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WHAT DRIVES INFLATION? LESSONS FROM DISAGGREGATED PRICE DATA

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ABSTRACT

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Abstract

The COVID pandemic triggered high inflation alongside large relative price movements. I document that this comovement is not unique to the COVID episode, and follows a systematic pattern: the relative price of industries with flexible prices and inelastic factor supply rises with the aggregate output gap, and, controlling for the output gap, aggregate inflation rises with the relative price of those same industries. I show that both facts arise naturally in a multi-sector New Keynesian model where industries have heterogeneous price adjustment frequencies and factor supply elasticities. Through the model, I compute the share of consumer price inflation due to aggregate demand misalignment versus relative price adjustments, and construct price indices which forecast inflation at different horizons. Applying this framework to US data, relative price adjustments account for 49% of inflation fluctuations between 2000 and 2019, but only for 31% over the COVID period (2020-2023).

1 Introduction

The aftermath of the COVID-19 pandemic saw the highest inflation rates in forty years. Supply chains and labor markets faced unprecedented disruptions, with markedly different consequences across industries. Disruptions in industry-specific demand and supply caused large relative price adjustments, and there was widespread belief that these relative price movements might be a driver of aggregate inflation. For example, as demand for manufacturing increased relative to services, the adjustment process might have led to temporarily higher inflation. Governments concurrently implemented unprecedented stimulus packages, while central banks maintained low interest rates, further increasing the inflationary pressure. How much of the surge in aggregate inflation can be attributed to

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aggregate expansionary policies versus industry-specific disruptions? Distinguishing these two components is essential to understanding what policy should do, and to forecasting what inflation will do.¹

Section 2 begins by noting that comovement between relative prices and aggregate inflation is not new to the COVID episode (Ball and Mankiw, 1995; Reis and Watson, 2010), and it follows a specific pattern. Classifying industries by their price adjustment frequency and their reliance on inelastically supplied inputs (such as structures and heavy equipment), I find a two-way correlation. On the one hand, the relative price of industries with flexible prices and inelastic supply is positively correlated with measures of aggregate demand. On the other hand, controlling for aggregate demand, aggregate inflation is positively correlated with the relative price of these same industries. The pattern holds during both expansions and contractions.

These correlations highlight that relative prices and aggregate inflation are intertwined. Shocks that trigger relative price adjustments may affect aggregate inflation, even if there is no excess demand. Likewise, relative prices may be endogenous to aggregate demand.² This makes it challenging to identify the effect of sectoral shocks on aggregate inflation, and using historical correlations could be misleading. Isolating the contribution of industry-specific shocks requires measuring them directly, or relying on structural assumptions.

I take a structural approach, noting that the empirical patterns documented in section 2 arise naturally in the context of disaggregated New Keynesian models. In section 3 I introduce a multi-sector economy with input-output linkages, industry-specific price rigidities, and multiple imperfectly substitutable primary factors with heterogeneous supply elasticities. As in Rubbo (2023), the model predicts that industries with more flexible prices are more sensitive to aggregate demand, and at the same time have a larger incidence on aggregate inflation. I show that the same is true of industries which rely on inelastically supplied factors.³

The key intuition is that industries not only face idiosyncratic shocks, but also respond to shocks with different elasticities, depending on their ability to adjust prices or scale up supply. Industries with more flexible prices or less scalable supply raise prices by more in response to both aggregate demand and their own cost shocks. As a result, whenever industries face different nominal rigidities or supply elasticities, the aggregate output gap affects relative prices, and conversely relative price adjustments affect aggregate inflation.

Building on this premise, in section 4 I propose a method to tease apart the components of inflation driven by aggregate demand misalignment versus relative price adjustments. These two drivers have natural model counterparts, given by the aggregate output gap and changes in the natural relative prices. I show how to infer them from

¹This paper is intended as a positive study of inflation drivers. My framework does not capture prominent policy considerations, such as the costs of firm-worker separations and labor market scarring. Therefore I refrain from normative statements. Rubbo (2023) provides normative results in a similar environment.

²Lorenzoni and Werning (2023) and Guerrieri et al. (2023) make a similar argument.

³Earlier multi-sector models capture only the first mechanism, letting price-adjustment frequencies differ across industries within an input-output network (Pasten et al., 2020; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). The second mechanism – heterogeneity in the ability to scale up supply – has been left aside so far. I argue it is equally important, from both a conceptual and a quantitative standpoint. Modeling it requires multiple, imperfectly substitutable primary factors, so that industries face different supply elasticities even under constant returns to scale. Earlier models, by contrast, assume a single unsegmented labor market, so that all industries face the same supply elasticity. Among other limitations, this precludes the modeling of supply bottlenecks.

observed sectoral inflation rates, by inverting the model. Changes in the output gap and natural relative prices arise from the interaction of several underlying shocks, which my method cannot identify separately. The output gap is the difference between realized output and the natural level of output that would prevail in an economy with flexible prices. Output gaps are jointly determined by aggregate shocks (productivity, markups, discount rates) and monetary-fiscal policies. Natural relative prices are driven by idiosyncratic productivity and markup shocks, and relative preference shifts across goods.⁴

I express inflation at each point in time as a discounted sum of current, past, and expected future output gaps and natural relative price changes. Discounting of past and future shocks is governed by a multidimensional discount factor, which captures persistent and anticipatory effects. Operationally, the dynamic path of output gaps and natural relative prices can be identified simply by tracking appropriately-weighted inflation indices. Likewise, I show that model-consistent inflation expectations at every horizon can also be computed by tracking appropriately weighted combinations of industry-level inflation rates.

My method is akin to a factor decomposition, in that it defines a set of inflation drivers and computes the loadings of industry-level inflation rates on those drivers. The difference from conventional decompositions consists in deriving the loadings from a structural model, rather than imposing model-free identification restrictions. For example, principal component analysis (PCA) restricts an industry’s loading on one factor to be uncorrelated with its loadings on other factors. Through the lens of my model, reading estimated principal components as aggregate and industry-specific drivers requires precisely this orthogonality – which, as the model establishes, only holds if all industries have identical network-adjusted nominal rigidities and factor supply elasticities.

My identification results also speak to a recent literature, which models the connection between industry-specific shocks and aggregate inflation as arising from non-linearities in production or pricing (Baqaae and Farhi, 2022; Comin et al., 2023; Bai et al., 2024). Non-linearities are one way of generating heterogeneous supply and/or pricing elasticities, and it is this heterogeneity that links relative prices with aggregate inflation. In non-linear models heterogeneity emerges ex-post, as shocks move industries toward flatter or steeper portions of their supply curves, whereas in linear settings it must be present ex-ante. Ex-ante and ex-post heterogeneity are not observationally equivalent. With ex-ante heterogeneity the aggregate output gap has a first-order effect on relative prices, while with ex-post heterogeneity it does not, and it only affects relative prices in combination with industry-specific shocks. Therefore, given the same observed inflation rates, a model in which heterogeneity arises only ex-post attributes a larger share of aggregate inflation to industry-specific shocks.

Section 5 applies my decomposition to inflation in the US over 2000–2024. I measure the average share of aggregate inflation changes explained by each of the two drivers. I find that relative price adjustments are an important driver

⁴In any multi-industry model, the natural relative prices respond mechanically to industry-level productivity and markups. Capturing the effect of relative preference shifts – a prevalent feature of the COVID pandemic – further requires multiple imperfectly substitutable primary factors. When demand shifts toward one industry, factor market segmentation prevents it from drawing resources away from other industries, so it must raise its relative price. With a single primary factor all industries draw on the same pool of resources, therefore changes in demand composition have no effect on relative prices.

between 2000 and 2019, explaining 49% of inflation fluctuations. Aggregate demand misalignment instead was the primary driver during the COVID episode, explaining 32% of the disinflation between 2020Q1–2020Q4 and 69% of the surge over 2021Q2–2023Q1. Inflation returned above the 2% target in 2021Q1, when relative price adjustments raised inflation 0.7 percentage points above target, while the output gap was still subtracting 0.3 percentage points.

I next turn to quantifying the interconnections between relative prices and aggregate inflation. I first compute a time series of sectoral relative price shocks, and find that the natural prices of oil, manufacturing, and construction increased significantly during the COVID pandemic, while the natural price of leisure and hospitality services increased moderately. Feeding estimated relative price shocks into the model reveals that the oil industry had the largest impact on aggregate inflation. I then estimate the component of relative price changes driven by the aggregate output gap, for a selected subset of industries (oil, utilities, food, vehicle dealers). This component is non-negligible for food and utilities, confirming that one cannot attribute relative price changes entirely to industry-specific shocks.

Counterfactual exercises show that both dimensions of industry heterogeneity – differences in how quickly cost shocks feed through into prices and differences in the ability to scale up supply – matter for the joint behavior of relative prices and aggregate inflation. Finally, I compute model-based inflation expectations and forecast revisions over time. I find that inflationary surprises continued to hit the economy until early 2023, driven by relative price shocks through early 2021 and by aggregate demand thereafter.

1.1 Related Literature

A long-standing empirical literature (Ball and Mankiw, 1995; Reis and Watson, 2010) established that episodes of high aggregate inflation are typically associated with large relative price changes.⁵ The COVID pandemic sparked renewed interest in the relationship between industry-specific shocks, relative price changes, and aggregate inflation. A large body of work sought to decompose aggregate inflation into its underlying drivers, through the lens of three main frameworks: VAR-based empirical analysis, linearized New Keynesian models, and non-linear models of supply-side disruptions.⁶ This paper sheds light on the economic mechanisms embedded in linearized, multi-sector New Keynesian models. It establishes that heterogeneous price adjustment frequencies or factor supply elasticities are necessary to rationalize the comovement between relative prices and aggregate inflation, and it shows how to identify their common drivers. While much of the literature focuses on the COVID episode, my analysis extends to the past twenty-five years, revealing that the importance of different inflation drivers varied substantially over time.

Several authors identified demand-, supply- and expectation-driven inflation components based on VAR or regression analysis (Beaudry et al., 2024; Giannone and Primiceri, 2024).⁷ Empirical models (Ball et al., 2022; Dao et al., 2024;

⁵Ball and Mankiw (1995) first documented that movements in aggregate inflation are associated with large price changes in a small number of industries, which they attributed to non-linearities in price adjustment due to menu costs. Reis and Watson (2010) found that a significant share of the variance of aggregate inflation is explained by principal components associated with relative price shocks.

⁶Pflueger (2023) takes a different approach, arguing that bond-stock betas are informative of beliefs about shock processes and the monetary policy rule.

⁷Beaudry et al. (2024) argue that inflation movements are almost entirely explained by inflation expectations. Giannone and Primiceri

Shapiro, 2020, 2022; Blanchard and Bernanke, 2023) typically treat relative price changes – such as increases in headline inflation relative to core, or food, car, and energy prices relative to the CPI – as exogenous.⁸ I emphasize that relative price movements not only depend on the industries’ own shocks, but also on aggregate demand and spillovers through input-output linkages. This leads me to attribute a smaller share of aggregate inflation to industry-specific shocks than reduced-form studies, as relative prices are partly driven by aggregate demand.

A second branch of the literature studies the connection between relative prices and aggregate inflation through the lens of linear, multi-sector New Keynesian frameworks.⁹ My paper bridges small-scale analytical models (Lorenzoni and Werning, 2023; Guerrieri et al., 2023; Gagliardone and Gertler, 2023) and large-scale quantitative models (Ruge-Murcia and Wolman, 2022), by providing analytical results that apply to multi-sector models of any size.¹⁰ I derive novel analytical results, including a sequence space representation of inflation in terms of past, current, and expected shocks to output gaps and natural relative prices,¹¹ and a set of price indices which allow us to identify inflation drivers and track model-consistent inflation expectations.

A third line of research considers various forms of non-linearity, such as downward rigid wages and a binding ZLB constraint (Guerrieri et al., 2022; Baqaee and Farhi, 2022; di Giovanni et al., 2022)¹² or non-linear Phillips curves (Ferrante et al., 2023; Comin et al., 2023; Harding et al., 2023; Benigno and Eggertsson, 2023; Bai et al., 2024; Dao et al., 2023).¹³ I argue that the interconnection between relative prices and aggregate inflation arises from heterogeneity, of which non-linearities are one source. Non-linearities create heterogeneity only ex-post – after shocks

(2024) estimate a VAR on aggregate GDP and aggregate CPI for the US and the Euro area, and identify demand and supply shocks imposing restrictions on the comovement of prices and quantities. In line with my results, they find that demand shocks were the dominant inflation driver throughout the pandemic.

⁸Blanchard and Bernanke (2023) propose a SVAR with exogenous shocks to the price of energy, food, and cars, finding that these shocks explain a large share of aggregate inflation in the aftermath of the COVID pandemic. Relatedly, Ball et al. (2022) document that changes in headline inflation had a significant pass-through into core inflation. Shapiro (2020, 2022) instead estimates industry-level demand and supply shocks based on the comovement between prices and quantities, and finds that industry-level shocks were a significant driver of aggregate inflation.

⁹Lorenzoni and Werning (2023) and Guerrieri et al. (2023) study the implications of decreasing returns to scale or heterogeneous nominal rigidity across industries for the dynamics of real wages and other relative prices, highlighting that these are endogenous to aggregate demand. Gagliardone and Gertler (2023) estimate the degree of complementarity between labor and oil in production, finding that the two are strong complements, and concluding that oil shocks generated a large inflation-output tradeoff in the aftermath of the COVID pandemic. Ruge-Murcia and Wolman (2022) instead estimate a medium-scale DSGE model, and perform a variance decomposition of industry-level inflation. They find that industry-specific shocks explain a sizable fraction of aggregate inflation, both before and during the COVID pandemic. Olivi et al. (2023) show that, with non-homothetic preferences, an increase in the price of necessities worsens the inflation-output tradeoff.

¹⁰My framework is similar to Rubbo (2023), extended to allow for multiple primary factors of production.

¹¹Afrouzi and Bhattarai (2023); Afrouzi et al. (2024) provide a different analytical characterization of the impulse responses of sectoral inflation to monetary and productivity shocks.

¹²In the early stages of the pandemic, Guerrieri et al. (2022) studied the interaction between supply and demand shortfalls in the presence of a binding ZLB constraint, downward rigid wages, and hand-to-mouth households. Baqaee and Farhi (2022) also considered an economy subject to downward rigid wages and a ZLB constraint, showing how demand and supply shortages contributed to a large decline in output and a relatively smaller decline in inflation. di Giovanni et al. (2022) applied the methodology in Baqaee and Farhi (2022) to provide a cross-country inflation decomposition, also covering the aftermath of the pandemic.

¹³Ferrante et al. (2023) document a large increase in the relative price of – and expenditure share on – manufacturing during the pandemic. They rationalize it through a multi-industry DSGE model with convex costs of increasing employment at the industry level. Comin et al. (2023) rationalize these same facts through a two-sector model with occasionally binding supply constraints. Bai et al. (2024) construct a direct measure of port congestion, and study the implications for output and inflation through a model with transport costs and search frictions between producers and retailers. Common to all these papers are convex supply curves, which imply that monetary policy faces a lower cost of reducing inflation when the economy is overheated. Harding et al. (2023) instead introduce a non-linear Phillips curve through variable demand elasticities. They find the opposite implication that reducing inflation is more costly when the economy is overheated and demand elasticities are lower.

Benigno and Eggertsson (2023) argue that historical evidence points at strong non-linearities in the Phillips curve. They model non-linearities as resulting from search and matching friction and downward wage rigidity. Through the lens of this framework, they attribute the rise in inflation during the COVID pandemic to a labor shortage.

hit – and ignoring ex-ante heterogeneity creates an upward bias in the share of aggregate inflation attributed to industry-specific shocks.

2 Relative Prices and Aggregate Inflation: Stylized Facts

It is a known fact ([Ball and Mankiw, 1995](#); [Reis and Watson, 2010](#)) that high aggregate inflation tends to be associated with large relative price movements – a phenomenon that was particularly prominent during the COVID pandemic. This section documents that the comovement between relative prices and aggregate inflation follows specific patterns, and suggests that these patterns point at the presence of common drivers.

I consider 66 private industries, as in the BEA input-output tables, and document that some industries are more cyclical than others. Their relative price is positively correlated with the aggregate output gap, and – conditional on the output gap – aggregate inflation is positively correlated with their relative price. Two characteristics of cyclical industries stand out: they adjust prices more frequently (as measured by [Pasten et al., 2020](#)), and a large share of their value added is accounted for by short-run fixed assets, such as structures and heavy equipment.

Figure 1 begins by showing that price adjustment probabilities and value added composition vary greatly across industries. Price adjustment probabilities range from 0.03 to nearly 1, and value-added shares of structures and heavy equipment vary from 0 to 0.79. The two are positively correlated but not collinear. Oil and farms are flexible and have high non-scalable shares, housing and equipment rental have high non-scalable shares but sticky prices, wholesale trade has flexible prices but low non-scalable shares.

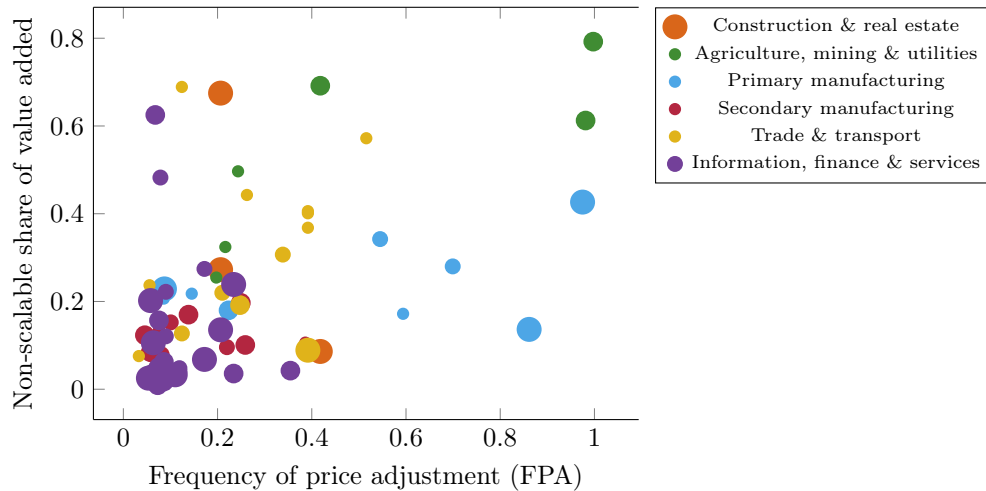


Figure 1 Price adjustment probabilities (horizontal axis) are from [Pasten et al. \(2020\)](#). Non-scalable share of value added (vertical axis) is defined as the share of value added paid to structures and heavy equipment. Non-labor shares of value added are from the BEA Use tables. Capital costs by industry and asset type are the product of BEA detailed capital stocks and asset-level rental rates from [Vom Lehn and Winberry \(2022\)](#). Each dot is one of 66 private BEA industries; marker size increases with the industry's sales share (quartile bins).

Prices in flexible and non-scalable industries are more sensitive to aggregate demand. Figure 2 shows that, when the output gap rises, inflation rises by more in industries with flexible prices and large non-scalable shares of value added. Both characteristics predict this sensitivity: one standard deviation of price flexibility raises an industry’s inflation response to a 1% output gap change by 0.13%, conditional on value added shares. Likewise, one standard deviation of the non-scalable share raises the inflation response by 0.16% conditional on price flexibility. Together, the two characteristics account for 34% of cross-industry variation in inflation cyclicality. Price flexibility, as measured here, largely identifies commodity producers. Excluding crude oil extraction, petroleum refining and farms reduces the price-flexibility loading to 0.04 (0.04). The non-scalable loading instead is largely unaffected by removing these industries, remaining at 0.12 (standard error 0.04). Appendix D reports the full set of exclusions and their effect on each fact. Figure 3 further illustrates this comovement pattern, over the years 2000–2024. The

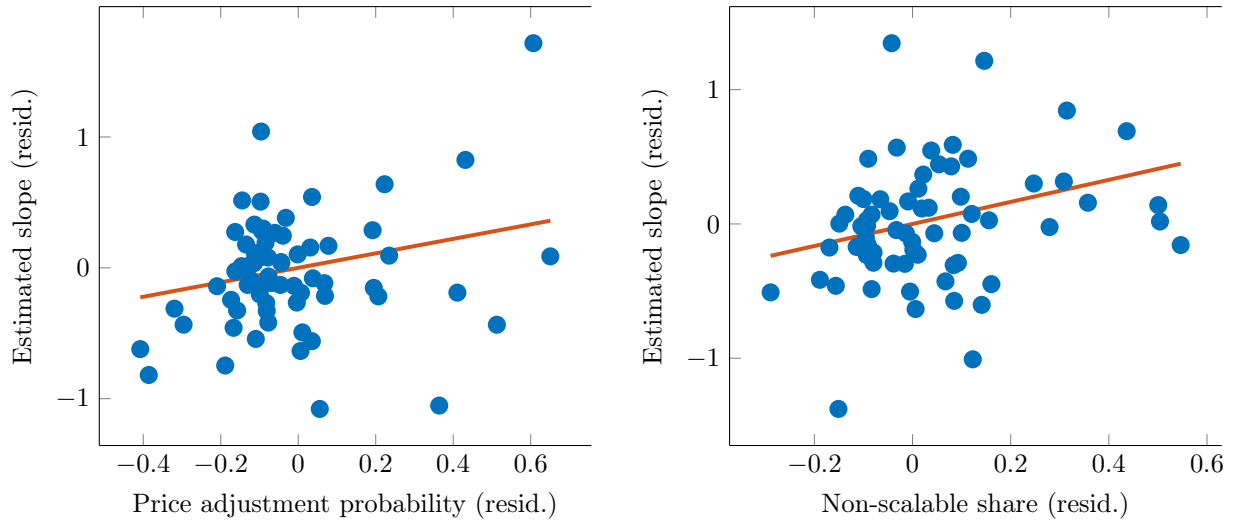


Figure 2 Industry inflation sensitivity to aggregate demand. Slopes are estimated industry by industry by regressing year-on-year industry inflation (measured from the BLS PPI and CPI datasets) on the CBO output gap, smoothed with a trailing 4-quarter moving average, over 2000Q1–2024Q1. The output gap is real GDP less CBO potential GDP, as a percent of potential. Each panel is an added-variable plot: slopes and the characteristic on the horizontal axis are residualized on the other characteristic, and both characteristics are expressed as z-scores over the 66 private industries. All regressions are weighted by industry value added. The pandemic quarters 2020Q1–2022Q4 are excluded from the estimation sample.

relative price of industries with flexible prices and large non-scalable shares rises and falls with the output gap.

Supply constraints dominate demand cyclicality in non-scalable industries. A potential concern is that industries with large non-scalable shares produce durable goods. Therefore higher price cyclicality in these industries might be due to higher demand cyclicality, rather than to steeper supply curves. Figure 4 shows that these industries increase prices by more and quantities by less as the output gap rises. Price flexibility instead strongly predicts price changes, but only weakly predicts quantity changes. This finding establishes that higher price cyclicality in non-scalable industries is not merely a consequence of more cyclical demand.

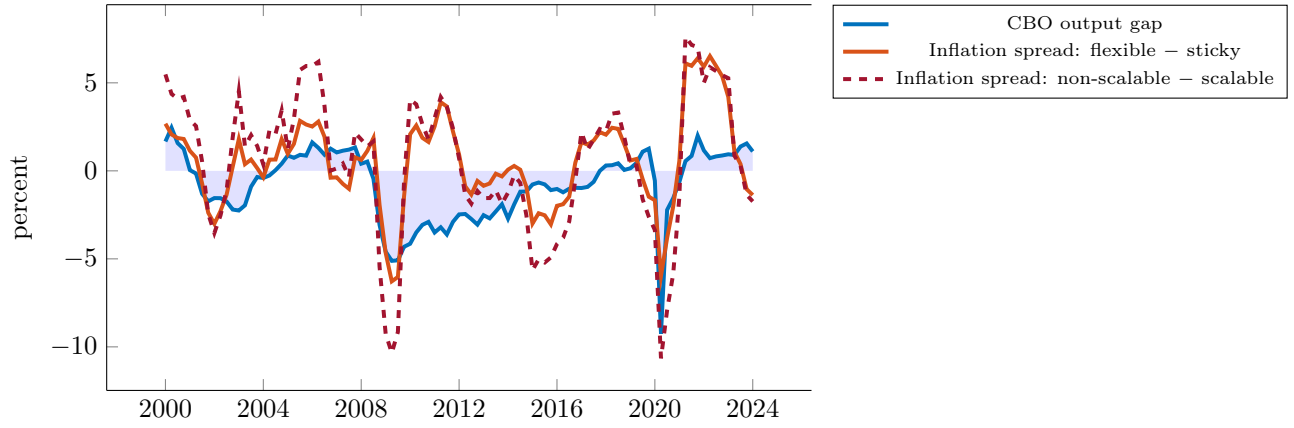


Figure 3 Relative price cyclicality, 2000–2024. Spreads are year-on-year inflation differences between value-added-weighted industry baskets. Industries are split at the median to construct the flex-price and sticky price baskets, and at the top tercile to construct the high- and low-non-scalable-share baskets. Correlation with the output gap (quarterly CBO series, not smoothed) is 0.49 and 0.55 respectively.

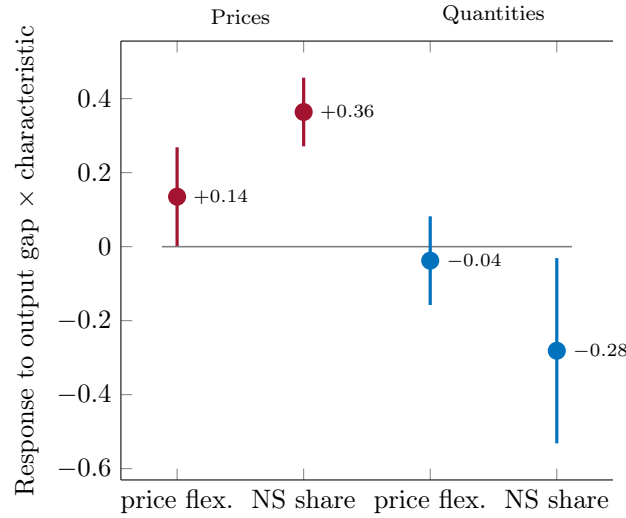


Figure 4 Response of industry-level prices and quantities to the aggregate output gap interacted with price flexibility and with the non-scalable share, estimated jointly. Panel of 23 mining, utility and manufacturing industries covered by the Federal Reserve G.17 industrial production release, 2000Q1–2024Q1, with industry and time fixed effects, weighted by industry value added. Prices are year-on-year industry inflation; quantities are year-on-year growth in industrial production. Price flexibility and non-scalable share enter as z-scores. The output gap is the trailing 4-quarter moving average; its level is absorbed by the time fixed effects. Whiskers are 95% confidence intervals; standard errors clustered by industry. The pandemic quarters 2020Q1–2022Q4 are excluded from the estimation sample.

Aggregate inflation is more sensitive to flexible and non-scalable industries. Figure 5 shows that, holding aggregate conditions fixed, aggregate inflation is systematically higher when the relative price of industries with high price adjustment frequency and high non-scalable share is rising. The figure scatters aggregate year-on-year inflation against the relative price of these industries, both residualized on a set of aggregate controls: the 4-quarter moving average of CBO output gaps, the unemployment rate, capacity utilization, log vacancies per unemployed, and one-year-ahead expected inflation from the Michigan survey and from the Cleveland Fed, each entered with 4

quarterly lags. I define the basket of flexible, high non-scalable industries to include the top value-added-weighted tercile sorted on the two characteristics, weighted by the estimated loadings from figure 2 ($0.13 \times \text{price flexibility} + 0.16 \times \text{non-scalable share}$).¹⁴

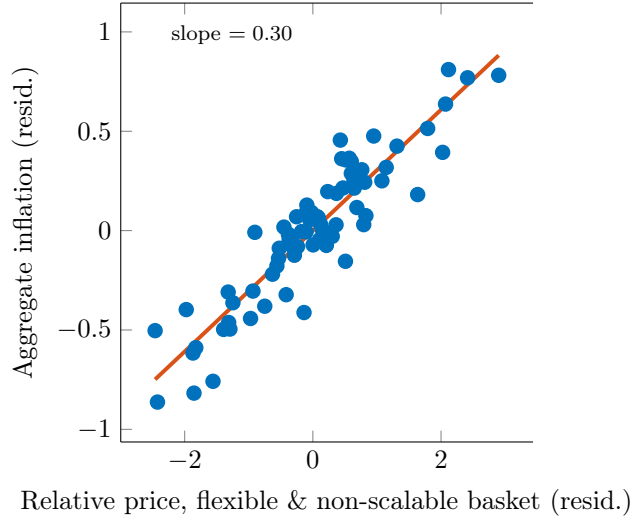


Figure 5 Aggregate inflation against the relative price of flexible, non-scalable industries, both residualized on the full set of aggregate controls; one dot per quarter, 2001Q4–2024Q1 ($N = 78$). The controls are the trailing 4-quarter moving average of the CBO output gap, the unemployment rate, capacity utilization and log vacancies per unemployed, each with 4 quarterly lags. Aggregate inflation is year-on-year inflation across 66 private industries, aggregated with value-added weights. The basket of flexible, non-scalable industries is defined in the text. The fitted slope is the regression coefficient on the relative price, and equals 0.30 (Newey–West standard error 0.03, 8 lags). The sample starts in 2001Q4 because the JOLTS vacancy series begins in 2000M12. The pandemic quarters 2020Q1–2022Q4 are excluded from the estimation sample.

COVID deflation and inflation were driven by the same industries. Figure 6 shows that industries with flexible prices and large non-scalable shares faced the largest deflation in 2020, and the largest inflation in 2021–22. The symmetry of this pattern points at persistent heterogeneity in supply slopes across industries, as opposed to asymmetrically binding constraints or non-linear supply curves.

Taking stock. Figures 1–6 report suggestive correlations, which point at a two-way causality: relative prices may respond to aggregate demand, and industry-specific shocks may affect the aggregate inflation-output tradeoff. The rest of the paper builds a structural framework where industries differ in their price flexibility and non-scalable shares, and uses it to identify the components of relative prices and aggregate inflation driven by industry-specific shocks versus aggregate demand.

¹⁴Conditional on the controls, inflation of cyclical industries has correlation -0.09 with inflation in the remaining industries and -0.54 with wage inflation. This confirms that figure 5 is not picking up a common aggregate factor that moves all prices together. Moreover, the price of non-cyclical industries enters the dependent variable with a negative sign, which biases against finding a positive slope. In the other direction, the basket’s own industries enter both variables, so measurement error would inflate the slope. Appendix figure 16 addresses this concern, re-estimating the regression on disjoint groups of industries and showing that the relationship remains.

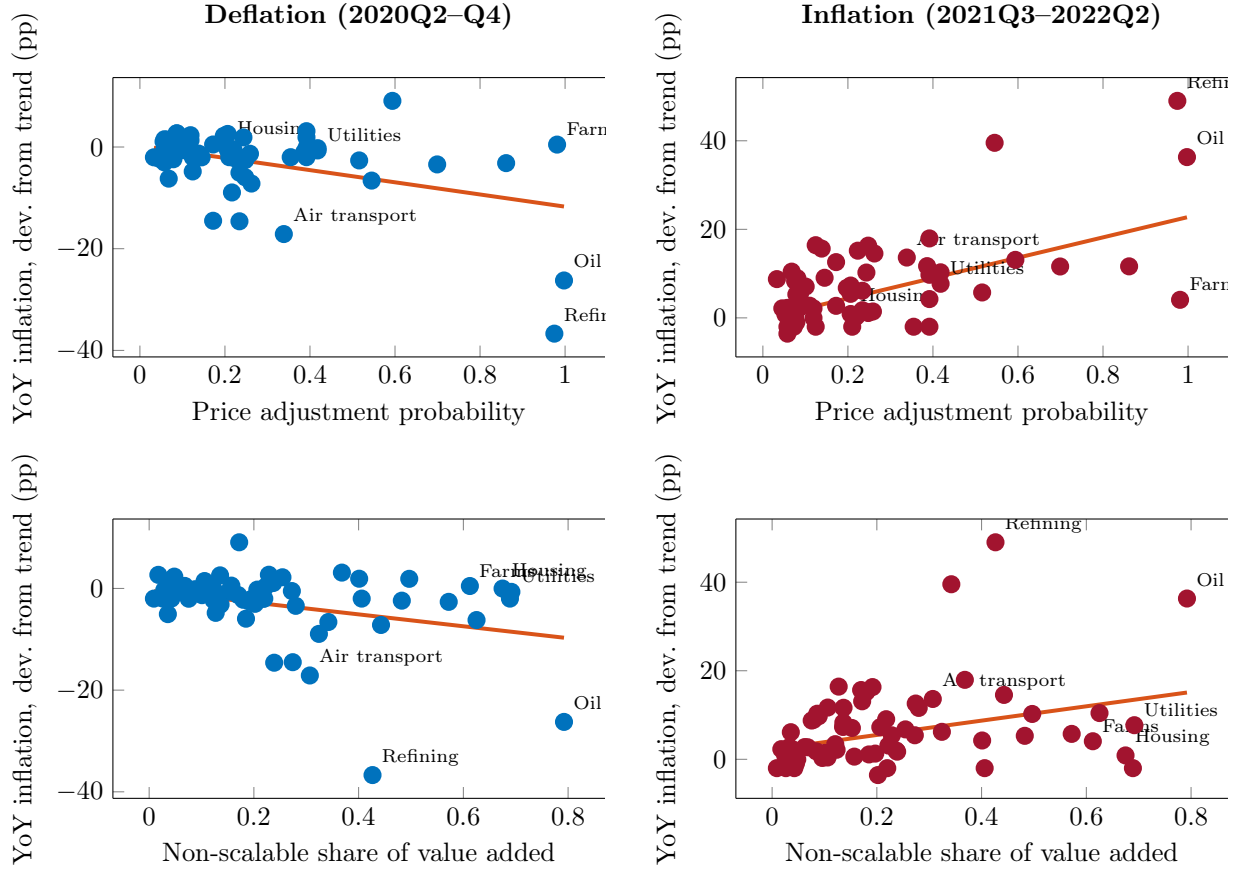


Figure 6 Industry year-on-year inflation in deviation from a 2% trend, averaged within each window, against price adjustment probabilities (top row) and non-scalable shares of value added (bottom row). Each dot is one of the 66 private industries. Fitted lines are value-added-weighted regressions on one characteristic at a time, in raw units. Unlike the rest of the section, this figure uses the full sample, including 2020Q1–2022Q4.

3 Model Environment

This section introduces the model environment, consisting of a New Keynesian framework with multiple industries and multiple primary factors of production.

Industries have constant returns to scale, and use different combinations of intermediate inputs and primary factors. Primary factors consist of differentiated labor occupations and non-labor assets, such as land, equipment, structures, intellectual property, etc. All producers and workers face Calvo-style nominal rigidities in price and wage setting. A representative household owns all industries and factors, and makes consumption-saving choices according to a standard Euler equation. The central bank sets monetary policy according to a conventional Taylor rule. The government chooses the total spending and its allocation across industries.

I allow for several shocks, to account for a comprehensive set of inflation drivers. Industries are subject to productivity shocks and shocks to their desired markups. The household is subject to three types of preference shocks: to the discount factor, to the relative preference for different consumption goods, and to the disutility of labor supply in

different occupations.

In order to account for the facts presented in section 2, I allow for industries to have different price adjustment probabilities and for primary factors to have different supply elasticities. The scalability of an industry's production is determined by the average supply elasticity of the factors it uses. In the quantitative section, factor supply elasticities are set to match relative price responses to identified monetary shocks across industries. Industries which rely more heavily on heavy equipment and structures have the largest price responses, suggesting that these factors are the least elastically supplied, followed by intellectual property, labor, and light equipment.

Section 3.1 introduces the model primitives, while section 3.2 presents the log-linearized equilibrium.

3.1 Primitives

Primary factors. There is a set $\mathcal{F} = \mathcal{L} \cup \mathcal{K}$ of primary factors in the economy, consisting of differentiated labor occupations (indexed by $f \in \mathcal{L}$) and non-labor assets – such as land, equipment, structures, and intellectual property products (indexed by $f \in \mathcal{K}$). Factor markets are segmented, meaning that labor occupations are imperfectly substitutable from the perspective of households and firms, and non-labor assets cannot be repurposed or transformed into labor. Employment of all factors f is denoted L_f , while factor prices are denoted W_f .

I represent the supply of non-labor factors through a stylized utilization model, described in detail in appendix A.1. My stylized setting abstracts from capital accumulation, while allowing me to parsimoniously account for heterogeneous supply elasticities and heterogeneous value added composition across industries. The model is designed so that non-labor factors have isoelastic supply curves, with inverse supply elasticities denoted φ_f , $f \in \mathcal{K}$.

Industries. There is a set $\mathcal{N}$ of good-producing industries, indexed by i, j . Within each industry there is a continuum of firms producing differentiated varieties, indexed by n . Customers buy CES bundles of these varieties, with time-varying elasticity of substitution ϵ_{it} . Industry output and prices are

$$Y_{it} = \left(\int Y_{int}^{\frac{\epsilon_{it}-1}{\epsilon_{it}}} dn \right)^{\frac{\epsilon_{it}}{\epsilon_{it}-1}}, \quad P_{it} = \left(\int P_{int}^{1-\epsilon_{it}} dn \right)^{\frac{1}{1-\epsilon_{it}}}$$

All firms n in industry i have the same constant returns to scale production function, taking as inputs intermediate goods and primary factors:

$$Y_{int} = A_{it} G_i(\{L_{ifnt}\}_{f \in \mathcal{F}}, \{X_{ijn}\}_{j=1}^N)$$

where L_{ifnt} is the quantity of primary factor f hired by firm n in industry i at time t , X_{ijn} is the quantity of intermediate input j used by the firm, and A_{it} is a Hicks-neutral productivity shifter.

All producers minimize costs given input prices. With constant returns to scale, firms within industry i use inputs

in the same proportions and have equal marginal costs

$$MC_i = \min_{\{X_{ij}\}, \{L_{if}\}} \sum_{f \in \mathcal{F}} W_f L_{if} + \sum_{j \in \mathcal{N}} P_j X_{ij} \quad \text{s.t.} \quad A_i G_i \left(\{L_{if}\}_{f \in \mathcal{F}}, \{X_{ij}\}_{j \in \mathcal{N}} \right) = 1 \quad (1)$$

Producers face Calvo-style price rigidities. In each period t , only a fraction δ_i of firms in industry i can update their price. Adjusting firms maximize expected discounted profits:

$$P_{int}^* = \frac{\epsilon_{it}}{\epsilon_{it} - 1} \frac{\mathbb{E}_t \sum_s [SDF_{t+s} (1 - \delta_i)^s Y_{int+s}(P_{int}^*) (1 - \tau_{it+s}) MC_{it+s}]}{\mathbb{E}_t \sum_s [SDF_{t+s} (1 - \delta_i)^s Y_{int+s}(P_{int}^*)]} \quad (2)$$

where P_{int}^* is the price chosen by adjusting firms, SDF_{t+s} is the stochastic discount factor, τ_{it+s} represents subsidies, and demand functions are $Y_{int+s}(P_{int}^*) = Y_{it+s} \left(\frac{P_{int}^*}{P_{it+s}} \right)^{-\epsilon_{it}}$. Non-adjusting firms keep prices fixed and absorb cost changes in their markups.

To model wage rigidity I add a set of fictitious industries, consisting of occupation-specific labor unions. Unions hire labor occupations and sell their services to good-producing industries. They are owned by the representative household, and face Calvo-style nominal rigidities like all other industries. Wage reset probabilities can differ across occupations. The presence of labor unions creates a wedge between the wage paid to workers and the price paid by firms to the labor unions. The difference is rebated lump-sum to the household through the unions' profits, so that in practice the unit price of labor equals the price paid to unions. Non-labor factors are likewise sold on factor-specific marketplaces, also facing nominal rigidities.

Following a standard practice in the literature, I assume that the government provides industry-specific subsidies to eliminate wedges between prices and marginal costs in steady state.

Assumption 3.1. Prices equal pre-subsidy marginal costs in steady state.

Household. A representative household maximizes the present discounted value of utility flows

$$\sum_{t=0}^{\infty} \rho^t \left[\frac{C_{ht}^{1-\gamma}}{1-\gamma} - \sum_{f \in \mathcal{L}} \left(\frac{1}{\epsilon_{ft}^{\mathcal{F}}} \right)^{\varphi_L} \frac{L_{ft}^{1+\varphi_L}}{1+\varphi_L} \right].$$

The household derives utility from consumption, and faces an additively separable disutility of supplying differentiated labor occupations. The parameter γ governs intertemporal substitution and wealth effects in labor supply, while the parameter φ_L is the inverse Frisch elasticity of labor supply. The shocks $\epsilon_{ft}^{\mathcal{F}}$ are occupation-specific labor supply shifters.

The consumption aggregator $C_{ht} \equiv \mathcal{C}_{ht}(c_{h1t}, \dots, c_{hNt})$ is homothetic over all goods produced in the economy. Consumption preferences are time-indexed to account for changes in the relative preference for different goods over

time, modeled as time-varying expenditure shares at constant prices.

The household freely trades a risk-free asset, in zero net supply, paying nominal interest rate i_t on bonds purchased at date t and redeemed at date $t + 1$. The per-period budget constraint is

$$P_{ht}^C C_{ht} \leq \sum_{f \in \mathcal{L}} W_{ft} L_{ft} + \sum_{f \in \mathcal{K}} \frac{\varphi_f}{1 + \varphi_f} W_{ft} L_{ft} + \sum_{i \in \mathcal{N}} \Pi_{it} + (1 + i_{t-1}) B_t - B_{t+1} - \hat{T}_{ht}, \quad (3)$$

where $\frac{\varphi_f}{1 + \varphi_f}$ is the share of income from non-labor factors rebated to the household,¹⁵ Π_{it} are profit rebates, $B_{t+1} - (1 + i_{t-1}) B_t$ is the change in asset holdings, and $\hat{T}_{ht}$ are taxes levied on the household.

Utility maximization yields the consumption-leisure tradeoff

$$C_{ht}^\gamma \left(\frac{L_{ft}}{\epsilon_{ft}^{\mathcal{F}}} \right)^{\varphi_L} = \frac{W_{ft}}{P_{ht}^C} \quad (4)$$

for each labor occupation f . Wage rigidities create a wedge between desired wages W_{ft} in (4) and the price paid by firms to the labor unions. The difference is rebated lump-sum to the household through the unions' profits.

Optimal consumption-saving choices follow a standard Euler equation:

$$\frac{C_{h,t+1}}{C_{h,t}} = \left[\rho (1 + i_t) \frac{P_{h,t}^C}{P_{h,t+1}^C} \right]^{\frac{1}{\gamma}} \quad (5)$$

Final users and ownership shares. This economy has three types of final users: the household, utilization producers of non-labor factors, and the government. I denote the set of final users by $\mathcal{Z} = \{h\} \cup \mathcal{U} \cup \{G\}$, indexed by z , with real consumption C_{zt} and price indices P_{zt} . I use the same notation for all of them, to emphasize that they are all final users.

Ownership shares in industries and primary factors are denoted $\hat{\Xi}_{zi}$, $i \in \mathcal{N}$, and $\hat{\Xi}_{zf}$, $f \in \mathcal{F}$. The household owns all industries and all labor occupations, so that $\hat{\Xi}_{hi} = 1$ for $i \in \mathcal{N}$ and $\hat{\Xi}_{hf} = 1$ for $f \in \mathcal{L}$. Income from non-labor factors is split between utilization costs and rebates to the household, with shares $\hat{\Xi}_{u_f f} = \frac{1}{1 + \varphi_f}$ and $\hat{\Xi}_{h_f} = \frac{\varphi_f}{1 + \varphi_f}$, as explained in appendix A.1. The government owns no assets, so that $\hat{\Xi}_{Gi} = \hat{\Xi}_{Gf} = 0$. I denote lump-sum transfers across final users relative to nominal GDP by T_{zt} . I assume $T_{ut} \equiv 0$ for utilization producers. The government is a net receiver of transfers, and the economy's resource constraint implies $-T_{Gt} = \frac{P_t^G C_t^G}{GDP} = T_{ht} = \frac{\hat{T}_{ht} + B_{t+1} - (1 + i_{t-1}) B_t}{GDP}$.

Policy. The central bank sets the nominal interest rate i_t following a Taylor rule. The government provides industry-level cost subsidies financed through lump-sum profit taxes. It also purchases a time-varying bundle of goods, with total spending G_t , allocated across industries with shares Ω_{Git} .

¹⁵This follows from my stylized utilization model. See appendix A.1.

Equilibrium. The equilibrium definition adapts Baqaee and Farhi (2020) to account for endogenous markups with pricing frictions. This definition nests the standard flexible-price equilibrium when $\delta_i = 1$ in all industries.

Definition 3.1. Given price adjustment probabilities δ_i and policies, general equilibrium consists of firm markups $\mathcal{M}_{int}$, sectoral prices P_{it} , factor wages W_{ft} , factor supplies L_{ft} , sectoral outputs Y_{it} , intermediate quantities X_{ijt} , and final demands C_{izt} such that: (i) a fraction δ_i of firms in sector i charge profit-maximizing prices (2); (ii) firms who cannot adjust prices absorb cost changes into their markups; (iii) the household maximizes utility subject to budget constraints; (iv) producers minimize costs and charge the relevant markups; (v) all markets clear.

3.2 Log-Linearized Equilibrium

For the rest of the analysis, I log-linearize the model around an efficient initial equilibrium (see assumption 3.1). Variables without a time subscript denote the initial equilibrium, while lower-case letters denote log-deviations from this equilibrium. Definitions 3.2 – 3.4 introduce expenditure and income shares, elasticities of substitution in production and in consumption, and changes in aggregate GDP and aggregate GDP deflator. Definitions 3.5 and 3.6 introduce the natural equilibrium and define gaps relative to this equilibrium. Lemma 3.1 summarizes the log-linearized equilibrium conditions.

Remark 3.1. For convenience, I extend the set $\mathcal{N}$ of producers to include labor unions, factor marketplaces, and retailers. There is a distinct retailer for each final user (household consumption, government spending, and utilization of non-labor factors), selling a bundle of goods from all industries to the respective final user. Ownership shares in unions and marketplaces equal those of the corresponding factors, and retailers are owned by the corresponding final users. This simplifies notation, ensuring that each primary factor is only purchased by its respective union or marketplace, and each final user only purchases from its respective retailer.

Definition 3.2. Expenditure and sales shares. For all industries $i, j \in \mathcal{N}$ denote

$$\Omega_{ij} \equiv \frac{P_j X_{ij}}{P_i Y_i} \quad \text{direct expenditure share of } i \text{ on } j \quad (6)$$

$$\Psi \equiv (I - \Omega)^{-1} \quad \text{total expenditure shares (Leontief inverse)} \quad (7)$$

$$\bar{\Psi}_i \equiv \frac{P_i Y_i}{GDP} \quad \text{sale share of } i. \quad (8)$$

Remark 3.2. Factor income shares coincide with the sales shares of labor unions and factor marketplaces. Likewise, the income shares of final users coincide with the sales shares of the respective retailers. With some abuse of notation, I use $\bar{\Psi}_f, f \in \mathcal{F}$ to denote factor income shares and $\bar{\Psi}_z, z \in \mathcal{Z}$ to denote final use shares. The set of final users $\mathcal{Z}$

includes the household, the government, and utilization of non-labor factors. All shares are expressed relative to nominal GDP, equal to total final expenditure $GDP \equiv \sum_{z \in \mathcal{Z}} P_z C_z$.

Definition 3.3. Substitution elasticities. Let subscripts denote derivatives of the cost function (1) with respect to input prices. The Allen–Uzawa elasticity of substitution between any two goods or factors j and k in the production of good i is defined as

$$\theta_{jk}^i \equiv \frac{MC_i MC_{i,jk}}{MC_{i,j} MC_{i,k}}, \quad j \neq k.$$

Likewise, denoting the unit cost function associated with $\mathcal{C}_{ht}$ by $P_h^C(P_1, \dots, P_N)$, the elasticity of substitution between goods i and j in consumption is

$$\theta_{ij}^C \equiv \frac{P_h^C \partial^2 P_h^C / \partial P_i \partial P_j}{(\partial P_h^C / \partial P_i) (\partial P_h^C / \partial P_j)}, \quad i \neq j.$$

Assumption 3.2. For each producer $i \in \mathcal{N}$, define $\theta_{jj}^i = -\sum_{k \neq j} \frac{\Omega_{ik}}{\Omega_{ij}} \theta_{jk}^i \forall j$ and $M^i = (M^i)_{jk} \equiv \Omega_{ij} \Omega_{ik} \theta_{jk}^i$. M^i must be negative semidefinite.

Assumption 3.2 is equivalent to concavity of the cost function. Given expenditure shares Ω_{ij} , assumption 3.2 is necessary and sufficient for the existence of a well-behaved technology G_i locally rationalizing the elasticities θ_{jk}^i . If G_i is CES with elasticity parameter θ^i , then $\theta_{jk}^i = \theta^i$ for all pairs.

Definition 3.4. Aggregates. Changes in aggregate real GDP and the aggregate GDP deflator are defined as

$$\bar{y}_t \equiv \sum_{z \in \mathcal{Z}} \bar{\Psi}_z C_{zt} = \sum_{f \in \mathcal{F}} \bar{\Psi}_f \ell_{ft} + \sum_{i \in \mathcal{N}} \bar{\Psi}_i a_{it} \quad \text{real GDP} \quad (9)$$

$$\bar{p}_t \equiv \sum_{z \in \mathcal{Z}} \bar{\Psi}_z p_{zt} \quad \text{GDP deflator.} \quad (10)$$

Definition 3.5. Natural equilibrium. The natural equilibrium is defined as the equilibrium which would prevail in an economy with flexible prices, subject to the same shocks and policies as the actual economy. Log-changes in the natural value of any variable x are denoted x^{nat} .

Definition 3.6. Gaps. For any variable x , the gap with respect to the natural equilibrium is defined as $\tilde{x}_t \equiv x_t - x_t^{nat}$.

Lemma 3.1. Following equation (4), denote shifts in the supply of each factor f at constant real wages by $\epsilon_{ft}^{\mathcal{F}}$. Let γ_f denote wealth effects in the supply of factor f , equal to γ for labor occupations $f \in \mathcal{L}$ and to zero for non-labor factors $f \in \mathcal{K}$. Also let $c_f^{\mathcal{F}}$ denote the relevant consumption index and $p_{ft}^{\mathcal{F}}$ denote the relevant price deflator for factor f , coinciding with household consumption c_h and the households' price index p_{ht}^C for labor occupations, and with utilization and its unit cost for non-labor factors (see appendix A.1). Further define expenditure-scaled ownership

shares $\Xi_{zi} \equiv (\hat{\Xi}_{zi} - T_z) \frac{\bar{\Psi}_i}{\bar{\Psi}_z}$. Denote the pass-through of marginal costs into prices $\hat{\delta}_i \equiv \frac{\delta_i(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)} \in (0, 1)$. Following (2), denote exogenous cost-push shocks to industry i by $\epsilon_{it}^p \equiv -\hat{\delta}_i \frac{d \log \epsilon_{it}}{\epsilon_i - 1}$. Also denote changes in the expenditure share of producer i on good j at constant prices by ϵ_{ijt}^d , satisfying $\sum_{j \in \mathcal{N}} \Omega_{ij} \epsilon_{ijt}^d = 0$. Finally, assume that the Taylor rule targets the output gap with weight ϕ_y , and an inflation index π^{TR} with weight ϕ_π . Denote discount factor shocks and monetary policy shocks by ϵ^ρ and ϵ^i .

The log-linearized equilibrium is defined by the following equations:

$$w_{ft} = \gamma_f c_{ht} + \varphi_f (\ell_{ft} - \epsilon_{ft}^{\mathcal{F}}) + p_{ft}^{\mathcal{F}} \quad \text{factor supply} \quad (11a)$$

$$mc_{it} = \sum_{j \in \mathcal{N}} \Omega_{ij} p_{jt} + \mathbb{I}_{i=f} w_{ft} - a_{it} \quad \text{marginal costs} \quad (11b)$$

$$\pi_{it} = \hat{\delta}_i (mc_{it} - p_{i,t-1}) + (1 - \hat{\delta}_i) \rho \mathbb{E}_t \pi_{i,t+1} + \epsilon_{it}^p \quad \text{pricing} \quad (11c)$$

$$y_{it} = \sum_{j \in \mathcal{N}} \Omega_{ij} x_{ijt} + \mathbb{I}_{i=f} \ell_{ft} + a_{it} \quad \text{industry output} \quad (11d)$$

$$x_{ijt} - (y_{it} - a_{it}) = \sum_{k \in \mathcal{N}} \Omega_{ik} \theta_{jk}^i (p_{kt} - p_{it}) + \epsilon_{ijt}^d \quad \text{input composition} \quad (11e)$$

$$y_{jt} = \sum_{i \in \mathcal{N}} \frac{\bar{\Psi}_i}{\bar{\Psi}_j} \Omega_{ij} x_{ijt} + \mathbb{I}_{j=z} c_{zt} \quad \text{market clearing} \quad (11f)$$

$$p_{zt}^C + c_{zt} = \sum_{f \in \mathcal{F}} \Xi_{zf} \ell_{ft} + \sum_{i \in \mathcal{N}} \Xi_{zi} \left(p_{it} - \sum_{j \in \mathcal{N}} \Omega_{ij} p_{jt} \right) - \frac{dT_{zt}}{\bar{\Psi}_z} \quad \text{budget constraints} \quad (11g)$$

$$\mathbb{E}_t c_{h,t+1} = c_{ht} + \frac{1}{\gamma} [(i_{t+1} + \log \rho) - \mathbb{E}_t \pi_{h,t+1}^C] + \epsilon_t^\rho \quad \text{Euler equation} \quad (11h)$$

$$i_{t+1} = \mathbb{E}_t r_{t+1}^{nat} + \phi_y \tilde{y}_t + \phi_\pi \pi_t^{TR} + \epsilon_{t+1}^i \quad \text{Taylor rule} \quad (11i)$$

Before moving to the analytical results, example 3.1 introduces a simplified two-industry setting. The example illustrates why adding inelastically supplied factors alongside labor is necessary to represent bottlenecks in production.

Example 3.1. Modeling bottlenecks in production. Consider an economy with two industries, services (S) and manufacturing (M), indexed by $i \in \{S, M\}$. Services use only labor, while manufacturing uses only capital. Price adjustment probabilities are $\delta_M > \delta_S$. The representative household and the capital utilization producer both have initial expenditure shares $\Omega_S = \Omega_M = \frac{1}{2}$ and substitution elasticity θ between the two goods.

To keep the closed-form expressions cleaner, in the example and the examples that build on it I abstract from wealth effects in labor supply (setting $\gamma = 0$ in the consumption-leisure condition (4)). All results still hold when $\gamma > 0$.

In this simplified setting, each industry uses a single production factor. With some abuse of notation, I index factors by the industry which employs them. The workers' wage is denoted w_{St} , while the rental rate of capital is denoted w_{Mt} . Employment of labor is denoted ℓ_S , and employment of capital is denoted ℓ_M , with inverse supply elasticities

$\varphi_M > \varphi_S$. Throughout this example, I use the notation $\bar{x} \equiv \frac{1}{2}(x_M + x_S)$ for the average of any variable x and $x^R \equiv \frac{1}{2}(x_M - x_S)$ for relative values. Following definition 3.4, log-changes in real GDP are $y = \frac{1}{2}(\ell_S + \ell_M) + \bar{a}$.

Using the notation of lemma 3.1, and holding desired markups constant ($\epsilon_{it}^p = 0$), the natural output is

$$y^{nat} = \bar{\epsilon}^{\mathcal{F}} + \bar{a} + \frac{1}{\bar{\varphi} \left[1 - \left(\frac{\varphi^R}{\bar{\varphi}} \right)^2 \frac{\theta \bar{\varphi}}{1 + \theta \bar{\varphi}} \right]} \left[\bar{a} + \frac{\varphi^R}{1 + \theta \bar{\varphi}} [(1 - \theta) a^R + \epsilon^{\mathcal{F}, R} - \epsilon^{d, R}] \right] \quad (12)$$

In equation (12), the natural level of output is lower when consumer preferences shift toward manufacturing ($\epsilon^{d, R} > 0$) or when manufacturing supply declines ($a^R < 0$ or $\epsilon^{\mathcal{F}, R} < 0$) and manufacturing and services are complementary ($\theta < 1$). Intuitively, all these shocks raise the demand for capital relative to labor, and the supply of capital is harder to ramp up ($\varphi^R > 0$). Therefore shifting demand toward capital reduces the overall production potential.

This result suggests a natural definition of bottlenecks in production: a bottleneck is a shock that increases the relative demand for inelastically supplied factors. Equation (12) shows that bottlenecks reduce the potential output of the economy, for given aggregate productivity. As we will see in section 4, these same shocks also cause a worse inflation-output tradeoff. Factor market segmentation is essential for this mechanism to arise. If both industries used primary factors in the same proportions, preferences or productivity shocks would have no effect on relative factor demand. Therefore we would have $\varphi^R = 0$, and production bottlenecks would never arise.

4 Inflation Drivers: Definition and Identification

This section presents analytical results. Section 4.1 starts by deriving industry-level Phillips curves, and illustrating how industry-level characteristics shape their responsiveness to the output gap and their incidence on aggregate inflation. Section 4.2 then discusses identification. It starts by deriving a Phillips Jacobian, which relates inflation at every point in time with current, past and expected future shocks to the output gap and natural relative prices. Building on this characterization I show how to identify the inflation components driven by aggregate output gap and natural relative prices. I conclude by deriving a new set of indicators, consisting in appropriately weighted price indices which track identified inflation drivers and inflation forecast at different horizons.

4.1 Relative Prices and Aggregate Inflation

Proposition 4.1 recasts the equilibrium system (11a)–(11i) in gaps relative to the natural equilibrium, and rearranges it to obtain a conventional three-equation representation of price and output dynamics.

Proposition 4.1. *The dynamic evolution of industry-level inflation, aggregate output gap, and nominal interest rate*

is characterized by the following system of difference equations:

$$\begin{cases} \pi_{it} - \rho \mathbb{E} \pi_{i,t+1} = \kappa_i \tilde{y}_t - \sum_j \mathcal{V}_{ij} (\tilde{p}_{j,t-1} - \pi_{jt}^{nat} + \rho \mathbb{E} \pi_{j,t+1}) & \text{Phillips curves} \\ \mathbb{E}_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\gamma} [i_{t+1} - (\mathbb{E}_t r_{t+1}^{nat} + \mathbb{E}_t \pi_{t+1}^{IS})] & \text{IS equation} \\ i_{t+1} - \mathbb{E}_t r_{t+1}^{nat} = \phi_y \tilde{y}_t + \phi_\pi \pi_t^{TR} + \epsilon_{t+1}^i, & \text{Taylor rule} \end{cases} \quad (13)$$

where κ and $\mathcal{V}$ are derived in lemmas 4.1-4.2 below, and the nominal rate deflator π^{IS} is defined in appendix A.4. It holds that $\sum_j \mathcal{V}_{ij} = 0 \forall i$, meaning that only relative natural prices matter for inflation dynamics.

Proof. The Taylor rule is given by (11i). Combining the supply equations (11a)–(11e) with the budget constraints (11g) yields the industry-level Phillips curves, while combining equations (11d)–(11g) with the Euler equation (11h) yields the aggregate IS curve. See appendix A.4 for full derivations. $\square$

Proposition 4.1 shows that inflation dynamics are driven by two sufficient statistics: the aggregate output gap $\tilde{y}$, and natural relative prices π^{nat} . In turn, these are determined by the interaction of several primitive shocks. Monetary policy shocks ϵ^i affect the output gap through the dynamic system (13), but they do not affect the natural output or the natural relative prices. All other shocks (TFP, factor supply, markups, subsidies, preferences, and government spending) affect both the output gap $\tilde{y}$ – by changing the natural output – and industry-level natural prices π_{it}^{nat} . For instance, an expansionary TFP shock in one industry decreases that industry’s natural relative price, while raising the economy’s natural output.

The response of sectoral inflation rates to the aggregate output gap is governed by industry-specific slopes κ_i , while the response to natural relative prices is governed by the cost-push elasticities $\mathcal{V}_{ij}$. Lemmas 4.1 and 4.2 connect slopes and cost-push elasticities with industry-level characteristics. Definitions 4.1-4.4 first introduce notation. For convenience, I stack variables into vectors and elasticities into matrices. Boldface denotes vectors, and indexing by $\mathcal{N}$, $\mathcal{F}$ and $\mathcal{Z}$ denotes subsets of their entries. As a convention, a colon in place of an index selects an entire block: for example, $\Psi_{\mathcal{Z}}$ denotes the rows corresponding to final users, while $\Psi_{:\mathcal{F}}$ denotes the columns corresponding to labor unions and factor marketplaces.

Definition 4.1. Price pass-through. Following definition 3.2, denote the input-output matrix by $\Omega = (\Omega_{ij})_{i,j \in \mathcal{N}}$. Also collect price flexibility parameters $\hat{\delta}_i$ into the diagonal matrix Δ . The pass-through of sector j ’s price into sector i ’s price, directly and through intermediate inputs, is defined as $\mathcal{P}_{ij} \equiv \left[\Delta \sum_{k=0}^{\infty} (\Omega \Delta)^k \right]_{ij}$. In matrix form, $\mathcal{P} \equiv \Delta (I - \Omega \Delta)^{-1}$.

Definition 4.2. Supply elasticities. Collect wealth effects in factor supply into the matrix $\Gamma = (\Gamma_{fz})_{f \in \mathcal{F}, z \in \mathcal{Z}}$, $\Gamma_{fz} \equiv \gamma \mathbb{I}_{f \in \mathcal{L}, z=h}$, and collect inverse supply elasticities for labor and non-labor factors into the matrix $\Phi =$

$(\Phi_{ff'})_{f,f' \in \mathcal{F}}$, with entries $\Phi_{ff'} \equiv \varphi_f \mathbb{I}_{f=f'} + \sum_{z \in \mathcal{Z}} \Gamma_{fz} \Xi_{zf'}$. The first term is the inverse Frisch elasticity, while the second term captures wealth effects in labor supply through changes in factor income.

Definition 4.3. Demand elasticities. Let the matrix

$$\Theta = (\Theta_{fi})_{f \in \mathcal{F}, i \in \mathcal{N}}, \quad \Theta_{fi} \equiv \frac{\partial \log(L_f / \bar{L})}{\partial \log(P_i / \bar{P})} \quad (14)$$

encode the response of factor f 's relative employment to a change in the relative price of good i . Appendix A.3 derives Θ from steady-state expenditure shares and substitution elasticities. The partial derivatives in Θ account for substitution by final users and producers at every stage of the production network, and for feedback effects between factor income and factor demand (which arise if factor ownership and factor contents in consumption are correlated).

Definition 4.4. Final use shares and factor price deflators. The expenditure shares of final users are encoded in the rows of the input-output matrix Ω corresponding to final use retailers, denoted $\Omega_{\mathcal{Z},\cdot}$. The industries' weights in the GDP deflator are then given by the row vector $\bar{\Omega}_{\mathcal{Z}} \equiv \bar{\Psi}_{\mathcal{Z}}^T \Omega_{\mathcal{Z},\cdot}$. The matrix $\Omega^{\mathcal{F}} \equiv (\Omega_{fi}^{\mathcal{F}})_{f \in \mathcal{F}, i \in \mathcal{N}}$ instead encodes the industries' shares in factor price deflators.¹⁶ The aggregate factor price deflator is given by $\bar{\Omega}^{\mathcal{F}} \equiv \bar{\Psi}_{\mathcal{F}}^T \Omega^{\mathcal{F}}$, which in general is different from the GDP deflator $\bar{\Omega}_{\mathcal{Z}}$.¹⁷

Lemma 4.1. *Industry-level Phillips curve slopes $\kappa = (\kappa_i)_{i \in \mathcal{N}}$ are given by*

$$\kappa = (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta))^{-1} \mathcal{P}_{:\mathcal{F}} \Phi \mathbf{1},$$

where $\mathbf{1}$ denotes a column vector with all entries equal to 1.¹⁸ Denoting the average slope $\bar{\kappa} \equiv \bar{\Omega}_{\mathcal{Z}} \kappa$, the weighted average pass-through of factor prices into good prices $\bar{\mathcal{P}}_{\mathcal{F}} \equiv \bar{\Omega}^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}} \mathbf{1}$, and the weighted average supply elasticity $\bar{\Phi} \equiv \bar{\Psi}_{\mathcal{F}}^T \Phi \mathbf{1}$, the response of relative prices to the aggregate output gap is given by

$$\begin{aligned} \kappa - \bar{\kappa} \mathbf{1} &= \bar{\mathcal{P}}_{\mathcal{F}} (I - \mathbf{1} \bar{\Omega}_{\mathcal{Z}}) (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta))^{-1} \left[\frac{\bar{\Phi}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} \left(\frac{\mathcal{P}_{:\mathcal{F}} \mathbf{1}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \mathbf{1} \right) \right. \\ &\quad \left. + \Psi_{:\mathcal{F}} (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) + \left(\frac{\mathcal{P}_{:\mathcal{F}}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \Psi_{:\mathcal{F}} \right) (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) \right]. \end{aligned} \quad (15)$$

¹⁶Because labor supply responds to household consumption, factor price deflators combine consumption prices with changes in the household's income:

$$\Omega^{\mathcal{F}} \equiv \left[\begin{pmatrix} \mathbf{1}_{\mathcal{L}} & \mathbf{0}_{\mathcal{LU}} & \mathbf{0}_{\mathcal{L}} \\ \mathbf{0}_{\mathcal{UL}} & I_{\mathcal{U}} & \mathbf{0}_{\mathcal{U}} \end{pmatrix} - \Gamma \right] \Omega_{\mathcal{Z},\cdot} + \Gamma \Xi_{:\mathcal{N}} \Psi^{-1}.$$

See appendix A.1 for a formal treatment.

¹⁷Factor price deflators depend on factor income shares, while the GDP deflator depends on consumption shares. The income share of non-labor factors is larger than their consumption share, because part of their income is rebated to the household. The household in turn pays part of its income to the government, reducing consumption relative to factor income.

¹⁸The matrix $I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)$ is invertible for all parameter values outside a measure-zero set, namely the set on which $\mathcal{P}_{:\mathcal{F}} (I - \Omega^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}})^{-1} \Phi \Theta$ has an eigenvalue equal to -1 ; see appendix A.4. The same qualification applies to lemma 4.2 and to every expression below that uses this inverse.

The first bracketed term captures heterogeneous factor price pass-through; the second term captures heterogeneous supply elasticities; the third term captures the interaction between the two. Changing the output gap has no effect on relative prices if $\Phi \mathbf{1} \propto \mathbf{1}$ (increasing employment raises desired real wages equally for all primary factors) and $\mathcal{P}_{:\mathcal{F}} \mathbf{1} \propto \mathbf{1}$ (increasing nominal wages equally across all primary factors raises prices equally across all industries).

Proof. See Appendix B. □

Lemma 4.2. The cost-push elasticity $\mathcal{V} \equiv (\mathcal{V}_{ij})_{i,j \in \mathcal{N}}$ is given by

$$\mathcal{V} = (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta))^{-1} (\mathcal{P} \Psi^{-1} - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)).$$

The response of the aggregate GDP deflator to changes in natural relative prices is

$$\begin{aligned} \bar{\Omega}_{\mathcal{Z}} \mathcal{V} \pi^{nat} = & \frac{\bar{\mathcal{P}}_{\mathcal{F}}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} \left[\left(\frac{\bar{\Omega}^{\mathcal{F}} \mathcal{P}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \bar{\Omega}^{\mathcal{F}} \Psi \right) \Psi^{-1} \pi^{nat} - \left(\frac{\bar{\Omega}^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) (I - \mathcal{V}) \pi^{nat} \right. \\ & \left. + \left(\bar{\Psi}_{\mathcal{F}}^T \Phi - \bar{\Phi} \bar{\Psi}_{\mathcal{F}}^T \right) \Theta (I - \mathcal{V}) \pi^{nat} \right] + \left(\bar{\Omega}_{\mathcal{Z}} - \bar{\Omega}^{\mathcal{F}} \right) \mathcal{V} \pi^{nat}. \end{aligned} \quad (16)$$

Changes in the natural relative prices do not affect the aggregate GDP deflator if all good and factor prices have the same pass-through into the aggregate factor price deflator ($\bar{\Omega}^{\mathcal{F}} \mathcal{P} = \bar{\mathcal{P}}_{\mathcal{F}} \bar{\Omega}^{\mathcal{F}} \Psi$ and $\bar{\Omega}^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}} = \bar{\mathcal{P}}_{\mathcal{F}} \bar{\Psi}_{\mathcal{F}}^T$), all primary factors have the same supply elasticity ($\bar{\Psi}_{\mathcal{F}}^T \Phi = \bar{\Phi} \bar{\Psi}_{\mathcal{F}}^T$), and the aggregate factor price deflator coincides with the GDP deflator ($\bar{\Omega}^{\mathcal{F}} = \bar{\Omega}_{\mathcal{Z}}$).

Proof. See Appendix B. □

Lemmas 4.1 and 4.2 answer two key questions: When does the aggregate output gap affect relative prices? And when do changes in natural relative prices affect aggregate inflation? In equation (15) the output gap affects realized relative prices – even though the natural relative prices remain unchanged – whenever industries have different slopes ($\kappa \not\propto \mathbf{1}$). In equation (16) changes in natural relative prices affect aggregate inflation – even though the output gap does not move – whenever $\bar{\Omega}_{\mathcal{Z}} \mathcal{V} \neq \mathbf{0}^T$. In both cases, the connection between aggregate and cross-sectional variables hinges on heterogeneity in factor supply elasticities and nominal rigidities.

In equation (15), industries with overall higher factor price pass-through ($\sum_{f \in \mathcal{F}} \mathcal{P}_{if} > \bar{\mathcal{P}}_{\mathcal{F}}$) or with overall more inelastic factor supply ($\sum_{f \in \mathcal{F}} \Psi_{if} (\Phi \mathbf{1})_f > \bar{\Phi}$) are more sensitive to the aggregate output gap. Differences in factor price pass-through and factor supply elasticities reinforce each other when they are positively correlated ($(\frac{\mathcal{P}_{:\mathcal{F}}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \Psi_{:\mathcal{F}}) (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) > 0$).

In equation (16), aggregate inflation under zero output gap depends on the distribution of relative price shocks. Inflation increases when three groups of industries face a worse cost shock: industries with a higher cost pass-through

into the aggregate factor price deflator ($(\bar{\Omega}^{\mathcal{F}} \mathcal{P})_i > \bar{\mathcal{P}}_{\mathcal{F}} (\bar{\Omega}^{\mathcal{F}} \Psi)_i$), industries with inelastic factor supply ($(\Phi \mathbf{1})_f > \bar{\Phi}$), or industries with a higher weight in the GDP deflator relative to the aggregate factor price deflator ($\bar{\Omega}_{\mathcal{Z},i} > \bar{\Omega}_i^{\mathcal{F}}$).

Example 4.1 illustrates relative prices and aggregate inflation in a simplified two-industry economy.

Example 4.1. Supply bottlenecks and inflation. Consider the two-sector economy introduced in example 3.1, where the manufacturing industry has more flexible prices and less elastic factor supply. Example 3.1 showed that any shock resulting in higher demand for manufacturing inputs reduces the economy's potential output. This example shows that the same shocks increase aggregate inflation, even when output is at potential. Moreover, inflation in the manufacturing industry is more sensitive to the aggregate output gap.

Using the same aggregate and relative notation as in example 3.1, and following equation (13) and lemmas 4.1-4.2, we can write aggregate and relative inflation rates as

$$\bar{\pi}_t - \rho \mathbb{E} \bar{\pi}_{t+1} = \bar{\kappa} \tilde{y} + v^R \left(\pi_t^{R,nat} - \tilde{p}_{t-1}^R - \rho \mathbb{E} \pi_{t+1}^R \right) \quad (17)$$

$$\pi_t^R - \rho \mathbb{E} \pi_{t+1}^R = \kappa^R \tilde{y}_t + \bar{v} \left(\pi_t^{R,nat} - \tilde{p}_{t-1}^R - \rho \mathbb{E} \pi_{t+1}^R \right). \quad (18)$$

Consider first a special case where industries face the same nominal rigidity ($\delta_M = \delta_S \equiv \delta$) but capital supply is less elastic than labor supply ($\varphi_M > \varphi_S$). Then the relative slope is $\kappa^R = \frac{\delta \varphi_R}{1 + \theta \delta \bar{\varphi}}$, indicating that raising the output gap increases the relative price of manufacturing. This occurs because manufacturing production is harder to ramp up, resulting in a larger price response. The effect is stronger when manufacturing and services are less substitutable (lower θ), because final users cannot shift expenditures away from manufacturing.

Bottleneck-like shocks, which increase the natural relative price of manufacturing, raise aggregate inflation by $v^R \pi^{R,nat} = \frac{\theta \delta}{1 + \theta \delta \bar{\varphi}} Cov(\varphi, \pi^{nat})$ under zero output gap. The mechanism is as follows. An increase in the natural relative price of manufacturing means that the relative demand for capital increases. Due to nominal rigidities, the relative price of manufacturing does not fully adjust to its natural value. Therefore, even if aggregate output is at potential, demand remains inefficiently skewed toward capital. The distortion vanishes only when goods are perfect complements ($\theta \rightarrow 0$), and relative price distortions have no effect on relative demand. Since capital has a steeper supply curve, inefficiently high demand raises manufacturing prices by more than it reduces the price of services. Overall, this raises aggregate inflation.

Notably, an increase in the natural price of manufacturing ($\pi^{R,nat} > 0$) reduces real wages, causing a real wage gap $-\frac{\theta(1-\delta)\varphi_S}{1+\theta\delta\bar{\varphi}} \pi^{R,nat}$. In practice, declining real wages are often attributed to rising markups. In our example, however, shocks that raise the relative price of capital reduce real wages even though desired markups remain constant. This suggests that, if changes in capital costs are not accurately measured, relative demand or productivity shocks might be erroneously interpreted as markup shocks.¹⁹

¹⁹Measuring capital costs is notoriously challenging (Karabarbounis and Neiman, 2014). For example, the shadow value of land and

Our reasoning so far implies that, when supply elasticities are heterogeneous, relative prices and aggregate inflation become intertwined. Increasing the output gap raises aggregate inflation and the relative price of manufacturing at the same time. Likewise, an increase in the natural relative price of manufacturing raises both aggregate inflation and the actual relative price of manufacturing, controlling for the output gap.

A similar situation arises when capital and labor have the same supply elasticity, but manufacturing prices are more flexible than service prices. Again, increasing the output gap raises the relative price of manufacturing by $\kappa^R = \frac{\varphi}{1+\theta\varphi\frac{\delta}{1-\delta}\left(1-\frac{\delta_M\delta_S}{2\delta}\right)}\frac{\delta^R}{1-\delta}$. Increasing the natural relative price of manufacturing raises aggregate inflation by $v^R\pi^{R,nat} = -\frac{1-\frac{\varphi\theta\delta}{1+\varphi\theta\delta}\frac{\delta^R}{\delta}}{1+\frac{\varphi\theta\delta}{1+\varphi\theta\delta}\frac{Var(\delta)}{\delta(1-\delta)}}Cov(\delta, \pi^{nat})$ at constant output gap. In this case, however, the inflation-output tradeoff arises from a different mechanism. While closing the output gap stabilizes average marginal costs, it does not stabilize industry-level costs, which increase in industries with higher π^{nat} and fall elsewhere. Industries with flexible prices, like manufacturing, pass through cost changes more rapidly, therefore they dominate aggregate inflation.²⁰

As example 4.1 highlights, the conditions of Lemmas 4.1 and 4.2 are closely related. On the one hand, industries with a steeper Phillips curve have more cyclical prices. On the other hand, negative shocks to industries with steeper Phillips curves tend to cause a worse inflation-output tradeoff. Aggregate and cross-sectional dynamics are disjoint only in the knife-edge case where all factors have identical supply elasticities and all industries have identical network-adjusted price pass-through $\mathcal{P}$. This makes it particularly challenging to tease inflation drivers apart. We turn to this problem in section 4.2.

4.2 Shock Identification and Forecast Revisions

This section shows how to identify the aggregate output gap and natural relative prices, by inverting the system (13). To build to this result, proposition 4.2 guarantees that the model is invertible for every parameter configuration outside a measure-zero subset of the parameter space: the vector π of industry-level inflation rates can be written as a linear combination of the Phillips curve slope κ and the cost-push elasticities $\mathcal{V}$. These directions are not collinear, and $\mathcal{V}$ has full rank on the space of relative prices.

Proposition 4.2. *For all parameter values outside a measure-zero subset of the parameter space, the slope κ and the cost-push elasticity $\mathcal{V}$ span the entire space $\mathbb{R}^N$ of industry-level inflation rates. Therefore the vector π of industry-level inflation rates can be decomposed into its projections along the slope κ and the cost-push space $\mathcal{V}$:*

$$\pi_t = \kappa\pi_t^{DC} + \left(I - \kappa\zeta_{DC}^T\right)\pi_t. \quad (19)$$

firm-specific assets are strongly affected by changes in preferences and productivity, yet difficult to measure accurately.

²⁰I make a similar argument in Rubbo (2023).

The projection along the slope is given by the inflation index $\pi^{DC} \equiv \zeta_{DC}^T \boldsymbol{\pi}$, with industry-level weights

$$\begin{aligned} \zeta_{DC}^T &\propto \bar{\Psi}^T (I - \Delta) \Delta^{-1} \\ &- \left[\text{Cov}_{\bar{\Psi}_{\mathcal{F}}}(\tilde{\Phi}, \tilde{\Theta}) + \text{Cov}_{\bar{\Omega}_{\mathcal{Z}}} \left(\frac{\bar{\Omega}_{\mathcal{Z}} - \bar{\Omega}^{\mathcal{F}}}{\bar{\Omega}_{\mathcal{Z}}}, I \right) \right] \Psi (I - \Delta) \Delta^{-1} \end{aligned} \quad (20)$$

such that $\zeta_{DC}^T \mathcal{V} = \mathbf{0}^T$. I impose the normalization $\zeta_{DC}^T \boldsymbol{\kappa} = 1$.

Proof. See appendix B for derivations and definitions of covariance operators, $\tilde{\Omega}^{\mathcal{F}}$, $\tilde{\Phi}$ and $\tilde{\Theta}$. $\square$

The first term in equation (20) coincides with the divine coincidence index in (Rubbo, 2023), which considers an economy with labor as the only primary factor of production. This term weighs industries according to sales shares, discounting those with more flexible prices. The remaining terms are new to economies with multiple primary factors. The first covariance discounts industries that rely on inelastic factors, either directly or through intermediate inputs. The second covariance discounts industries whose share in the GDP deflator ($\bar{\Omega}_{\mathcal{Z},i}$) is smaller than their incidence on the aggregate factor price deflator ($\tilde{\Omega}_i^{\mathcal{F}}$).

Proposition 4.2 guarantees that the loadings of sectoral inflation on current output gap and natural relative prices are never collinear. In the system (13), however, current inflation also depends on past and future shocks through inflation expectations and lagged price gaps. To clarify this issue, proposition 4.3 decomposes current inflation into two terms. The first depends on past, current, and expected future output gaps. The second depends on past, current, and expected natural relative price shocks.

Proposition 4.3. Inflation decomposition. Denote by $\Lambda \in \mathbb{R}^{N \times N}$ the stable solution of the Riccati equation

$$\Lambda^{-1} (I - \rho \Lambda) (I - \Lambda) = (I - \mathcal{V})^{-1} \mathcal{V}.$$

It holds that $\Lambda \mathbf{1} = \mathbf{1}$ and $\zeta_{DC}^T \Lambda = \zeta_{DC}^T$. If past shocks were unexpected at the time when they occurred, while current and future shocks were unexpected before the current period t , industry level inflation rates $\boldsymbol{\pi}_t$ are given by

$$\begin{aligned} \boldsymbol{\pi}_t &= \sum_{s=0}^{\infty} (\rho \Lambda)^s \Lambda (I - \mathcal{V})^{-1} \boldsymbol{\kappa} \mathbb{E} \tilde{y}_{t+s} - (I - \Lambda) \sum_{s=1}^{\infty} \Lambda^s (I - \mathcal{V})^{-1} \boldsymbol{\kappa} \tilde{y}_{t-s} \\ &+ (I - \Lambda) \left[\sum_{s=0}^{\infty} (\rho \Lambda)^s \mathbb{E} \boldsymbol{\pi}_{t+s}^{\text{nat}} + \sum_{s=1}^{\infty} \Lambda^s \boldsymbol{\pi}_{t-s}^{\text{nat}} \right]. \end{aligned} \quad (21)$$

Appendix B provides a general expression, where current inflation at time t depends on belief updates at all prior dates $t - \tau$ about output gaps and natural relative prices at dates $t + s$, with $s \geq -\tau$. Section 5 uses the more general expression to implement a quantitative inflation decomposition.

Proof. See appendix B. □

Proposition 4.3 derives an inflation decomposition grounded in theory. The first term captures aggregate demand misalignment, as measured by past, current, and future output gaps. The second component captures relative price adjustments, as measured by natural relative price changes. Proposition 4.3, however, does not tell us how to measure the two inflation components in practice. Lemma 4.3 makes a first step, showing that the present discounted value of output gaps can be directly identified from an appropriately-weighted inflation index.

Lemma 4.3. Present value of output gaps. *The inflation index π^{DC} is equal to the present discounted value of current and expected future output gaps:*

$$\pi_t^{DC} = \sum_{\tau=0}^{\infty} \rho^{\tau} \mathbb{E}_t \tilde{y}_{t+\tau}. \quad (22)$$

Proof. The result follows immediately from proposition 4.3. □

While the inflation index π^{DC} only depends on the present value of output gaps, equation (21) reveals that other price indices (including the CPI) depend on the full time profile of natural relative prices and output gaps. Remark 4.1 discusses implications for the inflation decomposition in proposition 4.3.

Remark 4.1. Robustness of the inflation decomposition. The two inflation components in proposition 4.3 depend on the Phillips curve slope κ , the cost-push elasticity $\mathcal{V}$, and the identified output gaps and natural price changes. Given that the decomposition adds up to actual inflation, knowing one of the two components is sufficient. Lemma 4.3 shows how to identify the present value of output gaps, which is informative of the first component. Notably, identified present values only depend on ζ_{DC} , which in turn is determined by supply-side parameters only (nominal rigidities, input-output linkages, supply and substitution elasticities). The monetary policy rule and the stochastic processes governing shocks only matter for identifying the time profile of output gaps. Consistent with this observation, the quantitative decomposition of section 5 is very sensitive to supply-side parameters, but not so sensitive to the policy rule and shock persistence.

Example 4.2 illustrates how persistent and anticipatory effects of output gaps translate into relative price changes.

Example 4.2. Phillips curves in sequence space. Equations (23) and (24) apply proposition 4.3 to our running two-industry example, assuming that past changes in output gaps and natural relative prices were unexpected at the

time when they occurred, and all current and future shocks were unexpected before the current period t .

$$\pi_t^{DC} = \sum_{s=0}^{\infty} \rho^s \mathbb{E}_t \tilde{y}_{t+s} \quad (23)$$

$$\begin{aligned} \left(I - \kappa \zeta_{DC}^T \right) \pi_t = & \begin{pmatrix} \frac{v_1}{\bar{v}(1-\bar{v})} \\ -\frac{v_2}{\bar{v}(1-\bar{v})} \end{pmatrix} \left\{ \sum_{s=0}^{\infty} \rho^s \left[(1-\bar{v})(1-\lambda) \lambda^s \pi_{R,t+s}^{nat} - (1-\bar{v}-\lambda^{s+1}) \kappa^R \tilde{y}_{t+s} \right] \right. \\ & \left. - (1-\lambda) \sum_{s=1}^{\infty} \lambda^{s-1} \left[\lambda \kappa^R \tilde{y}_{t-s} + [\lambda \bar{v} - (1-\rho\lambda(1-\lambda))(1-\bar{v})] \pi_{R,t-s}^{nat} \right] \right\}, \end{aligned} \quad (24)$$

where

$$\lambda = \frac{1}{2} \left[\left(1 + \frac{1}{\rho(1-\bar{v})} \right) - \sqrt{\left(1 + \frac{1}{\rho(1-\bar{v})} \right)^2 - \frac{4}{\rho}} \right].$$

In equation (23), the inflation index π^{DC} only depends on the present discounted value of future output gaps. The index $\left(I - \kappa \zeta_{DC}^T \right) \pi$ in (24) instead depends on current and expected relative price shocks $\pi_{R,t+s}^{nat}$, as well as on persistent and anticipated relative price distortions induced by past and future output gaps. This makes it impossible to use the indices π^{DC} and $\left(I - \kappa \zeta_{DC}^T \right) \pi$ directly for identification. I turn to this issue next.

Lemma 4.3 alone does not allow us to infer the expected time profile of output gaps. Likewise, lemma 4.3 and equation (21) by themselves are not sufficient to identify changes in natural relative prices. To separately identify the time paths of aggregate output gaps and natural relative prices we need additional information about the stochastic processes governing these variables. For intuition, start by assuming that the output gap and natural relative prices follow AR1 processes

$$\begin{aligned} \tilde{y}_t &= \eta^y \tilde{y}_{t-1} + \varsigma_t^y \\ \pi_t^{nat} &= \eta^{nat} \pi_{t-1}^{nat} + \varsigma_t^{nat}, \end{aligned} \quad (25)$$

where ς_t^y and ς_t^{nat} are iid innovations. This assumption holds under a modified Taylor rule which targets the inflation index π^{DC} : $i_t = \phi_y \tilde{y}_t + \phi_\pi \pi_t^{DC} + \mathbb{E} \pi_{t+1}^{IS} + \epsilon_t^i$. Lemma 4.4 shows that, under these assumptions, the inflation indices π_t^{DC} and $\left(I - \kappa \zeta_{DC}^T \right) \pi_t$ can be used directly to identify the innovations ς_t^y and ς_t^{nat} .

Lemma 4.4. Identification under output gap targeting. *Denote inflation surprises by $\hat{\pi}_t \equiv \pi_t - \mathbb{E}_{t-1} \pi_t$. When output gaps and natural relative prices follow the AR1 process (25), and apart from a measure-zero subset of the parameter space where $I - \Lambda + \mathbf{1} \bar{\Omega}_{\mathcal{Z}}$ is singular, innovations to the output gap and natural relative prices can be*

identified using the following aggregate and relative price indices:

$$\begin{aligned}\varsigma_t^y &= (1 - \rho\eta^y) \hat{\pi}_t^{DC} \\ \varsigma_t^{nat} &= (I - \Lambda + \mathbf{1}\bar{\Omega}_{\mathcal{Z}})^{-1} (I - \rho\eta^{nat}\Lambda) \left[(I - \kappa\zeta_{DC}^T) \hat{\pi}_t + \left[I - (1 - \rho\eta^y)(I - \rho\eta^y\Lambda)^{-1} \Lambda (I - \mathcal{V})^{-1} \right] \kappa \hat{\pi}_t^{DC} \right].\end{aligned}\tag{26}$$

The aggregate price index π^{DC} identifies output gaps, while the relative price index $(I - \kappa\zeta_{DC}^T) \pi$ identifies innovations to the natural relative prices. The identified value of ς^{nat} depends on π^{DC} , to control for the endogenous response of relative prices to current and future output gaps.

While useful for intuition, the policy rule in lemma 4.4 is counterfactual. The quantitative section 5 relies on a more general approach, derived in proposition 4.4, which holds under a generic Taylor rule.

Proposition 4.4. Identification under a generic Taylor rule. *Assume that ϵ^i and π^{nat} follow autoregressive processes:*

$$\begin{aligned}\epsilon_{t+1}^i &= \eta^D \epsilon_t^i + \varsigma_t^D \\ \pi_{t+1}^{nat} &= \eta^{nat} \pi_t^{nat} + \varsigma_t^{nat}.\end{aligned}$$

Under the [Blanchard and Kahn \(1980\)](#) determinacy conditions, there exist a transition matrix H and a jump matrix G such that

$$\begin{pmatrix} \tilde{y}_t \\ \epsilon_t^i \\ \pi_t^{nat} \end{pmatrix} = G \begin{pmatrix} \tilde{p}_{t-1} \\ \epsilon_t^i \\ \pi_t^{nat} \end{pmatrix}, \quad \begin{pmatrix} \tilde{p}_t \\ \epsilon_{t+1}^i \\ \pi_{t+1}^{nat} \end{pmatrix} = \left[H \begin{pmatrix} \tilde{p}_{t-1} \\ \epsilon_t^i \\ \pi_t^{nat} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \varsigma_{t+1}^D \\ \varsigma_{t+1}^{nat} \end{pmatrix} \right].\tag{27}$$

Let $G_{\pi\varsigma} \in \mathbb{R}^{N \times (N+1)}$ denote the rows of G corresponding to inflation and the columns corresponding to ϵ^i and π^{nat} . Denoting $\hat{\Omega}^{\mathcal{Z}} \equiv \begin{pmatrix} 0 & \bar{\Omega}_{\mathcal{Z}} \end{pmatrix}$, shocks to aggregate demand and natural relative prices can be identified using the aggregate and relative price indices introduced above:

$$\begin{pmatrix} \varsigma_t^D \\ \varsigma_t^{nat} \end{pmatrix} = \begin{pmatrix} \zeta_{DC}^T G_{\pi\varsigma} \\ (I - \kappa\zeta_{DC}^T) G_{\pi\varsigma} + \mathbf{1}\hat{\Omega}^{\mathcal{Z}} \end{pmatrix}^{-1} \begin{pmatrix} \hat{\pi}_t^{DC} \\ (I - \kappa\zeta_{DC}^T) \hat{\pi}_t \end{pmatrix}.\tag{28}$$

Proof. See appendix B □

The impulse-response representation in (27) can also be used to construct inflation forecasts, by combining estimated shocks with the dynamic system (13). Lemma 4.5 derives inflation indices that forecast aggregate inflation.

Lemma 4.5. Forecast revisions. Denoting by $H_{\cdot\varsigma}$ the columns of H corresponding to ϵ^i and π^{nat} , the following inflation indices provide forecast updates for aggregate inflation:

$$\mathbb{E}_t \bar{\pi}_{t+s} - \mathbb{E}_{t-1} \bar{\pi}_{t+s} = \bar{\Omega}_{\mathcal{Z}} G_{\pi: H_{\cdot\varsigma}}^s \left(\begin{array}{c} \zeta_{DC}^T G_{\pi\varsigma} \\ (I - \kappa \zeta_{DC}^T) G_{\pi\varsigma} + \mathbf{1} \hat{\Omega}^{\mathcal{Z}} \end{array} \right)^{-1} \left(\begin{array}{c} \hat{\pi}_t^{DC} \\ (I - \kappa \zeta_{DC}^T) \hat{\pi}_t \end{array} \right) \quad (29)$$

Notably, no single inflation index can be used to forecast aggregate inflation at every horizon. The powers H^s in (29) indicate that the inflation indices used for different time horizons s combine industry-level inflation surprises with different relative weights. This happens for two reasons. First, industries with flexible prices and/or inelastic factor supply respond quickly to shocks and in a less persistent way, while industries with stickier prices and/or elastic factor supply respond more slowly and persistently. Second, fundamental shocks themselves may have different persistence across industries. Inflation is shorter-lived when it originates in industries that face less persistent shocks.

Example 4.3. Shock identification and forecast revisions. In the two-industry example the relative price index is one-dimensional, so it is convenient to write it in scalar form as $(I - \kappa \zeta_{DC}^T) \pi_t = v \pi_t^{\chi}$, with $v \equiv (v_1, -v_2)^T$. Applying lemma 4.4 to our running two-industry example, we can construct inflation indices which identify innovations to the output gap and natural relative prices:

$$\begin{aligned} \varsigma_t^y &= (1 - \rho \eta^y) \hat{\pi}_t^{DC} \\ \varsigma_t^{nat} &= \frac{\bar{v} (1 - \rho \lambda \eta^{nat})}{1 - \lambda} \left[\hat{\pi}_t^{\chi} + \frac{\kappa^R}{\bar{v} (1 - \bar{v})} \left(1 - \bar{v} - \lambda \frac{1 - \rho \eta^y}{1 - \rho \lambda \eta^y} \right) \hat{\pi}_t^{DC} \right]. \end{aligned}$$

Combining this result with the sequence-space representation (23)-(24) allows us to express the time- t forecast revision for aggregate inflation at time $t + s$ as a combination of the inflation indices π_t^{DC} and π_t^{χ} :

$$\mathbb{E}_t \bar{\pi}_{t+s} - \mathbb{E}_{t-1} \bar{\pi}_{t+s} = (\bar{\kappa} + \mathcal{C}_s^{DC} \kappa^R) (\eta^y)^s \hat{\pi}_t^{DC} + \mathcal{C}_s^{\chi} (\eta^{nat})^s \hat{\pi}_t^{\chi}, \quad (30)$$

where we defined

$$\begin{aligned} \mathcal{C}_s^{\chi} &\equiv v^R \left[1 + \lambda (1 - \rho \eta^{nat}) \frac{1 - \left(\frac{\lambda}{\eta^{nat}} \right)^s}{\eta^{nat} - \lambda} \right] \\ \mathcal{C}_s^{DC} &\equiv -\frac{v^R}{\bar{v} (1 - \bar{v})} \left\{ \left(1 - \bar{v} - \lambda \frac{1 - \rho \eta^y}{1 - \rho \lambda \eta^y} \right) \left[1 - \left(\frac{\eta^{nat}}{\eta^y} \right)^s \frac{\mathcal{C}_s^{\chi}}{v^R} \right] + \frac{\lambda (1 - \lambda) (1 - \rho \eta^y)}{1 - \rho \lambda \eta^y} \frac{1 - \left(\frac{\lambda}{\eta^y} \right)^s}{\eta^y - \lambda} \right\}. \end{aligned}$$

The first term in (30) depends on the aggregate inflation index π^{DC} , which forecasts future output gaps. Future output gaps affect aggregate inflation directly through the slope $\bar{\kappa}$, and indirectly through their dynamic effects on relative prices. The response of relative prices to future output gaps is captured by $\mathcal{C}^{DC}$, scaled by the relative

slope κ^R . The first term in $\mathcal{C}^{DC}$ purges π^χ of the relative price movements that current and expected output gaps themselves induce. The second term accumulates the distortions past output gaps leave behind between t and $t + s$. The second term in (30) instead captures the contribution of industry-specific shocks, forecasted by the relative price index π^χ .

If the two industries face identical nominal rigidities and supply elasticities, it holds that $\kappa^R = \mathcal{C}_s^\chi = 0$. Forecast revisions for aggregate inflation only depend on the divine coincidence index π_t^{DC} , which becomes proportional to $\bar{\pi}_t$.

4.3 Broader Implications for Identification

Factor decomposition. The identification strategy proposed in this paper is akin to a factor decomposition, in that it projects industry-level inflation rates along specific directions, corresponding to aggregate and industry-specific drivers. Compared to standard model-free decompositions, such as principal component analysis (PCA), my identification method relies on knowing the primitive parameters of the economy, and it is sensitive to misspecification of these parameters. The two approaches differ in where identification comes from: rather than relying on a structural model, PCA imposes an orthogonality restriction, whereby an industry's loading on one factor does not predict its loadings on the other factors. Through the lens of the model, this restriction identifies aggregate and industry-specific drivers precisely when κ is orthogonal to $\mathcal{V}$. This is a knife-edge case, which arises only if industries have identical network-adjusted nominal rigidities and factor supply elasticities.

As we saw in lemmas 4.1-4.2 and example 4.1, when industries have different nominal rigidities or supply elasticities their loadings on aggregate and relative shocks become correlated. A structural reading of estimated principal components therefore requires industries to be identical along both dimensions in the data.

Figure 7 illustrates the problem in our running two-industry example. The panels simulate inflation responses to output gaps and natural relative prices, and estimate principal components. The top two panels represent an economy where $\kappa_M = \kappa_S$ and $\nu_M = \nu_S$, while the bottom panels represent an economy with $\kappa_M > \kappa_S$ and $\nu_M > \nu_S$. Shocks are drawn from the same distribution in both economies. The variance of natural relative prices is larger in the right two panels, while the variance of output gaps is larger in the left panels.

The top two panels show that PCA correctly recovers the directions of κ and $\mathcal{V}$ in the symmetric model, regardless of the shock distribution. The bottom two panels instead show that PCA never recovers both κ and $\mathcal{V}$ correctly in the asymmetric model. This happens because PCA imposes an orthogonal decomposition. PCA approximates well the slope κ when variation is mostly driven by relative price shocks, and vice versa.

Non-linear models. The COVID episode prompted many researchers to model the connection between relative prices and aggregate inflation, often relying on non-linearities in production or pricing.²¹ Non-linearities are a source

²¹See for example Comin et al. (2023); Ferrante et al. (2023); Alessandria et al. (2023); Bai et al. (2024).

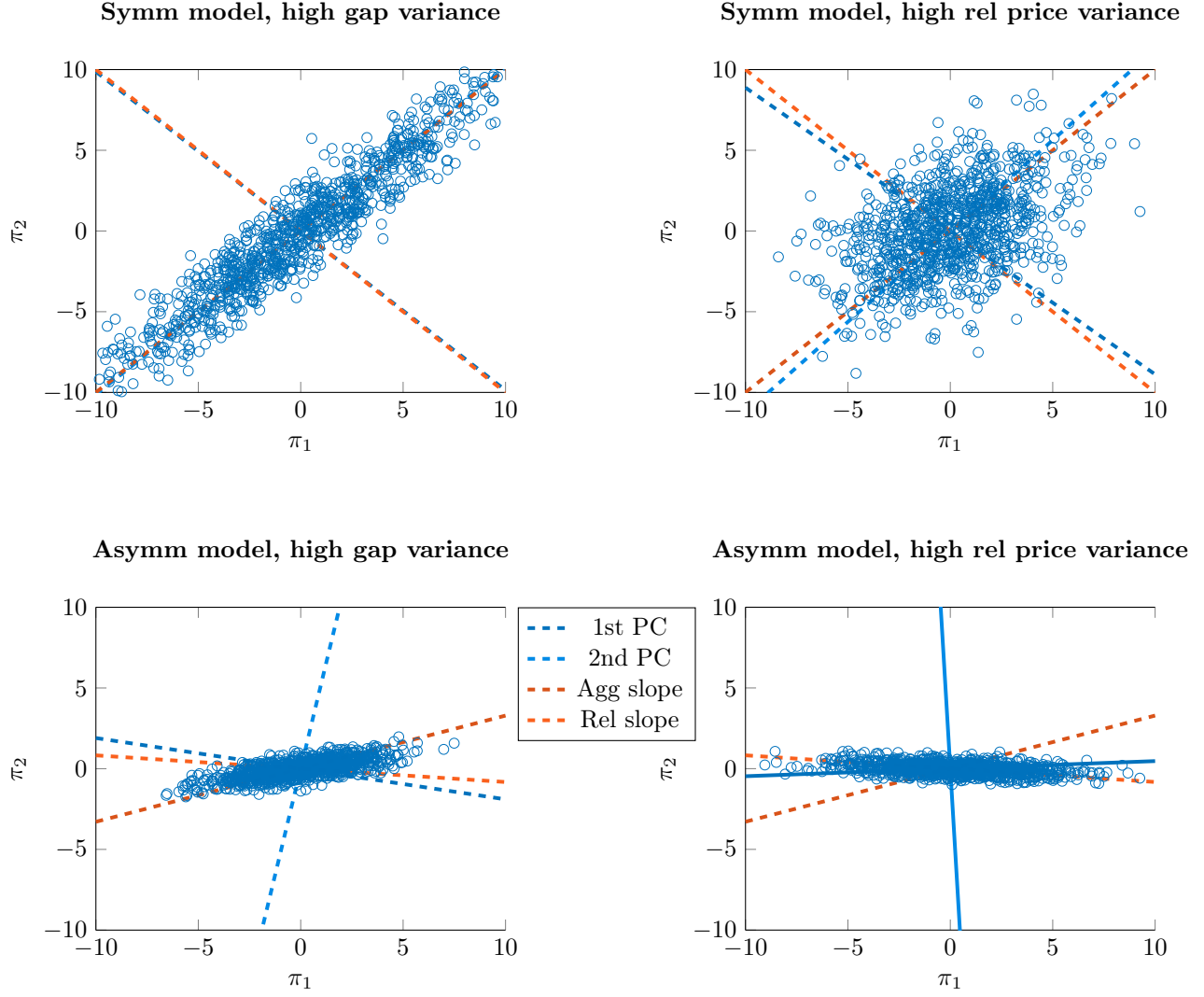


Figure 7 Estimated principal components (blue lines) versus structural aggregate and relative slopes (red lines). Top panels correspond to a two-industry economy where both industries face the same price adjustment frequency and factor supply elasticity. Bottom panels correspond to an economy where one industry uses inelastically supplied factors more intensively. The left panels illustrate the principal components recovered when variation is predominantly driven by the output gap, while the right panels illustrate the principal components recovered when variation is predominantly driven by natural relative price adjustments.

of heterogeneity in supply and pricing elasticities, and it is this heterogeneity – rather than non-linearity per se – that links industry-specific shocks to aggregate inflation. Non-linear models typically assume that Phillips curve slopes κ and cost-push elasticities $\mathcal{V}$ are homogeneous ex-ante, but they become heterogeneous in response to shocks, as industries move toward steeper or flatter portions of their supply curves. My linear framework assumes that they are heterogeneous ex-ante, but invariant to shocks. The two models are not observationally equivalent.

With ex-ante heterogeneous slopes the aggregate output gap moves relative prices at first order, whereas with ex-post heterogeneity alone it does so only at second order, and only in combination with industry-specific shocks. Given the same observed inflation rates, therefore, a model in which heterogeneity arises only ex-post attributes a larger share of aggregate inflation to industry-specific shocks than one in which heterogeneity is present ex-ante.

I illustrate these points by comparing a linear and a non-linear version of our running two-industry example. In the linear setting industries are heterogeneous ex-ante (prior to any perturbation). In the non-linear setting I assume that industries have identical nominal rigidities and supply elasticities ex-ante, but factor supply becomes less elastic as employment increases. For simplicity, I also assume that variation is entirely driven by productivity and output gaps. Appendix C.1 provides derivations under a larger set of shocks. A second-order approximation yields

$$\tilde{w}_f = \varphi_f^{nat} \tilde{\ell}_f + \frac{1}{2} \varphi_2 \tilde{\ell}_f^2 + \tilde{p}^C.$$

with $\varphi_f^{nat} \equiv \varphi + \varphi_2 \ell_f^{nat}$. With full discounting ($\rho = 0$), aggregate and relative inflation become

$$\begin{aligned} \bar{\pi} &= \frac{\delta}{1-\delta} \left[\bar{\varphi}^{nat} \tilde{y} + \frac{1}{2} \varphi_2 \tilde{y}^2 + \theta \varphi_2 \frac{1-\delta}{1+\theta \delta \bar{\varphi}^{nat}} \left(\frac{1-\theta}{1+\varphi} + \frac{1-\delta}{2} \frac{\theta}{1+\theta \delta \bar{\varphi}^{nat}} \right) Var(\pi^{nat}) \right] \\ \pi^R &= \frac{\delta(1+\theta \bar{\varphi}^{nat})}{1+\theta \delta \bar{\varphi}^{nat}} \pi^{R,nat} + \frac{\delta \varphi_2}{1+\theta \delta \bar{\varphi}^{nat}} \left(\frac{1-\theta}{1+\varphi} + \frac{\theta(1-\delta)}{1+\theta \delta \bar{\varphi}^{nat}} \right) \tilde{y} \pi^{R,nat}. \end{aligned} \quad (31)$$

Equation (31) highlights similarities and differences with the linear model in example 4.1. In the first line, aggregate inflation depends on the variance of industry-specific price gaps, whereas the linear model features a covariance between price gaps and supply elasticities. Both capture the same economic mechanism, but they differ in how heterogeneity arises. In the non-linear model, relative supply elasticities are endogenous: the industry whose natural relative price increases gets pushed toward a steeper portion of its supply curve, thus facing more inelastic supply. Therefore the covariance between natural relative prices and supply elasticities is necessarily negative, yielding a bad cost-push shock, as captured by the variance term in (31).

The second line shows that, absent ex-ante heterogeneity, the output gap has only a second-order effect on relative prices. Moreover, this effect only arises in combination with industry-specific shocks ($\pi^{R,nat} \neq 0$). With ex-ante identical supply elasticities the output gap does not affect relative prices, unless other shocks push industries toward different portions of their supply curves. By contrast, with ex-ante heterogeneity the output gap moves relative prices at first order, even absent industry-specific shocks.

These differences carry over to identification. A feature of ex-post heterogeneity is that the industry receiving a positive relative price shock is always the one pushed onto a steeper portion of its supply curve, so any relative price movement signals a worse inflation-output tradeoff. Ex-ante heterogeneity imposes no such restriction – whether the tradeoff worsens depends on which industry's relative price is rising. The two specifications therefore diverge most for shocks that improve the tradeoff, which the ex-post channel cannot represent.

The aggregate and relative inflation indices derived in proposition 4.2 can still provide useful intuition. Denote the industry weights in these two indices by $\hat{\zeta}_{DC}$ and $\hat{\zeta}_\chi$. In non-linear models the sectoral weights are not constant over time, and they depend on the unknowns that these indices are meant to identify. This gives rise to a fixed-point

problem:

$$\begin{aligned}\tilde{y}_t &= \hat{\zeta}_{DC}^T (\tilde{y}_t, \pi_{Rt}^{nat}) \boldsymbol{\pi}_t \\ \pi_{Rt}^{nat} &= \hat{\zeta}_{\chi}^T (\tilde{y}_t, \pi_{Rt}^{nat}) \boldsymbol{\pi}_t.\end{aligned}$$

Unlike its linear counterpart, this fixed-point problem has multiple solutions. Table 1 selects the solution closest to the linear one, and compares the inflation decompositions implied by various linear and non-linear models.

Throughout the table, I consider an increase in aggregate inflation. The first and second columns consider an increase in the relative price of manufacturing, while the third and fourth columns consider an increase in the relative price of services. The first and third columns report the component of aggregate inflation attributed to the aggregate output gap, while the second and fourth columns report the component attributed to industry-specific shocks. Each row corresponds to a different model. The first model is linear, with identical Phillips curve slopes in the two sectors. The second is linear with heterogeneous slopes. The third is non-linear with identical initial slopes.

Model	$\pi^{R,nat} = 3, \bar{\pi} = 4$		$\pi^{R,nat} = -3, \bar{\pi} = 4$	
	Aggregate	Industry-specific	Aggregate	Industry-specific
Linear, homogeneous slopes	4.00	0.00	4.00	0.00
Linear, heterogeneous slopes	3.42	0.58	5.03	-1.03
Non-linear, homogeneous slopes	1.89	2.11	1.89	2.11

Table 1 Inflation decomposition implied by alternative models, two-industry example. Each row reports the components of observed aggregate inflation $\bar{\pi} = 4$ attributed to the aggregate output gap and to industry-specific shocks, given observed inflation, when natural relative prices change by $\pi^{R,nat} = \pm 3$. The two components add up to $\bar{\pi}$ by construction.

In the linear and non-linear models with equal slopes, the decomposition of aggregate inflation does not depend on the sign of relative price changes. The linear model entirely attributes aggregate inflation to aggregate demand. The non-linear model attributes part of it to an adverse cost-push shock, signalled by relative price changes.

In the model with heterogeneous slopes, instead, the inflation decomposition depends on the sign of relative price changes. If the price of manufacturing increases, this model attributes part of aggregate inflation to an adverse cost-push shock, and part of the relative price change to the aggregate output gap. If the relative price of services increases, instead, this implies a good cost-push shock. Therefore industry-specific shocks had an offsetting effect on aggregate inflation, thus the model infers an aggregate component exceeding observed inflation.

In this example, then, the specification with only ex-post heterogeneity attributes more of aggregate inflation to industry-specific shocks than the specification with ex-ante heterogeneity, and the gap is largest when industry-specific shocks improve the inflation-output tradeoff. Comparing linear and non-linear models on US data would require calibrating the second-order terms φ_2 industry by industry, which is outside of the scope of this paper.

5 Inflation Drivers in the US, 2000 - 2024

This section provides a quantitative decomposition of inflation in the US, between 2000 and 2024. Section 5.1 describes the model calibration. Section 5.2 starts by presenting the headline inflation decomposition. It then turns to quantifying the interconnections between relative prices and aggregate inflation. On the one hand, it computes the impact of relative price shocks to various industries on aggregate inflation during the COVID pandemic. On the other hand, it quantifies the component of relative prices driven by the aggregate output gap. Finally, it computes model-based inflation forecasts between 2020 and 2023.

5.1 Data

The calibration features 53 primary factors (22 labor occupations, classified according to the Standard Occupation Classification by the BLS, and 31 semi-fixed assets corresponding to equipment, structures, and intellectual property in the NIPA tables), and 66 private industries, classified according to the input-output tables provided by the Bureau of Economic Analysis (BEA).²² Table 2 lists data sources and parameter values. A model period is one quarter, and parameters in table 2 are quarterly values.²³

Time series of prices and wages. I obtain time series for the Consumer Price Index (CPI) and the CBO output gap from the St Louis Fed database (FRED). Industry-level price series are from the BLS PPI and CPI datasets, and time series for occupation-level wages are from the Current Population Survey (CPS). All series are quarterly, and inflation rates are reported year-on-year in annual percentage points.

Supply elasticities of non-labor factors. Supply elasticities of non-labor factors are calibrated by matching empirical impulse-responses of industry-level prices to identified monetary shocks. I estimate local projections of industry-level inflation using the specification

$$\pi_{(i,t+h)} = \beta_0 + \beta_{1h} \widehat{gs1}_t + \sum_{f \in \mathcal{K}} \beta_{fh} (\widehat{gs1}_t \times S_i^f \times 100) + \theta(L)\pi_{(i,t)} + \gamma_i + u_{(i,t)}$$

where γ_i is an industry fixed effect, $\widehat{gs1}_t$ is a time series of identified monetary policy shocks, and S_i^f are demeaned value added shares of the four non-labor factor categories in industry i (structures, heavy equipment, intellectual property, and light equipment). I estimate inverse supply elasticities φ_f for the four non-labor factors by running

²²The 5 government industries (federal general government, split into defense and non-defense, federal government enterprises, and state-and-local general government and government enterprises) are excluded from every result reported at the industry level, because no producer price series exists for these industries. They are kept in the input-output network, since other industries buy from them.

²³The quantitative model adds interest-rate smoothing, as in Smets and Wouters (2007):

$$i_{t+1} - \mathbb{E}_t r_{t+1}^{nat} = \rho_i (i_t - \mathbb{E}_{t-1} r_t^{nat}) + (1 - \rho_i) [\phi_y \tilde{y}_t + \phi_\pi \pi_t^{TR}] + \epsilon_{t+1}^i.$$

Discount factor (quarterly)	$\rho = 0.99$
Intertemporal elasticity of substitution	$\gamma = 1$
Inverse Frisch elasticity (labor)	$\varphi_L = 1.83$ (Smets and Wouters, 2007)
Inverse supply elasticity (structures)	$\varphi_S = 5.70$
Inverse supply elasticity (heavy equipment)	$\varphi_{HE} = 9.26$
Inverse supply elasticity (intellectual property)	$\varphi_{IP} = 2.68$
Inverse supply elasticity (light equipment)	$\varphi_{LE} = 0$
Substitution between labor types	$\theta_{k,h}^i \in [0.10, 0.79]$, mean 0.40 $\forall k, h \in \mathcal{L}, i \in \{1, \dots, N\}$
Substitution between non-labor factors	$\theta_{k,h}^i = 0.50 \forall k, h \in \mathcal{K}, i \in \{1, \dots, N\}$
Substitution between labor and non-labor factors	$\theta_{k,h}^i = 0.10 \ k \in \mathcal{K}, h \in \mathcal{L}, i \in \{1, \dots, N\}$
Substitution between factors and intermediates	$\theta_{h,j}^i = 0.10 \ h \in \mathcal{F}, i, j \in \{1, \dots, N\}$
Substitution between intermediates	$\theta_{j,n}^i = 0.70$ across, 3.00 within 2-digit sectors, $i, j, n \in \{1, \dots, N\}$
Substitution between final goods	$\theta_{i,j}^h = 1.00$ across, 3.00 within 2-digit sectors, $h \in \mathcal{H}, i, j \in \{1, \dots, N\}$
Persistence of monetary shocks	$\eta^D = 0.15$
Persistence of shocks to π^{nat}	$\eta^{nat} = 0.10$
Taylor-rule response to aggregate inflation	$\phi_\pi = 1.60$, with $\pi^{TR} = \text{GDP deflator}$
Taylor-rule response to the output gap	$\phi_y = 0.33$
Taylor-rule interest-rate smoothing	$\rho_i = 0.80$
Industries' probabilities of price adjustment	Pasten et al. (2020)
Industries' expenditure on occupations	Census data (ACS)
Industries' expenditure on intermediate inputs	BEA input-output accounts
Expenditure shares of final consumers	BEA input-output accounts
Expenditure shares of utilization producers	Vom Lehn and Winberry (2022)
Ownership shares of firms and non-labor factors	Census data (ACS)

Table 2 Model calibration

the same regression in the model, and choosing these elasticities to match the peak local-projection coefficients β_{fh} , holding all other parameters fixed. Appendix D details the procedure and reports the model fit.

5.2 Identification and Forecast Revisions

Calibrated sectoral slopes and CPI incidences. Figure 8 plots calibrated industry-level slopes κ_i against CPI incidences $(\bar{\Omega}_Z \mathcal{V})_i$, coloring industries according to their price adjustment probabilities and non-scalable shares of value added. Both slopes and incidences are increasing in these two characteristics. Heterogeneous industry-level slopes imply that relative prices meaningfully respond to the aggregate output gap, while non-zero CPI incidences imply that aggregate inflation meaningfully responds to natural relative price changes. Disentangling these two inflation drivers thus becomes non-trivial. Slopes and incidences are positively correlated, although not perfectly, consistent with the invertibility result of proposition 4.2. The next paragraphs identify the drivers of relative prices

and aggregate inflation through the structural model.

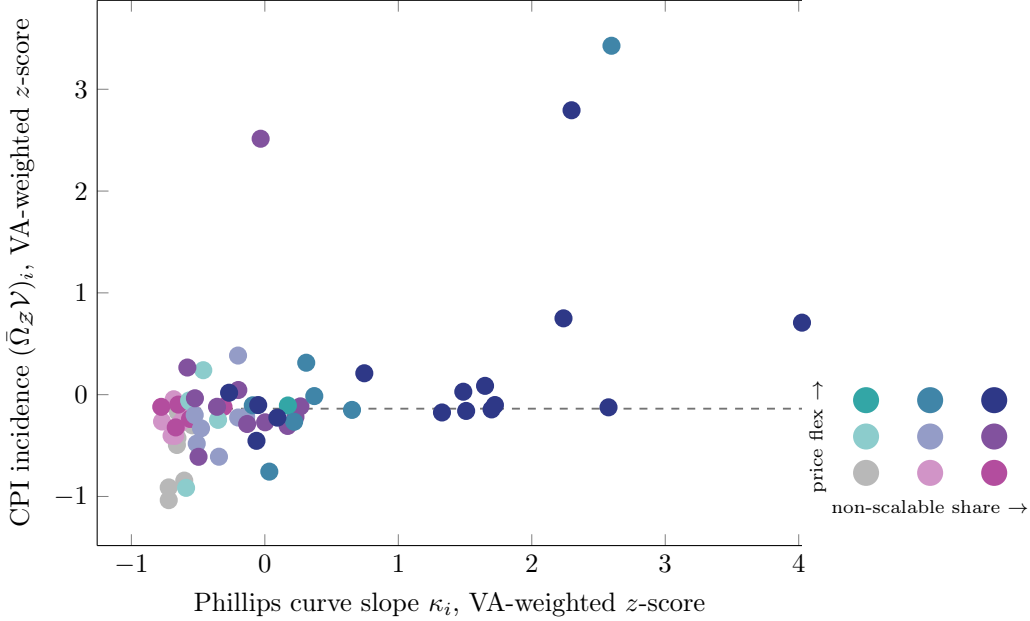


Figure 8 Correlation between model-implied industry-level Phillips curve slopes and incidences of relative price shocks on the aggregate CPI. Marker color varies by value-added-weighted tercile of the two characteristics. The dashed line marks zero CPI incidence (which differs from the value-added weighted mean).

Decomposition of aggregate inflation Following proposition 4.2, I isolate the components of aggregate inflation driven by aggregate demand misalignment and natural relative price adjustments.²⁴ The black line in figure 9 plots deviations of the CPI from its 2% average. The blue and orange bars decompose changes in the CPI into aggregate demand misalignment – as measured by shocks to the Taylor rule ϵ^i – and natural relative price changes.²⁵ Figure 9 reveals that both inflation drivers are important, and their relative importance varies over time. Industry-specific shocks accounted for 49% of consumer price inflation between 2000 and 2019. Aggregate demand instead was the main inflation driver during the COVID episode, accounting for 68% of the disinflation between 2020Q1–2020Q4 (light shaded window in figure 9) and for 69% of the inflation surge over 2021Q2–2023Q1 (dark shaded window). All percentages are shares of average absolute changes.²⁶ Inflation returned above the 2% target in 2021Q1, when relative price adjustments raised it 0.7 percentage points above target, while the output gap was still subtracting 0.3 percentage points.

²⁴The model inversion is numerically stable, with condition number of 1.1×10^3 . Perturbing every sectoral inflation series by relative noise of size 10^{-8} moves the recovered output gap by a relative 1.0×10^{-8} and the recovered natural prices by 3.0×10^{-8} .

²⁵The decomposition is implemented as follows. I initialize sectoral price gaps $\tilde{p}_{-1}$ at zero at the start of the sample. In each subsequent quarter I construct conditional inflation forecasts using the state transition matrix H of proposition 4.4; the difference between the observed sectoral inflation rates and model-implied forecasts gives the sectoral inflation surprises $\tilde{\pi}_t$. These surprises are then mapped into exogenous shocks ζ^D and ζ^{nat} using equation (28), and fed recursively into the belief-updating expression (46) of appendix B to obtain each quarter's inflation contributions. Notably, the orange bars include endogenous output-gap responses to relative price shocks through the Taylor rule. The two bars in figure 9 sum to the black line.

The inflation series are year-on-year. The model inversion is a linear function of the inflation path, therefore the components plotted in figure 9 are four-quarter moving sums of the components that would be obtained from quarter-on-quarter data.

²⁶Writing aggregate inflation as $\pi_t = \pi_t^{gap} + \pi_t^{nat}$, the share attributed to component k over a given window is $|\bar{\pi}^k| / (|\bar{\pi}^{gap}| + |\bar{\pi}^{nat}|)$, where a bar denotes the average over that window.

Figure 9 decomposes inflation in terms of exogenous shocks. Following proposition 4.3, figure 18 in appendix D provides an alternative decomposition in terms of output gaps and natural relative prices.

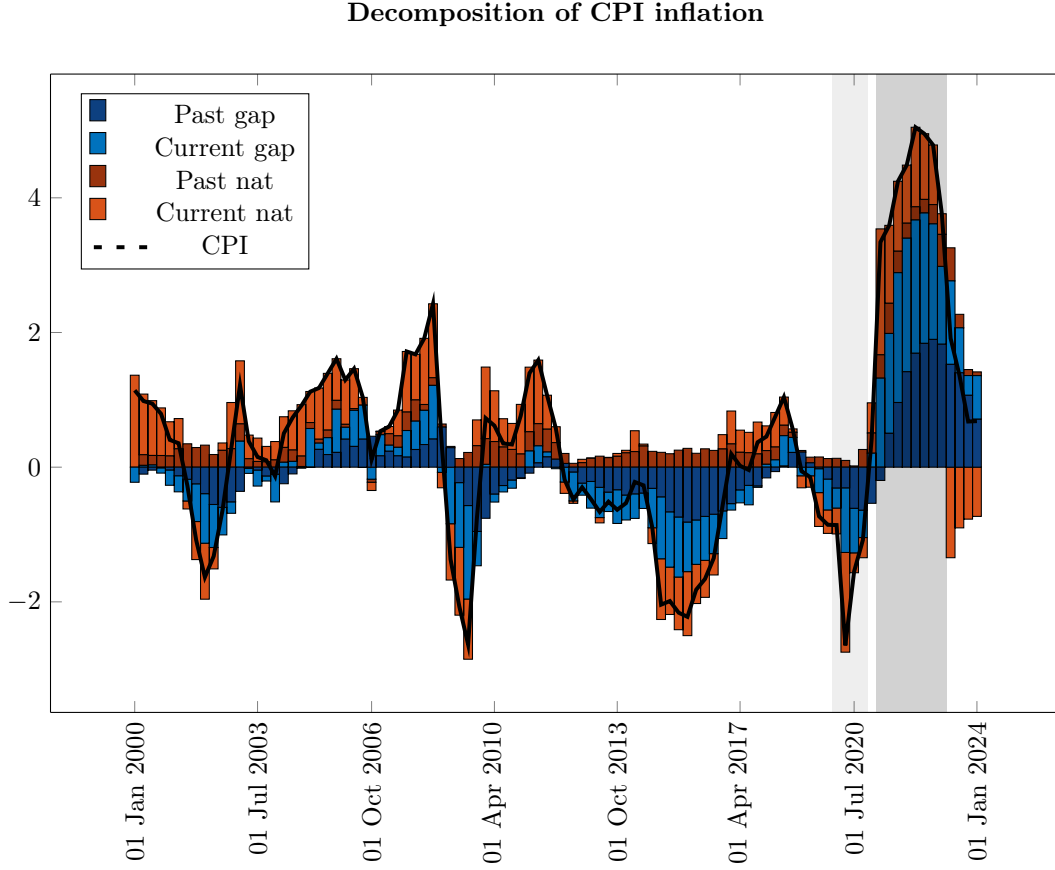


Figure 9 Decomposition of year-on-year CPI inflation, baseline calibration, 2000Q1–2024Q1, in percent. Blue bars correspond to the inflation component driven by aggregate demand shocks ϵ^i , orange bars correspond to the inflation component driven by natural relative price shocks. Each component results from the sum of past and current shocks. The shaded bands are the windows over which the decomposition shares reported in the text are computed: the light band is 2020Q1–2020Q4, when both components were negative, and the dark band is 2021Q2–2023Q1, when both were positive.

Industry-specific shocks and aggregate inflation. The left panel of figure 10 plots estimated shocks to the natural relative prices. The contribution of these shocks to aggregate inflation depends on each industry’s incidence on the aggregate CPI ($\bar{\Omega}_Z \mathcal{V}$), and it is shown in the right panel. Each set of bars corresponds to a different quarter, and bars of different colors represent selected broad industries. The oil sector faced the largest inflationary shocks, and contributed the most to aggregate inflation.

Output gaps and relative prices. Following lemma 4.1, figure 11 quantifies the contribution of output gaps to relative price changes for selected industries. The black lines plot industry-level relative prices. The blue bars represent the contribution of aggregate output gaps, while the orange bars represent the contribution of own industry-specific shocks. While not large, the output gap component is non-negligible for some industries, such as



Figure 10 Left panel: shocks to the natural relative price of ten broad industry groups, shown for five selected quarters. Right panel: contribution of natural relative price shocks to aggregate CPI inflation by industry group, each quarter from 2020Q1. Within a group G , the left panel is the Domar-weighted average $\sum_{i \in G} \frac{\bar{\Psi}_i}{\sum_{j \in G} \bar{\Psi}_j} \pi_{it}^{nat}$ and the right panel is the incidence-weighted sum $\sum_{i \in G} (\bar{\Omega}_Z \mathcal{V})_i \pi_{it}^{nat}$, where $\bar{\Psi}_i$ is the Domar weight of industry i and $(\bar{\Omega}_Z \mathcal{V})_i$ its incidence on the aggregate CPI. Government industries are excluded from both panels.

Utilities and Food and Beverage. This means that a regression of aggregate inflation on utility price changes would partly attribute to energy shocks what is in fact a consequence of output gaps.

Role of heterogeneity. The interconnection between relative and aggregate variables crucially depends on heterogeneity in price pass-through $\mathcal{P}$ (through input-output linkages and price adjustment frequencies) and factor supply elasticities Φ across industries. Figure 12 evaluates the importance of these two dimensions of heterogeneity for aggregate and relative prices.

The left panel represents the component of aggregate inflation attributed to the output-gap in the baseline calibration, against two counterfactuals. The first counterfactual (teal line) eliminates input-output linkages and assumes all industries have the same price reset probability.²⁷ The second counterfactual (orange line) assumes all industries use factors in the same proportions, thereby facing the same supply elasticity. The right panel instead feeds output gaps and natural relative prices estimated using the baseline model into the counterfactual models. The two panels show that both dimensions of heterogeneity matter, and heterogeneous price pass-through is more important quantitatively.

²⁷With input-output linkages, even uniform price adjustment frequencies would still generate heterogeneous price pass-through. Bundling input-output linkages and price adjustment probabilities into a single counterfactual allows me to make the price pass-through $\mathcal{P}$ uniform across industries.

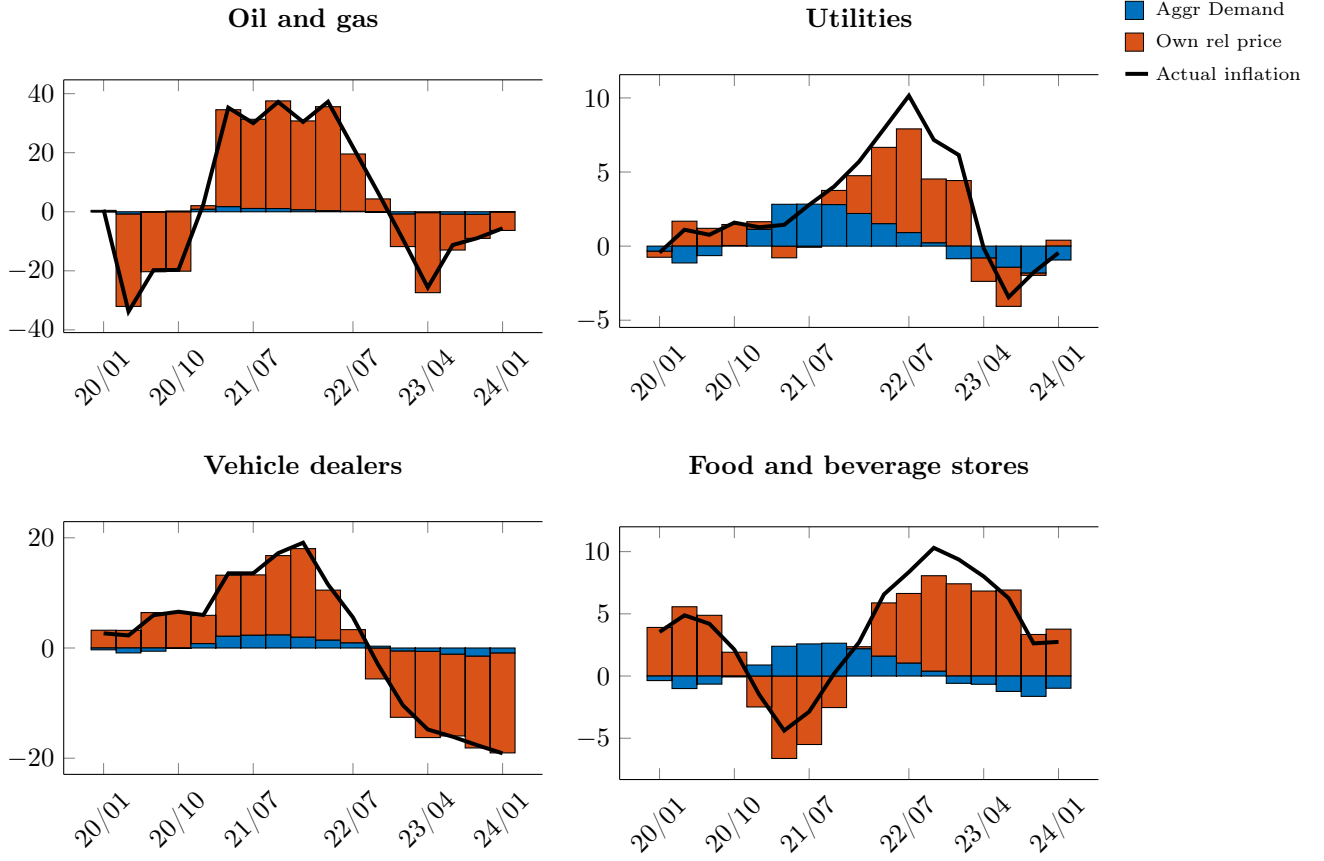
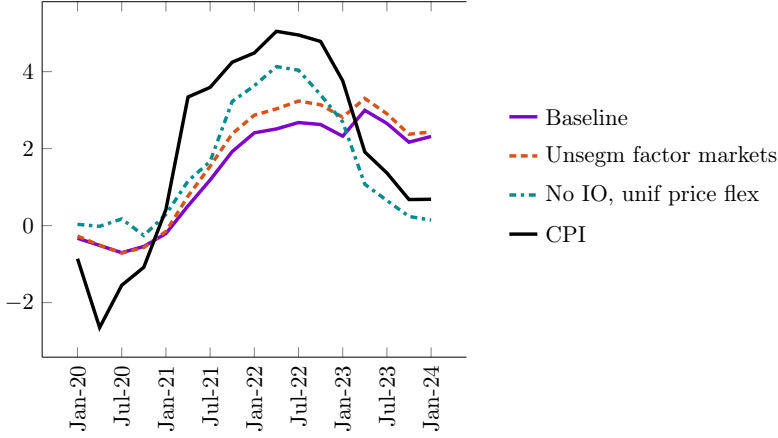


Figure 11 Decomposition of industry-level inflation. The black lines represent industry-level prices relative to aggregate CPI. The blue bars correspond to the component of industry-level relative inflation driven by aggregate output gaps. The orange bars correspond to the component driven by own natural relative price shocks.

Inflation surprises. Following lemma 4.5, figure 13 decomposes the evolution of model-implied inflation expectations over time. Each group of bars corresponds to a different quarter. The blue and orange bars represent forecast updates implied by innovations in aggregate demand and natural relative prices, while the dark gray bars correspond to expectations based on past shocks. The black line plots the actual CPI. As the figure shows, inflation surprises were mainly driven by relative price shocks until early 2021, and by aggregate demand from then onward.

Figure 14 plots sectoral weights in the inflation indices which provide model-consistent expectations of aggregate inflation, in levels and relative to CPI weights. Industries with flexible prices and high non-scalable share, such as utilities and primary manufacturing, get discounted throughout. Industries with stickier prices and more elastic supply, such as services and secondary manufacturing, are assigned a higher weight than in the CPI. Secondary manufacturing also has higher weight when predicting inflation further out in the future. This is because secondary manufacturing is fairly upstream, therefore changes in its costs take longer to propagate into final prices.

Output-gap component of CPI inflation



Relative inflation: baseline – counterfactual

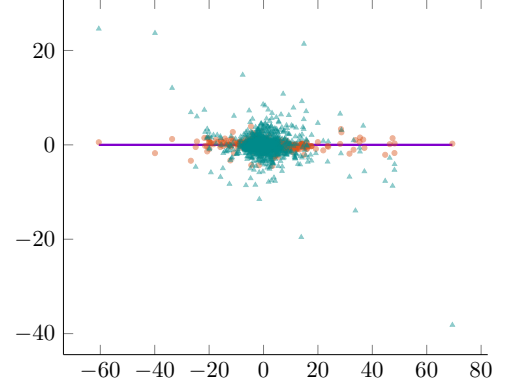


Figure 12 Left panel: component of CPI inflation attributed to aggregate output gaps under the baseline model and two counterfactuals, against actual CPI inflation. Right panel: relative price responses to the output gap and relative price shocks estimated through the baseline model, under each counterfactual. Responses are plotted as deviations from the baseline, one point per private industry and quarter from 2020Q1–2024Q1. Government industries are excluded.

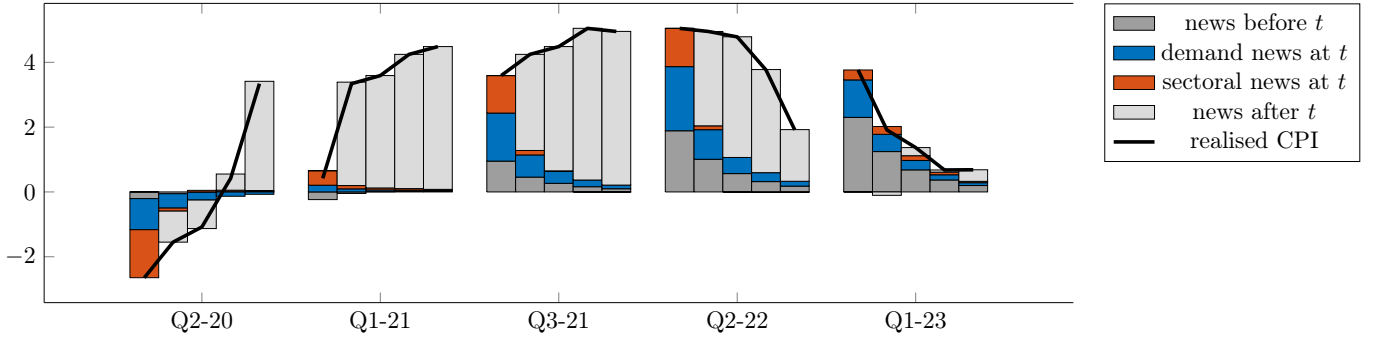


Figure 13 Decomposition of inflation surprises. Each group of bars corresponds to a forecast date t , over horizons $s = 0, \dots, 4$ quarters. Each bar splits realized CPI inflation at $t + s$ into the components attributable to news that arrived before t , to aggregate demand and natural relative price innovations at time t , and to innovations arriving after t .

6 Conclusion

This paper identifies the components of inflation driven by relative price adjustments versus aggregate demand misalignment. It first establishes that – whenever industries have heterogeneous network-adjusted supply or pricing elasticities – aggregate shocks affect relative prices and industry-specific shocks affect aggregate inflation.

This two-way causality makes it difficult to isolate aggregate and industry-specific inflation drivers. I do so through the lens of a disaggregated New Keynesian framework, constructing appropriately-weighted aggregate and relative price indices which serve as sufficient statistics for shock identification and forecast revision. The industries' weights in these indices depend on their supply and pricing elasticities, as well as on their size.

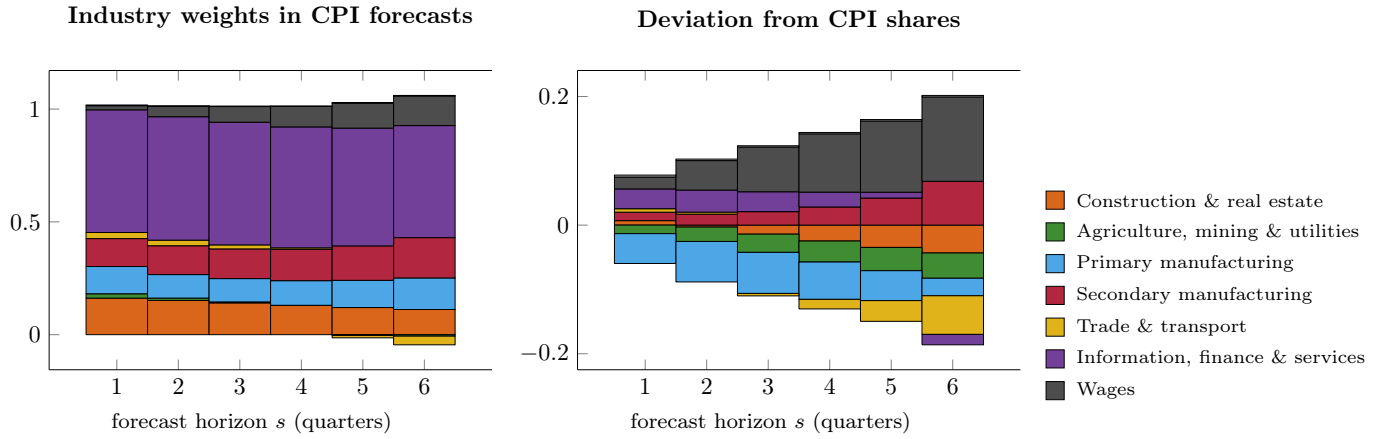


Figure 14 Sectoral weights in the inflation indices that track model-based expectations of aggregate inflation (vertical axis) against forecast horizon (horizontal axis). Each bar is the sum of weights over the industries in one of six broad groups, plus wages. Weights are normalized to sum to one at every horizon. The left panel plots raw weights, the right panel plots deviations from CPI shares.

I quantify the drivers of US inflation between 2000 and 2024, finding that they vary over time. Aggregate demand only accounted for 51% of consumer price inflation in the two decades before the COVID pandemic (2000-2019), against 68% of the disinflation between 2020Q1–2020Q4 and 69% of the inflation rebound between 2021Q2–2023Q1.

References

- AFROUZI, H. AND S. BHATTARAI (2023): “Inflation and GDP Dynamics in Production Networks: A Sufficient Statistics Approach,” Working Paper 31218, National Bureau of Economic Research.
- AFROUZI, H., S. BHATTARAI, AND E. WU (2024): “Relative-Price Changes as Aggregate Supply Shocks Revisited: Theory and Evidence,” Working Paper 32816, National Bureau of Economic Research.
- ALESSANDRIA, G., S. KHAN, A. KHEDERLARIAN, C. MIX, AND K. RUHL (2023): “The Aggregate Effects of Global and Local Supply Chain Disruptions: 2020 - 2022,” Working Paper 30849, National Bureau of Economic Research, Cambridge, MA.
- BAI, X., J. FERNANDEZ-VILLAYERDE, Y. LI, AND F. ZANETTI (2024): “The Causal Effects of Global Supply Chain Disruptions on Macroeconomic Outcomes: Evidence and Theory,” Working Paper 32098, National Bureau of Economic Research, Cambridge, MA.
- BALL, L., D. LEIGH, AND P. MISHRA (2022): “Understanding U.S. Inflation During the COVID Era,” Working Paper 30613, National Bureau of Economic Research, Cambridge, MA.
- BALL, L. AND N. G. MANKIW (1995): “Relative-Price Changes as Aggregate Supply Shocks,” *The Quarterly Journal of Economics*, 110, 161–193.

- BAQAEE, D. AND E. FARHI (2022): “Supply and Demand in Disaggregated Keynesian Economies with an Application to the COVID-19 Crisis,” *American Economic Review*, 112, 1397–1436.
- BAQAEE, D. R. AND E. FARHI (2020): “Productivity and Misallocation in General Equilibrium,” *The Quarterly Journal of Economics*, 135, 105–163.
- BEAUDRY, P., C. HOU, AND F. PORTIER (2024): “The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation,” Working Paper 32322, National Bureau of Economic Research.
- BENIGNO, P. AND G. EGGERTSSON (2023): “It’s Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve,” Working Paper 31197, National Bureau of Economic Research, Cambridge, MA.
- BLANCHARD, O. AND B. BERNANKE (2023): “What Caused the US Pandemic-Era Inflation?” Working Paper 31417, National Bureau of Economic Research, Cambridge, MA.
- BLANCHARD, O. J. AND C. M. KAHN (1980): “The Solution of Linear Difference Models under Rational Expectations,” *Econometrica*, 48, 1305.
- COMIN, D., R. JOHNSON, AND C. JONES (2023): “Supply Chain Constraints and Inflation,” Working Paper 31179, National Bureau of Economic Research, Cambridge, MA.
- DAO, M., A. DIZIOLI, C. JACKSON, D. LEIGH, AND P.-O. GOURINCHAS (2023): “Unconventional Fiscal Policy in Times of High Inflation,” IMF Working Paper, International Monetary Fund.
- DAO, M. C., P.-O. GOURINCHAS, D. LEIGH, AND P. MISHRA (2024): “Understanding the international rise and fall of inflation since 2020,” *Journal of Monetary Economics*, 148, 103658.
- DI GIOVANNI, J., S. KALEMLI-OZCAN, A. SILVA, AND M. A. YILDIRIM (2022): “Global Supply Chain Pressures, International Trade, and Inflation,” Working Paper 30240, National Bureau of Economic Research.
- FERRANTE, F., S. GRAVES, AND M. IACOVIELLO (2023): “The inflationary effects of sectoral reallocation,” *Journal of Monetary Economics*, 140, S64–S81.
- GAGLIARDONE, L. AND M. GERTLER (2023): “Oil Prices, Monetary Policy and Inflation Surges,” Working Paper 31263, National Bureau of Economic Research, Cambridge, MA.
- GIANNONE, D. AND G. PRIMICERI (2024): “The Drivers of Post-Pandemic Inflation,” Working Paper 32859, National Bureau of Economic Research, Cambridge, MA.
- GUERRIERI, V., G. LORENZONI, L. STRAUB, AND I. WERNING (2022): “Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?” *American Economic Review*, 112, 1437–1474.

- GUERRIERI, V., M. MARCUSSEN, L. REICHLIN, AND S. TENREYRO (2023): “The Art and Science of Patience: Relative prices and inflation,” *Geneva Reports on the World Economy*.
- HARDING, M., J. LINDE, AND M. TRABANDT (2023): “Understanding post-COVID inflation dynamics,” *Journal of Monetary Economics*, 140, S101–S118.
- KARABARBOUNIS, L. AND B. NEIMAN (2014): “The Global Decline of the Labor Share,” *The Quarterly Journal of Economics*, 129, 61–103.
- LA’O, J. AND A. TAHBAZ-SALEHI (2022): “Optimal Monetary Policy in Production Networks,” *Econometrica*, 90, 1295–1336.
- LORENZONI, G. AND I. WERNING (2023): “Wage-Price Spirals,” *Brookings Papers on Economic Activity*, 52, 317–393.
- OLIVI, A., V. STERK, AND D. XHANI (2023): “Optimal Monetary Policy during a Cost-of-Living Crisis,” Working paper.
- PASTEN, E., R. SCHOENLE, AND M. WEBER (2020): “The propagation of monetary policy shocks in a heterogeneous production economy,” *Journal of Monetary Economics*, 116, 1–22.
- PFLUEGER, C. (2023): “Back to the 1980s or Not? The Drivers of Inflation and Real Risks in Treasury Bonds,” Working Paper 30921, National Bureau of Economic Research, Cambridge, MA.
- REIS, R. AND M. W. WATSON (2010): “Relative Goods’ Prices, Pure Inflation, and The Phillips Correlation,” *American Economic Journal: Macroeconomics*, 2, 128–157.
- RUBBO, E. (2023): “Networks, Phillips Curves, and Monetary Policy,” *Econometrica*, 91, 1417–1455.
- RUGE-MURCIA, F. AND A. L. WOLMAN (2022): “Relative Price Shocks and Inflation,” *Federal Reserve Bank of Richmond Working Papers*, 22, 1–39.
- SHAPIRO, A. H. (2020): “A Simple Framework to Monitor Inflation,” *Federal Reserve Bank of San Francisco, Working Paper Series*, 1–34.
- (2022): “How Much Do Supply and Demand Drive Inflation?” *FRBSF Economic Letters*, 2022.
- SMETS, F. AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97, 586–606.
- VOM LEHN, C. AND T. WINBERRY (2022): “The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle,” *The Quarterly Journal of Economics*, 137, 387–433.

Appendix to What Drives Inflation?

A From the Log-Linearized Equilibrium to the Dynamic System

This appendix starts by detailing the utilization model for non-labor factors (section A.1). It then takes as its starting point the log-linearized equilibrium conditions of lemma 3.1, and derives the three-equation system (13). As an intermediate step, lemma A.1 derives a larger system of five equations – three supply-side equations (factor supply, pricing, and relative factor demand) and two aggregate demand equations (IS curve and Taylor rule) – which better illustrates the role of various primitive parameters.

A.1 Non-Labor Factors

Each non-labor factor $f \in \mathcal{K}$ – land, equipment, structures, intellectual property – is produced by combining a fixed endowment $\bar{K}_f$ with variable utilization U_f . The supply of factor f at time t is

$$L_{ft} = U_{ft} \bar{K}_f.$$

The utilization component U_f fully depreciates each period, while the endowment component $\bar{K}_f$ does not depreciate. This model abstracts from capital accumulation, while retaining heterogeneous supply elasticities and heterogeneous value added composition across industries.

The production function for utilization is

$$U_f \equiv \left[(1 + \varphi_f) \mathcal{E}_{ft}^{\varphi_f} C_f \right]^{\frac{1}{1+\varphi_f}},$$

where $\mathcal{E}_{ft}$ is a shock to the efficiency of utilization, and the utilization bundle C_f uses goods with constant returns to scale, $C_{ft} = C_{ft}(c_{f1}, \dots, c_{fN})$. The utilization producer solves

$$\max_{U_f, C_f} W_f L_f - P_f^U C_f \quad \text{s.t.} \quad L_f = U_{ft} \bar{K}_f, \quad U_f = \left[(1 + \varphi_f) \mathcal{E}_{ft}^{\varphi_f} C_f \right]^{\frac{1}{1+\varphi_f}}.$$

The first order condition yields a supply curve with constant elasticity $\frac{1}{\varphi_f}$:

$$U_f^{\varphi_f} = \frac{W_f}{P_f^U} \mathcal{E}_{ft}^{\varphi_f} \bar{K}_f. \tag{32}$$

Equation (32) is the exact counterpart of the workers' consumption-leisure tradeoff (4). Log-linearizing it gives

$$w_{ft} = \varphi_f (\ell_{ft} - \epsilon_{ft}^{\mathcal{F}}) + \Omega_{\mathcal{U}_f, :} \mathbf{p}_t, \quad f \in \mathcal{K},$$

with $\epsilon_{ft}^{\mathcal{F}} \equiv \log \mathcal{E}_{ft}$, while log-linearizing (4) gives

$$w_{ft} = \varphi_f (\ell_{ft} - \epsilon_{ft}^{\mathcal{F}}) + \Omega_{H,:} \mathbf{p}_t + \gamma c_{Ht}, \quad f \in \mathcal{L}.$$

The supply curves of labor and non-labor factors take the same form, differing only in the price deflator, which coincides with the household's consumption price for labor occupations, and with the utilization price index for non-labor factors, and in the wealth effect γc_{Ht} , which is present for labor occupations only.

Payments $W_f L_f$ to factor f are split between utilization costs and rebates to the household. Profit maximization by utilization producers implies that a constant share $\frac{1}{1+\varphi_f}$ of revenues covers utilization costs; the remainder, a share $\frac{\varphi_f}{1+\varphi_f}$, enters the household's budget constraint (3) as a rebate.

A.2 Notation

Vector-matrix notation. The derivations below collect variables pertaining to each group of agents – primary factors, industries, final users – into vectors, whose components correspond to individual agents. Consistent with this notation, the model elasticities are collected into matrices whose entries describe responses of the row variable to changes in the column variable.²⁸

As a convention, a colon in place of an index selects an entire block of a matrix. For example, $\Omega_{\mathcal{Z},:}$ and $\Psi_{\mathcal{Z},:}$ denote the rows corresponding to final users, and $\Omega_{:, \mathcal{F}}$, $\Psi_{:, \mathcal{F}}$, $\mathcal{P}_{:, \mathcal{F}}$ denote the columns corresponding to labor unions and factor marketplaces. Table 3 lists the model variables. Table 4 lists the matrices collecting income and expenditure shares, reproducing definition 3.2.

Variable	Definition
Aggregate output gap	$\tilde{y}_t \equiv \bar{y}_t - \bar{y}_t^{nat}$
Household consumption	$c_{Ht} \equiv \log \frac{C_{Ht}}{C_H^*}$
Employment of primary factors	$\ell_t = (\ell_{1t}, \dots, \ell_{Ft})^T$ where $\ell_{ft} \equiv \log \frac{L_{f,t}}{L_f^*}$
Factor prices	$\mathbf{w}_t = (w_{1t}, \dots, w_{Ft})^T$ where $w_{ft} \equiv \log \frac{W_{f,t}}{W_f^*}$
Industry-level prices	$\mathbf{p}_t = (p_{1t}, \dots, p_{Nt})^T$ where $p_{it} \equiv \log \frac{P_{i,t}}{P_i^*}$
Industry-level price gaps	$\tilde{\mathbf{p}}_t = \mathbf{p}_t - \mathbf{p}_t^{nat}$
Industry-level inflation	$\boldsymbol{\pi}_t = (\pi_{1t}, \dots, \pi_{Nt})^T$ where $\pi_{it} \equiv p_{it} - p_{i,t-1}$
Natural price changes	$\boldsymbol{\pi}_t^{nat} = \mathbf{p}_t^{nat} - \mathbf{p}_{t-1}^{nat}$
Nominal interest rate	i_{t+1}

Table 3 Model variables

²⁸When the row and column agents are the same, diagonal entries can be interpreted as own effects while off-diagonal entries describe spillovers across agents.

Parameter	Notation	Definition
Expenditure shares	$\Omega \in \mathbb{R}^{ \mathcal{N} \times \mathcal{N} }$	$\Omega_{ij} = \frac{P_j^* X_{ij}^*}{P_i^* Y_i^*}$
Leontief inverse	Ψ	$\Psi \equiv (I - \Omega)^{-1}$
Domar weights	$\bar{\Psi} \in \mathbb{R}^{ \mathcal{N} }$	$\bar{\Psi}_i \equiv \frac{P_i^* Y_i^*}{GDP^*}$
Final use shares	$\bar{\Psi}_{\mathcal{Z}} \in \mathbb{R}^{ \mathcal{Z} }$	$\bar{\Psi}_z \equiv \frac{P_z^{C^*} C_z^*}{GDP^*}$ (final-use block of $\bar{\Psi}$)
Primary factors' income shares	$\bar{\Psi}_{\mathcal{F}} \in \mathbb{R}^{ \mathcal{F} }$	$\bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*}$ (factors block of $\bar{\Psi}$)

Table 4 Income and expenditure shares

Covariance operator. Given a vector $\mathbf{z} \in \mathbb{R}^N$, $\mathbf{z} \geq \mathbf{0}$, and two matrices $X \in \mathbb{R}^{Q \times N}$, $Y \in \mathbb{R}^{N \times P}$, the covariance operator $Cov_{\mathbf{z}}(X, Y) \in \mathbb{R}^{Q \times P}$ is

$$Cov_{\mathbf{z}}(X, Y) \equiv \sum_i \frac{z_i}{\sum_j z_j} X_{:i} Y_{i:} - \left(\sum_i \frac{z_i}{\sum_j z_j} X_{:i} \right) \left(\sum_i \frac{z_i}{\sum_j z_j} Y_{i:} \right).$$

A.3 Equilibrium in vector-matrix notation

Lemma A.1 re-writes the equilibrium in vector-matrix notation, setting the stage for the following derivations.

Lemma A.1. Equilibrium in vector form. *The log-linearized equilibrium conditions (11a)–(11i) are equivalent to the system*

$$\mathbf{w}_t = \Phi \ell_t + \Omega^{\mathcal{F}} \mathbf{p}_t + \epsilon_t^w, \quad \epsilon_t^w \equiv -\text{diag}(\varphi_f)_{f \in \mathcal{F}} \epsilon_t^{\mathcal{F}} \quad (33a)$$

$$\mathbf{p}_t = \mathcal{P}_{:\mathcal{F}} \mathbf{w}_t - \mathcal{P} \mathbf{a}_t + (I - \mathcal{P} \Psi^{-1}) (\mathbf{p}_{t-1} + \rho \mathbb{E}_t \boldsymbol{\pi}_{t+1}) + \hat{\epsilon}_t^p, \quad \hat{\epsilon}_t^p \equiv (I - \Delta \Omega)^{-1} \epsilon_t^p \quad (33b)$$

$$\ell_t = \mathbf{1} \bar{y}_t - \Theta \mathbf{p}_t + \epsilon_t^d \quad (33c)$$

$$\mathbb{E}_t c_{H,t+1} = c_{H,t} + \frac{1}{\gamma} [(i_{t+1} + \log \rho^*) - \Omega_{H,:} \mathbb{E}_t \boldsymbol{\pi}_{t+1}] + \epsilon_t^p \quad (33d)$$

$$i_{t+1} = \mathbb{E}_t r_{t+1}^{nat} + \phi_y \tilde{y}_t + \phi_{\pi} \pi_t^{TR} + \epsilon_{t+1}^i \quad (33e)$$

where Φ , $\Omega^{\mathcal{F}}$ and $\mathcal{P}$ are as defined in steps 1 and 2 of the proof below, and Θ is defined in (36). Table 5 at the end of this appendix collects all definitions.

Proof. I derive each equation in turn.

Factor supply. Stacking the consumption-leisure tradeoff with optimal asset utilization implies the factor supply equation (11a):

$$\mathbf{w}_t = \text{diag}(\varphi_f) (\ell_t - \epsilon_t^{\mathcal{F}}) + \begin{pmatrix} \mathbf{1}_{\mathcal{L}} & \mathbb{O}_{\mathcal{L}\mathcal{U}} & \mathbf{0}_{\mathcal{L}} \\ \mathbb{O}_{\mathcal{U}\mathcal{L}} & I_{\mathcal{U}} & \mathbf{0}_{\mathcal{U}} \end{pmatrix} \Omega_{\mathcal{Z}:} \mathbf{p}_t + \Gamma \mathbf{c}_{\mathcal{Z}t},$$

where Γ is as in definition 4.2. The budget constraints (11g) give $\mathbf{c}_{\mathcal{Z}t} = \Xi_{:\mathcal{F}} \ell_t + (\Xi_{:\mathcal{N}} \Psi^{-1} - \Omega_{\mathcal{Z}:}) \mathbf{p}_t$, using $\Psi^{-1} = I - \Omega$ and $\Omega_{f:} = \mathbf{0}$ for the factor rows of the extended $\mathcal{N}$ (remark 3.1). Combining with the factor supply equation yields (33a), with Φ and $\Omega^{\mathcal{F}}$ as in definitions 4.2 and 4.4. Two properties follow: $\Phi \mathbf{1} = (\varphi_f + \gamma_f)_{f \in \mathcal{F}}$, and $\Omega^{\mathcal{F}} \mathbf{1} = \mathbf{1}$.

Pricing. Substituting the marginal cost equation of lemma 3.1 into the pricing equation (11c), and using $\pi_{it} = p_{it} - p_{i,t-1}$, gives

$$p_{it} = \hat{\delta}_i \left[\sum_{j \in \mathcal{N}} \Omega_{ij} p_{jt} + \mathbb{I}_{i=f} w_{ft} - a_{it} \right] + (1 - \hat{\delta}_i) (p_{i,t-1} + \rho \mathbb{E}_t \pi_{i,t+1}) + \epsilon_{it}^p.$$

Following remark 3.1, labor unions and factor marketplaces are themselves elements of $\mathcal{N}$, with $\Omega_{f:} = \mathbf{0}$ and marginal cost equal to w_f . Stacking and collecting terms in $\mathbf{p}_t$,

$$(I - \Delta \Omega) \mathbf{p}_t = \Delta [\mathbb{I}_{\mathcal{F}} \mathbf{w}_t - \mathbf{a}_t] + (I - \Delta) (\mathbf{p}_{t-1} + \rho \mathbb{E}_t \boldsymbol{\pi}_{t+1}) + \boldsymbol{\epsilon}_t^p.$$

The matrix $I - \Delta \Omega$ is invertible by the Perron–Frobenius theorem, because $\Delta \Omega$ is non-negative with row sums strictly less than one. Pre-multiplying by its inverse and using $\mathcal{P} \equiv (I - \Delta \Omega)^{-1} \Delta = \Delta (I - \Omega \Delta)^{-1}$ yields

$$\mathbf{p}_t = \mathcal{P}_{:\mathcal{F}} \mathbf{w}_t - \mathcal{P} \mathbf{a}_t + (I - \Delta \Omega)^{-1} (I - \Delta) (\mathbf{p}_{t-1} + \rho \mathbb{E}_t \boldsymbol{\pi}_{t+1}) + (I - \Delta \Omega)^{-1} \boldsymbol{\epsilon}_t^p.$$

Finally, note the identity

$$I - \mathcal{P} \Psi^{-1} = I - (I - \Delta \Omega)^{-1} \Delta (I - \Omega) = (I - \Delta \Omega)^{-1} [(I - \Delta \Omega) - \Delta + \Delta \Omega] = (I - \Delta \Omega)^{-1} (I - \Delta),$$

which delivers (33b). The coefficient $I - \mathcal{P} \Psi^{-1}$ measures the weight that industry i 's current price places on inherited prices and expected future inflation, cumulated through the production network.

Relative factor demand. To derive equation (33c) I aggregate the input composition equation (11e), the industry output equation (11d) and market clearing (11f) into a single relation between relative factor employment and relative prices. I also make extensive use of the accounting relation $\bar{\Psi}_{\mathcal{F}} = \Psi_{\mathcal{Z}\mathcal{F}}^T \bar{\Psi}_{\mathcal{Z}}$, which states that factor income shares are the factor content of final expenditure. Totally differentiating gives

$$d\bar{\Psi}_{\mathcal{F}} = d(\Psi_{\mathcal{Z}\mathcal{F}}^T) \bar{\Psi}_{\mathcal{Z}} + \Psi_{\mathcal{Z}\mathcal{F}}^T d\bar{\Psi}_{\mathcal{Z}}.$$

The first term captures substitution at every node of the network; the second captures the reallocation of income across final users with different consumption baskets.

For the first term, denote by $\boldsymbol{\mu}$ the vector of log-changes in industry-level markups and use $\Psi \equiv (I - \Omega)^{-1}$ to obtain

$$d(\Psi_{\mathcal{ZF}}^T) \bar{\Psi}_{\mathcal{Z}} = \Psi_{\mathcal{ZF}}^T d\Omega^T|_{Sub} \bar{\Psi} - \Psi_{\mathcal{ZF}}^T \text{diag}(\bar{\Psi}) \boldsymbol{\mu},$$

where the substitution term is

$$\begin{aligned} \Psi_{\mathcal{ZF}}^T d\Omega^T|_{Sub} \bar{\Psi} &\equiv \sum_{i,j,k} \bar{\Psi}_i \Omega_{ij} \Omega_{ik} (1 - \theta_{jk}^i) (\Psi_{\mathcal{F}})_{j,:} (p_j - p_k) \\ &= \left[\Psi_{\mathcal{F}}^T \left(I - \bar{\Omega}_{\mathcal{Z}}^T \mathbf{1}^T \right) \text{diag}(\bar{\Psi}) \Psi^{-1} - \text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{ZF}}^T, \Omega_{\mathcal{Z}:}) - \mathcal{X}(\Psi_{\mathcal{F}}, I) \right] \mathbf{p}, \end{aligned}$$

and where for any two matrices X, Y we define the bilinear form

$$[\mathcal{X}(X, Y)]_{hi} \equiv \frac{1}{2} \sum_{j,k,n=1}^N \bar{\Psi}_j \theta_{kn}^j \Omega_{jk} \Omega_{jn} (X_{kh} - X_{nh}) (Y_{ki} - Y_{ni}).$$

Solving for markups as a function of prices and wages, $\boldsymbol{\mu} = \Psi^{-1} \mathbf{p} - \mathbb{I}_{\mathcal{F}} \mathbf{w}$, gives

$$d(\Psi_{\mathcal{ZF}}^T) \bar{\Psi}_{\mathcal{Z}} = - [\bar{\Psi}_{\mathcal{F}} \bar{\Omega}_{\mathcal{Z}} + \text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{ZF}}^T, \Omega_{\mathcal{Z}:}) + \mathcal{X}(\Psi_{\mathcal{F}}, I)] \mathbf{p} + \text{diag}(\bar{\Psi}_{\mathcal{F}}) \mathbf{w}.$$

By definition, $d\bar{\Psi}_{\mathcal{F}} = \text{diag}(\bar{\Psi}_{\mathcal{F}}) (\mathbf{w} + \boldsymbol{\ell} - \mathbf{1}\bar{y}) - \bar{\Psi}_{\mathcal{F}} \bar{\Omega}_{\mathcal{Z}} \mathbf{p}$. Equating the two expressions, the wage terms cancel and

$$\text{diag}(\bar{\Psi}_{\mathcal{F}}) (\boldsymbol{\ell} - \mathbf{1}\bar{y}) = - [\text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{ZF}}^T, \Omega_{\mathcal{Z}:}) + \mathcal{X}(\Psi_{\mathcal{F}}, I)] \mathbf{p} + \Psi_{\mathcal{ZF}}^T d\bar{\Psi}_{\mathcal{Z}}. \quad (34)$$

It remains to solve for $d\bar{\Psi}_{\mathcal{Z}}$. This step uses the matrix of ownership shares $\Xi \in \mathbb{R}^{|\mathcal{Z}| \times |\mathcal{N}|}$ introduced in lemma 3.1, which tracks factor income and profit rebates to final users. The household pays net transfers T_H and receives all industry profits and all labor income, plus a share $\frac{\varphi_f}{1+\varphi_f}$ of non-labor factor income (see appendix A.1). This implies

$$\Xi_{H,i} = (1 - T_H) \frac{\bar{\Psi}_i}{\bar{\Psi}_H}, \quad i \in \mathcal{N} \cup \mathcal{L}, \quad \Xi_{H,f} = \left(\frac{\varphi_f}{1 + \varphi_f} - T_H \right) \frac{\bar{\Psi}_f}{\bar{\Psi}_H}, \quad f \in \mathcal{K}. \quad (35)$$

Utilization producers receive a share $\frac{1}{1+\varphi_f}$ of non-labor factor income and own nothing else, so $\Xi_{u_f, f'} = \mathbb{I}_{f=f'}$ and $\Xi_{u_f i} = 0$ for $i \in \mathcal{N}$.

Since income shares sum to one, it holds that $\Psi_{\mathcal{ZF}}^T d\bar{\Psi}_{\mathcal{Z}} = \text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{ZF}}^T, d \log \bar{\Psi}_{\mathcal{Z}})$, and accounting implies

$$d \log \bar{\Psi}_{\mathcal{Z}} = \Xi (\boldsymbol{\ell} - \mathbf{1}\bar{y}) + [\Xi \Psi^{-1} - \mathbf{1} \bar{\Omega}_{\mathcal{Z}}] \mathbf{p}.$$

Substituting into (34) and solving for $\ell - \mathbf{1}\bar{y}$ yields (33c), with

$$\begin{aligned}\Theta &\equiv (\text{diag}(\bar{\Psi}_{\mathcal{F}}) - \text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{Z}\mathcal{F}}^T, \Xi_{\mathcal{F}}))^{-1} \mathcal{D} \\ \mathcal{D} &\equiv \mathcal{X}(\Psi_{\mathcal{F}}, I) + \text{Cov}_{\bar{\Psi}_{\mathcal{Z}}}(\Psi_{\mathcal{Z}\mathcal{F}}^T, \Omega_{\mathcal{Z}} - \Xi\Psi^{-1}),\end{aligned}\tag{36}$$

The shifter ϵ_t^d collects the network-aggregated preference and government spending shocks,

$$\epsilon_{ft}^d \equiv \frac{1}{\bar{\Psi}_f} \sum_{i \in \mathcal{N}} \bar{\Psi}_i \sum_{h \in \mathcal{N}} \Omega_{ih} \Psi_{hf} \epsilon_{iht}^d.$$

Aggregate demand. The Euler equation (11b) and the Taylor rule (11i) simply restate (33d)–(33e).

The budget constraints (11g) do not appear separately in (33a)–(33e), because they have been substituted out into Φ and $\Omega^{\mathcal{F}}$ in the factor supply step, and into Θ in the relative factor demand step. $\square$

A.4 The Dynamic System

This section moves from lemma A.1 to derive the dynamic system (13) of proposition 4.1. Along the way it defines the industry-level slopes κ , the cost-push elasticities $\mathcal{V}$, and the nominal rate deflator π^{IS} .

Proof of proposition 4.1 I derive the Phillips curve block and the IS equation in turn.

Industry-level Phillips curves. Recast (33a)–(33c) in gaps relative to the natural equilibrium (definition 3.5). Because the natural equilibrium faces the same shocks and policies as the actual economy, the shifters $\epsilon^{\mathcal{F}}$, ϵ^d , $\mathbf{a}$ and $\hat{\epsilon}^p$ drop out of the gap representation. Combining (33a) and (33b),

$$(I - \mathcal{P}_{\mathcal{F}}\Omega^{\mathcal{F}})\tilde{\mathbf{p}}_t = \mathcal{P}_{\mathcal{F}}\Phi\tilde{\ell}_t + (I - \mathcal{P}\Psi^{-1})(\tilde{\mathbf{p}}_{t-1} - \pi_t^{nat} + \rho\mathbb{E}_t\pi_{t+1}).$$

Substituting the demand equation (33c) in gaps, $\tilde{\ell}_t = \mathbf{1}\tilde{y}_t - \Theta\tilde{\mathbf{p}}_t$,

$$(I - \mathcal{P}_{\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))\tilde{\mathbf{p}}_t = \mathcal{P}_{\mathcal{F}}\Phi\mathbf{1}\tilde{y}_t + (I - \mathcal{P}\Psi^{-1})(\tilde{\mathbf{p}}_{t-1} - \pi_t^{nat} + \rho\mathbb{E}_t\pi_{t+1}).$$

The coefficient matrix factors as

$$I - \mathcal{P}_{\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta) = (I - \mathcal{P}_{\mathcal{F}}\Omega^{\mathcal{F}})\left(I + \mathcal{P}_{\mathcal{F}}(I - \Omega^{\mathcal{F}}\mathcal{P}_{\mathcal{F}})^{-1}\Phi\Theta\right)$$

and is invertible for all parameter combinations except a measure-zero set on which $\mathcal{P}_{\mathcal{F}}(I - \Omega^{\mathcal{F}}\mathcal{P}_{\mathcal{F}})^{-1}\Phi\Theta$ has an

eigenvalue equal to -1 .²⁹ Outside that set, inverting and using $\boldsymbol{\pi}_t = \tilde{\boldsymbol{p}}_t - \tilde{\boldsymbol{p}}_{t-1} + \boldsymbol{\pi}_t^{nat}$ delivers the Phillips curve block of (13),

$$\boldsymbol{\pi}_t - \rho \mathbb{E}_t \boldsymbol{\pi}_{t+1} = \boldsymbol{\kappa} \tilde{y}_t - \mathcal{V} (\tilde{\boldsymbol{p}}_{t-1} - \boldsymbol{\pi}_t^{nat} + \rho \mathbb{E}_t \boldsymbol{\pi}_{t+1}), \quad (37)$$

with

$$\boldsymbol{\kappa} \equiv (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta))^{-1} \mathcal{P}_{:\mathcal{F}} \Phi \mathbf{1} \quad (38)$$

$$\mathcal{V} \equiv (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta))^{-1} (\mathcal{P} \Psi^{-1} - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)). \quad (39)$$

This proves the expressions for $\boldsymbol{\kappa}$ and $\mathcal{V}$ in lemmas 4.1 and 4.2; the decompositions (15) and (16) are derived in appendix B.

To see that $\mathcal{V} \mathbf{1} = \mathbf{0}$, note that $\mathcal{P} \Psi^{-1} \mathbf{1} = \mathcal{P} (I - \Omega) \mathbf{1}$ and $(\Omega^{\mathcal{F}} - \Phi \Theta) \mathbf{1} = \Omega^{\mathcal{F}} \mathbf{1}$.³⁰ A uniform increase in all natural prices therefore has no effect on inflation dynamics. In other words, only natural relative prices matter.

Aggregate IS equation. The Euler equation (33d) is expressed in terms of household consumption. To write it in terms of the aggregate output gap, take the household row of the budget constraints (11g) in gaps,

$$\tilde{c}_t = \Xi_{H\mathcal{F}} \tilde{\boldsymbol{\ell}}_t + (\Xi_{H,:} \Psi^{-1} - \Omega_{H,:}) \tilde{\boldsymbol{p}}_t,$$

and combine it with (33d) to obtain

$$\Xi_{H\mathcal{F}} \mathbb{E}_t \tilde{\boldsymbol{\ell}}_{t+1} = \Xi_{H\mathcal{F}} \tilde{\boldsymbol{\ell}}_t + \frac{1}{\gamma} [i_{t+1} + \log \rho^* - [(1 - \gamma) \Omega_{H,:} + \gamma \Xi_{H,:} \Psi^{-1}] \mathbb{E}_t \boldsymbol{\pi}_{t+1}] + \epsilon_t^\rho.$$

Substituting the demand equation $\tilde{\boldsymbol{\ell}}_t = \mathbf{1} \tilde{y}_t - \Theta \tilde{\boldsymbol{p}}_t$ then yields

$$\mathbb{E}_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\gamma} \left[i_{t+1} - \left(\mathbb{E}_t r_{t+1}^{nat} + \hat{\boldsymbol{\beta}}^T \mathbb{E}_t \boldsymbol{\pi}_{t+1} \right) \right] + \epsilon_t^\rho, \quad \hat{\boldsymbol{\beta}}^T \equiv (1 - \gamma) \Omega_{H,:} + \gamma (\Xi_{H,:} \Psi^{-1} - \Xi_{H\mathcal{F}} \Theta),$$

which is the IS block of (13) with $\pi^{IS} \equiv \hat{\boldsymbol{\beta}}^T \boldsymbol{\pi}$.

A.5 Elasticity Matrices (Summary)

For convenience, table 5 collects the elasticity matrices constructed above.

²⁹The matrix $I - \Omega^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}}$ is always invertible by the Perron–Frobenius theorem, because the elements of $\Omega^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}}$ lie in $(0, 1)$ and its row sums are always less than 1.

³⁰We used the property $\Theta \mathbf{1} = \mathbf{0}$: a uniform increase in all prices leaves relative factor demand unchanged.

Parameter	Notation	Definition
Wealth effects	$\Gamma \in \mathbb{R}^{ \mathcal{F} \times \mathcal{Z} }$	$\Gamma_{fz} \equiv \gamma \mathbb{I}_{f \in \mathcal{L}, z=H}$
Ownership shares	$\Xi \in \mathbb{R}^{ \mathcal{Z} \times \mathcal{N} }$	$\Xi_{zi} \equiv \left(\hat{\Xi}_{zi} - T_z \right) \frac{\bar{\Psi}_i}{\bar{\Psi}_z}$
Factor supply elasticities	$\Phi \in \mathbb{R}^{ \mathcal{F} \times \mathcal{F} }$	$\Phi \equiv \text{diag}(\varphi_f)_{f \in \mathcal{F}} + \Gamma \Xi_{:\mathcal{F}}$
Factor price deflators	$\Omega^{\mathcal{F}} \in \mathbb{R}^{ \mathcal{F} \times \mathcal{N} }$	$\Omega^{\mathcal{F}} \equiv \left[\begin{pmatrix} \mathbf{1}_{\mathcal{L}} & \mathbb{O}_{\mathcal{LU}} & \mathbf{0}_{\mathcal{L}} \\ \mathbb{O}_{\mathcal{UL}} & I_{\mathcal{U}} & \mathbf{0}_{\mathcal{U}} \end{pmatrix} - \Gamma \right] \Omega_{\mathcal{Z}:} + \Gamma \Xi_{:\mathcal{N}} \Psi^{-1}$
GDP deflator weights	$\bar{\Omega}_{\mathcal{Z}} \in \mathbb{R}^{1 \times \mathcal{N} }$	$\bar{\Omega}_{\mathcal{Z}} \equiv \bar{\Psi}_{\mathcal{Z}}^T \Omega_{\mathcal{Z}:}$
Aggregate factor price deflator	$\bar{\Omega}^{\mathcal{F}} \in \mathbb{R}^{1 \times \mathcal{N} }$	$\bar{\Omega}^{\mathcal{F}} \equiv \bar{\Psi}_{\mathcal{F}}^T \Omega^{\mathcal{F}}$
Nominal rate deflator	$\pi^{IS} \equiv \hat{\beta}^T \boldsymbol{\pi}$	$\hat{\beta}^T \equiv (1 - \gamma) \Omega_{H,:} + \gamma (\Xi_{H,:} \Psi^{-1} - \Xi_{H\mathcal{F}} \Theta)$
Marginal cost pass-through	$\Delta \equiv \text{diag}(\hat{\delta}_1, \dots, \hat{\delta}_N)$	$\hat{\delta}_i \equiv \frac{\delta_i(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)} \in (0, 1)$
Price pass-through	$\mathcal{P} \in \mathbb{R}^{ \mathcal{N} \times \mathcal{N} }$	$\mathcal{P} \equiv \Delta(I - \Omega\Delta)^{-1} = (I - \Delta\Omega)^{-1}\Delta$
Demand elasticity	$\Theta \in \mathbb{R}^{ \mathcal{F} \times \mathcal{N} }$	see (36)

Table 5 Elasticity definitions

B Section 4 Proofs

Proof of lemmas 4.1 and 4.2

The expressions for $\boldsymbol{\kappa}$ and $\mathcal{V}$ were derived in appendix A.4. It remains to derive the decompositions (15) and (16).

Relative slopes. The derivation rests on two identities. The rows of $\Omega^{\mathcal{F}}$ sum to one (step 1 of the proof of lemma A.1), and $\Theta \mathbf{1} = \mathbf{0}$ (a uniform increase in all prices leaves relative factor demand unchanged). Hence $(\Omega^{\mathcal{F}} - \Phi\Theta) \mathbf{1} = \mathbf{1}$, so that

$$(I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta)) \mathbf{1} = \mathbf{1} - \mathcal{P}_{:\mathcal{F}} \mathbf{1} \implies (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathcal{P}_{:\mathcal{F}} \mathbf{1} = (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathbf{1} - \mathbf{1}. \quad (40)$$

Split $\Phi \mathbf{1} = \bar{\Phi} \mathbf{1} + (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1})$ and apply (40):

$$\begin{aligned} \boldsymbol{\kappa} &= \bar{\Phi} (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathcal{P}_{:\mathcal{F}} \mathbf{1} + (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathcal{P}_{:\mathcal{F}} (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) \\ &= \bar{\Phi} \left((I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathbf{1} - \mathbf{1} \right) + (I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathcal{P}_{:\mathcal{F}} (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}). \end{aligned} \quad (41)$$

Next, split $\mathcal{P}_{:\mathcal{F}} \mathbf{1} = \bar{\mathcal{P}}_{\mathcal{F}} \mathbf{1} + (\mathcal{P}_{:\mathcal{F}} \mathbf{1} - \bar{\mathcal{P}}_{\mathcal{F}} \mathbf{1})$. Then (40) gives

$$(I - \mathcal{P}_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi\Theta)) \mathbf{1} = (1 - \bar{\mathcal{P}}_{\mathcal{F}}) \mathbf{1} - (\mathcal{P}_{:\mathcal{F}} \mathbf{1} - \bar{\mathcal{P}}_{\mathcal{F}} \mathbf{1}),$$

so that

$$(I - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \mathbf{1} - \mathbf{1} = \frac{\bar{\mathcal{P}}_{\mathcal{F}}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} \mathbf{1} + \frac{1}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} (I - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} (\mathcal{P}_{:\mathcal{F}} \mathbf{1} - \bar{\mathcal{P}}_{\mathcal{F}} \mathbf{1}).$$

Substituting into (41) yields the decomposition

$$\kappa = \underbrace{\frac{\bar{\Phi} \bar{\mathcal{P}}_{\mathcal{F}}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}}}_{\equiv \kappa_0} \mathbf{1} + \bar{\mathcal{P}}_{\mathcal{F}} (I - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} \left[\frac{\bar{\Phi}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} \left(\frac{\mathcal{P}_{:\mathcal{F}} \mathbf{1}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \mathbf{1} \right) + \Psi_{:\mathcal{F}}(\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) + \left(\frac{\mathcal{P}_{:\mathcal{F}}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \Psi_{:\mathcal{F}} \right) (\Phi \mathbf{1} - \bar{\Phi} \mathbf{1}) \right]. \quad (42)$$

The scalar κ_0 is the slope of the homogeneous benchmark in which all factors share the supply elasticity $\bar{\Phi}$ and all industries the pass-through $\bar{\mathcal{P}}_{\mathcal{F}}$. Demeaning with $(I - \mathbf{1}\bar{\Omega}_{\mathcal{Z}})$ yields equation (15).

Aggregate incidence of cost-push shocks. Pre-multiplying (39) by $\bar{\Omega}_{\mathcal{Z}}$ and applying $\bar{\Omega}_{\mathcal{Z}} = \bar{\Omega}^{\mathcal{F}} + (\bar{\Omega}_{\mathcal{Z}} - \bar{\Omega}^{\mathcal{F}})$ isolates the last term of (16). For the remainder, use the equality

$$(I - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))^{-1} = I + \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta) (I - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta))^{-1}$$

to obtain

$$\begin{aligned} \bar{\Omega}^{\mathcal{F}} \mathcal{V} \pi^{nat} &= \bar{\Omega}^{\mathcal{F}} [\mathcal{P} \Psi^{-1} - \mathcal{P}_{:\mathcal{F}}(\Omega^{\mathcal{F}} - \Phi\Theta) (I - \mathcal{V})] \pi^{nat} \\ &= \bar{\Omega}^{\mathcal{F}} (\mathcal{P} - \bar{\mathcal{P}}_{\mathcal{F}} \Psi) \Psi^{-1} \pi^{nat} - \bar{\Omega}^{\mathcal{F}} \left(\mathcal{P}_{:\mathcal{F}} - \bar{\mathcal{P}}_{\mathcal{F}} \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi\Theta) (I - \mathcal{V}) \pi^{nat} \\ &\quad + \bar{\mathcal{P}}_{\mathcal{F}} \bar{\Omega}^{\mathcal{F}} \mathcal{V} \pi^{nat} + \bar{\mathcal{P}}_{\mathcal{F}} \left(\bar{\Psi}_{\mathcal{F}}^T \Phi - \bar{\Phi} \bar{\Psi}_{\mathcal{F}}^T \right) \Theta (I - \mathcal{V}) \pi^{nat} \\ &= \frac{\bar{\mathcal{P}}_{\mathcal{F}}}{1 - \bar{\mathcal{P}}_{\mathcal{F}}} \left[\left(\frac{\bar{\Omega}^{\mathcal{F}} \mathcal{P}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \bar{\Omega}^{\mathcal{F}} \Psi \right) \Psi^{-1} \pi^{nat} - \left(\frac{\bar{\Omega}^{\mathcal{F}} \mathcal{P}_{:\mathcal{F}}}{\bar{\mathcal{P}}_{\mathcal{F}}} - \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi\Theta) (I - \mathcal{V}) \right. \\ &\quad \left. + \left(\bar{\Psi}_{\mathcal{F}}^T \Phi - \bar{\Phi} \bar{\Psi}_{\mathcal{F}}^T \right) \Theta (I - \mathcal{V}) \right] \pi^{nat}. \end{aligned}$$

Proof of proposition 4.2

Start from (37),

$$\pi_t - \kappa \tilde{y}_t - \rho \mathbb{E} \pi_{t+1} = -\mathcal{V} (\chi_t + \rho \mathbb{E} \pi_{t+1}), \quad \chi_t \equiv \tilde{p}_{t-1} - \pi_t^{nat}.$$

The matrices $\mathcal{P}$ and Ψ are always invertible,³¹ while

$$I - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi\Theta)$$

³¹This follows from the Perron–Frobenius theorem: Ω and $\Omega\Delta$ are non-negative with row sums less than 1, so their eigenvalues are less than 1 in absolute value and $I - \Omega$, $I - \Omega\Delta$ are invertible.

is invertible outside a measure-zero subset of the parameter space. On that set, pre-multiply both sides by

$$\left[I - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} [\Psi \mathcal{P}^{-1} - \Psi_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)]$$

to obtain

$$\begin{aligned} & \left[I - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} [\Psi \mathcal{P}^{-1} - \Psi_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)] [\boldsymbol{\pi}_t - \boldsymbol{\kappa} \tilde{y}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1}] \\ &= - \left[I - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T (\Omega^{\mathcal{F}} - \Phi \Theta) \right] (\boldsymbol{\chi}_t + \rho \mathbb{E} \boldsymbol{\pi}_{t+1}). \end{aligned}$$

The right hand side is zero if pre-multiplied by $\bar{\Psi}_{\mathcal{F}}^T (\Omega^{\mathcal{F}} - \Phi \Theta)$. Hence

$$\begin{aligned} \tilde{y}_t &= \boldsymbol{\zeta}_{DC}^T [\boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1}], \\ \boldsymbol{\zeta}_{DC}^T &\equiv \frac{\bar{\Psi}_{\mathcal{F}}^T (\Omega^{\mathcal{F}} - \Phi \Theta) \left[I - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} [\Psi \mathcal{P}^{-1} - \Psi_{:\mathcal{F}} (\Omega^{\mathcal{F}} - \Phi \Theta)]}{\bar{\Psi}_{\mathcal{F}}^T (\Omega^{\mathcal{F}} - \Phi \Theta) \left[I - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} \Psi_{:\mathcal{F}} \Phi \mathbf{1}}, \end{aligned}$$

up to a measure-zero parameter set on which the denominator vanishes. By construction $\boldsymbol{\zeta}_{DC}^T \mathcal{V} = \mathbf{0}^T$ and the normalization $\boldsymbol{\zeta}_{DC}^T \boldsymbol{\kappa} = 1$ holds. Since $\boldsymbol{\zeta}_{DC}^T \mathcal{V} = \mathbf{0}^T$ and $\boldsymbol{\zeta}_{DC}^T \boldsymbol{\kappa} \neq 0$, the slope $\boldsymbol{\kappa}$ does not lie in the column space of $\mathcal{V}$; together they span $\mathbb{R}^N$, which gives the decomposition (19).

The expansion (20) follows by substituting $\mathcal{P} = \Delta (I - \Omega \Delta)^{-1}$, so that $\Psi \mathcal{P}^{-1} = \Psi (I - \Omega \Delta) \Delta^{-1} = \Psi (I - \Delta) \Delta^{-1} + \Psi (I - \Omega)$, and collecting the covariance terms. The auxiliary matrices are

$$\begin{aligned} \tilde{\Omega}^{\mathcal{F}} &\equiv \bar{\Omega}^{\mathcal{F}} \left[I_N - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} \\ \tilde{\Theta} &\equiv \Theta \left[I_N - \left(\Psi_{:\mathcal{F}} - \mathbf{1} \bar{\Psi}_{\mathcal{F}}^T \right) (\Omega^{\mathcal{F}} - \Phi \Theta) \right]^{-1} \\ \tilde{\Phi}_f &\equiv \frac{\left(\bar{\Psi}_{\mathcal{F}}^T \Phi \right)_f}{\bar{\Psi}_f}, \quad f \in \mathcal{F}. \end{aligned}$$

□

Proof of proposition 4.3

Write (37) as the first-order system

$$\begin{pmatrix} \boldsymbol{\pi}_{t+1} \\ \tilde{\boldsymbol{p}}_t \end{pmatrix} = \mathcal{A} \begin{pmatrix} \boldsymbol{\pi}_t \\ \tilde{\boldsymbol{p}}_{t-1} \end{pmatrix} - \begin{pmatrix} \frac{1}{\rho} (I - \mathcal{V})^{-1} \boldsymbol{\kappa} \tilde{y}_t + \frac{1}{\rho} (I - \mathcal{V})^{-1} \mathcal{V} \boldsymbol{\pi}_t^{nat} \\ \boldsymbol{\pi}_t^{nat} \end{pmatrix}, \quad (43)$$

with

$$\mathcal{A} \equiv \begin{pmatrix} \frac{1}{\rho} (I - \mathcal{V})^{-1} & \frac{1}{\rho} (I - \mathcal{V})^{-1} \mathcal{V} \\ I & I \end{pmatrix}.$$

With N industries there are N forward-looking eigencombinations of $(\boldsymbol{\pi}, \tilde{\boldsymbol{p}}_{-1})$, corresponding to eigenvalues greater than one in absolute value, and N backward-looking ones. They satisfy

$$\begin{pmatrix} I & W \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_{t+1} \\ \tilde{\boldsymbol{p}}_t \end{pmatrix} = \Lambda \begin{pmatrix} I & W \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_t \\ \tilde{\boldsymbol{p}}_{t-1} \end{pmatrix}, \quad (44)$$

where the loading on current inflation is normalized to the identity without loss of generality. Substituting (44) into (43), the eigen-matrices Λ solve the quadratic equation

$$\Lambda^{-1} [I - (1 + \rho) \Lambda + \rho \Lambda^2] = (I - \mathcal{V})^{-1} \mathcal{V}, \quad (45)$$

which is the equation stated in the proposition, since $I - (1 + \rho) \Lambda + \rho \Lambda^2 = (I - \rho \Lambda) (I - \Lambda)$. Equation (45) has a unique backward-looking solution (Λ_B, W_B) with all eigenvalues weakly inside the unit circle, and $\Lambda_F \equiv \frac{1}{\rho} \Lambda_B^{-1}$ is part of the unique forward-looking solution. The loadings are

$$W_B = I - (\rho \Lambda_B)^{-1}, \quad W_F = I - \Lambda_B.$$

$\mathcal{V} \mathbf{1} = \mathbf{0}$ implies $\Lambda \mathbf{1} = \mathbf{1}$, and $\boldsymbol{\zeta}_{DC}^T \Lambda = \boldsymbol{\zeta}_{DC}^T$ from $\boldsymbol{\zeta}_{DC}^T \mathcal{V} = \mathbf{0}^T$. Iterating (43) forward and decomposing any vector in the $(\boldsymbol{\pi}, \tilde{\boldsymbol{p}}_{-1})$ space along the two eigen-blocks yields

$$\begin{pmatrix} \boldsymbol{\pi} \\ \tilde{\boldsymbol{p}}_{-1} \end{pmatrix} = \left[\begin{pmatrix} W_B \\ -I \end{pmatrix} (W_B - W_F)^{-1} \begin{pmatrix} I & W_F \end{pmatrix} - \begin{pmatrix} W_F \\ -I \end{pmatrix} (W_B - W_F)^{-1} \begin{pmatrix} I & W_B \end{pmatrix} \right] \begin{pmatrix} \boldsymbol{\pi} \\ \tilde{\boldsymbol{p}}_{-1} \end{pmatrix},$$

Imposing the initial condition $\tilde{\boldsymbol{p}}_{-1} = \tilde{\boldsymbol{p}}_{-1}^*$ and the boundary condition $\lim_{t \rightarrow \infty} \boldsymbol{\pi}_t = \mathbf{0}$ pins down

$$\boldsymbol{\pi}_0 = \sum_{s=0}^{\infty} (\rho \Lambda_B)^s \left[\Lambda_B (I - \mathcal{V})^{-1} \boldsymbol{\kappa} \tilde{y}_s + (I - \Lambda_B) \boldsymbol{\pi}_s^{nat} \right] - (I - \Lambda_B) \tilde{\boldsymbol{p}}_{-1}.$$

Iterating from this initial value gives the general expression, in which current inflation depends on belief updates at all prior dates:

$$\begin{aligned}
\pi_t = & (I - \rho\Lambda_B^2)^{-1} \left\{ \sum_{s=0}^{\infty} \sum_{\tau=0}^{\infty} \left[(I - \rho\Lambda_B) + \rho\Lambda_B (I - \Lambda_B) (\rho\Lambda_B^2)^{\tau} \right] (\rho\Lambda_B)^s \right. \\
& \times \left[\Lambda_B (I - \mathcal{V})^{-1} \kappa \tilde{y}_{t+s|\tau} + (I - \Lambda_B) \pi_{t+s|\tau}^{nat} \right] \\
& + (I - \Lambda_B) \sum_{s=1}^{\infty} \sum_{\tau=s}^{\infty} \left[\rho\Lambda_B^{s+1} (\rho\Lambda_B^2)^{\tau-s} \left[\Lambda_B (I - \mathcal{V})^{-1} \kappa \tilde{y}_{t-s|\tau} + (I - \Lambda_B) \pi_{t-s|\tau}^{nat} \right] \right. \\
& \left. \left. - \Lambda_B^s \left[(I - \mathcal{V})^{-1} \kappa \tilde{y}_{t-s|\tau} - (I - \rho\Lambda_B) \pi_{t-s|\tau}^{nat} \right] \right] \right\}, \tag{46}
\end{aligned}$$

where $x_{t+s|\tau}$ denotes the belief update about x_{t+s} formed at time $t - \tau$. If all past shocks were surprises at the time they occurred, and current and future shocks were unexpected before t , all belief updates collapse onto a single date and (46) reduces to (21) in the main text. $\square$

Proof of lemma 4.3

Pre-multiply (21) by ζ_{DC}^T . Since $\zeta_{DC}^T \Lambda = \zeta_{DC}^T$ and $\zeta_{DC}^T (I - \Lambda) = \mathbf{0}^T$, every term in π^{nat} drops out, as does the entire backward-looking sum. What remains is

$$\zeta_{DC}^T \pi_t = \sum_{s=0}^{\infty} \rho^s \zeta_{DC}^T \Lambda (I - \mathcal{V})^{-1} \kappa \mathbb{E}_t \tilde{y}_{t+s} = \sum_{s=0}^{\infty} \rho^s \mathbb{E}_t \tilde{y}_{t+s},$$

using $\zeta_{DC}^T \mathcal{V} = \mathbf{0}^T$ and the normalization $\zeta_{DC}^T \kappa = 1$. This is (22). Relative price indices are

$$\begin{aligned}
(I - \kappa \zeta_{DC}^T) \pi_t = & \sum_{s=0}^{\infty} \rho^s \left[(I - \Lambda) \Lambda^s \mathbb{E}_t \pi_{t+s}^{nat} - [I - \mathcal{V} - \Lambda^{s+1}] (I - \mathcal{V})^{-1} \kappa \mathbb{E}_t \tilde{y}_{t+s} \right] \\
& + (I - \Lambda) \sum_{s=1}^{\infty} \Lambda^s \left[\pi_{t-s}^{nat} - (I - \mathcal{V})^{-1} \kappa \tilde{y}_{t-s} \right].
\end{aligned}$$

This makes explicit that the relative price indices depend on the full path of natural relative prices intertwined with relative price movements induced by output gaps. $\square$

Proof of lemma 4.4

Under the AR1 processes (25), $\mathbb{E}_t \tilde{y}_{t+s} = (\eta^y)^s \tilde{y}_t$ and $\mathbb{E}_t \pi_{t+s}^{nat} = (\eta^{nat})^s \pi_t^{nat}$. Substituting into (22) gives $\pi_t^{DC} = \frac{\tilde{y}_t}{1 - \rho\eta^y}$, and taking innovations yields $\varsigma_t^y = (1 - \rho\eta^y) \hat{\pi}_t^{DC}$.

For the relative block, substitute the AR1 processes into the expression for $(I - \kappa \zeta_{DC}^T) \pi_t$ above and take innovations. The geometric sums evaluate to $(I - \rho\eta^{nat}\Lambda)^{-1}$ on the natural price terms and $(I - \rho\eta^y\Lambda)^{-1}$ on the output gap

terms. Since $(I - \kappa \zeta_{DC}^T)$ has rank $N - 1$, the level of ς^{nat} is pinned down by the normalization $\bar{\Omega}_{\mathcal{Z}} \varsigma^{nat} = 0$, which is without loss of generality because $\mathcal{V}\mathbf{1} = \mathbf{0}$. Adding $\mathbf{1}\bar{\Omega}_{\mathcal{Z}}$ to $I - \Lambda$ makes the system invertible outside a measure-zero subset of the parameter space and delivers (26). $\square$

Proof of proposition 4.4

Substituting the Taylor rule into the IS equation reduces (13) to two equations:

$$\begin{cases} \pi_t - \rho \mathbb{E} \pi_{t+1} = \kappa \tilde{y}_t - \mathcal{V}(\tilde{p}_{t-1} - \pi_t^{nat} + \rho \mathbb{E} \pi_{t+1}) \\ \mathbb{E}_t \tilde{y}_{t+1} - \left(1 + \frac{\phi_y}{\gamma}\right) \tilde{y}_t = \frac{1}{\gamma} \left(\phi_{\pi} \pi_t^{TR} - \hat{\beta}^T \mathbb{E}_t \pi_{t+1}\right) + \epsilon_{t+1}^i. \end{cases} \quad (47)$$

Inflation and the aggregate output gap are jump variables, while price gaps and shocks are state variables. Under the Blanchard and Kahn (1980) determinacy conditions there is a unique stable solution admitting the impulse-response representation (27), with G mapping states into jumps and H governing the transition of the state vector.

Equation (28) then follows directly from (27). Pre-multiplying the jump block by ζ_{DC}^T and by $I - \kappa \zeta_{DC}^T$ gives

$$\begin{pmatrix} \hat{\pi}_t^{DC} \\ (I - \kappa \zeta_{DC}^T) \hat{\pi}_t \end{pmatrix} = \begin{pmatrix} \zeta_{DC}^T G_{\pi_{\varsigma}} \\ (I - \kappa \zeta_{DC}^T) G_{\pi_{\varsigma}} \end{pmatrix} \begin{pmatrix} \varsigma_t^D \\ \varsigma_t^{nat} \end{pmatrix}. \quad (48)$$

Here $G_{\pi_{\varsigma}} \in \mathbb{R}^{N \times (N+1)}$ is the restriction of G_{π} to the (ϵ^i, π^{nat}) columns, defined after (27). The matrix in (48) is $(N + 1) \times (N + 1)$, but singular. Adding $\mathbf{1}\hat{\Omega}^{\mathcal{Z}}$ leaves the left-hand side unchanged, since $\bar{\Omega}_{\mathcal{Z}} \varsigma^{nat} = 0$, and repairs the rank. The result is invertible for all parameter values outside a measure-zero subset of the parameter space, which yields (28). $\square$

Proof of lemma 4.5

Iterating the transition equation in (27) forward s periods and pre-multiplying the jump block by the GDP deflator weights $\bar{\Omega}_{\mathcal{Z}}$ gives $\mathbb{E}_t \bar{\pi}_{t+s} - \mathbb{E}_{t-1} \bar{\pi}_{t+s} = \bar{\Omega}_{\mathcal{Z}} G_{\pi} H_{\varsigma}^s (\varsigma_t^D, \varsigma_t^{nat})^T$. Substituting the identified innovations from (28) delivers (29). The horizon dependence enters through H^s , which is why no single fixed-weight inflation index forecasts aggregate inflation at every horizon. $\square$

C Two-Industry Example: Derivations

This appendix collects the derivations behind the two-industry examples of sections 3 and 4. Throughout I use the notation of example 3.1: industries $i \in \{S, M\}$, services using only labor and manufacturing only capital, factors indexed by the industry that employs them, $\bar{x} \equiv \frac{1}{2}(x_M + x_S)$ and $x^R \equiv \frac{1}{2}(x_M - x_S)$.

Equilibrium of the Two-Industry Economy. Specialize lemma A.1 to this economy. Also use the fact that household and the utilization producer have identical expenditure shares, equal to $\frac{1}{2}$, so that both factor price deflators equal $\bar{p}$. The equilibrium conditions are

$$w_{it} = \varphi_i (\ell_{it} - \epsilon_{it}^{\mathcal{F}}) + \bar{p}_t \quad (49)$$

$$p_{it} = \hat{\delta}_i (w_{it} - a_{it}) + (1 - \hat{\delta}_i) (p_{i,t-1} + \rho \mathbb{E}_t \pi_{i,t+1}) + \epsilon_{it}^p \quad (50)$$

$$\ell_{Mt} - \ell_{St} = - (a_{Mt} - a_{St}) - \theta (p_{Mt} - p_{St}) + (\epsilon_{Mt}^d - \epsilon_{St}^d) \quad (51)$$

together with the Euler equation and Taylor rule. Equation (51) combines the input composition equation with market clearing: since each industry's entire output goes to final use and each industry uses a single factor, $y_{it} = \ell_{it} + a_{it}$, and relative final demand responds to relative prices with elasticity θ .

Natural Output. The natural equilibrium sets $\hat{\delta}_M = \hat{\delta}_S = 1$. Assuming constant desired markups ($\epsilon_{it}^p = 0$), equation (50) becomes $p_i = w_i - a_i$. Substituting (49),

$$p_{it} = \varphi_i (\ell_{it} - \epsilon_{it}^{\mathcal{F}}) + \bar{p}_t - a_{it}.$$

Averaging, the $\bar{p}$ terms cancel and

$$\bar{\varphi} (\bar{\ell}_t - \bar{\epsilon}_t^{\mathcal{F}}) + \varphi^R (\ell_t^R - \epsilon_t^{\mathcal{F},R}) = \bar{a}_t. \quad (52)$$

Differencing across industries,

$$p_t^R = \bar{\varphi} (\ell_t^R - \epsilon_t^{\mathcal{F},R}) + \varphi^R (\bar{\ell}_t - \bar{\epsilon}_t^{\mathcal{F}}) - a_t^R. \quad (53)$$

Substituting (51) into (53) and solving for p^R ,

$$p_t^R = \frac{-(1 + \bar{\varphi}) a_t^R + \bar{\varphi} (\epsilon_t^{d,R} - \epsilon_t^{\mathcal{F},R}) + \varphi^R (\bar{\ell}_t - \bar{\epsilon}_t^{\mathcal{F}})}{1 + \theta \bar{\varphi}}.$$

Substituting back, the relative term becomes

$$\ell_t^R - \epsilon_t^{\mathcal{F},R} = \frac{-(1 - \theta) a_t^R + (\epsilon_t^{d,R} - \epsilon_t^{\mathcal{F},R}) - \theta \varphi^R (\bar{\ell}_t - \bar{\epsilon}_t^{\mathcal{F}})}{1 + \theta \bar{\varphi}}.$$

Inserting this into (52) and solving for $\bar{\ell} - \bar{\epsilon}^{\mathcal{F}}$,

$$(\bar{\ell}_t - \bar{\epsilon}_t^{\mathcal{F}}) \bar{\varphi} \left[1 - \left(\frac{\varphi^R}{\bar{\varphi}} \right)^2 \frac{\theta \bar{\varphi}}{1 + \theta \bar{\varphi}} \right] = \bar{a}_t + \frac{\varphi^R}{1 + \theta \bar{\varphi}} \left[(1 - \theta) a_t^R + \epsilon_t^{\mathcal{F},R} - \epsilon_t^{d,R} \right].$$

Since $y^{nat} = \bar{\ell}^{nat} + \bar{a} = (\bar{\ell}^{nat} - \bar{\epsilon}^{\mathcal{F}}) + \bar{\epsilon}^{\mathcal{F}} + \bar{a}$, this is exactly equation (12).

Slopes and Cost-Push Elasticities. Recasting (49)–(51) in gaps and following the steps of appendix A.4 produces the industry-level Phillips curves

$$\pi_{it} - \rho \mathbb{E} \pi_{i,t+1} = \kappa_i \tilde{y}_t + v_i \left[(\pi_{it}^{nat} - \bar{\pi}_t^{nat}) - (\tilde{p}_{i,t-1} - \tilde{p}_{t-1}) - \rho \mathbb{E} (\pi_{i,t+1} - \bar{\pi}_{t+1}) \right], \quad (54)$$

whose average and difference are the aggregate and relative Phillips curves (17)–(18). Here $v_i \equiv 2\mathcal{V}_{ii}$ is the response of industry i 's inflation to its own natural relative price. Set $\hat{\delta}_M = \hat{\delta}_S \equiv \delta$ and $\varphi_M > \varphi_S$. Combining the gap versions of (49), (50) and (51) and differencing across industries,

$$\kappa^R = \frac{\delta \varphi^R}{1 + \theta \delta \bar{\varphi}}.$$

Raising the output gap raises the relative price of manufacturing: manufacturing production is harder to scale up, so a given increase in demand produces a larger price response and a smaller quantity response. The effect is stronger the lower is θ , because final users cannot shift expenditure away from the inelastically supplied good.

Averaging instead of differencing gives the incidence of natural relative prices on aggregate inflation,

$$v^R \pi_t^{R,nat} = \frac{\theta \delta}{1 + \theta \delta \bar{\varphi}} Cov(\varphi, \pi^{nat}).$$

Nominal rigidities prevent relative prices from adjusting fully, so relative demand for manufacturing – and hence for inelastically supplied capital – stays inefficiently high even when aggregate output is at potential. Because capital has the steeper supply curve, the resulting misallocation raises the price of manufacturing by more than it lowers the price of services, and aggregate inflation rises. The effect disappears as $\theta \rightarrow 0$, when relative price distortions no longer move relative demand.

The same algebra delivers the real wage response quoted in example 4.1. Labor is the factor employed by services, so the real wage is $w_S - \bar{p}$. At zero output gap, $\tilde{\ell}_S = -\tilde{\ell}^R$ and the labor supply curve in gaps gives $\tilde{w}_S - \tilde{p} = \varphi_S \tilde{\ell}_S = -\varphi_S \tilde{\ell}^R$. Solving for the employment gap, $\tilde{\ell}^R = -\theta \tilde{p}^R = \frac{\theta(1-\delta)}{1+\theta\delta\bar{\varphi}} \pi^{R,nat}$, so that

$$\tilde{w}_{St} - \tilde{p}_t = -\frac{\theta(1-\delta)\varphi_S}{1+\theta\delta\bar{\varphi}} \pi_t^{R,nat}, \quad \varphi_S = \bar{\varphi} - \varphi^R,$$

and real wages fall in response to an increase in the relative natural price of manufacturing.

Now set $\varphi_M = \varphi_S \equiv \varphi$ and $\delta_M > \delta_S$. The same steps give

$$\kappa^R = \frac{\varphi \frac{\delta^R}{1-\delta}}{1 + \theta \varphi \frac{\bar{\delta}}{1-\delta} \left(1 - \frac{\delta_M \delta_S}{2\delta}\right)}, \quad v^R \pi^{R,nat} = -\frac{1 - \frac{\varphi \theta \bar{\delta}}{1+\varphi \theta \bar{\delta}} \frac{\delta^R}{\bar{\delta}}}{1 + \frac{\varphi \theta \bar{\delta}}{1+\varphi \theta \bar{\delta}} \frac{Var(\delta)}{\bar{\delta}(1-\delta)}} Cov(\delta, \pi^{nat}).$$

Here the mechanism differs, and operates even when labor and capital are perfectly substitutable. Closing the output

gap stabilizes average marginal costs but not industry-level costs, which rise where π^{nat} is high and fall elsewhere. Industries with flexible prices pass cost changes through faster, so they dominate aggregate inflation.

Inflation Decomposition and Index Weights In this economy the cost-push elasticity has rank one,

$$\mathcal{V} = \begin{pmatrix} v_M \\ -v_S \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -1 \end{pmatrix},$$

where v_M and v_S are defined with opposite signs because the price of M rises and the price of S falls with $\pi^{R,nat}$. Solving the two-dimensional system $\pi_t = \kappa \pi_t^{DC} + \mathcal{V} \pi_t^\chi$ for the two projections gives, in terms of industry-level rates,

$$\pi_t^{DC} = \frac{v_S \pi_{Mt} + v_M \pi_{St}}{v_S \kappa_M + v_M \kappa_S}, \quad \pi_t^\chi = \frac{\kappa_S \pi_{Mt} - \kappa_M \pi_{St}}{v_S \kappa_M + v_M \kappa_S}.$$

Industries with a higher cost-push elasticity are downweighted in π^{DC} , and industries with a steeper slope are downweighted in π^χ . Since industries with flexible prices or inelastic factor supply have both a higher κ and a higher v , they carry a lower weight in both indices.

Specializing to the two cases considered above – uniform nominal rigidities with $\varphi_M > \varphi_S$, and uniform supply elasticities with $\delta_M > \delta_S$ – the relative weight in the divine coincidence index is

$$\zeta_M^{DC} - \zeta_S^{DC} = -\frac{\theta \varphi^R}{1 + \theta \bar{\varphi}}$$

under uniform nominal rigidities, and

$$\zeta_M^{DC} - \zeta_S^{DC} = -\frac{1 - \frac{\varphi \theta \bar{\delta}}{1 + \varphi \theta \bar{\delta}} \frac{\delta_M - \delta_S}{\bar{\delta}}}{1 + \frac{\varphi \theta \bar{\delta}}{1 + \varphi \theta \bar{\delta}} \frac{Var(\delta)}{\bar{\delta}(1 - \bar{\delta})}} (\delta_M - \delta_S)$$

under uniform supply elasticities. In the first case the index discounts the capital-intensive industry, because an increase in its relative price signals inefficiently high demand for inelastic capital and hence a worse inflation-output tradeoff. In the second it discounts the flexible-price industry, whose prices are more sensitive to cost changes.

With full discounting ($\rho = 0$) and $\tilde{p}_{t-1}^R = 0$, the Phillips curves (17)–(18) collapse to $\pi_t^{DC} = \tilde{y}_t$ and $\pi_t^\chi = \pi_t^{R,nat}$, so that in this special case π^{DC} directly identifies the output gap and π^χ the natural relative price change.

Sequence-Space Representation **Proposition C.1. Phillips curves in sequence space.** *Assume that past changes in output gaps and natural relative prices were unexpected at the time when they occurred, and that all*

current and future shocks are announced at time t . Then

$$\pi_t^{DC} = \sum_{s=0}^{\infty} \rho^s \mathbb{E}_t \tilde{y}_{t+s} \quad (55)$$

$$\begin{aligned} \mathbf{v} \pi_t^\chi = & \begin{pmatrix} \frac{v_M}{\bar{v}(1-\bar{v})} \\ -\frac{v_S}{\bar{v}(1-\bar{v})} \end{pmatrix} \left\{ \sum_{s=0}^{\infty} \rho^s [(1-\bar{v})(1-\lambda)\lambda^s \pi_{R,t+s}^{nat} - (1-\bar{v}-\lambda^{s+1})\kappa^R \tilde{y}_{t+s}] \right. \\ & \left. - (1-\lambda) \sum_{s=1}^{\infty} \lambda^{s-1} [\lambda \kappa^R \tilde{y}_{t-s} + [\lambda \bar{v} - (1-\rho\lambda(1-\lambda))(1-\bar{v})] \pi_{R,t-s}^{nat}] \right\}, \end{aligned} \quad (56)$$

where

$$\lambda = \frac{1}{2} \left[\left(1 + \frac{1}{\rho(1-\bar{v})} \right) - \sqrt{\left(1 + \frac{1}{\rho(1-\bar{v})} \right)^2 - \frac{4}{\rho}} \right].$$

Proof. Since $\mathcal{V}$ has rank one, the matrix quadratic (45) of proposition 4.3 reduces to the scalar equation $\lambda^{-1}(1-\rho\lambda)(1-\lambda) = \frac{\bar{v}}{1-\bar{v}}$ on the relative price direction, whose stable root is λ above, while Λ acts as the identity on the aggregate direction ($\Lambda \mathbf{1} = \mathbf{1}$). Equation (55) is then the two-industry case of lemma 4.3, and (56) is the two-industry case of the relative price block derived in the proof of that lemma, with Λ replaced by λ on the relative direction. $\square$

Equation (55) shows that π^{DC} depends only on the present discounted value of current and future output gaps. Equation (56) shows that π^χ depends on current and expected natural relative price changes, and also on the persistent and anticipated relative price distortions induced by past and future output gaps. This is example 4.2.

Identification and Forecasting Proposition C.2. Identification. Assume the Taylor rule targets π^{DC} rather than consumer prices, $i_t = \phi_y \tilde{y}_t + \phi_\pi \pi_t^{DC} + \mathbb{E} \pi_{t+1} + \epsilon_{it}$, so that the output gap and natural relative prices follow the AR1 processes (25). Denoting unexpected components by $\hat{\pi}_t \equiv \pi_t - \mathbb{E}_{t-1} \pi_t$, innovations are identified by

$$\begin{aligned} \varsigma_t^y &= (1 - \rho\eta^y) \hat{\pi}_t^{DC} \\ \varsigma_t^{nat} &= \frac{\bar{v}(1 - \rho\lambda\eta^{nat})}{1 - \lambda} \left[\hat{\pi}_t^\chi + \frac{\kappa^R}{\bar{v}(1-\bar{v})} \left(1 - \bar{v} - \lambda \frac{1 - \rho\eta^y}{1 - \rho\lambda\eta^y} \right) \hat{\pi}_t^{DC} \right]. \end{aligned}$$

Proof. Substitute $\mathbb{E}_t \tilde{y}_{t+s} = (\eta^y)^s \tilde{y}_t$ and $\mathbb{E}_t \pi_{t+s}^{R,nat} = (\eta^{nat})^s \pi_t^{R,nat}$ into (55) and (56), evaluate the geometric sums, and take innovations. The first line follows from $\pi_t^{DC} = \tilde{y}_t / (1 - \rho\eta^y)$. The second inverts the resulting expression for $\hat{\pi}^\chi$, whose $\hat{\pi}^{DC}$ term purges the relative price index of the response of relative prices to current and expected output gaps. $\square$

Lemma C.1. Forecasting in the two-industry economy. *Under the same assumptions, the time- t forecast revision for aggregate inflation at $t + s$ is*

$$\mathbb{E}_t \bar{\pi}_{t+s} - \mathbb{E}_{t-1} \bar{\pi}_{t+s} = (\bar{\kappa} + \mathcal{C}_s^{DC} \kappa^R) (\eta^y)^s \hat{\pi}_t^{DC} + \mathcal{C}_s^\chi (\eta^{nat})^s \hat{\pi}_t^\chi, \quad (57)$$

with

$$\begin{aligned} \mathcal{C}_s^\chi &\equiv v^R \left[1 + \lambda (1 - \rho \eta^{nat}) \frac{1 - \left(\frac{\lambda}{\eta^{nat}} \right)^s}{\eta^{nat} - \lambda} \right] \\ \mathcal{C}_s^{DC} &\equiv -\frac{v^R}{\bar{v}(1-\bar{v})} \left\{ \left(1 - \bar{v} - \lambda \frac{1 - \rho \eta^y}{1 - \rho \lambda \eta^y} \right) \left[1 - \left(\frac{\eta^{nat}}{\eta^y} \right)^s \frac{\mathcal{C}_s^\chi}{v^R} \right] + \frac{\lambda(1-\lambda)(1-\rho \eta^y)}{1 - \rho \lambda \eta^y} \frac{1 - \left(\frac{\lambda}{\eta^y} \right)^s}{\eta^y - \lambda} \right\}. \end{aligned}$$

At $s = 0$ both terms in braces vanish, so $\mathcal{C}_0^{DC} = 0$ and $\mathcal{C}_0^\chi = v^R$, as the accounting identity $\bar{\pi}_t = \bar{\kappa} \pi_t^{DC} + v^R \pi_t^\chi$ requires. If the two industries face identical nominal rigidities and supply elasticities then $\kappa^R = \mathcal{C}_s^\chi = 0$, and forecast revisions depend only on π^{DC} , which becomes proportional to $\bar{\pi}$.

Proof. From $\pi = \kappa \pi^{DC} + v \pi^\chi$, averaging gives $\bar{\pi} = \bar{\kappa} \pi^{DC} + v^R \pi^\chi$, so

$$\mathbb{E}_t \bar{\pi}_{t+s} - \mathbb{E}_{t-1} \bar{\pi}_{t+s} = \bar{\kappa} (\mathbb{E}_t \pi_{t+s}^{DC} - \mathbb{E}_{t-1} \pi_{t+s}^{DC}) + v^R (\mathbb{E}_t \pi_{t+s}^\chi - \mathbb{E}_{t-1} \pi_{t+s}^\chi).$$

By (55) the first revision equals $\bar{\kappa} (\eta^y)^s \hat{\pi}_t^{DC}$. Using $\pi_t^R = -(1-\lambda) \tilde{p}_{t-1}^R + f_t$, with $f_t \equiv \sum_{j \geq 0} (\rho \lambda)^j \left[\frac{\lambda \kappa^R}{1-\bar{v}} \tilde{y}_{t+j} + (1-\lambda) \pi_{t+j}^{R,nat} \right]$ and $\tilde{p}_t^R = \lambda \tilde{p}_{t-1}^R + f_t - \pi_t^{R,nat}$, iterating from a zero initial gap, and collecting coefficients gives $\mathcal{C}_s^\chi$ and $\mathcal{C}_s^{DC}$. Under symmetry $v^R = 0$, hence $\mathcal{C}_s^\chi = 0$. $\square$

C.1 Non-Linear Two-Industry Model

Consider the environment of example 3.1, but relax the assumption that the disutility of labor supply and the utilization cost of non-labor assets are isoelastic. Industries are identical ex ante (they have the same nominal rigidities and supply elasticities) but factor supply becomes less elastic as employment rises. Let the primitive schedule be $w_f = \varphi (\ell_f - \epsilon_f^F) + \frac{1}{2} \varphi_2 (\ell_f - \epsilon_f^F)^2 + p^C$, with $\varphi_2 > 0$ the super-elasticity of factor supply, and write $\ell_f = \ell_f^{nat} + \tilde{\ell}_f$. Factor supply curves in gaps become

$$\tilde{w}_f = \varphi \tilde{\ell}_f + \varphi_2 (\ell_f^{nat} - \epsilon_f^F) \tilde{\ell}_f + \frac{1}{2} \varphi_2 \tilde{\ell}_f^2 + \tilde{p}^C = \varphi_f^{nat} \tilde{\ell}_f + \frac{1}{2} \varphi_2 \tilde{\ell}_f^2 + \tilde{p}^C, \quad (58)$$

where $\varphi_f^{nat} \equiv \varphi + \varphi_2 (\ell_f^{nat} - \epsilon_f^F)$ is the local inverse supply elasticity at the natural allocation. Also denote the relative inverse supply elasticity at the natural allocation by $\varphi^{R,nat} = \varphi_2 (\ell^{nat,R} - \epsilon^{F,R})$.

Averaging and differencing (58) across the two factors, and using $\tilde{\ell} = \tilde{y}$, gives

$$\tilde{w}^R = \bar{\varphi}^{nat} \tilde{\ell}^R + \varphi^{R,nat} \tilde{y} + \varphi_2 \tilde{y} \tilde{\ell}^R \quad (59)$$

$$\tilde{\tilde{w}} = \bar{\varphi}^{nat} \tilde{y} + \varphi^{R,nat} \tilde{\ell}^R + \frac{1}{2} \varphi_2 \left[\tilde{y}^2 + \left(\tilde{\ell}^R \right)^2 \right] + \tilde{p}^C. \quad (60)$$

Price gaps satisfy $\tilde{p}_i = \delta \tilde{w}_i - (1 - \delta) \pi_i^{nat}$, while relative factor demand is $\tilde{\ell}^R = -\theta \tilde{p}^R$. Differencing the pricing equation across industries and substituting (59) then gives

$$\tilde{p}^R \left[1 + \theta \delta \bar{\varphi}^{nat} + \theta \delta \varphi_2 \tilde{y} \right] = \delta \varphi^{R,nat} \tilde{y} - (1 - \delta) \pi^{R,nat}. \quad (61)$$

Averaging the pricing equation and substituting (60) instead yields

$$\bar{\pi} = \frac{\delta}{1 - \delta} \left\{ \bar{\varphi}^{nat} \tilde{y} + \varphi^{R,nat} \tilde{\ell}^R + \frac{1}{2} \varphi_2 \left[\tilde{y}^2 + \left(\tilde{\ell}^R \right)^2 \right] \right\}. \quad (62)$$

Substituting $\tilde{\ell}^R$ yields

$$\tilde{p}^R = \frac{\delta \varphi^{R,nat} \tilde{y} - (1 - \delta) \pi^{R,nat}}{1 + \theta \delta \bar{\varphi}^{nat} + \theta \delta \varphi_2 \tilde{y}} \quad (63)$$

$$\pi^R = \pi^{R,nat} + \tilde{p}^R$$

$$\bar{\pi} = \frac{\delta}{1 - \delta} \left[\bar{\varphi}^{nat} \tilde{y} + \frac{1}{2} \varphi_2 \tilde{y}^2 - \theta \varphi^{R,nat} \tilde{p}^R + \frac{1}{2} \varphi_2 \theta^2 \left(\tilde{p}^R \right)^2 \right].$$

Let's now solve for $\bar{\varphi}^{nat}$ and $\varphi^{R,nat}$ as a function of shocks. Natural quantities only appear in second order terms, so a first-order solution is sufficient. Since industries are identical ex ante, (52)–(53) give

$$\bar{\ell}^{nat} - \bar{\epsilon}^{\mathcal{F}} = \frac{\bar{a}}{\varphi}, \quad \ell^{nat,R} - \epsilon^{\mathcal{F},R} = \frac{(\theta - 1) a^R + \epsilon^{d,R} - \epsilon^{\mathcal{F},R}}{1 + \theta \varphi}, \quad \pi^{R,nat} = \frac{-(1 + \varphi) a^R + \varphi (\epsilon^{d,R} - \epsilon^{\mathcal{F},R})}{1 + \theta \varphi}. \quad (64)$$

Hence

$$\bar{\varphi}^{nat} = \varphi + \varphi_2 \frac{\bar{a}}{\varphi}, \quad \varphi^{R,nat} = \varphi_2 \frac{(\theta - 1) a^R + \epsilon^{d,R} - \epsilon^{\mathcal{F},R}}{1 + \theta \varphi} \varphi^{R,nat} = \frac{\varphi_2}{\varphi} (a^R + \pi^{R,nat}).$$

Further substituting p^R and $\varphi^{R,nat}$ into (63) gives

$$\begin{aligned} \pi^R &= \frac{\delta (1 + \theta \bar{\varphi}^{nat})}{1 + \theta \delta \bar{\varphi}^{nat}} \pi^{R,nat} + \frac{\delta \varphi_2}{1 + \theta \delta \bar{\varphi}^{nat}} \left(\frac{1}{\varphi} + \frac{\theta (1 - \delta)}{1 + \theta \delta \bar{\varphi}^{nat}} \right) \tilde{y} \pi^{R,nat} + \frac{\delta \varphi_2}{\varphi (1 + \theta \delta \bar{\varphi}^{nat})} a^R \tilde{y} \\ \bar{\pi} &= \frac{\delta}{1 - \delta} \left[\bar{\varphi}^{nat} \tilde{y} + \frac{1}{2} \varphi_2 \tilde{y}^2 + \frac{\varphi_2 \theta (1 - \delta)}{1 + \theta \delta \bar{\varphi}^{nat}} \left(\frac{1}{\varphi} + \frac{\theta (1 - \delta)}{2 (1 + \theta \delta \bar{\varphi}^{nat})} \right) (\pi^{R,nat})^2 + \frac{\varphi_2 \theta (1 - \delta)}{\varphi (1 + \theta \delta \bar{\varphi}^{nat})} a^R \pi^{R,nat} \right]. \end{aligned}$$

Imposing $\epsilon^{d,R} = \epsilon^{\mathcal{F},R} = 0$ gives (31).

D Additional Quantitative Results

D.1 Stylized facts robustness

Excluding commodities. This paragraph revisits the stylized facts of section 2 excluding energy and food industries from the sample. Excluded industries are crude oil and gas extraction, petroleum and coal products, and farms, covering 4% of private value added. The cyclical basket's two largest members by value added, Banking and Chemicals, are not excluded. Controls, sample period, weighting and basket construction are otherwise unaffected.

Figure 15 shows that the inflation sensitivity of flex-priced industries to the output gap is driven by food and energy sectors. The regression coefficient in figure 2 drops from 0.13 (0.05) to 0.04 (0.04). The inflation sensitivity of non-scalable industries to the output gap instead survives the exclusion, going from 0.16 (0.05) to 0.12 (0.04).

The correlations of the two inflation spreads with the output gap are unaffected by the exclusion of food and energy, remaining at 0.49 and 0.57. The correlation between the relative price of the cyclical basket and aggregate inflation is also robust. Re-selecting the cyclical basket on the restricted sample, the slope of figure 5 becomes 0.25 (0.05) against 0.30 (0.03) in the baseline specification.

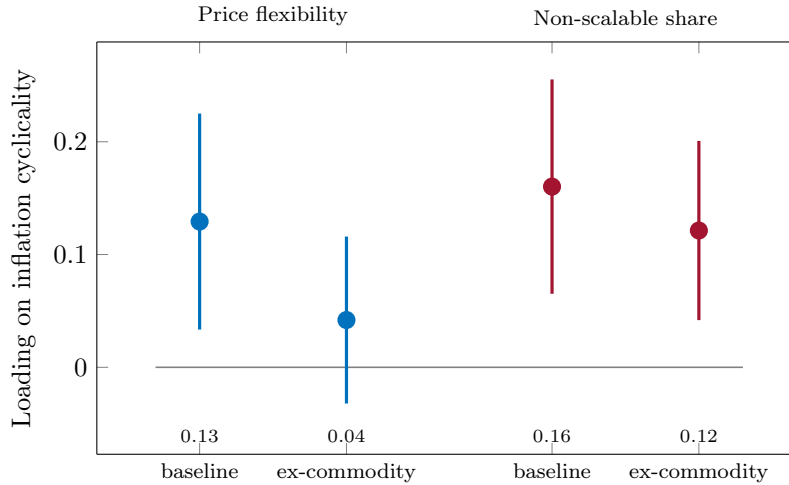


Figure 15 Fact 2 loadings re-estimated with energy and food industries removed from the private sample: crude oil and gas extraction, petroleum and coal products, and farms, together 4% of private value added. Both characteristics are re-standardized on the restricted sample. Points are value-added-weighted regression coefficients from the specification of figure 2; whiskers are 95% confidence intervals.

Measurement error. Figure 16 confirms that the positive slope estimated in figure 5 is not driven by common measurement error in the price of cyclical industries and aggregate inflation. In figure 5, the cyclical basket enters both the relative price on the horizontal axis, and aggregate inflation on the vertical axis. If the measured price of cyclical industries contains error, that error mechanically produces a positive slope. Figure 16 re-runs the regression on random splits of the industry sample in which no industry enters both sides, so no measurement error is shared

between them. The relative price on the horizontal axis uses one half of the cyclical basket and one half of the residual non-cyclical basket; aggregate inflation on the vertical axis uses the other half of each. The mean slope across random splits is 0.20.

The slope is different from figure 5, and it should not be interpreted as a revised estimate. The dependent variable is built from a subset of industries, making the slope smaller for two reasons. First, the value-added weight of the industries that had to be removed from the dependent variable to make the two sides disjoint no longer contributes to the slope. Second, a relative price built on half the industries is a noisier regressor, creating attenuation bias.

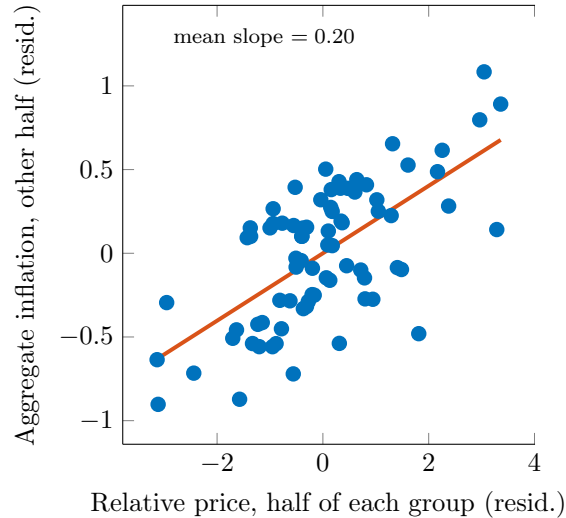


Figure 16 Figure 5 re-estimated on disjoint groups of industries: no industry’s price enters both the relative price and aggregate inflation. Controls, sample and construction are those of figure 5. The 28 cyclical industries and the 38 remaining industries are split in half separately, 2000 times; the relative price uses one half of each and aggregate inflation the other. The panel plots the draw whose slope is closest to the mean; the annotated slope is the mean across all draws, whose 5th–95th percentile range is $[0.02, 0.40]$. The level is not comparable to figure 5; see the text.

D.2 Quantitative model.

Supply elasticity of non-labor factors. I choose the four non-labor inverse supply elasticities $(\varphi_S, \varphi_{HE}, \varphi_{IP}, \varphi_{LE})$ so that the model reproduces peak local-projection coefficients across horizons (selecting the largest in absolute value). I hold other parameters fixed at their baseline values from table 2, and add a Taylor-rule interest-rate smoothing $\rho_i = 0.80$ as in Smets and Wouters (2007).

Empirical moments come from local projections on a monthly panel of 62 private industries, from 1981-12 to 2019-06.³² The dependent variable is year-on-year PPI inflation by industry. The monetary shock is the fed-funds-futures surprise, used as an instrument for the one-year rate GS1. The first stage projects GS1 on the instrument and its 12 lags; the fitted value is the surprise in policy-rate units. For each horizon $h = 0, \dots, 20$ months, I run one pooled regression of inflation at $t + h$ on a constant, the fitted shock, the four interactions of the fitted shock with the

³²Government industries are excluded, and four out of 66 private industries have no PPI series over this window.

category shares, 12 lags of the instrument, 12 lags of industry inflation, and industry fixed effects. The table reports the four interaction coefficients, which measure the differential response per percentage point of non-labor value added shares. Non-labor value added shares are built from the model’s calibration. Standard errors are clustered on industry, with the usual finite-sample degrees-of-freedom adjustment. All coefficients are scaled by the mean fitted shock and converted from annual to quarterly units. Monthly paths are averaged within quarter. Quarterly standard errors are the within-quarter average of the monthly ones, which treats adjacent horizons as perfectly correlated.

I run the same regression in the model, conditional on a monetary shock, for different values of φ . I then choose φ for the four non-labor categories to match empirical coefficients, obtaining four distinct moments. Each moment is weighted by the empirical standard error at the data’s peak quarter. Standard errors on $\hat{\varphi}$ are computed using the delta method, $\text{Var}(m(\hat{\varphi})) = G(G'\Omega^{-1}G)^{-1}G'$ with $G = \partial m/\partial \varphi$ and Ω the diagonal variance matrix of the empirical moments, evaluated over the elasticities not at a bound. Light equipment is at the lower bound, therefore its standard error is not defined.

Table 6 reports the resulting estimates. Estimated inverse supply elasticities φ_f need not be ordered like the regression coefficients β_f , because the model maps supply elasticities into price responses through value-added composition and network position as well. Running the same regression in the model as in the data shows that heavy equipment is the least elastically supplied factor, followed by structures. Intellectual property is somewhat less elastic than labor, while light equipment is the most elastically supplied. Given that empirical impulse-responses are imprecisely estimated, the matched φ values should be read as indicative.

Table 6 Inverse supply elasticities by factor category: estimates and moment fit.

Factor category	Data		Model	Estimate	
	β_f	(s.e.)	β_f	φ_f	(s.e.)
Heavy equipment	−0.0342	(0.0195)	−0.0320	9.26	(32.31)
Structures	−0.0128	(0.0132)	−0.0138	5.70	(20.90)
Intellectual property	−0.0246	(0.0178)	−0.0259	2.68	(6.94)
Light equipment	+0.0584	(0.0351)	+0.0200	0.00	—

Given that inverse supply elasticities for non-labor factors are imprecisely estimated, figure 17 repeats the inflation decomposition setting all factor supply elasticities equal to labor. The figure suggests that the inflation decomposition is robust to mismeasurement of these elasticities. Efficient relative price adjustments account for 49% of inflation fluctuations before 2020, like in the baseline, while aggregate demand accounts for 72% of the 2021Q2–2023Q1 surge, against 69% in the baseline.

Decomposition of CPI inflation

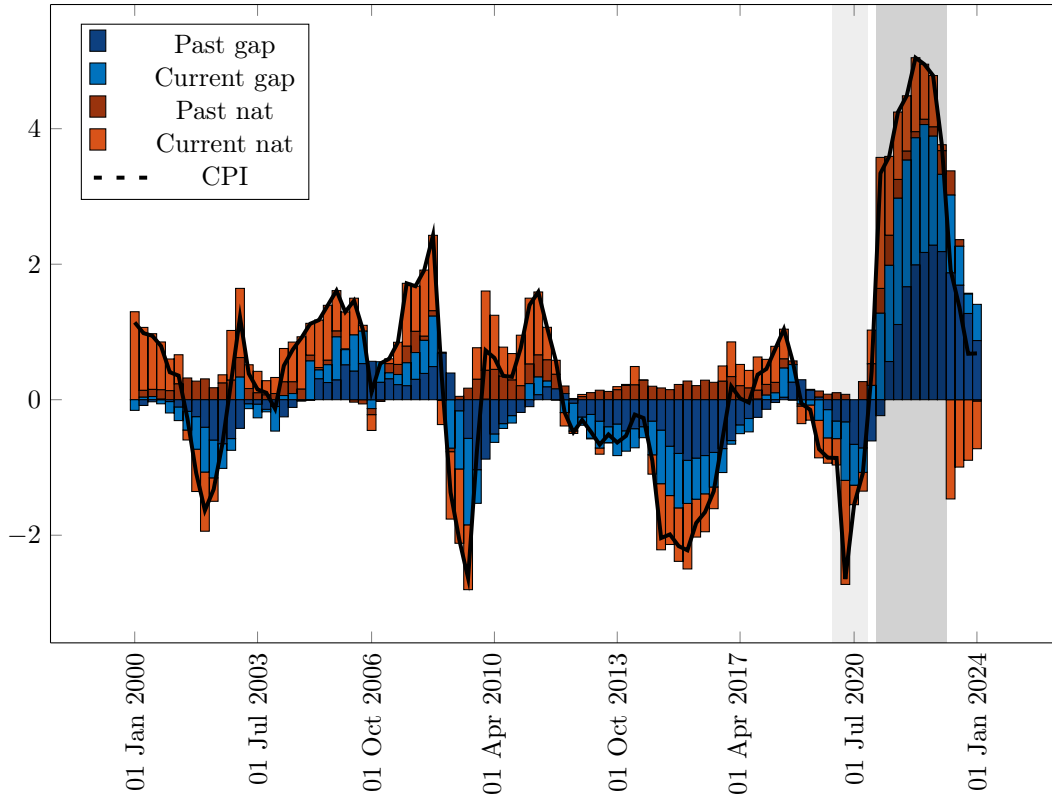


Figure 17 Decomposition of year-on-year CPI inflation, all factor supply elasticities set to the labor value, 2000Q1–2024Q1, in percent. Blue bars correspond to the inflation component driven by aggregate demand shocks ϵ^i , orange bars correspond to the inflation component driven by natural relative price shocks. Each component results from the sum of past and current shocks. The shaded bands are the windows over which the decomposition shares reported in the text are computed: the light band is 2020Q1–2020Q4, when both components were negative, and the dark band is 2021Q2–2023Q1, when both were positive.

Inflation decomposition in terms of output gap and natural relative prices. Following proposition 4.3, figure 18 presents an alternative decomposition of aggregate inflation in terms of output gaps and natural relative prices. Compared to figure 9, figure 18 attributes a larger share of inflation to relative price adjustments, equal to 58% before 2020, 66% of the 2020Q1–2020Q4 disinflation, and 53% of the 2021Q2–2023Q1 surge. This difference arises because relative price shocks affect the aggregate output gap, by moving aggregate inflation and therefore triggering a monetary response through the Taylor rule. Over the sample period, the endogenous gap response had an offsetting effect on the exogenous shocks ϵ^i to the Taylor rule. As a result, the overall output gap contribution is smaller than the contribution of exogenous demand shocks ϵ^i .

Decomposition of CPI inflation

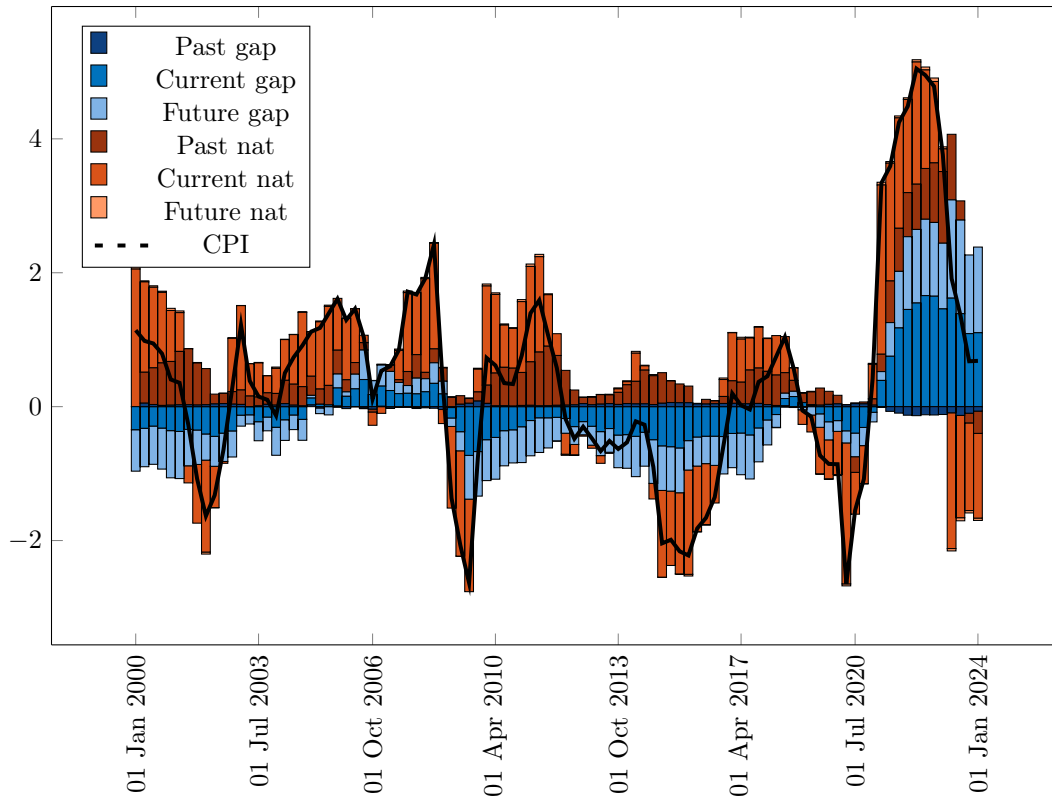


Figure 18 Decomposition of year-on-year CPI inflation, baseline calibration, 2000Q1–2024Q1, in percent. Following proposition 4.3 and equation (46), inflation is split into components driven by current, past, and future output gaps (blue bars) and natural relative prices (orange bars). Bars are stacked and sum to observed CPI inflation (black line).

Estimated Output Gap Against CBO Measure Figure 19 compares the aggregate output gap implied by the estimated model (solid line) with the CBO’s official measure (real GDP less CBO potential GDP, dashed line), over 2000Q1–2024Q1. Both are expressed in percentage changes. The two measures diverge from 2020 onward. This is because the model-based gap is estimated from price and wage changes. Wages and prices of sticky or scalable industries did not fall significantly in 2020Q2, while both prices and wages increased significantly from 2022 onward. This explains a muted behavior of the model-based gap in 2020, and a much larger estimate in 2022–2024 compared to official measures.

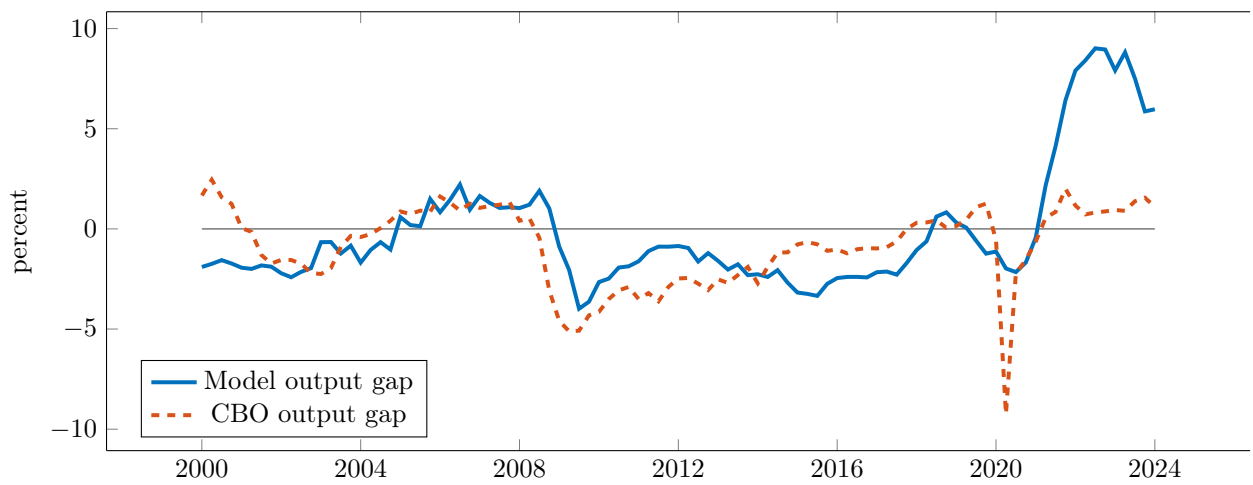


Figure 19 Aggregate output gap implied by the model against the CBO output gap, 2000Q1–2024Q1. The model gap $\hat{y}_t$ is recovered by inverting the industry inflation panel and is never targeted in the calibration.

Shock persistence and substitution elasticities. Figures 20, 21, and 22 illustrate the inflation decomposition under alternative assumptions about shock persistence and substitution elasticities. None of these variables significantly affects the aggregate inflation decomposition.

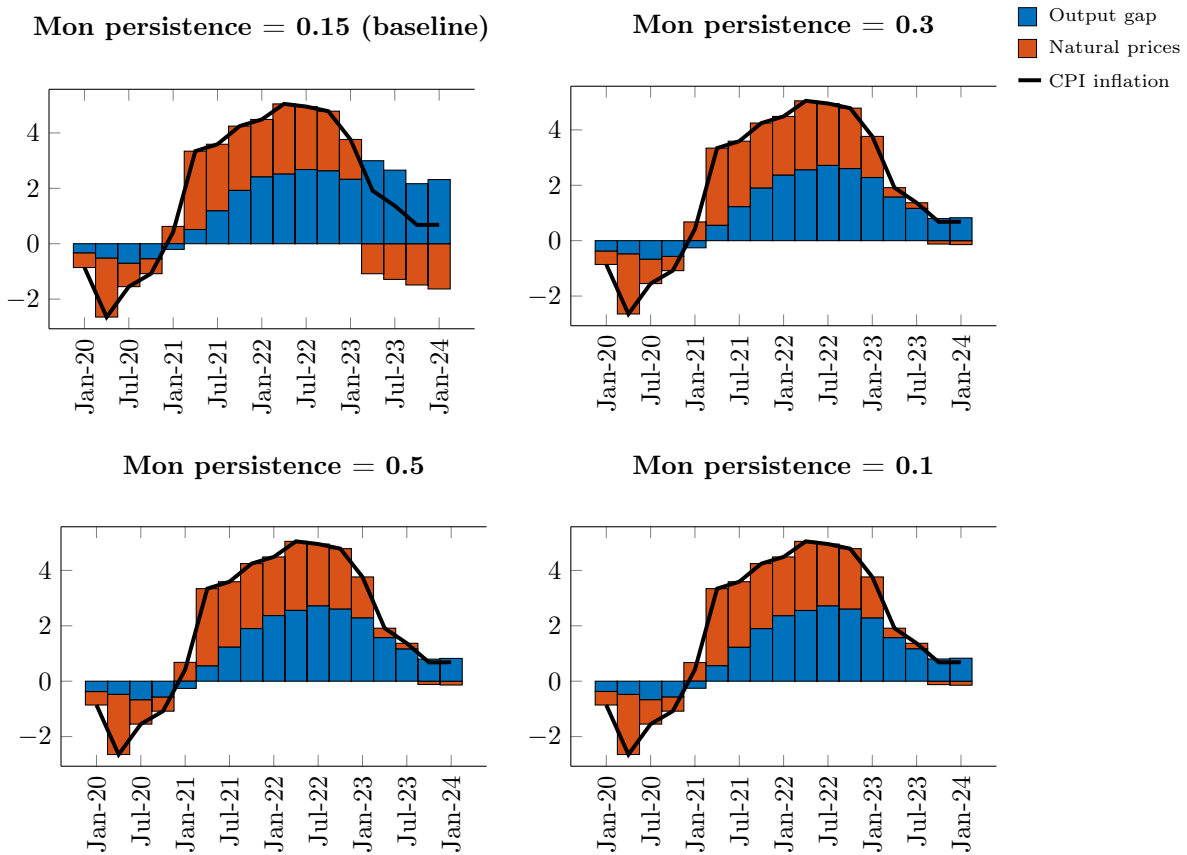


Figure 20 Inflation decomposition under different persistence of aggregate demand shocks.

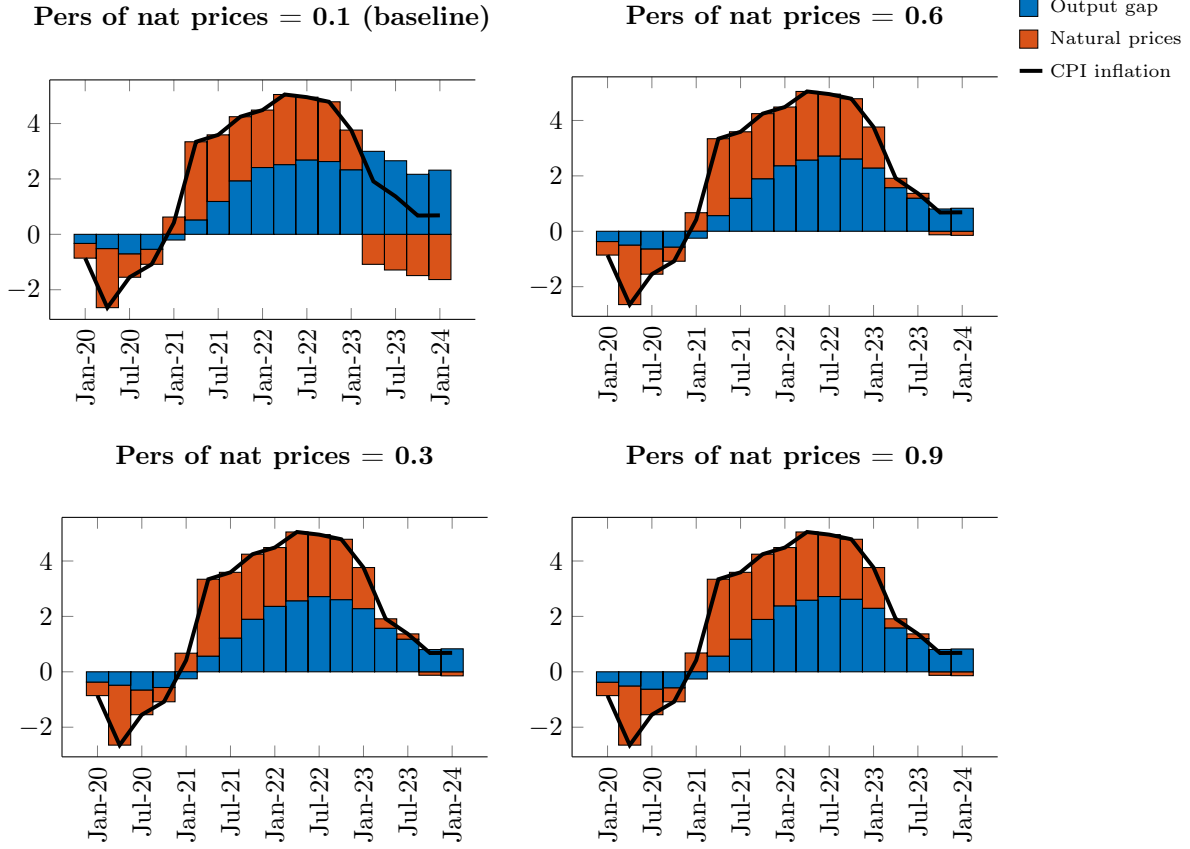


Figure 21 Inflation decomposition under different persistence of relative price shocks.

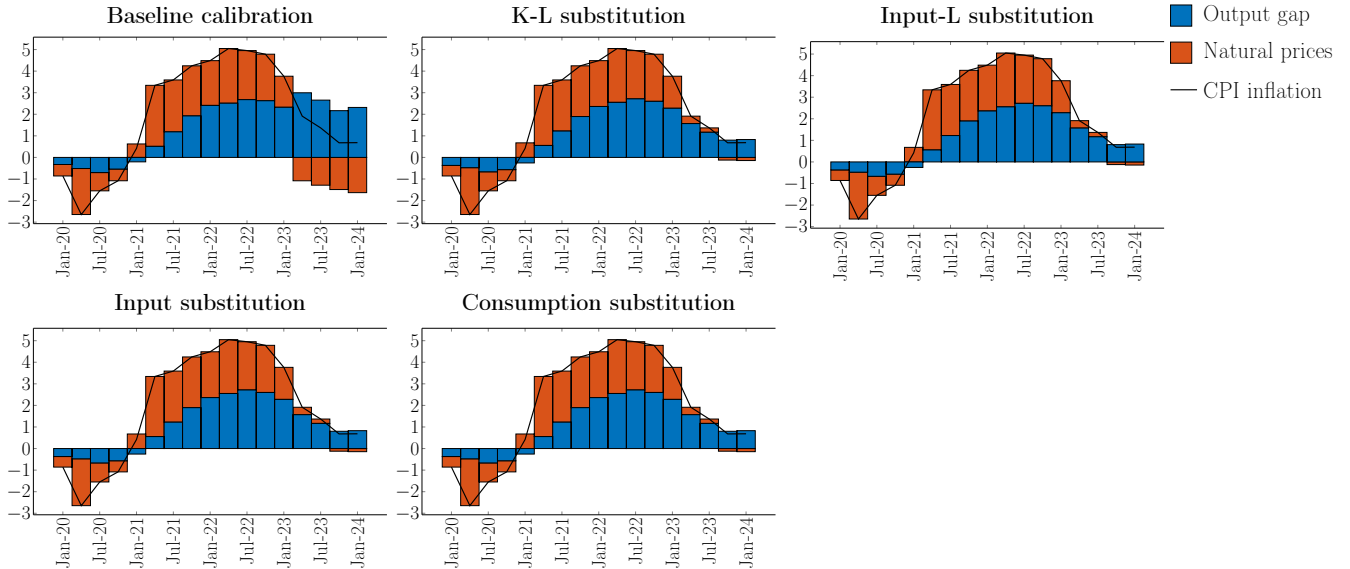


Figure 22 Comparison between inflation decompositions. Each counterfactual panel sets the relevant substitution elasticity = 3, instead of the baseline value.