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THE VOTING PREMIUM

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ABSTRACT

This paper develops a unified theory of blockholder governance and the voting premium, in a setting without takeovers and controlling shareholders. A voting premium emerges when a minority blockholder tries to influence the composition of the shareholder base by accumulating votes and buying shares from dissenting shareholders. Empirical measures of the voting premium do not reflect the value of voting rights or voting power. A negative voting premium results from free-riding by dispersed shareholders on the blockholder's trades. Conflicts between dispersed shareholders and the blockholder endogenously increase the liquidity of voting shares, but do not necessarily increase the voting premium.

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An Online Appendix is available at
https://www.dropbox.com/scl/fi/gzw0c6t3v3j39lfqaew0k/VotingPremium_OnlineAppendix.

1 Introduction

Voting is a central mechanism of corporate governance. It empowers shareholders of publicly traded companies to elect directors, approve major corporate transactions, and decide on governance, social, and environmental policies. Most corporations have blockholders that are large enough to influence voting outcomes (La Porta et al., 1999; Edmans and Holderness, 2017; Dasgupta, Fos, and Sautner, 2021; Lewellen and Lewellen, 2022). Blockholders' accumulation of voting power can affect stock prices and give rise to a voting premium.

The asset pricing implications of control rights have been studied extensively. The theoretical literature followed Grossman and Hart (1988) and Harris and Raviv (1988) and attributed the voting premium almost exclusively to takeovers and control contests. This is puzzling in light of a large empirical literature in this area. First, the most common measure of the voting premium is arguably the dual-class premium, which appears to be largest in economies in which contests for majority control are rare, and which does not disappear when regulation requires equal treatment of non-voting shares in takeovers.[1] Second, while all studies find the voting premium to be positive on *average*, many studies document negative voting premiums for some firms, which is difficult to explain in a model with bidding contests. Third, studies that construct non-voting shares synthetically to estimate the voting premium find that it is largest around shareholder meetings compared to other periods of the year (e.g., Kalay, Karakas, and Pant, 2014), which highlights the importance of voting on proposals for the existence of a voting premium. Moreover, recent studies estimate the voting premium from fees in equity lending markets or price changes around record dates. They usually find negligible values for voting rights, which is in conflict with the earlier literature and adds another puzzle.

Overall, these gaps and conflicting conclusions suggest that the theoretical underpinnings of the voting premium are still incomplete. To address these challenges, this paper develops a unified theory of blockholder governance, ownership structure, and the voting premium. We study how and why a voting premium emerges in the absence of takeovers or controlling shareholders, which is the empirically most relevant setting in most major economies.[2]

[1] See our extensive discussion of the empirical literature in Section 7.

[2] Our framework also captures cases in which the firm has a controlling shareholder but the corporate governance system requires the majority of the minority shareholders to approve the proposal (Atanasov, Black, and Ciccotello, 2011; Gözlügöl, 2021). Our analysis applies to such cases if we conceive of the shareholder base in our model as being all the minority shareholders in such a setting.

We analyze a model with a continuum of atomistic dispersed shareholders and one minority blockholder. The baseline model features one-share-one-vote. Shareholders first trade with each other in a competitive stock market. Those who own shares after trading then vote on a proposal at a shareholder meeting. Shareholders observe a public signal about the quality of the proposal before they cast a vote, and the proposal is approved if enough votes are cast in favor. Shareholders differ in their preferences for the proposal: While some shareholders are skeptical and need a lot of evidence to vote in favor, others are more disposed toward the proposal and generally support it. Such heterogeneity may arise from differences in investment horizons; tax status; social and political ideologies; corporate governance philosophies; ownership of other firms; differences in beliefs; and attitudes toward risk.[3]

In our framework, the composition of the shareholder base, voting outcomes, and asset prices are all endogenous. In equilibrium, the proposal is approved if and only if the public signal about its quality exceeds a certain cutoff. Shareholders are heterogeneous, so the proposal is accepted too often from the point of view of some, and rejected too often from the perspective of others. We call the shareholder who fully agrees with the decision rule implied by the cutoff the "median voter;" the median voter's identity completely characterizes the expected voting outcome. Importantly, the median voter can be either a dispersed shareholder or a blockholder, and his identity is determined by the composition of the shareholder base after trading. Hereafter, the term "median voter" is used interchangeably with the expected voting outcome.

The blockholder and dispersed shareholders trade in anticipation of the expected voting outcome and its impact on their valuations; shareholders' valuations could differ because of heterogeneous preferences. Trading reallocates cash flow rights and voting rights across shareholders, since shares are bundles of both. Price-taking dispersed shareholders trade only for cash flow reasons, i.e., if the share price differs from their private valuations. By contrast, the blockholder can be pivotal for the voting outcome, so he may also purchase shares to influence the voting outcome, that is, to push the median voter in his preferred direction.

[3] See the following literature on these issues: Investor time horizons: Bushee (1998) and Gaspar, Massa, and Matos (2005); tax status: Desai and Jin (2011); social and political ideologies and attitudes to corporate governance: Bolton et al. (2020) and Bubb and Catan (2022); conflicts of interest and differences in portfolio ownership: Cvijanovic, Dasgupta, and Zachariadis (2016) and He, Huang, and Zhao (2019); differences in beliefs: Li, Maug, and Schwartz-Ziv (2022). Hayden and Bodie (2008) provide a comprehensive overview of different sources of shareholder heterogeneity.

The equilibrium share price has two components. The first component captures the market clearing price in the hypothetical scenario in which all shareholders anticipate exactly the same decision rule as the one that actually arises, but take it as exogenously given. This price would emerge if the trading of shares did not reallocate voting rights across shareholders, e.g., if trade happened after the record date, or if shares did not have voting rights. The second component of the stock price arises exactly because the trading of shares reallocates voting rights across shareholders, moves the median voter, and thereby changes the value of the shares for small shareholders. We call this term the *voting premium* and show that it reflects the blockholder's net marginal payoff from buying one additional voting right.

Our theoretical definition of the voting premium has two appealing empirical counterparts. First, our definition of the voting premium captures the dual-class premium: In an extension to a dual-class setting, we show that the price differential between voting and non-voting shares reflects the blockholder's net marginal payoff from buying an additional voting right. Second, the voting premium can capture the difference between the share prices right before and right after the record date. The expected voting outcome is the same at both moments in time, but after the record date, trading no longer reallocates voting rights for decisions taken at the upcoming meeting. However, our definition of the voting premium is more general and extends to single-class firms and to dates other than the record date.

We show that a positive voting premium can arise in equilibrium even though there are no takeovers in our model and the blockholder does not obtain majority control. Intuitively, as the blockholder buys more shares, he moves the median voter closer to himself, which helps align the expected voting outcome with his preferences. If the blockholder accumulates enough shares, he even becomes the median voter himself. However, since voting rights are not traded separately from cash flow rights, this accumulation of voting power requires dispersed shareholders to sell some of their shares. Their heterogeneous valuations create an upward-sloping supply function to the blockholder, which results in price impact.

Moreover, these heterogeneous valuations imply that those shareholders who disagree more strongly with the blockholder have lower reservation prices and hence sell disproportionally more shares to him. Thus, the blockholder's trades affect the median voter and the composition of the shareholder base in two ways: directly, by the blockholder accumulating more votes, and indirectly, by reducing the weight of those dispersed shareholders who are less aligned with him. The indirect effect amplifies the blockholder's impact on the voting outcome.

The blockholder's payoff from buying an additional vote is determined by his benefit from one additional voting right, net of the price impact of this additional purchase. If this price impact is significant, the blockholder optimally limits his accumulation of shares; then the net marginal payoff from an additional voting right, and hence the voting premium, is positive. Conversely, if price impact is moderate, the blockholder buys sufficiently many shares to become the median voter. Then, any further purchases would leave the voting outcome unchanged, so the net marginal payoff from an additional voting right, and hence the voting premium, is zero. Therefore, our model can explain empirical studies that document a negligible voting premium (e.g., Christoffersen et al., 2007 and Fos and Holderness, 2022).

The case of a zero voting premium illustrates the general principle that the voting premium does not reflect the economic value of voting rights, because it captures only the blockholder's *marginal* value from an additional vote. In contrast, the blockholder's total value of voting rights reflects his *average* willingness to buy votes, which includes all the infra-marginal trades from his initial endowment to his equilibrium ownership. In this respect, the voting premium often underestimates the value of voting rights. This observation is important for interpreting empirical findings, since some proxies for the voting premium measure the marginal value of a vote (e.g., dual-class share premium; price drop on record days), whereas others are more related to the average value of voting rights (e.g., dual-class tender-offer premium).

For the same reasons, the voting premium is not a good measure of voting power. Voting power is related to the blockholder's likelihood of being pivotal and swinging the voting outcome. However, an increase in the blockholder's voting power decreases his distance from the median voter, and thus his valuation of a marginal vote. Therefore, the magnitude of the voting premium is generally unrelated to the blockholder's voting power, and this relationship can even be negative when the blockholder becomes the median voter himself: Then his voting power is large and the voting premium is zero. Thus, the voting premium emerges not from the blockholder's accumulation of voting power, but from his indirect influence on the voting outcome through the composition of the shareholder base.

Our model can also rationalize a negative voting premium, which has been documented in some firms in most empirical studies. A negative voting premium may appear puzzling; after all, the benefit of a marginal vote to the blockholder is always positive, since the value of his stake increases if he moves the median voter toward himself. However, if the price impact of buying an additional vote is sufficiently large, the blockholder ends up buying fewer shares compared to a scenario in which shares do not have voting rights, and the voting premium becomes negative. To see how this can happen, consider a scenario in which the blockholder opposes a management proposal, but small shareholders are even more strongly biased against this proposal than the blockholder himself. As the blockholder buys more shares, the proposal is less likely to be approved, increasing small shareholders' valuations. Hence, the price at which small shareholders supply shares increases, as they free ride on the blockholder's trades. This price impact can be so large that further purchases would increase the stock price even more than the blockholder's own valuation, inducing the blockholder to limit his purchases and giving rise to a negative voting premium.

The discussion above reflects the more general insight that the voting rights embedded in the shares can either amplify or attenuate the price impact of trades. If the blockholder's trades move the median voter in the direction preferred by dispersed shareholders, then his trades increase the price at which shares are supplied to the blockholder. Then his price impact is amplified compared to a scenario in which shares do not have voting rights. However, if the blockholder is in conflict with dispersed shareholders, then his trades push the median voter away from their desired point and thus reduce the price at which they are willing to sell, attenuating price impact. Overall, this argument implies that liquidity, if measured by price impact, is endogenous in our setting and generally differs between voting and non-voting shares. The differential liquidity of voting and non-voting shares in our model arises endogenously from the impact of the blockholder's trades on dispersed shareholders' valuations. It may even result in a negative voting premium, consistent with the empirical literature, which sometimes attributes the negative voting premium to the lower liquidity of superior voting shares.[4]

The literature often associates the voting premium with conflicts between blockholders and minority shareholders and sometimes uses it as a measure for private benefits of control. The idea is that a large voting premium may be associated with a lower payoff of small shareholders, since the blockholder uses his voting power to advance his agenda at the expense of others. However, we show that the relationship between the voting premium and conflicts between

[4] Neumann (2003), Odegaard (2007), and Broussard and Vaihekoski (2022) explain their observations of a negative voting premium by liquidity differences between voting and non-voting shares. Porras Prado, Saffi, and Sturgess (2016) show that voting shares have higher limits to arbitrage than non-voting shares.

shareholders is more nuanced. If the preferences of small shareholders are skewed, or with a supermajority requirement, a higher voting premium is associated with a higher payoff to both the blockholder and small shareholders. Hence, the voting premium is not always positively associated with the severity of conflicts between shareholders, and, therefore, probably not a good measure for them.

We extend the model in a number of ways to explore additional questions. First, we consider an extension in which firms issue voting as well as non-voting shares and show that our inferences from the baseline model with a single class of shares extend to this dual-class setting. Second, we analyze a setting in which voting rights are traded separately, e.g., through share lending, and show that the price of a separately traded vote can be zero even if the voting premium for a share in which voting and cash flow rights are combined is strictly positive. Thus, the price of a traded vote and the voting premium are conceptually different. Third, we consider a setting in which decisions are made not by voting, but by managers who internalize the preferences of their shareholder base. We show that the blockholder's trades can then give rise to an "influence premium" in the share price, which can be decomposed in exactly the same way as the voting premium in our baseline model. Thus, our insights regarding the value of voting rights extend to settings where blockholders exert influence through channels other than direct voting on proposals. Last, we introduce multiple blockholders and show that competition between blockholders could decrease the voting premium, while a conflict between blockholders could increase it.

After concluding our theoretical analysis, in Section 7 we use our insights to shed some light on the large number of empirical studies on the voting premium. We distinguish six different methodologies that have been used to measure the voting premium and discuss interpretations of the time-series and cross-sectional variation of the voting premium in light of our model.

Overall, our paper makes three contributions. First, it examines trading between small and large shareholders and the ownership structure of the firm in a context in which blockholders affect voting outcomes without majority control. Second, it contributes to our understanding of asset prices by showing how and when a voting premium emerges when blockholders can increase their influence through securities in which cash flow rights are bundled with voting rights. Third, it provides guidance to the empirical literature by showing how different proxies for the voting premium are related and why they may be different from each other.

2 Discussion of the literature

We contribute a new theory of the value of voting rights. The primary approach in the literature, pioneered by Grossman and Hart (1988) and Harris and Raviv (1988), considers settings with control contests of firms with dual-class shares. In this approach, rival bidders and incumbent managers differ in their ability to generate cash flows that are shared by all shareholders, and in their valuation of private benefits from controlling the firm. Bidders compete for control and pay a premium to the holders of the voting shares.[5] Studies in this literature have explored a range of alternative settings, including different types of admissible bids; variation in the ability to extract private benefits; settings without a free-rider problem; and frictions from asymmetric information.[6] Moreover, some studies have considered deviations from the one-share one-vote principle through trading in derivatives rather than in non-voting shares (e.g., Blair, Golbe, and Gerard, 1989; Kalay and Pant, 2010; Burkart and Lee, 2015; Dekel and Wolinsky, 2012). Independently of the details, a wide range of settings give rise to a voting premium in a bidding contest. Our theory contributes to this literature by showing how a voting premium emerges without takeovers and control contests. The critical point of departure is the way in which the blockholder's (respectively, bidder's) higher willingness to pay translates into a higher stock price. In control contests, this happens because competition among bidders or the free-rider problem force bidders to pay a higher price (see Bergström and Rydqvist, 1992 and Zingales, 1995 for similar observations). However, neither mechanism is present in our model, which does not feature majority control. Instead, in our setting, the blockholder's trades move the median voter and shift the upward-sloping supply curve of small shareholders by affecting their reservation prices.[7] This may give rise to a positive voting premium even if the blockholder does not acquire majority control, and a negative voting premium may result if this supply curve is sufficiently steep.

[5] Burkart and Lee (2008) survey theoretical work on the role of the security-voting structure and the control premium in the context of takeovers.

[6] Types of admissible bids: Vinaimont and Sercu (2003) and Dekel and Wolinsky (2012); variation in whether one party has private benefits: none in Bergström and Rydqvist (1992), one party is the main case in Grossman and Hart (1988), both parties in Vinaimont and Sercu (2003) and Burkart, Gromb, and Panunzi (1998); there is no free-rider problem in Bergström and Rydqvist (1992); asymmetric information: Burkart and Lee (2015). Some empirical contributions also include further modeling efforts to motivate specific empirical analyses, e.g., Zingales (1995) and Rydqvist (1996).

[7] An upward-sloping supply curve is also present in takeover models with majority control of Stulz (1988) and Burkart, Gromb, and Panunzi (1998, 2006).

A complementary literature analyzes a market in which votes trade separately from shares.[8] While these papers differ significantly regarding their chosen settings and normative conclusions, they all find that the value of separately traded votes is negligible, either because dispersed shareholders value votes in proportion to their probability of being pivotal (Neeman and Orosel, 2006; Brav and Mathews, 2011; Speit and Voss, 2020; Speit, Voss, and Danis, 2023) or because uninformed shareholders would like their votes to be picked up and cast by informed shareholders (Esö, Hansen, and White, 2014).[9] Our extension to a separate market for votes emphasizes that the price of a vote that is traded in conjunction with cash flow rights is different from the price of a vote traded separately: the former can be positive even though the latter is zero. Our focus on the voting premium also distinguishes our paper from a complementary study by Speit, Voss, and Danis (2023), who focus on the decoupling of voting and cash flow rights and show that separating votes from cash flows through equity lending is different from separating votes from the exposure to cash flows through hedging in derivatives markets.

The only approach that has derived a significant voting premium without control contests is Rydqvist (1987), who builds on Milnor and Shapley (1978) and introduces the notion of an oceanic Shapley value to the analysis of dual-class shares. The critical step here is that the ocean of atomistic shareholders can *collectively* become pivotal and thus value their voting power. However, this leaves open how atomistic shareholders resolve their collective action problem. In our setting, each dispersed shareholder maximizes only his individual payoff.

Our paper also contributes to the literature on the equilibrium ownership structure of firms and the analysis of blockholders. The vast majority of this literature considers the effects of blockholders through direct interventions ("voice") and trading ("exit").[10] By contrast, in our setting, the blockholder exercises influence by affecting the identity of the median voter. This is empirically important because many blockholders, notably financial institutions, rely on

[8] It is largely motivated by concerns about the incentives created by decoupling votes from cash flow rights ("empty voting"), triggered by Hu and Black (2007, 2015).

[9] These papers focus on vote trading by investors of a company. In the context of dynamic group decision-making, Garlappi, Giammarino, and Lazrak (2017, 2022) analyze how trading of votes (or shares) among group members can alleviate inefficiencies from differences in beliefs. A related literature in political science examines how vote trading allows agents with a higher intensity of preferences to buy votes from those who care about the decision less (e.g., Casella, Llorente-Saguer, and Palfrey, 2012).

[10] See, e.g., Admati, Pfleiderer, and Zechner (1994), Bolton and von Thadden (1998), Kahn and Winton (1998), and Maug (1998) for earlier contributions to the "voice" literature, and Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011) for the "exit" literature. See the surveys of Edmans (2014), Edmans and Holderness (2017), and Dasgupta, Fos, and Sautner (2021) for more recent work and further details.

voting to influence firms' policies.[11] Moreover, we emphasize that by trading, the blockholder affects voting outcomes not only by accumulating additional voting rights, but also by changing the composition of the shareholder base as he buys shares from those investors who disagree with him the most. Effectively, the market for voting shares helps shareholders to coordinate through trading — a mechanism that can complement explicit coordination, as analyzed by Brav, Dasgupta, and Mathews (2022), Doidge, Dyck, and Yang (2021), and Pi (2020). Dhillon and Rossetto (2015), Bar-Isaac and Shapiro (2020), Meirowitz and Pi (2022), and Pinnington (2023) also consider blockholder models with voting, but differently from our paper, they do not study the voting premium and instead focus on the effects of blockholders on, respectively, the risk taking of the firm and information aggregation. Last, Zwiebel (1995) applies cooperative game theory to study how a blockholder structure emerges endogenously, but does not derive implications for the voting premium.

More broadly, our paper is related to an earlier literature on the existence of equilibrium and the objectives of the firm in a context with incomplete markets and shareholders with heterogeneous preferences.[12] In particular, Drèze (1985) and DeMarzo (1993) develop models with the board of directors as a group of controlling blockholders. To this literature, we contribute by analyzing the voting premium and a richer characterization of the interplay between small shareholders and blockholders. This also distinguishes our paper from Levit, Malenko, and Maug (2022), who analyze trading and voting by atomistic shareholders – a setting in which the voting premium does not arise.

3 Model

Consider a publicly traded firm, which is initially owned by a continuum of measure one of dispersed shareholders and one large blockholder. The blockholder is endowed with $\alpha \in (0,1)$ shares, and each dispersed shareholder is endowed with $1-\alpha$ shares, so the total number of outstanding shares is 1.[13] In the baseline setting, each share has one vote. There is a proposal on which shareholders vote. The proposal could relate to director elections, M&As, executive

[11] See, e.g., McCahery, Sautner, and Starks (2016). Thus, our paper contributes to the broader literature on corporate voting (e.g., Maug and Rydqvist, 2009; Levit and Malenko, 2011; Van Wesep, 2014; Malenko and Malenko, 2019; and Zachariadis, Cvijanovic, and Groen-Xu, 2020).

[12] See Gevers (1974), Drèze (1985), DeMarzo (1993), and Kelsey and Milne (1996).

[13] In Section C.11 of the Online Appendix, we show that our main results also hold for $\alpha = 0$.

compensation, corporate governance, or social and environmental policies. The proposal can either be approved ($d = 1$) or rejected ($d = 0$).

Preferences. Shareholders' preferences over the proposal depend on two components, which reflect a common value and private values. The common value component depends on an unknown state $\theta \in \{-1, 1\}$: if $\theta = -1$ ($\theta = 1$), accepting the proposal is value-decreasing (increasing). In other words, the common value is maximized if the policy matches the state ($d = 1$ if $\theta = 1$), as is commonly assumed in the strategic voting literature (e.g., Austen-Smith and Banks, 1996 and Feddersen and Pesendorfer, 1996).

Shareholders also have private values from the proposal, which reflect the heterogeneity in their preferences. For simplicity, we refer to these private values as biases and denote them by b. A shareholder with bias $b > 0$ ($b < 0$) receives additional (dis)utility if the proposal is accepted. The distribution of biases b among the initial dispersed shareholders is given by a publicly known, twice differentiable cdf G, which has full support with a positive density function g on $\left[-\bar{b}, \bar{b}\right]$, where $\bar{b} \in (0, 1)$. Differences in shareholders' preferences can stem from a range of characteristics on which investors differ (see the Introduction and footnote 3). We expand on some of these sources of heterogeneity at the end of this section.

The value of a share from the perspective of a dispersed shareholder with bias b is

$$v\left(d, \theta, b\right) = v_0 + \left(\theta + b\right) d, \tag{1}$$

where $v_0 > 0$ is large enough to ensure that shareholder value is always non-negative. Because of heterogeneous preferences, shareholders apply different hurdle rates for accepting the proposal: a shareholder with bias b would like the proposal to be accepted if and only if his expectation of $\theta + b$ is positive. We will refer to shareholders with a higher b as being "more activist."

The blockholder's preferences have the same structure as those of dispersed shareholders, except that his bias is $\beta \in \left[-\bar{b}, \bar{b}\right]$. Thus, the value of a share from the perspective of a blockholder is $v\left(d, \theta, \beta\right)$.

Timeline. All shareholders are initially uninformed about the state θ and have the same prior about its distribution, which we specify below. Shareholders first trade and then vote on the proposal. This timing allows us to focus on how trading affects the composition of the voter base, which is crucial for the analysis of the voting premium. At the trading stage, each dispersed shareholder can buy any number of shares x, where $x < 0$ corresponds to the shareholder selling shares. A dispersed shareholder's utility from buying x shares given his endowment $1-\alpha$ is

$$u(d, \theta, b, x; \gamma, 1-\alpha) = (1-\alpha+x)\, v(d, \theta, b) - \frac{\gamma}{2}x^2, \tag{2}$$

where $\gamma > 0$ captures trading frictions, such as illiquidity, transaction costs, or wealth constraints, which limit shareholders' ability to build large positions in the firm (e.g., as in Vives, 1993).[14] Since the mass of investors is finite and $\gamma > 0$, our model features limits to arbitrage. We assume

$$\gamma > \overline{\gamma}, \tag{3}$$

where $\overline{\gamma} < \infty$ is formally defined at the beginning of the Appendix. Assuming that γ is sufficiently large guarantees several useful properties of the model, which we discuss below.

Similarly, the blockholder can trade any number of shares y, and his utility from buying y shares is $u(d, \theta, \beta, y; \eta, \alpha)$, where $u(\cdot)$ is given by (2) and $\eta \geq 0$ captures the blockholder's trading costs. All results hold for $\eta = 0$. While dispersed shareholders are price takers, the blockholder trades strategically, accounting for his expected price impact. For simplicity, we assume that the blockholder submits his order y first, and dispersed shareholders observe y and submit their orders next. Effectively, dispersed shareholders can condition their trades on y, which can be interpreted as them submitting limit orders.[15]

We denote the market clearing share price by p. After the market clears, but before voting takes place, all shareholders observe a public signal about the state θ, which may stem from disclosures by management, proxy advisors, or analysts.[16] Let $q = \mathbb{E}[\theta|\text{public signal}]$ be the shareholders' posterior expectation of the state following the signal. For simplicity, we assume that the public signal is q itself, and that q is distributed according to a cdf F with mean zero and full support with a positive twice differentiable density function f on $[-\Delta, \Delta]$, where

[14] Since each dispersed investor has a zero mass, his trade x and endowment $1-\alpha$ are infinitesimal quantities. With a slight abuse of notation, $1-\alpha$ also denotes the total endowment held by dispersed investors.

[15] Equation (35) shows that for γ large enough, the market clearing price is monotonic in y. Thus, whether the limit order is conditioned on the price or on the blockholder's trade is immaterial for our analysis.

[16] In practice, proxy advisors' recommendations (and management's response) are on average released about one month after the vote record date. See, for example, Fig. 1 in Li, Maug, and Schwartz-Ziv (2022).

$\Delta \in (\bar{b}, 1)$. Thus, the ex-ante expectation of θ is zero. The symmetry of the support of q around zero is not necessary for any of the main results. In what follows, we refer to $H(q^*) \equiv \Pr[q > q^*]$, rather than to the cdf.

After observing the public signal q, each shareholder votes the shares he owns after the trading stage. Hence, we assume that the record date, which determines who is eligible to participate in the vote, is after the trading stage. This timeline applies well to important votes, such as the votes on M&As, proxy fights, and high-profile shareholder proposals, which are typically known well ahead of the record date. The proposal is accepted if at least fraction $\tau \in (0, 1)$ of all shares are cast in favor; otherwise, the proposal is rejected. We assume that $\alpha < \min\{\tau, 1 - \tau\}$. As we show in Lemma 3 of the Online Appendix, this guarantees that if $\gamma > \overline{\gamma}$, then in equilibrium the blockholder does not have the power to veto or accept the proposal unilaterally. This lemma also shows that if $\gamma > \bar{\gamma}$, then neither dispersed shareholders nor the blockholder find it optimal to short sell in equilibrium.

We analyze subgame perfect Nash equilibria in undominated strategies of the voting game. The restriction to undominated strategies is common in voting games, which usually impose the equivalent restriction that dispersed shareholders vote as-if-pivotal.[17] This implies that an investor with bias b, whether he is a dispersed shareholder or the blockholder, votes in favor of the proposal if and only if

$$b + q \geq 0. \tag{4}$$

Applications of the model. Several applications can be mapped into our model (see Section B of the Online Appendix). In the first application, we consider investors with heterogeneous time horizons who vote between a short-termist and a long-termist investment strategy, such as in a proxy contest organized by a short-termist activist. Shareholders' valuations of the firm under each strategy depend on the likelihood that the strategy succeeds (common value) and on their time horizons (private values): long-termist shareholders put a higher weight on long-term cash flows relative to short-term cash flows. In this application, shareholders' trading and voting strategies, as well as the voting premium, are given by the same expressions (up to a constant) as in the baseline model.

In the second application we microfound shareholders' valuations in eq. (1) via a model

[17]See, e.g., Baron and Ferejohn (1989) and Austen-Smith and Banks (1996). This restriction helps rule out equilibria in which shareholders are indifferent between voting for or against because they are never pivotal.

in which biases b and β capture differences in beliefs ("sentiment") regarding the value of the proposal, rather than differences in preferences. The third application captures a setting with private benefits of control. The blockholder can dilute the assets of the firm if the proposal is approved, which leads him to favor the proposal relative to dispersed shareholders ($\beta > b$). Finally, the fourth application captures costly monitoring: if the proposal is approved, the blockholder can monitor the manager and increase firm value at a private cost. Since dispersed shareholders benefit from monitoring but do not incur its cost, they favor the proposal more than the blockholder ($b > \beta$).

4 Equilibrium

We begin by showing that for any trading outcome, proposal approval at the voting stage takes the form of a cutoff decision rule:

Lemma 1. *In any equilibrium, there exists q^* such that the proposal is approved if and only if $q > q^*$.*

The reason is that all shareholders value the proposal more if it is more likely to be value-increasing, i.e., if $\theta = 1$ is more likely.

We proceed in several steps. First, for any possible blockholder's trade y, Sections 4.1 and 4.2 characterize the trading of dispersed shareholders and the voting stage as a function of y. In Section 4.3, we solve for the optimal trade of the blockholder, y^*, and for the equilibrium share price.

4.1 Trading of dispersed shareholders

Given Lemma 1, suppose that dispersed shareholders expect the proposal to be accepted if and only if $q > q_e^*$ for some cutoff q_e^* (we later derive the equilibrium cutoff such that shareholders' expectations are rational). Let $v\left(b, q_e^*\right)$ denote the valuation of a shareholder with bias b prior to the realization of q, as a function of the cutoff q_e^*. Then

$$v\left(b, q_e^*\right) = \mathbb{E}\left[v\left(\mathbf{1}_{q>q_e^*}, \theta, b\right)\right], \tag{5}$$

where the indicator function $\mathbf{1}_{q>q_e^*}$ equals one if $q > q_e^*$ and zero otherwise, and $v(d, \theta, b)$ is defined by (1). We can rewrite (5) as

$$v(b, q_e^*) = v_0 + (b + \mathbb{E}[\theta | q > q_e^*]) H(q_e^*), \tag{6}$$

which increases in b. Notice that $v(b, q_e^*)$ is a hump-shaped function of q_e^* with a maximum at $q_e^* = -b$, i.e., the shareholder's valuation decreases in the distance between his bias and the expected decision rule q_e^*.

Dispersed shareholders are price takers, so for any price p, a dispersed shareholder solves

$$\max_x \left\{ (1 - \alpha + x) v(b, q_e^*) - xp - \frac{\gamma}{2} x^2 \right\} \tag{7}$$

and optimally chooses

$$x(b, q_e^*, p) = \frac{v(b, q_e^*) - p}{\gamma}. \tag{8}$$

Thus, shareholder b buys shares if his valuation exceeds the market price, $v(b, q_e^*) > p$, sells shares if $v(b, q_e^*) < p$, and does not trade otherwise. Given the blockholder's order y, the market clears if and only if

$$\int_{-\bar{b}}^{\bar{b}} x(b, q_e^*, p) g(b) db + y = 0, \tag{9}$$

which gives the market clearing price

$$p^*(y, q_e^*) = \gamma y + v(\mathbb{E}[b], q_e^*). \tag{10}$$

The equilibrium share price increases in y, and the price impact of the blockholder's trade increases in γ. Thus, we can interpret γ as measuring the illiquidity of the market, i.e., the inverse of γ reflects market depth. In addition, the share price (10) increases in the valuation of the average dispersed shareholder. Intuitively, if dispersed shareholders' valuations (conditional on q_e^*) are higher, they are willing to supply shares to the blockholder only at a higher price.

From (6), (8), and (10), dispersed shareholders' demand as a function of the blockholder's trade can be written as

$$x(b, y, q_e^*) = \frac{1}{\gamma}(b - \mathbb{E}[b]) H(q_e^*) - y. \tag{11}$$

The post-trade ownership structure. After the trading stage, the blockholder owns $\alpha+y$ shares, a dispersed shareholder with bias b owns $1-\alpha+x\left(b,y,q_e^*\right)$ shares, and all dispersed shareholders collectively own $1-\alpha-y$ shares. Thus, the proportion of shares owned post-trade by dispersed shareholders with bias b, conditional on the expected decision rule q_e^* and blockholder's trade y, is given by

$$r\left(b;y,q_e^*\right)\equiv g\left(b\right)\frac{1-\alpha+x\left(b,y,q_e^*\right)}{1-\alpha-y}. \tag{12}$$

Note that $r\left(b;y,q_e^*\right)$ is a density function, i.e., $\int_{-\bar{b}}^{\bar{b}}r\left(b;y,q_e^*\right)db=1$. Thus, the post-trade preferences of dispersed shareholders are distributed according to cdf $R\left(b;y,q_e^*\right)$ given by

$$\begin{aligned} R\left(b';y,q_e^*\right) &= \int_{-\bar{b}}^{b'} r\left(b;y,q_e^*\right)db \\ &= G\left(b'\right)\left(1-\frac{\mathbb{E}\left[b\right]-\mathbb{E}\left[b|b<b'\right]}{\gamma}\frac{H\left(q_e^*\right)}{1-\alpha-y}\right), \end{aligned} \tag{13}$$

where the second equality follows from (11) and (12). The cdf R characterizes the post-trade dispersed shareholder base, whereas G characterizes the pre-trade dispersed shareholder base. Note that $R\left(b\right)<G\left(b\right)$ for any b, i.e., R dominates G in the sense of first-order stochastic dominance. Hence, trading shifts the shareholder base in such a way that more activist shareholders own a larger proportion of the firm after trading. Moreover, $R\left(b';y,q_e^*\right)$ increases in q_e^*; hence, a more activist decision rule (lower q_e^*) makes the post-trade shareholder base more activist. Intuitively, shareholders' heterogeneous attitudes towards the proposal create gains from trade, so the shareholder base moves in the direction of the expected outcome.

4.2 Voting

The composition of the post-trade shareholder base determines the voting outcome. We first analyze the votes of dispersed shareholders. Denote by $s\left(q;y,q_e^*\right)$ the number of votes cast by dispersed shareholders in favor of the proposal if signal q is realized, the blockholder traded y shares, and the expected decision rule is q_e^*. Then,

$$s\left(q;y,q_e^*\right)\equiv\left(1-\alpha-y\right)\left(1-R\left(-q;y,q_e^*\right)\right), \tag{14}$$

which is the number of shares held by dispersed shareholders, $1-\alpha-y$, multiplied by the proportion of dispersed shareholders for whom $b>-q$.

The blockholder is pivotal for the outcome when his vote sways the decision on the proposal, which only happens if at least $\tau-(\alpha+y)$ but no more than τ dispersed shareholders vote to support the proposal, i.e., if and only if

$$\tau-(\alpha+y)\leq s\left(q;y,q_e^*\right)<\tau. \tag{15}$$

Otherwise, if $s\left(q;y,q_e^*\right)<\tau-(\alpha+y)$ ($s\left(q;y,q_e^*\right)\geq\tau$), the proposal fails (succeeds) independently of the vote of the blockholder. From (14), the support of dispersed shareholders is increasing in the signal q. Define the bounds $\underline{q}\equiv s^{-1}\left(\tau-\alpha-y;y,q_e^*\right)$ and $\overline{q}\equiv s^{-1}\left(\tau;y,q_e^*\right)$. Then the blockholder is pivotal if and only if the signal is in the intermediate range, $q\in[\underline{q},\overline{q})$. If $q<\underline{q}$ ($q\geq\overline{q}$), dispersed shareholders' support for the proposal is so low (high) that the proposal fails (succeeds) even if the blockholder supports (rejects) it.

We describe the voting outcome by characterizing the identity of the *median voter*, who we define as the shareholder whose individual vote coincides with the collective decision on the proposal for every possible realization of the signal q. In other words, whenever the median voter votes in favor (against), the proposal is accepted (rejected).

Let $b_{MV}\left(\beta,y,q_e^*\right)$ denote the bias of the median voter if the expected decision rule is q_e^*, the blockholder traded y shares, and his bias is β. There are three possible cases, which together define $b_{MV}\left(\beta,y,q_e^*\right)$:

(i) If $\beta\geq-\underline{q}$, the blockholder is very activist and supports the proposal whenever he is pivotal. The proposal is accepted if and only if $s\left(q;y,q_e^*\right)+\alpha+y\geq\tau$, i.e., whenever $q\geq\underline{q}$. Hence, the proposal passes if and only if the dispersed shareholder with bias $b=-\underline{q}$ votes in favor. This shareholder is then the median voter, i.e., $b_{MV}\left(\beta,y,q_e^*\right)=-\underline{q}$.

(ii) If $\beta\leq-\overline{q}$, the blockholder has a large bias against the proposal and votes against whenever he is pivotal. The proposal is accepted if and only if $s\left(q;y,q_e^*\right)\geq\tau$, i.e., whenever $q\geq\overline{q}$. Hence, it passes if and only if the dispersed shareholder with bias $b=-\overline{q}$ votes in favor. This shareholder is then the median voter, i.e., $b_{MV}\left(\beta,y,q_e^*\right)=-\overline{q}$.

(iii) If $-\overline{q}<\beta<-\underline{q}$, the blockholder is pivotal if $q\in[\underline{q},\overline{q})$ and votes in favor if and only if

$q \geq -\beta$. Then, the proposal is accepted if and only if the blockholder votes in favor, so the blockholder is the median voter, i.e., $b_{MV}(\beta, y, q_e^*) = \beta$.

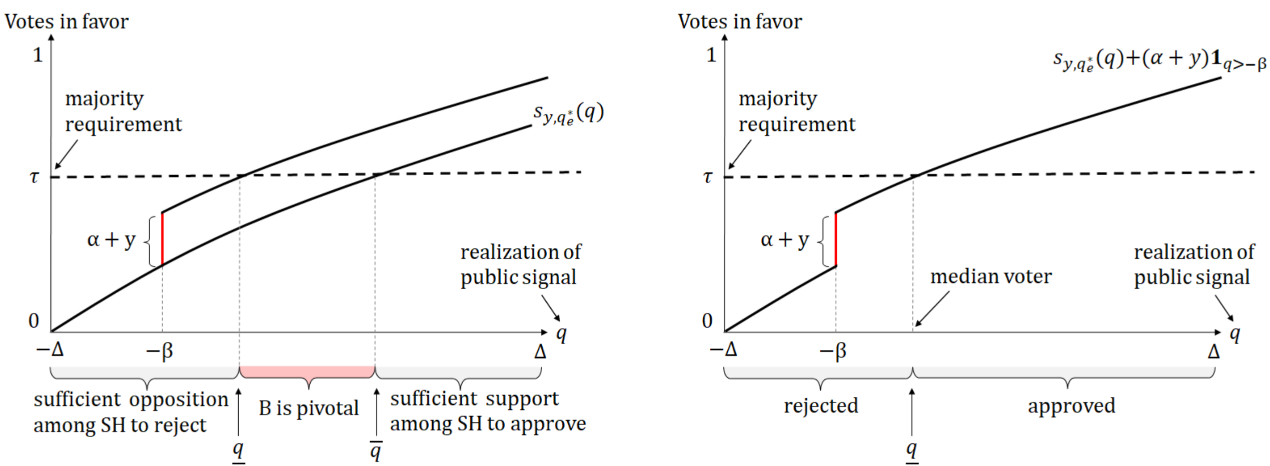

Figure 1. The median voter

We conclude that if shareholders anticipate decision rule q_e^* when trading, then the decision rule at the voting stage is characterized by the three cases above. The first case is illustrated in Figure 1, which plots the number of votes in favor of the proposal as a function of the signal q. The left panel indicates the range in which the blockholder is pivotal; the right panel indicates the approval range and the location of the median voter. Note that while a dispersed shareholder is never pivotal in our setting since he is atomistic, he can often be the median voter. For example, in Figure 1, the blockholder is extreme and the median voter is a dispersed shareholder closer to the center of the distribution.

In equilibrium, shareholders' expectations q_e^* must be consistent with the actual decision rule. Hence, an equilibrium can be found as a fixed point of q_e^*, such that $-b_{MV}(\beta, y, q_e^*) = q_e^*$, where $b_{MV}(\beta, y, q_e^*)$ is defined by the three cases above. Using this logic, the equilibrium at the voting stage is characterized as follows.

Proposition 1 (Voting stage). *If the blockholder trades y shares, then the proposal is approved if and only if $q > q^*(y)$, where $q^*(y)$ solves*

$$-b_{MV}(\beta, y, q^*) = q^*. \tag{16}$$

If $\gamma > \overline{\gamma}$, the solution of (16) is unique. In this case, there exists $\overline{y}$ such that if $y \geq \overline{y}$, the median voter is the blockholder ($-q^*(y) = \beta$), whereas if $y < \overline{y}$, the median voter is a dispersed shareholder with bias $-q^*(y) \neq \beta$, and $|\beta + q^*(y)|$ decreases in y.

In general, there can be multiple solutions to (16), and hence multiple equilibria at the voting stage. This is because for small γ, the shifts in the shareholder base are sensitive to the expected decision rule q_e^*, which can give rise to self-fulfilling expectations.[18] However, if γ is large enough, then dispersed shareholders trade less aggressively, the distribution of the post-trade shareholder base is less sensitive to q_e^*, and the equilibrium is unique.

Importantly, Proposition 1 shows that the blockholder can change the identity of the median voter, $-q^*(y)$, and thus the vote outcome, with his trades y. By buying more shares, the blockholder moves the bias of the median voter closer to β, which is captured by the result that $|\beta + q^*(y)|$ decreases. This can be seen in the left panel of Figure 2, which shows that a larger y pushes $-q^*(y)$ to the left, closer to $-\beta$. Once the blockholder buys enough shares ($y \geq \overline{y}$), the vote outcome exactly coincides with the blockholder's own voting rule, so the blockholder becomes the median voter (see the right panel in Figure 2). The accumulation of shares beyond $\bar{y}$ increases the probability of the blockholder being pivotal, but it does not change the expected vote outcome, that is, the identity of the median voter.

There are two complementary reasons why the accumulation of shares by the blockholder moves the median voter closer to him. First, more shares give the blockholder more voting rights. Second, as the blockholder buys more, the composition of the dispersed shareholder base changes towards those who are more aligned with the blockholder. For example, if the blockholder is very activist, then dispersed shareholders who hold the firm after trading are more activist. (Recall that $R(b'; y, q_e^*)$ increases in q_e^* in (13).)

[18] In particular, the cdf of the post-trade shareholder base, given by (13), increases in q_e^*, and hence a more activist *expected* decision rule (lower q_e^*) makes the post-trade shareholder base more activist. A more activist shareholder base, in turn, is more likely to approve the proposal for any given signal, leading to a lower *realized* cutoff for approving the proposal, confirming the ex-ante expectations.

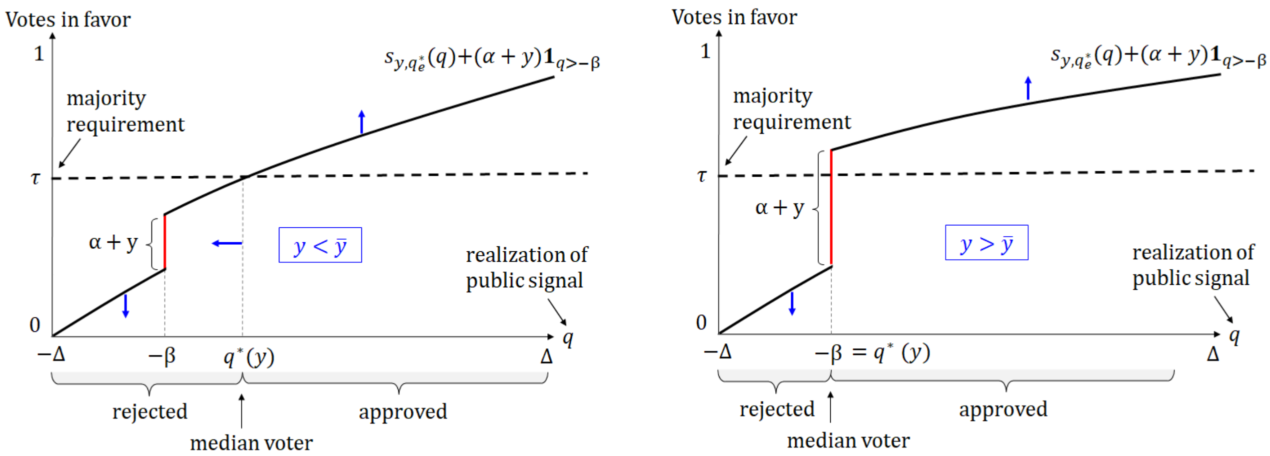

Figure 2. The effect of the blockholder's trade on the equilibrium median voter

4.3 Blockholder trading

Given the blockholder's trade y, all shareholders correctly anticipate that the decision rule at the voting stage will be $q^*(y)$, as given by (16), and that the market clearing price will be

$$p^*(y) = \gamma y + v(\mathbb{E}[b], q^*(y)) \tag{17}$$

from (10). In equilibrium, the blockholder chooses y to maximize

$$\Pi(y) \equiv (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2. \tag{18}$$

The marginal effect of buying additional shares on the blockholder's expected payoff is

$$\frac{d\Pi(y)}{dy} = \underbrace{\frac{\partial \Pi(y)}{\partial y}}_{MPC(y)} + \underbrace{\frac{\partial \Pi(y)}{\partial(-q^*(y))} \frac{\partial(-q^*(y))}{\partial y}}_{MPV(y)} = MPC(y) + MPV(y). \tag{19}$$

The term $MPC(y)$ is the *marginal payoff from buying cash flow rights*. It can be thought of as the blockholder's marginal payoff from trading in a hypothetical scenario in which the decision rule is set exogenously at the level $q^*(y)$ and is not affected by the blockholder's trades. This term equals

$$MPC(y) = (\beta - \mathbb{E}[b]) H(q^*(y)) - (2\gamma + \eta) y. \tag{20}$$

Intuitively, if $\beta > \mathbb{E}[b]$ ($\beta < \mathbb{E}[b]$), the blockholder values shares more (less) than the average dispersed shareholder, which creates gains from trade. The term $MPV(y)$ is the *marginal payoff from buying voting rights.*[19] It captures the blockholder's additional incentives to trade in order to change the decision rule, i.e., to shift the median voter $-q^*(y)$. In Section 5 below, we show that $MPV(y)$ is closely related to the voting premium.

The next proposition characterizes the equilibrium, including the blockholder's optimal trading strategy.

Proposition 2 (Equilibrium). *The equilibrium exists and is unique. In equilibrium:*

(i) *The blockholder trades y^* shares, where y^* is the unique solution of*

$$y^* = \frac{1}{2\gamma + \eta}(\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma + \eta}\sigma(\beta) MPV(y^*). \tag{21}$$

A dispersed shareholder with bias b trades $x^*(b)$ shares, where

$$x^*(b) = \frac{1}{\gamma}(b - b^*) H(q^*(y^*)) - \frac{1}{2\gamma + \eta}\sigma(\beta) MPV(y^*). \tag{22}$$

Parameter b^* is defined as

$$b^* = \frac{\gamma}{2\gamma + \eta}\beta + \left(1 - \frac{\gamma}{2\gamma + \eta}\right)\mathbb{E}[b], \tag{23}$$

and $\sigma(\beta) \in [0, 1]$ is defined by equation (70) in the Appendix.

(ii) *The share price is*

$$p^* = v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma + \eta}\sigma(\beta) MPV(y^*). \tag{24}$$

(iii) *The bias of the median voter is $-q^*(y^*)$, where $q^*(\cdot)$ is defined in Proposition 1.*

The blockholder's optimal trade y^* consists of two terms, which are related to decomposition (19). The first term reflects trading for cash flow reasons: the blockholder has an incentive to buy shares if and only if his valuation is higher than that of the average dispersed shareholder,

[19] The derivative $\frac{\partial \Pi(y)}{\partial(-q^*(y))}\frac{\partial(-q^*(y))}{\partial y}$ exists for all y except $\bar{y}$. If $y = \bar{y}$, we define $MPV(y)$ as the left derivative: $\lim_{y \nearrow \bar{y}} \frac{\partial \Pi(y)}{\partial(-q^*(y))}\frac{\partial(-q^*(y))}{\partial y}$.

$\beta > \mathbb{E}[b]$. The second term reflects the additional trading of voting shares because of the embedded voting rights and is proportional to $MPV(y^*)$.[20] The expression for dispersed shareholders' trades $x^*(b)$ follows directly from (11). Intuitively, trading shifts the dispersed shareholder base towards the expected outcome (see the discussion after eq. (13) above), and their combined supply of shares equals the blockholder's demand. If $MPV(y^*) = 0$, then the shareholder with bias b^* can be interpreted as the marginal trader who is indifferent between buying and selling shares at the equilibrium share price; in this scenario, all shareholders with a higher (lower) b than b^* buy (sell) shares. Finally, the equilibrium stock price consists of two terms, and we focus on this decomposition and its properties in the next section.

5 The voting premium

This section presents our main results. We proceed in several steps. We define the voting premium in Section 5.1 and clarify its determinants in Section 5.2. In Section 5.3 we characterize the equilibrium properties of the voting premium. Finally, in Section 5.4 we discuss the novel implications about the voting premium that emerge from our analysis.

5.1 Defining the voting premium

We start by defining the voting premium and relating it to MPV. Consider again the hypothetical scenario in which the voting rule is set exogenously at $q^*(y^*)$. Such a scenario may reflect cases in which trading does not reallocate voting rights, e.g., as in trading of non-voting shares, so that the median voter is unaffected.[21] Since the voting rule is exogenous in this hypothetical scenario, we have $\frac{\partial(-q^*(y^*))}{\partial y} = 0$, $MPV(y^*) = 0$, and the blockholder's first-order condition (19) reduces to $MPC(y^*) = 0$. A corollary of Proposition 2 is that the share price in this scenario reflects the valuation of the shareholder with bias b^*, defined by (23):

Corollary 1. *If the voting rule is set exogenously at $q^*(y^*)$, the equilibrium share price is*

$$p_{CF}(q^*(y^*)) = v(b^*, q^*(y^*)). \tag{25}$$

[20] The function $\sigma(\beta)$ is equal to 1 everywhere, except for a small interval that shrinks to zero as $\gamma \to \infty$. See Proposition 3 and Section C.10 in the Online Appendix for more details.

[21] This hypothetical scenario also captures cases in which the company's board and management have decision rights over the proposal and implement a decision rule $q^*(y^*)$ that is exogenous to the composition of the shareholder base.

Equation (25) can be interpreted as the price of a share without voting rights. Next, we define the *voting premium* as

$$VP\left(y^{*}\right) \equiv p^{*} - p_{CF}\left(q^{*}\left(y^{*}\right)\right), \tag{26}$$

i.e., the difference between the share price (24) that arises when the voting rule is determined endogenously by the post-trade shareholder base, and the share price in the hypothetical scenario when the voting rule is set exogenously at the same level $q^{*}\left(y^{*}\right)$. Proposition 2 and Corollary 1 imply that the voting premium is proportional to the blockholder's marginal payoff from buying voting rights:

$$VP\left(y^{*}\right) = \frac{\gamma\sigma\left(\beta\right)}{2\gamma+\eta} MPV\left(y^{*}\right). \tag{27}$$

Hence, the voting premium reflects the additional component of the stock price that arises from the blockholder's incentive to influence the voting outcome.

There are two empirical counterparts of our definition of the voting premium. First, we can think of it as the difference between the stock prices right before and after the record date (Fos and Holderness, 2022). These prices reflect the same expected voting rule, but shares traded right before the record date have voting rights for the upcoming shareholder meeting, whereas shares traded right after the record date do not.[22] The second empirical counterpart is the *dual-class premium*, which is defined as the price difference between voting and non-voting shares. In Section 6, we show how our definition extends to such a dual-class structure. However, our definition of the voting premium is broader than these two empirical measures and suggests a way to isolate the voting premium as a component of the stock price also in single-class firms and on dates other than the record date.

5.2 Determinants of the voting premium

To understand the determinants of the voting premium, we rewrite (27), using (19), as

[22] Our model is static in nature, so our analogy to trading around the record date abstracts from dynamic aspects of trading that could potentially affect the share price.

$$VP(y) = \underbrace{\frac{\partial(-q^*(y))}{\partial y}}_{\text{ability to move median voter}} \times \underbrace{\left[\underbrace{(\alpha+y)\frac{\partial v(\beta,q^*(y))}{\partial(-q^*)}}_{\text{marginal benefit of a vote}} - \underbrace{y\frac{\partial p^*(y)}{\partial(-q^*)}}_{\text{price impact of a vote}}\right]}_{\text{incentives to move median voter } = \frac{\partial \Pi(y)}{\partial(-q^*(y))}} \times \frac{\gamma\sigma(\beta)}{2\gamma+\eta}. \quad (28)$$

Thus, the voting premium can be decomposed into the blockholder's *ability* to influence the identity of the median voter, and his *incentives* to do so. The ability of the blockholder to influence the median voter depends on how his trades y affect $-q^*(y)$. According to Proposition 1, there exists a threshold $\overline{y}$ such that if $y \geq \overline{y}$, then the blockholder is the median voter and the accumulation of additional shares does not change the decision rule. Therefore, if $y \geq \overline{y}$, then $\frac{\partial(-q^*(y))}{\partial y} = 0$ and the blockholder cannot change the voting outcome even if he had the incentives to do so. According to (28), the voting premium is then zero. Intuitively, since the blockholder's trades have no impact on the voting outcome, he would not be willing to pay a premium for additional voting rights.

Proposition 1 also shows that if $y < \overline{y}$, then $\frac{\partial(-q^*(y))}{\partial y} \neq 0$ and the blockholder's trades change the identity of the median voter. In this case, the voting premium also depends on the incentives of the blockholder to move the median voter. Based on (28), these incentives consist of two components. The first component captures how a marginal change in the median voter affects the blockholder's valuation of his post-trade stake in the firm, $\alpha + y$, and we refer to it as the *marginal benefit of a vote*. From (6) and (28),

$$\textit{Marginal benefit of a vote} = (\alpha+y)\frac{\partial v(\beta,q^*(y))}{\partial(-q^*)} = (\alpha+y)(\beta+q^*(y))\,f(q^*(y)). \quad (29)$$

By buying additional shares, the blockholder moves the median voter $-q^*(y)$ closer to his own bias β (see Proposition 1), which increases the blockholder's valuation of his stake. Thus, the marginal benefit of a vote is always positive because the blockholder always values a marginal vote for its impact on his own stake.[23]

[23] This discussion simplifies by focusing on the case in which the blockholder is sufficiently activist, such that $\beta > -q^*(y)$ and (29) is positive. In the opposite case, in which $\beta < -q^*(y)$ and (29) is negative, the derivative $\frac{\partial(-q^*(y))}{\partial y}$ turns negative as well, so that the product of both expressions in (28) remains positive: The marginal benefit of a vote is always positive. Hence, this simplification is inconsequential.

The second component captures how the incentives of the blockholder to move the median voter depend on the *price impact* of his trades. The market clearing stock price is the reservation price at which dispersed shareholders are willing to supply one additional share. From (17), the effect of a marginal change in the median voter $-q^*$ on the stock price is

$$\textit{Price impact of a vote} = \frac{\partial p^*(y)}{\partial(-q^*)} = \left(\mathbb{E}[b] + q^*(y)\right) f\left(q^*(y)\right). \tag{30}$$

The sign and magnitude of the price impact of a vote depend on whether the resulting change in the median voter benefits or hurts dispersed shareholders, i.e., whether the distance between the bias of the average dispersed shareholder, $\mathbb{E}[b]$, and the median voter, $-q^*(y)$, decreases or increases. We explore this issue further in Section 5.4.4.

5.3 Properties of the voting premium

The discussion in Section 5.2 highlights that a key determinant of the voting premium is the location of the median voter. The next result characterizes the equilibrium median voter and the voting premium as functions of the blockholder's bias β.

Proposition 3. *There exist cutoffs $\beta^L_{non\text{-}mv} < \beta^L_{mv} < \beta^H_{mv} < \beta^H_{non\text{-}mv}$, all in the interval $\left(-\bar{b}, \bar{b}\right)$, such that:*

(i) If $\beta \in [\beta^L_{mv}, \beta^H_{mv}]$, then the median voter is the blockholder, the voting premium is zero, and $\sigma(\beta) = 1$.

(ii) If $\beta > \beta^H_{non\text{-}mv}$ ($\beta < \beta^L_{non\text{-}mv}$), then the median voter is a dispersed shareholder with a smaller (larger) bias toward the proposal than the blockholder, i.e., $-q^*(y^*) < \beta$ ($-q^*(y^*) > \beta$), the voting premium is strictly positive and increases (decreases) in β, and $\sigma(\beta) = 1$.

There are two cases to consider. First, if the blockholder is moderate, $\beta \in [\beta^L_{\text{mv}}, \beta^H_{\text{mv}}]$, then he becomes the median voter himself (hence, we use subscripts 'mv' for these cutoffs) and does not need to trade aggressively to achieve this. Since the blockholder is the median voter, he cannot further move the median voter at the margin, consequently, the equilibrium voting premium is zero. This observation highlights that the voting premium does not reflect the blockholder's voting power. In fact, if the blockholder's stake is large enough, he is likely to be the median voter, with an associated voting premium of zero.

Second, if the blockholder's preferences are extreme, $\beta > \beta^H_{\text{non-mv}}$ (or $\beta < \beta^L_{\text{non-mv}}$), then the blockholder does not become the median voter (hence, we use subscripts 'non-mv' for these cutoffs), as this would require buying too many additional shares. Then the median voter is a dispersed shareholder with a smaller (larger) bias toward the proposal than the blockholder. Now the blockholder does have the ability to move the median voter, and he benefits more from doing so the further away he is from the median voter (see equation (29)). Accordingly, the net marginal payoff from an additional voting right, and hence the voting premium, is positive and increases as β becomes more extreme. Importantly, even though the blockholder's marginal benefit of a vote is positive in equilibrium, he refrains from buying more voting shares, because he also considers the cost from his own price impact.

Proposition 3 does not characterize the voting premium and the median voter if $\beta \in [\beta^L_{\text{non-mv}}, \beta^L_{\text{mv}})$ or $\beta \in (\beta^H_{\text{mv}}, \beta^H_{\text{non-mv}}]$. In these regions, the median voter can switch back and forth between the blockholder and dispersed shareholders. In Proposition 5, we provide a partial characterization and show that the voting premium can be negative in the interval $[\beta^L_{\text{non-mv}}, \beta^L_{\text{mv}})$.

5.4 Implications

In this section we discuss the key implications of our analysis of the voting premium.

5.4.1 Voting premium vs. total value of voting rights

Our analysis highlights that the voting premium is likely to *underestimate* the overall value of voting rights. A zero voting premium does not imply that the blockholder does not value voting rights, or that he would not benefit from further influencing the voting outcome. Indeed, the incentives to move the median voter, captured by $\frac{\partial \Pi(y)}{\partial(-q^*)}$, will generally differ from zero, even if the blockholder is already the median voter.[24] Instead, a zero voting premium only implies that the blockholder cannot influence the position of the median voter through additional trades.

[24] To see this, note from (28)–(30) that if $-q^*(y) = \beta$, then $\frac{\partial \Pi(y)}{\partial(-q^*)} = y(\beta - \mathbb{E}[b]) f(-\beta)$, which generally differs from zero. Intuitively, by changing the median voter further, the blockholder affects his gains from trade with small shareholders since the effect of the median voter on their valuations differs from the effect of the median voter on his own valuation (see Section 5.4.4 for a more in-depth discussion).

Furthermore, the blockholder's overall benefits from accumulating voting rights can be positive even if his marginal benefits are zero, because these marginal benefits are evaluated at the blockholder's equilibrium trade y^*. By contrast, the overall benefits from owning voting rights also come from the blockholder's infra-marginal trades. These infra-marginal trades affect the voting outcome ($q^*(y^*) \neq q^*(0)$), even if the equilibrium voting premium is zero. Hence, empirical measures of the voting premium that measure the value of a marginal vote, such as the dual-class premium or the ex-record date price drop, are likely to underestimate the overall value of voting rights.

5.4.2 Direct vs. indirect influence on the voting outcome

The voting premium emerges because the blockholder pays for his influence on the voting outcome through affecting the composition of the shareholder base. This influence results not only directly from the increased weight of the blockholder in the shareholder base, but also indirectly, because he buys disproportionally more shares from those dispersed shareholders whose preferences are furthest away from his own. This reduces the weight of shareholders who are least aligned with him, so the indirect effect amplifies the impact of the blockholder on the voting outcome. Accordingly, the blockholder does not pay a voting premium for being directly in control of the voting outcome. Instead, he pays the voting premium for influencing the identity of the shareholder who is the median voter, and this influence comes from the direct and indirect effects noted above.

As such, the voting premium is generally unrelated, or can even be negatively related, to measures of voting power. If the blockholder's voting power is large, he is the median voter himself, so the voting premium is zero. In contrast, if the blockholder's voting power is small, his marginal payoff from moving the median voter is strictly positive. He has both the ability to move the median voter and, under the conditions of Proposition 3 (ii), the incentives to do so; accordingly, the voting premium is positive as well.

5.4.3 Conflicts of interest and the voting premium

Divergence between the blockholder and small shareholders. The literature often relates the voting premium to conflicts between blockholders and minority shareholders. The idea is that a higher voting premium may indicate a lower payoff of minority shareholders, since the blockholder exploits his voting power to extract private benefits and advance his own agenda at the expense of others. This section shows that this intuition is not always correct.[25]

We start by defining the aggregate equilibrium payoff of dispersed shareholders as

$$W^* \equiv \int_{-\bar{b}}^{\bar{b}} u^*(b)\, g(b)\, db, \tag{31}$$

where $u^*(b)$ is the expected payoff of a dispersed shareholder with bias b:

$$u^*(b) = (1 - \alpha + x^*(b))\, v(b, q^*(y^*)) - x^*(b)\, p^* - \frac{\gamma}{2} x^*(b)^2, \tag{32}$$

and $x^*(b)$, $q^*(y^*)$, and p^* are defined in Proposition 2. From (5), (22) and (24), W^* can be rewritten as

$$W^* = (1-\alpha)\, v(\mathbb{E}[b], q^*) + \frac{1}{2\gamma}\mathbb{E}\left[(v(b, q^*) - p^*)^2\right]. \tag{33}$$

The first term in (33) is dispersed shareholders' aggregate payoff from their endowment, which increases if the median voter $(-q^*)$ moves closer to the average dispersed shareholder, $\mathbb{E}[b]$.[26] The second term reflects the aggregate trading profits of dispersed shareholders, both from trades among themselves and with the blockholder.

Our main result in this section shows that a larger voting premium does not necessarily indicate a greater conflict between the blockholder and small shareholders.

Proposition 4. *If*

$$G^{-1}\left(\frac{1-\tau}{1-\alpha}\right) < \mathbb{E}[b] < \beta, \tag{34}$$

then both W^* and the voting premium strictly increase in β.

The intuition is illustrated in Figure 3. Condition (34) implies that β is high enough, so that the median voter is a dispersed shareholder who is less activist than the blockholder.[27] Then, as β increases and the blockholder becomes even more activist, his incentives to change the median voter become stronger, which increases the voting premium from Proposition 3.

[25] A related observation regarding the suitability of the block premium for estimating private benefits emanates from the analysis in Burkart, Gromb, and Panunzi (2000), Nicodano and Sembenelli (2004), and Albuquerque and Schroth (2010). However, the mechanisms in these papers are different from ours.

[26] Formally, $\frac{\partial v(\mathbb{E}[b], q^*)}{\partial(-q^*)} = (\mathbb{E}[b] + q^*)\, f(q^*)$, so $v(\mathbb{E}[b], q^*)$ is maximized when the median voter's bias is $\mathbb{E}[b]$.

[27] In particular, as we show in the Appendix, (34) implies that $\beta > \beta^H_{\text{non-mv}}$ defined in Proposition 3.

However, the payoff W^* of small shareholders increases at the same time. To see why, note that absent trading, $G^{-1}(\frac{1-\tau}{1-\alpha})$ is the median voter, so condition $G^{-1}(\frac{1-\tau}{1-\alpha}) < \mathbb{E}[b]$ implies that the median voter is less activist than the average small shareholder prior to trading. If γ is large enough, the same relationship also holds after trading, as shown in Figure 3. Then, the trades of a more activist blockholder also make the median voter more activist, which moves the median voter closer to the average of the small shareholders, thereby increasing their payoff.[28] As a result, the aggregate payoff of small shareholders and the voting premium move in the same direction.

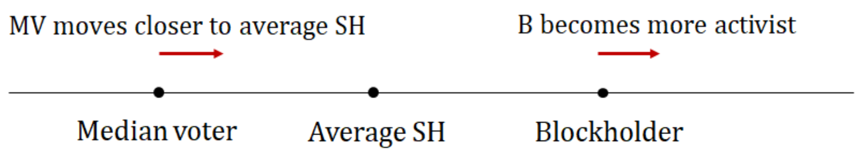

Figure 3. Small shareholders' welfare and voting premium move in the same direction

There are two scenarios that can lead to condition (34) and the situation described in Figure 3. First, suppose that there is a simple majority requirement but the distribution of small shareholders' preferences is right-skewed. This means that more activist small shareholders favor the proposal much more strongly than their less activist peers. The median voter does not reflect this asymmetric intensity of preferences and is less activist than the average small shareholder, but the intensity of preferences is relevant for shareholders' aggregate payoff W^*, which is based on the average and not the median payoff. Second, suppose that the distribution of preferences is symmetric, but the proposal is subject to a supermajority requirement ($\tau > 0.5$). The supermajority requirement introduces a conservative bias into the voting process and thereby reduces the bias of the median voter relative to the average shareholder.

In both scenarios, the voting rule is too conservative from the perspective of an average small shareholder: the median voter in Figure 3 is to the left of the average small shareholder. Then a more activist blockholder becomes a countervailing force against this conservative bias and increases the aggregate payoff of small shareholders. Notably, this happens even though the distance between the blockholder and the average small shareholder increases with β. Hence,

[28] This discussion abstracts from the welfare effects of trading profits (the second term in (33)). The condition $\mathbb{E}[b] < \beta$ in (34) guarantees that the trading profits of small shareholders also increase in β.

what ultimately matters for small shareholders is not whether the blockholder is closer to them, but whether he moves the median voter closer to them.

Divergence among dispersed shareholders. A positive voting premium emerges in our setting only if the blockholder is able to move the median voter. The next result shows that this can happen only if there is some heterogeneity of preferences among dispersed shareholders.

Corollary 2. *Suppose that all dispersed shareholders have the same bias b, which differs from that of the blockholder, $b \neq \beta$. Then the voting premium is zero.*

The situation described in Corollary 2 could arise in a setting in which all dispersed shareholders have the same valuation of cash flows and the blockholder can reduce these cash flows by diluting the assets of the firm if the proposal is accepted. Then all dispersed shareholders have the same b and $b < \beta$, since the blockholder benefits more from the proposal. This, as well as another example for the opposite case with $b > \beta$, are discussed in more detail in Sections B.3 and B.4 of the Online Appendix.

The key point is that in our setting, the blockholder does not acquire majority control. If dispersed shareholders are homogeneous, the median voter is always a dispersed shareholder and the blockholder does not have the ability to change his bias. Hence, heterogeneity among dispersed shareholders is critical for the existence of a positive voting premium in our setting with a minority blockholder. This discussion underscores our observation in Section 5.4.2 that the voting premium arises from the blockholder's ability to influence the composition of other shareholders, and not only from the direct increase in his holdings.

5.4.4 Liquidity of voting vs. non-voting shares

Price impact is often used as a measure of liquidity. Liquidity measured in this way is endogenous in our setting and arises because voting rights are bundled with cash flow rights in voting shares. Recall that the value of a non-voting share in our model is given by (25), which assumes that the decision is exogenous so that trades cannot move the median voter, i.e., $\frac{\partial(-q^*)}{\partial y} = 0$. To see how voting rights affect liquidity, note from (17) that the total price impact of the blockholder's trade in a voting share is

$$\frac{dp^*}{dy} = \gamma + \frac{\partial p^*}{\partial (-q^*)} \frac{\partial (-q^*(y))}{\partial y} = \gamma + \underbrace{(\mathbb{E}[b] + q^*) f(q^*)}_{\text{price impact of a vote}} \frac{\partial (-q^*(y))}{\partial y}. \tag{35}$$

The first term, γ, reflects dispersed shareholders' trading costs and would be present even for non-voting shares. The second term reflects the indirect effect through the influence of the blockholder's trades on the median voter, $-q^*(y)$, and is proportional to the price impact of a vote in equation (30). The sign of the indirect effect depends on whether the resulting change in the median voter benefits or hurts dispersed shareholders.

If the blockholder's and the average dispersed shareholder's interests are aligned (e.g., $-q^*(y^*) < \min\{\beta, \mathbb{E}[b]\}$, as in Figure 3), then the blockholder's trades move the median voter in the direction preferred by both (see Section 5.4.3 for a discussion of two scenarios in which this can occur). This increases the price at which dispersed shareholders supply additional shares; accordingly, the price impact of a vote (30) is positive. As a result, the liquidity of a voting share is smaller compared to that of a non-voting share. Essentially, dispersed shareholders *free-ride* on the blockholder's trades.

By contrast, if the blockholder's and dispersed shareholders' interests are in conflict (e.g., $\mathbb{E}[b] < -q^*(y^*) < \beta$), then the blockholder's trades move the median voter away from dispersed shareholders. This hurts dispersed shareholders and reduces the price at which they are willing to supply shares, so the price impact of a vote is negative. Accordingly, the supply function is flatter, and the liquidity of a voting share is now greater than that of a non-voting share.

5.4.5 Price impact and the sign of the voting premium

Based on the decomposition of the voting premium in (28), the incentive part of the voting premium is a combination of the marginal benefit of a vote (29) and the price impact of a vote (30). Hence, the voting premium is positive only if moving the median voter increases the value of the blockholder's stake by more than it increases the costs of his trades. This argument has important implications for the sign of the voting premium.

Conflict and a positive voting premium. If the blockholder's trades move the median voter in the direction that hurts dispersed shareholders, the voting premium is positive. This is because in this case, moving the median voter towards the blockholder not only increases the value of the blockholder's stake, but also reduces the price he has to pay to dispersed shareholders for their shares. Interestingly, this also implies that the voting rights embedded in the shares have opposite effects on the *price* of the shares and the *price impact* of trades: the embedded voting rights increase the share price but decrease the price impact of trades.

Free-riding and a negative voting premium. If the blockholder's trades move the median voter in the direction that benefits dispersed shareholders, dispersed shareholders free-ride on the blockholder's trades and increase the price at which they are willing to sell shares. As a result, the blockholder's incentives to move the median voter (28), and hence the voting premium, decrease. Proposition 3 shows that if $\beta < \beta^L_{\text{non-mv}}$ or $\beta > \beta^H_{\text{non-mv}}$, this force is not strong enough to prevent a positive voting premium. Intuitively, if the blockholder's preferences are extreme, then moving the median voter in his preferred direction, and thus the marginal benefit of a vote, is significantly more important for him than price impact considerations. However, if the blockholder's preferences are not extreme, a negative voting premium can arise:

Proposition 5. *There exist cutoffs $\underline{\beta}_{neg}$ and $\overline{\beta}_{neg}$, satisfying $\beta^L_{non\text{-}mv} < \underline{\beta}_{neg} < \overline{\beta}_{neg} < \beta^L_{mv}$, such that if*

$$\mathbb{E}\left[b\right] < G^{-1}(1 - \frac{\tau}{1-\alpha}) \text{ and } \beta \in (\underline{\beta}_{neg}, \overline{\beta}_{neg}), \tag{36}$$

then the blockholder buys shares ($y^* > 0$) and the voting premium is strictly negative: $p^* < p_{CF}\left(q^*\left(y^*\right)\right)$ in (26).

The intuition is illustrated in Figure 4. The first condition in (36), $\mathbb{E}\left[b\right] < G^{-1}(1 - \frac{\tau}{1-\alpha})$, ensures that for large enough γ, the average small shareholder is less activist than the median voter. The second condition, $\beta \in (\underline{\beta}_{\text{neg}}, \overline{\beta}_{\text{neg}})$, guarantees that the blockholder is more activist than the average small shareholder but less activist than the median voter, as in Figure 4. For example, the blockholder may be reluctant to support a certain management proposal, but small shareholders may be more strongly biased against this proposal than the blockholder himself. In this scenario, purchases of the blockholder make the median voter less activist and closer to his own preferences. However, since small shareholders are on average even less activist than the blockholder, they benefit even more from this change in the median voter than the blockholder himself. Thus, the change in the median voter affects the valuation of small shareholders and thereby the stock price more than it affects the valuation of the blockholder. Hence, the trading profits for the blockholder from buying shares are negative, since the price at which he buys increases by more than his own valuation. This negative effect on the blockholder's trading profits dominates the positive benefits from an increased valuation of his endowment α as long as α is not too large. (Note that the first condition in (36) implies an upper bound on α.) As a result, the blockholder buys fewer shares than he would if he could buy cash flow rights separately, without the attached voting rights. Thus, free-riding by dispersed shareholders results in a negative voting premium.[29]

The negative voting premium is directly related to the differential liquidity of voting and non-voting shares discussed in Section 5.4.4. If the price impact of trading voting shares is much stronger than the price impact of trading non-voting shares (the second term in (35) is large), then the blockholder's demand for voting shares can be smaller than his demand for non-voting shares, even though he values the voting rights per se. This results in a negative premium on the price of voting shares, i.e., in some sense, an illiquidity discount from the attached voting rights.

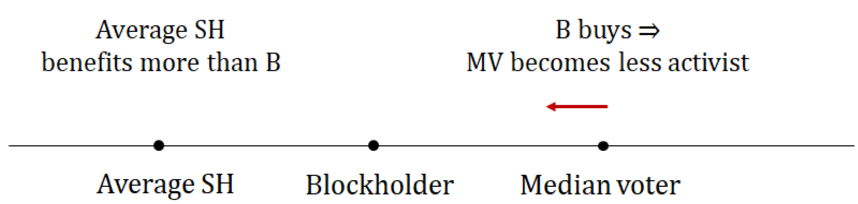

Figure 4. Negative voting premium

6 Extensions

In this section, we discuss several other implications of the baseline model and its extensions. The complete analysis of these results can be found in Section D of the Online Appendix.

[29]Proposition 7 in the Online Appendix shows that a negative voting premium can also emerge when the blockholder has no endowment, $\alpha = 0$. Intuitively, if $\alpha = 0$, the blockholder's marginal benefit of a vote is also small (see (29)), so the price impact considerations can dominate and result in a negative voting premium.

Dual-class shares. The empirical literature commonly measures the voting premium using firms with a dual-class share structure, by comparing the price of a share with superior voting rights and that with inferior voting rights (e.g., Zingales, 1995; Nenova, 2003). Our definition of the voting premium applies to all firms, including those with a single-class structure, but is closely related to this empirical measure. To show this, we consider an extension to dual-class shares in Section D.1 of the Online Appendix. There, investors can trade non-voting shares in addition to trading voting shares. In this setting, the blockholder's marginal payoff from buying voting rights translates into an actual price difference between voting and non-voting shares, i.e., the dual-class premium. In particular, similar to (27) in the baseline model,

$$p^*_{voting} - p^*_{non\text{-}voting} = \frac{\gamma\sigma(\beta)}{2\gamma+\eta} MPV(y^*, \hat{y}^*), \tag{37}$$

where $\hat{y}^*$ is the blockholder's equilibrium trade of non-voting shares and $MPV(y, \hat{y})$ is the analog of $MPV(y)$.[30] This dual-class premium has a similar structure to the voting premium in the baseline model and can be decomposed into the same components: the ability to move the median voter, the marginal benefit of a vote, and the price impact of a vote. Thus, our implications about the voting premium defined in the baseline model apply to the dual-class premium as well. Our analysis also implies that whenever the dual-class premium is positive (negative), dispersed shareholders have a larger (smaller) demand for non-voting shares than for more (less) expensive voting shares. Moreover, (37) shows that the dual-class premium exists only if there are trading frictions ($\gamma > 0$). Such limits to arbitrage are necessary for a voting premium to emerge.

Note also that the voting premium depends on $\hat{y}^*$, the volume of trades in the market for non-voting shares. Intuitively, the blockholder's position in non-voting shares gives him additional incentives to change the median voter and increases his marginal benefit of a vote. Moreover, the shift in the median voter changes dispersed shareholders' valuations and thus the price that the blockholder has to pay for both voting and non-voting shares.

Vote trading. In our baseline model, as well as in the extension to dual-class shares, voting rights can only be traded when bundled with cash flow rights. In practice, votes can be traded

[30] The expression for $MPV(y, \hat{y})$ is given by (139) in the Online Appendix, which shows that it is analogous to expression (28) for $MPV(y)$ in the baseline model.

separately from cash flow rights, for example, through share lending. In Section D.2 of the Online Appendix, we extend the model by adding a separate market for voting rights. The price of a vote is zero in this setting, since dispersed shareholders are never pivotal for the voting outcome and thus willing to supply their votes for an arbitrarily small price. However, the voting premium can still be strictly positive as long as the blockholder's ability to accumulate voting power through the market for votes is limited. Thus, the price of a separately traded vote is conceptually different from the voting premium for a share in which cash flow and voting rights are bundled. This is so because small heterogeneous shareholders value only cash flow rights, and the bundling gives rise to an upward-sloping supply function.

Influence premium. In practice, blockholders can exert influence even without a formal vote if they can influence the firm's management. In Section D.3 of the Online Appendix, we analyze a version of the model in which decisions are taken by management rather than by voting, and management puts some weight on the welfare of its current shareholders, represented by the preferences of the average post-trade shareholder. Then the blockholder's trades influence decisions by changing the average shareholder's preferences, and the blockholder values this influence, which may give rise to an "influence premium" on the share price.

Similar to the voting premium, the equilibrium influence premium is proportional to the blockholder's marginal payoff from buying shares to exert influence. Moreover, the influence premium can be decomposed in exactly the same way as the voting premium. Hence, many of our implications about the value of voting rights also extend to settings in which voting rights have not only a direct influence, but also an indirect one through other mechanisms that allow large shareholders to affect corporate policies. The key difference between the voting premium and the influence premium is that the former depends on the identity of the *median voter*, and the latter on the identity of the *average shareholder*. Since the median voter and the average shareholder are different from each other, these two premiums are not the same.

Multiple blockholders. Section D.4 of the Online Appendix generalizes our model to the case with multiple blockholders. It shows that the voting premium can be decomposed in exactly the same way as in our baseline model, and therefore, our main conclusions extend to this setting as well. Moreover, the magnitude of the voting premium depends on whether the blockholders' preferences are aligned or not. If blockholders are sufficiently heterogeneous, their trades pull the median voter in opposite directions. Then, as the blockholders' biases become more extreme, each blockholder tries harder to gain influence over the voting outcome, which results in a higher voting premium. In contrast, if the blockholders are homogeneous, Cournot competition between them drives up the share price and reduces their incentives to build up large stakes (e.g., Kyle, 1989; Edmans and Manso, 2011). With small stakes, blockholders can still influence the identity of the median voter, but none has incentives to do so in equilibrium. For this reason, the voting premium vanishes if blockholders are homogeneous and their number becomes sufficiently large.

Exit and a positive voting premium. In the context of our baseline model, we show that, although the blockholder can gain influence over the voting outcome by buying additional shares, he may nevertheless choose to do the opposite: sell shares to small shareholders and thereby give up his influence over the voting outcome, while demanding a positive premium from the small shareholders. Thus, the tension between exit and voice (e.g., Hirschman, 1970) also exists in our model, which shows that the incentives to exit can prevail even when the voting premium is positive. Hence, a positive voting premium does not necessarily indicate a more concentrated ownership structure. This analysis is presented in Section D.5 of the Online Appendix.

Vote participation. Implicit in our analysis is that dispersed shareholders participate in the vote even though they do not expect to be pivotal for the outcome. In practice, institutional investors vote their shares at a higher rate than retail investors (Brav, Cain, and Zytnick, 2022). In general, investors with stronger views about the proposal (i.e., larger $|b|$) are expected to participate and vote with a higher probability (Zachariadis, Cvijanovic, and Groen-Xu, 2020). We illustrate how such selective participation can be incorporated into our analysis in Section D.6 of the Online Appendix. We show that it changes the composition of the voter base and, by implication, the identity of the median voter. Other than that, our analysis can be performed as in the baseline model.

7 Empirical implications

The purpose of this section is to shed some light on the empirical discussion of the voting premium by exploring the implications of our model, and to locate the model in the context of the empirical literature. The literature employs six different strategies to estimate the value of a vote, which are summarized in Table 1 and are based on our review of 40 empirical studies. These studies cover data from a broad range of countries between 1940 and 2018 and are described in more detail in Tables A1 and A2 in Section A of the Online Appendix. However, we do not attempt a systematic survey here and only discuss some key patterns that emerge from these studies.[31]

The most salient feature of the estimates in Table 1 is their divergence, across methodologies as well as within-methodology. Next, we discuss why estimates of the voting premium may vary across methodologies (Section 7.1) and across firms (Section 7.2).

Methodology	Avg.	Median	Min	Max	# studies	# with neg.
Dual-class shares	22.68	13.58	4.07	81.50	23	18
Block trades	20.27	16.81	6.79	46.96	6	4
Dual-class tender offers	42.16	26.59	12.27	130.7	5	0
Option replication	0.20	0.16	0.09	0.37	5	5
Equity lending	0.02	0.02	0.01	0.02	3	0
Record-day trading	0.75	0.75	0.09	1.40	2	0

Table 1. Empirical estimates of the voting premium. The table summarizes the results of 40 studies, described in Tables A1 and A2 in Section A of the Online Appendix, which between them use six methodologies to estimate the voting premium. We first take the average estimate of the voting premium for each study and then calculate the averages, medians, minimums, and maximums of these estimates across all studies using a given methodology. We report these numbers (in percent of the stock price) in columns Avg., Median, Min, and Max, respectively. Of the seven studies that present evidence on dual-class tender offers (Table A2 in the Online Appendix), this table uses only the five studies that report at least one dual-class tender offer in their sample (two others report zero tender offers in their samples). Several studies report results on more than one method, so the number of studies (# studies) sums to more than 40. The last column (# with neg.) reports the number of studies within each methodology that document a negative voting premium for at least some firms in their sample.

[31] Some papers already contain surveys of different strands of this literature. Rydqvist (1992) provides a survey of earlier studies on dual-class shares, and Dittmann (2004), Adams and Ferreira (2008), and Kind and Poltera (2013) provide more recent updates.

7.1 Methodological differences across studies

Our model offers several ways to interpret the divergence of estimates across methodologies. Our comparison is only suggestive, since the samples across studies are different.

Marginal values vs. block values. The methods based on the dual-class share premium, option replication, equity lending, and record-day trading all measure the value of a *marginal* vote. In contrast, in block trades and dual-class tender offers, blockholders purchase an entire block of shares, which is related to the *average* value of a vote. Based on our theoretical argument, we expect this average value to be larger. To see this and relate the estimates in Table 1 to our model, let p_B be the price paid for the average voting share, either in a block trade or in a tender offer. Then the block premium can be expressed as $p_B - p^*$, the difference between the price in an entire block of voting shares, which reflects their *average* valuation, and the price of a *marginal* voting share.[32] The average block premium is 20.27%, which shows that the difference between the average and the marginal value of a vote is substantial.

By contrast, the dual-class premium is $p^* - p_{CF}$ in our notation, the difference between the price of a *marginal voting share* and the price of a *marginal non-voting share* in the stock market. Note that our model does not predict any relationship between the dual-class premium and the block premium. Finally, the dual-class tender-offer premium is $p_B - p_{CF}$, the difference between the price for an average *voting share* and the price of a *non-voting share.* Table 1 shows that the dual-class tender-offer premium (42.16%) significantly exceeds the dual-class premium (22.68%). This is consistent with our framework, because the latter is only the premium for a marginal vote, whereas the former also includes the additional value if shares are purchased in a block. Hence, our model helps to clarify the conceptual differences between methods to measure the voting premium, aiding in understanding systematic differences in the estimates.

Separate vs. joint trading of cash flow and voting rights. Another important difference between the methodologies is whether they estimate the price of the vote that is traded separately, as in the equity lending market, or the price of the vote that is traded in conjunction with cash flow rights, as in the estimates derived from comparing the price of a stock with superior voting rights to the price of a stock with inferior (or no) voting rights. Our

[32] We ignore that empirical studies express premiums as percentages by normalizing them by the share price. This difference is inconsequential for our qualitative assessment.

analysis emphasizes that estimates from these methodologies could be very different. Indeed, our extension to a separate market for votes shows that the premium on the price of voting shares could be strictly positive even if the price of a separately traded vote is zero, consistent with the equity lending methodology producing the smallest estimates among all.

Capitalized voting premiums. It is salient from the table that studies of dual-class shares, dual-class tender offers, and block trades obtain much larger estimates than the other three methods. We attribute this at least partly to the fact that the former three methods capitalize the value of the voting right over longer time horizons, which span potentially infinitely many future shareholder meetings. In contrast, the three other methods estimate the value of voting rights over a period of one year or less.

7.2 The cross-sectional variation in the voting premium

Negative values of the voting premium. Of the 40 studies we survey, no less than 27 provide evidence for a negative voting premium for some firms in their sample (see Table 1 and Table A1 in the Online Appendix for details). Many researchers note that these findings are puzzling, since they are difficult to interpret in the context of extant theories. Empirical studies often explain a negative voting premium by pointing out that voting shares may suffer from a liquidity discount relative to non-voting shares.[33] By contrast, a negative voting premium emerges naturally in our model because of the free-rider effect and the possibly substantial price impact of the blockholder's trades (see Proposition 5 and the related discussion). Moreover, our explanation also captures the liquidity difference between voting and non-voting shares, if we define liquidity as price impact: the voting premium becomes negative in our model only if the trading has a stronger price impact on voting shares than on non-voting shares.

Voting premiums, takeovers, and shareholder meetings. A standard explanation for how the blockholder's willingness to pay for voting control is translated into higher prices for voting shares is the takeover mechanism, and several empirical studies find support for this

[33] See the Introduction. Domowitz, Glen, and Madhavan (1997) and Gardiol, Gibson-Asner, and Tuchschmid (1997) are probably the first to show how liquidity differences between classes of stock differentiated by ownership and voting restrictions lead to price differentials.

explanation.[34] However, this theory has some limitations. First, since the 1990s, many countries have enacted coattail provisions, which mandate equal treatment of all classes of shares in control changes, yet the voting premium remains positive after such regulatory changes (e.g., Maynes, 1996; Nenova, 2003).[35] Second, in Table A2 of the Online Appendix we survey seven studies that provide evidence on the dual-class premium and the premium paid to superior-voting shares in dual-class tender offers. There, the dual-class tender-offer premium refers to the premium paid in actual tender offers, whereas the ex-ante premium weighs the dual-class tender-offer premium by the in-sample frequency with which dual-class firms are acquired, which should provide a reasonable approximation to the ex-ante expected tender-offer premium.[36] We also provide estimates of the premium paid for superior voting shares in the market, i.e., the dual-class premium. The average ex-ante voting premium accounts for only about one quarter of the dual-class premium (6.08% compared to 22.49%). Hence, the takeover explanation is probably only a partial explanation of market premiums on voting shares.

Differently from the takeover argument, our paper shows how the voting premium can arise without contests for majority control, and solely as a result of blockholders' desire to influence voting outcomes at shareholder meetings. This idea is consistent with the findings of the more recent literature, which analyzes the time-series variation and finds that the voting premium is largest around shareholder meetings compared to other periods of the year (Kind and Poltera, 2013; Kalay, Karakas, and Pant, 2014; Kind and Poltera, 2017; Fos and Holderness, 2022).

Voting premiums and ownership structure. Studies on the relationship between the voting premium and ownership concentration show that it is often non-monotonic: the value of voting rights is small both if ownership is very dispersed and if it is very concentrated with one blockholder who has majority control (Kind and Poltera, 2013).[37] Relatedly, Smith

[34] See Section 2 for details on these theories. For empirical evidence see Bergström and Rydqvist (1992); Zingales (1995); Rydqvist (1996); and Smith and Amoako-Adu (1995).

[35] Maynes (1996) performs an event study of such coattail implementations and shows that a dual-class premium of 8.22% declines by about two percentage points (see her Tables 2 and 3). Hence, about one-quarter of the voting premium could arguably be attributed to preferential treatment in takeovers.

[36] This is obviously a back-of-an-envelope calculation. Our calculations may overestimate the true ex-ante voting premium because we neglect discounting, or underestimate it because we do not account for the right-censoring of the data as acquisitions may occur after the sample period. We know of no study that provides rigorous estimates of the ex-ante voting premium.

[37] Commonly analyzed characteristics of the ownership structure are the prevalence and size of the first and the second-largest blockholders, oceanic Shapley values (following Rydqvist, 1987), and insider ownership. Of the 40 studies we survey, at least 18 provide evidence for the relevance of one of the indicators of ownership for

and Amoako-Adu (1995) show that block ownership (the presence of blockholders with more than 10% ownership of the votes) reduces the voting premium (see their Table 6, p. 237). Our analysis of multiple blockholders suggests a new empirical direction by showing that it is not only the concentration of ownership that matters, but also the preferences of blockholders. Specifically, if blockholders have similar preferences, then ownership concentration is positively correlated with the voting premium, and if blockholders disagree with each other, the voting premium increases the more they disagree (see Section D.4 of the Online Appendix).

8 Conclusion

We develop a unified theory of blockholder governance, ownership structure, and the voting premium. In equilibrium, the voting premium reflects the blockholder's marginal willingness to pay for a voting share and to change the preferences of the median voter. By trading, the blockholder affects the median voter both directly, by accumulating additional voting rights, and indirectly, by buying shares from investors who disagree with him the most.

Our analysis has several important takeaways for theoretical and empirical work in this area. First, a positive voting premium arises even in the absence of takeovers and acquisitions of controlling stakes, so future studies should pay more attention to voting at shareholder meetings. Second, a small voting premium does not imply that voting rights are worthless; instead, it can indicate that the accumulated voting block is relatively large. Third, the voting premium is not a good proxy for conflicts of interest between blockholders and dispersed shareholders. Fourth, the value of votes when they are bundled with cash flow rights is different from when they are traded separately. Last, the voting premium can be negative; this happens when the liquidity of voting shares is endogenously lower than that of non-voting shares because of dispersed shareholders free-riding on the blockholder's trades.

the respective measure of the voting premium.

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Appendix: Proofs

This appendix contains the proofs of the main results. The Online Appendix is available here.

Our proofs rely on γ being finite but large enough. In particular, in the proof of each result below we formally define a separate (finite) cutoff on γ such that if γ exceeds this cutoff, the statement of the result holds. Since the proofs of subsequent results build on the proofs of earlier results, in each subsequent proof we assume that γ is above the cutoffs defined in the previous proofs. We then define $\bar{\gamma}$ (introduced in equation (3)) as the maximum of the cutoffs in each of the proofs below. Then, for any finite $\gamma > \bar{\gamma}$, all the results hold.

Proof of Lemma 1. Given the realization of q, a shareholder with bias b votes for the proposal if and only if $q > -b$. Denote the fraction of post-trade shares voted in favor by $\Lambda(q)$, and note that $\Lambda(q)$ is weakly increasing. There are three cases. If $\Lambda(\Delta) \leq \tau$ for the highest possible $q = \Delta$, then q^* in the statement of the lemma equals Δ. If $\Lambda(-\Delta) > \tau$ for the lowest possible $q = -\Delta$, , then q^* in the statement of the lemma equals $-\Delta$. Finally, if $\Lambda(-\Delta) \leq \tau < \Lambda(\Delta)$, there exists $q^* \in [-\Delta, \Delta)$ such that the fraction of votes in favor of the proposal exceeds τ if and only if $q > q^*$, so the proposal is approved if and only if $q > q^*$. ■

Proof of Proposition 1. Recall that $\underline{q} = s^{-1}(\tau - \alpha - y; y, q_e^*)$ and $\overline{q} = s^{-1}(\tau; y, q_e^*)$, and denote the corresponding functions by $\underline{q}(y, q_e^*)$ and $\overline{q}(y, q_e^*)$. By the arguments in the main text prior to the proposition, the proposal is approved if and only if $q > -b_{MV}(\beta, y, q_e^*)$, where

$$b_{MV}(\beta, y, q_e^*) = \begin{cases} -\overline{q}(y, q_e^*) & \text{if } \beta < -\overline{q}(y, q_e^*) \\ \beta & \text{if } -\overline{q}(y, q_e^*) \leq \beta \leq -\underline{q}(y, q_e^*) \\ -\underline{q}(y, q_e^*) & \text{if } -\underline{q}(y, q_e^*) < \beta \end{cases} \tag{38}$$

is the bias of the median voter.

Since shareholders' expectations q_e^* at the trading stage have to be consistent with the actual decision rule at the voting stage, the equilibrium at the voting stage can be characterized as follows: the proposal is approved if and only if $q > q^*(y)$, where

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } \beta < \beta_L(y) \\ \beta & \text{if } \beta_L(y) \leq \beta \leq \beta_H(y) \\ \beta_H(y) & \text{if } \beta_H(y) < \beta \end{cases} \tag{39}$$

is the bias of the median voter, and $\beta_L(y)$ and $\beta_H(y)$ are the solutions of

$$\beta_L(y) = -\bar{q}(y, -\beta_L) \Leftrightarrow s(-\beta_L; y, -\beta_L) = \tau, \quad (40)$$

$$\beta_H(y) = -\underline{q}(y, -\beta_H) \Leftrightarrow s(-\beta_H; y, -\beta_H) = \tau - \alpha - y. \quad (41)$$

Using (14), conditions (40) and (41) can be rewritten as

$$R(\beta_L; y, -\beta_L) = 1 - \frac{\tau}{1 - \alpha - y}, \quad (42)$$

$$R(\beta_H; y, -\beta_H) = 1 - \frac{\tau - \alpha - y}{1 - \alpha - y}. \quad (43)$$

From (13), $R(b'; y, q^*)$ is a cdf, lies in the unit interval, and

$$\lim_{\beta \to -\bar{b}} R(\beta; y, -\beta) = 0 \text{ and } \lim_{\beta \to \bar{b}} R(\beta; y, -\beta) = 1. \quad (44)$$

According to Lemma 3, there exists $\bar{\gamma}_1$ such that for any $\gamma > \bar{\gamma}_1$, no shareholder short sells and the blockholder's optimal trade satisfies $1 - \alpha - y > \tau$ and $\alpha + y < \tau$, and hence, for such γ, we can restrict attention to y satisfying these properties. Then, the right-hand sides of both (42) and (43) lie in $(0, 1)$, and since R is continuous, (44) implies that solutions to (42) and (43), and therefore, to (40) and (41), exist. Note that the function R does not depend on β, and hence these solutions do not depend on β either. Using (13) and simplifying, we can show that the derivative of $R(\beta; y, -\beta)$ with respect to β is:

$$\frac{\partial R(\beta; y, -\beta)}{\partial \beta} = g(\beta)\left(1 + \frac{\beta - \mathbb{E}[b]}{\gamma} \frac{H(-\beta)}{1 - \alpha - y}\right) - G(\beta) \frac{f(-\beta)}{1 - \alpha - y} \frac{\mathbb{E}[b] - \mathbb{E}[b|b < \beta]}{\gamma}. \quad (45)$$

From (11) and (12), the first term in (45) equals $r(\beta; y, -\beta) > 0$. Since $\mathbb{E}[b] > \mathbb{E}[b|b < \beta]$, $R(\beta; y, -\beta)$ may be non-monotonic in β. From (45), $\frac{\partial R(\beta; y, -\beta)}{\partial \beta} > 0$ if and only if

$$\frac{\frac{G(\beta)}{g(\beta)} f(-\beta)(\mathbb{E}[b] - \mathbb{E}[b|b < \beta]) + H(-\beta)(\mathbb{E}[b] - \beta)}{1 - \alpha - y} < \gamma. \quad (46)$$

Since $g(\beta)$ is positive, (46) implies that there exists $\overline{\gamma} > \bar{\gamma}_1$ such that if $\gamma > \overline{\gamma}$, then $\frac{\partial R(\beta; y, -\beta)}{\partial \beta} > 0$ for all $y \geq -\alpha$ and all $\beta \in [-\bar{b}, \bar{b}]$. In this case, (44) implies that the solutions to (40)-(41) exist, are unique, and lie in $(-\bar{b}, \bar{b})$; as noted above, they do not depend on β.

Suppose $\gamma > \overline{\gamma}$, such that $\beta_L(y)$ and $\beta_H(y)$ exist and are unique. For $y = -\alpha$ (when the blockholder sells his entire endowment), the right hand sides of (42) and (43) are identical, so we can define

$$\beta^* \equiv \beta_H(-\alpha) = \beta_L(-\alpha), \quad (47)$$

or equivalently, $R(\beta^*; y, -\beta^*) = 1-\tau$. Auxiliary Lemma 2 in the Online Appendix shows that $\beta_L(y)$ is decreasing in y, $\lim_{y\nearrow 1-\tau-\alpha} \beta_L(y) = -\bar{b}$, $\beta_H(y)$ is increasing in y, and $\lim_{y\nearrow \tau-\alpha} \beta_H(y) = \bar{b}$. Using these properties, there are two cases to consider:

Case 1: $\beta \in (\beta^*, \bar{b}]$. Then $\beta > \beta_L(y)$ for all y, and there exists y_H such that $\beta_H(y) \geq \beta$ if and only if $y \geq y_H$, where from (43),

$$y_H = 1 - \alpha - \frac{1-\tau}{G(\beta)} - \frac{1}{\gamma}\left(\mathbb{E}[b] - \mathbb{E}[b|b<\beta]\right) H(-\beta). \tag{48}$$

Then, (39) implies that if $y \geq y_H$, the median voter is the blockholder (i.e., $-q^*(y) = \beta$), and if $y < y_H$, the median voter is a dispersed shareholder with bias $\beta_H(y)$. Suppose $y < y_H$. Since $\beta_H(y)$ is increasing in y and $\beta > \beta_H(y)$, then $|\beta + q^*(y)| = \beta - \beta_H(y)$ decreases in y.

Case 2: $\beta \in [-\bar{b}, \beta^*)$. Then $\beta < \beta_H(y)$ for all y, and there exists y_L such that $\beta_L(y) > \beta$ if and only if $y < y_L$, where from (42),

$$y_L = 1 - \alpha - \frac{\tau}{1-G(\beta)} + \frac{1}{\gamma}\frac{G(\beta)}{1-G(\beta)}\left(\mathbb{E}[b] - \mathbb{E}[b|b<\beta]\right) H(-\beta). \tag{49}$$

Then, (39) implies that if $y \geq y_L$, the median voter is the blockholder (i.e., $-q^*(y) = \beta$), and if $y < y_L$, the median voter is a dispersed shareholder with bias $\beta_L(y)$. Suppose $y < y_L$. Since $\beta_L(y)$ is decreasing in y and $\beta < \beta_L(y)$, then $|\beta + q^*(y)| = \beta_L(y) - \beta$ decreases in y.

Setting $\bar{y}$ as y_H if $\beta \in (\beta^*, \bar{b}]$ and as y_L if $\beta \in [-\bar{b}, \beta^*)$ completes the proof. Section C.2 in the Online Appendix visualizes the functions $\beta_L(y)$ and $\beta_H(y)$. ■

Proof of Proposition 2. Throughout the proof, we assume that γ is large enough, $\gamma > \bar{\gamma}$, where $\bar{\gamma}$ does not depend on β. In particular, we use the assumption of large γ in several places, defining several cutoffs on γ that do not depend on β (all formally defined below), above which our derivations hold. The overall cutoff $\bar{\gamma}$ is the maximum among these cutoffs.

By Lemma 3 in Section C of the Online Appendix, there exists $\overline{\gamma}_1$ such that if $\gamma > \overline{\gamma}_1$, then for all β, the blockholder's trade y lies in the interval $[-\varepsilon, \varepsilon] \subset (-\alpha, \min\{\tau-\alpha, 1-\tau-\alpha\})$ for some $\varepsilon > 0$. Given (5), (10) and (18), we have

$$\begin{aligned}\Pi(y,\beta) &= \alpha v(\beta, q^*(y)) + y(\beta - \mathbb{E}[b]) H(q^*) - (\gamma + 0.5\eta) y^2 \\ &= \alpha v_0 + \alpha \mathbb{E}[\theta | q > q^*(y)] H(q^*) + ((\alpha + y)\beta - y\mathbb{E}[b]) H(q^*) - (\gamma+0.5\eta) y^2,\end{aligned} \tag{50}$$

$$\frac{\partial \Pi(y,\beta)}{\partial y} = (\beta - \mathbb{E}[b]) H(q^*(y)) - (2\gamma+\eta) y + \frac{\partial(-q^*(y))}{\partial y}\left[\alpha(q^*(y) + \beta) + y(\beta - \mathbb{E}[b])\right] f(q^*(y)), \tag{51}$$

where $q^*(y)$ is given by (39). Both $\frac{\partial(-q^*(y))}{\partial y}$ and $\frac{\partial \Pi(y,\beta)}{\partial y}$ do not exist when $y \in \{y_L(\beta), y_H(\beta)\}$, where $y_L(\beta)$ and $y_H(\beta)$ are the trades above which the blockholder becomes the median voter if $\beta < \beta^*$ and if $\beta > \beta^*$, respectively; they are defined by (49) and (48) and depicted in Figure A1 in the Online Appendix.

By Proposition 1, there exists $\overline{\gamma}_2 > \overline{\gamma}_1$, which does not depend on β, such that, if $\gamma > \overline{\gamma}_2$, then $\beta_L(y)$ and $\beta_H(y)$ are well-defined continuous functions of y; these functions do not depend on β. Then $\Pi(y,\beta)$ is a continuous function of y, and the optimal y lies in $[-\varepsilon,\varepsilon]$. Hence, $\Pi(y,\beta)$ has a maximum on $[-\varepsilon,\varepsilon]$, although its maximizer is not necessarily unique. Recall from (47) that $\beta^* = \beta_H(-\alpha) = \beta_L(-\alpha)$, and that by Lemma 2 in the Online Appendix, $\lim_{\gamma\to\infty}\beta^* = G^{-1}(1-\tau) \in (-\bar{b},\bar{b})$. Consider three cases: $\beta < \beta^*$; $\beta = \beta^*$; and $\beta > \beta^*$.

First, suppose $\beta \in [-\bar{b},\beta^*)$. Since $\beta^* = \beta_L(-\alpha) = \beta_H(-\alpha) \le \beta_H(y)$ for all y, we have $\beta < \beta_H(y)$ for all y. Based on (39),

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } \beta < \beta_L(y) \\ \beta & \text{if } \beta_L(y) \le \beta, \end{cases}$$

and $\beta < \beta_L(y) \Leftrightarrow y < y_L$, where y_L is given by (49). We define it as a function of β: $y_L(\beta)$.

By Lemma 2 in the Online Appendix, $\beta_L'(y) < 0$, and hence $y_L'(\beta) < 0$. Define

$$\Pi_{\text{mv}}(y,\beta) \equiv \alpha v(\beta,-\beta) + y(\beta - \mathbb{E}[b])H(-\beta) - (\gamma+0.5\eta)y^2 \tag{52}$$

$$\Pi^L_{\text{non-mv}}(y,\beta) \equiv \alpha v(\beta,-\beta_L(y)) + y(\beta - \mathbb{E}[b])H(-\beta_L(y)) - (\gamma+0.5\eta)y^2, \tag{53}$$

which are the payoff functions of the blockholder if he becomes the median voter ($q^* = -\beta$) and if he does not become the median voter ($q^* = -\beta_L(y)$), respectively. Notice that

$$\Pi(y,\beta) = \begin{cases} \Pi^L_{\text{non-mv}}(y,\beta) & \text{if } y < y_L(\beta) \\ \Pi_{\text{mv}}(y,\beta) & \text{if } y \ge y_L(\beta). \end{cases} \tag{54}$$

Both functions are continuous. Since $\Pi_{\text{mv}}(y_L(\beta),\beta) = \Pi^L_{\text{non-mv}}(y_L(\beta),\beta)$, $\Pi(y,\beta)$ is also continuous. In addition, Lemma 3 and Lemma 4 in the Online Appendix imply that there exists $\overline{\gamma}_3 > \overline{\gamma}_2$ such that if $\gamma > \overline{\gamma}_3$, then for any β, the maximizers of $\Pi^L_{\text{non-mv}}(y,\beta)$, $\Pi_{\text{mv}}(y,\beta)$, and $\Pi^H_{\text{non-mv}}(y,\beta)$ (defined below by (64)) lie in some interval $[-\varepsilon,\varepsilon] \subset (-\alpha, \min\{\tau-\alpha, 1-\tau-\alpha\})$, and all three functions are concave. Consider such γ. Then, $\Pi_{\text{mv}}(y,\beta)$ has a unique maximizer, which solves the first-order condition

$$y_{\text{mv}}(\beta) \equiv \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])H(-\beta). \tag{55}$$

Similarly, the unique maximizer of $\Pi^L_{\text{non-mv}}(y,\beta)$ is $y^L_{\text{non-mv}}(\beta)$, which is the unique solution of

$$y = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta_L(y)) + \frac{1}{2\gamma+\eta} MPV^L(y,\beta), \tag{56}$$

where from (51),

$$MPV^L(y,\beta) \equiv \frac{\partial \beta_L(y)}{\partial y} f(-\beta_L(y))[\alpha(\beta - \beta_L(y)) + y(\beta - \mathbb{E}[b])]. \tag{57}$$

Thus, the maximizer of $\Pi(y,\beta)$ lies in the set $\{y_{\text{mv}}(\beta), y^L_{\text{non-mv}}(\beta), y_L(\beta)\}$. Note that $y^L_{\text{non-mv}}(\beta)$ could be potentially larger than $y_L(\beta)$ (in which case it is not the maximizer), and similarly, $y_{\text{mv}}(\beta)$ could be smaller than $y_L(\beta)$ (in which case it is not the maximizer).

Next, we define the right and left derivative of the blockholder's payoff function at $y_L(\beta)$:

$$\begin{aligned} \Delta^L_r(\beta) &\equiv \frac{\partial \Pi(y,\beta)}{\partial y}\Big|_{y \searrow y_L(\beta)} = \frac{\partial \Pi_{\text{mv}}(y,\beta)}{\partial y}\Big|_{y=y_L(\beta)} && (58)\\ &= (\beta - \mathbb{E}[b])\, H(-\beta) - (2\gamma+\eta) y_L(\beta); \\ \Delta^L_l(\beta) &\equiv \frac{\partial \Pi(y,\beta)}{\partial y}\Big|_{y \nearrow y_L(\beta)} = \frac{\partial \Pi^L_{\text{non-mv}}(y,\beta)}{\partial y}\Big|_{y=y_L(\beta)} && (59)\\ &= \Delta^L_r(\beta) + \frac{\partial \beta_L(y)}{\partial y}\Big|_{y=y_L(\beta)} y_L(\beta)(\beta - \mathbb{E}[b])\, f(-\beta), \end{aligned}$$

where we have used $\beta_L(y_L(\beta)) = \beta$ and $\frac{\partial v(\beta,q^*(y))}{\partial(-q^*)}\big|_{y=y_L(\beta)} = (\beta + q^*(y))\, f(q^*(y))\,\big|_{y=y_L(\beta)} = 0$.

In Section C.5.1 of the Online Appendix, we show that there exists a cutoff $\overline{\gamma}_4 > \overline{\gamma}_3$ such that if $\gamma > \overline{\gamma}_4$, then for all $\beta \in [-\overline{b}, \beta^*)$, $\frac{\partial \Delta^L_r(\beta)}{\partial \beta} > 0$, $\frac{\partial \Delta^L_l(\beta)}{\partial \beta} > 0$, $\Delta^L_r(\beta^*) > 0$, and $\Delta^L_l(\beta^*) > 0$. Consider such γ, and define κ^L_r and κ^L_l as solutions to

$$\begin{aligned} \Delta^L_r\left(\kappa^L_r\right) &= 0, \\ \Delta^L_l\left(\kappa^L_l\right) &= 0. \end{aligned}$$

Since Δ^L_r and Δ^L_l are increasing and $\Delta^L_r(\beta^*) > 0$, $\Delta^L_l(\beta^*) > 0$, these solutions, if they exist, are unique and are strictly smaller than β^*. In Section C.5.2 of the Online Appendix, we show that if $\gamma > \bar{\gamma}_4$, then $-\overline{b} < \kappa^L_r \le \kappa^L_l < \beta^*$,

$$\min\{\beta_L(0), \mathbb{E}[b]\} \le \kappa^L_r \le \kappa^L_l \le \max\{\beta_L(0), \mathbb{E}[b]\}, \tag{60}$$

and $\kappa^L_r < \kappa^L_l$ if and only if $\beta_L(0) \ne \mathbb{E}[b]$. We next consider three cases:

(1) If $\beta \in [-\overline{b}, \kappa^L_r)$, then $\Delta^L_l(\beta) < 0$, $\Delta^L_r(\beta) < 0$, so both the left and the right derivative

of $\Pi(y,\beta)$ are negative at $y_L(\beta)$, and since $\Pi(y,\beta)$ is continuous at $y_L(\beta)$, the optimal trade $y^* < y_L(\beta)$. In particular, $\Delta_l^L(\beta) < 0$ implies that $y^L_{\text{non-mv}}(\beta)$ in this case is smaller than $y_L(\beta)$, and hence $y^* = y^L_{\text{non-mv}}(\beta)$.

(2) If $\beta \in [\kappa_r^L, \kappa_l^L)$, then $\Delta_l^L(\beta) < 0 \le \Delta_r^L(\beta)$. Hence, $y_L(\beta)$ cannot be the maximizer, and $y^* \in \{y_{\text{mv}}(\beta), y^L_{\text{non-mv}}(\beta)\}$. Note that if $\beta = \kappa_r^L$, then $\Delta_l^L(\beta) < 0 = \Delta_r^L(\beta)$. Then, $y_{\text{mv}}(\beta) = y_L(\beta)$ and $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta) > \Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$. If $\beta = \kappa_l^L$, then $\Delta_l^L(\beta) = 0 < \Delta_r^L(\beta)$. Then, $y^L_{\text{non-mv}}(\beta) = y_L(\beta)$ and $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta) < \Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$. From the continuity of $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta)$ and $\Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$ in β, there exist $b^L_{\text{non-mv}}$ and b^L_{mv} that satisfy

$$\kappa_r^L < b^L_{\text{non-mv}} \le b^L_{\text{mv}} < \kappa_l^L,$$

such that if $\beta \in [\kappa_r^L, b^L_{\text{non-mv}})$, then $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta) > \Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$, and if $\beta \in (b^L_{\text{mv}}, \kappa_l^L]$, then $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta) < \Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$. Thus, $y^* = y^L_{\text{non-mv}}(\beta)$ for $\beta \in [\kappa_r^L, b^L_{\text{non-mv}})$, and $y^* = y^L_{\text{mv}}(\beta)$ for $\beta \in (b^L_{\text{mv}}, \kappa_l^L]$. If $\beta \in (b^L_{\text{non-mv}}, b^L_{\text{mv}})$ then y^* is either $y_{\text{mv}}(\beta)$ or $y^L_{\text{non-mv}}(\beta)$, depending on the sign of $\Pi^L_{\text{non-mv}}(y^L_{\text{non-mv}}(\beta), \beta) - \Pi_{\text{mv}}(y_{\text{mv}}(\beta), \beta)$. From (60), notice that the interval $[\kappa_r^L, \kappa_l^L]$, and hence $[b^L_{\text{non-mv}}, b^L_{\text{mv}}]$, collapses into a single point if $\beta_L(0) = \mathbb{E}[b]$.

(3) If $\beta \in [\kappa_l^L, \beta^*)$, then $\Delta_r^L(\beta) > 0$, $\Delta_l^L(\beta) \ge 0$, and since $\Pi(y,\beta)$ is continuous at $y_L(\beta)$, the optimal trade $y^* \ge y_L(\beta)$. In particular, since $\frac{\partial \Pi_{\text{mv}}(y,\beta)}{\partial y}|_{y=y_L(\beta)} = \Delta_r^L(\beta)$, then $\Delta_r^L(\beta) > 0$ implies that $y_{\text{mv}}(\beta)$ in this case is larger than $y_L(\beta)$, so $y^* = y_{\text{mv}}(\beta)$.

Combining the three cases above, we conclude that $y^* = y_{\text{mv}}(\beta)$ if $\beta \in [b^L_{\text{mv}}, \beta^*)$, $y^* = y_{\text{non-mv}}(\beta)$ if $\beta \le [-\bar{b}, b^L_{\text{non-mv}}]$, and y^* is either $y_{\text{mv}}(\beta)$ or $y_{\text{non-mv}}(\beta)$ in $(b^L_{\text{non-mv}}, b^L_{\text{mv}})$, where

$$\kappa_r^L < b^L_{\text{non-mv}} \le b^L_{\text{mv}} < \kappa_l^L.$$

Overall, there are two cases.

(1) If $y^* = y_{\text{mv}}(\beta)$, then $q^*(y^*) = -\beta$. Using the definition of $MPV(y)$ in (19), we get that in this region, $MPV(y^*) = \frac{\partial \Pi(y)}{\partial(-q^*(y))} \frac{\partial(-q^*(y))}{\partial y} = 0$. Next, based on (17),

$$\begin{aligned} p^*(y^*) &= \gamma y_{\text{mv}}(\beta) + v(\mathbb{E}[b], -\beta) \\ &= v_0 + \left(\frac{\gamma}{2\gamma+\eta}\beta + \left(1 - \frac{\gamma}{2\gamma+\eta}\right) \mathbb{E}[b] + \mathbb{E}[\theta | q > -\beta] \right) H(-\beta) \\ &= v(b^*, -\beta) = v(b^*, q^*(y^*)), \end{aligned} \tag{61}$$

where b^* is given by (23). Together, this gives (21) and (24) if we set $\sigma(\beta) = 1$ in this region.

(2) If $y^* = y^L_{\text{non-mv}}(\beta)$, then $q^*(y^*) = -\beta_L(y^*) < -\beta$. Using the definition of $MPV(y)$ in (19), $MPV(y^*) = \frac{\partial \Pi(y)}{\partial(-q^*(y))} \frac{\partial(-q^*(y))}{\partial y} = MPV^L\left(y^L_{\text{non-mv}}(\beta), \beta\right)$. Next, based on (17),

$$p^*(y^*) = \gamma y^L_{\text{non-mv}}(\beta) + v\left(\mathbb{E}[b], -\beta_L\left(y^L_{\text{non-mv}}(\beta)\right)\right) = \frac{\gamma}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta_L\left(y^L_{\text{non-mv}}(\beta)\right))$$

$$+\frac{\gamma}{2\gamma+\eta} MPV^L\left(y^L_{\text{non-mv}}(\beta), \beta\right) + v\left(\mathbb{E}[b], -\beta_L\left(y^L_{\text{non-mv}}(\beta)\right)\right) \tag{62}$$

$$= v\left(b^*, -\beta_L\left(y^L_{\text{non-mv}}(\beta)\right)\right) + \frac{\gamma}{2\gamma+\eta} MPV^L\left(y^L_{\text{non-mv}}(\beta), \beta\right) = v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*).$$

This gives (21) and (24) if we set $\sigma(\beta) = 1$ in this region. This concludes the case $\beta < \beta^*$.

Second, suppose $\beta = \beta^*$. Then, for any $y > -\alpha$, we have $\beta_L(y) < \beta_L(-\alpha) = \beta = \beta_H(-\alpha) < \beta_H(y)$, and hence, by (39), $-q^*(y) = \beta$ for all y. Thus, the blockholder maximizes $\Pi_{\text{mv}}(y, \beta)$, so his optimal trade is $y^* = y_{\text{mv}}(\beta^*)$. Thus, as in region (1) of the first case, $p^*(y^*) = v(b^*, q^*(y^*))$ and $MPV(y^*) = 0$, which gives (21) and (24) if we set $\sigma(\beta) = 1$ in this region. Combined with the first case, we get $y^* = y_{\text{mv}}(\beta)$, $q^*(y^*) = -\beta$ for all $\beta \in [b^L_{\text{mv}}, \beta^*]$.

Third, suppose $\beta \in (\beta^*, \bar{b}]$. Since $\beta^* = \beta_H(-\alpha) = \beta_L(-\alpha) > \beta_L(y)$ for all y, we have $\beta > \beta_L(y)$ for all y. Based on (39),

$$-q^*(y) = \begin{cases} \beta & \text{if } \beta \le \beta_H(y) \\ \beta_H(y) & \text{if } \beta_H(y) < \beta, \end{cases}$$

and $\beta > \beta_H(y) \Leftrightarrow y < y_H$, where y_H is given by (48). We define it as a function of β: $y_H(\beta)$.

By Lemma 2 in the Online Appendix, $\beta'_H(y) > 0$, and hence $y'_H(\beta) > 0$. Note that

$$\Pi(y, \beta) = \begin{cases} \Pi^H_{\text{non-mv}}(y, \beta) & \text{if } y < y_H(\beta) \\ \Pi_{\text{mv}}(y, \beta) & \text{if } y \ge y_H(\beta), \end{cases} \tag{63}$$

where $\Pi_{\text{mv}}(y, \beta)$ is defined as before (by (52)), and

$$\Pi^H_{\text{non-mv}}(y, \beta) \equiv \alpha v(\beta, -\beta_H(y)) + y(\beta - \mathbb{E}[b])\, H(-\beta_H(y)) - (\gamma + 0.5\eta)y^2. \tag{64}$$

Both functions are continuous. Since $\Pi_{\text{mv}}(y_H(\beta), \beta) = \Pi^H_{\text{non-mv}}(y_H(\beta), \beta)$, $\Pi(y, \beta)$ is also continuous. The unique maximizer of $\Pi_{\text{mv}}(y, \beta)$ is the same as before ($y_{\text{mv}}(\beta)$, given by (55)), and the unique maximizer of $\Pi^H_{\text{non-mv}}(y, \beta)$ is $y^H_{\text{non-mv}}(\beta)$, which is the unique solution of

$$y = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta_H(y)) + \frac{1}{2\gamma+\eta} MPV^H(y, \beta), \tag{65}$$

where

$$MPV^H(y, \beta) \equiv \frac{\partial \beta_H(y)}{\partial y} f(-\beta_H(y))[\alpha(\beta - \beta_H(y)) + y(\beta - \mathbb{E}[b])]. \tag{66}$$

Thus, the maximizer of $\Pi(y, \beta)$ is in the set $\{y_{\text{mv}}(\beta), y^H_{\text{non-mv}}(\beta), y_H(\beta)\}$. Note that $y^H_{\text{non-mv}}(\beta)$ could be potentially larger than $y_H(\beta)$ (in which case it is not the maximizer), and similarly,

$y_{\text{mv}}(\beta)$ could be smaller than $y_H(\beta)$ (in which case it is not the maximizer).

Next, we calculate the right and left derivative of the blockholder's payoff function at $y_H(\beta)$:

$$\begin{aligned}\Delta_r^H(\beta) &\equiv \frac{\partial \Pi(y,\beta)}{\partial y}|_{y\searrow y_H(\beta)} = \frac{\partial \Pi_{\text{mv}}(y,\beta)}{\partial y}|_{y=y_H(\beta)} \quad (67)\\ &= (\beta - \mathbb{E}[b])\,H(-\beta) - (2\gamma+\eta)y_H(\beta);\\ \Delta_l^H(\beta) &\equiv \frac{\partial \Pi(y,\beta)}{\partial y}|_{y\nearrow y_H(\beta)} = \frac{\partial \Pi^H_{\text{non-mv}}(y,\beta)}{\partial y}|_{y=y_H(\beta)} \quad (68)\\ &= \Delta_r^H(\beta) + \frac{\partial \beta_H(y)}{\partial y}|_{y=y_H(\beta)} y_H(\beta)(\beta - \mathbb{E}[b])\,f(-\beta),\end{aligned}$$

In Section C.5.3 of the Online Appendix, we show that there exists a cutoff $\overline{\gamma}_5 > \bar{\gamma}_4$ such that if $\gamma > \overline{\gamma}_5$, then $\frac{\partial \Delta_r^H(\beta)}{\partial \beta} < 0$, $\frac{\partial \Delta_l^H(\beta)}{\partial \beta} < 0$, $\Delta_r^H(\beta^*) > 0$, $\Delta_l^H(\beta^*) > 0$, $\Delta_r^H(\bar{b}) < 0$, and $\Delta_l^H(\bar{b}) < 0$. Consider such γ, and define κ_r^H and κ_l^H as solutions to

$$\begin{aligned}\Delta_r^H(\kappa_r^H) &= 0,\\ \Delta_l^H(\kappa_l^H) &= 0.\end{aligned}$$

Since Δ_r^H and Δ_l^H are decreasing and take opposite signs at β^* and $\bar{b}$, these solutions exist, are unique, and lie in $(\beta^*, \bar{b})$. In Section C.5.4 of the Online Appendix, we show that for $\gamma > \bar{\gamma}_5$, we have

$$\beta^* < \kappa_r^H \leq \kappa_l^H < \bar{b},$$

that either $\mathbb{E}[b] \leq \beta_H(0) \leq \kappa_r^H \leq \kappa_l^H < \bar{b}$ or $\beta^* < \kappa_r^H \leq \kappa_l^H < \beta_H(0) < \mathbb{E}[b]$, and that $\kappa_r^H < \kappa_l^H$ if and only if $\beta_H(0) \neq \mathbb{E}[b]$. Consider three cases:

(1) If $\beta \in (\beta^*, \kappa_r^H)$, then $\Delta_r^H(\beta) > 0$, $\Delta_l^H(\beta) > 0$, so both the left and the right derivatives of $\Pi(y,\beta)$ are positive at $y_H(\beta)$, and since $\Pi(y,\beta)$ is continuous at $y_H(\beta)$, the optimal trade $y^* > y_H(\beta)$. In particular, $\Delta_r^H(\beta) > 0$ implies that $y_{\text{mv}}(\beta)$ in this case is larger than $y_H(\beta)$, so $y^* = y_{\text{mv}}(\beta)$.

(2) If $\beta \in [\kappa_r^H, \kappa_l^H]$, then $\Delta_r^H(\beta) \leq 0 \leq \Delta_l^H(\beta)$. Hence, $y^* = y_H(\beta) \in [y_{\text{mv}}(\beta), y^H_{\text{non-mv}}(\beta)]$. Note also that $y_H(\beta)$ is strictly inside $(y_{\text{mv}}(\beta), y^H_{\text{non-mv}}(\beta))$ if $\beta \in (\kappa_r^H, \kappa_l^H)$.

(3) If $\beta \in (\kappa_l^H, \bar{b}]$, then $\Delta_r^H(\beta) < 0$, $\Delta_l^H(\beta) < 0$, so both the left and the right derivatives of $\Pi(y,\beta)$ are negative at $y_H(\beta)$, and since $\Pi(y,\beta)$ is continuous at $y_H(\beta)$, the optimal trade $y^* < y_H(\beta)$. In particular, $\Delta_l^H(\beta) < 0$ implies that $y^H_{\text{non-mv}}(\beta)$ in this case is smaller than $y_H(\beta)$, so $y^* = y^H_{\text{non-mv}}(\beta)$.

Combining the three cases above, we conclude that $y^* = y_{\text{mv}}(\beta)$ if $\beta \in [\beta^*, \kappa_r^H]$; $y^* = y_H(\beta)$ if $\beta \in [\kappa_r^H, \kappa_l^H]$; and $y^* = y^H_{\text{non-mv}}(\beta)$ if $\beta \geq \kappa_l^H$ (with the interval $[\kappa_r^H, \kappa_l^H]$ collapsing into a single point if $\beta_H(0) = \mathbb{E}[b]$). Consider each of the regions separately.

(1) If $\beta \leq \kappa_r^H$, then $y^* = y_{\text{mv}}(\beta)$, $q^*(y^*) = -\beta$, and $MPV(y^*) = 0$. Next, based on (17) and repeating the arguments used to derive (61),

$$p^*(y^*) = \gamma y_{\text{mv}}(\beta) + v(\mathbb{E}[b], -\beta) = v(b^*, -\beta) = v(b^*, q^*(y^*)).$$

This gives (21) and (24) if we set $\sigma(\beta) = 1$ in this region.

(2) If $\beta \geq \kappa_l^H$, then $y^* = y^H_{\text{non-mv}}(\beta)$, $q^*(y^*) = -\beta_H(y^*) \geq -\beta$, and by the definition of $MPV(y)$ in (19), $MPV(y^*) = \frac{\partial \Pi(y)}{\partial(-q^*(y))}\frac{\partial(-q^*(y))}{\partial y} = MPV^H\left(y^H_{\text{non-mv}}(\beta), \beta\right)$. Next, based on (17) and repeating the arguments above,

$$\begin{aligned} p^*(y^*) &= v\left(b^*, -\beta_H\left(y^H_{\text{non-mv}}(\beta)\right)\right) + \frac{\gamma}{2\gamma+\eta} MPV^H\left(y^H_{\text{non-mv}}(\beta), \beta\right) \\ &= v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*). \end{aligned}$$

Together, this gives (21) and (24) if we set $\sigma(\beta) = 1$ in this region.

(3) Last, if $\beta \in (\kappa_r^H, \kappa_l^H)$, then $y^* = y_H(\beta) \in \left(y_{\text{mv}}(\beta), y^H_{\text{non-mv}}(\beta)\right)$ and $q^*(y^*) = -\beta$. Note that $MPV^H(y_H(\beta), \beta) = \lim_{y \nearrow y_H(\beta)} \frac{\partial \Pi(y)}{\partial(-q^*(y))}\frac{\partial(-q^*(y))}{\partial y}$ since $\Pi(y) = \Pi^H_{\text{non-mv}}(y, \beta)$ for $y < y_H(\beta)$. Hence, by the definition of $MPV(y)$ (see the footnote after (20)), $MPV(y_H(\beta)) = MPV^H(y_H(\beta), \beta)$. Using (66) at $y = y_H(\beta)$ and the fact that $\beta_H(y_H(\beta)) = \beta$, we have

$$MPV^H(y_H(\beta), \beta) = \frac{\partial \beta_H(y)}{\partial y}|_{y=y_H(\beta)} f(-\beta) y_H(\beta)(\beta - \mathbb{E}[b]).$$

Recall that if $\kappa_r^H < \beta < \kappa_l^H$, two cases are possible: 1) either $\mathbb{E}[b] < \beta_H(0) < \kappa_r^H < \beta < \kappa_l^H$ and $y_H(\beta) > 0$, or 2) $\kappa_r^H < \beta < \kappa_l^H < \beta_H(0) < \mathbb{E}[b]$ and $y_H(\beta) < 0$. In both cases, $y_H(\beta)(\beta - \mathbb{E}[b]) > 0$, and hence $MPV^H(y_H(\beta), \beta) > 0$. Define

$$\sigma(\beta) = \frac{(2\gamma+\eta) y_H(\beta) - (\beta - \mathbb{E}[b]) H(-\beta)}{MPV^H(y_H(\beta), \beta)}. \tag{69}$$

Then

$$y_H(\beta) = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b]) H(q^*(y_H(\beta))) + \frac{1}{2\gamma+\eta}\sigma(\beta) MPV^H(y_H(\beta), \beta),$$

so (21) holds. Note that $\sigma(\beta) \in (0, 1)$: as shown above, whenever $\beta \in (\kappa_r^H, \kappa_l^H)$, we have $\Delta_r^H(\beta) < 0 < \Delta_l^H(\beta)$, and the latter is equivalent to $0 < \sigma(\beta) < 1$. Next, based on (17),

$$p^*(y^*) = \gamma y_H(\beta) + v(\mathbb{E}[b], -\beta) = \tfrac{\gamma}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta) + \tfrac{\gamma}{2\gamma+\eta}\sigma(\beta)\, MPV^H(y_H(\beta), \beta)$$
$$+v(\mathbb{E}[b], -\beta) = v(b^*, -\beta) + \tfrac{\gamma}{2\gamma+\eta}\sigma(\beta)\, MPV(y_H(\beta)) = v(b^*, q^*(y^*)) + \tfrac{\gamma}{2\gamma+\eta}\sigma(\beta)\, MPV(y^*).$$

Together, this gives (21) and (24) with $\sigma(\beta)$ given by (69). This concludes the case $\beta > \beta^*$.

Last, we combine cases $\beta \le \beta^*$ and $\beta > \beta^*$. Recall that for any $\gamma > \bar{\gamma}_5$, we have $-\bar{b} < \kappa_r^L < b_{\text{non-mv}}^L \le b_{\text{mv}}^L < \kappa_l^L < \beta^* < \kappa_r^H \le \kappa_l^H < \bar{b}$. Define

$$b_{\text{mv}}^H \equiv \kappa_r^H \text{ and } b_{\text{non-mv}}^H \equiv \kappa_l^H.$$

Then $b_{\text{non-mv}}^L \le b_{\text{mv}}^L \le b_{\text{mv}}^H < b_{\text{non-mv}}^H$. Moreover, $y^* = y_{\text{non-mv}}^L(\beta)$ for $\beta < b_{\text{non-mv}}^L$; $y^* \in \left\{y_{\text{non-mv}}^L(\beta), y_{\text{mv}}(\beta)\right\}$ for $\beta \in [b_{\text{non-mv}}^L, b_{\text{mv}}^L]$; $y^* = y_{\text{mv}}(\beta)$ for $\beta \in \left[b_{\text{mv}}^L, b_{\text{mv}}^H\right]$; $y^* = y_H(\beta)$ for $\beta \in [b_{\text{mv}}^H, b_{\text{non-mv}}^H]$; and $y^* = y_{\text{non-mv}}^H(\beta)$ for $\beta > b_{\text{non-mv}}^H$. Hence, (21), (23), and (24) hold for

$$\sigma(\beta) = \begin{cases} 1 & \text{if } \beta \notin (b_{\text{mv}}^H, b_{\text{non-mv}}^H), \\ \text{in } (0,1) \text{ and given by (69)} & \text{if } \beta \in (b_{\text{mv}}^H, b_{\text{non-mv}}^H). \end{cases} \tag{70}$$

In Lemma 5 in the Online Appendix we show that as $\gamma \to \infty$, the region $(b_{\text{mv}}^H, b_{\text{non-mv}}^H)$ becomes infinitesimally small, and hence $\sigma(\beta) = 1$ almost everywhere. Finally, expression (22) directly follows from (11). Note that the equilibrium of the game is unique, except for a knife-edge case in the region $\beta \in [\kappa_r^L, \kappa_l^L)$, where the trades $y_{\text{non-mv}}^L(\beta)$ and $y_{\text{mv}}(\beta)$ result in the same payoff for the blockholder. This completes the proof of the proposition for any $\gamma > \bar{\gamma}_5$. In subsequent proofs, we assume that $\gamma > \bar{\gamma}_5$, so that the proposition and its arguments apply. ∎

Proof of Corollary 1. Suppose $q^*(y^*)$ is fixed (denote it by q^* for simplicity). Denote by y_{CF}^* the optimal trade driven by the cash flow motive alone. Given $\alpha > 0$, the short-selling constraint does not bind for large enough γ, so (51) implies that $y_{CF}^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(q^*)$. Using (10), $p_{CF}(q^*(y^*)) = \gamma y_{CF}^* + v(\mathbb{E}[b], q^*) = v(b^*, q^*(y^*))$, as required. ∎

Proof of Proposition 3. To prove this result, we rely on the proof of Proposition 2, and in particular, refer to the cutoffs $b_{\text{non-mv}}^L \le b_{\text{mv}}^L < b_{\text{mv}}^H < b_{\text{non-mv}}^H$ defined in that proof.

Consider part (i). According to the proof of Proposition 2, if $\beta \in [b_{\text{mv}}^L, b_{\text{mv}}^H]$, then

$$y^* = y_{\text{mv}}(\beta) = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta)$$

and $-q^*(y^*) = \beta$. Since $MPV(y^*, \beta) = 0$, the voting premium is zero. Letting

$$\beta_{\text{mv}}^L \equiv b_{\text{mv}}^L \text{ and } \beta_{\text{mv}}^H \equiv b_{\text{mv}}^H \tag{71}$$

completes part (i). We prove part (ii) in several steps.

Step 1. First, define

$$\beta^{H}_{\text{non-mv}} \equiv b^{H}_{\text{non-mv}}. \tag{72}$$

We will show that for large enough γ, if $\beta > \beta^{H}_{\text{non-mv}}$, then (1) the median voter is a dispersed shareholder with a smaller bias than the blockholder; (2) the voting premium is strictly positive; and (3) the voting premium, the blockholder's trade, and the median voter all strictly increase in β. The first statement directly follows from the proof of Proposition 2, which shows that if $\beta > b^{H}_{\text{non-mv}}$, then $y^{*} = y^{H}_{\text{non-mv}}(\beta)$ and $-q^{*}(y^{*}) = \beta_{H}(y^{*}) < \beta$, i.e., the median voter is a dispersed shareholder with bias $\beta_{H}(y^{*}) < \beta$.

To prove the two other statements, recall that $y^{H}_{\text{non-mv}}(\beta)$ is the unique solution of (65)-(66). Recall also that $b^{H}_{\text{non-mv}} = \kappa^{H}_{l}$, which by definition is the solution to $\Delta^{H}_{l}\left(\kappa^{H}_{l}\right) = 0$. In addition, the proof of Proposition 2 shows that $\Delta^{H}_{l}\left(\kappa^{H}_{r}\right) \geq 0$, with a strict inequality whenever $\beta_{H}(0) \neq \mathbb{E}[b]$. Suppose that $\beta = \kappa^{H}_{l}$ $(= b^{H}_{\text{non-mv}})$. Since $\Delta^{H}_{l}\left(\kappa^{H}_{l}\right) = 0$, then $\frac{\partial \Pi^{H}_{\text{non-mv}}(y,\beta)}{\partial y}|_{y=y_{H}(\beta)} = 0$, and hence $y^{H}_{\text{non-mv}}(\beta) = y_{H}(\beta)$. Recall that in this case $y^{*} = y_{H}(\beta)$, and hence for this β, the equilibrium MPV satisfies

$$MPV^{H}\left(y_{H}\left(\kappa^{H}_{l}\right),\kappa^{H}_{l}\right) = \frac{\partial \beta_{H}(y)}{\partial y}|_{y=y_{H}\left(\kappa^{H}_{l}\right)} f(-\kappa^{H}_{l}) y_{H}\left(\kappa^{H}_{l}\right)\left(\kappa^{H}_{l} - \mathbb{E}[b]\right),$$

where we used (66) and the fact that $\beta - \beta_{H}(y_{H}(\beta)) = 0$. Next, (67)-(68) imply that

$$\Delta^{H}_{l}\left(\kappa^{H}_{l}\right) = \Delta^{H}_{r}\left(\kappa^{H}_{l}\right) + MPV^{H}\left(y_{H}\left(\kappa^{H}_{l}\right),\kappa^{H}_{l}\right). \tag{73}$$

Since $\Delta^{H}_{l}\left(\kappa^{H}_{r}\right) \geq 0 = \Delta^{H}_{l}\left(\kappa^{H}_{l}\right)$, then (73) implies $MPV^{H}\left(y_{H}\left(\kappa^{H}_{l}\right),\kappa^{H}_{l}\right) \geq 0$ (with a strict inequality whenever $\beta_{H}(0) \neq \mathbb{E}[b]$), i.e., the MPV is non-negative for $\beta = \kappa^{H}_{l} = b^{H}_{\text{non-mv}}$.

Thus, to prove that the voting premium is strictly positive and increases in β for $\beta > \beta^{H}_{\text{non-mv}}$, it is sufficient to prove that $MPV^{H}(y^{H}_{\text{non-mv}}(\beta),\beta)$ strictly increases in β for $\beta > b^{H}_{\text{non-mv}}$. We prove this next. Using (66) and noting that $\beta_{H}(y)$ does not depend on β, we have

$$\frac{\partial MPV^{H}(y,\beta)}{\partial \beta} = \frac{\partial \beta_{H}(y)}{\partial y} \times (\alpha + y) \times f(-\beta_{H}(y)) > 0, \tag{74}$$

$$\begin{aligned} \frac{\partial MPV^{H}(y,\beta)}{\partial y} &= \frac{\partial^{2} \beta_{H}(y)}{\partial y^{2}} \times \left[\alpha(\beta - \beta_{H}(y)) + y(\beta - \mathbb{E}[b])\right] \times f(-\beta_{H}(y)) \\ &+ \frac{\partial \beta_{H}(y)}{\partial y} \times \left(\begin{array}{c} \left[-\alpha \frac{\partial \beta_{H}(y)}{\partial y} + (\beta - \mathbb{E}[b])\right] \times f(-\beta_{H}(y)) \\ -\frac{\partial \beta_{H}(y)}{\partial y}\left[\alpha(\beta - \beta_{H}(y)) + y(\beta - \mathbb{E}[b])\right] \times f'(-\beta_{H}(y)) \end{array} \right). \end{aligned}$$

Thus, $\frac{\partial MPV^{H}(y,\beta)}{\partial \beta}$ does not depend on β, and $\frac{\partial MPV^{H}(y,\beta)}{\partial y}$ can be written as $\beta M(y) + S(y)$, where

$M(y)$ and $S(y)$ do not depend on β. Recall from the proof of Proposition 2 that $y^* \in [-\varepsilon, \varepsilon]$ for some $\varepsilon > 0$. The proofs of Lemma 2 and Lemma 4 in the Online Appendix imply that for a large enough γ, there exists $C > 0$ such that $\left|\frac{\partial \beta_H(y)}{\partial y}\right| < C$ and $\left|\frac{\partial^2 \beta_H(y)}{\partial y^2}\right| < C$ for all $y \in [-\varepsilon, \varepsilon]$. Since $f'(\cdot)$ is, by assumption, continuous and $|\beta_H(y)| \leq \bar{b}$, there exist $M > 0$ and $S > 0$ such that $|M(y)| < M$ and $|S(y)| < S$ for all $y \in [-\varepsilon, \varepsilon]$ and γ above some cutoff.

Since for $\beta > b^H_{\text{non-mv}}$ we have $y^* = y^H_{\text{non-mv}}(\beta)$, which satisfies (65), we differentiate both sides of (65) with respect to β and get

$$\frac{\partial y^*}{\partial \beta} = \frac{H(-\beta_H(y^*)) + \frac{\partial MPV^H(y^*,\beta)}{\partial \beta}}{2\gamma + \eta - (\beta - \mathbb{E}[b]) f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y} - \frac{\partial MPV^H(y^*,\beta)}{\partial y}} =$$
$$\frac{H(-\beta_H(y^*)) + \frac{\partial MPV^H(y^*,\beta)}{\partial \beta}}{2\gamma + \eta - \beta\left[M(y^*) + f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y}\right] + \mathbb{E}[b] f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y} - S(y^*)}, \tag{75}$$

where y^* stands for $y^*(\beta)$, the optimal trade given bias β. For all $y \in [-\varepsilon, \varepsilon]$, we have $H(-\beta_H(y)) > 0$ and $\frac{\partial MPV^H(y,\beta)}{\partial \beta} > 0$ (from (74)), and hence the numerator of (75) is positive for any β. In addition, as the arguments above imply, there exists $C > 0$ such that for all $y^* \in [-\varepsilon, \varepsilon]$, all the terms $|M(y^*)|$, $|S(y^*)|$, $\left|\frac{\partial \beta_H(y^*)}{\partial y}\right|$, and $f(-\beta_H(y^*)$ are bounded by C if γ is large enough. Since $\beta \in \left[-\bar{b}, \bar{b}\right]$, then together, these arguments imply that there exists $\bar{\gamma}_1$ such that for any $\gamma > \bar{\gamma}_1$, the denominator of (75) is positive as well, and hence $\frac{\partial y^*}{\partial \beta} > 0$. This implies that the blockholder's trade strictly increases in β.

Next, consider (65) evaluated at $y^*(\beta)$, and differentiate it w.r.to β, which gives:

$$\frac{dMPV^H(y^*(\beta), \beta)}{d\beta} = -H(-\beta_H(y^*(\beta)) + \frac{\partial y^*}{\partial \beta}\left[(2\gamma + \eta) - (\beta - \mathbb{E}[b]) f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y}\right]$$

Using (75) and simplifying, $\frac{dMPV^H(y^*(\beta),\beta)}{d\beta}$ equals

$$\frac{(\beta M(y^*) + S(y^*)) H(-\beta_H(y^*)) + \left[(2\gamma + \eta) - (\beta - \mathbb{E}[b]) f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y}\right] \frac{\partial MPV^H(y^*,\beta)}{\partial \beta}}{2\gamma + \eta - \beta(M(y^*) + f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y}) + \mathbb{E}[b] f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y} - S(y^*)}$$

or equivalently, $\frac{dMPV^H(y^*(\beta),\beta)}{d\beta} = \frac{N(y^*,\beta)}{D(y^*,\beta)}$, where

$$N(y^*, \beta) = \frac{\partial MPV^H(y^*,\beta)}{\partial \beta} + \frac{(\beta M(y^*) + S(y^*)) H(-\beta_H(y^*)) - (\beta - \mathbb{E}[b]) f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y} \frac{\partial MPV^H(y^*,\beta)}{\partial \beta}}{2\gamma + \eta},$$
$$D(y^*, \beta) = 1 - \frac{\beta(M(y^*) + f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y}) + \mathbb{E}[b] f(-\beta_H(y^*)) \frac{\partial \beta_H(y^*)}{\partial y} - S(y^*)}{2\gamma + \eta}.$$

Consider $D(y^*, \beta)$. Recall that $y^* \in [-\varepsilon, \varepsilon]$, $\beta \in \left[-\bar{b}, \bar{b}\right]$ and, as discussed above, $|M(y^*)|$, $|S(y^*)|$, $\left|\frac{\partial \beta_H(y^*)}{\partial y}\right|$, and $f(-\beta_H(y^*)$ are all bounded by some constant for all $y^* \in [-\varepsilon, \varepsilon]$ and large enough γ. Hence, for large enough γ, $D(y^*, \beta) > 0$ for all $y^* \in [-\varepsilon, \varepsilon]$ and $\beta \in \left[-\bar{b}, \bar{b}\right]$. Next, consider $N(y^*, \beta)$. From (74), we have $\frac{\partial MPV^H(y^*,\beta)}{\partial \beta} > 0$. In addition, since $\left|\frac{\partial \beta_H(y^*)}{\partial y}\right|$ is bounded by some constant for large enough γ, (74) implies that $\left|\frac{\partial MPV^H(y^*,\beta)}{\partial \beta}\right|$ is also bounded by some constant for large enough γ. The other functions in the numerator of the second term in $N(y^*, \beta)$ are bounded as well. Together, this implies that there is a cutoff on γ such that $N(y^*, \beta) > 0$ for all $y^* \in [-\varepsilon, \varepsilon]$ and $\beta \in \left[-\bar{b}, \bar{b}\right]$ for γ above this cutoff.

Combined, we have shown that there exists $\bar{\gamma}_2 > \bar{\gamma}_1$ such that $\frac{dMPV^H(y^*(\beta),\beta)}{d\beta} > 0$ for all $\beta > b^H_{\text{non-mv}}$ and all $\gamma > \bar{\gamma}_2$. Together with $MPV^H(y^H_{\text{non-mv}}(\beta), \beta) \geq 0$ for $\beta = b^H_{\text{non-mv}}$, this implies that $MPV^H(y^H_{\text{non-mv}}(\beta), \beta)$ is strictly positive and strictly increasing in β for $\beta > b^H_{\text{non-mv}}$ and $\gamma > \bar{\gamma}_2$. Since the voting premium in this region equals $\frac{\gamma}{2\gamma+\eta} MPV^H(y^H_{\text{non-mv}}(\beta), \beta)$, the voting premium is also strictly positive and strictly increasing in β.

Finally, note that the median voter in this range is $\beta_H\left(y^H_{\text{non-mv}}(\beta)\right)$. Since $y^H_{\text{non-mv}}(\beta)$ increases in β for $\gamma > \bar{\gamma}_2$ and $\beta_H(\cdot)$ is an increasing function, the median voter increases in β in this range as well. This completes the proof for the region $\beta > b^H_{\text{non-mv}}$.

The two other steps for part (ii) involve proving the following statements:

Step 2: If $\beta < b^L_{\text{non-mv}}$, then there exists $\bar{\gamma}_3 > \bar{\gamma}_2$ such that for any $\gamma > \bar{\gamma}_3$, the median voter is a dispersed shareholder with a larger bias toward the proposal than the blockholder and the voting premium strictly decreases in β.

Step 3: There exists $\bar{\gamma}_4 > \bar{\gamma}_3$ such that for any $\gamma > \bar{\gamma}_4$, there exists $\beta^L_{\text{non-mv}} \in (-\bar{b}, b^L_{\text{non-mv}})$ such that if $\beta < \beta^L_{\text{non-mv}}$, then $MPV^L(y^*(\beta), \beta) > 0$.

For brevity, the proof of these two steps is relegated to Section C.6 of the Online Appendix. This completes the proof of the proposition for any $\gamma > \bar{\gamma}_4$. In subsequent proofs, we assume that $\gamma > \bar{\gamma}_4$, so that the proposition and its arguments apply. ■

We relegate the proofs of Proposition 4, Corollary 2, and Proposition 5 to Sections C.7, C.8, and C.9 of the Online Appendix, respectively. The Online Appendix is available here.

Online Appendix for "The Voting Premium"

by Doron Levit[38], Nadya Malenko[39], and Ernst Maug[40]

The Online Appendix is organized as follows. Section A of the Online Appendix reviews 40 empirical studies that estimate the voting premium. Section B provides several microfoundations of the model. Section C contains supplemental results and proofs for the baseline model. Section D presents the analysis of extensions.

A Empirical studies

This section summarizes the findings of 40 studies that measure the voting premium using six different methodologies.[41] Five methodologies are surveyed in Table A1. The methodology based on dual-class tender offers is summarized in Table A2.

Table A1

Table A1 lists 40 studies that measure the voting premium.[42] Methods using dual-class share prices are tabulated in Panel A; studies on block trades, option replication, equity lending, and record-day price effects are tabulated in Panels B to E. The following descriptions provide short summaries of each of the five methodologies in Panels A to E. However, details of methodology and measurement within a given methodology sometimes differ across papers.

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[41] The studies are: Aggarwal, Saffi, and Sturgess (2015); Albuquerque and Schroth (2010); Albuquerque and Schroth (2015); Barak and Lauterbach (2011); Barclay and Holderness (1989); Bergstrom and Rydqvist (1992); Bigelli and Croci (2013); Broussard and Vaihekoski (2022); Caprio and Croci (2008); Christoffersen et al. (2007); Chung and Kim (1999); Cox and Roden (2002); DeAngelo and DeAngelo (1985); Dittmann (2003); Dodd and Warner (1983) Dyck and Zingales (2004); Fos and Holderness (2022); Franks and Mayer (2001); Gurun and Karakas (2022); Hauser and Lauterbach (2004); Horner (1988); Jang, Kim, and Mohseni (2023); Kalay, Karakas, and Pant (2014); Kind and Poltera (2013); Kind and Poltera (2017); Kunz and Angel (1996); Lease, McConnell, and Mikkelson (1983); Levy (1983); Maynes (1996); Megginson (1990); Moser, Ness, and Ness (2013); Muravyev (2004); Muus (1998); Nenova (2003); Neumann (2003); Odegaard (2007); Rydqvist (1996); Smith and Amoako-Adu (1995); Zingales (1994); Zingales (1995).

[42] Dittmann (2003) reports separate estimates for German and US firms, so we present the estimates from his paper in two separate rows.

(A) *Dual-class premium*: Difference in percent between the price of a stock with superior voting rights and the price of a stock with inferior (or no) voting rights.

(B) *Block trade premium*: Difference in percent between the price of a voting share paid in a block trade and the price of a voting share prevailing in secondary trading on an exchange.

(C) *Option replication*: Difference in percent between the price of a voting share and a position in derivatives that replicates a non-voting share.

(D) *Equity lending*: Fee in percent paid to borrow shares in the equity-lending market.

(E) *Record-day price effects*: Difference in percent between the prices of the same share before and after the record date.

The column "Average voting premium" measures the average voting premium reported in the corresponding study. The numbers in Table 1 (Avg., Median, Min, and Max) were calculated using this number. The table records "yes" in the column "negative voting premium" if the study provides evidence that the voting premium is negative for at least some firms in its sample (e.g., by showing that the minimum voting premium is negative), and it records a dash if there is not enough information about the sign of the smallest voting premium in the sample. The table records "yes" in the column "relevance of ownership structure" if the study discusses how ownership (e.g., block ownership, insider ownership, family ownership) affects the voting premium; otherwise, it records a dash. The table records "yes" in the column "liquidity effect" if the study reports evidence on how liquidity (e.g., bid-ask spread, turnover) affects the voting premium. Dashes in this column imply either that the study did not investigate liquidity, or that it did and the relevant coefficient is statistically insignificant.

Table A1, Panel A

Study		Sample		Average voting premium	Evidence provided for:		
Authors	Publication year	Country	Period		negative voting premium	relevance of ownership structure	liquidity effect
Panel A: Dual-class shares							
Lease et al.	1983	USA	1940-1978	4.07%	yes	-	-
Levy	1983	Israel	1974-1980	45.50%	yes	-	-
Horner	1988	Switzerland	1973-1983	20.79%	yes	yes	-
Megginson	1990	UK	1955-1982	13.30%	yes	yes	-
Bergström and Rydqvist	1992	Sweden	1980-1990	15.20%	yes	-	-
Zingales	1994	Italy	1987-1990	81.50%	yes	yes	-
Smith and Amoako-Adu	1995	Canada	1981-1992	10.40%	yes	yes	yes
Zingales	1995	USA	1984-1990	10.47%	yes	yes	yes
Rydqvist	1996	Sweden	1983-1990	12.00%	yes	yes	-
Kunz and Angel	1996	Switzerland	1990-1991	18.16%	yes	-	yes
Maynes	1996	Canada	1984	6.66%	-	-	-
Muus	1998	France	1986-1996	51.35%	yes	-	-
Chung and Kim	1999	Korea	1992-1993	9.60%	-	yes	-
Cox and Roden	2002	USA	1984-1999	7.70%	-	yes	-
Dittmann	2003	Germany	1960-2001	12.62%	-	yes	yes
Dittmann	2003	USA	2017	4.38%	-	yes	yes
Neumann	2003	Denmark	1992-1999	12.31%	yes	yes	yes
Nenova	2003	Cross-country	1997	13.85%	yes	yes	yes
Muravyev	2004	Russia	1997-2003	64.72%	yes	yes	yes
Ødegaard	2007	Norway	1988-2005	5.60%	yes	yes	yes
Caprio and Croci	2008	Italy	1977-2003	56.51%	yes	yes	-
Bigelli and Croci	2013	Italy	1999-2008	20.35%	yes	yes	-
Hauser and Lauterbach	2004	Israel	1990-2000	20.00%	-	yes	-
Broussard and Vaihekoski	2019	Finland	1982-2018	27.20%	yes	-	-
Total / average panel A				22.68%	18	17	9

Table A1, Panels B - E

Study		Sample		Average voting premium	Evidence provided for:		
Authors	Publication year	Country	Period		negative voting premium	relevance of ownership structure	liquidity effect
Panel B: Block trades							
Barclay and Holderness	1989	USA	1978-1984	20.40%	yes	-	-
Franks and Mayer	2001	Germany	1988-1997	13.85%	-	-	-
Dyck and Zingales	2004	Cross-country	1990-2000	14.00%	yes	-	-
Albuquerque and Schroth	2010	USA	1990-2006	19.62%	yes	-	-
Barak and Lauterbach	2011	Israel	1993-2005	46.96%	-	-	-
Albuquerque and Schroth	2015	USA	1990-2006	6.79%	yes	-	-
Total / average panel B				20.27%	4	0	0
Panel C: Option replication							
Kind and Poltera	2013	Cross-country	2003-2010	0.37%	yes	yes	-
Kalay et al.	2014	USA	1996-2007	0.16%	yes	-	-
Kind and Poltera	2017	USA	2002-2013	0.28%	yes	-	-
Jang et al.	2023	USA	1996-2015	0.10%	yes	-	-
Gurun and Karakas	2022	USA	1996-2015	0.09%	yes	-	-
Total / average panel C				0.20%	5	1	0
Panel D: Equity lending							
Christoffersen et al.	2007	USA	1998-1999	0.01%	-	-	-
Aggarwal et al.	2015	USA	2007-2009	0.02%	-	-	-
Moser, van Ness, van Ness	2013	USA	2005-2008	0.02%	-	-	-
Total / average panel D				0.02%	0	0	0
Panel E: Record-day price effects							
Dodd and Warner	1983	USA	1962-1978	1.40%	-	-	-
Fos and Holderness	2022	USA	1996-2018	0.09%	-	-	-
Total / average panel E				0.75%	0	0	0
Total panels A - E					27	18	9

Table A2

Table A2 tabulates seven studies that provide evidence on dual-class tender offers.[43] A tender offer is classified as "equal" if the prices offered to shares with superior voting rights and to those with inferior voting rights are identical. Otherwise the offer is classified as "differential."

The *dual-class tender-offer* premium is the difference in percent between the price of a stock with superior voting rights and the price of a stock with inferior (or no) voting rights in a tender offer for a dual-class firm. Column "Dual-class tender offer" reports the average dual-class tender-offer premium in completed transactions in each of the studies, where for equal tender offers, this difference is taken to be zero. The estimates in Table 1 (Avg., Median, Min, and Max) were calculated using this column across the five studies that report at least one tender offer in their sample.

We calculate the *ex ante* voting premium (column "Ex ante") as the product of the dual-class tender-offer premium and the frequency with which dual-class firms in the sample receive tender offers, calculated as the ratio of tender offers in a given sample to the total number of dual-class firms in that sample (the total number of dual-class firms in the sample is reported in column "# firms"). The *dual-class* voting premium (column "Dual-class") is the difference in percent between the *market* price of a stock with superior voting rights and the market price of a stock with inferior (or no) voting rights; it is the same as the estimate reported in column "Average voting premium" in Table A1, panel A for each of the six studies that estimate this number.

Study		Sample			# tender offers		Voting premium		
Authors	Publication year	Country	Period	# firms	equal	differential	Dual-class tender offer	Ex ante	Dual-class
Lease et al.	1983	USA	1940-1978	30	0	0	NA	0.00%	4.07%
DeAngelo and DeAngelo	1985	USA	1960-1980	45	0	4	130.75%	11.62%	NA
Megginson	1990	UK	1955-1982	152	6	37	27.60%	7.81%	13.30%
Bergström and Rydqvist	1992	Sweden	1980-1990	111	48	40	12.27%	9.73%	15.20%
Zingales	1994	Italy	1987-1990	96	0	0	NA	0.00%	81.50%
Smith and Amoako-Adu	1995	Canada	1981-1992	98	23	20	26.59%	11.67%	10.40%
Zingales	1995	USA	1984-1990	94	10	2	13.58%	1.73%	10.47%
Total / average				626	87	103	42.16%	6.08%	22.49%

The references for the 40 empirical studies, as well as Sections B, C, and D of the Online Appendix, are available here.

[43] These studies are: Bergstrom and Rydqvist (1992); DeAngelo and DeAngelo (1985); Lease, McConnell, and Mikkelson (1983); Megginson (1990); Smith and Amoako-Adu (1995); Zingales (1994); Zingales (1995). Two of these studies, Lease, McConnell, and Mikkelson (1983) and Zingales (1994), report zero tender offers for the firms in their sample.