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NEOCLASSICAL GROWTH IN AN INTERDEPENDENT WORLD

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ABSTRACT

We generalize the open-economy neoclassical growth model to allow for trade and capital market frictions and imperfect substitutability of goods and capital across countries. The multi-country model is tractable, amenable to quantitative analysis, and matches key empirical patterns such as gravity equations in trade and capital holdings. The degree of integration in trade and capital markets and their interaction shape adjustments to shocks and the speed of convergence to steady state. The model is well-suited to study counterfactual changes in both trade and capital market frictions, such as a decoupling between the United States and China.

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1 Introduction

The textbook neoclassical growth model remains a benchmark for thinking about cross-country income dynamics. In the closed-economy version of this model, each country converges along a transition path to its own steady-state level of income per capita, determined by its preferences and production technology. Open-economy versions of this model typically make strong assumptions about trade and capital markets and the substitutability of goods and capital across countries. The standard assumptions are homogeneous goods and frictionless trade between countries, whereas conventional quantitative trade models feature imperfect substitutability of goods across countries and bilateral trade frictions. Similarly, open-economy neoclassical growth models typically assume homogeneous capital and frictionless capital markets, implying perfectly elastic investment to equalize returns on capital across countries.

We develop an open-economy neoclassical growth model with trade and capital market frictions and imperfect substitutability of goods and capital across countries. The multi-country model is tractable, amenable to quantitative analysis, and matches key empirical patterns such as gravity equations in trade and capital holdings. We start with the Armington model of trade that features imperfect substitutability of goods across countries and bilateral trade frictions. Countries have preferences for variety and produce heterogeneous goods. After trading subject to bilateral trade frictions, each country's consumption index is an aggregate over all countries' goods. We then add an intertemporal consumption-saving decision and an intratemporal wealth allocation decision. The representative agent in each country chooses how much of the consumption index to consume and save. She also allocates wealth across countries, subject to bilateral capital market frictions and idiosyncratic shocks. The frictions make capital investments not fully respond to differences in rental rates of capital across countries. As in the closed-economy neoclassical growth model (CNGM), each country converges to its own steady state. However, the steady state as well as the transition path depend not only on domestic productivity but also on foreign productivity, trade frictions, and capital market frictions.

The model captures key features of the data on international trade and capital holdings, many of which are summarized in Obstfeld and Rogoff (2000). First, the model matches the well-known empirical finding that trade flows are well approximated by a gravity equation, such that bilateral trade increases with importer and exporter size, and decreases with bilateral trade frictions. We generate a similar gravity equation for bilateral capital holdings, which again provides a close approximation to the data. Second, the model allows for home bias in both trade and capital holdings if bilateral trade and investment frictions are sufficiently high. Third, the model generates a strong positive correlation between domestic saving and investment if capital market frictions are sufficiently high and the degree of substitution between domestic and foreign capital is suffi-

ciently low. Fourth, the model gives rise to gross capital holdings that exceed net capital holdings because the idiosyncratic shocks cause investors to disperse wealth across all countries. Finally, the model allows for limited capital investment from rich to poor countries because higher rental rates of capital in poor countries can be offset by higher capital market frictions.

With open goods and capital markets, our model implies richer and more realistic cross-country income dynamics than the CNGM. With open goods markets, as a country accumulates capital and increases output, its terms of trade deteriorate at a rate that depends on the substitutability of goods across countries. With open capital markets, a country can accumulate capital through both domestic saving and foreign investment. The country faces an upward-sloping supply curve for foreign investment, whose slope depends on the substitutability of capital across countries. Some countries specialize as importers of goods and exporters of capital services, while other countries specialize as exporters of goods and importers of capital services, even in the absence of current account imbalances. With an intertemporal consumption-saving decision, some countries spend more than their income and accumulate current account deficits, while other countries spend less than their income and accumulate current account surpluses.

The model delivers new insights on the impulse responses of wealth and capital to a productivity shock. In the CNGM, a positive productivity shock increases the steady-state capital stock, leading to a gradual accumulation of capital through domestic saving. In the conventional open-economy neoclassical growth model, a positive productivity shock leads to an immediate capital reallocation that equalizes the rental rate of capital across countries. Our model with trade and capital market frictions and imperfect substitutability of goods and capital across countries delivers a prediction between these two extremes. A positive productivity shock leads to a higher steady-state capital stock through *both* domestic saving *and* capital reallocation. The magnitude of the initial capital reallocation depends on the degree of substitution in capital across countries. Different investor countries earn different real returns along the transition path due to bilateral capital market frictions and different movements in goods prices. Consequently, some countries accumulate wealth more rapidly than other countries.

The model also delivers new insights on the speed of convergence to the steady state, depending on the degree of goods and capital market integration. In the CNGM, the speed of convergence depends on the degree of diminishing returns to capital investment. Opening the CNGM to *only* frictionless trade *increases* the speed of convergence. As a country grows, its output price decreases because it faces substitutes in world markets. The resulting faster decrease in the real return implies faster convergence relative to a closed economy. Opening the CNGM to *only* perfect capital markets also *increases* the speed of convergence. As a country's wealth grows from domestic and foreign investment, the domestic factor prices and the local consumption price index rise. The consequent faster decrease in the real return implies faster convergence relative to

a closed economy. In contrast, opening the CNGM to *both* frictionless trade and perfect capital markets *decreases* the speed of convergence. Opening trade and capital markets leads to an initial capital reallocation that equalizes the real return across countries. After this initial reallocation, there is no correlation between the real return and the initial levels of wealth or capital stock, which implies slower convergence relative to a closed economy. All countries accumulate wealth at the same rate, as determined by this common real return, and initial differences in wealth persist forever.

A key benefit of our framework is the ability to study static and dynamic welfare effects of counterfactual policies that involve disintegration of both trade and capital markets. We apply it to a counterfactual decoupling of the United States and China as an example. In the neoclassical growth model with open trade but capital autarky, the conventional static welfare gains from trade integration are magnified by dynamic welfare gains from capital accumulation. The fall in the consumption goods price index from reductions in trade frictions raises the real return in each country, which increases the rate of growth along the transition path, and the level of income per capita in the steady state. In our neoclassical growth model with open trade and capital markets, the static and dynamic effects of trade integration are more subtle. Reductions in trade frictions lead to a capital reallocation across countries, which affects income and consumption price indices, and hence the static welfare gains from trade. This change in consumption goods price indices in turn feeds back to influence the real return and the dynamic welfare gains from capital accumulation along the transition path to the steady state. Similarly, the static and dynamic effects of capital market frictions depend heavily on trade openness, highlighting the importance of jointly modeling these two dimensions of international integration.

We demonstrate how to study the quantitative implications of our model using a small number of standard parameter values from the literature and publicly available data on the national accounts, bilateral trade, and bilateral capital holdings. We develop technical tools to study the transition dynamics, which could be useful for future work that builds on our framework. First, we show how to undertake counterfactuals in the nonlinear model using observed data on bilateral trade and capital holdings—without taking a stance on initial country-level parameters—by extending “dynamic exact-hat algebra” techniques from the trade and spatial economics literature. Second, we show how to invert the nonlinear model to recover the fundamentals that rationalize these observed data as an equilibrium: the productivity and capital use efficiency of each producer, and the bilateral trade and capital market frictions. By conditioning on the observed data, we are able to undertake this model inversion without making assumptions about either the initial distance from steady-state or agents’ expectations about future fundamentals. Third, we linearize the model around the unobserved initial steady state to derive a first-order, closed-form solution for the economy’s transition path, and show that the dynamics of the sys-

tem are captured by a transition matrix that can be constructed from observed trade and capital holdings matrices. We use this linearization to decompose each country's economic growth into the contributions of initial conditions and shocks to domestic and foreign fundamentals. We also use this linearization to show analytically that the interaction between goods and capital market integration shapes impulse responses to productivity shocks, the impact of counterfactual policies, and the speed of convergence.

Our paper is related to a number of different strands of research. First, we connect with the large literature in macroeconomics on the CNGM following Ramsey (1928), Solow (1956) and Swan (1956). The CNGM's prediction of conditional convergence in income per capita finds strong empirical support in the cross-country growth literature following Barro (1991) and Mankiw, Romer, and Weil (1992). Much of the rapid growth of East Asian countries in recent decades is attributed to its mechanism of factor accumulation in the growth accounting exercise in Young (1995). One quantitative challenge for the CNGM is that empirical estimates of income convergence imply lengthy transitions to the steady state. In specifications with an endogenous saving rate, King and Rebelo (1993) argues that such lengthy transitions require implausibly low intertemporal elasticities of substitution. In response, a number of studies have explored extensions that generate slower convergence, including installation costs in Rappaport (2006), financial frictions in Barro, Mankiw, and Sala-i-Martin (1995) and multiple sectors in Buera et al. (2021).

A small number of papers have developed versions of the neoclassical growth model with open goods markets, while maintaining the assumption of autarky in capital markets. Ventura (1997) combines the neoclassical growth model with the factor price equalization theorem of the Heckscher-Ohlin model to rationalize both conditional convergence and episodes of rapid growth by developing countries. Cuñat and Maffezzoli (2004) allow for complete specialization and the resulting departures from factor price equalization. Acemoglu and Ventura (2002) show that specialization and trade can generate a stable world income distribution through terms of trade effects, even without diminishing returns in production. Relative to these studies, we generalize the neoclassical growth model to introduce open capital markets with imperfect substitutability, while also allowing for trade in differentiated goods and trade costs, so as to match the observed gravity equations for trade and capital holdings.

Second, our work is related to research in international trade. We consider the class of constant elasticity trade models, which includes differentiation by country of origin (Armington 1969), Ricardian technology differences (Eaton and Kortum 2002) and horizontally-differentiated firm varieties and increasing returns to scale (Krugman 1980 and Melitz 2003 with a Pareto distribution), as examined in Arkolakis, Costinot, and Rodriguez-Clare (2012). A key implication of these models is that bilateral trade exhibits a gravity equation, as highlighted in Anderson and Wincoop (2003) and Head and Mayer (2014). Manipulating the conditions for general equilibrium

in these static international trade models, Kleinman, Liu, and Redding (2023b) derive sufficient statistics for the impact of foreign productivity shocks on domestic welfare. Kleinman, Liu, and Redding (2023a) introduce capital accumulation into a dynamic model of migration within countries. But capital markets are assumed to be autarkic in each location and a separation is assumed between workers (who live hand to mouth) and capitalists (who can save) in order to tractably model migration.

Much of the quantitative international trade literature assumes exogenous trade imbalances, although Ju, Shi, and Wei (2014), Reyes-Heroles (2016), Eaton, Kortum, and Neiman (2016) and B. Ravikumar, Santacreu, and Sposi (2019) endogenize these imbalances following Obstfeld and Rogoff (1996). A related line of research examines the relationship between trade and growth through capital accumulation, including Anderson, Larch, and Yotov (2015), Alvarez (2017) and Mutreja, Ravikumar, and Sposi (2018). Within this line of research, Moll (2008) introduces bilateral production externalities between countries (e.g., from knowledge spillovers). Relative to these studies, we simultaneously model imperfect substitutability and frictions in goods and capital markets at a point in time and consumption-savings decisions over time.

Third, our analysis relates to several lines of research in international finance and macroeconomics. A first group of studies examines the origins of global imbalances, the exorbitant privilege of the United States, and the reasons why capital does not flow from rich to poor countries, including Lucas (1990), Jin (2012), Gourinchas and Rey (2007), Gourinchas and Jeanne (2006) and Gourinchas and Jeanne (2013), Maggiori, Neiman, and Schreger (2020), Auclert, Matthew Rognlie, and Straub (2020), Coppola et al. (2021), Davis, Valente, and Wincoop (2021), and Atkeson, Heathcote, and Perri (2022). A second series of studies examines imperfect substitutability in capital markets, including Kojien and Yogo (2019) and Kojien and Yogo (2020), Auclert, Matt Rognlie, et al. (2022) and Maggiori (2021). A third line of work evaluates the international propagation of shocks through goods and capital markets, including Backus, Kehoe, and Kydland (1992), Kose, Otrok, and Whiteman (2003) and Huo, Levchenko, and Pandalai-Nayar (2019).

A fourth body of papers provides evidence that the gravity equation provides a good approximation to international capital holdings, as in Portes and Rey (2005). A fifth vein of research explores home bias and the international risk diversification, including Cole and Obstfeld (1991), Obstfeld (1994), Martin and Rey (2004) and Martin and Rey (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Fitzgerald (2012), Pellegrino, Spolaore, and Wacziarg (2021), Jiang, Richmond, and Zhang (2022), Chau (2022), Hu (2022) and Kucheryavyy (2022). We abstract from international risk diversification by considering an environment with no aggregate uncertainty, in which unanticipated shocks to fundamentals are revealed under perfect foresight. Nevertheless, we show that our framework provides a natural explanation for a gravity equation for capital holdings and other features of observed data on bilateral capital holdings.

The remainder of the paper is structured as follows. Section 2 develops the theoretical framework. Section 3 develops the technical tools to study the transition dynamics. Section 4 describes the data and presents the quantitative analysis. Section 5 concludes.

2 Theoretical Framework

We develop a discrete-time, infinite-horizon production economy with many countries. We index time as $t \in \{1, \dots, \infty\}$ and countries as $n \in \{1, \dots, N\}$. Each country produces a differentiated good with labor and capital under constant returns to scale. Labor and capital markets are perfectly competitive. The representative agent in each country has a fixed endowment of labor. In each period, she chooses consumption and the allocation of wealth across countries to maximize discounted lifetime utility. Countries make international capital investments subject to bilateral capital market frictions. Countries trade goods subject to bilateral trade costs. All prices, wages, and rental rates of capital are in units of the world GDP as the numeraire. The economy has no aggregate risk, and the representative agent has perfect foresight of all aggregate variables.

2.1 Consumption-Saving Problem

The representative agent in each country n enters period t with a_{nt} units of wealth in local consumption units and an endowment of ℓ_n units of labor, and chooses a path of consumption and savings to maximize her intertemporal utility. We assume that intertemporal utility takes the constant relative risk aversion (CRRA) form:

$$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s} \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}, \quad (1)$$

where β is the discount rate and ψ is the intertemporal elasticity of substitution. c_{nt} is a consumption index that depends on the consumption of the goods produced by each country.

The representative agent's period-by-period budget constraint is

$$c_{nt} + a_{nt+1} = \mathcal{R}_{nt} a_{nt} + \frac{w_{nt} \ell_n}{p_{nt}}, \quad (2)$$

where p_{nt} is the price index dual to the consumption index; w_{nt} is the wage; and \mathcal{R}_{nt} is the gross return on period t wealth. On the left side is consumption plus wealth at the end of period t . On the right side is wealth at the beginning of period t times the gross return on wealth plus labor income in local consumption units.

The representative agent makes a portfolio decision to allocate a_{nit} units of wealth to each

producer country i , subject to $a_{nt} = \sum_{i=1}^N a_{nit}$. Each unit of wealth allocated to producer i earns a gross return \mathcal{R}_{nit} in period t . As we explain below, we index the return by n because different investor countries earn different returns from the same producer country due to bilateral capital market frictions and different inflation rates. Consequentially, the above gross return on period t wealth is given by $\mathcal{R}_{nt} = \sum_{i=1}^N a_{nit} \mathcal{R}_{nit} / a_{nt}$.

We define the present value of labor income as

$$h_{nt} = \sum_{s=1}^{\infty} \frac{w_{nt+s} \ell_n / p_{nt+s}}{\prod_{u=1}^s \mathcal{R}_{nt+u}}. \quad (3)$$

As we show in Online Appendix C, equilibrium consumption is linear in total wealth:

$$c_{nt} = \varsigma_{nt} \left(\mathcal{R}_{nt} a_{nt} + \frac{w_{nt} \ell_n}{p_{nt}} + h_{nt} \right), \quad (4)$$

where we define the saving rate $1 - \varsigma_{nt}$ recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^\psi \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}. \quad (5)$$

In the special case of log utility ($\psi = 1$), the saving rate is constant and equal to $1 - \varsigma_{nt} = \beta$ (Angeletos, 2007; Moll, 2014).

2.2 Allocation of Wealth

We make two assumptions to simplify the wealth allocation problem and to generate realistic implications for international capital holdings. First, we assume bilateral capital market frictions. We normalize the capital market frictions to $\kappa_{nnt} = 1$ for domestic producers and $\kappa_{nit} > 1$ for foreign producers $i \neq n$. These capital market frictions represent all costs related to foreign portfolio and direct investment, including trading costs, information acquisition costs, search costs, and regulatory costs.

Second, we assume that investors solve a discrete-choice problem for each unit of wealth. For each unit of wealth, investor n draws an idiosyncratic shock φ_{nit} for each producer i . The idiosyncratic shock is independent across investors and producers and is drawn from a Fréchet distribution:

$$F_{it}(\varphi) = e^{-(\varphi/\eta_{it})^{-\epsilon}}. \quad (6)$$

The scale parameter $\eta_{it} > 0$, which we refer to as the “capital use efficiency” in each investment destination i , determines the magnitude of the idiosyncratic shock and thus the average return from investments in producer i , which can depend for example on producer country institutions,

such as the protection of property rights. The shape parameter $\epsilon > 0$ determines its dispersion. The idiosyncratic shock represents all idiosyncratic risk related to foreign portfolio and direct investment, including idiosyncratic productivity shocks and stochastic search costs (Hortaçsu and Syverson, 2004). For example, when a U.S. investor makes a decision about investing in a Canadian or a Mexican firm, each choice is paired with an idiosyncratic shock that represents the productivity or quality of the match.

If investor n were to invest the unit of wealth in producer i , her return before depreciation would be $\varphi_{nit}r_{it}/\kappa_{nit}$. The unit of wealth becomes $\varphi_{nit}/\kappa_{nit}$ efficiency units of capital available for production by producer i , and r_{it} is the rental rate per efficiency unit of capital. The investor chooses producer i if its return is the highest given the realizations of the idiosyncratic shocks: $\varphi_{nit}r_{it}/\kappa_{nit} = \max_h \{\varphi_{nht}r_{ih}/\kappa_{nht}\}$. As we show in Online Appendix B, the probability that investor n allocates a unit of wealth to producer i in period t is

$$b_{nit} = \frac{a_{nit}}{a_{nt}} = \frac{(\eta_{it}r_{it}/\kappa_{nit})^\epsilon}{\sum_{h=1}^N (\eta_{ht}r_{ht}/\kappa_{nht})^\epsilon}. \quad (7)$$

In a population of investors, b_{nit} is also investor n 's portfolio share in producer i . The portfolio share (7) increases in the rental rate r_{it} with an elasticity of ϵ . We refer to ϵ as the capital elasticity because it plays a role that is similar to the trade elasticity in goods markets. The portfolio share (7) allocated to each producer i increases in the investment destination's capital use efficiency η_{it} . In the numerator of equation (7), the portfolio share decreases in the bilateral capital market friction κ_{nit} . In the denominator of equation (7), the portfolio share increases in the capital market frictions with all other producers ("multilateral resistance").

Equation (7) matches key facts about international capital holdings. First, the model matches the gravity equation in bilateral capital holdings (Portes and Rey, 2005) if the capital market frictions increase in bilateral distance (e.g., due to information acquisition costs). Second, the model allows for home bias in capital holdings (French and Poterba, 1991), because the capital market frictions on foreign investment are greater than those for domestic investment. Third, the model generates a strong positive correlation between domestic saving and investment (Feldstein and Horioka, 1980) if capital market frictions are sufficiently high and the elasticity of substitution between domestic and foreign capital is sufficiently low. Fourth, the model generates larger gross capital holdings than net capital holdings because each investor country holds strictly positive capital in each producer country if η_{it}/κ_{nit} is strictly positive. Fifth, the model rationalizes limited capital investment from rich to poor countries (Lucas, 1990) because higher rental rates of capital r_{it} in poor countries need not imply high investment if η_{it}/κ_{nit} is low.

The idiosyncratic shocks diversify the overall portfolio. Moreover, the investor has perfect foresight for all aggregate variables, including the capital use efficiency η_{it} in each investment

destination and the bilateral capital market frictions κ_{nit} . Therefore, with a continuous measure of units of wealth, the economy has no aggregate risk. Let v_{nit} denote the mean capital income rate, i.e., the average return per unit of capital investment by investor n in producer i after accounting for bilateral capital market frictions and idiosyncratic shocks:

$$v_{nit} = \mathbb{E} \left[\frac{\varphi_{nit} r_{it}}{\kappa_{nit}} \mid \frac{\varphi_{nit} r_{it}}{\kappa_{nit}} = \max_h \left\{ \frac{\varphi_{nht} r_{ht}}{\kappa_{nht}} \right\} \right] \quad (8)$$

As we show in Online Appendix B, investor n earns the same mean capital income rate from each producer i :

$$v_{nit} = v_{nt} = \gamma \left(\sum_{h=1}^N \left(\frac{\eta_{ht} r_{ht}}{\kappa_{nht}} \right)^\epsilon \right)^{\frac{1}{\epsilon}}, \quad (9)$$

where $\gamma = \Gamma(1 - 1/\epsilon)$ is the gamma function evaluated at $1 - 1/\epsilon$. Producers face an upward-sloping supply function for capital that depends on their rental rates r_{it} . However, producers with higher rental rates attract investors with lower realized efficiency units of capital $\varphi_{nit}/\kappa_{nit}$. Consequently, the mean capital income rate is the same across all producers for a given investor n . However, different investor countries earn different returns from the same producer country due to bilateral capital market frictions (i.e., $\kappa_{nht} \neq \kappa_{iht}$ for $n \neq i$).

The capital stock depreciates at a constant rate $\delta \in (0, 1)$. In units of the world GDP as our numeraire, investor n 's gross nominal portfolio return from period $t - 1$ to t is

$$\mathcal{R}_{nt}^{nom} = \frac{p_{nt}(1 - \delta) + v_{nt}}{p_{nt-1}}. \quad (10)$$

Dividing by the inflation rate of the local consumption price index, investor n 's gross real portfolio return from period $t - 1$ to t is

$$\mathcal{R}_{nt} = \frac{\mathcal{R}_{nt}^{nom}}{p_{nt}/p_{nt-1}} = 1 - \delta + \frac{v_{nt}}{p_{nt}}. \quad (11)$$

Different investor countries earn different real returns along the transition path due to bilateral capital market frictions and different inflation rates, although we show below that the steady-state real return is equalized across all countries.

As we show in Online Appendix B, the mean efficiency units of investor n 's capital investment in producer i and the capital stock in producer i taking into account efficiency units are

$$\bar{\varphi}_{nit} = \gamma \eta_{it} b_{nit}^{-\frac{1}{\epsilon}}, \quad k_{it} = \sum_{n=1}^N \bar{\varphi}_{nit} a_{nit}. \quad (12)$$

The mean efficiency units of capital decreases in the the portfolio share b_{nit} . Thus, we have a decreasing marginal efficiency of capital investment (Keynes, 1935), where a lower capital elasticity ϵ implies a higher rate of diminishing returns.

2.3 Production

Country i produces a differentiated good with labor and capital. The competitive, representative producer employs ℓ_{it} units of labor at the wage w_{it} and rents k_{it} units of capital at the rental rate r_{it} in period t . Producers take factor prices as given in perfectly competitive markets. Country i has a constant returns Cobb-Douglas production technology with productivity z_{it} in period t . Country i 's output in period t is

$$y_{it} = z_{it} \left(\frac{\ell_{it}}{\mu_i} \right)^{\mu_i} \left(\frac{k_{it}}{1 - \mu_i} \right)^{1 - \mu_i}, \quad (13)$$

where $\mu_i \in (0, 1)$ is the labor share.

With the Cobb-Douglas assumption, cost minimization and zero profits imply that the relative factor payments are proportional to the relative factor shares:

$$\frac{w_{it}\ell_{it}}{r_{it}k_{it}} = \frac{\mu_i}{1 - \mu_i}, \quad (14)$$

and the “free on board” price before trade costs is equal to marginal cost:

$$p_{iit} = \frac{w_{it}^{\mu_i} r_{it}^{1 - \mu_i}}{z_{it}}. \quad (15)$$

2.4 Consumption and Trade

Countries have preferences for consumption variety. Country n purchases c_{nit} units of the good produced by country i at the price p_{nit} in period t . We define country n 's consumption index in period t as a constant elasticity of substitution (CES) function over all goods:

$$c_{nt} = \left(\sum_{i=1}^N c_{nit}^{1 - 1/\sigma} \right)^{\frac{1}{1 - 1/\sigma}}, \quad (16)$$

where $\sigma > 1$ is the elasticity of substitution. Country n maximizes the consumption index (16), subject to the budget constraint $p_{nt}c_{nt} = \sum_{i=1}^N p_{nit}c_{nit}$. The first-order conditions imply that

country n 's expenditure share on the good produced by country i in period t is

$$s_{nit} = \frac{p_{nit}c_{nit}}{p_{nt}c_{nt}} = \frac{p_{nit}^{-\theta}}{\sum_{h=1}^N p_{nht}^{-\theta}}, \quad (17)$$

where $\theta = \sigma - 1 > 0$ is the trade elasticity. Substituting the expenditure share (17) in the budget constraint, the price index that is dual to the consumption index is

$$p_{nt} = \left(\sum_{i=1}^N p_{nit}^{-\theta} \right)^{-\frac{1}{\theta}}. \quad (18)$$

Countries trade goods subject to iceberg variable bilateral trade costs. When exporter i ships $\tau_{nit} \geq 1$ units of its good to importer n in period t , one unit arrives to be available for consumption. We normalize trade costs to $\tau_{nnt} = 1$ for domestic producers and $\tau_{nit} > 1$ for foreign producers $i \neq n$. The law of one price implies that the import prices satisfy $p_{nit} = p_{iit}\tau_{nit}$. Combined with equation (15) for the “free on board” price, the “cost inclusive of freight” price that importer n pays for exporter i 's good in period t is

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i}}{z_{it}}. \quad (19)$$

Substituting the import prices (19) in the expenditure share (17), we have

$$s_{nit} = \frac{(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it})^{-\theta}}{\sum_{h=1}^N (\tau_{nht} w_{ht}^{\mu_h} r_{ht}^{1-\mu_h} / z_{ht})^{-\theta}}. \quad (20)$$

In the numerator of equation (20), imports decrease in the bilateral trade cost τ_{nit} . Thus, the model matches the gravity equation in trade if the bilateral trade cost increases in bilateral distance. In the denominator of equation (20), imports increase in multilateral resistance through the trade costs with all other trading partners.

2.5 Market Clearing

Market clearing for each good i in period t is

$$p_{iit}y_{it} = \sum_{n=1}^N s_{nit}p_{nt} [c_{nt} + a_{nt+1} - (1 - \delta) a_{nt}]. \quad (21)$$

On the left side is country i 's output. On the right side is the aggregate demand for country i 's output for consumption and net investment. We next use zero profits ($p_{iit}y_{it} = w_{it}\ell_{it} + r_{it}k_{it}$)

and equation (14) to substitute for output on the left side of equation (21). We then use the intertemporal budget constraint (2) to substitute for net investment on the right side of equation (21). We also impose labor market clearing ($\ell_{it} = \ell_i$). We can then rewrite goods market clearing (21) as

$$\frac{w_{it}\ell_i}{\mu_i} = \sum_{n=1}^N s_{nit} (v_{nt}a_{nt} + w_{nt}\ell_n). \quad (22)$$

Market clearing for the capital used by producer i implies

$$r_{it}k_{it} = \sum_{n=1}^N v_{nt}b_{nit}a_{nt}. \quad (23)$$

On the left side is country i 's total rental payment on capital. On the right side is the total capital income earned by all investors. Using equation (14), we rewrite capital market clearing (23) as

$$\frac{(1 - \mu_i) w_{it}\ell_i}{\mu_i} = \sum_{n=1}^N v_{nt}b_{nit}a_{nt}. \quad (24)$$

As we show in Online Appendix D, the balance of payments identities hold in the model. The current account is the sum of the trade balance and net investment income and is also equal to minus the financial account.

2.6 Summary of General Equilibrium

For convenience, we summarize the equations that fully characterize the general equilibrium. The state variables at the beginning of period t are the wealth of all countries $\{a_{nt}\}_{n=1}^N$. The endogenous variables that are determined in period t are the wages $\{w_{it}\}_{i=1}^N$, the rental rates of capital $\{r_{it}\}_{i=1}^N$, the capital income rates $\{v_{nt}\}_{n=1}^N$, the consumption expenditure shares $\{s_{nit}\}_{n,i=1}^N$, and the portfolio shares $\{b_{nit}\}_{n,i=1}^N$. These variables are a solution to the following system of equations:

$$\frac{w_{it}\ell_i}{\mu_i} = \sum_{n=1}^N s_{nit} (v_{nt}a_{nt} + w_{nt}\ell_n), \quad (25)$$

$$\frac{(1 - \mu_i) w_{it}\ell_i}{\mu_i} = \sum_{n=1}^N v_{nt}b_{nit}a_{nt}. \quad (26)$$

$$v_{nt} = \gamma \left[\sum_{h=1}^N \left(\frac{\eta_{ht}r_{ht}}{\kappa_{nht}} \right)^\epsilon \right]^{1/\epsilon}, \quad (27)$$

$$s_{nit} = \frac{(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it})^{-\theta}}{\sum_{h=1}^N (\tau_{nht} w_{ht}^{\mu_h} r_{ht}^{1-\mu_h} / z_{ht})^{-\theta}}, \quad (28)$$

$$b_{nit} = \frac{(\eta_{it} r_{it} / \kappa_{nit})^\epsilon}{\sum_{h=1}^N (\eta_{ht} r_{ht} / \kappa_{nht})^\epsilon}. \quad (29)$$

In addition, we choose the world GDP as the numeraire in each period t :

$$\sum_{i=1}^N p_{it} y_{it} = \sum_{i=1}^N (w_{it} l_i + r_{it} k_{it}) = \sum_{i=1}^N \frac{w_{it} \ell_i}{\mu_i} = 1. \quad (30)$$

The wealth state variables evolve over time through equilibrium consumption-saving decisions. Substituting equation (19) in equation (18), the consumption price index is

$$p_{nt} = \left[\sum_{i=1}^N \left(\frac{\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i}}{z_{it}} \right)^{-\theta} \right]^{-1/\theta}. \quad (31)$$

We also have the gross portfolio return in equation (11) and the present value of labor income in equation (3). Then wealth evolves according to the law of motion:

$$a_{nt+1} = (1 - \varsigma_{nt}) \left(\mathcal{R}_{nt} a_{nt} + \frac{w_{nt} \ell_n}{p_{nt}} + h_{nt} \right) - h_{nt}, \quad (32)$$

where we define the saving rate $1 - \varsigma_{nt}$ recursively by equation (5), the present value of labor income h_{nt} as in equation (3), and the gross real return to capital \mathcal{R}_{nt} as in equation (11).

2.7 Steady-State Equilibrium

Let $\{\ell_n, z_n, \eta_n\}_{n=1}^N$ be time invariant values of the labor endowment, productivity, and capital use efficiency. Let $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$ be time-invariant values of trade costs and capital market frictions. We define the steady-state equilibrium as the corresponding time-invariant values of the wealth state variables $\{a_n^*\}_{n=1}^N$ and the other endogenous variables $\{w_i^*, r_i^*, v_n^*, s_{ni}^*, b_{ni}^*\}_{n,i=1}^N$. We denote the steady-state values of the endogenous variables with an asterisk.

Given the fixed labor endowment, a diminishing marginal product of capital implies a steady-state level of wealth a_n^* in each country n , just as in the Solow-Swan Model. However, the saving rates are endogenous through equilibrium consumption-saving decisions. As we show in Appendix F, the steady-state gross real return \mathcal{R}_n^* and the steady-state saving rate $1 - \varsigma_n^*$ are the same across investor countries and determined by the discount factor as

$$\mathcal{R}_n^* = \frac{1}{\beta}, \quad 1 - \varsigma_n^* = \beta. \quad (33)$$

This result implies a common steady-state real capital income rate

$$\frac{v_n^*}{p_n^*} = \frac{1}{\beta} - 1 + \delta. \quad (34)$$

3 Transition Dynamics

As in the conventional CNGM, our open-economy framework features conditional convergence in income per capita, in the sense that each country converges to its own steady-state level of income per capita. In contrast to this conventional framework, each country's steady-state level of income per capita and its growth rate along the transition path are influenced by fundamentals in other countries, where these fundamentals comprise trade frictions (τ_{ni}), capital market frictions (κ_{ni}), goods productivity (z_i), and capital use efficiency (η_i).

In Subsection 3.2, we show that we can solve for the economy's dynamic response to an anticipated sequence of changes in fundamentals in the nonlinear model using dynamic exact-hat algebra techniques. In Subsection 3.3, we linearize the model's general equilibrium conditions to obtain a closed-form solution for the economy's transition path in response to these changes in fundamentals. In Subsection 3.4, we use this linearization to quantify the contributions of convergence and changes in fundamentals to the evolution of the world income distribution. In Subsection 3.5, we undertake a spectral analysis to characterize analytically the speed of convergence to steady state and the evolution of the economy's state variables along the transition path. Finally, in Subsection 3.6, we use our closed-form solution to analyze the role of goods and capital market integration in shaping the speed of convergence.

For the remainder of the paper, we denote vectors in bold lowercase and matrices in bold uppercase. The derivations for all expressions and results in this section are reported in the Online Appendix.

3.1 Notation

We introduce additional notation to characterize the transition path. To simplify the notation, we suppress the time subscript throughout this subsection.

Let \mathbf{S} be an $N \times N$ *expenditure share* matrix, where the ni -th element is to importer n 's expenditure share on buying from exporter i ($S_{ni} \equiv [s_{ni}]$). Let \mathbf{T} be an $N \times N$ *income share* matrix, where the in -th element is exporter i 's income share from selling to importer n ($T_{in} \equiv \frac{s_{ni}(v_n a_n + w_n \ell_n)}{\sum_{h=1}^N s_{hi}(v_h a_h + w_h \ell_h)}$). Intuitively, S_{ni} captures the importance of exporter i as a seller to importer n , and T_{in} captures the importance of importer n as a buyer from exporter i . Note the order of the subscripts. The rows are importers and the columns are exporters in matrix \mathbf{S} , whereas the

rows are exporters and the columns are importers in matrix \mathbf{T} .

Similarly, let \mathbf{B} be an $N \times N$ *portfolio share* matrix, where the ni -th element is the share of investor n 's wealth in producer i ($B_{ni} \equiv [b_{ni}]$). Let \mathbf{X} be an $N \times N$ *payment share* matrix, where the in -th element is the share of producer i 's total rental payment on capital paid to investor n ($X_{in} \equiv \frac{v_n b_{ni} a_n}{\sum_{h=1}^N v_h b_{hi} a_h}$). Intuitively, B_{ni} captures the importance of producer i as a destination of capital investments from investor n , and X_{in} captures the importance of investor n as an origin of capital investments to producer i . Again, note the order of subscripts. The rows are investors and the columns are producers in matrix \mathbf{B} , whereas the rows are producers and the columns are investors in matrix \mathbf{X} .¹

Finally, let \mathbf{q} be an $N \times 1$ vector of labor income, where the n -th element is country n 's labor income ($q_n \equiv w_n \ell_n$). Let ζ be an $N \times 1$ vector of capital income, where the n -th element is country n 's capital income ($\zeta_n \equiv v_n a_n$).

3.2 Dynamic Exact-Hat Algebra

We suppose that we observe the world economy somewhere along the transition path towards an unobserved steady state. Given the initial observed endogenous variables of the model, we show that we are able to solve for the economy's transition path in time differences ($\dot{x}_{it+1} = x_{it+1}/x_{it}$) for any anticipated convergent sequence of future changes in fundamentals, without having to solve for the initial level of fundamentals.

Proposition 1. *Dynamic Exact Hat Algebra.* *Given observed initial populations $\{\ell_{i0}\}_{i=1}^N$, an initial observed allocation of the economy, $(\{a_{i0}\}_{i=1}^N, \{a_{i1}\}_{i=1}^N, \{S_{ni0}\}_{n,i=1}^N, \{T_{ni0}\}_{n,i=1}^N, \{B_{ni0}\}_{n,i=1}^N, \{X_{ni0}\}_{n,i=1}^N)$, and a convergent sequence of future changes in fundamentals under perfect foresight:*

$$\left\{ \left\{ \dot{z}_{it} \right\}_{i=1}^N, \left\{ \dot{\eta}_{it} \right\}_{i=1}^N, \left\{ \dot{\tau}_{ijt} \right\}_{i,j=1}^N, \left\{ \dot{\kappa}_{ijt} \right\}_{i,j=1}^N \right\}_{t=1}^{\infty},$$

the solution for the sequence of changes in the model's endogenous variables does not require information on the level of fundamentals:

$$\left\{ \left\{ z_{it} \right\}_{i=1}^N, \left\{ \eta_{it} \right\}_{i=1}^N, \left\{ \tau_{ijt} \right\}_{i,j=1}^N, \left\{ \kappa_{ijt} \right\}_{i,j=1}^N \right\}_{t=1}^{\infty}.$$

Proof. See Online Appendix G. □

¹For theoretical completeness, we maintain two assumptions on these matrices, which are satisfied empirically in all years of our data. First, we assume that the \mathbf{S} and \mathbf{B} matrices are irreducible, such that all locations are connected directly or indirectly by trade flows and capital holdings. For any i, n , there exists k such that $[\mathbf{S}^k]_{in} > 0$ and $[\mathbf{B}^k]_{in} > 0$. Second, we assume that each location consumes a positive amount of domestic goods and allocates a positive share of capital domestically. For all i , $S_{ii} > 0$ and $B_{ii} > 0$.

Intuitively, we use the initial observed endogenous variables and the equilibrium conditions of the model to control for the unobserved initial level of fundamentals. Applying this proposition, we can employ dynamic exact-hat algebra methods to solve for the unobserved initial steady state in the absence of any further changes in fundamentals. We can also use this approach to solve counterfactuals for the transition path of the global economy in response to assumed sequences of future changes in fundamentals.

In addition to these dynamic exact-hat algebra results in Proposition 1, we can invert the model to solve for the unobserved changes in goods productivity, capital use efficiency, trade frictions and capital market frictions that are implied by the observed changes of the endogenous variables of the model under perfect foresight, as shown in Online Appendix H. Importantly, we can undertake this model inversion along the transition path without making assumptions about the precise sequence of future fundamentals, because the observed changes in wealth capture agents' expectations about this sequence of future fundamentals.

3.3 Linearization

We now linearize the model to characterize analytically the speed of convergence and the evolution of the state variables along the transition path to steady state. We suppose that we observe population (ℓ), the wealth state variable (\mathbf{a}_t) for time $t = 0$ and $t = 1$, and the trade and capital share matrices (\mathbf{S} , \mathbf{T} , \mathbf{B} , \mathbf{X}) of the economy at time $t = 0$. The economy need not be in steady state at $t = 0$, but we assume that it is on a convergence path towards a steady state with constant fundamentals (z , η , τ , κ). We refer to the steady state implied by these initial fundamentals as the *initial* steady state. We use a tilde above a variable to denote a log deviation from this initial steady state (e.g., $\tilde{a}_{it+1} = \ln a_{it+1} - \ln a_i^*$).

We begin by totally differentiating the conditions for general equilibrium around this unobserved initial steady state, holding constant countries' labor endowments. We thus obtain a system of linear equations that fully characterizes the economy's transition path up to first-order, as reported in Online Appendix I.1. We next show that this system of linearized equations can be reduced to a second-order difference equation in the wealth state variables ($\tilde{\mathbf{a}}_t$) and changes to fundamentals. For expositional convenience, we focus here on the simplest form of changes in fundamentals, such that agents at time $t = 0$ learn about a one-time permanent shock to fundamentals from time $t = 1$ onwards. However, analogous results hold in the linearized model for any expected convergent sequence of future shocks to fundamentals under perfect foresight, and for the case in which agents observe an initial shock to fundamentals and form rational expectations about future shocks based on a known stochastic process for fundamentals.

We define measures of incoming and outgoing shocks to trade and capital frictions, which

aggregate bilateral changes across partner countries, using initial trade and capital share weights: $\tilde{\tau}_{nt}^{in} \equiv \sum_{i=1}^N S_{nit} \tilde{\tau}_{nit}$, $\tilde{\tau}_{nt}^{out} \equiv \sum_{n=1}^N T_{int} \tilde{\tau}_{nit}$, $\tilde{\kappa}_{nt}^{out} \equiv \sum_{i=1}^N B_{nit} \tilde{\kappa}_{nit}$, and $\tilde{\kappa}_{nt}^{in} \equiv \sum_{n=1}^N X_{int} \tilde{\kappa}_{nit}$. Using these definitions, we have the following result.

Proposition 2. State Variables. *Suppose that the economy at time $t = 0$ is on a convergence path toward an initial steady state with constant fundamentals (z, η, τ, κ) . At time $t = 0$, agents learn about one-time, permanent shocks to fundamentals $(\tilde{\mathbf{f}} \equiv [\tilde{z} \ \tilde{\eta} \ \tilde{\kappa}^{in} \ \tilde{\kappa}^{out} \ \tilde{\tau}^{in} \ \tilde{\tau}^{out}]')$ from time $t = 1$ onwards. The evolution of the economy's wealth state variables from time $t = 1$ onwards satisfies the following second-order difference equation:*

$$\Psi \tilde{\mathbf{a}}_{t+2} = \Gamma \tilde{\mathbf{a}}_{t+1} + \Theta \tilde{\mathbf{a}}_t + \Pi \tilde{\mathbf{f}}, \quad (35)$$

where the matrices $(\Psi, \Gamma, \Theta, \Pi)$ are functions of the trade and capital share matrices (S, T, B, X) and model parameters $(\psi, \theta, \beta, \delta, \epsilon, \mu_i)$, as defined in Online Appendix I.2.2.

Proof. See Online Appendix I.2.2. □

We solve this matrix system of equations using the method of undetermined coefficients following Uhlig (1999) to obtain a closed-form solution for the evolution of the state variables $\{\tilde{\mathbf{a}}_t\}_{t=1}^{\infty}$ in terms of an impact matrix (Q) , which captures the initial impact of the fundamental shocks, and a transition matrix (P) , which governs the updating of the state variables over time.

Proposition 3. Transition Matrix. *Suppose that the economy at time $t = 0$ is on a convergence path toward an initial steady state with constant fundamentals (z, η, τ, κ) . At time $t = 0$, agents learn about one-time, permanent shocks to fundamentals $(\tilde{\mathbf{f}} \equiv [\tilde{z} \ \tilde{\eta} \ \tilde{\kappa}^{in} \ \tilde{\kappa}^{out} \ \tilde{\tau}^{in} \ \tilde{\tau}^{out}]')$ from time $t = 1$ onwards. There exists a $N \times N$ transition matrix (P) and a $N \times 6N$ impact matrix (R) such that the second-order difference equation system in (35) has the closed-form solution:*

$$\tilde{\mathbf{a}}_t = P \tilde{\mathbf{a}}_{t-1} + R \tilde{\mathbf{f}}. \quad (36)$$

The transition matrix P satisfies:

$$P = U \Lambda U^{-1},$$

where Λ is a diagonal matrix of N stable eigenvalues $\{\lambda_k\}_{k=1}^N$ and U is a matrix stacking the corresponding N eigenvectors $\{\mathbf{u}_k\}_{k=1}^N$. The impact matrix (R) is given by:

$$R = (\Psi P + \Psi - \Gamma)^{-1} \Pi,$$

where $(\Psi, \Gamma, \Theta, \Pi)$ are the matrices from the system of second-order difference equations (35).

Proof. See Online Appendix I.3. □

The solutions for these impact and transition matrices (\mathbf{R} , \mathbf{P}) depend only on the trade and capital share matrices (\mathbf{S} , \mathbf{T} , \mathbf{B} , \mathbf{X}) and parameters (ψ , θ , β , δ , ϵ , μ_i). Given this closed-form solution for the wealth state variables $\{\tilde{\mathbf{a}}_t\}_{t=1}^{\infty}$, we can recover all other endogenous variables (including capital $\{\tilde{\mathbf{k}}_t\}_{t=1}^{\infty}$) as linear functions of these state variables, as shown in Online Appendix I.2.

3.4 Convergence Dynamics Versus Fundamental Shocks

Using Proposition 3, the transition path of the economy's state variables can be additively decomposed into the contributions of convergence dynamics given initial conditions and fundamental shocks. Applying equation (36) across time periods, we obtain:

$$\ln \mathbf{a}_t - \ln \mathbf{a}_{-1} = \underbrace{\sum_{s=0}^t \mathbf{P}^s (\ln \mathbf{a}_0 - \ln \mathbf{a}_{-1})}_{\text{convergence given initial fundamentals}} + \underbrace{\sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R} \tilde{\mathbf{f}}}_{\text{dynamics from fundamental shocks}} \quad \text{for all } t \geq 1. \quad (37)$$

In the absence of shocks to fundamentals ($\tilde{\mathbf{f}} = \mathbf{0}$), the second term on the right-hand side of equation (37) is zero. In this case, the evolution of the state variables is shaped solely by convergence dynamics given initial conditions, and converges over time to:

$$\ln \mathbf{a}_{\text{initial}}^* = \lim_{t \rightarrow \infty} \ln \mathbf{a}_t = \ln \mathbf{a}_{-1} + (\mathbf{I} - \mathbf{P})^{-1} (\ln \mathbf{a}_0 - \ln \mathbf{a}_{-1}), \quad (38)$$

where $(\mathbf{I} - \mathbf{P})^{-1} = \sum_{s=0}^{\infty} \mathbf{P}^s$ is well-defined under the condition that the spectral radius of \mathbf{P} is smaller than one.

In contrast, if the economy is initially in a steady state at time 0, the first term on the right-hand side of equation (37) is zero. In this case, the transition path of the state variables is solely driven by the second term for fundamental shocks, and follows:

$$\tilde{\mathbf{a}}_t = \ln \mathbf{a}_t - \ln \mathbf{a}_0 = \sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R} \tilde{\mathbf{f}} = (\mathbf{I} - \mathbf{P}^t) (\mathbf{I} - \mathbf{P})^{-1} \mathbf{R} \tilde{\mathbf{f}} \quad \text{for all } t \geq 1. \quad (39)$$

In period $t = 1$ when the shocks occur, the response of the state variables is $\tilde{\mathbf{a}}_1 = \mathbf{R} \tilde{\mathbf{f}}$. Taking the limit as $t \rightarrow \infty$ in equation (39), the comparative steady-state response is

$$\lim_{t \rightarrow \infty} \tilde{\mathbf{a}}_t = \ln \mathbf{a}_{\text{new}}^* - \ln \mathbf{a}_{\text{initial}}^* = (\mathbf{I} - \mathbf{P})^{-1} \mathbf{R} \tilde{\mathbf{f}}. \quad (40)$$

A key implication of this additive separability in equation (37) is that we can examine the

economy's dynamic response to fundamental shocks separately from its convergence towards an initial steady state with unchanged fundamentals. Therefore, without loss of generality, we focus in the remainder of this section on an economy that is initially in steady state.

3.5 Spectral Analysis of the Transition Matrix P

We now provide a further analytical characterization of the economy's dynamic response to fundamental shocks using a spectral analysis of the transition matrix. We show that the speed of convergence to steady state and the evolution of the wealth state variables along the transition path can be written solely in terms of the eigenvalues and eigenvectors of this transition matrix. Since all other endogenous variables are linear functions of the wealth state variables, we also obtain impulse responses of each of the endogenous variables with respect to shocks to productivity and goods and capital market frictions.

3.5.1 Eigendecomposition of the Transition Matrix

We begin by using the eigendecomposition of the transition matrix, $P \equiv U\Lambda V$, where Λ is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and $V = U^{-1}$. For each eigenvalue λ_h , the h -th column of U (\mathbf{u}_h) and the h -th row of V (\mathbf{v}'_h) are the corresponding right- and left-eigenvectors of P , respectively, such that

$$\lambda_h \mathbf{u}_h = P \mathbf{u}_h, \quad \lambda_h \mathbf{v}'_h = \mathbf{v}'_h P.$$

That is, \mathbf{u}_h (\mathbf{v}'_h) is the vector that, when left-multiplied (right-multiplied) by P , is proportional to itself but scaled by the corresponding eigenvalue λ_h .² We refer to \mathbf{u}_h simply as eigenvectors. Both $\{\mathbf{u}_h\}$ and $\{\mathbf{v}'_h\}$ are bases that span the N -dimensional state space.

3.5.2 Eigen-shock

We next introduce a particular type of shock to fundamentals that proves useful for characterizing the model's transition dynamics. We define an *eigen-shock* as a shock to fundamentals ($\tilde{\mathbf{f}}_{(h)}$) for which the initial impact of the shock on the state variables ($R\tilde{\mathbf{f}}_{(h)}$) coincides with a real eigenvector of the transition matrix (\mathbf{u}_h). With N state variables ($\tilde{\mathbf{a}}$) and $6 \times N$ fundamental shocks ($\tilde{z}, \tilde{\eta}, \tilde{\kappa}^{in}, \tilde{\kappa}^{out}, \tilde{\tau}^{in}, \tilde{\tau}^{out}$), the space of fundamental shocks is of higher dimension than the space of state variables. Therefore, many fundamental shocks generate identical time paths for the state variables. In fact, for any fundamental shock vector ($\tilde{\mathbf{f}}$), there exists a productivity shock

²Note that P need not be symmetric. This eigendecomposition exists if the transition matrix has distinct eigenvalues. We construct the right-eigenvectors such that the 2-norm of \mathbf{u}_h is equal to 1 for all h , where note that $\mathbf{v}'_i \mathbf{u}_h = 1$ for $i = h$ and $\mathbf{v}'_i \mathbf{u}_h = 0$ otherwise.

vector ($\tilde{\mathbf{z}}$) that generates the same time path of the state variables. For expositional simplicity, we define the eigen-shocks in terms of shocks to productivity ($\tilde{\mathbf{z}}$), setting all other shocks equal to zero.³ Consequentially, the impact of the eigen-shocks $\{\tilde{\mathbf{f}}_{(h)}\}_{h=1}^N$ form a basis that spans the N -dimensional state space. Each eigenvector of \mathbf{P} (\mathbf{u}_h) has a corresponding eigen-shock for which $\mathbf{R}\tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$.

In general, there is no reason why any vector of empirical shocks to fundamentals across countries should correspond to an eigen-shock. But we can use these eigen-shocks to characterize the impact of any empirical shock using the following two properties. First, we can solve for these eigen-shocks from the observed data, because the impact matrix (\mathbf{R}) and the transition matrix (\mathbf{P}) depend solely on our observed trade and capital share matrices ($\mathbf{S}, \mathbf{T}, \mathbf{B}, \mathbf{X}$) and the structural parameters of the model $\{\psi, \theta, \beta, \delta, \epsilon, \mu_i\}$. Second, the initial and dynamic impact on the state variables from any vector of empirical shocks to fundamentals ($\tilde{\mathbf{f}}$) can be equivalently expressed as a linear combination of the impact from eigen-shocks ($\tilde{\mathbf{f}}_{(h)}$), where the weights or loadings in this linear combination can be recovered from a linear projection (regression) of the initial impact from the observed shocks ($\mathbf{R}\tilde{\mathbf{f}}$) on the initial impact from the eigen-shocks ($\mathbf{R}\tilde{\mathbf{f}}_{(h)}$). Using this property, the transition path of the state variables in response to any vector of empirical shocks to fundamentals can be expressed solely in terms of the eigenvalues and eigenvectors of the transition matrix, as summarized in the following proposition.

Proposition 4. Spectral Analysis. *Consider an economy that is initially in steady state at time $t = 0$ when agents learn about one-time, permanent shocks to fundamentals ($\tilde{\mathbf{f}} \equiv [\tilde{\mathbf{z}} \ \tilde{\boldsymbol{\eta}} \ \tilde{\boldsymbol{\kappa}}^{in} \ \tilde{\boldsymbol{\kappa}}^{out} \ \tilde{\boldsymbol{\tau}}^{in} \ \tilde{\boldsymbol{\tau}}^{out}]'$) from time $t = 1$ onwards. The transition path of the state variables (\mathbf{a}_t) can be written as a linear combination of the eigenvalues (λ_h) and eigenvectors (\mathbf{u}_h) of the transition matrix:*

$$\tilde{\mathbf{a}}_t = \sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R}\tilde{\mathbf{f}} = \sum_{h=1}^{2N} \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \mathbf{v}_h' \mathbf{R}\tilde{\mathbf{f}} = \sum_{h=2}^{2N} \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \varrho_h, \quad (41)$$

where the weights in this linear combination (ϱ_h) can be recovered as the coefficients in a linear projection (regression) of the initial impact from the observed shocks ($\mathbf{R}\tilde{\mathbf{f}}$) on the initial impact from the eigen-shocks ($\mathbf{R}\tilde{\mathbf{f}}_{(h)}$).

Proof. The proposition follows from the eigendecomposition of the transition matrix: $\mathbf{P} \equiv \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}$, as shown in Online Appendix I.4. \square

³Recall from Proposition 2 that the dimension of the state space is N , whereas the dimension of the fundamental shocks is $6 \times N$, because $\tilde{\mathbf{f}}$ includes shocks to goods and capital productivities ($\tilde{\mathbf{z}}, \tilde{\boldsymbol{\eta}}$) and aggregations of bilateral shocks to trade and capital frictions ($\tilde{\boldsymbol{\kappa}}^{in}, \tilde{\boldsymbol{\kappa}}^{out}, \tilde{\boldsymbol{\tau}}^{in}, \tilde{\boldsymbol{\tau}}^{out}$). Therefore, defining our eigen-shocks in terms of productivity shocks ($\tilde{\mathbf{z}}$) ensures that each eigenvector is associated with a unique eigen-shock (up to scale).

Another important property of an eigen-shock is that the speed of convergence to steady state, as measured by the half-life of convergence to steady state, depends solely on the associated eigenvalue of the transition matrix, as summarized in the following proposition.

Proposition 5. Speed of Convergence. *Consider an economy that is initially in steady state at time $t = 0$ when agents learn about one-time, permanent shocks to fundamentals ($\tilde{\mathbf{f}} \equiv [\tilde{z} \ \tilde{\eta} \ \tilde{\kappa}^{in} \ \tilde{\kappa}^{out} \ \tilde{\tau}^{in} \ \tilde{\tau}^{out}]'$) from time $t = 1$ onwards. Suppose that these shocks are an eigen-shock ($\tilde{\mathbf{f}}_{(h)}$), for which the initial impact on the state variables at time $t = 1$ coincides with a real eigenvector (\mathbf{u}_h) of the transition matrix (\mathbf{P}): $\mathbf{R}\tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$. The transition path of the state variables (\mathbf{a}_t) in response to such an eigen-shock ($\tilde{\mathbf{f}}_{(h)}$) is :*

$$\tilde{\mathbf{a}}_t = \sum_{j=2}^{2N} \frac{1 - \lambda_j^t}{1 - \lambda_j} \mathbf{u}_j \mathbf{v}_j' \mathbf{u}_h = \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \quad \implies \quad \ln \mathbf{a}_{t+1} - \ln \mathbf{a}_t = \lambda_h^t \mathbf{u}_h,$$

and the half-life of convergence to steady state is given by:

$$t_h^{(1/2)}(\tilde{\mathbf{f}}) = - \left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil,$$

for all state variables $h = 2, \dots, 2N$, where $\tilde{a}_{i\infty} = a_{i,new}^* - a_{i,initial}^*$ and $\lceil \cdot \rceil$ is the ceiling function.

Proof. The proposition follows from the eigendecomposition of the transition matrix ($\mathbf{P} \equiv \mathbf{U}\mathbf{\Lambda}\mathbf{V}$), for the case of an eigen-shock in which the initial impact of the shocks to fundamentals on the state variables at time $t = 1$ coincides with a real eigenvector ($\mathbf{R}\tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$) of the transition matrix (\mathbf{P}), as shown in Online Appendix I.5. \square

We focus on the speed of convergence for wealth in Proposition 5, because the log deviations in wealth from the initial steady state ($\tilde{\mathbf{a}}$) are the state variables of our dynamical system. Nevertheless, since the log deviations in all other endogenous variables from steady state (including capital, $\tilde{\mathbf{k}}$) are linear functions of these log deviations in wealth, these other endogenous variables have the same convergence properties as wealth.

From Proposition 5, the impact of an eigen-shock ($\tilde{\mathbf{f}}_{(h)}$) on the state variables in each time period is always proportional to the corresponding eigenvector (\mathbf{u}_h), and decays exponentially at a rate determined by the associated eigenvalue (λ_h), as the economy converges to the new steady state.⁴ These eigenvalues fully summarize the economy's speed of convergence in response to

⁴In general, these eigenvectors and eigenvalues can be complex-valued. If the initial impact is the real part of a complex eigenvector \mathbf{u}_h ($\tilde{\mathbf{R}}\tilde{\mathbf{f}} = \text{Re}(\mathbf{u}_h)$), then $\ln \mathbf{a}_{t+1} - \ln \mathbf{a}_t = \text{Re}(\lambda_h^t \mathbf{u}_h) \neq \text{Re}(\lambda_h) \cdot \text{Re}(\lambda_h^{t-1} \mathbf{u}_h)$. That is, the impact no longer decays at a constant rate λ_h . Instead, the complex eigenvalues introduce oscillatory motion as the dynamical system converges to the new steady state. In our empirical application, the imaginary components of \mathbf{P} 's eigenvalues are small, implying that oscillatory effects are small relative to the effects that decay exponentially.

eigen-shocks, even in our setting with many asymmetric countries and a rich geography of trade and capital market frictions.

In general, each eigen-shock ($\tilde{\mathbf{f}}_{(h)}$) has a different speed of convergence (as captured by the associated eigenvalue λ_h), which reflects the fact that the speed of convergence to steady state does not only depend on the structural parameters of the model ($\psi, \theta, \beta, \delta, \epsilon, \mu_i$), but also on the incidence of the fundamental shock on the state variables in each country (as captured by $\mathbf{u}_h = \mathbf{R}\tilde{\mathbf{f}}_{(h)}$). From Proposition 4, any empirical shock ($\tilde{\mathbf{f}}$) can be expressed as a linear combination of the eigen-shocks. Therefore, the speed of convergence also varies across these empirical shocks with their incidence on the state variables in each country, reflecting the extent to which they load on eigen-shocks with slow versus fast convergence.

3.6 Goods and Capital Market Integration and Convergence

We now use our analytical results for the economy's transition path to examine the role of goods and capital market integration in determining the speed of convergence in response to fundamental shocks. To simplify the exposition, we begin by considering the special case of the model with a separation between (i) workers, who earn wage income and live hand to mouth, and (ii) capitalists, who have log utility and make forward-looking consumption-saving decisions. We later generalize our analysis to a representative agent and CRRA preferences.

In this special case of a separation between workers and capitalists, capitalists with log utility consume a constant fraction $(1 - \beta)$ of their capital wealth every period. Therefore, the evolution of the log deviations in the wealth state variables from steady state simplifies as follows:

$$\tilde{a}_{nt+1} - \tilde{a}_{nt} = (1 - \beta + \beta\delta)(\tilde{v}_{nt} - \tilde{p}_{nt}), \quad (42)$$

where the derivations for this subsection are reported in Online Appendix J.

A common measure of the speed of convergence is the slope coefficient from a regression of log changes on log initial levels of a variable, as in a conventional β -convergence regression from the growth literature. From equation (42), this measure of the speed of convergence for log deviations in wealth depends on the covariance between the log deviation in the real return ($\tilde{v}_{nt} - \tilde{p}_{nt}$) and the log deviation in the initial level of wealth (\tilde{a}_{nt}):

$$\frac{\text{Cov}(\tilde{a}_{nt+1} - \tilde{a}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = (1 - \beta + \beta\delta) \frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})}. \quad (43)$$

These log deviations in the real return ($\tilde{v}_{nt} - \tilde{p}_{nt}$) depend on both capital market integration (through the nominal return (\tilde{v}_{nt})) and goods market integration (through the consumption price index (\tilde{p}_{nt})). The log deviations in the nominal return (\tilde{v}_{nt}) in turn depend on log deviations

in rental rates (\tilde{r}_{nt}). Using the first-order condition for cost minimization in production, and assuming a common labor share (μ) across countries and a constant labor endowment in each country (ℓ_n), we have the following relationship between log deviations in the rental rate (\tilde{r}_{nt}) and log deviations in the capital stock (\tilde{k}_{nt}):

$$\tilde{r}_{nt} = \tilde{p}_{nnt} - \mu \tilde{k}_{nt}, \quad (44)$$

where \tilde{p}_{nnt} is the log deviation in the local price of a country's own good from steady state (recall $\tau_{nnt} = 1$), and in general differs from the log deviation in the consumption price index (\tilde{p}_{nt}) that is a CES aggregate of the goods produced by all countries.

To provide economic intuition for the impact of goods and capital market integration on the speed of convergence, we evaluate this measure of the speed of convergence (43) for the limiting cases of completely open and completely closed goods and capital markets.

3.6.1 CNGM (Trade and Capital Autarky)

Under capital autarky ($\kappa_{nit} \rightarrow \infty$ for $n \neq i$), each country's wealth equals its capital stock ($\tilde{a}_{nt} = \tilde{k}_{nt}$), and the nominal return equals the domestic rental rate ($\tilde{v}_{nt} = \tilde{r}_{nt}$). Under trade autarky ($\tau_{nit} \rightarrow \infty$ for $n \neq i$), the consumption price index equals the local price of a country's own good ($\tilde{p}_{nt} = \tilde{p}_{nnt}$). Using these results in equations (42)-(44), we find that with a Cobb-Douglas production technology and a common labor share (μ), the speed of convergence to steady state depends solely on this labor share:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\mu. \quad (45)$$

Intuitively, there is diminishing marginal physical productivity of capital in the production technology. The larger the labor share (μ), the stronger these diminishing marginal returns to capital, and the faster the rate of convergence in capital and wealth towards steady state.

3.6.2 Frictionless Trade and Capital Autarky

Under capital autarky ($\kappa_{nit} \rightarrow \infty$ for $n \neq i$), each country's wealth equals its capital stock ($\tilde{a}_{nt} = \tilde{k}_{nt}$), and the nominal return equals the domestic rental rate ($\tilde{v}_{nt} = \tilde{r}_{nt}$). With frictionless trade ($\tau_{nit} = 1$ for all n, i), the consumption price index takes the same value across all countries ($\tilde{p}_{nt} = \tilde{p}_t$ for all n). But the local price of a country's own good can differ from the consumption price index ($\tilde{p}_{nnt} \neq \tilde{p}_{nt}$), because countries' goods are imperfect substitutes ($1 < \sigma < \infty$). Using these results in equations (42)-(44), frictionless trade alone implies *faster* convergence than in the

CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\frac{1}{\sigma} (1 - \mu) - \mu. \quad (46)$$

Intuitively, with autarkic capital markets, wealth accumulation in a given country expands its capital stock, which increases output of its good. With frictionless trade, in order for consumers worldwide to demand more of this good instead of the substitutes produced by other countries, the price of this good must fall. Therefore, wealth accumulation not only leads to a decline in the marginal physical product of capital as in the closed economy (captured by μ), but also leads to a fall in the price of a country's good (with an elasticity determined by σ), which implies a larger decline in the value marginal product of capital, and faster convergence to steady state.

3.6.3 Trade Autarky and Perfect Capital Markets

Under trade autarky ($\tau_{nit} \rightarrow \infty$ for $n \neq i$), the consumption price index equals the price of a country's domestic good ($\tilde{p}_{nt} = \tilde{p}_{nit}$). Under perfect capital markets ($\kappa_{nit} = 1$ for all n, i), the nominal return takes the same value across all countries ($\tilde{v}_{nt} = \tilde{v}_t$ for all n). But the domestic capital stock can differ from domestic wealth ($\tilde{k}_{nt} \neq \tilde{a}_{nt}$), and the domestic rental rate can differ from the nominal return ($\tilde{r}_{nt} \neq \tilde{v}_{nt}$), because of imperfect substitutability of capital between countries ($1 < \epsilon < \infty$). Using these results in equations (42)-(44), perfect capital markets alone imply *faster* convergence than in the CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\frac{1}{\epsilon} (1 - \mu) - \mu. \quad (47)$$

Intuitively, with perfect capital markets, capital reallocates across countries to equalize the nominal return for a given world stock of wealth. Nevertheless, countries accumulate wealth at different rates, because of differences in the real consumption price index under trade autarky, which lead to differences in the real return. Under perfect capital markets, wealth accumulation in a given country expands investment at home and abroad, which increases the country's income from these investments. Under trade autarky, this increased country income is spent on domestic goods, which bids up the domestic factor prices, where the elasticity of the domestic rental rate with respect to this expenditure depends on ϵ . Higher domestic factor prices increase the price of the domestic good, and hence the domestic consumption price index, which reduces the real return, and speeds up convergence to steady state.

3.6.4 Frictionless Trade and Perfect Capital Markets

Under frictionless trade ($\tau_{nit} = 1$ for all n, i), the consumption price index takes the same value across all countries ($\tilde{p}_{nt} = \tilde{p}_t$ for all n). Under perfect capital markets ($\kappa_{nit} = 1$ for all n, i), the

nominal return takes the same value across all countries ($\tilde{v}_{nt} = \tilde{v}_t$ for all n). Using these results in equations (42)-(44), the real return takes the same value across all countries ($\tilde{v}_{nt} - \tilde{p}_{nt} = \tilde{v}_t - \tilde{p}_t$ for all n), and hence is uncorrelated with the initial level of wealth in each country (\tilde{a}_{nt}). Therefore, frictionless trade and perfect capital markets together imply *slower* convergence to steady state than in the CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = 0. \quad (48)$$

Intuitively, with frictionless trade and perfect capital markets, movements of goods and capital between countries equalize the real return for a given world stock of wealth. Wealth accumulation in each country only affects this common real return ($\tilde{v}_{nt} - \tilde{p}_{nt} = \tilde{v}_t - \tilde{p}_t$) through the world stock of wealth. Therefore, each country accumulates wealth at the same rate, as determined by this common real return, and initial differences in wealth persist forever, as the world economy gradually converges to the world steady-state level of wealth.

For expositional simplicity, we have examined the speed of convergence in this section by comparing autarky and frictionless trade and capital markets for the special case of a separation of workers and capitalists with logarithmic utility. The following proposition extends these results for a representative agent and CRRA utility.

Proposition 6. Goods and Capital Market Integration. *The speed of convergence to steady state is **faster** than in the CNGM with **either** (i) frictionless trade and capital autarky **or** (ii) trade autarky and perfect capital markets. This speed of convergence is **slower** than in the CNGM with (iii) **both** frictionless trade and perfect capital markets.*

Proof. See Online Appendix J. □

More generally, for intermediate levels of trade and capital market frictions in between autarky and frictionless trade, we again find that reductions in *both* goods *and* capital market frictions slow convergence, whereas reductions in *either* goods *or* capital market frictions *alone* accelerate convergence.⁵ The intuition is that reductions in both frictions lead to a capital reallocation across countries that reduces differences in the real return, and hence leads to slower convergence to steady state after this reallocation. In contrast, when *either* goods *or* capital market frictions *alone* are reduced, this closure of gaps in the real return does not occur, and the real return becomes more sensitive to local wealth, implying faster convergence.

⁵Proposition 7 in Online Appendix J.2.1 further extends the results in this section by considering this case of intermediate levels of trade and capital market frictions, and evaluating the impact of marginal changes in these frictions on the speed of convergence.

3.7 Three-Country Example

We now illustrate these general results using a simple example. Consider a world with three economies—call them US, EU, and China—and a common labor share across countries. The first two economies (US and EU) are relatively more integrated in terms of both trade and capital markets. The third economy, China, is relatively more remote. Suppose the trade expenditure share and capital allocation share matrices (\mathbf{S} and \mathbf{B}) take the following form:

$$\mathbf{S} = \mathbf{B} = \begin{bmatrix} .8 - x & .1 + x & .1 \\ .1 + x & .8 - x & .1 \\ .1 & .1 & .8 \end{bmatrix}, \quad x \in [0, 0.35]$$

That is, 80% of China's expenditure and capital wealth are allocated domestically, with the remaining 20% equally split between the US and the EU. The US and the EU are symmetric. They each allocate 10% + x expenditure and capital wealth to each other, 80% - x domestically, and the remaining 10% to China. The value of x captures the degree of integration between the EU and the US. When $x = 0$, all three economies are symmetric. When $x = 0.35$, there is frictionless trade and capital markets between the US and the EU.

The three eigenvectors in this example take the following forms:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -\gamma \\ -\gamma \\ 1 \end{bmatrix}.$$

The associated eigenshocks, when expressed in terms of shocks to productivities, take the following forms:

$$\tilde{\mathbf{f}}_1 \equiv \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \quad \tilde{\mathbf{f}}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{f}}_3 = \begin{bmatrix} \xi \\ \xi \\ -1 \end{bmatrix}.$$

The associated eigenvalues, as well as the values of γ and ξ (in \mathbf{u}_3 and $\tilde{\mathbf{f}}_3$), all depend on the model parameters $\{\beta, \psi, \delta, \theta, \epsilon\}$, the labor share μ , and the degree of integration between the US and the EU, x .

We now interpret these eigencomponents. The first component captures a global productivity shock that is common to all countries in the world. If the world economy was originally in a steady state, the shock $\tilde{\mathbf{f}}_1$ would induce capital dynamics around the globe in a homogeneous manner—capital wealth would evolve proportionally across countries, with the share of global capital wealth belonging to each country staying constant along the entire transition path. In

other words, the shock \tilde{f}_1 does not lead to capital reallocation across countries. Instead, the total global capital wealth behaves as in the CNGM, with the associated eigenvalue coinciding with that of the CNGM.

The second eigen-shock \tilde{f}_2 captures rising dispersion in productivities of the two well-integrated economies, while holding economic fundamentals in the more remote country (China) constant. This shock induces dynamic reallocation of wealth within the integrated block, while leaving wealth in China unchanged along the entire transition path.

Finally, the third eigen-shock \tilde{f}_3 captures rising dispersion in productivities and the resulting reallocation of wealth between the integrated block (US and EU) and China. The relative wealth between the US and EU remains unchanged along the entire transition path following this shock.

The eigenvalues associated with the second and third components capture the rate at which these shocks affect the distribution of capital across countries. Numerically, we find that $|\lambda_2| > |\lambda_3|$, meaning the reallocation within the integrated block (i.e., between US and EU) is slower than the reallocation between the integrated block and China. Moreover, we find that $|\lambda_2|$ increases in x , meaning that the reallocation of wealth between the US and the EU gets slower the more integrated they are. The reason is again that greater integration leads to a larger initial reallocation of wealth, which weakens the correlation between the real return and the initial level of wealth, and hence leads to slower convergence towards steady state.

4 Quantitative Analysis

We now examine the quantitative implications of the neoclassical growth model with trade and capital market frictions and imperfect substitutability of goods and capital across countries. In Subsection 4.1, we choose the parameters for our calibration. In Subsection 4.2, we summarize the data construction. In Subsection 4.3, we estimate the gravity equations for trade and capital holdings to show that these well-known relations hold in our data. In Subsection 4.4, we analyze the impulse responses of wealth and capital stock to productivity shocks in our model as well as the conventional closed- and open-economy neoclassical growth models. In Subsection 4.5, we compare the speed of convergence to the steady state in our model and the CNGM. In Subsection 4.6, we study a counterfactual decoupling of goods and capital markets between the United States and China.

4.1 Parameterization

We calibrate the model using standard parameter values from the literature. We set the depreciation rate to $\delta = 0.05$. We set the discount factor to $\beta = 0.95$. We set the elasticity of intertemporal

substitution to $\psi = 0.5$, which is the upper range of the confidence intervals for a cross section of countries (Yogo, 2004). We set the trade elasticity to $\theta = 5$, which is the baseline value in Costinot and Rodríguez-Clare (2014) and is within the range of 2 to 12 in Eaton and Kortum (2002). We assume a capital elasticity of $\epsilon = 4$, based on the estimate for international equity markets in Kojien and Yogo (2019).

4.2 Data Construction

We use publicly available data on the national accounts, bilateral trade, and bilateral capital holdings. Our sample covers 46 countries plus the rest of the world for the period 2013 to 2017. Online Appendix L contains further details about the construction of the bilateral capital holdings.

4.2.1 National Accounts

The data on gross domestic product (GDP), the labor share (i.e., labor compensation as a share of GDP), the capital stock, and the population are from the Penn World Tables (Feenstra, Inklaar, and Timmer, 2015). We map the model to these data as μ_{it} for the labor share, k_{it} for the capital stock, and ℓ_i for the population. We construct the wage w_{it} as the labor compensation divided by the population. We construct the capital payment $r_{it}k_{it}$ as GDP minus the labor compensation. Then the rental rate of capital r_{it} is the capital payment divided by the capital stock.

4.2.2 Bilateral Trade

The data on bilateral trade are from the Comtrade Database (United Nations, 2013–2017). Following Feenstra, Lipsey, et al. (2005), we use the import data, which are more likely to be accurate than the export data because importers typically levy trade policies. We construct domestic expenditure as the gross output from Timmer et al. (2015) minus total exports. Thus, we have a matrix of bilateral expenditure E_{nit} by importer n on goods produced by exporter i , including domestic expenditure as the diagonal elements. We construct the bilateral expenditure shares of importer n as $S_{nit} = E_{nit} / \sum_{h=1}^N E_{nht}$. We construct the bilateral income shares of exporter i as $T_{int} = E_{nit} / \sum_{h=1}^N E_{hit}$.

4.2.3 Bilateral Capital Holdings

We construct a comprehensive measure of bilateral capital holdings as the sum of bilateral portfolio holdings (in debt securities, equity securities, and fund shares) and bilateral direct investment. We construct the total amounts outstanding in debt securities, equity securities, and fund shares, based on OECD (2013–2017) for the OECD countries and Bank for International Settlements (2013–2017) and World Bank (2013–2017) for the non-OECD countries. The availability of

the data on total amounts outstanding limits our sample to 46 countries. Our data on bilateral portfolio holdings are from the Coordinated Portfolio Investment Survey (International Monetary Fund, 2013–2017). To account for investments through tax havens, we restate both the total amounts outstanding and bilateral portfolio holdings from the issuer’s residency to nationality, using the restatement matrices of the Global Capital Allocation Project (Coppola et al., 2021). We construct domestic portfolio holdings as the amount outstanding minus the sum of foreign portfolio holdings. Our data on bilateral direct investment, restated from residency to nationality accounting, are from Damgaard, Elkjaer, and Johannesen (2019). The availability of the data on restated bilateral direct investment limits our sample period to 2013 to 2017.

We construct bilateral ownership shares as bilateral portfolio and direct investment as a share of the total investment in each producer country. We multiply the capital payment $r_{it}k_{it}$ by the bilateral ownership shares to construct the capital income $a_{nit}v_{nit}$ earned by investor n in producer i . We then construct capital income earned by investor n in the rest of the world (ROW) as the residual: $a_{n,ROW,t}v_{n,ROW,t} = a_{nt}v_{nt} - \sum_{i \neq ROW} a_{nit}v_{nit}$. Thus, the bilateral capital income sums to total capital income $a_{nt}v_{nt}$ by investor country and to total capital payment $r_{it}k_{it}$ by producer country. We construct the bilateral portfolio shares of investor n as $B_{nit} = a_{nit}v_{nit} / \sum_{h=1}^N a_{nht}v_{nht}$. We construct the bilateral capital payment shares of producer i as $X_{int} = a_{nit}v_{nit} / \sum_{h=1}^N a_{hit}v_{hit}$.

4.3 Gravity Equations in Trade and Capital Holdings

We estimate the gravity equations for trade and capital holdings to show that these well-known relations hold in our data. Using the cross section of 46 countries in 2017, we estimate the gravity equation:

$$Y_{ni} = D_{ni}^{\delta} \vartheta_n \vartheta_i \nu_{ni} \quad (49)$$

for all observations $n \neq i$. The variable Y_{ni} is either the expenditure E_{ni} of importer n on exporter i or the capital income $a_{nit}v_{nit}$ of investor n from producer i . The variable D_{ni} is the bilateral distance from the GeoDist Database (Mayer and Zignago, 2011). We use the simple distance, defined as the weighted distance between the most populous cities. The parameters ϑ_n and ϑ_i are origin and destination fixed effects. The variable ν_{ni} is the error term.

In column (1) of Table 1, we estimate the gravity equation (49) for trade in logarithms by ordinary least squares (OLS) with origin and destination fixed effects. We find a negative and highly significant relation between bilateral trade and distance with an elasticity of -1.18 and an R^2 of 0.88. The estimation in logarithms drops observations with a zero value for bilateral trade. We also estimate the gravity equation in levels by Poisson pseudo maximum likelihood (PPML), following Santos Silva and Tenreyro (2006) and Head and Mayer (2014). In column (2),

we again find a negative and highly significant relation between bilateral trade and distance with an elasticity of -0.79 . Thus, the gravity equation in trade is a robust relation in both levels and logarithms.

Table 1: Estimated Gravity Equations

	(1)	(2)	(3)	(4)
	Log Trade	Trade	Log Capital	Capital
Distance	-1.18 (0.02)	-0.79 (0.03)	-1.41 (0.05)	-0.63 (0.05)
Estimator	OLS	PPML	OLS	PPML
Observations	2,069	2,070	2,042	2,070
R^2	0.88		0.82	
Pseudo R^2		0.91		0.92

Note: Columns (1) and (3) are estimated in logarithms by ordinary least squares (OLS), and Columns (2) and (4) are estimated in levels by Poisson pseudo maximum likelihood (PPML). All specifications include importer and exporter country fixed effects. Robust standard errors, two-way clustered by importer and exporter, are reported in parentheses. The sample consists of the cross section of 46 countries in 2017.

In Column (3) of Table 1, we estimate the gravity equation (49) for capital holdings in logarithms by OLS with origin and destination fixed effects. We find a negative and highly significant relation between bilateral capital holdings and distance with an elasticity of -1.41 and an R^2 of 0.82. In Column (4), we estimate the gravity equation in levels by PPML and find a negative and highly significant relation between bilateral capital holdings and distance with an elasticity of -0.63 . The gravity equation for capital holdings suggests the presence of bilateral capital market frictions, perhaps due to information frictions (Portes and Rey, 2005), that are analogous to bilateral trade frictions.

To separate the explanatory power of distance from the origin and destination fixed effects, we apply the Frisch-Waugh-Lovell Theorem. We estimate two separate OLS regressions of log trade and log distance on origin and destination fixed effects. We then estimate an OLS regression of the residuals for log trade on the residuals for log distance. As we report in Online Appendix K.1, we find an R^2 of 0.54. When we repeat the same procedure for log capital holdings, we find an R^2 of 0.32. Thus, we find that distance has significant explanatory power for trade and capital holdings, even after removing the origin and destination fixed effects.

These estimated gravity equations suggest the presence of bilateral trade and capital market frictions if these frictions increase in bilateral distance. Based on this evidence, we proceed to study the quantitative implications of our model with trade and capital market frictions and imperfect substitutability of goods and capital across countries.

4.4 Impulse Responses to a Productivity Shock

Based on the closed-form solutions for the economy's transition path in Proposition 4, we analyze the impulse responses of wealth and capital to productivity shocks in our model as well as the conventional closed- and open-economy neoclassical growth models.

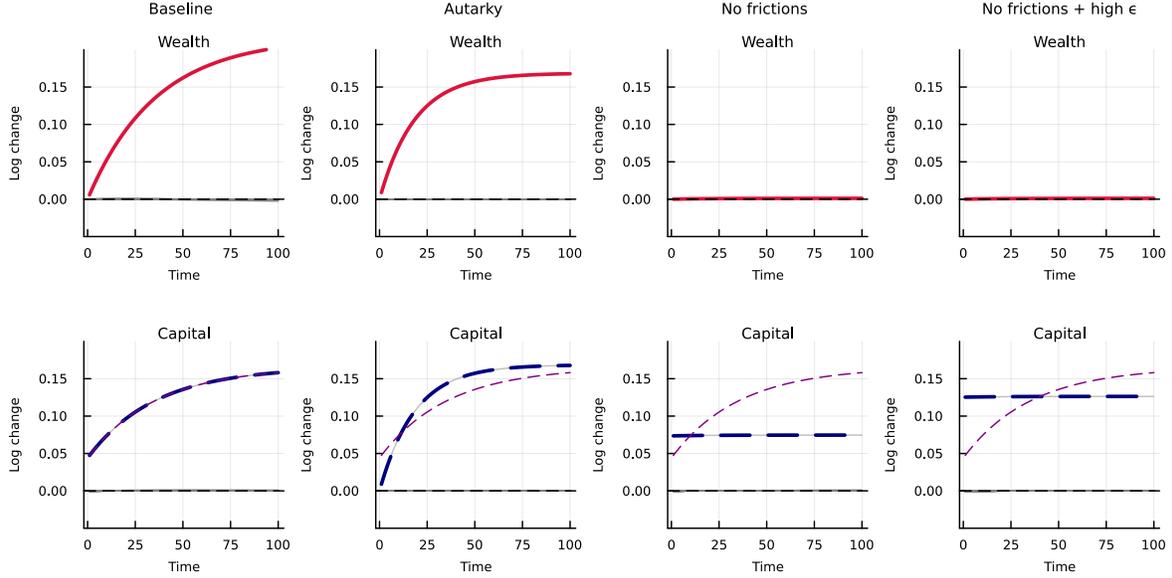
Figure 1 shows the impulse responses of wealth in red (top row) and capital stock in dashed blue (bottom row) to a 10 percent productivity shock in a relatively small country, Belgium. The first column is our baseline model with trade and capital market frictions and imperfect substitutability of goods and capital (i.e., $\theta = 5$ and $\epsilon = 4$). The second column is the CNGM. The third column is a special case of our baseline model with no trade and capital market frictions (i.e., $\tau_{ni} = \kappa_{ni} = 1$) and imperfect substitutability of goods and capital (i.e., $\theta = 5$ and $\epsilon = 4$). The last column is a version of the conventional open-economy neoclassical growth model, which is a special case of our baseline model with no trade and capital market frictions (i.e., $\tau_{ni} = \kappa_{ni} = 1$), imperfect substitutability of goods (i.e., $\theta = 5$), and perfect substitutability of capital (i.e., $\epsilon \rightarrow \infty$). The gray lines are the impulse responses of wealth and capital for all other countries, which are barely distinguishable from the horizontal axis because a productivity shock in a relatively small country has negligible impact on the global economy. For ease of comparison, the dashed purple line in all panels of the bottom row reproduces the impulse response for Belgium's capital stock in our baseline model.

In the CNGM (second column), a positive productivity shock in Belgium increases its steady-state wealth and capital stock. Capital market autarky implies that Belgium can accumulate capital only through domestic saving. By consumption smoothing, Belgium gradually accumulates wealth and capital at a rate that depends on the decreasing returns to capital investment. Goods and capital market autarky implies that the productivity shock in Belgium has no impact on the steady-state wealth and capital stock in the other countries.

In the special case of our model with no trade and capital market frictions and imperfect substitutability of goods and capital (third column), a positive productivity shock in Belgium leads to an immediate international capital reallocation. Without capital market frictions, all countries hold the same global portfolio. Because Belgium is a relatively small country, the productivity shock has a negligible impact on the composition of the global portfolio and hence the steady-state wealth in the other countries. In addition, the international capital reallocation implies that Belgium immediately converges to its new steady-state capital stock. Because Belgium is a relatively small country, the international capital reallocation has a negligible impact on the steady-state capital stock in the other countries (i.e., the gray lines are barely distinguishable from the horizontal axis).

In the conventional open-economy neoclassical growth model (last column), we have no trade

Figure 1: Impulse Responses to a Small-Country Productivity Shock



Note: The figure shows the impulse responses of wealth in red (top row) and capital stock in dashed blue (bottom row) to a 10 percent productivity shock in Belgium. The first column is the baseline model with trade and capital market frictions and imperfect substitutability of goods and capital (i.e., $\theta = 5$ and $\epsilon = 4$). The second column is the CNGM. The third column is a special case with no trade and capital market frictions (i.e., $\tau_{ni} = \kappa_{ni} = 1$) and imperfect substitutability of goods and capital (i.e., $\theta = 5$ and $\epsilon = 4$). The last column is the conventional open-economy neoclassical growth model, which is a special case with no trade and capital market frictions (i.e., $\tau_{ni} = \kappa_{ni} = 1$), imperfect substitutability of goods (i.e., $\theta = 5$), and perfect substitutability of capital (i.e., $\epsilon \rightarrow \infty$). For ease of comparison, the dashed purple line in all panels of the bottom row reproduces the impulse response for Belgium's capital stock in the baseline model.

and capital market frictions, imperfect substitutability of goods, and perfect substitutability of capital. Again, the international capital reallocation implies that Belgium immediately converges to its new steady-state capital stock. Relative to the third column, a higher substitutability of capital implies a larger increase in the steady-state capital stock in Belgium.

Our baseline model with trade and capital market frictions and imperfect substitutability of goods and capital (first column) has impulse responses that are between the extremes of the closed- and the open-economy neoclassical growth models. On impact, a positive productivity shock in Belgium increases its rental rate of capital, leading to an immediate international capital reallocation. However, the initial capital reallocation is smaller than that in the open-economy neoclassical growth model because of capital market frictions, and the rental rate of capital remains high relative to the steady state. Consequently, Belgian investors' real return on domestic investment remains high relative to the steady state. Just as in the CNGM, Belgium gradually accumulates wealth and capital to reach the new steady state, at which the real return is equalized across all investor countries. Interestingly, this accumulation of wealth and capital is slower than that in the CNGM, because the initial capital reallocation dampens the impact of the productivity

shock on the real return (i.e., the dashed purple line starts above but then lies below the blue line in the second column).

Although we use Belgium as an example of a relatively small country in Figure 1, we find similar results for China as an example of a relatively large country in Online Appendix K.3. An important difference is that a productivity shock in China has a larger impact on the composition of the global portfolio and hence the steady-state wealth and capital stock in the other countries. Bilateral trade and capital market frictions imply heterogeneous effects across countries through the global network of trade and capital holdings.

4.5 Speed of Convergence

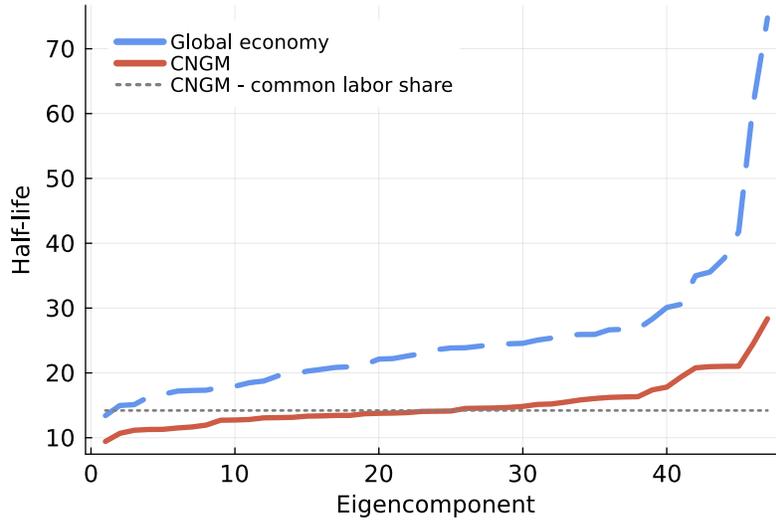
Using our closed-form solutions for the economy’s transition path, we now analyze the speed of convergence to steady state in our model relative to that in the CNGM. We begin by using our eigendecomposition in Proposition 3 to recover the eigenvectors (\mathbf{u}_h) and eigenvalues (λ_h) of the transition matrix (\mathbf{P}) that governs the updating of the wealth state variables over time. Using Proposition 5, we define an eigen-shock as a shock to fundamentals ($\tilde{\mathbf{f}}_{(h)}$) for which the initial impact on the state variables ($\mathbf{R}\tilde{\mathbf{f}}_{(h)}$) coincides with a real eigenvector of this transition matrix (\mathbf{u}_h). Each of these eigen-shocks corresponds to a different incidence of fundamental shocks on the wealth state variables ($\tilde{\mathbf{a}}_t$) for each country and is characterized by a speed of convergence that is determined by the corresponding eigenvalue (λ_h).

In Figure 2, the long-dashed blue line shows the implied half lives of convergence to steady state for each eigen-shock using the observed trade and capital share matrices for the year 2017. The vertical axis displays the half life for each eigen-shock, while the horizontal axis sorts these eigen-shocks in terms of increasing half-lives. With open goods and capital markets, these half-lives depend on the entire network of bilateral trade and capital frictions (as captured in the observed trade and capital share matrices) and the parameters of the model.

As a point of comparison, the solid red line displays half-lives of convergence to steady state for the CNGM, which corresponds to the special case of our model with autarky in both goods and capital markets. In this special case, the half-life of convergence only varies across eigen-shocks, because of differences across countries in the labor share (μ_i). As a further benchmark, the short-dashed black line shows the common half-life of convergence for the CNGM with a common labor share ($\mu_i = \mu$), equal to the average labor share across countries.

We find a substantially slower rate of convergence to steady state in our neoclassical growth model with open goods and capital markets and imperfect substitutability than in the CNGM. This speed of convergence also displays considerable heterogeneity across the eigen-shocks, ranging from around 15 to 75 years, compared to a range from 10 to 30 years in the CNGM with country-

Figure 2: Half Lives of Convergence to Steady State for each Eigen-shock



Note: Vertical axis shows half life of convergence to steady state in years for each eigen-shock. Horizontal axis shows the rank of the eigen-shocks in terms of increasing half life in years. Long-dashed blue line shows these half lives for our baseline model with trade and capital market frictions and imperfect substitutability between countries for the year 2017. Solid red line shows these half lives of convergence for the special case of the closed-economy neoclassical growth model (CNGM) with a country-specific labor share. Short-dashed black line shows these half-lives for the special case of the CNGM with a common labor share.

specific labor shares. Since any vector of empirical shocks to fundamentals can be written as a linear combination of the eigen-shocks, these results imply slow rates of convergence to steady state in response to vectors of empirical shocks. Therefore, our open-economy framework with imperfect substitutability provides a natural approach to addressing the concern that the speed of convergence in the CNGM is too fast relative to empirical transitions for plausible values of the elasticity of intertemporal substitution (as discussed, for example, in King and Rebelo 1993).

In Section K.4 of the Online Appendix, we use our eigendecomposition of the transition matrix (P) to examine how the speed of convergence to steady state depends on model parameters. We find an intuitive pattern of comparative statics. For example, a higher capital elasticity (ϵ) or a higher trade elasticity (θ) imply a longer half-life (slower convergence), because greater substitutability for either capital or goods reduces the absolute value of the covariance between the real return and the initial level of wealth (see equations (46) and (47)). A lower labor share (μ) also implies a longer half-life (slower convergence), because it implies a greater role for wealth accumulation, which again magnifies the impact of fundamental shocks, and hence requires a greater length of time for adjustment to occur.

4.6 Counterfactuals for U.S.-China Decoupling

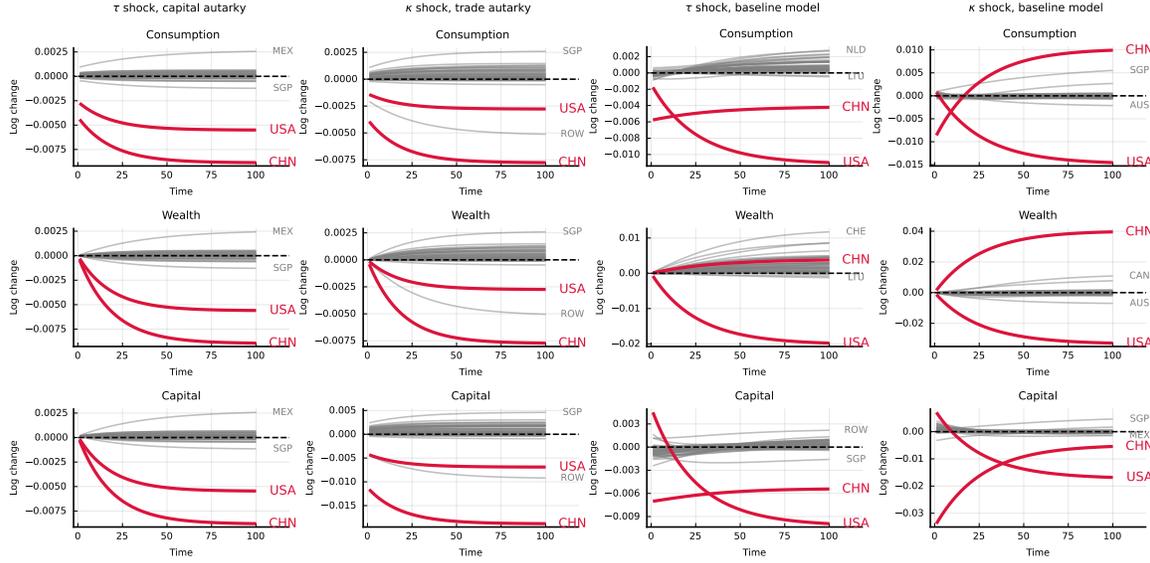
Since our framework matches the observed gravity equation relationships for bilateral trade and capital holdings, and allows for intertemporal consumption-savings decisions, it is particularly well suited for evaluating counterfactual policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling).

We now use our framework to evaluate two counterfactuals for U.S.-China decoupling: (i) a 50 percent increase in bilateral trade frictions alone between China and the United States, (ii) a 50 percent increase in bilateral capital frictions alone between these two countries. We undertake these counterfactuals for our baseline model using our linearization and the observed trade and capital share matrices (\mathbf{S} , \mathbf{T} , \mathbf{B} , \mathbf{X}) for 2017. We assume that agents at time $t = 0$ learn about a permanent increase in bilateral frictions from time $t = 1$ onwards. Using Propositions 3 and 4, we solve for the entire transition path of the wealth state variables and all other endogenous variables of the model from time $t = 1$ onwards.

We also compare the counterfactual predictions of our baseline open economy model to special cases with either capital autarky (and open trade) or trade autarky (and open capital markets), in order to highlight the interaction between goods and capital market integration. When we consider the special case with capital autarky, we replace the observed capital share matrices (\mathbf{B} , \mathbf{X}) with identity matrices, such that each country only invests domestically. Thus, we make sure to match the observed trade data in both cases, and only vary the degree of capital openness. Similarly, when we consider the special case with trade autarky, we replace the observed trade share matrices (\mathbf{S} , \mathbf{T}) with identity matrices, such that each country only consumes its own goods, while exactly matching observed data on capital holdings.

In Figure 3, we show the results of these four counterfactuals: (i) Higher bilateral trade frictions with open goods markets and capital autarky (far-left panel), (ii) Higher bilateral capital frictions with trade autarky and open capital markets (middle-left panel), (iii) Higher bilateral trade frictions with open goods and capital markets (middle-right panel), (iv) Higher bilateral capital frictions with open goods and capital markets (far-right panel). In each panel, we show the transition path for consumption in the top row, the transition path for the wealth state variables in the middle row, and the transition path for capital in the bottom row. We show results for China and the United States by the solid red lines labeled with three-letter international standards organization (ISO) country codes (CHN and USA). We show results for all other countries by the solid gray lines. We label the results for the other countries characterized by the largest and smallest changes in a variable with three-letter ISO country codes.

Figure 3: Counterfactuals for an Increase in Bilateral U.S.-China Trade and Capital Frictions



Note: Counterfactuals for permanent increase in bilateral frictions between China and the United States at time $t = 1$ using our closed-form solution for the economy's transition path. The first and the third columns show counterfactuals for 50% increase in trade frictions, and the second and fourth columns show counterfactuals for 50% increase in capital frictions. The first column considers the special case of the model with international trade but no international capital holdings. The second column considers the special case of the model with international capital holdings but no international trade. The last two columns consider our baseline model with both trade and capital holdings. Each row shows log deviations from the initial steady state. The first row shows these deviations for consumption (\tilde{c}_{it}). The second row shows these deviations for wealth (\tilde{a}_{it}). The bottom row shows these log deviations for capital (\tilde{k}_{it}).

4.6.1 Higher Trade Frictions with Open Goods Markets and Capital Autarky

With capital autarky and open goods markets (far-left panel), the financial account and the current account of the balance of payments are necessarily equal to zero, but there can be a trade imbalance that is offset by net investment income from domestic assets.

Higher U.S.-China trade frictions lead to an initial drop in consumption in both countries (top row), which captures foregone static welfare gains from trade. This initial drop in consumption is larger for China than for the United States, since the United States is a more central market for China's exports than China is for the United States.

With open goods markets, there are cross-substitution and market size effects on consumption in third countries. On the one hand, the higher cost of Chinese goods in the U.S. market, and the higher cost of U.S. goods in the Chinese market, increases the demand for other countries' goods. This cross-substitution effect implies that Mexico (MEX) enjoys the largest immediate increase in consumption from higher U.S.-China trade frictions. On the other hand, the reduction in income in China and the United States from higher bilateral trade frictions reduces the demand for other countries' goods. This market-size effect leads Singapore (SGP) to experience the largest

immediate reduction in consumption from higher U.S-China trade frictions.

In addition to conventional static welfare losses, the higher consumption price index in China and United States from higher bilateral trade frictions reduces the real return to investment, which leads to a gradual decumulation of wealth and capital (middle and bottom rows). This decumulation of wealth further reduces consumption in these two countries (top row), and gives rise to dynamic welfare losses, as wealth in these two countries gradually converges to its new lower steady-state level. With open goods markets, third countries can experience either increases or decreases in the real return to investment, depending on the balance of cross-substitution and market size effects. Therefore, Mexico experiences dynamic welfare gains from increased wealth accumulation, while Singapore experiences dynamic welfare losses.

4.6.2 Higher Capital Frictions with Trade Autarky and Open Capital Markets

With trade autarky and open capital markets (middle-left panel), imports, exports and the trade balance all equal zero, but there can be imbalances in the current and financial accounts, reflecting net investment income from wealth allocated at home and abroad.

Higher U.S.-China capital market frictions lead to an initial drop in consumption in both countries (top row). This initial decline in consumption reflects static welfare losses from capital market disintegration and the resulting international capital reallocation. The initial fall in consumption is again larger for China than for the U.S., because the U.S. is more important as a capital supplier for China than China is for the U.S.. Higher bilateral capital market frictions between the U.S. and China make these two countries less attractive for capital holdings, which causes a capital reallocation towards third countries (bottom row), with Singapore (SGP) and the Rest of the World (ROW) experiencing the largest and smallest inflows of capital, respectively (bottom row).

The increase in U.S.-China capital market frictions also reduces the real return to investment in both countries, which leads to a gradual decumulation of wealth and capital (middle and bottom rows). This decumulation of wealth further reduces consumption in both countries (top row), as wealth gradually converges to its new lower steady-state level. Again the effects are larger for China than for the United States. As third countries become more attractive for capital holdings, this increases the real return to investment in those countries, and induces wealth and capital accumulation (middle and bottom rows). Again Singapore (SGP) and the Rest of the World (ROW) experience the largest and smallest increases in the real return to investment and wealth accumulation, respectively.

4.6.3 Higher Trade Frictions with Open Goods and Capital Markets

We now examine higher bilateral trade frictions with open goods and capital markets (middle-right panel), in which case trade and investment income flows can be imbalanced, and there can be offsetting imbalances in the current and financial accounts.

Higher U.S.-China trade frictions again lead to an initial drop in consumption in both countries from foregone static welfare gains from trade (top row). As for changes in bilateral trade frictions under capital autarky (far-left panel), the initial decline in consumption is larger for China than for the United States, because the United States is a more central market for China's exports than China is for the United States.

However, with open capital markets, changes in bilateral trade frictions now lead to initial capital reallocation across countries. In particular, China becomes relatively less attractive for capital holdings, because of the decline in demand for its output in U.S. markets, while the United States becomes more attractive for capital holdings, as it substitutes away from consumption of Chinese goods towards local production. As a result, there is an initial capital reallocation from China and some third countries towards the United States (bottom row). While the static welfare effects on third countries depended only on the cross-substitution and market size effects under capital autarky (far-left panel), they now also depend on this international capital reallocation through open capital markets (middle-right panel), with the majority of other countries experiencing a reduction in the short-run supply of capital.

In addition to these static welfare effects, higher U.S.-China trade frictions also affect the real return to investment, which has dynamic welfare effects through the accumulation of wealth and capital. Here again we find a starkly different pattern of results from under capital autarky. With open goods and capital markets, the impact on China's real return depends not only on the deterioration in local capital market conditions, but also on the improvement in investment opportunities in other countries. Moreover, the reduction in demand for its goods in the U.S. market decreases China's consumption price index, leading to a cheaper cost of investment goods. Consequentially, its real return on investment rises, which leads to mild accumulation of wealth over time. In contrast, the U.S. experiences an increase in its consumption price index and the cost of investment goods, and a decline in the return in one of its major investment destinations (China), leading to a decumulation of wealth.

Comparing consumption in China and United States in the new steady state (top row), we find a reversal of fortune between China and the United States over time, with China losing more consumption in the short run, whereas the United States loses more consumption in the long run. This is in contrast to our findings in the case of capital autarky and open goods markets (far-left panel), in which the greater reduction in China's consumption persists over time.

Therefore, we find that the effects of changes in goods market integration depend heavily

on the degree of capital market integration, emphasizing the importance of studying these two dimensions of international integration in tandem.

4.6.4 Higher Capital Frictions with Open Goods and Capital Markets

Finally, we examine higher bilateral capital frictions with open goods and capital markets (far-right panel), in which case trade and investment income flows again can be imbalanced, and there can be offsetting imbalances in the current and financial accounts.

Higher U.S.-China capital market frictions lead to a decline in the supply of capital in China (bottom row), and reallocation of existing capital to the United States and to other countries, stemming from the position of the United States as a major supplier of capital to China. Consequentially, consumption drops in China, while the United States experiences a small increase in consumption (top row). Most third countries also experience an increase in consumption, due to the greater availability of cheap capital in the short run.

The increase in U.S.-China capital market frictions also increases the real return to investment in China, because of the decline in the supply of capital from the U.S.. In contrast, the real return to investment in the U.S. decreases, because of the capital reallocation previously invested in China back to the home market. Both effects follow from the position of the United States as a major supplier of capital to China. As a result, China accumulates wealth (middle panel), and its consumption gradually increases over time (top panel), whereas the opposite occurs in the United States. Moreover, since with open goods and capital markets the United States specializes in exporting capital services, it is more sensitive to rising frictions in capital markets, with this negative income effect further lowering the real return and inducing the decumulation of wealth.

Comparing consumption in China and United States in the new steady state (upper row), we again find a reversal of fortune. Whereas China is more adversely affected than the U.S., in the short run, it is less adversely affected (and even gains) in the long run. This is in contrast to our findings under goods autarky (second column), in which the adverse effect on China relative to the U.S. persists over time.

Therefore, we find that the effects of changes in capital market integration also depend heavily on the level of goods market integration, again highlighting the importance of simultaneously modeling both these dimensions of international integration.

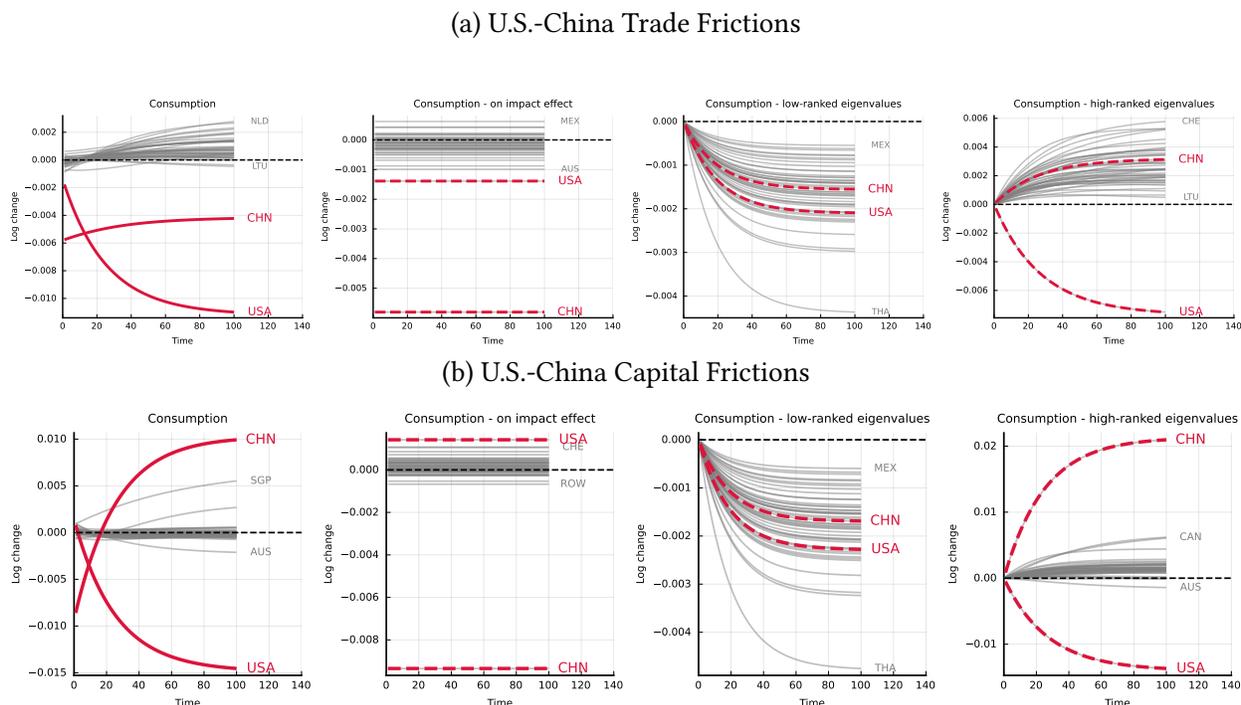
4.6.5 Decomposing the Impact of Changes in Goods and Capital Market Frictions

We next use our spectral analysis to further decompose the effects of changes in U.S.-China trade and capital market frictions. We use our result from Proposition 4 that the impact of any shock to fundamentals (\tilde{f}) on the wealth state variables can be expressed as a linear combination of

the impacts of the eigen-shocks ($\tilde{f}_{(h)}$), for which the initial impact on the state variables equals a real eigenvector of the transition matrix ($R\tilde{f}_{(h)} = \mathbf{u}_h$). Using this result, we can decompose the overall impact of any fundamental shock on the state variables at each point along the transition path into the contributions of each of the eigen-shocks. Since all of the model's endogenous variables are linear functions of the state variables, we can similarly decompose the impact of the fundamental shock on any endogenous variable at each point along the transition path.

In Figure 4, we implement this decomposition for consumption. The top row shows the effect of changes in bilateral trade frictions, while the bottom row shows the effect of changes in bilateral capital frictions. In the far-left panel, we show the overall impact on consumption (which replicates the consumption results from the middle-right and far-right panels of Figure 3). In the middle-left-panel, we show the immediate impact on consumption for the initial values of the state variables, which corresponds to the effect in a static trade or capital allocation model. In the middle-right panel, we show the impact on consumption for the lowest-ranked eigenshock with the smallest half life (fast convergence). In the far-right panel, we show the impact on consumption for all higher-ranked eigenshocks with larger half life (slow convergence).

Figure 4: Impulse Responses of Consumption to 50 Percent Increase in Bilateral U.S.-China Trade and Capital Frictions and Their Eigencomponents



Note: Impulse responses of consumption to a permanent 50 percent increase in bilateral U.S.-China trade frictions (top row) and bilateral U.S.-China capital frictions (bottom row); consumption measured as log deviations from the initial steady state (\tilde{c}_{it}). Far-left panel shows overall impulse response of consumption. Middle-left panel shows on impact effect on consumption. Middle-right panel shows lowest-ranked eigencomponent (fastest convergence). Far-right panel shows all other eigencomponents (slower convergence).

The middle-left panel shows the immediate adjustment of consumption following higher U.S.-China trade frictions (top row) or higher U.S.-China capital market frictions (bottom row). As discussed above, China is particularly negatively affected in both cases, given its position as a major importer of capital from the U.S. and a major exporter of goods to the U.S. Adjustment in other countries reflects on-impact shifts in the terms-of-trade between countries and the supply of the existing stock of global capital.

From the middle-right panel, short-run adjustment is relatively similar in both countries and in third countries, reflecting global capital decumulation in response to increases in trade or capital market frictions. However, from the far-right panel, long-run adjustment is substantially different, and tends to favor China relative to the United States, especially for capital-market shocks. These patterns of long-run adjustment include a reallocation of wealth away from the U.S. to China and third countries, such as Canada, in order to serve the U.S. market without having to incur the higher U.S.-China trade frictions, or to supply China with capital, without having to incur the higher U.S.-China capital frictions.

This decomposition highlights the important distinction between the static and dynamic effects of higher trade and capital frictions, and the rich dynamics that occur as the relative importance of eigencomponents with short versus long half lives changes along the transition path towards steady state.

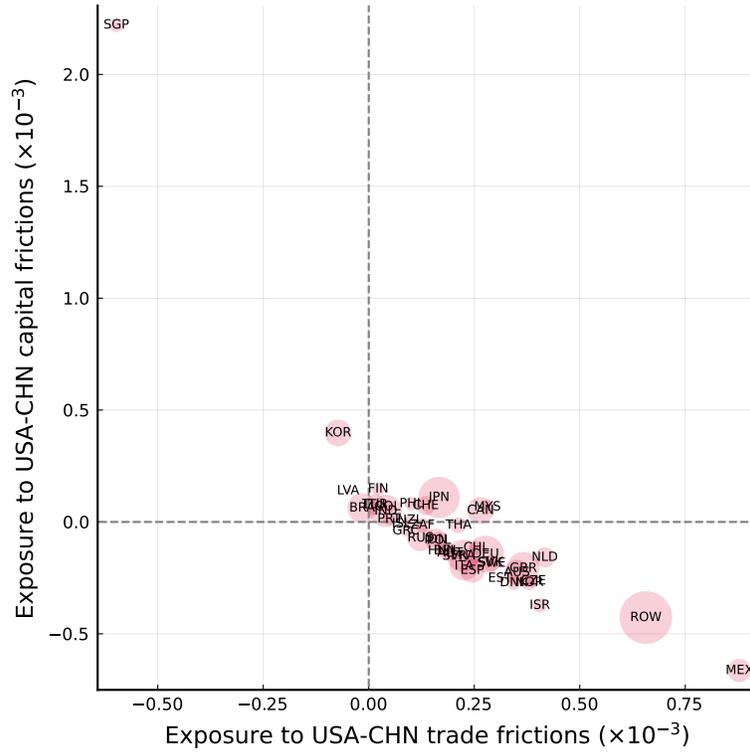
4.6.6 Goods Versus Capital Market Frictions

Finally, we show that changes in U.S.-China trade and capital market frictions tend to have quite different effects on consumption in third countries, because of differences in the initial networks of trade and capital shares and patterns of capital reallocation.

Figure 5 shows the welfare exposure of each third country to changes in U.S.-China goods market frictions (horizontal axis) and U.S.-China capital market frictions (vertical axis). Welfare is measured as the net present value of the discounted stream of utility along the transition path to the new steady state. Welfare exposure equals the elasticity of welfare with respect to a change in goods or capital market frictions, as computed using our closed-form solutions for the economy's transition path from Proposition 4. The circles for each country are proportional to their GDP and are labeled with their three-letter ISO codes.

Countries with values above [below] zero on the vertical axis gain [lose] from higher U.S.-China capital frictions, while those with values above [below] zero on the horizontal axis gain [lose] from higher U.S.-China trade frictions. Mexico gains the most from increases in trade frictions between the U.S. and China, through cross-substitution effects in goods markets. Singapore gains the most from increases in capital frictions between the U.S. and China, through the capital reallocation to serve the U.S. and Chinese markets without incurring the higher frictions. More

Figure 5: Welfare Effects of Rising U.S.-China Trade and Capital Frictions



Note: Welfare changes across countries following a 50 percent increase in bilateral trade frictions between China and the United States (x-axis) and 50 percent increase in bilateral capital frictions between China and the United States (y-axis). Welfare is measured as the net present value of the discounted stream of utility along the transition path to the new steady state. Welfare exposure equals the elasticity of welfare with respect to a change in goods or capital market frictions, as computed using our closed-form solutions for the economy's transition path from Proposition 4. Results are derived using the baseline version of our model with open goods and capital markets based on data from 2017. Each point represents a country in our data and the size of the circle stands for country GDP. China and the United States are excluded.

generally, we find a negative correlation between welfare exposure to higher trade or capital market frictions between China and the United States.

Therefore, we find that changes in these two different dimensions of international integration can have heterogeneous effects across countries, depending on initial trade and capital shares, and the extent to which countries initially specialize as exporters of capital or exporters of goods. Again these findings highlight the importance of jointly modeling these two forms of international integration, particularly for evaluating counterfactual policies that effect both goods and capital market integration.

5 Conclusion

The textbook CNGM remains central to our understanding of cross-country income dynamics. But the open-economy versions of this model make strong assumptions about substitutability in goods and capital markets. We generalize this canonical framework to allow for trade and capital market frictions and imperfect substitutability of goods and capital across countries. We develop a tractable, multi-country model that is amenable to quantitative analysis, which simultaneously incorporates international trade and capital allocations across countries, as well as intertemporal savings decisions over time.

Our framework captures a number of key features of observed international trade and capital holdings. It generates gravity equations for bilateral trade and capital holdings, because trade and capital frictions increase in bilateral distance. It predicts home bias in international capital allocations if managing capital is more costly abroad than at home. It implies a positive correlation between domestic saving and investment, because foreign capital is an imperfect substitute for domestic capital and is subject to greater capital market frictions. It generates gross capital holdings that are substantially larger than net capital holdings, because of idiosyncratic shocks to returns. It gives rise to limited capital investment from rich to poor countries, because of imperfect capital substitutability, and even if poor countries offer higher rental rates, they can have lower capital use efficiency or higher capital market frictions.

Incorporating imperfect substitutability and goods and capital market frictions yields new insights for impulse responses to productivity shocks and the speed of convergence to steady state. In the CNGM, the higher steady-state capital stock implied by a positive productivity shock only can be achieved through domestic wealth accumulation. In contrast, in the conventional open-economy neoclassical growth model, a positive productivity shock induces an initial capital reallocation that equalizes the rental rate on capital across all countries.

Our framework generates predictions in between these two extremes. The higher steady-state capital stock implied by a positive productivity shock is achieved through a combination of *both* domestic wealth accumulation *and* an initial capital reallocation. This initial capital reallocation dampens the variation in the real return across countries in response to the productivity shock, thereby implying slower convergence to steady state. Our open-economy framework thus provides a natural explanation for the apparent puzzle that empirical estimates of income convergence imply longer transitions than predicted by the CNGM for plausible intertemporal elasticities of substitution.

Since our framework matches the observed gravity equation relationships for bilateral trade and capital holdings, and allows for intertemporal consumption-savings decisions, it is particularly well suited for evaluating counterfactual policies that affect bilateral frictions in both goods

and capital markets (e.g., U.S.-China decoupling). Higher bilateral trade frictions give rise to cross-substitution and market size effects in goods markets, as in conventional trade models with capital autarky. However, they also lead to a global capital reallocation, because they alter the geography of market access between countries. Furthermore, the resulting movements in the real return to investment in each country give rise to a rich pattern of dynamic welfare gains and losses along the global economy's transition path to steady state.

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