NBER WORKING PAPER SERIES

MISALLOCATION AND ASSET PRICES

Winston Wei Dou
Yan Ji
Di Tian
Pengfei Wang

Working Paper 32147
http://www.nber.org/papers/w32147

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2024, Revised January 2025

We thank Paco Buera, Vincenzo Quadrini, Paul Romer, and Rob Townsend for detailed comments. We are grateful for helpful comments from Andy Abel, Hengjie Ai, Patrick Bolton, Georgy Chabakauri, Alex Corhay, Joel M. David, Shaun Davies, Sebastian Di Tella, Jan Eberly, Andrea Eisfeldt, Lorenzo Garlappi, Vincent Glode, Joao Gomes, Lars Peter Hansen, Ken Judd, Felix Kubler, Michael Roberts, Nick Roussanov, Tom Sargent, Lukas Schmid, Rob Stambaugh, Kjetil Storesletten, Neng Wang, Tom Winberry, and Wei Xiong as well as the participants in AFA, AsianFA, CEPR Summer Asset Pricing Meeting, CFRC, CICF, CICM, CUHK Greater Bay Area Finance Conference, EFA, European Winter Finance Summit, Financial Market and Financial Technology Conference, FIRS, HK Joint Finance Research Workshop, HKUST, HKUST-JINAN Macro Workshop, International Congress on Industrial and Applied Mathematics, MFA, NFA, PKU (Guanghua), PKU/PHBS Sargent Institute Workshop, Purdue University, SAFE Asset Pricing Workshop, SFS Cavalcade North America, Shanghai Macro Workshop, SHUFE, SMU, Stanford SITE (New Frontier in Asset Pricing), SWUFE International Macro-Finance Conference, Texas Finance Festival, the Macroeconomy and Finance in China Conference, Tsinghua SEM Alumni Conference, University of Pennsylvania (Wharton), UNSW Asset Pricing Workshop, and USC. Wang acknowledges the financial support from China National Science Foundation (#7215003 and #72125007). Dou is grateful for the support of the Golub Faculty Scholar Award at University of Pennsylvania. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed additional relationships of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w32147

Misallocation and Asset Prices
Winston Wei Dou, Yan Ji, Di Tian, and Pengfei Wang
NBER Working Paper No. 32147
February 2024, Revised January 2025
JEL No. L11, O30, O40

ABSTRACT

We develop an endogenous growth model with heterogeneous firms facing financial frictions, where misallocation emerges explicitly as a crucial endogenous state variable and significantly affects economic growth through the valuation channel. The model illustrates that transient macroeconomic shocks affecting misallocation can yield persistent effects on aggregate growth. In equilibrium, slow-moving misallocation endogenously generates long-run uncertainty about economic growth by distorting innovation decisions. When agents hold recursive preferences, misallocation-driven low-frequency growth fluctuations result in substantial risk premia in capital markets and large losses in welfare. We provide evidence that misallocation effectively captures low-frequency fluctuations in both aggregate growth and asset returns.

Winston Wei Dou
The Wharton School
University of Pennsylvania
2318 Steinberg-Dietrich Hall
3620 Locust Walk
Philadelphia, PA 19104
and NBER
wdou@wharton.upenn.edu

Yan Ji
Hong Kong University of Science
and Technology
Room 5005, Department of Finance
Lee Shau Kee Business Building
Clear Water Bay, Kowloon
Hong Kong
jiy@ust.hk

Di Tian
Hong Kong University of Science
and Technology
Room 5074, Department of Finance
Lee Shau Kee Business Building
Clear Water Bay, Kowloon
Hong Kong
ditian@ust.hk

Pengfei Wang
HSBC Business School
Peking University
Shenzhen, Guan 518055
China
pfwang@phbs.pku.edu.cn

A data appendix is available at http://www.nber.org/data-appendix/w32147

1 Introduction

Misallocation significantly impacts economic growth, both during economic transitions (e.g., Buera and Shin, 2013; Moll, 2014) and in long-run steady states (e.g., Jovanovic, 2014; Acemoglu et al., 2018). Various empirical measures of cross-sectional dispersion indicate that the allocation efficiency of capital displays strong pro-cyclical patterns.[1]

This paper examines the critical interplay between the time-series variations in capital misallocation and growth prospects, highlighting its central importance in explaining the forces behind low-frequency growth fluctuations.[2] These fluctuations constitute a systematic risk that quantitatively explains many asset pricing phenomena in capital markets (e.g., Bansal and Yaron, 2004; Hansen, Heaton and Li, 2008) and justifies the significant welfare costs of economic fluctuations. At its core, our analysis introduces a misallocation-driven asset pricing mechanism, emphasizing the valuation channel as a crucial amplifier of misallocation's impact on economic growth.

Specifically, we quantitatively examine the interplay between misallocation, growth prospects, and the systematic risk that influences asset prices in capital markets. To this end, we construct an analytically tractable general equilibrium model featuring heterogeneous firms and endogenous stochastic growth, in which the misallocation of production capital is endogenously slow-moving and leads to low-frequency fluctuations in economic growth. Our model builds upon Moll (2014), where misallocation arises endogenously due to financial frictions. It incorporates persistent firm-level idiosyncratic productivity, ensuring that the misallocation of production resources serves as a critical determinant in the aggregation of output across firms in the economy; without such persistence, the potential for misallocation across different firms would not exist (similar to, e.g., Moll, 2014; Di Tella, Maglieri and Tonetti, 2024). We extend this framework in three key ways while preserving its analytical tractability within a continuous-time model setting. First, we incorporate heterogeneous, publicly traded firms producing final goods. These firms are owned by shareholders with homogeneous recursive preferences but are managed by corporate managers whose objectives differ from those of the shareholders, generating agency conflicts that serve as the microfoundation for financial frictions. Second, we incorporate intermediate goods and R&D sectors alongside the final goods sector. The R&D sector expands the variety of intermediate goods, driving technological progress and endogenous growth, as described by Romer (1986, 1990). Third, we introduce transitory

[1]For empirical evidence, see Eisfeldt and Rampini (2006), Bloom (2009), Kehrig (2015), and Bloom et al. (2018), among others.

[2]These low-frequency growth fluctuations are intrinsically tied to the medium-term business cycle identified by Comin and Gertler (2006).

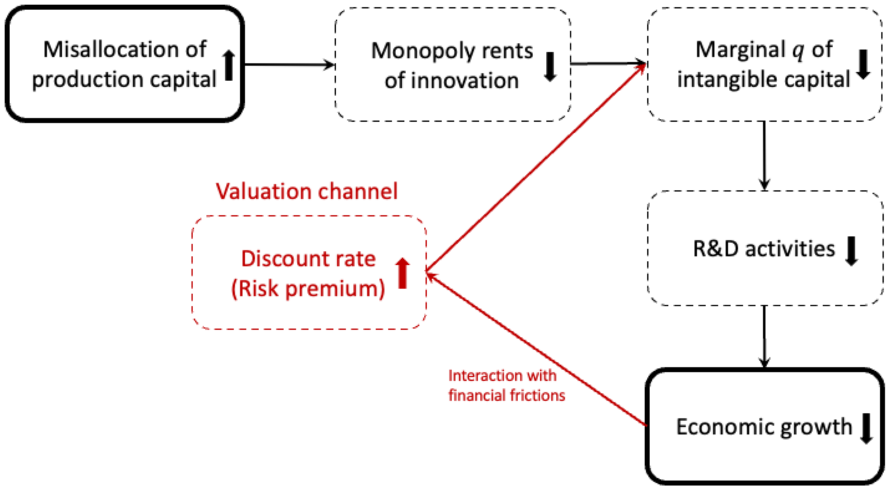


Figure 1: Our model elucidates a mechanism that quantitatively links capital misallocation to economic growth via a valuation channel.

aggregate shocks that endogenously generate the slow-moving dynamics of misallocation.

The combination of these three components enables us to illustrate an economic mechanism featuring a novel "valuation channel", as illustrated in Figure 1. When misallocation of production capital in the final goods sector rises persistently, typically as an endogenous response to transitory shocks, aggregate demand for intermediate goods experiences a persistent decline. Each producer in the intermediate good sector holds monopoly power over a specific type of intermediate good by acquiring blueprints from the R&D sector. The persistent drop in demand leads to a gradual, long-lasting decline in monopoly rents, which results in a persistent reduction in the value of blueprints produced by the R&D sector. This, in turn, causes a significant and persistent decline in the marginal q of intangible capital, which is the present value of marginal profits derived from intangible capital (e.g., Crouzet and Eberly, 2023). As a result, incentives for innovation within the R&D sector diminish persistently, leading to a long-term drag on economic growth.

Importantly, a novel valuation channel operates on top of the mechanism described above. The endogenously persistent and slow-moving misallocation of production capital implies that transitory aggregate shocks affecting misallocation can generate low-frequency fluctuations in economic growth. When agents have recursive preferences, these low-frequency growth fluctuations emerge as a fundamental source of systematic risk, playing a critical role in shaping the discount rate, particularly the risk premium. Moreover, As a result of the interaction between aggregate fluctuations and financial frictions, economic growth becomes both depressed and more volatile during downturns characterized by heightened misallocation. Intuitively, this heightened volatility of aggregate output growth during downturns arises from the amplification effect of financial frictions. The same aggregate shocks cause larger percentage changes in output when its level is low, primarily due to these frictions. This increased volatility elevates the risk premium required to discount the future rents of innovation during downturns, further depressing the marginal q of intangible capital across firms. Consequently, the valuation channel amplifies the impact of production capital misallocation on growth prospects, with significant implications for asset prices.

Below, we elaborate on the key elements of our model, which comprises three sectors. First, the R&D sector drives knowledge creation by using final goods and existing knowledge to generate new blueprints. Second, the intermediate goods sector utilizes these blueprints, along with final goods, to produce differentiated intermediate goods. Each producer in this sector holds a monopoly over a specific type of intermediate good, secured through the blueprint acquired from the R&D sector. Third, the final goods sector combines production capital, labor, and intermediate inputs to produce final goods. A representative agent owns all firms across all sectors, with firms in the final goods sector being heterogeneous, while firms in the intermediate goods and R&D sectors remain homogeneous.

Firms in the final goods sector differ in productivity and their stock of production capital. However, due to agency conflicts, they face both collateral constraints on borrowing and equity market constraints on payouts and issuances. These financial frictions contribute to the misallocation of production capital among firms. Following the established literature (e.g., Gertler and Kiyotaki, 2010; Gourio, 2012; Brunnermeier and Sannikov, 2017), we introduce aggregate capital quality shocks, interpreted as "liquidity shocks" when firms use capital as collateral for borrowing. Firms endogenously choose their capacity utilization intensity, with higher intensity enabling greater output but increasing exposure to aggregate liquidity shocks. In equilibrium, more productive firms utilize their capital more intensively, making them more exposed to these shocks. As a result, aggregate liquidity shocks drive fluctuations in capital misallocation, which in turn cause fluctuations in the economy's growth rate. Because the evolution of misallocation is endogenously slow-moving, fluctuations in aggregate growth driven by misallocation are both significant and persistent, even with i.i.d. aggregate liquidity shocks in the model. These low-frequency growth fluctuations are intrinsically linked to the medium-term business cycle (Comin and Gertler, 2006) and the growth cycle (Kung and Schmid, 2015).

The standard approach to solving general equilibrium models with heterogeneous firms and aggregate fluctuations relies on numerical approximations using key moments of the cross-sectional firm distribution. We depart from the standard approach by proposing a parametric approximation of the distribution of log productivity and log capital using a bivariate normal distribution. This method offers two key advantages. First, it enables us to derive a covariance-type measure for the misallocation of production capital in the final goods sector, which emerges as a crucial endogenous state variable summarizing the cross-sectional distribution of firms and characterizing the equilibrium in closed form. Specifically, in our model, misallocation is captured by the covariance between the log marginal revenue product of capital (MRPK) and log capital, normalized by the variance of log MRPK. This covariance-based measure of misallocation is intuitive and aligns with metrics commonly used in empirical studies to assess capital allocation efficiency (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013). Second, this parametric approximation makes the model highly tractable and transparent, allowing the economy's evolution to be analytically characterized by two endogenous state variables: misallocation and the knowledge stock-capital ratio. This approach allows for a clear analysis of the interplay between misallocation dynamics, aggregate growth fluctuations, and the systematic risk shaping asset prices.

We validate the parametric approximation in two ways. First, on the deterministic balanced growth path without aggregate shocks, we analytically justify the approximation using the Berry-Esseen bound (Tikhomirov, 1980; Bentkus, Gotze and Tikhomoirov, 1997). Second, in the presence of aggregate shocks, we compare the solution derived from the parametric approximation with the solution from standard global numerical methods, which use a finite set of moments to capture the infinite-dimensional cross-sectional distribution of firms, as in Krusell and Smith (1998). Our results show that, under baseline calibration, the model-implied cross-sectional distribution of firms and key statistics for various variables are similar both on the deterministic balanced growth path and in the stochastic steady state with aggregate shocks.

To illustrate the key theoretical mechanism, we begin our analysis by focusing on the deterministic balanced growth path in the absence of aggregate shocks. We show that a one-time shock that increases misallocation can exert a persistently adverse effect on economic growth. Specifically, due to financial frictions, the reallocation of capital across firms takes time. As a result, the shock not only escalates misallocation at the moment of impact but also prolongs this heightened level into the long-term future. Therefore, through its influence on the marginal q of intangible capital, and consequently on R&D incentives, what begins as a temporary shock to misallocation can result in a prolonged downturn in economic growth. This underscores the profound and lasting effects that misallocation has on the economy. Furthermore, we show that the persistence of both misallocation and economic growth is closely related to the persistence of firms' idiosyncratic productivity.

This augments the key insight from Moll (2014), which states that an increase in the persistence of firms' idiosyncratic productivity leads to a longer time for the economy to reach its steady state. In our model, the persistence of idiosyncratic productivity emerges as a crucial determinant of the persistence of misallocation-driven aggregate economic growth. This is primarily because misallocation naturally adjusts more slowly when idiosyncratic productivity becomes more persistent.

Building on this mechanism, we show that in the full model with aggregate shocks, misallocation evolves slowly, leading to low-frequency fluctuations in economic growth. Quantitatively, the annual autocorrelation of misallocation is 0.75, while that of consumption growth is 0.46, closely aligning with empirical estimates. Our model thus demonstrates a novel misallocation-based mechanism that explains the low-frequency covariation in the time series of consumption and output growth (e.g., Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Müller and Watson, 2008, 2018). At the heart of this mechanism is the valuation channel, which amplifies the effects of production capital misallocation in the final goods sector on economic growth, particularly its low-frequency component. During downturns marked by heightened misallocation and reduced growth, firms in the final goods sector face tighter financial constraints. In such periods, economic growth is not only low but also highly volatile. Consequently, low expected consumption growth typically coincides with high macroeconomic volatility, leading to an elevated risk premium. As a result, the marginal q of intangible capital takes a dual hit from heightened misallocation: it is depressed both by reduced profits and by a higher discount rate on future profits due to the elevated risk premium.

Furthermore, we show that our model not only rationalizes several important asset pricing moments but also suggests significant welfare costs associated with misallocation fluctuation. This complements the existing literature, which primarily focuses on quantifying the welfare costs of the level of misallocation rather than its time-series variations. Specifically, the model implies a high Sharpe ratio of 0.39 for the aggregate consumption claim, accompanied by a low and stable risk-free rate, aligning with empirical observations. Eliminating misallocation fluctuations would provide the representative agent with a welfare gain of approximately 10%. The large quantitative effects of misallocation fluctuations hinge on two key properties of the model: the low-frequency growth fluctuations driven by slow-moving misallocation and the recursive preferences of the representative agent. Without either, the Sharpe ratio and welfare gain would be negligible. Intuitively, recursive preferences ensure that the representative agent's marginal utility depends not only on current consumption growth but, more importantly, on expectations of future consumption growth. As a result, fluctuations in anticipated consumption growth significantly influence valuations through the stochastic discount factor (SDF). Additionally, the persistent nature of consumption growth implies that even transitory shocks can have lasting effects on future growth. This persistence amplifies the role of future consumption growth in determining current marginal utility, thereby magnifying the impact of capital misallocation on economic growth.

Although our main contribution is theoretical, we empirically test the main predictions of our model. Motivated by our theory, we construct a misallocation measure based on the covariance between log MRPK and log capital using U.S. Compustat data. We show that the misallocation measure is persistent, with a yearly autocorrelation of 0.84. Moreover, our empirical measure of misallocation is strongly countercyclical, and increases in misallocation causally lead to reductions in R&D intensity as well as declines in the growth rate of aggregate consumption and output over long horizons. In Online Appendix 2, we show that, as a macroeconomic factor, the empirical misallocation measure has significant cross-sectional asset pricing implications.

Related Literature. Our paper contributes to the asset pricing literature by offering a novel perspective on low-frequency growth fluctuations as a source of systematic risk in capital markets. Influential theoretical studies have provided microfoundations for these fluctuations (e.g., Ai, 2010; Kaltenbrunner and Lochstoer, 2010; Nicolae, Panageas and Yu, 2012; Croce, 2014; Kung and Schmid, 2015; Collin-Dufresne, Johannes and Lochstoer, 2016; Ai, Li and Yang, 2020; Gârleanu and Panageas, 2020; Croce, Nguyen and Raymond, 2021). In closely related work, Kung and Schmid (2015) show that R&D endogenously generates long-run uncertainty in economic growth, serving as a source of long-run risk in asset pricing. Our model differs primarily by incorporating the misallocation of production capital across firms, which affects both aggregate total factor productivity (TFP) and total demand for intermediate inputs. Importantly, due to the interaction with financial frictions, the economic growth rate is not only reduced but also subject to greater volatility in times of economic downturns, which elevates the risk premium, thereby amplifying the effect of production capital misallocation on economic growth prospects through the valuation channel. This difference allows our theory to rationalize low-frequency growth fluctuations through the equilibrium interactions between endogenous slow-moving misallocation, marginal q of intangible capital, and R&D activities — a mechanism supported by the data.

Our paper is also closely related to the literature on financial frictions and misallocation. Most studies in this area focus on long-run TFP and welfare losses due to misallocation in the deterministic steady state (e.g., Amaral and Quintin, 2010; Greenwood, Sanchez and Wang, 2010, 2013; Caselli and Gennaioli, 2013; Midrigan and Xu, 2014; Buera, Kaboski and

Shin, 2015), while some also examine transitional dynamics (e.g., Jeong and Townsend, 2007; Buera and Shin, 2011, 2013; Moll, 2014; Buera and Moll, 2015; Gopinath et al., 2017; Itskhoki and Moll, 2019). Building on the model of Moll (2014), our paper develops a stochastic growth model in which misallocation evolves endogenously, is slow-moving, and drives low-frequency growth cycles, giving rise to systematic risk that shapes asset prices. The mechanism linking misallocation and growth is distinct from that of Acemoglu et al. (2018), who emphasize the misallocation of R&D inputs in determining equilibrium economic growth. In our model, it is the misallocation of production capital within the final goods sector, rather than within the R&D sector, that drives aggregate growth. Thus, our model shares similarities with Peters (2020), where firms' innovation rates are negatively impacted by the misallocation of labor for production. However, it differs from Peters (2020) in three key ways: (i) the source of misallocation in our model arises from financial frictions due to agency conflicts, rather than product market imperfections; (ii) our model focuses on the dynamics of stochastic growth, rather than deterministic steady-state growth; and (iii) our model highlights the valuation channel, a critical aspect absent in his framework. We demonstrate that with recursive preferences, misallocation caused by financial frictions can generate substantial risk premia and welfare losses through endogenous low-frequency growth fluctuations. Furthermore, the persistence of firm-level idiosyncratic productivity plays a crucial role in driving slow-moving misallocation, which, in turn, generates low-frequency growth fluctuations. By linking the persistence of idiosyncratic productivity to the persistence of aggregate consumption growth, our model suggests that low-frequency growth fluctuations can be identified using granular firm-level cross-sectional data. This approach addresses concerns about the "dark matter" in macro asset pricing models (Chen, Dou and Kogan, 2024; Cheng, Dou and Liao, 2022), enhancing their robustness. Our findings complement the key insight of Moll (2014), who shows that greater persistence in idiosyncratic productivity slows the transition from a distorted initial state to the steady state, leading to potentially large welfare losses during transitions.

Our paper is also related to the broad literature on the role of misallocation in shaping economic growth and development. In the context of economic growth, it connects to research by Banerjee and Duflo (2005), Jones (2013), Jovanovic (2014), Acemoglu et al. (2018), Peters (2020), and Glode and Ordonez (2023), among others. For economic development, it relates to studies such as Foster, Haltiwanger and Syverson (2008), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Jones (2011), Bartelsman, Haltiwanger and Scarpetta (2013), Asker, Collard-Wexler and Loecker (2014), David, Hopenhayn and Venkateswaran (2016), and David and Venkateswaran (2019). In contrast to the extensive growth and development literature on misallocation, relatively few studies in finance have focused on the role of misallocation. This gap presents a valuable opportunity to examine how misallocation interacts with financial market dynamics. Recent advances include Eisfeldt and Rampini (2006, 2008), Rampini and Viswanathan (2010), Fuchs, Green and Papanikolaou (2016), van Binsbergen and Opp (2019), Ai et al. (2020), and Ai, Li and Yang (2020), among others. Our paper is closely related to studies exploring the relationship between financial risk and misallocation. Di Tella, Maglieri and Tonetti (2024) analyze misallocation driven by heterogeneous markups resulting from endogenous risk premia in an economy with idiosyncratic risk and incomplete markets, focusing on optimal policy design. David, Schmid and Zeke (2022) study the effects of macroeconomic risk on misallocation using an exogenous SDF. In contrast to both studies, we examine the reverse relationship, analyzing the impact of misallocation on macroeconomic risk. Our model shows that misallocation can serve as a macroeconomic risk factor in asset pricing by influencing the investors' SDF through its effect on low-frequency consumption growth.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates model solutions and key mechanisms. Section 4 calibrates the model and undertakes quantitative analyses. Section 5 concludes. The internet appendix contains proofs of the theoretical results, additional empirical evidence, and assessment of the parametric approximation method. Moreover, a note on additional materials can be found in Dou et al. (2024), which is available on the authors' personal websites.

2 Model

There are three sectors: a final goods sector with heterogeneous firms, an intermediate goods sector, and an R&D sector. A representative agent owns firms across all these sectors.

2.1 Final Goods Sector

In the final goods sector, there is a continuum of firms of measure one, indexed by $i \in \mathcal{I} \equiv [0,1]$ and operated by managers. Firms are different from each other in their idiosyncratic productivity $z_{i,t}$ and capital $a_{i,t}$. The distribution of firms at t is characterized by the joint probability density function (PDF), $\varphi_t(a,z)$.

The firm produces output at intensity $y_{i,t}$ over $[t, t+\mathrm{d}t)$ using a technology with constant returns to scale (CRS):

$$y_{i,t} = \left[(z_{i,t}u_{i,t}k_{i,t})^{\alpha}\ell_{i,t}^{1-\alpha}\right]^{1-\varepsilon} x_{i,t}^{\varepsilon}, \quad \text{with } \alpha,\ \varepsilon \in (0,1), \tag{1}$$

where labor $\ell_{i,t}$ is hired in a competitive labor market at the equilibrium wage w_t. The variable $k_{i,t} = a_{i,t} + \widehat{a}_{i,t}$ is the total amount of capital installed in production, which includes the firm's own capital $a_{i,t}$ and the leased capital $\widehat{a}_{i,t}$ borrowed from a competitive rental market at the equilibrium risk-free rate $r_{f,t}$. The final goods are the numeraire.

As specified in (1), the firm's output $y_{i,t}$ increases with its idiosyncratic productivity $z_{i,t}$ and endogenous choice of capacity utilization intensity $u_{i,t} \in [0,1]$. Utilizing capital at intensity $u_{i,t}$ leads to depreciation of $u_{i,t}k_{i,t}\mathrm{d}\Delta_t$ over $[t, t+\mathrm{d}t)$. In this expression, $\mathrm{d}\Delta_t = \delta \mathrm{d}t + \sigma \mathrm{d}W_t$ represents the stochastic depreciation rate, where δ and σ are positive constants. In our framework, the standard Brownian motion, denoted by W_t, is employed to represent the aggregate capital quality shock, consistent with the established literature (e.g., Gertler and Kiyotaki, 2010; Gourio, 2012; Brunnermeier and Sannikov, 2017). These shocks are interpreted as liquidity shocks, particularly when firms use capital as collateral for borrowing.

The firm's own capital stock evolves according to

$$\mathrm{d}a_{i,t} = \mathrm{d}I_{i,t} - \delta a_{i,t}\mathrm{d}t + \sigma a_{i,t}\mathrm{d}W_t, \tag{2}$$

where $\mathrm{d}I_{i,t}$ denotes the firm's investment over $[t, t+\mathrm{d}t)$, with its modeling detailed in (16).

The composite $x_{i,t}$ in (1) consists of differentiated intermediate goods, given by the constant elasticity of substitution (CES) aggregation, $x_{i,t} = \left(\int_0^{N_t} x_{i,j,t}^{\nu}\mathrm{d}j\right)^{1/\nu}$, where $x_{i,j,t}$ is the quantity of intermediate goods $j \in [0, N_t]$. The elasticity of substitution among intermediate goods is $1/(1-\nu) > 0$. At any time t, the economy's stock of knowledge, encapsulated in the variety of intermediate goods, is quantified as N_t. It is through the expansion of N_t that technological advances occur and drive economic growth.

The firm's idiosyncratic productivity $z_{i,t}$ evolves according to

$$\mathrm{d}\ln z_{i,t} = -\theta \ln z_{i,t}\mathrm{d}t + \sigma_z\sqrt{\theta}\mathrm{d}W_{i,t}, \tag{3}$$

where the standard Brownian motion $W_{i,t}$ captures idiosyncratic productivity shocks, and θ and σ_z are parameters governing the persistence and volatility of $z_{i,t}$.

2.2 Intermediate Goods Sector

There is a continuum of homogeneous intermediate goods producers indexed by $j \in [0, N_t]$. Each producer j holds monopoly power in pricing a specific type of intermediate goods, a power that is guaranteed by the blueprint obtained from the R&D sector. These producers purchase final goods and convert them into intermediate goods following the blueprints. In this process, one unit of final goods is utilized to produce one unit of intermediate goods. Let $p_{j,t}$ denote the price of intermediate good j. The producer solves the following problem to maximize monopoly profit:

$$\pi_{j,t} = \max_{p_{j,t}} \quad p_{j,t} e_{j,t} - e_{j,t}, \tag{4}$$

subject to the downward-sloping demand curve:

$$e_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{\frac{1}{\nu-1}} X_t, \;\; \text{with } p_t = \left(\int_0^{N_t} p_{j,t}^{\frac{\nu}{\nu-1}} \mathrm{d}j \right)^{\frac{\nu-1}{\nu}}, \tag{5}$$

where $X_t \equiv \int_{i \in \mathcal{I}} x_{i,t} \mathrm{d}i$ is the aggregate demand for the composite of intermediate goods.

Let $q_{j,t}$ be the value of owning the exclusive rights to produce intermediate good j. Because intermediate goods producers are homogeneous, in a symmetric equilibrium, it must hold that $q_{j,t} \equiv q_t$ and $\pi_{j,t} \equiv \pi_t$, for all producers $j \in [0, N_t]$. Intermediate goods producers, while engaging in monopolistic competition in the intermediate goods market dealing with final goods producers, operate under perfect competition in the blueprint market with innovators. As a result, the price of a blueprint, q_t, equates to the present value of future monopoly rents that a blueprint can generate, discounted by the SDF of the representative agent. Thus, the value of q_t satisfies the Hamilton-Jacobi-Bellman equation:

$$0 = \Lambda_t \left(\pi_t - \delta_b q_t \right) \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t q_t) \right], \tag{6}$$

where Λ_t is the representative agent's SDF, as specified in (12); δ_b is the patent obsolescence rate; and q_t can be interpreted as the marginal q of intangible capital in the economy.

Intuitively, equation (6) indicates that the value of q_t is determined by both time-varying profits π_t and SDF Λ_t, both of which are, in turn, determined by the capital allocation efficiency within the final goods sector in general equilibrium. In Section 3.5, we show that variations in Λ_t result in large variations in the marginal q of intangible capital through the valuation channel, significantly amplifying the impact of capital misallocation on economic growth prospects.

2.3 R&D Sector

Innovators in the model are atomistic. Each one is capable of inventing a single blueprint through an R&D experiment over $[t, t + \mathrm{d}t)$ with a success rate $\vartheta_t > 0$. Each R&D experiment requires the use of final goods as R&D expenditure with unity intensity over $[t, t + \mathrm{d}t)$.

Each innovator in the model can optimally decide to engage in an R&D experiment without incurring any entry costs. Let S_t represent the total number of innovators who choose to participate over $[t, t + \mathrm{d}t)$. As a result, the total number of newly created blueprints over $[t, t + \mathrm{d}t)$ is given by $\vartheta_t S_t \mathrm{d}t$, which contributes to the evolution of the aggregate knowledge stock, N_t, as follows:

$$\mathrm{d}N_t = \vartheta_t S_t \mathrm{d}t - \delta_b N_t \mathrm{d}t. \tag{7}$$

Importantly, the success rate of R&D experiments, ϑ_t, is influenced by both the aggregate stock of knowledge N_t and the total R&D expenditure S_t. In line with Comin and Gertler (2006), we model the success rate as $\vartheta_t = \chi \left(N_t/S_t\right)^h$, where $h \in (0, 1)$. This formulation captures the positive spillover effect of the aggregate knowledge stock, N_t, as emphasized by Romer (1990), and the congestion or competition effect of the total R&D activities, S_t, on the success rate.[3]

In equilibrium, the free-entry condition dictates that the expected return from R&D for the marginal innovator choosing to engage in an R&D experiment must be equal to the expenditure incurred for the R&D experiment. This implies that

$$q_t \vartheta_t = 1. \tag{8}$$

The free-entry condition implies an investment-q relation for intangible capital at the aggregate level (e.g., Peters and Taylor, 2017; Crouzet and Eberly, 2023) as follows:

$$q_t = \chi^{-1} \left(S_t/N_t\right)^h. \tag{9}$$

Intuitively, equation (9) indicates that a higher q_t increases the total R&D expenditure S_t relative to aggregate knowledge stock N_t, thereby boosting the economy's growth rate.

2.4 Agents

There is a continuum of agents, including workers and managers. Each manager operates a firm in the final goods sector that is subject to agency problems.[4] Workers in the model lend funds to firms and additionally hold equity claims on all of them. We assume the existence of a complete set of Arrow-Debreu securities, allowing agents to fully insure against idiosyncratic consumption risks, ensuring the existence of a representative agent.

[3]The production of non-rival knowledge stock through R&D is the core engine of long-run growth (Romer, 1986, 1990). Recently, Crouzet et al. (2022) develop a model to show that the degree of nonrivalry in intangible capital has non-monotonic effects on growth.

[4]Managers, including executives, directors, entrepreneurs, and, more broadly, controlling shareholders, exercise control over firms' investment and payout policies (e.g., Albuquerue and Wang, 2008).

The aggregate labor supply is inelastic and normalized to be $L_t \equiv 1$.

Preferences. The representative agent has stochastic differential utility as in Duffie and Epstein (1992):

$$U_0 = \mathbb{E}_0 \left[\int_0^\infty f(C_t, U_t) \mathrm{d}t \right], \tag{10}$$

where

$$f(C_t, U_t) = \left(\frac{1-\gamma}{1-\psi^{-1}} \right) U_t \left[\left(\frac{C_t}{[(1-\gamma)U_t]^{1/(1-\gamma)}} \right)^{1-\psi^{-1}} - \rho \right]. \tag{11}$$

This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The felicity function f is an aggregator over the current consumption rate C_t of final goods and future utility level U_t. The coefficient ρ is the subjective discount rate, the parameter ψ is the elasticity of intertemporal substitution (EIS), and the parameter γ captures risk aversion.

The representative agent's SDF is

$$\Lambda_t = \exp \left[\int_0^t f_U(C_s, U_s) ds \right] f_C(C_t, U_t). \tag{12}$$

Limited Enforcement. Constraints in the equity market for payouts/issuances and in the credit market for borrowing emerge endogenously due to limited enforcement problems associated with equity and debt contracts.

Manager i extracts pecuniary rents $\tau a_{i,t} \mathrm{d}t$ over $[t, t + \mathrm{d}t)$ while running firm i.[5] Shareholders have the option to intervene and take control of the firm by replacing the manager. However, this intervention is costly due to the need for collective action, as noted by Myers (2000), and it can also damage the firm's talent-dependent customer capital, as detailed in Dou et al. (2021). In particular, we assume that upon shareholder intervention, a fraction τ/ϖ of the capital $a_{i,t}$ is lost, with $\tau < \varpi$, and the shareholders then become the firm's new manager. In equilibrium, to prevent such an intervention, the manager optimally pays out dividends at the minimum amount necessary to dissuade shareholders from intervening. This leads to a payout intensity policy of $d_{i,t} = \varpi a_{i,t}$ over $[t, t + \mathrm{d}t)$.[6]

[5] Managers can extract rents due to imperfections in corporate governance. Preventing them from diverting cash flows for private benefit remains challenging for shareholders, even when cash flows are observable and property rights to firm assets are protected. Consistent with the corporate finance literature (e.g., Myers, 2000; Lambrecht and Myers, 2008, 2012), we conceptualize rents primarily as cash compensation. However, managerial rents can also take various forms, including above-market salaries, generous pensions, perks, and enhanced job security.

[6] Technically, since the dividend intensity is a constant fraction of the firm's capital, the model has linear solutions and tractable aggregation. A similar feature is observed in the model of Moll (2014), which results

Moreover, the manager can divert a fraction $1/\lambda$ of leased capital $\widehat{a}_{i,t}$ with $\lambda \geq 1$. As a punishment, the firm would lose its own capital $a_{i,t}$. In equilibrium, the manager is able to borrow up to the point where he has no incentive to divert leased capital, implying a collateral constraint of $\widehat{a}_{i,t} \leq \lambda a_{i,t}$, as in Buera and Shin (2013) and Moll (2014).

The financial frictions described above are formally encapsulated in the following proposition, with its proof provided in Online Appendix 4.1.

Proposition 1. *Because of the agency problem with limited enforcement, the firm's payout/issuance policy is subject to the following equity market constraint:*

$$d_{i,t} = \varpi a_{i,t}. \tag{13}$$

Moreover, the firm's leased capital is subject to the following collateral constraint:

$$-a_{i,t} \leq \widehat{a}_{i,t} \leq \lambda a_{i,t}. \tag{14}$$

Several points are worth further discussion. First, there are other agency problems that can lead to the equity market and collateral constraints (e.g., Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Second, the equity market constraint, widely studied in the corporate finance literature (e.g., Myers, 2000; Lambrecht and Myers, 2008, 2012), essentially means that firms cannot freely move funds in and out of themselves. Third, our model's formulation of capital market imperfections, which is analytically tractable, captures the fact that firms face restrictions and costs in accessing external funds. Fourth, one specific interpretation of interfirm borrowing and lending is through a competitive rental market, where firms can rent capital from each other (e.g., Jorgenson, 1963; Hall and Jorgenson, 1969; Buera and Shin, 2013; Moll, 2014).

Managers' Problem. The manager of firm i makes decisions for all $s \geq t$ to maximize the present value $J_{i,t}$ of future managerial rents, as in Lambrecht and Myers (2008, 2012),

$$J_{i,t} = \max_{\widehat{a}_{i,s}, u_{i,s}, \ell_{i,s}, \{x_{i,j,s}\}_{j=0}^{N_t}} \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau a_{i,s} \mathrm{d}s \right], \tag{15}$$

subject to the equity market constraint (13), the collateral constraint (14), and the intertemporal budget constraint (2) with $\mathrm{d}I_{i,t}$ given by

$$\mathrm{d}I_{i,t} = y_{i,t}\mathrm{d}t - \int_0^{N_t} p_{j,t} x_{i,j,t} \mathrm{d}j\mathrm{d}t - w_t \ell_{i,t}\mathrm{d}t - u_{i,t} k_{i,t} \mathrm{d}\Delta_t - r_{f,t} \widehat{a}_{i,t}\mathrm{d}t - d_{i,t}\mathrm{d}t, \tag{16}$$

from the logarithmic preferences of entrepreneurs and the presence of CRS technology.

where profits are reinvested, similar to Pástor and Veronesi (2012).

By exploiting the homogeneity of $J_{i,t}$ in capital $a_{i,t}$, we derive the manager's optimal decisions, as summarized in Lemma 1, with the proof provided in Online Appendix 4.2.

Lemma 1. *There is a cutoff $\underline{z}_t$ for being active, and factor demands are linear in $k_t(a,z)$:*

$$u_t(z) = \begin{cases} 1, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases}, \qquad k_t(a,z) = \begin{cases} (1+\lambda)a, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases}, \tag{17}$$

$$\ell_t(a,z) = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} \varkappa_t^{\frac{1}{\alpha}} z u_t(z) k_t(a,z), \text{ and} \tag{18}$$

$$x_{j,t}(a,z) = \left(p_{j,t}/p_t\right)^{\frac{1}{\nu-1}} (\varepsilon/p_t)^{\frac{1-(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)}} \varkappa_t^{\frac{1-\alpha}{\alpha}} z u_t(z) k_t(a,z), \text{ for } j \in [0, N_t], \tag{19}$$

where $\varkappa_t = (1-\alpha)(1-\varepsilon)/w_t$. The productivity cutoff $\underline{z}_t$ is determined by:

$$\underline{z}_t \kappa_t = r_{f,t} + \delta + \sigma[\sigma_{\xi,t}(\underline{z}_t) - \eta_t], \text{ with } \kappa_t = \alpha(1-\varepsilon)\,(\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} \varkappa_t^{\frac{1-\alpha}{\alpha}}. \tag{20}$$

At time t, only firms with $z_{i,t} \geq \underline{z}_t$ produce, and these firms rent the maximal amount $\widehat{a}_{i,t} = \lambda a_{i,t}$ allowed by the collateral constraint. In equation (20), the cutoff $\underline{z}_t$ is determined such that the marginal return $\underline{z}_t \kappa_t$ is equal to the marginal cost of leased capital, which includes the locally deterministic user cost of capital $r_{f,t} + \delta$ and a stochastic term $\sigma\left[\sigma_{\xi,t}(\underline{z}_t) - \eta_t\right]$, reflecting the firm's exposure to aggregate risk (see Online Appendix 4.2).

Using Lemma 1, equation (16) can be simplified as[7]

$$\mathrm{d}I_{i,t} = (1+\lambda)\left(\kappa_t z_{i,t}\mathrm{d}t - \mathrm{d}\Delta_t - r_{f,t}\mathrm{d}t\right) a_{i,t} \mathbb{1}_{z_{i,t} \geq \underline{z}_t} + (r_{f,t} - \varpi) a_{i,t}\mathrm{d}t. \tag{21}$$

2.5 Equilibrium and Aggregation

Definition 2.1 (Competitive Equilibrium)**.** *At any given time t, the competitive equilibrium of the economy is defined by a set of prices w_t, $r_{f,t}$, and $\left\{p_{j,t}\right\}_{j=0}^{N_t}$, along with their corresponding quantities, such that*

(i) each firm i in the final goods sector maximizes (15) by choosing $\widehat{a}_{i,t}$, $u_{i,t}$, $\ell_{i,t}$, and $\{x_{i,j,t}\}_{j=0}^{N_t}$, subject to (13), (14), and (16), given the equilibrium prices;

(ii) each intermediate goods producer j maximizes (4) by choosing $p_{j,t}$ for $j \in [0, N_t]$;

(iii) the equilibrium R&D expenditure S_t is determined by (8);

[7] Similar to the equation in Moll (2014), the drift term in the capital accumulation equation is proportional to the firm's capital $a_{i,t}$. This relationship directly results from the constant payout ratio as specified in equation (13) and the CRS production technology, outlined in equation (1), given a specific N_t.

(iv) the SDF Λ_t is given by (12) and the risk-free rate $r_{f,t}$ is determined by

$$r_{f,t} = -\frac{1}{dt}\mathbb{E}_t\left[\frac{d\Lambda_t}{\Lambda_t}\right]; \tag{22}$$

(v) the labor market-clearing condition determines w_t:

$$L_t = \int_{\underline{z}_t}^{\infty}\int_0^{\infty} \ell_t(a,z)\varphi_t(a,z)dadz; \tag{23}$$

(vi) the leased capital market-clearing condition determines the representative agent's bond holdings B_t, which is the amount of capital lent to the final goods sector:

$$B_t = \int_0^{\infty}\int_0^{\infty} \widehat{a}_t(a,z)\varphi_t(a,z)dadz. \tag{24}$$

The aggregate capital K_t is given by

$$K_t = \int_0^{\infty}\int_0^{\infty} k_t(a,z)\varphi_t(a,z)dadz = A_t + B_t, \tag{25}$$

where A_t is the aggregate capital held by firms in the final goods sector, given by

$$A_t = \int_0^{\infty}\int_0^{\infty} a\varphi_t(a,z)dadz. \tag{26}$$

(vii) the resource constraint is satisfied because of Walras's law.

Because firms' problem is linear in capital $a_{i,t}$, we introduce the capital share $\omega_t(z)$ to fully characterize the distribution of firms in the final goods sector:

$$\omega_t(z) \equiv \frac{1}{A_t}\int_0^{\infty} a\varphi_t(a,z)\mathrm{d}a \;\text{ and }\; \Omega_t(z) \equiv \int_0^{z} \omega_t(z')\mathrm{d}z'. \tag{27}$$

Intuitively, the capital share $\omega_t(z)$ plays the role of a density and captures the share of firms' capital held by each productivity type z. The corresponding cumulative distribution function (CDF) is $\Omega_t(z)$.

Proposition 2. *At time $t \geq 0$, given $\omega_t(z)$, the equilibrium aggregate output is*

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{28}$$

where Z_t is the economy's TFP, given by

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} H_t N_t^{1-\alpha} \ \ with \ \ H_t = \left[\frac{1}{1-\Omega_t(\underline{z}_t)} \int_{\underline{z}_t}^{\infty} z\omega_t(z)dz\right]^{\alpha}, \tag{29}$$

where H_t captures the endogenous productivity of the final goods sector. Factor prices are

$$p_{j,t} = 1/\nu \ \ for \ \ j \in [0, N_t], \ \ p_t = N_t^{\frac{\nu-1}{\nu}}/\nu, \ \ and \ \ w_t = (1-\alpha)(1-\varepsilon)Y_t/L_t. \tag{30}$$

The aggregate profits of the intermediate goods sector and R&D expenditure are,

$$N_t\pi_t = (1-\nu)\varepsilon Y_t \ \ and \ \ S_t = (\chi q_t)^{\frac{1}{h}} N_t, \ respectively. \tag{31}$$

Equation (29) shows that TFP depends on both the knowledge stock N_t and the final goods sector's productivity H_t, which is the average firm-level productivity z weighted by $\omega_t(z)$.[8] The value of H_t is higher when more productive firms are associated with more capital, which reflects a more efficient allocation of capital across firms.

3 Model Solution and Mechanism

In this section, we present a parametric approximation of the firm distribution and characterize the mechanism that links misallocation and growth.

3.1 Parametric Approximation: Misallocation as a State Variable

The model is not analytically tractable due to the simultaneous presence of aggregate shocks and forward-looking heterogeneous firms. The key challenge lies in tracking the cross-sectional distribution of capital share, $\omega_t(z)$, an infinite-dimensional object that evolves endogenously. A standard approach to solving such a model involves using numerical approximation methods that specify a few moments to approximate $\omega_t(z)$ (e.g., Krusell and Smith, 1998). Instead of adopting these methods, we propose a parametric approximation of $\omega_t(z)$. Our method shares a similar philosophy with standard numerical approximation methods in that it uses a small number of moments to encapsulate the infinite-dimensional cross-sectional distribution of firms. The key distinction, however, lies in our approach's

[8]Equation (29) is related to the industry-level TFP formula derived by Hsieh and Klenow (2009). The key difference is that in our model, firms in the final goods sector produce homogeneous goods, whereas firms in the model of Hsieh and Klenow (2009) produce differentiated goods. In Online Appendix 5, we show that by driving the elasticity of substitution among goods to infinity and wedges to 0, the industry-level TFP formula of Hsieh and Klenow (2009) coincides with our productivity H_t in equation (29).

direct application of a parametric functional form to delineate the distribution at any given time. This approach enables us to derive closed-form equations for the evolution of these moments.

Our proposed analytical approximation serves three purposes. First, it yields a simple endogenous state variable that intuitively captures the misallocation of capital in the final goods sector. Second, it enables us to clearly illustrate the relationship between misallocation dynamics and aggregate growth dynamics, thereby making it easier to demonstrate the pivotal mechanism linking production capital misallocation with the low-frequency component of economic growth.[9] Third, it facilitates an analytical characterization of the model economy's evolution, rendering the computation of model dynamics highly tractable.

Specifically, at any time $t \geq 0$, we approximate the distribution of log capital $\widetilde{a}_{i,t} = \ln a_{i,t}$ and log productivity $\widetilde{z}_{i,t} = \ln z_{i,t}$ across firms in the final goods sector using a bivariate normal distribution. This assumption is similar in spirit to the bivariate log-normal distribution of the skills of matched young and old agents in the model of Jovanovic (2014). With this parametric assumption, Jovanovic (2014) derives analytical transitional dynamics to cleanly characterize the link between misallocation in the labor market and growth.

The approximation is intuitive because according to equation (3), we have $\widetilde{z}_{i,t} \sim N(0, \sigma_z^2/2)$ in the cross section of firms. Moreover, using the Berry-Esseen bound, we can prove that $\widetilde{a}_{i,t}$ across firms approximately follows a normal distribution on the deterministic balanced growth path without aggregate shocks (see Online Appendix 6). In Section 4.6, we further assess the accuracy of our parametric approximation by comparing our solutions to those obtained using standard global solution methods based on numerical approximations and show that the two sets of solutions are quite similar to each other under the baseline calibration. The joint log-normal approximation enables us to derive a closed-form formula for $\omega_t(z)$.

Proposition 3. *For any $t \geq 0$, the capital share $\omega_t(z)$ can be approximated by the PDF of a log-normal distribution,*

$$\omega_t(z) = \frac{1}{z\sigma_z\sqrt{\pi}} \exp\left[-\frac{(\ln z + \sigma_z^2 M_t/2)^2}{\sigma_z^2}\right], \tag{32}$$

where $M_t \equiv -Cov_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}_t(\widetilde{z}_{i,t}) = -2Cov_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\sigma_z^2$.

[9] The use of tractable parametric approximations to capture key model mechanisms shares similarities in spirit with several influential works in the finance literature. For example, Campbell and Shiller (1988) and Campbell and Vuolteenaho (2004) use log-linear present value approximations to disentangle the effects of discount-rate news and cash-flow news on stock valuations. Likewise, Gabaix (2012) develops the class of "linearity-generating" processes to improve analytical tractability in addressing macro-finance puzzles.

Intuitively, Proposition 3 implies that under our approximation, the endogenous state variable $M_t \equiv -\text{Cov}_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\text{var}_t(\widetilde{z}_{i,t})$ is a sufficient statistic that characterizes the evolution of $\omega_t(z)$. The parametric functional form (32) for $\omega_t(z)$ coincides with the initial wealth shares specified in equation (29) of Moll (2014) for conducting transition experiments. As noted by Moll (2014), M_t essentially captures the allocation efficiency of production capital. Below, we provide further discussions on the role of M_t and its relation to existing empirical measures of misallocation.

We characterize the economy's TFP Z_t in closed form, as follows.

Proposition 4. *Under our approximation, the aggregate TFP Z_t is*

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} N_t^{1-\alpha} \left[(1+\lambda)\frac{A_t}{K_t} \exp\left(-\frac{\sigma_z^2}{2}M_t + \frac{\sigma_z^2}{4}\right) \Phi\left(\Phi^{-1}\left(\frac{1}{1+\lambda}\frac{K_t}{A_t}\right) + \frac{\sigma_z}{\sqrt{2}}\right)\right]^{\alpha}, \quad (33)$$

where $\Phi(\cdot)$ represents the CDF of a standard normal variable.

Equation (33) shows that the economy's TFP, Z_t, strictly decreases with the endogenous state variable M_t, holding aggregate variables A_t, K_t, and N_t fixed. Thus, M_t reflects the degree of misallocation in our model economy. In fact, M_t also directly reflects the distribution of MRPK. To elaborate, substituting out labor and intermediate inputs in firms' technology using Lemma 1, we obtain

$$y_{i,t} = v_{i,t}k_{i,t}, \quad \text{with} \quad v_{i,t} = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} \varkappa_t^{\frac{1-\alpha}{\alpha}} z_{i,t}. \quad (34)$$

Because final goods are the numeraire, $v_{i,t}$ measures firm i's MRPK at t. Define $\widetilde{v}_{i,t} = \ln v_{i,t}$. We obtain a theoretically motivated measure for misallocation:

$$M_t \equiv -\frac{\text{Cov}_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})}{\text{var}_t(\widetilde{z}_{i,t})} = -\frac{\text{Cov}_t(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})}{\text{var}_t(\widetilde{v}_{i,t})}, \quad \forall\ t \geq 0. \quad (35)$$

Intuitively, in our model, the covariance $\text{Cov}_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})$ is fundamentally akin to the covariance between MRPK and capital, $\text{Cov}_t(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})$, given that firms produce homogeneous goods using a CRS technology. A higher M_t reflects that firms with higher productivity ($z_{i,t}$) or MRPK ($v_{i,t}$) are linked to a lower level of production capital ($a_{i,t}$), which, according to Proposition 4, results in a diminished TFP.

Because firms have CRS technology, the dispersion in MRPK (i.e., $\sigma_t^2(\widetilde{v}_{i,t})$) remains constant at $\sigma_z^2/2$, rendering it an invalid metric for misallocation in the model economy here. In models where firms' revenue exhibits decreasing returns to scale, a positive relationship between $\sigma_t^2(\widetilde{v}_{i,t})$ and M_t can arise under general conditions, although it is not unconditionally guaranteed theoretically. For example, consider the models of Buera and Shin (2011, 2013), where misallocation arises from financial frictions similar to our framework. In their baseline calibration, the steady state satisfies $M_t < 0$, reflecting a positive covariance between MRPK and production capital across firms. With the distribution of capital unchanged, tighter funding liquidity constraints increase both $\sigma_t^2(\widetilde{v}_{i,t})$ and M_t. Similarly, in the model of David, Hopenhayn and Venkateswaran (2016), higher information frictions exacerbate misallocation, as reflected in increases in both $\sigma_t^2(\widetilde{v}_{i,t})$ and M_t.

We emphasize that misallocation in our model, represented by the endogenous state variable M_t, arises from firms' funding liquidity constraints due to financial frictions, as in the model of Moll (2014). Under our parametric approximation, M_t fully summarizes the firm distribution, $\omega_t(z)$. Consequently, the endogenous time variation in $\omega_t(z)$ in response to aggregate shocks is entirely captured by the time variation in M_t, which serves as a sufficient statistic, as characterized by equation (38) below.

Relation to Existing Empirical Measures of Misallocation. Our model-implied misallocation metric, M_t, is conceptually similar to the capital allocation efficiency measure based on the cross-sectional covariance between size and productivity, used in several seminal empirical studies (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013).[10] The covariance-based misallocation measure is highly intuitive and does not rely on specific functional form assumptions. Compared to dispersion-based measures, such as the dispersion of revenue TFP or MRPK (e.g., Foster, Haltiwanger and Syverson, 2008; Hsieh and Klenow, 2009), it offers several advantages for analyzing variations in misallocation over economic cycles. In particular, Bartelsman, Haltiwanger and Scarpetta (2013) provide evidence that the relationship between size and productivity across firms is more resilient to multiplicative measurement errors than dispersion-based misallocation measures. They argue that classical measurement errors in MRPK or productivity tend to inflate dispersion-based misallocation measures but leave covariance-based measures unaffected. Furthermore, the magnitude of these measurement errors varies over economic cycles, undermining the precision of dispersion-based measures in capturing time variations in misallocation. Similarly, Eisfeldt and Shi (2018) argue that the inherent noisiness

[10] Olley and Pakes (1996) decompose total productivity into the unweighted average of plant-level productivities and the cross-sectional covariance between productivity and output share, positing that this covariance captures capital allocation efficiency. A higher covariance implies that more productive firms account for a larger share of output. Bartelsman, Haltiwanger and Scarpetta (2009, 2013) extend this approach by analyzing the cross-sectional covariance between firm-level log productivity and size, where productivity is measured by physical TFP, revenue TFP, or labor productivity, and size is measured by output, revenue, or input. They demonstrate that the relationship between size and productivity holds consistently across these measures in a broad class of models.

of productivity dispersion measures limits their effectiveness in capturing business cycle variations in misallocation. This insight is particularly relevant to our research, as we focus on examining the implications of misallocation fluctuations rather than its level for growth fluctuations and asset pricing. Accordingly, in our empirical and quantitative analysis in Section 4.1, we construct a model-consistent covariance-based measure, M_t.

We emphasize that the contribution of this paper is not to propose a new measure of capital misallocation. Instead, it lies in showing that the endogenous state variable M_t, defined in equation (35), provides strong theoretical support for using the size and productivity covariance as a measure of capital allocation efficiency, a metric already widely used in the literature. Specifically, our model analytically demonstrates that a higher M_t (i.e., a lower covariance) reduces aggregate TFP (see equation (33)). Furthermore, under the parametric approximation of our model, M_t serves as a sufficient statistic summarizing the cross-sectional distribution of firms $\omega_t(z)$, underscoring the central role of production capital misallocation as an endogenous state variable mediating the interaction between macroeconomic and capital market dynamics.

3.2 Evolution of the Economy

Under the parametric approximation, the economy's transitional dynamics are characterized by the evolution of aggregate capital A_t in the final goods sector, the knowledge stock N_t, and misallocation M_t, as summarized in the proposition below.

Proposition 5. *Under our parametric approximation, for all $t \geq 0$, the economy is fully characterized by the evolution of A_t, N_t, and M_t, as follows*

$$dA_t = \left[\alpha(1-\varepsilon)Y_t - \delta K_t - r_{f,t}B_t - (\varpi + \delta)A_t\right] dt - \sigma B_t dW_t, \tag{36}$$

$$dN_t = \chi\left(\chi q_t\right)^{\frac{1-h}{h}} N_t dt - \delta_b N_t dt, \tag{37}$$

$$dM_t = -\theta M_t dt - \ Cov_t(\widetilde{z}_{i,t}, d\widetilde{a}_{i,t})/\mathrm{var}_t(\widetilde{z}_{i,t}), \tag{38}$$

where $K_t = (1+\lambda)\left[1 - \Omega_t(\underline{z}_t)\right] A_t$ and $B_t = K_t - A_t$.

Define $E_t = N_t/A_t$ as the knowledge stock-capital ratio. Because the economy is homogeneous of degree one in A_t, the state variables (A_t, N_t, M_t) can be reduced to (E_t, M_t). Equation (38) shows that the evolution of M_t depends on two terms. The first term $-\theta M_t dt$ is linked to the evolution of $z_{i,t}$ through the persistence parameter θ (see equation (3)). Intuitively, a higher θ implies less persistent $z_{i,t}$, causing misallocation $M_t = -\mathrm{Cov}_t(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}_t(\widetilde{z}_{i,t})$ to revert to its long-run mean more quickly, thereby reducing the persistence of M_t. The second term, $-\mathrm{Cov}_t(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})/\mathrm{var}_t(\widetilde{z}_{i,t})$, captures the effect of heterogeneous changes in $\widetilde{a}_{i,t}$, represented by $\mathrm{d}\widetilde{a}_{i,t}$ (as defined in equation (2)), across firms of different $z_{i,t}$ on misallocation M_t. A higher $\mathrm{Cov}_t(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ implies that more productive firms accumulate their capital at a higher rate, which reduces misallocation M_t. Under our parametric approximation, $\mathrm{Cov}_t(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ has a closed-form expression (see equation (IA.66) in Online Appendix 4.6), which reveals its negative dependence on the aggregate shock $\mathrm{d}W_t$. A positive shock ($\mathrm{d}W_t > 0$) increases the depreciation rate of capital $k_{i,t}$, reducing the capital accumulation of more productive firms (i.e., $z_{i,t} \geq \underline{z}_t$) but not that of less productive firms (i.e., $z_{i,t} < \underline{z}_t$), which do not produce (see equation (17)). As a result, a positive shock lowers $\mathrm{Cov}_t(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$, increases misallocation M_t, and reduces aggregate output and consumption. This dynamic highlights the countercyclical nature of M_t.

3.3 Deterministic Balanced Growth Path

To clearly illustrate the equilibrium relationship between misallocation and long-run growth, we characterize the economy's deterministic balanced growth path in the absence of aggregate shocks (i.e., $\mathrm{d}W_t \equiv 0$).

Proposition 6. *There is a deterministic balanced growth path on which $E_t \equiv E$, $M_t \equiv M$, and $H_t \equiv H$ are constant. The aggregate capital A_t, knowledge stock N_t, output Y_t, TFP Z_t, and consumption C_t grow at the same constant rate g, and their ratios are constant.*

The values of these variables and the growth rate g are determined by the system of equations presented in Online Appendix 4.7. We highlight that g is directly related to the marginal q of intangible capital as follows:

$$g = \chi(\chi q)^{\frac{1-h}{h}} - \delta_b. \tag{39}$$

The next proposition clearly shows that on the deterministic balanced growth path, there is a negative relationship between misallocation M and the marginal q of intangible capital.

Proposition 7. *Under our parametric approximation, the marginal q of intangible capital is negatively related to misallocation M on the deterministic balanced growth path:*

$$\begin{aligned} \ln q = &- \frac{\alpha\sigma_z^2}{2} M + \frac{\alpha\sigma_z^2}{4} + \ln\left[\frac{(1-\nu)\varepsilon(\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}}}{r_f + \delta_b}\right] + \alpha\ln(1+\lambda) - \alpha\ln E \\ &+ \alpha\ln\left[\Phi\left(\Phi^{-1}\left(\frac{K/A}{1+\lambda}\right) + \frac{\sigma_z}{\sqrt{2}}\right)\right], \end{aligned} \tag{40}$$

where K/A is the constant ratio of K_t to A_t on the deterministic balanced growth path.

3.4 Key Mechanism: Persistence of Misallocation and Growth

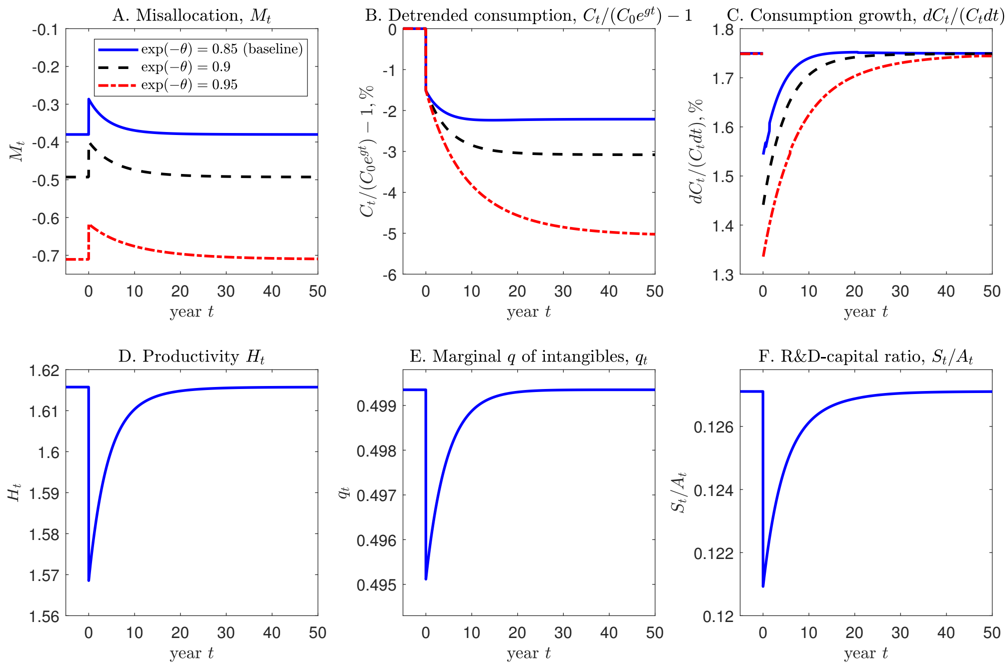
In this section, we focus on the deterministic balanced growth path to illustrate the model's core mechanism. We show that a one-time shock, increasing the misallocation level at $t = 0$, induces an endogenous and persistent effect on misallocation M_t from $t = 0$ onwards. This effect, in turn, triggers a long-lasting influence on aggregate growth by affecting the marginal q of intangible capital (see Proposition 7), and consequently, the R&D-capital ratio – a critical driver of economic growth. Moreover, we show that the persistence of aggregate growth depends on the persistence of misallocation, which depends largely on the persistence of idiosyncratic productivity.

Impulse Response Function. Consider a scenario involving a one-time, unexpected shock that exogenously increases misallocation M_t at $t = 0$.[11] From $t = 0$ onward, it will gradually converge back to the deterministic balanced growth path. The blue solid lines in Figure 2 illustrate the transitional dynamics of several key variables from $t = 0$ onward, based on our baseline calibration (see Table I). To render the quantitative effects informative, the magnitude of the shock is set to 0.09, aligning with the standard deviation of M_t in our calibration. As depicted in Panel A, misallocation M_t will experience an extended endogenous transitional period, lasting about 20 years, before it reaches the level on the deterministic balanced growth path.

In the absence of aggregate shocks, aggregate consumption would follow $C_0 e^{gt}$, growing at a constant annual rate of $g = 1.75\%$ for all $t \geq 0$. To focus on the change in growth rates relative to the deterministic trend in C_t, we consider detrended consumption, defined as $C_t/(C_0 e^{gt}) - 1$. The blue solid line in Panel B indicates that $C_t/(C_0 e^{gt}) - 1$ is 0 before the shock, jumps to approximately -1.6% at the moment the shock hits at $t = 0$, and gradually decreases until reaching the level on the deterministic balanced growth path. Although the shock to misallocation is transitory, the economy shifts to a steady state with permanently lower consumption, driven by the reduced accumulation speed of knowledge stock N_t. Panel C demonstrates a similar concept by displaying the contemporaneous consumption growth rate over the interval $[t, t + \mathrm{d}t)$, calculated as $\mathrm{d}C_t/(C_t \mathrm{d}t)$. The blue solid line illustrates that the consumption growth rate sharply decreases to about 1.55% at $t = 0$ and then slowly adjusts to the level on the deterministic balanced growth path as misallocation persists.

The mechanism connecting misallocation to growth is depicted by the arrows in Figure

[11] This unexpected shock to M_t in our experiment effectively represents a shock to the distribution of firms, $\omega_t(z)$, as M_t serves as a sufficient statistic fully characterizing $\omega_t(z)$ under our parametric approximation (see Proposition 3).



Note: Panels A, B, and C consider different calibrated values of θ. For each choice of θ, we recalibrate the parameter χ so that the consumption growth rate on the deterministic balanced growth path is the same as our baseline calibration. All other parameters are set according to our calibration in Table I. Panels D, E, and F focus on the baseline calibration with $e^{-\theta} = 0.85$.

Figure 2: Transitional dynamics after a one-time shock to misallocation M_t.

1. An increase in misallocation, M_t, directly reduces the productivity, H_t, of the final goods sector (see Panel D of Figure 2). A lower H_t reduces aggregate output, Y_t, which in turn reduces the marginal q of intangible capital (see Panel E of Figure 2 and equations (6) and (31)), leading less R&D activities (see Panel F of Figure 2). This chain of effects culminates in a lower growth rate via the reduced accumulation speed of knowledge stock, N_t.

Role of the Persistence of Idiosyncratic Productivity. As discussed above, it is the persistence of misallocation M_t, particularly through its impact on R&D, that drives the persistent excess consumption growth relative to the deterministic balanced growth path. As shown in equation (38), the persistence of misallocation depends on θ, which governs the persistence of $z_{i,t}$. To further illustrate the relationship between the persistence of misallocation and the persistence of aggregate consumption growth, we study the transitional dynamics under different values of θ. Specifically, according to equation (3), the yearly autocorrelation in $\ln z_{i,t}$ is $e^{-\theta}$. In Panels A, B and C of Figure 2, we compare our baseline calibration of $e^{-\theta} = 0.85$ with two alternative calibrations in which the yearly autocorrelation in $\ln z_{i,t}$ is 0.9 (dashed line) and 0.95 (dash-dotted line), respectively.

Panel A demonstrates that calibrations with a higher persistence of $z_{i,t}$ result in lower misallocation M_t on the deterministic balanced growth path, aligning with the insights provided by Buera and Shin (2011) and Moll (2014). Crucially, the convergence speed of M_t to its deterministic balanced growth path slows as the persistence of $z_{i,t}$ increases. As a measure to capture this phenomenon, we compute the half-life of transitions, which is the time it takes for M_t to revert to half of its long-term value post-shock. The half-life of M_t is 3.0, 4.1, and 6.7 years for $e^{-\theta} = 0.85$, 0.9, and 0.95, respectively, indicating that misallocation becomes more persistent when idiosyncratic productivity is more persistent. Comparing the three curves in Panels B and C, it is clear that the economy with a higher persistence of $z_{i,t}$ has more persistent consumption growth after the shock to M_t.

Thus, our model suggests that the persistence of $z_{i,t}$ plays an important role in determining the persistence of the growth rate of aggregate consumption, $\mathrm{d}C_t/(C_t\mathrm{d}t)$. The persistence levels of these two variables are connected via the persistent endogenous misallocation M_t. This result generalizes the key insight of Moll (2014) to an economy with stochastic growth. In a model without long-run growth or aggregate shocks, Moll (2014) shows that the transition to steady states slows down as idiosyncratic productivity becomes more persistent. Building on this insight, we additionally demonstrate that in a model with endogenous stochastic growth, the persistence of idiosyncratic productivity shapes the persistence of aggregate growth by affecting the persistence of endogenous misallocation. In Sections 4.4 and 4.5, we further show that the endogenous low-frequency component of growth fluctuations, driven by misallocation fluctuations, has first-order implications for asset prices and welfare.

3.5 Growth Fluctuations and Discount Rates

As illustrated by the arrows in Figure 1 and the impulse responses in Figure 2, on the deterministic balanced growth path without aggregate shocks, misallocation affects growth through its impact on the marginal q of intangible capital, which determines aggregate R&D expenditure. In the full model with aggregate shocks, the link between the marginal q of intangible capital and growth is amplified by countercylical discount rates (risk premia) through the valuation channel, as illustrated by Figure 1.

In our model, economic downturns are characterized by high misallocation M_t, during which high-productivity firms in the final goods sector face severe financial constraints due to insufficient capital, making their funding liquidity constraints more binding. Thus, aggregate output growth is not only low but also highly volatile during downturns, as financial constraints amplify the effects of aggregate liquidity shocks on percentage changes in output. Consequently, downturns are characterized by low expected long-term consumption growth and heightened macroeconomic uncertainty, leading to increased conditional volatility of the SDF and, therefore, a higher risk premium. Indeed, equations in (41) show that the conditional volatility of the SDF Λ_t is strongly positively correlated with misallocation M_t and negatively correlated with one-year expected consumption growth rate:

$$corr\left[\sigma_t(\Delta\widetilde{\Lambda}_{t+1}),\ M_t\right] = 0.93 \ \text{ and } \ corr\left[\sigma_t(\Delta\widetilde{\Lambda}_{t+1}), \mathbb{E}_t(\Delta\widetilde{C}_{t+1})\right] = -0.89, \tag{41}$$

where $\Delta\widetilde{X}_t \equiv \ln X_t - \ln X_{t-1}$ represents the difference in $\ln X_t$ between year t and year $t-1$, and the yearly value of X_t is computed by integrating $X_t\mathrm{d}t$ in continuous time.

Since the conditional volatility of the SDF Λ_t at t directly determines the market price of risk for the aggregate liquidity shock $\mathrm{d}W_t$ at t, the model generates countercylical risk premium. The countercyclical risk premium amplifies the variation in the marginal q of intangible capital, q_t. To see this, note that q_t is determined by equation (6). During downturns with high misallocation M_t, q_t is depressed not only because of reduced profits π_t but also because future profits are discounted at a higher discount rate, reflecting the market price of risk for the aggregate liquidity shock $\mathrm{d}W_t$. Conversely, during periods with low misallocation M_t, q_t increases both because of higher profits π_t and a lower discount rate. Together, these forces create significant fluctuations in q_t over economic cycles, which, in turn, lead to substantial variation in the low-frequency component of aggregate consumption growth rates through the effect of q_t on R&D expenditure (see equations (9) and (39)). This mechanism constitutes the valuation channel illustrated in Figure 1.

Quantitatively, more than half of the volatility of q_t is attributed to the countercyclical risk premium while the remaining is due to procycical profits π_t. Following the theoretical mechanism elaborated in Section 3.4, this valuation channel is quantitatively significant because the fluctuations in misallcocation driven by the aggregate shocks $\mathrm{d}W_t$ are persistent, when the parameter θ is calibrated to match the persistence of idiosyncratic productivity shocks in the data (see Table I). The slow-moving misallocation in turn generates low-frequency fluctuations in economic growth and macroeconomic uncertainty, thereby implying a high Sharpe ratio in the capital market (see Section 4.4) and significant welfare losses (see Section 4.5).

4 Quantitative Analysis

In this section, we analyze the quantitative effects of misallocation fluctuations, rather than its level, on low-frequency variations in economic growth, asset prices, and welfare.

Importantly, none of the quantitative implications depend on whether the model is solved using our parametric approximation or a standard global numerical approximation method. We solve the model using both approaches and obtain similar quantitative results. In Section 4.6, we systematically evaluate the accuracy of our parametric approximation for the model.

4.1 Data and Empirical Measures

We obtain annual consumption and GDP data from the U.S. Bureau of Economic Analysis (BEA) and stock return data from the Center for Research in Security Prices (CRSP). Output and consumption growth are measured by the log growth rate of per-capita real GDP and per-capita real personal consumption expenditures on nondurable goods and services. The nominal variables are converted to real terms using the consumer price index (CPI). We obtain data on private business R&D investment from the National Science Foundation (NSF) and on R&D stock from the Bureau of Labor Statistics (BLS). These two time series are considered empirical counterparts for S_t and N_t, respectively. The ratio of the two (i.e., S_t/N_t) is our empirical measure of R&D intensity. The risk-free rate is constructed using the yield of 3-month Treasury Bills, obtained from CRSP. Firms' dividend yield is computed as the ratio of total dividends over market capitalization, obtained from Compustat.

Model-Consistent Empirical Measure of Misallocation. We construct a model-consistent empirical measure of misallocation according to equation (35), $M_t = -\frac{\mathrm{Cov}_t(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})}{\mathrm{var}_t(\widetilde{v}_{i,t})}$. Specifically, we construct empirical measures of log capital $\widetilde{a}_{i,t}$ and log MRPK $\widetilde{v}_{i,t}$ (see Online Appendix 1) and run the following regression using the cross section of firms in each year t in U.S. Compustat data from 1965 to 2016:[12]

$$\widetilde{a}_{i,t} = \alpha_t + \beta_t \widetilde{v}_{i,t} + \varepsilon_{i,t}, \tag{42}$$

where the estimated coefficient $\widehat{\beta}_t$ directly captures $\mathrm{Cov}_t(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}_t(\widetilde{v}_{i,t})$.

The empirical measure of M_t is constructed using the filtered time series of $-\widehat{\beta}_t$ from 1965 to 2016. Following Comin and Gertler (2006), we apply a band-pass filter to extract frequencies up to 50 years, capturing the cyclical component of capital misallocation fluctuations corresponding to medium-term business cycles as defined by Comin and Gertler (2006).

[12]Because our theory mainly applies to manufacturing firms, we exclude firms from financial, utility, public administration, and non-tradable industries, where non-tradable industries are defined according to Mian and Sufi (2014). The empirical results are robust if non-tradable industries are included in the sample.

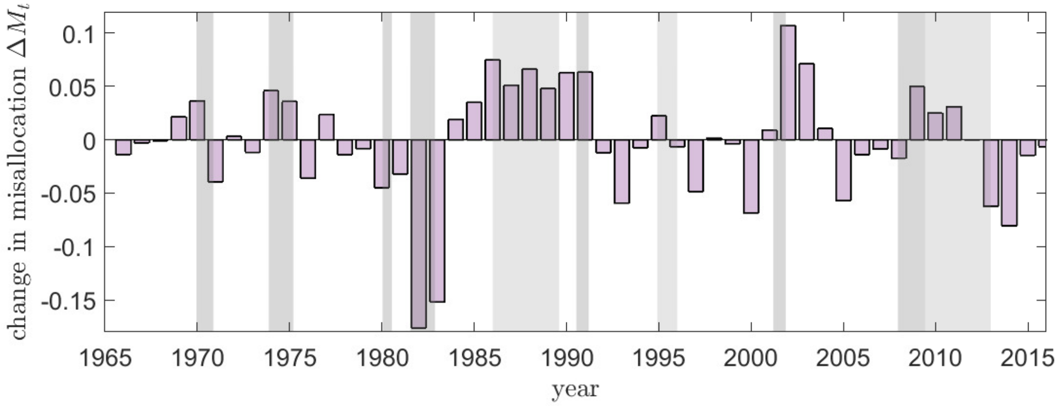


Figure 3: Time series of yearly changes in the empirical measure of misallocation, ΔM_t.

Figure 3 plots the time series of year-on-year changes in the empirical measure of misallocation, denoted as ΔM_t. The shaded areas represent periods of economic downturns, including economic recessions and three financial crises.[13] Aligned with our theoretical framework and empirical evidence from the literature, capital misallocation typically escalates during economic downturns. Our empirical measure of misallocation significantly increases in seven out of the nine economic downturns we examined. This stylized pattern is consistent with the model's prediction that misallocation typically increases during a period involving macroeconomic recessions or financial turmoil.

4.2 Calibration and Validation of the Model

Panel A of Table I presents the externally calibrated parameters. Following standard practice, we set the capital share in production technology at $\alpha = 0.33$. We set the capital depreciation rate at $\delta = 3\%$. We set the share of intermediate inputs at $\varepsilon = 0.5$ according to the choice of Comin and Gertler (2006) and Kung and Schmid (2015). The inverse markup is set at $\nu = \epsilon/[\epsilon + (1-\alpha)(1-\epsilon)] = 0.6$ to guarantee the existence of a balanced growth path.[14] Following standard practice in the asset pricing literature, we set risk aversion at $\gamma = 8$. Consistent with Kung and Schmid (2015), we set the EIS at $\psi = 1.85$, the patent obsolescence rate at $\delta_b = 15\%$, and $h = 0.17$ so that the elasticity of new blueprints with respect to R&D is 0.83. We set the volatility of idiosyncratic productivity $z_{i,t}$ at $\sigma_z = 1.39$

[13] The three crises are the savings and loan crisis from 1986 to 1989, the Mexican peso crisis from 1994 to 1995, and the European sovereign debt crisis from 2008 to 2012.

[14] This parametric restriction ensures that the deterministic balanced growth path, analyzed in Section 3.3, is well defined, consistent with the majority of the endogenous growth literature. Without this restriction, the economy could exhibit decreasing or increasing returns to scale, resulting in either zero growth or explosive growth rates in the long run. A similar parametric restriction is imposed in the model of Kung and Schmid (2015), for example.

Table I: Parameter calibration and targeted moments.

Panel A: Externally determined parameters

Parameter	Symbol	Value	Parameter	Symbol	Value
Capital share	α	0.33	Capital depreciation rate	δ	0.03
Share of intermediate inputs	ε	0.5	1− R&D elasticity	h	0.17
EIS	ψ	1.85	Risk aversion	γ	8
Patent obsolescence rate	δ_b	0.15	Volatility of idio. productivity	σ_z	1.39
Inverse markup	ν	0.6	Rent extraction rate	τ	0.01
Collateral constraint	λ	1.1	Persistence of idio. productivity	θ	0.1625

Panel B: Internally calibrated parameters and targeted moments

Parameter	Symbol	Value	Moments	Data	Model
Subjective discount rate	ρ	0.01	Real risk-free rate (%)	1.11	1.58
R&D productivity	χ	1.35	Consumption growth rate (%)	1.76	1.75
Volatility of aggregate shocks	σ	0.19	Consumption growth vol. (%)	1.50	1.67
Dividend payout rate	ϖ	0.037	Dividend yield (%)	2.35	2.14

according to the calibration of Moll (2014). The persistence of $z_{i,t}$ is set at $\theta = 0.1625$, which implies that $\ln z_{i,t}$ has a yearly autocorrelation of $e^{-\theta} = 0.85$, consistent with the estimate of Asker, Collard-Wexler and Loecker (2014) based on U.S. census data, as well as with the calibration in the macroeconomics literature (e.g., Moll, 2014). We set the collateral constraint parameter at $\lambda = 1.1$, which is within the range of calibration values in the macroeconomics literature (e.g., Jermann and Quadrini, 2012; Buera and Shin, 2013; Moll, 2014; Dabla-Norris et al., 2021). The rent extraction rate τ is a scaling parameter and normalized to 1%; its value does not affect firm decisions.

The remaining parameters are calibrated by matching the relevant moments summarized in Panel B of Table I. When constructing the model moments, we simulate a sample for $1,000$ years with a 100-year burn-in period, which is long enough to guarantee the stability of these moments. The discount rate is set at $\rho = 0.01$ to generate a real risk-free rate of 1.58%. R&D productivity is set at $\chi = 1.35$ to generate an average consumption growth rate of 1.75%. We calibrate $\sigma = 0.19$ so that the model-implied volatility of consumption growth is 1.67%, as in Storesletten, Telmer and Yaron (2007). We set the payout rate at $\varpi = 3.7\%$ so that the dividend yield is 2.14%.

Table II presents the untargeted moments. Panel A shows that the moments reflecting the persistence of consumption growth implied by the model are roughly consistent with those in the data, even though these moments are not directly targeted in our calibration. Panel B shows that the yearly autocorrelation of R&D expenditure growth $\Delta\widetilde{S}_t$ and misallocation

Table II: Untargeted moments in the data and model.

Moments	Data	Model	Moments	Data	Model
		Panel A: Consumption moments			
$AC1(\Delta\widetilde{C}_t)$ (%)	0.44	0.46	$AC2(\Delta\widetilde{C}_t)$ (%)	0.08	0.28
$AC5(\Delta\widetilde{C}_t)$ (%)	−0.01	0.00	$AC10(\Delta\widetilde{C}_t)$ (%)	0.06	−0.06
$VR2(\Delta\widetilde{C}_t)$ (%)	1.52	1.46	$VR5(\Delta\widetilde{C}_t)$ (%)	2.02	2.21
		Panel B: Other moments			
$AC1(\Delta\widetilde{S}_t)$ (%)	0.30	0.42	$AC1(M_t)$ (%)	0.84	0.75
$SR[R_{m,t}]$	0.36	0.39	$\sigma[r_{f,t}]$ (%)	2.06	0.47

Note: With slight abuse of notations, $\Delta\widetilde{X}_t = \ln X_t - \ln X_{t-1}$ represents the difference in $\ln X_t$ between year t and $t-1$, where the yearly value of X_t is computed by integrating $X_t \mathrm{d}t$ in continuous time. $ACk(\Delta\widetilde{C}_t)$ refers to the autocorrelation of log consumption growth with a k-year lag. $VRk(\Delta\widetilde{C}_t)$ refers to the variance ratio of log consumption growth with a k-year horizon. $AC1(\Delta\widetilde{S}_t)$ is the yearly autocorrelation of log private business R&D investment growth. $AC1(M_t)$ is the yearly autocorrelation of misallocation M_t. $SR[R_{m,t}] = \mathbb{E}[R_{m,t} - r_{f,t}]/\sigma[R_{m,t} - r_{f,t}]$ is the Sharpe ratio of the consumption claim.

M_t have comparable values in the model and data. The model implies a smooth risk-free rate and a high Sharpe ratio of the consumption claim, consistent with the Sharpe ratio of the market portfolio in our data sample.

4.3 Misallocation, R&D, and Growth

In this section, we show that misallocation M_t robustly captures low-frequency growth fluctuations in both the data and the model. Predictive regressions over long horizons confirm M_t's predictive power in tracking long-term growth trends. Additionally, in Online Appendix 3, we exploit industries' differential exposure to the policy shock from the American Jobs Creation Act (AJCA) using a difference-in-differences (DID) framework. This analysis provides direct causal evidence for the model's core mechanism, demonstrating that misallocation drives long-run growth through its impact on R&D investment.

In Panel A of Table III, we study the relationship between misallocation M_t and R&D intensity. In both the data and model (i.e., the simulated data), we regress R&D intensity in the current year (t) and the next year ($t+1$) on misallocation M_t, as follows:

$$\frac{S_{t+h}}{N_{t+h}} = \alpha + \beta M_t + \varepsilon_{t+h}, \quad \text{with } h = 0, 1. \tag{43}$$

The results indicate that higher misallocation is associated with a decline in contemporaneous R&D intensity and predicts a lower R&D intensity in the next year.

Table III: Misallocation, R&D, and growth in the data and model.

	Panel A: R&D intensity (S_t/N_t)			
	t		$t+1$	
	Data	Model	Data	Model
β	-0.090	-0.039	-0.088	-0.042
	(0.014)	(0.004)	(0.011)	(0.004)

	Panel B: Consumption growth ($\Delta\widetilde{C}_t$)									
	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	-0.048	-0.140	-0.085	-0.201	-0.116	-0.246	-0.141	-0.275	-0.157	-0.276
	(0.014)	(0.017)	(0.021)	(0.033)	(0.029)	(0.047)	(0.032)	(0.064)	(0.036)	(0.080)

	Panel C: Output growth ($\Delta\widetilde{Y}_t$)									
	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	-0.049	-0.109	-0.080	-0.243	-0.100	-0.218	-0.120	-0.225	-0.135	-0.233
	(0.024)	(0.032)	(0.038)	(0.037)	(0.049)	(0.054)	(0.053)	(0.064)	(0.059)	(0.075)

Note: The data sample is yearly and spans the period from 1965 to 2016. In the model, we simulate a sample of 52 years as in the data. Robust standard errors are reported in brackets.

Next, we examine whether misallocation M_t covaries with the slow-moving component of expected growth by testing whether misallocation negatively predicts future consumption growth in the data and model. We run the following regression:

$$\Delta\widetilde{C}_{t,t+1} + \cdots + \Delta\widetilde{C}_{t+h-1,t+h} = \alpha + \beta M_t + \varepsilon_{t,t+h}, \tag{44}$$

where $h = 1, \cdots, 5$ and $\Delta\widetilde{C}_{t+h-1,t+h} = \ln C_{t+h} - \ln C_{t+h-1}$ is the one-year log consumption growth from year $t+h-1$ to $t+h$. Panel B of Table III presents the results of projecting future consumption growth over horizons of 1 to 5 years on misallocation M_t. In both the data and model, the slope coefficients are negative and statistically significant. The coefficients are more negative for longer horizons because consumption growth is persistent. Our estimates indicate that misallocation M_t comoves with the slow-moving component of expected consumption growth. We further run regressions similar to (44) using future log output growth as the dependent variable. Panel C of Table III presents the results of projecting future output growth over horizons of 1 to 5 years on misallocation M_t. The patterns are similar to those of consumption growth in Panel B.

Taken together, we find empirical evidence that the aggregate growth rates of consumption and output can be predicted by our empirical measure of misallocation M_t, especially

Table IV: Asset pricing implications under different model specifications.

	(1) Baseline	(2) $\mathrm{d}N_t \equiv 0$	(3) $e^{-\theta}$	(4)	(5) CRRA ($\gamma = 1/\psi$)	(6)	(7) $M_t \equiv \mathbb{E}[M_t]$
			$= 0.2$	$= 0.45$	$= 1.5$	$= 3$	
$\mathbb{E}[R^e_{m,t}]$ (%)	0.54	0.02	0.01	0.08	0.02	0.02	0.02
$\sigma[R^e_{m,t}]$ (%)	1.39	0.72	1.17	1.09	1.01	0.57	0.31
$SR[R_{m,t}]$	0.39	0.02	0.01	0.08	0.02	0.04	0.06
$\mathbb{E}[r_{f,t}]$ (%)	1.58	0.98	1.93	1.88	3.60	6.17	1.78
$\sigma[r_{f,t}]$ (%)	0.47	0.34	0.33	0.41	0.47	0.57	0.02
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0.03	0.06	0.10	0.03	0.05	0.08

Note: In the table, $R^e_{m,t} = R_{m,t} - r_{f,t}$ is the consumption claim's return $R_{m,t}$ in excess of the risk-free rate $r_{f,t}$; $SR[R_{m,t}] = \mathbb{E}[R^e_{m,t}]/\sigma[R^e_{m,t}]$ is the Sharpe ratio of the consumption claim; and $\sigma[\Lambda_{t+1}/\Lambda_t]/\mathbb{E}[\Lambda_{t+1}/\Lambda_t]$ is the ratio of the volatility of 1-year SDF to its mean. Column (1) presents the results under the baseline calibration. In column (2), we adopt the same baseline calibration but eliminate the growth of knowledge stock N_t by imposing $\mathrm{d}N_t \equiv 0$ exogenously. In columns (3) and (4), we use alternative values of parameter θ. In columns (5) and (6), we impose $\gamma = 1/\psi$ and set different values of parameter γ. In column (7), we adopt the same baseline calibration but eliminate fluctuations in misallocation by imposing $M_t \equiv \mathbb{E}[M_t]$ exogenously. For columns (3) to (6), we calibrate χ and σ to generate the same model-implied average consumption growth rate and volatility as those reported in Panel B of Table I. Other parameters are set at the same values as the baseline calibration.

over long horizons. Our findings lend empirical support to the notion of misallocation-driven low-frequency growth fluctuations. In the simulated data, similar patterns are observed due to the mechanism elaborated in Section 3.4. Thus, our model helps rationalize and identify misallocation as an economic source of low-frequency growth fluctuations in the data.

4.4 Asset Pricing Implications of Misallocation

We now evaluate the asset pricing implications of misallocation. In Table IV, column (1) presents the implications in the baseline model. The aggregate consumption claim has a high Sharpe ratio of 0.39, which is similar to that of the market portfolio in the data. Because the model is calibrated to match an annualized volatility of consumption growth of 1.5%, the excess return of the consumption claim has an annualized volatility of only 1.39%. Thus, the average excess return is low due to low volatility. The risk-free rate has an average value of 1.58% and low volatility, as in the data. We also compute the ratio of the volatility of 1-year SDF to its mean, $\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$, which determines the maximal Sharpe ratio in the model. The baseline calibration implies a high value of 0.61.

Next, we study different model specifications. To study the role of economic growth, we consider a specification with no economic growth in column (2), setting $\mathrm{d}N_t \equiv 0$.[15] Compared with the baseline model in column (1), the volatility of the consumption claim's excess returns drops by about half, from 1.39% to 0.72%. The average excess return declines even more significantly, resulting in a Sharpe ratio of only 0.02.

In columns (3) and (4), we further show that fluctuations in economic growth are not sufficient to rationalize a high Sharpe ratio; it is important for misallocation fluctuations to generate low-frequency growth fluctuations. Specifically, following the insight illustrated in Figure 2, the persistence of idiosyncratic productivity determines the persistence of growth. We set $e^{-\theta}$ at 0.2 and 0.45 in columns (4) and (5), respectively, which results in a lower yearly autocorrelation of consumption growth than that in the baseline calibration, where $e^{-\theta} = 0.85$. Compared with column (1), the Sharpe ratio of the consumption claim drops significantly when idiosyncratic shocks are not persistent. These results highlight the importance of low-frequency growth fluctuations in amplifying the impacts of misallocation fluctuations on risk premia. Our findings complement the main insights of Buera and Shin (2011) and Moll (2014), who analyze the impacts of the persistence of idiosyncratic productivity on TFP, welfare, and the speed of transition through the self-financing channel.

In columns (5) and (6), we adopt a specification where the representative agent is characterized by preferences with constant relative risk aversion (CRRA), setting $\gamma = 1/\psi$. In this setup, the Sharpe ratio predicted by the model turns out to be notably low, whereas the risk-free rate is exceptionally high, a consequence of the low EIS. When considering a (non-recursive) CRRA preference structure, the valuation effects of low-frequency fluctuations in consumption growth are negligible. This occurs because the representative agent effectively prices the risk of the shock driving expected future consumption growth at zero.

Finally, in column (7), we exogenously fix misallocation M_t at its long-run mean $\mathbb{E}[M_t]$. The volatility of the consumption claim's excess returns falls to 0.31 and the Sharpe ratio drops to 0.06. This occurs because, within our model, the aggregate shock $\mathrm{d}W_t$ drives economic fluctuations mainly through its effect on M_t, while the aggregate knowledge stock-capital ratio E_t has small conditional volatility.[16]

[15] Under this specification, the economy's aggregate output and consumption still fluctuate due to aggregate shocks. However, there is no long-run growth as the average growth rates of Y_t and C_t are 0.

[16] This property differentiates our mechanism from those of Kaltenbrunner and Lochstoer (2010) and Kung and Schmid (2015), whose models generate low-frequency growth fluctuations through time-varying aggregate capital stock or R&D expenditure, rather than the covariance between capital and productivity across firms (i.e., M_t).

Table V: Welfare gains from removing consumption fluctuations.

	(1) Baseline	(2) $\mathrm{d}N_t \equiv 0$	(3) $e^{-\theta}$	(4)	(5) CRRA ($\gamma = 1/\psi$)	(6)
			$= 0.2$	$= 0.45$	$= 1.5$	$= 3$
Welfare gains (%)	10.34	0.33	0.24	0.98	0.58	0.65

Note: The specification in each column is described in Table IV. We focus on consumption-equivalent welfare. Specifically, we solve a similarly parameterized model without aggregate shocks (i.e., $\sigma = 0$). We compute the percentage change in lifetime consumption required to give the representative agent facing aggregate fluctuations the same expected lifetime utility as the representative agent on the deterministic balanced growth path without aggregate shocks. That is, we compute the percentage welfare gain, ζ, according to $U_0((1+\zeta)C_0) = \overline{U}_0(C_0)$, where $U_0(C_0)$ and $\overline{U}_0(C_0)$ represent the representative agent's utility at $t = 0$ in a model with and without aggregate shocks, respectively.

4.5 Welfare Costs of Misallocation-Driven Growth Fluctuations

In our model, consumption fluctuations are almost entirely driven by fluctuations in misallocation. Therefore, by evaluating the welfare costs associated with consumption fluctuations, we are able to offer a quantitative analysis of the welfare implications of misallocation-driven growth fluctuations within our theoretical framework. It is acknowledged that, in real-world scenarios, consumption fluctuations may result from a variety of aggregate variables. Bearing this in mind, our objective is not to precisely isolate the welfare costs directly attributable to misallocation fluctuations. Rather, we aim to demonstrate that fluctuations in misallocation have the potential to inflict significant welfare costs by causing consumption fluctuations, within a model that is calibrated to align with observed aggregate consumption moments (see Panel A of Table II).

Table V reports the results. Column (1) shows that the welfare gain from removing all consumption fluctuations is 10.34% under the baseline calibration. Moreover, in columns (2) through (6), we compute the welfare gains from removing consumption fluctuations under different specifications, similar to those in Table IV. Columns (2) through (4) show that the welfare gains will be small if misallocation cannot affect economic growth (i.e., setting $\mathrm{d}N_t \equiv 0$) or if misallocation is not persistent enough to generate low-frequency growth fluctuations (i.e., $e^{-\theta} = 0.2$ or $e^{-\theta} = 0.45$).[17] Columns (5) and (6) show that if the agent's preference is non-recursive (i.e., setting $\gamma = 1/\psi$), the welfare gains are also small.

[17] Columns (3) and (4) show that as idiosyncratic productivity becomes more persistent (i.e., higher $e^{-\theta}$), the welfare gain from removing consumption fluctuations increases. This finding is related to the key insight of Moll (2014), who shows that as the persistence of idiosyncratic productivity increases, the transition speed from a distorted initial state to the steady state slows down, resulting in potentially larger welfare losses during transitions. In our model with stochastic growth, the slow "transition" in response to aggregate shocks generates endogenous low-frequency growth fluctuations, which result in large welfare costs under the recursive preference of the representative agent.

Taken together, our findings suggest that the model posits significant welfare costs arising from misallocation-driven consumption fluctuations, attributable to a combination of two distinct properties. First, as elaborated in Section 3.4, the model is able to generate low-frequency growth fluctuations through slow-moving misallocation. Second, given the representative agent's recursive preferences, news about future consumption growth impacts his current marginal utility. As illustrated in Table IV, these two properties also allow the model to account for the observed high Sharpe ratio in the capital market. Within our model framework, there is a direct link between the welfare costs associated with consumption fluctuations and the Sharpe ratio observed in the capital markets. Intuitively, both metrics are elevated when variations in the representative agent's marginal utility in response to aggregate shocks are more pronounced. This connection is exploited by Alvarez and Jermann (2004) to estimate the welfare gains from eliminating all consumption fluctuations by directly applying the no-arbitrage principles on financial market data without specifying consumer preferences. We implement the method proposed by Alvarez and Jermann (2004) in our 1965-2016 sample and estimate that the welfare gain from eliminating all consumption fluctuations ranges from 6.03% to 23.97%, which nests the value implied by our structural model.[18]

The results in Tables IV and V show that misallocation-driven growth fluctuations can have significant implications for asset prices and welfare. As misallocation arises from firms' financial constraints in our model, our results are related to the literature on the connection between financial frictions and misallocation (e.g., Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014). A direct comparison of our model's quantitative implications with these models in the literature is difficult due to the differences in model setups. For example, our model involves stochastic growth driven by misallocation fluctuations, whereas these models quantify losses from misallocation in steady states or transitions, without aggregate shocks. In addition, although our model incorporates both the final goods and intermediate goods sectors, we only consider misallocation in the final goods sector.

Despite the differences in model setups, our findings in Table V are broadly consistent with the literature. For example, consistent with the calibration of Buera and Shin (2013) and Moll (2014), our calibration of large idiosyncratic shocks implies that firm-level productivity is not very persistent. As a result, purely through the variation in misallocation M_t, the model is able to generate a TFP volatility of 2.48%, as in the data. This result is consistent with the finding of Buera and Shin (2013) that misallocation resulting from financial frictions

[18] Alvarez and Jermann (2004) propose different estimation methods to demonstrate robustness. We use their first method, which projects consumption growth onto the payoff space spanned by a set of tradable assets.

can generate sizable TFP losses.[19]

While Buera and Shin (2013) focus on quantifying misallocation across the intensive margin (that is, differences in MRPK among active firms due to financial frictions), other research (e.g., Banerjee and Moll, 2010; Buera, Kaboski and Shin, 2011; Midrigan and Xu, 2014) underscores the significance of misallocation at the extensive margin (that is, productive firms may stay inactive or refrain from entering the market due to financial frictions). Depending on the calibration and model setup, Buera, Kaboski and Shin (2011) quantify that both extensive and intensive margins are important, whereas Midrigan and Xu (2014) estimate large TFP losses through the extensive margin rather than the intensive margin. In our model, misallocation due to financial frictions reduces the final goods sector's productivity H_t, which captures the intensive margin effect. A lower H_t, in turn, reduces the profits of innovators. Through the free-entry condition (8), this further leads to a lower growth rate of the variety of intermediate goods, $\mathrm{d}N_t/N_t$ (see equation (7)), which can be seen as capturing the extensive margin effect.[20] The results in column (2) of Tables IV and V indicate that the extensive margin plays a crucial role in rationalizing the high Sharpe ratio in the capital market and in generating a large welfare cost of misallocation-driven growth fluctuations. These findings support the significant role of extensive-margin misallocation quantified by Midrigan and Xu (2014).

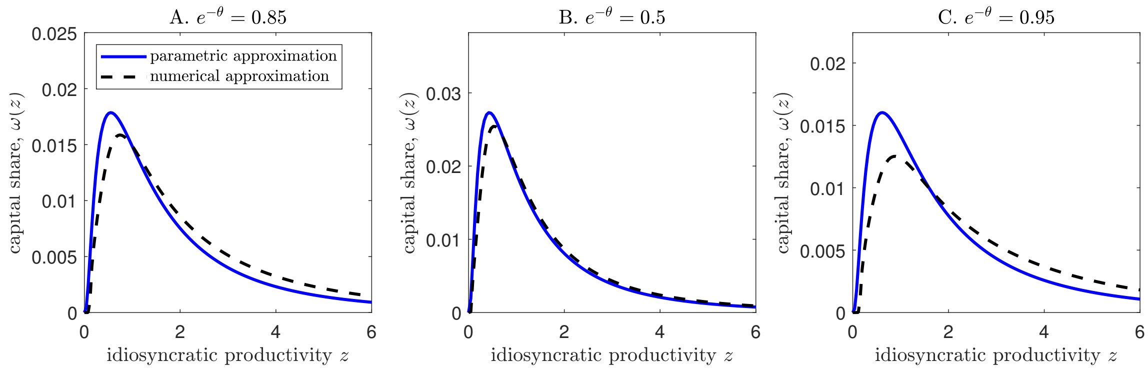
4.6 Assessing the Performance of Parametric Approximation Methods

In this section, we evaluate the accuracy of our parametric approximation by comparing the model solution with the solution of standard global numerical approximation methods. In particular, we solve the calibrated model using standard numerical methods by directly tracking the capital share $\omega_t(z)$ using a selected number of moments. We show that, under the baseline calibration, our solution closely matches those obtained through standard numerical approximation methods both on the deterministic balanced growth path and in the full model with aggregate shocks.

Deterministic Balanced Growth Path. We begin by evaluating the performance of our parametric approximation method on the deterministic balanced growth path without

[19]Several papers measure the importance of financing costs in generating misallocation. For example, Gilchrist, Sim and Zakrajsek (2013) find that the costs of debt play a limited role in generating misallocation based on a sample consisting of about 500 (mostly large) firms that issue corporate bonds. David, Schmid and Zeke (2022) find that the costs of equity are important in generating misallocation. Whited and Zhao (2021) find significant variations in the costs of debt and equity across U.S. firms.

[20]There is no misallocation along the intensive margin in the intermediate goods sector because all firms in this sector are homogeneous.



Note: This figure compares the capital share $\omega(z)$ on the deterministic balanced growth path solved by our parametric approximation method and that solved by the histogram-based numerical approximation method. All the parameter values are taken from the baseline calibration in Table I except for θ (consider three values, $\exp(-\theta) = 0.5, 0.85, 0.95$) and ψ (set its value to match a growth rate of 1.75% for corresponding θ).

Figure 4: Capital share distributions on the deterministic balanced growth path.

aggregate shocks. We analytically justify the validity of this approximation using the Berry-Esseen bound (Tikhomirov, 1980; Bentkus, Gotze and Tikhomoirov, 1997) in Online Appendix 6. Furthermore, we verify the validity of our parametric approximation method by comparing its results with those obtained from a global solution method. On the deterministic balanced growth path, the capital share distribution, $\omega(z)$, is time invariant. We solve the model numerically by approximating $\omega(z)$ non-parametrically using a fine histogram following the solution method of Moll (2014). To ensure accuracy, we use 251 equally spaced grids for idiosyncratic productivity z over the interval $[z_{min}, z_{max}]$, with $z_{min} = 0$ and $z_{max} = \exp(\Phi^{-1}(0.99)\sigma_z/\sqrt{2})$. The choice of z_{max} corresponds to the 99th percentile of the steady-state distribution of $z_{i,t}$. We discretize the time horizon using a short time period, $\Delta t = 1/200$, and verify that the solution does not change when finer grid points are chosen.

Figure 4 compares the capital share $\omega(z)$ on the deterministic balanced growth path solved by our parametric approximation method with that solved by the histogram-based numerical approximation method. Panel A shows that under the baseline calibration, the two methods produce similar solutions of $\omega(z)$. Compared with parametric approximation, numerical approximation generates a larger capital share at higher levels of z. Intuitively, this is because with a yearly autocorrelation of $\exp(-\theta) = 0.85$, idiosyncratic productivity z is persistent, allowing productive firms to accumulate significant amounts of capital in steady states. This results in a capital share with a fat right tail, which cannot be perfectly approximated by the log-normal density function under our parametric specification. Compared with the numerical approximation method with a fine histogram, this is the main approximation error produced by our parametric approximation method. In panels

Table VI: Key endogenous variables on the deterministic balanced growth path.

Variables	Parametric	Numerical	Variables	Parametric	Numerical
Firm profitability, κ	0.027	0.028	Wage-capital ratio, w/A	0.214	0.231
Productivity cutoff, $\underline{z}$	1.729	1.709	Dividend-capital ratio, D/A	0.038	0.039
Marginal q of intangible capital	0.473	0.473	R&D-capital ratio, S/A	0.127	0.137
Productivity, H	1.618	1.557	Growth rate, g (%)	1.750	1.709
Flow profit of innovators, π	0.080	0.080	Risk-free rate, r_f (%)	1.946	1.924

Note: The columns labeled “Parametric” and “Numerical” present the values of corresponding variables on the deterministic balanced growth path solved by our parametric approximation method and the histogram-based numerical approximation method, respectively. All the parameter values are set according to Table I.

B and C, we further compare $\omega(z)$ solved by the two methods under two alternative calibrations, with $\exp(-\theta) = 0.5$ and $\exp(-\theta) = 0.95$, respectively. It is clear that the $\omega(z)$ solved by the two methods are closer to each other when idiosyncratic productivity is less persistent (see panel B). By contrast, the $\omega(z)$ solved by the two methods diverge more significantly when idiosyncratic productivity become more persistent (see panel C).

Overall, Figure 4 shows that under our baseline calibration with $\exp(-\theta) = 0.85$, corresponding to the persistence of idiosyncratic productivity shock estimated by Asker, Collard-Wexler and Loecker (2014) based on U.S. census data, the parametric approximation method can capture the capital share $\omega(z)$ with sufficient accuracy. Table VI further shows that various key endogenous aggregate variables implied by the two solution methods have similar magnitudes under the baseline calibration.

Stochastic Steady State with Aggregate Shocks. On the deterministic balanced growth path, we theoretically establish the effectiveness of the parametric approximation method using the Berry-Esseen bound and verify its consistency with the solution obtained through the numerical approximation method. When aggregate shocks are incorporated, we expect the solutions from the parametric and numerical approximation methods to remain aligned, provided the shocks are Brownian in nature and moderate in magnitude. To validate this formally, we use standard global numerical approximation techniques to solve the full model with aggregate shocks under our baseline calibration. Following Krusell and Smith (1998), we address the challenge of the infinite dimensionality of the cross-sectional distribution by approximating it with a finite set of moments. Specifically, we use the first few moments to represent the firm distribution (see Online Appendix 7 for details) and solve the model using a globally accurate projection technique.

In Table VII, we compare the key variables solved by our baseline parametric approximation method and those solved by 2nd-, 3rd, and 4th-order numerical approximation

Table VII: Accuracy of our parametric approximation in stochastic steady states.

Variables	Parametric	Numerical		
		2nd-order	3rd-order	4th-order
Firm profitability, $\mathbb{E}[\kappa_t]$	0.027	0.027	0.027	0.027
Productivity cutoff, $\mathbb{E}[\ln(\underline{z}_t)]$	1.526	1.514	1.393	1.308
Marginal q of intangible capital, $\mathbb{E}[q_t]$	0.483	0.483	0.483	0.483
Productivity, $\mathbb{E}[\ln H_t]$	0.664	0.679	0.660	0.645
Flow profit of innovators, $\mathbb{E}[\pi_t]$	0.081	0.081	0.081	0.081
Wage-capital ratio, $\mathbb{E}[\ln(w_t/A_t)]$	−1.900	−1.901	−1.861	−1.834
Dividend-capital ratio, $\mathbb{E}[\ln(D_t/A_t)]$	−3.337	−3.334	−3.314	−3.303
R&D-capital ratio, $\mathbb{E}[\ln(S_t/A_t)]$	−2.412	−2.413	−2.376	−2.351
Consumption-capital ratio, $\mathbb{E}[\ln(C_t/A_t)]$	−1.696	−1.697	−1.659	−1.634
Knowledge stock-capital ratio, $\mathbb{E}[\ln(N_t/A_t)]$	0.104	0.103	0.140	0.165
Capital growth, $\mathbb{E}[\Delta\widetilde{A}_t]$ (%)	1.754	1.750	1.743	1.730
Consumption growth, $\mathbb{E}[\Delta\widetilde{C}_t]$ (%)	1.753	1.750	1.743	1.730
Volatility of consumption growth, $\text{var}[\Delta\widetilde{C}_t]$ (%)	1.668	1.671	1.492	1.358
Risk-free rate, $\mathbb{E}[r_{f,t}]$ (%)	1.580	1.590	1.697	1.745
Consumption claim's return, $\mathbb{E}[R_{m,t}]$ (%)	2.124	2.112	2.074	2.040

Note: The column labeled "Parametric" presents the values of the corresponding variables in the stochastic steady state, obtained using our parametric approximation method. The columns labeled "2nd-order," "3rd-order," and "4th-order" show the results for various orders of numerical approximations of the capital share distribution $\omega_t(z)$. All the parameter values are taken from our baseline calibration in Table I.

methods. The results of our parametric approximation method are very similar to the results of the 2nd-order numerical approximation method, because our parametric approximation method essentially keeps track of the first and second moments of $\omega_t(z)$.[21] Table VII also shows that the differences between our baseline parametric approximation method and the 4th-order numerical approximation method are generally within 10% for most variables. These results suggest that the model implications based on parametric approximation are quantitatively similar to those based on numerical approximation methods.

5 Conclusion

This paper develops an analytically tractable general equilibrium model with heterogeneous firms and endogenous stochastic growth to quantitatively explore the relationship between

[21]The first moment is $m_{1,t} = -M_t\sigma_z^2/2$ and the second moment is $m_{2,t} = \sigma_z^2/2$, which is a constant under our parametric approximation method. However, the results of our parametric approximation method do not exactly match the results of the 2nd-order numerical approximation method due to a subtle difference in implementation procedures (see footnote 9 in Online Appendix 7 for details).

misallocation, growth prospects, and the systematic risk that shapes asset prices in capital markets. In our model, increased misallocation reduces economic growth by depressing the marginal q of intangible capital and thus R&D incentives. Misallocation evolves slowly, leading to low-frequency fluctuations in economic growth. Central to this mechanism is the valuation channel, which significantly magnifies the effects of misallocation on economic growth. When agents have recursive preferences, the low-frequency growth fluctuations driven by slow-moving misallocation not only rationalize several crucial asset pricing moments but also suggest significant welfare costs associated with misallocation fluctuations.

In the data, we construct a misallocation measure motivated by our theory and provide supporting evidence for the model predictions. We show that the value of our empirical measure of misallocation is persistent and increases during economic downturns. Moreover, an increase in misallocation predicts declines in R&D intensity and reductions in the growth of aggregate consumption and output over long horizons.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. 2018. "Innovation, Reallocation, and Growth." *American Economic Review*, 108(11): 3450–3491.

Ai, Hengjie. 2010. "Information Quality and Long-Run Risk: Asset Pricing Implications." *Journal of Finance*, 65(4): 1333–1367.

Ai, Hengjie, Jun E Li, Kai Li, and Christian Schlag. 2020. "The Collateralizability Premium." *Review of Financial Studies*, 33(12): 5821–5855.

Ai, Hengjie, Kai Li, and Fang Yang. 2020. "Financial intermediation and capital reallocation." *Journal of Financial Economics*, 138(3): 663–686.

Albuquerue, Rui, and Neng Wang. 2008. "Agency Conflicts, Investment, and Asset Pricing." *Journal of Finance*, 63(1): 1–40.

Alvarez, Fernando, and Urban J. Jermann. 2004. "Using Asset Prices to Measure the Cost of Business Cycles." *Journal of Political Economy*, 112(6): 1223–1256.

Amaral, Pedro S., and Erwan Quintin. 2010. "Limited Enforcement, Financial Intermediation, And Economic Development: A Quantitative Assessment." *International Economic Review*, 51(3): 785–811.

Asker, John, Allan Collard-Wexler, and Jan De Loecker. 2014. "Dynamic Inputs and Resource (Mis)Allocation." *Journal of Political Economy*, 122(5): 1013–1063.

Banerjee, Abhijit V., and Benjamin Moll. 2010. "Why Does Misallocation Persist?" *American Economic Journal: Macroeconomics*, 2(1): 189–206.

Banerjee, Abhijit V., and Esther Duflo. 2005. "Growth Theory through the Lens of Development Economics." In *Handbook of Economic Growth*. Vol. 1 of *Handbook of Economic Growth*, , ed. Philippe Aghion and Steven Durlauf, Chapter 7, 473–552. Elsevier.

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance*, 59(4): 1481–1509.

Bansal, Ravi, Robert F. Dittmar, and Christian T. Lundblad. 2005. "Consumption, Dividends, and the Cross Section of Equity Returns." *The Journal of Finance*, 60(4): 1639–1672.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta. 2009. "Measuring and Analyzing Cross-country Differences in Firm Dynamics." In *Producer Dynamics: New Evidence from Micro Data*. *NBER Chapters*, 15–76. National Bureau of Economic Research, Inc.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta. 2013. "Cross-Country Differences in Productivity: The Role of Allocation and Selection." *American Economic Review*, 103(1): 305–334.

Bentkus, V., F. Gotze, and A. Tikhomoirov. 1997. "Berry-Esseen bounds for statistics of weakly dependent samples." *Bernoulli*, 3(3): 329–349.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2018. "Really Uncertain Business Cycles." *Econometrica*, 86(3): 1031–1065.

Brunnermeier, Markus K., and Yuliy Sannikov. 2017. "Macro, Money and Finance: A Continuous-Time Approach." In *Handbook of Macroeconomics*. 1497–1546. North-Holland.

Buera, Francisco J., and Benjamin Moll. 2015. "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity." *American Economic Journal: Macroeconomics*, 7(3): 1–42.

Buera, Francisco J., and Yongseok Shin. 2011. "Self-insurance vs. self-financing: A welfare analysis of the persistence of shocks." *Journal of Economic Theory*, 146(3): 845–862.

Buera, Francisco J., and Yongseok Shin. 2013. "Financial Frictions and the Persistence of History: A Quantitative Exploration." *Journal of Political Economy*, 121(2): 221–272.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review*, 101(5): 1964–2002.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2015. "Entrepreneurship and Financial Frictions: A Macrodevelopment Perspective." *Annual Review of Economics*, 7(1): 409–436.

Campbell, John Y., and Robert J. Shiller. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies*, 1(3): 195–228.

Campbell, John Y., and Tuomo Vuolteenaho. 2004. "Bad Beta, Good Beta." *The American Economic Review*, 94(5): 1249–1275.

Caselli, Francesco, and Nicola Gennaioli. 2013. "Dynastic Management." *Economic Inquiry*, 51(1): 971–996.

Cheng, Xu, Winston Wei Dou, and Zhipeng Liao. 2022. "Macro-Finance Decoupling: Robust Evaluations of Macro Asset Pricing Models." *Econometrica*, 90(2): 685–713.

Chen, Hui, Winston Wei Dou, and Leonid Kogan. 2024. "Measuring "Dark Matter" in Asset Pricing Models." *The Journal of Finance*, 79(2): 843–902.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer. 2016. "Parameter Learning in General Equilibrium: The Asset Pricing Implications." *American Economic Review*, 106(3): 664–98.

Comin, Diego, and Mark Gertler. 2006. "Medium-Term Business Cycles." *American Economic Review*, 96(3): 523–551.

Croce, Mariano Massimiliano. 2014. "Long-run productivity risk: A new hope for production-based asset pricing?" *Journal of Monetary Economics*, 66(C): 13–31.

Croce, Mariano Max, Thien T. Nguyen, and Steve Raymond. 2021. "Persistent Government Debt and Aggregate Risk Distribution." *Journal of Financial Economics*, forthcoming.

Crouzet, Nicolas, and Janice Eberly. 2023. "Rents and Intangible Capital: A Q+ Framework." *Journal of Finance*, 78(4): 1873–1916.

Crouzet, Nicolas, Janice Eberly, Andrea L. Eisfeldt, and Dimitris Papanikolaou. 2022. "Intangible capital, non-rivalry, and growth." Working Papers.

Dabla-Norris, Era, Yan Ji, Robert M. Townsend, and D. Filiz Unsal. 2021. "Distinguishing constraints on financial inclusion and their impact on GDP, TFP, and the distribution of income." *Journal of Monetary Economics*, 117(C): 1–18.

David, Joel M., and Venky Venkateswaran. 2019. "The Sources of Capital Misallocation." *American Economic Review*, 109(7): 2531–2567.

David, Joel M., Hugo A. Hopenhayn, and Venky Venkateswaran. 2016. "Information, Misallocation, and Aggregate Productivity." *The Quarterly Journal of Economics*, 131(2): 943–1005.

David, Joel M., Lukas Schmid, and David Zeke. 2022. "Risk-adjusted capital allocation and misallocation." *Journal of Financial Economics*, 145(3): 684–705.

Di Tella, Sebastian, Cedomir Maglieri, and Christopher Tonetti. 2024. "Risk Markups ." Stanford University Working Papers.

Dou, Winston, Yan Ji, David Reibstein, and Wei Wu. 2021. "Inalienable customer capital, corporate liquidity, and stock returns." *Journal of Finance*, 76(1): 211–265.

Dou, Winston, Yan Ji, Di Tian, and Pengfei Wang. 2024. "A Note on Additional Materials for 'Misallocation

and Asset Prices'."
Duffie, Darrell, and Larry G. Epstein. 1992. "Stochastic differential utility." *Econometrica*, 60(2): 353–94.
Eisfeldt, Andrea L., and Adriano A. Rampini. 2006. "Capital reallocation and liquidity." *Journal of Monetary Economics*, 53(3): 369–399.
Eisfeldt, Andrea L., and Adriano A. Rampini. 2008. "Managerial incentives, capital reallocation, and the business cycle." *Journal of Financial Economics*, 87(1): 177 – 199.
Eisfeldt, Andrea L., and Yu Shi. 2018. "Capital Reallocation." *Annual Review of Financial Economics*, 10: 361–386.
Epstein, Larry G., and Stanley E. Zin. 1989. "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework." *Econometrica*, 57(4): 937–969.
Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98(1): 394–425.
Fuchs, William, Brett Green, and Dimitris Papanikolaou. 2016. "Adverse selection, slow-moving capital, and misallocation." *Journal of Financial Economics*, 120(2): 286–308.
Gabaix, Xavier. 2012. "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance." *The Quarterly Journal of Economics*, 127(2): 645–700.
Gârleanu, Nicolae B., and Stavros Panageas. 2020. "Heterogeneity and Asset Prices: A Different Approach." National Bureau of Economic Research, Inc.
Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In . Vol. 3 of *Handbook of Monetary Economics*, , ed. Benjamin M. Friedman and Michael Woodford, 547 – 599. Elsevier.
Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58(1): 17 – 34.
Gilchrist, Simon, Jae W. Sim, and Egon Zakrajsek. 2013. "Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs." *Review of Economic Dynamics*, 16(1): 159–176.
Glode, Vincent, and Guillermo Ordonez. 2023. "Technological progress and rent seeking." University of Pennsylvania Working Paper.
Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez. 2017. "Capital Allocation and Productivity in South Europe." *The Quarterly Journal of Economics*, 132(4): 1915–1967.
Gourio, Francois. 2012. "Disaster Risk and Business Cycles." *American Economic Review*, 102(6): 2734–2766.
Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang. 2010. "Financing Development: The Role of Information Costs." *American Economic Review*, 100(4): 1875–1891.
Greenwood, Jeremy, Juan Sanchez, and Cheng Wang. 2013. "Quantifying the Impact of Financial Development on Economic Development." *Review of Economic Dynamics*, 16(1): 194–215.
Hall, Robert E., and Dale W. Jorgenson. 1969. "Tax Policy and Investment Behavior: Reply and Further Results." *The American Economic Review*, 59(3): 388–401.
Hansen, Lars Peter, John C. Heaton, and Nan Li. 2008. "Consumption Strikes Back? Measuring Long-Run Risk." *Journal of Political Economy*, 116(2): 260–302.
Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *The Quarterly Journal of Economics*, 124(4): 1403–1448.
Itskhoki, Oleg, and Benjamin Moll. 2019. "Optimal Development Policies With Financial Frictions." *Econometrica*, 87(1): 139–173.
Jeong, Hyeok, and Robert Townsend. 2007. "Sources of TFP growth: occupational choice and financial deepening." *Economic Theory*, 32(1): 179–221.
Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–271.
Jones, Charles I. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics*, 3(2): 1–28.
Jones, Charles I. 2013. "Misallocation, Economic Growth, and Input-Output Economics." , ed. Daron Acemoglu, Manuel Arellano and Eddie Dekel Vol. II of *Advances in Economics and Econometrics*, 419–454. Cambridge University Press.
Jorgenson, Dale. 1963. "Capital Theory and Investment Behavior." *American Economic Review*, 53(2): 247–259.
Jovanovic, Boyan. 2014. "Misallocation and Growth." *American Economic Review*, 104(4): 1149–1171.
Kaltenbrunner, Georg, and Lars A. Lochstoer. 2010. "Long-Run Risk through Consumption Smoothing." *Review of Financial Studies*, 23(8): 3190–3224.

Kehrig, Matthias. 2015. "The Cyclical Nature of the Productivity Distribution." Working Papers 11-15.

Kreps, David M., and Evan L. Porteus. 1978. "Temporal resolution of uncertainty and dynamic choice theory." *Econometrica*, 46(1): 185–200.

Krusell, Per, and Anthony A. Smith. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy*, 106(5): 867–896.

Kung, Howard, and Lukas Schmid. 2015. "Innovation, Growth, and Asset Prices." *Journal of Finance*, 70(3): 1001–1037.

Lambrecht, Bart M., and Stewart C. Myers. 2008. "Debt and managerial rents in a real-options model of the firm." *Journal of Financial Economics*, 89(2): 209 – 231.

Lambrecht, Bart M., and Stewart C. Myers. 2012. "A Lintner Model of Payout and Managerial Rents." *The Journal of Finance*, 67(5): 1761–1810.

Mian, Atif, and Amir Sufi. 2014. "What Explains the 2007–2009 Drop in Employment?" *Econometrica*, 82: 2197–2223.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2): 422–458.

Moll, Benjamin. 2014. "Productivity losses from financial frictions: Can self-financing undo capital misallocation?" *American Economic Review*, 104(10): 3186–3221.

Müller, Ulrich K., and Mark W. Watson. 2008. "Testing Models of Low-Frequency Variability." *Econometrica*, 76(5): 979–1016.

Müller, Ulrich K., and Mark W. Watson. 2018. "Long-Run Covariability." *Econometrica*, 86(3): 775–804.

Myers, Stewart C. 2000. "Outside Equity." *The Journal of Finance*, 55(3): 1005–1037.

Nicolae, Gârleanu B., Stavros Panageas, and Jianfeng Yu. 2012. "Technological Growth and Asset Pricing." *Journal of Finance*, 67(4): 1265–1292.

Olley, G Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica*, 64(6): 1263–1297.

Pástor, L'uboš, and Pietro Veronesi. 2012. "Uncertainty about Government Policy and Stock Prices." *The Journal of Finance*, 67(4): 1219–1264.

Peters, Michael. 2020. "Heterogeneous mark-ups, growth and endogenous misallocation." *Econometrica*, 88(5): 2037–2073.

Peters, Ryan H., and Lucian A. Taylor. 2017. "Intangible capital and the investment-q relation." *Journal of Financial Economics*, 123(2): 251–272.

Rampini, Adriano A., and S. Viswanathan. 2010. "Collateral, Risk Management, and the Distribution of Debt Capacity." *Journal of Finance*, 65(6): 2293–2322.

Restuccia, Diego, and Richard Rogerson. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants." *Review of Economic Dynamics*, 11(4): 707–720.

Romer, Paul M. 1986. "Increasing Returns and Long-run Growth." *Journal of Political Economy*, 94(5): 1002–1037.

Romer, Paul M. 1990. "Endogenous Technological Change." *Journal of Political Economy*, 98(5): 71–102.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron. 2007. "Asset Pricing with Idiosyncratic Risk and Overlapping Generations." *Review of Economic Dynamics*, 10(4): 519–548.

Tikhomirov, A. N. 1980. "On the Convergence Rate in The Central Limit Theorem for Weakly Dependent Random Variables." *Theory of Probability and Its Applications*, XXV(4): 790–809.

van Binsbergen, Jules H., and Christian C. Opp. 2019. "Real Anomalies." *Journal of Finance*, 74(4): 1659–1706.

Weil, Philippe. 1990. "Nonexpected utility in macroeconomics." *Quarterly Journal of Economics*, 105(1): 29–42.

Whited, Toni M., and Jake Zhao. 2021. "The Misallocation of Finance." *Journal of Finance*, forthcoming.