\psi = \psi^*$, but without accounting for the endogenous reduction in risk when constructing the counterfactual. Solid Line: posterior density for the counterfactual with FCI targeting counterfactual.

	Baseline			No i_t smoothing term		
Loss	Median	10th Perc.	90th Perc.	Median	10th Perc.	90th Perc.
Data	5.45			5.09		
Dual Mandate ($\psi = 0$)	4.47	4.14	4.80	4.12	3.74	4.50
FCI T. ($\psi^* = 19.37$)	4.25	3.95	4.62	3.90	3.59	4.25
Simple Target	4.38	4.10	4.78	4.02	3.74	4.39
Taylor Rule + Simple Target	4.63	4.43	5.05	4.14	3.97	4.47

Table 1: Central bank loss function in the data, and median, 10th and 90th percentiles under the counterfactual policy rules. The policy rules always include the interest rate smoothing term. The first set of columns shows the baseline loss, $E[\mathcal{L}] = \sigma^2_{\tilde{y}} + \sigma^2_{\pi} + \lambda_{\Delta i}\sigma^2_{\Delta i}$. The second set of columns shows the values of $E[\tilde{\mathcal{L}}] = \sigma^2_{\tilde{y}} + \sigma^2_{\pi}$

variance of the output gap, inflation, and interest rates fall by 36%, 2%, and 6%, respectively, and the conditional variance of the FCI falls by 55%. The small reduction on inflation variance relative to output gap variance is due to the small and delayed the response of inflation to monetary policy shocks, consistent with a flat Phillips curve. Conversely, the real effects of monetary policy are significant, so most of the variance gains come from output gaps.

When compared to FDM, the reductions are more modest, with the output gap and inflation decreasing by 8% and 2% respectively (when comparing medians), while the interest rate variance reduction is still 6% (recall that FDM is calibrated to fit the observed interest rate volatility). However, the decrease in financial conditions variance remains substantial, at approximately 34%.

Table 1 presents the loss (53) with and without the interest rate smoothing term. As expected, the loss is lower under FCI targeting, and this is not driven by the interest rate smoothing term.

5.2.3. FCI targeting vs interest rate forward guidance

Following the discussion of Section 4.5, we now compare the performance of FCI targeting with a version of interest rate forward guidance. In particular, we consider losses of the form:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{y}_t^2 + \psi_{if} (i_t^f - \bar{i}_t^f))^2 + \psi (FCI_t - \overline{FCI}_t))^2 \right], \tag{54}$$

and compare interest rate forward guidance ($\psi_{if} > 0, \psi = 0$) with FCI targeting ($\psi_{if} = 0, \psi > 0$). We pick ψ_{if} and ψ^* to minimize the pure dual mandate loss $\sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \tilde{y}_t^2]$. Notice that we omit the interest rate smoothing term, in order to make the comparison between both "pure" regimes.

The inflation variance is roughly the same, but the output gap variance is 21% lower under FCI targeting compared to interest rate forward guidance. Unsurprisingly, FCI is less volatile under FCI targeting, while interest rates are less volatile under interest rate targeting. Overall, this shows that FCI targeting is superior (in terms of the volatility of macroeconomic outcomes) than standard forward guidance in interest rates. Appendix C.3.1 contains additional details.

5.3. Evaluation of Simple FCI Targets and Simple Rules

The results in Section 5.2 underscore the potential improvements that could have been achieved under optimal FCI targeting. However, the reader may worry that these gains depend strongly on the ability to implement the (potentially complex) loss-minimizing plan and committing to the optimal FCI target.

To address this concern, we simplify the implementation of FCI targeting progressively in two steps. First, we consider a simpler proxy-target: a reduced form approximation to the fully optimal FCI target. We show that this proxy-target tracks the actual target reasonably well.

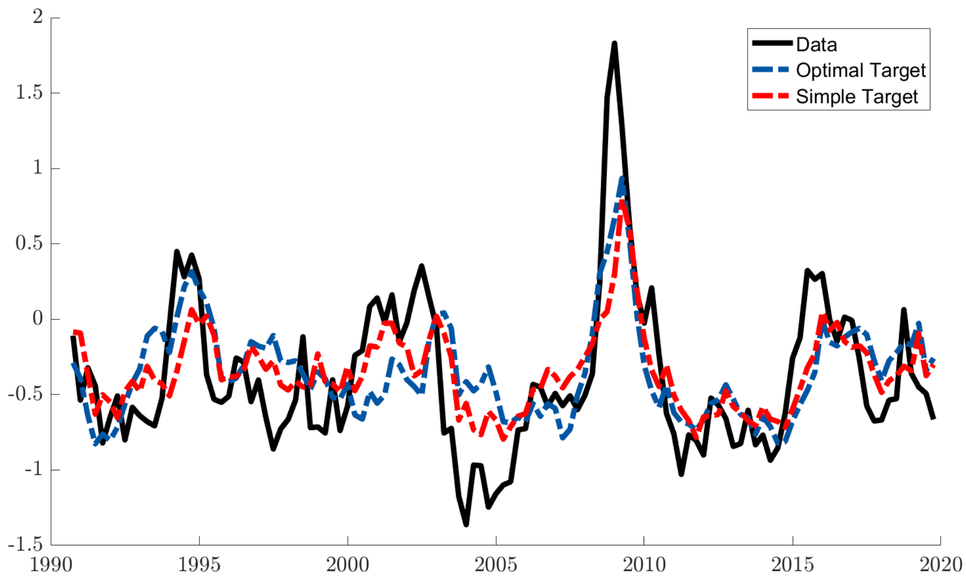

Figure 11: Black: Financial Conditions Index (FCI). Dashed blue: Optimal FCI target computed following the procedure outlined in Section 5.2, with $\psi^* = 19.37$. Dashed red: simple target, constructed as the fitted value of the OLS regression in (55).

Second, we consider a simple rule: we fit a Taylor rule to the data, and add an FCI term to it, using the proxy-target described below. Our results suggest that a large coefficient on the FCI term is optimal, and substantial stabilization gains can still be achieved even with this simplified rule.

5.3.1. Simple Approximation to the Optimal Target

In order to assess the robustness to the exact target used, we compare the optimal target derived in subsection 5.2 to a reduced-form approximation to the optimal target (a proxy-target), constructed as:

$$\overline{FCI}_t = \hat{\phi}_{fci} F_{t-2}(FCI) + \hat{\phi}_{\tilde{y}} F_{t-2}(\tilde{y}) \tag{55}$$

where $F_{t-2}(x) = \frac{1}{4}\sum_{j=0}^{3} x_{t-2+j}$ is the cumulative one-year-ahead forecast for variable x_t made at time $t-2$. The two lags come from the implementation of the FCI target: 1 lag is the information lag to all shocks, and the second one reflects the fact that the target is committed one period in advance. We use the forecasts implied by the VAR described in the implementation section of Section 5.1. We use forecasts instead of the actual levels of the variables because, given the forward-looking nature of optimal policy, they provide a significantly better fit. We obtain the coefficients on the forecasts by fitting equation (55) by OLS. The estimated coefficients are $\hat{\phi}_{fci} = 0.66$, $\hat{\phi}_{\tilde{y}} = 0.02$, both significant. The R^2 is around 0.69.[21]

[21]We omit inflation forecasts since the fit is essentially the same.

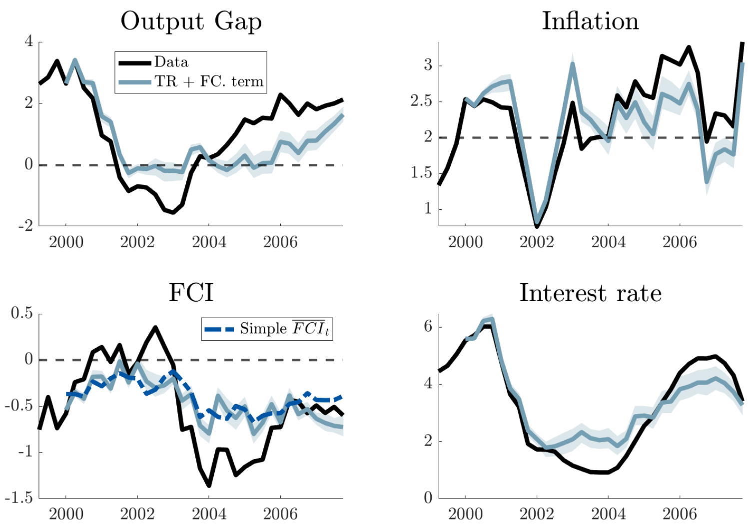

Figure 12: Counterfactual Historical Evolution for 2000Q1-2007Q4. Black: data. Blue: Taylor rule (56) with $\psi_{TR} = 40$ and simple target constructed from 55. Solid: median. Shaded area: 16 and 84 confidence bands. Dashed line in the FCI panel indicates the simple target $\overline{FCI}_t$.

Under the estimated simple target, a higher output gap forecast makes the Fed target tighter financial conditions, as implied by the model. Furthermore, for a given output gap forecast, higher FCI forecasts imply higher FCI targets, but the response is less than 1 to 1. Since the target is adjusted less than proportionally, this implies that there is a "leaning against the wind" component in the optimal FCI target.

Figure 11 depicts the fully optimal target in blue, and the simple target in red. The fit is reasonably good: the approximate target tracks the overall patters in the optimal target closely, although there are some small discrepancies that sometimes persist for up to a few years.

5.3.2. FCI-expanded Taylor Rule

In a second step, we replace the full optimal policy for a Taylor rule augmented with an FCI term. In particular, we consider rules of the form:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_y \tilde{y}_t - \psi_{TR}(FCI_t - \overline{FCI}_t)). \tag{56}$$

Setting $\psi_{TR} = 0$ in (56), we obtain the benchmark with no FCI targeting term. We pick parameters ρ_i, ϕ_π, ϕ_y, such that the unconditional variance of inflation, interest rates and output gaps under the benchmark are the same as in the data.[22] We compare this benchmark with a Taylor rule that features $\psi_{TR} > 0$. We select a large coefficient of $\psi_{TR} = 40$, because it turns out to be the best in terms of minimizing the unconditional loss (see Appendix C.3.2). This

[22]We obtain $\hat{\rho}_i = 0.76$, $\hat{\phi}_\pi = 1.53$ and $\hat{\phi}_{\tilde{y}} = 0.76$. Results are robust to using different coefficients on the Taylor Rule, such as the ones obtained via direct OLS estimation.

policy is close to a policy that directly targets $FCI_t = \overline{FCI}_t$.

The bottom two rows of Table 1 report the unconditional losses under i) loss mimimization but using the proxy-target; ii) the previously discussed Taylor rule with the proxy-target. As we can see, the loss becomes progressively worse as we add each layer of approximation. However, even under the Taylor Rule, the distribution of losses is significantly below the observed loss in the data.

To illustrate how this operates during a specific episode, Figure 12 presents the Counterfactual Historical Evolution under the FCI-augment Taylor Rule, using the proxy-target constructed in the previous subsection. Compared to the fully optimal FCI targeting: the gains in terms of reduced output gap and inflation volatility are similar. The volatility in financial conditions is somewhat lower, as the target moves less and the deviations from target are smaller. Given the relatively higher stability of financial conditions, the interest rate also fluctuates less.[23]

Overall, these findings demonstrate that an expanded Taylor rule that gives a large weight to the financial conditions gap can notably stabilize *macroeconomic* outcomes, even if the target used is just a simple approximation to the true optimal target.

6. Final Remarks

This paper theoretically and empirically investigates how monetary policy should respond to macroeconomic fluctuations driven by financial noise.

We motivate our analysis by using (identified) vector-autoregression (VAR) models to demonstrate that financial noise shocks can account for up to 55% of the variance in financial conditions and up to 50% of the variance in output gaps in the U.S.

We then develop a model with financial noise and limits to arbitrage wherein it is optimal for the central bank to stabilize financial conditions beyond their direct impact on output gaps, even though stable financial conditions themselves are not a social objective. Our primary finding reveals that an FCI targeting framework—in which the central bank announces a (soft) FCI target and tries to keep the actual FCI close to this target—reduces FCI volatility and stabilizes the output gap. This improvement occurs because FCI targeting reduces the macroeconomic data-dependency of monetary policy. This reduces FCI volatility and recruits arbitrageurs to mitigate the impact of noise shocks on the FCI, which reduces FCI volatility even more, and so on. We further demonstrate that in our model FCI targeting is more effective than interest rate forward guidance, because it retains the flexibility of monetary policy to respond to post-guidance noise shocks. Importantly, FCI targeting does *not* require the central bank to distinguish in real time whether asset price movements are driven by noise or fundamentals.

Finally, we extend recent policy counterfactual methods to incorporate our model's endogenous risk reduction mechanism. We use this method to perform a series of counterfactual experiments to assess the potential effects of FCI targeting in the U.S. Our findings indicate

[23] Appendix C.3.3 contains additional details on counterfactual second moments under the Taylor Rule.

that an FCI targeting policy could have substantially stabilized the macroeconomic effects of noise shocks and that the gains would have primarily come from arbitrageurs trading more aggressively against noise shocks. Consequently, we find that FCI targeting could have decreased the variance of the FCI, the output gap, and inflation by 55%, 36% and 2%, respectively, while also reducing the variance of the interest rate by 6%. We also empirically confirm that FCI targeting outperforms interest rate forward guidance.

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Online Appendices: Not for Publication

A. Theory Appendix

This appendix contains details related to the theoretical model. Section A.1 provides the microfoundations for the model. Section A.2 contains the proofs omitted from the main text. The remaining Sections A.3-A.5 provide the details of various extensions that we discuss in the main text.

A.1. Microfoundations of the model

In this section, we provide the microfoundations of the model that we summarize in Section 3.1 and use throughout the paper. The real side of the economy is the same as the baseline model in Caballero and Simsek (2023). The financial market side is different and allows for noise shocks.

The economy is set in discrete time $t \in \{0, 1, ..\}$. The model consists of four agent types: asset-holding households (households), hand-to-mouth agents, portfolio managers, and the central bank. The hand-to-mouth agents primarily serve to decouple labor supply decisions from household consumption behavior. The asset-holding households are the main drivers of aggregate demand through their consumption and savings choices. The portfolio managers act on behalf of these households by making portfolio allocation decisions that determine asset prices in financial markets. The central bank conducts monetary policy.

A.1.1. Supply side

Hand-to-mouth agents provide all of the labor supply and spend all of their income (they do not save). Their problem is

$$\begin{aligned} &\max_{L_t} \log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi}, \\ &Q_t C_t^{HM} = W_t L_t + T_t. \end{aligned} \tag{A.1}$$

Here, φ denotes the Frisch elasticity of labor supply, Q_t denotes the nominal price for the final good, W_t denotes the nominal wage, and T_t denotes lump-sum transfers from the government (described subsequently). The optimality condition implies a standard labor supply equation

$$\frac{W_t}{Q_t} = \chi L_t^{\varphi} C_t^{HM}. \tag{A.2}$$

The rest of the supply side is similar to the standard New Keynesian Model. A competitive final goods producer combines the intermediate goods according to the CES technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} d\nu \right)^{\varepsilon_t/(\varepsilon_t - 1)} \quad \text{where } Y_t(\nu) = Z_t L_t(\nu)^{1-\nu}. \tag{A.3}$$

Here, $\varepsilon_t > 1$ denotes the elasticity of substitution that determines the firm markups in equilibrium. We assume it is stochastic around a steady-state level $\varepsilon^* > 1$, which allows us to accommodate cost-push shocks. With these technologies, the demand for intermediate good firms satisfies,

$$
\begin{aligned}
Y_t(\nu) &\leq \left(\frac{Q_t(\nu)}{Q_t}\right)^{-\varepsilon_t} Y_t, && \text{(A.4)} \\
\text{where } Q_t &= \left(\int_0^1 Q_t(\nu)^{1-\varepsilon_t} d\nu\right)^{1/(1-\varepsilon_t)}. && \text{(A.5)}
\end{aligned}
$$

$Q_t(\nu)$ denotes the nominal price set by the intermediate good firm ν and Q_t is the ideal price index.

The goods market clearing condition is:

$$Y_t = C_t^H + C_t^{HM}. \tag{A.6}$$

Here, C_t^H and C_t^{HM} denote consumption by the asset holding households and the hand-to-mouth agents, respectively.

The labor market clearing condition is

$$\int_0^1 L_t(\nu)\, d\nu = L_t. \tag{A.7}$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the firms' profits lump-sum and redistributes to the hand-to-mouth agents to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T_t = (1-\nu)\, Q_t Y_t - W_t L_t. \tag{A.8}$$

This ensures that in equilibrium hand-to-mouth agents receive and spend their production share of output, $(1-\nu)\, Q_t Y_t$, and consume [see $(A.1)$]

$$C_t^{HM} = (1-\nu)\, Y_t. \tag{A.9}$$

Substituting this into the goods market clearing condition $(A.6)$, we further obtain

$$Y_t = \frac{C_t^H}{\nu}. \tag{A.10}$$

Hand-to-mouth agents create a Keynesian multiplier effect, but output is ultimately determined by (asset-holding) *households'* spending, C_t^H.

Flexible-price equilibrium. Consider a benchmark without nominal rigidities. In this benchmark, the intermediate good firm ν solves

$$
\begin{aligned}
\Pi &= \max_{Q,L} QY - W_t L - T_t, && \text{(A.11)} \\
&\text{where } Y = Z_t L^{1-\nu} = \left(\frac{Q}{Q_t}\right)^{-\varepsilon_t} Y_t.
\end{aligned}
$$

The firm takes as given the aggregate price, wage, and output, Q_t, W_t, Y_t, and chooses its price, labor input, and output Q, L, Y.

The optimal price is given by

$$Q = \frac{\varepsilon_t}{\varepsilon_t - 1} W_t \frac{1}{(1-\nu) Z_t L^{-\nu}}. \tag{A.12}$$

The firm sets an optimal markup over the marginal cost, where the markup depends (inversely) on the elasticity of substitution and the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q_t = Q$ and $L_t = L$. Substituting this into $(A.12)$, we obtain a labor demand equation,

$$\frac{W_t}{Q_t} = \frac{\varepsilon_t - 1}{\varepsilon_t} (1-\nu) Z_t L_t^{-\nu}. \tag{A.13}$$

Combining this with the labor supply equation $(A.2)$, and substituting the hand-to-mouth consumption $(A.9)$, we obtain the equilibrium labor as the solution to,

$$\chi (L_t^*)^{\varphi} (1-\nu) Y_t^* = \frac{\varepsilon_t - 1}{\varepsilon_t} (1-\nu) Z_t (L_t^*)^{-\nu}.$$

In equilibrium, output is given by $Y_t^* = Z_t (L^*)^{1-\nu}$. Therefore, the flexible-price equilibrium conditions are given by:

$$\begin{aligned} \chi (L_t^*)^{1+\varphi} &= \frac{\varepsilon_t - 1}{\varepsilon_t}, \\ Y_t^* &= Z_t (L_t^*)^{1-\nu}. \end{aligned} \tag{A.14}$$

Potential output. Consider the flexible-price allocation in which the firms' markups are at their steady-state level, $\varepsilon_t = \varepsilon^*$, that is:

$$\begin{aligned} \chi (L^*)^{1+\varphi} &= \frac{\varepsilon^* - 1}{\varepsilon^*}, \\ Y_t^* &= Z_t (L^*)^{1-\nu}. \end{aligned} \tag{A.15}$$

We refer to L^* as the potential labor supply and $Y^* = Z_t (L^*)^{1-\nu}$ as the potential output. In the main text, we assume the central bank attempts to keep labor and output demand at these levels. In particular, the central bank attempts to stabilize the output fluctuations driven by shocks to ε_t (or markups), because these shocks are distortionary. This enables us to accommodate cost-push shocks that create a trade-off for the central bank for stabilizing inflation and output.[24]

Sticky prices and demand-driven output. We next describe the equilibrium with nominal rigidities. We start with the special case with full price stickiness and then extend the analysis to partially flexible prices. With fully sticky prices, intermediate good firms have a preset nominal price that remains fixed over time, $Q_t(\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q_t = Q^*$. Then, each intermediate good firm ν at time t solves the following version of

[24]The central bank does not attempt to stabilize the distortions generates by the *average* markup, because this would induce an average inflationary bias. In practice, these average distortions should ideally be corrected by other policy tools rather than monetary policy.

problem $(A.11)$,

$$\Pi = \max_L Q^* Y - W_t L - T_t \tag{A.16}$$
$$\text{where } Y = AL^{1-\nu} \leq Y_t.$$

Since the firm operates with a markup, for small aggregate demand shocks (which we assume) it optimally chooses to meet the demand for its goods, $Y = ZL^{1-\nu} = Y_t$. Therefore, each firm's output is determined by aggregate demand, which is driven by households' spending C_t^H according to $(A.10)$.

Partially flexible prices and the New Keynesian Phillips curve. We next allow for partially flexible prices. With partially flexible prices, each firm still optimally serves the demand and output is still determined by aggregate demand. However, inflation is also endogenous and reacts to output gaps as well as other (cost-push) shocks. We derive a Phillips curve that describes inflation.

We consider the setup in the textbook New Keynesian model in which in each period a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period t, set their labor input to meet the demand for their goods.

Consider the firms that adjust their price in period t. Let Q_t^{adj} denote the optimal price set by these firms. We assume Q_t^{adj} solves the following version of problem $(A.11)$

$$\max_{Q_t^{adj}} \sum_{h=0}^{\infty} \theta^h E_t \left\{ M_{t,t+h} \left(Y_{t+h|t} Q_t^{adj} - W_{t+h} L_{t+h|t} - T_t \right) \right\}, \tag{A.17}$$
$$\text{where } Y_{t+h|t} = Z_{t+h} L_{t+h|t}^{1-\nu} = \left(\frac{Q_t^{adj}}{Q_{t+h}} \right)^{-\varepsilon_{t+h}} Y_{t+h}$$
$$\text{and } M_{t,t+h} = \beta^h \frac{1/P_{t+h}}{1/P_t} \frac{Q_t}{Q_{t+h}}.$$

The terms $L_{t+h|t}$ and $Y_{t+h|t}$ denote the input and the output of the firm (that resets its price in period t) in a future period $t+h$. The term $M_{t,t+h}$ is the stochastic discount factor (SDF) between periods t and $t+h$. Here, P_t denotes the end-of-period price of the market portfolio which we describe later in the appendix.[25]

The optimality condition for problem $(A.17)$ is given by

$$\sum_{h=0}^{\infty} \theta^h E_t \left\{ M_{t,t+h} Q_{t+h}^{\varepsilon_{t+h}} Y_{t+h} \left(Q_t^{adj} - \frac{\varepsilon_{t+h}}{\varepsilon_{t+h}-1} \frac{W_{t+h}}{(1-\nu) Z_{t+h} L_{t+h|t}^{-\nu}} \right) \right\} = 0, \tag{A.18}$$
$$\text{where } L_{t+h|t} = \left(\frac{Q_t^{adj}}{Q_{t+h}} \right)^{\frac{-\varepsilon_{t+h}}{1-\nu}} \left(\frac{Y_{t+h}}{Z_{t+h}} \right)^{\frac{1}{1-\nu}}.$$

We next combine Eq. $(A.18)$ with the remaining equilibrium conditions to derive the New Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real

[25]Consistent with the financial market side of our model, we assume the SDF is determined by asset-holding households' wealth rather than their consumption. In equilibrium, asset-holding households' wealth is equal to the value of the market portfolio. The exact specification does not affect our analysis because we log-linearize the equation and the interaction of the SDF and prices, $M_{t,t+h} Q_t^{adj}$, generates second-order terms that drops out of the log linearization.

potential outcomes (with constant markups) and zero inflation, that is, $L_t = L^*, Y_t = Y_t^*, \frac{\varepsilon_t}{\varepsilon_t - 1} = \frac{\varepsilon^*}{\varepsilon^* - 1}, Q_t = Q^*$ for each t, where recall that L^* is given by $(A.15)$ and $Y_t^* = Z_t (L^*)^{1-\nu}$. Throughout, we use the notation $\tilde{x}_t = \log(X_t / X_t^*)$ to denote the log-linearized version of the corresponding variable X_t and we use $\tilde{\mu}_t = \frac{\varepsilon_t}{\varepsilon_t - 1} - \frac{\varepsilon^*}{\varepsilon^* - 1}$ to denote the deviation of the desired markup from its steady-state level level. We also let $W_t^{norm} = \frac{W_t}{Z_t Q_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize the labor-supply equilibrium condition $(A.2)$ and use $C_t^{HM} = (1 - \nu) Y_t$ to obtain

$$\tilde{w}_t^{norm} = \varphi \tilde{l}_t + \tilde{y}_t. \tag{A.19}$$

Log-linearizing Eqs. $(A.3)$ and $(A.7)$, we also obtain

$$\tilde{y}_t = (1 - \nu) \tilde{l}_t. \tag{A.20}$$

Finally, we log-linearize Eq. $(A.18)$ (and linearize for $\tilde{\mu}_t$) to obtain

$$\sum_{h=0}^{\infty} (\theta\beta)^h E_t \left\{ \tilde{q}_t^{adj} - \left(\tilde{w}_{t+h}^{norm} + \nu \tilde{l}_{t+h|t} + \tilde{q}_{t+h} \right) - \tilde{\mu}_{t+h} \right\} = 0, \tag{A.21}$$
$$\text{where } \tilde{l}_{t|t+h} = \frac{-\varepsilon^* \left(\tilde{q}_t^{adj} - \tilde{q}_{t+h} \right)}{1 - \nu} + \tilde{l}_{t+h}.$$

The second line uses $\tilde{y}_t = (1 - \nu) \tilde{l}_t$.

We next combine Eqs. $(A.19 - A.21)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms

$$\begin{aligned} \tilde{q}_t^{adj} &= (1 - \theta\beta) \sum_{h=0}^{\infty} (\theta\beta)^h E_t \left[\Theta \tilde{y}_{t+h} + \tilde{q}_{t+h} + \tilde{\mu}_{t+h} \right], \\ \text{where } \Theta &= \frac{1 + \varphi}{1 - \nu + \nu\varepsilon} \end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{q}_t^{adj} = (1 - \theta\beta) (\Theta \tilde{y}_t + \tilde{q}_t + \tilde{\mu}_t) + \theta\beta E_t \left[\tilde{q}_{t+1}^{adj} \right]. \tag{A.22}$$

Here, we have used the law of iterated expectations, $E_t[\cdot] = E_t[E_{t+1}[\cdot]]$.

Next, we consider the aggregate price index $(A.5)$

$$\begin{aligned} Q_t &= \left((1 - \theta) \left(Q_t^{adj} \right)^{1-\varepsilon} + \int_{S_t} (Q_{t-1}(\nu))^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)} \\ &= \left((1 - \theta) \left(Q_t^{adj} \right)^{1-\varepsilon} + \theta Q_{t-1}^{1-\varepsilon} \right)^{1/(1-\varepsilon)}, \end{aligned}$$

where we have used the observation that a fraction θ of prices are the same as in the last period. The term, S_t, denotes the set of sticky firms in period t, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{q}_t = (1 - \theta) \tilde{q}_t^{adj} + \theta \tilde{q}_{t-1}$. After substituting inflation, $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$, this implies

$$\pi_t = (1 - \theta) \left(\tilde{q}_t^{adj} - \tilde{q}_{t-1} \right). \tag{A.23}$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.22)$ can be written in terms of the price change of adjusting firms as

$$\tilde{q}_t^{adj} - \tilde{q}_{t-1} = (1-\theta\beta)(\Theta\tilde{y}_t + \tilde{\mu}_t) + \tilde{q}_t - \tilde{q}_{t-1} + \theta\beta E_t\left[\tilde{q}_{t+1}^{adj} - \tilde{q}_t\right].$$

Substituting $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$ and combining with Eq. $(A.23)$, we obtain the New Keynesian Phillips curve (40) that we use in the main text

$$\begin{aligned}
\pi_t &= \kappa\tilde{y}_t + \beta E_t[\pi_{t+1}] + u_t, \\
\text{where } \kappa &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1+\varphi}{1-\nu+\nu\varepsilon} \\
\text{and } u_t &= \frac{1-\theta}{\theta}(1-\theta\beta)\tilde{\mu}_t, \text{ where } \tilde{\mu}_t = \frac{\varepsilon_t}{\varepsilon_t - 1} - \frac{\varepsilon^*}{\varepsilon^* - 1}.
\end{aligned} \tag{A.24}$$

A.1.2. Demand side and financial markets

We next describe households' consumption-savings and portfolio allocation decisions. In equilibrium, together with monetary policy, these decisions determine aggregate demand, asset prices, and output.

Financial assets. There are two assets. There is a market portfolio, which is a claim on firms' profits νY_t (the firms' share of output). We let P_t denote the ex-dividend price of the market portfolio (which we also refer to as "the aggregate asset price" or "aggregate asset prices"). The gross return of the market portfolio is

$$R_{t+1} = \frac{\nu Y_{t+1} + P_{t+1}}{P_t}. \tag{A.25}$$

There is also a risk-free asset in zero net supply. Its gross return R_t^f is set by the central bank, as we describe in the main text.

Households' consumption-savings decisions. Households have standard preferences:

$$E_t\left[\sum_{h=0}^{\infty}\beta^{t+h}\log C_{t+h}^H\right], \tag{A.26}$$

along with the budget constraint

$$\begin{aligned}
W_{t+1} + C_{t+1}^H &= W_t\left((1-\omega_t)R_t^f + \omega_t R_{t+1}\right) \\
&= D_{t+1} + K_{t+1}, \\
\text{where } D_{t+1} &= W_t\left[(1-\omega_t)\left(R_t^f - 1\right) + \omega_t\frac{\nu Y_{t+1}}{P_t}\right] \\
\text{and } K_{t+1} &= W_t\left[1 - \omega_t + \omega_t\frac{P_{t+1}}{P_t}\right].
\end{aligned} \tag{A.27}$$

W_t denotes the end-of-period wealth and ω_t denotes the market portfolio weight in period t. The term $W_t\left((1-\omega_t)R_t^f + \omega_t R_{t+1}\right)$ is the beginning-of-period wealth in period $t+1$. The second line breaks this term into a component that captures the interest and dividend income (D_{t+1}) and a residual component that captures the capital (K_{t+1}).

Households take their portfolio allocation as given (delegated to the portfolio managers) and make a consumption-savings decision. We assume their consumption follows the rule

$$C_t^H = (1-\beta)\left(D_t + K_t \exp\left(\delta_t\right)\right). \tag{A.28}$$

When $\delta_t = 0$, this is the optimal rule given the log preferences in ($A.26$). When $\delta_t > 0$ (resp. $\delta_t < 0$), households spend more (resp. less) than the optimal rule. We refer to δ_t as an *aggregate demand shock* and view it as a modeling device to capture various factors that affect aggregate spending in practice, e.g., a consumer sentiment shock, a fiscal policy shock, or a discount rate shock. Having the demand shock multiply K_t rather than $D_t + K_t$ does not play an important role beyond simplifying the expressions.[26]

The portfolio managers (the market) and the portfolio allocation. Households delegate their portfolio choice to managers. The portfolio managers are infinitesimal and they do not consume themselves; they simply make a portfolio choice decision for households. For simplicity, each household invests with a continuum of managers randomly sampled from all managers. This assumption ensures that there is no portfolio heterogeneity across households and thus no individual wealth dynamics.

In each period, a fraction η of portfolio managers are "noise traders" and their portfolio weight is given by $\omega_t^N = 1 + \frac{1}{\eta}\mu_t$. That is, they deviate from the optimal portfolio benchmark by an amount given by $\frac{1}{\eta}\mu_t$. We refer to μ_t as the aggregate noise—the total amount of flow that needs to be absorbed by other investors. Among the remaining managers, a mass $1-\eta-\alpha$ represent "inelastic funds" and their portfolio weight is given by $\omega_t^I = 1$. Finally, a mass α of managers are "arbitrageurs" (or elastic funds) who choose their portfolio weights to maximize expected log assets-under-management, after observing the risk-free rate $r_t^f = \log R_t^f$ and the current noise μ_t:[27]

$$\max_{\omega_t^A} E_t\left[\log\left(\alpha W_t\left(R_t^f + \omega_t\left(R_{t+1} - R_t^f\right)\right)\right)\right].$$

As we describe in the main text, the optimality condition is approximately given by (14)

$$\omega_t^A \sigma_{t,r_{t+1}} = \frac{E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f}{\sigma_{t,r_{t+1}}}.$$

Financial market clearing. Financial markets are in equilibrium when the households in the aggregate hold the market portfolio, both before and after the portfolio allocation:

$$W_t = P_t \quad \text{and} \quad \omega_t = \alpha\omega_t^A + \eta\left(1 + \frac{\mu_t}{\eta}\right) + (1-\eta-\alpha) = 1. \tag{A.29}$$

Output-asset price relation. We next derive the equilibrium condition (9) that we use in the main text. Combining Eqs. ($A.27$) and ($A.29$), we obtain $D_t = \nu Y_t, K_t = P_t$. In equilibrium, dividends are

[26] We could alternatively capture demand shocks as shocks to households' discount factor β in a fully optimizing framework. We prefer our approach where we view demand shocks as small consumption "mistakes" because doing so simplifies the analysis and gives us greater flexibility in specifying the process for δ_t.

[27] We assume arbitrageurs maximize log-wealth in line with the households' preferences in ($A.26$). In the special case where households follow the optimal rule ($\sigma_\delta^2 = 0$), this problem results in portfolio allocations that maximize the households' utility. We formulate the portfolio problem in terms of wealth, rather than consumption, because we allow consumption to deviate from the optimal rule. In our setup, wealth is a more accurate representation of welfare, as it captures the *ideal* consumption a household could choose if she followed the optimal rule.

equal to the firms' share of output. Capital is equal to the (ex-dividend) value of the market portfolio. Substituting these observations into the consumption rule in $(A.28)$, we obtain

$$C_t^H = (1-\beta)\left(\nu Y_t + P_t \exp\left(\delta_t\right)\right).$$

Substituting Eq. $(A.10)$ $(C_t^H = \nu Y_t)$ into this expression yields Eq. (9)

$$\begin{aligned} Y_t &= (1-\beta)\frac{1}{\nu\beta}P_t \exp\left(\delta_t\right) \\ &\implies y_t = m + p_t + \delta_t, \quad \text{where } m \equiv \log\left(\frac{1-\beta}{\nu\beta}\right). \end{aligned} \tag{A.30}$$

We refer to this as *the output-asset price relation.* In equilibrium, output depends on aggregate wealth, P_t, the MPC out of wealth, $1-\beta$, the demand shock, δ_t, and the Keynesian multiplier, $1/(\nu\beta)$. The second line describes the relation in logs and obtains the derived parameter m.

Financial market equilibrium condition. We next derive the equilibrium condition (15) that we use in the main text. Eq. $(A.29)$ implies $\omega_t^A = 1 - \frac{\mu_t}{\alpha}$. Substituting this into (14), we obtain (15)

$$E_t\left[r_{t+1}\right] = r_t^f + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} + \mu_t \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}.$$

In equilibrium, the expected return on the market portfolio depends on the risk premium, return variance, and noise. The impact of noise is increasing in the return variance and decreasing in the mass of arbitrageurs.

Campbell-Shiller approximation to the equilibrium return. We next derive the Campbell-Shiller approximation in (11). First note that Eq. $(A.25)$ implies

$$\begin{aligned} r_{t+1} &= \log\left(\frac{\alpha Y_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t} + \frac{P_{t+1}}{P_t}\right) \\ &= \log\left(\frac{\alpha Y_{t+1}}{P_{t+1}} + 1\right) + \log\left(\frac{P_{t+1}}{P_t}\right) \\ &= \log\left(1 + X_{t+1}\right) + p_{t+1} - p_t. \end{aligned} \tag{A.31}$$

Here, we have defined the dividend price ratio, $X_t = \alpha Y_t / P_t$.

Setting the demand shifter to zero ($\delta_t = 0$) and output equal to its potential $Y = Y^*$, Eq. $(A.30)$ implies $Y^* = (1-\beta)\frac{1}{\alpha\beta}P^*$. This implies $X^* = \alpha Y_t^* / P_t^* = \frac{1-\beta}{\beta}$.

Finally, log-linearize $(A.31)$ around $X_{t+1} = X^*$. Let $x_{t+1} = \log\left(X_{t+1}/X^*\right)$ denote the log deviation of the dividend price ratio from its steady-state level. Consider the term, $\log\left(1+X_{t+1}\right) = \log\left(1 + X^* \exp\left(x_{t+1}\right)\right)$. Using a Taylor approximation around $x_{t+1} = 0$, we obtain

$$\begin{aligned} \log\left(1+X_{t+1}\right) &\approx \log\left(1+X^*\right) + \frac{X^*}{1+X^*}x_{t+1} \\ &\approx \log\left(\frac{1}{\beta}\right) + (1-\beta)\left(\log\left(\frac{\alpha Y_{t+1}}{P_{t+1}}\right) - \log\left(\frac{1-\beta}{\beta}\right)\right). \end{aligned}$$

Substituting this into $(A.31)$ and collecting the constant terms, we obtain Eq. (11)

$$\begin{aligned} r_{t+1} &= \rho - (1-\beta)\, m + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t \\ &= \rho + p_{t+1} + (1-\beta)\, \delta_{t+1} - p_t, \end{aligned}$$

where the second line substitutes the output asset price relation (9) to simplify the expression.

Present discounted value relation. Substituting Eq. (11) into the financial market equilibrium condition (15), we also obtain the present discounted value relation (16) that describes the equilibrium asset price

$$p_t = \rho + E_t\left[p_{t+1}\right] + (1-\beta)\, E_t\left[\delta_{t+1}\right] - \left(r_t^f + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2\right) + \mu_t \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}.$$

A.2. Omitted proofs

This section contains the proofs omitted from the main text.

Proof of Proposition 1. To characterize the equilibrium, first observe that the central bank's problem is

$$G_t = \min_{r_t^f} \underline{E}_t\left[(y_t - y_t^*)^2\right] + \beta \underline{E}_t\left[G_{t+1}\right].$$

The expected *future* gaps $\underline{E}_t\left[G_{t+1}\right]$ do not depend on the current policy rate r_t^f, because the model is forward looking without any endogenous state variables. Thus, the optimality condition is given by $\underline{E}_t\left[\frac{dy_t}{dr_t^f}\tilde{y}_t\right] = 0$. We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr_t^f} = -1$. Consequently, the optimality condition implies

$$\underline{E}_t\left[\tilde{y}_t\right] = 0 \Longrightarrow \underline{E}_t\left[y_t\right] = \underline{E}_t\left[y_t^*\right] = y_t^*. \tag{A.32}$$

Since the central bank sets policy *before* observing the noise, it cannot ensure output is equal to its potential in every state, $y_t = y_t^*$. Instead, it does so in expectation. Combining this with Eq. (9), we also obtain

$$\underline{E}_t\left[m + p_t + \delta_t\right] = y_t^* \Longrightarrow \underline{E}_t\left[p_t\right] = p_t^* \equiv y_t^* - m - \delta_t. \tag{A.33}$$

That is, the central bank sets the asset price equal to "pstar" in expectation.

We next conjecture (and verify) that there is an equilibrium in which the return volatility σ^2 is constant and the aggregate asset price is given by (19). Substituting this into the output asset price relation (9), we obtain (20). Note that Eqs. $(19-20)$ satisfy the optimality conditions $(A.32 - A.33)$ since $\underline{E}_t\left[\varepsilon_{\mu,t}\right] = 0$. Substituting (19) into (11), we also obtain

$$\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\
&= \rho + p_{t+1}^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} - \left(p_t^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + y_{t+1}^* - m - \delta_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} - \left(y_t^* - m - \delta_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + \delta_t + \varepsilon_{z,t+1} - \beta\delta_{t+1} + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right).
\end{aligned}$$

The third line substitutes for p_{t+1}^* and p_t^*, the fourth line uses (8) and simplifies the expressions. This proves (22). Combining this with (15), we also characterize the interest rate as

$$\begin{aligned}
r_t^f &= E_t\left[r_{t+1}\right] - \frac{1}{2}\sigma^2 + \frac{\sigma^2}{\alpha}\mu_t \\
&= \rho + \delta_t - \beta\varphi_\delta\delta_t + \frac{\sigma^2}{\alpha}E_t\left[\mu_t - \varepsilon_{\mu,t}\right] \\
&= \rho + (1-\beta\varphi_\delta)\,\delta_t + \varphi_z z_t + \frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1}.
\end{aligned}$$

The second line substitutes the AR(1) process for δ_t and the last line substitutes the AR(1) process for μ_t. This proves (21).

We next characterize the return volatility corresponding to this equilibrium. Using (22), along with the observation that all shocks are conditionally independent, we obtain

$$\sigma^2 = var_t\left(r_{t+1}\right) = \sigma^2_{macro} + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_{\mu}, \quad \text{where } \sigma^2_{macro} = \sigma^2_z + \beta^2\sigma^2_{\delta}.$$

In particular, the conditional volatility is a root of a quadratic, $P\left(\sigma^2\right) = 0$, given by

$$P\left(x\right) = \frac{\sigma^2_{\mu}}{\alpha^2}x^2 - x + \sigma^2_{macro}. \tag{A.34}$$

As long as the parameters satisfy $\alpha^2 > 4\sigma^2_{\mu}\sigma^2_{macro}$, which we assume, this polynomial has two positive roots. The larger root is unstable in the sense that small changes in volatility induce further changes in volatility that move the equilibrium away from this point. The smaller root corresponds to a stable equilibrium. This verifies that the equilibrium volatility is the smaller solution to the fixed point equation in (23). To assist with the calibrations, we observe that the smaller root is associated with a negative derivative for the polynomial,

$$P'\left(x\right) = 2\frac{\sigma^2_{\mu}}{\alpha^2}x - 1\Big|_{x=\sigma^2} \leq 0.$$

This shows that a candidate solution that satisfies $P\left(\sigma^2\right) = 0$ is stable as long as it also satisfies $2\sigma^2_{\mu}\sigma^2 \leq \alpha^2$. In contrast, the positive root has $2\sigma^2_{\mu}\sigma^2 \geq \alpha^2$.

It remains to verify our conjecture that $\frac{dy_t}{dr^f_t} = -1$. Along the equilibrium path, output satisfies $y_t = m + p_t + \delta_t$, where the asset price satisfies (16)

$$p_t = \rho + E_t\left[p_{t+1}\right] + \left(1-\beta\right)E_t\left[\delta_{t+1}\right] - \left(r^f_t + \frac{1}{2}\sigma^2\right) + \frac{\sigma^2}{\alpha}\mu_t.$$

This shows $\frac{dy_t}{dr^f_t} = -1$ and completes the characterization of equilibrium.

We next establish the comparative statics with respect to the noise variance σ^2_{μ}. Observe that $P\left(x\right)$ in $(A.34)$ corresponds to an upward-sloping parabola with two positive roots. Observe also that increasing σ^2_{μ} increases $P\left(x\right)$ for each x and therefore lifts the parabola upward. Therefore, increasing σ^2_{μ} increases the smaller root (while reducing the larger root). Since the equilibrium volatility σ^2 corresponds to the smaller root, increasing σ^2_{μ} increases σ^2.

Finally, we characterize the output-gap loss $G_t = \underline{E}_t\left[\sum_{h=0}^{\infty}\beta^h\tilde{y}^2_{t+h}\right]$ along the equilibrium path. Note that the output gaps are given $\tilde{y}_{t+h} = \varepsilon_{\mu,t+h}\frac{\sigma^2}{\alpha}$. This implies $G_t = \frac{\sigma^2_{\mu}\left(\sigma^2\right)^2}{1-\beta}$. In particular, increasing σ^2_{μ} also increases G_t both directly by increasing noise and also indirectly by increasing the impact of noise. This completes the proof of the proposition. □

Proof of Proposition 2. To characterize the equilibrium, observe that the central bank's modified problem can be written as

$$G^{FCI}_t\left(\overline{p}_t\right) = \min_{r^f_t,\overline{p}_{t+1}} \underline{E}_t\left[\left(y_t - y^*_t\right)^2 + \psi\left(p_t - \overline{p}_t\right)^2\right] + \beta\underline{E}_t\left[G^{FCI}_{t+1}\left(\overline{p}_{t+1}\right)\right].$$

The expected future gaps $\underline{E}_t\left[G_{t+1}\left(\overline{p}_{t+1}\right)\right]$ depend on the announced target $\overline{p}_{t+1}$ but not on the current policy rate r_t^f (because the model is forward looking). Thus, the optimality condition for r_t^f is given by

$$\underline{E}_t \left[\frac{dy_t}{dr_t^f} (y_t - y_t^*)^2 + \psi \frac{dp_t}{dr_t^f} (p_t - \overline{p}_t)^2 \right] = 0.$$

We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr_t^f} = \frac{dp_t}{dr_t^f} = -1$. Therefore, the optimality condition implies

$$\underline{E}_t [y_t - y_t^*] + \psi \underline{E}_t [p_t - \overline{p}_t] = 0.$$

Substituting $y_t = m + p_t + \delta_t$ and $y_t^* = p_t^* + m + \delta_t$, we obtain

$$\underline{E}_t [p_t] - p_t^* + \psi (\underline{E}_t [p_t] - \overline{p}_t) = 0.$$

After rearranging, we obtain the optimality condition

$$\underline{E}_t [p_t] = \frac{1}{1+\psi} p_t^* + \frac{\psi}{1+\psi} \overline{p}_t = p_t^* + \frac{\psi}{1+\psi} (\overline{p}_t - p_t^*) . \tag{A.35}$$

Under FCI targeting, the central bank's expected asset price is a weighted average of its pre-announced target $\overline{p}_t$ and the current "pstar" p_t^*.

We next conjecture an equilibrium in which $\sigma^2_{t,r_{t+1}} \equiv \sigma^2$ is constant over time, the central bank announces the expected "pstar" as its target $\overline{p}_t = \underline{E}_{t-1} [p_t^*]$, and the aggregate asset price satisfies

$$p_t = \frac{\psi}{1+\psi} \overline{p}_t + \frac{1}{1+\psi} p_t^* + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}.$$

Taking the expectation of this expression and using $\underline{E}_t [\varepsilon_{\mu,t}] = 0$, we obtain $(A.35)$. Hence, the conjectured allocation satisfies the optimality condition. Note also that this expression implies

$$\begin{aligned} p_t &= \underline{E}_{t-1} [p_t^*] + \frac{1}{1+\psi} \left(p_t^* - \underline{E}_{t-1} [p_t^*]\right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \\ &= \underline{E}_{t-1} [p_t^*] + \frac{1}{1+\psi} \left(y_t^* - \delta_t - \underline{E}_{t-1} [y_t^* - \delta_t]\right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \\ &= \underline{E}_{t-1} [p_t^*] + \frac{1}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}. \end{aligned}$$

The first line substitutes the optimal target for period $t-1$ using (26), $\overline{p}_t = \underline{E}_{t-1} [p_t^*]$, the second line substitutes for p_t^*, and the last line uses the definition of supply and demand surprises, $y_t^* = \underline{E}_{t-1} [y_t^*] + \varepsilon_{z,t}$ and $\delta_t = \underline{E}_{t-1} [\delta_t] + \varepsilon_{\delta,t}$. This proves Eq. (27).

Substituting (27) into (9), we further obtain

$$\begin{aligned} y_t &= m + \delta_t + E_{t-1} [y_t^* - \delta_t - m] + \frac{1}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} x \\ &= \varepsilon_{\delta,t} + y_t^* - \varepsilon_{z,t} + \frac{1}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \\ &= y_t^* + \frac{\psi}{1+\psi} (\varepsilon_{\delta,t} - \varepsilon_{z,t}) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \end{aligned}$$

The last line substitutes the definition of demand shocks $\varepsilon_{\delta,t} = \delta_t - E_{t-1} [\delta_t]$. This proves (28).

We substitute the aggregate asset price into (11) to characterize the equilibrium return,

$$\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\
&= \rho + \underline{E}_t\left[p^*_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} \\
&\quad - \left(\underline{E}_{t-1}\left[p^*_t\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + E_t\left[y^*_{t-1} + \varepsilon_{z,t} + \varepsilon_{z,t+1} - \delta_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} \\
&\quad - \left(E_{t-1}\left[y^*_{t-1} + \varepsilon_{z,t} - \delta_t\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + \left(\varphi_\delta\delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi}\right) - \left(\varphi_\delta\delta_t + \frac{\varepsilon_{\delta,t+1}}{1+\psi}\right) + (1-\beta)\left(\varphi_\delta\delta_t + \varepsilon_{\delta,t+1}\right) \\
&\quad + \frac{\varepsilon_{z,t+1}}{1+\psi} + \left(\varepsilon_{z,t} - \frac{1}{1+\psi}\varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right).
\end{aligned}$$

The third equation substitutes p^*_{t+1} and p^*_t. We also replace $\underline{E}\left[\cdot\right]$ with $E\left[\cdot\right]$ since the realization of noise does not affect the terms inside the expectation. The last equation substitutes the AR(1) process for δ_t and collects similar terms together. This proves (30) where the expected return is given by

$$E_t\left[r_{t+1}\right] = \rho + \varphi_\delta\delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi} - \beta\varphi_\delta\delta_t + \frac{\psi\varepsilon_{z,t}}{1+\psi} - \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}. \tag{A.36}$$

We next combine the expression for the expected return with (15) to calculate the interest rate,

$$r^f_t = \rho - \frac{1}{2}\sigma^2 + \varphi_\delta\delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi} - \beta\varphi_\delta\delta_t + \frac{\psi\varepsilon_{z,t}}{1+\psi} + \frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1},$$

where we substituted the AR(1) process for μ_t from (12). This proves (29).

We next use (30) to calculate the conditional return volatility as

$$\begin{aligned}
\sigma^2 &= var_t\left(r_{t+1}\right) = \sigma^2_{macro}\left(\psi\right) + \left(\frac{\sigma^2}{\alpha}\right)^2\sigma^2_\mu \\
\text{where } \sigma^2_{macro}\left(\psi\right) &= \sigma^2_z\left(\frac{1}{1+\psi}\right)^2 + \sigma^2_\delta\left(\frac{1}{1+\psi} - (1-\beta)\right)^2.
\end{aligned}$$

In particular, the conditional volatility is a root of a quadratic, $P\left(\sigma^2;\psi\right) = 0$, given by

$$P\left(x;\psi\right) = \frac{\sigma^2_\mu}{\alpha^2}x^2 - x + \sigma^2_{macro}\left(\psi\right). \tag{A.37}$$

Note that $\sigma^2_{macro}\left(\psi\right)$ is convex, minimized at some $\overline{\psi} > 0$, and satisfies $\sigma^2_{macro}\left(0\right) = \sigma^2_z + \beta^2\sigma^2_\delta$ and $\lim_{\psi\to\infty}\sigma^2_{macro}\left(\psi\right) = \left(1-\beta\right)^2\sigma^2_\delta$. Since $\beta > 1-\beta$, this implies $\sigma^2_{macro}\left(0\right) \geq \sigma^2_{macro}\left(\psi\right)$ for each $\sigma^2_{macro}\left(\psi\right)$ Therefore, the assumed parametric condition $\alpha^2 > 4\sigma^2_\mu\left(\sigma^2_z + \beta^2\sigma^2_\delta\right)$ implies that $\alpha^2 > 4\sigma^2_\mu\sigma^2_{macro}\left(\psi\right)$ for each $\psi \geq 0$. Consequently, the polynomial in $(A.37)$ has two positive roots for each $\psi \geq 0$. The smaller root corresponds to the stable equilibrium. This proves (31).

Along the equilibrium path, output satisfies $y_t = m + p_t + \delta_t$ where the asset price satisfies (16)

$$p_t = \rho + E_t[p_{t+1}] + (1-\beta) E_t[\delta_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma^2\right) + \frac{\sigma^2}{\alpha}\mu_t.$$

This verifies our conjectures $\frac{dy_t}{dr_t^f} = \frac{dp_t}{dr_t^f} = -1$.

It remains to verify our conjecture that it is optimal for the central bank to announce the target in (26). Fix period $t-1$ and consider the optimal choice of $\overline{p}_t$. This is chosen to minimize the objective function $\underline{E}_{t-1}\left[G_t^{FCI}(\overline{p}_t)\right]$ (since $\overline{p}_t$ does not affect the gaps in period $t-1$). To characterize this, note that Eq. $(A.35)$ applies for an arbitrary target $\overline{p}_t$,

$$\underline{E}_t[p_t] = p_t^* + \frac{\psi}{1+\psi}(\overline{p}_t - p_t^*) = \overline{p}_t + \frac{1}{1+\psi}(p_t^* - \overline{p}_t).$$

Combining this with $y_t = m + p_t + \delta_t$ and $y_t^* = p_t^* + m + \delta_t$, we also obtain the following expression for output that applies for an arbitrary target $\overline{p}_t$,

$$\underline{E}_t[y_t] = y_t^* + \frac{\psi}{1+\psi}(\overline{p}_t - p_t^*).$$

Substituting these expressions into the objective function, we obtain

$$\begin{aligned}\underline{E}_{t-1}\left[G_t^{FCI}(\overline{p}_t)\right] &= \underline{E}_{t-1}\left[(y_t - y_t^*)^2 + \psi(p_t - \overline{p}_t)^2\right] + \beta \underline{E}_{t-1}\left[G_{t+1}^{FCI}(\overline{p}_{t+1})\right] \\ &= \left(\left(\frac{\psi}{1+\psi}\right)^2 + \psi\left(\frac{1}{1+\psi}\right)^2\right)\underline{E}_{t-1}\left[(\overline{p}_t - p_t^*)^2\right] + \beta \underline{E}_{t-1}\left[G_{t+1}^{FCI}(\overline{p}_{t+1})\right].\end{aligned}$$

Taking the derivative with respect to $\overline{p}_t$ and observing that $G_{t+1}^{FCI}(\overline{p}_{t+1})$ does not depend on $\overline{p}_t$, we find $\overline{p}_t = \underline{E}_{t-1}[p_t^*]$. This verifies (26) and completes the characterization of equilibrium.

Next consider the comparative statics of return variance with respect to ψ. Recall that $x = \sigma^2$ corresponds to the smaller (positive) root of the polynomial $P(x;\psi)$ in $(A.37)$. This is an upward sloping parabola with two positive roots and the solution corresponds to the smaller root. Note that $P(0;\psi) = \sigma^2_{macro}(\psi)$. Note also that $\sigma^2_{macro}(\psi)$ is convex with a minimum that satisfies $\overline{\psi} > \frac{\beta}{1-\beta} > 0$. Therefore, increasing ψ over the range $[0, \overline{\psi}]$ shifts the parabola upward and reduces the smaller root. This proves that increasing ψ over the range $[0, \overline{\psi}]$ reduces both $\sigma^2_{macro}(\psi)$ and σ^2. Conversely, increasing ψ over the range $(\overline{\psi}, \infty)$ increases both $\sigma^2_{macro}(\psi)$ and σ^2. □

Proof of Proposition 3. Note that Eq. (28) implies (34). After substituting this into (33) and calculating the variances, we further obtain (35). Differentiating with respect to ψ, we obtain $\frac{dG^e_{macro}(\psi)}{d\psi} = 0$ and

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} = \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} = \frac{2}{1-\beta}\left(\sigma_\mu^2 \sigma^2 \frac{d\sigma^2}{d\psi}|_{\psi=0}\right) < 0.$$

The inequality follows since Proposition 2 shows that $\frac{d\sigma^2}{d\psi} < 0$ over the range $\psi \in [0, \overline{\psi}]$. This completes the proof. □

A.3. FCI targeting vs interest rate targeting

This section analyzes the extension we discuss in Section 4.5 where the central bank targets the future interest rate rather than the future FCI. We show that FCI targeting is strictly superior to interest rate targeting.

To capture interest rate targeting, consider the baseline model from Section 4 but suppose the central bank solves problem (39), which we replicate here:

$$G_t^{R\text{-target}} = \min_{r_t^f, \bar{r}_{t+1}^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(r_{t+h}^f - \bar{r}_{t+h}^f \right)^2 \right] \right],$$

where $\bar{r}_{t+h}^f$ denotes an interest rate target that the central bank announces in the previous period, $t+h-1$ (the initial target $\bar{r}_0^f$ is given). Similarly to FCI targeting, the central bank penalizes the deviations of *the interest rate* (rather than the FCI) from a pre-announced target. As before, the central bank's true objective function is unchanged and still given by (33).

The following result characterizes the equilibrium with interest rate targeting. We focus on the case in which there are no supply shocks, $\varepsilon_{z,t} = 0$, and demand shocks are transitory, $\varphi_\delta = 0$. This case makes the analysis tractable and directly comparable to FCI targeting, but the qualitative results also hold with supply shocks and more general processes for demand shocks.

Proposition 5 (Equilibrium with Interest Rate Targeting)**.** *Suppose the planner follows the interest rate targeting policy in* (39) *with $\psi \geq 0$, there are no supply shocks $\varepsilon_{z,t} = 0$ and demand shocks are transitory $\varphi_\delta = 0$, the parameters satisfy $\alpha^2 > 4\sigma_\delta^2 \left(\frac{1}{1+\psi} - (1-\beta) \right)^2 \sigma_\mu^2 \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right)^2$, and the initial target satisfies $\bar{r}_0 = \underline{E}_{-1}\left[r_0^f \right]$. There is a (stable) equilibrium in which the planner announces the expected interest rate for the next period as its target $\bar{r}_{t+1}^f = \underline{E}_t \left[r_{t+1}^f \right]$. The equilibrium, asset price, output, and interest rate are given by*

$$p_t = y_t^* - m - \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t} \right), \tag{A.38}$$

$$y_t = y_t^* + \frac{\psi \varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t} \right), \tag{A.39}$$

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\varphi_\mu^2 \mu_{t-1} + \frac{1}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} \right). \tag{A.40}$$

The equilibrium return is

$$\begin{aligned} r_{t+1} &= E_t\left[r_{t+1} \right] - \varepsilon_{\delta,t+1} \left(\frac{1}{1+\psi} - (1-\beta) \right) + \frac{\sigma^2}{\alpha} \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t+1}, \qquad \text{(A.41)} \\ \textit{where } E_t\left[r_{t+1} \right] &= \rho + \frac{\varepsilon_{\delta,t}}{1+\psi} - \left[\varepsilon_{\mu,t} + \frac{\psi}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha}. \end{aligned}$$

The return variance $\sigma^2 = var_t\left(r_{t+1} \right)$ is the smaller positive solution to the fixed point problem

$$\begin{aligned} \sigma^2 &= \sigma_{macro}^2(\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2} \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right)^2 \sigma_\mu^2, \qquad \text{(A.42)} \\ \textit{where } \sigma_{macro}^2(\psi) &= \sigma_\delta^2 \left(\frac{1}{1+\psi} - (1-\beta) \right)^2 \end{aligned}$$

For a fixed ψ, the solution satisfies $\sigma^2 > \left(\sigma^{FCI}\right)^2$ where $\left(\sigma^{FCI}\right)^2$ is the equilibrium return variance with FCI targeting characterized in Proposition 2.

Comparing Eqs. $(A.39)$ and (28) shows that for a given return variance σ^2 interest rate targeting generates greater output gap volatility than FCI targeting. The last part of the result shows that interest rate targeting also induces greater return variance σ^2, which further increases output gap volatility. It follows that interest rate targeting is inferior to FCI targeting (it achieves higher expected squared output gaps). Intuitively, as we discuss in the main text, interest rate targeting stabilizes the incorrect financial variable and reduces the central bank's flexibility to respond to recent noise shocks, $\varphi_\mu \varepsilon_{\mu,t-1}$. This reduced flexibility implies that recent noise shocks affect asset prices and output gaps (captured by the term $\varphi_\mu \varepsilon_{\mu,t-1}$). Moreover, current noise shocks have a larger price impact, because financial markets anticipate that the central bank will not fully offset noise shocks (captured by the term $1 + \frac{\psi}{1+\psi}\varphi_\mu$).

Proof of Proposition 5. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future rate as its target $\overline{r}_t^f = \underline{E}_{t-1}\left[r_t^f\right]$, and the asset price and the interest rate satisfies

$$\begin{aligned} p_t &= y_t^* - m + D_p \varepsilon_{\delta,t} + \left(M_{p,1}\varphi_\mu \varepsilon_{\mu,t-1} + M_{p,0}\varepsilon_{\mu,t}\right)\frac{\sigma^2}{\alpha}, \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + D_r \varepsilon_{\delta,t} + \left(\varphi_\mu^2 \mu_{t-2} + M_{r,1}\varphi_\mu \varepsilon_{\mu,t-1}\right)\frac{\sigma^2}{\alpha} \end{aligned} \tag{A.43}$$

for undetermined coefficients $D_p, D_r, M_{p,1}, M_{p,0}, M_{r,1}$. Note that we allow the asset price and interest rate to react to the past period noise surprise as well as the current-period noise surprise $\varepsilon_{\mu,t}$. However, the interest rate cannot respond to the current noise surprise $\varepsilon_{\mu,t}$.We also conjecture that the interest rate will fully stabilize the current price impact of the noise shock from two periods before.

The optimality condition for r_t^f is given by

$$\underline{E}_t\left[\frac{dy_t}{dr_t^f}\left(y_t - y_t^*\right)^2 + \psi\left(r_t^f - \overline{r}_t^f\right)^2\right] = 0.$$

As before, we conjecture (and verify later) that $\frac{dy_t}{dr_t^f} = -1$. Therefore, the optimality condition implies

$$\underline{E}_t\left[y_t\right] - y_t^* = \psi \underline{E}_t\left[r_t^f - \overline{r}_t^f\right].$$

Using the conjecture $\underline{E}_{t-1}\left[\overline{r}_t^f\right] = r_t^f$, observing that $\underline{E}_t\left[r_t^f\right] = r_t^f$, this further implies

$$\underline{E}_t\left[y_t\right] - y_t^* = \psi\left(r_t^f - \underline{E}_{t-1}\left[r_t^f\right]\right).$$

The pre-noise output is centered around y_t^* but it shifts with the information that shifts r_t^f between periods $t-1$ and t due to the policy pre-commitment. Substituting $y_t = m + p_t + \varepsilon_{\delta,t}$ (since demand shocks are i.i.d.), we further obtain

$$\underline{E}_t\left[p_t\right] = y_t^* - m - \varepsilon_{\delta,t} + \psi\left(r_t^f - \underline{E}_{t-1}\left[r_t^f\right]\right). \tag{A.44}$$

Combining this with $(A.43)$, we find

$$D_p \varepsilon_{\delta,t} + M_{p,1} \varphi_\mu \varepsilon_{\mu,t-1} \frac{\sigma^2}{\alpha} = -\varepsilon_{\delta,t} + \psi \left(D_r \varepsilon_{\delta,t} + M_{r,1} \varphi_\mu \varepsilon_{\mu,t-1} \frac{\sigma^2}{\alpha} \right).$$

The optimality condition holds for all shocks if the undetermined coefficients satisfy

$$\begin{aligned} D_p &= -1 + \psi D_r, \\ M_{p,1} &= \psi M_{r,1}. \end{aligned} \tag{A.45}$$

We next substitute the conjectured price into (11) to calculate the equilibrium return

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\, \varepsilon_{\delta,t+1} - p_t \\ &= \rho + [D_p + 1 - \beta]\, \varepsilon_{\delta,t+1} - D_p \varepsilon_{\delta,t} \\ &\quad + \left[\left(M_{p,1} \varphi_\mu - M_{p,0} \right) \varepsilon_{\mu,t} + M_{p,0} \varepsilon_{\mu,t+1} - M_{p,1} \varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha} \\ &= E_t [r_{t+1}] + \varepsilon_{\delta,t+1} [D_p + 1 - \beta] + \varepsilon_{\mu,t+1} M_{p,0} \frac{\sigma^2}{\alpha}, \end{aligned}$$

where the expected return is given by

$$E_t [r_{t+1}] = \rho - D_p \varepsilon_{\delta,t} + \left[\left(M_{p,1} \varphi_\mu - M_{p,0} \right) \varepsilon_{\mu,t} - M_{p,1} \varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha}.$$

We combine this expression with (15) to calculate the interest rate,

$$\begin{aligned} r_t^f &= \rho - \frac{1}{2}\sigma^2 - D_p \varepsilon_{\delta,t} + \left[\mu_t + \left(M_{p,1} \varphi_\mu - M_{p,0} \right) \varepsilon_{\mu,t} - M_{p,1} \varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha} \\ &= \rho - \frac{1}{2}\sigma^2 - D_p \varepsilon_{\delta,t} + \left[\varphi_\mu^2 \mu_{t-1} + \left(1 + M_{p,1} \varphi_\mu - M_{p,0} \right) \varepsilon_{\mu,t} + (1 - M_{p,1})\, \varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha}. \end{aligned}$$

Here, the second line substitutes $\mu_t = \varphi_\mu^2 \mu_{t-1} + \varphi_\mu \varepsilon_{\mu,t-1} + \varepsilon_{\mu,t}$ and collects terms. Comparing this with the conjectured interest rate in $(A.43)$, the undetermined coefficients must satisfy

$$\begin{aligned} D_r &= -D_p, \\ M_{r,1} &= 1 - M_{p,1}, \\ 1 + M_{p,1} \varphi_\mu - M_{p,0} &= 0. \end{aligned} \tag{A.46}$$

Combining Eqs. $(A.45)$ and $(A.46)$, we solve for the equilibrium coefficients

$$\begin{aligned} D_p &= -\frac{1}{1+\psi} \text{ and } D_r = \frac{1}{1+\psi}, \\ M_{p,1} &= \frac{\psi}{1+\psi} \text{ and } M_{r,1} = \frac{1}{1+\psi}, \\ M_{p,0} &= 1 + \varphi_\mu \frac{\psi}{1+\psi}. \end{aligned}$$

Substituting the solution into $(A.43)$ verifies that the equilibrium asset price and interest rate are given by $(A.38)$ and $(A.40)$. Combining the asset price expression with $y_t = m + p_t + \varepsilon_{\delta,t}$ verifies that output is given by $(A.39)$. Substituting the solution into the expression for the return verifies that the return is given by $(A.41)$.

Finally, observe that Eq. (30) implies that σ^2 solves the fixed point problem $(A.42)$. Under the assumed parametric condition, this problem has two positive roots. The smaller root corresponds to the stable equilibrium.

It remains to verify our conjectures that $\frac{dy_t}{dr_t^f} = -1$ and the central bank optimally announces the expected interest rate as its target $\underline{E}_{t-1}\left[\overline{r}_t^f\right] = r_t^f$. These follow from similar steps as in the proof of Proposition 2.

Finally, consider the comparative statics exercise. Note that σ^2 and $\left(\sigma^{FCI}\right)^2$ are the smaller root of the following two polynomials, respectively:

$$\begin{aligned} P(x) &= \left(1 + \frac{\psi}{1+\psi}\varphi_\mu\right)^2 \frac{\sigma_\mu^2}{\alpha^2} x^2 - x + \sigma_{macro}^2(\psi), \\ P^{FCI}(x) &= \frac{\sigma_\mu^2}{\alpha^2} x^2 - x + \sigma_{macro}^2(\psi). \end{aligned}$$

Observe that $P(x) > P^{FCI}(x)$ for each $x > 0$. Since $P\left(\sigma^2\right) = 0$, this implies $P^{FCI}\left(\sigma^2\right) < 0$. This in turn implies $\left(\sigma^{FCI}\right)^2 < \sigma^2$ because $\left(\sigma^{FCI}\right)^2$ is the smaller positive root of $P^{FCI}(x)$. $\square$

A.4. FCI targeting with policy lags to all current shocks

This section analyzes the extension we discuss in Section 4.6.1 the central bank sets policy before observing all current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$ (rather than only $\varepsilon_{\mu,t}$). We show that macroeconomic shocks still induce asset price volatility due to the *anticipated* policy reaction to these shocks. Therefore, as in the main text, an appropriate FCI targeting policy reduces return volatility and the price impact of noise shocks.

Formally, consider the baseline model from Section 4 but suppose the central bank solves the following modified problem

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} E_{t-1} \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^*\right)^2 + \psi \left(p_{t+h} - \overline{p}_{t+h}\right)^2 \right] \right]. \tag{A.47}$$

The policy sets the interest rate r_t^f and next-period's price target $\overline{p}_{t+1}$ under the information set of period $t-1$, before observing the shocks in period t. The following result characterizes the equilibrium for the case without supply shocks, $\varepsilon_{z,t} = 0$ (results qualitatively hold also with supply shocks).

Proposition 6 (Equilibrium with policy lags to all current shocks)**.** *Suppose the planner sets policy before observing all current-period shocks and it follows the FCI targeting policy in (A.47) with $\psi \in [0, \overline{\psi})$ where $\overline{\psi}$ is defined below. Suppose there are no supply shocks $\varepsilon_{z,t} = 0$, the parameters satisfy $\alpha^2 \geq 4\sigma_\delta^2 \sigma_\mu^2 \left(\beta\varphi_\delta - (1-\beta)\right)^2$, and the initial target satisfies $\overline{p}_0 = E_{-2}\left[p_0^*\right]$. There is a (stable) equilibrium in which the planner announces the expected "pstar" for the next period as its target $\overline{p}_{t+1} = E_{t-1}\left[p_{t+1}^*\right]$ where $p_{t+1}^* = y_{t+1}^* - m - \delta_{t+1}$. The equilibrium asset price, output, and interest rate are given by*

$$\begin{aligned} p_t &= y_t^* - m - \varphi_\delta^2 \delta_{t-2} - \frac{1}{1+\psi} \varphi_\delta \varepsilon_{\delta,t-1} - \left(\frac{1}{1+\psi} - (1-\beta) \right) \varphi_\delta \varepsilon_{\delta,t} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, & \text{(A.48)} \\ y_t &= y_t^* + \frac{\psi}{1+\psi} \varphi_\delta \varepsilon_{\delta,t-1} + \left[1 - \left(\frac{1}{1+\psi} - (1-\beta) \right) \varphi_\delta \right] \varepsilon_{\delta,t} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, & \text{(A.49)} \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta) \varphi_\delta^2 \delta_{t-2} + \left(\frac{1}{1+\psi} - \beta\varphi_\delta \right) \varphi_\delta \varepsilon_{\delta,t-1} + \mu_{t-1} \frac{\sigma^2}{\alpha}. & \text{(A.50)} \end{aligned}$$

The equilibrium return is

$$\begin{aligned} r_{t+1} &= E_t\left[r_{t+1}\right] - \varepsilon_{\delta,t+1} \left[\varphi_\delta \left(\frac{1}{1+\psi} - (1-\beta) \right) - (1-\beta) \right] + \varepsilon_{\mu,t+1} \frac{\sigma^2}{\alpha}, \qquad \text{(A.51)} \\ \textit{where } E_t\left[r_{t+1}\right] &= \rho + (1-\beta\varphi_\delta) \varphi_\delta^2 \delta_{t-2} + \left(\frac{1}{1+\psi} - \beta\varphi_\delta \right) \varphi_\delta \varepsilon_{\delta,t-1} - \varepsilon_{\mu,t} \frac{\sigma^2}{\alpha}. \end{aligned}$$

The return variance $\sigma^2 = var_t\left(r_{t+1}\right)$ is the smaller positive solution to the fixed point problem

$$\begin{aligned} \sigma^2 &= \sigma_{macro}^2(\psi) + \sigma_\mu^2 \frac{\left(\sigma^2\right)^2}{\alpha^2}, & \text{(A.52)} \\ \textit{where } \sigma_{macro}^2(\psi) &= \sigma_\delta^2 \left[\varphi_\delta \left(\frac{1}{1+\psi} - (1-\beta) \right) - (1-\beta) \right]^2. \end{aligned}$$

Let $\overline{\psi} = \arg\min_{\psi \geq 0} \sigma_{macro}^2(\psi)$. Over the range $\psi \in [0, \overline{\psi})$, increasing ψ strictly reduces σ^2. That is, stronger FCI targeting reduces the return variance.

Proof of Proposition 6. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future "pstar" as its target $\overline{p}_t = E_{t-2}[p_t^*]$, and the asset price and the interest rate satisfies

$$\begin{aligned} p_t &= y_t^* - m - \varphi_\delta^2 \delta_{t-2} + D_{p,1}\varphi_\delta \varepsilon_{\delta,t-1} + D_{p,0}\varepsilon_{\delta,t} + \varepsilon_{\mu,t}\frac{\sigma^2}{\alpha}, \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\varphi_\delta^2 \delta_{t-2} + D_{r,1}\varphi_\delta \varepsilon_{\delta,t-1} + \mu_{t-1}\frac{\sigma^2}{\alpha} \end{aligned} \tag{A.53}$$

for appropriate coefficients $D_{p,1}, D_{p,0}, D_{r,1}$. Note that we allow the asset price to react to the past period demand shocks as well as the current-period demand shock. However, the interest rate cannot react to the current period demand shock since the central bank sets the policy before observing δ_t. We also conjecture that the central bank will fully stabilize the current price impact of the demand shock from two periods before as well as the noise shock from the last period.

Following similar steps as in the proof of Proposition 2, we obtain

$$E_{t-1}[p_t] = \frac{1}{1+\psi}E_{t-1}[p_t^*] + \frac{\psi}{1+\psi}\overline{p}_t.$$

Substituting $\overline{p}_t = E_{t-2}[p_t^*], p_t^* = y_t^* - m - \delta_t$ and the AR(1) process for δ_t, we further obtain

$$\begin{aligned} E_{t-1}[p_t] &= \frac{1}{1+\psi}E_{t-1}[p_t^*] + \frac{\psi}{1+\psi}E_{t-2}[p_t^*] \\ &= y_t^* - m - \frac{1}{1+\psi}E_{t-1}[\delta_t] + \frac{\psi}{1+\psi}E_{t-2}[\delta_t] \\ &= y_t^* - m - \varphi_\delta^2\delta_{t-2} - \frac{1}{1+\psi}\varphi_\delta\varepsilon_{\delta,t-1}. \end{aligned}$$

The central bank's expected asset price partially incorporates the recent demand shock $\varepsilon_{\delta,t-1}$. Combining this with $(A.53)$, we find that the optimality condition holds if the coefficient on past demand satisfies:

$$D_{p,1} = -\frac{1}{1+\psi}. \tag{A.54}$$

We next substitute the conjectured price into (11) to calculate the equilibrium return

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\delta_{t+1} - p_t \\ &= \rho - \varphi_\delta^2\delta_{t-1} + D_{p,1}\varphi_\delta\varepsilon_{\delta,t} + D_{p,0}\varepsilon_{\delta,t+1} + \varepsilon_{\mu,t+1}\frac{\sigma^2}{\alpha} \\ &\quad + \varphi_\delta^2\delta_{t-2} - D_{p,1}\varphi_\delta\varepsilon_{\delta,t-1} - D_{p,0}\varepsilon_{\delta,t} - \varepsilon_{\mu,t}\frac{\sigma^2}{\alpha} \\ &\quad + (1-\beta)\left[\varphi_\delta^2\delta_{t-1} + \varphi_\delta\varepsilon_{\delta,t} + \varepsilon_{\delta,t+1}\right] \\ &= E_t[r_{t+1}] + \varepsilon_{\delta,t+1}[D_{p,0} + 1 - \beta] + \varepsilon_{\mu,t+1}\frac{\sigma^2}{\alpha}, \end{aligned}$$

where the expected return is given by

$$\begin{aligned}
E_t[r_{t+1}] &= \rho + \varphi_\delta^2 \delta_{t-2} - \beta \varphi_\delta^2 \delta_{t-1} \\
&\quad + (D_{p,1}\varphi_\delta - D_{p,0} + (1-\beta)\varphi_\delta)\,\varepsilon_{\delta,t} - D_{p,1}\varphi_\delta \varepsilon_{\delta,t-1} - \varepsilon_{\mu,t}\frac{\sigma^2}{\alpha} \\
&= \rho + (1-\beta\varphi_\delta)\,\varphi_\delta^2 \delta_{t-2} + (D_{p,1}\varphi_\delta - D_{p,0} + (1-\beta)\varphi_\delta)\,\varepsilon_{\delta,t} \\
&\quad - \left(D_{p,1}\varphi_\delta + \beta\varphi_\delta^2\right)\varepsilon_{\delta,t-1} - \varepsilon_{\mu,t}\frac{\sigma^2}{\alpha}.
\end{aligned}$$

Here, the second line substitutes $\delta_{t-1} = \varphi_\delta \delta_{t-2} + \varepsilon_{\delta,t-1}$ and collects terms. We combine this expression with (15) and substitute $\mu_t - \varepsilon_{\mu,t} = \varphi_\mu \mu_{t-1}$ to calculate the interest rate

$$\begin{aligned}
r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\,\varphi_\delta^2\delta_{t-2} + (D_{p,1}\varphi_\delta - D_{p,0} + (1-\beta)\varphi_\delta)\,\varepsilon_{\delta,t} \\
&\quad - \left(D_{p,1}\varphi_\delta + \beta\varphi_\delta^2\right)\varepsilon_{\delta,t-1} + \varphi_\mu \mu_{t-1}\frac{\sigma^2}{\alpha}.
\end{aligned}$$

Comparing this with the equilibrium conjecture in $(A.53)$, we solve for the undetermined coefficients as

$$\begin{aligned}
D_{p,0} &= (1-\beta)\,\varphi_\delta + D_{p,1}\varphi_\delta = \varphi_\delta\left(1 - \beta - \frac{1}{1+\psi}\right), \\
D_{r,1} &= -\left(D_{p,1} + \beta\varphi_\delta\right) = \frac{1}{1+\psi} - \beta\varphi_\delta.
\end{aligned} \tag{A.55}$$

Substituting $(A.54)$ and $(A.55)$ into $(A.53)$ verifies that the equilibrium asset price and interest rate are given by $(A.48)$ and $(A.50)$. Combining the asset price expression with $y_t = m + p_t + \delta_t$ verifies that output satisfies $(A.49)$. Substituting the solution into the expression for the return, we also find that the return satisfies $(A.51)$.

Finally, observe that Eq. $(A.51)$ implies that σ^2 solves the fixed point problem $(A.52)$. Under the assumed parametric condition, this problem has two positive roots for each $\psi \in [0, \overline{\psi})$. The smaller root corresponds to the stable equilibrium. The rest of the proof follows from similar steps as in the proof of Proposition 2. □

A.5. FCI targeting with inflation and output trade-off

This section analyzes the extension we discuss in Section 4.6.2 where prices are partially flexible and the central bank might face a trade-off between stabilizing inflation and output. In this case, cost-push shocks result in positive inflation and negative output gaps and create a new source of aggregate asset price volatility that further deters arbitrageurs. Moreover, noise shocks affect inflation gaps as well as output gaps. FCI targeting reduces the aggregate return volatility and enables arbitrageurs to absorb noise more effectively, reducing the impact of noise on inflation and output. Moreover, some degree of FCI targeting is still optimal and enables the central bank to achieve lower output gap and inflation losses. Intuitively, while cost-push shocks induce nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by these shocks. Therefore, small deviations from this policy generate only second-order losses, while still inducing first-order gains via the noise-reduction mechanism.

Environment with inflation. Formally, consider the baseline model from Section 4 but suppose inflation is not necessarily zero and follows the New Keynesian Phillips Curve (NKPC) that we derived in Appendix A.1 (see ($A.24$))

$$\begin{aligned}\pi_t &= \kappa \tilde{y}_t + \beta E_t\left[\pi_{t+1}\right] + u_t, \\ \text{where } u_t &= \varphi_u u_{t-1} + \varepsilon_{u,t} \text{ and } \sigma_u^2 \equiv var\left(\varepsilon_{u,t}\right).\end{aligned}$$

Here, $\pi_t \simeq \log \frac{Q_t}{Q_{t-1}}$ denotes inflation measured as the log change of the nominal price index Q_t. We assume the cost-push shocks u_t follow an AR(1) process that is independent from all other (supply, demand, and noise) shocks.

We adjust the financial market side of the model to allow for a *nominal* interest rate (which is what the Fed sets) in addition to the real interest rate. There is a *nominal* risk-free asset with nominal rate denoted by $\exp\left(i_t^f\right)$, in addition to the real risk-free asset with real rate $\exp\left(r_t^f\right)$, and the market portfolio with real return R_{t+1}. Both risk-free assets are in zero net supply. There are three sets of investors as in Section 3: noise traders, arbitrageurs, and inelastic funds. Noise traders and arbitrageurs are the same as before; in particular, they do not trade the nominal bonds. Likewise, inelastic funds are constrained to hold the average market portfolio weight $\omega_t^I = 1$. These assumptions ensure that we still have the financial market equilibrium condition in (15)

$$E_t\left[r_{t+1}\right] + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2 = r_t^f + \left(\sigma_{t,r_{t+1}}\right)^2\left(1 - \frac{\mu_t}{\alpha}\right).$$

There is a second financial equilibrium condition that describes the relationship between the nominal and the real rates. To derive this condition, we assume for simplicity that *only* the inelastic funds can trade the nominal bond in exchange for the real bond. They maximize the expected wealth under management similar to arbitrageurs. In equilibrium, their optimization problem implies

$$E_t\left[M_{t+1}^I\left(\frac{\exp\left(i_t^f\right)}{Q_{t+1}/Q_t} - \exp\left(r_t^f\right)\right)\right] = 0, \text{ where } M_{t+1}^I = \frac{1}{R_{t+1}}.$$

Assuming R_{t+1} and inflation $\frac{Q_{t+1}}{Q_t}$ are (approximately) log-normally distributed, we obtain

$$i_t^f = r_t^f + \left[E_t\left[\pi_{t+1}\right] - \frac{1}{2}\sigma_t^2\left(\pi_{t+1}\right)\right] - cov_t\left(\pi_{t+1}, r_{t+1}\right). \tag{A.56}$$

This equation is like the Fisher equation except that it also accounts for inflation risk. The nominal interest rate is equal to the real rate plus the expected inflation (adjusted for a Jensen's term) and an inflation risk premium. The latter depends on the covariance between the inflation and the real return, $-cov_t\left(\pi_{t+1}, r_{t+1}\right)$. Our assumption that nominal bonds are traded only by the inelastic funds ensures that *the current noise* μ_t does not affect the wedge between the nominal and the real rate (future noise can still affect the wedge via the covariance term). Thus, even though the Fed decides before observing μ_t, it can effectively still target a particular real interest rate r_t^f by setting the nominal rate i_t^f according to ($A.56$). In the rest of this appendix, we will assume the Fed "sets" the real interest rate r_t^f and verify that the implied nominal rate i_t^f does not depend on μ_t.

Finally, we modify the central bank's (true) objective function to capture the costs of inflation:

$$G_t = \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\tilde{y}_{t+h}^2 + \zeta \pi_{t+h}^2 \right] \right]. \tag{A.57}$$

We normalize the inflation target to zero. The parameter ζ captures the cost of inflation gaps relative to output gaps. The rest of the environment is the same as in Sections 3 and 4. The baseline model is the special case with $\kappa = u_t = 0$.

Equilibrium with discretionary policy. We first characterize the discretionary equilibrium. Suppose the central bank (effectively) sets r_t^f to maximize $(A.57)$ subject to the equilibrium conditions and taking its future actions as given. The solution is as in the textbook New Keynesian model (see Clarida et al. (1999)) with the difference that noise also affects the equilibrium outcomes. In particular, the central bank may no longer target a zero output gap on average. Its optimality condition is given by:

$$\underline{E}_t \left[\tilde{y}_t \right] = -\kappa \zeta \underline{E}_t \left[\pi_t \right]. \tag{A.58}$$

With a positive cost-push shock, the central bank targets a negative average output gap to stabilize inflation. The output gap is more negative when it has a greater impact on inflation (higher κ) and when the central bank puts a greater weight on inflation (high ζ). To solve for the equilibrium, we conjecture that the (pre-noise) output and inflation gaps are linear functions of the cost-push shock

$$\underline{E}_t \left[\pi_t \right] = \Pi_u u_t \text{ and } \underline{E}_t \left[\tilde{y}_t \right] = -Y_u u_t.$$

Combining this conjecture with the NKPC (and the AR(1) process for the cost-push shocks), we obtain the closed-form solutions

$$\Pi_u = \frac{1}{1 + \kappa^2 \zeta - \beta \varphi_u} \text{ and } Y_u = \frac{\kappa \zeta}{1 + \kappa^2 \zeta - \beta \varphi_u}. \tag{A.59}$$

The rest of the equilibrium is similar to Section 3.2 and is given by:

$$\begin{aligned}
p_t &= p_t^o + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \quad \text{ where } p_t^o \equiv y_t^* - m - \delta_t - Y_u u_t, && \text{(A.60)} \\
y_t &= y_t^* - Y_u u_t + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, \\
\pi_t &= \Pi_u u_t + \kappa \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, \\
r_t^f &= \rho - \frac{1}{2} \sigma^2 + (1 - \beta \varphi_\delta) \delta_t + (1 - \varphi_u) Y_u u_t + \frac{\sigma^2}{\alpha} \varphi_\mu \mu_{t-1}.
\end{aligned}$$

p_t^o is the central bank's optimal asset price target, which is different from p_t^* due to cost-push shocks. Noise creates additional gaps from central bank's targets and its impact depends on σ^2, which is the smaller solution to:

$$\sigma^2 = \sigma_{macro}^2 + \frac{\left(\sigma^2\right)^2}{\alpha^2} \sigma_\mu^2, \text{ where } \sigma_{macro}^2 = \sigma_z^2 + Y_u^2 \sigma_u^2 + \beta^2 \sigma_\delta^2.$$

In this case, the impact of noise is higher because cost-push shocks create a new source of asset price and return volatility.

Equilibrium with FCI targeting. We next consider the equilibrium with FCI targeting. In particular, suppose the central bank instead solves

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} E_{t-1}\left[\sum_{h=0}^{\infty}\beta^h\left[\left(y_{t+h}-y_{t+h}^*\right)^2+\zeta\pi_{t+h}^2+\psi\left(1+\kappa^2\zeta\right)\left(p_{t+h}-\overline{p}_{t+h}\right)^2\right]\right]. \tag{A.61}$$

Here, the term $1+\kappa^2\zeta$ is a normalizing factor for the FCI targeting objective that helps to simplify the expression. The next result characterizes the equilibrium.

Proposition 7 (Equilibrium with Inflation and FCI Targeting)**.** *Consider the setup with inflation described above and suppose the planner follows the FCI targeting policy in (A.61) with $\psi \geq 0$. Let Π_u, Y_u denote the coefficients in (A.59), and suppose the parameters satisfy $\alpha^2 \geq 4\sigma_\mu^2\left(\sigma_z^2+Y_u^2\sigma_u^2+\beta^2\sigma_\delta^2\right)$ (and $\beta > 1-\beta$) and the initial target satisfies $\overline{p}_0 = E_{-1}\left[p_0^o\right]$. Then, there is a (stable) equilibrium in which the planner announces as its target the expected optimal asset price for the next period*

$$\overline{p}_{t+1} = \underline{E}_t\left[p_{t+1}^o\right] \text{ where } p_{t+1}^o = y_{t+1}^* - m - \delta_{t+1} - Y_u u_{t+1}. \tag{A.62}$$

The equilibrium asset price, output, inflation, and real and nominal interest rates are

$$\begin{aligned}
p_t &= \underline{E}_{t-1}\left[p_t^o\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}-Y_u\varepsilon_{u,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, && \text{(A.63)}\\
y_t &= y_t^* - Y_u u_t - \frac{\psi}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}-Y_u\varepsilon_{u,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, && \text{(A.64)}\\
\pi_t &= \Pi_u u_t - \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}-Y_u\varepsilon_{u,t}\right)+\frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}, && \text{(A.65)}\\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1-\beta\varphi_\delta\right)\delta_t + Y_u\left(1-\varphi_u\right)u_t + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}-\varepsilon_{u,t}\right)+\frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1}. && \text{(A.66)}
\end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t\left[r_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1}-Y_u\varepsilon_{u,t+1}\right) - \left(\frac{1}{1+\psi}-(1-\beta)\right)\varepsilon_{\delta,t+1}+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \tag{A.67}$$

where $E_t\left[r_{t+1}\right]$ is given by (A.36). The return variance $\sigma^2 = var_t\left(r_{t+1}\right)$ is the smaller positive solution to the following fixed point problem

$$\begin{aligned}
&\sigma^2 = \sigma_{macro}^2(\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, && \text{(A.68)}\\
&\text{where } \sigma_{macro}^2(\psi) = \left(\sigma_z^2+Y_u^2\sigma_u^2\right)\left(\frac{1}{1+\psi}\right)^2+\sigma_\delta^2\left(\frac{1}{1+\psi}-(1-\beta)\right)^2.
\end{aligned}$$

Let $\overline{\psi} = \arg\min_{\psi\geq 0}\sigma_{macro}^2(\psi)$. Over the range $\psi \in [0,\overline{\psi})$, increasing ψ strictly reduces σ^2 as well as $\sigma_{macro}^2(\psi)$ and $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2$. The equilibrium nominal interest rate i_t^f is given by (A.74) and it does not depend on the current noise shock μ_t.

We relegate the proof of this result to the end of the theory appendix. The equilibrium with FCI targeting has a similar structure as before, with the difference that FCI targeting mitigates the policy response to cost-push shocks u_t as well as to supply and demand shocks (cf. Proposition 2). Consequently, cost-push shocks have a greater effect on inflation than with discretion. Moreover, since supply and demand shocks affect the output gaps, they also affect inflation unlike the case with discretion (cf. $(A.60)$). On the other hand, FCI targeting exerts a stabilizing influence on inflation, by mitigating the return volatility and the impact of noise on inflation as well as on output gaps.

Macro-stabilization effects of FCI targeting. We next explore the macro-stabilization effects of FCI targeting more systematically. As before, we evaluate the policy performance with the true loss function G_t in $(A.57)$. This function might depend on the current supply, demand, and cost-push shocks as well as the expected level of the cost-push shock, $\varepsilon_{z,t}, \varepsilon_{\delta,t}, \varepsilon_{u,t}, \varphi_u u_{t-1}$. To evaluate performance across a variety of shocks, we consider the unconditional expectation of this function given by

$$G^e(\psi) = E\left[G_t(\psi)\right] = E\left[\sum_{h=0}^{\infty} \beta^h \left[\tilde{y}_{t+h}^2(\psi) + \zeta \pi_{t+h}^2(\psi)\right]\right]. \tag{A.69}$$

Using Eqs. $(A.64)$ and $(A.65)$, output and inflation gaps are given by:

$$\begin{aligned}
\tilde{y}_t &= -Y_u\left(\varphi_u u_{t-1} + \varepsilon_{u,t}\right) + \frac{\psi}{1+\psi} Y_u \varepsilon_{u,t} - \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
\pi_t &= \Pi_u\left(\varphi_u u_{t-1} + \varepsilon_{u,t}\right) + \frac{\psi}{1+\psi}\kappa Y_u \varepsilon_{u,t} - \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}.
\end{aligned}$$

We substitute $\tilde{y}_t$ and π_t into $(A.69)$ to calculate and decompose $G^e(\psi)$ into two components:

$$G^e(\psi) = G^e_{macro}(\psi) + G^e_{noise}(\psi). \tag{A.70}$$

$G^e_{noise}(\psi)$ is the expected loss driven by noise shocks, which is given by a similar expression as before (cf. (35))

$$(1-\beta)\, G^e_{noise}(\psi) = \sigma_\mu^2 \left(\frac{\sigma^2}{\alpha}\right)^2 \left(1 + \zeta\kappa^2\right). \tag{A.71}$$

$G^e_{macro}(\psi)$ is the expected loss driven by macroeconomic shocks, which is given by

$$\begin{aligned}
(1-\beta)\, G^e_{macro}(\psi) = {} & \left(Y_u^2 + \zeta \Pi_u^2\right)\frac{\varphi_u^2 \sigma_u^2}{1-\varphi_u^2} \\
& + \left[\left(Y_u - \frac{\psi}{1+\psi} Y_u\right)^2 + \zeta\left(\Pi_u + \frac{\psi}{1+\psi}\kappa Y_u\right)^2\right]\sigma_u^2 \\
& + \left[\left(\frac{\psi}{1+\psi}\right)^2 \left(\sigma_z^2 + \sigma_\delta^2\right)\right]\left(1 + \zeta\kappa^2\right).
\end{aligned} \tag{A.72}$$

The first line uses the observation that the *unconditional* distribution of $\varphi_u u_{t-1}$ is given by $N\left(0, \frac{\varphi_u^2 \sigma_u^2}{1-\varphi_u^2}\right)$ to evaluate the losses driven by the conditionally expected level of the cost-push shock $\varphi_u u_{t-1}$. The second line evaluates the losses driven by the surprise component of cost-push shocks $\varepsilon_{u,t}$. The last line evaluates the losses driven by the supply and demand shocks. Our next result describes how FCI targeting affects $G^e(\psi)$ and its components.

Proposition 8 (Macrostabilization Effects of FCI Targeting with Inflation)**.** *Consider the equilibrium in Proposition 2. Then, a small degree of FCI targeting reduces the output-gap loss*

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} < 0, \text{ with } \frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0 \text{ and } \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} < 0.$$

Thus, $\psi^* = \arg\min_{\psi \geq 0} G_t(\psi) > 0$*, i.e., the gap loss minimizing policy features FCI targeting.*

Proof of Proposition 8. We differentiate Eq. $(A.72)$ with respect to ψ to obtain

$$\frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = \frac{2\sigma_u^2}{1-\beta}\left[-Y_u^2 + \zeta\kappa Y_u \Pi_u\right] = 0,$$

where we have used the observation that the coefficients satisfy $Y_u = \zeta\kappa\Pi_u$ in view of the central bank's optimality condition [see $(A.59)$ and $(A.58)$]. It follows that

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} = \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} = \frac{2\left(1+\zeta\kappa^2\right)}{1-\beta}\left(\sigma_\mu^2\sigma^2\frac{d\sigma^2}{d\psi}|_{\psi=0}\right) < 0.$$

The inequality follows since Proposition 7 shows that $\frac{d\sigma^2}{d\psi} < 0$ over the range $\psi \in \left[0, \overline{\psi}\right]$. □

In this case, unlike in the baseline model without inflation, $G^e_{macro}(0)$ is not necessarily zero: even absent FCI targeting, macroeconomic (cost-push) shocks induce some gap losses. Nonetheless, it is still the case that small degrees of FCI targeting has a second-order effect on these losses, $\frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0$. Intuitively, while cost-push shocks create nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by the cost-push shocks, u_t, captured by the condition $Y_u = \zeta\kappa\Pi_u$ [see $(A.58)$ and $(A.59)$]. Thus, small deviations from this policy generate only second-order losses, while still inducing first-order gains by reducing the impact of noise on inflation and output.

Proof of Proposition 7. The central bank's modified problem is given by

$$G_t^{FCI}(\overline{p}_t) = \min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t\left[(y_t - y_t^*)^2 + \zeta\pi_t^2 + \psi\left(1+\kappa^2\zeta\right)(p_t - \overline{p}_t)^2\right] + \beta\underline{E}_t\left[G_{t+1}^{FCI}(\overline{p}_{t+1})\right].$$

The optimality condition for r_t^f is given by

$$\underline{E}_t\left[\frac{dy_t}{dr_t^f}(y_t - y_t^*)^2 + \zeta\frac{d\pi_t}{dr_t^f}\pi_t + \psi\left(1+\kappa^2\zeta\right)\frac{dp_t}{dr_t^f}(p_t - \overline{p}_t)^2\right] = 0.$$

We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr_t^f} = \frac{dp_t}{dr_t^f} = -1$ and $\frac{d\pi_t}{dr_t^f} = \kappa$. Therefore, the optimality condition implies

$$\underline{E}_t\left[y_t - y_t^*\right] + \kappa\zeta\underline{E}_t\left[\pi_t\right] + \psi\left(1+\kappa^2\zeta\right)\underline{E}_t\left[p_t - \overline{p}_t\right] = 0. \tag{A.73}$$

We next conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future asset price target $\overline{p}_t = E_{t-1}[p_t^o]$, the expected next-period inflation is the same as in the case with discretion $E_t[\pi_{t+1}] = \Pi_u\varphi_u u_t$ [see $(A.60)$], and the equilibrium asset price is given by

$$p_t = \underline{E}_{t-1}[p_t^o] + P_z\varepsilon_{z,t} - P_\delta\varepsilon_{\delta,t} - P_u Y_u\varepsilon_{u,t} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},$$

for appropriate coefficients P_δ, P_z, P_u that describes the central bank's response new information. Substituting this conjecture into the output and asset price relation and using $p_t^o = y_t^* - m - \delta_t - Y_u u_t$ we obtain

$$y_t = y_t^* - Y_u u_t - (1 - P_z)\, \varepsilon_{z,t} + (1 - P_\delta)\, \varepsilon_{\delta,t} + (1 - P_u)\, Y_u \varepsilon_{u,t} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t},$$

where we have used $y_t^* = \underline{E}_{t-1}[y_t^*] + z_t, \delta_t = \underline{E}_{t-1}[\delta_t] + \varepsilon_{\delta,t}$ and $u_t = \underline{E}_{t-1}[u_t] + \varepsilon_{u,t}$. Substituting this into the NKPC and using $E_t[\pi_{t+1}] = \Pi_u \varphi_u u_t$, we further obtain

$$\begin{aligned} \pi_t &= -\kappa Y_u u_t + \beta \Pi_u \varphi_u u_t \\ &\quad -\kappa (1 - P_z)\, \varepsilon_{z,t} + \kappa (1 - P_\delta)\, \varepsilon_{\delta,t} + \kappa (1 - P_u)\, Y_u \varepsilon_{u,t} + \kappa \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t} \\ &= \Pi_u u_t - \kappa (1 - P_z)\, \varepsilon_{z,t} + \kappa (1 - P_\delta)\, \varepsilon_{\delta,t} + \kappa (1 - P_u)\, Y_u \varepsilon_{u,t} + \kappa \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}. \end{aligned}$$

Here, we have used $-\kappa Y_u + \beta \varphi_u \Pi_u = \Pi_u$ which holds from the definition of Y_u, Π_u (see $(A.59)$). Substituting these expressions into the optimality condition $(A.73)$, and using $Y_u = \kappa \zeta \Pi_u$, we obtain

$$\left[\begin{array}{c} \left(1 + \kappa^2 \zeta\right) \left(-(1 - P_z)\, \varepsilon_{z,t} + (1 - P_\delta)\, \varepsilon_{\delta,t} + (1 - P_u)\, Y_u \varepsilon_{u,t}\right) \\ +\psi \left(1 + \kappa^2 \zeta\right) \left(P_z \varepsilon_{z,t} - P_\delta \varepsilon_{\delta,t} - P_u Y_u \varepsilon_{u,t}\right) \end{array} \right] = 0.$$

Solving for the undetermined coefficients, we obtain

$$P_z = P_\delta = P_u = \frac{1}{1+\psi}.$$

This proves Eqs. $(A.63 - A.65)$. We verify that the solution for inflation satisfies the conjecture for expected inflation since $E_t[\pi_{t+1}] = \Pi_u E_t[u_{t+1}] = \Pi_u \varphi_u u_t$.

We next substitute the aggregate asset price into (11) to characterize the equilibrium return,

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\, \delta_{t+1} - p_t \\ &= \rho + \underline{E}_t\left[p_{t+1}^o\right] + \frac{1}{1+\psi} \left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1} - Y_u \varepsilon_{u,t+1}\right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t+1} + (1-\beta)\, \delta_{t+1} \\ &\quad - \left(\underline{E}_{t-1}\left[p_t^o\right] + \frac{1}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}\right) \\ &= E_t[r_{t+1}] + \frac{1}{1+\psi} \left(\varepsilon_{z,t+1} - Y_u \varepsilon_{u,t+1}\right) - \left(\frac{1}{1+\psi} - (1-\beta)\right) \varepsilon_{\delta,t+1} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t+1} \end{aligned}$$

where

$$E_t[r_{t+1}] = \rho + (1-\beta)\, \varphi_\delta \delta_t + Y_u (1 - \varphi_u)\, u_t + \frac{\psi}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - \varepsilon_{u,t}\right) - \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}.$$

This proves Eq. $(A.67)$. Combining this with (15) proves $(A.66)$.

Eq. (30) implies the conditional return volatility is the solution to the following quadratic

$$\begin{aligned} \sigma^2 &= var_t(r_{t+1}) = \sigma_{macro}^2(\psi) + \left(\frac{\sigma^2}{\alpha}\right)^2 \sigma_\mu^2, \\ \text{where } \sigma_{macro}^2(\psi) &= \left(\sigma_z^2 + Y_u^2 \sigma_u^2\right) \left(\frac{1}{1+\psi}\right)^2 + \sigma_\delta^2 \left(\frac{1}{1+\psi} - (1-\beta)\right)^2. \end{aligned}$$

Under the assumed parametric condition, this quadratic has two positive roots for each $\psi \geq 0$. The smaller root corresponds to the stable equilibrium. This proves $(A.68)$.

We verify the conjectures $\frac{dy_t}{dr_t^f} = \frac{dp_t}{dr_t^f} = -1$ and $\overline{p}_t = \underline{E}_{t-1}\left[p_t^o\right]$ as in the proof of Proposition 2. To verify the conjecture $\frac{d\pi_t}{dr_t^f} = \kappa$, observe that along the equilibrium path inflation satisfies the NKPC

$$\pi_t = \kappa \tilde{y}_t + \beta E_t\left[\pi_{t+1}\right] + u_t,$$

where the expected inflation $E_t\left[\pi_{t+1}\right] = \Pi_u \varphi_u u_t$ is exogenous to the current policy rate. Therefore, we have $\frac{d\pi_t}{dr_t^f} = \frac{dy_t}{dr_t^f} = \kappa$, verifying the remaining conjecture.

Finally, we characterize the equilibrium nominal interest rate i_t^f. Combining Eqs. $(A.56)$ with $(A.65)$ and $(A.67)$, we have

$$\begin{aligned} i_t^f &= r_t^f + \left[E_t\left[\pi_{t+1}\right] - \frac{1}{2}\sigma_t^2\left(\pi_{t+1}\right)\right] - cov_t\left(\pi_{t+1}, r_{t+1}\right), \qquad \text{(A.74)} \\ \text{where } E_t\left[\pi_{t+1}\right] &= \Pi_u \varphi_u u_t \\ \sigma_t^2\left(\pi_{t+1}\right) &= \left(\Pi_u + \kappa Y_u \frac{\psi}{1+\psi}\right)^2 \sigma_u^2 + \left(\frac{\psi}{1+\psi}\right)^2 \kappa^2 \left(\sigma_z^2 + \sigma_\delta^2\right) + \left(\frac{\sigma^2}{\alpha}\right)^2 \kappa^2 \sigma_\mu^2 \\ -cov_t\left(\pi_{t+1}, r_{t+1}\right) &= \left(\Pi_u + \kappa Y_u \frac{\psi}{1+\psi}\right) \frac{1}{1+\psi} Y_u \\ &\quad + \frac{\psi}{1+\psi}\kappa \left[\frac{1}{1+\psi}\sigma_z^2 + \left(\frac{1}{1+\psi} - (1-\beta)\right)\sigma_\delta^2\right] - \left(\frac{\sigma^2}{\alpha}\right)^2 \kappa \sigma_\mu^2. \end{aligned}$$

Note that i_t^f does not depend on the current noise shock μ_t (although it depends on the variance of the future noise shocks σ_μ^2). This verifies that the central bank can implement the equilibrium by setting the nominal rate i_t^f under its information set and completes the proof. $\square$

A.6. FCI targeting with time-varying beliefs

In this appendix, we consider a version of our model in which the aggregate asset price can fluctuate due to the market's (arbitrageurs') average beliefs about future productivity in addition to noise shocks. We accommodate cases in which these beliefs are informative about future productivity (driven by news) as well as cases they are not informative. For either case, we show that some FCI targeting is still optimal. In fact, belief-driven asset price fluctuations create an additional mechanism by which FCI targeting stabilizes the aggregate asset price and output gaps.

Model with time-varying beliefs. We consider the baseline model in Section 3.1 with two changes. First, we modify Eq. (8) slightly so that:

$$y_{t+1}^* = y_t^* + \varepsilon_{z,t+1} \text{ where } \varepsilon_{z,t+1} \sim N\left(0, \overline{\sigma}_z^2\right). \qquad \text{(A.75)}$$

We assume $\varepsilon_{z,t}$ is i.i.d. Normally distributed and we denote the variance with $\overline{\sigma}_z^2$ (we reserve the notation σ_z^2 for the variance of $\varepsilon_{z,t+1}$ conditional on news). Second, we assume the arbitrageurs receive a signal (news) that *they believe* is informative about future productivity

$$n_{z,t} =^A \varepsilon_{z,t+1} + e_{zt} \text{ where } e_{zt} \sim N\left(0, \tilde{\sigma}_z^2\right). \qquad \text{(A.76)}$$

Here, e_{zt} is i.i.d. Normally distributed and the notation $=^A$ describes arbitrageurs' beliefs. For simplicity, we assume the central bank has the same belief as the arbitrageurs (we could relax this assumption with minor adjustments, since the central bank's beliefs about future productivity does not play a central role).[28]

We remain agnostic about whether these beliefs are correct. For concreteness, suppose $n_{z,t}$ is actually (in the data) drawn according to the relation $n_{z,t} = \eta_t + \varepsilon_{z,t+1} + e_{zt}$ where $\eta_t \sim N\left(\overline{\eta}_t, \sigma_\eta^2\right)$. When $\overline{\eta}_t = \sigma_\eta^2 = 0$, the arbitrageurs' beliefs are correct. When $\overline{\eta}_t = 0$ but $\sigma_\eta^2 > 0$, the arbitrageurs have unbiased beliefs but they are overconfident (the signal is less informative than what they think). When $\overline{\eta}_t \neq 0$, arbitrageurs' beliefs are also biased. As we will see, the parameters $\overline{\eta}_t$ and σ_η^2 do *not* affect the equilibrium. Therefore, our model accommodates these possibilities.

The rest of the model is the same as in Section 3.1. In particular, demand shocks and noisy flow shocks follow the same processes as before (see (10) and (12)) and agents do not have any signal about these processes.

To characterize the equilibrium, observe that Eqs. $(A.75)$ and $(A.76)$ imply that the arbitrageurs' posterior belief is given by

$$\begin{aligned} \varepsilon_{z,t+1} &\sim {}^A N\left(b_t, \sigma_z^2\right) \text{ where} \\ b_t &= \frac{1/\tilde{\sigma}_z^2}{1/\tilde{\sigma}_z^2 + 1/\overline{\sigma}_z^2} n_{zt} \quad \text{and} \quad \sigma_z^2 = \frac{1}{1/\tilde{\sigma}_\delta^2 + 1/\overline{\sigma}_\delta^2}. \end{aligned} \tag{A.77}$$

Observe also that ex-ante (before observing the news) agents believe arbitrageurs' posterior belief will be distributed according to

$$b_t \sim^A N\left(0, \sigma_b^2\right) \text{ where } \sigma_b^2 = \overline{\sigma}_z^2 - \sigma_z^2. \tag{A.78}$$

The ex-ante mean of beliefs is zero as the news is unpredictable, and the ex-ante variance of beliefs is equal to the variance reduction induced by (perceived) news.

Arbitrageurs choose their portfolio allocations to maximize expected log assets-under-management under this belief. This implies that Eqs. (11) and (16) in the main text still hold but with beliefs driven by (perceived) news about future productivity. In particular, the equilibrium asset price satisfies

$$p_t = \rho + E_t^A\left[p_{t+1}\right] + (1-\beta) E_t^A\left[\delta_{t+1}\right] - \left(r_t^f + \frac{1}{2} var_t^A\left[r_{t+1}\right]\right) + \frac{var_t^A\left[r_{t+1}\right]}{\alpha}\mu_t.$$

Benchmark equilibrium without policy reaction lags. As before, we start by describing the benchmark equilibrium without reaction lags:

$$\begin{aligned} p_t &= p_t^* \equiv y_t^* - m - \delta_t, \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\delta_t + b_t + \frac{\sigma^2}{\alpha}\mu_t, \\ r_{t+1} &= \rho + \delta_t - \beta\delta_{t+1} + \varepsilon_{z,t+1} - b_t, \\ \text{where } \sigma^2 &\equiv var_t^A\left(r_{t+1}\right) = \sigma_z^2 + \beta^2\sigma_\delta^2. \end{aligned}$$

[28] We remain agnostic about the households' beliefs. The households' belief does not affect the equilibrium in our model as they do not make a portfolio choice decision and their consumption is driven by asset prices and demand shocks, which we take as exogenous (in practice, households' beliefs can be one driver of demand shocks—we leave this extension for future work).

These equations are similar to their counterparts in (17). The aggregate asset price depends on neither noisy flows *nor* beliefs about future productivity. Both factors are absorbed by the interest rate, $b_t + \frac{\sigma^2}{\alpha}\mu_t$. The "pstar" that ensures zero output gaps is determined by the *current (near-term)* supply and demand. Therefore, the central bank needs to adjust the interest rate to insulate economic activity from asset price fluctuations driven by beliefs about future productivity. Observe also that the return expression is slightly different and depends on the productivity surprise, $\varepsilon_{z,t+1} - b_t$. Since we have redefined $\sigma_z^2 = var^A(\varepsilon_{z,t+1} - b_t)$, the expression for total variance is unchanged.

Equilibrium with policy reaction lags and FCI targeting. We next consider the main setup in which the central bank sets r_t^f before observing the current-period noisy flows μ_t *and* the current period beliefs of the arbitrageurs b_t. We allow for FCI targeting: that is, the central bank solves the same problem as before

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t^A \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(p_{t+h} - \overline{p}_{t+h} \right)^2 \right] \right]. \tag{A.79}$$

Discretionary policy is the special case with $\psi = 0$. The following result generalizes Proposition 2 to this setting.

Proposition 9 (Equilibrium with FCI Targeting and Time-Varying Beliefs)**.** *Suppose the planner follows the FCI targeting policy in (25) with $\psi \geq 0$, the parameters satisfy $\alpha^2 \geq 4\sigma_\mu^2 \left(\sigma_z^2 + \sigma_b^2 + \beta^2 \sigma_\delta^2\right)$ (and $\beta > 1 - \beta$), and the initial target satisfies $\overline{p}_0 = E_{-1}^A [p_0^*]$. Then, there is a (stable) equilibrium in which the planner announces the expected "pstar" for the next period as its target*

$$\overline{p}_{t+1} = \underline{E}_t^A \left[p_{t+1}^* \right] \text{ where } p_{t+1}^* = y_{t+1}^* - m - \delta_{t+1}. \tag{A.80}$$

The equilibrium asset price, output, and interest rate are

$$\begin{aligned} p_t &= \underline{E}_{t-1}^A [p_t^*] + \frac{1}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, && \text{(A.81)} \\ y_t &= y_t^* + \frac{\psi}{1+\psi} (\varepsilon_{\delta,t} - \varepsilon_{z,t}) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, && \text{(A.82)} \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1 - \beta\varphi_\delta)\, \delta_t + \frac{\sigma^2}{\alpha} \varphi_\mu \mu_{t-1} + \frac{\psi}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}). && \text{(A.83)} \end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t^A [r_{t+1}] + \left[\begin{array}{c} \frac{1}{1+\psi} (\varepsilon_{z,t+1} - b_t) \\ - \left(\frac{1}{1+\psi} - (1-\beta) \right) \varepsilon_{\delta,t+1} \end{array} \right] + \frac{1}{1+\psi} b_{t+1} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t+1} \tag{A.84}$$

where the expected return is

$$E_t^A [r_{t+1}] = \rho + \delta_t (1 - \beta\varphi_\delta) + \frac{\psi}{1+\psi} (\varepsilon_{z,t} - \varepsilon_{\delta,t}) - \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}. \tag{A.85}$$

The return variance $\sigma^2 = var_t^A(r_{t+1})$ is the smaller positive solution to the following fixed point problem

$$\sigma^2 = \sigma^2_{macro}(\psi) + \frac{(\sigma^2)^2}{\alpha^2}\sigma^2_\mu, \tag{A.86}$$
$$\text{where } \sigma^2_{macro}(\psi) = \left(\frac{1}{1+\psi}\right)^2 (\sigma_z^2 + \sigma_b^2) + \left(\frac{1}{1+\psi} - (1-\beta)\right)^2 \sigma_\delta^2.$$

Let $\overline{\psi} = \arg\min_{\psi \geq 0} \sigma^2_{macro}(\psi)$. Over the range $\psi \in [0, \overline{\psi})$, increasing ψ strictly reduces σ^2 as well as $\sigma^2_{macro}(\psi)$ and $\frac{(\sigma^2)^2}{\alpha^2}\sigma^2_\mu$.

First consider the case with discretionary policy $\psi = 0$. For this case, the result shows the equilibrium is similar to its counterpart in Proposition 1 with the main difference that the asset price and output are also influenced by the arbitrageurs' belief shocks

$$p_t = \underline{E}^A_{t-1}[p_t^*] + \varepsilon_{z,t} - \varepsilon_{\delta,t} + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \tag{A.87}$$
$$y_t = y_t^* + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \tag{A.88}$$

Consequently, these belief shocks also generate return variance, in addition to the previous drivers of return variance

$$r_{t+1} = E_t^A[r_{t+1}] + (\varepsilon_{z,t+1} - b_t) - \beta\varepsilon_{\delta,t+1} + b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}. \tag{A.89}$$

The rest of the equilibrium is as before. In particular, the return variance is the solution to a fixed point problem. Observe that the "macro-induced" variance $\sigma^2_{macro}(\psi)$ is driven by not only supply and demand shocks but also the belief shocks about future supply.

Next consider the case with FCI targeting $\psi > 0$. In this case, the asset price and output are given by

$$p_t = \underline{E}^A_{t-1}[p_t^*] + \frac{1}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}$$
$$y_t = y_t^* + \frac{\psi}{1+\psi}(\varepsilon_{\delta,t} - \varepsilon_{z,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

As before, the supply and demand shocks have a smaller effect on the asset price (and some effect on output), because the central bank is partially committed to the previously announced FCI target. Importantly, belief shocks about future supply have a smaller effect on the asset price and output. This is because FCI targeting implies the central bank will not allow the *future* asset price to adjust to the *future* supply immediately, $p_{t+1} = \underline{E}^A_t[p_t^*] + \frac{1}{1+\psi}\varepsilon_{z,t+1} + ...$ This reduces the impact of arbitrageurs' beliefs about future supply on the aggregate asset price and output. It follows that FCI targeting reduces the macro-induced variance by reducing the impact of belief shocks as well as supply and demand shocks. This in turn reduces the noise-induced variance as in the main text.

In this setup FCI targeting ($\psi > 0$) is beneficial for two distinct reasons. To see this, consider the expression for the output. As before, FCI targeting reduces variance σ^2 and mitigates the impact of noise shocks on output gaps. Moreover, FCI targeting also reduces the effect of belief shocks on the output gap through the mechanism we described. FCI targeting also creates a new source of gaps through supply and demand shocks, but as before these costs are second order in ψ. In contrast, the benefits through the belief shocks is first order in ψ, similar to the benefits through the noise shocks. It follows that FCI targeting is robust to allowing for belief-driven fluctuations in asset prices. In fact, these fluctuations create an additional channel by which FCI targeting stabilizes the aggregate asset price and output gaps.

Proof of Proposition 9. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future "pstar" as its target $\overline{p}_t = \underline{E}_{t-1}^A [p_t^*]$, and the asset price and the interest rate satisfies $(A.81)$ and $(A.83)$, which can be written as

$$\begin{aligned}
p_t &= y_{t-1}^* - m - \varphi_\delta \delta_{t-1} + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right)\delta_t + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}
\end{aligned}$$

Following similar steps as in the proof of Proposition 2, the central bank's optimality condition satisfies

$$\underline{E}_t [p_t] = \frac{1}{1+\psi} p_t^* + \frac{\psi}{1+\psi}\overline{p}_t.$$

Substituting $\overline{p}_t = \underline{E}_{t-1}[p_t^*], p_t^* = y_t^* - m - \delta_t$ and the AR(1) process for δ_t, we obtain

$$\begin{aligned}
\underline{E}_t [p_t] &= \frac{1}{1+\psi}\left(y_t^* - m - \delta_t\right) + \frac{\psi}{1+\psi}\underline{E}_{t-1}\left[y_{t-1}^* + \varepsilon_{z,t} - m - \delta_t\right] \\
&= y_{t-1}^* - m - \varphi_\delta \delta_{t-1} + \frac{\psi}{1+\psi}\varepsilon_{z,t} - \frac{1}{1+\psi}\varepsilon_{\delta,t} \\
&= y_t^* - m - \varphi_\delta \delta_{t-1} + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right).
\end{aligned}$$

This verifies that the conjectured price satisfies the optimality condition.

We next substitute the conjectured price into (11) to calculate the equilibrium return

$$\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\delta_{t+1} - p_t \\
&= \rho + y_t^* - \varphi_\delta \delta_t + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{1}{1+\psi} b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} \\
&\quad + (1-\beta)\delta_{t+1} - \left(y_{t-1}^* - \varphi_\delta \delta_{t-1} + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + \varepsilon_{z,t} + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} + b_{t+1} - \left(\varepsilon_{z,t} + b_t\right)\right) + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right) \\
&\quad + \delta_t\left(1 - \beta\varphi_\delta\right) - \frac{\psi}{1+\psi}\varepsilon_{\delta,t} + \left(1 - \beta - \frac{1}{1+\psi}\right)\varepsilon_{\delta,t+1} \\
&= E_t^A [r_{t+1}] + \left(\varepsilon_{z,t+1} - b_t + b_{t+1}\right)\frac{1}{1+\psi} + \varepsilon_{\delta,t+1}\left(1 - \beta - \frac{1}{1+\psi}\right) + \varepsilon_{\mu,t+1}\frac{\sigma^2}{\alpha},
\end{aligned}$$

where the expected return is given by $(A.36)$

$$E_t^A [r_{t+1}] = \rho + \delta_t\left(1 - \beta\varphi_\delta\right) + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) - \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

We combine this expression with (15) and substitute $\mu_t - \varepsilon_{\mu,t} = \varphi_\mu \mu_{t-1}$ to calculate the interest rate

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + (1 - \beta\varphi_\delta)\,\delta_t + \frac{\psi}{1+\psi}\,(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}.$$

This verifies the expression for the interest rate.

Combining the conjecture for the price with $y_t = m + p_t + \delta_t$ verifies that output satisfies $(A.82)$. Substituting the solution into the expression for the return, we also find that the return satisfies $(A.84)$.

Finally, observe that Eq. $(A.84)$ implies that σ^2 solves the fixed point problem $(A.86)$. Under the assumed parametric condition, this problem has two positive roots for each $\psi \in [0, \overline{\psi})$. The smaller root corresponds to the stable equilibrium. The rest of the proof follows from similar steps as in the proof of Proposition 2. $\square$

B. Empirical Appendix

B.1. Data

B.1.1. Macroeconomic data

We download the following data from FRED (FRED series name in parenthesis): nominal potential GDP (NGDPPOT), nominal GDP (GDP), nominal investment (GPDI), nominal personal consumption expenditures (PCEC), GDP deflator (GDPDEF), PCE price index (PCEPI), the Chicago Fed National Financial Conditions Index (NFCI), the 3-month yield (TB3), labor productivity (OPHNFB), labor share (PRS85006173), weekly hours (PRS85006023), employment (CE16OV), population (CNP16OV) and consumer sentiment (UMCSENT). We obtain the updated series for the Excess Bond Premium from Favara et al. (2016). In order to compute real variables, we divide the nominal variables by the GDP deflator. For the new FCI index from Ajello et al. (2023b), we use the baseline construction, that allows shocks to have effects up to 3 years. Using the 1-year version does not alter the results. We compute hours per worker as weekly hours times employment divided over population. Inflation is computed as 400 times the log-difference in the PCE price index. Since the FCI index is available from 1990 onwards, we use the Chicago Fed FCI (NFCI) for the sample period 1973-1990 when computing the IRFs of monetary policy shocks. Ajello et al. (2023b) show that their index is similar in sample to the Chicago Fed FCI, and estimated IRFs are similar if we use that FCI for the full sample.

B.1.2. Construction of the financial noise shock

In order to construct the shock, we follow Gabaix and Koijen (2021) closely. We use quarterly data (sample: 1990Q1 to 2024Q2) from the Flow of Funds.[29] We use unadjusted flows (FU), and for the levels we use unadjusted market values when available (LM), and otherwise the estimated level. We collect data on flows for the following sectors: 15 (households), 21 (state and local governments), 22 (state and local retirement funds), 26 (rest of the world), 34 (federal retirement funds), 51 (property and casualty insurance), 54 (life insurance companies), 55 (closed end funds), 56 (ETFs), 57 (private pension funds), 63 (money market funds), 65 (mutual funds), 66 (securities brokers and dealers), and 76 (us chartered deposit institutions). As in Gabaix and Koijen (2021), we use data on three asset classes: 30611 (treasury securities), 30630 (corporate and foreign bonds), 30641 (corporate equities). Notice that the monetary authority does not hold equity in our data, so we drop it to build the flows into equity. For returns data, we use ex-dividend returns on the CRSP value-weighted market portfolio. For GDP growth, we use the log difference of real GDP obtained from FRED. We adjust the data on flows for foreign holdings following Appendix C.1.3 in Gabaix and Koijen (2021).

We follow the same notation and conventions as Appendix C.1.2 in Gabaix and Koijen (2021). We construct a measure of the proportional change of the quantity of equity in sector i between $t-1$ and t ($\Delta q_{it}^{\mathcal{E}}$) as follows: In the FoF, equity flows are defined by $\Delta F_{it}^{\mathcal{E}} = W_{it}^{\mathcal{E}} - W_{i,t-1}^{\mathcal{E}} R_t^X$. We assume the securities are adjusted at the end of the period, so $\Delta F_{it}^{\mathcal{E}} = (\Delta Q_{it}^{\mathcal{E}}) P_t^{\mathcal{E}}$, where Q_{it} is the amount of equities held by sector i at time t, and $P_t^{\mathcal{E}}$ is the price of each share. The relative flow in equities is $\Delta f_{it}^{\mathcal{E}} = \dfrac{\Delta F_{it}^{\mathcal{E}}}{W_{i,t-1}^{\mathcal{E}}}$.

The proportional change in quantity of equity is $\Delta q_{it}^{\mathcal{E}} = \Delta f_{it}^{\mathcal{E}} (R_t^X)^{-1} = \dfrac{\Delta Q_{it}^{\mathcal{E}}}{Q_{i,t-1}^{\mathcal{E}}}$.

[29] Raw data is downloaded from https://www.federalreserve.gov/datadownload/Build.aspx?rel=Z1.

With our measure of $\Delta q_{it}^{\mathcal{E}}$ in hand, we proceed exactly as in Appendix B of Gabaix and Koijen (2021) in order to construct the financial flow shock series. We briefly expand on each of the steps below:

1. Construct pseudo-equal value weights $\tilde{E}_i$ as in Gabaix and Koijen (2021).
2. Run the panel regression:
$$\Delta q_{it} = \alpha_i + \beta_t + \gamma_i \Delta y_t + \delta_i t + \Delta \check{q}_{it} \tag{B.1}$$
using $\tilde{E}_i$ as weights. Here Δy_t is quarter-on-quarter real GDP growth. In an alternative specification, we also control for changes in the consumer sentiment index ($Sent_t$), i.e we add $\Delta Sent_t$ as an additional control. We implement the weighting scheme by multiplying each observation by $\tilde{E}_i^{1/2}$ and then running a normal regression. Denote the residuals of this transformed regression as $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$
3. We run PCA on $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$. In our baseline specification, we control for aggregate factors by removing the first N principal components (ordered in terms of share of variance explained) from $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$. That is, we construct:
$$\Delta \tilde{q}_{it} = \tilde{E}_i^{1/2} \Delta \check{q}_{it} - \sum_{n=1}^{N} \lambda_{i,n} \eta_{t,n}^{PC} \tag{B.2}$$
where $\eta_{t,n}^{PC}$ is principal component n at time t, and $\lambda_{i,n}$ is the loading of sector i on that principal component. Our baseline uses $N = 2$. Our results are essentially unchanged if we use $N = 3$ or $N = 4$ instead.
4. Finally, we construct the financial flow shock as:
$$Z_t^{\mu} = \sum_{i=1}^{I} S_{i,t-1} \Delta \tilde{q}_{it}$$
where $S_{i,t-1} = \frac{W_{it}^{\mathcal{E}}}{\sum_{j=1}^{I} W_{jt}^{\mathcal{E}}}$ is the share of total equity held by sector i at time $t-1$.

B.2. Additional Empirical results

B.2.1. Robustness for impulse response estimation

Figures 13 and 14 show the estimated IRFs when we control for 3 and 4 Principal components (respectively) in equation (B.2). As we can see, results are virtually identical, which is strong evidence that the procedure followed adequately controls for aggregate factors in this setting (Gabaix and Koijen, 2024). Figure 15 shows the results when we further residualize by changes in consumer sentiment in the construction of the shock. Figure 16 depicts the IRF estimated using an SVAR-IV procedure. Results are similar to the baseline, but with tighter confidence bands, which is expected.

B.2.2. Robustness: drivers of the financial noise shock

As it is well known, news shocks about future productivity or earnings may drive financial trade and look similar to financial noise shocks. In order to test for this competing interpretation, we look at the responses of TFP, stock prices and price-to-earnings ratios (all in logs) to the identified shock. Given that some of this shocks may take several years to unfold, we use local projections since VARs are biased

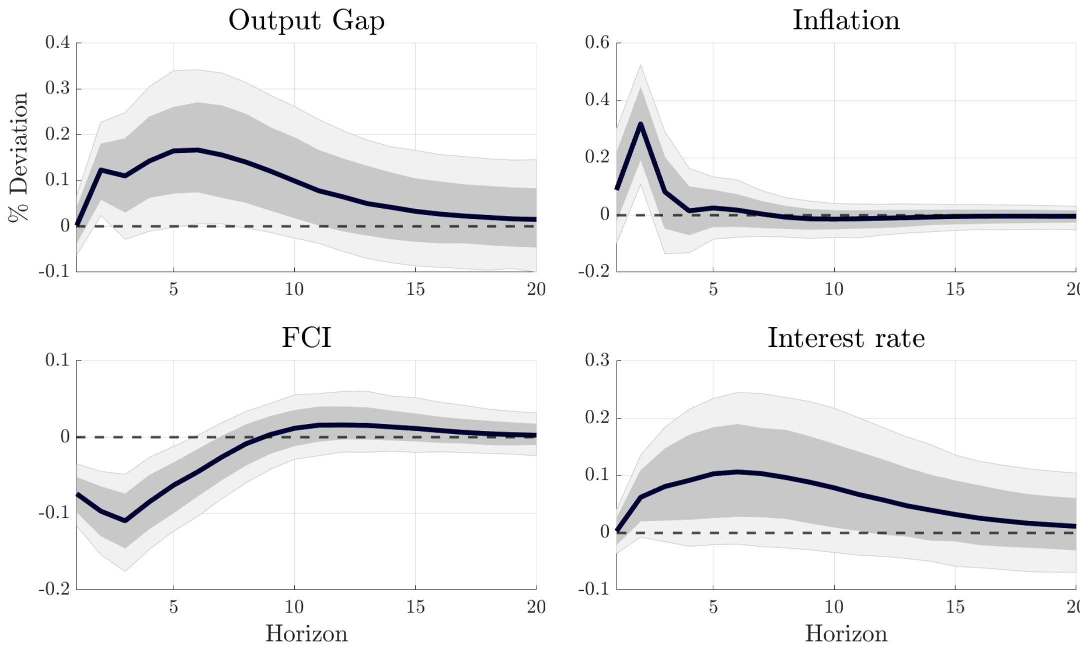

Figure 13: Impulse response to a financial noise shock, where the shock is identified controlling for 3 Principal Components in (B.2). Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

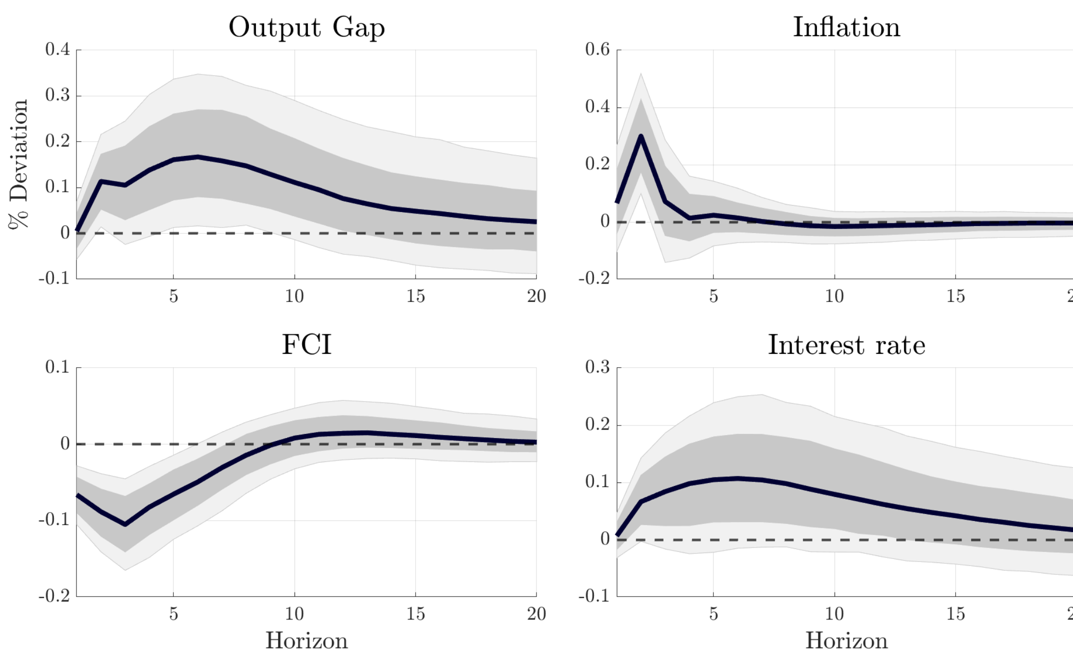

Figure 14: Impulse response to a financial noise shock, where the shock is identified controlling for 4 Principal Components in (B.2). Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

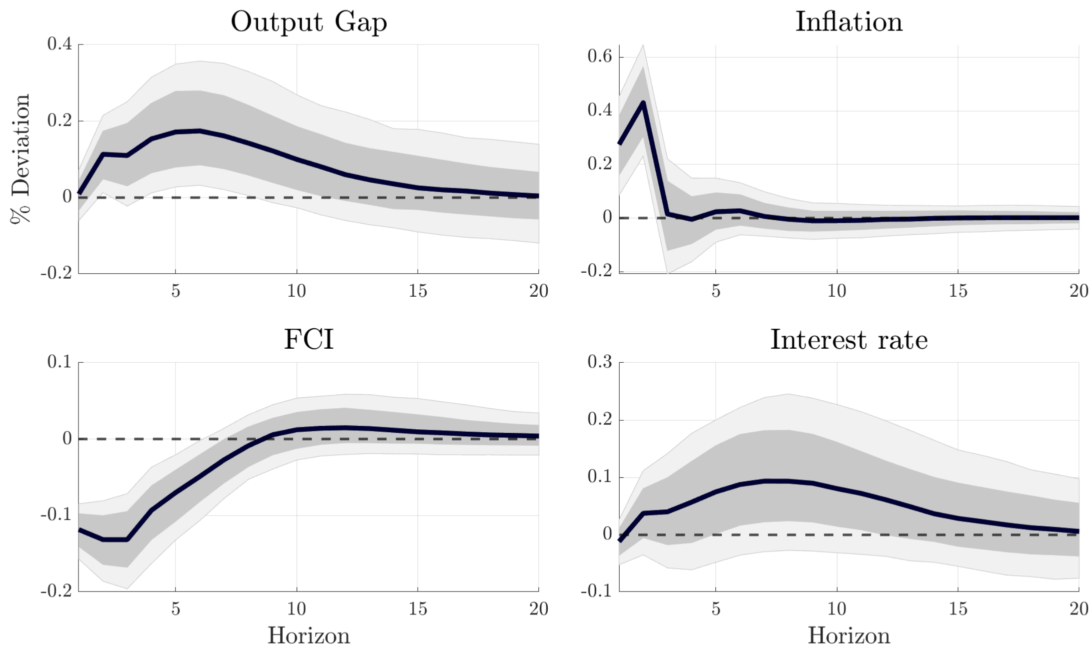

Figure 15: Impulse response to a financial noise shock, where we also residualize by changes in consumer sentiment when constructing the shock. Light shaded grey bands indicate 90 confidence sets.

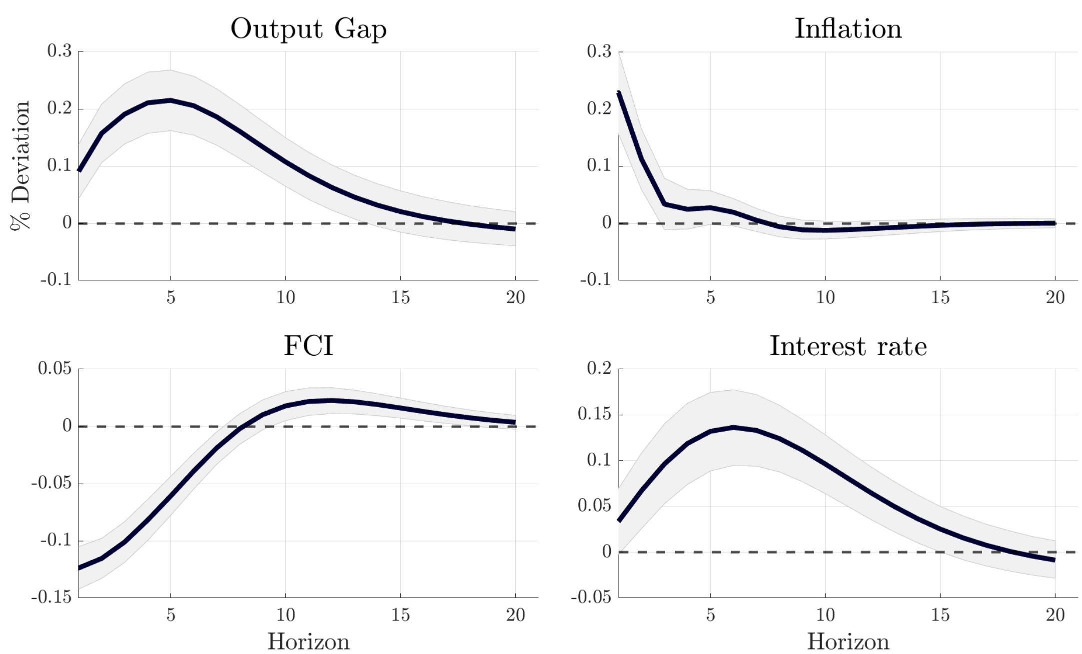

Figure 16: Impulse response to a financial noise shock, where the shock is identified using and SVAR-IV procedure. Light shaded grey bands indicate 90 confidence sets.

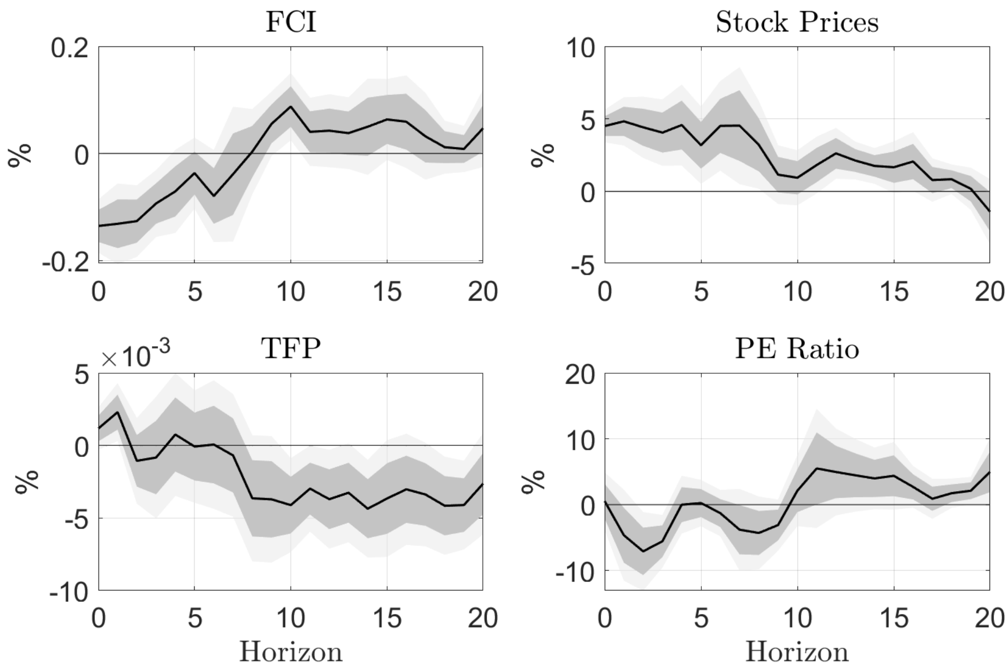

Figure 17: Impulse response to a financial noise shock for FCI, S&P 500, TFP and Price-to-earnings ratio (all in logs)

at longer horizons (Li et al., 2024). As controls, we use two lags of TFP, stock prices, price-to-earnings ratios, FCI, output gap, inflation and interest rates. We use the same controls for all equations. For TFP, we use the utilization adjusted measure by Fernald (2014), available up to 2023 in Fernald's website. For earning and stock prices, we use the real values reported in the Shiller database. We convert to quarterly by taking the value at the end of the quarter.

Figure 17 shows the results. The estimated path for FCI after the shock is essentially the same as the one estimated with the main VAR specification. The shock induces an increase in stock prices, that mean reverts back to zero in the following years. This is consistent with the shock being financial noise, that generates a transitory increase in stock prices that then reverts. TFP basically does not react to the shock in short horizons, and then reacts *negatively* after two years. This is consistent with a financial noise shock that is unrelated to TFP, and inconsistent with the view that these shocks are generated by positive news about future TFP. Finally, the price-to-earnings ratio has an initially negative reaction, which quickly reverts to zero. This is consistent with the boom in aggregate demand generated by the shock, that increases earnings in the first few quarters. Notice that if the shock was generated by news about future earnings, then the price-to-earnings ratio should increase in the short run, since prices will lead earnings. Empirically, the opposite happens. Overall, this additional empirical evidence supports the notion that the shock is related to financial noise, as opposed to some news about future fundamentals.

B.2.3. Robustness: extending the sample to include Covid

In order to assess the stability of our estimates to including the recent Covid period, we extend the sample to 2024Q2. In order to deal with Covid during the construction of the shock, we dummy out the observations for 2020Q2 and 2020Q3, which are the two big outliers in quarterly GDP growth.

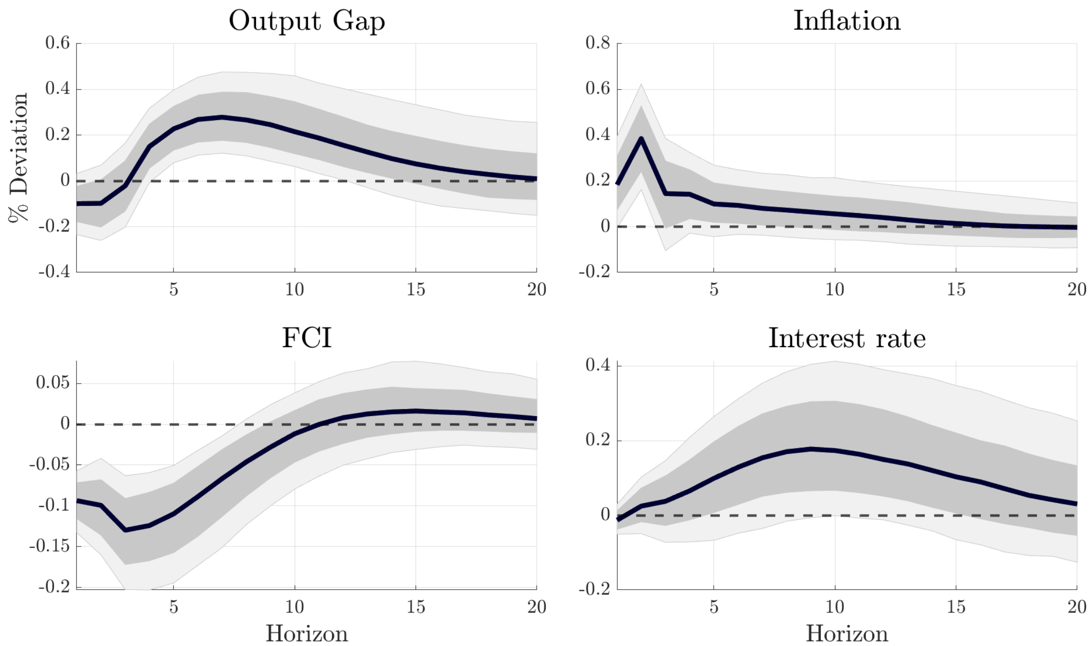

Figure 18: Impulse response to a financial noise shock, sample: 1990Q1-2024Q2. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

Figure 18 shows the estimated impulse response. The overall patterns are unchanged: upon the shock, there are a delayed responses in the output gap and policy rates, and a faster response in inflation and FCI. There is a mild initially *negative* response, although this is not significantly different from zero.[30] Figure 19 depicts the forecast variance ratios. The importance for output gaps is lower, in the order of 15-20% for the SVAR or recoverability based estimates. This outcome is expected: the large COVID shock increases the total forecast variance in the sample. However, since this shock is orthogonal to financial noise shocks, the estimated proportional significance of the latter is reduced.

B.2.4. Monetary Policy Shocks IRFs

Figures 20 and 21 contain the impulse-response to monetary policy shocks identified by Aruoba and Drechsel (2022) and Romer and Romer (2004) respectively. The responses are standard. The time pattern of the response of FCI is different in both specifications, with FCI spiking more strongly for the Romer and Romer (2004) IRF. Using two shocks that generate different paths allows for extra degrees of freedom when implementing the counterfactuals.

B.2.5. Counterfactual propagation of monetary policy shocks

One of the key observation of risk-centric models (Caballero and Simsek (2020)) is that monetary policy affects the economy via asset prices. Given our setting, we can test this claim empirically using the tools developed in McKay and Wolf (2023b).

[30]If we add exogenous dummies for the 2020Q2 and 2020Q3 quarters in the VAR, this initially negative response disappear.

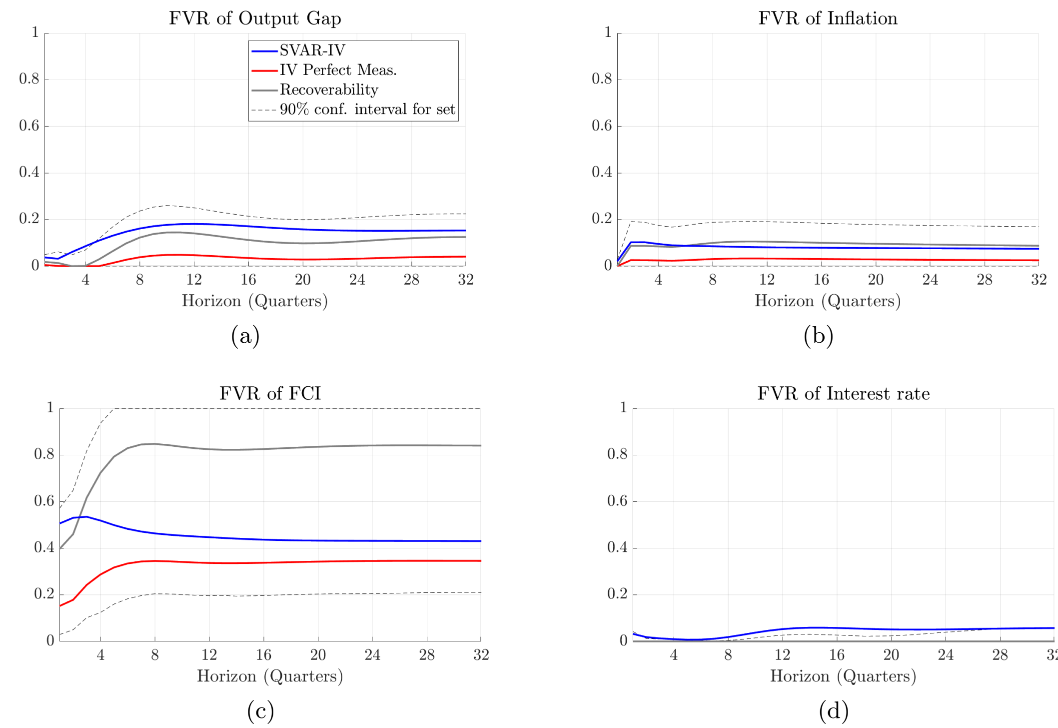

Figure 19: Identified Forecast Variance Ratios of the noise shock. Blue: SVAR-IV, assuming invertibility. Red: lower bound, assumes perfect measurement of the shocks. Grey: recoverability-based FVR. VAR includes the full set of macro outcomes (baseline + labor market variables). Dashed lines are 90% confidence intervals of the identified set, (Plagborg-Møller and Wolf, 2022) computed via bootstrap with 1000 repetitions.

Specifically, we examine the following counterfactual question: how would a monetary policy shock propagate if financial conditions were unresponsive to monetary policy? To answer that question, we take the impulse-response of the Aruoba and Drechsel (2022) monetary policy shock, and use the identified response to a financial flow shock to approximately enforce $FCI_t = 0$ on impact and in expectation. Importantly, although the methodology is the same as in McKay and Wolf (2023b), this is not a *policy* counterfactual. Instead, we are asking how a given policy shock would have propagated under a different mapping between monetary policy and financial conditions.[31]

Figure 22 shows the results. As we can see, the approximation is good for the first 12 quarters, but we still get some delayed response of financial conditions at longer horizons approximation error. Crucially, the path of interest rates is basically unchanged. Turning to output gap, the response is essentially zero at all horizons.[32] The real effect of the monetary policy shock is much smaller for the first two years. Regarding inflation, except for a *positive* initial response attributable in part to a price-puzzle-type response in the original monetary impulse-response, the path for inflation is essentially zero at all horizons. Overall, the result is consistent with the key tenet of the risk-centric view of monetary policy:

[31]The assumptions required for this to yield the correct counterfactual are analogous to the ones in McKay and Wolf (2023b): we need that financial conditions enter in the rest of the private sector equations and in the monetary policy rule only through its expected values.

[32]Only the response on impact is marginally significant at the 90% level.

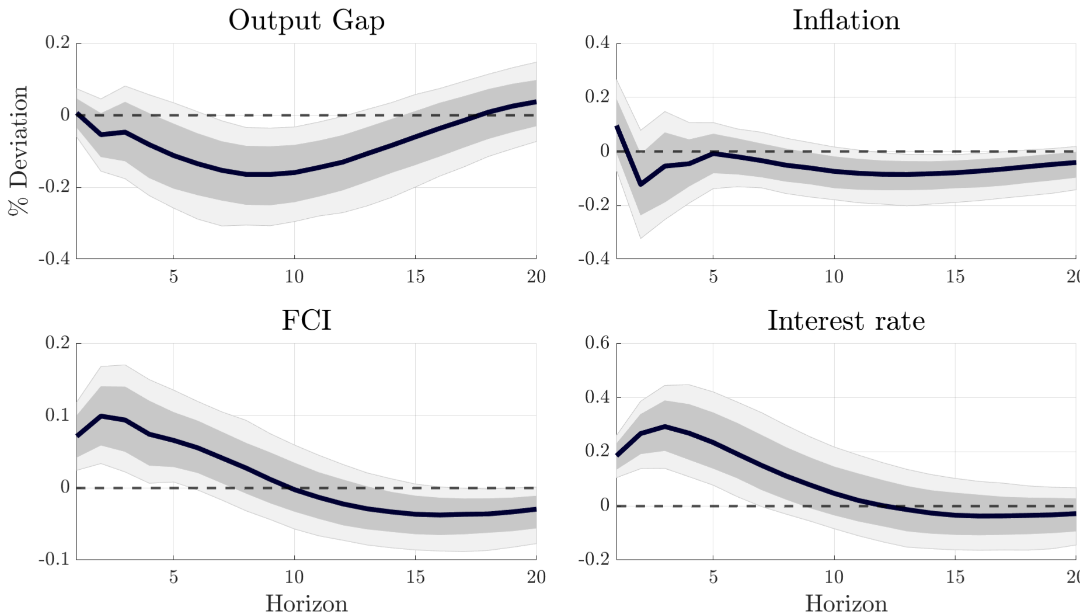

Figure 20: Impulse response to the Aruoba and Drechsel (2022) monetary policy shock. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

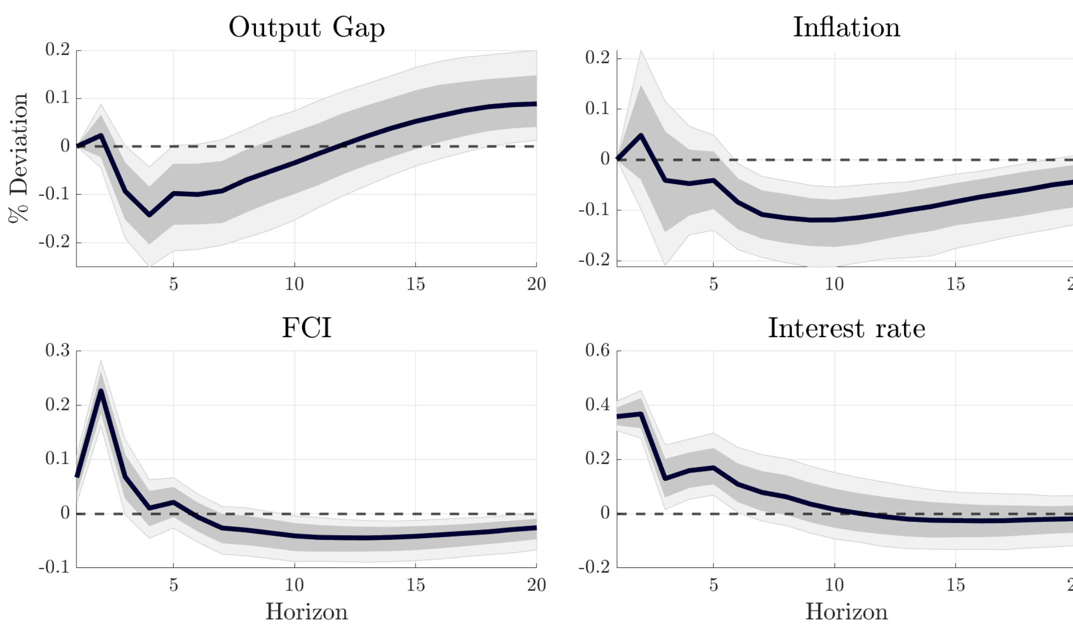

Figure 21: Impulse response to the Romer and Romer (2004) monetary policy shock. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

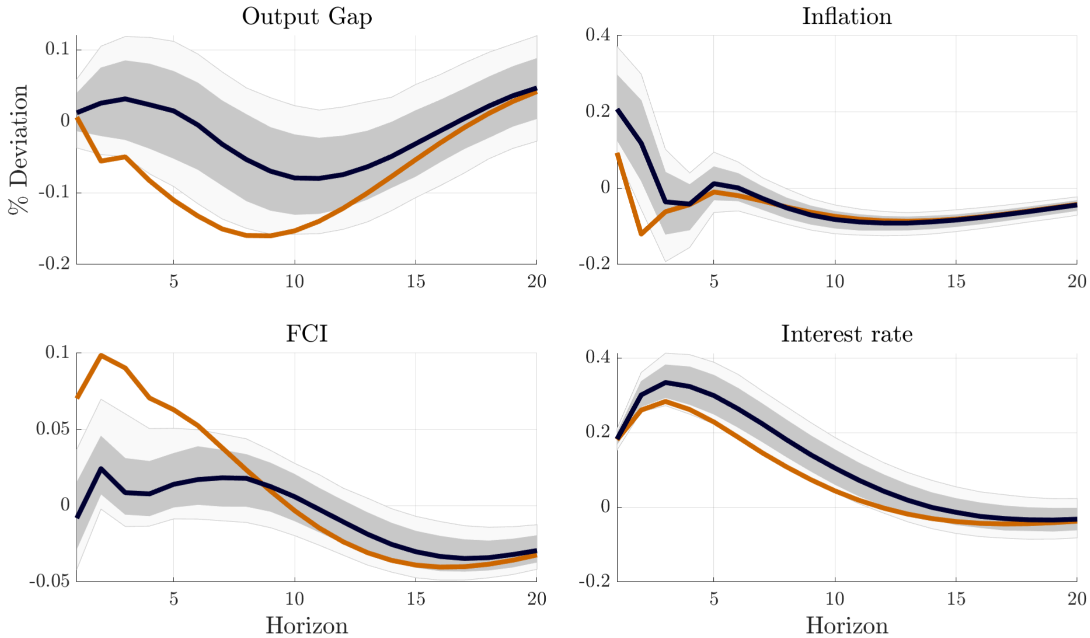

Figure 22: Counterfactual impulse response to a monetary policy shock, identified following Aruoba and Drechsel (2022). The orange line is the original point estimate, black line is the best approximation to the counterfactual impulse response. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

monetary policy affects the economy via financial conditions. Our results indicate that in a counterfactual economy where short-term interest rates and broader Financial Conditions are disconnected, monetary policy shocks would have essentially no impact in output gap or inflation.

C. Policy Counterfactuals

C.1. Proof of Proposition 4

Without loss of generality, assume that the noise shock is ordered first in the ε_t vector. Using the time series of the shock, we can partition the rotation matrix P' in two. The first column (known) $(P^{'})_{\bullet,1}$, and the rest of the matrix $(P^{'}_{\bullet,-1}$, which does not need to be fully identified. The first column identifies the financial flow shock, which is the only one whose transmission is affected by risk. This column is identifiable up to scale by regressing the Wold residuals on ε_t^{μ}. For the remaining macroeconomic shocks, we simply identify a rotation of these shocks, as argued in Caravello et al. (2024), which suffices for our purposes.

We then apply the same procedure as in Caravello et al. (2024). This yields the correct counterfactual for (a rotation of) $\tilde{\Theta}_{-\mu,\ell}$. We only have left to construct the correct $\tilde{\Theta}_{\mu,\ell}$. Applying McKay and Wolf (2023b) to $\Theta_{\mu,\ell}$ yields $\hat{\Theta}_{\mu,\ell}$, which is the solution to a unit-size shock for the impulse response system that satisfies:

$$\begin{aligned}
\mathcal{F}_w\hat{\Theta}_{\mu,w} + \mathcal{F}_x\hat{\Theta}_{\mu,x} + \mathcal{F}_z\hat{\Theta}_{\mu,z} + \mathcal{F}_\mu(\sigma_r^2 \times 1) &= \mathbf{0}, \\
\mathcal{H}_w\hat{\Theta}_{\mu,w} + \mathcal{H}_x\hat{\Theta}_{\mu,x} + \mathcal{H}_z\hat{\Theta}_{\mu,z} &= \mathbf{0}, \\
\tilde{\mathcal{A}}_x\hat{\Theta}_{\mu,x} + \tilde{\mathcal{A}}_z\hat{\Theta}_{\mu,z} = \mathbf{0}.
\end{aligned}$$

However, the true counterfactual solves that system with $\tilde{\sigma}_r^2$ instead of σ_r^2. By linearity of the solution, if we knew $\tilde{\sigma}_r^2$, we can obtain the true counterfactual as $\tilde{\Theta}_{\mu,\ell} = \hat{\Theta}_{\mu,\ell}\frac{\tilde{\sigma}_r^2}{\sigma_r^2}$. Finally, in order to obtain $\tilde{\sigma}_r^2$, note that the true conditional volatility satisfies:

$$\begin{aligned}
\sigma_r^2 &= \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2\sigma_r^4 + \Theta_{r,-\mu,0}\Theta'_{r,-\mu,0} \\
&= \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2\sigma^4 + \Psi_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\Psi'_{r,0}
\end{aligned}$$

where $\theta_{r,\mu,0}$ is the response on impact of returns to the financial noise shock, $\Theta_{r,-\mu,0}$ is a $1\times(n_\varepsilon - 1)$ row vector that contains all the responses to structural shocks other than ε_μ, $\Psi_{r,-\mu,0}$ is the analogous object for Wold innovations, and $P_{\bullet,-1}$ is a $n_y \times (n_y - 1)$ matrix obtained by taking P and deleting the first column, which corresponds to the financial noise shock. Note, therefore, that the original volatility is the root of a quadratic of the form $P(x) = ax^2 - x + c$ where $a = \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2$, and $c = \Psi_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\Psi'_{r,0}$, and that c is the same for any rotation of the Wold shocks since $P_{\bullet,-1}P'_{\bullet,-1}$ always equals a matrix that has a zero in the (1,1) element, ones along the rest of the diagonal and zeros everywhere else, given that P is orthogonal and because of our identification assumption on $\varepsilon_{\mu,t}$, the first column and row of P are equal to the n_y vector $(1, 0, \dots)$.[33]

In order to find the counterfactual conditional variance, we solve the quadratic:

$$\tilde{a}x^2 - x + \tilde{c} = 0$$

where $\tilde{a} = \left(\tilde{\theta}_{r,\mu,0}/\sigma_r^2\right)^2$ and $\tilde{c} = \tilde{\Psi}_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\tilde{\Psi}'_{r,0}$ can be both constructed from the initial step.

Given that, we obtain the correct $\tilde{\Theta}_{\mu,\ell}$, and a rotation of the correct $\Theta_{-\mu,\ell}$ as in Caravello et al.

[33] In our implementation, we pick the first shock to correspond to ε_μ, and then build the rest of the rotation recursively by imposing that the shock ε_n has to be orthogonal to the past $n-1$ shocks.

(2024). This yields the counterfactual impulse response of interest. With the full $\tilde{\Theta}_\ell$ matrix, we can proceed as in Caravello et al. (2024) to obtain the counterfactuals second moments, our second object of interest. Note that, for the counterfactual historical evolution, the treatment of the initial condition is the same as in Caravello et al. (2024): since the policy change is unanticipated, whatever extra volatility the reaction to the initial condition generates is unanticipated, and moving forward future conditional volatility is not affected by this term.

C.2. Policy with a time-varying target set one period in advance

The building block of our counterfactuals is the counterfactual response to a particular shock. Intuitively, once we know how to obtain this, we can collect the response to multiple shocks to obtain a full counterfactual.

Consider the response to a shock. We assume the policy minimizes a quadratic loss. Define $\lambda_i \tilde{W}_i$ as a matrix that collects proper discount factors (in $\tilde{W}_i$) and weights (in λ_i) for variable i. For example, using $\tilde{W}_i$ with terms β^t along the diagonal defines the standard loss as in McKay and Wolf (2023a). Putting a zero in the first element of such matrix means that the planner ignores that variable in the first period. As explained, we account for the transmission lags by having a planner that targets i_0 at the natural rate in the first period (no reaction), and then optimal policy from then onwards. Let $\boldsymbol{y} = (\tilde{y}_0, \tilde{y}_1, \dots)$ denote the sequence of output gaps, $\boldsymbol{\pi}$ denote the sequence of inflation, $\boldsymbol{i}$ denote the sequence of interest rates, and $\boldsymbol{f}$ denote the sequence of FCI.

The problem of the central bank can be written in two steps. First, an "operational" central bank, who picks policy to minimize its loss subject to an FCI target. Second, a "long run" central bank who optimally chooses the target for the future periods.

First, the operational central bank solves:[34]

$$
\begin{aligned}
&\min_{\boldsymbol{v}} \frac{1}{2} \sum_i \lambda_i [\boldsymbol{x}_i' \tilde{W}_i \boldsymbol{x}_i + 2 c_i' x_i] + \lambda_f (\boldsymbol{f} - \bar{\boldsymbol{f}})' \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) \\
s.t. \quad & \boldsymbol{x}_i = \Theta_{x_i,v} \boldsymbol{v} + \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon}, \\
& \boldsymbol{f} = \Theta_{f,v} \boldsymbol{v} + \Theta_{f,\varepsilon} \boldsymbol{\varepsilon}.
\end{aligned}
$$

The first order condition is:

$$
\sum_i \lambda_i \Theta_{x_i,v}' [\tilde{W}_i \boldsymbol{x}_i + c_i] + \lambda_f \Theta_{f,v}' \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) = 0, \tag{C.1}
$$

[34] In our application, the $c_i' x_i$ term only shows up in order to account for the initial conditions when we start the historical episode counterfactual. After the initial period, it plays no role.

and the shock that solves this is:

$$\tilde{v}^*(\bar{\boldsymbol{f}}) = -\underbrace{\left(\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_i\Theta_{x_i,v} + \lambda_f \Theta'_{f,v}\tilde{W}_i\Theta_{f,v}\right)^{-1}}_{A^{-1}} \left(\sum_i \lambda_i \Theta'_{x_i,v}[\tilde{W}_i\Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon} + c_i] + \lambda_f \Theta'_{f,v}\tilde{W}_f(\Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{\boldsymbol{f}})\right)$$

$$\tilde{v}^*(\bar{\boldsymbol{f}}) = -A^{-1}\left((\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_i\Theta_{x_i,v}) \underbrace{(\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_i\Theta_{x_i,v})^{-1}(\sum_i \lambda_i \Theta'_{x_i,v}[\tilde{W}_i\Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon} + c_i])}_{-v^{**}} + \lambda_f \Theta'_{f,v}\tilde{W}_f(\Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{\boldsymbol{f}}) \right)$$

$$\tilde{v}^*(\bar{\boldsymbol{f}}) = -A^{-1}\left(-Av^{**} + \lambda_f \Theta'_{f,v}\tilde{W}_f\Theta_{f,v}v^{**} + \lambda_{fci}\Theta'_{f,v}\tilde{W}_f(\Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{\boldsymbol{f}})\right)$$

$$\tilde{v}^*(\bar{\boldsymbol{f}}) = \tilde{v}^{**} - \left(\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_1\Theta_{x_i,v} + \lambda_f \Theta'_{f,v}\tilde{W}_f\Theta_{f,v}\right)^{-1} \lambda_{fci}\Theta'_{f,v}\tilde{W}_f(f^{**} - \bar{\boldsymbol{f}})$$

where $\tilde{v}^{**} = -\left(\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_i\Theta_{x_i,v}\right)^{-1}\left(\sum_i \lambda_i \Theta'_{x_i,v}[\tilde{W}_i\Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon} + c_i]\right)$ is the shock that would solve the pure dual mandate problem, and $f^{**} = \Theta_{f,\varepsilon}\boldsymbol{\varepsilon} + \Theta_{f,v}v^{**}$ is the value of f^{**} that a pure dual mandate CB would choose. From now on, denote $\Theta_{\bar{f}} = \left(\sum_i \lambda_i \Theta'_{x_i,v}\tilde{W}_i\Theta_{x_i,v} + \lambda_f \Theta'_{f,v}\tilde{W}_f\Theta_{fci,v}\right)^{-1} \lambda_f \Theta'_{f,v}\tilde{W}_f$.

Secondly, we have the "long-run" central bank, who chooses $\bar{\boldsymbol{f}}$ in order to minimize the loss, conditional on their timing constraints:

$$\begin{aligned}
&\min_{\bar{\boldsymbol{f}}} \frac{1}{2}\sum_i \lambda_i[\boldsymbol{x}_i'\tilde{W}_i\boldsymbol{x}_i + 2c_i'x_i] + \lambda_f(\boldsymbol{f} - \bar{\boldsymbol{f}})'\tilde{W}_f(\boldsymbol{f} - \bar{\boldsymbol{f}}) \\
s.t. \quad & \boldsymbol{x}_i = \Theta_{x_i,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}, \\
& \boldsymbol{f} = \Theta_{f,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{f,\varepsilon}\boldsymbol{\varepsilon}, \\
& R\bar{\boldsymbol{f}} = 0,
\end{aligned}$$

where R is a $N \times T$ matrix that incorporates timing restrictions, in this case that $\bar{\boldsymbol{f}}$ has to be equal to zero in the first $N+1$ periods.[35] Forming a Lagrangian with vector of multipliers $\gamma' = (\gamma_1, \gamma_2, \dots)$, the first order condition is:

$$R'\gamma + \sum_i \lambda_i \Theta'_{\bar{f}}\Theta'_{x,v}[\tilde{W}_i\left(\Theta_{x_i,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}\right) + c_i] +$$
$$\lambda_f \Theta'_{\bar{f}}\Theta'_{f,v}\tilde{W}_f(\Theta_{f,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{f}) - \lambda_f\tilde{W}_f(\Theta_{f,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{f}) - \bar{f}) = 0$$

[35] Take, for example, $N = 1$. In the first period, the target is preset at the SS value of 0. The target in the second period is chosen in the first period *before* observing any shock, thus it also equals zero.

and the constraint. Working with the first equation:

$$R'\gamma + \Theta'_{\bar f}\left(\sum_i \lambda_i \Theta'_{x,v}[\tilde W_i(\boldsymbol{x}_i^{**} - \Theta_{x_i,v}\Theta_{\bar f}(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}})) + c_i] + \lambda_f \Theta'_{f,v}\tilde W_f(I - \Theta_{f,v}\Theta_{\bar f})((\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}))\right) + \\ -\lambda_f \tilde W_f(I - \Theta_{f,v}\Theta_{\bar f})(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) = 0$$

$$R'\gamma + \Theta'_{\bar f}\left(\sum_i \lambda_i \Theta'_{x,v}[\tilde W_i \boldsymbol{x}_i^{**} + c_i]\right) - \Theta'_{\bar f}(\sum_i \lambda_i \Theta'_{x,v}\tilde W_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v}\tilde W_f \Theta_{f,v})\Theta_{\bar f}(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) + \\ \lambda_f \Theta'_{\bar f}\Theta'_{f,v}\tilde W_f(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) - \lambda_f \tilde W_f(I - \Theta_{f,v}\Theta_{\bar f})(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) = 0$$

Define the following matrices:

$$A_1 = -\left(\Theta'_{\bar f}(\sum_i \lambda_i \Theta'_{x,v}\tilde W_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v}\tilde W_f \Theta_{f,v})\Theta_{\bar f} + \lambda_f(\tilde W_f - \tilde W_f \Theta_{f,v}\Theta_{\bar f} - \Theta'_{\bar f}\Theta'_{f,v}\tilde W_f)\right),$$

$$A_2 = \Theta'_{\bar f}\left(\sum_i \lambda_i \Theta'_{x,v}[\tilde W_i \boldsymbol{x}_i^{**} + c_i]\right),$$

then the equations can be written more compactly as:

$$-A_1(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) + A_2 + R'\gamma = 0, \tag{C.2}$$
$$A_1(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) = (A_2 + R'\gamma). \tag{C.3}$$

If the matrix A_1 is invertible, we can solve for $\bar f$ as:

$$\bar{\boldsymbol{f}} = \boldsymbol{f}^{**} + A_1^{-1}[A_2 + R'\gamma],$$

and then using the constraint:

$$0 = R\bar{\boldsymbol{f}} = R\boldsymbol{f}^{**} + RA_1^{-1}[A_2 + R'\gamma] \tag{C.4}$$
$$\gamma = -\left[RA_1^{-1}R'\right]^{-1}[R\boldsymbol{f}^{**} + RA_1^{-1}A_2] \tag{C.5}$$

which fully characterizes the target.

If transmission lags are included, then A_1 is not invertible. In particular, if the Central Bank reacts with a lag of N periods, the first N rows and columns are zeros. Furthermore, the first $N \times N$ submatrix of A_2 is also full of zeros. Thus, this implies that we can solve this by setting $\gamma_1, \ldots, \gamma_N = 0$, and then in find γ_{N+1} by deleting the rows and columns of zeros of A_1, A_2 and the first N equations in R. We obtain:

$$A_{1,(N+1:\bullet,N+1:\bullet)}(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) = (A_{2,(N+1:\bullet)} + R'_{\bullet,N+1}\gamma_{N+1})$$

and then we can proceed as before to find γ. With γ, the elements $N+1, \ldots$ of $\bar f$ are uniquely determined by C.3, and the first N elements are zeros thanks to the constraint.

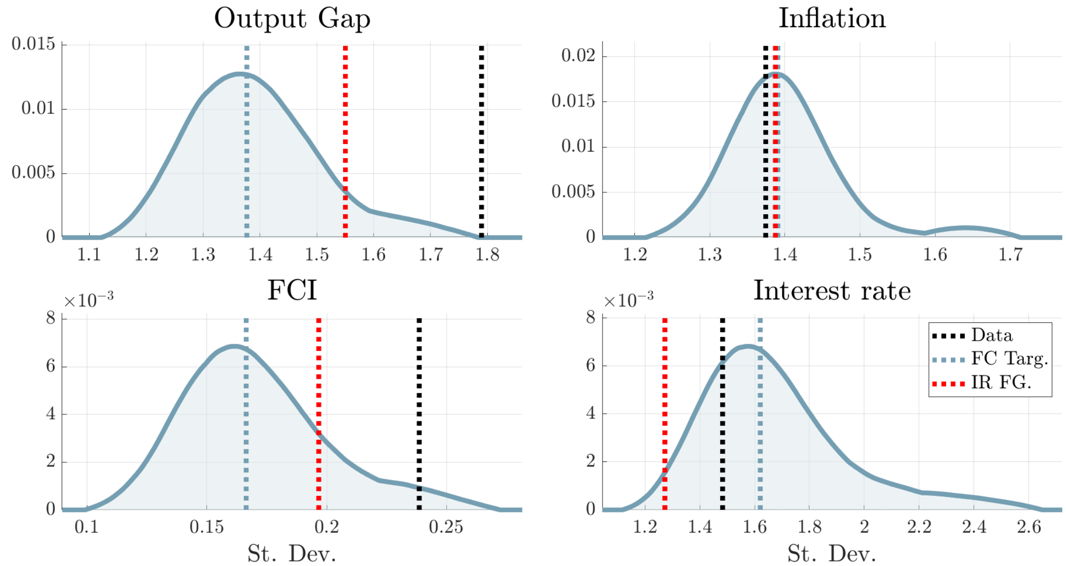

Figure 23: Counterfactual Standard Deviations. For Output Gap, Inflation and interest rates, this is the unconditional standard deviation, for FCI this is the conditional SD. Black dashed: data. Dashed lines the median. Red: Pure Dual Mandate with Forward Guidance in interest rates. Blue: FCI targeting, i.e minimize 54 with $\psi = \psi^*$. Solid Line: posterior density for the counterfactual with FCI targeting counterfactual.

C.3. Additional Results

C.3.1. FCI vs interest rate targeting

Figure 23 compares of unconditional second moments under optimal FCI targeting and interest rate forward guidance as described in subsection 5.2.3.

C.3.2. Loss as a function of Taylor Rule coefficient

Figure 24 depicts the loss $E[\mathcal{L}] = \sigma_{\tilde{y}}^2 + \sigma_{\pi}^2 + \lambda_{\Delta i}\sigma_{\Delta i}^2$ as a function of the parameter ψ_{TR} for i) the optimal FCI target consirered in section 5.2, ii) the simple FCI target constructed in Section 5.3.1 and iii) a constant FCI target. For all three cases, initially the loss declines sharply with ψ_{TR}. The loss is essentially monotone in ψ_{TR} and it levels off for high values of ψ_{TR}. This shows that, compared to the benchmark Taylor Rule, adding some reaction to financial conditions, even if the target is not exactly optimal, leads to better outcomes. However, the size of the possible gains is sensitive to the construction of the target: there are non-trivial differences in the gains between using the optimal target with respect to using an approximate or constant target.

Notice also that as ψ_{TR} grows, the loss levels off but, for the range we plot, does not start to increase. This is driven by how the approximation to the counterfactual is constructed: when ψ_{TR} grows large, we are essentially trying to enforce $FCI_t = \bar{FCI}_t$ as well as possible given the monetary policy shocks we have. Thus, an alternative interpretation of the rule for high ψ_{TR} is simply that the Fed is setting its interest rate to enforce $FCI_t = \bar{FCI}_t$ as well as possible. When $\bar{FCI}_t$ is constructed optimally, then $\psi_{TR} \to \infty$ is very close to the optimal policy, since we are enforcing $FCI_t = \bar{FCI}_t$ in expectation, but that is also what the optimal policy does. For that reason, if we tried to pick the ψ_{TR} that minimizes

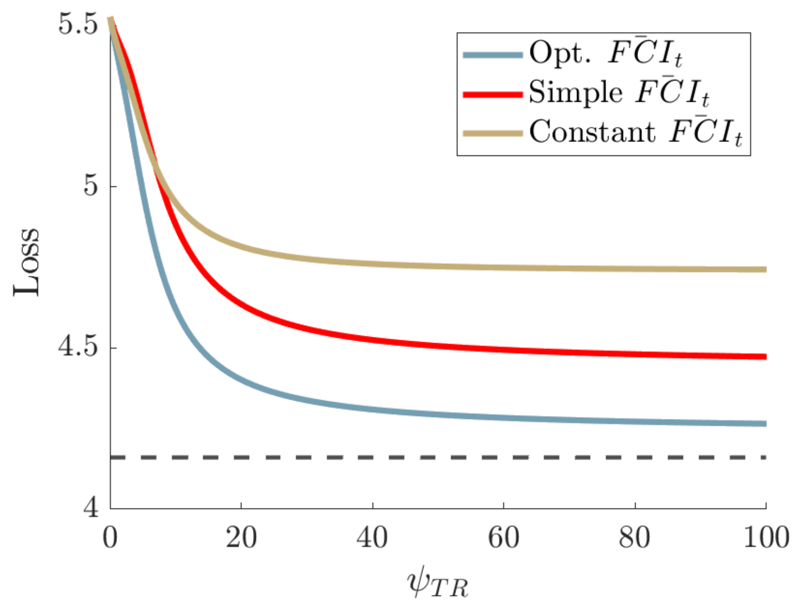

Figure 24: Loss $E[\mathcal{L}] = \sigma^2_{\tilde{y}} + \sigma^2_{\pi} + \lambda_{\Delta i}\sigma^2_{\Delta i}$ as a function of coefficient ψ_{TR} in (56), for different assumptions about $\bar{FCI}_t$. Blue: optimal $\bar{FCI}_t$. Red: approximate $\bar{FCI}_t$ constructed via equation (55). Beige: constant $\bar{FCI}_t$. Dashed: loss under optimal FCI targeting.

the loss using the optimal target, we get $\psi_{TR} \to \infty$.[36]

C.3.3. Counterfactual Second Moments under FCI-augmented Taylor Rule

Figure 25 presents (in blue) the counterfactual second moments for a Taylor Rule with $\psi_{TR} = 40$, where the target is constructed according to equation (55). The value of ψ_{TR} is chosen because the loss function is essentially flat after $\psi_{TR} = 40$.

Figure 25 also compares the results with the benchmark Taylor Rule, in red. As expected, the benchmark Taylor Rule (red dashed) aligns closely with the targeted unconditional standard deviations observed in the data (black dashed), though it predicts somewhat lower FCI conditional variance. Relative to this benchmark, adding a FCI target substantially reduces the variance of macroeconomic outcomes. Comparing medians, the unconditional variance of the output gap, inflation and the interest rate drop by $27\%, 4\%$ and 6% respectively. Regarding the conditional variance of the FCI, it sees a 27% reduction compared to the benchmark Taylor rule, which is already markedly lower than the levels observed in the data.

[36]For the approximate or constant targets, the minimum loss is attained for a finite but very large ψ_{TR}.

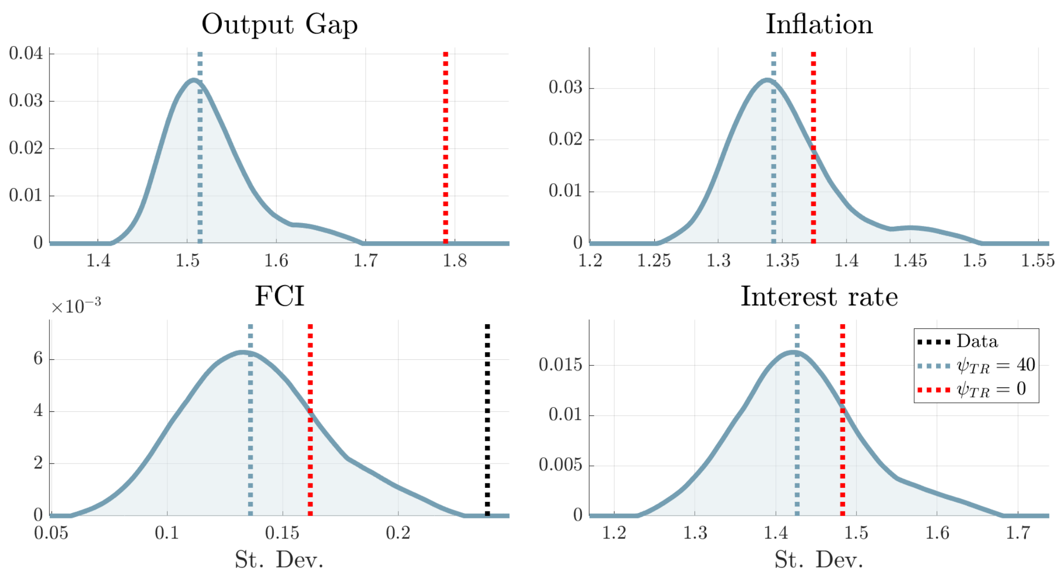

Figure 25: Counterfactual Standard Deviations. For Output Gap, Inflation and interest rates, this is the unconditional standard deviation, for FCI this is the conditional SD. Black dashed: data. Dashed lines the median. Red: baseline, Taylor Rule in 56 with $\psi = 0$. Blue: Taylor Rule with $\psi_{TR} = 20$ and $F\bar{C}I_t$ constructed suing (55). Solid Line: posterior density for the counterfactual, same Taylor Rule as blue.