NBER WORKING PAPER SERIES

A FRAMEWORK FOR GEOECONOMICS

Christopher Clayton
Matteo Maggiori
Jesse Schreger

Working Paper 31852
http://www.nber.org/papers/w31852

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2023, Revised 2024

We thank David Baqaee, Luigi Bocola, Saki Bigio, Fernando Broner (discussant), Laura Doval, Ana Fostel (discussant), Jeff Frieden (discussant), Xavier Gabaix, Ben Golub, Bengt Holmstrom, Alberto Martin, Zhengyang Jiang, Reka Juhasz, Ernest Liu, Pablo Ottonello, Diego Perez, Mathias Thoenig, Stephen Redding, Joe Stiglitz, Jean Tirole, and Christoph Trebesch, for helpful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Framework for Geoeconomics
Christopher Clayton, Matteo Maggiori, and Jesse Schreger
NBER Working Paper No. 31852
November 2023, Revised 2024
JEL No. F02,F10,F5

ABSTRACT

Governments use their countries' economic strength from financial and trade relationships to achieve geopolitical and economic goals. We provide a model of the sources of geoeconomic power and how it is wielded. The source of this power is the ability of a hegemonic country to coordinate threats across disparate economic relationships as a mean of enforcement on foreign entities. The hegemon wields this power to demand costly actions out of the targeted entities, including mark-ups, import restrictions, tariffs, and political concessions. The hegemon uses its power to change targeted entities' activities to manipulate the global equilibrium in its favor and increase its power. A sector is strategic either in helping the hegemon form threats or in manipulating the world equilibrium via input-output amplification. The hegemon acts a global enforcer, thus adding value to the world economy, but destroys value by distorting the equilibrium in its favor.

Christopher Clayton
Yale School of Management
165 Whitney Avenue
P.O. Box 208200
New Haven, CT 06520-8200
and NBER
christopherdclayton@gmail.com

Jesse Schreger
Columbia Business School
665 W 130th St
Kravis Hall, 551
New York, NY 10027
and NBER
jesse.schreger@columbia.edu

Matteo Maggiori
Stanford University
Graduate School of Business
655 Knight Way
Stanford, CA 94305
and NBER
maggiori@stanford.edu

1. INTRODUCTION

Hegemonic countries use their financial and economic strength to extract economic and political surplus from other countries around the world. This practice, referred to as geoeconomics, is not as blunt as the direct threat to go to war, as it operates through commercial channels like the threat to interrupt the supply or purchase of goods, the sharing of technology, or financial relationships and services. Despite its importance and practical relevance, the deeper foundations of geoeconomic power have remained elusive.

We provide a formal model of the sources of geoeconomic power and how it is wielded. We identify the source of the power to be the ability of countries like the US (or China), which we refer to as hegemons, to coordinate threats across disparate economic relationships as a mean of enforcement for their demands on foreign entities over which they have no direct legal control. Such coordinated "joint threats" – for example, suspending access to the dollar-based financial system and blocking technological inputs such as semiconductors – are particularly effective because they threaten punishment across many relationships for deviations on any one of them. Indeed, geoeconomic power operates in areas in which complete contracts are not feasible either because of limited enforceability or for political and legal reasons formal contracts are unpalatable (e.g. government to government relationships). The hegemon's ability to act as a global enforcer using joint threats can add value by reducing commitment issues and expanding the set of feasible economic activity.

The hegemon wields its power to demand costly actions from the targeted entities. This notion of power is broader than market power and also includes the ability to demand changes in economic activities and political concessions. We show how the hegemon uses its demands not only to extract direct monetary benefits but also to shape the global equilibrium in its favor by asking targeted entities to alter their activities vis à vis other entities. For example, the hegemon may demand that foreign banks stop lending to a geopolitical rival, such as when the US demanded that European commercial banks stop financing trade between Iran and third-party countries. A hegemon may also ask a foreign firm to stop using sensitive technology sourced from a rival, such as when the US pressured European firms to stop purchasing telecommunication technology and infrastructure supplied by Huawei.

Formally, we model a collection of countries and productive sectors with an input-output network structure. Sectors are collections of firms operating in a specific country and industry (e.g. Russian oil extraction and American oil extraction are two distinct sectors). The model features limited enforceability of contracts, as well as externalities both in production functions and in the objective functions of country-level representative consumers. Production externalities, whereby an individual sector's productivity can depend of what other sectors are producing both within and across countries, can capture external economies of scale and strategic complementarities. The externalities entering directly in the representative consumer's objective help us capture political affinity between countries' governments as well as externalities that are traditionally outside of the domain of economics, such as national security. We model threats as trigger strategies that firms and governments employ to punish other entities for deviating from contracts through exclusion from an economic relationship in the future. Joint threats are trigger strategies in which the punishment of exclusion from multiple economic relationships is triggered by an entity's deviation on any one of them. In our model, a hegemon is a country that is able to coordinate many such threats both via its national entities and via their economic network abroad.

We allow targeted entities to be either firms or governments. In practice both are relevant: hegemons pressure foreign governments to obtain political concessions or pressure foreign firms for specific actions often against the wishes of those firms' governments. A key feature of our model is that the targeted foreign entities voluntarily comply with the hegemon's demands. They do so if the value of commitment derived from the hegemon's joint threats outweighs the costs of acceding to the hegemon's demands. In practice, these threats are crucial in the conduct of secondary sanctions to induce foreign entities to stop activities that are legal in their own jurisdictions. For example, foreign banks comply with US secondary sanctions given the value generated by their business with US based institutions. Formally, voluntary compliance is described by the participation constraint of the targeted entity that tracks the limits to the hegemons' power: i.e. the maximal private cost to the entity of the actions the hegemon can demand. We refer to this as the hegemon's Micro-Power.

We show that the hegemon always maximizes global enforcement by coordinating punishment along as many relationships as feasible. In doing so, the hegemon maximizes its Micro-Power. From a micro perspective, a sector is strategic to the extent that the hegemon can use it to build its Micro-Power by forming threats on other entities. In this sense, strategic sectors are those that supply inputs that are widely used, with high value added for targets, and with poor available substitutes. Some goods may have these properties due to physical constraints: rare earths, oil and gas. Others have them in equilibrium due to increasing returns to scale and natural monopolies. For example, the dollar-based financial infrastructure of payment and clearing systems (like SWIFT) is a strategic asset that the US often uses in geoeconomic threats.

We allow for a rich set of costly actions that the hegemon can demand. Formally, they include both monetary transfers and a complete set of revenue neutral taxes (wedges) on targeted entities' input purchases. These instruments can be specialized to take the form of mark-ups, bilateral import-export quantity restrictions, tariffs, and political concessions. Many of these instruments are used in practice in economic coercion and sanctions policy. Given its limited power, the hegemon optimally trades-off the use of each of the instruments to maximize its country's welfare. All else equal, it favors monetary extraction from sectors that have little influence on the global equilibrium. It favors wedges to alter a target's economic activities whenever those activities impact other sectors that the hegemon cares about. We show that this input-output propagation of the production externalities is summarized by a generalized Leontief inverse matrix and that the hegemon manipulates the transmission in its favor. We define Macro-Power to be the social value to the hegemon of the costly actions it demands of the targeted entities. From a macro perspective, a sector is strategic if demanding costly actions from it is particularly effective at shaping the world equilibrium in the hegemon's favor. In this sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification (in the generalized Leontief-inverse). Sectors like finance, research and development, and information technology are good candidates for being strategic in this sense.

Crucially, Micro- and Macro-Power interact since the hegemon can use demands on one part of the network to shape the equilibrium in ways that increase its power over other parts. The hegemon values having Micro-Power over sectors that generate its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon's demands, the targeted entities consider only their private costs, but the hegemon enjoys the social benefits of the outcomes of these actions. As a result, we show that allocations with a hegemon are constrained inefficient from a global perspective. The hegemon acts as a global enforcer, echoing the public good provision highlighted in "hegemonic stability theory" in political science, and some of its policies correct negative externalities. The global planner also provides the same enforcement (maximal joint threats) and, in some dimension, corrects externalities similarly to the hegemon. However, the hegemon destroys value at the global level compared to the global planner by demanding transfers and manipulating the equilibrium in its favor. The equilibrium with the hegemon can even be worse for some targeted entities than the equilibrium without the hegemon depending on whether the enforcement and positive correction of externalities are more then offset by the externality and price manipulation.

Finally, we specialize the model to two simple applications that illustrate recent examples of geoeconomics in practice. In the first example, we focus on the US demand to European governments and firms that they stop using information technology (IT) infrastructure produced by China's Huawei. Since this technology has strategic complementarities in its adoption, the example illustrates the Macro-Power notion of a strategic sector. Indeed, we show that the pressure that the US applied to European sectors that it could influence was higher because, by causing these sectors not to adopt the technology, the US can also induce lower adoption by sectors and countries that it could not directly pressure.

Our second example focuses on the Chinese Belt and Road Initiative (BRI), an official lending program that aims to join borrowing and trade decisions. The example illustrates the value of joint threats in an economic relationship, government to government lending, in which enforcement is typically limited. In this example, profitable trade relationships act as an endogenous cost of default. Our model explains how China's BRI can enhance borrowing capacity in developing countries, while allowing China to demand political concessions from these governments in return.

Literature Review. In two landmark contributions Hirschman (1945, 1958) relates the structure of international trade to international power dynamics and sets up forward and backward linkages in input-output structures as a foundation for structural economic development. Much of our model is inspired by this work and aims to provide a formal framework for the power structures. We connect to three broad strands of literature.

First, the paper connects to the literature in political science on economic statecraft. The notion of economic statecraft, or the use of economic means for political ends, was explored in depth by Baldwin (1985) and Blackwill and Harris (2016). The literature on hegemonic stability theory debated whether hegemons, by providing public goods globally, can generate better world outcomes than multipolar configurations (Kindleberger (1973), Gilpin

(1981), Keohane (1984)). Keohane and Nye (1977) analyze the relationship between power and economic interdependence. Farrell and Newman (2023) and Drezner et al. (2021) investigate how interdependence can be "weaponized."

Second, the paper relates to the literature on networks, industrial policy, and trade. The literature on networks includes Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Blanchard et al. (2016), Bigio and La'O (2020), Baqaee and Farhi (2019, 2022), Liu (2019), Elliott, Golub, and Leduc (2022), Bachmann et al. (2022), and Hausmann et al. (2024). In trade, we relate to the study of global value chains (Grossman et al. (2021), Antràs and Chor (2022)), optimal tariffs and trade agreements (Bagwell and Staiger (1999), Grossman and Helpman (1994)), issue linkage (Limão (2005), Maggi (2016)), and sanctions (Eaton and Engers (1992)). Antràs and Miquel (2023) explore how foreign influence affects tariff and capital taxation policy, and Kleinman, Liu, and Redding (2020) explore whether countries become more politically aligned as they trade more with each other. We also relate to the literate on whether closer trade relationships promote peace (Martin, Mayer, and Thoenig (2008, 2012), Thoenig (2023)).

Third, the paper uses tools developed in economic theory and macroeconomics. We employ grim trigger strategies to build a subgame perfect equilibrium building on Abreu et al. (1986, 1990). Our notion of joint triggers relates to the literature on multi-market contact (Bernheim and Whinston (1990)) and multitasking (Holmstrom and Milgrom (1991)) in which the presence of multiple activities or tasks can help to provide higher powered incentives. We introduce externalities a la Greenwald and Stiglitz (1986) and our study of the hegemon's optimal usage of wedges and transfers is related to the analysis of inefficiency in the presence of externalities (Geanakoplos and Polemarchakis (1985)) and the macro-prudential tools that can be used to improve welfare (Farhi and Werning (2016)).

2. MODEL SETUP

Time is discrete and infinite, $t = 0, 1, \ldots$ Each period is a stage game, described below. All agents have subjective discount factor β.

2.1. *Stage Game*

There are N countries in the world. Each country n is populated by a representative consumer and a set of productive sectors $\mathcal{I}_n$, and is endowed with a set of local factors $\mathcal{F}_n$. We define $\mathcal{I}$ to be the union of all productive sectors across all countries, $\mathcal{I} = \bigcup_{n=1}^{N} \mathcal{I}_n$, and define $\mathcal{F}$ analogously. Each sector produces a differentiated good indexed by $i \in \mathcal{I}$ out of local factors and intermediate inputs produced by other sectors. Each sector is populated by a continuum of identical firms. The good produced by sector i is sold on world markets at price p_i. Factor f has price p^ℓ_f. Factors are internationally immobile. We take the good produced by sector 1 as the numeraire, so that $p_1 = 1$. We define the vector of intermediate goods prices as p, the vector of factor prices as p^ℓ, and the vector of all prices as $P = (p, p^\ell)$. To help keep track of notation, Online Appendix Table B.1 references the most frequently used notation in the paper.

Representative Consumer. The representative consumer in country n has preferences $U_n(C_n) + u_n(z)$, where $C_n = \{C_{ni}\}_{i\in\mathcal{I}}$ and where z is a vector of aggregate variables which we use to capture externalities a la Greenwald and Stiglitz (1986). Consumers take z as given. We assume U_n is increasing, concave, and continuously differentiable. We assume that the representative consumer in each country owns all domestic firms and the endowments of local factors. The representative consumer of country n faces a budget constraint given by:

$$\sum_{i\in\mathcal{I}} p_i\, C_{ni} \leq \sum_{i\in\mathcal{I}_n} \Pi_i + \sum_{f\in\mathcal{F}_n} p^\ell_f \bar{\ell}_f,$$

where Π_i are the profits of sector i and $p^\ell_f \bar{\ell}_f$ is the compensation earned by the local factor of production f. We define the consumer's Marshallian demand function $C_n(p, w_n)$, where $w_n = \sum_{i\in\mathcal{I}_n} \Pi_i + \sum_{f\in\mathcal{F}_n} p^\ell_f \bar{\ell}_f$, and the consumer's indirect utility function from consumption in the stage game as $W_n(p, w_n) = U_n(C_n(p, w_n))$. The consumer's total indirect utility in the stage game is $W_n(p, w_n) + u_n(z)$.

Firms. A firm in sector i located in country n produces output y_i using a subset $\mathcal{J}_i \subset \mathcal{I}$ of intermediate inputs and the set of local factors of country n, $\mathcal{F}_n$. Firm i's production is $y_i = f_i(x_i, \ell_i, z)$, where $x_i = \{x_{ij}\}_{j\in\mathcal{J}_i}$ is the vector of intermediate inputs used by firm i, x_{ij} is use of intermediate input j, $\ell_i = \{\ell_{if}\}_{f\in\mathcal{F}_n}$ is the vector of factors used by firm i, and ℓ_{if} is use of local factor f. Firms take the aggregate vector z as given. For simplicity, we assume that for production functions that can use both factors and intermediate inputs we have $f_i(0, \ell_i, z) = 0$, so that a firm that has no ability to source intermediate inputs cannot produce.[1] We assume that f_i is increasing, strictly concave, and satisfies the Inada conditions in (x_i, ℓ_i), and also is continuously differentiable in all its arguments. The sector-specific production function f_i can capture technology but also transport costs. We use the language of firms and sectors, but this is not meant to restrict the focus to private actors exclusively. Indeed many of these entities might be part of, owned, or operated by the government (e.g., a state-owned enterprise).

The stage game has three subperiods: Beginning, Middle, and End. Since each sector has a continuum of identical firms and we restrict to symmetric equilibria, we consider a representative firm per sector. We refer to firm i when clarity necessitates distinguishing an individual firm from the rest of the firms in the same sector, and sector i when describing representative firm outcomes. The game described below unfolds between an individual firm in sector i and the continuum of firms (suppliers) in sector j.

In the Beginning, firm i places an order x_{ij} to suppliers in sector $j \in \mathcal{J}_i$ and an order ℓ_i for local factors. The order x_{ij} is placed in equal proportion to each firm in sector j. Factor orders are always accepted and factors cannot be stolen.

In the Middle, each firm in sector j decides to Accept, $a_{ij} = 1$, or Reject, $a_{ij} = 0$, the order of firm i. We assume all firms within a given sector j play the same pure strategy. If the order x_{ij} is Rejected by suppliers in sector j, firm i receives none of that input and owes no payment to suppliers in sector j. If the order is Accepted by suppliers in sector j, the suppliers immediately deliver the entire order x_{ij} to firm i.

In the End, if the order was Accepted, firm i owes the payment $p_j x_{ij}$ to suppliers in sector j. Firm i can choose to Pay suppliers, or Steal from them. If firm i chooses to Steal, suppliers in sector j are only able to recover an exogenous fraction $1 - \theta_{ij} \in [0, 1]$ of the sale order value $p_j x_{ij}$. We denote $S_i \subset \mathcal{J}_i$ the subset of sectors from which firm i steals. For example, $S_i = \{1, 2\}$ denotes the action of stealing inputs provided by suppliers in sectors 1 and 2 and not any others, and $S_i = \emptyset$ denotes no stealing.

[1] We allow for the presence of sectors that simply repackage the factors and use no intermediate inputs. As we describe below, since factors cannot be stolen, these sectors are treated separately from the main analysis and only used in some examples to sharpen the characterization.

For an order (x_i, ℓ_i) in the Beginning, a vector $a_i \in \{0,1\}^{J_i}$ of acceptance choices in the Middle, and a stealing action $S_i \subset \mathcal{J}_i$ in the End, the stage game payoff to firm i is:

$$p_i f_i(x_i \cdot a_i, \ell_i, z) - \sum_{j \in \mathcal{J}_i} p_j a_{ij} x_{ij} - \sum_{f \in \mathcal{F}_n} p_f^{\ell} \ell_{if} + \sum_{j \in S} \theta_{ij} p_j a_{ij} x_{ij}.$$

Correspondingly, suppliers in sector j lose $\theta_{ij} p_j a_{ij} x_{ij}$ if firm i steals from them. The stage game captures many economic relationships that are based on repeated transactions and limited enforceability: a lender-borrower relationship in finance or a supplier-customer relationship in goods or services. The enforceability parameters θ_{ij} are flexible: a typical parametrization might set lower enforceability for international than domestic relationships, and the lowest when involving a foreign government. Indeed, the application in Section 4.2 studies cross-border official lending and sets enforceability to zero.

2.2. *Repeated Game*

We assume suppliers play trigger strategies, defined below, that involve switching to Rejecting any future order by an individual firm following some Stealing actions by that firm. We track permanent exclusion by $B_{ij} \in \{0,1\}$. If $B_{ij} = 0$, then suppliers in sector j will Reject any order placed by firm i. If $B_{ij} = 1$, then suppliers in sector j will Accept an incentive compatible order (defined below) and Reject an order that is not incentive compatible. For expositional convenience, we say that suppliers in j "Trust" firm i if $B_{ij} = 1$ and "Distrust" firm i if $B_{ij} = 0$. We define $\mathcal{B}_i = \{j \mid B_{ij} = 1\}$ to be the set of supplying sectors that Trust firm i. Note that exclusion off-path is tracked at the level of the specific firm within a sector that deviates, taking as given that on path the other firms in the same sector did not deviate and thus retained access. This means that equilibrium prices and quantities do not change based on the deviation of an individual atomistic firm.

We study subgame perfect equilibria that are Markov in $\mathcal{B}_i$, and restrict attention to pure strategies that are symmetric within a sector. A strategy of firm i in the Beginning is $\sigma_i^-(\mathcal{B}_i) \in \mathbb{R}_+^{J_i + F_n}$, mapping $\mathcal{B}_i$ into an order (x_i, ℓ_i). A strategy of suppliers in sector j in the Middle with regard to firm i is $\sigma_{ij}(x_i, \ell_i, \mathcal{B}_i) \in \{0,1\}$, mapping an order size and $\mathcal{B}_i$ into an acceptance decision a_{ij}. A strategy of firm i in the End is $\sigma_i^+(a_i, x_i, \ell_i, \mathcal{B}_i) \in P(\mathcal{J}_i)$, mapping acceptance decisions of its suppliers, its order size, and $\mathcal{B}_i$ into stealing action S_i.

We conjecture and verify a value function $\mathcal{V}_i(\mathcal{B}_i)$ of firm i in the repeated game that is non-decreasing in $\mathcal{B}_i$, that is $\mathcal{V}_i(\mathcal{B}_i) \leq \mathcal{V}_i(\mathcal{B}'_i)$ if $\mathcal{B}_i \subset \mathcal{B}'_i$. We build this function below starting from an exogenous continuation value $\nu_i(\mathcal{B}_i)$ assumed to be non-decreasing and with $\nu_i(\emptyset) = 0$.

2.2.1. *Trigger Strategies and Incentive Compatibility*

We assume that suppliers employ trigger strategies that define the evolution of $\mathcal{B}_i$ following a stealing action S_i of the firm. We study triggers that take two forms: individual and joint. The proof of Lemma 1 in Appendix A formally characterizes trigger strategies and we focus below on an intuitive presentation. In the case of an individual trigger, if firm i Steals from suppliers in sector j, then suppliers in sector j Distrust individual firm i in all future periods. Formally, $B'_{ij}(S_i) = 0$ if $j \in S_i$, that is suppliers in sector j Distrust firm i starting in the next stage game, following the firm i stealing action S_i in the current stage game.

In the case of a joint trigger between suppliers in sectors j and k with respect to firm i, if firm i Steals from suppliers in either sector j or k, then suppliers in both sectors j and k Distrust individual firm i in all future periods. Formally, $B'_{ij}(S_i) = 0$ and $B'_{ik}(S_i) = 0$ if either $k \in S_i$ or $j \in S_i$. We assume that joint triggers are symmetric and note that they can be chained. For example, firm i stealing from suppliers h triggers suppliers j if h has a joint trigger with k and k has a joint trigger with j.

We denote by K_{ij} the set of individual and joint trigger relationships (including chaining) for suppliers in j with respect to firm i's stealing actions. The suppliers that Trust firm i following Stealing action S_i are therefore $\mathcal{B}'_i(S_i) = \mathcal{B}_i \backslash (\bigcup_{j \in S_i} K_{ij})$. In building the incentive compatibility constraint for firm i, we know by backward induction that suppliers never Accept an order that will be stolen since their payoff is strictly negative from doing so. Hence, we focus on a constraint for orders that are Accepted and not stolen.

Potentially, the set of possible stealing actions is very large. Fortunately, the lemma below shows that the dimensionality of the problem can be reduced: rather than checking all possible stealing actions, it suffices to check sets of actions that are not generically dominated given the trigger strategies. Let $P(\mathcal{J}_i)$ denote the power set of $\mathcal{J}_i$, that is all subsets of $\mathcal{J}_i$, and let $\Sigma(\mathcal{S}) = \{\bigcup_{X \in \mathcal{X}} X \mid \emptyset \neq \mathcal{X} \subset \mathcal{S}\}$ be all possible unions of elements of $\mathcal{S}$.

LEMMA 1: *There is a partition $\mathcal{S}_i$ of $\mathcal{J}_i$ such that the order (x_i, ℓ_i) is incentive compatible with respect to all stealing actions, $P(\mathcal{J}_i)$, if and only if it is incentive compatible with respect to $\Sigma(\mathcal{S}_i)$. The incentive compatibility constraint for $S_i \in \Sigma(\mathcal{S}_i)$ is*

$$\sum_{j \in S_i} \theta_{ij} p_j x_{ij} \leq \beta \bigg[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S_i) \bigg]. \tag{1}$$

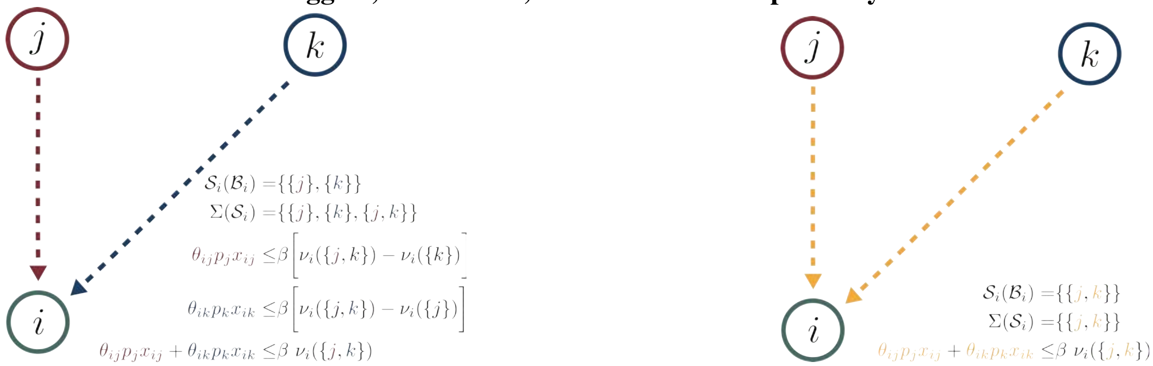
Lemma 1 is presented for $\mathcal{B}_i = \mathcal{J}_i$ and its proof shows a counterpart holds for each $\mathcal{B}_i \subset \mathcal{J}_i$ that can occur off path. Figure 1 illustrates the lemma in the simple case of two sectors j and k supplying to firm i. In Panel (a), the suppliers in sectors j only have individual triggers, resulting in an IC constraint $\theta_{ij} p_j x_{ij} \leq \beta[\nu_i(\{j,k\}) - \nu_i(\{k\})]$. Firm i compares the one-off Stealing gain $\theta_{ij} p_j x_{ij}$ with the continuation value loss of not being able to use input j again. Suppliers in sector k have an identical set-up and constraint. Finally, the firm could Steal from both suppliers generating the constraint $\theta_{ij} p_j x_{ij} + \theta_{ik} p_k x_{ik} \leq \beta \nu_i(\{j,k\})$.

Panel (b) illustrates joint triggers between sectors j and k. Intuitively, firm i would never Steal from only one of sectors j or k, since both would retaliate anyway. $\mathcal{S}_i$ is the set of the smallest undominated stealing actions. In Panel (a) this included stealing from j and k separately, but in panel (b) only stealing from both at the same time is undominated. The set $\Sigma(\mathcal{S}_i)$ then considers all possible combinations of these undominated actions. In Panel (a) this includes Stealing from j and k separately and Stealing from both at the same time. In Panel (b) this only includes Stealing from both. Therefore, under joint triggers in Panel (b) there is only one IC left, the joint stealing constraint: $\theta_{ij} p_j x_{ij} + \theta_{ik} p_k x_{ik} \leq \beta \nu_i(\{j,k\})$.

It is convenient to track the "action set" $\mathcal{S}_i$ directly, instead of tracking the sets of triggers K_{ij}. We denote $\mathcal{S}_i(\mathcal{B}_i)$ the action set arising from Lemma 1 off-path at $\mathcal{B}_i$ (see the proof). Given the firm's incentive problems, suppliers' strategy in the Middle is to Accept an order if and only if equation (1) is satisfied for all $S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$.[2]

[2]In the SPE that we construct, suppliers that Distrust individual firm i, i.e. $j \notin \mathcal{B}_i$, Reject any positive order. If hypothetically suppliers in $j \notin \mathcal{B}_i$ Accepted a positive order, firm i would still believe that suppliers in j will reject every future order, given $B_{ij} = 0$. Firm i would then Steal from suppliers in j. Hence, suppliers in j Reject the order. For $\theta_{ij} = 0$ this is an assumption given indifference for the suppliers, and otherwise a strict preference.

FIGURE 1.—**Triggers, Action Sets, and Incentive Compatibility Constraints**



(a) Individual Triggers (b) Joint Triggers

Notes: Panels focus on a firm in sector i with suppliers in sectors j and k. Action sets and related incentive constraints are from the perspective of firm i under different configurations. Panel (a) illustrates the case in which suppliers in sectors j and k have individual triggers only. Panel (b) illustrates the case in which suppliers in sectors j and k have a joint trigger.

2.2.2. *Firm i Optimal Production and Value Function*

Since continuation value $\nu_i(\mathcal{B}_i)$ is non-decreasing, firm i's strategy in the Beginning is an order size (x_i, ℓ_i) to maximize its stage game payoff $\Pi_i(x_i, \ell_i, \mathcal{B}_i) = p_i f_i(x_i, \ell_i, z) - \sum_{j\in\mathcal{B}_i} p_j x_{ij} - \sum_{f\in\mathcal{F}_n} p_f^{\ell} \ell_{if}$, subject to incentive compatibility (equation (1)), and where $x_{ij} = 0$ for $j \notin \mathcal{B}_i$. Since Π_i is a concave function and equation (1) describes a convex set, the optimization problem of firm i is convex.

We complete construction of a subgame perfect equilibrium (SPE) for firm i by constructing the associated value function $\mathcal{V}_i(\mathcal{B}_i)$ at each set $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$. This construction follows an iterative process (Abreu et al. (1990)), which is derived in detail in Online Appendix B.2.2.1 and outlined here.[3] First, we have $\mathcal{V}_i(\emptyset) = 0$. The next step is to consider $\mathcal{B}_i \in \mathcal{S}_i$, so that firm i is Trusted by the smallest subsets of suppliers that enter a joint trigger. We construct $\mathcal{V}_i(\mathcal{B}_i)$ using the fact that if firm i Steals it then reverts to $\mathcal{V}_i(\emptyset) = 0$. The iteration then progresses by constructing $\mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i = S_1 \cup S_2$ for $S_1, S_2 \in \mathcal{S}_i$, and so

[3]In principle, one could allow for non-stationary (front-loaded) punishments in an attempt to worsen the off-path equilibrium and sustain a better equilibrium than Markov and potentially implement the Ramsey plan. Our purpose is not to explore the best sustainable equilibrium but to focus on a simple Markov one that provides much economics while minimizing the theoretical complexity.

forth. In each step, the value function $\mathcal{V}_i(\mathcal{B}_i)$ is given as a fixed point of the equation

$$\begin{aligned}\mathcal{V}_i(\mathcal{B}_i) \;=\; & \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i)+\beta\mathcal{V}_i(\mathcal{B}_i)\\ & s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij}\le\beta\Big[\mathcal{V}_i(\mathcal{B}_i)-\mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}_i(\mathcal{B}_i)).\end{aligned}$$

In this iterative process, the value function constructed in the SPE with no stealing in steps $n=0,\dots,N$ is subsequently used as the off-path continuation values of the SPE at step $N+1$, until the final step with $\mathcal{B}_i=\mathcal{J}_i$ is reached.[4]

2.3. *Market Clearing, Externalities, and Equilibrium*

Denote $D_j=\{i\in\mathcal{I}\mid j\in\mathcal{J}_i\}$ the set of sectors that source from sector j, i.e. the sectors immediately downstream from j. Market clearing for good j is given by: $\sum_{n=1}^{N}C_{nj}+\sum_{i\in D_j}x_{ij}=y_j$. Market clearing for factor f in country n is $\sum_{i\in\mathcal{I}_n}\ell_{if}=\overline{\ell}_f$. We assume that the vector of aggregates takes the form $z=\{z_{ij}\}$. In equilibrium $z^*_{ij}=x^*_{ij}$, where we use the $*$ notation to stress it is an equilibrium value. That is z-externalities are based on the quantities of inputs in bilateral sectors i and j relationships. This general formulation can be specialized to cover pure size externalities, in which it is the total output of a sector that matters, but also thick market externalities, in which it is the extent to which an input is widely used by many sectors that matters.

An equilibrium of the model is prices for goods and factors P and allocations $\{x_i,C_n,y_i,\ell_i,z_{ij}\}$ such that: (i) firms maximize profits, given prices; (ii) households maximize utility, given prices; (iii) markets clear.

3. HEGEMONIC POWER

Our main analysis focuses on when and how a hegemon can build power and wield it to demand costly actions. We begin this section by defining and characterizing *pressure*

[4]If an element $\mathcal{B}_i\in\Sigma(\mathcal{S}_i)$ has no SPE associated with no stealing, then we assume that at the beginning of a period in which firm i faces $\mathcal{B}_i$, the suppliers that Trust firm i automatically updates to an element $\hat{\mathcal{B}}_i\in\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ such that $\hat{\mathcal{B}}_i$ results in an SPE with no stealing. As a result, $V_i(\mathcal{B}_i)=V_i(\hat{\mathcal{B}}_i)$. That is to say, suppliers understand that if the suppliers that Trust firm i were $\mathcal{B}_i$, the firm would in fact Steal from a subset with probability 1, and therefore suppliers update accordingly. We assume throughout the paper that $\mathcal{B}_i=\mathcal{J}_i$ has an SPE with no stealing.

points on firms, which denote a set of off-equilibrium-path threats on a firm that, when consolidated into a single joint threat, generate an increase in profits earned by that firm on the equilibrium path.

A joint threat in our model is a coordination of trigger strategies among multiple supplying sectors of the same firm. As an example, returning to Figure 1, a joint threat on a firm in sector i is the suppliers in j and k adopting a joint trigger (essentially moving from the configuration in Panel (a) to that in Panel (b)). Joint threats generically generate value for the firm being threatened because they relax incentive constraints. This is natural in set-ups in which trigger strategies can be used to threaten agents with punishments in order to induce good behavior. Consider a firm i that faces an exogenous continuation value function ν_i and is trusted by all of its suppliers, that is $\mathcal{B}_i = \mathcal{J}_i$. We define the firm's current value as a function of its action set $\mathcal{S}_i$ as[5]

$$
\begin{aligned}
V_i(\mathcal{S}_i) \;&=\; \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{J}_i) + \beta\nu_i(\mathcal{J}_i) \\
&\quad s.t. \quad \sum_{j\in S}\theta_{ij}p_j x_{ij} \le \beta\Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}_i).
\end{aligned}
$$

Then for any joint threat action set $\mathcal{S}_i'$ formed from $\mathcal{S}_i$, we have $V_i(\mathcal{S}_i') \ge V_i(\mathcal{S}_i)$. Of course, in many cases the value creation is zero, for example when incentive constraints are all not binding, but our main interest is in the cases of strictly positive value. We define a pressure point for firm i as a joint threat that *strictly* increases the profits of firm i.

DEFINITION 1: *A* ***joint threat*** *$\mathcal{S}_i'$ is a partition of $\mathcal{J}_i$ such that $\mathcal{S}_i'$ is coarser than $\mathcal{S}_i$. A* ***pressure point*** *of firm i is a joint threat $\mathcal{S}_i'$ that strictly increases firm i's profits, that is $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$.*

3.1. *Hegemon Contract Terms*

We consider a single country m that is a hegemon. Online Appendix B.1 provides an extension to competition between multiple hegemons. The hegemon country has the ability

[5]Note that $V_i(\mathcal{S}_i)$ defines a value function of firm i over its action set $\mathcal{S}_i$ for an exogenous continuation value function and assuming the firm is trusted by all its suppliers ($\mathcal{B}_i = \mathcal{J}_i$), whereas $\mathcal{V}_i(\mathcal{B}_i)$ defines the equilibrium (fixed point) value function of firm i when Trusted by suppliers $\mathcal{B}_i$ and keeping constant the joint threats.

to coordinate its firms, including the ability to create joint threats. It can propose take-it-or-leave-it offers to *all* downstream sectors of its firms, where contract terms specify joint threats, transfers, and restrictions on inputs purchased. Unlike individual firms and consumers, the hegemon internalizes how the terms of its contract affect the aggregates z and prices P.

Since we focus on Markov equilibria, the hegemon offers a contract only for the current stage game and takes the future decisions of itself and of firms as given (i.e., the hegemon cannot commit to future contracts). As in Section 2.2, we start by taking $\nu_i(\mathcal{B}_i)$ to be an exogenous continuation value function of firm i.

Recalling that D_i is the set of sectors downstream from sector i, let $\mathcal{D}_m = \bigcup_{i\in\mathcal{I}_m} D_i \backslash \mathcal{I}_m$ denote the set of foreign sectors that source at least one input from the sectors in the hegemon's country. We assume that the hegemon can contract with all its domestic sectors and their foreign downstream sectors, and denote $\mathcal{C}_m = \mathcal{I}_m \cup \mathcal{D}_m$ to be this set. Let $\mathcal{J}_{im} = \mathcal{I}_m \cap \mathcal{J}_i$ denote the set of inputs that sector i sources from (sectors in) country m. Recall that some of the sectors the hegemon contracts with might be foreign governments or their state-owned enterprises, so the contract encompasses government-to-government and government-to-private sectors relationships. Relationships with foreign governments typically have low enforceability (high θ_{ij}) and, therefore, potentially the most gain from the hegemon acting as a global enforcer.

Hegemon m proposes a take-it-or-leave-it contract to each firm $i \in \mathcal{C}_m$ with three terms: (i) a joint threat $\mathcal{S}_i'$; (ii) nonnegative transfers $\mathcal{T}_i = \{T_{ij}\}_{j\in\mathcal{J}_{im}}$ from firm i to the hegemon's representative consumer; (iii) revenue-neutral taxes $\tau_i = \{\{\tau_{ij}\}_{j\in\mathcal{J}_i}, \{\tau_{if}^{\ell}\}_{f\in\mathcal{F}_n}\}$ on purchases of inputs and factors, with equilibrium revenues $\tau_{ij}x_{ij}^*$ and $\tau_{if}^{\ell}\ell_{if}^*$ raised from sector i rebated lump sum to firms in sector i. We denote $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}$ the contract offered to firm i, and denote $\Gamma = \{\Gamma_i\}_{i\in\mathcal{C}_m}$ the set of all contracts.

Taxes adjust the effective price firm i faces to $p_j + \tau_{ij}$ for inputs and $p_f^{\ell} + \tau_{if}^{\ell}$ for factors. Because taxes are revenue-neutral, without loss of generality we assume that tax payments and rebates do not enter the Pay/Steal decision. Transfers occur contemporaneously with the Pay/Steal decision. Under the contract, if firm i Pays suppliers in sector j, then it pays $p_j x_{ij}$ to suppliers in sector j and pays $\tau_{ij}(x_{ij} - x_{ij}^*) + T_{ij}$ to the hegemon's consumer. If firm i Steals from suppliers in sector j, its only payment is $\tau_{ij}(x_{ij} - x_{ij}^*)$ to the hegemon's consumer (which is zero in equilibrium). In this case, suppliers in sector j only recover an amount $(1-\theta_{ij})p_j x_{ij}$, while hegemon m's representative consumer recovers none of the transfer.

Transfers T_{ij} can cover different interpretations: direct monetary payments, a firm-specific mark-up charged by the hegemon on sales of its goods, or the extraction of value in some other action the firm takes on behalf of the hegemon (see later discussion of lobbying and political concessions). The revenue-neutral taxes τ_{ij} are typical in the macro-prudential literature that focuses on pecuniary and demand externalities (Farhi and Werning (2016)). They can capture either quantity restrictions or taxes/subsidies (see for example Clayton and Schaab (2022)). Importantly, we allow these instruments to target relationships between two sectors. This covers, for example, restricting energy imports from Russia but not from other countries; or tariffs and quantity restrictions on imports of Chinese goods.[6]

Feasible Joint Threats. We restrict the joint threats that the hegemon can make to involve sectors that are at most one step removed from the hegemon. We impose this restriction to prevent unrealistic situations in which the hegemon threatens a firm that it has no (immediate) relationship with.

DEFINITION 2: *It is feasible for the hegemon to use $S \in \mathcal{S}_i$ in forming a joint threat $\mathcal{S}_i'$ if $\exists j \in S$ with $j \in \mathcal{C}_m$.*

Intuitively, Definition 2 says that the hegemon can create a joint threat using action $S \in \mathcal{S}_i$ if either the hegemon directly supplies a good $j \in S$ to firm i, or if the hegemon supplies a good to a foreign sector j that in turn is a direct supplier to firm i with $j \in S$. In the former case, the hegemon coordinates a joint threat between two or more of its domestic firms. In the latter case, the hegemon creates a joint threat via a foreign downstream supplier of its firms, by requiring this supplier, as part of its contract, to adopt the trigger strategy associated with the joint threat. Online Appendix Figure B.2 provides an illustration along the line of Figure 1 of which threats by the hegemon are feasible.

[6] We focus on restrictions (costly actions) imposed on firms on buying inputs from other suppliers. In principle, we could also allow for bilateral taxes on sales by firm i. In equilibrium, any sales taxes would be fully passed through to the buyer and, in this sense, would be captured by the input taxes that we already consider. However, a difference is that the input taxes on firm i that arise from sales taxes on firm j would not in principle require firm i to agree to the contract. Similarly, we could also allow bilateral taxes on sales by firm i to consumers.

Firm Participation Constraint. Firm $i \in \mathcal{C}_m$ chooses whether or not to accept the take-it-or-leave-it offer made by the hegemon. Firm i, being small, does not internalize the effect of its decision to accept or reject the contract on the prevailing aggregate vector z and prices. If firm i rejects the hegemon's contract, it retains its original action set and achieves the value $V_i(\mathcal{S}_i)$. If instead firm i accepts the offer, it chooses allocations to maximize profits given the contract terms. Given a contract Γ_i, the value to firm i of accepting the contract is given by[7]

$$V_i(\Gamma_i) = \max_{x_i,\ell_i} \; \Pi_i(x_i,\ell_i,\mathcal{J}_i) - \sum_{j\in\mathcal{J}_i}[\tau_{ij}(x_{ij}-x^*_{ij}) + T_{ij}] - \sum_{f\in\mathcal{F}_m} \tau^{\ell}_{ij}(\ell_{if}-\ell^*_{if}) + \beta\nu_i(\mathcal{J}_i)$$

$$s.t. \sum_{j\in S}\Big[\theta_{ij}p_j x_{ij} + T_{ij}\Big] \le \beta\Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}'_i) \tag{2}$$

Recall that transfers are associated with the firm decision to Pay and, thus, enter the incentive constraint. Transfers T_{ij} tighten the incentive constraint, all else equal. At the level of the individual firm, taxes have two effects: (i) they affect the firm's optimal allocation because they alter the perceived price of the input good; (ii) they affect firm profits directly. In equilibrium, this latter effect washes out since taxes are rebated lump sum (i.e., $x_{ij} = x^*_{ij}$). The optimal allocation x^*_{ij}, and hence remitted revenues, are defined implicitly as a function of contract terms, prices, and z-externalities by the above optimization problem.

For firm i to accept the contract, it must be better off under the contract than by rejecting it. This gives rise to the **participation constraint** of firm i,

$$V_i(\Gamma_i) \ge V_i(\mathcal{S}_i), \tag{3}$$

where recall that $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i, \tau_i\}$ so that the participation constraint is comparing the hegemon's contract with joint threats, transfers, and wedges to the outside option. Slackness in the participation constraint when the hegemon applies the joint threats but demands no costly actions out of the target means that the hegemon has a pressure point on firm i (Definition 1). This is the source of hegemon's power over firm i. Online Appendix B.2.1 shows

[7] We extend the previous definition of firm i value function $V_i(\mathcal{S}_i)$ to incorporate the full terms of the hegemon contract $V_i(\Gamma_i)$ where $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i, \tau_i\}$. We abuse notation and write $V_i(\mathcal{S}_i)$ as short hand for $V_i(\Gamma_i)$ when $\Gamma_i = \{\mathcal{S}_i, 0, 0\}$.

how to extend the model to allow the hegemon to also generate slack by making the outside option worse by threatening to cut off firms that reject the contract from its inputs.

Hegemon Maximization Problem. The hegemon's objective function is the utility of its representative consumer, to whom all domestic firm profits and all transfers accrue:

$$\mathcal{U}_m = W_m(p, w_m) + u_m(z), \quad w_m = \sum_{i \in \mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i \in \mathcal{D}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}, \tag{4}$$

where as in Section 2.1, w_m is the consumer's wealth. Since transfers from domestic sectors to the hegemon's consumer net out from the consumer's wealth, we need only keep track of operating profits $\Pi_i(\Gamma_i) = V_i(\Gamma_i) + \sum_{j \in \mathcal{J}_{im}} T_{ij}$ of the hegemon's domestic sectors. Similarly, taxes on all sectors are revenue neutral for the hegemon, and therefore net out. However, transfers from foreign sectors do not net out, precisely because the hegemon's consumer has no claim to foreign sectors' profits.

The hegemon's maximization problem is choosing a contract Γ to maximize its consumer utility (equation (4)), subject to the participation constraints of firms (equation (3)), the feasibility of joint threats (Definition 2), the determination of aggregates $z_{ij}^* = x_{ij}^*$, and determination of prices via market clearing.

3.2. *Optimality of Maximal Joint Threats*

We solve the hegemon's problem in two steps. First, we prove that the hegemon offers a "maximal" joint threat that joins together all feasible threats. Second, we characterize transfers and wedges under the optimal contract.

Starting from the existing set $\mathcal{S}_i$, we show that hegemon optimally consolidates all feasible threats at its disposal, $\mathcal{S}_i^D = \{S \in \mathcal{S}_i \,|\, \exists j \in S \; s.t. \; j \in \mathcal{C}_m\}$, into a single stealing action $S_i^D = \bigcup_{S \in \mathcal{S}_i^D} S$. The maximal joint threat is then the single action S_i^D and the remaining threats that the hegemon could not feasibly consolidate: $\overline{\mathcal{S}}_i' = S_i^D \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$.

PROPOSITION 1: *It is weakly optimal for the hegemon to offer a contract with maximal joint threats to every firm it contracts with, that is $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ for all $i \in \mathcal{C}_m$.*

Intuitively, Proposition 1 follows from the observation that joint threats expand the set of feasible allocations, and so weakly increase targeted entities' profits. A hegemon that chose a contract that did not involve maximal joint threats could always implement the same transfers and allocations while offering a contract with maximal joint threats. Hence offering maximal joint threats can increase value to the hegemon but cannot decrease it. The hegemon, therefore, wants to maximize its global enforcer capabilities.

Since the hegemon's contract involves all of its domestic sectors that supply to sector i entering a single joint threat, transfers can be tracked in total at the sector level, that is $\overline{T}_i = \sum_{j \in \mathcal{J}_{im}} T_{ij}$, rather than at the bilateral supplier level T_{ij}. We therefore abuse notation and track only $\overline{T}_i$ in the contract, rather than the full vector $\mathcal{T}_i$.

3.3. *Leontief Inverse and Network Propagation with Externalities*

In demanding costly actions and transfers out of targeted entities, the hegemon takes into consideration their impact on aggregate prices P and quantity based externalities z. Therefore, to analyze the hegemon optimal contract we need to first characterize how changes in firms' allocations x_{ij} propagate through the global network. The proposition below shows that the entire propagation can be characterized in terms of a generalized Leontief inverse.

PROPOSITION 2: *The aggregate response of z^* and P to a perturbation in an exogenous variable e is*

$$\frac{dz^*}{de} = \Psi^z \left(\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P} \frac{dP}{de} \right)$$

$$\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial P} \right)^{-1} \left(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial e} \right),$$

where $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*} \right)^{-1}$ and ED is the vector of excess demand in every good and factor.

The matrix Ψ^z keeps track of all the successive amplification via the z-externalities of the original perturbation. The term $\frac{dP}{de}$ keeps track of the input-output amplification occurring via changes in equilibrium prices. To provide intuition here and in the rest of the paper, it is sometimes useful to consider special cases which we define formally below. First, consider an environment in which all prices are constant in equilibrium as defined below:

DEFINITION 3: *The* ***constant prices*** *environment assumes that consumers have identical linear preferences over goods, $U_n = \sum_{i\in\mathcal{I}} \tilde{p}_i C_{ni}$, and that each country has a local-factor-only firm with linear production $f_i(\ell_i) = \sum_{f\in\mathcal{F}_n} \frac{1}{\tilde{p}_i}\tilde{p}^\ell_f \ell_{if}$. It assumes that consumers are marginal in every good and factor-only firms are marginal in every local factor so that $p_i = \tilde{p}_i$ and $p^\ell_f = \tilde{p}^\ell_f$.*

In this simplified environment the term $\frac{\partial x^*}{\partial P}\frac{dP}{de}$ would be zero and amplification would only occur via the z-externalities: $\frac{dz^*}{de} = \Psi^z \frac{\partial x^*}{\partial e}$. Here the matrix Ψ^z captures all endogenous amplification since prices are constant, and is akin to a Leontief inverse. Intuitively, the perturbation to e changes production in a sector, leading to re-optimization in other sectors given the production externalities, which in turn filters to other sectors, and so on.

Second, consider switching off the z-externalities as defined below:

DEFINITION 4: *The* ***no z-externalities*** *environment assumes that $u_n(z)$ and $f_i(x_i, \ell_i, z)$ are constant in z.*

In this simplified environment the term $\frac{\partial x^*}{\partial z^*}$ would be zero and the matrix Ψ^z would reduce to the identity matrix. Amplification would only occur via prices: $\frac{dz^*}{de} = \frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de}$, where $\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P}\right)^{-1}\frac{\partial ED}{\partial e}$. Intuitively, the perturbation to e changes excess demand in each market as a result of reoptimization by firms and consumers. These changes in excess demand must then be counteracted through price changes to equilibrate markets, with $\frac{\partial ED}{\partial P}$ giving the response of excess demand to prices.

The general result puts together both types of amplification, price and z-externality based, and tracks their interacted effect throughout the network.

3.4. *Hegemon's Optimal Contract and Efficiency*

In characterizing the hegemon's optimal contract, we set up the following notation (see the proof of Proposition 3 for details). Letting $\mathcal{L}_m$ be the hegemon's Lagrangian, we denote $\eta_i \geq 0$ the Lagrange multiplier on the participation constraint of firm i, and $\Lambda_{iS} \geq 0$ the Lagrange multiplier on the incentive constraint of firm i for stealing action S. We also define $\overline{\Lambda}_i = \sum_{S\in\Sigma(\overline{\mathcal{S}}'_i)|S^D_i\subset S} \Lambda_{iS}$, which sums all multipliers involving a stealing action included in the hegemon's maximal joint threat. We define $\mathcal{E}_{ij} = \frac{d\mathcal{L}_m}{dz^*_{ij}}$ to be the hegemon's perceived externalities from an increase in z^*_{ij}, and $\Xi_{mn} = \frac{\partial \mathcal{L}_m}{\partial z^*}\left[\frac{dz^*}{dw_m} - \frac{dz^*}{dw_n}\right] + \frac{\partial \mathcal{L}_m}{\partial P}\left[\frac{dP}{dw_m} - \frac{dP}{dw_n}\right]$ to be the hegemon's perceived externalities from a transfer of wealth from consumers in country n to consumers in country m. An optimal contract is characterized by the proposition below.[8]

PROPOSITION 3: *An optimal contract of the hegemon has the following terms:*

1. *For foreign firms $i \in \mathcal{D}_m$ located in country n, if $\overline{\mathcal{S}}'_i$ is a pressure point on i:*
   (a) *Input wedges satisfy: $\eta_i \tau^*_{ij} = -\mathcal{E}_{ij}$.*
   (b) *Transfers satisfy: $\overline{\Lambda}_i + \eta_i \geq \frac{\partial W_m}{\partial w_m} + \Xi_{mn}$, with equality if $\overline{T}^*_i > 0$.*
2. *For domestic firms $i \in \mathcal{I}_m$, if $\overline{\mathcal{S}}'_i$ is a pressure point on i:*
   (a) *Input wedges satisfy: $(\frac{\partial W_m}{\partial w_m} + \eta_i)\tau^*_{ij} = -\mathcal{E}_{ij}$.*
   (b) *Transfers are zero: $\overline{T}^*_i = 0$.*
3. *If $\overline{\mathcal{S}}'_i$ is not a pressure point of firm i, then $\overline{T}_i = 0$ and $\tau_i = 0$.*

To provide intuition for the hegemon's optimal contract, consider a foreign firm i with a binding participation constraint. We can expand the optimal tax formula in 1(a) above to:

$$\tau^*_{ij} = -\frac{1}{\eta_i}\mathcal{E}_{ij} = -\frac{1}{\eta_i}\Bigg[\underbrace{\varepsilon^z_{ij}}_{\text{Direct Impact}} + \underbrace{\overbrace{\varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}}}^{\text{Aggregate Quantities}} + \overbrace{\varepsilon^{P^m}\frac{dP^m}{dz_{ij}}}^{\text{Prices}}}_{\text{Indirect Impact: Input-Output Amplification}}\Bigg]. \tag{5}$$

The hegemon uses the wedges to manipulate externalities in its favor. Activities that generate positive (negative) externalities $\mathcal{E}_{ij} > 0$ are subsidized (taxed). We decompose $\mathcal{E}_{ij}$ in three terms (see proof of Proposition 3).

The first term in equation (5), ε^z_{ij}, measures the direct value to the hegemon of increasing sector i's use of input j and arises from two sub-components:

[8] Proposition 3 provides necessary conditions for optimality. Formally, if for a foreign firm i we have $\eta_i = 0$, it instead characterizes the limit of a sequence of wedges, each of which is part of a (different) optimal contract (see the proof for details). For technical reasons, we assume that if $\overline{\mathcal{S}}'_i$ is not a pressure point on firm i at the optimal (z^*, P), then it is also not a pressure point on i in a neighborhood of (z^*, P). Finally to streamline analysis we assume that every foreign country contains at least one firm that the hegemon cannot contract with, meaning that the hegemon cannot directly mandate factor prices in foreign countries.

$$\varepsilon^z_{ij} = \underbrace{\frac{\partial W_m}{\partial w_m} \sum_{k \in \mathcal{I}_m} \frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}}}_{\text{Externalities on Hegemon's Economy}} + \underbrace{\sum_{k \in \mathcal{C}_m} \eta_k \left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}} \right]}_{\text{Building Power}} \tag{6}$$

The hegemon wants to increase foreign activity x_{ij} if it directly benefits one of the sectors in the hegemon's economy or if the consumer directly cares about that activity. An example of the first is a foreign firm R&D activity that has a positive knowledge spillover on the productivity of domestic sectors. An example of the second is a foreign firm R&D activity that is uses by the military of a country hostile to the hegemon. The hegemon also cares about how its demands on activity x_{ij} affect the amount of power it has over the all sectors. All else equal, the hegemon asks for actions that make it less attractive on the margin for a firm to reject its contract (decrease the outside option $V_k(\mathcal{S}_k)$ or increase on-path profits Π_k) thus binding it tighter to the hegemon and increasing its power. As an example, in the presence of strategic complementarities, the US demands that more foreign firms rely on US financial institutions, making it harder for any one firm to deviate from US demands.

The second term in equation (5), $\varepsilon^{zNC} \frac{dz^{*NC}}{dz_{ij}}$, measures the indirect value of altering production via input-output amplification in sectors that the hegemon does not control. The term ε^{zNC} is analogous to equation (6) but for firms not in the hegemon's network. The term $\frac{dz^{*NC}}{dz_{ij}}$ summarizes the Leontief amplification impact and is given by Proposition 2 taking z_{ij} to be the exogenous variable e. The hegemon demands more action in the x_{ij} relationship the more, via the network, these actions propagate and affect activities that the hegemon does not control but values. An example is the US demanding European banks to curb financing of legitimate (from a European regulatory perspective) commercial activities of Iranian entities in order to affect the overall Iranian economy and in particular Iran's government budget and military sector.

The third term in equation (5), $\varepsilon^{P^m} \frac{dP^m}{dz_{ij}}$, is the indirect value of the induced changes in equilibrium prices. The term $\frac{dP^m}{dz_{ij}}$ summarizes the Leontief amplification impact and is given by Proposition 2 taking z_{ij} to be the exogenous variable e. Isolating the component of the vector ε^{P^m} corresponding to the value from changes in the price of input j, we have

$$\varepsilon_j^{P^m} = \underbrace{\frac{\partial W_m}{\partial w_m} X_{m,j}}_{\text{Terms of Trade}} - \underbrace{\sum_{k \in \mathcal{C}_m} \overline{\Lambda}_{kj} \theta_{kj} x_{kj}}_{\text{Pecuniary Externalities (IC)}} + \underbrace{\sum_{k \in \mathcal{C}_m} \eta_k \left[\frac{\partial \Pi_k}{\partial p_j} - \frac{\partial V_k(\mathcal{S}_k)}{\partial p_j} \right]}_{\text{Building Power}}$$

where $X_{m,j}$ is exports of good j by country m (negative, i.e. imports, if $j \notin \mathcal{I}_m$). Much of the trade and international macroeconomics literature has focused on terms of trade manipulation as the motive for imposing tariffs, capital controls, and entering multilateral trade agreements. Similarly, the macro-finance literature has focused on pecuniary externalities, which are also present in our framework since prices enter the incentive constraints. The last term, "Building Power", is analogous to the last term in equation (6) and key to our analysis of international power. The hegemon takes into consideration how its demands change prices and how those affect the marginal willingness of firms to accept its demands.

Proposition 3 part 1(b) shows that the hegemon has an incentive to extract transfers from foreign firms but, in the presence of externalities, there are countervailing forces. Charging a higher transfer to a firm has the cost of tightening both the participation constraint and the incentive constraint, valued by the multipliers $\eta_i + \overline{\Lambda}_i$. The marginal benefit to the hegemon of the transfer includes the direct marginal benefit, given by the marginal value of wealth $\frac{\partial W_m}{\partial w_m}$, and the indirect (externality) term Ξ_{mn} because reallocating wealth from consumers in country n to those in the hegemon country m, alters equilibrium prices and aggregates z as long as these consumers have different marginal expenditures. Despite the hegemon having all the bargaining power, in the presence of externalities the optimal contract might leave surplus to the foreign entities (slack participation constraint) whenever the indirect benefits to the hegemon from these sectors not shrinking are sufficiently high. For example, a hegemon might leave surplus to a friendly sector in order to maximize the benefits arising indirectly from its positive externalities. In this cases the hegemon has power but optimally decides no to fully exert it, as in a liberal world order. In the spirit of Nye (2004), our framework has some elements of hard economic power, as in the economic coercion “stick”, but also elements of soft economic power since being under the hegemon’s influence adds value (a “carrot") to the targeted entities that participate voluntarily and might retain some of the surplus.

Consider next a domestic firm. The hegemon's optimal wedge formula (Proposition 3 part 2(a)) is almost identical to that for foreign firms, except that the magnitude of wedges (whether tax or subsidy) is lower. Intuitively, this occurs because the hegemon values the profits of domestic firms and wedges erode these profits. The term $\frac{\partial W_m}{\partial w_m}$ is added in 2(a) compared to 1(a) to capture the marginal value of profits. Domestic firms are never charged transfers since the firms are owned by the hegemon's consumers and transfers tighten the incentive constraints.

The wedges applied to domestic firms are akin to industrial policy, and in our framework this policy can be driven by domestic (e.g. education and R&D) or foreign considerations. In particular the hegemon uses the wedges to building up domestic industries that increase the country's power. Through our framework, one can understand recent U.S. export restrictions on U.S. semiconductor firms (such as Nvidia and Intel) selling their output to certain Chinese sectors. While the U.S. government overall subsidizes the American semiconductor industry to build hegemonic power, it also restricts its exports to Chinese firms given the technology (even indirect) usage in the military sector.

3.5. *Strategic Sectors and The Nature of Geoeconomic Power*

Controlling, defending from foreign influence, and growing strategic sectors is a core government policy in democracies and autocracies alike. While governments frequently protect or control industries claiming they are strategic for "national interest", there is a concern that the "strategic" label is in reality a cover for protectionism or for subsidies to politically connected entities. This ambiguity is possible because of a lack of clarity on what it means for an economic activity to be strategic and a clear framework against which policies are to be evaluated.[9]

In our framework, a sector is strategic in two dimensions. First, a sector can be strategic because the hegemon can use it to form (off-path) threats on other entities. Second, a sector can be strategic because the hegemon can demand (on-path) costly actions from this

[9]See Baldwin (1985)["Strategic Goods" section, pages 223-233] for a review of many informal definitions of strategic goods, including a quote from Soviet leader Nikita Khrushchev: "Anything one pleases can be regarded as strategic material, even a button, because it can be sewn onto a soldier's pants. A solider will not wear pants without buttons, since otherwise he would have to hold them up with his hands. And then what can he do with his weapon?".

sector that shape the world equilibrium in the hegemon's favor. Control, either directly via ownership or indirectly via other economic relationships, of a sector enables the hegemon to build power by making joint threats. We distinguish two notions of power that are what makes sectors strategic: Micro-Power and Macro-Power.

3.5.1. *Micro-Power: Strategic Sectors in Threatening Target Output*

Micro power is the maximum private cost to the target of the hegemon's demanded actions. It is the most the hegemon could demand before its contract gets rejected. The source of this power is the value to the targeted entity of the hegemon's threats, that is whether the hegemon has a pressure point on that entity (Definition 1). The amount of Micro-Power is given by $V_i(\overline{\mathcal{S}}_i') - V_i(\mathcal{S}_i)$, taking as given all equilibrium aggregate quantities and prices. The hegemon builds as much Micro-Power as it can by making maximal joint threats (Proposition 1), and then uses this power to demand costly actions (Proposition 3).

To isolate micro power, the corollary below considers a special case of Proposition 3 in which equilibrium prices are constant and z-externalities are switched off both in the firms production function and consumer utility function.

COROLLARY 1: *With constant prices (Definition 3) and no z-externalities (Definition 4), an optimal contract of the hegemon has the following terms:*

1. *All wedges are zero on all sectors, $\tau_{ij}^* = \tau_{if}^{\ell *} = 0$ for all $i \in \mathcal{C}_m,\ j \in \mathcal{J}_i,\ f \in \mathcal{F}_n$.*
2. *All transfers are zero for domestic sectors, that is $\overline{T}_i^* = 0$ for all $i \in \mathcal{I}_m$.*
3. *Foreign sector i is charged a positive transfer $\overline{T}_i^* > 0$ if and only if $\overline{\mathcal{S}}_i'$ is a pressure point on i. The transfer is then set so that the participation constraint binds, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$ and $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^*, 0\}$.*

Once endogenous amplification through the network is switched off, the hegemon has no reason to impose wedges: they decrease targeted firm profits with no corresponding benefit. Instead, the hegemon uses all its Micro-Power to extract transfers from foreign firms until their participation constraint binds. Note that this requires not only the absence of z-externalities, but also constant prices in order to shut off terms of trade manipulation motive and the classic input-output amplification via prices.

A crucial source of micro-power arises from the loss for the target from the hegemon cutting off access to some of it inputs. Despite the hegemon threats being off-path, this loss in continuation value can be computed, using the model structure, as a counterfactual based on the observed on-path data. While a full empirical analysis is beyond the scope of this paper, in Online Appendix B.2.4, we offer some initial empirical guidance by specializing the production function to be Cobb-Douglass across industries and CES within industries. With this standard production function, the counterfactual loss can be measured using available estimates of the elasticity of substitution within sectors and trade data on a country's expenditure share on a sector and the expenditure share on goods each country buys from the hegemon as a share of total spending within each sector. These losses are in the spirit of Hirschman (1945) notion of asymmetric power coming from trade relationships with the hegemon.

Goods that are strategic in this micro-sense are those widely used, with high value added for targets, and with poor substitutes. Some goods have these properties due to physical constraints like rare earths, oil, and gas. However, in identifying Micro-Power it is necessary, but not sufficient, to know the parameters of the production function. It is also necessary to know which inputs the hegemon controls. As emphasized by Schelling (1958), the notion of strategic has to be defined in the context of an equilibrium and cannot be determined solely from ex-ante characteristics of a sector. For example, controlling one variety of natural gas is ineffective since there is a high degree of substitutability in production with other types of natural gas. However, if the hegemon controls a joint threat among all varieties of natural gas, that threat is very valuable since the input is essential for many sectors. This logic also applies to joint threats for inputs that might seem rather unrelated without guidance from a theoretical framework. For example, the a joint threat involving loans and manufacturing inputs. Section 4.2 provides an application along these lines for China's Belt and Road Initiative.

3.5.2. *Macro-Power: Strategic Sectors in General Equilibrium*

Macro power is the social value to the hegemon's country of the costly actions it demands of targeted entities. It arises from the hegemon's ability to extract value from the world economy indirectly, via shaping the externalities and prices. By collectively asking entities that it can pressure to take costly actions, such as curbing the usage of some inputs, the hegemon indirectly influences a larger part of the input-output network than what it directly controls. The propagation and amplification through the network structure, our externality based Leontief-inverse, is key to this effect. In this macro sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification (in the Leontief-inverse). Sectors like research and development, finance, and information technology are good candidates for being strategic.

Proposition 3 shows that the marginal value to the hegemon of having more power over sector i is given by the Lagrange multiplier η_i on that sector's participation constraint. This multiplier reflects the benefit to exerting both Micro- and the Macro-Power over sector i. A hegemon particularly values having Micro-Power over sectors that increase its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon's demands the targeted entities consider only their private costs, but the hegemon internalizes the social benefits of the outcomes of these actions. Section 4.1 highlights these forces in an application on telecommunication infrastructure and national security, and characterizes how the hegemon can extract value indirectly by using network amplification to contain an hostile country.

Re-arranging equation (5) into $\eta_i = -\frac{\mathcal{E}_{ij}}{\tau^*_{ij}}$ highlights that the marginal value of power over a sector, η_i, is related to the ratio of how much the hegemon wants to control activities in that sector, $\mathcal{E}_{ij}$, versus how much the hegemon actually controls activities in that sector, τ^*_{ij}. When desired control $\mathcal{E}_{ij}$ is high relative to actual control τ^*_{ij}, the hegemon has little correction in place over an activity that it perceives to have high general equilibrium influence. Macro-Power is thus highly valuable in such circumstances.[10]

Finally, the theory helps interpret a type of reduced-form empirical analysis that has become common in both economics and political science: regressing measures of political affinity among countries on bilateral trade or investment. The loose prediction being that as geopolitical tensions rise between two countries, one must observe a fall in bilateral economic activity. In terms of equation (5), the loose prediction appears to rely on the direct term ε^z_{ij} and in particular the direct representative consumer disliking activity in an geopolitical rival (the $u(z)$ term). Our analysis makes clear that indirect effects might well

[10]Our framework can be extended to allow the hegemons to buy controlling stakes (FDI) in foreign sectors. We think of purchasing a controlling stake as a way to bypass the participation constraint since then the hegemon can simply dictate the actions. Interestingly, the private market value of such stake should be lower than the social value to the hegemon that internalizes its geoeconomic use thus providing a rationale for the investment screening policies such as CFIUS in the US.

dominate the direct ones and increases in geopolitical rivalry might still generate more bilateral trade in some sectors.

3.6. *Efficient Allocations*

We provide an efficiency benchmark by taking the perspective of a global planner that has exactly the same powers and constraints as the hegemon, but cares about global welfare. Formally, the planner chooses a contract Γ to maximize global welfare:

$$\sum_{n=1}^{N} \Omega_n \left[W_n(p, w_n) + u_n(z) \right], \quad w_n = \sum_{i \in \mathcal{I}_n} V_i(\Gamma_i) + \sum_{f \in \mathcal{F}_i} p_f^{\ell} \bar{\ell}_f + \mathbf{1}_{n=m} \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}, \tag{7}$$

subject to the participation constraints of firms (equation (3)), the feasibility of joint threats (Definition 2), the determination of aggregates, and the determination of prices via market clearing. The Pareto weight placed on the welfare of country n's consumer is Ω_n. As is common in the literature, we mute the planner's motive to redistribute wealth between countries by setting the welfare weights to equalize the social marginal value of wealth across consumers. The following proposition characterizes the global planner's solution.

PROPOSITION 4: *An optimal contract of the hegemon from the global planner's perspective features maximal joint threats $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$, zero transfers $\overline{T}_i = 0$, and wedges given by $(\Omega_n \frac{\partial W_n}{\partial w_n} + \eta_i)\tau_{ij}^* = -\mathcal{E}_{ij}^p$ for all sectors $i \in \mathcal{C}_m$ on which the hegemon has a pressure point. Wedges and transfers are zero if $\overline{\mathcal{S}}_i'$ is not a pressure point on i.*

The planner and the hegemon agree that supplying maximal joint threats is optimal since it relaxes the targeted entities' incentive problems and in principle allows more economic activity to take place. The planner and the hegemon, however, disagree on the value of transfers and on the optimal wedges to be applied.

Both the planner and the hegemon understand that the transfers are negative-sum globally since they tighten incentive problems. The planner, therefore, chooses never to demand transfers. The hegemon, instead, values receiving positive transfers from foreign firms.[11]

[11] If we allowed hegemon consumers to own foreign sectors this would contribute to aligning the hegemons' incentives with those of the planner by making the hegemon care about the profits of foreign sectors that it owns. Exogenous ownership of foreign sectors would be easy to introduce in this framework.

Both the global planner and the hegemon want to use the wedges in equilibrium to affect externalities. However, the global planner implements wedges that are different from those implemented by the hegemon. Intuitively, a sector might have a negative externality on the hegemon country but a positive one on other countries. The hegemon manipulates the the externalities in its favor, while the global planner corrects externalities putting welfare weights on all countries. Formally, this can be seen in the proposition above in which $\mathcal{E}_{ij}^{p}$ tracks the impact of activity x_{ij} on the planner's Lagrangian rather than the hegemon's one.

Proposition 4 highlights some crucial features of our model that relate to the concept of hegemonic stability in political science (Kindleberger (1973), Gilpin (1981), Keohane (1984)). That literature debated whether hegemons by providing public goods globally can generate better world outcomes than configurations with no hegemons (or multiple hegemons). In our framework, the hegemon acts as a global enforcer, echoing the public good provision, and some of its policies correct negative externalities. Indeed, the global planner also provides the same enforcement (maximal joint threats) and, in some dimensions, might correct externalities similarly to the hegemon. However, the hegemon destroys value at the global level compared to the global planner by demanding transfers and manipulating the externalities in its favor. Because of the externalities, the equilibrium with the hegemon can even be worse for some entities than the equilibrium without the hegemon depending on whether the enforcement and positive correction of externalities are more then offset by the externality manipulation.

4. APPLICATIONS

We specialize the model to capture two leading applications of geoeconomics in practice.

4.1. *National Security Externalities*

In this application we take as inspiration the US government demand to European governments and firms that they stop using information technology infrastructure produced by China's Huawei (Farrell and Newman (2023)). We assume the hostile technology is a national security threat from the perspective of the hegemon, but a positive production externality for firms in third party countries. This captures the notion that this infrastructure could be used for spying and or military uses, but that for a private firm the technology is attractive (privately profitable) and the more so the more other firms are using it. That is, the technology has a strategic complementary in its adoption capturing interoperability. The application is both of practical interest and helps us illustrate the importance of production externalities and network amplification in how a hegemon pressures strategic sectors.

There are three regions: the hegemon country m, a hostile foreign country h, and "rest of world" RoW which may comprise multiple countries. Figure 2 illustrates the set-up of this application. We assume constant prices (Definition 3). The hostile foreign country h has a single sector, which we denote by H. We take the output of this sector to be the numeraire, $p_H = 1$. Sector H and sectors in the hegemon country are not subject to externalities from z, that is $f_H(x_H, \ell_H, z)$ and $f_k(x_k, \ell_k, z)$ for $k \in \mathcal{I}_m$ are constant in z. We assume that sectors in the hegemon country do not source from the hostile country's sector H and vice-versa, ensuring that H cannot be used by the hegemon as part of a joint threat.

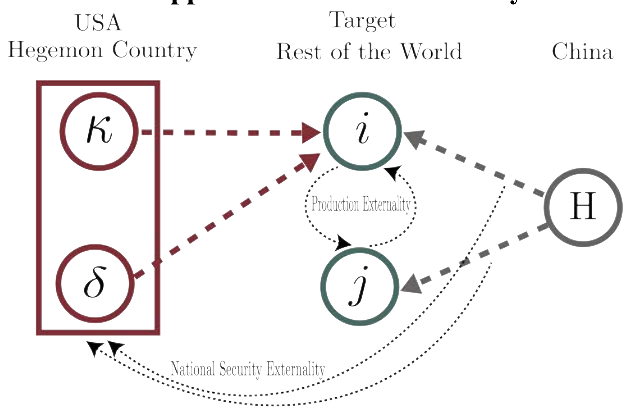
The main action in this application comes from RoW sectors. We assume that all RoW sectors source from H, and define $z^H \equiv \{z_{iH}\}_{i \in \mathcal{I}_{RoW}}$ to be the vector of purchases by RoW sectors of input H. For simplicity, we assume sectors in RoW have production that is separable in H: $f_i(x_i, \ell_i, z) = f_{i,-H}(x_{i,-H}, \ell_i) + f_{iH}(x_{iH}, z^H)$, where $x_{i,-H}$ denotes the vector of all inputs except input H. We introduce external economies of scale by setting:

$$f_{iH}(x_{iH}, z^H) = A_{iH}(z^H) g_{iH}(x_{iH}). \tag{8}$$

We assume that $\frac{\partial A_{iH}}{\partial z_{jH}} > 0$ for all $i, j \in \mathcal{I}_{RoW}$, so that there are positive spillovers from greater usage of H. This helps us capture technologies, such as 5G infrastructure, that have strategic complementarities in adoption and usage. We further assume that $A_{iH}(z^H) g_{iH}(z_{iH})$ is concave in z^H. Observe that $f_{i,-H}$ is constant in z. For simplicity we assume $\theta_{iH} = 0$, so that firms are unconstrained in their use of input H. We assume that in absence of a hegemon, there are no joint triggers.

Hegemon Negative Externality from H. We assume that the hegemon's representative consumer's utility function has a negative externality from rest-of-world production using H, that is $u_m(z) = u_m(z^H)$ and $\frac{\partial u_m}{\partial z_{iH}} < 0$ for all $i \in \mathcal{I}_{RoW}$. This simple reduced-form utility term in the objective function of the hegemon helps us capture a direct disutility from the RoW usage of the technology of a hostile country. In practice, the US government concerns regarding Huawei technology stemmed from the possibility that it could be used for spying

FIGURE 2.—**Application: National Security Externality**



Notes: Figure depicts the model set-up for the application on national security as described in Section 4.1.

or in military applications; we capture the direct US government goal of shrinking the usage of the technology.

From Proposition 1, maximal joint threats are optimal for the hegemon. Since there are no z-externalities in production by domestic firms and prices are constant, Proposition 3 tells us $\overline{T}_i = 0$ and $\tau_i = 0$ is an optimal contract for all domestic sectors. To characterize the optimal contracts for sectors in the RoW, the relevant part of the objective function (equation (4)) reduces to $\mathcal{U}_m = u_m(z^H) + \sum_{i \in \mathcal{D}_m} \overline{T}_i$.

Network Amplification. Network amplification occurs due to the strategic complementarity in the use of H. We can capture the interesting economics even considering only two sectors in RoW: one sector, which we denote i, that the hegemon can contract with; and one sector, which we denote j, that the hegemon cannot contract with. In this environment, employing Proposition 2 we have $\Psi^{z,NC} = \left(1 - \frac{\partial x^*_{jH}}{\partial z_{jH}}\right)^{-1} = \frac{\gamma_j}{\gamma_j - \xi_{jj}}$, where $\xi_{ij} = \frac{z_{jH}}{A_{iH}(z^H)} \frac{\partial A_{iH}(z^H)}{\partial z_{jH}}$ is the elasticity of productivity A_{iH} with respect to the externality z_{jH}, so that ξ_{jj} are sector j external economies of scale, and where $\gamma_i = \frac{-x^*_{iH} g''_{iH}(x^*_{iH})}{g'_{iH}(x^*_{iH})}$. Applying Proposition 2 we have that the total transmission of a change in the targeted sector i usage of input H to the usage by sector j of the same input is given by:

$$\frac{dz^{NC}}{dz_{iH}} = \frac{dz_{jH}}{dz_{iH}} = \Psi^{z,NC} \frac{\partial x^*_{jH}}{\partial z_{iH}} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}.$$

Optimal Contract. The hegemon's optimal tax formula of Proposition 3 and equation (5) reduces to $\tau_{iH} = \frac{1}{\eta_i}\varepsilon^z_{iH} + \varepsilon^z_{jH}\frac{dz_{jH}}{dz_{iH}}$ since the term $\varepsilon^{P^m}\frac{dP^m}{dz_{ij}}$ is zero given constant prices. Using equation (6) we can further unpack this formula to write:

$$\tau_{iH} = -\overbrace{\frac{1}{\eta_i}\frac{\partial u_m}{\partial z_{iH}}}^{\text{Externalities on Hegemon's Economy}} \overbrace{+ p_i A_{iH}(z^H)\Big[g_{iH}(x^o_{iH}(z^H)) - g_{iH}(x^*_{iH})\Big]\xi_{ii}\frac{1}{z_{iH}}}^{\text{Building Power}}$$
$$+ \underbrace{\left[-\frac{1}{\eta_i}\frac{\partial u_m}{\partial z_{jH}} + p_i A_{iH}(z^H)\Big[g_{iH}(x^o_{iH}(z^H)) - g_{iH}(x^*_{iH})\Big]\xi_{ij}\frac{1}{z_{jH}}\right]\frac{\xi_{ji}}{\gamma_j - \xi_{jj}}\frac{z_{jH}}{z_{iH}}}_{\text{Network Amplification}}$$

where $x^o_{iH}(z^H)$ is what firm i's optimal usage of input H would be if it rejected the hegemon contract. In the presence of national security externalities, the optimal tax is positive, $\tau_{iH} > 0$, reflecting the hegemon's desire to mitigate the negative externality. Three key forces underlie the tax formula.

The first term in the tax formula is the direct externality from an increase in z_{iH} on representative consumer m. The negative externality contributes to a positive tax. This tax is higher when η_i is lower, that is when the marginal cost of using the hegemon's power over firm i (the slack in that firm participation constraint) is lower.

The second term captures the hegemon's desire to build micro-power over firms in sector i by leveraging the external economies of scale. Each firm that accepts the hegemon's contract and reduces its usage of input of H increases on the margin the hegemon's power over other firms in the same sector by lowering productivity A_{iH} and making it less attractive to reject the contract to use more of the H input. The hegemon is manipulating the external economies of scale to get firms to downscale the undesirable technology. Once it is successfully downscaled, no individual firm has a high desire to use it on the margin.

Finally, the third term is the indirect effect of the hegemon's demands on the sector it can pressure (sector i) on the sector it cannot pressure (sector j). As sector i usage of input H falls, that is z_{iH} falls, the productivity A_{jH} of firms in sector j in using input H also falls, prompting firms in sector j to reduce the use of H. This leads to a fall in z_{jH}, which has a positive externality effect on the hegemon consumer and also increases micro-power over firms in sector i. Both effects mirror those described in the previous two paragraphs but are now arising from the equilibrium choices of a sector the hegemon does not directly control. The Leontief amplification $\frac{dz_{jH}}{dz_{iH}} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}}$ captures the magnitude of this response by sector j. This effect contributes towards a higher tax rate, since reducing usage by sector i of input H has a positive externality by also reducing demand by sector j for input H.

In this application, sector i is strategic from a Macro-Power perspective because by influencing its actions the hegemon impacts the actions of sectors it could not pressure directly. As a consequence the hegemon makes higher demands (more positive τ_{iH}) and manipulates the difference between the private cost to the target of the actions (Micro-Power) and the social value to the hegemon (Macro-Power) to build more power over the targeted sector. In practice, this explains that the strong pressure applied by the US on European firms to prevent usage of Huawei technology aimed at making the technology less valuable to adopt for other entities, that the US couldn't directly pressure, once European entities were also not using it.

4.2. *Official Lending, Infrastructure Projects, and Political Concessions*

China's flagship Belt and Road Initiative (BRI) has sought to jointly provide official loans and manufacturing inputs often in exchange for political concessions (Dreher et al. (2022)). Our model explains how China acts as a global enforcer in emerging economies by providing pressure jointly across lending and manufacturing relationships while extracting surplus in terms of political concessions.

We specialize the model to the configuration in Figure 3. The hegemon country, in this application China, has two sectors: sector k is a lender and sector j is a manufacturer. For simplicity, both sectors produce only using local factors. The target country, in this application a developing economy, has a single sector i that uses both inputs from China to produce. To focus the application on the essentials, we further assume constant prices (Definition 3), no z-externalities (Definition 4), and that sector i has a separable production function $f_i(x_i) = f_{ij}(x_{ij}) + f_{ik}(x_{ik})$.

We think of the lending sector, k, as providing a loan to or buying a bond issued by sector i. The loan is for amount $x_{ik} = b$ and the gross interest rate is $p_k = R$. Like in the sovereign default literature, we assume that the loan is not legally enforceable, so that $\theta_{ik} = 1$.

If there are only individual triggers on j and k, lending can be sustained by the future surplus of the lending relationship, along the lines of the sovereign default model of Eaton and Gersovitz (1981). In particular, we have $Rb \leq \beta\left[\mathcal{V}_i(\{j,k\}) - \mathcal{V}_i(\{j\})\right] = \beta\mathcal{V}_i(\{k\})$, where the latter equality follows from the separable production function and individual triggers. The Markov equilibrium value of $\mathcal{V}_i(\{k\}) = \frac{p_i f_{ik}(b^*) - Rb^*}{1-\beta}$ is the present discounted value of all future borrowing by sector i. Solving for the borrowing limit, we obtain $b \leq \left(\beta\frac{p_i}{R}\right)^{\frac{1}{1-\xi}}$ under the assumption that $f_{ik}(b) = b^{\xi}$ for $\xi \in (0,1)$. The IC (borrowing limit) binds whenever $\xi > \beta$, which we assume to be the case.

To sharpen the application, we assume that $\theta_{ij} = 0$ so that firms in sector i can never steal input j. Thus the incentive constraint for stealing j does not bind. Without a hegemonic China, the equilibrium features limited lending and an unconstrained manufacturing relationship. As a hegemon, China can impose a joint threat that links together the provision of lending and manufacturing goods. If the target country defaults on either input, both are withdrawn in the future. Under the joint threat the incentive constraint of the target country sector i is:

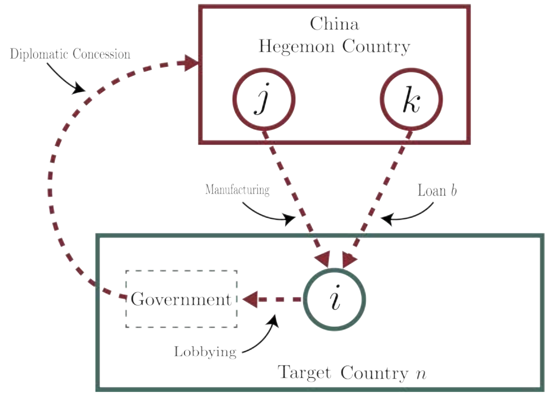
$$Rb \leq \beta\mathcal{V}_i(\{j,k\}) = \frac{p_i f_{ik}(b^*) - Rb^*}{1-\beta} + \frac{p_i f_{ij}(x^*_{ij}) - p_j x^*_{ij}}{1-\beta}.$$

The present value of the manufacturing relationship provides additional incentives to repay the debt in the joint threat, an endogenous cost of default on the loan. Under the joint threat, the equilibrium features the same level of manufacturing activity but an increase in the borrowing. The surplus is extracted by China via a transfer $\overline{T}^*_i > 0$ (Corollary 1).

Our mechanism is related to that proposed in Bulow and Rogoff (1989), whereby lenders seize the exports of a country conditional on a default, thereby generating a cost of default.[12] It is also related to Cole and Kehoe (1998), where government reputation is common across multiple relationships. In Mendoza and Yue (2012) a country faces an endogenous productivity loss in case of default due to being shut off from trade finance, hence losing the ability to import intermediate goods and being forced to switch to imperfect do-

[12]Under isolated threats, our model features positive borrowing. The impossibility result of Bulow and Rogoff (1989) does not kick in because we are not allowing inter-temporal saving and up-front payment contracts as in Eaton and Gersovitz (1981).

FIGURE 3.—**Application: Belt and Road Initiative**



Notes: Figure depicts the model set-up for the application on the Belt and Road Initiative as described in Section 4.2.

mestic substitutes. In our framework, joint threats offer a means for a country to voluntarily raise its cost of default, thereby allowing it to borrow more. In particular, the more input varieties and the more profitable those input varieties that are sourced from China, the more the borrowing constraint is relaxed.

One interpretation of the transfers is mark-ups on the manufacturing goods being sold by China to the target country, or equivalently an interest rate on the loan above the market rate R. This application cautions against empirical work that assesses China's lending programs in isolation: i.e. focusing only on the loans and their returns. Both the sustainability of the loans and the economic returns from the lending have to be assessed jointly with other activities, such as manufacturing exports, that are occurring jointly with the lending. The benefits to China might not even accrue in monetary form as we explore below.

Transfers as Costly Actions and Political Concessions. Our framework could be extended to allow for a rich model of political lobbying and influence (Grossman and Helpman (1994), Bombardini and Trebbi (2020)). The costly actions that the hegemon demands can take the form of political lobbying or diplomatic concessions. In this case, the transfer $\overline{T}_i$ represents the private cost to the firm of an action. Here we focus on a leading example for geoeconomics in which China asks the firms to lobby their governments for a political concession. We necessarily keep the modeling reduced form, but it provides a starting point for future research interested in introducing a deeper model of lobbying.

We assume that a bilateral geopolitical concession can be made from country n to China. We let the concession, be the element $z^c_n \in \{0, 1\}$ of aggregate vector z and assume that it enters positively in China's utility, $u_m(z_n^c)$ with $u_m(1) > u_m(0)$, and negatively in the target's country utility, $u_n(z_n^c)$ with $u_n(0) > u_n(1)$. We assume that no utility is derived by either country from all other elements of z. Governments care about consumer welfare and therefore internalize these utility costs and benefits. Governments also care about the profits of the firms in their country net of transfers. We assume that a hegemon asking a firm to make a positive transfer can alternatively ask that firm to transfer part or all of that transfer to the government in exchange for the government undertaking the geopolitical action, with any money not transferred being paid as usual to the hegemon. The geopolitical action is feasible to implement as long as country level transfer exceed the government utility cost of the concession. These concessions can account, for example, for China asking countries that are part of the Belt and Road Initiative not to recognize Taiwan (Dreher et al. (2022)).

5. CONCLUSION

Geoeconomics is a topic of practical importance but for which a formal treatment has proven elusive. This paper provides a general and formal framework that derives precise economic concepts to analyze this important topic. We show how concepts such as pressure, economic coercion, power, interdependence, strategic sectors, and third party sanctions emerge based on three core ingredients: limited enforceability and trigger punishments, input-output amplification, and externalities. We show how the framework can be used to make sense of many geoeconomic activities in practice like the US demands that European firms not use Huawei's technology, or China's flagship Belt and Road Initiative. The framework is flexible and can be extended for future analyses of a rich set of issues in geoeconomics as well as guide the necessary empirical measurement.

REFERENCES

ABREU, DILIP, DAVID PEARCE, AND ENNIO STACCHETTI (1986): "Optimal cartel equilibria with imperfect monitoring," *Journal of Economic Theory*, 39 (1), 251–269. []

——— (1990): "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica: Journal of the Econometric Society*, 1041–1063. []

ACEMOGLU, DARON, VASCO M CARVALHO, ASUMAN OZDAGLAR, AND ALIREZA TAHBAZ-SALEHI (2012): "The network origins of aggregate fluctuations," *Econometrica*, 80 (5), 1977–2016. []

ANTRÀS, POL AND DAVIN CHOR (2022): "Global Value Chains," in *Handbook of International Economics: International Trade, Volume 5*, vol. 5, 297–376. []

ANTRÀS, POL AND GERARD PADRÓ I MIQUEL (2023): "Exporting Ideology: The Right and Left of Foreign Influence," Tech. rep., National Bureau of Economic Research. []

BACHMANN, RUEDIGER, DAVID BAQAEE, CHRISTIAN BAYER, MORITZ KUHN, ANDREAS LÖSCHEL, BENJAMIN MOLL, ANDREAS PEICHL, KAREN PITTEL, AND MORITZ SCHULARICK (2022): "What if? The economic effects for Germany of a stop of energy imports from Russia," . []

BAGWELL, KYLE AND ROBERT W STAIGER (1999): "An economic theory of GATT," *American Economic Review*, 89 (1), 215–248. []

BALDWIN, DAVID A (1985): *Economic Statecraft*, Princeton University Press. []

BAQAEE, DAVID REZZA AND EMMANUEL FARHI (2019): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 87 (4), 1155–1203. []

——— (2022): "Networks, Barriers, and Trade," . []

BERNHEIM, B DOUGLAS AND MICHAEL D WHINSTON (1990): "Multimarket contact and collusive behavior," *The RAND Journal of Economics*, 1–26. []

BIGIO, SAKI AND JENNIFER LA'O (2020): "Distortions in production networks," *The Quarterly Journal of Economics*, 135 (4), 2187–2253. []

BLACKWILL, ROBERT D AND JENNIFER M HARRIS (2016): *War by other means: Geoeconomics and statecraft*, Harvard University Press. []

BLANCHARD, EMILY J, CHAD P BOWN, AND ROBERT C JOHNSON (2016): "Global supply chains and trade policy," Tech. rep., National Bureau of Economic Research. []

BOMBARDINI, MATILDE AND FRANCESCO TREBBI (2020): "Empirical models of lobbying," *Annual Review of Economics*, 12, 391–413. []

BULOW, JEREMY AND KENNETH ROGOFF (1989): "A constant recontracting model of sovereign debt," *Journal of political Economy*, 97 (1), 155–178. []

CLAYTON, CHRISTOPHER AND ANDREAS SCHAAB (2022): "Multinational banks and financial stability," *The Quarterly Journal of Economics*, 137 (3), 1681–1736. []

COLE, HAROLD L. AND PATRICK J. KEHOE (1998): "Models of Sovereign Debt: Partial Versus General Reputations," *International Economic Review*, 39 (1), 55–70. []

DREHER, AXEL, ANDREAS FUCHS, BRADLEY PARKS, AUSTIN STRANGE, AND MICHAEL J TIERNEY (2022): *Banking on Beijing: The Aims and Impacts of China's Overseas Development Program*. []

DREZNER, DANIEL W, HENRY FARRELL, AND ABRAHAM L NEWMAN (2021): *The uses and abuses of weaponized interdependence*, Brookings Institution Press. []

EATON, JONATHAN AND MAXIM ENGERS (1992): "Sanctions," *Journal of political economy*, 100, 899–928. []

EATON, JONATHAN AND MARK GERSOVITZ (1981): "Debt with potential repudiation: Theoretical and empirical analysis," *The Review of Economic Studies*, 48 (2), 289–309. []

ELLIOTT, MATTHEW, BENJAMIN GOLUB, AND MATT V. LEDUC (2022): "Supply Network Formation and Fragility," *American Economic Review*, 112 (8), 2701–2747. []

FARHI, EMMANUEL AND IVÁN WERNING (2016): "A theory of macroprudential policies in the presence of nominal rigidities," *Econometrica*, 84 (5), 1645–1704. []

FARRELL, HENRY AND ABRAHAM NEWMAN (2023): *Underground Empire: How America Weaponized the World Economy*, Henry Holt and Co. []

GABAIX, XAVIER (2011): "The granular origins of aggregate fluctuations," *Econometrica*, 79 (3), 733–772. []

GEANAKOPLOS, JOHN AND HERACLES M POLEMARCHAKIS (1985): "Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete," . []

GILPIN, ROBERT (1981): *War and change in world politics*, Cambridge University Press. []

GREENWALD, BRUCE C AND JOSEPH E STIGLITZ (1986): "Externalities in economies with imperfect information and incomplete markets," *The quarterly journal of economics*, 101 (2), 229–264. []

GROSSMAN, GENE, ELHANAN HELPMAN, AND STEPHEN REDDING (2021): "When tariffs disrupt global supply chains," Tech. rep. []

GROSSMAN, GENE M AND ELHANAN HELPMAN (1994): "Protection for Sale," *The American Economic Review*, 84 (4), 833–850. []

HAUSMANN, RICARDO, ULRICH SCHETTER, AND MUHAMMED A YILDIRIM (2024): "On the design of effective sanctions: The case of bans on exports to Russia," *Economic Policy*, 39 (117), 109–153. []

HIRSCHMAN, ALBERT (1945): *National power and the structure of foreign trade*, Univ of California. []

HIRSCHMAN, ALBERT O (1958): *The Strategy of Economic Development*, Yale University Press. []

HOLMSTROM, BENGT AND PAUL MILGROM (1991): "Multitask principal–agent analyses: Incentive contracts, asset ownership, and job design," *The Journal of Law, Economics, and Organization*, 7, 24–52. []

KEOHANE, R.O. AND J.S. NYE (1977): *Power and Interdependence*, Little Brown. []

KEOHANE, ROBERT O (1984): *After hegemony: Cooperation and discord in the world political economy*, Princeton university press. []

KINDLEBERGER, CHARLES POOR (1973): *The world in depression, 1929-1939*, Univ of California. []

KLEINMAN, BENNY, ERNEST LIU, AND STEPHEN J REDDING (2020): "International friends and enemies," Tech. rep., National Bureau of Economic Research. []

LIMÃO, NUNO (2005): "Trade policy, cross-border externalities and lobbies: do linked agreements enforce more cooperative outcomes?" *Journal of International Economics*, 67 (1), 175–199. []

LIU, ERNEST (2019): "Industrial policies in production networks," *Quarterly Journal of Economics*, 134 (4), 1883–1948. []

MAGGI, GIOVANNI (2016): "Issue linkage," in *Handbook of commercial policy*, Elsevier, vol. 1, 513–564. []

MARTIN, PHILIPPE, THIERRY MAYER, AND MATHIAS THOENIG (2008): "Make trade not war?" *The Review of Economic Studies*, 75 (3), 865–900. []

——— (2012): "The geography of conflicts and regional trade agreements," *American Economic Journal: Macroeconomics*, 4 (4), 1–35. []

MENDOZA, ENRIQUE G AND VIVIAN Z YUE (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127 (2), 889–946. []

NYE, JOSEPH S (2004): *Soft power: The means to success in world politics*, Public affairs. []

SCHELLING, T.C. (1958): *International Economics*, Allyn and Bacon. []

THOENIG, MATHIAS (2023): "Trade Policy in the Shadow of War: Quantitative Tools for Geoeconomics," . []

APPENDIX A: PROOFS

Proof of Lemma 1. Trigger strategies are formally defined by

$$B'_{ij}(S) = \begin{cases} B_{ij}, S \cap K_{ij} = \emptyset \\ 0, \qquad o.w. \end{cases}, \quad K_{ij} = \{j\} \cup \bigcup_{k \in M_{ij}} K_{ik} \tag{9}$$

where M_{ij} represents the joint triggers of suppliers in j. We first construct the smallest sets consistent with equation (9), that is involving minimal retaliation. Let $\{X_{ij}^n\}_{n=0}^{\infty}$ be a sequence of sets constructed iteratively as follows. Let $X_{ij}^0 = \{j\}$ and, for $n \geq 1$, let $X_{ij}^n = X_{ij}^{n-1} \cup \bigcup_{x \in X_{ij}^{n-1}} M_{ix}$.[13] Since $\mathcal{J}_i$ is a finite set, since $X_{ij}^{n-1} \subset X_{ij}^n \subset \mathcal{J}_i$, and since $X_{ij}^n = X_{ij}^{n-1} \Rightarrow X_{ij}^{n+1} = X_{ij}^n$, then $\exists \overline{N}_{ij} > 0$ such that $X_{ij}^{\overline{N}_{ij}} = X_{ij}^n$ for all $n \geq \overline{N}_{ij}$. We define the *minimum retaliation set* of suppliers in j for firm i as $X_{ij}^* = X_{ij}^{\overline{N}_{ij}}$.

We first show that $k \in X_{ij}^*$ if and only if $X_{ik}^* = X_{ij}^*$. The if statement is immediate since $k \in X_{ik}^*$ by construction. Consider then only if and let $k \in X_{ij}^*$. Since $k \in X_{ij}^*$, then $\exists N > 0$ s.t. $k \in X_{ij}^N$ and therefore $X_{ik}^* \subset X_{ij}^*$. Moreover since $k \in X_{ij}^*$, by construction there is a sequence $x_0, .., x_N$, with $x_0 = j$ and $x_N = k$, such that $x_n \in M_{ix_{n-1}}$ for $n = 1, ..., N$. Reversing that sequence and using symmetry of joint triggers, we have a sequence $x_N, ..., x_0$ such that $x_{n-1} \in M_{ix_n}$. Hence, $j \in X_{ik}^N$, and hence $j \in X_{ik}^*$. But then $X_{ij}^* \subset X_{ik}^*$, and hence $X_{ij}^* = X_{ik}^*$.

Next, we define $K_{ij} = X_{ij}^*$ and $\mathcal{S}_i(\mathcal{B}_i) = \bigcup_{j \in \mathcal{B}_i} \{K_{ij}\}$. Observe that $\mathcal{S}_i(\mathcal{B}_i)$ is a partition of $\mathcal{B}_i$, since: (i) $\bigcup_{j \in \mathcal{B}_i} X_{ij}^* = \mathcal{B}_i$; (ii) $\forall j, k \in \mathcal{B}_i$, either $X_{ij}^* = X_{ik}^*$ or $X_{ij}^* \cap X_{ik}^* = \emptyset$.

The incentive compatibility constraint associated with firm i preferring no stealing over stealing action $S \in P(\mathcal{B}_i)$ is

$$\Pi_i(x_i, \ell_i, \mathcal{B}_i) + \sum_{j \in S} \theta_{ij} p_j x_{ij} + \beta \nu_i(\mathcal{B}'_i(S)) \leq \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \nu_i(\mathcal{B}_i),$$

which reduces to $\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}'_i(S))]$.

We now complete the proof of the Lemma. The only if statement holds trivially since $\Sigma(\mathcal{S}_i(\mathcal{B}_i)) \subset \Sigma(\mathcal{B}_i) = P(\mathcal{B}_i) \backslash \{\emptyset\}$ since $\mathcal{S}(\mathcal{B}_i)$ is a partition of $\mathcal{B}_i$. Thus consider the if

[13] The first element $X_{ij}^0 = \{j\}$ is the individual trigger. The second element, $X_{ij}^1 = \{j\} \cup M_{ij}$, adds in the joint triggers of suppliers in j, and so on.

statement. Suppose that (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Let $S \in P(\mathcal{B}_i)$. If $S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ then incentive compatibility holds by assumption, so let $S \notin \Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Given a stealing action S, all suppliers $k \in \bigcup_{j \in S} X^*_{ij}$ Distrust firm i. Since elements of $\mathcal{S}_i(\mathcal{B}_i)$ are disjoint, there is a unique subset $\mathcal{X}_i(S) \subset \mathcal{S}_i(\mathcal{B}_i)$ of elements such that $\bigcup_{X \in \mathcal{X}_i(S)} X = \bigcup_{j \in S} X^*_{ij}$. Define $\Xi_i(S) = \bigcup_{X \in \mathcal{X}_i(S)} X$. Now, observe that for any $S \in P(\mathcal{B}_i)$, the stealing choice S is weakly dominated by the stealing choice $\Xi_i(S)$, since S and $\Xi_i(S)$ yield the same continuation value $\nu_i(\mathcal{B}_i \backslash \Xi_i(S))$ but $\Xi_i(S)$ yields higher flow payoff. Since $\Xi_i(S) \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ and since $\Xi_i(S)$ weakly dominates S, then if (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ it is also incentive compatible with respect to S. But since S was generic, then incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ implies incentive compatibility with respect to $P(\mathcal{B}_i)$, completing the proof.

Proof of Proposition 1. Consider a hypothetical optimal contract $\Gamma^o = \{\mathcal{S}'^o_i, \mathcal{T}^o_i, \tau^o_i\}_{i \in \mathcal{C}_m}$ that is feasible and satisfies firms' participation constraints, and suppose that $\mathcal{S}'_i \neq \overline{\mathcal{S}}'_i$. We use (x^o, ℓ^o) to denote firm allocations under this contract (and so on). The proof is one of implementability: we show that the hegemon can achieve the same allocations, prices, and transfers using a feasible contract with maximal joint threats, $\Gamma^* = \{\overline{\mathcal{S}}'_i, \mathcal{T}^o_i, \tau^*_i\}$.

We first construct τ^* by $\tau^*_{ij} = \frac{\partial \Pi_i(x^o_i, \ell^o_i, \mathcal{J}_i)}{\partial x_{ij}}$ and $\tau^{\ell *}_{if} = \frac{\partial \Pi_i(x^o_i, \ell^o_i, \mathcal{J}_i)}{\partial \ell_{if}}$. The relaxed problem (not subject to incentive compatibility) of firm i is

$$\max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau^*_{ij}(x_{ij} - x^*_{ij}) + T_{ij}] - \sum_{f \in \mathcal{F}_m} \tau^{\ell *}_{if}(\ell_{if} - \ell^*_{if}),$$

which yields solution $\frac{\partial \Pi_i}{\partial x_{ij}} = \tau^*_{ij}$ and $\frac{\partial \Pi_i}{\partial \ell_{if}} = \tau^*_{if}$, that is $x_i = x^o_i$ and $\ell_i = \ell^o_i$. It remains to verify this allocation is incentive compatible. Since $\overline{\mathcal{S}}'_i$ is a joint threat of $\mathcal{S}'^o_i$, then $\Sigma(\overline{\mathcal{S}}'_i) \subset \Sigma(\mathcal{S}'^o_i)$, and hence (x^o_i, ℓ^o_i) is incentive compatibility under contract Γ^*_i. Since (x^o_i, ℓ^o_i) solves firm i's relaxed problem and is incentive compatible, it is optimal for firm i.

Next, conjecturing $(z^*, P^*) = (z^o, P^o)$, then every firm $i \notin \mathcal{C}_m$ and every consumer n faces the same decision problem as under the original contract. Hence, every firm and every consumer has the same optimal policy. Hence $x^* = z^o$ and markets clear at prices $P^* = P^o$, consistent with the conjecture.

Finally, since allocations, transfers, and prices are the same, then since firm i's participation constraint is satisfied under contract Γ^o it is also satisfied under contract Γ^*. Since prices, allocations, and transfers are unchanged, the hegemon's objective attains the same value as under the original contract. Thus the hegemon is indifferent between feasible contracts Γ^o and Γ^*, completing the proof.

Proof of Proposition 2. To clarify the ordering for matrix algebra, $z_i^* = (z^*_{i,\min \mathcal{J}_i}, \ldots, z^*_{i,\max \mathcal{J}_i})^T$ is a $|\mathcal{J}_i| \times 1$ vector and $z^* = (z_1^{*T}, \ldots, z_{|\mathcal{I}|}^{*T})^T$ is a $\sum_{i\in\mathcal{I}} |\mathcal{J}_i| \times 1$ vector. Let $|z^*| = \sum_{i\in\mathcal{I}} |\mathcal{J}_i|$. We stack x^* from x^*_{ij} in the same manner. Since $x^*(\Gamma, z^*, P) = z^*$, then totally differenting yields $\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de} + \frac{\partial x^*}{\partial z^*}\frac{dz^*}{de} = \frac{dz^*}{de}$, where $\frac{\partial x^*}{\partial e}$ is a $|z^*| \times 1$, and $\frac{\partial x^*}{\partial z^*}$ is a $|z^*| \times |z^*|$ matrix with each row corresponding to the vector $\frac{\partial x^*_{ij}}{\partial z^*}$. Rearranging yields $\frac{dz^*}{de} = \Psi^z \frac{\partial x^*}{\partial e} + \Psi^z \frac{\partial x^*}{\partial P}\frac{dP}{de}$, where $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$.

Next, define the excess demand for good i as $ED_i = \sum_{n=1}^N C_{ni} + \sum_{j\in\mathcal{D}_i} x_{ji} - y_i$ and the excess demand for market f as $ED_f^\ell = \sum_{i\in\mathcal{I}_n} \ell_{if} - \overline{\ell}_f$. Define $ED = (ED_1, \ldots, ED_{|\mathcal{I}|}, ED_1^\ell, \ldots, ED_{|\mathcal{F}|})^T$ which is a $(|\mathcal{I}| + |\mathcal{F}|) \times 1$ vector. Market clearing requires $ED(\Gamma, z^*, P) = 0$, so that totally differentiating in e yields $\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\frac{dz^*}{de} + \frac{\partial ED}{\partial P}\frac{dP}{de} = 0$. Substituting in for $\frac{dz^*}{de}$, rearranging, and inverting yields $\frac{dP}{de} = -(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial P})^{-1}(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial e})$, which completes the proof.

Proof of Proposition 3. For any prices and aggregates $Q = (P, z^*)$, define the subset $\mathcal{P}(Q) \subset \mathcal{C}_m$ of sectors that the hegemon contracts with and has pressure points on. We divide the proof into the four regions in which the hegemon's optimal contract could lie: (i) the hegemon has no pressure points, $\mathcal{P} = \emptyset$; (ii) the hegemon has pressure points on all sectors, $\mathcal{P} = \mathcal{C}_m$; (iii) the hegemon has pressure points on all domestic sectors but not on some foreign sectors, $\mathcal{I}_m \subset \mathcal{P}$; (iv) the hegemon does not have pressure points on some domestic sectors, $\mathcal{I}_m \cap \mathcal{P} \neq \mathcal{I}_m$. Note that some of these regions may be empty and some points Q cannot be part of an equilibrium.

Case (i): Pressure points on no sectors. Suppose that $\mathcal{P}(Q) = \emptyset$. Then $V_i(\overline{\mathcal{S}}_i') = V_i(\mathcal{S}_i)$ for all $i \in \mathcal{C}_m$, and hence the hegemon must set $\overline{T}_i = 0$ and $\tau_i = 0$ for all i.

Case (ii): Pressure points on all sectors $i \in \mathcal{C}_m$. Suppose that $P(Q) = \mathcal{C}_m$. Since the hegemon has complete instruments for $i \in \mathcal{C}_m$, we adopt the primal approach whereby the hegemon directly selects allocations of firms $i \in \mathcal{C}_m$, and derive the wedges that implement these allocations. The Lagrangian of firm i, with choice variables (x_i, ℓ_i), is

$$\begin{aligned}\mathcal{L} =& \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x^*_{ij})] - \sum_{f \in \mathcal{F}_m} \tau^\ell_{if}(\ell_{if} - \ell^*_{if}) - \overline{T}_i + \beta \nu_i(\mathcal{J}_i) \\ &+ \sum_{S \in \Sigma(\overline{S}'_i)} \lambda_{iS} \left[\beta \Big(\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\Big) - \sum_{j \in S} \left[\theta_{ij} p_j x_{ij} - \mathbf{1}_{S^D_i \subset S} \overline{T}_i\right]\right.\end{aligned}$$

Denoting $\overline{\lambda}_{ij} \equiv \sum_{S \in \Sigma(\overline{\mathcal{S}}'_i) | j \in S} \lambda_{iS}$, the FOCs are

$$\tau_{ij} = \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\lambda}_{ij} \theta_{ij} p_j; \qquad \tau^\ell_{if} = \frac{\partial \Pi_i}{\partial \ell_{if}}$$

Given that the firm's optimization problem is convex, given an incentive compatible allocation (x_i, ℓ_i), and given nonnegative Lagrange multipliers $\lambda_{iS} \geq 0$ such that complementary slackness holds, these equations define wedges that implement (x_i, ℓ_i).

Next consider the hegemon's Lagrangian. Given the hegemon has complete factor wedges on domestic firms, we can treat p^ℓ_f as a direct choice variable of the hegemon (subject to market clearing).[14] Under the primal approach of choosing $\{x_i, \ell_i, \overline{T}_i\}_{i \in \mathcal{C}_m}, \{p^\ell_f\}_{f \in \mathcal{F}_m}$

$$\begin{aligned}\mathcal{L}_m =& W_m \Bigg(p, \sum_{i \in \mathcal{I}_m} \Pi_i(x_i, \ell_i) + \sum_{f \in \mathcal{F}_m} p^\ell_f \bar{\ell}_f + \sum_{i \in \mathcal{D}_m} \overline{T}_i \Bigg) + u_m(z) \\ &+ \sum_{i \in \mathcal{C}_m} \eta_i \Big[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta \nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i)\Big] + \sum_{f \in \mathcal{F}_m} \kappa_f \Big[\bar{\ell}_f - \sum_{i \in \mathcal{I}_m} \ell_f\Big] \\ &+ \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{\mathcal{S}}'_i)} \Lambda_{iS} \Big[\beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\Big] - \sum_{j \in S} \theta_{ij} p_j x_{ij} - \mathbf{1}_{S^D_i \subset S} \overline{T}_i \Big]\end{aligned}$$

Define $z^{NC} = \{z_{ij}\}_{i \notin \mathcal{C}_m}$, and define $P^m = (p, p^\ell_{-m})$ factor prices except those in country m. The direct spillovers $\varepsilon^z_{ij} = \frac{\partial \mathcal{L}_m}{\partial z_{ij}}$ and $\varepsilon^{P^m} = \frac{\partial \mathcal{L}}{\partial P^m}$ are

$$\varepsilon^z_{ij} = \frac{\partial W_m}{\partial w_m} \sum_{k \in \mathcal{I}_m} \frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}} + \sum_{k \in \mathcal{C}_m} \eta_k \left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right].$$

[14] If the hegemon contracts with every firm in a foreign country n, we could treat factor prices in n analogously.

$$\varepsilon^{P^m} = \frac{\partial W_m}{\partial P^m} + \frac{\partial W_m}{\partial w_m}\frac{\partial w_m}{\partial P^m} + \sum_{i\in\mathcal{C}_m}\eta_i\left[\frac{\partial \Pi_i}{\partial P^m} - \frac{\partial V_i}{\partial P^m}\right] - \sum_{i\in\mathcal{C}_m}\sum_{S\in\Sigma(\overline{\mathcal{S}}_i')}\Lambda_{iS}\sum_{j\in S}\theta_{ij}\frac{\partial p_j}{\partial P^m}x_{ij}.$$

Observe that by Envelope Theorem $\frac{\partial W_m}{\partial P^m} = -\frac{\partial W_m}{\partial w_m}\sum_{i\in\mathcal{I}}\frac{\partial p_i}{\partial P^m}C_{mi}$ and $\frac{\partial w_m}{\partial P^m} = \sum_{i\in\mathcal{I}_m}\frac{\partial \Pi_i}{\partial P^m} = \sum_{i\in\mathcal{I}_m}[\frac{\partial p_i}{\partial P^m}y_i - \sum_{j\in\mathcal{J}_i}\frac{\partial p_j}{\partial P_m}x_{ij}]$, from which $\varepsilon_j^{P^m}$ obtains with $X_{m,j} = \mathbf{1}_{j\in\mathcal{I}_m}y_j - \sum_{i\in\mathcal{I}_m}x_{ij} - C_{mj}$. Lastly, the direct spillover of factor prices p_f^ℓ for $f\in\mathcal{F}_m$ is

$$\varepsilon_f = \sum_{i\in\mathcal{I}_m}\eta_i\left[\frac{\partial \Pi_i(x_i,\ell_i,\mathcal{J}_i)}{\partial p_f^\ell} - \frac{\partial V_i(\mathcal{S}_i)}{\partial p_f^\ell}\right] = \sum_{i\in\mathcal{I}_m}\eta_i\left[\ell_{if} - \ell_{if}^{\text{Outside}}\right]$$

where the second equality follows by Envelope Theorem, and $\ell_{if}^{Outside}$ is factor usage of a firm that deviates to the outside option.

FOC for p_f^ℓ for $f\in\mathcal{F}_m$. The hegemon's FOC for domestic factor price p_f^ℓ is $0 = \sum_{i\in\mathcal{I}_m}\eta_i[\ell_{if} - \ell_{if}^{Outside}]$, which follows because the wealth of consumer m does not locally change in the domestic factor price, $\frac{\partial w_m}{\partial p_\ell^f} = \sum_{i\in\mathcal{I}_m}\ell_{if} - \overline{\ell}_f = 0$. Thus excess demand in every good and factor market is unaffected, that is $\frac{\partial ED}{\partial p_f^\ell} = 0$, so other equilibrium prices and quantitites do not change.

FOC for ℓ_{if} for a Domestic Firm. The hegemon's FOC for (domestic) ℓ_{if} is

$$0 = \frac{\partial W_m}{\partial w_m}\frac{\partial \Pi_i}{\partial \ell_{if}} + \eta_i\frac{\partial \Pi_i}{\partial \ell_{if}} - \kappa_f + \mathcal{E}_{if}^\ell,$$

where $\mathcal{E}_{if}^\ell = \varepsilon^{zNC}\frac{dz^{*NC}}{d\ell_{if}} + \varepsilon^{P^m}\frac{dP}{d\ell_{if}}$, where $\varepsilon^{zNC} = \{\varepsilon_{ij}^z\}_{i\notin\mathcal{C}_m}$, where $\frac{dz^{*NC}}{d\ell_{if}}$ and $\frac{dP^m}{d\ell_{if}}$ are defined as in Proposition 2 for the subset of aggregates z^{*NC} and prices P^m. Since the firm's problem yields a tax rate $\tau_{if}^\ell = \frac{\partial \Pi_i}{\partial \ell_{if}}$, then we have $(\frac{\partial W_m}{\partial w_m} + \eta_i)\tau_{if}^\ell = -\mathcal{E}_{if}^\ell + \kappa_f$.

FOC for ℓ_{if} for a Foreign Firm. The hegemon's FOC for (foreign) ℓ_{if} is $0 = \eta_i\frac{\partial \Pi_i}{\partial \ell_{if}} + \mathcal{E}_{if}^\ell$, so $\eta_i\tau_{if}^\ell = -\mathcal{E}_{if}^\ell$.

FOC for x_{ij} for a domestic firm. Let $\mathcal{E}_{ij} = \varepsilon_{ij}^z + \varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}} + \varepsilon^{P^m}\frac{dP}{dz_{ij}}$ and let $\overline{\Lambda}_{ij} = \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')|j\in S}\Lambda_{iS}$. For a domestic sector, the hegemon's FOC for x_{ij} is

$$0 = \frac{\partial W_m}{\partial w_m}\frac{\partial \Pi_i}{\partial x_{ij}} + \eta_i\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j + \mathcal{E}_{ij}$$

To obtain the implementing taxes, construct the firm nonnegative Lagrange multiplier as $\lambda_{iS} = \frac{\Lambda_{iS}}{\frac{\partial W_m}{\partial w_m}+\eta_i}$. The firm's FOC is therefore $\tau_{ij}(\frac{\partial W_m}{\partial w_m} + \eta_i) = (\frac{\partial W_m}{\partial w_m} + \eta_i)\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j$, which combined with the planner's FOC yields $\tau_{ij}(\frac{\partial W_m}{\partial w_m} + \eta_i) = -\mathcal{E}_{ij}$.

FOC for x_{ij} for a foreign sector. The hegemon's FOC for (foreign) x_{ij} is

$$0 = \eta_i \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j + \mathcal{E}_{ij}.$$

For a positive constant $\alpha > 0$, we add and subtract $\alpha\frac{\partial \Pi_i}{\partial x_{ij}}$ to obtain $(\eta_i + \alpha)\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j = -\mathcal{E}_{ij} + \alpha\frac{\partial \Pi_i}{\partial x_{ij}}$. Constructing the nonnegative firm Lagrange multiplier $\lambda_{iS} = \frac{\Lambda_{iS}}{\eta_i+\alpha}$ and combining the firm's FOC with the planner's FOC obtains $\tau_{ij}(\eta_i + \alpha) = -(\mathcal{E}_{ij} - \alpha\frac{\partial \Pi_i}{\partial x_{ij}})$. Taking the limit $\alpha \to 0$ yields $\tau_{ij}\eta_i = -\mathcal{E}_{ij}$ (The limiting argument is used because if $\eta_i = 0$, then $\lambda_{iS} \to +\infty$ as $\alpha \to 0$.)

FOC for $\overline{T}_i$ for a domestic sector. Holding fixed allocations, a transfer $\overline{T}_i$ for a domestic sector has no impact on excess demand in any market, since it redistributes from country m's firms to country m's consumer. The FOC is $0 \geq -\eta_i - \overline{\Lambda}_i$, so that $\overline{T}_i = 0$.

FOC for $\overline{T}_i$ for a foreign sector in country n. Holding fixed allocations, a transfer $\overline{T}_i$ reallocates wealth from consumers in country n to consumers in country m. The FOC is $0 \geq \frac{\partial W_m}{\partial w_m} - \eta_i - \overline{\Lambda}_i + \Xi_{mn}$ (for Ξ_{mn} defined as in main text).

Case (iii): Pressure points on all domestic firms but not some foreign firms. Suppose $\mathcal{D}_m^p \subset \mathcal{D}_m$ with $\mathcal{D}_m^p \cap \mathcal{P}(Q) = \emptyset$. As in case (i), $\overline{T}_i = 0$ and $\tau_i = 0$ for $i \in \mathcal{D}_m^p$. Redefining the contractible set as $\mathcal{C}_m^{new} = \mathcal{C}_m \backslash \mathcal{D}_m^p$, analysis proceeds as in case (ii).

Case (iv): Pressure points on some or no domestic firms. Suppose $\mathcal{I}_m^p \subset \mathcal{I}_m$ and $\mathcal{D}_m^p \subset \mathcal{D}_m$ with $(\mathcal{I}_m^p \cup \mathcal{D}_m^p) \cap \mathcal{P}(Q) = \emptyset$. As in case (i), $\overline{T}_i = 0$ and $\tau_i = 0$ for $i \in \mathcal{I}_m^p \cup \mathcal{D}_m^p$. Redefine the contractible set as $\mathcal{C}_m^{new} = \mathcal{C}_m \backslash (\mathcal{I}_m^p \cup \mathcal{D}_m^p)$ and $\mathcal{D}_m^{new} = \mathcal{D}_m \backslash \mathcal{D}_m^p$. Redefine $P^m = P$. The hegemon's Lagrangian over $\{x_i, \ell_i, \overline{T}_i\}_{i \in \mathcal{C}_m^{new}}$ is

$$\begin{aligned}\mathcal{L}_m =& W_m\left(p, \sum_{i \in \mathcal{I}_m \backslash \mathcal{I}_m^p} \Pi_i(x_i, \ell_i, \mathcal{J}_i) + \sum_{i \in \mathcal{I}_m^p} V_i(\mathcal{S}_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i \in \mathcal{D}_m^{new}} \overline{T}_i\right) + u_m(z) \\ &+ \sum_{i \in \mathcal{C}_m^{new}} \eta_i \left[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta(\nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i))\right]\end{aligned}$$

$$+\sum_{i\in\mathcal{C}_m}\sum_{S\in\Sigma(\overline{\mathcal{S}}'_i)}\Lambda_{iS}\left[\beta\left[\nu_i(\mathcal{J}_i)-\nu_i(\mathcal{J}_i\backslash S)\right]-\sum_{j\in S}\theta_{ij}p_jx_{ij}-\mathbf{1}_{S_i^D\subset S}\overline{T}_i\right]$$

Analysis parallels case (ii) and we highlight the differences. We have

$$\varepsilon^z_{ij}=\frac{\partial W_m}{\partial w_m}\left[\sum_{k\in\mathcal{I}_m\backslash\mathcal{I}^p_m}\frac{\partial\Pi_k}{\partial z_{ij}}+\sum_{k\in\mathcal{I}^p_m}\frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]+\frac{\partial u_m(z)}{\partial z_{ij}}+\sum_{k\in\mathcal{C}^{new}_m}\eta_k\left[\frac{\partial\Pi_k}{\partial z_{ij}}-\frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right].$$

$\varepsilon^{P^m}=\varepsilon^P$ is formally defined as before (with $\mathcal{C}^{new}_m$ replacing $\mathcal{C}_m$) but includes all price spillovers. Under the new definitions, the first order conditions for ℓ_{if} are the same as case (ii) with $\kappa_f=0$, while the first order conditions for x_{ij} and $\overline{T}_i$ are identical to case (ii).

Proof of Proposition 4. Proposition 1 holds for the global planner by the same argument. The firm Lagrangian and first order conditions are same as in the proof of Proposition 3. The global planner's Lagrangian is the same as the hegemon's up to the new objective function, $\sum_{n=1}^N\Omega_n[W_n(p,w_n)+u_n(z)]$, which includes welfare of every country. Formal analysis proceeds analogously to the proof of Proposition 3 up to the new objective function. Absent a pressure point on sector i, $\overline{T}_i=0$ and $\tau_i=0$. For any sector i located in country n, the same derivations yield input wedges satisfying $(\Omega_n\frac{\partial W_n}{\partial w_n}+\eta_i)\tau_{ij}=-\mathcal{E}^p_{ij}$ (note this sector is valued by n's consumer). The externality vector $\mathcal{E}^p_{ij}$ is formally defined by the same equation, but replacing ε^z_{ij} and ε^{P^m} with:

$$\varepsilon^{zp}_{ij}=\sum_{n=1}^N\Omega_n\left[\frac{\partial W_n}{\partial w_n}\frac{\partial w_n}{\partial z_{ij}}+\frac{\partial u_n}{\partial z_{ij}}\right]+\sum_{k\in\mathcal{C}_m}\eta_k\left[\frac{\partial\Pi_k}{\partial z_{ij}}-\frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]$$

$$\varepsilon^{P^mp}=\sum_{n=1}^N\Omega_n\frac{dW_n}{dP^m}+\sum_{i\in\mathcal{C}_m}\left[\eta_i\left[\frac{\partial\Pi_i}{\partial P^m}-\frac{\partial V_i(\mathcal{S}_i)}{\partial P^m}\right]-\sum_{S\in\Sigma(\overline{\mathcal{S}}'_i)}\Lambda_{iS}\sum_{j\in S}\theta_{ij}\frac{\partial p_j}{\partial P^m}x_{ij}\right]$$

where $\frac{dW_n}{dP^m}=\frac{\partial W_n}{\partial P^m}+\frac{\partial W_n}{\partial w_n}\frac{\partial w_n}{\partial P^m}$. The spillover Ξ^p_{mn} is defined as before, replacing $\varepsilon^z_{ij},\varepsilon^{P^m}$ with $\varepsilon^{zp}_{ij},\varepsilon^{P^mp}$ The condition for no redistributive motive is therefore $\Omega_m\frac{\partial W_m}{\partial w_n}-\Omega_n\frac{\partial W_n}{\partial w_n}+\Xi^p_{mn}=0$. Finally, the FOC for a transfer $\overline{T}_i$ for a firm in country n is $0\geq-\eta_i-\overline{\Lambda}_i+\Omega_m\frac{\partial W_m}{\partial w_n}-\Omega_n\frac{\partial W_n}{\partial w_n}+\Xi^p_{mn}=-\eta_i-\overline{\Lambda}_i$, so that $\overline{T}_i=0$. This completes the proof.

ONLINE APPENDIX FOR "A FRAMEWORK FOR GEOECONOMICS"

Christopher Clayton Matteo Maggiori Jesse Schreger

June 2024

Not for Publication

B.1. EXTENDING THE FRAMEWORK: HEGEMONIC COMPETITION FOR DOMINANCE

We now consider the possibility that multiple countries are hegemons. For simplicity, we focus on the case in which two countries, m_1 and m_2, are hegemons. To streamline analysis, we focus on competition over transfers, and assume constant prices and no z-externalities (Definitions 3 and 4).

Each hegemon offers a contract as described in Section 3, taking as given the contract offered by the other hegemon. As usual, we begin by taking as given continuation value functions ν_i of firms.

B.1.1. *Competition Setup*

Let $\mathcal{C} = \mathcal{C}_{m_1} \cup \mathcal{C}_{m_2}$ be the set of firms that contract with at least one hegemon. Hegemon $m \in \{m_1, m_2\}$ offers a contract $\{\Gamma_i^m\}_{i \in \mathcal{C}_m}$, where $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, \tau_i^m\}_{i \in \mathcal{C}_m}$ denotes the contract offered to firm $i \in \mathcal{C}_m$. As in Section 3, the joint threat $\mathcal{S}_i'$ must be feasible under direct transmission. In the analysis that follows, it will be notationally convenient to designate a hypothetical trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ offered by hegemon m to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$. This reduces cumbersome notation of tracking which firms are offered one or two contracts by ensuring all firms in $\mathcal{C}$ are offered two contracts (one of which may be trivial, purely hypothetical, and equivalent to their outside option). We let $\Gamma^m = \{\mathcal{S}_i'^m, \mathcal{T}_i, \tau_i^m\}_{i \in \mathcal{C}}$ be the hegemon's contract, including trivial contracts offered to firms $i \notin \mathcal{C}_m$.

Firm i faces revenue-neutral wedges and transfers from both hegemons that are added together when both contracts are accepted.[1] Anticipating that a best response to hegemon

[1]Each hegemon takes as given the other hegemon's equilibrium rebates when both contracts are accepted. If firm i chooses to only accept one contract, equilibrium rebates by the hegemon whose contract is accepted are those that maintain revenue neutrality under the single contract, while there are no rebates by the hegemon whose contract was rejected. If neither contract is accepted, there are no rebates.

$-m$ setting $\tau_i^{-m} = 0$ is for hegemon m to set $\tau_i^m = 0$, we will solve the model assuming all wedges to be zero, and then verify that neither hegemon has an incentive to deviate to nonzero wedges. Therefore, we write the contract $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^{m_1} + \mathcal{T}_i^{m_2}, 0\}$ as the combined contract when firms accept both contracts.

The joint threat $\mathcal{S}_i'$ arising when firm i accepts both contracts is constructed by taking the union of joint trigger sets and applying Lemma 1 (see the proof of Lemma 1 for details on triggers). Here we detail the special case where both hegemons offer maximal joint threats, as indeed they will in equilibrium. Recalling that $S_i^{Dm} = \bigcup_{S \in \mathcal{S}_i^{Dm}} S$ and $\overline{\mathcal{S}}_i'^m = \{S_i^{Dm}\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, where we define $S_i^{Dm} = \emptyset$ if $i \notin \mathcal{C}_m$. Then, maximal (combined) joint threats, $\overline{S}_i'$, is given by

$$\overline{S}_i' = (\mathcal{S}_i \backslash (\mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2})) \cup \mathcal{X}_i, \quad \mathcal{X}_i = \begin{cases} \{S_i^{Dm_1}, S_i^{Dm_2}\} & S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset \\ \{S_i^{Dm_1} \cup S_i^{Dm_2}\} & \text{otherwise} \end{cases} \tag{B.1}$$

Intuitively, $\overline{\mathcal{S}}_i'$ combines both hegemon's maximal joint threats into a single maximal joint threat if the two have any common threats. If there are no common threats, the two hegemon's maximal joint threats are separate actions within $\overline{\mathcal{S}}_i'$.

Finally, we define the participation constraints of all firms. In particular, hegemon m's contract is accepted by firm i if

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.2}$$

Both contracts are accepted by firm i if

$$V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}. \tag{B.3}$$

B.1.2. *Existence of an Equilibrium*

We show existence of an equilibrium in which both hegemons offer maximal joint threats, and both hegemon's contracts are accepted. We then discuss how competition shapes the transfers extracted.

The model with two hegemons has to account for the fact that if hegemon m's contract is rejected by firm i, then hegemon m can no longer use firm i in joint threats.[2] This is important because a best response of hegemon m to a contract Γ^{-m} might involve offering a contract to firm i that leads firm i to reject the contract of hegemon $-m$. To make progress, we restrict the form of the network structure as follows. Let $\mathcal{P} = \{i \in \mathcal{C} \mid V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)\}$ denote the set of firms for which the two hegemons can, possibly only jointly, generate a pressure point.

DEFINITION 5: *Hegemon pressure points are* ***isolated*** *if:* $i \in \mathcal{P} \Rightarrow \mathcal{J}_i \cap \mathcal{P} = \emptyset$.

Definition 5 states that if the two hegemons can generate a pressure point on i, then the two hegemons cannot generate a pressure point on any firm $j \in \mathcal{J}_i$ that is immediately upstream from i. It ensures that two firms with pressure points from the set of hegemons they contract with are not directly linked to one another. Using this condition, we can now prove that an equilibrium exists in which both hegemons offer maximal joint threats with no wedges.

PROPOSITION 5: *Suppose that hegemon pressure points are isolated. An equilibrium of the model with competition exists in which each hegemon m offers a contract featuring maximal joint threats and no wedges, $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^{m*}, 0\}$, to each $i \in \mathcal{C}_m$. Transfers from all firms $i \notin \mathcal{P}$ are zero. Each firm $i \in \mathcal{C}$ accepts the contract(s) it is offered.*

The proof of Proposition 5 proceeds by constructing transfers $\overline{T}_i^{m*}$ such that each contract Γ_i^m is a best response to contract Γ_i^{-m}, and such that both contracts are accepted, that is $V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2}), V_i(\mathcal{S}_i)\}$.

The transfers extracted by each hegemon from a foreign firm $i \notin \mathcal{I}_{m_1} \cup \mathcal{I}_{m_2}$ depend on the degree to which they can provide different threats. In the limit where hegemon threats have no overlap, $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2} = \emptyset$, competition is limited because each hegemon offers a different set of threats. By contrast when threats have full overlap, $\mathcal{S}_i^{Dm_1} = \mathcal{S}_i^{Dm_2}$, the two hegemons offer the same set of threats, and so bid each other down to zero transfers, $\overline{T}_i^m = 0$. In this case, firms receive full surplus from the relationships. This result is reminiscent

[2]This was not an issue in the model with a single hegemon because that hegemon always ensured its contract satisfied the participation constraint.

of the Bertrand paradox, in which two firms competing on prices bid each other down to the perfect competition price. This outcome is also efficient ex post, since all joint threats are supplied and no transfers are extracted.

For a firm that is domestic to hegemon m, that is $i \in \mathcal{I}_m$, it remains optimal for hegemon m to demand no transfers, $\overline{T}_i^{m*} = 0$. Hegemon $-m$ then extracts the largest transfer that leaves firm i indifferent between accepting both contracts and accepting only that of hegemon m: $V_i(\overline{S}_i', \overline{T}_i^{-m*}) = V_i(\overline{\mathcal{S}}_i'^m)$. Thus the joint threats that the firm's own hegemon can provide become that firm's outside option, to which that firm is held by the other hegemon.

Proof of Proposition 5. Given constant prices and no z externalities (Definitions 3 and 4), the objective function of hegemon m is to maximize its country's wealth level,

$$w_m = \sum_{i \in \mathcal{I}_m} V_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \sum_j T_{ij}.$$

We assume that $\tau_i = 0$ for both hegemons, and then verify that neither hegemon has an incentive to deviate.

Given hegemons do not have a pressure point on firm $i \notin \mathcal{P}$, both hegemons must offer a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ to such firms to avoid having their contract rejected. Since all firms $i \notin \mathcal{P}$ therefore trivially accept the contracts they are offered, given Definition 5 then the decision problem of each hegemon becomes separable across sectors $i \in \mathcal{P}$. This is due not only to separability of the objective function, but also because under Definition 5, a joint threat is feasible if it is feasible under direct transmission, even if some firms in $\mathcal{P}$ reject hegemon m's contract, given that every firm $i \in \mathcal{P}$ has $\mathcal{J}_i \cap \mathcal{P} = \emptyset$ (i.e., direct transmission links satisfy $S_i^D \subset \mathcal{J}_i \backslash \mathcal{P}$).

We begin by providing the analog of Proposition 1: both hegemons offer contracts featuring maximal joint threats to all firms $i \in \mathcal{P}$

LEMMA 2: *Fix a contract Γ^{-m} of hegemon $-m$. Then for all $i \in \mathcal{P}$, it is weakly optimal for hegemon m to offer maximal joint threats, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$.*

Proof of Lemma 2. Fix a contract $\Gamma_i^{-m} = \{\mathcal{S}_i'^{-m}, \mathcal{T}_i^{-m}, 0\}$ of hegemon $-m$. The proof strategy is to show that if a contract $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, 0\}$ is accepted by firm i, then the contract $\Gamma_i^{m\prime} = \{\overline{\mathcal{S}}_i^{\prime m}, \mathcal{T}_i^m, 0\}$ is also accepted by firm i. Let $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers Γ_i^m, and $\Gamma_i' = \{\mathcal{S}_i'', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ the joint contract if hegemon m offers $\Gamma_i^{m\prime}$. Since the contract Γ_i^m is accepted by firm i, then

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

Since $\overline{\mathcal{S}}_i^{\prime m}$ is a joint threat of $\mathcal{S}_i^{\prime m}$, then $\mathcal{S}_i''$ is a joint threat of $\mathcal{S}_i'$. Therefore, $V_i(\Gamma_i^{m\prime}) \geq V_i(\Gamma_i^m)$ and $V_i(\Gamma_i') \geq V_i(\Gamma_i)$. Therefore,

$$\max\{V_i(\Gamma_i'), V_i(\Gamma_i^{m\prime})\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\},$$

and hence contract $\Gamma_i^{m\prime}$ is also accepted by firm i. Finally, firm i is weakly better off (which is valued by hegemon m if firm i is domestic). Thus, maximal joint threats is a weak best response, concluding the proof. □

From Lemma 2, $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ is a best response to any contract Γ_i^{-m}, and therefore all transfers of m appear under the joint threat. Thus we will focus on the total transfer $\overline{T}_i$ for firms $i \in \mathcal{P}$. The optimal contract for firm i is characterized by Corollary 1 if only one hegemon contracts with i, so assume $i \in \mathcal{C}_{m_1} \cap \mathcal{C}_{m_2}$.

Let $\Gamma_i^m = \{\overline{\mathcal{S}}_i^{\prime m}, \overline{T}_i^m, 0\}$ be a candidate optimal contract of hegemon m, and let $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^{m_1} + \overline{T}_i^{m_2}, 0\}$ be the joint contract.

Foreign Firms. Let $i \in \mathcal{P} \backslash (\mathcal{I}_{m_1} \cup \mathcal{I}_{m_2})$ be a firm foreign to both hegemons. We begin with the following intermediate result.

LEMMA 3: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium is which firm i accepts both contracts if and only if one of the following holds:*

1. *Firm i is held to its outside option, with*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.4}$$

B.6

2. *Firm i exceeds its outside option, with*

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i) \tag{B.5}$$

Proof of Lemma 3. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition. Then, if either hegemon increased its transfer, the firm would reject both contracts and revert to the outside option. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}.$$

Then, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Suppose then that (without loss)

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) > V_i(\Gamma_i^{-m}).$$

Then again, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore,

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i).$$

Finally, supposing this condition holds, then if either hegemon increased its transfer, the firm would reject its contract and accept only that of the other hegemon. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof of Lemma 3. □

We use Lemma 3 to construct an equilibrium. Since $i \in \mathcal{P}$, $V_i(\overline{\mathcal{S}}'_i) > V_i(\mathcal{S}_i)$. Without loss of generality, let $V_i(\overline{\mathcal{S}}'^m_i) \geq V_i(\overline{\mathcal{S}}'^{-m}_i)$. We begin by constructing the minimal transfer $t_0^m \geq 0$ such that $V_i(\overline{\mathcal{S}}'^m_i, t_0^m) = V_i(\overline{\mathcal{S}}'^{-m}_i, 0)$. Since $\overline{\mathcal{S}}'_i$ is a joint threat of $\overline{\mathcal{S}}'^m_i$, and therefore $V_i(\overline{\mathcal{S}}'_i, t_0^m) \geq V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$. If $V_i(\overline{\mathcal{S}}'_i, t_0^m) = V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$, then we have found contracts such that $V_i(\Gamma_i) = V_i(\Gamma_i^m) = V_i(\Gamma_i^{-m})$, and hence either equation (B.4) or (B.5) is satisfied. Thus we have an equilibrium.

Suppose instead $V_i(\overline{\mathcal{S}}'_i, t_0^m) > V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$. Then, we construct a function $t^{-m}(t)$ by

$$V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t) = V_i(\overline{\mathcal{S}}'^{-m}_i, t^{-m}(t)).$$

We can construct this function from $t = 0$ to $t = \bar{t}$, where $\bar{t}$ solves $V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t) = V_i(\mathcal{S}_i)$ (note it is possible for $\bar{t} = 0$).

Suppose first $\exists t^* \in [0, t]$ such that

$$V_i(\overline{\mathcal{S}}'_i, t_0^m + t^* + t^{-m}(t^*)) = V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t^*).$$

Then, equation (B.5) is satisfied if $t^* < \bar{t}$, and equation (B.4) is satisfied if $t^* = \bar{t}$. Therefore, by Lemma 3 we have found an equilibrium.

Suppose instead that no such t^* exists, and therefore $V_i(\overline{\mathcal{S}}'_i, t_0^m + \bar{t} + t^{-m}(\bar{t})) > V_i(\mathcal{S}_i)$. Then, define T^* such that $V_i(\overline{\mathcal{S}}'_i, T^*) = V_i(\mathcal{S}_i)$, and define $\overline{T}_i^m$ and $\overline{T}_i^{-m}$ such that $\overline{T}_i^m + \overline{T}_i^{-m} = T^*$, $\overline{T}_i^{-m} \geq t_0^m + \bar{t}$, and $\overline{T}_i^{-m} \geq t^{-m}(\bar{t})$. Then, equation (B.4) is satisfied, and hence we have found an equilibrium.

Therefore, an equilibrium exists as described, assuming both hegemons impose zero wedges. Observe that imposing nonzero wedges cannot increase the value of its objective, and leads to its contract being (weakly) rejected. Thus, zero wedges is a best response of each hegemon, concluding this portion of the proof.

Domestic Firms. Let $i \in \mathcal{P} \cap \mathcal{I}_m$ be a domestic firm of hegemon m. We obtain the following result, which parallels Lemma 3.

LEMMA 4: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm* $i \in \mathcal{P} \cap \mathcal{I}_m$ *accepts both contracts if and only if one of the following holds:*

1. *Firm* i *is held to its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.6}$$

2. *Firm* i *exceeds its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.7}$$

Proof of Lemma 4. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition and that $\overline{T}_i^m = 0$. If hegemon $-m$ increased its transfer, then its contract would be rejected. If hypothetically hegemon m had a positive transfer, it could decrease the transfer, have its contract remain accepted, and increase value of its domestic firm i. Therefore, we have an equilibrium if $\overline{T}_i^m = 0$.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > V_i(\Gamma_i^m).$$

Then, hegemon $-m$ could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = V_i(\Gamma_i^m)$, and therefore

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

If this condition holds, and $\overline{T}_i^m > 0$, then hegemon m could decrease its transfer for its domestic firm without its contract being rejected, and so be strictly better off. Therefore, $\overline{T}_i^m = 0$. Finally, suppose this condition holds and $\overline{T}_i^m = 0$. Then, if hegemon $-m$ increased its transfer, its contract would be rejected. Hegemon m cannot further decrease its transfer. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof. $\square$

Lemma 4 shows that $\overline{T}_i^m = 0$ in any equilibrium, that is a domestic firm is not charged a transfer by its hegemon. Since $\overline{T}_i^m = 0$, then $V_i(\Gamma_i^{-m}) \leq V_i(\Gamma_i)$. We can construct the transfer of hegemon $-m$ as the solution to $V_i(\overline{\mathcal{S}_i}', \overline{T}_i^{-m}) = V_i(\overline{\mathcal{S}}_i'^m)$. If $V_i(\overline{\mathcal{S}}_i'^m) = V_i(\mathcal{S}_i)$, then equation (B.6) is satisfied and we have an equilibrium. If $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$, then equation (B.7) is satisfied and we have an equilibrium. In both cases, zero wedges is part of an optimal policy for both hegemons. Therefore, we have an equilibrium.

This concludes the proof of existence.

B.2. ADDITIONAL RESULTS AND DERIVATIONS

B.2.1. *Manipulating the Outside Option*

We show how to extend our setup to allow the hegemon to make threats conditional on a firm rejecting the contract. This amounts to manipulating the outside option of targeted entities by threatening to cut off access to the inputs controlled by the hegemon if the contract is rejected.

In addition to specifying a joint threat $\mathcal{S}_i'$, transfers $\mathcal{T}_i$, and wedges τ_i, the hegemon can also impose a *punishment* $\mathcal{P}_i$, which is a restriction that the firm permanently loses access to inputs contained in $\mathcal{P}_i \subset \mathcal{S}_i$ if it rejects the hegemon's contract.[3] As with feasibility of joint threats, it is feasible for the hegemon to use $S \in \mathcal{S}_i$ to form a punishment if $\exists j \in S$ such

[3] It is also straight-forward to allow for the punishment to entail only loss of access for the current period.

that $j \in \mathcal{C}_m$.[4] Let $\mathcal{B}_i(\mathcal{P}_i) = \mathcal{J}_i \backslash (\bigcup_{S \in \mathcal{P}_i} S)$ be the set of retained inputs given punishment $\mathcal{P}_i$. The outside option of firm i is therefore

$$\begin{aligned} V_i^o(\mathcal{P}_i) \;&=\; \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i(\mathcal{P}_i)) + \beta \nu_i(B_i(\mathcal{P}_i)) \\ &s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \Big[\nu_i(B_i(\mathcal{P}_i)) - \nu_i(\mathcal{B}_i(\mathcal{P}_i) \backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i(\mathcal{P}_i))). \end{aligned}$$

which leaves implicit that $x_{ij} = 0$ for $j \notin \mathcal{B}_i(\mathcal{P}_i)$. The participation constraint is therefore

$$V_i(\Gamma_i) \geq V_i^o(\mathcal{P}_i).$$

Similar in spirit to Proposition 1, the optimal punishment is the one that minimizes the outside option, and we denote this minimal value $\underline{V}_i^o$.[5] From here, formal analysis of the optimal contract would proceed analogously to the baseline model with $\underline{V}_i^o$ replacing $V_i(\mathcal{S}_i)$.

A one-off threatened punishment at date t is (weakly) effective for the hegemon taking continuation values as given. However, in a Markov equilibrium the threat would be made in every period. For exposition, suppose the participation constraint binds. Lowering the future value of retaining access to the hegemon's inputs therefore lowers the continuation value $\nu_i(\mathcal{J}_i) = V_i^o(\mathcal{P}_i)$, which tightens the participation constraint this period. Therefore, threats of punishment for contract rejection are partially self-defeating. In contrast, with joint threats the hegemon first increases the inside option to $V_i(\overline{\mathcal{S}}_i')$, and then uses demands for costly actions to lower it to $V_i(\mathcal{S}_i)$, making the continuation value $\nu_i(\mathcal{J}_i) = V_i(\mathcal{S}_i)$ the same (for the same equilibrium prices and aggregates) as if the firm had not engaged with the hegemon.

[4] For brevity we state punishments as losing access to elements of $\mathcal{S}_i$, but it is easy to extend to allow for punishments of losing access to specific inputs (while potentially retaining acess to other inputs of $S \in \mathcal{S}_i$).

[5] That is, $\underline{V}_i^o = \min_{\mathcal{P}_i \subset \mathcal{S}_i | \mathcal{P}_i \text{ is feasible}} V_i^o(\mathcal{P}_i)$. Although the economically intuitive case is the one in which the outside is minimized with the threat to cut off as many inputs as possible, translating optimality of a maximal utility punishment into cutting off the most varities requires that V_i^o be a nonincreasing function, which cannot be guaranteed in general due to incentive problems.

B.2.2. *Additional Derivations on Constructing Value Functions*

B.2.2.1. *Additional Derivations on Construction of $\mathcal{V}$*

For an action set (basis), in this appendix we show details how to construct $\mathcal{V}_i(\mathcal{B}_i)$ for all $\mathcal{B}_i \in \Sigma_i(\mathcal{S}_i)$. Since f_i is increasing, concave, and satisfies Inada conditions, then defining

$$\overline{v}_i \equiv \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{J}_i)$$

we have $\overline{v}_i < +\infty$. Thus we must have $\mathcal{V}_i(\mathcal{B}_i) \leq \frac{1}{1-\beta}\overline{v}_i$ for all $\mathcal{B}_i$.

That $\mathcal{V}_i(\emptyset) = 0$ follows trivially from $f_i(0,\ell_i,z) = 0$. Consider first an element $B_i \in \mathcal{S}_i$, so that the continuation value from the stealing action is zero. To construct an SPE, we define for $u \geq 0$ the equation

$$\mathcal{V}_i(\mathcal{B}_i|u) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) + \beta\mathcal{V}_i(\mathcal{B}_i|u) \quad s.t. \quad \sum_{j\in\mathcal{B}_i} \theta_{ij}p_j x_{ij} \leq \beta u. \tag{B.8}$$

Since $\overline{v}_i < +\infty$, we can for any $u \geq 0$ define the unique finite value $v_i(u)$ by

$$v_i(u) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) \quad s.t. \quad \sum_{j\in\mathcal{B}_i} \theta_{ij}p_j x_{ij} \leq \beta u$$

Then, $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta}v_i(u)$ is the unique solution to equation (B.8). Therefore, there is an SPE without stealing with value $\mathcal{V}_i(\mathcal{B}_i) = u$ if

$$\frac{1}{1-\beta}v_i(u) = u.$$

Consider the function $\Delta(u) = \frac{1}{1-\beta}v_i(u) - u$. Zeros of this function provide values in SPEs with no stealing. First, $\Delta(0) = 0$ (which is thus an SPE). There is also a positive SPE: from the Inada condition, $\Delta'(0+) = +\infty$, and hence $\Delta(\epsilon) > 0$ for sufficiently small ϵ. Likewise since $v_i(u) \leq \overline{v}_i$, then $\Delta(u) < 0$ for $u > \frac{1}{1-\beta}\overline{v}_i$. Hence by continuity, there is at least one positive SPE $u > 0$. Finally, since f is concave and $\sum_{j\in\mathcal{B}_i} \theta_{ij}p_j x_{ij} \leq u$ describes a convex set, then $v_i(u)$ is increasing and concave in u, and hence $\Delta(u)$ is concave. Therefore, there is exactly one positive value of u.

Next consider the induction. Suppose we have constructed, either as SPEs or with reversion, values for all $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))\backslash\{\mathcal{B}_i\}$. That is we know the continuation values $\mathcal{V}_i(\mathcal{B}_i\backslash S)$ $\forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ from previously constructed SPEs. Then we construct the value

$$\begin{aligned}\mathcal{V}_i(\mathcal{B}_i|u) = &\max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) + \beta\mathcal{V}_i(\mathcal{B}_i|u)\\ &\qquad s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[u - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))\end{aligned}$$

Thus defining stage game payoff as

$$v_i(u) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) \quad s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[u - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

then we have $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta}v_i(u)$. We construct the fixed points, if any, of $\frac{1}{1-\beta}v_i(u) = u$. As before, $v_i(u)$ is increasing and concave, and therefore there are at most two positive fixed points. If instead no fixed point exists, the suppliers that Trust firm i update to an element $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))\backslash\{\mathcal{B}_i\}$ and we set $\mathcal{V}_i(\mathcal{B}_i) = \mathcal{V}_i(\hat{\mathcal{B}}_i)$.

B.2.2.2. *Continuation Value Functions in Hegemon Problem*

The hegemon's optimal contract was characterized in Section 3 for a given set of continuation value functions ν_i. We now provide the equilibrium consistency conditions for a Markov equilibrium. Consider a set of continuation value functions $\nu = \{\nu_i\}$ for firms. Given these continuation value functions, let (Γ, P, z) be the hegemon's optimal contract, prices, and aggregates when the continuation value functions are ν. Then, (Γ, P, z, ν) is an equilibrium if: (i) $\nu_i(\mathcal{B}_i) = \mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)\backslash\{\mathcal{J}_i\}$; and, (ii) $\nu_i(\mathcal{J}_i) = V_i(\Gamma_i)$.

B.2.3. *Specializing the Model to Nested CES Production Functions*

Assume there are only two periods and that in the second period there are no incentive problems (i.e. all θ's are set to zero in the second period). Each sector uses a two-tier nested constant elasticity of substitution (CES) production function. Firm i produces using input vector x_i with length $|\mathcal{J}_i|$ and, for simplicity, no local factors. The inputs are partitioned into bundles, where $\tilde{x} \in \tilde{X}_i$ denotes the varieties of inputs used in a given bundle, and $\tilde{X}_i$

is the set of all bundles. We assume each input only enters one bundle. The production function is then given by:

$$f_i(x_i) = \left(\sum_{\tilde{x} \in \tilde{X}_i} \tilde{\alpha}_{i\tilde{x}} \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{\rho_i}{\chi_{i\tilde{x}}}} \right)^{\frac{\xi_i}{\rho_i}}. \tag{B.9}$$

We allow CES parameters $\chi_{i\tilde{x}}$ to vary across bundles. At time zero, the loss in continuation value arising from stealing variety k is given by:

$$\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i \backslash \{k\}) = -\frac{\xi_i}{1-\xi_i} \frac{1-\rho_i}{\rho_i} \log \left[1 - \Omega_{i\tilde{x}_k} \left(1 - \left(1 - \omega_{ik} \right)^{\frac{1-\chi_{i\tilde{x}_k}}{\chi_{i\tilde{x}_k}} \frac{\rho_i}{1-\rho_i}} \right) \right], \tag{B.10}$$

where $\Omega_{i\tilde{x}_k}$ is the expenditure share of firm i on the bundle that contains input k denoted by $\tilde{x}_k$, and ω_{ik} is the expenditure share on input k within that bundle. We provide a step-by-step derivation of this equation below and definitions of the expenditure shares, but we first provide some intuition.

All else equal, losing varieties with bigger expenditure shares leads to a greater loss. Intuitively, losing inputs that are cheap (low p_k) or are technologically a large fraction of production (i.e. high related α's) increases the loss. Losing a variety k is more costly the closer the production function is to constant returns to scale $\xi \uparrow 1$ because a more scalable production suffers more from one of its inputs being constrained at zero.

To understand the role of substitutability within and across buckets, consider the specific bucket that contains variety k. Fix a within-bundle expenditure share ω_{ik}. If that bucket has a parameter $\chi_{i\tilde{x}_k} \leq 0$ (i.e. more complementarity than Cobb-Douglas), then losing variety k amounts to the same as losing the entire bucket. Intuitively, this occurs because the absence of input k makes strictly positive production from that bucket impossible. For parameters $\chi_{i\tilde{x}_k} > 0$, the loss decreases the more the varieties are substitutable. A similar logic applies across baskets and is governed by the parameter ρ_i.

This example illustrates the role of "alternatives" in diminishing the value of threats to shut off a firm from a particular input. Intuitively, the existence of closely substitutable inputs or the fact that a particular input accounts for a small expenditure share, decreases this input's strategic value in threats.

Derivation of Equation (B.10). Starting from the nested CES production function in equation (B.9), we first solve the expenditure minimization problem associated with bundle $\tilde{x}$, given by

$$\min \sum_{j\in\tilde{x}} p_j x_{ij} \quad s.t. \quad \left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \geq \overline{x}$$

Letting λ denote the Lagrange multiplier on the production constraint, the FOCs are

$$0 = p_j - \lambda \left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}-1} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}-1}$$

$$\Rightarrow \left(\frac{p_j}{\alpha_{ij}} \frac{\alpha_{ik}}{p_k}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ij} = x_{ik}$$

Substituting into the production constraint yields

$$\overline{x} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \left(\frac{p_k}{\alpha_{ik}}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ik}.$$

Therefore, the expenditure function is

$$e_i(p, \overline{x}) = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}} \overline{x}.$$

We therefore define the price index $P_{i\tilde{x}} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$ associated with total consumption of basket $\tilde{x}$.

The optimization problem thus reduces to an optimization problem over bundles. We abuse notation and use $\tilde{x}$ as aggregate consumption of bundle $\tilde{x}$, so that we have

$$\max p_i \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}} - \sum_{\tilde{x}\in\tilde{X}_i} P_{i\tilde{x}} \tilde{x}$$

This yields FOCs

$$p_i\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}\tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}-1}\alpha_{i\tilde{x}}\xi_i\tilde{x}^{\rho_i-1}=P_{i\tilde{x}}$$

$$\Rightarrow \tilde{x}=\left(\frac{P_{i\tilde{x}_k}}{P_{i\tilde{x}}}\frac{\alpha_{i\tilde{x}}}{\alpha_{i\tilde{x}_k}}\right)^{\frac{1}{1-\rho_i}}\tilde{x}_k$$

Substituting the second equation into the first, we obtain

$$\tilde{x}_k=\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i-\rho_i}{\rho_i(1-\xi_i)}}\left(\frac{\alpha_{i\tilde{x}_k}}{P_{i\tilde{x}_k}}\right)^{\frac{1}{1-\rho_i}}(p_i\xi_i)^{\frac{1}{1-\xi_i}}.$$

Therefore, expenditures are

$$\sum_{\tilde{x}}P_{i\tilde{x}}\tilde{x}=(p_i\xi_i)^{\frac{1}{1-\xi_i}}\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}$$

while revenues from production are

$$p_i\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}\tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}}=p_i(p_i\xi_i)^{\frac{\xi_i}{1-\xi_i}}\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

If firm i has all inputs left, we therefore have

$$\nu_i(\mathcal{J}_i)=p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}}-(\xi_i)^{\frac{1}{1-\xi_i}}\right]\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

Now consider a firm that only has inputs $\mathcal{B}_i$ remaining. The price index for such a firm can be written as

$$P_{i\tilde{x}}(\mathcal{B}_i)=\left(\sum_{j\in\tilde{x}\cap\mathcal{B}_i}\alpha_{ij}^{\frac{1}{1-\overline{\chi}_{i\tilde{x}}}}p_j^{-\frac{\overline{\chi}_{i\tilde{x}}}{1-\overline{\chi}_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$$

B.16

and therefore we can write

$$\nu_i(\mathcal{B}_i) = p_i^{\frac{1}{1-\xi_i}} \left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}} \right] \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i}}.$$

Therefore, we have

$$\begin{aligned}
&\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i \backslash \{k\}) \\
&= \frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i} \log \left(\frac{\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{J}_i \backslash \{k\})^{-\frac{\rho_i}{1-\rho_i}}} \right) \\
&= -\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i} \log \left(1 - \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}_k}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}} \left[1 - \left(\frac{P_{i\tilde{x}_k}(\mathcal{J}_i \backslash \{k\})}{P_{i\tilde{x}_k}(\mathcal{J}_i)} \right)^{-\frac{\rho_i}{1-\rho_i}} \right] \right) \\
&= -\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i} \log \left(1 - \Omega_{i\tilde{x}_k} \left[1 - \left(1 - \omega_{ik} \right)^{\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}} \frac{\rho_i}{1-\rho_i}} \right] \right)
\end{aligned}$$

given the definitions of expenditure shares,

$$\Omega_{i\tilde{x}_k} = \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}_k}(\mathcal{J}_i))^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{J}_i))^{-\frac{\rho_i}{1-\rho_i}}}$$

$$\omega_{ik} = \frac{\alpha_{ik}^{\frac{1}{1-\chi_{i\tilde{x}_k}}} p_k^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}{\sum_{j \in \tilde{x}_k} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}_k}}} p_j^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}$$

Now, consider the case of Cobb-Douglas ($\rho = 0$), then

$$\log \nu_i(\mathcal{B}_i) = \log\left(p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}}\right]\right) - \frac{\xi_i}{1-\xi_i}\sum_{\tilde{x}\in\tilde{X}_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\log P_{i\tilde{x}}(\mathcal{B}_i)$$

For the illustrative empirical work below in Appendix B.2.4, we assume that each nested basket has exactly one hegemon input $|\mathcal{I}_m \cap \tilde{x}| = 1$, and then

$$\sum_{k\in\mathcal{I}_m}\left[\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i\backslash\{k\})\right] = \log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i\backslash\mathcal{I}_m).$$

Therefore, we can recover $\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i\backslash\mathcal{I}_m)$ by adding up

$$\begin{aligned}\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i\backslash\{k\}) &= \frac{\xi_i}{1-\xi_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\left[\log P_{i\tilde{x}}(\mathcal{J}_i\backslash\{k\}) - \log P_{i\tilde{x}}(\mathcal{J}_i)\right]\\ &= \frac{\xi_i}{1-\xi_i}\Omega_{i\tilde{x}_k}\log\left(1-\omega_{ik}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}\\ &\approx \frac{\xi_i}{1-\xi_i}\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}\Omega_{i\tilde{x}_k}\omega_{ik}\end{aligned}$$

The final step follows from the elasticity $\sigma_{i\tilde{x}} = \frac{1}{1-\chi_{i\tilde{x}}}$.

B.2.4. *Measuring The Loss in Continuation Value*

From the previous subsection, with a Cobb-Douglass outer nest, we can write the loss to country n from losing access to all the hegemon's goods as

$$\tilde{\nu}_n \equiv \frac{\xi}{1-\xi}\sum_{i\in\mathcal{I}_n}\sum_{k\in\mathcal{I}_m}\left(\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i\backslash\{k\})\right) \tag{B.11}$$

To measure this loss, we start by considering loss from losing all $k \in \mathcal{I}_m$, that is all firms based in country m, and abstract from threats using firms based outside of m. We measure the losses that the hegemon can induce on country n relative to a base country (so that $\xi/(1-\xi)$ divides out).

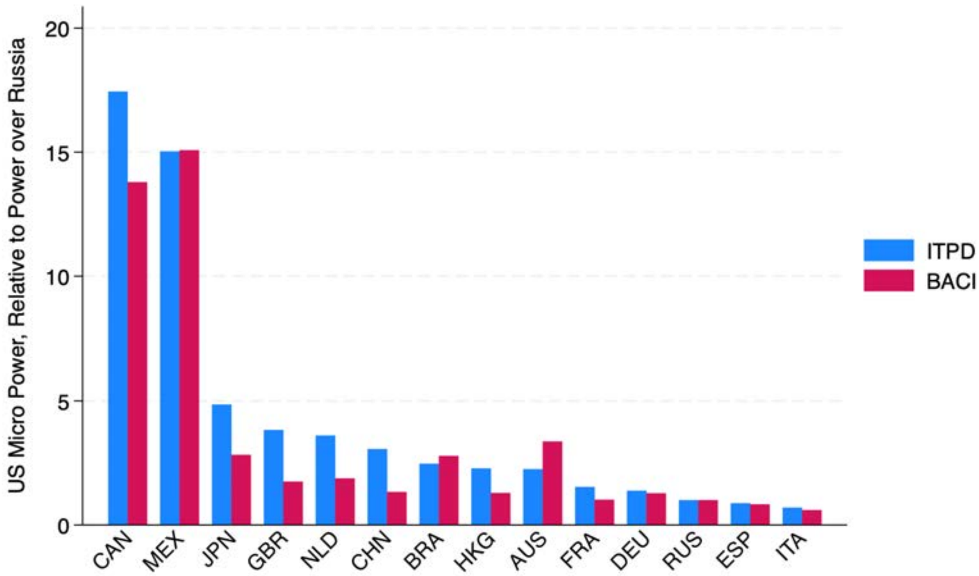
B.18

We use trade elasticities from Fontagné et al. (2022). These are estimated based on tariff rates at the HS06 level. We use trade data from two sources. First, we use the UN Comtrade Data as organized by BACI. The shortcoming of this dataset for this exercise is that it misses domestic production. This potentially biases the estimates because it may overstate the market share controlled by the hegemon exporters, as other countries may be able to switch toward domestic production. To address this issue, we also use the International Trade and Production Database for Estimation (ITPD-E) Version 2.0 from Borchert et al. (2022). This has the advantage of containing domestic production data, improving the calculation of ω, but is available for more aggregated industries than the BACI data. In this case, we match each elasticity at the HS06 level to an ISIC rev. 3 industry code, and then match ITPD-E industries to the ISIC level. We assume that the elasticity of substituion within ITPD-E industry is the mean elasticity of substitution of the HS06/ISIC matched to that industry.

Our exercise is in the spirit of Hirschman (1945), evaluating which countries a hegemon has power over based on the nature of the bilateral trade relationship. We focus here on the case where only the hegemon can cut off goods. For every country n, we estimate the loss in continuation value that the United States can cause as in equation B.11. In order to avoid taking a stand on the returns to scale ξ and to make the measures comparable across the ITPD and BACI estimates, we can estimate relative losses as $\tilde{\nu}_I^{Rel} \equiv \tilde{\nu}_I / \tilde{\nu}_{Base}$, where $Base$ is a reference country. For this exercise, we pick Russia to be the base country, and so $\tilde{\nu}_{RUS}^{Rel} = 1$ and $\tilde{\nu}_I^{Rel}$ is defined relative to the losses the US can cause to Russia. In Figure B.1, we plot two estimates of the losses for the year 2019 (the last year for which ITPD-E is available). Regardless of whether we use ITPD or BACI, the rankings across major countries are relatively similar, with the United States having the potential to cause much higher losses to Canada and Mexico than China or Russia. Our measure of the loss of continuation value is related to the Hausmann et al. (2024) estimation of the economic costs that the United States and Europe could impose on Russia via export controls in the Baqaee and Farhi (2022) framework. More generally, our measure parallels the sufficient statistics for welfare gains from international trade in Arkolakis et al. (2012) while focusing on the loss of exports from a single country.

While this measure takes a first step towards measuring the losses that the hegemon threats could induce, one key shortcoming of the current estimates is that they remain restricted to manufacturing sectors with estimated trade elasticities. While the United States

FIGURE B.1.—**USA Ability to Induce Continuation Value Losses in Various Countries, 2019**



Notes: This plots the loss in continuation value calculation following Equation B.11. Trade and production data from ITPD-E and BACI, and trade elasticities from Fontagné et al. (2022). All losses in continuation are defined relative the loss that could be imposed on Russia.

has seen a significant decline in its importance in goods trade, it retains a commanding position as a service exporter, most notably in financial services. Because of the way in which trade in services enters export data and the absence of elasticity estimates within these sectors, these estimates exclude potentially the most powerful lever in the American geoeconomic toolkit.

B.2.5. *Identifying Pressure Points: A Special Case*

In this appendix, we consider an environment in which firms have separable production and provide a necessary and sufficient condition for identifying pressure points. We start by defining the environment:

DEFINITION 6: *The **separable production** environment assumes that firms that use intermediate inputs have* $f_i(x_i, \ell_i, z) = \sum_{j \in \mathcal{J}_i} f_{ij}(x_{ij}, z)$.

We assume separable production. We write $\Pi_i(x_i, \mathcal{B}_i) = \sum_{j \in \mathcal{B}_i} \pi_{ij}(x_{ij})$, where $\pi_{ij}(x_{ij}) = p_i f_{ij}(x_{ij}, z) - p_j x_{ij}$. Now, suppose that continuation value ν_i is separable across elements of $\mathcal{S}_i(\mathcal{B}_i)$, that is we can write $\nu_i(\mathcal{B}_i) = \sum_{S \in \mathcal{S}_i(\mathcal{B}_i)} v_i(S)$. Then, the incentive constraint associated with $S \in S_i(\mathcal{B}_i)$ is

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S).$$

Therefore, if the incentive constraint holds for $S_1, S_2 \in \mathcal{S}_i(\mathcal{B}_i)$, it also holds for $S_1 \cup S_2$. Thus incentive compatibility with respect to $\mathcal{S}_i(\mathcal{B}_i)$ implies incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Thus the decision problem of firm i becomes separable over elements of the action set $\mathcal{S}_i(\mathcal{B}_i)$, leading to a value function that is separable over elements of the basis, consistent with the assumption.

Now, we move to characterizing pressure points. As a preliminary, the optimization problem of firm i has a corresponding Lagrangian

$$\mathcal{L}(x_i, \lambda | \mathcal{S}_i) \equiv \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{S}_i} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right],$$

where $\lambda_{iS} \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint associated with $S \in \mathcal{S}_i$. We obtain the following result.

PROPOSITION 6: *$S_1, \ldots, S_n \in \mathcal{S}_i$ is a pressure point of firm i if and only if $\lambda_{iS} \neq \lambda_{iS'}$ for some $S, S' \in \{S_1, \ldots, S_n\}$.*

Proposition 6 proves that a necessary and sufficient condition for a pressure point is that the Lagrange multipliers of the existing equilibrium differ among those input relationships that enter the joint threat. To build intuition, return to the example in Figure 1. Consider the equilibrium under individual triggers $\mathcal{S}_i = \{\{j\}, \{k\}\}$, then firms in sector i have a pressure point resulting from the joint threat actions $\{j\}, \{k\}$ if and only if $\lambda_{ij} \neq \lambda_{ik}$. Intuitively, if $\lambda_{ij} > \lambda_{ik}$, then the marginal value of slack in the incentive compatibility constraint for (stealing) good j is higher than for slack in the incentive compatibility constraint for good k. The joint threat creates value by consolidating the two constraints and altering relative production of the two goods, a means of redistributing slack. Heuristically, the joint threat facilitates a *decrease* in production using k in order to create slack that allows for an *in-*

crease in production using j under the joint threat. By contrast if $\lambda_j = \lambda_k$, then slack is equally valuable across goods j and k, even when both multipliers are strictly positive and both constraints bind. In this case, no value is created by forming a joint threat: production under the joint threat is precisely the same as under isolated threats. The proof of Proposition 6 formalizes these intuitions for more general action sets $\mathcal{S}_i$. This result is useful because, in separable production environments, it is possible to identify pressure points based on the existing equilibrium without having to re-compute the firm's optimization problem.

Proof of Proposition 6. We break the proof into the if and only if statements.

If. Suppose that there exist $S', S'' \in \{S_1, \ldots, S_n\}$ such that $\lambda_{iS'} > \lambda_{iS''}$ (without loss of generality). Suppose that we augment the incentive compatibility constraint for S to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S) + \tau_S,$$

where τ_S is a constant (that is set equal to zero in the baseline). Observe that since $S' \cap S'' = \emptyset$, then joint threat constructed from S' and S'' yields the incentive constraint

$$\sum_{j \in S' \cup S''} \theta_{ij} p_j x_{ij} \leq \beta[v_i(S') + v_i(S'')] + \tau_{S'} + \tau_{S'\prime}.$$

Therefore, a weaker expansion of incentive compatible allocations than achieved by a joint threat is to instead increase $\tau_{S'}$ and decrease $\tau_{S''}$ in such a manner that $\tau_{S'} + \tau_{S''} = 0$. If such a perturbation strictly increases value, then creating a joint threat also strictly increases value.

Since $V_i(\mathcal{S}_i, \tau) = \mathcal{L}$, then the welfare effect of a perturbation to τ_S, by Envelope Theorem, is

$$\frac{\partial V_i}{\partial \tau_S} = \lambda_{iS}$$

Therefore, the total profit impact on firm i of the perturbation $d\tau_{S'} = 1$ and $d\tau_{S''} = -1$ is

$$\frac{\partial V_i}{\partial \tau_{S'}} - \frac{\partial V_i}{\partial \tau_{S'\prime}} = \lambda_{iS'} - \lambda_{iS''} > 0.$$

Therefore, there is an $\epsilon > 0$ such that when defining τ by $\tau_{S'} = \epsilon$, $\tau_{S''} = -\epsilon$, and $\tau_S = 0$ otherwise, we have $V_i(\mathcal{S}_i, \tau) > V_i(\mathcal{S}_i, 0)$. But since $V_i(\mathcal{S}'_i) \geq V_i(\mathcal{S}_i, \tau)$, then $V_i(\mathcal{S}'_i) > V_i(\mathcal{S}_i)$, and hence $(S_1, \ldots, S_n)$ is a pressure point on i.

Only If. Because the decision problem of firm i is separable across elements of the action set, and because elements $S \notin \{S_1, \ldots, S_n\}$ are unchanged, the same allocations x^*_{ij} for $j \in \bigcup_{S \in \mathcal{S}_i \setminus \{S_1, \ldots, S_n\}} S$ remain optimal. It remains to show that optimal allocations are unchanged for $j \in \bigcup_{S \in \{S_1, \ldots, S_n\}} S$.

Suppose first that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} = 0$. Then, x_{ij} is produced at first-best scale, $x_{ij} = x^u_{ij}$. But then since $x^*_{ij} = x^u_{ij}$ is also implementable under joint threats, then the optimal allocation under joint threats is again $x^*_{ij} = x^u_{ij}$, and hence $(S_1, \ldots, S_n)$ is not a pressure point on i.

Suppose next that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} > 0$ and let x^*_i be optimal production under $\mathcal{S}_i$. Because the decision problem of firm i is separable across elements of the action set, let us focus on the subset $\mathcal{X} = \{S_1, \ldots, S_n\}$ of elements in the joint threat. Denoting $\mathcal{L}(x_i, \hat{\lambda}|\mathcal{X})$ the Lagrangian associated with elements $\mathcal{X}$,

$$\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X}) = \sum_{j \in \bigcup_{S \in \mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \hat{\lambda}_{iS} \left[\beta v_i(S) - \sum_{j \in \mathcal{S}} \theta_{ij} p_j x_{ij}\right].$$

Recalling that the firm's objective function is concave while each constraint is convex, the Lagrangian has a saddle point at (x^*_i, λ_i).

Next, consider the decision problem of firm i when faced with a joint threat, so that $\mathcal{S}'_i$ has an element $S' = \bigcup_{S \in \mathcal{X}} S$. As again the decision problem of the firm is separable across elements of $\mathcal{S}'_i$, then we can define the Lagrangian of firm i with respect to element S' by

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \mu_{iS'} \left[\beta \sum_{S \in \mathcal{X}} v_i(S) - \sum_{j \in S'} \theta_{ij} p_j x_{ij}\right].$$

Observe that once again, the objective function is concave while the constraint is convex. Since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{X}$, then we can write

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \mu_{iS'} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij}\right].$$

Finally, let us define $\mu_{iS'} = \lambda_{iS_1}$. Since $\lambda_{iS_1} = \ldots = \lambda_{i,S_n}$, then we have

$$\mathcal{L}(x_i, \mu_i | S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

As a result, we have $\mathcal{L}(x_i, \mu_i | S') = \mathcal{L}(x_i, \lambda_i | \mathcal{X})$ for all x_i. More generally since for any μ_i' there is a corresponding vector $\lambda_{iS}' = \mu_i'$, then since $\mathcal{L}(x_i, \hat{\lambda}_i | \mathcal{X})$ has a saddle point at (λ_i, x_i^*), then $\mathcal{L}(x_i, \hat{\mu}_i | S')$ has a saddle point at (μ_i, x_i^*). Therefore, x_i^* is also an optimal policy under joint threat $\mathcal{S}_i'$. Therefore, $V_i(\mathcal{S}_i') = V_i(\mathcal{S}_i)$ and hence $(S_1, \ldots, S_n)$ is not a pressure point. This concludes the proof.

REFERENCES FOR ONLINE APPENDIX

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New trade models, same old gains?," *American Economic Review*, 2012, *102* (1), 94–130. []

Baqaee, David Rezza and Emmanuel Farhi, "Networks, Barriers, and Trade," 2022. []

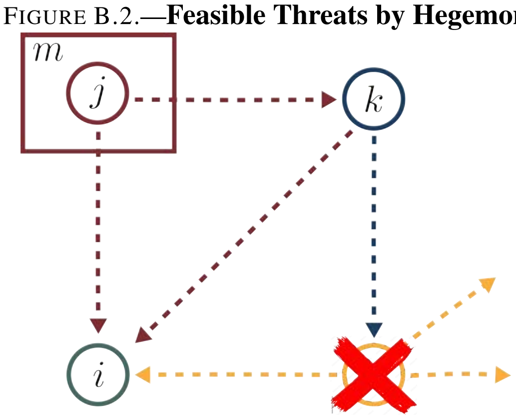
Borchert, Ingo, Mario Larch, Serge Shikher, and Yoto V Yotov, "Disaggregated gravity: Benchmark estimates and stylized facts from a new database," *Review of International Economics*, 2022, *30* (1), 113–136. []

Fontagné, Lionel, Houssein Guimbard, and Gianluca Orefice, "Tariff-based product-level trade elasticities," *Journal of International Economics*, 2022, *137*, 103593. []

Hausmann, Ricardo, Ulrich Schetter, and Muhammed A Yildirim, "On the design of effective sanctions: The case of bans on exports to Russia," *Economic Policy*, 2024, *39* (117), 109–153. []

Hirschman, Albert, *National power and the structure of foreign trade*, Univ of California, 1945. []

FIGURE B.2.—**Feasible Threats by Hegemon**



Notes: The figure illustrates the following configuration: sector j is located in the hegemon country and supplies to sector k and i. Sector k supplies to sector i and to another sector (orange and crossed-out), which itself supplies to sector i. The hegemon has a feasible joint threat on sector i via controlling the threats of j and k. The hegemon does NOT have a feasible joint threat on the orange crossed-out sector.

TABLE B.1

Summary of Notation

Symbol	Meaning	Symbol	Meaning
General Set-up		B_{ij}	Dummy for whether suppliers j Trusts firm i
$\mathcal{I}_n$	Set of sectors in country n. $\mathcal{I}$ set of all sectors	$\mathcal{B}_i$	Set of suppliers that Trust firm i
$\mathcal{F}_n$	Set of factors in country n. $\mathcal{F}$ set of all factors	**Continuation and Value Functions**	
Equilibrium Objects		$\nu_i(\mathcal{B}_i)$	Exogenous continuation value
p_i	Price of good produced by sector i	$\mathcal{V}_i(\mathcal{B}_i)$	Eqm value function of firm i in repeated game
p_f^ℓ	Price of local factor ℓ	$V_i(S_i)$	Firm's current value as a function of its action set $\mathcal{S}_i$
p, p_ℓ, P	Vector of intermediate goods, factor, all prices	$V_i(\Gamma_i)$	Value of firm i when accepting contract Γ_i
z	z vector of all externalities z_{ij}	**Hegemon**	
Consumer		D_i	Set of sectors downstream from Sector i
$U_n(C_n)$	Utility of rep.agent in country n from consumption	$\mathcal{D}_m$	Set of foreign sectors that source at least one input from hegemon's country
$u_n(z)$	Utility of rep. agent in country n from z	$\mathcal{C}_m$	Set of firms hegemon can contract with.
Π_i	Profits of sector i	$\mathcal{J}_{im}$	Set of inputs that sector i sources from sectors in country m
w_n	Income of consumer in country n	T_{ij}	Transfers from i to hegemon in relationship with j. Vector $\mathcal{T}_i$. Sum $\overline{T}_i$
$W_n(p, w_n)$	Indirect utility function from consumption	τ_{ij}	Revenue-neutral tax i faces on purchases of goods from sector j
Firms		τ_{if}^ℓ	Revenue-neutral tax i faces on purchases of factor ℓ
x_{ij}	Intermediate input j used by firm i. Vector x_i	τ_i	Vector of revenue-neutal input and factor taxes faced by i
ℓ_{if}	Local factor $f$used by firm i. Vector ℓ_i	Γ_i	Hegemon's contract $\Gamma_i = \left\{\mathcal{S}_i^{'}, \mathcal{T}_i, \tau_i\right\}$. Vector Γ
y_i	Output of firm i, $y_i = f_i(x_i, \ell_i, z)$	Ψ^z	Matrix capturing endogenous externality amplification
$\mathcal{J}_i$	Set of suppliers to firm i	$\mathcal{L}^m$	Hegemon's Lagrangian
Stealing		η_i	Lagrange multiplier on the participation constraint of firm i
θ_{ij}	Share of order that can be stolen	Λ_{iS}	Lagrange multiplier on the IC constraint of firm i for action S
a_{ij}	Dummy for whether suppliers j Accepts order of firm i	$\bar{\Lambda}_i$	Sum of multipliers for stealing actions in hegemon's threat
$S_i \subset \mathcal{J}_i$	The subset of sectors from which firm i steals	$\mathcal{E}_{ij}$	Hegemon's perceived externalities from increase in z_{ij}^*
$\mathcal{S}_i$	Action Set: Set of firms' possible stealing decisions	Ξ_{mn}	Hegemon's perceived externalities from a transfer from in n to m
$\mathcal{S}_i^{'}$	Joint threat, Coarser partition of $\mathcal{S}_i$	ε_{ij}^z	Direct value to hegemon of increasing sector $i's$ use of input j
$\overline{\mathcal{S}}_i'$	Maximal joint threat	ε_{ij}^{zNC}	Indirect value to hegemon of increasing sector $i's$ use of input j
S_i^D	Inputs in hegemon's maximal joint threat	$\varepsilon_j^{P^m}$	Value to hegemon from changes in the price of input j
$P(\mathcal{J}_i)$	Power set of $\mathcal{J}_i$	Ω_n	Global planner's welfare weight on country n
$\Sigma(\mathcal{S})$	Set of all possible unions of elements of $\mathcal{S}$	β	Discount Factor