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PRIVATE EQUITY FOR PENSION PLANS? EVALUATING PRIVATE EQUITY PERFORMANCE FROM AN INVESTOR'S PERSPECTIVE

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ABSTRACT

We evaluate private equity (PE) performance using investor-specific stochastic discount factors, and examine whether investors could benefit from changing their allocation to PE. Plans invest in PE funds with higher average risk-adjusted performance. This is mainly due to access to successful PE managers, not superior selection skill. Decomposing returns into risk-compensation and "alpha", we find that some plans obtain higher PE returns by taking more risk without earning higher, and in some cases earning lower, risk-adjusted returns, broadly consistent with agency problems within plans.

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The performance evaluation of investments is one of the most actively researched topics in finance. The typical approach is to compare historical rates of return against the return of similarly risky investments as predicted by various asset pricing models. In illiquid asset classes like private equity (PE), returns are not observed at regular intervals and valuations are stale and potentially biased, complicating this task (see Korteweg, 2023, for a review of the literature). The most commonly used performance measures in PE, such as the internal rate of return (IRR) and cash multiples, therefore rely primarily on fund cash flows (and do so exclusively in the case of fully liquidated funds). However, a high IRR or cash multiple could simply indicate a very risky investment, rather than an investment that raises a portfolio's Sharpe ratio upon inclusion. These limitations led to the development of performance metrics that use a stochastic discount factor (SDF) to discount cash flows. The key feature is the use of the cumulative rate of return of some portfolio (or a "levered" version of that portfolio) for discounting. For example, the Kaplan and Schoar (2005) public market equivalent (PME) metric uses the public stock market return to discount cash flows back to the fund's inception date.

The SDF approach to performance evaluation has several theoretical advantages, as we discuss in detail in section 1.1. For example, if the cash flows of the PE fund can be replicated by some dynamic trading strategy in publicly traded assets, then the present value of the private equity fund's cash flows has a value of zero for all SDFs that price these same public assets. However, in the more realistic case where such a replication is not possible, there could be multiple SDFs that price all the publicly traded assets but assign different values to the "unspanned" risks of private equity. This problem becomes particularly important if investors hold different optimal portfolios (e.g., due to non-participation in certain markets, or portfolio constraints), as they are likely to assign very different values to these unspanned risks.

In this paper we propose a pragmatic approach to determining the SDF that is to be used for discounting. The key idea is to use a given investor's own portfolio return to form the stochastic discount factor. We provide several theoretical arguments why using the investor's own return to form the SDF has some appealing properties, even if the financial market is incomplete. Specifically, we examine two measures. The first measure, the "investor portfolio equivalent" (IPE), is essentially the same measure as the PME, except that we use the investor's own portfolio return rather than the return of the market portfolio when discounting the cash flows of the private equity fund. We show that when this measure has a positive value, it indicates that an investor could raise the (logarithmic) growth rate of her investment portfolio by allocating a marginal dollar towards PE. While simple to compute and easy to interpret, the IPE measure has the disadvantage that it could be affected by different risk attitudes, which are reflected in an investor's optimal mix of stocks and bonds. To overcome this shortcoming, we develop a generalized version (GIPE), which takes into account different investor risk aversions. In effect, the GIPE uses an investor-specific, appropriately levered version of the investor's portfolio to form the SDF. An attractive property of the GIPE is that it is zero if a PE investment can be replicated by some, possibly levered, trading strategy in publicly traded stocks and bonds. In addition, the GIPE *does not depend* on an investor's risk aversion: If two investor portfolios differ in the shares that they invest in public equities and bonds, they still assign the same GIPE to a given PE investment. Because of this invariance property, the GIPE is the main measure that we use for our empirical analysis. In addition, we use the (G)IPE measure to compute an annualized "alpha". This alpha can be interpreted as the component of the internal rate of return of the investment that is not due to risk, but rather due to a meaningful expansion of the investment opportunity set for a specific investor.

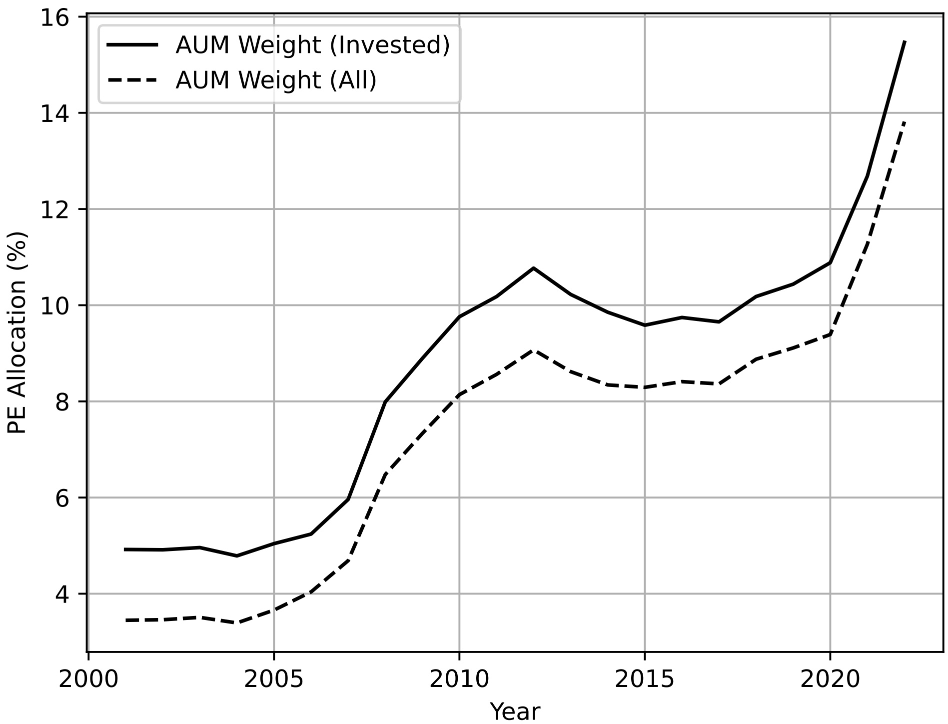
We also develop diagnostics to determine whether simple, *long-only* public market strategies can produce the same gains as a given PE investment (e.g., a value strategy for buyout funds). The comparison to long-only investment alternatives is important, because many large investors in private equity are constrained or altogether prohibited from shorting.

To illustrate our approach, we compute the (G)IPE of several PE strategies from the perspective of U.S. public pension plans.[1] We focus on public pension plans for several reasons. First, as Figure 1 illustrates, public pension plans have been very active investors in private equity, and many plans now have double-digit percentage allocations to this asset class (see also Ivashina and Lerner, 2018; Begenau et al., 2024). Second, underfunding and corporate governance concerns about pension plans make it especially important to risk-adjust their investments. Third, there is large cross-sectional heterogeneity in portfolios. For example, in 2018, the Public School Employees Retirement System of Pennsylvania had an allocation to public equities of 17.9%, compared to 61.9% for the Employees' Retirement System of Georgia. The cross-sectional mean across all pension plans for

[1] For disambiguation, we use the nomenclature pension *plans* (rather than pension funds) to distinguish them from private equity *funds*. Also, when we write private equity we mean all forms of PE, including but not limited to venture capital, buyout, and real estate funds.

Figure 1: **Average Public Pension Plan Allocation to Private Equity**

This figure shows the time series of portfolio allocations to private equity (PE) by U.S. public pension plans from 2001 to 2022. The solid line represents the assets under management (AUM) weighted average of allocations to PE across plans that have positive PE allocations in each year. The dashed line represents the AUM-weighted average across all plans, assigning zero PE allocations to the plans without reported PE investments. Source: Comprehensive Annual Financial Report (CAFR) data from the Center for Retirement Research at Boston College, the Mission Square Research Institute, the National Association of State Retirement Administrators, and the Government Finance Officers Association (available at `https://publicplansdata.org`).



the year was 46.2%, and the standard deviation was 9.8%. Dispersion in portfolio holdings was similar in other years. Fourth, data on pension plan investment returns are readily available, which helps in forming our SDFs.

Our results can be summarized as follows. First, for our sample period, which runs from 1995 to 2018, the GIPE measure (averaged across pension plan and PE fund combinations) is approximately zero, indicating that the representative pension plan would not have benefited from changing its allocation to PE. A notable exception is buyout funds, which show a positive GIPE. This positive GIPE is partly, but not entirely, due to their value exposure. By contrast, VC funds underperformed on average (i.e., its GIPE was negative), but less so than publicly traded small-growth firms.

Second, we do not find evidence that pension plans had market timing skill. We do find that the PE funds that were selected by public pension plans outperform the average PE fund of the same vintage year, but this does not seem to be due to a genuine ability to select the better-performing PE investments. Rather, it appears that the PE funds give certain investors preferential access to the better PE investments: pension plan "selectivity skill" disappears when we confine attention to the universe of private equity investments that are continuations of ongoing relationships with a given pension plan, or when we consider first time funds (which are likely to be less selective about their clients).

Third, we decompose the IRR of a pension plan's private equity investments into a risk-compensation and an alpha component and examine whether plan characteristics that correlate with a high IRR do so because of the riskiness of these PE investments, or because of a meaningful expansion of the investment opportunity set (alpha). We find that underfunded pension plans, and plans with boards that have a larger fraction of state officials and members of the public appointed by a government official, take more risk but earn lower risk-adjusted returns in their PE investments. One potential channel to explain the lower risk-adjusted returns of underfunded plans is that they tend to invest in PE managers whose portfolio firms reduce labor productivity, as shown in Mittal (2022). Investments in PE funds that are located in the pension plan's state do not differ from out-of-state funds in their level of risk, but they do earn lower risk-adjusted returns. Taken together, these results are suggestive of agency problems, such as gambling for resurrection and political influence, playing an important role in the selection of PE investments by pension plans.

The paper is related to several strands of literature. First, our SDF-based metrics are related to the public market equivalent measure of Kaplan and Schoar (2005) and, especially, the generalized PME (GPME) of Korteweg and Nagel (2016). The key difference is that we use investor-specific SDFs, which allows for the possibility of heterogeneous investors with different investment opportunity sets. Compared to GPME, we show that our GIPE measure produces a performance distribution that is more stable over time, consistent with relatively constant investment skill (or lack thereof). The relative objectivity of GIPE-type metrics to assess the PE performance for an individual investor is also important, as Augustin et al. (2024) find that the choice of PE investment benchmarks by pension plans is subject to agency problems with respect to investment consultants.

The second related strand is the literature on limited partner performance in private equity.

Prior work has shown that different types of limited partners experience different performance (e.g., Lerner et al., 2007; Sensoy et al., 2014; Dyck and Pomorski, 2015; Cavagnaro et al., 2019; Goyal et al., 2022). Korteweg and Westerfield (2022) survey this literature. For pension plans specifically, several papers consider the gambling for resurrection (also known as risk-shifting) concern for underfunded plans by studying the relation between underfunding and the share of risky assets in the portfolio. The evidence is mixed. Most studies find evidence of risk-shifting (Pennacchi and Rastad, 2011; Mohan and Zhang, 2014; Bradley et al., 2016; Andonov et al., 2017; Lu et al., 2019; Myers, 2022), but Begenau et al. (2024) find weaker support, and Lucas and Zeldes (2009); Rauh (2009) find the opposite result (although Rauh studies private, not public, pension plans). Andonov et al. (2017) find that underfunding is associated with lower overall plan returns. Our contribution is to separate risk and excess return within PE investments, which allows us to examine whether the documented heterogeneity in PE performance is simply due to a difference in risk-taking. Similarly, the literature that considers home bias (e.g., Lerner et al., 2007; Hochberg and Rauh, 2013; Bradley et al., 2016; Andonov et al., 2018) and board structure (Bradley et al., 2016; Andonov et al., 2017, 2018) in public pension investing only considers broad plan performance (not specific to PE) or only total (not risk-adjusted) PE performance.

The paper is organized as follows. Section 1 develops the theory behind our performance measures. Section 2 describes the pension plan and private equity fund data. Sections 3 and 4 present the empirical results on PE fund performance and the heterogeneity in performance across pension plans, respectively. Section 5 concludes.

1 Theoretical framework

Private equity funds are structured as limited partnerships. Investors, such as pension funds, are the limited partners (LPs) of the fund. The LPs are pure capital providers and have no control over which deals are invested or exited. Capital is committed at fundraising but not immediately transferred to the fund. Instead, the fund manager (general partner, or GP) searches for deals, and "calls" capital from the LPs when they have identified an investment. Money from the sale of investments is distributed to the LPs, after fees to the GP. The fund has a limited lifetime to invest its committed capital and realize exits (typically 10 years, with limited extension options in case of unexited investments). When all portfolio investments have been sold, the fund is liquidated. For an in-depth description of the PE industry, see, for example, Korteweg and Westerfield (2022).

From an LP's perspective, committing to a PE fund may be viewed as producing a sequence of future net-of-fee fund cash flows $C = \{C_{t_0}, .., C_{t_K}\}$ of random magnitude, timing, and number. Capital calls are negative flows for the LP ($C_{t_k} < 0$) whereas distributions are positive ($C_{t_k} > 0$). Typically, the first cash flows are capital calls, with distributions occurring later in the fund's life, but we place no restriction on the sign of each flow.

The fund's "net asset position", A_t, is the *economic value* of the assets that help finance the cash flow series C. At t_0, the time of fund inception, $A_{t_0} = 0$. The asset value increases when a capital call occurs and decreases upon a distribution, that is, $A_{t_k^+} = A_{t_k} - C_{t_k}$. After the final cash flow (i.e., at fund liquidation), the asset value is zero again: $A_{t_K^+} = 0$. Between capital calls and distributions, A_t evolves randomly according to some diffusion process.

We assume throughout that an econometrician cannot observe the true A_t. This is a realistic assumption in PE, because quarterly fund net asset values (NAVs) reported by GPs are subject to staleness and manipulation.[2] Therefore, as is typical in the literature and in practice, our performance measures rely on the observed cash flows, only using the final reported NAV as a pseudo-distribution in cases where the fund is not yet liquidated at the end of the sample period.

In addition to the cash flow sequence C, investors have access to N other risky investments (public equities, long term bonds, commodities, etc.) and a risk-free security yielding the instantaneous interest rate r_t. The vector of risky security returns follows a diffusion process

$$\underbrace{dR_t}_{N\times 1} = \underbrace{\mu_t}_{N\times 1} dt + \underbrace{\sigma_t}_{N\times d} \underbrace{dB_t}_{d\times 1}, \tag{1}$$

where μ_t is the vector of expected returns and σ_t is an $N \times d-$dimensional matrix of exposures to the $d-$dimensional Brownian motion, dB_t. We assume that $d \geq N$ to allow for the possibility that the market is incomplete.

[2] As of 2007, Accounting Standards Code (ASC) Topic 820 (formerly known as FAS 157) requires the disclosure of fair values. However, there is no market to mark PE investments to. GPs and auditors usually rely on the pricing of recently traded comparable assets, but this is a subjective process. For empirical evidence of staleness and manipulation of reported NAVs, see Phalippou and Gottschalg (2009); Jenkinson et al. (2013); Barber and Yasuda (2017); Chakraborty and Ewens (2018); Brown et al. (2019); Jenkinson et al. (2020).

1.1 SDF-based performance evaluation

A common approach to evaluating whether an investment in the cash-flow stream C is an attractive investment opportunity is to test whether its net present value is zero, using a stochastic discount factor (SDF) to discount the sequence of payments in C.

The justification for the SDF-approach is a replication argument. Specifically, suppose that there exists a portfolio process ϕ_t of the N risky assets such that the fund's net asset position evolves as

$$\frac{dA_t}{A_t} = r_t dt + \phi_t' \left(dR_t - r_t 1_N dt\right), \tag{2}$$

where 1_N is a column vector of ones. The position in the risk-free asset is $1 - 1_N'\phi_t$, ensuring that the portfolio is self-financing. An implication of (2) is that the cash flows C can be exactly replicated with the dynamic, self-financing strategy ϕ_t in existing assets. Letting H_t denote any SDF that prices the N risky assets, a standard no-arbitrage argument implies that the net present value (NPV) of C should be zero:

$$0 = A_{t_0} = E_{t_0} \sum_{k=0}^{K} \left(\frac{H_{t_k}}{H_{t_0}}\right) C_{t_k}. \tag{3}$$

Matters become more complicated in the more realistic situation where C cannot be replicated by some dynamic trading strategy. In this case,

$$\frac{dA_t}{A_t} = r_t dt + \phi_t' \left(dR_t - r_t 1_N dt\right) + d\widetilde{R}_t, \tag{4}$$

where $d\widetilde{R}_t$ is a residual; equivalently, $d\widetilde{R}_t$ can be viewed as the excess return of a fictitious asset that is orthogonal to all N risky asset excess returns.[3] Now, choosing an SDF that prices the N existing risky assets no longer implies that the zero-NPV equation (3) holds. The sign and magnitude of the NPV will depend on the choice of SDF, since the usual no-arbitrage replication argument cannot provide a unique and unambiguous way of pricing the unreplicable return component $d\widetilde{R}_t$.[4]

[3] To obtain (4), regress $\frac{dA_t}{A_t} - r_t dt$ on a constant and the excess return vector $dR_t - r_t 1_N$:

$$\frac{dA_t}{A_t} - r_t dt = a dt + \phi_t' \left(dR_t - r_t 1_N dt\right) + d\eta_t,$$

and define $d\widetilde{R}_t \equiv d\eta_t + a dt$. Note that the residual $d\eta_t$ is orthogonal to the excess returns on all N risky assets.

[4] For example, there could be one or more PE-specific (risk) factors in the unspanned component $d\widetilde{R}_t$. But, even

1.2 Investor Portfolio Equivalent

To account for the fact that in incomplete markets different investors may value the unspanned return components differently, we propose a relatively simple modification to the NPV criterion that incorporates investor-specific SDFs. To fix ideas, and in order to relate our performance measures to the ones conventionally used in the PE literature, we first consider an investor who is interested in maximizing expected logarithmic wealth:

$$V(t_0, W_{t_0}) \equiv E_{t_0} \log W_T, \tag{5}$$

where W_t is the value of their portfolio at time t, and $T > t_K$ is some distant time. There is no intermediate consumption financed by the portfolio, and therefore maximizing the expectation of the logarithm of terminal wealth is equivalent to maximizing the expected logarithmic return of the portfolio. Absent PE, the investor maximizes (5) over the $N \times 1$ vector of portfolio weights in the existing risky assets, $w_t \in \mathcal{W}$, with the weight on the risk-free asset being $1 - 1_N' w_t$. The set $\mathcal{W}$ captures any constraints placed on the investor's portfolio. For example, if an investor is prohibited from investing in the asset i, then $w_i = 0$ for all $w \in \mathcal{W}$. When portfolio choice is unconstrained, $\mathcal{W} = \mathbb{R}^N$.

We next present the investor with the opportunity to invest a small amount $\varepsilon > 0$ in the PE cash flow sequence C, and ask whether this investment improves her objective function. The following proposition is an implication of the envelope theorem (all proofs are in Appendix A).

Proposition 1 *Suppose that an investor maximizes (5) over $w_t \in \mathcal{W}$. Let εC denote an investment of $\varepsilon > 0$ in the cash flow process C, and define the Investor Portfolio Equivalent (IPE)*

$$IPE \equiv E_{t_0} \sum_{k=0}^{K} \left(\frac{W_{t_0}}{W_{t_k}} \right) C_{t_k}, \tag{6}$$

if $d\widetilde{R}_t$ were independent across PE funds, the usual APT-like asymptotic arbitrage argument that a well-diversified PE portfolio would be spanned by the N risky factors is unlikely to hold. Unlike in publicly traded securities, diversification in PE is difficult and expensive. Funds are not traded publicly and the market in LP stakes has low liquidity and high transaction costs (e.g., Nadauld et al. (2019)). Moreover, GPs require minimum commitments, and negotiations between GPs and LPs are time-consuming. Indeed, Gredil et al. (2023) document that most LPs invest in only one or two PE funds per year. The low correlation of returns of alternative-asset positions across investors, which we document below, further strengthens this argument: if it was simple to create a well-diversified portfolio, these returns should be highly correlated, as investors should only retain the factor risk(s), but not idiosyncratic risk.

where $\frac{W_{t_0}}{W_{t_k}}$ *is the inverse of the cumulative return on the investor's portfolio.*

Then $\frac{dV}{d\varepsilon} = V_W(t_0, W_{t_0}) \times IPE$*, where* $V_W(t_0, W_{t_0})$ *is the marginal value of a dollar at time* t_0*.*

The IPE resembles the PME as defined by Korteweg and Nagel (2016), except that the return on the investor's portfolio is used in place of the stock market return. Thus, instead of a "one-size-fits-all"pricing assumption, the IPE uses each investor's own investment return to price the unspanned return components. Its theoretical advantage is its direct connection to the investor's objective: Per Proposition 1, the change in the investor's objective for an ε commitment equals the IPE times the investor's marginal value of wealth at time t_0, $V_W(W_{t_0})$. This means that the investor is indifferent between committing to the cash flow sequence C or receiving a lump sum of IPE dollars at time t_0. As such, the investor finds it worthwhile to commit to the fund if and only if $IPE > 0$.

Remark 1 *Under the assumptions that (a) the investor's objective function* (5) *is correctly specified and (b) portfolio constraints are not binding,* $H_t = W_t^{-1}$ *is also a valid stochastic discount factor for all assets that comprise the investor's portfolio.*[5] *Per the result of the previous section, this implies that if the cash flow sequence* C *can be replicated with a trading strategy using the assets that the investor is already invested in, then the IPE will be zero and* C *does not present a meaningful expansion of the investor's investment opportunity set.*

Before proceeding, it is useful to relate the IPE to the PME measure of Kaplan and Schoar (2005) One might suspect that if investors maximize (5), then the cumulative return on public equities provides a valid SDF,[6]and therefore the PME approach is valid. This conclusion relies on the implicit assumption that the stock market is the only positive supply asset in the representative investor's portfolio. In the more realistic case where there are multiple positive-supply assets in the investor's portfolio (with PE being one of them), then the PME will be a biased away from unity, even if all assets are fairly priced (see Gârleanu and Panageas (2024) for a proof[7]). Unlike the PME, the IPE does not suffer from such a bias. An implication of Remark 1 is that the PME

[5] The fact that the return of the expected-logarithmic-growth-maximizing portfolio is a valid SDF is a well-known result (see Long (1990) and papers cited therein).

[6] See Sørensen and Jagannathan (2015).

[7] The argument in Gârleanu and Panageas (2024) is easy to sketch. Consider a one-period model and let R_1 denote the gross return on public equities ("the stock market") and R_2 the gross return on private equity, which is defined as the ratio of the time-1 PE cash flow dividend by period 0 commitment. Suppose an investor's portfolio return, R_w, is a weighted average of two investments, $R_w = a_1R_1 + a_2R_2$, with weights a_1 and a_2 that are positive and sum to one,

has an expectation of zero in a market where PE is fairly priced (assuming that investors maximize the objective (5)).

While the IPE can better handle situations where the representative investor's portfolio differs from the stock market, it is sensitive to the assumption that all investors maximize (5) (i.e., that they have unit risk aversion.) If one drops this assumption, the IPE will depend not only on the PE investment, but also the risk aversion of the investor. For example, the IPE of the same PE investment will generally be higher for a relatively more risk averse investor, whose portfolio is heavily invested in bonds, as compared to a more risk tolerant investor, whose portfolio is invested in stocks. We next develop a measure that is designed to tackle hererogenous risk aversion.

1.3 Generalized IPE

In this section we generalize the IPE by considering a power utility investor with a constant relative risk aversion (CRRA) coefficient $\gamma > 0$, whose objective is to maximize

$$V(t_0, W_{t_0}) = E_{t_0} \frac{W_T^{1-\gamma}}{1-\gamma}. \tag{8}$$

To underpin the objective function of (8), in Appendix B we outline a standard, stochastic, overlapping generations (OLG) model with agents maximizing a CRRA utility function. We show that if pension plans are viewed as delegated portfolio managers tasked with maximizing the welfare of their participants, then they must evaluate investment projects with a utility function identical to that of plan participants, effectively adopting the CRRA objective function of their investors. Moreover, if pension plans (a) choose investment allocations so as to maximize a CRRA objective, (b) set an appropriate contribution rate for workers, and (c) specify pensions in proportion to end-of-work-life wages, then in equilibrium the pension obligations match the pension liabilities.[8] We

$a_1 + a_2 = 1$. The assumption of unit risk aversion implies that the SDF is R_w^{-1}. Assume that all assets are "fairly priced" in the sense that the SDF-discounted returns have an expectation equal to one: $E\left[\frac{R_i}{R_w}\right] = 1$ for $i = 1, 2$. We obtain the following:

$$1 = \frac{1}{E\left[\frac{R_1}{R_w}\right]} = \frac{1}{E\left[\frac{R_1}{a_1 R_1 + a_2 R_2}\right]} \leq E\left[\frac{a_1 R_1 + a_2 R_2}{R_1}\right] = a_1 + (1 - a_1) E\left[\frac{R_2}{R_1}\right], \tag{7}$$

where the inequality follows from Jensen's inequality applied to the convex function $x \to x^{-1}$. Since $a_1 < 1$, the inequality is equivalent to PME$\equiv E\left[\frac{R_2}{R_1}\right] \geq 1$, despite the fact that all assets are fairly priced.

[8]This result requires only that the labor share of output is constant and that capital depreciates fully between periods. If capital depreciates only partially, then pensions needs to be specified as a linear combination of the wage

note that the log-wealth objective (5) in the previous section can be viewed as the special, limiting case when $\gamma = 1$.

A straightforward adaptation of Proposition 1 yields a similar result, namely a generalized IPE (GIPE) metric that represents the marginal improvement in wealth from investing in PE:

Proposition 2 *Consider the same investment εC as in Proposition 1, except that the investor maximizes (8) over risky-asset weights $w_t \in \mathcal{W}$. Also assume (a) the investment opportunity set is constant, $\mu_t = \mu, \sigma_t = \sigma, r_t = r$ and (b) the investor's choice of risk-free bonds is interior.*[9] *Define*

$$GIPE \equiv E_{t_0} \sum_{k=0}^{K} e^{-r(t_k - t_0)} \frac{\left(\frac{W_{t_k}}{W_{t_0}}\right)^{-\gamma}}{E_0\left[\left(\frac{W_{t_k}}{W_{t_0}}\right)^{-\gamma}\right]} C_{t_k}. \tag{9}$$

Then $\frac{dV}{d\varepsilon} = V_W(t_0, W_{t_0}) \times GIPE$, where $V_W(t_0, W_{t_0})$ is the marginal value of a dollar at time t_0.

Proposition 5 in Appendix A includes a generalization of Proposition 2 to time-varying μ_t, σ_t, and r_t, that shows that, under certain conditions, (G)IPE and $\frac{dV}{d\varepsilon}$ have the same sign, so that (G)IPE is the correct metric to determine whether the investment in C should be undertaken.

Under the assumptions of Proposition 2, and the additional assumption that portfolio choice is unconstrained ($\mathcal{W} = \mathbb{R}^N$), the process $H_t = e^{-rt} \frac{(W_t)^{-\gamma}}{E_0\left[W_t^{-\gamma}\right]}$ is a valid SDF for the assets in the investor's portfolio. Therefore, if C can be replicated by some self-financing trading strategy in these assets, then (G)IPE equals zero.

The (G)IPE is closely related to the familiar beta-based approach to performance evaluation:

Proposition 3 *Maintain the same assumptions as in Proposition 2 and assume that the portfolio choice of the investor is not subject to constraints, $\mathcal{W} = \mathbb{R}^N$. Suppose the return process $\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dB_t$ is observed, where μ_A is the expected return of $\frac{dA_t}{A_t}$ and σ_A is a row vector of exposures to the $d-$dimensional Brownian motion of equation (1). Let β denote the regression coefficient from regressing $\frac{dA_t}{A_t}$ on $\frac{dW_t}{W_t}$, and let $\mu^W - r \equiv w'(\mu - r)$ denote the expected excess return on the investor's portfolio. Then the GIPE is zero if*

$$\mu_A - r = 0 + \beta\left(\mu^W - r\right). \tag{10}$$

when young and the wage when old.

[9] A sufficient (but not necessary) condition for assumption (b) to hold is the following: If $w \in \mathcal{W}$ then $\kappa w \in \mathcal{W}$ for any scalar κ.

Equation (10) is in the form of a familiar "Jensen's alpha" regression. The left-hand side is the expected excess return of investing in the asset with return process A_t. The right-hand side is the sum of a "Jensen's alpha" of zero and a "risk compensation component" which is the product of the expected excess return on the investor's overall portfolio times the "beta" from a regression of the returns of A_t on the investor's overall portfolio return. Equation (10) shows that a zero GIPE is essentially the same as a zero "Jensen's alpha" in a regression of $\frac{dA_t}{A_t}$ on the investor's portfolio return. The advantage of the GIPE is that it does not require A_t to be observed. Instead, it can be computed from the cash flow sequence C alone.

An important implication of (10) is that if investors are unconstrained in their portfolio choice, then the risk compensation component $\beta\left(\mu^W - r\right)$ does not depend on the risk aversion of an investor.[10] Since the risk compensation component does not depend on the investor's risk aversion, two investors with different risk aversions (and accordingly different stock-bond portfolio composition) will assign the same Jensen's "alpha" to a given PE investment. Equivalently, two investors with different risk aversion coefficients will assign the same GIPE value to a given PE investment.

1.4 Portfolio constraints and mimicking funds

The results sofar can be summarized as follows: A positive GIPE signals a beneficial investment, irrespective of whether the investor's portfolio choice in existing assets is unconstrained or not (as long as Assumption (b) of Proposition 2 holds). Under the additional assumption that portfolio choice is unconstrained, a positive GIPE implies that the investor cannot attain the benefits of the PE investment by some dynamic trading strategy in the assets she is already investing in. When portfolio constraints are binding, then a positive GIPE still signals a beneficial investment opportunity, but this opportunity could be because it helps relax some of the investor's portfolio constraints in existing risky assets. In effect, the PE fund could provide a "backdoor" way to increase exposure to risk factors that the investor cannot obtain directly, similar to the argument

[10] To see this, note that neither side of equation (10) depends on γ. The left-hand side follows by inspection. For the right-hand side, note that an investor's optimal portfolio is proportional to $\frac{1}{\gamma}$,

$$w = \frac{1}{\gamma}\left(\sigma\sigma'\right)^{-1}(\mu - r).$$

Accordingly,

$$\beta\left(\mu^W - r\right) = \frac{\left(w'\sigma\sigma'_A\right)\left(w'(\mu - r)\right)}{w'\sigma\sigma' w}$$

is independent of γ.

that certain financial instruments (such as options and leveraged ETFs) may alleviate borrowing constraints for investors (see Frazzini and Pedersen, 2022).

It is therefore useful to diagnose whether a positive (G)IPE is due to the attractiveness of PE's nonreplicable return component, or whether it reflects an investor's portfolio constraints. To this end, we construct a cash-flow process, $\widehat{C}$, which mimics the cash-flows of a PE fund by using existing assets, with cash flows occurring at the same times t_k as C. Specifically, capital calls are equal to those of C,:

$$\widehat{C}_{t_k} \equiv C_{t_k}, \text{ for } C_{t_k} < 0. \tag{11}$$

The capital calls are invested in a benchmark portfolio of the existing assets $1, ..., N$. The benchmark portfolio earns an (instantaneous) return dR_t^b. We define G_t to be the cumulative return on investing a dollar in the benchmark, so that $G_{t_0} = 1$, $\frac{dG_t}{G_t} = dR_t^b$.

Whenever the PE fund has a distribution, the mimicking fund distributes[11]

$$\widehat{C}_{t_k} = \widehat{V}_0 \omega_{t_k} G_{t_k}, \tag{12}$$

where

$$\widehat{V}_0 \equiv \sum_{k=0}^{K} \frac{|C_{t_k}|}{G_{t_k}} 1_{\{C_{t_k}<0\}} \qquad \text{and} \qquad \omega_{t_k} \equiv \frac{\frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}}{\sum_{k=0}^{K} \frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}}.$$

The quantity $\widehat{V}_0$ is the present value of all capital commitments, and the ratio ω_{t_k} is the present value of the distribution at time t_k as a fraction of the present value of all distributions, with the benchmark return serving as the discount rate.

The mimicking fund has desirable characteristics, as established in the following proposition. The most important one is that C and $\widehat{C}$ coincide when the return on the PE fund's net asset position, $\frac{dA_t}{A_t}$, is the same as the return on the benchmark portfolio.

Proposition 4 *a) The distributions (12) are non-negative. b) The cash flows $\widehat{C}_{t_k}$ given by (11) and (12) can be financed by the trading strategy of investing the capital calls at the benchmark return R_t^b and making distributions equal to (12). c) Under the null hypothesis that the net asset position*

[11] The construction of $\widehat{C}$ is similar to the modified PME proposed by Cambridge Associates. Both approaches consider a mimicking fund that invests the capital calls in the benchmark portfolio. In both approaches the distributions are positive. But our approach enforces that under the null hypothesis (13) the sequences C and $\widehat{C}$ coincide. Moreover, our approach does not rely on GP-provided NAVs, which may be problematic, as discussed above.

associated with the cash flow C grows at the benchmark return,

$$\frac{dA_t}{A_t} = dR_t^b, \tag{13}$$

the cash flows of the mimicking fund $\widehat{C}$ are identically equal to the cash flows of the fund C.

The mimicking fund $\widehat{C}$ allows us to perform an exercise similar in spirit to the popular "style" analysis that Sharpe (1988, 1992) introduced for mutual funds. Specifically, we choose a benchmark portfolio, construct the mimicking cash-flow sequence $\widehat{C}$, and examine its (G)IPE, as well as that of the differential cash flows $C - \widehat{C}$.[12] As a practical matter, we are particularly interested whether the PE cash flows can be replicated by simple, long-only strategies in publicly-traded benchmark portfolios (e.g., a value portfolio return could be used as a benchmark for buyout funds, or a small-growth portfolio return could be used as a benchmark for venture capital funds).

Table 1 summarizes the interpretation of the (G)IPE of $\widehat{C}$ and that of $C - \widehat{C}$. If the (G)IPE of $\widehat{C}$ is zero, then the replicable component of PE cash flows does not improve the investor's objective function, suggesting that portfolio constraints are not binding. In this case, the (G)IPE of C is equal to the (G)IPE of $C - \widehat{C}$, and its sign indicates whether the nonreplicable component of the PE fund improves the investor's objective function. If instead the (G)IPE of $\widehat{C}$ is positive (resp. negative), then even a marginal increase (decrease) in the allocation to the publicly-traded benchmark portfolio would improve the investor's objective, indicating a portfolio constraint to publicly traded assets. In those situations, the (G)IPE of $C - \widehat{C}$ reveals whether the nonreplicable component of the PE cash-flows raises the investor's objective function relative to investing in (or shorting) the benchmark portfolio.

One other (not mutually exclusive) reason why the (G)IPE of the mimicking fund $\widehat{C}$ could deviate from zero is market timing ability of the PE manager in the benchmark asset. For example, suppose that the manager simply invests in the public stock market index. Suppose also that the expected return of the public stock market, μ_t, can have one of two values, $\mu^H > \mu^L$, that switch according to some regime-switching process. The manager has the ability to predict regime switches, which allows her to collect a capital call at the beginning of regime H, invest it in the stock market, and liquidate it immediately before the regime is about to switch to L. By contrast, the investor

[12]Since (G)IPE is linear in cash flows, the (G)IPE of $C - \widehat{C}$ is equal to the (G)IPE of C minus that of $\widehat{C}$.

Table 1: **Interpretation of Performance with Private Equity-mimicking Funds**
The first column of this table describes the interpretation of the (G)IPE of the PE-mimicking fund cash flows, $\widehat{C}$. These funds have identical capital calls to the PE funds, invest in a chosen benchmark portfolio, and distribute cash at the same times as the PE funds (see the main text for a complete description of their construction). The second column describes the interpretation of the cash flow component of the PE funds that is not replicable by investing in the benchmark, i.e., the difference between the PE fund cash flows, C, and the mimicking flows $\widehat{C}$. The columns are to be considered independently, for example, $\widehat{C}$ could have a (G)IPE of zero while $C - \widehat{C}$ has a positive (G)IPE.

(G)IPE of	$\widehat{C}$	$C - \widehat{C}$
< 0	Portfolio constraints prevent reducing/shorting exposure to existing (non-PE) assets and/or the PE manager has negative timing ability.	The nonreplicable component of the PE payoff has negative value for the investor.
$= 0$	The replicable component of PE neither adds nor destroys value for the investor.	The nonreplicable component of the PE payoff neither raises nor lowers the investor's objective function.
> 0	PE relaxes binding (long) portfolio constraints and/or the PE manager has positive timing ability.	The nonreplicable component of the PE payoff has positive value for the investor.

(LP) does not know which regime the economy is in and holds a constant portfolio in the stock market. In this example, the cash flow stream C has a positive GIPE, but the GIPE of $C - \widehat{C}$ is zero, since assumption (13) holds by construction. This example shows that the (G)IPE of $C - \widehat{C}$ is tailored to identify whether the nonreplicable component of the PE investment is valuable for the investor above and beyond any timing ability the manager may possess.

2 Data

We use data on pension plans and private equity funds to compute (G)IPE metrics. We describe each data source in turn.

2.1 Pension plans

We use pension plan data collected from Comprehensive Annual Financial Reports (CAFRs) of U.S. defined benefit public pension plans. These reports contain balance sheet and income statement information, returns, valuations, actuarial data, and other key statistics, audited to conform to Government Accounting Standards Board (GASB) reporting requirements.

We start with a CAFR data set developed and maintained by a collaboration of the Center for Retirement Research at Boston College, the Center for State and Local Government Excellence, and the National Association of State Retirement Administrators.[13] The data includes 179 state and local pension plans for the years 2001 to 2018, and cover 95% of U.S. public pension membership and assets. We extend coverage by hand-collecting additional CAFRs going back to 1995, when most start to be available online. Furthermore, for some plans we can impute pre-2001 annual returns from reported longer-horizon returns.

We manually consolidate pension plans whose assets are jointly owned and managed, since their reported returns are virtually identical. For example, Maine's local employee plan that covers participating districts and its state employee and teacher plan are both managed by the Maine Public Employees' Retirement System (PERS), even though they are treated separately for accounting purposes and publish separate CAFRs. Maine PERS is also the investor listed in the private equity fund commitment data described below. To ensure completeness of information for the pension plans in our data set, we add the information of 6 additional plans that were not in the original data download, but that are jointly managed with plans that were. Furthermore, there are three cases where plans consolidate during our sample period, and one case where a plan splits off and becomes separately managed. In the former case, we treat the plans as separate entities pre-consolidation, and as one new, combined entity afterwards. In the latter case, the separating plan is considered to be a new entity, while the remaining ones continue to be consolidated in an ongoing, albeit smaller, entity. After these adjustments, we arrive at a final sample of 150 pension plans.

Panel A of Table 2 reports descriptive statistics of the pension plans. The first four columns show characteristics across all 150 plans. The most common plan covers state and/or local employees (70% of the sample), followed by teachers (29%) and police or fire personnel (21%). Note that these categories are not mutually exclusive due to the merging of some jointly managed plans that cover multiple employee types. A majority of plans (67%) are administered at the state level, with the remainder administered locally (either by a county, city, or school district). About two thirds cover multiple employers, and the vast majority of these plans have a cost-sharing agreement that pools the employers' pension assets and obligations.

The size distribution of pension plans is highly skewed. Across all plan-years, the median assets under management (AUM) is \$7.20 billion. The average, \$19.06 billion, is pulled upwards by a few very large plans, such as CalPERS, which had \$354 billion in AUM in 2018. Most plans are

[13] This data set is freely downloadable from: `https://publicplansdata.org`.

Table 2: **Descriptive Statistics**

Panel A shows descriptive statistics for public pension plans between 1995 and 2018. The first four columns show statistics for the full sample of 150 pension plans, the next four columns show the statistics for the sample of 117 plans for which we observe at least one private equity (PE) fund commitment in the Preqin data, and the column labeled "Diff. test p" shows the p-value for the test of equal proportions or means across the 117-plan subsample with commitment data and the 33-plan subsample without commitment data (for panel variables, *Assets*, *Funded ratio*, and *Annual return*, the tests include year fixed effects and double-cluster residuals by year and pension plan state). Numbers in the "%" column refer to the percentage of plans that belong to the category described in that row. The covered employee categories (*State/local employees*, *Teachers*, *Police/fire*) are not mutually exclusive. *State-administered plan* refers to a plan administered at the state level (rather than at the county, city, or school level). A *Multiple employer plan* covers employees for more than one employer, and *Cost sharing across employers* is the percentage of multiple employer plans in which employers are jointly responsible for each others' pension liabilities. *Assets* is the market value of pension assets in billions of U.S. dollars. *Funded ratio* is the ratio of actuarial assets to liabilities under GASB 25 accounting standards. *Annual return* is the one-year return on the pension plan (in %). Statistics for these three panel variables are calculated across all plan-years. *PE fund commitments per plan* is the plan's number of observed commitments to PE funds in the Preqin data over the sample. Panel B shows descriptive statistics for 1,303 North American private equity funds raised between 1995 and 2013, by strategy (venture capital (VC), buyout, and real estate). *Number of GPs* is the number of unique fund managers. *Fund size* is the fund's committed capital in millions of U.S. dollars. *Percentage of funds liquidated* is the percentage of funds that are fully liquidated (*100% liquidated*) or that have less than 5% of committed capital left in residual net asset value (*95% liquidated*). *Fund effective years* is the number of years between the first and the last observed cash flow for the fund, where cash flows are observed until the end of June 2018. *IRR* is the internal rate of return, *TVPI* the total value to paid-in capital, and *PME(KS)* and *PME(KN)* are the Public Market Equivalent of Kaplan and Schoar (2005) and Korteweg and Nagel (2016). *Size-weighted* is the NAV-weighted average performance metric. *Funds with matched LP data* is the number of funds for which we observe at least one public pension plan commitment in our data set. *Number of matched LPs / fund* is the number of observed pension plans commitments in a given fund in our data.

Panel A: Pension plans.

	Full Sample (N = 150)				Commitment Sample (N = 117)				Diff.
	%	Mean	Median	St.Dev.	%	Mean	Median	St.Dev.	Test p
Plans covering									
State/local employees	70.00				71.79				0.370
Teachers	28.67				29.91				0.528
Police/fire	21.33				20.51				0.647
State-administered plan	67.33				70.09				0.178
Inception year		1943.55	1944.00	18.37		1943.54	1944.00	18.46	0.139
Multiple employer plan, of which:	66.19				71.15				0.033
Cost sharing across employers	88.04				86.49				0.356
Assets (market value, $b)		19.06	7.20	32.38		22.05	9.10	34.53	0.000
Funded ratio (actuarial)		0.81	0.82	0.19		0.81	0.82	0.20	0.803
Annual return (%)		8.07	10.60	10.51		8.16	10.81	10.44	0.854
PE fund commitments per plan						49.88	26.00	59.49	N/A

Panel B: Private equity funds.

	VC funds (N = 527)			Buyout funds (N = 527)			Real Estate funds (N = 249)		
	Mean	Median	St.Dev.	Mean	Median	St.Dev.	Mean	Median	St.Dev.
Number of GPs	261			256			134		
Funds per GP	2.02	2.00	1.37	2.06	2.00	1.22	1.86	1.00	1.33
Fund size ($m)	348.20	250.00	356.60	1,588.85	700.00	2,634.58	929.05	535.00	1,429.91
Percentage of funds liquidated:									
100% liquidated	36.81			34.91			28.92		
95% liquidated	43.64			42.45			37.75		
Fund effective years	12.22	12.26	4.30	11.13	11.13	4.14	8.33	7.51	3.55
IRR (%)	7.95	3.69	36.94	12.92	13.14	16.21	12.47	12.18	11.28
Size-weighted	7.31		27.89	14.21		11.56	12.81		12.24
TVPI	1.54	1.22	1.85	1.67	1.63	0.65	1.45	1.44	0.40
Size-weighted	1.45		1.37	1.69		0.47	1.46		0.44
PME(KS)	1.00	0.79	1.22	1.18	1.13	0.49	0.98	1.01	0.31
Size-weighted	0.94		0.92	1.17		0.37	0.99		0.33
PME(KN)	-0.03	-0.17	1.01	0.16	0.11	0.42	-0.03	0.00	0.30
Size-weighted	-0.07		0.78	0.15		0.34	-0.02		0.33
Funds with matched LP data	465			488			211		
Number of matched LPs / fund	3.43	2.00	2.90	5.90	4.00	5.88	4.45	3.00	4.25

underfunded on an actuarial basis, with the average (median) funded ratio across plan-years equal to 81% (82%).

The average (median) annual reported return across plan-years is 8.07% (10.60%), with a standard deviation of 10.51%. Most of the variance is coming from the time-series, but there is also an economically meaningful degree of cross-sectional dispersion. This can be seen in Panel A of Figure 2, which shows the time series of the average return across plans, as well as the 10th and 90th percentiles. To mitigate any concerns about reporting differences across plans, Appendix C.1 shows that our results are robust to using an alternate return construction from Andonov and Rauh (2022) that has a clear and consistent definition but has the drawback of only being available from 2001 onwards.

For a subset of plans we have data on their private equity fund commitments (described in detail in the next section). Panel A of Table 2 shows the descriptive statistics for the 117 pension plans that we can match to at least one investment in a PE fund (we call this the "commitment sample"). The commitment sample is statistically indistinguishable from the subsample of 33 plans without commitment data on most dimensions, as shown by the p-values of the difference in proportions and means tests in the final column of Panel A. The exceptions are that the commitment sample has a higher proportion of multiple-employer plans and a higher mean AUM.[14] Notably, the subsamples of plans with and without commitments are not statistically different in terms of their returns. This can be confirmed visually in Panel B of Figure 2, which plots the time series of average returns for the two subsamples. As such, any differences in our (G)IPE metric across these two subsamples is not due to differences in discount rates.

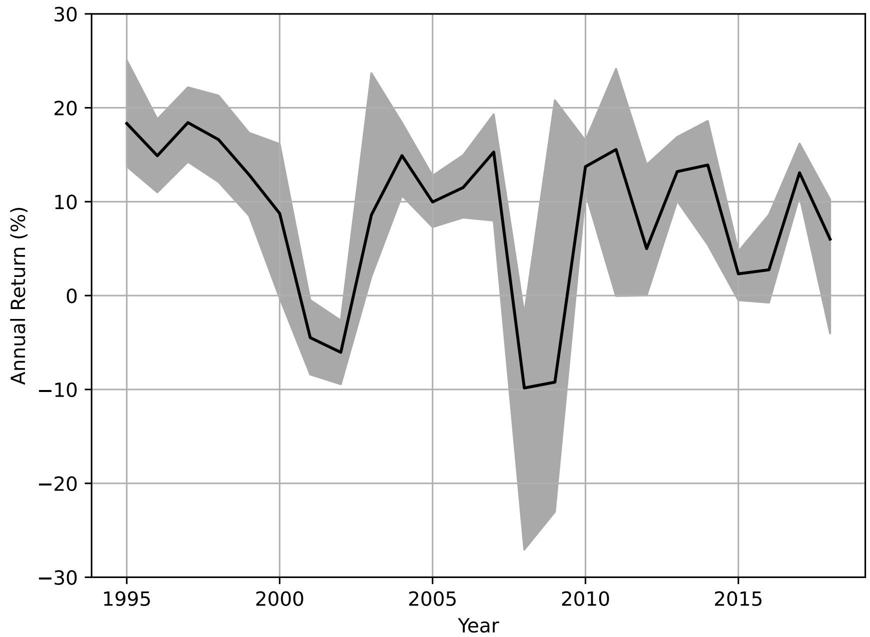
Over the period from 1995 to 2013, the commitment sample plans made an average (median) of 49.88(26) commitments to PE funds in our data set (we drop PE funds with vintages post-2013 for reasons explained below). Similar to the size distribution, the number of commitments is highly skewed with a long right tail.

[14]For the three panel variables in Table 2 (AUM, funded ratio, and annual returns), the tests of differences in means allow for year fixed effects, and double-cluster residuals by year and pension plan state. That is, these tests show whether, *within the same year*, the commitment sample plans have a statistically significant different mean from the plans that are not in the commitment sample.

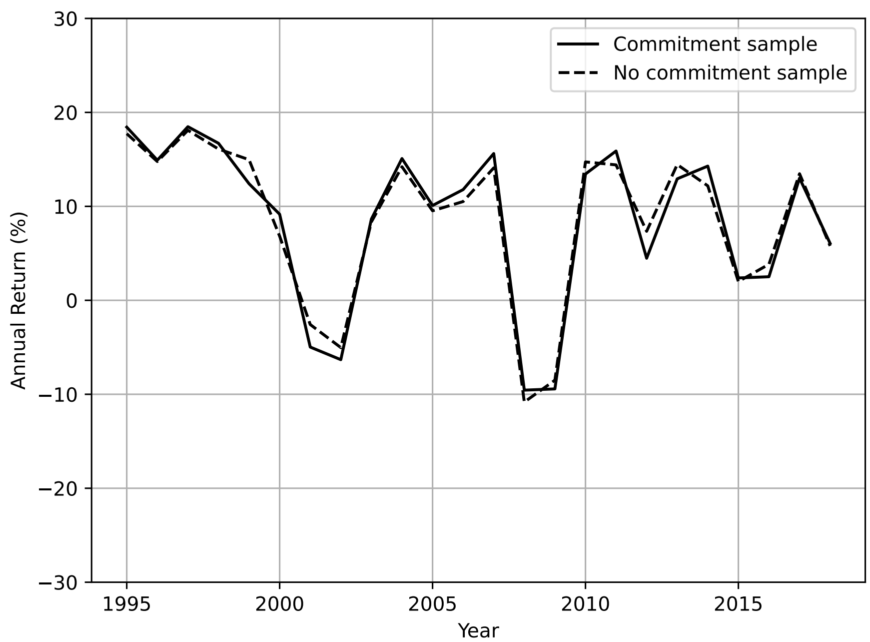
Figure 2: **Pension Plan Returns**

Panel A shows the time series of the average one-year return across pension plans, for the full sample of 150 plans described in Table 2. The shaded area represents the region between the 10th and the 90th percentile of plan returns. Panel B graphs the time series of average one-year plan returns for the subsample of 117 plans with at least one observed commitment to a PE fund (the solid line; this is the "commitment sample" described in Table 2) and the subsample of 33 plans without observed commitment data (the dashed line, labeled "No commitment sample").

Panel A: Pension plan returns for the full sample.



Panel B: Average pension plan returns for subsamples with and without PE fund commitment data.



2.2 Private equity funds

Our private equity data is sourced from Preqin, and contains fund capital calls and distributions net of fees paid to the GPs, as well as quarterly net asset values (NAVs). Preqin primarily sources data through Freedom of Information Act (FOIA) requests. Begenau and Siriwardane (2024) find the data to be accurate when compared to a sample of their own FOIA requests. Harris et al. (2014) suggest that the overall performance of funds in Preqin is comparable to Burgiss and Cambridge Associates, two other well-known data providers. However, cash flow data in Preqin is available for only a subset of funds. The online appendix of Korteweg and Nagel (2016) reports that VC funds with reported cash flows tend to show somewhat lower average performance than the broader Preqin dataset. This discrepancy might not be as large for funds invested in by pension plans, where coverage is likely more comprehensive due to their frequent involvement in FOIA requests. Korteweg and Nagel also do not consider buyout or real estate funds.[15]

Following the literature, we limit the sample to North American funds with at least $5 million in committed capital. We focus on the three main strategies in PE; venture capital (VC), buyout (BO), and real estate (RE). Given the data limitations for pension plans described above, we only include funds raised since 1995, and we use all cash flows until the end of June 2018. Our final filter drops fund vintages after 2013, so we observe at least 5 years of cash flow data for each fund. The final sample contains 1,303 funds.

Panel B of Table 2 reports descriptive statistics by strategy. We observe 527 VC funds managed by 261 unique GPs, with the median GP raising two funds during 1995 to 2013 period. The median VC fund has $250 million in committed capital, whereas the average is higher at $348 million due to a few very large funds. For buyout, the number of funds and GPs are similar to VC (527 funds by 256 GPs), but funds are substantially larger, at an average (median) size of $1,589 million ($700 million). There are fewer real estate funds (249 funds by 134 GPs, with the median GP raising just one fund). The average (median) real estate fund size of $929 million ($535 million) is between VC and buyout fund sizes.

For VC and buyout, just over a third of funds have been fully liquidated by the end of the sample

[15] The availability of cash flows in Preqin also reduces the number of pension plans in the commitment sample, from 118 plans for which we observe at least one commitment to 104 plans where we have cash flow data for at least one of those committed funds.

period (June 2018). The liquidation rate increases to roughly 43% for both strategies if we include funds that have less than 5% of committed capital in remaining NAV, as it is not uncommon for funds to be extended after their original 10-year life if any un-exited portfolio companies remain. As Table 2 shows, the time between the first and final observed cash flow for the median VC (buyout) fund is 12.3 (11.1) years. The proportion of liquidated real estate funds is lower, with only 29% fully liquidated, and the time between first and final cash flow is shorter (7.5 years for the median fund), in large part because PE real estate is a younger strategy with a higher proportion of funds raised in more recent times.

With respect to performance, we compute the standard metrics in the literature; total value to paid-in capital (TVPI), which is a cash multiple of total fund distributions to date divided by total capital calls, internal rate of return (IRR), and public market equivalent (PME). We compute two versions of the PME. The first is the Kaplan and Schoar (2005) defined as the sum of fund distributions discounted to fund inception at the public equity market rate of return, divided by the similarly discounted sum of capital calls. The second version is defined by Korteweg and Nagel (2016) as the sum (not the ratio) of discounted net cash flows (distributions minus capital calls, normalized by the fund's committed capital). Interpreting the PME as a benchmark against the public equity market, a PE fund has outperformed public equities if the Kaplan-Schoar (Korteweg-Nagel) PME is above one (zero). For funds that are not yet liquidated by the end of June 2018, we follow standard practice and include their final reported net asset value (NAV) as a pseudo-distribution in all return metrics. Panel B shows that VC funds had the worst performance during the sample period by most measures, and buyout funds experienced the best performance. VC also has by far the highest variance in fund outcomes (and real estate is the least variable), and its performance is the most skewed, as indicated by the difference between mean and median metrics.

Finally, our Preqin data includes commitments to PE funds by LPs, which we match to our pension plan data. For roughly 9 out of 10 PE funds we observe at least one investment by a pension plan in our data, depending on the strategy (465 out of 527 VC funds, 488 out of 527 buyout funds, and 211 out of 249 real estate funds). For the median VC fund we see 2 pension plan investments, 4 for the median buyout fund, and 3 for the median real estate fund. The averages are higher (3.43, 5.90, and 4.45, respectively) due to a number of large funds for which we see many commitments.

3 Private equity fund performance

3.1 IPE

We start the presentation of our results with the IPE. Similar to the popular PME measure, the IPE effectively imposes a unit risk aversion (logarithmic objective function). As we show in the next section, the assumption of unit risk aversion is empirically questionable. Nonetheless, we find it useful to set the stage with the IPE results, since (a) the IPE is simple to compute and (b) by comparing the IPE with the GIPE results (of the next section) we can illustrate the quantitative importance of estimating the risk-aversion coefficient from the data, rather than imposing it.

We compute the IPE for each combination of private equity fund i and pension plan j, regardless of whether plan j invested in fund i. Specifically, following equation (6), $IPE_{i,j}$ is the sum of the net cash flows (distributions minus capital calls) of private equity fund i, discounted to fund inception by the compounded cumulative return of pension plan j. For non-liquidated funds we include the final reported NAV at the end of the sample period as a pseudo-distribution, similarly discounted. Since pension plan returns are only available at the annual frequency, we discount a cash flow on day d of year τ by the factor $\left(\prod_{t=1..\tau-1} R_{j,t}\right)^{-1} e^{-(\log R_{j,\tau})\frac{d}{365}}$, where $R_{j,t}$ is plan j's return in year t. We report the IPE per dollar of commitment.

The first row of Table 3 Panel A shows that the average IPE is 0.132, which is statistically significant at the 1% level. We double-cluster standard errors by pension plan and vintage year.[16] The positive IPE implies that a marginal investment in the average PE fund would have increased the average logarithmic growth rate of assets of a typical pension plan over the sample period. With the additional assumption of log-utility, an IPE of 0.132 represents a net present value of 13.2 cents on a marginal one dollar commitment, for the average pairing of a pension plan and PE fund.

To facilitate comparison with IRR and traditional performance metrics in public equities, Table 3 also shows performance expressed in "alpha", the additional annualized return (not due to a risk premium) that is required to make the IPE equal to zero. Specifically, we define α as the number that sets $\sum_{k=0}^{K} e^{-\alpha(t_k-t_0)} \frac{W_{t_0}}{W_{t_k}} C_{t_k} = 0$.[17]

[16] We double-cluster the standard errors to account for the fact that each fund and each plan may be represented in multiple observations (since a given pension plan can price multiple funds, and a given fund can be evaluated by multiple plans), and to allow for other sources of cross-correlation within vintage year, such as correlated idiosyncratic shocks.

[17] The relation between alpha and $(G)IPE$ is analogous to the relation between direct alpha and PME as described

The overall IPE alpha across all strategies is 3.1% per year, which, like its IPE counterpart, is significant at the 1% level. The average difference between IRR and alpha of 7.6% can be interpreted as the investment's risk premium in a world where investors maximize the expected logarithmic growth rate of their investments. IRRs and alphas cannot be computed for a small proportion (less than 0.5%) of plan-fund combinations. These funds have an average PME(KN) of −0.627 and a TVPI of 0.144, indicating poor performance with minimal distributions, such that there is no discount rate at which their net present value equals zero. We set the missing IRRs and alphas to −100%. In Appendix C.2 of the robustness section, we show that if we drop the missing observations instead (an overly rosy treatment), average IRRs and alphas are 30 to 60 basis points higher, the exact magnitude depending on the metric and PE strategy. This is primarily a level effect, and it does not materially impact the regression results below. Note that this issue only affects IRRs and alphas, but not (G)IPEs, which are always available.

Figure 3 shows the time series of average IPE across PE fund vintage years. The IPE is high initially (around 1995) but quickly drops to hover around zero from the late 1990s vintages until the mid 2000s, after which it is mildly positive until the end of our sample period.

With respect to individual PE strategies, Table 3 shows that the average IPE (alpha) for buyout funds is 0.225 (5.7%). Both are statistically significant at the 1% level. Venture capital has a statistically insignificant average IPE (alpha) of 0.058 (−0.01%). Real estate average IPE and alpha are 0.090 and 4.4%, which are significant at the 10% and 5% level, respectively. The time series patterns also varies by PE strategy, as shown in Figure 3, with a positive average buyout IPE throughout the sample, VC being high initially but showing a marked decline after the bursting of the dot-com bubble, and real estate showing negative IPE for the mid-to-late 2000 vintages (due to the global financial crisis).

3.2 GIPE

GIPE is a more general metric than IPE, allowing for a coefficient of relative risk aversion, γ_j, that may be different from one and may vary by pension plan. For each plan, we determine its γ_j such that the plan correctly prices the average excess return on the CRSP value-weighted market

in Gredil et al. (2022).

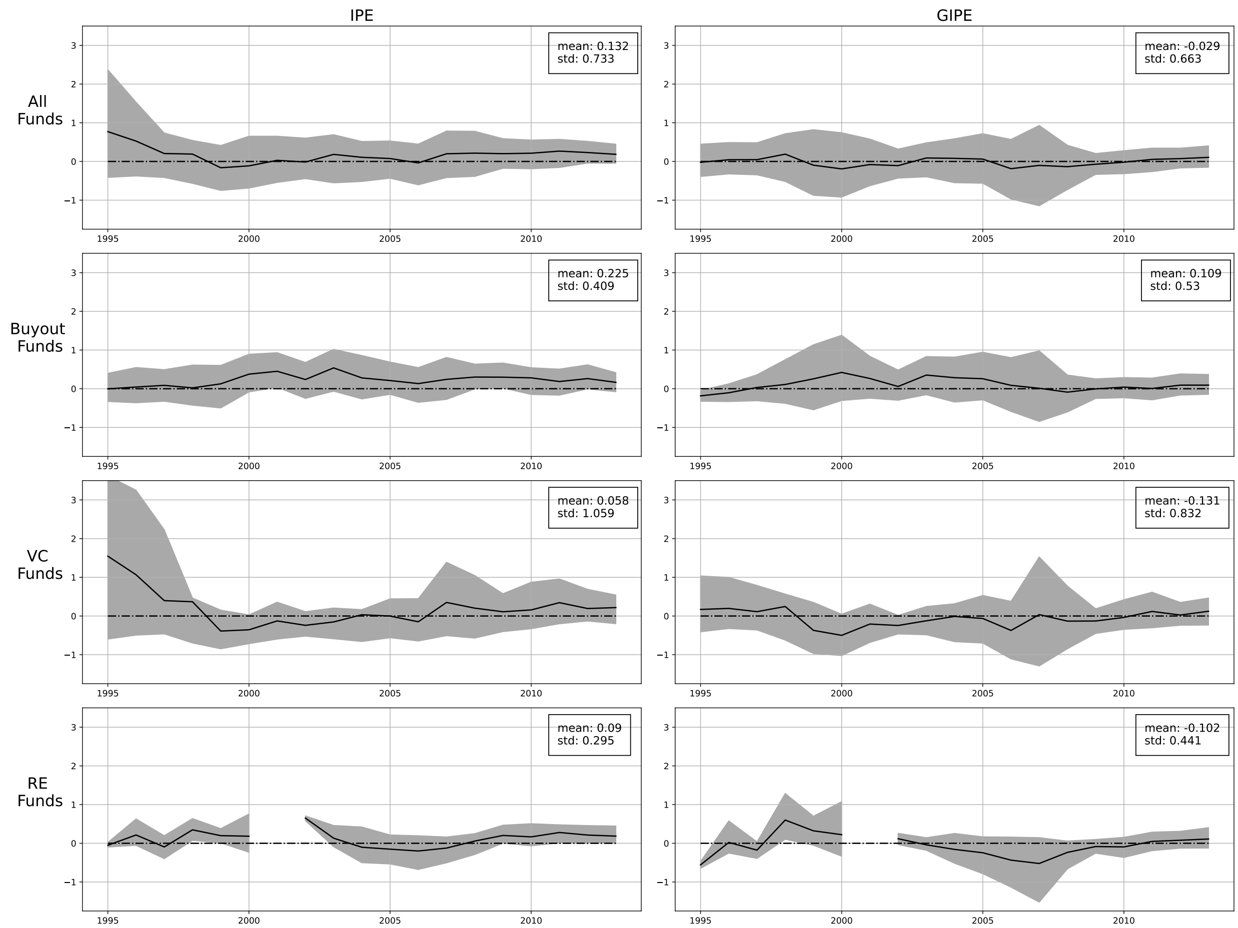
Table 3: **Private Equity Fund Performance Metrics**

This table reports performance results for private equity funds. The first column shows the average performance across all possible pairs of pension plans and PE funds. The second column shows the standard error (s.e.) for the test of the null hypothesis that the mean is zero, double-clustered by pension plan and vintage year. The second, third, and fourth sets of two columns shows the average performance and its standard error separated by PE strategy: buyout, venture capital, and real estate funds. Panel A shows IPE-type metrics. The first row is the *IPE* of equation (6). The second row, *PME(KN)*, shows the PME of Korteweg and Nagel (2016), for comparison. The *IPE(market-repl.)* is the IPE of PE-mimicking funds with the same capital calls as the PE fund, and the same timing of distributions, but invested in the CRSP value-weighted market portfolio. Similarly, *IPE(value-repl.)* refers to mimicking funds that invest in value stocks, defined as stocks in the top quintile of book-to-market ratios, and *IPE(growth-repl.)* refer to mimicking funds that invest in stocks at the intersection of the lowest size and book-to-market quintiles. Panel B shows similar results for GIPE-type metrics, where *GIPE* is as defined in equation (9), and *GPME* is the Generalized PME of Korteweg and Nagel (2016). The rows labeled α show the annualized excess return that makes the (G)IPE equal to zero, computed as described in the text. The α is in percentage points (an α of 1 is an excess return of 1% per year). ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	All funds		Buyout		VC		Real Estate	
	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
Panel A: IPE-type metrics.								
IPE	0.132	0.036***	0.225	0.024***	0.058	0.083	0.090	0.054*
PME (KN)	0.037	0.028	0.154	0.036***	-0.047	0.067	-0.035	0.053
IPE (mkt-repl.)	0.091	0.025***	0.076	0.023***	0.083	0.031***	0.134	0.016***
IPE (value-repl.)	0.187	0.026***	0.182	0.032***	0.212	0.029***	0.149	0.016***
IPE (growth-repl.)	-0.103	0.027***	-0.097	0.027***	-0.124	0.032***	-0.072	0.023***
α(IPE)	3.149	1.167***	5.666	0.960***	-0.008	1.993	4.351	1.770**
IPE - PME(KN)	0.095	0.024***	0.071	0.030**	0.104	0.024***	0.125	0.015***
IPE - IPE(mkt-repl.)	0.041	0.032	0.149	0.029***	-0.025	0.069	-0.045	0.062
IPE - IPE(value-repl.)	-0.056	0.049	0.043	0.045	-0.154	0.098	-0.059	0.063
IPE - IPE(growth-repl.)	0.234	0.026***	0.323	0.033***	0.182	0.063***	0.161	0.037***
IRR - α(IPE)	7.552	0.322***	7.489	0.346***	7.281	0.417***	8.218	0.361***
Panel B: GIPE-type metrics.								
GIPE	-0.029	0.032	0.109	0.035***	-0.131	0.064**	-0.102	0.075
GPME	-0.122	0.133	0.164	0.205	-0.288	0.109***	-0.367	0.147**
GIPE(mkt-repl.)	-0.043	0.021**	-0.040	0.023*	-0.050	0.021**	-0.037	0.022*
GIPE(value-repl.)	0.069	0.055	0.069	0.051	0.111	0.079	-0.016	0.024
GIPE(growth-repl.)	-0.235	0.041***	-0.213	0.040***	-0.259	0.039***	-0.231	0.052***
α(GIPE)	-2.720	1.210**	0.453	1.192	-5.870	1.535***	-2.853	1.796
GIPE-GPME	0.093	0.127	-0.055	0.181	0.157	0.106	0.265	0.085***
GIPE - GIPE (mkt-repl.)	0.015	0.027	0.149	0.033***	-0.082	0.066	-0.065	0.070
GIPE - GIPE(value-repl.)	-0.097	0.063	0.040	0.045	-0.242	0.126*	-0.086	0.070
GIPE - GIPE(growth-repl.)	0.206	0.024***	0.322	0.046***	0.128	0.057**	0.129	0.030***
IRR - α(GIPE)	13.420	1.352***	12.702	1.482***	13.143	1.582***	15.422	1.381***
N	170,355		68,649		67,696		34,010	

Figure 3: **Time Series of IPE and GIPE**

This figure presents the average IPE (left column) and GIPE (right column) for pension plan-PE fund pairs, categorized by fund vintage year. The first row shows data for all funds, followed by buyout funds, venture capital funds, and real estate funds. The horizontal axis represents fund vintage years, with solid lines indicating the mean and shaded areas showing the 10th-90th percentile range. The mean and standard deviation across all vintages are reported in the top right corner of each graph. Note that there is no North American real estate fund for the 2001 vintage year in the data, hence the gap for real estate funds.



portfolio, $R_t^m - \left(1 + r_{t-1}^f\right)$. Specifically, we solve the Euler equation

$$\frac{1}{n_T - n_0 + 1} \sum_{t=n_0 \ldots n_T} (R_{j,t})^{-\gamma_j} \left(R_t^m - \left(1 + r_{t-1}^f\right)\right) = 0, \tag{14}$$

using all available annual plan returns. The histogram in Figure 4 shows that the mass of the γ_j distribution is centered around 5, and all but one plan have γ_j below 10.[18] Importantly, the estimated risk aversion coefficient for all funds is above one. As we discuss next, this is the reason why the IPE numbers are higher than the GIPE numbers.

Panel B of Table 3 shows that the GIPE results are indeed quite different from IPE. The average GIPE across all strategies is economically small at −0.029, and statistically insignificant. The corresponding alpha, which is defined in an analogous fashion as for IPE, equals −2.7% per year and is significant at the 5% level. Across strategies, only buyout has a positive and significant average GIPE, equal to 0.109. However, its alpha of 0.45% is not statistically significant. Venture capital and real estate have a negative GIPE (alpha) of −0.131 (−5.87%) and −0.102 (−2.85%) respectively. These numbers are statistically significant for venture capital. For real estate, only alpha is marginally significant, at the 10% level. The time series plots in the right column of Figure 3 show that GIPE tends to be closer to zero than IPE. Most striking is that the strong IPE performance of VC in the mid-1990s does not show up in GIPE.

Unlike the results for IPE, the GIPE numbers suggest that a typical pension plan would not have benefited from an increased allocation to an average PE fund over the sample period. This underscores the importance of estimating rather than imposing the risk-aversion coefficient. Intuitively, the significant difference between IPE and GIPE arises because GIPE effectively discounts the cash flows of the PE fund using a "levered" version of the investor's portfolio,[19] whereas the IPE uses the unlevered return.

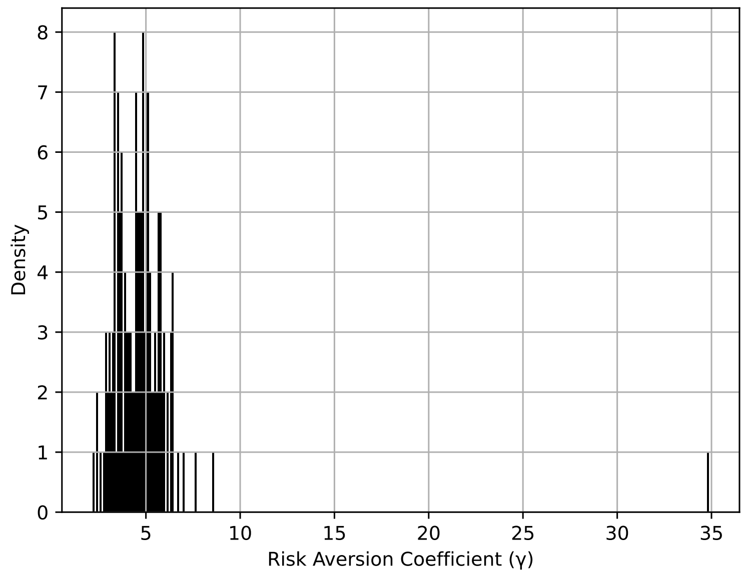
It is useful to compare GIPE to the Korteweg and Nagel (2016) GPME, because both measures are constructed with the aim of assigning a zero value to a fund whose cash flows are replicable with some (possibly) levered trading strategy in stocks and bonds. Table 3 shows that, across all

[18] The one outlier is the Texas Municipal Retirement System, with a γ_j around 35. Excluding this plan does not materially change the empirical results.

[19] Rebalancing a portfolio continuously to maintain a constant leverage ratio implies that the gross return of the levered portfolio (over any discrete interval of time) is equal to the gross return of the unlevered portfolio raised to a power.

Figure 4: **Histogram of Pension Plan Risk Aversion Coefficients**

This figure shows the distribution of estimated risk aversion coefficients (γ) for U.S. public pension plans. Each plan's risk aversion coefficient is estimated to match the pricing of excess returns on the public stock market, satisfying the Euler equation (14) based on the plan's available annual returns.

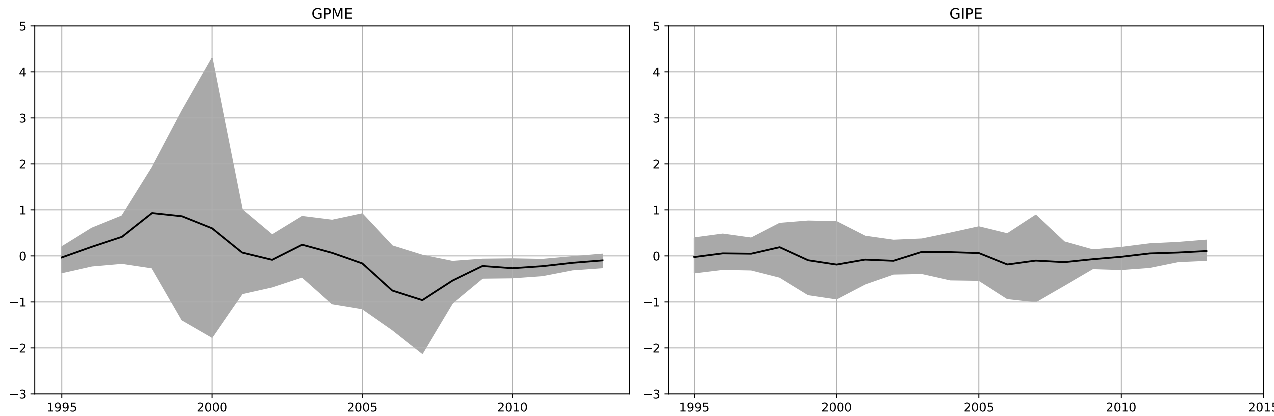


plan-fund combinations, the average GIPE-GPME is 0.093. While economically meaningful, this difference is statistically insignificant. The only PE strategy for which the difference is significant is real estate, where GIPE is 0.265 higher than GPME. The table also shows that average GPME has a standard error that is approximately four times larger than GIPE. This is due to the fact that the cross-sectional dispersion in GPME within a given vintage is much higher than for GIPE, as seen in Figure 5. The figure also shows that the cross-sectional variation in GPME changes dramatically from year to year. It is reassuring that the distribution of GIPE is more stable across vintage years, as one would expect that the cross-sectional distribution of risk-adjusted performance (e.g., from manager skill) does not change dramatically from one year to the next.

Motivated by the discussion in Section 1.4, we consider next the breakdown of the GIPE between the component of PE payoffs that is replicable using publicly-traded securities versus the nonreplicable component. We focus the discussion on the more general GIPE results, but Table 3 also reports the IPE results for completeness. The row labeled "GIPE(market-repl.)" reports results for mimicking funds that make capital calls that are identical to the PE funds but invest

Figure 5: **Time Series of GPME and GIPE**

This figure presents the time series of GPME (left) and GIPE (right) across fund vintage years (horizontal axis). The solid lines indicate the means: for GPME, the mean is computed across PE funds, and for GIPE, across all pension plan and PE fund combinations, regardless of commitments. The shaded areas represent the 10th to 90th percentile range of observations for each vintage.



them in the CRSP value-weighted market portfolio instead, with distributions following equation (12). The average GIPE is -0.043, which is significant at the 5% level, and quite consistent across strategies. The negative values imply that, if anything, PE fund managers have inferior market-timing ability. The difference in the GIPE of a PE fund and that of its market-replicating portfolio is informative in determining whether there is added value to investors in the PE-specific component of payoffs.[20] The row labeled "GIPE - GIPE (market-repl.)" shows that the only strategy for which this difference is statistically significant is buyout. The positive difference of 0.149 shows that – at least historically – buyout funds gave pension funds access to investments that on average dominated the alternative of investing the same capital calls in the stock market and making similar distributions to the buyout fund.

Buyout funds historically tended to invest in value stocks (Stafford, 2022) and venture capital in (small) growth firms. Given the strong performance of buyout, we are especially interested in examining whether buyout funds just represent "covert" value strategies. To that end, we construct mimicking funds that invest in publicly traded portfolios of value (the equally-weighted top quintile of book-to-market stocks) and small-growth stocks (those in the lowest size and book-to-market

[20]Note that the difference in $(G)IPE$ of the two cash flow streams is identical to the $(G)IPE$ of the difference in the two cash flow streams (i.e., the $(G)IPE$ of $C_{i,t} - \widehat{C}_{i,t}$), since these performance metrics are linear in cash flows.

quintile, also equally weighted).[21] The row labeled "GIPE(value-repl.)" in Table 3 shows that the average value-mimicking buyout fund GIPE is 0.069. While positive, this number is statistically not different from zero, so one cannot reject the null hypothesis that the SDF of a typical pension plan can price a publicly traded value portfolio with capital calls and distributions that mimic those of an average buyout fund. The results do suggest, however, that buyout owes its success partly due to its value exposure. The difference between the GIPE of the actual cash flows of buyout funds and their mimicking cash flows using the value return as a benchmark is positive at 0.040, but not statistically significant (see the row labeled "GIPE - GIPE(value-repl.)"). In that sense, the historical outperformance of buyout funds is due to both: a) their value exposure and; b) their ability to select better investments. Combining a) and b) leads to a positive and significant GIPE, even though individually the two components are not significant.

It is useful to contrast the buyout value results with the GIPE of the mimicking funds that invest in small-growth stocks (see the row labeled "GIPE(growth-repl.)" in Table 3). The GIPEs are significantly negative for all PE strategies, ranging from -0.213 to -0.259. While we also found negative performance for the market-replicating funds, the numbers here are a factor five to six larger. This implies that the SDF of a given pension plan cannot "price" the strategy of investing in small growth stocks. The existence of these large and negative GIPEs may be due to shorting constraints or investment mandates that prevent pension plans from reducing an existing exposure, preventing them from capitalizing on strategies with negative long-only excess returns. Interestingly, the poor performance of VC is explained away by the poor performance of small-growth companies. Relative to the publicly traded small-growth mimicking funds, VC performance was positive and significant.

To summarize, some mimicking funds exhibit negative and statistically significant GIPEs, but none of the positive GIPEs are significant. Thus, while a plan's portfolio decisions may be constrained with respect to shorting (or reducing exposures), our proposed SDF can price certain long-only strategies in publicly-traded equities, such as value strategies. Therefore, the fact that buyout strategies have a significantly positive GIPE, whereas the GIPE of their value-mimicking fund is insignificant, is noteworthy.

[21]Size and value portfolios are downloaded from Kenneth French's website: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

4 Pension plan heterogeneity and PE performance

In this section, we dig deeper into the drivers of PE performance across pension plans. Unlike the prior literature, which has focused on IRR as the primary performance metric, our measure can disentangle whether a high IRR is due to high risk or genuine outperformance. This allows us to provide a more detailed decomposition of the sources of heterogeneity in the performance of public pension plan investments in private equity.

We first test whether (risk-adjusted) returns are different for funds that pension plans actually invested in. Then we turn our attention to performance differences between pension plans as a function of their funding status and governance structure, and we consider whether there is home bias in investments.

4.1 Invested funds

Table 4 shows estimates from regressions with either the IRR or the GIPE-implied alpha of all plan-fund combinations as the dependent variable. These two performance metrics are useful to compare, since the former measures a fund's total annualized return and the latter its risk-adjusted annualized return (we omit the IPE-implied alpha results for brevity). Both IRR and alpha are measured in percentage points per year (for example, an IRR of 1.0 means a return of 1 percent per year). Panel A shows specifications with vintage fixed effects, and Panel B adds pension plan fixed effects. Standard errors are double-clustered by vintage and pension plan.

The regressions in Table 4 include covariates that are intended to capture performance heterogeneity due to pension plans' PE investment decisions. We first consider the coefficient on the indicator variable *Active*, which measures if a pension plan was an active PE investor in a given year: It equals one if the plan made a commitment to any PE fund for the year, and zero otherwise. For IRR, the estimated coefficient is negative in specifications with vintage fixed effects only (Panel A). However, the estimates are economically small at less than 20 basis points across the board. When adding pension plan fixed effects in Panel B, the economic and statistical magnitudes become negligible, on the order of a few basis points. This means that pension plans do not tend to enter PE when IRRs are expected to be higher, or to get out of PE when future returns are lower.

Results are different for alpha. Without plan fixed effects, plans that invest in PE have average

Table 4: **Pension Plan Commitments and Performance**

Each column shows the coefficients of a regression of private equity fund performance on a set of indicator variables plus fixed effects controls. The observations are all possible combinations of pension plans and PE funds. The indicator *Active* equals one if the pension plan made a commitment to any PE fund in that vintage year and zero otherwise. *Commit* equals one when the pension plan committed to this particular PE fund, and zero otherwise. *Relationship* equals one for all plan-fund combinations where a prior commitment has been made between the pension plan and PE fund manager. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. The odd-numbered columns use IRR as the performance metric, and the even-numbered columns use the GIPE-implied α (both measured in percentage points per year, e.g., an IRR of 1 means one percent per year). All regressions include vintage fixed effects. The regressions in Panel B also include pension plan fixed effects. Standard errors are double-clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

Panel A: Without pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	-0.198***	2.168*	-0.195***	2.466**	-0.147	2.085*	-0.076	2.025*
	(0.052)	(1.048)	(0.064)	(1.139)	(0.090)	(1.065)	(0.063)	(0.982)
Commit=1	2.199***	2.504***	1.255**	1.882***	3.358	2.465**	0.121	0.210
	(0.655)	(0.544)	(0.583)	(0.614)	(2.072)	(1.123)	(0.692)	(0.454)
Relationship=1	3.212***	3.817***	3.482**	3.490**	4.048**	5.508***	0.929	1.671
	(0.985)	(0.971)	(1.502)	(1.319)	(1.674)	(1.513)	(1.268)	(1.438)
Relationship x Commit	-2.694*	-2.682*	-2.796**	-2.455*	-5.245	-4.957*	0.539	-0.066
	(1.447)	(1.319)	(1.239)	(1.329)	(3.734)	(2.587)	(1.046)	(1.048)
N	170,355	170,355	68,649	68,649	67,696	67,696	34,010	34,010
adj. R^2	0.056	0.027	0.094	0.047	0.100	0.045	0.328	0.131
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	N	N	N	N	N	N	N	N

Panel B: With pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	-0.007	-0.015	-0.041	0.046	0.110	0.030	-0.017	0.007
	(0.043)	(0.394)	(0.029)	(0.439)	(0.091)	(0.357)	(0.024)	(0.296)
Commit=1	2.306***	2.098***	1.364**	1.448**	3.623*	1.950	0.164	0.186
	(0.641)	(0.609)	(0.620)	(0.581)	(2.083)	(1.338)	(0.709)	(0.638)
Relationship=1	3.351***	2.870***	3.621**	2.765*	4.363**	4.605***	0.965	0.553
	(1.005)	(0.930)	(1.569)	(1.353)	(1.746)	(1.564)	(1.306)	(1.254)
Relationship x Commit	-2.793*	-2.029	-2.898**	-2.105	-5.521	-4.144	0.521	0.464
	(1.446)	(1.341)	(1.264)	(1.300)	(3.780)	(2.774)	(1.059)	(0.827)
N	170,355	170,355	68,649	68,649	67,696	67,696	34,010	34,010
adj. R^2	0.055	0.227	0.092	0.334	0.098	0.166	0.327	0.613
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	Y	Y	Y	Y	Y	Y	Y	Y

alphas of 2 to 2.5 percentage points higher (depending on the strategy) than plans that do not. This could indicate that some plans make a rational decision to stay out of PE investments, given the low alphas that they expect to earn. Indeed, the result is due to heterogeneity across plans: with plan fixed effects, the *Active* coefficient is close to zero and insignificant across the board. This insignificant within-plan coefficient also suggests that pension plans do not have skill in timing their entry or exit from PE investing altogether.

To test whether plans experience better performance in the PE funds that they actually invest in, the regressions include *Commit*, an indicator that equals one only if, in a given plan-fund combination, the pension plan made a commitment to that specific PE fund. In the regressions that use all PE funds, the coefficients are statistically significant and similar in magnitude at around 2 to 2.5 percentage points, for both IRR and alpha and both with and without pension plan fixed effects. There is heterogeneity across PE strategies, but all estimates are positive.

This outperformance could be either because public pension plans have selection skill or because they enjoy differential access to better PE funds. To disentangle these two channels, we consider the performance of funds of GPs with which the pension plan has invested before, measured by the indicator *Relationship*. This indicator equals one only if the pension plan has made a commitment to a prior fund of the same GP. Prior investors are usually given the option to reinvest in follow-on funds (Lerner et al., 2007), such that access is not a concern in the performance of reinvestment decisions. The large, positive coefficient on *Relationship* shows that such funds tend to perform well, both in terms of IRR and alpha, possibly due to the fact that these GPs have survived to raise follow-on funds. More important for our purpose is the interaction between *Relationship* and *Commit*. The coefficient is negative and of similar magnitude (if not larger) to *Commit*, with the exception of real estate, where the coefficients are small and statistically insignificant. The bottom line is that, conditional on a pre-existing relationship with a GP, a commitment is not associated with higher fund performance. This suggests that the unconditional higher performance from committed funds is driven by access, not skill. To further strengthen this result, we also find no evidence of outperformance in a sub-sample of pension plan investments in first-time PE funds, which typically accept investments from any limited partner that is interested in investing (Lerner et al., 2011). For brevity, these results are reported in Appendix C.3.

The prior literature also uses reinvestments and first-time funds to distinguish access and skill

(Lerner et al., 2007; Sensoy et al., 2014; Andonov et al., 2018; Cavagnaro et al., 2019). The empirical evidence is mixed. For public pension plans, Lerner et al. (2007) and Cavagnaro et al. (2019) conclude that performance is due to more than just access (but not as much as for endowments), whereas Sensoy et al. (2014) find no evidence of skill after controlling for access, especially post-1999. Given that competition in PE has increased, and performance persistence has weakened for some strategies (e.g., Harris et al. (2022)), the fact that our sample period is more recent than these prior studies may help explain why we find little evidence of skill. Dyck et al. (2022) offer another potential explanation: Lower quality managers end up being hired in equilibrium if there is fear of public outrage if pension plan managers were to be compensated at market level salaries.[22]

4.2 Funding status, governance, and home bias

Next, we introduce additional plan-specific covariates to further explore performance differences between pension plans. We are especially interested in agency problems that may arise from a plan's funding status, its governance structure, and the location of the PE fund that the plan invests in. We focus on the subsample of plan-fund combinations (i, j) for which the plan j actually invested in the PE fund i. Furthermore, since many of the explanatory variables, such as the structure of a pension plan's board of trustees, do not vary much (or at all) over time, we do not include pension plan fixed effects. Instead, we use plan-state fixed effects (in addition to vintage fixed effects), so that we are effectively comparing the outcome of investments made by different plans within the same state in the same year. All standard errors are double-clustered by vintage and pension plan.

Table 5 reports the estimated coefficients of regressions of IRR and alpha on plan characteristics. To determine the effect of geography we include an indicator variable, *Home State*, that equals one if the PE fund has the same state of domicile as the pension plan, and zero otherwise. Across all plan-fund investments (column 1), we find that home-state investments have about 2.1 percentage points lower IRR, which is statistically significant at the 5% level. This result is consistent with Hochberg and Rauh (2013), and could be the result of political pressure to invest in the home state. An alternative explanation is that plans are able to identify less risky investments in their

[22] Another source of performance heterogeneity is differences in fees paid by investors in the same PE fund, as shown by Begenau and Siriwardane (2024). They also use Preqin data, focusing on cash multiples, because most funds do not have a fully balanced panel of cash flows across investors that is needed to compute most other performance metrics. As such, an analysis of the impact of fee heterogeneity on (G)IPEs is not feasible at this time, but it is an important topic for future research.

home state, possibly as a result of superior information, and these investments consequently have lower expected returns. We can distinguish between these two explanations, by using alpha as the dependent variable. As column 2 shows, home state investments have a 1.6 percentage points lower alpha, significant at the 10% level. Thus, local underperformance appears to be due to investments being worse in a risk-adjusted sense, rather than being less risky. The coefficient estimates for VC (in columns 5 and 6) are of similar sign and magnitude as PE overall. For real estate, home state investments have lower IRRs, but unlike VC, this is because they are less risky, while being no worse in a risk-adjusted sense (columns 7 and 8). Finally, for buyout, the coefficients on IRR and alpha are both positive and of similar magnitude, suggesting that pension funds may have an information advantage when it comes to investing in local buyout funds. However, while economically meaningful, the coefficients are statistically insignificant.

Turning to the funding status of the plan, we find that better funded plans, measured by the ratio of actuarial assets to actuarial liabilities, have lower IRR but higher alpha when we consider all plan-fund investments (in columns 1 and 2). The coefficients are insignificant, but economically meaningful: a one standard deviation increase in the funded ratio lowers the expected annual IRR by 0.55 percentage points, and raises the expected alpha by 0.94 percentage points. Across the strategies, the coefficient estimates imply that IRR and alpha are closer together for plans with higher funded ratios, an observation that we will return to below when we consider risk-taking.[23]

To analyze the impact of governance structure on performance, we follow Andonov et al. (2018) and use the fraction of trustees on the pension plan's board that are state officials, plan participants, or members of the public, and whether they were elected by plan members, appointed by a government official, or serving ex-officio. Across these 9 categories, the most common ones are elected plan participants (27% of the average board), state officials, such as the state treasurer, serving ex-officio (25%), appointed members of the public (25%), appointed plan participants (12%), and

[23]We use the actuarial funded ratio, which represents the publicly observed funded status that pension plans arguably base decisions on. True funded ratios are lower due to the fact that actuarial numbers discount future liabilities at the expected rate of return on pension assets, which are higher than discount rates considered appropriate in the literature (see, for example, Brown and Wilcox, 2009; Novy-Marx and Rauh, 2009, 2011, 2014). Based on state-level estimates reported in Table 1 of Novy-Marx and Rauh (2009), changing the discount factor for liabilities to the U.S. Treasury yield curve cuts the average funded ratio roughly in half, but does not have a large impact on the cross-sectional variation in funded ratios: A univariate regression of the Treasury rate-adjusted funded ratio on the reported (actuarial) funded ratio has an R^2 of 92.8% (results are available upon request). As such, the conclusions from our regressions that use actuarial funded ratios are unlikely to materially change if we instead were to use a plan-level measure that better approximates "true" funded ratios (which are not easily computed).

Table 5: **Pension Plan Characteristics and Performance**

Each column shows the coefficients of a regression of private equity fund performance on pension plan characteristics plus controls. The observations are all plan-fund combinations for which the pension plan invested in the PE fund. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. The odd-numbered columns use IRR as the performance metric, and the even-numbered columns use the GIPE-implied α (both measured in percentage points). *Home state = 1* is an indicator variable that equals one if the PE fund and pension plan are located in the same state, and zero otherwise. *Funded ratio* is the ratio of actuarial assets to actuarial liabilities. *Public Equity Wt* is the fraction of the plan's portfolio allocated to public equity. *State Appointed*, *State Ex-officio*, *Member Elected*, and *Public appointed* are the fraction of the plan's board members who are state officials appointed by government official, state officials serving ex-officio, plan members elected by their peers, and members of the public appointed by a government official, respectively. *Other Trustees* is the fraction of the board who were installed by other means, with the omitted category being plan members who were appointed by an official. *Board Size* is the number of pension plan board members. *Log(AUM)* is the natural logarithm of the pension plan's assets under management, and *Log(commitment %)* is the natural logarithm of the commitment of the plan to the fund, as a percentage of the plan's assets under management. All regressions include vintage year and plan-state fixed effects. Standard errors are double-clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Home State=1	-2.089**	-1.607*	1.094	1.141	-2.225	-2.263	-2.582**	1.101
	(0.971)	(0.890)	(0.945)	(1.093)	(1.801)	(1.777)	(1.040)	(1.998)
Funded Ratio	-2.908	4.937	0.241	7.889**	-3.129	4.151	0.115	6.344
	(2.827)	(3.340)	(3.493)	(3.279)	(6.498)	(5.851)	(1.840)	(3.684)
Public Equity Wt	-17.032**	-25.161***	-4.713	-13.863***	-27.455	-18.263	-10.067***	-27.495***
	(6.819)	(5.411)	(5.750)	(4.117)	(16.348)	(13.715)	(3.015)	(7.359)
Log(AUM)	0.889	1.411*	0.951**	1.035*	-1.100	-0.899	-0.466	0.227
	(0.625)	(0.744)	(0.430)	(0.493)	(1.601)	(1.366)	(0.434)	(0.854)
State Appointed	-6.944	-15.045	-7.532	-0.834	-0.867	-13.626	0.002	-24.219
	(4.807)	(13.920)	(7.118)	(6.492)	(7.409)	(18.886)	(5.060)	(26.332)
State Ex-officio	-5.755	-9.213	-6.341	-8.558*	-2.694	-7.956	3.767	-2.528
	(3.629)	(5.403)	(4.932)	(4.101)	(4.648)	(7.395)	(3.379)	(7.952)
Member Elected	-2.694	-0.762	-3.075	-2.402	-3.720	-7.415	1.923	7.970
	(2.282)	(3.525)	(2.654)	(2.579)	(3.798)	(4.308)	(3.348)	(8.132)
Public Appointed	0.098	-3.481	-3.042	-7.024*	3.368	-5.640	0.410	5.491
	(2.094)	(3.556)	(2.589)	(3.993)	(4.054)	(5.175)	(2.443)	(5.788)
Board Size	-0.094	0.149	-0.136*	0.115*	0.071	0.169	0.016	0.364*
	(0.095)	(0.119)	(0.070)	(0.063)	(0.239)	(0.218)	(0.059)	(0.200)
Log (Commitment %)	1.195	1.387**	1.087*	1.165*	-1.153	-1.294	-0.752	-0.148
	(0.732)	(0.577)	(0.586)	(0.615)	(1.081)	(0.852)	(0.847)	(1.078)
Other Trustees	11.150*	5.083	5.368	0.055	18.919	8.159	-0.884	-3.748
	(6.098)	(7.589)	(4.913)	(5.274)	(13.961)	(15.031)	(2.974)	(8.012)
N	4,741	4,741	2,597	2,597	1,255	1,255	880	880
adj. R^2	0.104	0.086	0.173	0.148	0.128	0.080	0.387	0.229
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Plan State FE	Y	Y	Y	Y	Y	Y	Y	Y

appointed state officials (8%). The other four categories are less than 2% of the average board each, and we lump them together as *Other trustees*.

Andonov et al. (2018) find that plans experience lower IRRs on their PE investments when they have a higher fraction of trustees who are state-appointed, serving ex-officio, member-elected, or appointed members of the public (relative to appointed plan participant trustees, the omitted category). The effect is largest for the two categories of state official trustees. Despite covering a different sample period and using a different regression specification, we find the same result, except for appointed members of the public, who have a negligible relation to IRR (see column 1 of Table 5). Unlike Andonov et al. (2018), we can also consider risk-adjusted returns. With alpha as the dependent variable, we find even larger (i.e., more negative) coefficients for all but member-elected trustees (column 2). Whereas the point estimates are economically meaningful, and of similar magnitude to those in Andonov et al. (2018), we only find statistical significance for some trustee categories in the individual PE strategies. Specifically, results for buyout and VC (columns 3-6) look broadly similar to PE overall, and the negative relation between state ex-officio and public appointed trustees and buyout funds' alpha is weakly significant. In real estate, the IRR coefficients on state appointed, ex-officio, and member elected trustees (column 7) have the opposite sign from overall PE (column 1). What is similar to the other strategies, is the lower alpha of investments by plans with more state trustees, both appointed and ex-officio. However, none of the coefficients are statistically significant, making it difficult to draw strong conclusions for real estate.

Interestingly, larger boards realize lower IRR and higher alpha, after conditioning on a plan's assets under management, though neither coefficient is significant when we consider all PE funds. For alpha, the coefficient is significant (at the 10% level) for buyout and real estate. A more striking difference is the wedge between IRR and alpha, which we will return to below.

With respect to plan size, we find that larger pension plans (measured by the natural logarithm of assets under management) have better performance. The relation is stronger for alpha compared to IRR. This result is consistent with Dyck and Pomorski (2015), who find that plans that have more dollars invested in PE, realize higher returns. Possible explanations are that larger LPs have access to better GPs (Lerner et al., 2007), and a wider scope of due diligence, monitoring, and other related activities (Da Rin and Phalippou, 2017). However, this result is driven by buyout, where coefficients for both IRR and alpha are positive and statistically significant. The signs reverse in VC and real estate investments (but the coefficients are insignificant).

Finally, we control for the fraction of the plan's portfolio that is allocated to public equity (*Public Equity Wt*). Plans with a higher allocation to public equity, indicative of lower risk aversion, have worse performance in private equity, both in terms of IRR and alpha, for all strategies. This echoes the results in Andonov et al. (2017), who find that pension plans with a higher allocation to risky assets have lower benchmark-adjusted plan returns.

The relation between performance and the funded ratio, as well as the equity share, suggest that risk-taking by pension plans may play an important role. A unique advantage of our approach is that we can measure the degree of risk-taking by pension plans more directly, at the individual investment level, by the difference between PE fund IRR and alpha. Table 6 reports regression results with $IRR - \alpha(GIPE)$ as the dependent variable.[24] For several covariates, such as the funded ratio and board size, we find that, even though the coefficients on IRR and alpha are individually not significantly different from zero, their difference is significant. Regarding funded ratio, a key result is that better-funded pension plans take less risk in their private equity investments, consistent with gambling for resurrection by underfunded plans. At the same time, plans with a higher public equity allocation take more PE risk (with the exception of VC), consistent with these plans having lower risk aversion. As to governance, the point estimates are economically meaningful but noisy. The estimates suggest that plans with a higher fraction of trustees who are ex-officio state officials or appointed members of the public take more risk, especially in VC. Boards with a higher fraction of appointed state officials also tend to take more risk, except in buyout. Larger plans (in terms of AUM) are associated with lower risk-taking, but the coefficients are not significant, especially when controlling for governance variables. The results on board size are stronger: Larger boards (conditional on plan AUM) have a small negative, but statistically significant, relation with risk-taking. Finally, home-state investments are not related to risk-taking, except for real estate, where local investments tend to be less risky.

To summarize, the main results in this section are as follows. Pension plans experience better risk-adjusted performance in PE funds that they invest in compared to funds in which they do not

[24] Note that the coefficients in the even-numbered columns of Table 6 are equal to the difference in the corresponding coefficients of the IRR and alpha regressions in Table 5.

Table 6: **Pension Plan Risk-taking**

This table shows results for regressions of the difference between IRR and the GIPE-implied α, our measure for risk-compensation, on pension plan characteristics plus controls. The observations are all plan-fund combinations for which the pension plan invested in the PE fund. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. All explanatory variables are as described in Table 5. All regressions include vintage year and plan-state fixed effects. Standard errors are double-clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home State=1	-0.515	-0.483	-0.050	-0.046	-0.082	0.038	-3.556*	-3.683**
	(0.662)	(0.629)	(0.781)	(0.719)	(0.288)	(0.329)	(1.726)	(1.609)
Funded Ratio	-6.190***	-7.844***	-7.838***	-7.648**	-4.946**	-7.280**	-4.329**	-6.229**
	(1.978)	(2.355)	(2.714)	(3.257)	(2.020)	(3.321)	(1.724)	(2.275)
Public Equity Wt	7.297	8.129	11.710**	9.151**	-8.765	-9.192	10.864	17.429*
	(6.203)	(4.888)	(5.405)	(4.201)	(10.885)	(10.843)	(7.436)	(8.325)
Log(AUM)	-0.500	-0.522	-0.519	-0.084	-0.703*	-0.201	0.224	-0.692
	(0.432)	(0.609)	(0.420)	(0.629)	(0.398)	(0.591)	(0.579)	(0.867)
State Appointed		8.100		-6.697		12.759		24.222
		(14.030)		(7.513)		(17.598)		(26.653)
State Ex-officio		3.457		2.217		5.263		6.296
		(4.977)		(3.896)		(7.196)		(9.652)
Member Elected		-1.932		-0.674		3.694		-6.047
		(3.225)		(2.764)		(2.540)		(5.714)
Public Appointed		3.579		3.982		9.009*		-5.081
		(3.004)		(2.328)		(5.000)		(5.662)
Board Size		-0.242***		-0.252**		-0.098		-0.347*
		(0.080)		(0.088)		(0.080)		(0.183)
Log (Commitment %)		-0.192		-0.078		0.142		-0.604
		(0.337)		(0.412)		(0.355)		(0.860)
Other Trustees		6.067		5.314		10.760		2.864
		(4.223)		(3.325)		(7.046)		(7.006)
N	4,741	4,741	2,597	2,597	1,255	1,255	880	880
adj. R^2	0.314	0.320	0.454	0.461	0.286	0.289	0.226	0.265
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Plan State FE	Y	Y	Y	Y	Y	Y	Y	Y

invest, but this appears to be driven primarily by access to high-quality PE managers. Across all PE investments, underfunded plans take more risk, which translates into higher IRRs but lower alphas. PE funds in a pension plan's home state tend to underperform, and this is not due to these investments being less risky. Finally, pension plan boards composed of a higher fraction of (ex-officio) state officials and appointed members of the public tend to invest in more risky PE funds, but earn lower alphas on these investments, especially in buyout.

5 Conclusion

Distinguishing between risk-taking and genuine outperformance is a fundamental issue in the theory and practice of investments. A key step in addressing such questions is to separate the compensation for bearing risk from excess returns. With heterogeneous agents, the assessed riskiness of the same investment opportunity can vary across investors. The measures developed in this paper extend existing metrics to allow for the estimation of investor-specific risk-adjusted performance in long-dated investments, without relying on stale or potentially biased valuations.

We apply our method to U.S. public pension plan investments in private equity funds. We find that for the 1995 to 2018 sample period, pension plans would not have experienced a significant benefit from changing their allocation to the representative PE fund. Looking at individual PE strategies, the average plan could have benefited from a higher allocation to buyout funds, although this is at least in part due to buyout's value exposure. While there is no evidence of market timing skill, pension plans did realize higher risk-adjusted returns in the funds they chose to invest in, compared to an average PE fund of the same vintage. However, this appears to be due to differences in access rather than skill in picking outperforming funds.

We find systematic differences across plans: Underfunded plans take more risk, which yields higher total returns, but lower risk-adjusted returns. Similarly, pension plan boards that have a higher fraction of (ex-officio) state officials and appointed members of the public tend to invest in riskier funds, but earn lower alpha. With the exception of real estate, home-state PE investments also earn lower alpha but do not differ in risk compensation, so that total returns are lower. These results are broadly consistent with agency problems within pension plans, such as gambling for resurrection and political influence, playing an important role in investment decisions. Our findings augment the prior literature that considers total, but not risk-adjusted, plan returns and papers that only consider pension returns at a highly aggregated level.

The paper does not explore the mechanisms by which pension plans identify attractive PE funds, or the underlying reasons why certain types of board members have a preference for riskier investments. These issues are left for future research.

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Appendix

A Proofs

Proof of Proposition 1. The dynamics of wealth are given by

$$\frac{dW_t}{W_t} = r_t dt + w_t'(dR_t - r_t 1_N dt).$$

Applying Ito's Lemma to compute the dynamics of $d \log W_t$ and taking expectations, implies that for any t_k the value function $V(W_{t_k}, t_k)$ satisfies

$$\begin{aligned} V(W_{t_k}, t_k) &= E_{t_k} \log W_T \\ &= \log(W_{t_k}) + E_{t_k} \int_{t_k}^{T} \max_{w_u} \left(r_u + w_u' (\mu_u - r_u 1_N) - \frac{1}{2} w_u' \sigma_u \sigma_u' w_u \right) du. \end{aligned}$$

Accordingly, an implication of the envelope theorem is

$$\frac{d(E_{t_0} \log W_T)}{d\varepsilon} = E_{t_0} \sum_{k=0}^{K} V_W(W_{t_k}, t_k) \cdot C_{t_k} = E_{t_0} \sum_{k=0}^{K} \frac{C_{t_k}}{W_{t_k}}. \tag{A.1}$$

Therefore

$$\frac{d(E_{t_0} \log W_T)}{d\varepsilon} = \frac{1}{W_{t_0}} \times IPE = V_W(W_{t_0}, t_0) \times IPE.$$

■

Proof of Proposition 2. For any t_k we have that that the value function $V(W_{t_k}, t_k)$ satisfies

$$V(W_{t_k}, t_k) = \frac{E_{t_k} W_T^{1-\gamma}}{1-\gamma} = \frac{W_{t_k}^{1-\gamma}}{1-\gamma} f(t_k), \tag{A.2}$$

where

$$f(t_k) = E_{t_k} \exp \left\{ \begin{array}{c} (1-\gamma)\left(r + w'(\mu - r 1_N) - \frac{\gamma}{2} w' \sigma \sigma' w\right)(T - t_k) \\ + (1-\gamma) w' \sigma (B_T - B_{t_k}) \end{array} \right\},$$

where w is the optimal portfolio in $\mathcal{W}$.

Accordingly, an implication of the envelope theorem is

$$\frac{d\left(\frac{E_{t_0}W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon}=E_{t_0}\sum_{k=0}^{K}V_W\left(W_{t_k},t_k\right)C_{t_k}=E_{t_0}\sum_{k=0}^{K}W_{t_k}^{-\gamma}f\left(t_k\right)C_{t_k}. \tag{A.3}$$

Sine the risk-free asset choice is unconstrained (Assumption (b) of the Proposition), the Euler equation for bonds implies that

$$e^{r(t_k-t_0)}E_{t_0}\frac{V_W\left(W_{t_k},t_k\right)}{V_W\left(W_{t_0},t_0\right)}=1. \tag{A.4}$$

Using (A.2) inside (A.4) leads to

$$e^{r(t_k-t_0)}\frac{f\left(t_k\right)}{f\left(t_0\right)}E_{t_0}\left(\frac{W_{t_k}}{W_{t_0}}\right)^{-\gamma}=1, \tag{A.5}$$

and using (A.5) inside (A.3) gives

$$\frac{d\left(\frac{E_{t_0}W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon}=f\left(t_0\right)W_{t_0}^{-\gamma}\times GIPE=V_W\left(t_0,W_{t_0}\right)\times GIPE.$$

■

Proposition 5 *Maintain Assumption (b) of Proposition 2. Assume that* μ_t, r_t, σ_t *are functions of some vector of state variables* X_t *that follow some diffusion* $dX_t=\mu_X dt+\sigma_X dB_t$. *Assume also that* $\sigma_X\sigma_A'=0$, *i.e., innovations to* dX_t *and* $\frac{dA_t}{A_t}$ *are independent. Define the process* Z_t *as* $\frac{dZ_t}{Z_t}\equiv\frac{dW_t^{-\gamma}}{W_t^{-\gamma}}-E_t\left[\frac{dW_t^{-\gamma}}{W_t^{-\gamma}}\right]=-\gamma w_t'\sigma_t dB_t$ *subject to* $Z_0=1$. *Assume that* $\mu_{A,t}-r_t+\frac{\left\langle dW_t^{-\gamma};dA_t\right\rangle}{W_t^{-\gamma}A_t}$ *has the same sign for all* t *and define*

$$GIPE^{(\gamma)}\equiv E_{t_0}\sum_{k=0}^{K}\exp\left(-\int_{t_0}^{t_k}r_u du\right)Z_{t_k}C_{t_k}. \tag{A.6}$$

Then $sign\left(\frac{dV}{d\varepsilon}\right)=sign\left(GIPE^{(\gamma)}\right)=sign\left(\mu_{A,t}-r_t+\frac{\left\langle dW_t^{-\gamma};dA_t\right\rangle}{W_t^{-\gamma}A_t}\right)$.

Remark 2 *Note that when* $r_t=r,\mu_t=\mu,\sigma_t=\sigma$ *the definition of the GIPE in equation* (A.6) *coincides with the definition of the GIPE in* (9), *since then* $Z_t=\frac{W_t^{-\gamma}}{E_{t_0}\left(W_t^{-\gamma}\right)}$.

Proof of Proposition 5. With a (Markovian) time-varying opportunity set, the value function is multiplicatively separable in $W_{t_k}^{1-\gamma}$ and X_{t_k} :

$$V\left(W_{t_k}, X_{t_k}, t_k\right) = \frac{E_{t_k} W_T^{1-\gamma}}{1-\gamma} = \frac{W_{t_k}^{1-\gamma}}{1-\gamma} f\left(X_{t_k}, t_k\right). \tag{A.7}$$

Noting that $A_{t_0} = A_{t_K^+} = 0$ the marginal valuation of C is now

$$\begin{aligned}
\frac{d\left(\frac{E_{t_0} W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon} &= E_{t_0} \sum_{k=0}^{K} V_W\left(W_{t_k}, t_k\right) C_{t_k} \qquad\qquad (A.8)\\
&= E_{t_0} \sum_{k=0}^{K} V_W\left(W_{t_k}, t_k\right) \left(A_{t_k} - A_{t_k^+}\right) \\
&= E_{t_0} \sum_{k=0}^{K-1} \left(V_W\left(W_{t_{k+1}}, t_{k+1}\right) A_{t_{k+1}} - V_W\left(W_{t_k}, t_k\right) A_{t_k^+}\right) \\
&= E_{t_0} \sum_{k=0}^{K-1} \int_{t_k^+}^{t_{k+1}} d(V_W\left(W_t, t\right) A_t).
\end{aligned}$$

Using Ito's Lemma and the assumption that $\sigma_X' \sigma_A = 0$, gives

$$\begin{aligned}
\frac{d(V_W\left(W_t, t\right) A_t)}{V_W\left(W_t, t\right) A_t} &= \frac{d(V_W\left(W_t, t\right))}{V_W\left(W_t, t\right)} + \frac{dA_t}{A_t} + \frac{\langle dV_W\left(W_t, t\right); dA_t \rangle}{V_W\left(W_t, t\right) A_t} \\
&= -r_t dt + \frac{dA_t}{A_t} + \frac{\left\langle dW_t^{-\gamma}; dA_t \right\rangle}{W_t^{-\gamma} A_t}.
\end{aligned}$$

It follows that $E_{t_0} \sum_{k=0}^{K-1} \int_{t_k^+}^{t_{k+1}} d(V_W\left(W_t, t\right) A_t)$ has the same sign as $\mu_{A,t} - r_t + \frac{\langle dW_t^{-\gamma}; dA_t \rangle}{W_t^{-\gamma} A_t}$. Defining $H_t = e^{-\int_{t_0}^{t} r_u du} Z_t$ and repeating the same steps as in (A.8) leads to

$$GIPE^{(\gamma)} = E_{t_0} \sum_{k=0}^{K} H_{t_k} C_{t_k} = E_{t_0} \sum_{k=0}^{K-1} \int_{t_k^+}^{t_{k+1}} d(H_t A_t). \tag{A.9}$$

Applying Ito's Lemma to $d(H_t A_t)$, and noting that Z_t is a martingale gives

$$\frac{d(H_t A_t)}{H_t A_t} = -r_t dt + \frac{dA_t}{A_t} + \frac{\langle dZ_t; dA_t \rangle}{Z_t A_t} = -r_t dt + \frac{dA_t}{A_t} + \frac{\left\langle dW_t^{-\gamma}; dA_t \right\rangle}{W_t^{-\gamma} A_t}. \tag{A.10}$$

Substituting (A.10) into (A.9) shows that $GIPE^{(\gamma)}$ has the same sign as $\mu_{A,t} - r_t + \frac{\langle dW_t^{-\gamma}; dA_t \rangle}{W_t^{-\gamma} A_t}$. ■

Proof of Proposition 3. Let $H_t = e^{-rt} \frac{W_t^{-\gamma}}{E_{t_0}\left(W_t^{-\gamma}\right)}$. Noting that $\frac{W_t^{-\gamma}}{E_{t_0}\left(W_t^{-\gamma}\right)}$ is a martingale[25], an application of Ito's Lemma yields

$$\frac{dH_t}{H_t} = -rdt - \gamma w' \sigma dB_t,$$

where the optimal portfolio $w = \frac{1}{\gamma} \left(\sigma\sigma'\right)^{-1} \left(\mu - r1_N\right)$. Applying Ito's Lemma again shows that

$$\frac{d\left(H_t A_t\right)}{H_t A_t} = \left(\mu_A - r - \gamma w' \sigma \sigma_A'\right) dt - \left(\gamma w' \sigma - \sigma_A\right) dB_t. \tag{A.11}$$

Next note that

$$\begin{aligned} \beta\left(\mu^W - r\right) &= \beta w' \left(\mu - r1_N\right) = \frac{w'\sigma\sigma_A'}{w'\sigma\sigma' w} w' \left(\mu - r1_N\right) \\ &= \frac{\gamma w' \sigma \sigma_A'}{\left(\mu - r1_N\right)' \left(\sigma\sigma'\right)^{-1} \left(\mu - r1_N\right)} \left(\mu - r1_N\right)' \left(\sigma\sigma'\right)^{-1} \left(\mu - r1_N\right) \\ &= \gamma w' \sigma \sigma_A'. \end{aligned}$$

Accordingly, assumption (10) implies that $\mu_A - r = \beta\left(\mu^W - r\right)$ is equivalent to $\mu_A - r - \gamma w'\sigma\sigma_A' = 0$. Accordingly, (A.11) implies that $H_t A_t$ is a martingale, and therefore

$$H_{t_k^+} A_{t_k^+} = E_{t_k}\left(H_{t_{k+1}} A_{t_{k+1}}\right) = E_{t_k}\left(H_{t_{k+1}} A_{t_{k+1}^+}\right) + E_{t_k}\left(H_{t_{k+1}} C_{t_{k+1}}\right).$$

Iterating forward, using the law of the iterated expetation and noting that $A_{t_0} = A_{t_K^+} = 0$ implies that $GIPE = 0$. ■

Proof of Proposition 4. a) All terms in (12) are non-negative for all t_k. b) It suffices to show that the present value of the cash flows $\widehat{C}_{t_k}$ discounted at the benchmark rate of return is zero, $\sum_{k=0}^{K} \frac{\widehat{C}_{t_k}}{G_{t_k}} = 0$. We have that

$$\sum_{k=0}^{K} \frac{\widehat{C}_{t_k}}{G_{t_k}} 1_{\left\{\widehat{C}_{t_k} > 0\right\}} = \widehat{A}_0 \times \sum_{k=0}^{K} \omega_{t_k} = \widehat{A}_0. \tag{A.12}$$

[25] Recall that since the investment opportunity set (interest rate, excess returns, volatility matrix, etc.) is constant, W_t is log-normal.

Equation (A.12) implies $\sum_{k=0}^{K} \frac{\widehat{C}_{t_k}}{G_{t_k}} = 0$, which shows that the strategy can be financed by investing the funds in the benchmark portfolio. c) By construction, $C_{t_k} = \widehat{C}_{t_k}$ whenever $\widehat{C}_{t_k} < 0$. So we focus on $C_{t_k} > 0$ to obtain

$$\begin{aligned} \widehat{C}_{t_k} &= \widehat{A}_0 \omega_{t_k} G_{t_k} \\ &= \frac{\widehat{A}_0}{\sum_{k=0}^{K} \frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}} C_{t_k} \\ &= C_{t_k}, \end{aligned} \tag{A.13}$$

where the last equality follow from assumption (13) and (A.12). ∎

B Micro-foundations for the GIPE

In this appendix we outline a basic stochastic overlapping-generations model with multiple types of capital.[26] The goal is to illustrate that an equilibrium with young workers choosing their saving and investment allocations optimally can be implemented by delegating these choices to a pension plan that evaluates investment projects according to the GIPE rule.

In the standard overlapping-generations model a consumer lives for two periods. In the first period of her life, she is endowed with units of labor, L. For simplicity, assume that she supplies labor inelastically. In the second period of life she relies on her investment income to support her consumption. The consumer's Euler equation is

$$u'\left(c_t^y\right) = \zeta E_t\left[u'\left(c_{t+1}^o\right) R_{j,t+1}\right], \tag{B.1}$$

where c_t^y is the consumption of a young consumer at time t, c_{t+1}^o is the consumption of the consumer in her old age (time $t+1$), $\zeta < 1$ is a subjective discount factor, and $R_{j,t+1}$ is the gross return from investing in (any) asset j.

In the typical overlapping-generations model, the consumption when old is equal to her (young-

[26]For more details on some of the notation and equations used in this section see e.g., Abel and Panageas (2022).

period) savings, S_t, times the gross rate of return on those savings, $R_{p,t+1}$:

$$c_{t+1}^o = S_t \times R_{p,t+1}, \tag{B.2}$$

where $R_{p,t+1} \equiv \omega_t' R_{t+1}$, with ω_t the vector of portfolio holdings and R_{t+1} the vector of gross returns.

Assuming that the consumer has constant relative risk aversion, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, equations (B.1) and (B.2) imply[27]

$$E_t\left[R_{p,t+1}^{-\gamma} \cdot (R_{j,t+1} - R_{f,t})\right] = 0, \tag{B.3}$$

where $R_{f,t}$ is the risk-free rate. We note that (B.3) can be equivalently re-written as

$$1 = E_t\left[\frac{1}{R_{f,t}} \frac{(R_{p,t+1})^{-\gamma}}{E_t\left(R_{p,t+1}^{-\gamma}\right)} R_{j,t+1}\right]. \tag{B.4}$$

We assume that risk-free bonds are in zero net supply. The positive-supply assets are different forms of capital, $K_{j,t}$.We make the standard assumption that one unit of the consumption good can be converted to a unit of the capital j (for any j). Finally, we assume that output, Y_t is produced according to constant returns to scale (CRS) technology. The implication of the CRS assumption is that $Y_t = F\left(\overrightarrow{K}_t, L_t; \varepsilon_t\right) = \sum_j F_{K_{j,t}}(\varepsilon_t) K_{j,t} + F_{L,t}(\varepsilon_t) L$, where ε_t captures (a vector) of random productivity shocks due to production uncertainty. Having noted that production is subject to random shocks, henceforth, we suppress ε_t to save notation. Other than the CRS assumption, we also assume that the labor share is constant: $\frac{F_{L,t} L}{Y_t} = \alpha$. This assumption is automatically true if the production function is Cobb-Douglas.

The return to investing in the capital of type j is given by

$$R_{K_j,t+1} = F_{K_{j,t+1}} + (1 - \delta), \tag{B.5}$$

where δ is the depreciation rate of capital. For simplicity, we assume that all forms of capital have the same depreciation rate. $F_{K_{j,t+1}}$ is the marginal product of capital j at time $t+1$, and is subject to the random productivity shock(s) that will materialize in period $t+1$. Equation (B.5) is the

[27]Note that equation (B.1) applies to any return, including the risk-free rate. Accordingly, $u'(c_t^y) = \zeta E_t\left[u'(c_{t+1}^o) R_{j,t+1}\right] = \zeta E_t\left[u'(c_{t+1}^o)\right] R_{f,t}$, which implies that $E_t\left[(c_{t+1}^o)^{-\gamma}(R_{j,t+1} - R_{f,t})\right] = 0$. Using (B.2) to express $c_{t+1}^o = S_t \times R_{p,t+1}$ leads to (B.3).

well-known Jorgenson identity, which states that the return on capital is the sum of its marginal product plus its un-depreciated part.

Since the different forms of capital are the only positive-supply assets, in equilibrium the value-weighted return on all capital investments is equal to the total return on the portfolio. Letting $K_{t+1} \equiv \sum_j K_{j,t+1}$, and recalling that all units of capital have the same purchase price, we obtain

$$\begin{aligned} R_{p,t+1} &= \sum_j \omega_j R_{K_j,t+1} = \sum_j \frac{K_{j,t+1}}{K_{t+1}} R_{K_j,t+1} = \\ &= \sum_j \frac{K_{j,t+1}}{K_{t+1}} \left(F_{K_{j,t+1}} + (1-\delta) \right) = (1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + (1-\delta), \end{aligned}$$

where the last equality follows from the assumptions of CRS and a constant labor share.

In equilibrium, the savings of the young generation equal the next period's total capital stock, $S_t = K_{t+1}$. Letting $s_t = \frac{S_t}{w_t L}$ denote the (young-generation) savings rate (that is, the ratio of savings to labor income) allows us to express $R_{p,t+1}$ as

$$\begin{aligned} R_{p,t+1} &= (1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) = \left(\frac{1-\alpha}{\alpha} \right) \frac{L w_{t+1}}{K_{t+1}} + (1-\delta) \\ &= \left(\frac{1-\alpha}{\alpha} \right) \frac{L}{S_t} w_{t+1} + (1-\delta) = \left(\frac{1-\alpha}{\alpha} \right) \frac{1}{s_t} \frac{w_{t+1}}{w_t} + (1-\delta) . \end{aligned} \tag{B.6}$$

Equation (B.6) implies that the total portfolio return is an affine function of wage-growth, $R_{p,t+1} = (1-\delta) + \xi_t \frac{w_{t+1}}{w_t}$, where $\xi_t \equiv \left(\frac{1-\alpha}{\alpha} \right) \frac{1}{s_t}$ is a known constant at time t. In the realistic case, where a "period" is taken to be 30 years, it is not unreasonable to assume full depreciation, $\delta = 1$, in which case $R_{p,t+1}$ is simply proportional to $\frac{w_{t+1}}{w_t}$.

One way of implementing the above equilibrium outcome is by delegating the portfolio decision to pension funds. These pension funds are "pass-through" entities that collect total contributions $s_t w_t L$ from young workers at time t, make portfolio choices according to equation (B.3) (or, equivalently, (B.4)) and promise to deliver a pension equal to $S_t R_{p,t+1} = s_t w_t L R_{p,t+1} = L \left(\left(\frac{1-\alpha}{\alpha} \right) w_{t+1} + s_t (1-\delta) w_t \right)$ to workers at time $t+1$. Note that the promised pension is a linear combination of current and next-period's wages. In the realistic case where $\delta = 1$, the pension is simply proportional to w_{t+1}, the *wages of the next period.* To compare the stylized model's prediction with the real world, note that most real-world pension funds promise pensions based on a worker's highest wage, which in a growing economy is very likely to correspond to the wage *at the end* of her work-life, or equivalently, the beginning of the next period in the context of our model.

For our purposes, the most important feature of the equilibrium with delegation is that the pension fund's portfolio choice must conform with equation (B.4), which is the equation that underlies the definition of the GIPE (to see this, note that $R_{j,t+1} = \frac{\text{Distributions}_{t+1}}{\text{Commitments}_t}$ in a two-period example. Therefore, equation (B.4) is identical to equation 9 in our two-period example).

While the model of this appendix section is stylized, the insight that the pension fund should inherit the preferences of the worker, and in particular that they should satisfy the portfolio equation (B.4), is general. It is also useful to observe that — in equilibrium — the evolution of the pension fund's assets, $R_{p,t+1}$, "matches" the evolution of the fund's liabilities, which are an affine function of w_{t+1}. In other words, if the pension fund chooses its portfolio to conform with equation (B.4), then the pension fund's assets match its liabilities in equilibrium.[28]

C Robustness

This appendix describes three additional exercises designed to check the robustness of our findings.

C.1 Alternate pension plan return measurement

First, as an alternative to the reported pension plan returns from annual reports, we calculate returns following the methodology outlined in Andonov and Rauh (2022). Specifically, annual plan returns are computed as the total net investment income divided by the beginning-of-the-year assets. This alternate measure has the advantage of being more robust to differences in accounting for fees across pension plans. Its drawbacks are that the measure is only available from 2001 onwards (the starting year of the pension plan CAFR data), and it does not account for the timing of contributions and pension benefit payments within a given year. Figure C.1 plots a time series of the average one-year reported returns from the paper and the alternate returns. On average, the reported return is 30 basis points higher than the alternate return over the same sample period. This difference is significant at the 1% level and is likely due to some reported returns being gross of fees. The average correlation between the two return series is 97% (equally weighted across plans).

[28]For this outcome to obtain, the ratio of pensions-to-contributions must be set exactly equal to $\frac{\xi_t w_{t+1} + (1-\delta) w_t}{w_t}$.

Table C.1 shows PE performance metrics computed using the reported and alternate returns. For each of the performance metrics (IPE, GIPE, and their alphas), we show three values: *Full*, *Alt*, and *Match*. The *Full* measure is computed using the full sample of available reported returns for all pension plan and PE fund combinations (regardless of commitment) and is identical to the performance measures reported in Table 3. The *Alt* measure is computed using the alternate returns discussed above. The *Match* measure uses the reported returns from the paper, but from 2001 onwards only, to match the sample period of the alternate return measure.

We focus on the difference between *Match* and *Alt*, as these are most comparable. The mean IPE (*Alt*) is 0.011 higher than IPE (*Match*), as shown in Panel A of Table C.1. This difference, while statistically significantly different from zero, is economically very small. Similarly, α-IPE (*Alt*) is 23.5 basis points higher than α-IPE (*Match*) and significant at the 1% level. To compute the GIPE measures in Panel B, we separately estimate pension plan γ_j for each of the measures. The average γ_j for the *Full*, *Alt*, and *Match* samples are 4.78, 4.00, and 4.05, respectively. The difference in γ_j between the *Alt* and *Match* sample is statistically not different from zero. The difference between the *Full* and *Alt* sample average γ_j is 0.73 and significant at the 1% level. The differences in GIPE performance between the *Full* and *Alt* samples arises predominantly due to the difference in γ_j. The differences between the *Alt* and *Match* GIPE-metrics are statistically indistinguishable from zero. We conclude that the (G)IPE values computed using the alternate measure are quite similar to those derived from reported returns.

C.2 Missing IRRs and alphas

Our second set of robustness checks uses an alternative assumption for funds for which an IRR or alpha could not be computed.[29] This is quite rare. For example, $\alpha(GIPE)$ cannot be computed for four PE funds and one pension plan (which has a risk aversion coefficient of 35), representing fewer than 0.5% of all possible pension plan-PE fund combinations. In the main text we assign these observations a value of -100%, since they tend to have very low (G)IPEs (their average PME(KN) is -0.627 and their average TVPI is 0.144). As an alternative, we omit the missing values, and we show how this changes the reported statistics and regressions. While assigning a value of -100%

[29] For instance, a project with three cash flows of $-\$3$ initially, $+\$2$ one year later, and $-\$1$ in year two fails to produce an IRR because its net present value remains non-zero at all possible discount rates. Note that (G)IPE is always computable.

may understate performance, this alternative assumption should provide an upper bound on the effect of missing values on IRRs and alphas.

Table C.2 reports the IRR and alpha statistics under the two missing values assumptions. Rows labeled "*adj*" represent performance values when missing values are replaced with −100%, and these results are identical to those presented in Table 3. Rows labeled "*obs*" exclude the missing observations. The difference in average IRR across all funds is 39 basis points. The largest difference is in VC, at 52 basis points. For alpha, the differences are around 40 to 60 basis points. As with IRR, the largest difference is in VC, where the GIPE-based alpha is 59 basis points higher when omitting the missing values, compared to assigning a −100% value. Since the direction of change is the same for both alpha and IRR, the wedge between IRR and alpha is close to equal between the two alternatives.

Finally, Tables C.3 to C.5 show the results from rerunning the regressions in Tables 4 to 6 when we omit the missing alphas and IRRs. We conclude that omitting the missing values instead of assigning them a placeholder value of −100% does not materially impact the results in the paper.

C.3 Access versus skill: First-time funds

In our final robustness check we use an alternative approach to testing the "access versus skill" channel as an explanation for the higher returns of invested funds in section 4.1. Instead of using evidence from re-investments as in Table 4, we use investments in the first fund raised by the GPs in our data. These first-time funds typically attract capital from any investor willing to invest. We observe a total of 209 first-time funds, comprising 76 buyout, 80 venture capital, and 53 real estate funds. Table C.6 shows the coefficient on the indicator variable *Commit* (which equals one when the pension plan committed to this particular PE fund) is negative in about half of the specifications, and only statistically significant (at the 10% level) in one specification. Consistent with the results in the main text, this suggests that pension plans do not have skill in selecting well-performing PE funds.

Table C.1: **PE Performance Metrics with Alternate Pension Plan Returns**

This table reports performance results for private equity funds computed using returns from pension plan annual reports (as in the main text) and a set of alternate returns. The alternate returns are calculated using the methodology in Andonov and Rauh (2022). The first column shows the average performance across all possible pairs of pension plans and PE funds. The second column shows the standard error (s.e.) for the test of the null hypothesis that the mean is zero, double-clustered by pension plan and vintage year. The second, third, and fourth sets of two columns shows the average performance and its standard error separated by PE strategy: buyout, venture capital, and real estate funds. PE fund performance metrics computed using the entire data set of annual report returns are denoted by *Full* in parentheses, and match the results in Table 3. Metrics computed using the alternate return measure are denoted by *Alt.*. Metrics computed using only annual report returns from 2001 onwards, to match the shorter sample period of alternate returns, are denoted by *Match*. Panels A and B shows IPE and GIPE-type metrics, respectively, computed irrespective of whether a pension plan actually invested in the PE fund. The α metric is an annualized excess return that makes the (G)IPE equal to zero, computed as described in the text and reported in percentage points (e.g., an α of 1 is an excess return of 1% per year). ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	All funds		Buyout		VC		Real Estate	
	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
Panel A: IPE-type metrics.								
IPE (Full)	0.132	0.036***	0.225	0.024***	0.058	0.083	0.090	0.054*
IPE (Alt.)	0.147	0.031***	0.258	0.024***	0.056	0.064	0.096	0.057*
IPE (Match)	0.136	0.032***	0.246	0.023***	0.045	0.063	0.089	0.057
IPE (Alt.) - IPE (Match)	0.011	0.002***	0.012	0.002***	0.011	0.002***	0.007	0.001***
α(IPE)(Full)	3.149	1.167***	5.666	0.960***	-0.008	1.993	4.351	1.770**
α(IPE)(Alt.)	3.474	1.222***	6.620	1.069***	-0.777	1.560	4.641	1.866**
α(IPE)(Match)	3.239	1.232***	6.373	1.072***	-1.012	1.568	4.427	1.874**
α(IPE)(Alt.) - α(IPE)(Match)	0.235	0.035***	0.247	0.036***	0.235	0.032***	0.214	0.038***
Panel B: GIPE-type metrics.								
GIPE (Full)	-0.029	0.032	0.109	0.035***	-0.131	0.064**	-0.102	0.075
GIPE (Alt.)	-0.072	0.033**	0.038	0.025	-0.158	0.048***	-0.128	0.071*
GIPE (Match)	-0.074	0.033**	0.033	0.024	-0.158	0.048***	-0.128	0.071*
GIPE (Alt.) - GIPE (Match)	0.002	0.003	0.005	0.004	0.000	0.002	0.000	0.002
α(GIPE)(Full)	-2.720	1.210**	0.453	1.192	-5.870	1.535***	-2.853	1.796
α(GIPE)(Alt.)	-3.166	0.958***	0.117	0.888	-6.742	1.197***	-3.271	1.597**
α(GIPE)(Match)	-3.517	1.022***	-0.286	0.932	-7.083	1.239***	-3.547	1.661**
α(GIPE)(Alt.) - α(GIPE)(Match)	0.351	0.235	0.403	0.258	0.341	0.216	0.276	0.227
N (*Full*)	170,355		68,649		67,696		34,010	
N (*Alt & Match*)	133,333		53,645		48,323		31,365	

Table C.2: **Comparison of Missing Value Treatments for IRRs and Alphas**

This table reports descriptive statistics of IRRs and alphas for two treatments of missing observations. The rows labeled "*adj*" (short for adjusted) assign a placeholder of -100% for missing IRR and alpha values, as in the main text (these numbers are identical to Table 3). The rows labeled "*obs*" (short for observed) exclude these missing observations. The first column shows the average performance across all possible pairs of pension plans and PE funds regardless of commitment. The second column shows the standard error (s.e.) for the test of the null hypothesis that the mean is zero, double-clustered by pension plan and vintage year. The second, third, and fourth sets of two columns shows the average performance and its standard error separated by PE strategy: buyout, venture capital, and real estate funds. Both IRR and α are in percentage points (e.g., an α of 1 is an excess return of 1% per year). Finally, $N(adj)$ represents the count of all available observations. $N(\alpha(GIPE)(obs))$ is the number of observations after removing missing data for $\alpha(GIPE)$. Similarly,$N(\alpha(IPE)(obs))$ and $N(IRR(obs))$ are number of observations after excluding missing data for $\alpha(IPE)$ and IRR respectively. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	All funds		Buyout		VC		Real Estate	
	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
Panel A: IRR-type metrics.								
IRR (*adj*)	10.700	1.429***	13.154	1.228***	7.273	2.371***	12.569	2.029***
IRR (*obs*)	11.094	1.405***	13.608	1.186***	7.799	2.475***	12.569	2.029***
Panel B: $\alpha(IPE)$-type metrics.								
$\alpha(IPE)$ (*adj*)	3.149	1.167***	5.666	0.960***	-0.008	1.993	4.351	1.770**
$\alpha(IPE)$ (*obs*)	3.516	1.147***	6.089	0.927***	0.482	2.094	4.351	1.770**
IRR - $\alpha(IPE)$ (*adj*)	7.579	0.320***	7.519	0.344***	7.317	0.423***	8.218	0.361***
IRR - $\alpha(IPE)$ (*obs*)	7.552	0.322***	7.489	0.346***	7.281	0.417***	8.218	0.361***
Panel C: $\alpha(GIPE)$-type metrics.								
$\alpha(GIPE)$ (*adj*)	-2.720	1.210**	0.453	1.192	-5.870	1.535***	-2.853	1.796
$\alpha(GIPE)$ (*obs*)	-2.262	1.154*	0.985	1.134	-5.272	1.578***	-2.827	1.784
IRR - $\alpha(GIPE)$ (*adj*)	13.372	1.317***	12.634	1.444***	13.097	1.563***	15.398	1.372***
IRR - $\alpha(GIPE)$ (*obs*)	13.420	1.352***	12.702	1.483***	13.143	1.582***	15.422	1.381***
N (*adj*)	170,355		68,649		67,696		34,010	
N ($\alpha(GIPE)(obs)$)	169,557		68,287		67,269		34,001	
N ($\alpha(IPE)(obs)$)	169,751		68,375		67,366		34,010	
N (IRR (*obs*))	169,751		68,375		67,366		34,010	

Table C.3: **Pension Plan Commitments and Performance: Missing IRRs and Alphas**

This table repeats the regressions in Table 4 but omitting observations where IRR and alpha cannot be computed (instead of setting them to −100% as in the original table). See Table 4 for a detailed description of the regressions.

Panel A: Without pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	-0.179***	1.982**	-0.144*	2.289**	-0.133	1.848*	-0.076	1.967*
	(0.052)	(0.918)	(0.072)	(0.993)	(0.089)	(0.893)	(0.063)	(0.955)
Commit=1	2.090***	2.352***	0.990	1.571**	3.382	2.417**	0.121	0.188
	(0.613)	(0.517)	(0.611)	(0.626)	(1.991)	(1.047)	(0.692)	(0.449)
Relationship=1	2.989***	3.632***	2.885*	2.979**	3.881**	5.368***	0.929	1.669
	(0.975)	(0.964)	(1.571)	(1.380)	(1.641)	(1.503)	(1.268)	(1.437)
Relationship x Commit	-2.705*	-2.667*	-2.692*	-2.293	-5.511	-5.197*	0.539	-0.043
	(1.420)	(1.313)	(1.293)	(1.414)	(3.800)	(2.647)	(1.046)	(1.037)
N	169,751	169,557	68,375	68,287	67,366	67,269	34,010	34,001
adj. R^2	0.062	0.031	0.126	0.061	0.120	0.058	0.328	0.133
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	N	N	N	N	N	N	N	N

Panel B: With pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	-0.001	-0.010	-0.033	0.050	0.112	0.038	-0.017	0.002
	(0.049)	(0.397)	(0.022)	(0.437)	(0.105)	(0.361)	(0.024)	(0.297)
Commit=1	2.187***	1.995***	1.068	1.180*	3.635*	1.962	0.164	0.184
	(0.600)	(0.579)	(0.629)	(0.578)	(2.001)	(1.274)	(0.709)	(0.638)
Relationship=1	3.117***	2.655***	2.987*	2.189	4.180**	4.435***	0.965	0.554
	(0.995)	(0.905)	(1.616)	(1.385)	(1.708)	(1.539)	(1.306)	(1.254)
Relationship x Commit	-2.795*	-2.037	-2.769**	-1.990	-5.777	-4.378	0.521	0.466
	(1.421)	(1.334)	(1.318)	(1.400)	(3.843)	(2.823)	(1.059)	(0.827)
N	169,751	169,557	68,374	68,287	67,366	67,269	34,010	34,001
adj. R^2	0.061	0.227	0.125	0.360	0.118	0.171	0.327	0.609
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	Y	Y	Y	Y	Y	Y	Y	Y

Table C.4: **Pension Plan Characteristics and Performance: Missing IRRs/Alphas**

This table repeats the regressions in Table 5 but omitting observations where IRR and alpha cannot be computed (instead of setting them to -100% as in the original table). See Table 5 for a detailed description of the regressions.

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Home State=1	-1.839**	-1.367	1.251	1.276	-1.675	-1.741	-2.582**	1.101
	(0.839)	(0.823)	(0.887)	(1.025)	(1.598)	(1.584)	(1.040)	(1.998)
Funded Ratio	-3.354	4.562	-2.073	5.795*	-0.650	6.530	0.115	6.344
	(2.541)	(2.913)	(3.205)	(3.123)	(5.350)	(4.474)	(1.840)	(3.684)
Public Equity Wt	-15.673**	-23.864***	-4.179	-13.375***	-24.655	-15.604	-10.067***	-27.495***
	(6.379)	(5.054)	(5.663)	(4.332)	(15.266)	(12.860)	(3.015)	(7.359)
Log(AUM)	0.688	1.217*	0.940**	1.023**	-1.941	-1.703	-0.466	0.227
	(0.559)	(0.685)	(0.414)	(0.480)	(1.379)	(1.108)	(0.434)	(0.854)
State Appointed	-7.644	-15.645	-9.469	-2.557	0.840	-11.971	0.002	-24.219
	(4.513)	(13.670)	(6.067)	(6.087)	(7.298)	(18.206)	(5.060)	(26.332)
State ex-officio	-6.368*	-9.762*	-7.930*	-9.976**	-1.416	-6.737	3.767	-2.528
	(3.460)	(5.282)	(4.062)	(3.753)	(4.429)	(7.235)	(3.379)	(7.952)
Member Elected	-2.464	-0.533	-3.035	-2.359	-2.281	-6.041	1.923	7.970
	(2.279)	(3.427)	(2.532)	(2.500)	(3.176)	(3.718)	(3.348)	(8.132)
Public Appointed	-1.008	-4.536	-3.523	-7.450*	0.713	-8.187	0.410	5.491
	(1.972)	(3.364)	(2.265)	(3.713)	(3.673)	(4.782)	(2.443)	(5.788)
Board Size	-0.057	0.184*	-0.121*	0.129*	0.205	0.297*	0.016	0.364*
	(0.070)	(0.094)	(0.064)	(0.063)	(0.171)	(0.152)	(0.059)	(0.200)
Log (Commitment %)	0.989	1.194**	0.694	0.801	-1.349	-1.481*	-0.752	-0.148
	(0.689)	(0.533)	(0.441)	(0.505)	(1.021)	(0.773)	(0.847)	(1.078)
Other Trustees	6.084*	0.286	2.612	-2.421	4.842	-5.234	-0.884	-3.748
	(3.135)	(5.299)	(3.875)	(4.838)	(5.666)	(7.491)	(2.974)	(8.012)
N	4,733	4,733	2,592	2,592	1,252	1,252	880	880
adj. R^2	0.108	0.095	0.219	0.182	0.125	0.077	0.387	0.229
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Plan State FE	Y	Y	Y	Y	Y	Y	Y	Y

Table C.5: **Pension Plan Risk-taking: Missing IRRs/Alphas**

This table repeats the regressions in Table 6 but omitting observations where IRR and alpha cannot be computed (instead of setting them to −100% as in the original table). See Table 6 for a detailed description of the regressions.

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home State=1	-0.492	-0.473	-0.025	-0.025	-0.012	0.066	-3.556*	-3.683**
	(0.660)	(0.629)	(0.790)	(0.728)	(0.266)	(0.324)	(1.726)	(1.609)
Funded Ratio	-6.314***	-7.916***	-8.088***	-7.868**	-5.071**	-7.180**	-4.329**	-6.229**
	(1.977)	(2.370)	(2.683)	(3.245)	(2.025)	(3.348)	(1.724)	(2.275)
Public Equity Wt	7.300	8.191	11.711**	9.196**	-8.670	-9.051	10.864	17.429*
	(6.205)	(4.864)	(5.405)	(4.179)	(10.888)	(10.839)	(7.436)	(8.325)
Log(AUM)	-0.499	-0.529	-0.512	-0.083	-0.711*	-0.238	0.224	-0.692
	(0.432)	(0.612)	(0.419)	(0.629)	(0.397)	(0.603)	(0.579)	(0.867)
State Appointed		8.001		-6.912		12.811		24.222
		(14.059)		(7.474)		(17.658)		(26.653)
State ex-officio		3.394		2.046		5.322		6.296
		(4.988)		(3.838)		(7.217)		(9.652)
Member Elected		-1.931		-0.676		3.760		-6.047
		(3.229)		(2.760)		(2.562)		(5.714)
Public Appointed		3.528		3.927		8.900*		-5.081
		(3.017)		(2.347)		(5.014)		(5.662)
Board Size		-0.241***		-0.250**		-0.092		-0.347*
		(0.081)		(0.088)		(0.081)		(0.183)
Log (Commitment %)		-0.205		-0.108		0.131		-0.604
		(0.335)		(0.407)		(0.358)		(0.860)
Other Trustees		5.797		5.033		10.076		2.864
		(4.261)		(3.300)		(7.217)		(7.006)
N	4,733	4,733	2,592	2,592	1,252	1,252	880	880
adj. R^2	0.314	0.320	0.455	0.462	0.286	0.287	0.226	0.265
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Plan State FE	Y	Y	Y	Y	Y	Y	Y	Y

Table C.6: **Pension Plan Commitments and Performance: First-time Funds**

Each column shows the coefficients of a regression of private equity fund performance on two indicator variables plus fixed effects controls. The observations are combinations of pension plans and PE funds, but only for the first-time funds of the PE managers in our sample. The indicator *Active* equals one if the pension plan made a commitment to any PE fund in that vintage year and zero otherwise. *Commit* equals one when the pension plan committed to this particular PE fund, and zero otherwise. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. The odd-numbered columns use IRR as the performance metric, and the even-numbered columns use the GIPE-implied α. All regressions include vintage fixed effects. The regressions in Panel B also include pension plan fixed effects. Both IRR and α are in percentage points (e.g., an α of 1 is an excess return of 1% per year). Standard errors are double clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level

Panel A: Without pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	-0.040	2.285**	0.036	2.559**	-0.017	2.208*	0.016	2.198**
	(0.071)	(1.016)	(0.135)	(1.166)	(0.115)	(1.091)	(0.017)	(0.963)
Commit=1	1.112	2.474*	-0.577	1.867	0.601	1.234	-1.035	-0.448
	(1.385)	(1.199)	(2.066)	(1.669)	(1.915)	(1.926)	(0.689)	(1.068)
N	27,145	27,145	9,665	9,665	10,254	10,254	7,226	7,226
adj. R^2	0.056	0.063	0.137	0.140	0.089	0.095	0.516	0.177
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	N	N	N	N	N	N	N	N

Panel B: With pension plan fixed effects

	All Funds		Buyout Funds		Venture Capital Funds		Real Estate Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$	IRR	$\alpha(GIPE)$
Active=1	0.040	-0.029	-0.003	0.063	0.153	-0.006	0.014	0.008
	(0.057)	(0.391)	(0.084)	(0.427)	(0.104)	(0.280)	(0.020)	(0.313)
Commit=1	1.178	1.682	-0.605	1.235	0.810	-0.060	-1.102	-1.211
	(1.436)	(1.446)	(2.180)	(1.920)	(2.186)	(2.097)	(0.733)	(0.742)
N	27,145	27,145	9,665	9,665	10,254	10,254	7,222	7,222
adj. R^2	0.051	0.230	0.124	0.275	0.077	0.226	0.506	0.722
Vintage FE	Y	Y	Y	Y	Y	Y	Y	Y
Pension Plan FE	Y	Y	Y	Y	Y	Y	Y	Y

Figure C.1: **Pension Plan Reported and Alternate Returns**

This figure presents the time series of average one-year returns across the full sample of 150 pension plans described in Table 2. The solid line represents reported returns, while the dashed line depicts an alternative return measure, calculated as total net investment income divided by beginning-of-year assets, following Andonov and Rauh (2022). The alternative measure is available from 2001 onward.