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REGULATING ARTIFICIAL INTELLIGENCE

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ABSTRACT

Advances in AI offer substantial benefits but also pose societal risks. We analyze optimal regulation under uncertainty about societal costs, differing expectations regarding risks, and opportunities to reduce uncertainty through experimentation like beta testing and red teaming. Pigouvian taxes fail to achieve the first-best outcome due to heterogeneous beliefs about risks and the regulator's inability to observe developers' expectations. We propose a two-stage optimal policy: first, deciding between immediate release or sandbox experimentation; second, using gathered information to determine whether to publicly release or withdraw the algorithm. This approach achieves the socially optimal outcome.

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1 Introduction

In 1950, Isaac Asimov published *I, Robot*, a collection of short stories about the dilemmas of a world where robots powered by artificial intelligence (AI) interact with humans. Recent advances in AI have brought these dilemmas from the realm of science fiction to the pages of newspapers and the halls of parliaments.

AI algorithms promise great benefits but also pose substantial risks. Some risks stem from the alignment problem (Wiener, 1960), where AI systems optimize narrowly defined objectives while neglecting broader human values. For example, social media algorithms may maximize user engagement at the cost of user well-being (Russell, Dewey, and Tegmark, 2015, Amodei, Olah, Steinhardt, Christiano, Schulman, and Mané, 2016). Other dangers arise from AI's potential to facilitate harmful activities, such as generating deepfakes for fraud, automating cyberattacks, manipulating users through hyper-personalization, and aiding the design of biological or chemical weapons.

There is substantial uncertainty about these risks. In a recent interview (Tyrangiel, 2025), Sam Altman, the CEO of OpenAI says "I still expect that on cybersecurity and bio stuff we'll see serious, or potentially serious, short-term issues that need mitigation. Long term, as you think about a system that really just has incredible capability, there's risks that *are probably hard to precisely imagine and model*. But I can simultaneously think that these risks are real and also believe that the only way to appropriately address them is to ship product and learn."

There is also considerable disagreement about AI's societal risks, even among AI pioneers. Geoffrey Hinton resigned from Google to openly discuss the potential threats AI poses to humanity (Heaven, 2023). In contrast, Yann LeCun, Meta's Chief AI Scientist from 2013 to 2025, and Richard Sutton, a professor at the University of Alberta and, like Hinton and LeCun, a Turing Award recipient, have both dismissed these concerns as overblown (Hart, 2024, Scott, 2025).

Uncertainty and disagreement about AI risks are reflected in recent efforts by major insurers to exclude AI-related liabilities from corporate insurance policies (see, e.g., Harris and Criddle, 2025).

This paper studies the optimal AI regulatory policy under uncertainty and disagreement about its societal costs. We classify these costs into two categories. The first is negative externalities, such as fueling political polarization, facilitating fraud, disseminating false information, jeopardizing financial stability, and weakening democracies (see, e.g., Acemoglu, 2021, and Beraja, Kao, Yang, and Yuchtman, 2023). The second is "internalities," where individuals make harmful choices due to cognitive biases or misinformation (Herrnstein, Loewenstein, Prelec, and Vaughan Jr, 1993).

Beta testing and red-teaming can help identify AI risks before deployment. Beta testing exposes the AI algorithm to a limited group of users to assess societal costs. Red-teaming involves hiring experts to actively probe for vulnerabilities.[1] To simplify, we use the expression beta testing to encompass both approaches. In our model, beta testing emerges endogenously as a response to uncertainty. Under certain circumstances and government policies, developers are willing to forego short-term profits to gain information that informs their decision on whether to release the algorithm to the broader population.

Pigouvian taxes are commonly used to align private and social incentives. These taxes can be levied ex-ante, based on expected external costs, or ex-post, determined by actual social damages. Applying these taxes to AI faces a key challenge: developers and regulators may have different expectations about AI's risks. When developer expectations are private information, Pigouvian taxes fail to implement the first-best allocation. We show that charging developers for the realized external damages (ex-post taxes) is insufficient to align incentives both in the short run (beta testing) and in

[1]The term "red teaming" originated in Cold War military strategy, where red teams simulated adversarial attacks to test defenses. It has since been adopted in cybersecurity and AI safety to describe efforts to uncover vulnerabilities.

the long run (deployment). When beliefs are private information, ex-ante taxes can be used to align incentives in the long run but may induce excessive experimentation in the short run.

The social optimum can be achieved through a two-stage regulatory policy. In the first stage, the regulator either approves the AI algorithm for release or requires beta testing, specifying the sample size used in the test. In the second stage, based on the information obtained in the first stage, the regulator decides whether to permit full deployment or withdraw the algorithm.

Even though our paper focuses on AI regulation, our findings offer broader insights applicable to any industry marked by significant uncertainty and heterogeneous expectations about externalities. The regulation policy that emerges as optimal from our analysis has been used in two prominent domains that share these attributes.

The first domain is financial regulation. Since the UK Financial Conduct Authority introduced regulatory sandboxes in 2015, more than 50 regulators, including the U.S. Consumer Financial Protection Bureau, have adopted this approach (Cornelli, Doerr, Gambacorta, and Merrouche, 2024). By allowing firms to test products with limited users in controlled environments, sandboxes reveal consumer protection, cybersecurity, regulatory, and systemic risks before broader market introduction.

The second domain is autonomous vehicle regulation. The U.S. National Highway Traffic Safety Administration (Burd, 2021) and Germany's Federal Ministry of Transport run sandbox programs that test autonomous vehicles in controlled environments to generate data, reduce uncertainty, and guide deployment decisions.

Our paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces our benchmark model and 4 discusses optimal policy. In Section 5, we analyze scenarios where AI algorithms create internalities. In Section 6, we discuss other frictions which might be relevant for the design of AI regulation and how our results relate to current regulatory approaches in the U.S. and the European

Union. Finally, we summarize our results and discuss additional considerations relevant to the design of AI regulation.

2 Related literature

Our paper relates to four important strands of research. The first is a nascent economics literature on AI regulation. Acemoglu and Lensman (2024) analyze optimal AI adoption when uncertainty about potential disasters arrives exogenously over time, creating incentives for delayed adoption. Gans (2024) studies a model in which information arrives endogenously through adoption and shows that learning about AI's social costs may justify accelerating adoption, provided it is reversible. Both papers abstract from heterogeneous and unobservable beliefs held by AI developers and from the endogenous design of beta-testing and conditional-approval mechanisms, which are central to our analysis. In work subsequent to ours, Gans (2025) studies the regulation of innovation direction across technological paths with uncertain externalities, showing that unrestricted ex-ante Pigouvian taxation achieves the first-best, while ex-post liability can dominate when policy instruments are constrained to respond only to proven damages. In our setting, which features heterogeneous and privately held beliefs, Pigouvian taxation, whether ex-ante or ex-post, is generally insufficient to achieve the first-best.e show that, when beliefs are heterogeneous and private information, Pigouvian taxation, ex-ante and ex-post, cannot achieve the first-best.

Our emphasis on beta testing as a means of resolving uncertainty about potential societal harms is related to work that highlights the use of random audits to evaluate algorithmic decision-making. For example, Kleinberg, Ludwig, Mullainathan, and Sunstein (2018) discuss audit studies that assign otherwise equivalent applicants different race or gender attributes to detect discriminatory hiring and lending patterns. Relatedly, Rambachan, Kleinberg, Mullainathan, and Ludwig (2020) develop a reg-

ulatory framework that emphasizes algorithmic disclosure and auditability as tools for detecting and limiting discrimination ex-post.

Callander and Li (2024) study a setting in which firms have superior information about a technology's potential harms and regulators attempt to extract this information, showing that greater competition weakens information extraction and reduces the approval of beneficial innovations. Blattner, Nelson, and Spiess (2021) analyze a delegation game in which an agent designs a prediction algorithm under regulatory constraints, showing that mandating fully transparent, simple algorithms is inefficient when the divergence with the regulator is limited, and complex models deliver superior performance. Chen and Hua (2024) discuss conditions under which, in the presence of limited liability for catastrophic harms, setting liability above realized damages in non-catastrophic cases can increase social welfare.

Our work makes three key contributions to this literature. First, we examine how uncertainty about AI's internal and external effects, and disagreement about their likelihood, shape optimal regulation. Second, we highlight the role of beta testing in mitigating AI's downside risk. Third, we use our model to shed light on the regulatory approaches pursued by the United States and the European Union.

A second related literature examines the impact of AI and automation on the labor market (e.g., Burstein, Morales, and Vogel, 2019, Martinez, 2021, Acemoglu and Restrepo, 2022, Guerreiro, Rebelo, and Teles, 2022, Costinot and Werning, 2023, Thuemmel, 2023, and Ide and Talamàs, 2025), the critical role of data in AI algorithms (e.g., Jones and Tonetti, 2020, and Farboodi and Veldkamp, 2021), and the potential existential risks associated with AI (Jones, 2024). Our contribution relative to this literature is to characterize optimal policy responses to AI's externalities and internalities.

A third strand of research investigates the value of experimentation (e.g., Callander, 2011). We contribute to this literature by studying an environment in which pre-launch experimentation emerges endogenously as a distinct stage of product

development. Firms choose both whether to experiment and the scale of the experimental trial.

Finally, a fourth related area is clinical trial design, often modeled as a multi-armed bandit problem (e.g., Thompson, 1933 and Gittins, 1974). Our emphasis on optimal beta testing relates to work on conservative or safe exploration that constrains experimentation relative to a baseline policy to limit downside risk during learning (e.g., Wu, Shariff, Lattimore, and Szepesvári, 2016; Kazerouni, Ghavamzadeh, Abbasi-Yadkori, and Van Roy, 2017; Jagerman, Roijers, Hennes, and de Rijke, 2020). We contribute to this literature by considering settings where private and social incentives diverge and offering policies that equate these incentives in settings with heterogeneous expectations about external social costs.

In addition to these four strands of research, our results on the dominance of mandatory beta testing and regulatory approval over Pigouvian taxation relate to the literature on instrument choice. Weitzman (1974, 1978) studies whether a planner should regulate through prices or quantities under uncertainty when firms observe cost shocks that the planner does not. He shows that quantity regulation is preferred when marginal costs are relatively flat, because price errors then generate large and inefficient quantity responses. In our model, price-based instruments fail for a different reason: firms' beliefs about external harms are unobservable, so the planner can only set taxes based on its own expectations of firms' beliefs.

In related work, Farhi and Gabaix (2020) show that quantity regulation can dominate Pigouvian taxes when agents differ in their attention to taxes. In our setting, heterogeneity arises from disagreement about risks rather than behavioral biases, making optimal Pigouvian taxation informationally demanding. Our key contribution is to endogenize information generation through beta testing, which informs regulatory approval and yields implementable policies.

In Section 6, we further relate our results to the literature on regulation policy, drawing in particular on the analyses of regulation under limited liability and im-

perfect verifiability developed by Tirole (2010) and Kolstad and Ulen (1983).

3 Benchmark model

We consider a two-period model with a continuum of identical households and a single AI developer. We interpret the first period as the short run and the second as the long run.[2] In our model, using AI carries inherent risks, as it can create misalignments or facilitate activities that generate significant social costs. When the algorithm is deemed too risky for full initial release, the developer may choose to conduct beta testing by distributing the algorithm to a limited subset of the population. Based on the outcome of this beta testing, the developer can then decide whether to release the algorithm in the second period.

We allow for disagreement between society and AI developers about the likelihood of societal risks. This disagreement can arise because developers may be overly optimistic, expecting negative external effects to be small.

We now discuss the household problem, the AI developer's problem, and the unregulated equilibrium. Then, we characterize the social optimum and compare it with the unregulated equilibrium.

3.1 Unregulated equilibrium

Household problem The economy has a continuum of households indexed by $j \in [0, N]$, where N denotes the total number of households in the population. Each household lives for two periods.

Household j's momentary utility in period t, $v_{j,t}$, has a quasi-linear form:

$$v_{j,t} = y_t + [u - \mathbb{E}_t^s(i_t^2) - p_t] \times \mathcal{I}_{j,t} - \mathbb{E}_t^s(e_t^2), \tag{1}$$

[2] We omit time subscripts throughout the text whenever doing so does not compromise clarity.

where y_t is the exogenous income earned in period t, u is the utility of using the algorithm and p_t is the price of the algorithm in period t. The indicator function $\mathcal{I}_{j,t}$ takes the value one if household j buys the AI license and zero otherwise. The mass of AI users at time t is $\mu_t \equiv \int_0^N \mathcal{I}_{j,t} dj$. We now discuss the variables i_t and e_t.

Alignment and other problems In the introduction, we classify AI risks and misalignments into two types: internal and external. Internal risks and misalignments arise when AI algorithms manipulate households into making decisions that reduce their welfare. This effect, i_t, decreases momentary utility by i_t^2, which is measured in units of output.

External risks and misalignments occur when an AI algorithm affects a household indirectly through the use of the AI algorithm by other households. For example, AI-driven social media may polarize public opinion and distort election outcomes. This effect, e_t, reduces momentary utility by e_t^2, which is measured in units of output. This reduction is increasing in the number of users, μ_t.

Households can control internal risks and misalignments by choosing not to purchase the algorithm. In this section, we assume they account for the expected welfare reduction from the internal effect $\left(\mathbb{E}^s(i_t^2)\right.$ in equation (1)$\left.\right)$ when making their purchase decision. In Section 5, we examine a scenario where behavioral biases lead households to overlook these internal effects when deciding whether to adopt the algorithm.

In contrast, households have no control over external misalignments and risks, as these depend on the adoption decisions made by other households.

Expectations of short- and long-run risks and misalignments We assume that the short-run impact of internal and external risks and misalignments on utility is equal to the long-run impact, ϕ_x^2, plus a mean-zero random variable, ξ_x for $x \in \{i, e\}$:

$$i_1^2 = \phi_i^2 + \xi_i, \qquad i_2^2 = \phi_i^2,$$
$$e_1^2 = (\phi_e^2 + \xi_e)\mu_1, \qquad e_2^2 = \phi_e^2 \mu_2.$$

Positive and negative values of the random variable ϕ_x represent undesirable risks and misalignments. The random variables ξ_x capture the idea that the full consequences of AI usage may not be fully realized in the short run but emerge over the long run.

We allow developers to have different beliefs over the likelihood of misalignments than the rest of society. We assume that developers and society hold these different beliefs dogmatically. The superscript d denotes the developer's beliefs, $\mathbb{E}_t^d(\cdot)$, and the superscript s denotes societal beliefs, $\mathbb{E}_t^s(\cdot)$. For each $k = d, s$, we assume that the expected value of ϕ_x is zero for $x \in \{i, e\}$:

$$\mathbb{E}_1^k(\phi_x) = 0.$$

Let $\sigma_{k,x}^2$ denote uncertainty at time one about the algorithm's potential misalignment:

$$\sigma_{k,x}^2 = \mathbb{E}_1^k(\phi_x^2).$$

The perceived distributions of ϕ_x can include very large realizations, corresponding to catastrophic events.

To assess internal and external risks and misalignments, the developer can release the algorithm to a sample of μ_1 users in period one. The outcomes from this partial release inform the developer's decision about a full-scale release in the second period.

We assume the probability of generating information from the initial release, denoted by $\pi(\mu_1)$, depends on the number of licenses μ_1,

$$\pi(\mu_1) = \begin{cases} (\mu_1/\kappa)^\alpha & \text{if } \mu_1 < \kappa, \\ 1 & \text{if } \mu_1 \geq \kappa. \end{cases}$$

Here, κ denotes the minimal number of participants required to obtain information with certainty. If $\kappa = N$, information is generated with probability one only when the algorithm is released to the whole population. The parameter $\alpha < 1$ determines the effectiveness of information generation. As $\alpha \to 0$, $\pi(\mu_1) \to 1$ if $\mu_1 > 0$ and $\pi(\mu_1) = 0$ if $\mu_1 = 0$. In this limiting case, testing on an infinitesimally small sample generates information with certainty.

We define an indicator function $\mathcal{B}$ such that $\mathcal{B} = 1$ if information is generated and $\mathcal{B} = 0$ otherwise. If information is generated, a public signal about the algorithm's risks and misalignments becomes available. Rather than detailing the distributions of the random variables ξ_x, we model the effect of this information directly on posterior beliefs. In particular, the posterior beliefs about ϕ_x at the beginning of the second period become:

$$\mathbb{E}_2^k(\phi_x) = \hat{\phi}_{k,x}, \quad \mathbb{VAR}_2^k(\phi_x) = \hat{\sigma}_{k,x}^2 < \sigma_{k,x}^2.$$

Thus, while uncertainty is reduced, some residual uncertainty ($\hat{\sigma}_{k,x}^2 > 0$) persists, requiring decisions to be made under incomplete information. Posterior beliefs satisfy the consistency conditions:

$$\mathbb{E}_1^k[\hat{\phi}_{k,x}] = 0, \quad \mathbb{E}_1^k[\hat{\phi}_{k,x}^2 + \hat{\sigma}_{k,x}^2] = \sigma_{k,x}^2.$$

If no information is generated ($\mathcal{B} = 0$), initial priors remain unchanged for both the developer and society.

Developer optimism We make the natural assumption that developers are more optimistic than society about the both the internal ($x = i$) and external ($x = e$) effects of the AI algorithm:

$$\mathbb{E}_t^d(\phi_x^2) \leq \mathbb{E}_t^s(\phi_x^2), \tag{2}$$

and

$$\mathbb{E}_1^d[\mathbb{E}_2^s(\phi_x^2)] \leq \mathbb{E}_1^s[\phi_x^2]. \tag{3}$$

Household decisions Household j chooses whether to purchase an algorithm license in each period to maximize their expected lifetime utility, given by

$$\mathcal{U}_j = (1-\beta)v_{j,1} + \beta \mathbb{E}_1^s(v_{j,2}). \tag{4}$$

The household buys an AI license in period t if the expected private benefits, net of the expected internal effects from the algorithm, exceed the algorithm's price. In period one, this condition is

$$u - \sigma_{s,i}^2 \geq p_1.$$

A similar condition applies in period two:

$$u - \mathbb{E}_2^s(\phi_i^2) \geq p_2.$$

The expected negative welfare consequences of internal misalignments ($\sigma_{s,i}^2$ in period one and $\mathbb{E}_2^s(\phi_i^2)$ in period two) reduce the price households are willing to pay for the algorithm in both periods.

The AI developer's problem There is a single AI developer who has designed an algorithm. In period one, the developer releases the algorithm to a fraction $\mu_1 \in [0, N]$ of the population. We assume that the decision to deploy the algorithm in period one can be reversed in period two. If this reversal occurs, the algorithm does not impact period two utility.

The developer's problem in period two At the beginning of period two, the developer decides whether to release the algorithm to the population, choosing the number of AI licenses to offer for sale (μ_2) and the price of each license (p_2). At the end of period two, uncertainty about internal and external misalignments is realized.

The developer's utility in the second period is,

$$\mathcal{V}_2 = \begin{cases} p_2\mu_2 - \mathbb{E}_2^d(\phi_e^2)\mu_2, & \text{if } p_2 \leq u - \mathbb{E}_2^s(\phi_i^2) \text{ and } \mathcal{B} = 1, \\ p_2\mu_2 - \sigma_{d,e}^2\mu_2, & \text{if } p_2 \leq u - \sigma_{s,i}^2 \text{ and } \mathcal{B} = 0, \\ 0, & \text{otherwise.} \end{cases}$$

where $p_2\mu_2$ is the developer's revenue when p_2 is sufficiently low for sales to be positive, that is, when p_2 is weakly below the household's utility net of the expected internality effect.

We assume that the developer is immune to the algorithm's internal effect but experiences disutility from the externality in the same way households do. The variables $\mathbb{E}_2^d(\phi_e^2)\mu_2$ and $\sigma_{d,e}^2\mu_2$ represent the expected externality borne by the developer with and without testing in period one, respectively.

The developer, acting as a monopolist, sets the price to extract the expected household surplus.[3] Whenever the developer markets the algorithm, it charges the highest price the household is willing to pay, that is, the utility of the algorithm net of the expected internality effect,

$$p_2 = \begin{cases} u - \mathbb{E}_2^s(\phi_i^2), & \text{if } \mathcal{B} = 1, \\ u - \sigma_{s,i}^2, & \text{if } \mathcal{B} = 0. \end{cases}$$

In period two, the developer releases the algorithm if the maximum price the household is willing to pay is greater than the reduction in the developer's utility caused by the externality associated with the algorithm, i.e., if $p_2 \geq \mathbb{E}_2^d(\phi_e^2)$.

If new information is generated in period one ($\mathcal{B} = 1$) the posterior means of ϕ_x^2, for $x \in \{i, e\}$, are given by $\mathbb{E}_2^d(\phi_x^2) = \hat{\phi}_{d,x}^2 + \hat{\sigma}_{d,x}^2$. The developer releases the algorithm in the second period if the maximum price it can charge exceeds the reduction in utility caused by the external effect on the developer,

$$u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) \geq \hat{\phi}_{d,e}^2 + \hat{\sigma}_{d,e}^2.$$

Otherwise, the algorithm is not released ($\mu_2 = 0$).

In the unregulated equilibrium, household expectations regarding internal risks and misalignments matter because they determine their willingness to pay for the

[3] This pricing strategy generates no deadweight losses. It simply redistributes resources from the households to the monopolist.

algorithm, while the developer's expectations regarding internal risks are inconsequential.

Conversely, household expectations about external risks and misalignments have no impact. Instead, the developer's expectations about external effects matter because they determine the release decision.

If no information is generated in period one ($\mathcal{B} = 0$), the algorithm is released in period two to the whole population ($\mu_2 = N$) if

$$u - \sigma_{s,i}^2 \geq \sigma_{d,e}^2,$$

and not released otherwise ($\mu_2 = 0$).

The optimized developer utility in period two is,

$$\mathcal{V}_2^* = \begin{cases} \max\{u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) - (\hat{\phi}_{d,e}^2 + \hat{\sigma}_{d,e}^2), 0\}N, & \text{if } \mathcal{B} = 1, \\ \max\{u - \sigma_{s,i}^2 - \sigma_{d,e}^2, 0\}N, & \text{if } \mathcal{B} = 0. \end{cases}$$

The asterisk indicates that the value function is evaluated using the developer's optimal pricing and implementation strategy in period two.

To make the problem interesting, we assume that the distributions of ϕ_x are such that there is a strictly positive probability that both $u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) > \hat{\phi}_{d,e}^2 + \hat{\sigma}_{d,e}^2$, in which case the developer releases the algorithm, and $u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) < \hat{\phi}_{d,e}^2 + \hat{\sigma}_{d,e}^2$ in which case the algorithm is not released. This assumption means that the probability of the algorithm being implemented in period two is strictly positive but less than one.

The following proposition states that, from the perspective of period two, a higher μ_1 has a strictly positive value. The intuition for this result is that the developer is better off because it can make decisions based on the acquired information.

Lemma 1 (Private benefits of releasing in period one)**.** *The developer's expected utility in the second period increases with the number of licenses sold in period* 1 *if* $\mu_1 < \kappa$.

$$\frac{d\mathbb{E}_1^d(\mathcal{V}_2^*)}{d\mu_1} > 0.$$

This lemma will be useful in characterizing the developer's optimal release policy in period one, to which we turn next.

The developer's problem in period one In period one, the developer chooses the number of licenses, μ_1, and the price per license, p_1. The developer's objective function is given by:

$$\mathcal{V} = (1-\beta)\left(\begin{cases} p_1\mu_1 - \sigma^2_{d,e}\mu_1, & \text{if } p_1 \leq u - \sigma^2_{s,i} \\ 0, & \text{if } p_1 > u - \sigma^2_{s,i} \end{cases}\right) + \beta\mathbb{E}^d_1(\mathcal{V}^*_2).$$

The optimal price for the developer is the maximum price the household is willing to pay: $p = u - \sigma^2_{s,i}$.

If $u - \sigma^2_{s,i} \geq \sigma^2_{d,e}$, it is optimal to release the algorithm to the entire population, $\mu_1 = N$. Instead, if $u - \sigma^2_{s,i} < \sigma^2_{d,e}$, the expected benefit of releasing the algorithm in period one is negative. Still, it might be optimal to release the algorithm to at least part of the population to obtain information that can be used in period two (see Lemma 1). We call this type of experimentation *beta testing*. Since $\alpha < 1$, the developer's utility is increasing around $\mu_1 = 0$, so the optimal solution features positive beta testing: $\mu_1 > 0$. The intuition for this result is that the benefits from learning increase sufficiently fast with μ_1 to offset the costs of testing, which are given by $(u - \sigma^2_{s,i} - \sigma^2_{d,e})\mu_1$.

Learning through beta testing is costly because it sacrifices the period-one revenue that could be collected from a population-wide release to generate information through a limited rollout.

Proposition 1 summarizes the developer's optimal release policy. To describe this policy, it is useful to define the developer's information benefit-cost ratio, Λ^d:

$$\Lambda^d \equiv \frac{\beta}{1-\beta}\frac{\mathbb{E}^d_1\left[\max\left\{u - \mathbb{E}^s_2(\phi^2_i) - \mathbb{E}^d_2(\phi^2_e), 0\right\}\right]}{\sigma^2_{s,i} + \sigma^2_{d,e} - u}. \tag{5}$$

This ratio compares the expected benefits of increasing the probability of learning the external effects of the AI algorithm, $\beta\mathbb{E}^d_1\left[\max\left\{u - \mathbb{E}^s_2(\phi^2_i) - \mathbb{E}^d_2(\phi^2_e), 0\right\}\right]$, to the

immediate cost to the developer of selling the AI algorithm to an additional person today. This cost is the external effect on the developer minus the sale price of the algorithm, $(1-\beta)[\sigma^2_{d,e} - (u - \sigma^2_{s,i})]$.

Proposition 1 (Uncertainty, beta testing, and algorithm release)**.** *In an unregulated equilibrium, the number of user licenses offered by the developer in the first period depends on the level of uncertainty, the effectiveness of beta testing, and the information benefit-cost ratio. The equilibrium has the following properties:*

1. *If uncertainty about external effects is low, $\sigma^2_{d,e} \leq u - \sigma^2_{s,i}$, the developer foregoes beta testing and releases the AI algorithm to the entire population in the first period ($\mu_1 = N$).*

2. *If uncertainty about external effects is relatively high. $\sigma^2_{d,e} > u - \sigma^2_{s,i}$, then the developer beta tests the algorithm on a sample of size,*

$$\mu_1 = \min\left\{\left[\alpha\Lambda^d \frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}, 1\right\}\kappa. \tag{6}$$

The developer may opt to withdraw the product from the market even when the expected misalignment, $\hat{\phi}_{k,x}$, is relatively small in absolute value, provided that the residual uncertainty, $\hat{\sigma}^2_{k,x}$, remains substantial. This outcome highlights the role of uncertainty in decision-making, as the long-term negative consequences are not immediately apparent, prompting both households and developers to exercise caution regarding the algorithm's future impact.

3.2 The first-best solution (planner's problem)

We consider a central planner who, in the first period, chooses the number of households that can use the algorithm. The planner may obtain information about its internal and external effects when this number is positive. In the second period, the

planner decides whether to make the algorithm available and how many licenses to issue.

We define social welfare as the sum of the households' and developer's utilities, $\int_0^N \mathcal{U}_j dj + \mathcal{V}$. With quasi-linear utility, maximizing this social welfare function is equivalent to maximizing efficiency. Any distribution of utilities can be achieved using lump-sum transfers.

To compute the socially optimal allocations, we describe the solution to the second-period problem, contingent upon the choices made in the first period about μ_1.

The planner's problem in period two The expected social welfare in the second period, considering the available information, is given by:

$$\mathcal{W}_2 = \begin{cases} Ny_2 + \left[u - (\hat{\phi}^2_{s,i} + \hat{\sigma}^2_{s,i}) - (N+1)\left(\hat{\phi}^2_{s,e} + \hat{\sigma}^2_{s,e}\right)\right]\mu_2, & \text{if } \mathcal{B} = 1, \\ Ny_2 + \left[u - \sigma^2_{s,i} - (N+1)\,\sigma^2_{s,e}\right]\mu_2, & \text{if } \mathcal{B} = 0. \end{cases}$$

We now determine the optimal value of μ_2. If $\mathcal{B} = 1$, the posterior expectation is given by $\mathbb{E}^s_2(\phi^2_x) = \hat{\phi}^2_{s,x} + \hat{\sigma}^2_{s,x}$. In this case, releasing the algorithm is optimal if

$$\frac{u - (\hat{\phi}^2_{s,i} + \hat{\sigma}^2_{s,i})}{N+1} \geq \hat{\phi}^2_{s,e} + \hat{\sigma}^2_{s,e},$$

ensuring that the household's utility from using the algorithm, net of expected internal effects, exceeds the external effect. If this condition is not satisfied, then $\mu_2 = 0$.

If $\mathcal{B} = 0$, then $\mu_2 = N$ if

$$\frac{u - \sigma^2_{s,i}}{N+1} \geq \sigma^2_{s,e},$$

and otherwise $\mu_2 = 0$.

In period two, the planner only releases AI algorithms that are expected to be socially beneficial, taking into account the expected external effects on the entire population, $(N+1)\mathbb{E}^s_2(\phi^2_e)$. In contrast, the developer considers only its own expected loss of utility due to the externality, $\mathbb{E}^d_2(\phi^2_e)$. This difference implies that the developer is willing to commercialize AI algorithms that are detrimental to society.

The social welfare in period two is given by:

$$\mathcal{W}_2^* = Ny_2 + \begin{cases} \max\{u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) - (N+1)(\hat{\phi}_{s,e}^2 + \hat{\sigma}_{s,e}^2), 0\}N, & \text{if } \mathcal{B} = 1 \\ \max\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2, 0\}N, & \text{if } \mathcal{B} = 0, \end{cases}$$

where the asterisk indicates that the value function has been maximized with respect to the choice of implementation in period two.

We assume that there is a strictly positive probability that $u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) > (N+1)(\hat{\phi}_{s,e}^2 + \hat{\sigma}_{s,e}^2)$, in which case it is optimal to release the algorithm, and $u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) < (N+1)(\hat{\phi}_{s,e}^2 + \hat{\sigma}_{s,e}^2)$, in which case it is not. This assumption means that the probability that the planner releases the algorithm in the second period, given the information obtained in the first period, is strictly positive but less than one.

The equivalent of Lemma 1 for the planner is as follows.

Lemma 2 (Social benefits of beta testing in period one)**.** *Expected social welfare in the second period increases with the size of the sample used for beta testing in the first period for $\mu_1 < \kappa$:*

$$\frac{\mathbb{E}_1^s(d\mathcal{W}_2^*)}{d\mu_1} > 0.$$

This lemma will be useful in characterizing the planner's optimal release policy in period one, to which we turn next.

The planner's problem in period one Expected social welfare is given by

$$\mathcal{W} = (1-\beta)\left[Ny_1 + \left\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\right\}\mu_1\right] + \beta\mathbb{E}_1^s[\mathcal{W}_2^*].$$

From a static perspective, it is optimal to set $\mu_1 = 0$ if $u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2 < 0$. However, beta testing in the first period ($\mu_1 > 0$) creates value by generating information that the planner can use in the second period (see Lemma 2).

Proposition 2 summarizes the planner's optimal release policy. To describe this solution, it is useful to define the planner's information benefit-cost ratio, Λ^s:

$$\Lambda^s \equiv \frac{\beta}{1-\beta}\frac{\mathbb{E}_1^s\left[\max\left\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\right\}\right]}{\sigma_{s,i}^2 + (N+1)\sigma_{s,e}^2 - u}. \tag{7}$$

With homogeneous beliefs $\Lambda^s < \Lambda^d$ because the developer does not take into account external effects on the population. Because of differences in beliefs and weights assigned to external damages, the planner's information benefit-cost ratio is lower than the developer's, $\Lambda^s < \Lambda^d$.

Proposition 2 (Uncertainty, beta testing, and algorithm release)**.** *In the first best, the number of user licenses offered in the first period depends on the level of uncertainty, the effectiveness of beta testing, and the information benefit-cost ratio. The solution is as follows:*

1. *If uncertainty is low, $\sigma^2_{s,e} \leq \frac{u-\sigma^2_{s,i}}{N+1}$, the planner always foregoes beta testing and releases the AI algorithm to the entire population in the first period ($\mu_1 = N$)*

2. *If uncertainty is relatively high $\sigma^2_{s,e} > \frac{u-\sigma^2_{s,i}}{N+1}$, the planner beta tests the algorithm on a sample of size*

$$\mu_1 = \min\left\{\left[\alpha\Lambda^s\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}, 1\right\}\kappa. \tag{8}$$

Figure 1 depicts the optimal values of μ_1 (Panel A) and μ_2 (Panel B) chosen by the planner and the developer for various levels of uncertainty. In the first period, both the developer and the planner agree to release the algorithm to the entire population when uncertainty about the external effect is low $\left(\sigma^2_{s,e} \leq \frac{u-\sigma^2_{s,i}}{N+1}\right)$. At higher levels of uncertainty, the planner is more cautious than the developer, releasing the algorithm to fewer users. The reason is that the planner takes into account the impact of the externalities generated by the algorithm on the entire population.

In the second period, the developer and planner make the same release decisions when external effects are low or high. However, when external effects are in an intermediate range, they disagree: the developer opts to release the algorithm, whereas the planner chooses not to. This disparity occurs because the developer disregards the algorithm's external effects on the population.

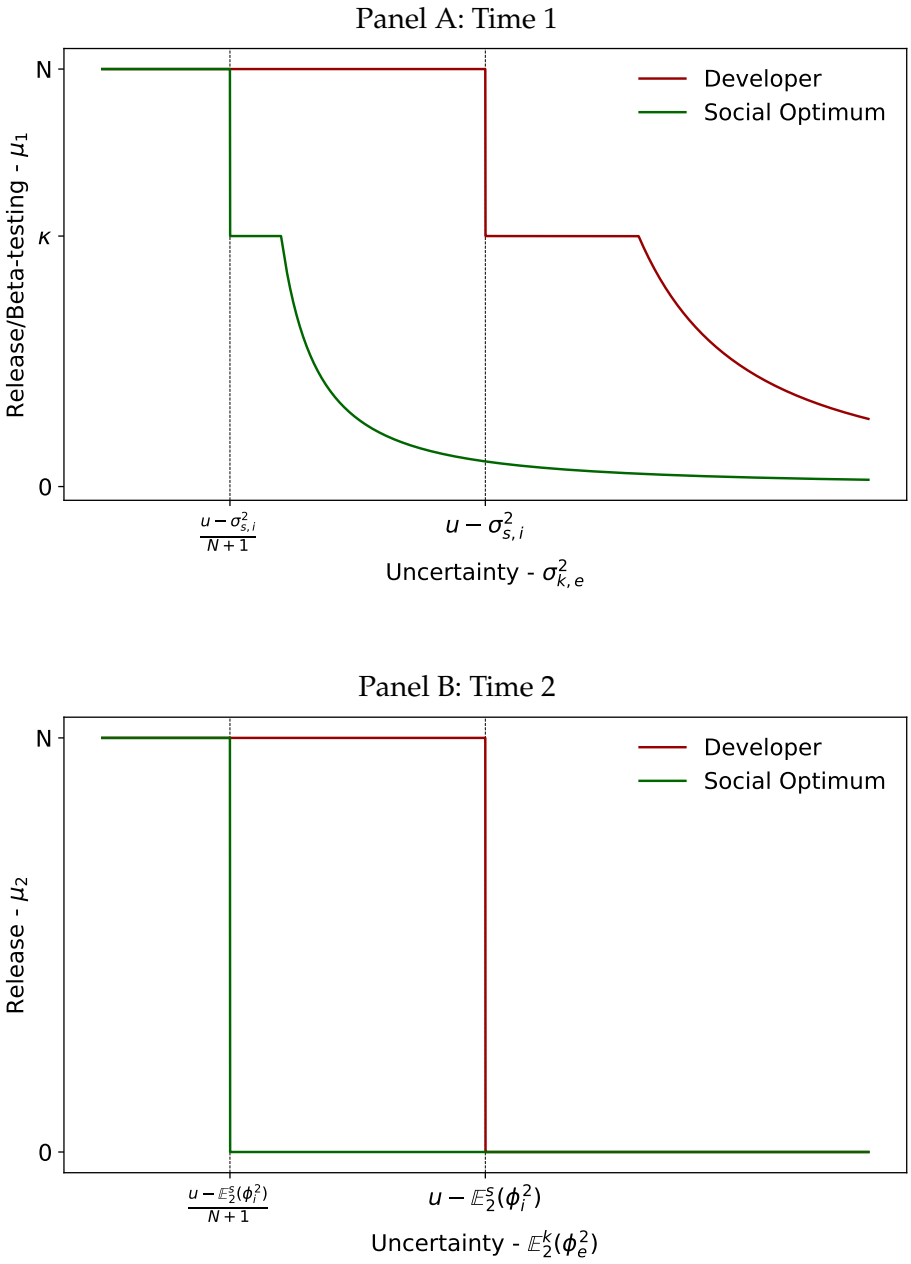
4 Regulating AI

Social welfare in the unregulated equilibrium falls short of the social optimum because developers overlook AI's external impact on households. Aligning private and social incentives through Pigouvian taxes is challenging because of uncertainty and disagreement over AI's external effects.

We consider two types of Pigouvian taxes: ex-post, which hold developers fully liable for realized damages, and ex-ante, which charge them based on expected external effects. Ex-post taxes work when expectations are homogeneous but fail when expectations are heterogeneous. Ex-ante taxes remain effective with heterogeneous expectations but must account for differing expectations between regulators and developers. Eliciting developers' true expectations is inherently difficult because they have incentives to feign pessimism about external effects to reduce ex-ante Pigouvian taxes. So, when the developer's expectations are unobservable to the planner, ex-ante Pigouvian taxes fail to implement the first best.

In this model, the optimal policy includes a prescription about the release in period one, which is the extent of beta testing, as well as conditional approval in period two. In particular, the regulator determines whether to release the algorithm or subject it to beta testing to assess societal risks. Second, based on the information obtained, the regulator either approves or withdraws the algorithm.

Figure 1: Release decisions in the first and second periods



4.1 Pigouvian taxes

We first consider ex-post Pigouvian taxes in an economy with homogeneous beliefs. Throughout, we assume that any tax revenue is redistributed to households as lump-sum transfers.

4.1.1 Ex-post Pigouvian taxes

Suppose that regulators levy liability taxes on developers equal to the welfare cost of the realized external effects imposed on the households:

$$T_t = N \times e_t^2. \tag{9}$$

The developer understands that selling the AI to a larger population (higher μ_t) makes them more likely to face higher taxes: $\mathbb{E}_t^d(T_t) = N\mathbb{E}_t^d(\phi_e^2)\mu_t$. The welfare properties of these taxes are summarized by the following proposition.

Proposition 3 (Ex-Post Pigouvian Taxes under Homogeneous Beliefs)**.** *Suppose that developers and society have the same beliefs. Then, ex-post Pigouvian taxes in equation* (9) *align private and social incentives. As a result, the developer's decisions regarding testing, implementation, and innovation are socially optimal.*

Proof. If beliefs are homogeneous, then $\mathbb{E}_t^d(\phi_e^2) = \mathbb{E}_t^s(\phi_e^2)$, and for notational convenience we drop the indices s and d. It is still optimal for the developer to set $p_t = u - \mathbb{E}_t^s(\phi_i^2)$. Replacing this price and the expected taxes into the utility of the developer we find that $\mathcal{V} = (1-\beta)\mathcal{V}_1 + \beta\mathbb{E}_1(\mathcal{V}_2)$, where

$$\begin{aligned}
\mathcal{V}_1 &= [u - \sigma_i^2 - (N+1)\sigma_e^2]\mu_1 \\
\mathcal{V}_2 &= \begin{cases} [u - (\hat{\phi}_i^2 + \hat{\sigma}_i^2) - (N+1)(\hat{\phi}_e^2 + \hat{\sigma}_e^2)]\mu_2, & \text{if } \mathcal{B} = 1, \\ [u - \sigma_i^2 - (N+1)\sigma_e^2]\mu_2, & \text{if } \mathcal{B} = 0. \end{cases}
\end{aligned}$$

It follows that $\mathcal{V}_t = \mathcal{W}_t - Ny_t$. Private and social incentives are aligned, so privately optimal decisions coincide with the social optimum. □

Heterogeneous beliefs When the developer and society hold different beliefs about the algorithm's potential risks, ex-post Pigouvian taxes fail to align the developer's incentives with those of society. This divergence in incentives arises because the developer, being more optimistic, assigns a lower probability to negative external effects and is therefore more inclined to release the algorithm than the planner.

To see this result formally, note that in period t, the developer's expectation of their tax liability is lower than the planner's $\mathbb{E}_t^d(T_t) = N\mathbb{E}_t^d(\phi_e^2)\mu_t < N\mathbb{E}_t^s(\phi_e^2)\mu_t$. It follows that

$$\mathcal{V}_t = \mathcal{W}_t - Ny_t + (N+1)\left\{\mathbb{E}_t^s(\phi_e^2) - \mathbb{E}_t^d(\phi_e^2)\right\}\mu_t$$

This expression shows that the developer is willing to release the algorithm in cases where the planner would not. In the extreme case where $\mathbb{E}_t^d(\phi_e^2) = 0$, the developer is always willing to release the algorithm, while the planner is more cautious.

4.1.2 Ex-ante Pigouvian taxes

We now show that ex-ante Pigouvian taxes can achieve the efficient outcome when beliefs are heterogeneous, but only if beliefs are publicly known and contractible. Suppose the regulator sets taxes at the beginning of each period based on the expected external damages caused by the algorithm:

$$T_t^{ex-ante} = \mathbb{E}_t^s(Ne_t^2) = N\mathbb{E}_t^s(\phi_e^2)\mu_t. \tag{10}$$

When beliefs are heterogeneous, the ex-ante taxes in equation (10) do not implement the social optimum because we need to correct for differences in beliefs. For instance, the taxes in period two need to be corrected as follows:

$$T_2^{ex-ante} = \begin{cases} N\mathbb{E}_2^s(\phi_e^2)\mu_2 + \left[\mathbb{E}_2^s(\phi_e^2) - \mathbb{E}_2^d(\phi_e^2)\right]\mu_2 & \text{if } \mathcal{B} = 1, \\ N\sigma_{s,e}^2\mu_2 + \left(\sigma_{s,e}^2 - \sigma_{d,e}^2\right)\mu_2 & \text{if } \mathcal{B} = 0. \end{cases} \tag{11}$$

The first term internalizes the externality according to the regulator's expectations. The second term corrects for the difference in expectations between the developer and the regulator.

Note that the developer pays a higher tax when they are relatively optimistic (lower $\mathbb{E}_t^d(\phi_e^2)$), and a lower tax when they are relatively pessimistic (higher $\mathbb{E}_t^d(\phi_e^2)$).[4]

However, the analog of taxes (11) applied to period one fail to implement the socially optimal level of μ_1. The reason is that the developer's information benefit-cost ratio is higher than that of the regulator,

$$\Lambda^d = \Lambda^s \times \frac{\mathbb{E}_1^d\left[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}\right]}{\mathbb{E}_1^s\left[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}\right]} > \Lambda^s. \tag{12}$$

To implement the first-best outcome, the period-one taxes must correct not only for differences in expectations about the externality in that period but also for differences in expectations regarding the externality in period two. The latter correction is necessary because the developer's decision to beta test in period one has informational value in period two. The period-one taxes that achieve the first-best are given by:

$$\begin{aligned} T_1^{ex-ante} = {} & N\mathbb{E}_1^s(\phi_e^2)\mu_1 + \left[\mathbb{E}_1^s(\phi_e^2) - \mathbb{E}_1^d(\phi_e^2)\right]\mu_1 \\ & + \frac{\beta}{1-\beta}\pi(\mu_1)N\Bigg(\mathbb{E}_1^d\left[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}\right] \\ & \qquad - \mathbb{E}_1^s\left[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}\right]\Bigg), \end{aligned} \tag{13}$$

Proposition 4 (Ex-Ante Pigouvian Taxes under Heterogeneous Beliefs)**.** *Suppose the developer and society have different beliefs about the algorithm's potential external effects. If the regulator imposes the ex-ante Pigouvian taxes specified in equations* (11) *and* (13)*, the developer's decisions regarding release, withdrawal, and the sample size used in beta testing align with the socially optimal outcomes.*

[4] Laffont (1977) explores a similar result in a version of Weitzman (1974)'s model in which the firm and the planner have different expectations about fundamentals. When prices are used as incentives, they must correct for belief differences. As we discuss below, in our model, there is a dynamic element to the belief correction because information generated by beta testing or releasing at time one has value at time two.

Unfortunately, this policy is impractical because the developer's expectations required to construct the taxes (11) and (13) are generally unobservable. What is the optimal policy when the developer's beliefs are private information? If the regulator applies the taxes (11) and (13) to the developer's self-reported beliefs, then the developers have an incentive to pretend to be more pessimistic than they are to pay lower taxes.

4.2 Optimal policy when beliefs are private information

In this subsection, we study the optimal policy in a setting where developer beliefs are private information. Our main result is that, under these informational constraints, Pigouvian taxes fail to implement the social optimum. We show that the first best can be implemented by combining mandatory beta testing with regulatory approval.

The developer draws their belief type θ from a set $\Theta = [\underline{\theta}, \overline{\theta}]$, with cumulative density function F. We denote by $\underline{\theta}$ and $\overline{\theta}$ the most pessimistic and optimistic beliefs in the set Θ. We assume that the most optimistic developer assigns zero probability to non-zero external effects, so $\mathbb{E}_1^{\overline{\theta}}(\phi_e^2) = 0$. The most pessimistic developer has the same expectations as society $\mathbb{E}_1^{\underline{\theta}}(\phi_e^2) = \mathbb{E}_1^{s}(\phi_e^2)$. Under these conditions, developers are weakly more optimistic than society.

We consider two sets of policies without commitment: (i) linear tax on the number of licenses sold and (ii) regulation that controls release and beta testing decisions. Timing is as follows. First, nature draws the developer's belief type θ, which is private information. Next, the regulator designs the optimal policy for period one. Given this policy, the developer decides their optimal price and release strategy. In the second period, all individuals observe the beta test results from period one, and beliefs are updated. The regulator then chooses the optimal policy for period two, after which the developer decides their optimal release strategy.

As is standard in the Pigouvian taxation literature, we model taxes as linear func-

tions of the number of licenses sold, μ_t. Given that developer type is private information, taxes cannot depend directly on developers' beliefs. Consequently, a tax policy is defined as a set of state-contingent tax rates per unit sold $\boldsymbol{\tau} \equiv \{\tau_t\}$, for $t = 1, 2$. So total taxes are $T_t = \tau_t \mu_t$.

The second policy framework we consider consists of mandatory beta testing and regulatory approval. These policies set limits on the maximum number of licenses that can be sold in each period.

In period one, the regulator can either mandate a beta test involving up to $\overline{\mu}_1$ users or permit full release, in which case $\overline{\mu}_1 = N$. In period two, the regulator can approve full commercialization by setting $\overline{\mu}_2 = N$, require the developer to withdraw the algorithm by setting $\overline{\mu}_2 = 0$, or approve a number of users, $\overline{\mu}_2$, strictly between 0 and N.

A mandatory beta-testing and regulatory approval policy is a set of state-contingent restrictions on license sales, denoted by $\overline{\boldsymbol{\mu}} \equiv \{\overline{\mu}_t\}$ for $t = 1, 2$. These restrictions impose an upper limit on number of licenses that can be sold in each period such that $\mu_t \leq \overline{\mu}_t$.[5]

Optimal tax policy For any tax policy $\boldsymbol{\tau}$, the developer's beta testing and release strategies follow the same policy as before. Lemma 3 characterizes the developer's behavior for an arbitrary tax policy.

Lemma 3 (Optimal Developer Behavior Under Tax Policy)**.** *For any tax policy $\boldsymbol{\tau}$, the developer's release, beta-testing and withdrawal policies are as follows.*
At time $t = 2$:

1. *If $\mathbb{E}_2^{\theta}(\phi_e^2) < u - \mathbb{E}_2^{s}(\phi_i^2) - \tau_2$, the developer releases the algorithm to the entire population, $\mu_2^{\theta} = N$.*

[5]In the appendix, we discuss the case of non-linear taxes. We show that there is a non-linear, discontinuous tax schedule that replicates the outcomes obtained under the optimal mandatory beta-testing and regulatory approval policy. This tax schedule confiscates the developer's revenues when their choices deviate from the efficient allocations.

2. *If* $\mathbb{E}_2^{\theta}(\phi_e^2) \geq u - \mathbb{E}_2^{s}(\phi_i^2) - \tau_2$, *the developer withdraws the algorithm,* $\mu_2^{\theta} = 0$.

At time $t = 1$*:*

1. *If* $\sigma_{\theta,e}^2 < u - \sigma_{s,i}^2 - \tau_1$, *the developer foregoes beta testing and releases the algorithm to the entire population,* $\mu_1^{\theta} = N$.

2. *If* $\sigma_{\theta,e}^2 \geq u - \sigma_{s,i}^2 - \tau_1$, *the developer beta tests the algorithm on*

$$\mu_1^{\theta}(\tau_1, \tau_2) = \min \left\{ \left[\alpha \Lambda^{\theta}(\tau_1, \tau_2) \frac{N}{\kappa} \right]^{\frac{1}{1-\alpha}}, 1 \right\} \kappa, \tag{14}$$

where $\Lambda^{\theta}(\tau_1, \tau_2)$ *is the developer's information benefit-to-cost ratio defined in equation* (A.9).

Taxes affect the release decision at time two and the sample size used for beta testing at time one.

Proposition 5 characterizes the optimal tax policy when beliefs are private information.

Proposition 5 (Optimal Tax Policy When Beliefs Are Private Information)**.** *The optimal tax policy is as follows.*
At time $t = 2$*:*

1. *If uncertainty is small,* $\mathbb{E}_2^{s}(\phi_e^2) \leq \frac{u - \mathbb{E}_2^{s}(\phi_i^2)}{N+1}$, *the regulator sets the tax to zero* $\tau_2 = 0$ *and the developer releases the algorithm to the entire population,* $\mu_2^{\theta} = N$.

2. *If uncertainty is large,* $\mathbb{E}_2^{s}(\phi_e^2) > \frac{u - \mathbb{E}_2^{s}(\phi_i^2)}{N+1}$, *the regulator sets the tax* $\tau_2 = p_2$ *and the developer withdraws the algorithm,* $\mu_2^{\theta} = 0$.

At time $t = 1$*:*

1. *If uncertainty is small,* $\sigma_{s,e}^2 \leq \frac{u - \sigma_{s,i}^2}{N+1}$, *the regulator sets the tax to zero* $\tau_1 = 0$ *and the developer releases the algorithm to the entire population,* $\mu_1^{\theta} = N$ *for all* $\theta \in \Theta$.

2. *If uncertainty is large,* $\sigma^2_{s,e} > \frac{u-\sigma^2_{s,i}}{N+1}$*, the regulator sets the tax so that*

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{d\mu_1^\theta/d\tau_1}{\int_{\underline{\theta}}^{\overline{\theta}} d\mu_1^\theta/d\tau_1 dF(\theta)} \pi'(\mu_1^\theta)\Lambda^s N dF(\theta) = 1, \tag{15}$$

and the developer sells μ_1^θ *licenses as given in equation* (14)*.*

In the second period, the tax is set to zero if the algorithm is socially beneficial. If the algorithm is socially harmful, all revenue is taxed away, ensuring that developers of all types choose not to release the algorithm. This tax policy ensures that the release of the algorithm in period two is optimal.

In the first period, the planner chooses a tax rate that equates the expected value of beta testing across developer types with the welfare cost of conducting beta testing. Because this tax policy adjusts developer incentives based on average values across different belief types rather than individual beliefs, it generally falls short of achieving the efficient outcome.

Next, we describe the implementation of the efficient outcome using mandatory beta testing and regulatory approval of the algorithm, conditional on the results of the beta test.

Optimal regulation policy Lemma 4 characterizes the developer's behavior for an arbitrary policy regulating beta testing and release.

Lemma 4 (Optimal Developer Behavior Under Regulation Policy)**.** *For any regulation policy* $\overline{\boldsymbol{\mu}}$*, the developer's release, beta-testing and withdrawal policies are as follows.*
At time $t = 2$*:*

1. *If* $\mathbb{E}_2^\theta(\phi_e^2) < u - \mathbb{E}_2^s(\phi_i^2)$*, the developer releases the algorithm to the maximum number of people allowed,* $\mu_2^\theta = \overline{\mu}_2$*.*

2. *If* $\mathbb{E}_2^\theta(\phi_e^2) \geq u - \mathbb{E}_2^s(\phi_i^2)$*, the developer withdraws the algorithm,* $\mu_2^\theta = 0$*.*

At time $t = 1$:

1. *If $\sigma^2_{\theta,e} < u - \sigma^2_{s,i}$, the developer sells the maximum number of licenses allowed, $\mu^\theta_1 = \overline{\mu}_1$.*

2. *If $\sigma^2_{\theta,e} \geq u - \sigma^2_{s,i}$, the developer beta tests the algorithm on*

$$\mu^\theta_1 = \min\left\{\min\left\{\left[\alpha\Lambda^\theta\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}, 1\right\}\kappa, \overline{\mu}_1\right\}, \tag{16}$$

where Λ^θ is the developer's information benefit-to-cost ratio, θ, given by equation (A.17).

Proposition 6 characterizes the optimal regulation policy when beliefs are private information.

Proposition 6 (Optimal regulation policy under private information)**.** *The optimal regulation policy is as follows. At time $t = 2$:*

1. *If uncertainty is low, $\mathbb{E}^s_2(\phi^2_e) \leq \frac{u - \mathbb{E}^s_2(\phi^2_i)}{N+1}$, the regulator sets a non-binding limit on the number of licenses $\overline{\mu}_2 = N$ and the developer releases the algorithm to the entire population, $\mu^\theta_2 = N$ for all θ.*

2. *If uncertainty is high, $\mathbb{E}^s_2(\phi^2_e) > \frac{u - \mathbb{E}^s_2(\phi^2_i)}{N+1}$, the regulator mandates the withdrawal of the algorithm, setting $\overline{\mu}_2 = 0$ and so $\mu^\theta_2 = 0$ for all θ.*

At time $t = 1$:

1. *If uncertainty is low, $\sigma^2_{s,e} \leq \frac{u - \sigma^2_{s,i}}{N+1}$, the regulator sets a non-binding limit on the number of licenses $\overline{\mu}_1 = N$ and the developer releases the algorithm to the entire population, $\mu^\theta_1 = N$ for all θ.*

2. *If uncertainty is high, $\sigma^2_{s,e} > \frac{u - \sigma^2_{s,i}}{N+1}$, the regulator sets the following upper bound*

$$\overline{\mu}_1 = \min\left\{\left[\alpha\Lambda^s\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}, 1\right\}\kappa. \tag{17}$$

The developer sells $\mu_1^\theta = \overline{\mu}_1$ licenses for all θ.

This regulation policy implements the socially optimal allocation in this economy, so it is superior to the best tax policy.[6] Overall, the optimal regulatory policy follows a simple threshold rule: intervention occurs only when uncertainty is high. This approach prevents the premature deployment of risky algorithms while allowing efficient learning through beta testing, ensuring that release decisions are made with adequate information. At time one, if uncertainty is low, the regulator does not restrict the algorithm's release. When uncertainty is high, the regulator limits the number of beta licenses issued, balancing the benefits of acquiring more information with the risk of significant negative external effects. At time two, the developer decides whether to release or withdraw the algorithm. If uncertainty about externalities is low, the regulator imposes no restrictions, allowing full deployment. If uncertainty is high, the regulator mandates withdrawal to prevent potential harm.

We now briefly discuss the impact of limited liability on our results.

4.3 Limited liability

In practice, limited liability protects developers from bearing the full cost of large social damages. The model analyzed here abstracts from this consideration. In the Appendix, we analyze how this constraint affects our results, focusing, for simplicity, on the case of homogeneous beliefs. We model limited liability by assuming that taxes in period t cannot exceed the sales revenue generated in that period. Under this constraint, ex-post taxes do not achieve the first-best allocation in either period one or two because the developer does not take into account damages that exceed their limited liability. In contrast, ex-ante taxes succeed in implementing the first-

[6] It is possible to formulate a discontinuous tax policy that replicates the effects of the optimal regulatory framework. This policy involves granting a subsidy to developers who select the socially optimal value of μ_1. The subsidy can be set at a sufficiently high level to ensure that all types of developers opt for the socially optimal μ_1.

best allocation in period two but fail to do so in period one. The regulatory policy described in Proposition 6 remains effective in that model.

5 A model with internalities

This section considers a model that incorporates deviations from rational behavior, known as internalities. These deviations lead households to make decisions that are not in their self-interest because of misinformation, self-control issues, cognitive biases, or time inconsistency problems, all of which can be exploited by AI algorithms.

For example, recent experimental evidence suggests that reliance on Large Language Models may generate cognitive internalities. Users are drawn to interfaces that minimize immediate effort and maximize short-run output quality, but systematically neglect the delayed costs for memory formation, cognitive engagement, and autonomy induced by repeated reliance on the tool (Kosmyna, Hauptmann, Yuan, Situ, Liao, Beresnitzky, Braunstein, and Maes, 2025).

5.1 Unregulated equilibrium

Household's problem In Section 3, we assume that households take the expected welfare reduction caused by internal effects, $\mathbb{E}_t^s(i_t^2)$, into account when deciding whether to use the algorithm. Here, we consider the case in which, due to behavioral biases, households disregard these internal effects when making their purchase decisions.

We formalize this idea by assuming that $\mathcal{U}_j$, defined in equation (4), is the household's "experienced utility," but that households base their choices on a different, misspecified, objective function that we refer to as the "decision utility."[7] Lifetime

[7] This terminology is common in behavioral price theory (e.g., Farhi and Gabaix, 2020).

decision utility takes the form:

$$\mathcal{U}_j^b = (1-\beta)v_{j,1}^b + \beta \mathbb{E}_1^s(v_{j,2}^b),$$

where momentary decision utility is

$$v_{j,t}^b = y_t + [u - p_t] \times \mathcal{I}_{j,t} - \mathbb{E}_t^s(e_t^2). \tag{18}$$

The household decides whether to purchase the AI algorithm to maximize $\mathcal{U}_j^b$. The resulting decision rule is to buy the algorithm whenever $p_t \leq u$. Recall that without behavioral biases, the decision rule is to buy the algorithm when $p_t \leq u - \mathbb{E}_t(\phi_i^2)$.

We assume that the developer is immune to the algorithm's internal effects, either because it does not use the algorithm or is more sophisticated than the households.[8]

What are the key differences between this model and our benchmark model? Because households ignore expected negative internal effects on utility, the developer can charge them a higher price: $p_t = u$ instead of $p_t = u - \mathbb{E}_t^s(\phi_i^2)$.

Internalities widen the gap between the unregulated equilibrium and the social optimum. In period one, the developer beta tests the algorithm when $\sigma_{d,e}^2 > u$ and releases the algorithm otherwise. In contrast, the planner has a lower threshold for the level of uncertainty required for beta testing. It is socially optimal to beta test whenever $\sigma_{s,e}^2 > (u - \sigma_{s,i}^2)/(N+1)$.

In period two, the developer withdraws the algorithm only when $\hat{\phi}_{d,e}^2 + \sigma_{d,e}^2 > u$. The planner uses a lower uncertainty threshold for withdrawal. It is socially optimal to withdraw the algorithm whenever $\hat{\phi}_{s,e}^2 + \sigma_{s,e}^2 > (u - \hat{\phi}_{s,i}^2 - \sigma_{s,i}^2)/(N+1)$.

The developer overlooks the external impacts on the broader population but personally experiences these effects, just like any household. These external effects increase with the number of algorithm users. Consequently, when externalities are

[8]Extending our analysis to the case where the algorithm's internal effects also affect the developer is straightforward. Such an extension would not significantly alter our findings.

high, the developer is dissuaded from releasing the algorithm. This restraining factor is absent with respect to internalities because the developer is not personally affected by internalities and the price does not reflect the internal effects experienced by households.

Figure 2 shows that the developer and the planner select the same value of μ_1 only when uncertainty is low ($\sigma^2_{s,e} < (u - \sigma^2_{s,i})/(N+1)$). Under higher uncertainty, the developer opts for a larger μ_1 than the planner, as it does not account for the algorithm's external effects on the population. The choice of μ_2 coincides under both low and high uncertainty but diverges at moderate levels of uncertainty. This divergence is more pronounced than when households consider internal effects in deciding to purchase the algorithm.

The policy outcomes in the presence of internalities resemble those without internalities. Pigouvian taxes fail to implement the first-best allocation. Achieving the first-best instead requires a policy combining mandatory beta testing with regulatory approval contingent on the beta-test results.

6 AI Regulation in Practice

Our analysis focuses on a specific friction that we view as central to AI regulation: systematic differences in beliefs about societal risks between developers and regulators. Neither ex-ante nor ex-post Pigouvian taxes generally implement the social optimum when beliefs diverge and are private information. In this setting, mandatory beta testing combined with regulatory approval, i.e., quantity regulation, can implement the social optimum.

Ex-post Pigouvian taxes perform poorly if the perception of damages by developers differs widely from that of society. In this case, ex-ante policies based on society's perceived damages are superior to ex-post taxes. Within ex-ante policies, quantity regulation is still superior to ex-ante taxes, since the latter would have to correct for

differences in beliefs, which are private information.

Of course, belief heterogeneity is not the only friction that matters in practice. One potentially important consideration is information asymmetry. Developers may possess superior information about risks, for example, due to proprietary data, internal testing, or privileged access to system architectures. When developers have superior information, ex-post Pigouvian taxes become relatively more effective than ex-ante policies, since developers are better able to internalize the expected damages they may ultimately face.

Another relevant friction is limited liability, which we analyze in Appendix B. When liability is capped, ex-post damages lose much of their deterrent power, since harms beyond the cap are not borne by the developer. In this case, the ranking of policies is again tilted towards ex-ante policies. Ex-ante instruments, such as mandatory insurance, ex-ante taxes, or testing and regulatory approval, are more effective and can come closer to implementing the social optimum. This point has been emphasized in the literature on liability and regulation under imperfect financial markets. In particular, Tirole, 2010 shows that limited liability fundamentally alters the optimal design of Pigouvian taxation and can justify regulatory interventions that extend beyond standard ex-post liability.

Ex-post verifiability is another potentially important friction. Damages may be difficult to measure and attribute to a specific model or developer, weakening the effectiveness of regulatory regimes that rely primarily on ex-post enforcement. These informational constraints reduce deterrence and tilt the balance toward ex-ante interventions, including precautionary requirements imposed before deployment (see Kolstad and Ulen, 1983).

The optimal regulatory framework depends on which frictions dominate. When developers have substantially superior information, there is limited disagreement in beliefs with regulators, and enforcement of ex-post damage liability is effective, price-based instruments such as ex-post Pigouvian taxes perform well. By contrast,

when differences in beliefs between regulators and industry are large and beliefs are private information, and limits to liability or verifiability are particularly severe, quantity-based regulation, particularly when combined with beta testing and regulatory approval, becomes more attractive.

This distinction helps interpret the divergent regulatory paths currently being pursued in practice. Europe has moved toward quantity regulation, adopting a risk-based framework that includes outright bans on certain high-risk applications, testing both using regulatory sandboxes as well as real world conditions prior to full deployment (see articles 57 and 60 of the European Union Artificial Intelligence Act, European Union, 2024).

By contrast, the United States has leaned toward an ex-post, liability-based approach, relying on existing tort law, consumer protection, and sector-specific enforcement after damages materialize. This regulatory stance is consistent with an environment in which developers are presumed to have superior information about risks, making ex-post Pigouvian taxes a relatively effective tool.

7 Conclusion

In this paper, we study optimal regulation in environments characterized by two salient features of artificial intelligence: substantial uncertainty about potential social harms and persistent disagreement between developers and regulators about their likelihood and magnitude. We show that these features fundamentally shape the relative performance of regulatory instruments.

Our main result is that optimal regulation in this environment takes the form of a two-stage process. In the first stage, regulators decide whether an algorithm should undergo beta testing or be approved for full deployment. The information generated during this experimental phase then informs the second-stage decision on whether to permit broad release or withdraw the algorithm to prevent anticipated

social harm. When belief disagreement is significant and beliefs are private information, this combination of mandatory experimentation and regulatory approval can strictly dominate both ex-ante and ex-post Pigouvian tax regimes.

While our analysis emphasizes belief heterogeneity, real-world regulation is shaped by multiple frictions, including information asymmetries, limited liability, and constraints on verifiability. These frictions help explain cross-jurisdictional differences in regulatory approaches.

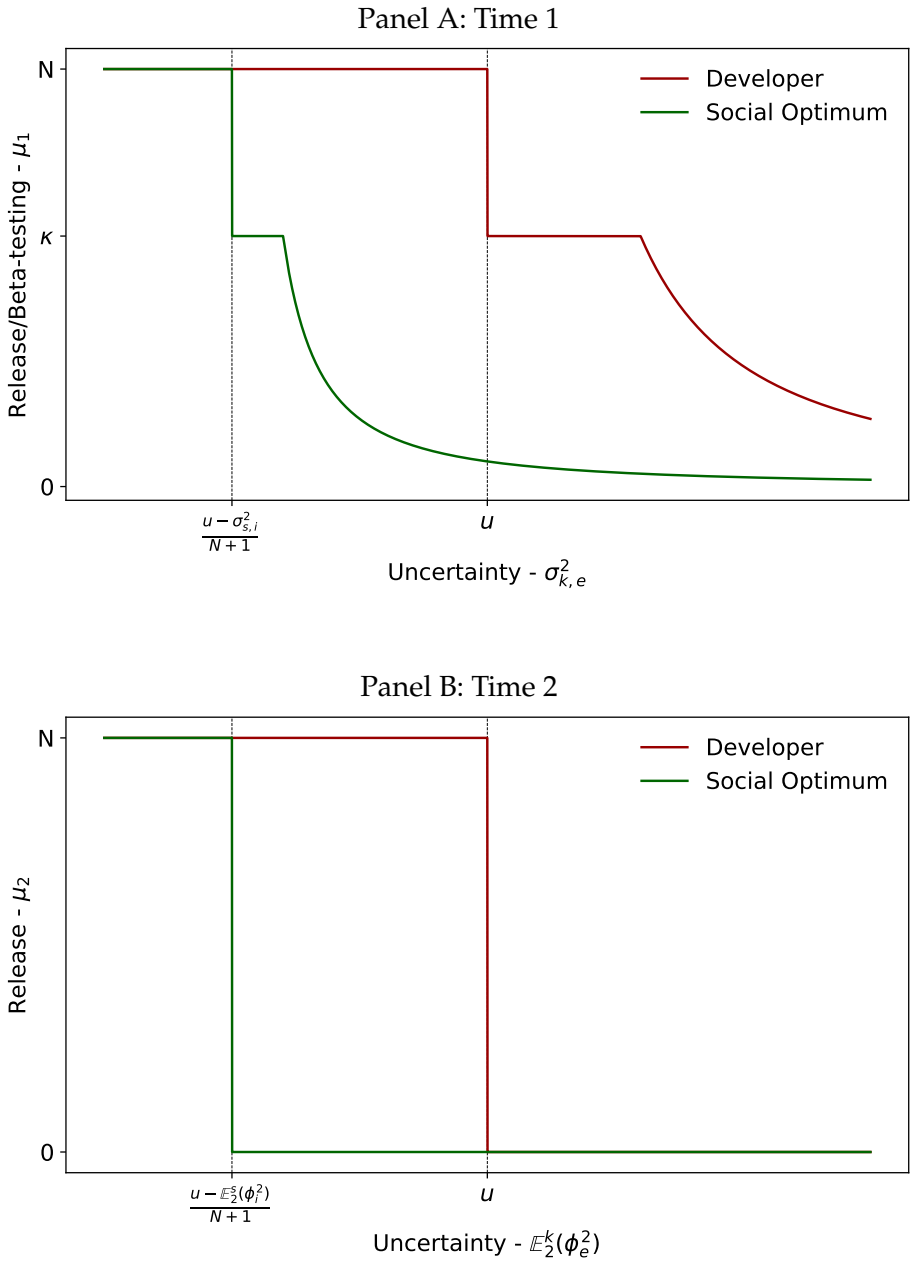
Europe's reliance on banning high-risk applications and regulatory sandboxes closely mirrors the quantity-based regulation that is optimal in our model when belief disagreement is central. By contrast, the U.S. emphasis on ex-post liability enforced by existing legal frameworks is consistent with environments in which developers are presumed to have superior information about risks.

AI regulation faces broader challenges that are common to other industries. Regulatory capture, where dominant firms shape policies to serve their interests (Stigler, 1971 and Peltzman, 1976), is a significant risk. Additionally, measuring externalities and internalities is complex, and high compliance costs could stifle innovation.

While our analysis focuses on the regulatory framework in a single country, international cooperation is essential when AI systems generate cross-border externalities (see Choi, Jeon, and Menicucci, 2026) for a recent analysis.

Isaac Asimov wrote, "The saddest aspect of life right now is that science gathers knowledge faster than society gathers wisdom." Continued work on AI regulation is essential to build the wisdom required to harness the benefits of this new technology while managing its risks.

Figure 2: Release decisions in the first and second periods



References

ACEMOGLU, D. (2021): "Harms of AI," in *The Oxford Handbook of AI Governance*, Oxford University Press.

ACEMOGLU, D. AND T. LENSMAN (2024): "Regulating Transformative Technologies," *American Economic Review: Insights*, 6, 359–376.

ACEMOGLU, D. AND P. RESTREPO (2022): "Tasks, Automation, and the Rise in U.S. Wage Inequality," *Econometrica*, 90, 1973–2016.

AMODEI, D., C. OLAH, J. STEINHARDT, P. CHRISTIANO, J. SCHULMAN, AND D. MANÉ (2016): "Concrete Problems in AI Safety," *arXiv preprint arXiv:1606.06565*.

BERAJA, M., A. KAO, D. Y. YANG, AND N. YUCHTMAN (2023): "Exporting the Surveillance State via Trade in AI," Working paper, Brookings Center on Regulation and Markets.

BLATTNER, L., S. NELSON, AND J. SPIESS (2021): "Unpacking the Black Box: Regulating Algorithmic Decisions," *arXiv preprint arXiv:2110.03443*.

BURD, J. T. (2021): "Regulatory Sandboxes for Safety Assurance of Autonomous Vehicles," *U. Pa. JL & Pub. Aff.*, 7, 194.

BURSTEIN, A., E. MORALES, AND J. VOGEL (2019): "Changes in Between-Group Inequality: Computers, Occupations, and International Trade," *American Economic Journal: Macroeconomics*, 11, 348–400.

CALLANDER, S. (2011): "Searching and Learning by Trial and Error," *American Economic Review*, 101, 2277–2308.

CALLANDER, S. AND H. LI (2024): "Regulating an Innovative Industry," Working paper.

CHEN, Y. AND X. HUA (2024): "Product Safety in the Age of AI: Autonomy, R&D, and Liability," *HKUST Business School Research Paper*.

CHOI, J. P., D.-S. JEON, AND D. MENICUCCI (2026): "AI Safety and Competition," Manuscript.

CORNELLI, G., S. DOERR, L. GAMBACORTA, AND O. MERROUCHE (2024): "Regulatory Sandboxes and Fintech Funding: Evidence from the UK," *Review of Finance*, 28, 203–233.

COSTINOT, A. AND I. WERNING (2023): "Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation," *The Review of Economic Studies*, 90, 2261–2291.

EUROPEAN UNION (2024): "Regulation (EU) 2024/1689 of the European Parliament and of the Council of 13 June 2024 laying down harmonised rules on artificial intelligence (Artificial Intelligence Act)," Official Journal of the European Union, oJ L, 2024/1689, 12 July 2024.

FARBOODI, M. AND L. VELDKAMP (2021): "A Model of the Data Economy," Tech. rep., National Bureau of Economic Research.

FARHI, E. AND X. GABAIX (2020): "Optimal Taxation with Behavioral Agents," *American Economic Review*, 110, 298–336.

GANS, J. S. (2024): "How Learning About Harms Impacts the Optimal Rate of Artificial Intelligence Adoption," *Economic Policy*, 40, 199–219.

——— (2025): "Regulating the Direction of Innovation," *Journal of Public Economics*, 246, 105375.

GITTINS, J. (1974): "A Dynamic Allocation Index for the Sequential Design of Experiments," *Progress in statistics*, 241–266.

GUERREIRO, J., S. REBELO, AND P. TELES (2022): "Should Robots Be Taxed?" *The Review of Economic Studies*, 89, 279–311.

HARRIS, L. AND C. CRIDDLE (2025): "Insurers Retreat from AI Cover as Risk of Multibillion-Dollar Claims Mounts," *Financial Times*, accessed: 2025-11-29.

HART, R. (2024): "AI Models Like ChatGPT Won't Reach Human Intelligence, Meta's AI Chief Says," *Forbes*.

HEAVEN, W. D. (2023): "Geoffrey Hinton Tells Us Why He's Now Scared of the Tech He Helped Build," *MIT Technology Review*.

HERRNSTEIN, R. J., G. F. LOEWENSTEIN, D. PRELEC, AND W. VAUGHAN JR (1993): "Utility Maximization and Melioration: Internalities in Individual Choice," *Journal of behavioral decision making*, 6, 149–185.

IDE, E. AND E. TALAMÀS (2025): "Artificial Intelligence in the Knowledge Economy," *Journal of Political Economy*, 133, 3762–3800.

JAGERMAN, R., D. M. ROIJERS, D. HENNES, AND M. DE RIJKE (2020): "Safe Exploration for Optimizing Contextual Bandits," in *Proceedings of the 43rd International ACM SIGIR Conference on Research and Development in Information Retrieval*, 1613–1616.

JONES, C. I. (2024): "The AI Dilemma: Growth Versus Existential Risk," *American Economic Review: Insights*, 6, 575–590.

JONES, C. I. AND C. TONETTI (2020): "Nonrivalry and the Economics of Data," *American Economic Review*, 110, 2819–2858.

KAZEROUNI, A., M. GHAVAMZADEH, Y. ABBASI-YADKORI, AND B. VAN ROY (2017): "Conservative Contextual Linear Bandits," in *Advances in Neural Information Processing Systems*, vol. 30.

KLEINBERG, J., J. LUDWIG, S. MULLAINATHAN, AND C. R. SUNSTEIN (2018): "Discrimination in the Age of Algorithms," *Journal of Legal Analysis*, 10, 113–174.

KOLSTAD, C. D. AND T. S. ULEN (1983): "Ex Post Liability for Harm vs. Ex Ante Safety Regulation: Substitutes or Complements?" *American Economic Review Papers and Proceedings*, 73, 277–280.

KOSMYNA, N., E. HAUPTMANN, Y. T. YUAN, J. SITU, X.-H. LIAO, A. V. BERESNITZKY, I. BRAUNSTEIN, AND P. MAES (2025): "Your Brain on ChatGPT: Accumulation of Cognitive Debt when Using an AI Assistant for Essay Writing Tasks," *arXiv preprint*, preprint, under review.

LAFFONT, J. J. (1977): "More on Prices vs. Quantities," *The Review of Economic Studies*, 44, 177–182.

MARTINEZ, J. (2021): "Putty-Clay Automation," *DP16022*.

PELTZMAN, S. (1976): "Toward a More General Theory of Regulation," *Journal of Law and Economics*, 19, 211–240.

RAMBACHAN, A., J. KLEINBERG, S. MULLAINATHAN, AND J. LUDWIG (2020): "An Economic Approach to Regulating Algorithms," NBER Working Paper 27111, National Bureau of Economic Research, revised January 2021.

RUSSELL, S., D. DEWEY, AND M. TEGMARK (2015): "Research Priorities for Robust and Beneficial Artificial Intelligence," *AI Magazine*, 36, 105–114.

SCOTT, J. (2025): "New Turing Award Winner Richard Sutton Calls Doomers "Out of Line," Talks Path to Human-Like AI," *BetaKit*, accessed: 2025-03-09.

STIGLER, G. J. (1971): "The Theory of Economic Regulation," *The Bell Journal of Economics and Management Science*, 2, 3–21.

THOMPSON, W. R. (1933): "On the Likelihood that One Unknown Probability Exceeds Another in View of the Evidence of Two Samples," *Biometrika*, 25, 285–294.

THUEMMEL, U. (2023): "Optimal Taxation of Robots," *Journal of the European Economic Association*, 21, 1154–1190.

TIROLE, J. (2010): "From Pigou to Extended Liability: On the Optimal Taxation of Externalities Under Imperfect Financial Markets," *The Review of Economic Studies*, 77, 697–729.

TYRANGIEL, J. (2025): "Sam Altman on ChatGPT's First Two Years, Elon Musk and AI Under Trump," *Bloomberg Businessweek*, accessed: 2025-02-09.

WEITZMAN, M. L. (1974): "Prices vs. Quantities," *The Review of Economic Studies*, 41, 477–491.

——— (1978): "Optimal Rewards for Economic Regulation," *The American Economic Review*, 68, 683–691.

WIENER, N. (1960): "Some Moral and Technical Consequences of Automation," *Science*, 131, 1355–58.

WU, Y., R. SHARIFF, T. LATTIMORE, AND C. SZEPESVÁRI (2016): "Conservative Bandits," in *Advances in Neural Information Processing Systems*, vol. 29.

Online Appendix
Regulating Artificial Intelligence

A Proofs

A.1 Proof of Lemma 1

First, note that

$$\begin{aligned}\mathbb{E}_1^d[\mathcal{V}_2^*] =& \pi(\mu_1)\mathbb{E}_1^d\left[\max\{u-(\hat{\phi}_{s,i}^2+\hat{\sigma}_{s,i}^2)-(\hat{\phi}_{d,e}^2+\hat{\sigma}_{d,e}^2),0\}N\right]\\ &+(1-\pi(\mu_1))\max\{u-\sigma_{s,i}^2-\sigma_{d,e}^2,0\}N\end{aligned}$$

So, assuming $\mu_1 < \kappa$,

$$\begin{aligned}\frac{d\mathbb{E}_1^d[\mathcal{V}_2^*]}{d\mu_1} &= \pi'(\mu_1)\left\{\mathbb{E}_1^d\left[\max\{u-(\hat{\phi}_{s,i}^2+\hat{\sigma}_{s,i}^2)-(\hat{\phi}_{d,e}^2+\hat{\sigma}_{d,e}^2),0\}N\right]-\max\{u-\sigma_{s,i}^2-\sigma_{d,e}^2,0\}N\right\}\\ &> \pi'(\mu_1)\left\{\max\{u-\mathbb{E}_1^d\left[(\hat{\phi}_{s,i}^2+\hat{\sigma}_{s,i}^2)\right]-\mathbb{E}_1^d\left[\hat{\phi}_{d,e}^2+\hat{\sigma}_{d,e}^2\right],0\}N-\max\{u-\sigma_{s,i}^2-\sigma_{d,e}^2,0\}N\right\}\\ &\geq \pi'(\mu_1)\left\{\max\{u-\sigma_{s,i}^2-\sigma_{d,e}^2,0\}N-\max\{u-\sigma_{s,i}^2-\sigma_{d,e}^2,0\}N\right\}=0.\end{aligned}$$

The inequality holds because the expected value of the maxima is higher than the maximum of the expected value. The inequality is strict because the probability that the algorithm is implemented in period two, given the information obtained in period one, is strictly positive but less than one.

A.2 Proof of Proposition 1

The developer chooses μ_1 to maximize

$$\mathcal{V} \equiv (1-\beta)\{u-\sigma_{s,i}^2-\sigma_{d,e}^2\}\mu_1+\beta\mathbb{E}_1^d(\mathcal{V}_2^*). \tag{A.1}$$

From Lemma 1, we know that $\mathbb{E}_1^d(\mathcal{V}_2^*)$ is increasing in μ_1. So, if $\sigma_{d,e}^2 \leq u-\sigma_{s,i}^2$, then the developer chooses $\mu_1 = N$ since $\mathcal{V}$ is always increasing in μ_1.

Suppose instead that $\sigma^2_{d,e} > u - \sigma^2_{s,i}$. In this case, if $\mathcal{B} = 0$, the developer does not release the algorithm at time two. Since $\alpha \in (0,1)$, then $\pi'(\mu_1) \to \infty$ as $\mu_1 \to 0$. It follows that the optimal $\mu_1 > 0$.

$$\frac{d\mathbb{E}^d_1(\mathcal{V}^*_2)}{d\mu_1} = \pi'(\mu_1)\mathbb{E}^d_1\left[\max\{u - (\hat{\phi}^2_{s,i} + \hat{\sigma}^2_{s,i}) - (\hat{\phi}^2_{d,e} + \hat{\sigma}^2_{d,e}), 0\}N\right], \tag{A.2}$$

so, the first order condition is given by

$$(1-\beta)\{u - \sigma^2_{s,i} - \sigma^2_{d,e}\} + \beta\pi'(\mu_1)\mathbb{E}^d_1\left[\max\{u - (\hat{\phi}^2_{s,i} + \hat{\sigma}^2_{s,i}) - (\hat{\phi}^2_{d,e} + \hat{\sigma}^2_{d,e}), 0\}N\right] = 0$$

$$\Leftrightarrow \pi'(\mu_1)\Lambda^d N = 1 \Leftrightarrow \alpha\Lambda^d\frac{N}{\kappa} = \left(\frac{\mu_1}{\kappa}\right)^{1-\alpha} \Leftrightarrow \mu_1 = \left[\alpha\Lambda^d\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa,$$

or $\mu_1 = \kappa$ if $\left[\alpha\Lambda^d\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}} > 1$.

A.3 Proof of Lemma 2

As before, assuming $\mu_1 < \kappa$,

$$\begin{aligned}\frac{d\mathbb{E}^s_1(d\mathcal{W}^*_2)}{d\mu_1} &= \pi'(\mu_1)\left\{\mathbb{E}^s_1\left(\max\left\{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e), 0\right\}N\right)\right.\\ &\quad\left. - \max\left\{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}, 0\right\}N\right\}\\ &> \pi'(\mu_1)\left\{\max\left\{u - \mathbb{E}^s_1(\phi^2_i) - (N+1)\mathbb{E}^s_1(\phi^2_e), 0\right\}N\right.\\ &\quad\left. - \max\left\{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}, 0\right\}N\right\} = 0.\end{aligned}$$

A.4 Proof of Proposition 2

The planner chooses μ_1 to maximize

$$\mathcal{W} \equiv (1-\beta)\{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}\}\mu_1 + \beta\mathbb{E}^s_1(\mathcal{W}^*_2). \tag{A.3}$$

From Lemma 2, we know that $\mathbb{E}_1^s(\mathcal{W}_2^*)$ is increasing in μ_1. So, if $\sigma_{s,e}^2 \leq \frac{u-\sigma_{s,i}^2}{N+1}$, then the planner chooses $\mu_1 = N$ since $\mathcal{W}$ is always increasing in μ_1.

Suppose instead that $\sigma_{s,e}^2 > \frac{u-\sigma_{s,i}^2}{N+1}$. In this case, if $\mathcal{B} = 0$, then the planner does not release the algorithm at time two. Since $\pi'(\mu_1) \to \infty$ as $\mu_1 \to 0$, then $\mu_1 > 0$.

$$\frac{d\mathbb{E}_1^s[\mathcal{W}_2^*]}{d\mu_1} = \pi'(\mu_1)\mathbb{E}_1^s\left[\max\{u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) - (N+1)(\hat{\phi}_{s,e}^2 + \hat{\sigma}_{s,e}^2), 0\}N\right]. \tag{A.4}$$

So, the first order condition is given by

$$(1-\beta)\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\} + \beta\pi'(\mu_1)\mathbb{E}_1^s\left[\max\{u - (\hat{\phi}_{s,i}^2 + \hat{\sigma}_{s,i}^2) - (N+1)(\hat{\phi}_{s,e}^2 + \hat{\sigma}_{s,e}^2), 0\}N\right] = 0$$

$$\Leftrightarrow \pi'(\mu_1)\Lambda^s N = 1 \Leftrightarrow \alpha\Lambda^s\frac{N}{\kappa} = \left(\frac{\mu_1}{\kappa}\right)^{1-\alpha} \Leftrightarrow \mu_1 = \left[\alpha\Lambda^s\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa,$$

or $\mu_1 = \kappa$ if $\left[\alpha\Lambda^s\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}} > 1$.

A.5 Ex-ante Pigouvian Taxes with Heterogeneous Beliefs and Proof of Proposition 4

Assume that the regulator enforces the ex-ante taxes specified in equation (11). Consider the problem of period two. Substituting the optimal license price, the developer's utility is given by

$$\begin{cases} [u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^d(\phi_e^2)]\mu_2 - T_2^{ex-ante} = [u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2)]\mu_2, & \text{if } \mathcal{B} = 1 \\ [u - \sigma_{s,i}^2 - \sigma_{d,e}^2]\mu_2 - T_2^{ex-ante} = [u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2]\mu_2, & \text{if } \mathcal{B} = 0. \end{cases}$$

It follows that private and social incentives are always aligned in period two.

Turning to the problem of the first period, with the taxes given by equation (11), the problem becomes:

$$\max_{\mu_1}\left\{(1-\beta)\left\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\right\}\mu_1 + \beta\mathbb{E}_1^d[\mathcal{W}_2^*]\right\}. \tag{A.5}$$

We immediately see that if the risk is not very large, $\sigma^2_{s,e} \leq \frac{u-\sigma^2_{s,i}}{N+1}$, then the developer chooses $\mu_1 = N$. If the risk is large, $\sigma^2_{s,e} > \frac{u-\sigma^2_{s,i}}{N+1}$, then the developer's choice satisfies

$$\pi'(\mu_1)N\Lambda^s \frac{\mathbb{E}^d_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e), 0\}]}{\mathbb{E}^s_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e), 0\}]} = 1$$

$$\Leftrightarrow \mu_1 = \left[\alpha\Lambda^s \frac{\mathbb{E}^d_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e), 0\}]}{\mathbb{E}^s_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e), 0\}]} \frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}} \kappa.$$

So, in general, the developer prefers to beta test on a larger population sample than what would be socially optimal.

Suppose that the tax in period one is given by equation (13). Then, again, if the uncertainty is small enough, $\sigma^2_{s,e} \leq \frac{u-\sigma^2_{s,i}}{N+1}$, then the developer chooses $\mu_1 = N$. Instead, if the risk is large, $\sigma^2_{s,e} > \frac{u-\sigma^2_{s,i}}{N+1}$, the developer solves the same problem as the planner:

$$\max_{\mu_1} \left\{(1-\beta)\left\{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}\right\}\mu_1 + \beta\mathbb{E}^s_1(\mathcal{W}^*_2)\right\}. \tag{A.6}$$

So private and social incentives are aligned.

A.6 Proof of Lemma 3

Given the tax policy, the developer's problem at time two is:

$$\mathcal{V}_2 = \begin{cases} (p_2 - \tau_2)\mu_2 - \mathbb{E}^\theta_2(\phi^2_e)\mu_2, & \text{if } p_2 \leq u - \mathbb{E}^s_2(\phi^2_i) \text{ and } \mathcal{B} = 1, \\ (p_2 - \tau_2)\mu_2 - \sigma^2_{\theta,e}\mu_2, & \text{if } p_2 \leq u - \sigma^2_{s,i} \text{ and } \mathcal{B} = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the optimal price is

$$p_2 = \begin{cases} u - \mathbb{E}^s_2(\phi^2_i), & \text{if } \mathcal{B} = 1, \\ u - \sigma^2_{s,i}, & \text{if } \mathcal{B} = 0. \end{cases}$$

If $\mathcal{B} = 1$, the developer releases the algorithm to the entire population if $u - \mathbb{E}^s_2(\phi^2_i) - \tau_2 > \mathbb{E}^\theta_2(\phi^2_e)$ and sets $\mu_2 = 0$ otherwise. If $\mathcal{B} = 0$, the developer releases the algorithm to the entire population if $u - \sigma^2_{s,i} - \tau_2 > \sigma^2_{\theta,e}$ and sets $\mu_2 = 0$ otherwise.

At time one, it is optimal to set $p_1 = u - \sigma^2_{s,i}$. The developer's problem at time one (replacing p_1) is given by

$$\max_{\mu_1}(1-\beta)\{u - \sigma^2_{s,i} - \sigma^2_{\theta,e} - \tau_1\}\mu_1 + \beta\mathbb{E}^\theta_1[\mathcal{V}^*_{2,\theta}], \tag{A.7}$$

where $\mathbb{E}^\theta_1(\mathcal{V}^*_{2,\theta})$ is increasing in μ_1 as in Lemma 1.

So, if $\sigma^2_{\theta,e} \leq u - \sigma^2_{s,i} - \tau_1$, then the developer relases the algorithm to the entire population $\mu_1 = N$.

If $\sigma^2_{\theta,e} > u - \sigma^2_{s,i} - \tau_1$, it is optimal to beta test the algorithm. In this case, the optimal μ_1 solves

$$\max_{\mu_1} -\mu_1 + \pi(\mu_1)\frac{\beta}{1-\beta}\frac{\mathbb{E}^\theta_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - \mathbb{E}^\theta_2(\phi^2_e) - \tau_2, 0\}] - \max\{u - \sigma^2_{s,i} - \sigma^2_{\theta,e} - \tau_2, 0\}}{\sigma^2_{s,i} + \sigma^2_{\theta,e} + \tau_1 - u}N. \tag{A.8}$$

Let

$$\Lambda^\theta(\tau_1, \tau_2) \equiv \frac{\beta}{1-\beta}\frac{\mathbb{E}^\theta_1[\max\{u - \mathbb{E}^s_2(\phi^2_i) - \mathbb{E}^\theta_2(\phi^2_e) - \tau_2, 0\}] - \max\{u - \sigma^2_{s,i} - \sigma^2_{\theta,e} - \tau_2, 0\}}{\sigma^2_{s,i} + \sigma^2_{\theta,e} + \tau_1 - u}. \tag{A.9}$$

The first order condition is,

$$\pi'(\mu_1)\Lambda^\theta(\tau_1, \tau_2)N = 1 \Leftrightarrow \mu_1 = \left[\alpha\Lambda^\theta(\tau_1, \tau_2)\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa.$$

A.7 Proof of Proposition 5

We solve for the optimal tax policy without commitment. At time two, the regulator chooses τ_2 to maximize

$$\begin{cases} \{u - \mathbb{E}^s_2(\phi^2_i) - (N+1)\mathbb{E}^s_2(\phi^2_e)\}\mathbb{E}^s_2(\mu^\theta_2), & \text{if } \mathcal{B} = 1, \\ \{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}\}\mathbb{E}^s_2(\mu^\theta_2), & \text{if } \mathcal{B} = 0, \end{cases}$$

where $\mathbb{E}^s_2(\mu^\theta_2)$ denotes the regulator's expectations of the developer's choice of μ_2 given the regulator's beliefs over the developer's type at time two. These beliefs are influenced by the developer's decisions observed at time one.

If given the regulator's beliefs the algorithm should be released, then it is optimal to set $\tau_2 = 0$, since under this tax $\mu_2^\theta = N$ for all θ:

$$\begin{cases} 0 \leq u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2) < u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^s(\phi_e^2) \leq u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^\theta(\phi_e^2), \quad \forall \theta \in \Theta \\ 0 \leq u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2 < u - \sigma_{s,i}^2 - \sigma_{s,e}^2 \leq u - \sigma_{s,i}^2 - \sigma_{\theta,e}^2, \quad \forall \theta \in \Theta. \end{cases}$$

If the regulator's beliefs are such that the algorithm should be withdrawn, then by setting $\tau_2 = p_2$ the regulator ensures that all developer types withdraw the algorithm.[9] Welfare at time two coincides with the social optimum.

At time one, the regulator chooses τ_1 to maximize

$$(1-\beta)\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\} \int_{\underline{\theta}}^{\overline{\theta}} \mu_1^\theta dF(\theta) + \beta \mathbb{E}_1^s(\mathcal{W}_2^*). \tag{A.10}$$

First, suppose that uncertainty about the externality is small $\sigma_{s,e} \leq \frac{u-\sigma_{s,i}^2}{N+1}$. In this case, it is efficient to release the algorithm to the entire population. So, the regulator sets $\tau_1 = 0$ and $\mu_1^\theta = N$ for all θ, since

$$0 \leq u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2 < u - \sigma_{s,i}^2 - \sigma_{s,e}^2 \leq u - \sigma_{s,i}^2 - \sigma_{\theta,e}^2, \quad \forall \theta \in \Theta.$$

Suppose that uncertainty is large $\sigma_{s,e} > \frac{u-\sigma_{s,i}^2}{N+1}$. Then, the regulator chooses τ_1 to maximize

$$(1-\beta)\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\} \int_{\underline{\theta}}^{\overline{\theta}} \mu_1^\theta dF(\theta) + \beta \int_{\underline{\theta}}^{\overline{\theta}} \pi(\mu_1^\theta) dF(\theta) \times \mathbb{E}_1^s[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}]N. \tag{A.11}$$

Equivalently, τ_1 solves

$$\max_{\tau_1} \left(- \int_{\underline{\theta}}^{\overline{\theta}} \mu_1^\theta dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi(\mu_1^\theta) dF(\theta) \Lambda^s N \right). \tag{A.12}$$

The first order condition with respect to τ_1 is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{d\mu_1^\theta / d\tau_1}{\int_{\underline{\theta}}^{\overline{\theta}} d\mu_1^\theta / d\tau_1 dF(\theta)} \pi'(\mu_1^\theta) \Lambda^s N dF(\theta) = 1, \tag{A.13}$$

[9] We assume that, in indifference, developers decide in favor of the planner.

where μ_1^θ is given by

$$\mu_1^\theta = \min\left\{\left[\alpha\Lambda^\theta\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}, 1\right\}\kappa \tag{A.14}$$

and

$$\Lambda^\theta \equiv \frac{\beta}{1-\beta}\frac{\mathbb{E}_1^\theta\left[\{u-\mathbb{E}_2^\theta(\phi_i^2)-\mathbb{E}_2^\theta(\phi_e^2)\}\mathbb{1}_{\{u-\mathbb{E}_2^s(\phi_i^2)-(N+1)\mathbb{E}_2^s(\phi_e^2)\geq 0\}}\right]}{\sigma_{s,i}^2+\sigma_{\theta,e}^2+\tau_1-u}.$$

Since Λ^θ is increasing in θ, the developer's optimal beta test sample size is also increasing in θ.

A.8 Proof of Lemma 4

For any regulation policy, the problem of the developer at time two is given by:

$$\mathcal{V}_2 = \begin{cases} p_2\mu_2 - \mathbb{E}_2^\theta(\phi_e^2)\mu_2, & \text{if } p_2 \leq u - \mathbb{E}_2^s(\phi_i^2) \text{ and } \mathcal{B}=1, \\ p_2\mu_2 - \sigma_{\theta,e}^2\mu_2, & \text{if } p_2 \leq u - \sigma_{s,i}^2 \text{ and } \mathcal{B}=0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the optimal price is

$$p_2 = \begin{cases} u - \mathbb{E}_2^s(\phi_i^2), & \text{if } \mathcal{B}=1, \\ u - \sigma_{s,i}^2, & \text{if } \mathcal{B}=0. \end{cases}$$

So, if $\mathcal{B}=1$, the developer releases the algorithm to the maximum $\mu_2=\overline{\mu}_2$ if $u - \mathbb{E}_2^s(\phi_i^2) > \mathbb{E}_2^\theta(\phi_e^2)$ and sets $\mu_2=0$ otherwise. Analogously, if $\mathcal{B}=0$, the developer releases the algorithm to to the maximum $\mu_2=\overline{\mu}_2$ if $u-\sigma_{s,i}^2 > \sigma_{\theta,e}^2$ and sets $\mu_2=0$ otherwise.

At time one, it is optimal to set $p_1 = u - \sigma_{s,i}^2$. The developer's problem at time one (replacing p_1) is given by

$$\max_{\mu_1}(1-\beta)\{u-\sigma_{s,i}^2-\sigma_{\theta,e}^2\}\mu_1 + \beta\mathbb{E}_1^\theta[\mathcal{V}_{2,\theta}^*]. \tag{A.15}$$

Where $\mathbb{E}_1^\theta(\mathcal{V}_{2,\theta}^*)$ is increasing in μ_1 as in Lemma 1.

If $\sigma^2_{\theta,e} \leq u - \sigma^2_{s,i}$, then the developer releases the algorithm to the maximum number of people $\mu_1 = \overline{\mu}_1$.

If $\sigma^2_{\theta,e} > u - \sigma^2_{s,i}$, it is optimal to beta test the algorithm. In this case, the optimal μ_1 solves

$$\max_{\mu_1} -\mu_1 + \pi(\mu_1)\frac{\beta}{1-\beta}\frac{\mathbb{E}_1^\theta[\max\{u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^\theta(\phi_e^2), 0\}\overline{\mu}_2] - \mathbb{E}_1^\theta[\max\{u - \sigma^2_{s,i} - \sigma^2_{\theta,e}, 0\}\overline{\mu}_2]}{\sigma^2_{s,i} + \sigma^2_{\theta,e} - u}. \tag{A.16}$$

Let

$$\Lambda^\theta \equiv \frac{\beta}{1-\beta}\frac{\mathbb{E}_1^\theta[\max\{u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^\theta(\phi_e^2), 0\}\overline{\mu}_2] - \mathbb{E}_1^\theta[\max\{u - \sigma^2_{s,i} - \sigma^2_{\theta,e}, 0\}\overline{\mu}_2]}{\sigma^2_{s,i} + \sigma^2_{\theta,e} - u}. \tag{A.17}$$

Then, the first order condition is given by

$$\pi'(\mu_1)\Lambda^\theta = 1 \Leftrightarrow \mu_1 = \left[\alpha\Lambda^\theta\frac{1}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa.$$

In this case, the developer sells to this number of people if $\left[\alpha\Lambda^\theta\frac{1}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa \leq \overline{\mu}_1$ and $\mu_1 = \overline{\mu}_1$ otherwise.

A.9 Proof of Proposition 6

We solve for the optimal regulation policy without commitment. At time two, the regulator chooses $\overline{\mu}_2$ to maximize

$$\begin{cases} \{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2)\}\mathbb{E}_2^s[\mu_2^\theta], & \text{if } \mathcal{B} = 1, \\ \{u - \sigma^2_{s,i} - (N+1)\sigma^2_{s,e}\}\mathbb{E}_2^s[\mu_2^\theta], & \text{if } \mathcal{B} = 0, \end{cases}$$

where $\mathbb{E}_2^s[\mu_2^\theta]$ denotes the regulator's expectations over the developer's behavior given the regulator's beliefs over the developer's type at time two (which are influenced by the observed decisions at time-one).

If given the regulator's beliefs the algorithm should be released, it is optimal to set $\overline{\mu}_2 = N$, since under this cap $\mu_2^\theta = N$ for all θ:

$$\begin{cases} 0 \leq u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2) < u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^s(\phi_e^2) \leq u - \mathbb{E}_2^s(\phi_i^2) - \mathbb{E}_2^\theta(\phi_e^2), & \forall \theta \in \Theta \\ 0 \leq u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2 < u - \sigma_{s,i}^2 - \sigma_{s,e}^2 \leq u - \sigma_{s,i}^2 - \sigma_{\theta,e}^2, & \forall \theta \in \Theta. \end{cases}$$

Instead, if the regulator's beliefs are such that the algorithm should be withdrawn, then by setting $\overline{\mu}_2 = 0$ the regulator ensures that no developer type releases the algorithm. Note that welfare at time two coincides with the efficient level.

Turning to the problem at time one, the regulator chooses $\overline{\mu}_1$ to maximize

$$(1-\beta)\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\} \int_{\underline{\theta}}^{\overline{\theta}} \mu_1^\theta dF(\theta) + \beta \mathbb{E}_1^s[\mathcal{W}_2^*]. \tag{A.18}$$

First, suppose that uncertainty about the externality is small $\sigma_{s,e} \leq \frac{u - \sigma_{s,i}^2}{N+1}$. In this case, it is efficient to release the algorithm to the entire population. So, the regulator sets $\overline{\mu}_1 = N$, which imples that $\mu_1^\theta = N$ for all θ, since

$$0 \leq u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2 < u - \sigma_{s,i}^2 - \sigma_{s,e}^2 \leq u - \sigma_{s,i}^2 - \sigma_{\theta,e}^2, \quad \forall \theta \in \Theta.$$

Suppose that uncertainty is large $\sigma_{s,e}^2 > \frac{u - \sigma_{s,i}^2}{N+1}$. Then, the regulator chooses τ_1 to maximize

$$(1-\beta)\{u - \sigma_{s,i}^2 - (N+1)\sigma_{s,e}^2\} \int_{\underline{\theta}}^{\overline{\theta}} \mu_1^\theta dF(\theta) + \beta \int_{\underline{\theta}}^{\overline{\theta}} \pi(\mu_1^\theta) dF(\theta) \\ \times \mathbb{E}_1^s[\max\{u - \mathbb{E}_2^s(\phi_i^2) - (N+1)\mathbb{E}_2^s(\phi_e^2), 0\}]N. \tag{A.19}$$

Let μ_1^* denote the efficient size of the beta-test in this case. We have already established that unconstrained

$$\mu_1^\theta > \mu_1^*.$$

It follows that setting $\overline{\mu}_1 = \mu_1^*$ implies that $\mu_1^\theta = \mu_1^*$ for all θ. This policy implements the efficient outcome and is therefore optimal.

A.10 Optimal Non-Linear Taxes on Developer with Private Information

In the main text, we restrict attention to linear Pigouvian taxes. In this appendix, we generalize the analysis to considering non-linear taxes on license sales. Let the total tax payment as a function of μ_t be $t_t(\mu_t)$. The case of linear taxes obtains when $t_t(\mu_t) = \tau_t \mu_t$.

Definition 1. *A tax policy is a set of (state-contingent) tax functions* $\boldsymbol{t} \equiv \{t_t(\mu_t)\}$ *for* $t = 1, 2$ *that determine the taxes imposed on the developer as a function of their release strategy* μ_t*.*

We allow the tax functions to be arbitrarily non-linear, so solving the developer's general problem becomes more complex. However, it is possible to show that there is a non-linear tax policy that implements efficient allocation.

It is easy to construct tax functions that implement the efficient allocation. For example, consider the following tax policy. At time 2, set the tax function

$$t_2(\mu_2) = \begin{cases} p_2 \mu_2, & \text{if } \mu_2 \neq \mu_2^*, \\ 0, & \text{if } \mu_2 = \mu_2^*, \end{cases}$$

where μ_2^* is the efficient level of release. At time 1, if the AI is sufficiently risky so that beta testing is socially optimal, the regulator announces the taxes

$$t_1(\mu_1) = \begin{cases} p_1 \mu_1 + \frac{\beta}{1-\beta} \pi(\mu_1) \mathbb{E}_1^{\overline{\theta}}[\max\{u - \mathbb{E}_2^s[\phi_i^2], 0\} N], & \text{if } \mu_1 \neq \mu_1^*, \\ 0, & \text{if } \mu_1 = \mu_1^*, \end{cases}$$

where μ_1^* is the efficient level of beta testing. If the AI is not risky enough to warrant beta testing, the regulator sets $t_1(\mu_1) = 0$.

This tax policy implements the efficient allocation. Effectively, this non-linear tax policy implements the same allocation as the MBR policy.

B Limited Liability

B.1 The Impact of Limited liability on Ex-post Pigouvian Taxes

To study the consequences of limited liability for ex-post Pigouvian taxes, we consider the simple case in which the taxes paid by the developer in each period cannot exceed their revenue ($p_t\mu_t$):

$$T_t = \min\{Ne_t^2, p_t\mu_t\}.$$

This limited liability constraint is a cash-flow constraint that limits the developer's ability to pay taxes.

Under limited liability, the developer's optimal algorithm release policy differs from the social optimum even if beliefs are homogeneous. This divergence arises because the developer's potential losses are capped, encouraging it to release moderately risky algorithms relying on limited liability to protect itself if significant adverse external effects occur.

In this case, we can show that

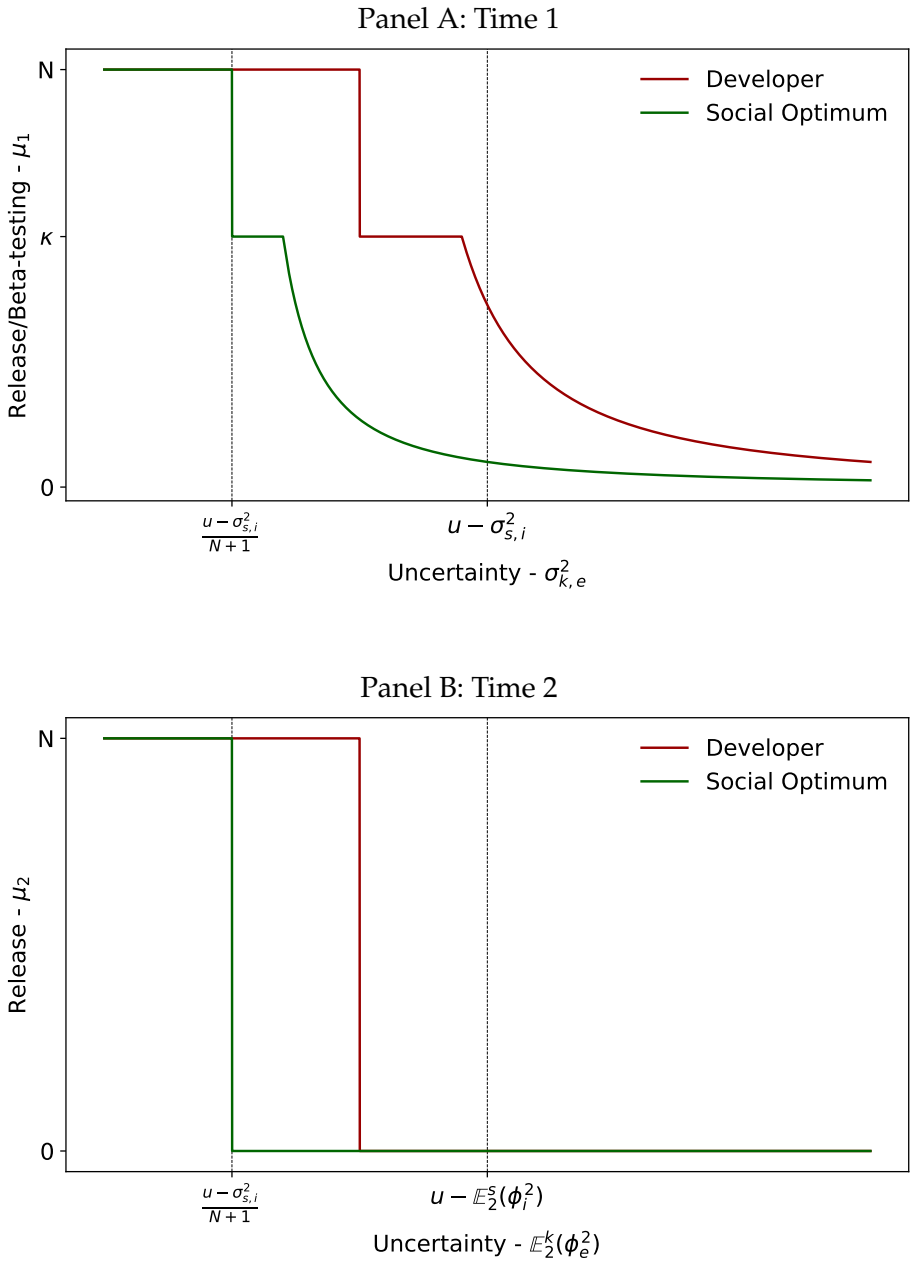
$$\mathcal{V}_t = \mathcal{W}_t - Ny_t + \mathbb{E}_t[\max\{N(\phi_e^2 + \xi_{e,t}) - p_t, 0\}]\mu_t, \tag{B.20}$$

where $\xi_{e,1} = \xi_e$ and $\xi_{e,2} = 0$. To see this, note that if beliefs are homogeneous, then $\mathbb{E}_t^d(\phi_e^2) = \mathbb{E}_t^s(\phi_e^2)$. It is still optimal for the developer to set $p_t = u - \mathbb{E}_t^s(\phi_i^2)$. Replacing this price and the expected taxes into the utility of the developer we find that $\mathcal{V} = (1-\beta)\mathcal{V}_1 + \beta\mathbb{E}_1(\mathcal{V}_2)$, where

$$\begin{aligned}
\mathcal{V}_t &= \mathcal{W}_t - Ny_t - \mathbb{E}_t\left[\min\{Ne_t^2, p_t\mu_t\}\right] + N\mathbb{E}_t\left[e_t^2\right] \\
&= \mathcal{W}_t - Ny_t + \mathbb{E}_t\left[Ne_t^2 + \max\{-Ne_t^2, -p_t\mu_t\}\right] \\
&= \mathcal{W}_t - Ny_t + \mathbb{E}_t\left[\max\{0, Ne_t^2 - p_t\mu_t\}\right] \\
&= \mathcal{W}_t - Ny_t + \mathbb{E}_t\left[\max\{0, N(\phi_e^2 + \xi_{e,t}) - p_t\}\mu_t\right].
\end{aligned}$$

The expected value of the taxes on the developer is lower than the expected social welfare cost of the externality. It follows that the developer is more likely to release the algorithm in period two than the planner, being protected by limited liability should very negative external effects materialize. The same logic implies that the developer may forgo beta testing in period one and release the algorithm immediately, knowing it is protected by limited liability if dire external effects materialize. As a result, the developer may act with less caution than would be socially optimal. Figure 3 illustrates the release decisions under ex-post Pigouvian taxes with limited liability.

Figure 3: Release decisions in the first and second periods under Ex-post Pigouvian Taxes with Limited Liability



B.2 The Impact of Limited liability on Ex-ante Pigouvian Taxes

We now show that ex-ante Pigouvian taxes fail to implement the social optimum when limited liability is present, even under homogeneous beliefs. While they ensure that the release policy in period 2 remains optimal, they lead to beta test sample sizes in period one that exceed the optimal level.

Imposing limited liability means that

$$T_t^{ex-ante} = \min\left\{N\mathbb{E}_t(\phi_e^2)\mu_t, p_t\mu_t\right\}. \tag{B.21}$$

Consider the problem at time two, after a successful beta test ($\mathcal{B} = 1$). If limited liability is not binding, then private and social incentives coincide. What happens when limited liability does bind? Given that $p_2 = u - \mathbb{E}_2^s(\phi_i^2)$, limited liability binds whenever $u - \mathbb{E}_2^s(\phi_i^2) < N\mathbb{E}_2^s\left(\phi_e^2\right)$. In this scenario, the developer makes no profit from selling the algorithm but still experiences the consequences of the negative externality the algorithm creates. Consequently, the developer has a strict preference not to release the algorithm when limited liability is binding. Limited liability binds only in cases where the regulator would also strictly prefer not to release the algorithm, since $N\mathbb{E}_t^s(\phi_e^2) < (N+1)\mathbb{E}_t^s(\phi_e^2)$. In other words, the developer and the regulator agree not to release the algorithm whenever limited liability binds.

If the beta test is unsuccessful ($\mathcal{B} = 0$), limited liability binds in period two whenever $u - \sigma_{s,i}^2 < N\sigma_{s,e}^2$. In this case, the developer strictly prefers not to release the algorithm. The regulator strictly prefers not to release the algorithm whenever $N\mathbb{E}_t^s(\phi_e^2) < (N+1)\sigma_{s,e}^2$. Therefore, whenever limited liability is binding, both the developer and the regulator agree that the algorithm should not be released.

With ex-ante Pigouvian taxes $\mathbb{E}_1^s(\mathcal{V}_2^*) = \mathbb{E}_1^s(\mathcal{W}_2^*) - Ny_2$, even in the presence of limited liability, so private and social incentives are aligned.

Turning to the problem at time one, private and social incentives coincide if limited liability does not bind. when limited liability binds, both the developer and the planner choose strictly positive values for μ_1, but they select different values. To see

this result, consider the developer's information benefit-cost ratio

$$\Lambda^d = \Lambda^s \times \frac{\sigma_{s,i}^2 + (N+1)\sigma_{s,e}^2 - u}{\frac{T_t^{ex-ante}}{\mu_t} + \sigma_{s,i}^2 + \sigma_{s,e}^2 - u} = \Lambda^s \times \frac{\sigma_{s,i}^2 + (N+1)\sigma_{s,e}^2 - u}{\sigma_{s,e}^2}, \tag{B.22}$$

where Λ^s is given by (7). Since limited liability binds, we have $\sigma_{s,i}^2 + N\sigma_{s,e}^2 - u > 0$, which implies

$$\frac{\sigma_{s,i}^2 + (N+1)\sigma_{s,e}^2 - u}{\sigma_{s,e}^2} > \frac{\sigma_{s,e}^2}{\sigma_{s,e}^2} = 1.$$

Thus, under limited liability, the developer's information benefit-cost ratio exceeds that of the regulator. Consequently, the developer adopts a more aggressive approach, choosing to beta test the algorithm on a larger sample than the regulator would.

When limited liability binds, the developer's after-tax sales revenue is zero. However, since the tax does not fully internalize the externality, the developer still has an incentive to conduct beta testing on a larger sample to increase the likelihood of obtaining valuable information for period two (see Figure 4).

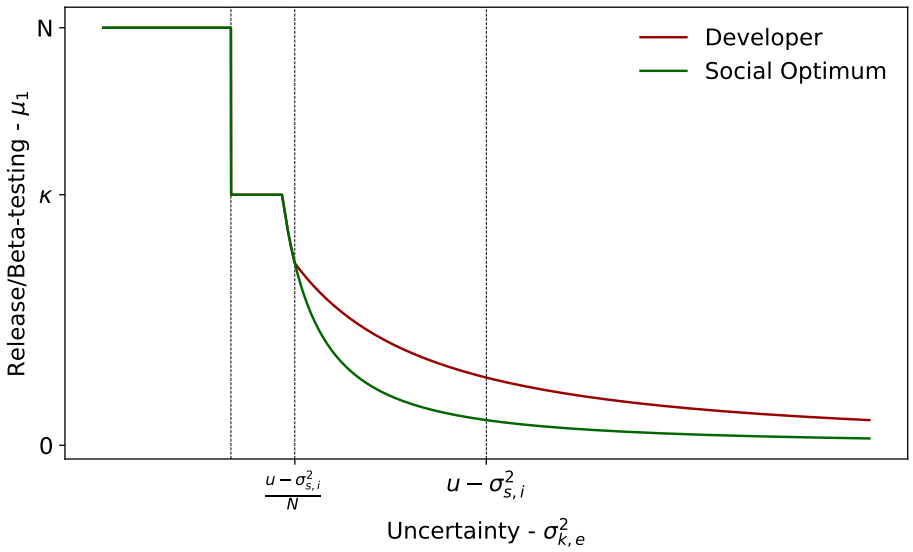
We summarize these results in the following proposition.

Proposition 7 (Ex-Ante Pigouvian Taxes with Homogeneous Beliefs and Limited Liability)**.** *Suppose developers and society share the same beliefs. If the regulator implements ex-ante Pigouvian taxes subject to the limited liability constraints in equation* (B.21)*, then:*

- *In the first period, the developer's choices align with the socially optimal outcomes only if limited liability does not bind. When it binds, the developer has an incentive to beta test the algorithm on a sample that is larger than socially optimal.*
- *In the second period, the developer's decisions regarding release and withdrawal are socially optimal.*

Figure 4 illustrates the release decisions under ex-ante Pigouvian taxes with limited liability for the first period. We ommit the case of the second period, since they coincide.

Figure 4: Time 1 testing decisions with Ex-Ante Taxes and Limited Liability



B.2.1 Proof of Proposition 7

Consider first the problem in the second period. If limited liability does not bind, then release and withdrawal incentives are aligned. Instead, suppose that limited liability binds. Then, the developer makes zero profits from releasing the algorithm, but still suffers from the externality. It follows that, if limited liability binds, the developer withdraws the algorithm. Limited liability binds when

$$\begin{cases} u - \mathbb{E}_2^s(\phi_i^2) = N\mathbb{E}_2^s(\phi_e^2) < (N+1)\mathbb{E}_2^s(\phi_e^2), & \text{if } \mathcal{B}=1 \\ u - \sigma_{s,i}^2 = N\sigma_{s,e}^2 < (N+1)\sigma_{s,e}^2, & \text{if } \mathcal{B}=0. \end{cases}$$

So, when limited liability binds, the regulator also wants the algorithm to be withdrawn.

Turning to the problem of the first period, if limited liability does not bind, then incentives are aligned as before. If limited liability binds, then the developer's prob-

lem is

$$\max_{\mu_1}\left\{(1-\beta)\left\{u-\sigma_{s,i}^2-\sigma_{s,e}^2-\frac{T_1^{ex-ante}}{\mu_1}\right\}\mu_1+\beta\pi(\mu_1)\mathbb{E}_1^s[\max\{u-\mathbb{E}_2^s(\phi_i^2)-(N+1)\mathbb{E}_2^s(\phi_e^2),0\}]N\right\} \tag{B.23}$$

The first order condition is given by

$$(1-\beta)\left\{u-\sigma_{s,i}^2-\sigma_{s,e}^2-\frac{T_1^{ex-ante}}{\mu_1}\right\}+(1-\beta)\pi'(\mu_1)\Lambda^s N\left\{\sigma_{s,i}^2+(N+1)\sigma_{s,e}^2-u\right\}=0$$

$$\Leftrightarrow\pi'(\mu_1)N\Lambda^s\frac{\sigma_{s,i}^2+(N+1)\sigma_{s,e}^2-u}{\frac{T_1^{ex-ante}}{\mu_1}+\sigma_{s,i}^2+\sigma_{s,e}^2-u}=1\Leftrightarrow\pi'(\mu_1)N\Lambda^s\frac{\sigma_{s,i}^2+(N+1)\sigma_{s,e}^2-u}{\sigma_{s,e}^2}=1$$

$$\Leftrightarrow\mu_1=\left[\alpha\Lambda^s\frac{\sigma_{s,i}^2+(N+1)\sigma_{s,e}^2-u}{\sigma_{s,e}^2}\frac{N}{\kappa}\right]^{\frac{1}{1-\alpha}}\kappa.$$

So, when limited liability binds, the developer generally beta tests the algorithm in a larger pool than the social optimal size.