NBER WORKING PAPER SERIES

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Working Paper 33056
http://www.nber.org/papers/w33056

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2024, Revised April 2026

This is a revised version of NBER WP w33056, under the title "Revisiting the private value of scientific inventions". We thank participants at the Boston University IP Day-2024 Conference, the NBER Productivity Seminar, and Duke University for comments and feedback. We thank Hansen Zhang for providing excellent research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 33056
November 2024, Revised April 2026
JEL No. O3

ABSTRACT

Estimating the private value of patents is important, yet challenging. By developing a method based on stock market returns to produce estimates of individual patent values, Kogan, Papanikolaou, Seru, and Stoffman (2017) (KPSS) opened venues for new research. We characterize the measurement error in KPSS – the difference between the true patent value and the corresponding KPSS estimate – and show it is negatively correlated with the true patent value. We then investigate the use of KPSS estimates in two different applications. First, we show that using KPSS values to gauge differences in value between different patent groups is internally inconsistent and introduces attenuation bias. We offer two solutions: extending the original KPSS method to allow for patents to be drawn from two distinct value distributions, and using abnormal stock market returns. We compare both to the original KPSS estimates in several contexts relevant to the organizational scholars, such as patents by large and small teams, scientific and non-scientific patents, and offshored and domestically invented patents. Second, we show that KPSS yield unbiased estimates when used as explanatory variables. These analyses allow us to characterize the main trade-offs associated with each approach, and offer practical guidance to researchers.

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1 Introduction

Innovation scholars often need measures of the private economic value of patents. Griliches (1981) introduced stock market signals as an indicator of innovation value by estimating the stock market returns to R&D stock. The pioneering work by Hall, Jaffe, and Trajtenberg (2005) introduced firm-level patent data to the field, allowing researchers to estimate the stock market returns to patenting activity by firms. While very influential, this approach estimates the average patent value at the firm or firm-year level (rather than at the individual patent level), which is problematic because most patents have no or limited private (or social) value (Scherer & Harhoff, 2000). Kogan et al. (2017) (henceforth KPSS) made an important contribution to the field by developing a method that measures the value to each patent separately. This method relies on stock market returns, as in previous research, but filters out noise from those returns by imposing distributional assumptions to yield the expected value of a patent given the observed stock return. The availability of individual patent values makes KPSS very attractive to empirical researchers.

This paper characterizes the properties of KPSS estimates, shows that these properties have important implications for empirical research, and introduces two alternative stock-return-based methods to study the private value of patents. In so doing, we identify the types of questions and research designs that are aligned with the KPSS estimates, and those that may instead require alternative methods. Thus, our contribution is to offer a better understanding of the strengths and weaknesses of KPSS measures of patent value, and offer guidance about when different measures are more appropriate.

Our analysis proceeds in three steps. First, we note that the KPSS estimate of patent value has measurement error – the difference between KPSS estimates and the "true" value of the patent – for two reasons: (i) noise in the relation between patent value and stock returns and (ii) distortion introduced by the assumption that patent values must be non-negative. We show that while KPSS estimates have the same mean of the underlying patent value, their distribution has a smaller variance. We then show that the measurement error in KPSS estimates covaries negatively with the true patent value, implying that KPSS tends to undervalue more valuable patents and overvalue less valuable ones. The presence of measurement error introduces a trade-off between using stock returns without filtering out noise (henceforth, "unfiltered" stock returns) and the KPSS method to estimate patent value: the KPSS method filters out considerable noise in stock returns, thereby allowing for more precise estimation, but it also introduces non-classical measurement error. We evaluate this trade-off in the next two steps of our analysis.

The second step of our analysis evaluates the use of KPSS estimates as the outcome variable to examine differences in value between groups of patents, such as patents invented by large and small teams of inventors, or scientific and non-scientific patents. We note that this approach suffers from inconsistency between the assumptions used to estimate patent values and the downstream hypothesis tests conducted using those estimates. Researchers compare the means of different patent types using KPSS estimates. Yet, the KPSS method assumes that all patent types are drawn from the same distribution, implying that the expected values of different patent groups are the same. We confirm this intuition analytically by showing that there can be no systematic differences in the means for any two groups of patents under the assumptions used to estimate KPSS values. We further show that even if patent groups have different distributions KPSS values will underestimate the difference in value, and that the KPSS formula is invalid when patents from different groups are granted to the same firm on the same day.

We then evaluate two potential solutions. We introduce a generalized version of the KPSS method that allows patent values to be drawn from distinct distributions while accommodating simultaneous grants of different types to the same firm. Implementing this model, however, requires sufficient "variation in bundles" — specifically, enough days where one patent type is granted without the other — to separately identify the parameters of each distribution. If different patent types are granted in tandem too often, precision suffers. Second, we consider "unfiltered" stock returns as a simpler proxy for value. While this approach avoids complex filtering, it is inherently noisier and cannot assign individual values to patents granted on the same day. Ultimately, both methods allow us to resolve a common contradiction in the literature, where researchers investigate value differences between patent groups while using an empirical framework that assumes those differences do not exist.

We deploy these two approaches (together with standard KPSS estimates) to test for differences in group means at three levels of analysis: innovation team characteristics (team size and composition), patent characteristics (science-based versus non-science based patents), and organization characteristics (vertical integration). The different methods produce consistent evidence that patents awarded to large teams, science-based patents, and patents awarded to vertically integrated firms have higher private value, confirming previous findings in the literature. At the same time, the different methods yield mixed evidence regarding the value implications of R&D offshoring, suggesting that previous findings may be method-sensitive. Moreover, we find that unfiltered returns produce larger differences in group means relative to our generalized version of KPSS, which itself produces larger differences in group means relative to KPSS.

The third and final step of our analysis considers the implications of measurement error in KPSS when these estimates are used as an explanatory variable or as a control. We show analytically that though measurement error in KPSS estimates is non-classical (correlated with the true patent value), using these estimates as an independent variable does not introduce bias, such as attenuation bias typical with classical measurement error. Yet, using KPSS will bias standard errors downward because the variance of the patent value estimates is smaller than the variance of patent values. We then deploy the three approaches to measure patent value based on stock market signals – KPSS, our generalized version of KPSS, and stock returns – to investigate whether and how patent value at award date correlates with future patent-level outcomes. Across a multitude of future patent-level outcomes, we observe that stock returns are associated with these outcomes as predicted by economic theory, but these associations are small and not precisely estimated, most likely because stock returns are too noisy and therefore suffer from attenuation bias. At the same time, KPSS and our generalized version of KPSS produce estimated association with future patent-level outcomes that are not only consistent with economic theory but also precisely estimated even after adjusting standard errors. This result lends support to the intuition in Kogan et al. (2017) that de-noising stock returns may be necessary when estimated patent values are used as an independent variable. We also observe that our generalized version of KPSS produces only marginally different point estimates (relative to KPSS) that vary with the partitioning variable, suggesting that our approach is not necessary when KPSS estimates are used as an independent variable.

Based on the results of our analysis, we offer the following recommendations:

- Researchers interested in studying differences in value between patent groups *should not* use KPSS estimates because doing so is internally inconsistent. In these cases, researchers should use unfiltered stock returns or our generalized KPSS approach (particularly when most days feature the award of both patent groups). The two options – unfiltered returns and generalized KPSS – have distinctive strengths and weaknesses, so finding consistent evidence using both methods would produce more reliable results.

- Researchers interested in studying the correlation between patent private value and future patent-level outcomes can continue using KPSS estimates, provided they adjust the standard errors, for instance by bootstrapping. In these cases, the noise in stock price movements will bias estimates towards zero, so filtering out such noise is necessary to produce precise estimates of the correlations of interest.

To put our contribution into context, this paper does not argue that Kogan et al. (2017)'s method is flawed. What is flawed is how the resulting estimates are used in certain applications. Thus, scholars should carefully consider whether KPSS is appropriate for the research question of interest to them. If the intent is to explore how the private value of patents correlates with future patent outcomes – or, more generally, use patent value estimates as an independent variable – then KPSS estimates will do the job. If, instead, the intent is to explore the determinants of patent value, rather than using patent value estimates that assume a single distribution and then comparing conditional means, as common in the literature, researchers should estimate the distribution of patent value (i) by using unfiltered stock returns (e.g., by regressing the stock market response around the patent award date on the variable of interest) or (ii) separately by the variable of interest using our generalized KPSS method. In all cases, it is important to explore the robustness of results using different ways of obtaining patent values. To help researchers choose among the various approaches, we summarize the tradeoffs with each approach and our recommendations in Table 1.

In addition to offering the above guidance to empirical researchers interested in the private value of patents, we also speak to work investigating the link between theoretical constructs and empirical proxies for those constructs (including Lieberman et al., 2021; Gibbs, Simcoe, and Waguespack, 2025; and Souder, Shaver, Harris, and Alrashdan, 2024) and work on methodological challenges in strategy (e.g., Shaver, 1998). We revive stock market responses to patent awards as a simple, viable approach to compare patent values across patent groups, and introduce a generalized version of the KPSS method that allows for two separate distributions of patent value and for different patent types granted on the same day. Furthermore, we conclude that measurement error in KPSS estimates may have prevented researchers from documenting many relationships of interest between patent value and other constructs.

The paper is organized as follows. Section 2 revisits the KPSS methodology. Section 3 formalizes the measurement error in KPSS-based patent value estimates. Section 4 considers the consequences of using KPSS as an outcome variable, discusses strategies to address the resulting measurement error, and presents empirical applications on inventor team size, R&D offshoring, reliance on basic science, and vertical integration. Section 5 analyzes the implications of using KPSS as an explanatory variable and evaluates its performance relative to alternative stock-market-based measures. Conclusions are in Section 6.

2 Stock Market Signals as a Lens for Patent Valuation

2.1 Background

Estimating the private economic value of patents has been a key question in the economics of innovation scholarship. Researchers have used different approaches to measure patent value, each with its own strengths and weaknesses.

Starting with Sanders, Rossman, and Harris (1958), some scholars have directly surveyed inventors or firms to elicit valuations of individual patents (e.g., Gambardella, Harhoff, & Verspagen, 2008; Nagaoka & Walsh, 2009). These measures are both subjective and sensitive to how the responses are elicited. (See also Hall and Harhoff, 2012 for a more systematic review.) Another stream of literature measures the value of inventions using forward citations received by each patent (e.g., Trajtenberg, 1990). Although patent citation trails offer valuable insights, they mainly measure technical value, namely the influence of an invention on follow-ons, which is both conceptually and empirically different from the private value of the invention (Arora, Cohen, Lee, & Sebastian, 2023). A third stream of literature uses patent renewals (e.g., Harhoff, Scherer, & Vopel, 2003; Hegde & Sampat, 2009; Pakes, Simpson, Judd, & Mansfield, 1989; Schankerman, 1998; Serrano, 2010). This literature focuses on the resolution of uncertainty about the private value of either the underlying invention or perhaps of continued patent protection. Further, renewal data cannot provide fine-grained estimates of the value of individual patents.

Finally, a growing literature initiated by Griliches (1981) relies on stock market value to measure the average return to the invention, namely its private value. Such estimates were based on patent stocks and thus primarily at the firm or firm-year level, not at the individual patent (or patent-day) level, until Kogan et al. (2017) introduced methodology that allows researchers to obtain the value for each individual patent.[5] We characterize the nature and

[5] The KPSS method has also been applied in other contexts, including new product announcements (Mukherjee, Pellegrino, Žaldokas, Ren, & Thornquist, 2025), government R&D contracts (Arora, Belenzon, Cioaca, & Ferracuti, 2025), and startup acquisitions (Masclans, 2025).

direction of measurement error in KPSS estimates. We then extend the original Kogan et al. (2017) method by allowing different patent groups to be drawn from different value distributions. Moreover, we revive the use of "unfiltered" stock returns in certain contexts. Last, we lend support to certain findings in the literature, while calling into question the robustness of others.

These contributions notwithstanding, our methodology shares some limitations with prior approaches to measuring patent value. It is broadly recognized that these approaches potentially confound the value of patent protection and the value of the underlying invention. The value of the invention and of patent protection may also vary depending on the ability of the firm to commercialize and capture value from the invention. Our methodology also does not allow researchers to distinguish whether any estimated difference in the value of different types of patented inventions (e.g., scientific inventions versus non-scientific inventions) is because the inventions types are differentially valuable, or because patents on those inventions provide differentially effective protection, or a combination of the two. In addition, the values of individual patents may be interdependent. There are often multiple patents covering a commercialized innovation. This makes it conceptually difficult to assign value to a given patent because it may depend on the portfolio of patents available to the firm. Finally, as reflected in the patent-renewal literature, the value of a patent evolves over time, as technology and market conditions change. Therefore, the stock-market-based patent values, which have built-in option value, in addition to the expected value of the patented invention itself, will tend to differ from retrospective measures that survey measures typically reflect.

2.2 The Kogan et al. (2017) approach

Kogan et al. (2017) introduced a method that produces estimates of patent-level estimates of private value. It relies on the idea that stock prices reflect the market's expectations of a company's future profitability, including that due to patents. The authors use the short-term stock market response to patent grants to gauge their value implications for shareholders.

An important advantage of this approach is that asset prices are forward-looking and hence provide us with a signal of the private value to the patent holder. While conceptually straightforward, the approach is empirically challenging because stock prices can move for many reasons even over short periods around patent grants. KPSS overcome this difficulty by filtering stock returns of the noise through distributional assumptions on the value of patent grants and other events. In KPSS's approach, which we fully characterize in Appendix A1, the private value of patent j granted to firm f on day t is given by:

$$\hat{A}_{jft} = \frac{1}{K_{ft}}(1-\pi)^{-1} \cdot S \cdot \mathbb{E}[x_j|r_j] \tag{1}$$

Where K_{ft} represents the number of patents granted to firm f on day t, π is the ex-ante probability that the patent application is successful, and S denotes the market capitalization of the firm the day prior to the patent grants.[6]

The key estimand is $\mathbb{E}[x_j|r_j]$, the expected value of the patent (x_j) conditional on the observed stock market reaction around the patent grant announcement date (r_j). To estimate it, KPSS separate the stock return around the time patent j is granted (r_j) into two components: the change in firm value induced by the patent grant announcement, x_j, and the contemporaneous change in value induced by other value-relevant events that are unrelated to the patent, ε_j. These two components are unobservable, so the authors impose distributional assumptions to estimate them from observable variables. First, because the market value of a patent is a positive random variable, the authors assume that x_j, the stock price change due to the patent grant, follows a normal distribution truncated at zero: $x_j \sim \mathcal{N}^+(0, \sigma^2_{xft})$. Second, because value-relevant events other than patent grants can be both value-enhancing (e.g., new product announcement) or value-destroying (e.g., lawsuit announcement), ε is assumed to be normally distributed: $\varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon ft})$. Under these assumptions, $\mathbb{E}[x_j|r_j]$ is given by:[7]

[6]To reduce notation, we omit the length of the time window over which the stock market response to the patent grant is measured.

[7]To simplify notation, subscripts denoting patent, year, and firm are omitted.

$$\mathbb{E}[x|r] = \delta r + \sqrt{\delta}\sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)} = \delta r + \lambda(R, \delta, \sigma_\varepsilon) \tag{2}$$

where r is the market return corresponding to that patent, $R = -\sqrt{\delta}\frac{r}{\sigma_\varepsilon}$, $\delta = \frac{\sigma_x^2}{\sigma_\varepsilon^2 + \sigma_x^2}$, $\lambda(R, \delta, \sigma_\varepsilon) = \sqrt{\delta}\sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)}$. Equation 2 has two attractive features: (i) the estimated patent value is higher in the presence of higher stock returns at grant date (r_{jft}); and (ii) the relation between estimated patent value and stock returns at grant date is stronger when stock markets are more informative about patent value (namely they have larger δ_{ft}).

Implementing this formula requires researchers to estimate two parameters: δ_{ft} (the signal-to-noise ratio) and $\sigma^2_{\varepsilon ft}$ (the variance of returns due to other contemporaneous unobservable events, also called the noise). To economize on the number of parameters to be estimated, KPSS assume that the signal-to-noise ratio is constant over time and across firms, so that $\delta_{ft} = \delta$, estimated using the following regression:

$$ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma I_{fd} + \mu_{fd} \tag{3}$$

Where r_{fd} denotes the three-day idiosyncratic return for firm f starting on the day d, I_{fd} is an indicator that identifies days with patent grants, and a_{ft} and b_d indicate controls for firm-year and the day of the week fixed effects, respectively, to account for the time-varying nature of firm volatility and its seasonal fluctuations. The coefficient of interest is γ, which denotes the effects of a patent grant announcement on idiosyncratic stock return volatility and can be used to recover $\hat{\delta} = 1 - e^{-\hat{\gamma}}$ with some approximation. Finally, the authors estimate $\sigma_{\varepsilon ft}$ non-parametrically using the sum of squared market-adjusted returns.

3 Measurement error in KPSS estimates

In light of its popularity, it is important to examine the strengths and limitations of the KPSS measure for the private value of patents. We characterize the measurement error analytically in this Section, and discuss the implications of this measurement error when KPSS is used as dependent variable and explanatory variable in Sections 4 and 5, respectively.

We identify two properties of KPSS estimates that have important implications for empirical research. First, while KPSS estimates have the same mean as the underlying distribution of patent value x, the two distributions differ in both variance and skewness. By the law of total variance, the overall dispersion of patent values can be decomposed into the average dispersion of values conditional on the observed signal and the dispersion of the conditional expectation itself. Because the conditional variance is strictly positive whenever patent values are not perfectly revealed by the signal, the variance of the conditional expectation must be strictly smaller than the variance of true patent values. As a result, KPSS values, defined as conditional expectations, necessarily exhibit lower dispersion than the underlying distribution of patent values. A similar compression arises for higher moments, and leads to a distribution of KPSS values that is less right-skewed than the distribution of true patent values. Numerical simulations reported in Appendix A1.1 confirm that KPSS values have lower variance and reduced positive skewness relative to the underlying patent value distribution.[8]

This shrinkage also implies that measurement error in KPSS estimates covaries negatively with true patent value: KPSS will tend to undervalue more valuable patents and overvalue less valuable patents. The measurement error in the patent value is $\omega = \mathbb{E}[x|r] - x$. Thus, we can rewrite equation 2 as:

$$\begin{aligned} &\mathbb{E}[x|r] = r + (\lambda(R) - (1-\delta)r) = x + \epsilon + (\lambda(R) - (1-\delta)r) \\ &\implies \omega = \mathbb{E}[x|r] - x = \epsilon + (\lambda(R) - (1-\delta)r) \end{aligned} \tag{4}$$

The measurement error ω is equal to $\epsilon + (\lambda(R) - (1-\delta)r)$. By the law of iterated expectations, $\mathbb{E}_r[\mathbb{E}[x|r]] = E[x]$, which implies that $\mathbb{E}[\omega] = 0$. Further, by the monotone hazard rate property of the normal distribution, $\lambda(R)$ increases with R and therefore decreases with r.

[8]Taking expectations conditional on the signal smooths extreme realizations of patent value and attenuates the right tail that generates skewness. High-value patents are pulled toward the mean when mapped into KPSS values, while low-value realizations are correspondingly lifted.

Hence, $\lambda(R) - (1-\delta)r$ decreases with r. It follows that $\lambda(R) - (1-\delta)r$ decreases with x. This implies that the covariance between x and $\lambda(R) - (1-\delta)r$ is negative. Letting $\Lambda = \lambda(R) - (1-\delta)r$, the covariance between ω and x is given by $\text{Cov}(x, \omega) = \text{Cov}(x, e) + \text{Cov}(x, \Lambda) = \text{Cov}(x, \Lambda) < 0$. That is, measurement error in KPSS estimates (ω) covaries negatively with patent value (x).

The presence of measurement error introduces a trade-off between using the KPSS method or "unfiltered" stock returns to estimate patent value: the KPSS method filters out some of the noise in stock returns, thus allowing for more precise estimation, but comes at the cost of introducing non-classical measurement error. We evaluate this trade off below, where we explore the implications of the two properties for research using KPSS as an explanatory and outcome variable, respectively.

Before doing so, we note that KPSS entails additional assumption that affect patent value estimates. First, KPSS assumes that all patents issued to a firm on a given day are identical in value. More specifically, KPSS deal with the presence of multiple patents granted to a firm on the same day by first estimating the total value attributable to patent grant on a given day ($\mathbb{E}[x_j|r_j]$), and then by dividing it equally among all patents granted on that day. Alternatively, one could assume that all patents issued are identically (and independently) *distributed*, which then requires an adjustment when estimating the signal-to-noise ratio to account for the number of patents granted on a particular day. Our analysis in Appendix Section A2.1 shows that KPSS overestimates patent value when the number of patents granted on that day is low, while adjusting for the number of patents underestimates the patent value in the same scenario. This issue is investigated in greater depth in Federle, Harhoff, and Kreyer (2025). The authors apply a hedonic patent value regression to assign weights within weekly patent grant bundles using publicly known quality characteristics; these weights are then applied to allocate patent value among individual patents granted simultaneously to a firm.

Second, insofar as patent grants are partially anticipated by investors, using stock market responses to grants raises additional complications. Because the USPTO announces patent grants on Tuesdays, one needs additional assumptions on how investors update if there is no patent grant on a Tuesday. KPSS assumes that all patents share the same ex-ante probability that the patent application is successful, and that absence of a patent grant does not provide any additional information to investors. More specifically, KPSS assume that all firms have the same probability of award, p, so that the market return on the day of award is $r = (1-p)x + \epsilon$, and on days without patents is $r = \epsilon$. If researchers want to allow award probabilities to differ across firms, they can empirically estimate the grant probability p_f for firm f and, following KPSS, divide estimated values by $\frac{1}{(1-p_f)}$. For firms with few patent applications, researchers can group firms, or conduct sensitivity analyses. A similar approach may be used for differences in award probabilities across groups of patents for a given firm. Differences in grant probabilities is one, but not the only reason to be wary of comparing KPSS values across firms or industries. The KPSS assumption of a common δ implies that firms with higher volatility in stock returns i.e., a higher σ_ε, will mechanically have higher σ_x and therefore, more valuable patents. Both of these problems are likely to be less important when comparing patent values within a firm. We therefore caution against comparisons of patent values across firms or industries.

4 KPSS as outcome variable

Several papers use KPSS estimates as outcome variable to examine differences in value between patent groups. These studies generally aim to determine whether the expected value of patents varies with a particular characteristic, such as firm size, legal environment, or reliance on scientific knowledge. We show that comparing the means of different patent types ex-post after implementing the KPSS method, as done in those papers, poses both conceptual and empirical challenges. We then introduce and evaluate two solutions that overcome these challenges, and highlight their respective strengths and weaknesses.

4.1 Implications of measurement error in KPSS

The use of KPSS as an outcome variable to examine differences in value between patent groups presents a conceptual flaw: The KPSS method assumes that all patent types granted to the focal firm in a given year have a common signal-to-noise ratio (δ), which is equivalent to assuming that all patent *groups* granted to focal firm in a given year are drawn from the same distribution. In turn, this implies that all patent groups for a given firm in a given year have the same expected value, and any observable difference between their sample means observed ex post should reflect noise.

More formally, assume two groups of patents, S and N. The focal firm receives S patents of type S and N patents of type N throughout the year. The KPSS estimate of the i^{th} S type patent is $y_{is} = \delta r_{is} + \sqrt{\delta}\sigma_\varepsilon \lambda(R_{is})$, where $\lambda(R_{is}) = \dfrac{\phi(R_{is})}{1 - \Phi(R_{is})}$ and $R_{is} = -r_{is}\dfrac{\sqrt{\delta_s}}{\sigma_\varepsilon}$. The sample mean of all S type patents for that firm and year is:[9]

$$\bar{y}_s = \delta \frac{1}{S} \sum_{i=1}^{i=S} r_{is} + \sqrt{\delta}\sigma_\varepsilon \frac{1}{S} \sum_{i=1}^{i=S} \lambda(R_{is}) \tag{5}$$

Therefore, the sample difference between the mean patent value of groups S and N is:

$$\bar{y}_s - \bar{y}_n = \delta \left(\frac{1}{S} \sum_{i=1}^{i=S} r_{is} - \frac{1}{N} \sum_{i=1}^{i=N} r_{in} \right) + \sqrt{\delta}\sigma_\varepsilon \left(\frac{1}{S} \sum_{i=1}^{i=S} \lambda(R_{is}) - \frac{1}{N} \sum_{i=1}^{i=N} \lambda(R_{in}) \right) \tag{6}$$

Under the standard KPSS assumption that x_s and x_n have the same distribution, each bracketed term is the difference in the means of two samples from a common distribution. Thus, the market return to the granting of the two patent types, r_s and r_n, have the same distribution (and therefore $\lambda(R_{is})$ and $\lambda(R_{in})$ will also have identical distributions). The differences in sample means for samples drawn from the same distribution will approach zero as the sample size increases. So, for large samples, both bracketed terms in Equation 6 will vanish, and *there can be no systematic differences in the means for any two groups of patents of a given firm and year under the assumptions used to generate KPSS values.*

[9] In large samples, the sample mean converges to $\mathbb{E}_{r_s}\, \mathbb{E}(x_s|r_s) = \mathbb{E}(x_s)$

Therefore, there is an inherent contradiction in empirical strategy research that relies on KPSS as outcome variable because it investigates differences in value between patent groups while also assuming these differences do not exist.

If instead x_s and x_n *have different distributions*, KPSS suffers from two empirical problems. First, comparing the means of S and N patent types will underestimate the (absolute) difference in value between these patent types. Suppose that S type patents have a higher value than N type patents, i.e., $\sigma_s > \sigma_n$, but the researcher follows the standard KPSS approach and computes patent values assuming a single distribution. Because this approach imposes a single parameter δ_x, the researcher is implicitly imposing a common σ_x for all the patents granted to a given firm in a year. This parameter is the weighted mean of the true parameters $\delta_s \geq \delta_x \geq \delta_n$. As Appendix C1 formally shows, the sample mean for S type patents using patent values computed from KPSS will underestimate the true mean of S type patents and, conversely, the sample mean for N type patents will overestimate the true mean of N type patents.

Second, if x_s and x_n *have different distributions* and *both patent types can be granted on the same day*, the observed stock market response on that day arises from two separate patent values and two different signal-to-noise ratios. Thus, the observed return contains two signals and one noise component, and we aim to estimate $\mathbb{E}[x_{sj} \mid x_{sj} + x_{nj} + \varepsilon_j]$. As a consequence, equation 2 is not longer valid. This further causes KPSS to underestimate difference in group means.

In sum, if researchers are interested in testing for differences in the mean value of two groups of patents, comparing the means of different patent types ex-post after having implemented the KPSS method is conceptually inconsistent. Furthermore, if one does find systematic differences between types of patents (within a firm-year) in patent value, the estimated differences will understate the true differences, more so if patents from different groups are awarded on the same day.

4.2 Alternatives to KPSS for patent values as outcome variable

We discuss two alternatives that researchers can use to address some of the limitations in KPSS discussed above when using patent values as outcome variable: our generalized version of KPSS and "unfiltered" stock returns.

4.2.1 Generalized KPSS Method

We extend the KPSS estimation method to allow the value of two different patent types to be drawn from different distributions and accommodate for the possibility that both different patent types are granted on the same day.

As before, consider two patent types denoted by S and N. Assume that the values of these patents, x_{sj} and x_{nj}, are distributed according to a normal distribution truncated at zero as in KPSS but with different variances (σ^2_{xsft} and σ^2_{xnft}). Assume additionally (as in KPSS) that the value of other contemporaneous unobservable events is normally distributed with variance $\sigma_{\varepsilon ft}$, and that the three variances may vary across firms and over time, but in constant proportions. As discussed above, in the baseline KPSS setup with only one type of patent, the observed abnormal return for patent j is given by $r_j = x_{sj} + \varepsilon_j$, where x_{sj} is the true (unobserved) signal and ε_j is the noise. However, when there are two types of patents granted on the same day, the observed abnormal return becomes $r_j = x_{sj} + x_{nj} + \varepsilon_j$. In this case, the observed return contains two signals and one noise component, and we aim to estimate $\mathbb{E}[x_{sj} \mid x_{sj} + x_{nj} + \varepsilon_j]$, which invalidates equation 2.

We adopt the method from Papadopoulos (2015) and Papadopoulos (2021) and derive the adjusted formula for the value of S-type patents when different patent types are granted on the same day. A step-by-step derivation is available in to Appendix B1. The resulting equation (Appendix Equation 32) is a generalized version of Equation 2 from KPSS. Importantly, Appendix Equation 32 accommodates the presence of two signals on the same day, with both S and N type patents, and these signals are distributed differently, albeit independently.

Our approach also requires empirical counterparts for parameters related to signal-to-noise ratios, δ_s and δ_n, and for the noise variance, $\sigma_{\varepsilon ft}$. We recover the signal-to-noise ratios with the following equation:

$$ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_s I_{sfd} + \gamma_n I_{nfd} + \nu_{fd} \tag{7}$$

where r_{fd} denotes the three-day idiosyncratic return of firm f following the patent grant date d. I_{sfd} indicates announcement days that include S type patents, defined as a dummy variable set to one if at least one S type patent is granted on that day, zero otherwise; while I_{nfd} identifies announcement days that do not include S type patents, defined as a dummy variable set to one if at least one N type patent but no S type patent is granted on that day, zero otherwise. Estimated values of γ_s and γ_n can be used to recover estimates of δ_s and δ_n respectively. Finally, $\sigma_{\varepsilon ft}$ can be calculated as in KPSS with:

$$\sigma^2_{\varepsilon ft} + \mu_{sft}\sigma^2_{xsft}(1+l) + \mu_{nft}\sigma^2_{xnft}(1+l) = \sigma^2_{ft}(1+l) \tag{8}$$

μ_{sft} represents the share of days when S-type patents are granted and μ_{nft} represents the share of days when only N-type patents are granted in a given firm-year. σ^2_{ft} is the estimated realized mean idiosyncratic squared returns of firm f at time t, and l is the window of time in days over which the market reaction is calculated.[10]

We run a simulation analysis – discussed in Appendix D1 – to illustrate the differences between KPSS and our procedure. We simulate 5,000 firms, and for each firm, we generate daily data for 40 years, or 365 × 40 days. We first compute $\mathbb{E}[x_j|r_j]$ following the KPSS methodology, where we apply equation 3 to our simulated data to estimate the common signal-to-noise ratio. We also compute $\mathbb{E}[x_j|r_j]$ separately for S-type patents and N-type patents, where we estimate separate signal-to-noise ratios using equation 7 applied to our

[10]The term $1+l$ represents the time window (in days) over which the stock market's reaction is calculated, indicating how long it takes for the stock market to incorporate information about the patent's value into stock prices. In contrast, noise is continuously integrated into stock prices throughout the year.

data. The results of this simulation confirm our analytical predictions that measurement error in KPSS has the potential to affect inference when KPSS estimates are used as outcome variable to investigate differences in value between patent groups.

These simulations notwithstanding, our generalized KPSS method is not without limitations. First and foremost, our generalized KPSS approach is more complex to implement than the standard KPSS approach. For this reason, we offer researchers a detailed step-by-step guide on how implement this approach in Appendix B2, include an illustrative example of this implementation in Section 4.3 below, and make data on patent values by category for the categories we study in our manuscript and the code necessary to calculate these values available in our website.[11] Second, this method can only be implemented when comparing two patent groups or firm groups. Accordingly, researchers interested in using our methodology for continuous variables or more than two groups would have to reduce the dimensionality of their data by converting those variables into binary indicators. While this practice is commonly employed in the literature (for example, see pp. 4 and 5 of Higham, De Rassenfosse, and Jaffe (2021)), we acknowledge this is a limitation of our method. Third, application of this method produces patent values that are not invariant to the partitioning variable. As a consequence, the same patent will have different dollar value depending on the group categorization the researcher is interested in studying. The implication is that this method is likely to be most useful when the researcher is interested in comparing the mean value of two types of patents.

4.2.2 "Unfiltered" Stock Returns

The second alternative we consider is the use of "unfiltered" stock returns, namely returns before applying the Kogan et al. (2017) filtering to remove noise. Researchers can directly use r, the stock market idiosyncratic return corresponding to that patent, calculated as the firm's

[11]Detailed step-by-step code for implementing the baseline and generalized KPSS estimators, covering applications to team size, R&D offshoring, reliance on science, and vertical integration, is available at https://github.com/jaypnagar/ABFN_Relication.

return minus the return of the market portfolio (to remove broad stock market movements). To further isolate the patent value component from the unrelated news component in stock returns, researchers can (i) compare the idiosyncratic stock returns on days when the firm receives the award of a patent group to the same returns on days when the firm receives the award of a different patent group; and (ii) use the idiosyncratic stock returns on days when the firm does not receive any patent award as a benchmark to account for differences in the frequency and value implication of unrelated news across firms.[12]

This approach, while conceptually sound for testing differences in the mean value between two groups of patents, is not without limitations. The USPTO issues patents on Tuesdays (unless there is a federal holiday), and stock markets respond to the award of a patent over three days (see Figure 1 of Kogan et al. (2017)). Thus, finding an appropriate benchmark return in the near proximity to the patent award date is nearly impossible. Moreover, patent awards induce increased stock return volatility, which introduces challenges for statistical testing (e.g., Kolari and Pynnönen (2010)). Finally, "unfiltered" returns approach will assign the same value to different patent types granted on the same day.

4.3 Applications

We have established that using KPSS to investigate differences in the private value of patents between patent groups is internally inconsistent and presents empirical challenges, and have offered two possible alternatives. However, it remains unclear whether and to what extent our findings "matter" for empirical organizational research. In this section, we show that our findings matter for the literature investigating differences in the private value of patents between groups of patents based on innovation team characteristics (team size and composition), patent characteristics (science-based versus non-science-based patents), and organi-

[12]The KPSS estimation applies adjustments for the number of patents granted ($\frac{1}{K_{ft}}$), firm size (S), and the ex-ante probability that the patent application is successful ($(1-\pi)^{-1}$. We do not apply this adjustment to "unfiltered" stock returns. Instead, we control for the number of patents granted and firm size in our regression specification. We do not adjust for grant probability because this adjustment does not extend to days without a patent award. Table F5 of Appendix F shows that our inference is robust to this adjustment.

zation characteristics (vertical integration). All of our analyses follow the same structure. We start by estimating patent value using the standard KPSS method and our generalized KPSS method, and compare the estimates using both univariate and multivariate analysis. Finally, we test for the difference in private value between patent groups using "unfiltered" returns.

Overall, the different methods produce consistent evidence that patents awarded to large teams, to vertically integrated firms, and science-based patents have higher private value, confirming previous findings in the literature. At the same time, the different methods yield mixed evidence regarding the value implications of R&D offshoring, suggesting that previous findings may be method-sensitive. Moreover, we find that unfiltered returns yield larger differences in group means than our generalized version of KPSS, which in turn yields larger differences in group means than KPSS, consistent with KPSS suffering from attenuation bias.

We would like to emphasize that, similar to Gibbs et al. (2025), the goal of our analyses in this section is not to replicate previous work (Bettis, Helfat, & Shaver, 2016). Rather, our goal is to show that performing the same analysis on the same data but using different stock-market-based approaches to estimate patent value can affect inference. Nonetheless, because our analyses are close to prior work, our evidence can offer new insights into the determinants of patent value.

4.3.1 Sample Construction and Description

To perform our analyses, we assemble a dataset of U.S. Patent and Trademark Office (USPTO) patents assigned to U.S.-based firms between 1980 and 2019, restricting our sample to 2019 to avoid COVID-19 disruptions. This dataset is constructed by merging various sources. The starting point is the DISCERN 2.0 dataset, which connects all USPTO patents granted to Compustat firms (Arora, Belenzon, & Sheer, 2021) and provides significant enhancements and expansions to the historical NBER patent dataset.

We complement this database by gathering data on the patent filing date, International Patent Classification (IPC) classes, patent applicants, inventors, and other relevant details from the USPTO patent database and the European Patent Office (EPO) PATSTAT database. PATSTAT data are also used to construct patent family measures, including indicators for triadic patent families. We further incorporate forward citation counts, the number of claims, patent family size, and related patent quality indicators from the OECD Patent Quality Indicators database (Squicciarini, Dernis, & Criscuolo, 2013). We incorporate firm–year level measures of vertical integration (Frésard, Hoberg, & Phillips, 2020), patent reassignment data from the USPTO patent assignment dataset (Graham, Marco, & Myers, 2018), and patent litigation indicators (Toole, Miller, & Sichelman, 2024).

Our sample consists of the patents granted by the United States Patent and Trademark Office (USPTO) from 1980 to 2019. To focus on the innovating firms, we restrict the analysis to a subset of 4,977 firms that were assigned at least one patent during the sample period. Our final sample comprises 1,444,649 patents, assigned to publicly traded firms headquartered in the United States. We also merge this sample with a database of daily stock returns that we use to estimate the signal-to-noise ratio, necessary for the standard and generalized KPSS methods, and to perform analyses using "unfiltered" stock returns.

4.3.2 Patent Private Value and Inventor Team Size

A substantial body of research highlights the important role of team size in shaping the outcomes of innovative work (Wuchty, Jones, & Uzzi, 2007). Larger teams may signal greater organizational commitment of resources toward promising projects, resulting in higher-quality outcomes (Breitzman & Thomas, 2015; Wu, Wang, & Evans, 2019; Wuchty et al., 2007). Collaboration among inventors—particularly within organizations—not only reduces the risk of failure but also enhances the chances of producing high-impact innovations. A key finding across these studies is that larger teams tend to produce innovations with higher impact. We revisit this evidence by analyzing how team size relates to the private value of patents, estimated using the three approaches laid out above.

We define a patent as having a large team if its number of inventors falls within the top 25% of the team size distribution among all patents granted in the same year and 4-digit IPC class; we show in Appendix F that our inference is robust to alternative definitions. Table 2 shows that 17.9% of the patents are awarded to a large team, while the remaining patents are awarded to small teams. Conditional on patent activity, 68.1% of days feature only small-team patents, 9.4% feature only large-team patents, and 22.5% feature a mix of large- and small-team patents.

We start by estimating the private value of patents using the standard KPSS method described in Section 2.2, which entails the following steps. First, we estimate the signal-to-noise ratio. As shown in Column (1) of Table 3, Equation 3 estimated on our sample produces a value of of $\hat{\gamma}$ equal to 0.019, which is higher than the value of 0.015 reported in Kogan et al. (2017).[13] Second, we use this parameter estimate together with Equations 1 and 2 to calculate the private value of patents. Table 4 Panel A shows that patent value amounts to \$16.117 million, on average, with a standard deviation of \$26.003 million. Patents granted to larger teams are more valuable: the average patent is worth \$17.710 million for large teams and \$15.770 million for small teams, and the difference of \$1.941 million (12% relative to the value of patents awarded to small teams) is statistically significant.

Next, we estimate patent values using our generalized KPSS estimation method. First, we estimate separate signal-to noise-ratios for patents awarded to large and small teams by estimating Equation 7. Column (2) of Table 3 displays $\gamma_{\hat{Large}} = 0.025$ for patents awarded to large teams and $\gamma_{No\hat{Large}} = 0.017$ for patents awarded to small teams, suggesting that the assumption of equal distribution for patents granted to large and small teams is unwarranted. Second, we use this parameter estimate together with methodology described in Section 4.2.1 to calculate the private value of patents. Table 4 Panel A shows that patent value amounts

[13] This difference likely arises because our sample period (1980–2019) is shorter and more recent compared to theirs (1926–2010), and also because we use DISCERN, which matches patents to Compustat firms more accurately than the NBER patent file.

to \$15.548 million, on average, with a standard deviation of \$25.257 million, similar to the values obtained with the standard KPSS method. Importantly, we find that the difference in value between patents granted to large and small teams increases to \$5.102 million (35% relative to the value of patents awarded to small teams), consistent with measurement error in KPSS inducing an attenuation bias.

We accompany these univariate analyses with multivariate tests. Our regression specification is as follows:

$$Y_{ijkt} = \beta_0 + \beta_1(\mathbb{1}[TeamSize : Large]_{it}) + \eta X_{it} + \alpha_t + \delta_j + \psi_k + \epsilon_i \tag{9}$$

Y_{ijkt} is the log-transformed dollar value of patent i belonging to the technology class j (defined by IPC 4-digit classification), granted in year t, and owned by firm k. $\mathbb{1}[TeamSize : Large]_{it})$ is a dummy variable equal to 1 if the patent has large team. We estimate increasingly stringent specifications that include patent IPC class and year fixed effects, a control for firm market capitalization, and firm fixed effects.

When we measure patent value using the standard KPSS method (Column (1) of Table 5, we find that patents awarded to large teams are 2.53% more valuable than patents awarded to small teams.[14] When we replace estimates based on the standard KPSS method with estimates from our generalized method (Column (2) of Table 5), we observe that patents awarded to large team are 26.1% more valuable than patent awarded to small teams. Thus, measurement error in KPSS introduces severe attenuation bias: the difference between the two estimates is 0.207 (Column (3)), which indicates that the estimated effect is 9.3 times larger when using our generalized estimation method.

Lastly, we repeat the exercise in which we measure patent value using "unfiltered" stock returns. To reduce the confounding effect of noise (namely, the value implications of unrelated news), we (i) restrict the sample to firms that have both patent and non-patent days in the estimation period, and (ii) compare idiosyncratic returns during weeks when a

[14]We report the full tables from these estimations in Appendix E1.

firm receives a patent (Tuesday to Thursday, the period over which stock markets respond to patent awards as in Figure 1 of Kogan et al. (2017)) to the idiosyncratic returns during weeks when the same firm does not receive a patent. We then regress idiosyncratic returns on indicators for whether any patent is awarded to the firm on that day and whether any large-team patent is awarded to the firm on that day. We include in the specification controls for market value (to account for differences in firm size) and for the number of patents awarded on any given day (to account for any difference in the average number of patents awarded to large versus small teams on any given day), in addition to firm and date fixed effects. We report our coefficient estimates in Column (1) of Table 6. We find that days when patents are awarded to small teams are associated with a positive and statistically significant stock market response, indicating that patents, on average, create private value for firms. Importantly, we also observe that days when patents are awarded to larger teams are associated with an incrementally positive stock market response, indicating that patents awarded to large teams are, on average, more valuable. The difference is economically meaningful, as it suggests that the stock market response to a patent award is 67% larger (0.0166/0.0250) for patents awarded to large teams.

Taken together, these findings show that (i) patents from large inventor teams carry a clear premium across methodologies even when using two conceptually more attractive alternatives to KPSS; (ii) the KPSS method underestimates this value gap by a large margin.

4.3.3 Patent Private Value and R&D offshoring

Firms are increasingly outsourcing their R&D work overseas. For instance, the share of USPTO patents invented by overseas-based inventors more than doubled from 8% in 1980 to 19% in 2020 (Sebastian, 2025). The coordination of overseas operations in multinational firms can be fraught with frictions, which may result in the loss of valuable information and reduce the realized value of a firm's inventions (Teece, 1977, 1986). Furthermore, these operations entail higher risk of leakage of intellectual property, particularly in countries with weak intellectual property rights (IPR) (Nandkumar & Srikanth, 2016; Zhao, 2006). As a consequence, inventions originating in weak IPR regimes could be of lower value because firms cannot fully appropriate the returns from their innovations (Zhao, 2006). We revisit evidence that the private value of patents invented in the U.S. is higher than the private value of patents invented abroad.

For this purpose, we classify a patent as invented abroad if all listed inventors are located outside the United States; we show in Appendix F that our inference is robust to alternative definitions. This classification is based on country-level inventor information from the EPO PatStat database, which covers all patents granted to firms in the DISCERN dataset. Table 2 shows that 11.0% of the patents are awarded to teams made of all inventors located outside of the U.S., while the remaining patents are awarded to teams with at least one U.S. inventor. Conditional on patent activity, 81.6% of days feature patents awarded only to teams with at least one U.S.-based inventor, 4.9% of days feature patents awarded exclusively to teams composed entirely of non-U.S. inventors, and 13.5% of days feature a mix of fully-abroad and non-fully-abroad inventor teams.

The analysis follows the same structure we used for team size. We start with KPSS-based patent value estimates, which indicate that patents from foreign inventors are less valuable, with an average value of \$13.097 million compared to \$16.489 million for patents from domestic inventors. Next, we estimate patent values using our generalized KPSS estimation method. Column (3) of Table 3 displays $\hat{\gamma_{Foreign}} = 0.015$ for patents awarded to teams made of all inventors located outside of the U.S. and $\hat{\gamma_{NoForeign}} = 0.020$ for patents awarded to to teams made of at least one inventor located in the U.S., suggesting that the assumption of equal distribution is unwarranted. Table 4 Panel B shows that when we use our generalized KPSS method, the difference in value between patent groups increases to \$4.980 million, consistent with measurement error in KPSS inducing an attenuation bias.

Multivariate analysis using the standard KPSS method (Column (1) of Table 5) suggests that patents awarded to foreign teams are 0.8% less valuable than patents awarded to non-foreign teams.[15] When we replace estimates based on the standard KPSS method with estimates from our generalized method (Column (2) of Table 5), we observe that patents awarded to foreign teams are 13% less valuable than patents awarded to non-foreign teams. Thus, measurement error in KPSS introduces extreme attenuation bias: the difference between the two estimates is 0.132 (Column (3)), which indicates that the estimated effect is 17.5 times larger when using our generalized estimation method.

Inference changes when we rely on "unfiltered" stock returns. Column (2) of Table 6 shows that patent award days are associated with a positive and statistically significant stock market response. However, days when patents are awarded to teams of foreign-only inventors are not associated with a differential stock market response, indicating that patents awarded to foreign and non-foreign inventors have similar value implications.

Overall, these results suggest that the evidence that patents with only foreign inventors are less valuable than those with domestic inventors is sensitive to the methodology employed.

4.3.4 Patent Private Value and Patent Reliance on Basic Science

Corporate innovation increasingly relies on science (Marx & Fuegi, 2020). Science guides the creative process, reducing exploration costs, eliminating unproductive avenues, and expediting the invention timeline (Ahmadpoor & Jones, 2017; Arora, Belenzon, Kosenko, Suh, & Yafeh, 2024; Kline & Rosenberg, 1986; Rosenberg, 1990; Sorenson & Fleming, 2004). Because scientific patents are technically more important, as measured in terms of forward citations (Arora, Belenzon, & Dionisi, 2023), and more novel, as indicated by the use of new words (Ahmadpoor & Jones, 2017), than non-scientific patents, researchers have empirically explored whether the private value of scientific patents is higher than that of non-scientific patents (e.g., Arora, Belenzon, & Dionisi, 2023; Krieger, Schnitzer, & Watzinger, 2024). We revisit whether scientific patents are more valuable than non scientific patents.

For this purpose, we link the DISCERN 2.0 database with the "Reliance on Science"

[15] We report the full tables from these estimations in Appendix E2.

dataset, which includes all patent citations to scientific publications (Marx & Fuegi, 2020).[16] We complement this data by sourcing information on the scientific articles, including publication year, journal details and publication-publication citations, from Microsoft Academic (Sinha et al., 2015), a large-scale scholarly database, and OpenAlex (Priem, Piwowar, & Orr, 2022), an open-access dataset that builds on the Microsoft Academic Graph (MAG) and contains scholarly publications such as journal articles, conference papers, books and book chapters. We classify patents as science-based and non-science-based by relying on non-patent literature (NPL) citations in patent documents. Over the years, both the total number of patents and the proportion of patents citing at least one Non-Patent Literature (NPL) have grown significantly. Accordingly, we define "science-based patents" as those in the top three quartiles for the number of NPL citations to science within a given IPC class (at 4-digit level) and year, provided they cite at least one NPL; we show in Appendix F that our inference is robust to alternative definitions. Table 2 shows that 25.0% of awarded patents are science-based, while the remaining patents are not. Conditional on patent activity, 62.1% of days feature only non-science patents, 15.8% of days feature only science patents, and 22.1% of days feature a mix of science and non-science patents.

The analysis follows the same structure we used before. We start with KPSS-based patent value estimates, which indicate that science-based patents are more valuable, with an average value of $19.024 million compared to $15.197 million for non-science patents. Next, we estimate patent values using our generalized KPSS estimation method. Column (4) of Table 3 displays $\gamma_{Sci\hat{e}nce} = 0.022$ for science-based patents and $\gamma_{Non\hat{S}cience} = 0.017$ for non-science-based patents, suggesting that the assumption of equal distribution is unwarranted. Table 4 Panel C shows that when we use our generalized KPSS method, the difference in value between patent groups increases to $5.748 million, consistent with measurement error in KPSS inducing an attenuation bias.

Multivariate analysis using the standard KPSS method (Column (1) of Table 5) suggests

[16]https://zenodo.org/records/8278104.

that science-based patents are 1.5% more valuable than non-science-based patents.[17] When we replace estimates based on the standard KPSS method with estimates from our generalized method (Column (2) of Table 5), science-based patents are 15.0% more valuable than non-science-based patents. Thus, measurement error in KPSS introduces severe attenuation bias: the difference between the two estimates is 0.132 (Column (3)), which indicates that the estimated effect is 9.3 times larger when using our generalized estimation method.

We obtain the same inference with "unfiltered" stock returns. Column (3) of Table 6 shows that the award of non-science-based patents is associated with a positive and statistically significant stock market response. This response is twice as large for science-based patents, indicating that science-based patents are more valuable than non-science-based ones.

4.3.5 Patent Private Value and Vertical Integration

In a seminal study, Armour and Teece (1980) examine the relationship between technological innovation and vertical integration. The authors argue that vertical integration facilitates the sharing of technological information across different stages of production, enhances the implementation of new technologies, and leads to more informed research objectives. Their empirical investigation documents a positive correlation between research and development (R&D) investments and vertical integration, suggesting that integrated firms are better positioned to innovate. Later research confirms and expands on this finding (Frésard et al., 2020; Helfat & Campo-Rembado, 2016; Zhang & Tong, 2021). We re-investigate whether vertically integrated firms have a higher private value of patented inventions compared to non-vertically integrated firms.

For this purpose, we use the vertical integration measure developed by Frésard et al. (2020). This measure is derived from the textual analysis of firms' annual 10-K filings and their alignment with product descriptions from the Bureau of Economic Analysis (BEA) Input-Output tables. This methodology offers a refined, firm-level measure of vertical in-

[17] We report the full tables from these estimations in Appendix E3.

tegration that effectively identifies supply chain relationships beyond conventional industry classifications. Using this measure, we categorize firms into highly and not highly vertically integrated groups based on whether their vertical integration score falls in the top quartile for each industry-year; we show in Appendix F that our inference is robust to alternative definitions.[18]

As before, we start with KPSS-based patent value estimates, which indicate that patents awarded to vertically integrated firms are more valuable, with an average value of $19.695 million compared to $17.819 million for patents awarded to non-vertically integrated firms. Next, we estimate patent values using our generalized KPSS estimation method. Column (5) of Table 3 displays $\gamma_{V\hat{IFirm}} = 0.037$ for patents awarded to vertically integrated firms and $\gamma_{NoV\hat{IFirm}} = 0.016$ for patents awarded to non-vertically integrated firms, suggesting that the assumption of equal distribution is unwarranted. Table 4 Panel D shows that when we use our generalized KPSS method, the difference in value between patent groups increases to $6.786 million, consistent with measurement error in KPSS inducing an attenuation bias.

Multivariate analysis using the standard KPSS method (Column (1) of Table 5) suggests patents awarded to vertically integrated firms are 0.6% more valuable than patents awarded to non-vertically integrated firms.[19] When we replace estimates based on the standard KPSS method with estimates from our generalized method (Column (2) of Table 5), we observe that patents awarded to vertically integrated firms are 13.9% more valuable than patents awarded to non-vertically integrated firms. Thus, measurement error in KPSS introduces extreme attenuation bias: the difference between the two estimates is 0.124 (Column (3)), which indicates that the estimated effect is 21.7 times larger when using our generalized estimation method.

[18] The reader may wonder whether there is enough within-firm variation in vertical integration to include fixed effects in our analyses. Our investigation indicates that this is the case: firm fixed effects absorb only 40% of the total variation in vertical integration, leaving considerable variation in this construct. With firm fixed effects, the coefficient on $\mathbb{1}$[Vertical Integration: High] is identified solely from within-firm variation over time. In the estimation sample, 932 of 4,240 firms (22.0%) experience at least one change in vertical integration status and thus contribute to identification, accounting for 721,927 patent observations (55.7%).

[19] We report the full tables from these estimations in Appendix E4.

We obtain the same inference with "unfiltered" stock returns. Column (4) of Table 6 shows that the award of patents to non-vertically integrated firms is associated with a positive and statistically significant stock market response. This response is 158% (0.0179/0.0113) larger for patents awarded to vertically integrated firms, indicating that patents awarded to vertically integrated firms are dramatically more valuable than patents awarded to non-vertically integrated ones.

5 KPSS as explanatory variable

Other research has used KPSS estimates as an explanatory variable, either as the variable of interest or as a control. These studies generally aim to determine whether and how the expected value of patents correlates with various future outcomes at the patent, firm, industry or economy level (as in Kogan et al. (2017)). We show that using KPSS estimates as an explanatory variable yields consistent estimates. Our generalized KPSS approach yields limited benefits, while "unfiltered" stock returns are too noisy, leading to coefficient estimates biased towards zero.

5.1 Implications of measurement error in KPSS

Consider a regression with patent value as an explanatory variable.

$$Y = \alpha + \beta x + \eta \tag{10}$$

Using KPSS values as the proxy for the patent value implies replacing x with $\mathbb{E}(x|r)$. Equation 10 can be rewritten as

$$\begin{aligned} Y &= \alpha + \beta(\mathbb{E}(x|r) + \omega) + \eta \\ &= \alpha + \beta\mathbb{E}(x|r) + (\eta + \beta\omega) \\ &= \alpha + \beta\mathbb{E}(x|r) + (\nu), \text{ where } \nu = \eta + \beta\omega \end{aligned} \tag{11}$$

The OLS estimate of β, represented by b_{OLS}, is unbiased if $\text{Cov}(\mathbb{E}(x|r), \nu) = 0$, which is indeed the case. Formally, $\text{Cov}(\mathbb{E}(x|r), \nu) = \text{Cov}(\mathbb{E}(x|r), \eta) + \text{Cov}(\mathbb{E}(x|r), \beta\omega)$. Further, $\text{Cov}(\mathbb{E}(x|r), \eta) = 0$ because η is independent of x and ϵ by assumption, and hence, independent of $\mathbb{E}(x|r)$. Moreover, $cov(\mathbb{E}(x|r), \beta\omega) = 0$ because ω is orthogonal to $\mathbb{E}(x|r)$ by construction. Though unbiased, the OLS standard errors understate the true standard errors because ν has a larger variance than η and because $\mathbb{E}(x|r)$ has a compressed distribution compared to x. Therefore, using KPSS values as explanatory variables in regressions is viable as long as researchers adjust the standard errors, for instance by bootstrapping.

To summarize, stock returns are an inherently noisy signal of patent value. Using them as proxies for patent value as an explanatory variable leads to classical attenuation bias towards zero in OLS estimates. KPSS values are constructed by filtering stock market reactions to patent grant announcements. This filtering compresses the variance of the resulting value estimates. Despite this, when used as a right-hand-side variable, KPSS-based patent values yield unbiased estimates, albeit the standard errors are too low and have to be adjusted or estimated using other means such as bootstrapping. If, however, the KPSS assumptions (such as all patents for a given firm-year are drawn from the same distribution) are violated, KPSS estimates may also suffer attenuation bias as well as having incorrect standard errors.

5.2 Applications

In this section, we compare the relationship between alternative patent value measures and a broad set of future patent-level outcomes, and report our findings in Table 7.[20] We estimate the following regression:

$$Y_i = \alpha + \beta\, PatVal_i + \gamma \log(MktCap_{f(i),t(i)-1}) + \alpha_{f(i)} + \delta_{t(i)} + \varepsilon_i, \tag{12}$$

where Y_i denotes a future patent-level outcome, $PatVal_i$ is a standardized patent value measure, $\alpha_{f(i)}$ and $\delta_{t(i)}$ denote firm and year fixed effects, respectively.

[20] Detailed regression results for each outcome are reported in Appendix E.

Our future patent-level outcome variables include patent forward citations, measured as citations received within five years of grant; an indicator for top 1 percent cited patents, defined relative to patents granted in the same year and IPC class; renewal through the full statutory term based on observed maintenance fee payments; and indicators for reassignment, membership in trilateral patent families with filings at the USPTO, EPO, and JPO, and involvement in patent litigation. All of these are widely interpreted as markers of economic and strategic importance. We measure patent value using unfiltered returns, KPSS, and our generalized KPSS approach for different partitioning variables. We further include firm and year fixed effects, and estimate bootstrapped standard errors.[21]

Across all outcomes, coefficients obtained using unfiltered stock market reactions are consistently the smallest in magnitude and estimated imprecisely in most cases, reflecting the substantial noise in "unfiltered" stock returns. KPSS-based estimates, by contrast, are both larger in magnitude and more precisely estimated. For example, the association between patent value and forward citations is 0.12 (significant at the 10% level) when using unfiltered returns, but 0.33 (significant at the 1% level) when using KPSS-based estimates. The same pattern appears for top 1 percent cited patent status, full term renewal, reassignment, trilateral filings, and litigation, outcomes. We also observe that the generalized KPSS approach yields coefficient estimates that can be both larger or smaller than KPSS estimates depending on the partitioning variable selected, but always precisely estimated.

Taken together, the evidence in Table 7 confirms the analytical results derived above. When KPSS values are used as explanatory variables, the resulting coefficients are unbiased estimates of the true relationship between patent value and subsequent outcomes. Unfiltered stock market returns, though also unbiased, may be too noisy, leading to attenuation bias and lack of statistical precision. Overall, researchers interested in correlating patent value with future patent level outcomes can continue to rely on KPSS estimates, provided they

[21] In these analyses, we do not include IPC fixed effects because when we do, the variance-covariance matrix does not allow us to estimate bootstrapped standard errors. Nonetheless, we find a similar inference if we demean all variables by IPC class before estimating our regressions with bootstrapped standard errors. These analyses are reported in Table E11 to E16.

accept the KPSS assumptions and adjust the standard errors.

6 Discussion and Conclusion

Estimating the private economic value of patents is important to investigate important questions in organizational scholarship, yet challenging. By developing a methodology that uses stock market returns to produce a distribution of patent values, rather than just an estimate of the mean of that distribution, Kogan et al. (2017) has advanced the field considerably and opened multiple avenues for new research.

We characterize the properties of KPSS estimates, show that these properties have important implications for empirical strategy research, and introduce alternative stock-return based methods to study the private value of patents. Our analyses produce the following insights. First, researchers interested in studying differences in value between patent groups *should not* use KPSS estimates because doing so is internally inconsistent. In these cases, researchers should instead use unfiltered stock returns and our generalized KPSS approach (particularly when most days feature the award of both patent groups). Each of these two methods presents distinctive strengths and weaknesses, so finding consistent evidence across methods would produce the strongest inference. Second, researchers interested in studying the correlation between patent private value and future patent-level outcomes can continue using KPSS estimates (but must adjust standard errors through, for example, bootstrapping). In these cases, stock prices may be too noisy, so filtering out such noise is necessary to produce precise estimates of the correlations of interest.

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Table 1: Practical Recommendations to Researchers

Panel A: studies on the difference in value between patent groups

Estimation method	Strengths	Weaknesses	Recommendation
KPSS	Removes noise	Internally inconsistent Attenuation bias No concurrent award of different patent types	Do not use
Generalized KPSS	Removes noise Internally consistent Concurrent award of different patent types	Complex	Use
“Unfiltered” stock returns	Internally consistent Simple	Noisy No concurrent award of different patent types	Use

Panel B: studies on the correlation between patent value and future patent-level outcomes

Estimation method	Strengths	Weaknesses	Recommendation
KPSS	Removes noise	Underestimate SEs	Use
Generalized KPSS	Removes noise	Complex	Targeted use
“Unfiltered” stock returns	Noisy	Simple	Targeted use

Table 2: Summary Statistics: Patent Level

	Obs.	Mean	Std. Dev.	10%	50%	90%
$\mathbb{1}$[Team Size: Large]	1444649	0.179	0.383	0.000	0.000	1.000
$\mathbb{1}$[All Foreign Inventor]	1444649	0.110	0.312	0.000	0.000	1.000
$\mathbb{1}$[Patent Science Dummy]	1444649	0.240	0.427	0.000	0.000	1.000
$\mathbb{1}$[Vertical Integration: High]	1295446	0.285	0.452	0.000	0.000	1.000
Patent forward cites	1444649	14.406	50.331	0.000	5.000	28.000
$\mathbb{1}$[Top 1% of Cited Patents]	1444649	1.284	11.257	0.000	0.000	0.000
$\mathbb{1}$[Renewed (Full Term)]	774499	0.540	0.498	0.000	1.000	1.000
$\mathbb{1}$[Reassignment]	1445143	0.074	0.262	0.000	0.000	0.000
$\mathbb{1}$[Trilateral Patent]	1444649	0.209	0.407	0.000	0.000	1.000
$\mathbb{1}$[Litigation]	1445143	0.590	7.660	0.000	0.000	0.000

Note: $\mathbb{1}$[Team Size: Large] equals 1 if the inventor team size is in the top 25% of the team-size distribution within the same IPC class-year. $\mathbb{1}$[All Foreign Inventor] equals 1 if all inventors listed on the patent are non-U.S.-based. $\mathbb{1}$[Patent Science Dummy] equals 1 if the patent is science-based. We define science-based patents as those in the top three quartiles of non-patent literature (NPL) citations within an IPC class-year, conditional on citing at least one NPL. $\mathbb{1}$[Vertical Integration: High] equals 1 for assignees in the top 25% of vertical integration within their industry-year, based on the Frésard, Hoberg, and Phillips (2020) dataset. *Patent forward cites* measures the number of forward citations received by the patent over a five-year period. $\mathbb{1}$[Top 1% of Cited Patents] equals 1 if a patent belongs to the top 1 percent of the forward-citation distribution among patents granted in the same year and IPC class, and 0 otherwise; this variable is multiplied by 100 for ease of interpretation, as sourced from Squicciarini, Dernis, and Criscuolo (2013). $\mathbb{1}$[Renewed (Full Term)] equals 1 if the patent is renewed through the full statutory term, and 0 otherwise, based on observed maintenance fee payments. $\mathbb{1}$[Reassignment] equals 1 if the patent experiences at least one ownership reassignment during its lifetime. $\mathbb{1}$[Trilateral Patent] equals 1 if the patent belongs to a triadic patent family with filings at the USPTO, EPO, and JPO. $\mathbb{1}$[Litigation] equals 1 if the patent is involved in at least one patent litigation event (Toole, Miller, & Sichelman, 2024), this variable is multiplied by 100 for ease of interpretation.

Table 3: Signal to Noise ratio

	Baseline	Team Size	R&D Offshoring	Science	Vertical Integration	
	(1)	(2)	(3)	(4)	(5)	(6)
Patent day	0.019***				0.023***	
	(0.005)				(0.005)	
Patent day: Large Team Pat		0.025***				
		(0.008)				
Patent day: No Large Team Pat		0.017***				
		(0.006)				
Patent day: Foreign Invt Pat			0.015			
			(0.010)			
Patent day: No Foreign Invt Pat			0.020***			
			(0.005)			
Patent day: Science Pat				0.022***		
				(0.006)		
Patent day: No Science Pat				0.017**		
				(0.006)		
Patent day: High VI Firm						0.037***
						(0.008)
Patent day: Low VI Firm						0.016**
						(0.006)
Constant	-7.622***	-7.622***	-7.622***	-7.622***	-7.561***	-7.561***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Firm*Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Date FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.169	0.169	0.169	0.169	0.177	0.177
N	16,637,718	16,637,718	16,637,718	16,637,718	13,277,826	13,277,826

Notes: This table reports estimates from regressions of firm-level abnormal stock returns around patent grant dates. The dependent variable is the three-day idiosyncratic return of firm f following patent grant date d, denoted r_{fd}. All specifications include firm-by-year fixed effects and grant-date fixed effects. Column (1) and (5) implements the baseline KPSS specification, which estimates a single patent-day effect and implicitly assumes a common signal-to-noise ratio across patent types. Columns (2), (3), (4), and (6) relax this restriction by allowing patent-day effects to vary across dimensions of heterogeneity: inventor team size, R&D offshoring, reliance on basic science, and vertical integration. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 4: Patent-Level Private Value: Group Mean Comparisons

	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**	**Difference**
Panel A: Team Size	(Full Sample)		(Large Team)		(Small Team)		(Diff.)
Pat Val: KPSS (const. usd)	16.117	26.003	17.710	27.902	15.770	25.557	1.941***
Pat Val: Teamsize (mixed)	15.548	25.257	19.737	29.748	14.634	24.070	5.102***
Abnormal return on day $t(0-2)$	0.053	3.225	0.056	3.142	0.053	3.242	0.003
Observations			258,679		1,185,970		1,444,649
Panel B: R&D Offshoring	(Full Sample)		(All Foreign Inventors)		(Domestic Inventors)		(Diff.)
Pat Val: KPSS (const. usd)	16.117	26.003	13.097	22.589	16.489	26.368	-3.392***
Pat Val: Foreign (mixed)	16.207	26.214	11.772	20.969	16.752	26.737	-4.980***
Abnormal return on day $t(0-2)$	0.053	3.225	0.048	2.921	0.054	3.260	-0.006
Observations			158,194		1,286,455		1,444,649
Panel C: Reliance on Science	(Full Sample)		(Science-Based Patents)		(Other Patents)		(Diff.)
Pat Val: KPSS (const. usd)	16.117	26.003	19.024	30.358	15.197	24.391	3.827***
Pat Val: Sci&NoSci (mixed)	15.836	25.788	20.202	31.548	14.453	23.505	5.748***
Abnormal return on day $t(0-2)$	0.053	3.225	0.056	3.307	0.052	3.198	0.003
Observations			347,403		1,097,246		1,444,649
Panel D: Vertical Integration	(Full Sample)		(High Vertical Integration)		(Low Vertical Integration)		(Diff.)
Pat Val: KPSS (VI)	18.355	29.868	19.695	31.113	17.819	29.338	1.876***
Pat Val: Vertical Integration (sep)	19.941	33.060	24.789	37.763	18.004	30.770	6.786***
Abnormal return on day $t(0-2)$	0.053	3.225	0.010	2.927	0.080	3.375	-0.069***
Observations			369,838		925,608		1,295,446

This table compares average patent value across four dimensions of heterogeneity: inventor team size (Panel A), R&D offshoring (Panel B), reliance on basic science (Panel C), and vertical integration (Panel D). For each dimension, we report the mean and standard deviation of patent value for the full sample and for the relevant subgroups.Patent value is measured using the baseline KPSS method and generalized KPSS estimators that allow for group-specific signal-to-noise ratios and explicitly account for multiple patent types granted on the same day (mixed days). The final column reports the difference in mean patent value between the two subgroups within each panel. All variable definitions and sample construction follow Table 2. Patent values are expressed in constant million U.S. dollars. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 5: Patent-Level Regression: Coefficient on Patent Indicator Dummy Variables

Indicators	(1) Baseline KPSS	(2) Generalised KPSS	(3) Difference	(4) Ratio
$\mathbb{1}$[Team Size: Large]	0.025***	0.232***	0.207	9.280
	(0.001)	(0.001)		
$\mathbb{1}$[All Foreign Inventor]	-0.008***	-0.140***	-0.132	17.500
	(0.002)	(0.002)		
$\mathbb{1}$[Patent Science Dummy]	0.015***	0.140***	0.125	9.333
	(0.001)	(0.001)		
$\mathbb{1}$[Vertical Integration: High]	0.006 ***	0.130***	0.124	21.667
	(0.002)	(0.002)		

Notes: Column (1) reports coefficients using Baseline KPSS assuming a common signal-to-noise ratio across two type of patents. Column (2) uses the Generalised KPSS with heterogeneous signal-to-noise ratios and a modified valuation formula. Column (3) reports the difference between Columns (2) and (1). Standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

Table 6: Regression: Abnormal return as the dependent variable

	Abnormal Return			
	(1)	(2)	(3)	(4)
Patent day	0.0250***	0.0248***	0.0217***	0.0113*
	(0.0047)	(0.0046)	(0.0048)	(0.0066)
Patent day: Large Team Pat	0.0166***			
	(0.0060)			
Patent day: Foreign Invt Pat		0.0023		
		(0.0068)		
Patent day: Science Pat			0.0224***	
			(0.0067)	
Patent day: High VI Firm				0.0179*
				(0.0106)
Number of Patent	0.0003	0.0000	0.0002	0.0007
	(0.0006)	(0.0004)	(0.0006)	(0.0007)
Log(Market Cap (const. usd))	-0.1479***	-0.1194***	-0.1549***	-0.1621***
	(0.0038)	(0.0040)	(0.0041)	(0.0067)
Constant	2.9414***	2.4481***	3.0989***	3.2688***
	(0.0749)	(0.0805)	(0.0810)	(0.1341)
Avg DV	0.039	0.036	0.041	0.039
Firm Fixed Effects	Yes	Yes	Yes	Yes
Business Date	Yes	Yes	Yes	Yes
R^2	0.018	0.020	0.019	0.016
N	11,963,634	7,532,889	10,861,460	5,479,710

Note: This table reports firm-level regressions of abnormal stock returns around patent grant dates. The dependent variable is the cumulative abnormal return over the three-day window following the patent grant date. The indicator *Patent day* equals one on days when a firm is granted at least one patent. All columns restrict the sample to firms that have both patent and non-patent days in the estimation period. Columns (1)–(4) further restrict the sample to firms that exhibit within-firm variation in the corresponding patent-type indicator: large-team patents (Column 1), patents with all non-U.S. inventors (Column 2), science-based patents (Column 3), and patents granted by firms with high vertical integration (Column 4). All specifications include firm fixed effects and business-date fixed effects. Control variables include the log of firm market value and the number of patents granted on the event day. Standard errors are clustered at the firm level.

Table 7: Patent Value Measures and Innovation Outcomes (Standardized)

	Standardized Patent Value Measure (independent variable)						
	Return	KPSS	KPSS TeamSize	KPSS Foreign	KPSS Science	KPSS VI	KPSS VI (Sep. SNR)
Innovation Outcomes (dependent variables)							
Patent forward cites	0.1255*	0.3288***	0.6390***	0.3724***	0.6522***	0.3321***	0.2960***
	(0.0654)	(0.0584)	(0.0592)	(0.0592)	(0.0614)	(0.0661)	(0.0619)
$\mathbb{1}$[Top 1% of Cited Patents]	0.0214**	0.0767***	0.1267***	0.0829***	0.1197***	0.0678***	0.0549***
	(0.0104)	(0.0164)	(0.0167)	(0.0164)	(0.0166)	(0.0171)	(0.0165)
$\mathbb{1}$[Litigation]	0.0005	0.0849***	0.0933***	0.0864***	0.0920***	0.0908***	0.0897***
	(0.0045)	(0.0131)	(0.0129)	(0.0131)	(0.0132)	(0.0146)	(0.0143)
$\mathbb{1}$[Reassignment]	0.0002	0.0039***	0.0056***	0.0036***	0.0042***	0.0033***	0.0031***
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
$\mathbb{1}$[Renewed (Full Term)]	-0.0009	0.0255***	0.0264***	0.0261***	0.0252***	0.0260***	0.0257***
	(0.0013)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)
$\mathbb{1}$[Trilateral Patent]	-0.0003	0.0129***	0.0142***	0.0123***	0.0137***	0.0063***	0.0063***
	(0.0004)	(0.0007)	(0.0008)	(0.0007)	(0.0008)	(0.0007)	(0.0007)

Notes: Each cell reports the coefficient on a standardized (mean 0, standard deviation 1) patent value measure from a separate regression, where the dependent variable is the outcome listed in the row. Outcome variables correspond to Appendix Tables E5–E10. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we derive bootstrap-based standard errors from the reported confidence intervals and use these for inference. Significance levels are denoted by * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

Part 1: Appendix A
Estimating the value of patents with single patent type: the KPSS estimation

A1 KPSS patent value estimation methodology

KPSS make the following assumptions:

1. x_j is normally distributed truncated at zero $\mathcal{N}(0, \sigma^2_{xft})$
2. ϵ_j is normally distributed $\mathcal{N}(0, \sigma^2_{\varepsilon ft})$
3. σ^2_{xft} and $\sigma^2_{\varepsilon ft}$ vary across firms and across time but in constant proportions
4. x_j and ϵ_j are independent.

Under these assumptions, we can calculate $\mathbb{E}[x_j|r_j]$ using the distribution of x_j conditional on r_j. By independence, the joint density of $f(x_j, \epsilon_j)$ is product of their density:

$$f(x_j, \epsilon_j) = \frac{1}{(\pi\sigma_{xft}\sigma_{\varepsilon ft})} \exp\Big[\frac{-1}{2\sigma^2_{xft}} x_j^2 - \frac{1}{\sigma^2_{\varepsilon ft}}\epsilon_j^2\Big]; x_j > 0 \tag{13}$$

using the transformation $\epsilon_j = r_j - x_j$:

$$f(x_j, r_j) = \frac{1}{(\pi\sigma_{xft}\sigma_{\varepsilon ft})} \exp\Big[\frac{-1}{2\sigma^2_{vft}} x_j^2 - \frac{1}{2\sigma^2_{\varepsilon ft}}(r_j - x_j)^2\Big] \tag{14}$$

Using Aigner, Lovell, and Schmidt (1977), the density function of $f(r_j)$ is given by:

$$f(r_j) = \frac{2}{(\sqrt{2\pi}\sigma)}(\Phi(\frac{r_j\lambda}{\sigma}))\exp\Big[\frac{-1}{2\sigma^2}(r_j)^2\Big] \tag{15}$$

where $\sigma = \sigma^2_{xft} + \sigma^2_{\varepsilon ft}$, $\lambda = \frac{\sigma_{xft}}{\sigma_{\varepsilon ft}}$, and $\Phi(.)$ is CDF of standard normal distribution. As shown by Jondrow, Lovell, Materov, and Schmidt (1982), letting $\sigma^2_* = \frac{\sigma^2_{xft}\sigma^2_{\varepsilon ft}}{\sigma^2}$ we can calculate the conditional distribution and then expected values from:

$$f(x_j|r_j) = \frac{1}{(\Phi(\frac{r_j\lambda}{\sigma}))}\frac{1}{(\sqrt{2\pi}\sigma_*)}\exp\Big[\frac{-1}{2\sigma^2_*}(-x_j + \frac{\sigma^2_{xft}r_j}{\sigma^2})^2\Big] \tag{16}$$

The distribution of $f(x_j|r_j)$ is the same as a Normal distribution with $\mathcal{N}(\mu_*, \sigma^2_*)$ multiplied by $\frac{1}{(\Phi(\frac{r_j\lambda}{\sigma}))}$, where $\mu_* = \frac{\sigma^2_{xft}r_j}{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}$. The density is equivalent to the $\mathcal{N}(\mu_*, \sigma^2_*)$ distribution truncated at zero.

Using the expectation formula for truncated normal distribution:

$$\mathbb{E}[x_j|r_j] = \mu_* + \sigma_*\frac{\phi(R_j)}{1 - \Phi(R_j)} \tag{17}$$

where $R_j = \mu_*/\sigma^2_* = -\sqrt{\delta_j}\frac{r_j}{\sigma_{\varepsilon ft}}$ and $\delta_j = \frac{\sigma^2_{xft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}$. Thus, μ_* can be written as $\delta_j r_j$ and σ^2_* becomes $\sqrt{\delta_j}\sigma_{\varepsilon ft}$, leading to the following formula:

$$\mathbb{E}[x_j|r_j] = \delta_j r_j + \sqrt{\delta_j}\sigma_{\varepsilon ft}\frac{\phi(R_j)}{1-\Phi(R_j)} \tag{18}$$

To estimate the value of patents from the formula, the parameters δ_j and $\sigma_{\varepsilon ft}$ need to be estimated. KPSS assume that δ_j, which is the ratio of the variance of x_j to the sum of the variance of x_j and ϵ_j, is constant across firms and over time. To compute δ, they estimate:

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma I_{fd} + \mu_{fd} \tag{19}$$

where r^l_{fd} is the idiosyncratic return of firm f centered on day d with a window of length l, a_{ft} is the firm-year fixed effect, and b_d is the day-of-week fixed effect. γ is equal to :

$$\mathbb{E}[ln(x_j + \epsilon_j)^2] - \mathbb{E}[ln(\epsilon_j^2)] = \gamma \tag{20}$$

Approximating the distribution of $x_j + \epsilon_j$ as a normal distribution, the square of a standard normal variable is distributed as $\chi^2(1)$.

$$\mathbb{E}\Big[\Big(ln(x_j+\epsilon_j)^2\Big)\Big(\frac{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}\Big)\Big] - \mathbb{E}\Big[ln\Big(\epsilon_j^2\Big)\Big(\frac{\sigma^2_{\varepsilon ft}}{\sigma^2_{\varepsilon ft}}\Big)\Big] = \gamma \tag{21}$$

Solving this and adjusting for the truncated variance of x_j leads to:[22]

$$\ln\Big[\frac{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}(1-(\frac{\phi(0)}{1-\Phi(0)})^2)}{\sigma^2_{\varepsilon ft}}\Big] = \gamma \tag{22}$$

$$\Big[\frac{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}(1-(\frac{\phi(0)}{1-\Phi(0)})^2)}{\sigma^2_{\varepsilon ft}}\Big] = e^\gamma \tag{23}$$

simplifying this using $\delta_j = \frac{\sigma^2_{xft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}$ leads to:

$$\hat{\delta} = 1-\Bigg(1+\frac{1}{(1-(\frac{\phi(0)}{1-\Phi(0)})^2)}(e^\gamma-1)\Bigg)^{-1} = 1-e^{-\gamma}$$

Next, KPSS need to recover the $\sigma^2_{\varepsilon ft}$. This is done non-parametrically using the sum of squared market-adjusted returns σ_{ft} if m_{ft} is the fraction of trading days with a patent grant in a firm-year.

$$\sigma^2_{\epsilon_{ft}} + m_{ft}\sigma^2_{xft}(1+l) = \sigma^2_{ft}(1+l)$$

[22]The variance of $\mathcal{N}(0,\sigma^2)$ truncated at d is $\sigma^2\lambda(d)(1-\lambda(d))$ where λ is the inverse mill ratio $(\frac{\phi(d)}{(1-\Phi(d)})$. The square of standard Normal is distributed as $\chi^2(1)$.

using further simplification of equation 23 we get:

$$\sigma^2_{\epsilon_{ft}} = \frac{\sigma^2_{ft}(1+l)}{(1+m_{ft}(1+l)(e^{\hat{\gamma}}-1))}$$

A1.1 Distribution of KPSS values

Note that the KPSS values are distributed differently than the distribution of x itself. For a given firm-year, the covariance between $\mathbb{E}[x|r]$ & x is given by $\delta\sigma_x^2 + \sqrt{\delta}\sigma_\varepsilon Cov(\lambda(R),\ x)$. The term $\sqrt{\delta}\sigma_\varepsilon Cov(\lambda(R),\ x) \leq 0$ because $\lambda(R)$ is a decreasing function of r whereas x is an increasing function. Therefore, $Cov(\mathbb{E}[x|r], x) \leq \delta\sigma_x^2$.

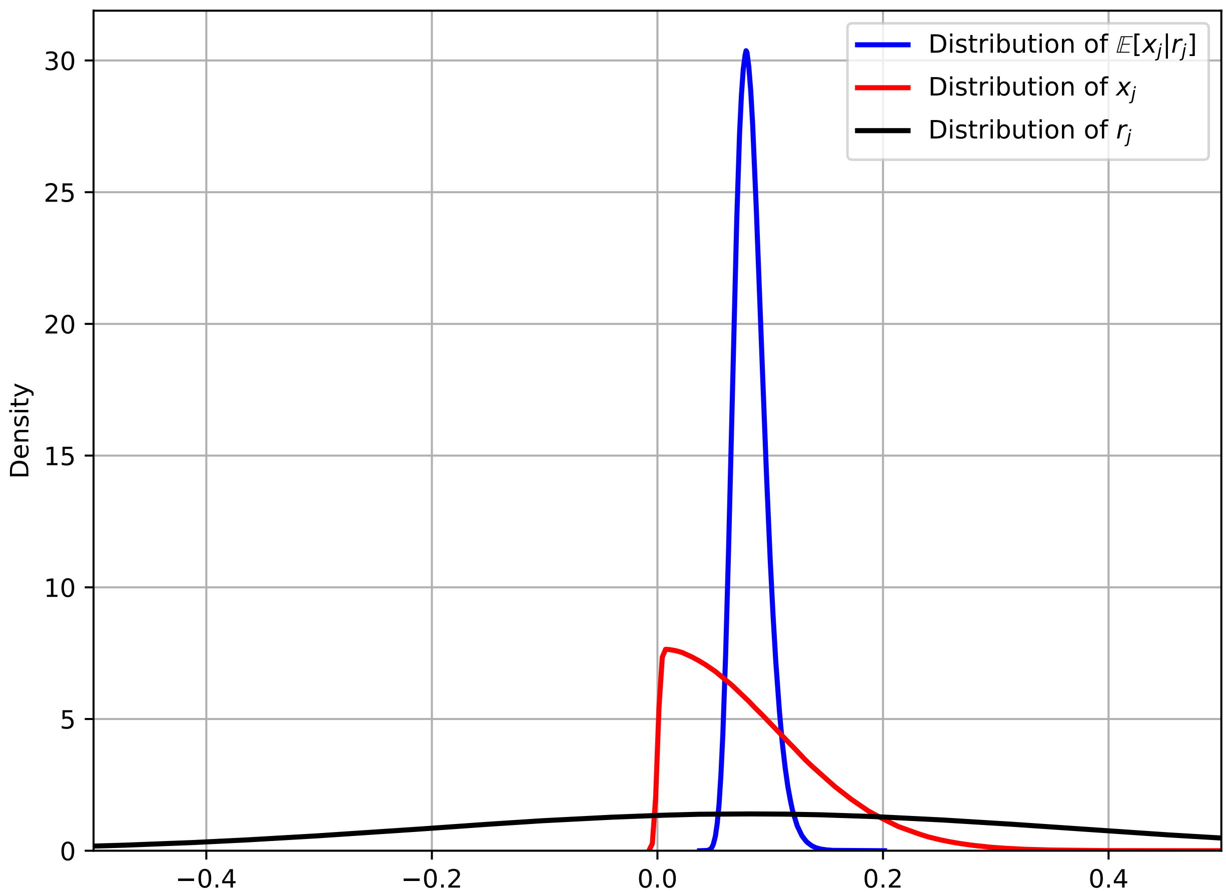

Figure 1: Simulation distribution of x, r, and $\mathbb{E}(x|r)$

Numerical simulations indicate that $\mathbb{E}[x|r]$ has a has a less skewed distribution and smaller variance than x.[23] The median of the distribution of KPSS values is greater than the median of x but also has less probability mass in the tails.

[23]Where $x \sim \mathcal{N}^+(0, \sigma_x^2)$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$, with $\sigma_x^2 = 0.011$ and $\sigma_\epsilon^2 = 0.077$. More details on the simulation are provided in Appendix D, where we extend to simulation to two types of patents.

A1.2 Variance of $\mathbb{E}[x \mid r]$

By the law of total variance,

$$\mathrm{Var}(x) = \mathbb{E}[\mathrm{Var}(x \mid r)] + \mathrm{Var}(\mathbb{E}[x \mid r]) \,.$$

Since the first term is strictly positive whenever patent values are not perfectly revealed by r, it follows immediately that

$$\mathrm{Var}(\mathbb{E}[x \mid r]) < (x).$$

Thus, KPSS values, defined as conditional expectations, necessarily exhibit lower dispersion than the underlying distribution of true patent values.

Numerical simulations confirm the mechanical compression implied by conditional expectation. In our baseline calibration, the variance of KPSS values is an order of magnitude smaller than the variance of true patent values. In addition, while the underlying patent value distribution is strongly right-skewed, mapping values into conditional expectations substantially attenuates skewness. The right tail is compressed as extreme realizations of patent value are shrunk toward the mean, although positive skewness remains. These patterns illustrate how KPSS values smooth both dispersion and tail asymmetry relative to the underlying distribution.

A2 Revisiting the KPSS method to handle multiple patents granted in a single day

$$r = x_1 + x_2 + \varepsilon$$

Where x_1 and x_2 are two patents drawn from the same distribution (e.g., both are science-based or non-science-based).[24]

First, assuming only two patents are granted on a given day. Following the KPSS method as described in Appendix A section A1.

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1 I_{1fd} + \gamma_2 I_{2fd} + \mu_{fd} \tag{24}$$

I_{1fd} indicates that there is only 1 patent on grant day; similarly, I_{2fd} indicates two patents on the grant day.

$$\ln\Big[\frac{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}{\sigma^2_{\varepsilon ft}}\Big] = \gamma_1, \ln\Big[\frac{\sigma^2_{\varepsilon ft} + 2 * \sigma^2_{xft}}{\sigma^2_{\varepsilon ft}}\Big] = \gamma_2 \tag{25}$$

[24] We dropped the index for patent day etc. to keep the notation simple.

This implies

$$1 + \theta^2 = e^{\gamma_1}$$
$$1 + 2\theta^2 = e^{\gamma_2}$$

where $\theta = \frac{\sigma^2_{xft}}{\sigma^2_{\varepsilon ft}}$

We solve for θ^2 from the first equation:

$$\theta^2 = e^{\gamma_1} - 1$$

Substitute θ^2 into the second equation:

$$1 + 2(e^{\gamma_1} - 1) = e^{\gamma_2}$$

Simplify the left-hand side:

$$1 + 2e^{\gamma_1} - 2 = e^{\gamma_2} 2e^{\gamma_1} - 1 = e^{\gamma_2}$$

Thus, we have the relationship between γ_1 and γ_2:

$$e^{\gamma_2} = 2e^{\gamma_1} - 1$$

Alternatively, we can express γ_2 in terms of γ_1:

$$\gamma_2 = \ln(2e^{\gamma_1} - 1)$$

Approximation for Small γ

Consider the relationship:

$$e^{\gamma_2} = 2e^{\gamma_1} - 1$$

Using the Taylor expansion for e

$$1 + \gamma_2 = 2(1 + \gamma_1) - 1$$

So, for small γ_1, we can approximate:

For a more exact representation, the original expression $\gamma_2 = \ln(2e^{\gamma_1} - 1)$ is the most accurate, but the approximation $\gamma_2 \approx 2\gamma_1$ provides a useful simplification under certain conditions.

We can impose the restriction $\gamma_2 = \ln(2e^{\gamma_1} - 1)$ in equation 24 which becomes

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1 I_{1fd} + (\ln(2e^{\gamma_1} - 1))I_{2fd} + \mu_{fd} \tag{26}$$

This can be estimated by GMM or non-linear least squares. If we impose the approximation $\gamma_2 = 2\gamma_1$, equation 24 simplifies to

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1(I_{1fd} + \frac{2}{1}I_{2fd}) + \mu_{fd} \tag{27}$$

This can also extend to n-patent granted on a given day.

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma n + \mu_{fd} \tag{28}$$

A2.1 Correcting the bias in KPSS estimates if more than one patent is granted on a single day

Suppose n patents are granted on a particular day, where n is a random variable. As before, we assume that the probability that any patent is granted in a given day is $p > 0$. We show that the KPSS version of equation 27 yields an upward biased estimate, where the bias is given by $\gamma_1(1 - N)$, where $N = E(n|I_{1fd} = 1) = np$. Put differently, one needs to divide the KPSS estimate of γ_1 by the expected number of patents conditional on a patent being granted.

To see this, consider a simplified version of of equation 27, where we dispense with the subscripts to reduce notational clutter.

$$Y = a_0 + \gamma n + \zeta \tag{29}$$

where Y is the square of the log returns, and n is the number patents granted, $n = 0, 1, 2, ...$ Consider the analog of the KPSS estimating equation

$$Y = a_0 + \beta I + \zeta \tag{30}$$

where I is a dummy variable that takes value one if there is a patent granted and zero otherwise.

Then the expected value of b, the OLS estimate of β is given by

$$
\begin{aligned}
\mathbb{E}(b) &= \frac{Cov(Y,I)}{Var(I)} \\
Cov(Y,I) &= E(YI) - E(Y)E(I) = \gamma np - (a_0 + \gamma np)p \\
\implies \mathbb{E}(b) &= \gamma N
\end{aligned} \tag{31}
$$

This implies that KPSS estimates of γ should be divided by the mean number of patents per day, conditional on patents being granted.

Figure 2: Distribution of average patent value by number of patents on the grant day

Notes: The following plot shows the average patent value calculated using the KPSS assumption (where the total invention value of all patents granted on a given day follows a half-normal distribution) compared to a scenario where we assume a patent value distribution and adjust for the number of patents when calculating the signal-to-noise ratio. The x-axis represents the number of patents granted on a given day. The KPSS method tends to overestimate the patent value compared to the average when the number of patents granted is low, while the modified approximation, which adjusts for the number of patents, underestimates the patent value compared to the average in the same scenario.

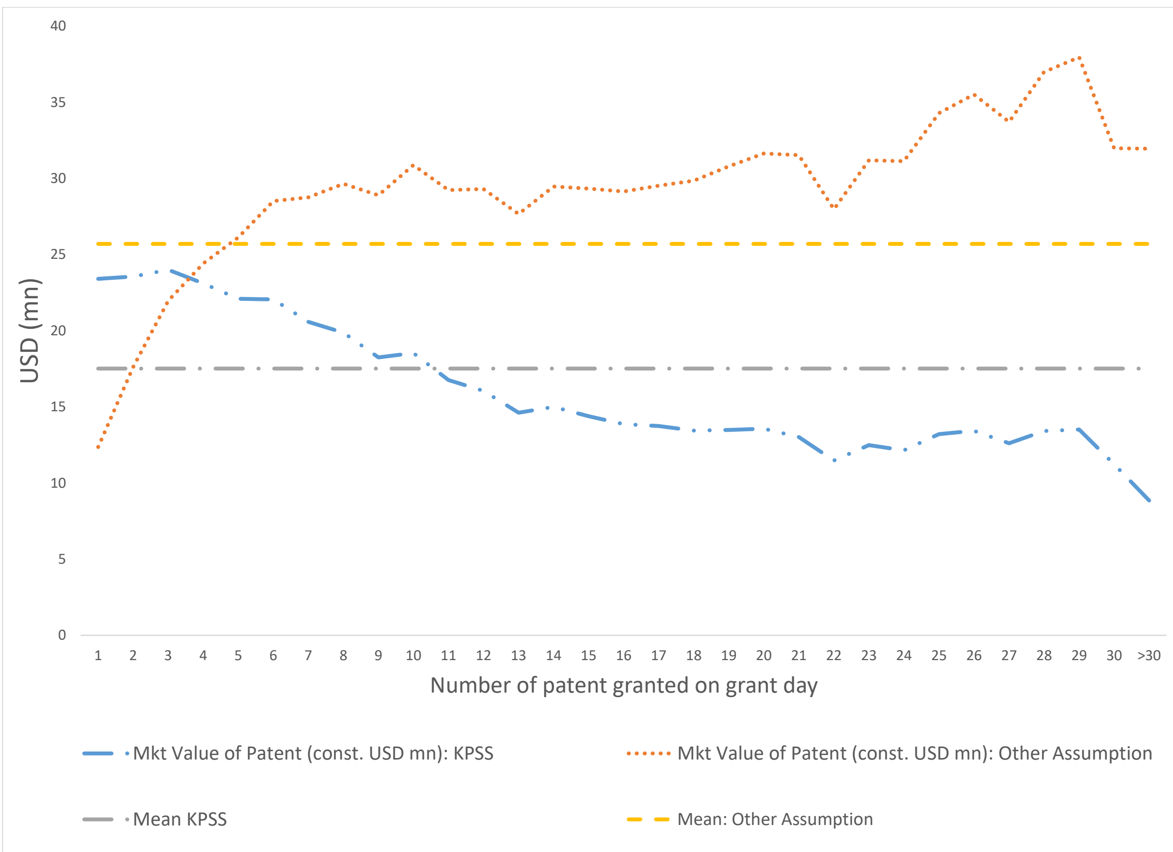

Table A1: Patent private value estimation and KPSS assumptions

	Log(1+Fwd citations)							Breakthrough				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log (Pat val KPSS (Baseline))		0.053***			0.083***			0.059***			0.149***	
		(0.001)			(0.001)			(0.008)			(0.013)	
Log (Pat val KPSS (New Assump.))			0.011***			0.103***			-0.079***			0.166***
			(0.001)			(0.001)			(0.008)			(0.016)
Constant	1.826***	0.981***	1.648***	1.826***	0.512***	0.154***	1.283***	0.357***	2.574***	1.283***	-1.073***	-1.413***
	(0.001)	(0.013)	(0.012)	(0.001)	(0.019)	(0.024)	(0.009)	(0.133)	(0.133)	(0.009)	(0.207)	(0.262)
Avg of DV	1.826	1.826	1.826	1.826	1.826	1.826	1.283	1.283	1.283	1.283	1.283	1.283
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
R^2	0.1317	0.1345	0.1318	0.2080	0.2105	0.2105	0.0052	0.0052	0.0053	0.0418	0.0419	0.0419
N	1,443,877	1,443,866	1,443,866	1,443,262	1,443,251	1,443,251	1,443,877	1,443,866	1,443,866	1,443,262	1,443,251	1,443,251

Note: The dependent variable in odd columns (1-6) is the *Log (1 + patent Forward citation)*, which is the natural log of the patent's forward citations garnered over a five-year period. The dependent variable in even columns (7-12) is a binary variable equal to one if the focal patent received a number of forward citations in the top 99th percentile among the patents granted in the same year and within the same patent class, as sourced from Squicciarini et al. (2013). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Appendix B
Estimating the value of patents with multiple patent types

B1 Estimating the Value of Patents with Multiple Patent Types

We make a similar assumption to KPSS, with a slight variation:

- x_{sj} & x_{nj} denote science-based and non science patent values respectively for firm f and time t.
- x_{sfti} & x_{nftk} are normally distributed truncated at zero but have different variance ($\mathcal{N}^+(0, \sigma^2_{xsft})$ and $\mathcal{N}^+(0, \sigma^2_{xnft})$).
- ε_j is normally distributed $\mathcal{N}(0, \sigma^2_{\varepsilon ft})$.
- σ^2_{xft}, σ^2_{xnft} and $\sigma^2_{\varepsilon ft}$ vary across firms and across time but in constant proportions.
- $\delta_{sft} = \frac{\sigma^2_{xsft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xsft}} = \delta_s$ and $\delta_{nft} = \frac{\sigma^2_{xnft}}{\sigma^2_{\varepsilon ft}+\sigma^2_x nft} = \delta_n$
- x_{sift}, x_{njft} and ϵ_{ft} are independent for a given firm and time.

we can derive the conditional expectation of the scientific contribution to a patent's value.

$$E(x_{sj}|r_{sj}) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)} r_j + \frac{\left\{2\frac{(1+\theta_2^2)\sigma^2_{\varepsilon ft}}{\omega_1}\phi(r_j/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r_j\right) - \frac{2\theta_1^2\sigma^2_{\varepsilon ft}}{\omega_2}\phi\left(\frac{r_j}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r_j\right)\right\}}{\left[\Phi\left(\frac{r_j}{\omega_1}\right) - 2T\left(\frac{r_j}{\omega_1},\lambda_1\right) + \Phi\left(\frac{r_j}{\omega_2}\right) - 2T\left(\frac{r_j}{\omega_2},\lambda_2\right)\right]} \tag{32}$$

Where:

- $\theta_1 = \frac{\sigma_{xsft}}{\sigma_{\varepsilon ft}}$ and $\theta_2 = \frac{\sigma_{xnft}}{\sigma_{\varepsilon ft}}$
- $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$ and $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
- $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$ and $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$
- $s = \sqrt{\sigma^2_{\varepsilon ft} + (\sigma_{xsft})^2 + (\sigma_{xnft})^2} = \sigma_{\varepsilon j}\sqrt{1+\theta_1^2+\theta_2^2}$
- T = Owen's T function (Owen, 1980)

To derive the $\mathbb{E}[x_j|r_j]$ it is useful to analyze the sum of random variables. Let $r = u+v+w$ where v and w are normally distributed, truncated at zero, but have different variances: $v \sim \mathcal{N}^+(0, \sigma_1^2)$ and $w \sim \mathcal{N}^+(0, \sigma_2^2)$ and u is normally distributed $u \sim \mathcal{N}(0, \sigma_u^2))$.[25] We follow Papadopoulos (2015), which entails these steps:

[25]The relationship to patent values is readily apparent when r represents market returns, u represents the noise term, u represents value of science-based patents, and v represents the value of non science-based patents. This substitution simplifies notation, particularly subscripts.

- Derive the distribution of $z = v + w$

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) + \Phi\left(\frac{\sigma_2}{\sigma_1}(z/s_h)\right) - 1\right] \tag{33}$$

- Derive the distribution of r

$$f_r(r) = \frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right] \tag{34}$$

Where:

  - $s^2 = \sigma_u^2 + \sigma_1^2 + \sigma_2^2$
  - $s_h^2 = \sigma_2^2 + \sigma_1^2$
  - $\lambda_1 = \frac{s\lambda}{\sigma_u}$ & $\lambda_2 = \frac{s}{\lambda\sigma_u}$ where $\lambda = \frac{\sigma_1}{\sigma_2}$
  - $a = \frac{s_h}{s\sigma_u}$

- Derive the conditional density $f(v|r)$

$$f_{v|r}(v|r) = A^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right) \tag{35}$$

Where:

  - $\frac{\sigma_1}{ss_2} = \frac{\theta_1}{s\sqrt{1+\theta_2^2}} = \frac{1}{\omega_1}$
  - $\frac{\theta_2}{s_2} = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}\frac{1}{\omega_1} = \frac{\lambda_1}{\omega_1}$
  - $\omega_v = \frac{\sigma_1 s_2}{s}$

- Derive the condition expectation $\mathbb{E}(v|r)$

$$E(v|r) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)}r + \frac{\left\{2\frac{(1+\theta_2^2)\sigma_u^2}{\omega_1}\phi(r/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2\theta_1^2\sigma_u^2}{\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}}{\left[\Phi\left(\frac{r}{\omega_1}\right) - 2T\left(\frac{r}{\omega_1}, \lambda_1\right) + \Phi\left(\frac{r}{\omega_2}\right) - 2T\left(\frac{r}{\omega_2}, \lambda_2\right)\right]} \tag{36}$$

We will use the following parameterization, where:

  - $\theta_1 = \frac{\sigma_1}{\sigma_u}$
  - $\theta_2 = \frac{\sigma_2}{\sigma_u}$
  - $s = \sqrt{\sigma_u^2 + \sigma_1^2 + \sigma_2^2} = \sigma_u\sqrt{1+\theta_1^2+\theta_2^2}$, with
  - $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$

– $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
– $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$
– $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$

- Derive the two signal-to-noise ratios. We can estimate the γ_s and γ_n and recover δ_s as $1-e^{-\gamma_s}$ and δ_n as $1-e^{-\gamma_n}$ from the following regression:

$$ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_s I_{sfd} + \gamma_n I_{nfd} + \nu_{fd} \tag{37}$$

- Derive the variance of ϵ_j. We can estimate it as follows:

$$\sigma^2_{\varepsilon ft} + \mu_s \sigma^2_{sft}(1+l) + \mu^N \sigma^2_{nft}(1+l) = \sigma^2_{ft}(1+l) \tag{38}$$

Derive the distribution of $v + w$

$z = v + w$. Both v and w are truncated normal at zero with variance σ_1 & σ_2, so z is the sum of two truncated normal distributions. Further, both v and w are iid so the distribution of z is:

$$F_z(z) = \mathbb{P}(v + w \leq z)$$
$$f_Z(z) = \int_0^z f(z-w)f(w)\,dw$$
$$f_Z(z) = \frac{2}{\pi\sigma_1\sigma_2}\int_0^z \left(\exp\left(-\frac{(z-w)^2}{2\sigma_1^2}\right)\right)\left(\exp\left(-\frac{w^2}{2\sigma_2^2}\right)\right) dw$$

The inside term can written as:

$$\frac{2}{\pi\sigma_1\sigma_2}\exp\left[-\frac{1}{2}\left(\frac{z}{\sigma_1}\right)^2\right]\exp\left[-\frac{1}{2}\frac{s_h^2}{\sigma_1^2\sigma_2^2}w^2 + \frac{z}{\sigma_1^2}w\right]$$

where $s_h^2 = \sigma_1^2 + \sigma_2^2$

Imposing restriction on the domain of w, with the integral between 0 and z, we have to calculate the integral from

$$\int_0^z \left(\exp\left(-\frac{1}{4\delta}\omega^2 + \gamma\omega\right)\right) d\omega$$

Where $\delta = \frac{\sigma_1^2\sigma_2^2}{2s_h^2}$ and $\gamma = \frac{z}{\sigma_1}$

We can write the above expression in the following form:

$$f_Z(z) = \frac{2}{\pi\sigma_1\sigma_2} \exp\left[\frac{1}{2}\left(\frac{z}{s_h^2}\right)^2\right] \sqrt{2\pi\left(\frac{\sigma_1\sigma_2}{s_h}\right)^2} \int_0^z \frac{1}{\sqrt{2\pi\left(\frac{\sigma_1\sigma_2}{s_h}\right)^2}} \exp\left[-\frac{1}{2}\frac{(-(w-\frac{z\sigma_1^2}{s_h^2})^2}{(\frac{\sigma_1\sigma_2}{s_h})^2}\right] dw$$

The density function can be simplified to

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) + \Phi\left(\frac{\sigma_2}{\sigma_1}(z/s_h)\right) - 1\right]$$

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) - \Phi\left(-\frac{\sigma_2}{\sigma_1}(z/s_h)\right)\right]$$

Derive the distribution of $r = u + v + w$

$r = u + z$ (where u is normal distribution with variance σ_u). The domain of u is $(-\infty, \infty)$, while the domain of z is $(0, \infty)$:

$$F_r(r) = \int_0^\infty \int_{-\infty}^{r-z} f_{u,z}(u, z) du dz$$

$$f_r(r) = \frac{d}{dr}F_r(r) = \int_0^\infty f_{u,z}(r-z, z)dz$$

Since the variables are independent, joint density is the product of the two marginal densities (one normal and one derived in the previous section). We use $1 - \Phi(x) = \Phi(-x)$

$$f_r(r) = \int_0^\infty \frac{1}{\sigma_u}\phi\left(\frac{r-z}{\sigma_u}\right)\frac{4}{s_h}\phi(z/s_h)\left[1 - \Phi\left(-\frac{\sigma_1}{\sigma_2}(z/s_h)\right) - \Phi\left(-\frac{\sigma_2}{\sigma_1}(z/s_h)\right)\right] dz$$

Following Papadopoulos (2015) we get

$$f_r(r) = \frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]$$

Where:

- $s^2 = \sigma_u^2 + \sigma_1^2 + \sigma_2^2$
- $s_h^2 = \sigma_2^2 + \sigma_1^2$

- $\lambda_1 = \frac{s\lambda}{\sigma_u}$ & $\lambda_2 = \frac{s}{\lambda\sigma_u}$
- $\lambda = \frac{\sigma_1}{\sigma_2}$
- $a = \frac{s_h}{s\sigma_u}$
- T = Owen's T function (Owen, 1980)

Derive the conditional density $f(v|r)$

$$f_{v|r}(v|r) = \frac{f_{v,u+w}(v, u+w)}{F_r(r)}$$

Note: v and w are independent of each other.
$R = \zeta + v$, where $\zeta = u + w$

$$f_{v|r}(v|r) = \frac{f_v(v)f_\zeta(r-v)}{F_r(r)}$$

$$f_{v|r}(v|r) = \frac{\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_1}\exp\left(\frac{-1}{2}(\frac{v}{\sigma_1})^2\right)\frac{2}{s_2}\phi((r-v)/s_2)\Phi\left(\theta_2\frac{r-v}{s_2}\right)}{\frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]}$$

Where $s_2^2 = \sigma_2^2 + \sigma_u^2$ and $\theta_2 = \frac{\sigma_2}{\sigma_u}$ & $\theta_1 = \frac{\sigma_1}{\sigma_u}$

$$A = \left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]$$

We can write this in the following form:

$$f_{v|r}(v|r) = A^{-1}\sqrt{\frac{2}{\pi}}\frac{s}{\sigma_1 s_2}\exp\left\{-\frac{1}{2}\left(\frac{v}{\sigma_1}\right)^2 - \frac{1}{2}\left(\frac{r-v}{s_2}\right)^2 + \frac{1}{2}\left(\frac{r}{s}\right)^2\right\}\Phi\left(\theta_2\frac{r-v}{s_2}\right)$$

Following Papadopoulos (2015), we can write
$\frac{\sigma_1}{ss_2} = \frac{\theta_1}{s\sqrt{1+\theta_2^2}} = \frac{1}{\omega_1}$

$\frac{\theta_2}{s_2} = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}\frac{1}{\omega_1} = \frac{\lambda_1}{\omega_1}$

$\omega_v = \frac{\sigma_1 s_2}{s}$

$$f_{v|r}(v|r) = A^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right)$$

The conditional expected value $\mathbb{E}(v|r)$

$$\mathbb{E}(v|r) = \int_0^\infty v f_{v|r}(v|r)dv = \int_0^\infty vA^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right)dv$$

Using the substitution:$v^* = \left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)$

$v = \omega_v V^* + (\omega_v/\omega_1)r$

$dv = \omega_v dv^*$

$v = 0 \rightarrow -\frac{r}{\omega_1}$

Substituting:

$$\mathbb{E}(v|r) = (\omega_v/\omega_1)r + (2\omega_v A^{-1})\int_{-\frac{r}{\omega_1}}^\infty (v^*\phi(v^*)\Phi\left\{-\left(\frac{\lambda_1 r(\omega_v - \omega_1)}{\omega_1^2} + \frac{\lambda_1\omega_v}{\omega_1}v^*\right)\right\}dv^*$$

Similar to Papadopoulos (2015), this integral can be written as

$$\int_{-\frac{r}{\omega_1}}^\infty x\phi(x)\Phi(a+bx)dx$$

From Owen (1980):

$$\int_c^\infty x\phi(x)\Phi(a+bx) = \frac{b}{\sqrt{1+b^2}}\phi\left(\frac{a}{\sqrt{1+b^2)}}\right)\Phi\left(\frac{c+b(a+bc)}{-\sqrt{1+b^2}}\right) + \phi(c)\Phi(a+bc)$$

Where:

- $a = \frac{-\lambda_1(\omega_v - \omega_1)}{\omega_1^2}$
  $b = \frac{-\lambda_1\omega_v}{\omega_1}$
  $c = -r/\omega_1$
- $a + bc = \frac{\lambda_1}{\omega_1}r$
- $c + b(a+bc) = -\frac{1}{\omega_1}(1+\omega_2^2)r = \frac{-\theta_1\sqrt{(1+\theta_2^2)}}{s}r$
- $\sqrt{1+b^2} = \frac{s_2}{s}\sqrt{1+\theta_1^2}$

Solving for $\frac{c+b(a+bc)}{\sqrt{1+b^2}} = \frac{\lambda_2}{\omega_2}r$:

- $\frac{a}{\sqrt{1+b^2}} = -\frac{r}{\omega_2}$
- $\frac{b}{\sqrt{1+b^2}} = -\frac{s^2\omega_v}{s_2^2\omega_2}$

Finally, we obtain.

$$E(v|r) = (\omega_v/\omega_1)r + A^{-1}\left\{2\omega_v\frac{\omega_1}{\omega_1}\phi(r/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2s^2\omega_v^2}{s_2^2\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}$$

Note, this can be further simplified to
$\omega_v/\omega_1 = \frac{\sigma_1^2}{s^2}$
$\omega_v * \omega_1 = \sigma_u^2 + \sigma_2^2$
$\frac{s^2\omega_v^2}{s_2^2} = \sigma_1^2$

$$E(v|r) = \frac{\sigma_1^2}{s^2}r + A^{-1}\left\{(s^2 - \sigma_1^2)g_1 - \sigma_1^2 g_2\right\}$$

using $\frac{a}{\sqrt{1+\lambda_1^2}} = \frac{1}{\omega_1}$ & $\frac{a}{\sqrt{1+\lambda_2^2}} = \frac{1}{\omega_2}$

$$E(v|r) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)}r + \frac{\left\{2\frac{(1+\theta_2^2)\sigma_u^2}{\omega_1}\phi(R/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2\theta_1^2\sigma_u^2}{\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}}{\left[\Phi\left(\frac{r}{\omega_1}\right) - 2T\left(\frac{r}{\omega_1},\lambda_1\right) + \Phi\left(\frac{r}{\omega_2}\right) - 2T\left(\frac{r}{\omega_2},\lambda_2\right)\right]} \quad (39)$$

Let
$\theta_1 = \frac{\sigma_1}{\sigma_u}$
$\theta_2 = \frac{\sigma_2}{\sigma_u}$
$s = \sqrt{\sigma_u^2 + \sigma_1^2 + \sigma_2^2} = \sigma_u\sqrt{1+\theta_1^2+\theta_2^2}$

Following Papadopoulos (2015) we get:

- $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$
- $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
- $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$
- $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$

If $\sigma_2^2 \to 0$, we obtain $\theta_1 = \frac{\sigma_1}{\sigma_u}$; $\theta_1 = 0$; $s = \sqrt{\sigma_u^2 + \sigma_1^2}$; $\frac{1}{\omega_1} = \frac{\sigma_1}{\sigma_u \sqrt{\sigma_u^2+\sigma_1^2}}$. It follows that:

$$E(v|r) = \frac{\sigma_1^2}{\sigma_u^2 + \sigma_1^2} r + \frac{\sigma_1}{\sqrt{\sigma_u^2 + \sigma_1^2}} \sigma_u \frac{\phi\left(\frac{\sigma_1}{\sigma_u \sqrt{\sigma_u^2+\sigma_1^2}} r\right)}{\Phi\left(\frac{\sigma_1}{\sigma_u \sqrt{\sigma_u^2+\sigma_1^2}} r\right)}$$

This is same as Kogan et al. (2017) upon noting the $1 - \Phi(-x) = \Phi(x)$ & $\phi(x) = \phi(-x)$ and $\delta = \frac{\sigma_1^2}{\sigma_u^2+\sigma_1^2}$:

$$E(v|r) = \delta r + \sqrt{\delta} \sigma_u \frac{\phi\left(\frac{\sqrt{\delta}}{\sigma_u} r\right)}{\Phi\left(\frac{\sqrt{\delta}}{\sigma_u} r\right)}$$

B2 Step-by-step implementation guide for Generalised KPSS

B2.1 Baseline KPSS: Estimating Private Patent Value

In KPSS's approach, which we fully characterize in Appendix A1, the private value of patent j granted to firm f on day t is given by:

$$\hat{A}_{jft} = \frac{1}{K_{ft}}(1-\pi)^{-1} \cdot S \cdot \mathbb{E}[x_j \mid r_j], \tag{40}$$

where K_{ft} represents the number of patents granted on the day, π is the ex-ante probability that the patent application is successful, and S denotes the market capitalization of the firm the day prior to the patent grants.[26]

The key estimand is $\mathbb{E}[x_j \mid r_j]$, the expected value of the patent (x_j) conditional on the observed stock market reaction around the patent grant announcement date (r_j). KPSS decompose the observed return into two unobserved components:

$$r_j = x_j + \varepsilon_j, \tag{41}$$

where x_j is the patent-induced change in firm value and ε_j captures contemporaneous, non-patent value-relevant events. KPSS impose the following distributional assumptions:

$$x_j \sim \mathcal{N}^+(0, \sigma^2_{xft}), \tag{42}$$
$$\varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon ft}). \tag{43}$$

Under these assumptions, the conditional expectation takes the form:[27]

$$\mathbb{E}[x \mid r] = \delta\, r + \sqrt{\delta}\, \sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)} = \delta\, r + \lambda(R, \delta, \sigma_\varepsilon), \tag{44}$$

where

$$R = -\sqrt{\delta}\,\frac{r}{\sigma_\varepsilon}, \qquad \delta = \frac{\sigma_x^2}{\sigma_\varepsilon^2 + \sigma_x^2}, \qquad \lambda(R, \delta, \sigma_\varepsilon) = \sqrt{\delta}\,\sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)}. \tag{45}$$

Equation 44 has two attractive features: (i) the estimated patent value is higher for larger stock-market reactions at the grant date (r_j); and (ii) this relation is stronger when markets are more informative about patent value (i.e., when δ is larger).

[26] We use a 3-day time window over which the stock market response to the patent grant is measured.
[27] To simplify notation, subscripts denoting patent, year, and firm are omitted.

Step 1: Estimate the signal-to-noise ratio. Implementing (44) requires estimating (i) δ_{ft} (signal-to-noise) and (ii) $\sigma^2_{\varepsilon ft}$ (noise variance). KPSS assume δ_{ft} is constant across firms and time, $\delta_{ft} = \delta$, and estimate it using:

$$\ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma I_{fd} + \mu_{fd}, \tag{46}$$

where r_{fd} denotes the three-day idiosyncratic return for firm f starting on day d, I_{fd} is an indicator for patent-grant days, and a_{ft} and b_d are firm-year and day-of-week fixed effects. The coefficient γ captures the patent announcement effect on idiosyncratic volatility and can be used to recover

$$\hat{\delta} = 1 - e^{-\hat{\gamma}}, \tag{47}$$

using the approximation described in KPSS.

Step 2: Estimate the noise variance. KPSS estimate $\sigma^2_{\varepsilon ft}$ non-parametrically using realized idiosyncratic squared returns. Denote the resulting estimate by $\widehat{\sigma^2_{\varepsilon ft}}$. Specifically, this is done using the sum of squared market-adjusted returns σ^2_{ft}, where m_{ft} is the fraction of trading days with a patent grant in a firm-year:

$$\sigma^2_{\varepsilon ft} + m_{ft}\sigma^2_{xft}(1+l) = \sigma^2_{ft}(1+l).$$

Using further simplification of the volatility decomposition, we obtain:

$$\sigma^2_{\varepsilon ft} = \frac{\sigma^2_{ft}(1+l)}{1 + m_{ft}(1+l)\left(e^{\hat{\gamma}} - 1\right)}.$$

Step 3: Compute baseline KPSS values. Given $\hat{\delta}$ and $\widehat{\sigma^2_{\varepsilon ft}}$, compute $\mathbb{E}[x_j \mid r_j]$ from (44), and then compute $\hat{A}_{jft}$ from (40) [28].

B2.2 Generalised KPSS: Multiple Patent Types (Science vs. Non-science)

We extend KPSS to settings where a firm may receive multiple patent types on a given grant day. Let x_{sj} and x_{nj} denote the latent values of science-based and non-science patents, respectively, granted to firm f on day t. We maintain KPSS's distributional structure with a slight variation:

- x_{sj} and x_{nj} denote science-based and non-science patent values for firm f at time t.

[28] Similar to KPSS, we set the ex-ante probability of patent grant to $\pi = 0.55$.

- $x_{sj} \sim \mathcal{N}^+(0, \sigma^2_{xsft})$ and $x_{nj} \sim \mathcal{N}^+(0, \sigma^2_{xnft})$.
- $\varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon ft})$.
- σ^2_{xsft}, σ^2_{xnft}, and $\sigma^2_{\varepsilon ft}$ vary across firms and time but in constant proportions.
- $\delta_{sft} = \frac{\sigma^2_{xsft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xsft}} = \delta_s$ and $\delta_{nft} = \frac{\sigma^2_{xnft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xnft}} = \delta_n$.
- x_{sj}, x_{nj}, and ε_j are independent for a given firm and time.

B2.2.1 Different-day case: only one type is granted on a day

When patent grant days are "pure" (only S-type patents or only N-type patents are granted on a given day), the problem reduces to baseline KPSS applied separately by type.

Step 1: Estimate separate signal-to-noise ratios. We recover δ_s and δ_n using:

$$\ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_s I_{sfd} + \gamma_n I_{nfd} + \nu_{fd}, \tag{48}$$

where $I_{sfd} = 1$ if at least one S-type patent is granted on day d and 0 otherwise, while $I_{nfd} = 1$ if at least one N-type patent and no S-type patent is granted on day d and 0 otherwise. We then recover:

$$\hat{\delta}_s = 1 - e^{-\hat{\gamma}_s}, \qquad \hat{\delta}_n = 1 - e^{-\hat{\gamma}_n}. \tag{49}$$

Step 2: Estimate the noise variance. We compute $\sigma^2_{\varepsilon ft}$ using the accounting identity:

$$\sigma^2_{\varepsilon ft} + \mu_{sft}\sigma^2_{xsft}(1+l) + \mu_{nft}\sigma^2_{xnft}(1+l) = \sigma^2_{ft}(1+l), \tag{50}$$

where μ_{sft} is the share of firm-year days with S-type patent grants, μ_{nft} is the share of firm-year days with only N-type patent grants, σ^2_{ft} is the realized mean idiosyncratic squared returns of firm f in year t, and l is the event window length in days.[29]

Step 3: KPSS on S-only days (or N-only days). On S-only days, compute the conditional expectation using the baseline KPSS formula with $(\delta, \sigma_\varepsilon) = (\delta_s, \sigma_{\varepsilon ft})$:

$$\mathbb{E}[x_s \mid r] = \delta_s r + \sqrt{\delta_s}\,\sigma_{\varepsilon ft}\frac{\phi(R_s)}{1-\Phi(R_s)}, \qquad R_s = -\sqrt{\delta_s}\frac{r}{\sigma_{\varepsilon ft}}. \tag{51}$$

[29] The term $(1+l)$ (3-day window in our case) reflects the event window over which the market incorporates patent information, whereas noise is integrated continuously throughout the year.

Analogously, on N-only days, replace δ_s with δ_n.

B2.2.2 Same-day case: both S and N patents are granted on the same day

When both S- and N-type patents are granted on the same day, the observed stock reaction loads on both latent components:

$$r_j = x_{sj} + x_{nj} + \varepsilon_j. \tag{52}$$

The object of interest becomes the conditional expectation of the *scientific* contribution:

$$\mathbb{E}[x_{sj} \mid r_j]. \tag{53}$$

Step 1: Separate signal-to-noise ratios. Estimate $\hat{\delta}_s$ and $\hat{\delta}_n$ as above using (48).

Step 2: Noise variance. Recover $\widehat{\sigma^2_{\varepsilon ft}}$ using the firm-year identity above.

Step 3: Generalised KPSS formula for $\mathbb{E}[x_{sj} \mid r_j]$. Define:

$$\theta_1 = \frac{\sigma_{xsft}}{\sigma_{\varepsilon ft}}, \qquad \theta_2 = \frac{\sigma_{xnft}}{\sigma_{\varepsilon ft}}, \tag{54}$$

$$s = \sqrt{\sigma^2_{\varepsilon ft} + \sigma^2_{xsft} + \sigma^2_{xnft}} = \sigma_{\varepsilon ft}\sqrt{1 + \theta_1^2 + \theta_2^2}, \tag{55}$$

$$\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}, \qquad \omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}, \tag{56}$$

$$\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1 + \theta_1^2 + \theta_2^2}, \qquad \lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1 + \theta_1^2 + \theta_2^2}. \tag{57}$$

Then the conditional expectation of the scientific contribution is:

$$\mathbb{E}(x_{sj} \mid r_j) = \frac{\theta_1^2}{1+\theta_1^2+\theta_2^2}\, r_j + \frac{2\left\{\frac{(1+\theta_2^2)\sigma^2_{\varepsilon ft}}{\omega_1}\phi\left(\frac{r_j}{\omega_1}\right)\Phi\left(\frac{\lambda_1}{\omega_1}r_j\right) - \frac{\theta_1^2\sigma^2_{\varepsilon ft}}{\omega_2}\phi\left(\frac{r_j}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r_j\right)\right\}}{\left[\Phi\left(\frac{r_j}{\omega_1}\right) - 2T\left(\frac{r_j}{\omega_1}, \lambda_1\right) + \Phi\left(\frac{r_j}{\omega_2}\right) - 2T\left(\frac{r_j}{\omega_2}, \lambda_2\right)\right]}, \tag{58}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal pdf and cdf, respectively, and $T(\cdot,\cdot)$ is Owen's T function.

Step 4: Private value scaling (generalised case). Given $\mathbb{E}[x_{sj} \mid r_j]$ from (58), compute the private value contribution using the same scaling as KPSS:

$$\hat{A}_{jft}^{(s)} = \frac{1}{K_{ft}}(1-\pi)^{-1} \cdot S \cdot \mathbb{E}[x_{sj} \mid r_j], \tag{59}$$

and analogously for the non-scientific component $\hat{A}_{jft}^{(n)}$.

Replication code. Detailed step-by-step code for implementing the baseline and generalized KPSS estimators—covering applications to team size, R&D offshoring, reliance on science, and vertical integration—is available at https://github.com/jaypnagar/ABFN_Relication.

Appendix C
Measurement error in KPSS

C1 Comparing differences in value of groups of patents using KPSS values

Consider two types of patents, N and S. Letting r_s represent the market returns for S type patents and r_n for N patents, so that $r_{is} = x_{is} + \epsilon_i$, and $r_{jn} = x_{jn} + \epsilon_j$. If ε_i and ε_j have a normal distribution with mean zero and standard deviation σ_ε, we have

$$\mathbb{E}[r_s] = \mathbb{E}[x_s] = \sigma_s \sqrt{\frac{2}{\pi}} > \mathbb{E}[r_n] = \mathbb{E}[x_n] = \sigma_n \sqrt{\frac{2}{\pi}} \tag{60}$$

The sample mean of S type patents for that firm and year is:

$$\bar{y}_s = \delta \frac{1}{S} \sum_{i=1}^{i=S} r_{is} + \sqrt{\delta} \sigma_\varepsilon \frac{1}{S} \sum_{i=1}^{i=S} \lambda(R_{is}) \tag{61}$$

The other term in equation 61 involves $\lambda(R)$. Taking expectations, equation 2 implies that

$$\mathbb{E}[\lambda(R_s)] = \mathbb{E}[r_s] \frac{(1-\delta_s)}{\sqrt{\delta_s}\sigma_\varepsilon}; \quad \mathbb{E}[\lambda(R_n)] = \mathbb{E}[r_n] \frac{(1-\delta_n)}{\sqrt{\delta_n}\sigma_\varepsilon} \tag{62}$$

Using equation 5 one gets

$$\begin{aligned}
\bar{y}_s &= \delta \frac{1}{S} \sum_{i-1}^{i=S} r_{is} + \sqrt{\delta} \sigma_\varepsilon \frac{1}{S} \sum_{i-1}^{i=S} \lambda \left(R_s \sqrt{\frac{\delta}{\delta_s}} \right) \\
\mathbb{E}[\bar{y}_s] &= \delta \, \mathbb{E}[r_s] + \sqrt{\delta} \sigma_\varepsilon \, \mathbb{E} \left[\lambda \left(R_s \sqrt{\frac{\delta}{\delta_s}} \right) \right] \\
&\leq \delta \, \mathbb{E}[r_s] + \sqrt{\delta} \sigma_\varepsilon \, \mathbb{E}[\lambda\,(R_s)] \text{ (because } \lambda \text{ increases with } R \text{ and } \delta_s > \delta) \\
&= \delta \, \mathbb{E}[r_s] + (1-\delta_s) \, \mathbb{E}[r_s] \frac{\sqrt{\delta}}{\sqrt{\delta_s}} (\text{ by using equation 62}) \\
&\leq \mathbb{E}[r_s] \text{ because } \delta_s \geq \delta \\
\implies \mathbb{E}[\bar{y}_s] &\leq \mathbb{E}[x_s]
\end{aligned} \tag{63}$$

That is, the sample mean for S type patents using KPSS values will underestimate their true mean. A similar logic implies that the sample mean for N type patents using KPSS values with over-estimate their true value, $\mathbb{E}[\bar{y}_n] > \mathbb{E}[x_n]$. It follows that

$$\mathbb{E}[x_s] - \mathbb{E}[x_n] \geq \mathbb{E}[\bar{y}_s] - \mathbb{E}[\bar{y}_n] \tag{64}$$

Appendix D
Simulated data where signal values are drawn from two different distributions

D1 Simulated Data with Heterogeneous Signal Distributions

We conduct a simulation to compare the performance of the KPSS method with our proposed generalization when patent values are drawn from different signal distributions. The goal is to assess how imposing a common signal-to-noise ratio affects the estimated difference in average patent values across patent types.

D1.1 Data Generating Process

We simulate data for 5,000 firms observed daily over 40 years, yielding 365×40 observations per firm. For each firm i, we draw a firm-specific scale parameter σ_i uniformly from the interval $(0, 1)$. Patent-specific signals and noise are then generated as follows.

For N-type (non-science) patents, the latent signal is

$$x_n \sim \mathcal{N}^+(0, \sigma_i),$$

while for S-type (science-based) patents, the signal is

$$x_s \sim \mathcal{N}^+(0, 1.1\,\sigma_i),$$

implying $\sigma_{si} = 1.1\,\sigma_i$. The noise term follows

$$\epsilon \sim \mathcal{N}(0, 5\,\sigma_i).$$

On each day, a firm generates an N-type patent with probability 15% and an S-type patent with probability 10%. The realized return is given by

$$r_j = I(n)\,x_n + I(s)\,x_s + \epsilon,$$

where $I(n)$ and $I(s)$ are indicators for N-type and S-type patents, respectively.

D1.2 Estimation Procedure

Using the simulated returns, we first compute $\mathbb{E}[x_j \mid r_j]$ following the KPSS methodology, imposing a common signal-to-noise ratio estimated via equation 3. We then estimate separate signal-to-noise ratios for N-type and S-type patents using equation 7, and compute $\mathbb{E}[x_j \mid r_j]$ separately by patent type.

D1.3 True and Estimated Signal-to-Noise Ratios

In the simulated data, the true coefficients implied by the data-generating process are

$$\gamma_s = \ln\left(\frac{(1.1)^2 + 5^2}{5^2}\right) = 0.047 \quad \text{and} \quad \gamma_n = \ln\left(\frac{1^2 + 5^2}{5^2}\right) = 0.039.$$

When imposing a common signal-to-noise ratio (KPSS), the estimated coefficient is $\hat{\gamma} = 0.042$. In contrast, estimating separate ratios yields $\hat{\gamma}_s = 0.047$ and $\hat{\gamma}_n = 0.038$, closely matching the true values.

D1.4 Results

The simulation illustrates that imposing a common signal-to-noise ratio substantially attenuates differences in estimated patent values across types. In the data-generating process, the true difference in mean latent values satisfies

$$\mathbb{E}[x_s] - \mathbb{E}[x_n] = 0.04$$

Under the KPSS restriction of a common signal-to-noise ratio, the estimated difference is close to zero. Allowing signal-to-noise ratios to differ across patent types recovers a difference of 0.0447 , closely aligned with the true value.

Appendix Tables D1–D5 report the estimated signal-to-noise ratios and the correlations between true and estimated latent values. Overall, the simulation confirms the analytical prediction that measurement error induced by pooling heterogeneous signal distributions can materially bias inference when using KPSS values.

We note that allowing for heterogeneous signal-to-noise ratios produces estimates that are slightly larger than the true effect in the simulation. This difference does not reflect systematic overestimation, but instead arises from estimation error in recovering the signal-to-noise ratio and noise variance from realized returns. As shown in Appendix A, both KPSS and our extension rely on simplifying approximations when mapping return moments to these parameters. When the true signal-to-noise ratios and noise variances used in the data-generating process are supplied directly, as in Column (4) of Appendix Table D3, the estimated science patent premium is statistically indistinguishable from the true value. This confirms that the small upward deviation observed when parameters are estimated reflects finite-sample parameter estimation noise rather than bias introduced by our approach.

Table D1: Estimated coefficients of signal-to-noise ratio: One distribution (KPSS) vs. two distributions

	KPSS: One Distribution	Modified: Two distribution
	(1)	(2)
Science Patent Dummy		0.0475***
		(0.0010)
Patent Dummy	0.0420***	
	(0.0006)	
No Science Patent Dummy		0.0384***
		(0.0006)
Constant	-0.0319***	-0.0319***
	(0.0001)	(0.0001)
Firm*Year Fixed Effects	Yes	Yes
Return Day	Yes	Yes
R^2	0.443	0.443
N	72,995,000	72,995,000

Note: In column 1, we estimate the singal-to-noise ratio using equation 3. In column 2 we have two different signal dummies based on the distribution type and used the equation similar to equation 7 To simulate data where signal values are drawn from two different distributions, we proceed as follows. We create 5,000 firms and, for each firm, generate daily data for 365×40 days. For each firm, the ε values are drawn from a normal distribution with a mean of 0 and a standard deviation defined as a random number between 0 and 1 for each firm, multiplied by five, denoted as $\mathcal{N}(0, \sigma_i \times 5)$. The patent value x values are then generated as follows: on most days, the signal value $r = 0$. For 15% of the days, r is drawn from a half-normal distribution $\mathcal{N}^+(0, \sigma_i)$, representing nonscience patents. For 10% of the days, r is drawn from a half-normal distribution $\mathcal{N}^+(0, \sigma_i \times 1.1)$, representing science patents. The actual Science Patent dummy coefficient in our simulated data $ln(\frac{(1.1)^2+5^2}{5^2}) = 0.047$ and the No Science Patent dummy coefficient is $ln(\frac{(1)^2+5^2}{5^2}) = 0.039$ Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table D2: Difference in True vs. Estimated patent values: One distribution vs. two distributions

	True value (x_i)		Estimated value of $\mathbb{E}(x_i \mid r_i)$			
			One distribution		Two distribution	
	(1)	(2)	(3)	(4)	(5)	(6)
Science Patent Dummy	0.0401***	0.0399***	0.0008***	0.0006***	0.0449***	0.0447***
	(0.0002)	(0.0002)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
Constant	0.3992***	0.3992***	0.4131***	0.4132***	0.3950***	0.3950***
	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	Yes	No	Yes	No	Yes
R^2	0.002	0.303	0.000	0.960	0.008	0.957
N	18,247,117	18,247,117	18,247,117	18,247,117	18,247,117	18,247,117
Mean DV	0.415	0.415	0.413	0.413	0.413	0.413

Note: In columns 1 and 2, the dependent variable is the simulated values of x. In columns 3 and 4, we estimate $\mathbb{E}(x|r)$ using the KPSS methodology, while in columns 5 and 6, the dependent variable is $\mathbb{E}(x|r)$ estimated using two different signal-to-noise ratios for x_s and x_n. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table D3: Difference Between True and Estimated Patent Values: One vs. Two Distributions and Signal-to-Noise Assumptions

	Simulated Data	One distribution	Two distribution	Using Actual SNR and Variance
	(1)	(2)	(3)	(4)
Science Patent Dummy	0.040***	0.001***	0.045***	0.041***
	(0.000)	(0.000)	(0.000)	(0.000)
Constant	0.399***	0.413***	0.395***	0.403***
	(0.000)	(0.000)	(0.000)	(0.000)
Year Fixed Effects	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes
R^2	0.303	0.960	0.957	0.962
N	18,247,117	18,247,117	18,247,117	18,247,117
Mean DV	0.415	0.413	0.413	0.420

Notes: In columns 1, the dependent variable is the simulated values of x. In columns 2 , we estimate $\mathbb{E}(x|r)$ using the KPSS methodology, while in columns 3, the dependent variable is $\mathbb{E}(x|r)$ estimated using two different signal-to-noise ratios for x_s and x_n. Robust standard errors in parentheses.In the final column (4), $\mathbb{E}(x \mid r)$ is constructed using the *true* signal-to-noise ratio and the true variance of the noise term for each firm used in the data-generating process, rather than estimating these parameters from the data. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D4: Summary Statistics: True and estimated patent value

	Obs.	Mean	Std. Dev.	10%	50%	90%
True value (x_i)	18247117	0.415	0.433	0.029	0.271	1.011
Estimated Value $\mathbb{E}(x_i|r_i)$ (One distribution)	18247117	0.413	0.241	0.085	0.412	0.740
Estimated Value $\mathbb{E}(x_i|r_i)$ (two distribution)	18247117	0.413	0.242	0.085	0.410	0.740

Note: The *simulated values* are the actual simulated values of x as described in D1. The *Estimated Value (One Distribution)* represents the estimated values of $\mathbb{E}(x|r)$ following the KPSS methodology, while the *Estimated Value (Two Distributions)* refers to the estimated values of $\mathbb{E}(x|r)$ obtained using separate signal-to-noise ratios for s-type and n-type patents.

Table D5: Correlation: True and estimated patent value

	True value (x_i)	Estimated Value $\mathbb{E}(x_i|r_i)$	
		One Distribution	Two Distribution
True value (x_i)	1.000		
Estimated Value $\mathbb{E}(x_i|r_i)$ (One distribution)	0.557	1.000	
Estimated Value $\mathbb{E}(x_i|r_i)$ (two distribution)	0.559	0.995	1.000

Note: The *simulated values* are the actual simulated values of x as described in D1. The *Estimated Value (One Distribution)* represents the estimated values of $\mathbb{E}(x|r)$ following the KPSS methodology, while the *Estimated Value (Two Distributions)* refers to the estimated values of $\mathbb{E}(x|r)$ obtained using separate signal-to-noise ratios for s-type and n-type patents.

Part 2: Appendix E

Individual regressions corresponding to Tables 4 and 6

Table E1: Patent Level Regression: Inventing Team Size

	KPSS Estimates		Large and Small team Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
1[Team Size: Large]	0.034***	0.025***	0.107***	0.098***	0.241***	0.232***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Log (Market cap (const. usd))		0.751***		0.760***		0.750***
		(0.001)		(0.001)		(0.001)
Constant	15.814***	-1.407***	15.865***	-1.561***	15.734***	-1.477***
	(0.001)	(0.028)	(0.001)	(0.028)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.884	15.884	15.777	15.777
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.693	0.792	0.682	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS methodology. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Patent Value: Teamsize (separate), estimated using separate distribution assumptions for patents based on inventor team size. The value is equally distributed among all patents granted on a given day. In columns (5) and (6), the dependent variable is Patent Value: Teamsize (mixed), where we further differentiate large-team patents from small-team patents using our modified formula and estimate the patent value for the days when both types of patents were granted. A patent is classified as having a "large team" if its inventor team size falls in the top quartile within its IPC class-year. Robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Table E2: Patent Level Regression: R&D Offshoring

	KPSS Estimates		Forein and Domestic Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[All Foreign Inventor]	-0.025***	-0.008***	-0.071***	-0.055***	-0.157***	-0.140***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log (Market cap (const. usd))		0.751***		0.745***		0.751***
		(0.001)		(0.001)		(0.001)
Constant	15.823***	-1.404***	15.781***	-1.313***	15.839***	-1.382***
	(0.001)	(0.028)	(0.001)	(0.029)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.773	15.773	15.822	15.822
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.676	0.771	0.682	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS method. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Pat Value: Foreign (separate), estimated using separate distribution assumptions for patents with and without U.S.-based inventors. The value is equally distributed among all patents granted on a given day. In columns (5) and (6), the dependent variable is Pat Value: Foreign (mixed), where we further differentiate between patents with only foreign inventors and those with at least one U.S.-based inventor using our modified formula and estimate the patent value for the days when both types of patents were granted. A patent is defined as "foreign" if all inventors listed on the patent are non-U.S.-based. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table E3: Patent Level Regression: Reliance on Science

	KPSS Estimates		Science and Non-Science Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[Patent Science Dummy]	0.008***	0.015***	0.042***	0.049***	0.133***	0.140***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Log (Market cap (const. usd))		0.751***		0.756***		0.751***
		(0.001)		(0.001)		(0.001)
Constant	15.818***	-1.411***	15.847***	-1.489***	15.764***	-1.458***
	(0.001)	(0.028)	(0.001)	(0.028)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.857	15.857	15.796	15.796
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.687	0.786	0.681	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS method. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Patent Value: Sci&Non-Sci (separate), estimated using separate distribution assumptions for science-based and other patents. However, the patent value is equally distributed among all patents granted on the science patent day—days when at least one science-based patent is granted. In columns (5) and (6), the dependent variable is Patent Value: Sci&Non-Sci (mixed), where we further differentiate science-based patents from non-science-based patents using our modified formula and estimate the patent value for the days when both types of patents were granted. The "science-based patents" are defined as those ranking in the top three quartiles for the number of non-patent literature (NPL) citations within a specific IPC class and year, provided they cite at least one NPL. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table E4: Patent Level Regression: Vertical Integration

	KPSS Estimates			Vetical Intgration		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}$[Vertical Integration: High]	0.170***	0.050***	0.006***	0.403***	0.282***	0.130***
	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)
Log (Market cap (const. usd))		0.383***	0.728***		0.385***	0.748***
		(0.000)	(0.001)		(0.000)	(0.001)
Constant	15.891***	7.089***	-0.867***	15.885***	7.028***	-1.300***
	(0.001)	(0.010)	(0.030)	(0.001)	(0.010)	(0.031)
Avg DV	15.939	15.939	15.940	16.000	16.000	16.001
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	Yes	No	No	Yes
R^2	0.143	0.467	0.786	0.161	0.476	0.785
N	1,294,402	1,294,402	1,293,900	1,294,402	1,294,402	1,293,900

The dependent variable in columns (1), (2) and (3) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS methodology. The dependent variable in columns (4), (5) and (6) is the natural log transformation of the patent private value: Patent Value: Vertical Integration (separate), estimated using separate distribution assumptions for patents assigned to firms with high versus low vertical integration. The value is equally distributed among all patents granted on a given day. A firm is classified as highly vertically integrated if it falls in the top quartile of vertical integration within its industry-year, based on the Frésard, Hoberg, and Phillips (2020) dataset. Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1

Table E5: Regression All: Dependent variable: Forward Citation

	Forward Citation:All					Forward Citation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.1255*						
	(0.0654)						
Pat Val: KPSS (std.)		0.3288***					
		(0.0584)					
Pat Val: Teamsize (mixed, std.)			0.6390***				
			(0.0592)				
Pat Val: Foreign (mixed, std.)				0.3724***			
				(0.0592)			
Pat Val: Sci & NoSci (mixed, std.)					0.6522***		
					(0.0614)		
Pat Val: KPSS (VI, std.)						0.3321***	
						(0.0661)	
Pat Val: Vertical Integration (std.)							0.2960***
							(0.0619)
Log (Market cap (const. usd))	1.5276***	1.3963***	1.2726***	1.3792***	1.2679***	1.4936***	1.5091***
	(0.0705)	(0.0776)	(0.0769)	(0.0774)	(0.0764)	(0.0872)	(0.0869)
Constant	-25.4466***	-22.4650***	-19.6527***	-22.0768***	-19.5402***	-22.9733***	-23.3195***
	(1.5342)	(1.6980)	(1.6825)	(1.6943)	(1.6725)	(1.9476)	(1.9423)
Avg DV	14.407	14.407	14.407	14.407	14.407	15.661	15.661
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-6.90e+06	-6.90e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable *Patent Forward Cites* measures the number of forward citations received by a patent within five years of grant. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are reported in parentheses. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E6: Regression All: Dependent variable: Breakthrough Citation

	Breakthrough:All					Breakthrough: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0214**						
	(0.0104)						
Pat Val: KPSS (std.)		0.0767***					
		(0.0164)					
Pat Val: Teamsize (mixed, std.)			0.1267***				
			(0.0167)				
Pat Val: Foreign (mixed, std.)				0.0829***			
				(0.0164)			
Pat Val: Sci & NoSci (mixed, std.)					0.1197***		
					(0.0166)		
Pat Val: KPSS (VI, std.)						0.0678***	
						(0.0171)	
Pat Val: Vertical Integration (std.)							0.0549***
							(0.0165)
Log (Market cap (const. usd))	0.0349*	0.0042	-0.0157	0.0018	-0.0128	-0.0114	-0.0062
	(0.0194)	(0.0204)	(0.0204)	(0.0204)	(0.0204)	(0.0222)	(0.0222)
Constant	0.7046	1.4018***	1.8545***	1.4564***	1.7892***	2.2403***	2.1232***
	(0.4360)	(0.4616)	(0.4603)	(0.4611)	(0.4601)	(0.5229)	(0.5225)
Avg DV	1.283	1.283	1.283	1.283	1.283	1.295	1.295
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.52e+06	-5.52e+06	-5.52e+06	-5.52e+06	-5.52e+06	-4.95e+06	-4.95e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Top 1% of Cited Patents] equals one if a patent belongs to the top 1 percent of the forward-citation distribution among patents granted in the same year and IPC class, and zero otherwise; this variable is multiplied by 100 for ease of interpretation (Squicciarini, Dernis, & Criscuolo, 2013). The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E7: Regression All: Dependent variable: Renew

	Renewed Full Term:All					Renewed Full Term: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	-0.0009						
	(0.0013)						
Pat Val: KPSS (std.)		0.0255***					
		(0.0009)					
Pat Val: Teamsize (mixed, std.)			0.0264***				
			(0.0009)				
Pat Val: Foreign (mixed, std.)				0.0261***			
				(0.0009)			
Pat Val: Sci & NoSci (mixed, std.)					0.0252***		
					(0.0009)		
Pat Val: KPSS (VI, std.)						0.0260***	
						(0.0009)	
Pat Val: Vertical Integration (std.)							0.0257***
							(0.0009)
Log (Market cap (const. usd))	-0.0073***	-0.0191***	-0.0194***	-0.0193***	-0.0189***	-0.0189***	-0.0184***
	(0.0015)	(0.0016)	(0.0016)	(0.0016)	(0.0016)	(0.0016)	(0.0016)
Constant	0.7221***	0.9892***	0.9955***	0.9945***	0.9841***	0.9860***	0.9765***
	(0.0342)	(0.0352)	(0.0350)	(0.0351)	(0.0352)	(0.0352)	(0.0350)
Avg DV	0.540	0.540	0.540	0.540	0.540	0.540	0.540
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.91e+05	-4.90e+05	-4.90e+05	-4.90e+05	-4.90e+05	-4.88e+05	-4.88e+05
N	774,044	774,044	774,044	774,044	774,044	770,304	770,304

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Renewed (Full Term)] equals one if the patent is renewed through the full statutory term, and zero otherwise, based on observed maintenance fee payments. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E8: Regression All: Dependent variable: Litigation Citation

	Litigation:All					Litigation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0005						
	(0.0045)						
Pat Val: KPSS (std.)		0.0849***					
		(0.0131)					
Pat Val: Teamsize (mixed, std.)			0.0933***				
			(0.0129)				
Pat Val: Foreign (mixed, std.)				0.0864***			
				(0.0131)			
Pat Val: Sci & NoSci (mixed, std.)					0.0920***		
					(0.0132)		
Pat Val: KPSS (VI, std.)						0.0908***	
						(0.0146)	
Pat Val: Vertical Integration (std.)							0.0897***
							(0.0143)
Log (Market cap (const. usd))	-0.0706***	-0.1048***	-0.1080***	-0.1053***	-0.1074***	-0.1190***	-0.1179***
	(0.0158)	(0.0166)	(0.0166)	(0.0166)	(0.0166)	(0.0185)	(0.0185)
Constant	1.8465***	2.6242***	2.6970***	2.6353***	2.6835***	3.4168***	3.3959***
	(0.3490)	(0.3667)	(0.3663)	(0.3668)	(0.3662)	(0.4237)	(0.4255)
Avg DV	0.589	0.589	0.589	0.589	0.589	0.631	0.631
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.47e+06	-4.47e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Litigation] equals one if the patent is involved in at least one patent litigation event. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E9: Regression All: Dependent variable: Reassign Citation

	Reassign: All					Reassign: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0002						
	(0.0003)						
Pat Val: KPSS (std.)		0.0039***					
		(0.0004)					
Pat Val: Teamsize (mixed, std.)			0.0056***				
			(0.0004)				
Pat Val: Foreign (mixed, std.)				0.0036***			
				(0.0004)			
Pat Val: Sci & NoSci (mixed, std.)					0.0042***		
					(0.0004)		
Pat Val: KPSS (VI, std.)						0.0033***	
						(0.0004)	
Pat Val: Vertical Integration (std.)							0.0031***
							(0.0004)
Log (Market cap (const. usd))	0.0012**	-0.0004	-0.0011**	-0.0003	-0.0005	-0.0003	-0.0002
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
Constant	-0.0181	0.0173	0.0334***	0.0148	0.0204*	0.0691***	0.0672***
	(0.0113)	(0.0119)	(0.0118)	(0.0119)	(0.0119)	(0.0138)	(0.0138)
Avg DV	0.074	0.074	0.074	0.074	0.074	0.078	0.078
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-6.09e+04	-6.08e+04	-6.07e+04	-6.08e+04	-6.08e+04	-8.85e+04	-8.85e+04
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Reassignment] equals one if the patent experiences at least one ownership reassignment during its lifetime. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E10: Regression All: Dependent variable: Trilateral Patent Citation

	Trilateral Patent:All					Trilateral Patent:VI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	-0.0003						
	(0.0004)						
Pat Val: KPSS (std.)		0.0129***					
		(0.0007)					
Pat Val: Teamsize (mixed, std.)			0.0142***				
			(0.0008)				
Pat Val: Foreign (mixed, std.)				0.0123***			
				(0.0007)			
Pat Val: Sci & NoSci (mixed, std.)					0.0137***		
					(0.0008)		
Pat Val: KPSS (VI, std.)						0.0063***	
						(0.0007)	
Pat Val: Vertical Integration (std.)							0.0063***
							(0.0007)
Log (Market cap (const. usd))	0.0276***	0.0224***	0.0219***	0.0226***	0.0221***	0.0149***	0.0149***
	(0.0009)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)
Constant	-0.5292***	-0.4114***	-0.3999***	-0.4164***	-0.4047***	-0.0422**	-0.0429**
	(0.0192)	(0.0183)	(0.0182)	(0.0183)	(0.0183)	(0.0178)	(0.0178)
Avg DV	0.209	0.209	0.209	0.209	0.209	0.210	0.210
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.92e+05	-5.91e+05	-5.91e+05	-5.91e+05	-5.91e+05	-5.16e+05	-5.16e+05
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable is $\mathbb{1}$[Trilateral Patent] equals one if the patent belongs to a triadic patent family with filings at the USPTO, EPO, and JPO, and zero otherwise. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E11: Regression All: Dependent variable: Forward Citation

	Forward Citation:All					Forward Citation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.1425**						
	(0.0659)						
Pat Val: KPSS (within-IPC std.)		0.4804***					
		(0.0533)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.7630***				
			(0.0549)				
Pat Val: Foreign (mixed, within-IPC std.)				0.5207***			
				(0.0537)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.7715***		
					(0.0564)		
Pat Val: KPSS (VI, within-IPC std.)						0.4082***	
						(0.0594)	
Pat Val: Vertical Integration (within-IPC std.)							0.2565***
							(0.0557)
Within-IPC demeaned market cap	1.5001***	1.3731***	1.2985***	1.3629***	1.2968***	1.3988***	1.4369***
	(0.0537)	(0.0567)	(0.0562)	(0.0566)	(0.0563)	(0.0625)	(0.0624)
Constant	-2.8561***	-2.8047***	-2.7737***	-2.8015***	-2.7666***	-1.9646***	-1.9874***
	(0.1013)	(0.1012)	(0.1015)	(0.1012)	(0.1016)	(0.1358)	(0.1359)
Avg DV	0.001	0.001	0.001	0.001	0.001	0.678	0.678
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-6.90e+06	-6.90e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable *Patent Forward Cites* measures the number of forward citations received by a patent within five years of grant. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are reported in parentheses. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E12: Regression All: Dependent variable: Breakthrough Citation

	Breakthrough:All					Breakthrough: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0231**						
	(0.0103)						
Pat Val: KPSS (within-IPC std.)		0.1048***					
		(0.0153)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.1496***				
			(0.0154)				
Pat Val: Foreign (mixed, within-IPC std.)				0.1103***			
				(0.0153)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.1441***		
					(0.0153)		
Pat Val: KPSS (VI, within-IPC std.)						0.0964***	
						(0.0152)	
Pat Val: Vertical Integration (within-IPC std.)							0.0789***
							(0.0149)
Within-IPC demeaned market cap	-0.0140	-0.0418***	-0.0536***	-0.0431***	-0.0520***	-0.0437***	-0.0389***
	(0.0130)	(0.0134)	(0.0134)	(0.0134)	(0.0134)	(0.0144)	(0.0144)
Constant	0.2432**	0.2543***	0.2593***	0.2547***	0.2599***	0.7281***	0.7279***
	(0.0983)	(0.0982)	(0.0981)	(0.0982)	(0.0981)	(0.1331)	(0.1330)
Avg DV	-0.000	-0.000	-0.000	-0.000	-0.000	0.006	0.006
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.51e+06	-5.51e+06	-5.51e+06	-5.51e+06	-5.51e+06	-4.95e+06	-4.95e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Top 1% of Cited Patents] equals one if a patent belongs to the top 1 percent of the forward-citation distribution among patents granted in the same year and IPC class, and zero otherwise; this variable is multiplied by 100 for ease of interpretation (Squicciarini, Dernis, & Criscuolo, 2013). The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E13: Regression All: Dependent variable: Renew

	Renewed Full Term:All					Renewed Full Term: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	-0.0009						
	(0.0013)						
Pat Val: KPSS (within-IPC std.)		0.0235***					
		(0.0008)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0242***				
			(0.0008)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0240***			
				(0.0008)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0231***		
					(0.0008)		
Pat Val: KPSS (VI, within-IPC std.)						0.0232***	
						(0.0008)	
Pat Val: Vertical Integration (within-IPC std.)							0.0229***
							(0.0008)
Within-IPC demeaned market cap	-0.0157***	-0.0226***	-0.0228***	-0.0228***	-0.0224***	-0.0225***	-0.0222***
	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)
Constant	0.0171***	0.0177***	0.0178***	0.0177***	0.0178***	0.0192***	0.0203***
	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0039)	(0.0039)
Avg DV	0.000	0.000	0.000	0.000	0.000	-0.000	-0.000
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.88e+05	-4.87e+05	-4.87e+05	-4.87e+05	-4.87e+05	-4.84e+05	-4.84e+05
N	774,044	774,044	774,044	774,044	774,044	770,304	770,304

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Renewed (Full Term)] equals one if the patent is renewed through the full statutory term, and zero otherwise, based on observed maintenance fee payments. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E14: Regression All: Dependent variable: Litigation Citation

	Litigation:All					Litigation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0000						
	(0.0045)						
Pat Val: KPSS (within-IPC std.)		0.1165***					
		(0.0115)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.1239***				
			(0.0114)				
Pat Val: Foreign (mixed, within-IPC std.)				0.1177***			
				(0.0116)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.1229***		
					(0.0116)		
Pat Val: KPSS (VI, within-IPC std.)						0.1173***	
						(0.0129)	
Pat Val: Vertical Integration (within-IPC std.)							0.1124***
							(0.0127)
Within-IPC demeaned market cap	-0.1554***	-0.1866***	-0.1884***	-0.1868***	-0.1880***	-0.1954***	-0.1934***
	(0.0103)	(0.0107)	(0.0107)	(0.0107)	(0.0106)	(0.0117)	(0.0117)
Constant	-0.3638***	-0.3515***	-0.3505***	-0.3516***	-0.3497***	0.1237	0.1281
	(0.0270)	(0.0271)	(0.0271)	(0.0271)	(0.0271)	(0.0784)	(0.0785)
Avg DV	-0.001	-0.001	-0.001	-0.001	-0.001	0.033	0.033
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.47e+06	-4.47e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Litigation] equals one if the patent is involved in at least one patent litigation event. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E15: Regression All: Dependent variable: Reassign Citation

	Reassign: All					Reassign: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0002						
	(0.0003)						
Pat Val: KPSS (within-IPC std.)		0.0051***					
		(0.0003)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0067***				
			(0.0004)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0049***			
				(0.0003)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0054***		
					(0.0003)		
Pat Val: KPSS (VI, within-IPC std.)						0.0043***	
						(0.0004)	
Pat Val: Vertical Integration (within-IPC std.)							0.0040***
							(0.0004)
Within-IPC demeaned market cap	-0.0021***	-0.0035***	-0.0039***	-0.0034***	-0.0035***	-0.0036***	-0.0035***
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
Constant	-0.0636***	-0.0630***	-0.0629***	-0.0631***	-0.0630***	-0.0100***	-0.0099***
	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0025)	(0.0025)
Avg DV	-0.000	-0.000	-0.000	-0.000	-0.000	0.004	0.004
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.96e+04	-5.95e+04	-5.93e+04	-5.95e+04	-5.94e+04	-8.71e+04	-8.71e+04
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Reassignment] equals one if the patent experiences at least one ownership reassignment during its lifetime. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E16: Regression All: Dependent variable: Trilateral Patent Citation

	Trilateral Patent:All					Trilateral Patent:VI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	-0.0003						
	(0.0004)						
Pat Val: KPSS (within-IPC std.)		0.0216***					
		(0.0007)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0228***				
			(0.0007)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0211***			
				(0.0007)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0224***		
					(0.0007)		
Pat Val: KPSS (VI, within-IPC std.)						0.0148***	
						(0.0007)	
Pat Val: Vertical Integration (within-IPC std.)							0.0143***
							(0.0006)
Within-IPC demeaned market cap	-0.0006	-0.0064***	-0.0066***	-0.0062***	-0.0065***	-0.0122***	-0.0119***
	(0.0006)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0005)	(0.0005)
Constant	-0.1472***	-0.1450***	-0.1448***	-0.1451***	-0.1447***	0.0727***	0.0732***
	(0.0033)	(0.0034)	(0.0034)	(0.0034)	(0.0034)	(0.0048)	(0.0048)
Avg DV	0.000	0.000	0.000	0.000	0.000	0.006	0.006
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.96e+05	-5.94e+05	-5.94e+05	-5.94e+05	-5.94e+05	-5.18e+05	-5.18e+05
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable is $\mathbb{1}$[Trilateral Patent] equals one if the patent belongs to a triadic patent family with filings at the USPTO, EPO, and JPO, and zero otherwise. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at t–1) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Appendix F

Robustness Analysis

Robustness tests, repeating Tables 2–6 using alternative measures of team size, offshoring, science, and vertical integration

Table F1: Summary Statistics: Patent Level

	Obs.	Mean	Std. Dev.	10%	50%	90%
$\mathbb{1}$[Large Team (median)]	1444649	0.375	0.484	0.000	0.000	1.000
$\mathbb{1}$[2/3 Foreign]	1444649	0.123	0.329	0.000	0.000	1.000
$\mathbb{1}$[Science Dummy (median)]	1444649	0.160	0.367	0.000	0.000	1.000
$\mathbb{1}$[Vertical Integration(median)]: High	1295446	0.550	0.498	0.000	1.000	1.000
Pat Val: KPSS (const. usd)	1444638	16.117	26.003	1.450	7.433	37.405
Pat Val: Teamsize (median) (mixed)	1444638	15.764	25.539	1.415	7.233	36.686
Pat Val:2/3 Foreign (mixed)	1444638	16.120	26.050	1.443	7.428	37.425
Pat Val: Sci&NoSci (median) (mixed)	1444638	15.762	25.629	1.413	7.207	36.550
Pat Val: KPSS (Verti Ineg.)	1295089	19.941	47.919	1.609	8.346	42.862
Pat Val: Vertical Integration (median) (sep)	1295089	19.139	46.096	1.335	7.802	40.695

Notes: This table reports summary statistics at the patent level. $\mathbb{1}$[Large Team (median)] equals one if the inventor team size is above the median within the same IPC class–year. $\mathbb{1}$[2/3 Foreign] equals one if at least two-thirds of inventors listed on the patent are non-U.S. based. $\mathbb{1}$[Science Dummy (median)] equals one if the patent is science-based, defined as having above-median non-patent literature (NPL) citations within an IPC class–year, conditional on citing at least one NPL. $\mathbb{1}$[Vertical Integration (median): High] equals one if the assignee's vertical integration measure is above the median within the same industry–year, based on (Frésard, Hoberg, & Phillips, 2020). Patent value measures are expressed in constant U.S. dollars. *Pat Val: KPSS (const. USD)* reports baseline patent values estimated using the standard KPSS methodology. *Pat Val: Team Size (median) (mixed)*, *Pat Val: 2/3 Foreign (mixed)*, and *Pat Val: Sci&NoSci (median) (mixed)* report patent values estimated using the generalized KPSS approach with mixed distributions corresponding to the indicated patent characteristics. *Pat Val: KPSS (Vertical Integration)* and *Pat Val: Vertical Integration (median) (sep)* report patent values for the subsample with available vertical-integration data, using baseline KPSS and characteristic-specific (separate) signal to noise ratio, respectively.

Table F2: Patent Level Regression: Alternate Definition

	Teamsize		R&D Offshoring		Science		Vertical Integration	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\mathbb{1}$[Large Team (median)]	0.010***	0.121***						
	(0.001)	(0.001)						
$\mathbb{1}$[2/3 Foreign]			-0.009***	-0.064***				
			(0.002)	(0.002)				
$\mathbb{1}$[Science Dummy (median)]					0.017***	0.167***		
					(0.001)	(0.001)		
$\mathbb{1}$[Vertical Integration(median)]: High							-0.008***	0.567***
							(0.002)	(0.002)
Log (Market cap (const. usd))	0.751***	0.751***	0.751***	0.751***	0.751***	0.751***	0.729***	0.729***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Constant	-1.409***	-1.469***	-1.403***	-1.391***	-1.409***	-1.453***	-0.876***	-1.276***
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.030)	(0.030)
Avg DV	15.820	15.795	15.820	15.818	15.820	15.792	15.940	15.857
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.778	0.778	0.778	0.778	0.778	0.778	0.786	0.800
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,293,900	1,293,900

Notes: This table reports patent-level regressions using alternate definitions of key explanatory variables and patent-value estimation methods. The dependent variable in Columns (1), (3), (5) and (7) is the natural logarithm of patent private value estimated using the baseline KPSS methodology, which assumes a common signal-to-noise ratio across all patents. The dependent variable in Columns (2), (4), (6) and (8) is the natural logarithm of patent private value estimated using the generalized KPSS methodology, which allows the signal-to-noise ratio to vary by patent type. $\mathbb{1}$[Large Team (median)] equals one if the inventor team size is above the median within the same IPC class–year. $\mathbb{1}$[2/3 Foreign] equals one if at least two-thirds of inventors listed on the patent are non-U.S. based. $\mathbb{1}$[Science Dummy (median)] equals one if the patent is science-based, defined as having above-median non-patent literature (NPL) citations within an IPC class–year, conditional on citing at least one NPL. $\mathbb{1}$[Vertical Integration (median): High] equals one if the assignee's vertical integration measure is above the median within the same industry–year, based onFrésard, Hoberg, and Phillips (2020). All specifications include year fixed effects, four-digit IPC fixed effects, and firm fixed effects. Robust standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table F3: Regression: Abnormal return as the dependent variable (Alternate Definition)

	Abnormal Return			
	(1)	(2)	(3)	(4)
Patent day	0.0261***	0.0240***	0.0247***	0.0239***
	(0.0051)	(0.0045)	(0.0047)	(0.0062)
Patent day: Large Team Pat (median)	0.0154***			
	(0.0059)			
Patent day: Foreign Invt Pat (2/3)		0.0073		
		(0.0069)		
Patent day: Science Pat (median)			0.0167**	
			(0.0069)	
Patent day: High VI Firm (median)				0.0286***
				(0.0089)
Number of Patent	0.0005	0.0000	0.0002	0.0007
	(0.0007)	(0.0004)	(0.0006)	(0.0006)
Log(Market Cap (const. usd))	-0.1597***	-0.1214***	-0.1505***	-0.1760***
	(0.0037)	(0.0039)	(0.0042)	(0.0050)
Constant	3.1375***	2.4842***	3.0247***	3.5003***
	(0.0722)	(0.0795)	(0.0839)	(0.0981)
Avg DV	0.039	0.037	0.040	0.044
Firm Fixed Effects	Yes	Yes	Yes	Yes
Business Date	Yes	Yes	Yes	Yes
R^2	0.017	0.020	0.019	0.016
N	14,163,165	7,853,417	9,919,551	9,318,888

Note: This table presents robustness checks using alternate definitions of patent characteristics. The table reports firm-level regressions of abnormal stock returns around patent grant dates. Large-team, science-based, and vertical-integration indicators are redefined using median-based thresholds within IPC class–year (or industry–year for vertical integration), while the foreign-inventor indicator equals one if at least two-thirds of inventors listed on the patent are located outside the United States. The dependent variable is the cumulative abnormal return over the three-day window following the patent grant date. The indicator *Patent day* equals one on days when a firm is granted at least one patent. All columns restrict the sample to firms that have both patent and non-patent days in the estimation period. Columns (1)–(4) further restrict the sample to firms that exhibit within-firm variation in the corresponding patent-type indicator: large-team patents (Column 1), patents with all non-U.S. inventors (Column 2), science-based patents (Column 3), and patents granted by firms with high vertical integration (Column 4). All specifications include firm fixed effects and business-date fixed effects. Control variables include the log of firm market value and the number of patents granted on the event day. Standard errors are clustered at the firm level.

Table F4: Signal to Noise ratio

	Baseline	Team Size	R&D Offshoring	Science	Vertical Integration	
	(1)	(2)	(3)	(4)	(5)	(6)
Patent day	0.019***				0.023***	
	(0.005)				(0.005)	
Patent day: Large Team Pat (median)		0.021***				
		(0.007)				
Patent day: No Large Team Pat (median)		0.017***				
		(0.006)				
Patent day: Foreign Invt Pat (2/3)			0.018*			
			(0.010)			
Patent day: No Foreign Invt Pat (2/3)			0.019***			
			(0.005)			
Patent day: Science Pat (median)				0.024***		
				(0.008)		
Patent day: No Science Pat (median)				0.017***		
				(0.006)		
Patent day: High VI Firm (median)						0.032***
						(0.007)
Patent day: Low VI Firm (median)						0.010
						(0.007)
Constant	-7.622***	-7.622***	-7.622***	-7.622***	-7.561***	-7.561***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Avg DV	-7.622	-7.622	-7.622	-7.622	-7.561	-7.561
Firm*Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Date FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.169	0.169	0.169	0.169	0.177	0.177
N	16,637,718	16,637,718	16,637,718	16,637,718	13,277,826	13,277,826

Notes: This table reports estimates from regressions of firm-level abnormal stock returns around patent grant dates. The dependent variable is the three-day idiosyncratic return of firm f following patent grant date d, denoted r_{fd}. All specifications include firm-by-year fixed effects and grant-date fixed effects. Column (1) and (5) implements the baseline KPSS specification, which estimates a single patent-day effect and implicitly assumes a common signal-to-noise ratio across patent types. Columns (2), (3), (4) and (6) relax this restriction by allowing patent-day effects to vary across dimensions of heterogeneity: inventor team size, R&D offshoring, reliance on basic science, and vertical integration. This table also reports robustness checks using alternate definitions of patent characteristics; large-team, science-based, and vertical-integration indicators are defined using median-based thresholds within IPC class–year (or industry–year for vertical integration), while R&D offshoring is defined based on the share of foreign inventors. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table F5: Regression: Abnormal Returns (Adjusted for Grant Probability) as the Dependent Variable

	Abnormal Return			
	(1)	(2)	(3)	(4)
Patent day	0.0694***	0.0764***	0.0637***	0.0471***
	(0.0090)	(0.0085)	(0.0089)	(0.0121)
Patent day: Large Team Pat	0.0232*			
	(0.0127)			
Patent day: Foreign Invt Pat		-0.0117		
		(0.0143)		
Patent day: Science Pat			0.0380***	
			(0.0138)	
Patent day: High VI Firm				-0.0041
				(0.0213)
Number of Patent	-0.0005	-0.0005	-0.0006	0.0004
	(0.0005)	(0.0004)	(0.0005)	(0.0006)
Log(Market Cap (const. usd))	-0.1550***	-0.1284***	-0.1624***	-0.1701***
	(0.0040)	(0.0042)	(0.0043)	(0.0069)
Constant	3.0803***	2.6293***	3.2467***	3.4282***
	(0.0778)	(0.0859)	(0.0840)	(0.1379)
Avg DV	0.043	0.042	0.045	0.042
Firm Fixed Effects	Yes	Yes	Yes	Yes
Business Date	Yes	Yes	Yes	Yes
R^2	0.017	0.020	0.018	0.016
N	11,963,634	7,532,889	10,861,460	5,479,710

Note: This table reports firm-level regressions of abnormal stock returns around patent grant dates, where returns are adjusted for the probability of patent grant. The dependent variable is the cumulative abnormal return over the three-day window following the patent grant date. On grant days, returns are scaled by the inverse of the ex-ante probability of patent success, $(1-\pi)^{-1}$. The indicator *Patent day* equals one on days when a firm is granted at least one patent. All columns restrict the sample to firms that have both patent and non-patent days in the estimation period. Columns (1)–(4) further restrict the sample to firms that exhibit within-firm variation in the corresponding patent-type indicator: large-team patents (Column 1), patents with all non-U.S. inventors (Column 2), science-based patents (Column 3), and patents granted by firms with high vertical integration (Column 4). All specifications include firm fixed effects and business-date fixed effects. Control variables include the log of firm market value and the number of patents granted on the event day. Standard errors are clustered at the firm level.