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RISK PREFERENCES IMPLIED BY SYNTHETIC OPTIONS

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ABSTRACT

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Risk preferences implied by synthetic options

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Abstract

The historical returns on equity index options are well known to be strikingly negative. That is typically explained either by investors having convex marginal utility over stock returns (e.g. crash/variance aversion) or by intermediaries demanding a premium for hedging risk. This paper examines the consistency of those explanations with returns on dynamically replicated, or synthetic, options. Theoretically, it derives conditions under which convex marginal utility leads synthetic options to also have negative excess returns. Empirically, synthetic options have CAPM alphas near zero over the period 1926–2022, in stark contrast to exchange-traded options. Over the last 15 years, returns on traded options have converged to those on synthetic options – with the variance risk premium shrinking towards zero – while various drivers of the cost and risk of hedging options exposures have declined, consistent with a model in which intermediaries drive option prices.

1 Introduction

Background

A major empirical fact in financial markets is that equity index options have been overpriced historically relative to simple benchmark models. Investors who purchase options have, on average, earned significant negative returns and negative CAPM alphas.¹ Bates

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¹For analyses of returns, see Coval and Shumway (2001), Bakshi and Kapadia (2003), Broadie, Chernov, and Johannes (2007), Constantinides, Jackwerth and Savov (2013), Chambers, Foy, Liebner, and Lu (2014)

(2022) discusses two classes of explanations for that fact. The first is that marginal utility for some hypothetical representative investor is convex in market returns. Periods with large negative returns (and possibly also large positive returns) have state prices that are higher than would be expected just based on a model like the CAPM in which marginal utility rises linearly as the market drops. That can be due, for example, to aversion to crashes, aversion to high volatility, time-varying risk aversion, or behavioral factors.²

It is well known, though, that option prices have puzzling implications that are difficult to reconcile with standard utility theory, for example sometimes implying *negative* risk aversion. The second class of explanations therefore focuses on intermediaries, explaining option overpricing as the result of intermediaries being net short options and charging a premium for their concentrated risk.³ In that case, option prices reveal the preferences and constraints of the specialist investors that trade in options markets, and *not* necessarily those of the typical equity investor. The paper shows that in such a model, as segmentation declines over time the returns on traded and synthetic options should converge.

Understanding which of the two explanations is correct is important because, in addition to being intrinsically interesting, option prices are used to measure many features of financial markets, including investor expectations of various moments of the conditional distribution of returns, investor preferences across market return states, and the drivers of risk premia.⁴ They can also reveal potential amplification mechanisms for macroeconomic shocks (e.g., investors becoming more risk averse when the market drops),⁵ and are a key input in understanding the importance of stabilization policy, as optimal policy depends on agents'

, Dew-Becker et al. (2017), and Muravyev and Ni (2020), among many others. For structural models, see Backus, Chernov, and Martin (2011), Drechsler and Yaron (2011), Gabaix (2012), Drechsler (2013), Seo and Wachter (2019), and Schreindorfer (2020). Note, though, that those models are almost exclusively endowment economies.

²See Cuesdeanu and Jackwerth (2018) for an extensive review of such models and their empirical support, along with a discussion of models of intermediary frictions. On risk aversion, see Ait-Sahalia and Lo (2000), Jackwerth (2000), Rosenberg and Engle (2002), and Schreindorfer and Sichert (2022). On the variance risk premium, see, in addition to work cited elsewhere here, see Lamoreaux and Lastrapes (1993), Coval and Shumway (2001), Bakshi and Kapadia (2003), Du (2011), Dew-Becker et al. (2017), and Ait-Sahalia, Karaman, and Mancini (2020).

³See Jackwerth (2000), Bollen and Whaley (2004), Bates (2008), Han (2008), Garleanu, Pedersen, and Poteshman (2008), Jurek and Stafford (2015), Haddad and Muir (2021), Frazzini and Pedersen (2022), among many others

⁴On preferences over market states, see Ait-Sahalia and Lo (2000), Jackwerth (2000), and Rosenberg and Engle (2002). For conditional moments, see the CBOE VIX index and, among many others, Carr and Madan (1998), Carr and Wu (2009), and Martin (2017). Options have also been used to measure jump risk (e.g. Bollerslev and Todorov (2014)), micro uncertainty (Dew-Becker and Giglio (2020)), and option implied skewness (Kozhan, Neuberger, and Schneider (2013), Dew-Becker (2022)). Bollerslev and Todorov (2011) and Beason and Schreindorfer (2022) use option prices to measure the drivers of risk premia.

⁵E.g. He and Krishnamurthy (2013), Hall (2017), and Muir (2017).

subjective valuations of different possible states of the world.⁶

Contribution

This paper theoretically develops a novel approach to measuring the average investor’s risk preferences by studying *synthetic* options – dynamic portfolios that attempt to replicate returns on traded options by dynamically trading the underlying. Empirically, it measures returns on synthetic options over nearly a century of data and compares them to traded option returns. To interpret the results, the paper extends the intermediary-based asset pricing model of Garleanu, Pedersen, and Poteshman (2009) to examine how the gap in returns between true and synthetic options relates to the magnitude of trading and hedging frictions.

Methods and results

The paper first provides a simple theoretical framework to ground the interpretation of option returns. The results show how the CAPM alpha of a traded option on the stock market measures curvature in marginal utility with respect to the market return.⁷ The CAPM appears as a benchmark here not because we want to make any claim that marginal utility is necessarily “truly” linear in any sense, but rather to take an empiricist’s approach of starting with a linear null and then measuring the degree of nonlinearity.

Since options have payoffs that are convex in the market return, when marginal utility is also convex options have relatively high prices and consequently low returns – negative CAPM alphas. The paper’s more important theoretical contribution, though, is to give circumstances under which convexity in marginal utility also implies negative CAPM alphas for *synthetic* options. While the required conditions are strong, the paper provides some empirical evidence in their favor and also gives bounds on the potential magnitude of the bias due to their violation.

Empirically, the paper examines monthly returns on synthetic and traded options (motivated by the fact that models are typically calibrated and estimated at a monthly or quarterly frequency). The synthetic options are constructed back to 1926 using data on the CRSP market return, while monthly traded option returns are available since August, 1987. The first empirical question is how well replication works. The answer is quite well: synthetic options have returns that are over 90-percent correlated with traded option returns and hedge realized crashes in the data effectively. That does not mean that options could have been synthesized in real time historically, though – trading costs and other frictions would have

⁶Alvarez and Jermann (2004) is an example of how asset prices can be used to measure the cost of fluctuations. De Paoli and Zabczyk (2013) study optimal policy under time-varying risk aversion.

⁷Importantly, this is a reduced-form statement. Option prices do not measure a *structural* effect of the market return on marginal utility. They measure the average value of marginal utility conditional on a return state.

made that infeasible (and in fact replication frictions are central to our intermediary-based explanation of the results).

The next question, and one of the paper’s two central results, is what alphas the synthetic options earn. Whereas traded options have strongly negative CAPM alphas, *synthetic options have historical alphas that are zero or even positive*, with confidence bands that are economically narrow (and half the width of those for traded options). That result is robust over time, across strikes, across maturities, and to modifying various details in the construction.

That result is in fact consistent with the reduced-form option pricing literature – the finding of a large negative return on delta-hedged returns going back to Bakshi and Kapadia (2003) is equivalent to the statement that traded options have more negative returns than synthetic options. The novelty in what has been described so far is in pointing out that the small alpha for synthetic options extends back to 1926.

While synthetic options have consistently earned near-zero CAPM alphas, the alphas of traded options have *not* been so consistent (see also Bates (2022)). The paper’s second key empirical result is that according to various estimation methods, there is a break in the returns somewhere around 2010. In the period since 2010, in fact, *the alphas of the traded options have converged to zero*, consistent with the synthetic options. Since the gap between traded and synthetic option returns is literally a delta-hedged return, another way to state the result is that the alpha of delta-hedged options has gone to zero. The paper also shows that the variance risk premium has shrunk towards zero, as has the gap between the VIX and realized volatility.

The final section of the paper asks what might have caused such a shift. One might take the view that synthetic options, because they were not really feasible to construct for most of the sample (e.g. due to the lack of index futures), were not a good way to measure investor preferences. In that case, as those frictions decline and daily rebalancing becomes feasible, the returns on synthetic options would converge (down) to those on traded options. But that is not what occurred empirically.

A similar problem arises if one takes the view that the synthetic and traded options hedge different states. Synthetic option returns depend on the path the market takes, so the gap between true and synthetic returns is a function of realized volatility and other higher-order factors like jumps. If that gap is related to marginal utility, then it will be priced and drive a wedge between the traded and synthetic returns.⁸ But those factors do not appear to have shrunk over time. The volatility of the gap between traded and synthetic option returns

⁸No-arbitrage option prices typically rely on this type of mechanism, e.g. Pan (2002).

has been stable, jump variation shows no trend, and skewness in market returns has become significantly *more* negative.⁹

Based on the evidence, our preferred explanation of the results is that the stock and options markets are segmented. We extend the segmented-markets model of Garleanu, Pedersen, and Poteshman (2009) to allow for trading costs and an index-futures basis (i.e. imperfect tracking of the underlying index by the futures market). In the model, the difference in alphas for synthetic and traded options depends on the magnitude of hedging frictions that intermediaries face. When they shrink, the alpha on traded options converges to that on synthetic options. Empirically, both trading costs and risk due to the index-futures basis have declined over time. The index-futures basis is particularly notable. During the 1987 crash, S&P 500 futures were at times 20% underpriced relative to the level of the S&P 500 index, which would represent an enormous cost to a dealer with net short options positions who must sell futures as the underlying falls. In the crash in the fall of 2008, though, the basis was never more than about 5% and was centered on zero. One explanation for the decline in the option premium soon after the crash, then, is that intermediaries learned that they faced less hedging risk during crashes than they had thought based on past data. And in fact the paper shows that the spread in returns between synthetic and traded options is closely related to hedging risk.

Broader implications

The paper's basic findings have three additional implications beyond what has been discussed so far. First, significant care must be taken when using option prices or returns to estimate or test models since equity and options are not frictionlessly integrated, which is often assumed in structural models. Second, and relatedly, the paper's results imply that derivatives prices, up until relatively recently, were distorted away from those implied by the preferences of whoever is the typical investor pricing equities. In thinking about calibration and estimation of structural models, then, one must decide whose preferences exactly are being modeled. As one alternative path forward, this paper's results suggest the use of synthetic options for calibration and estimation (e.g. ask whether a given model explains prices of synthetic options, instead of traded options). And that can be done not only for the overall equity market – synthetic options can be created on any underlying.

Finally, the analysis finds clear evidence of nonstationarity. So when studying traded option returns, attention must be paid to the exact sample being used and how the results may have changed over time. Financial markets have gone through many massive changes over the past century. An interesting fact here is that through all of that, the returns

⁹Instability of parameters in no-arbitrage models is a well understood issue in the option pricing literature. See, for example, Bollen and Whaley (2004) and Andersen, Fusari, and Todorov (2015).

implied by option replication have been largely stable, while there are much larger changes in derivatives markets.

While the paper’s analysis ultimately favors a frictional model, there are always frictionless models that can rationalize any deviation in the pricing of traded and synthetic options. At a high level, though, this paper’s contribution is simply to ask whether any conclusions are changed if one focuses on synthetic options – whose pricing depends only on behavior of equity prices – instead of traded options, whose pricing depends on a separate derivatives market.

Outline

The remainder of the paper is organized as follows. Section 2 discusses the theoretical framework and how it is applied to the data in practice. Section 3 describes the data and empirical methods and section 4 reports our main empirical results on the returns of synthetic and traded options and the shape of marginal utility. Finally, section 5 presents a model of intermediary frictions that can explain theoretically and empirically both why there was a gap between synthetic and traded option returns and why it has shrunk, and section 6 concludes.

2 Theory

This section provides some theoretical structure to help interpret average returns on traded and synthetic options. Its key result is proposition 3, which yields three conditions needed in order to use returns on synthetic options to estimate curvature in marginal utility with respect to the market return.

2.1 Definitions and notation

The market return between periods t and $t + j$ is $R_{t,t+j}^m$. The change in marginal utility is $M_{t,t+j}$ (i.e. u'_{t+j}/u'_t , where u is utility over consumption).¹⁰ Since $M_{t,t+j}$ is a ratio of marginal utilities, we immediately have that $M_{t,t+2} = M_{t,t+1}M_{t+1,t+2}$, etc. Since any deviations of investors’ subjective probability measure from the truth can also be incorporated into $M_{t,t+j}$, we refer to it more generally as subjective marginal utility, or *SMU*.

Definition 1 *We say that a return $R_{t,t+j}$ is priced by subjective marginal utility, denoted $M_{t,t+j}$, over $t \rightarrow t + j$ if*

$$1 = E_t [M_{t,t+j}R_{t,t+j}] \tag{1}$$

¹⁰If agents have biased probability measures, then the bias will also be part of $M_{t,t+j}$. We would in that case just require that there are internally consistent in that $M_{t,t+2} = M_{t,t+1}M_{t+1,t+2}$.

where E_t is the expectation operator under the physical probability measure conditional on information available on date t .

Note that this definition may potentially only hold for certain t and j – i.e. only on some dates or over just some horizons. The definition does not imply that markets are complete, meaning that marginal utility need not be identical across agents. Rather, all agents must just agree on equation (1) for whichever assets are (universally) priced by marginal utility. In addition, it does not require rational expectations – irrational beliefs can be accommodated by $M_{t,t+j}$ as long as they satisfy basic axioms for probability measures.

For simplicity, the theoretical analysis takes the risk-free rate to be zero (equivalently, all returns can be interpreted as on forward contracts). That implies that $E_t M_{t,t+j} = 1$ for all t and j . The analysis is straightforward to recapitulate in the case where the risk-free rate is nonzero, and the empirical analysis accounts for nonzero interest rates.

The paper’s goal is to understand how marginal utility varies with the state of the equity market. To that end, define, for an arbitrary variable $X_{t,t+j}$, the nonlinear projection on the market return,

$$\bar{X}_{t,t+j} \equiv E[X_{t,t+j} | R_{t,t+j}^m], \quad (2)$$

$$\text{and } \hat{X}_{t,t+j} = X_{t,t+j} - \bar{X}_{t,t+j} \quad (3)$$

$\bar{X}_{t,t+j}$ is the component of $X_{t,t+j}$ that can be written as a function of the market return and $\hat{X}_{t,t+j}$ is the residual. We say $\bar{X}_{t,t+j}$ is the (nonlinearly) **spanned** part and $\hat{X}_{t,t+j}$ the **unspanned** part.¹¹

For $M_{t,t+j}$ the decomposition is into a part related to the total market return and a residual. $\hat{M}_{t,t+j}$ does not affect the pricing of the market portfolio, and is not correlated with *any* function of the market return, so it also cannot affect the pricing of traded options, as shown in the next section. It can, though, affect the pricing of synthetic options and other securities.

$\bar{M}_{t,t+j}$ is the paper’s primary object of interest – how marginal utility varies with the market return (again, not causally, just in a conditional expectation sense). It is what the past literature on option-implied pricing kernels has focused on, since, as the next section shows, it is what options carry information about.

¹¹Throughout the paper, the term “span” is used in the Hilbert space sense of a conditional expectation.

2.2 Interpreting traded option returns

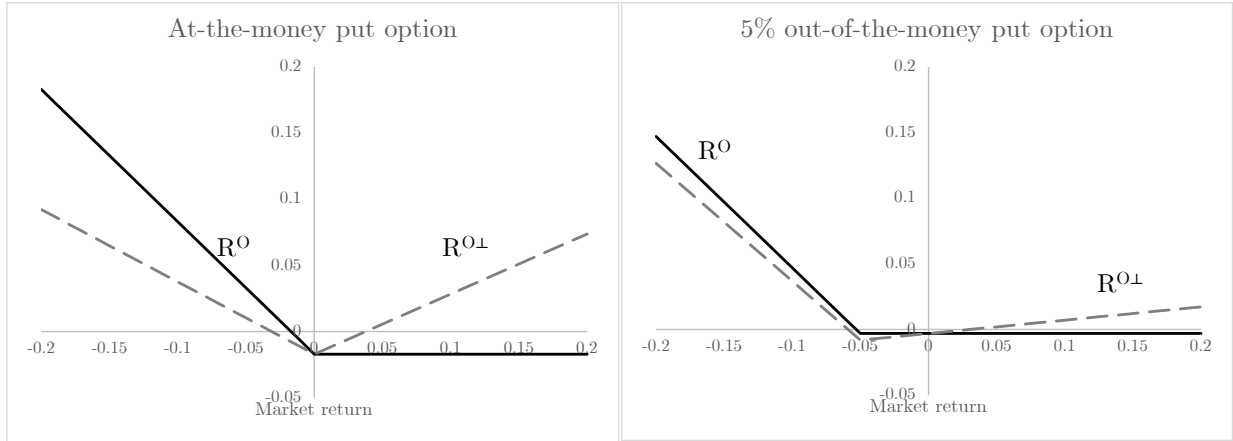
Define the gross return on some arbitrary option on the market (or even a portfolio of options) to be $R_{t,t+j}^O$. The part of that return (linearly) correlated with the market can always be subtracted, and we have

$$R_{t,t+j}^{O\perp} \equiv R_{t,t+j}^O - \frac{\text{cov}_t(R_{t,t+j}^O, R_{t,t+j}^m)}{\text{var}_t(R_{t,t+j}^O, R_{t,t+j}^m)} (R_{t,t+j}^m - 1) \quad (4)$$

$$\alpha_{t,t+j}^O = E_t [R_{t,t+j}^{O\perp} - 1] \quad (5)$$

where $\alpha_{t,t+j}^O$ is the CAPM alpha of $R_{t,t+j}^O$. It will become clear in a moment why we create the objects. First, though, note that $R_{t,t+j}^{O\perp}$ is not a delta-hedged (i.e. a dynamically hedged) return; rather, one might say it is beta-hedged, where the hedge is a *fixed* position in the underlying. The hedge is conditional on date- t information. Since it just adds a static position in the market, $R_{t,t+j}^{O\perp}$ has the usual kinked relationship with $R_{t,t+j}^m$, just tilted compared to $R_{t,t+j}^O$. Figure 1 gives example for an out-of-the-money and at-the-money put option.

Figure 1: Hypothetical option payoffs



Note: Hypothetical net payoffs $R_{t,t+j}^O$ and $R_{t,t+j}^{O\perp}$ for two different strikes.

Proposition 2 If $R_{t,t+j}^O$ and $R_{t,t+j}^m$ are priced over $t \rightarrow t + j$, then $\bar{M}_{t,t+j}$ has the representation,

$$\bar{M}_{t,t+j} = \text{const.} - \frac{E_t [R_{t,t+j}^m - 1]}{\text{var}_t [R_{t,t+j}^m]} R_{t,t+j}^m - \frac{\alpha_{t,t+j}^O}{\text{var}_t (R_{t,t+j}^{O\perp})} R_{t,t+j}^{O\perp} + \text{resid.} \quad (6)$$

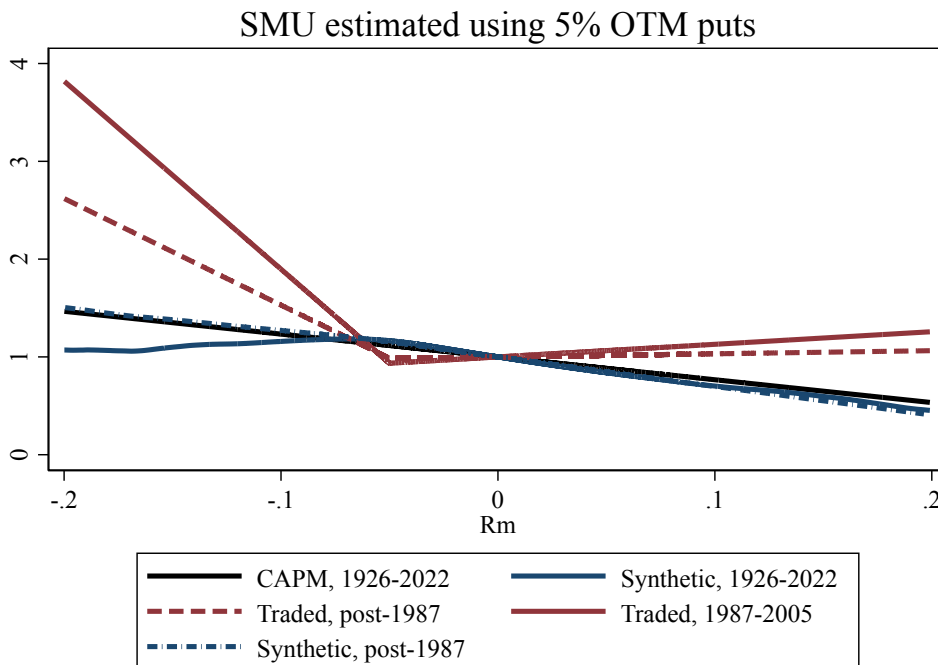
where the residual term is orthogonal to $R_{t,t+j}^m$ and $R_{t,t+j}^{O\perp}$.

Proposition 2 is important because it shows that the alpha of an option measures nonlinearity in marginal utility relative to the market return. Specifically, equation (6) is a regression of $\bar{M}_{t,t+j}$ on two functions of the market return: a linear term ($R_{t,t+1}^m$) and a nonlinear term ($R_{t,t+j}^{O\perp}$, which is, conditionally, an exact nonlinear function of the market return). The result comes from the fact that the covariances in the numerators of the regression coefficients can be replaced here by the two risk premia – e.g. $E_t [R_{t,t+j}^m - 1] = -\text{cov}_t (R_{t,t+j}^m, M_{t,t+j})$.

$\frac{-E_t [R_{t,t+j}^m - 1]}{\text{var}_t [R_{t,t+j}^m]}$ therefore measures the average slope of marginal utility with respect to the market return. $R_{t,t+j}^{O\perp}$ is a piecewise linear function of the market return, so its coefficient, $\frac{-\alpha_{t,t+j}^O}{\text{var}_t (R_{t,t+j}^{O\perp})}$, measures how the slope of $\bar{M}_{t,t+j}$ changes across the strike.

Figure 2 illustrates that idea, plotting SMU, normalized to equal 1 for $R_{t,t+j}^m = 1$, relative to the market return. Under the CAPM (the black line), SMU is linear in the market return, all alphas are zero, there is no convexity, and the slope is recovered simply as $\frac{-E_t [R_{t,t+j}^m - 1]}{\text{var}_t [R_{t,t+j}^m]}$.

Figure 2: SMU estimated using exchange-traded and synthetic puts



Note: The figure shows estimated SMU under different models and estimated in different samples. The solid black line reports the estimated SMU as a function of the market alone (as in the CAPM). The other lines model SMU as a function of the market and the orthogonalized returns on traded and synthetic options in various samples.

The dashed red line plots the SMU implied by the alphas observed for 5% out-of-the-money listed S&P 500 puts between 1987 and 2022. Historical put returns imply that

effective risk aversion – as measured by the slope of SMU – is significantly higher when the market falls. The non-monotonicity here is a typical, if surprising, empirical finding.

2.3 Interpreting synthetic option returns

It is well known that option returns can be approximated through dynamic trading in the underlying asset – the market return in this case. Do the approximated returns then also (approximately, perhaps) measure nonlinearity in $\bar{M}_{t,t+j}$?

By *synthetic* option return, we simply mean a return that is replicated via dynamic trading in the underlying. Denote the weight on the underlying each day by δ_t^S (which depends only on information up to date t , ensuring feasibility, at least if trading is frictionless). In the Black–Scholes (1973) replication, for example, δ_t^S is exactly the delta of the option being replicated, which depends on the level of the market index and its volatility.

The return on the synthetic option from t to $t + j$ is then

$$R_{t,t+j}^S \equiv \sum_{s=t}^{t+j-1} \delta_s^S (R_{s,s+1}^m - 1) + 1 \quad (7)$$

Note that in general $R_{t,t+j}^S \neq R_{t,t+j}^O$ and the replication will not be perfect (it is not hard to construct examples in which the replication is, in fact, useless). Nevertheless, we have the following result:

Proposition 3 *If $R_{t,t+1}^m$ is priced by SMU for all $s \rightarrow s + 1$ for $t \leq s < t + j$, then*

$$\bar{M}_{t,t+j} = \text{const.} - \frac{E_t [R_{t,t+j}^m - 1]}{\text{var}_t [R_{t,t+j}^m]} R_{t,t+j}^m - \frac{\left(\alpha_{t,t+j}^S + \text{cov}_t \left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S \right) \right)}{\text{var}_t (R_{t,t+j}^{S\perp})} \bar{R}_{t,t+j}^{S\perp} + \text{resid.} \quad (8)$$

where $\alpha_{t,t+j}^S$ is the CAPM alpha of $R_{t,t+j}^S$ and the residual is orthogonal to $R_{t,t+j}^m$ and $\bar{R}_{t,t+j}^{S\perp}$.¹²

There is again an expression for SMU in terms of two returns, with coefficients depending on their risk premia. Proposition 3 gives three conditions under which $\alpha_{t,t+j}^S$ can be used to measure convexity in SMU:

1. $\bar{R}_{t,t+j}^S$ is a convex function of the market return
2. $R_{t,t+1}^m$ is priced by SMU for all $t \rightarrow t + 1$
3. $\text{cov}_t \left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S \right)$ is zero

¹²Recall that overbars denote components spanned by the market return and hats the unspanned components.

The first condition just says that synthetic options have returns that are convex in the market, which can be checked empirically. It will be violated if, for example, large declines in the market are driven by intraday jumps.

The second and third conditions are harder to evaluate because they are statements about SMU, which is not directly observable. The pricing condition might fail at the daily level if there are frictions allowing market prices to deviate from their fundamental values at high frequency (see section 4.4).

The third condition requires that the unspanned part of synthetic option returns not be priced. Section 4.5 discusses it extensively, both looking at what variables $\hat{R}_{t,t+j}^S$ is correlated with and also using the method of Cochrane and Saa-Requejo (2000) to bound the covariance. Condition 3 holds in any model where the stock market return $R_{t,t+j}^m$ is a sufficient statistic for SMU (so that $\hat{M}_{t,t+j} = 0$).¹³ If the exact path of the market return matters (e.g. through realized volatility) or if there are other state variables that affect marginal utility and are independent of market returns (e.g. perhaps unspanned volatility), they will appear in $\hat{M}_{t,t+j}$. Condition 3 also holds when nonlinear payoffs on the market can be replicated via dynamic trading, e.g. when the market follows a binomial tree or in continuous time (with continuous hedging) when the market is a single-dimensional diffusion, so that $\hat{R}_{t,t+j}^S = 0$. In reality neither of those conditions holds literally, and the question becomes how large the bias from $\text{cov}(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)$ is.

Those three conditions are the key point of the theoretical analysis. They show what is required in order to use synthetic options to measure nonlinearity in marginal utility. More generally, they show that what is needed is not necessarily a traded or synthetic option, but just an investment whose payoff is a nonlinear function of the market return (ideally where additionally $\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S) = 0$).

It may initially be puzzling that synthetic options could earn an alpha when they are constructed from positions in the market itself. Section 4.2.3 shows that the alpha comes from timing – the synthetic option has positions changing every day, while we are looking at alpha over a longer horizon (monthly, in the data), and consequently measuring nonlinearity in marginal utility over that horizon.

¹³As examples, this would hold if marginal utility is a function of current wealth and that wealth returns are perfectly correlated with stock market returns, or if investors have Epstein–Zin preferences in the limit where the intertemporal elasticity of substitution is infinite and the stock market is equivalent to a claim on consumption (potentially levered).

2.3.1 Feasibility

A surprising feature of the three conditions for interpreting synthetic option returns is that none of them directly requires that options synthesis be feasible in practice. The requirement that the market be priced correctly every day, for example, does not say that every investor can trade every day. In many models, the market is correctly priced even though there is no trade in equilibrium.

There is no question that for the vast majority of investors over the vast majority of the empirical sample, replicating options dynamically would have been expensive and time-consuming. And even for professionals it may not have worked at critical moments, like the 1987 crash. That in fact will be a critical part of the paper’s intermediary-based explanation of the returns on traded and synthetic options in section 5. The model shows that it is in fact *precisely* the difficulty of replicating options via dynamic trading that leads intermediaries to demand a premium.

Furthermore, in segmented-markets models, frictions lead there to be different SMU for different agents, since they do not have access to the same investments. For example, if the typical equity investor cannot freely trade options, then their SMU may not price options, violating the requirement of proposition 2. And if hedging options is expensive for intermediaries, that drives a wedge between traded option prices and the prices implied by the marginal utility of equity investors – that are reflected in synthetic options.

Finally, just like there are frictions in trading equities, there are also frictions in trading options, including very large ticks and bid/ask spreads, transaction fees, large amounts of floor trading, market opacity, and the simple fact that options are more technically sophisticated than simple equity investments.

3 Data and methods

3.1 Option synthesis

Throughout the analysis, t is taken to be a day. The weight δ_t^S is equal to the delta of the option – the partial derivative of the value with respect to the price of the underlying. Different models give different exact expressions for delta. The main analysis uses a method from Hull and White (2017) that corrects the Black–Scholes delta for the leverage effect, but the unadjusted Black–Scholes delta delivers similar results (see section 4.7). There is no single “true” or “correct” δ_t^S for our purposes. Again, what is important is just that it generates a nonlinear payoff with minimal unspanned variation.

Section 4.4 discusses potential effects of market microstructure biases. There is evidence that stale prices affect the results in the earlier part of the sample (see also Bates (2012)). The bias can be reduced or eliminated by choosing the weights δ_t^S based only on information lagged by a day, and the main results use that method.¹⁴ That makes the estimates of mean returns robust, but it is also conservative in terms of fit – delta hedging is less effective when using stale information.

The return volatility needed to calculate delta is obtained from a heterogeneous autoregressive model (Corsi (2009)) that forecasts 1-month volatility as a function of past two-week volatility and the past three months of volatility (with the lags chosen based on the Bayesian information criterion). The model is estimated on an expanding window, so that when the delta is computed only past information is used. Robustness to the various choices here is examined in section 4.7.

The market return is measured as the (daily) CRSP value-weighted stock market return and the risk-free rate is the one-month Treasury bill rate from Kenneth French’s website.

3.2 Traded options

The dataset for traded options splices together CME futures options for the period 1987–1995 with CBOE SPX options from Optionmetrics for 1996–2022. Following Broadie, Chernov, and Johannes (2009), we study a monthly rolling strategy, where options are purchased on the third Friday of every month and then held to their maturity on the following month’s third Friday.

In parts of the analysis that involve direct comparisons of synthetic and traded options, we align the returns – comparing returns over the same third-Friday-to-third-Friday period. However, when looking at univariate statistics, the analysis uses 21-day overlapping windows for the synthetic options to maximize statistical power (since there is no need to only use a single return per month).

For both the synthetic and traded options, excess returns are scaled with the price of the underlying in the denominator, rather than the price of the option, as in Büchner and Kelly (2022). The scaling is the return perceived by an investor who is buying options in proportion to the underlying. It is a payoff per unit of insurance, rather than per unit of the insurance premium that is paid. See appendix B for further discussion.

¹⁴That is, in all of the benchmark results δ_t^S – the investment in the market on day $t + 1$ (e.g. held during the day on Wednesday) – is set based only on information available at the end of day $t - 1$ (Monday afternoon).

4 Empirical results

4.1 The relationship between $R_{t,t+j}^S$ and $R_{t,t+j}^m$

The top panel of Figure 3 plots $R_{t,t+j}^S$ for put options against $R_{t,t+j}^m$, where $j = 21$ days and the strike used to construct δ_t^S is 95% of the initial level of the market (corresponding to approximately a unit standard deviation decline). The plot for call options with the same strike is identical but rotated 45 degrees counterclockwise.

There is clearly significant nonlinearity – for values of the market return above the strike the slope is near zero, while for values below it the slope is approximately -1, consistent with the fact that $R_{t,t+j}^S$ is constructed to mimic a put option. The red line plots the nonparametric estimates of $\bar{R}_{t,t+j}^S \equiv E [R_{t,t+j}^S | R_{t,t+j}^m]$.¹⁵ They formally quantify the relevant nonlinearity, showing that $\bar{R}_{t,t+j}^S$ is close to piecewise linear in the market return.

Importantly, $\bar{R}_{t,t+j}^S$ rises with a consistent slope as $R_{t,t+j}^m$ falls, regardless of how large the decline is. If it was not possible to span large declines in the market with time-varying weights, e.g. due to large jumps, $\bar{R}_{t,t+j}^S$ would flatten out for the very negative values of $R_{t,t+j}^m$. Figure A.1 replicates figure 3 across strikes and shows the results are highly similar.

The table below reports the most extreme 21-day returns in the five most extreme events in the US stock market in the sample. As a benchmark, the 5% OTM synthetic puts would ideally generate a return that is 5% lower than the negative of the market return, minus the initial cost of the position. In all five cases, the synthetic puts have highly positive returns, providing economically meaningful insurance against these crashes.

Returns on the market and synthetic puts, five most extreme events

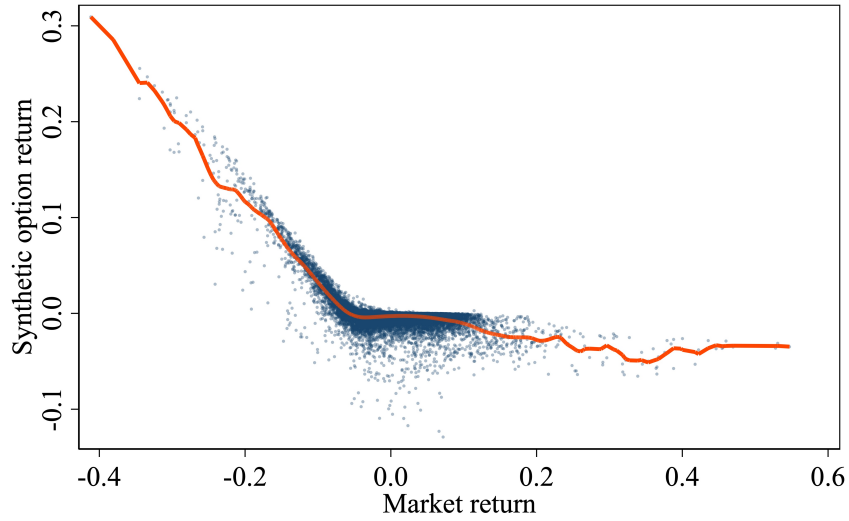
	$R_{t,t+j}^m$	$\bar{R}_{t,t+j}^S$	$\bar{R}_{t,t+j}^S$ (no lag in δ_t^S)	Ideal return
Nov. 1929	-41%	31%	32%	35%
Mar. 2020	-33%	25%	25%	28%
Oct. 2008	-31%	22%	22%	24%
Oct. 1987	-30%	17%	23%	24%
Oct. 1931	-29%	22%	22%	24%

Recall that in the benchmark results, δ_t^S depends on data only up to date $t - 1$ in order to avoid microstructure biases. The third column in the table shows that when δ_t^S uses date- t information, the hedge becomes noticeably better, particularly in 1987.¹⁶ This shows that

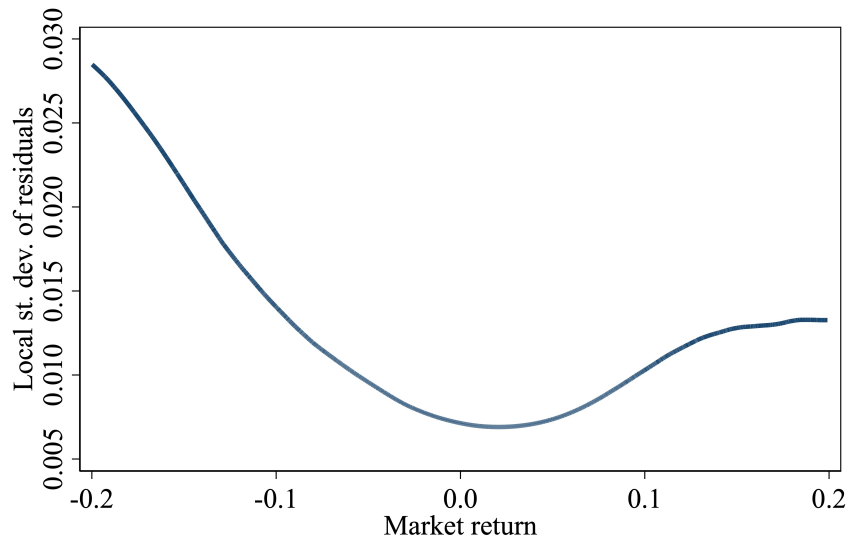
¹⁵Conditional expectations are calculated via a local linear regression on $R_{t,t+j}^m$ with a Gaussian kernel and the bandwidth set to 0.01.

¹⁶Using date t information means that the investment held during day $t + 1$ (e.g. Wednesday) is chosen based on information at the end of date t (Tuesday afternoon; as opposed to the benchmark using info from

Figure 3: Synthetic put returns as a function of the market



(a) R^S vs. R^m



(b) Local standard deviation of residuals ($\hat{R}_{t,t+j}^S$)

Note: Panel (a) plots returns on synthetic options, R^S , against the returns of the market, R^m . The red line is a kernel estimate of the local mean. Panel (b) plots the local standard deviation of the residuals.

using lagged information for hedging is conservative for fit.

Even though the synthetic options fit well, the claim is not that the replication was implementable. The results just show that the hypothetical returns are nonlinear in the market, so that the first required condition from section 2.3 is satisfied empirically.

To begin to evaluate the third condition from section 2.3, that the residual risk is small, the bottom panel of figure 3 plots the conditional standard deviation of the residuals $\hat{R}_{t,t+j}^S \equiv R_{t,t+j}^S - \bar{R}_{t,t+j}^S$.¹⁷ That standard deviation is always less than 3%, and in many cases less than 1%. In addition, we have the following variance decomposition:

$$\underbrace{\text{var} \left(R_{t,t+j}^S \right)}_{3.1 \times 10^{-4}} = \underbrace{\text{var} \left(\bar{R}_{t,t+j}^S \right)}_{2.5 \times 10^{-4}} + \underbrace{\text{var} \left(\hat{R}_{t,t+j}^S \right)}_{0.6 \times 10^{-4}} \quad (10)$$

21% of the variation in synthetic option returns in this case are unspanned by the market return. The amount of residual risk measured here is again a conservative estimate due to the fact that δ_t^S is constructed using only lagged information. If δ_t^S uses date- t information, the fraction of the variance of $R_{t,t+j}^S$ from $\hat{R}_{t,t+j}^S$ falls to 15%. The more important question, though, is whether that variation is priced, which we return to in section 4.5.

Finally, to directly compare synthetic and traded option returns, figure A.2 compares standard deviations and betas for traded and synthetic options across strikes and finds they are highly similar. Figure A.7 plots synthetic against traded option returns for different strikes and reports pairwise correlations, which range between 0.89 and 1.00.

4.2 Risk premia

4.2.1 Varying strikes at the monthly maturity

Figure 4 reports the paper’s key results for long-run average option risk premia. It includes results for three different periods: the full sample available for synthetic returns (1926–2022), the full sample available for both synthetic and traded returns (1987–2022), and the 1987–2005 sample used by Broadie, Chernov, and Johannes (BCJ; 2009), who report an extensive analysis of the performance of traded options. In all cases, the figure gives results for put options. For alphas and information ratios, results for puts and calls are guaranteed to be

Monday afternoon).

¹⁷The local volatility is estimated from a second kernel regression,

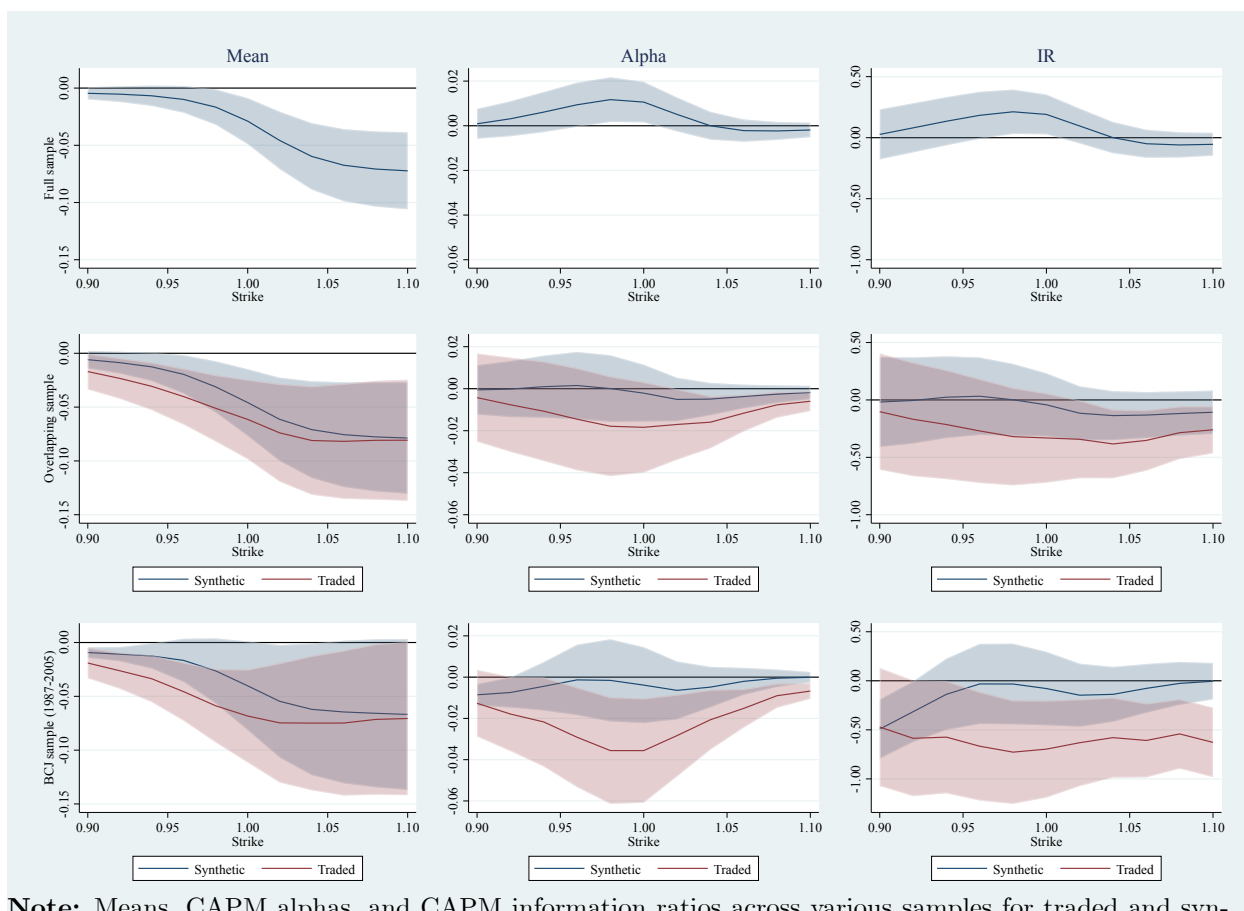
$$\eta_{t,t+j}^2 = h \left(R_{t,t+j}^m \right) + \text{residual} \quad (9)$$

where for the function h we set the bandwidth to 0.05 due to there being greater variation in the squared residuals around the fitted value.

identical for synthetic options at a given strike, and they are highly similar for traded options (in both cases due to put-call parity).

The left column of Figure 4 plots average returns. In all three panels, returns decline as the strike rises, which is to be expected as the betas also become more negative for higher strikes. In the periods of overlap between synthetic and traded options, traded options always have lower average returns than synthetic.

Figure 4: Average option returns across strikes



Note: Means, CAPM alphas, and CAPM information ratios across various samples for traded and synthetic put options. Shaded regions represent 95-percent confidence intervals. In the top panel, for the full post-1926 period, only synthetic options are available. The overlapping sample in the middle row is 1987–2022.

The middle column reports CAPM alphas, which are the paper’s key object of interest based on the theoretical analysis. The bottom two panels show that the estimated alphas of traded options are negative across all strikes in both the 1987–2022 and BCJ samples. The statistical evidence for the alphas being negative is stronger in the earlier BCJ sample, with the magnitudes falling by half in the longer sample.

In the same post-1987 period, though, and regardless of whether the post-2005 period is included, synthetic options have estimated alphas very close to zero, with no evidence of mispricing relative to the CAPM. The top panel shows that the same result holds in the full sample and is if anything actually stronger – in that case the alphas are statistically significantly *positive* for the middle strikes. The paper’s claim that synthetic options have been fairly priced historically relative to the CAPM is based on the results reported here for alphas.

The right-hand column of Figure 4 plots information ratios – the Sharpe ratio of the part of option returns uncorrelated with the market. Again, they are if anything statistically significantly *positive* in the full sample, peaking at values of about +0.2. The traded options, on the other hand, have information ratios as negative as -0.75 in the BCJ sample, which is larger than the Sharpe ratio of the overall stock market.

Figure A.3 replicates 4, but varying the maturity instead of the strike price, figure A.4 scales the moneyness in volatility units, and figure A.5 puts the price of the option in the denominator instead of the level of the underlying. The results are similar to the baseline in all three cases.

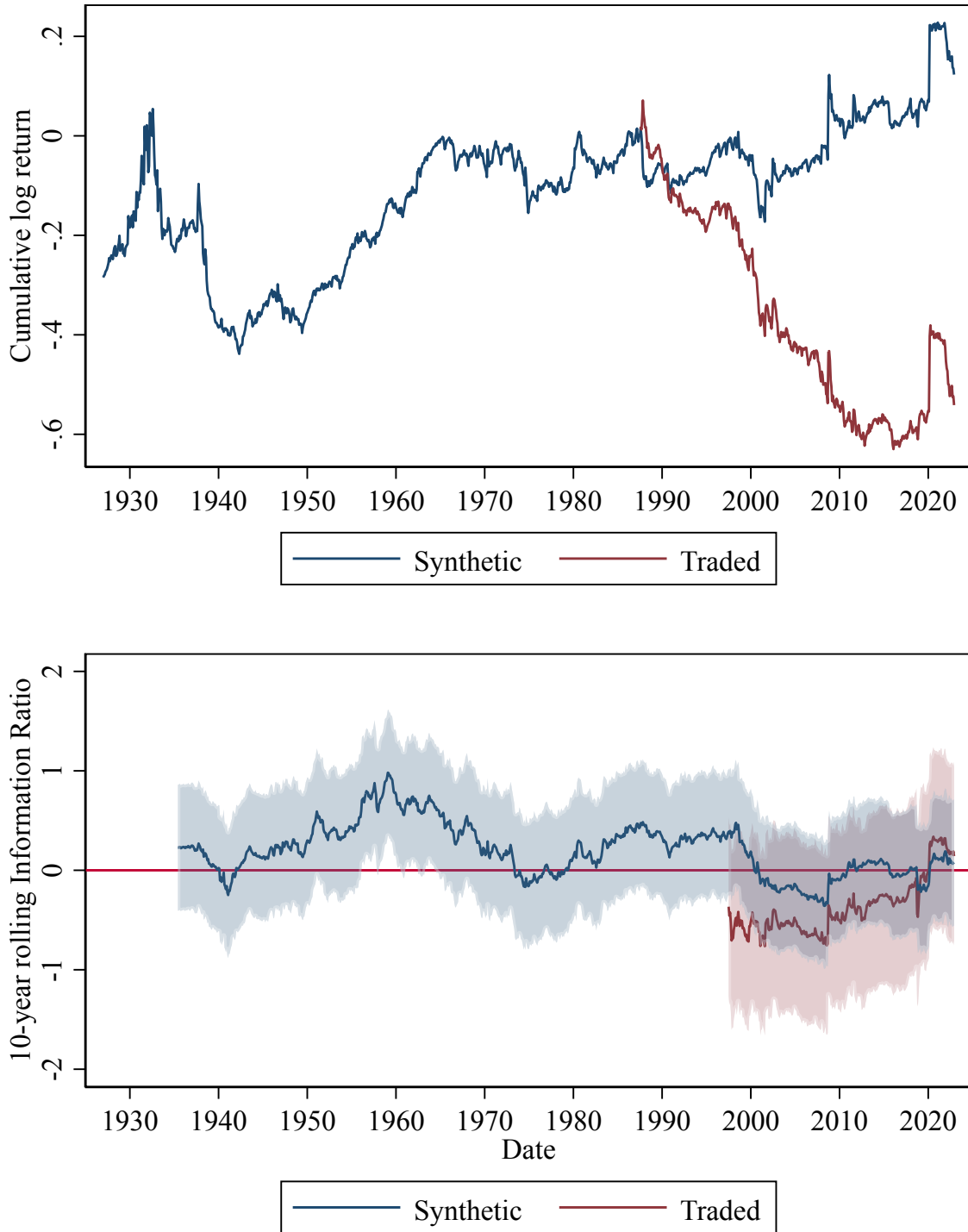
4.2.2 Cumulative alphas and variation in risk premia over time

The top panel of figure 5 plots cumulative CAPM alphas for synthetic 5% OTM options over the period 1926–2022 and for traded put options over the period 1987–2022 (again, for alpha the choice of put versus call is irrelevant for synthetic options; and it has only very minor effects for traded options conditional on strike). For readability, the cumulative returns are normalized to zero in July 1987 when the data for the traded options begins.

For synthetic options, the figure reinforces the result that over the full sample the alphas have been slightly positive. Covid jumps out in 2020 as a large positive innovation, due to the significant decline in the level of the stock market. The fact that the synthetic portfolio captures that gives clear evidence that it is able to capture economically significant large declines in the market. The bottom panel of figure 5 plots information ratios over rolling 10-year windows and again shows that the returns on synthetic options have been stable, with a brief period in the 1930’s when the returns were statistically significantly positive. In no period were they significantly negative.

Traded put returns have two striking features. On the one hand, the month-to-month variation appears very similar to that for the synthetic options, consistent with the results presented so far. On the other hand, the average return is drastically different. The returns are highly negative, especially in the period up to 2010. The returns were roughly flat from

Figure 5: Cumulative returns



Note: The top panel plots cumulative CAPM alphas for traded and synthetic -5% put options. The lines are constructed to equal zero in July, 1987, when the true put options become available. The bottom panel plots 10-year rolling CAPM information ratios, with the shaded regions representing 95-percent confidence intervals.

then until the large market decline with Covid. In fact, the overall cumulative return on traded puts is zero between March, 2009 and the end of the sample in December, 2022. The bottom panel of figure 5 shows how, over ten-year windows, the return on traded puts actually turned positive at the end of the sample.

Since a synthetic put is essentially a delta hedge, the difference between the returns on the traded and synthetic put returns is the return on a delta-hedged put, which is a measure of the variance risk premium (Bakshi and Kapadia (2003)). To see that, figure A.9 in the appendix plots cumulative alphas for delta-hedged puts and straddles. They have been approximately zero since 2010. Section 5.1 revisits this point in more detail and reports formal tests for a structural break.

4.2.3 A conditional CAPM interpretation

Since synthetic options are created by trading the market dynamically, any monthly CAPM alpha they earn has to come from timing the market risk premium at the daily level – their daily alpha is identically zero. For both synthetic puts and calls, the investment in the market, δ_t^S , declines – becoming more negative for a put and less positive for a call – when the market declines and rises when the market rises. Synthetic options are therefore bets on momentum. To get a negative alpha (which would be consistent with traded option returns) would require mean reversion in returns.

Formally, one can derive from results in Lewellen and Nagel (2006) that

$$\alpha_{t,t+j}^S \approx \text{cov} \left(\delta_t^S, [E_t [R_{t,t+1}^m] - E [R_{t,t+1}^m]] - \frac{E [R_{t,t+1}^m - 1]}{\text{var} (R_{t,t+1}^m)} [\text{var}_t (R_{t,t+1}^m) - \text{var} (R_{t,t+1}^m)] \right) \quad (11)$$

The first part of the covariance is the usual conditional CAPM intuition, which says that if δ_t^S covaries positively with the market risk premium, then $\alpha_{t,t+j}^S$ will be positive. The second part is a contribution from the comovement of δ_t^S with conditional volatility – the movement of deltas with volatility is second-order (and its sign is ambiguous), so this term is relatively small quantitatively. The equation can also be interpreted in the opposite direction: if synthetic puts earn a negative alpha then (holding volatility fixed) expected returns must be countercyclical. That is, convexity in SMU implies countercyclical risk premia.¹⁸

¹⁸How does time-varying volatility affect this analysis? If the CAPM holds period-by-period with constant risk aversion (in the sense of the pricing kernel being linear in $R_{t,t+1}^m$), then $E_t [R_{t,t+1}^m - 1] \propto \text{var}_t (R_{t,t+1}^m)$, which would imply that the covariance in (11) is identically zero. If risk aversion is countercyclical, as with a convex pricing kernel, then even if volatility rises when the market falls, $E_t [R_{t,t+1}^m - 1]$ will rise by enough to offset that effect, so that the covariance term is negative and $\alpha_{t,t+j}^S < 0$. In other words, if both volatility and risk aversion are countercyclical, equation (11) implies that $\alpha_{t,t+j}^S$ is negative.

One intuition for these results is the following. If marginal utility is convex, then a drop in the market moves the representative investor to a steeper part of the SMU function, causing them to demand higher expected returns. Returns then mean-revert somewhat on average, inducing a negative alpha for synthetic puts.

4.3 Effects of conditioning on betas

In the theoretical analysis, all alphas and betas are conditional, and hence potentially time-varying. Since option returns are non-linear functions of the market return, conditional betas of both true and synthetic options will necessarily change over time as the conditional distribution of the market return varies. That naturally then affects estimation of alphas.¹⁹

Figure A.6 examines possible ways of accounting for time-variation in conditional betas. The left-hand column of panels plots the baseline results. In the middle column, betas are estimated from a rolling three-month window. The right-hand panels model conditional beta as a function of lagged (i.e., end of previous month) variables: the market return volatility forecast (which is most important), the market return itself, industrial production growth, and the corporate bond default spread. In both cases, the results are highly similar to the benchmark qualitatively and quantitatively.

4.4 Effects of daily mispricing

Recall the second condition for using synthetic options to measure curvature in marginal utility from section 2.3 that the market is priced correctly every day. Obviously no such condition is literally true, so the question is how pricing errors might bias the results.

Suppose that the “true” market return that satisfies $1 = E_t [M_{t,t+1} R_{t,t+1}^m]$ every day, $R_{t,t+1}^m$, is unobservable and instead we can only see some contaminated return $R_{t,t+1}^{m*}$, with

$$R_{t,t+1}^{m*} = R_{t,t+1}^m + \varepsilon_{t+1} \tag{12}$$

where ε_{t+1} is the contamination. Depending on the properties of ε_{t+1} , it may be that $1 \neq E_t [M_{t,t+1} R_{t,t+1}^{m*}]$ even if $R_{t,t+1}^m$ is in fact priced each day.

If we define $R_{t,t+j}^{S*}$ to be the return on an option synthesized from the contaminated

¹⁹Note that in the theoretical analysis, all conditioning for an option return between dates t and $t + j$ is taken as of date t . That is, for monthly returns, we must condition on information available at the *beginning* of the month, not over the course of the month (if we did the latter, then the alphas of synthetic options would be identically equal to zero).

market return, $R_{t,t+1}^{m*}$, then

$$E [R_{t,t+j}^{S*}] = E [R_{t,t+j}^S] + \sum_{s=0}^{j-1} E [\delta_{t+s}^S \varepsilon_{t+s+1}] \quad (13)$$

In order for the contamination, ε_{t+1} , to not affect measured risk premia, it must have zero mean and be uncorrelated with past values of the weights δ_t^S . On the other hand, the errors need not be i.i.d., for example, or necessarily independent of anything else.

Recall that in the analysis above δ_t^S is set based on information about returns only up to date $t - 1$. That choice is made to address the potential bias identified in this section. In particular, if ε_t is an MA(1) process, so that $E[\varepsilon_t \varepsilon_{t-k}] = 0 \forall k > 1$, then choosing δ_t^S based on information from date $t - 1$ is enough to ensure that $E[\delta_t^S \varepsilon_{t+1}] = 0$.

As discussed in Bates (2012), observed positive serial correlation in daily market index returns is evidence of stale prices.^{20,21} When there is a positive return in the underlying, the hedge weight δ_t^S rises, both for puts and calls. That leads to the result that $\text{cov}(\varepsilon_{t+1}, \delta_t^S) > 0$ when ε_t is positively serially correlated, which would lead to a positive bias, $E[R_{t,t+j}^{S*}] > E[R_{t,t+j}^S]$. That is why the main results use lagged information, so that δ_t^S does not depend on R_t^{m*} .²²

To examine this effect in the data, the top panel of figure 6 reports the autocorrelations of daily returns in the full sample and pre- and post-1973 separately. In the pre-1973 sample, there is clear evidence of one-day positive serial correlation, consistent with the presence of stale prices. The two-day autocorrelation, on the other hand, is negative, which is one reason why the analysis only uses a single-day lag in constructing the weights.

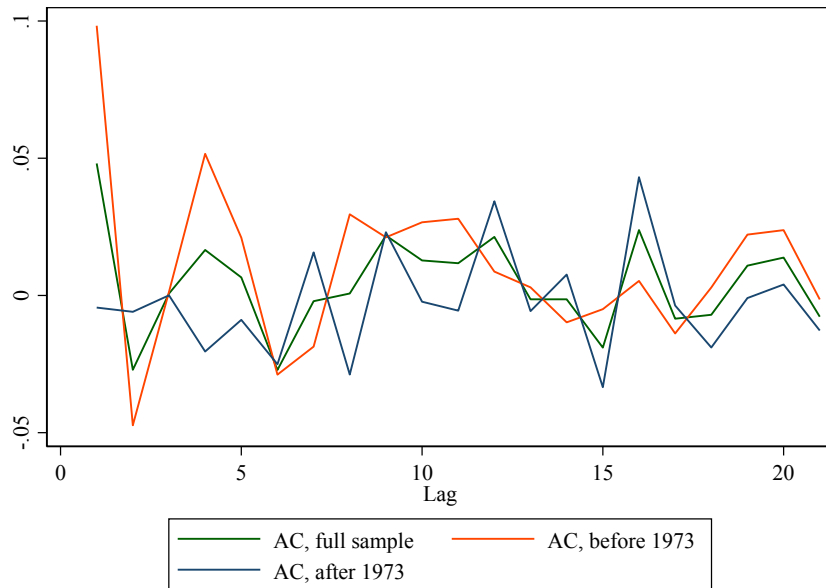
To see the effects of different choices for the information set for δ_t^S , the bottom panel of figure 6 plots three versions of the cumulative alphas for synthetic options: with the baseline one-day lag, with no lag, and with a two-day lag. Switching from the baseline to no lag causes a large increase in alphas, entirely due to the positive autocorrelation, which is why

²⁰For example, suppose on each day 50% of stock prices are updated. If there is good news on date t , that will be impounded into half of stock prices on date t , 1/4 on $t+1$, 1/8 on $t+2$, etc., inducing positive serial correlation in the “errors” ε_t relative to the “true” market return that would be observed if all stock prices were updated every day.

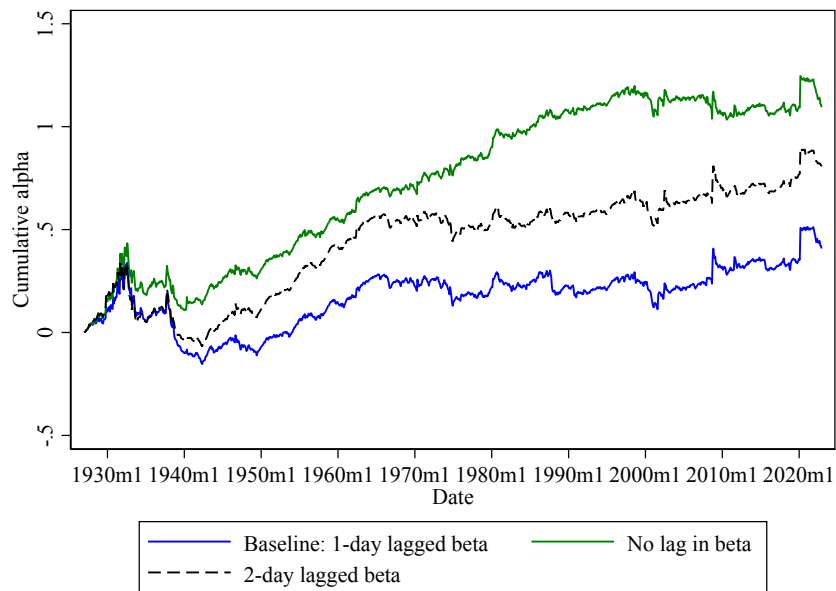
²¹Another type of error that can arise is bid-ask bounce, which, as discussed in Jegadeesh and Titman (1995), creates negative serial correlation in returns. In contrast to stale prices, that would bias the estimate of synthetic option returns *downward*, implying the true alphas for synthetic options are even more positive and in even stronger conflict with the alphas on traded options than is suggested by the baseline results.

²²One might also ask about the effect of mispricing on estimated betas. First, as an empirical matter, recall that the data shows that the betas of synthetic and traded options are highly similar, implying that there is not a severe bias. Second, while stale prices and bid-ask bounce can affect betas at high frequencies, those effects tend to shrink at lower frequencies, hence the more common focus on, say, monthly data in studies of equity returns.

Figure 6: Daily autocorrelation of returns



(a) Autocorrelations



(b) Cumulated returns of synthetic options

Note: Panel (a) plots the autocorrelations of daily returns up to 21 lags, using the full sample (1926-2022), and using the pre- and post-1973 data separately. Panel (b) plots cumulative CAPM alphas on the synthetic puts built using one-day lagged weights (the benchmark case), no lag, and two-day lagged weights.

it shows up only in the first half of the sample. Going from a one-day to a two-day lag also increases the alphas (by a lower amount). None of the lag choices leads to negative average alphas.

4.5 The effect of unspanned variation – $\hat{R}_{t,t+j}^S$ and $\hat{M}_{t,t+j}$

Recall from section 2.3 that the curvature of marginal utility is determined by $\alpha_t^S + \text{cov}_t \left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S \right)$ where the variables with hats are the components unexplained by any nonlinear function of the market return. So far the analysis has ignored the covariance term. The questions are whether that covariance is zero and if not, how large it might be. This section examines what risk factors $\hat{R}_{t,t+j}^S$ might be correlated with and then bounds the magnitude of $\text{cov}_t \left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S \right)$ using the method of Cochrane and Saa-Requejo (2000).

4.5.1 Relationship of $\hat{R}_{t,t+j}^S$ with risk factors

Table 1 reports correlations between the unspanned part of synthetic option returns, $\hat{R}_{t,t+j}^S$, and statistical innovations in prominent macro and financial variables.²³ Since $\hat{R}_{t,t+j}^S$ is orthogonal to the market return by construction, we also orthogonalize the innovations in all of the other macro and financial time series with respect to the market return.

Among the macro time series, the correlations are all economically small and statistically insignificant. The only notable correlations are for financial series: the excess bond premium (EBP), the VIX, and realized volatility. In months in which shocks to these financial series, after orthogonalizing with respect to the market return, are unexpectedly high, $\hat{R}_{t,t+j}^S$ tends to be low.²⁴ If those are bad states of the world, that would make $\hat{R}_{t,t+j}^S$ risky, which proposition 3 shows would imply more convexity in SMU than implied by $\alpha_{t,t+j}^S$.

The results suggest that options do not look risky to an investor whose marginal utility depends on either the level of the stock market or to macroeconomic variables. They *do* look risky, though, to an investor who cares about the *path* that the market return takes, suggesting that intermediaries might be relevant for pricing.

The bottom row of table 1 reports the maximum correlation of $\hat{R}_{t,t+j}^S$ with any linear

²³For $\hat{R}_{t,t+j}^S$ we take the return from the beginning to the end of a month. For the variables that are measured at a fixed point in time, we take the statistical innovation in the value at the end of the month relative to the lags of the variable (information available at the beginning of the month). For the other variables, we take the statistical innovation in the monthly value relative to data available in the previous month. The different time series are available for different time periods and each correlation is computed using the longest period available for that variable.

²⁴The correlations of the unspanned option returns with the SMB and HML factors are also very small – 0.02 and -0.05, respectively.

Table 1: Correlation of residuals with macro variables

	All data	Excluding 2020
Unemployment	-0.08	-0.06
Ind. Pro. Growth	0.06	0.07
Employment growth	0.07	0.06
FFR	-0.01	-0.01
Term Spread	0.02	0.02
Default Spread	-0.01	-0.01
EBP	-0.11	-0.14
VIX	-0.21	-0.21
VXO	-0.14	-0.12
rv	-0.35	-0.38
Maximal corr	0.35	0.41

Note: Table reports the correlations between the residuals of the nonlinear fit of R^S onto the market and various macroeconomic variables: unemployment, industrial production growth, employment growth, the federal funds rate, the term spread (10 year minus 1 year), the default spread (BAA-AAA spread), the excess bond premium (EBP) from Gilchrist et al. (2021), the VIX, the VXO, and realized volatility. All variables are orthogonalized to the market. The last row reports the maximal correlation between any linear combination of these variables and the residuals. The second column replicates the results excluding 2020.

combination of the innovations. It is less than 0.5, so we take that as an upper end for a reasonable estimate of the correlation of $\hat{R}_{t,t+j}^S$ with $\hat{M}_{t,t+j}$, but we also examine results with the correlation set to 1.

4.5.2 Robust uncertainty bands

If $\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)$ is not equal to zero, how large of a bias does it create in measuring curvature in marginal utility? To bound the magnitude of $\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)$, following Cochrane and Saa-Requejo (2000), start from the identity,

$$\left| \text{cov}_t(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}) \right| = \left| \text{corr}_t(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}) \right| \text{std}_t(\hat{R}_{t,t+j}^S) \text{std}_t(\hat{M}_{t,t+j}) \quad (14)$$

$\text{std}(\hat{R}_{t,t+j}^S)$ can be estimated based on the empirical time-series of $\hat{R}_{t,t+j}^S$. $\left| \text{corr}(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}) \right|$ is not observable, but the results in the previous section imply 0.5 as an estimate, and the upper bound is 1. Finally, to get $\text{std}(\hat{M}_{t,t+j})$ we assume that the volatility of the unspanned part of SMU, $\hat{M}_{t,t+j}$, is no greater than that from the part of SMU spanned by the market,

$\bar{M}_{t,t+j}$. That implies that

$$std\left(\hat{M}_{t,t+j}\right) \leq E\left[R_{t,t+j}^m - 1\right] / std\left(R_{t,t+j}^m\right) \quad (15)$$

Intuitively, that restriction says that the Sharpe ratio available from any investment independent of the market return can be no greater than that of the market itself, similar to Cochrane and Saa-Requejo (2000) (see also references therein).²⁵

The parameter of interest, which measures convexity in SMU, is

$$\alpha_{t,t+j}^{S,adjusted} \equiv \alpha_{t,t+j}^S + cov_t\left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S\right) \quad (16)$$

It has two sources of uncertainty: estimation uncertainty in $\alpha_{t,t+j}^S$ and the unobservable value of $cov_t\left(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S\right)$. Appendix C shows how those two sources of uncertainty can be combined geometrically, essentially treating $cov\left(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}\right)$ as another Gaussian source of error.

Figure 7 reports an alternative version of figure 4 that now incorporates these robust uncertainty bands instead of the original confidence bands based only on estimation error. The left-hand panels assume that $corr\left(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}\right)$ based on the results in the previous section, while the right-hand panels use the most conservative possible value of 1.²⁶

The uncertainty bands in figure 7 are guaranteed to be wider than in the baseline. However, they can still reject information ratios of -0.5 in all but a few cases with the shortest sample. In the top-center panel, which is the most powerful case, using the longest sample and $corr\left(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j}\right) = 0.5$, the bound can reject even small negative information ratios. So even when accounting for unspanned risk, the curvature of SMU implied by synthetic options remains small.

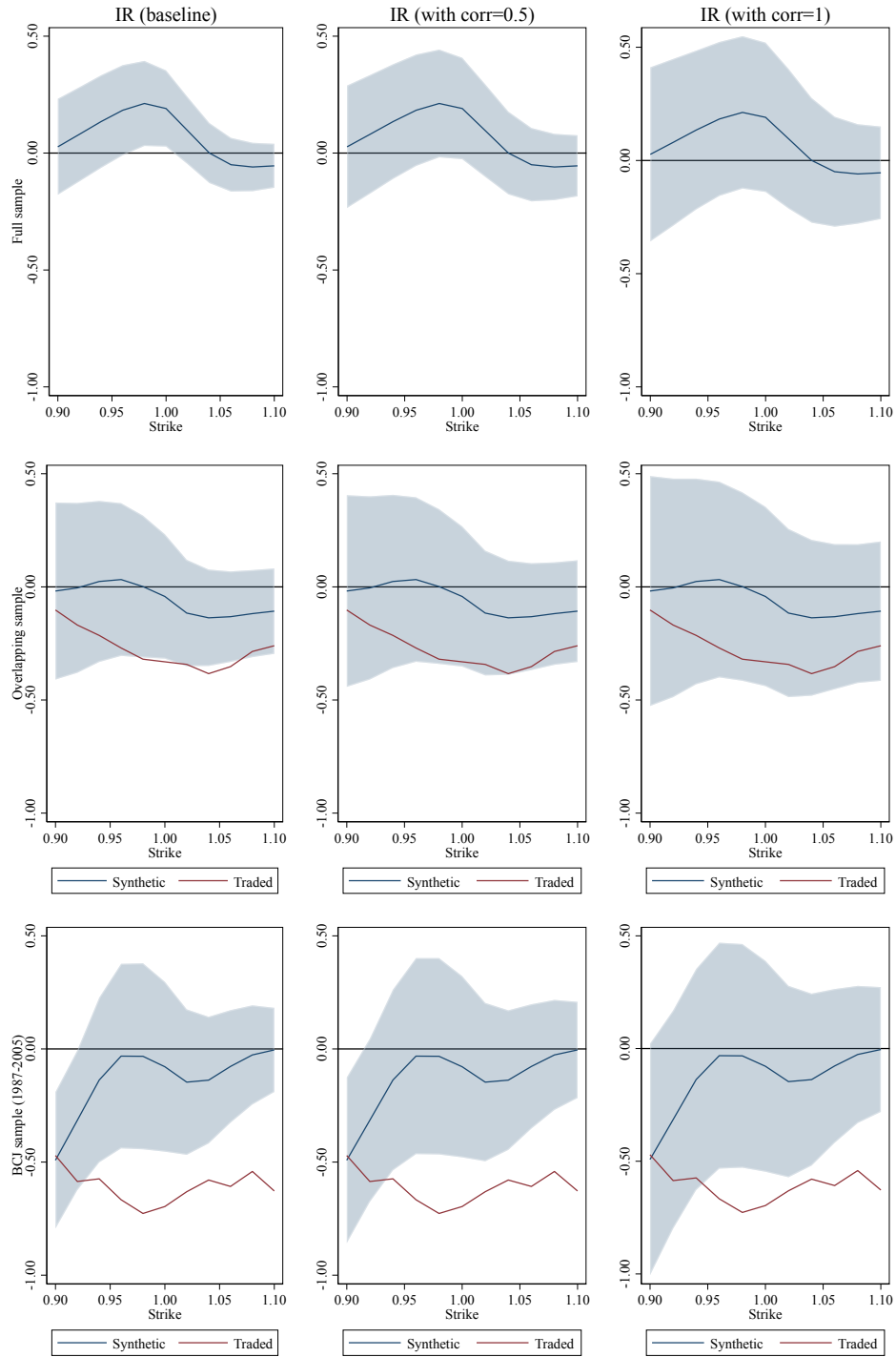
4.6 Implications for marginal utility

Figure 2 plots, in blue, the shape of SMU implied by the synthetic option returns for a strike of -5% at maturity of one month for various samples. In all cases, the synthetic options imply that marginal utility is if anything weakly concave, consistent with the positive measured alphas and implying risk aversion falls slightly in bad times. While there are no confidence bands plotted in figure 2, the change in the slope across the strike is measured by the alphas

²⁵The 1 here represents the gross-risk-free rate. Again, in the empirical analysis the actual risk-free rate is used.

²⁶Equivalently, the right-hand panel can be taken as treating the market Sharpe ratio as 1 instead of 0.5 (and leaving the correlation at 0.5).

Figure 7: Average option returns with robust uncertainty intervals



Note: These graphs replicate the main results, but the shaded uncertainty intervals here incorporate the bound on the effect from potential pricing of unspanned risk in the synthetic option returns.

reported above, and so the confidence bands apply here.

As discussed above, the red lines corresponding to traded options imply significant convexity due to the large negative estimated alphas, so that effective risk aversion rises strongly as market returns fall.

The analysis in section 2 of how to estimate SMU based on the returns on the market and a single option is a special case of the minimum-variance SDF of Hansen and Jagannathan (1991) and naturally extends to using multiple options simultaneously.

Figure 8 plots the minimum-variance SMU for the traded and synthetic options separately using strikes at 5% moneyness intervals from 90 to 110 percent (for 1926–2022 for synthetic options and 1987–2022 for traded). The mean-variance optimal portfolio is calculated based on the full-sample estimates of mean returns and covariances, with both a lasso- and ridge-type adjustment for robustness.²⁷

As in the benchmark case, estimated SMU is convex for the traded options and concave for the synthetic options. For the synthetic options, the concavity is fairly consistent, though it may change signs at large positive strikes. For the traded options, the convexity appears strongest around market returns of zero. While traded options again imply non-monotone marginal utility, the synthetic options do not.

4.7 Robustness

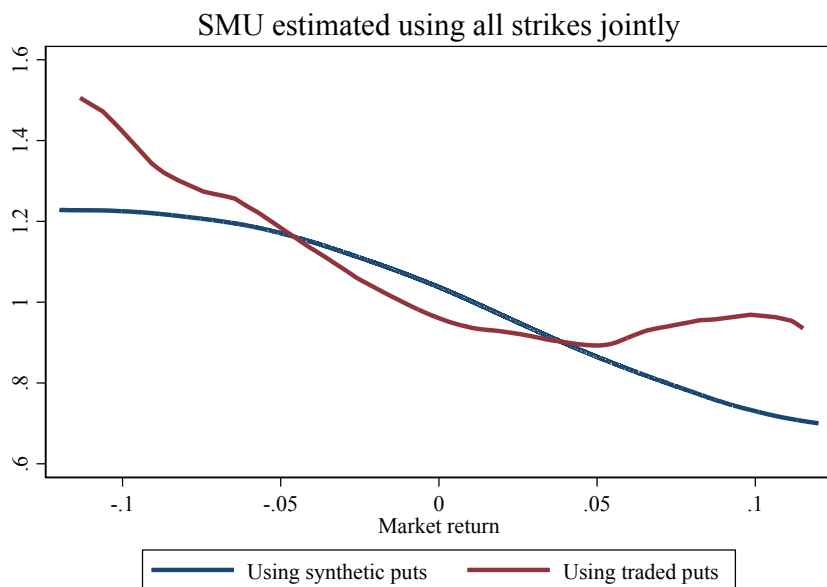
The baseline results use deltas from the method of Hull and White (2017). The left column of figure A.8 shows (for different sample periods) that the results are highly similar simply using the standard Black–Scholes delta.

Constructing the weights for the synthesis requires a forecast of volatility. The benchmark analysis uses a recursively estimated HAR model (Corsi (2009)). The middle column of figure A.8 shows that the alphas are very similar to those in the baseline if the volatility used to calculate the hedge weights is simply set to 0.15 on all dates. That shows that the results are driven by how δ_t^S depends on the level of the market, rather than its volatility.

The benchmark analysis uses asymptotic standard errors based on the Hansen–Hodrick method. Since option returns can be highly non-normal, convergence to the asymptotic distribution might be slow, which can be addressed via bootstrapping. Results with block-bootstrapped standard errors with block length equal to two months are reported in right

²⁷Mean-variance optimization requires inverting the covariance matrix. We make two modifications to ensure that the inversion is well behaved. First, we inflate the main diagonal of the covariance matrix by 10%, which corresponds to a ridge-type adjustment. Second, in constructing the inverse, we drop any eigenvalue smaller than 0.01 times the largest (in practice this eliminates one eigenvalue). These adjustments only affect the weights in the tangency portfolio.

Figure 8: Estimated SMU using all options jointly



Note: The figure reports the estimated SMU obtained by using options of all strikes jointly. The blue line uses synthetic puts (sample 1926–2022), and the red line uses traded puts (sample 1987–2022).

column of figure A.8, and they are highly similar to the baseline.

5 Intermediary frictions and the change in option returns over time

The theoretical analysis examined two ways to estimate curvature in marginal utility – using traded options and using synthetic options – and in the data we find that they give different answers. There were three theoretical conditions needed for estimating curvature from synthetic options, and the empirical analysis provided some evidence in their favor. And even allowing for a conservative bound on the pricing of unspanned risk in synthetic options (i.e. relaxing condition 3), the implied convexity in SMU is far smaller than what is measured from traded options.

There is also a required condition for using traded options to measure the curvature of SMU, though, which is that they are priced by marginal utility. This section considers a model in which the prices of traded options do not align with the marginal utility of a representative investor, but rather are driven by intermediary frictions. It shows that it can rationalize the gap in returns between the synthetic and traded options, including its changes

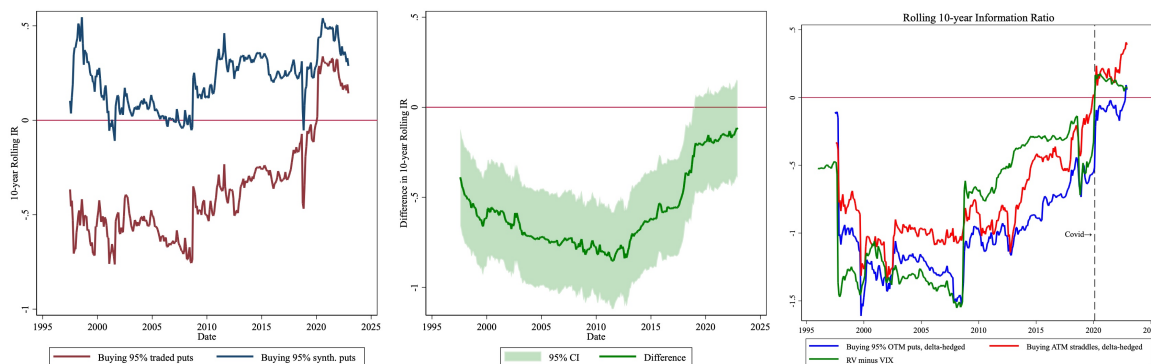
over time.

5.1 The decline of option overpricing

Figure 5 above already showed some suggestive evidence that the returns on traded options may have trended towards zero in recent years. Figure 9 examines that behavior in greater detail. The left-hand panel of figure 9 plots information ratios over rolling 10-year windows for traded and synthetic puts along with their difference for the 1987–2022 sample.²⁸ The synthetic options consistently had zero or positive returns, while the traded options consistently had strongly negative returns until 2009, when they begin to trend up, eventually turning starkly positive in 2020. In the early periods, the information ratio for the traded options was very large – about equal to the size of the market risk premium itself.

The difference between the information ratios on traded and synthetic options, plotted in the middle panel of figure 9, has also nearly converged to zero over the same period. The confidence bands show that the change in the information ratio appears to be highly statistically significant, which is tested more formally in section 5.3.4.

Figure 9: Changes in premia over time



Note: The left-hand panel plots 10-year rolling CAPM information ratios for traded and synthetic 5The middle panel plots the difference between those two series for each 10-year window along with a 95-percent confidence band. The right-hand panel plots information ratios for three other related measures of the difference between traded and synthetic options.

The difference in the information ratios between traded and synthetic options is very closely related to the return on delta-hedged options, which have been studied widely in the

²⁸The main results use the overlapping 21-day returns for synthetic option. In this section, all results use synthetic option returns that match exactly the roll dates of the traded options, in order to ensure comparability. This has only minor effects, and in any case they run against the main conclusions of this section.

past literature and used as a proxy for the variance risk premium (e.g. Bakshi and Kapadia (2003)). To examine that idea more directly, the right-hand panel of figure 9 plots the rolling 10-year information ratios for delta-hedged 5% OTM put options and at-the-money straddles. Both have risen over time, converging to zero in 2020 (with the convergence for the ATM straddles coming even before Covid). As an even simpler test, the green line in the same plot proxies for the payoff of a variance swap simply as the gap between realized variance and the squared VIX (Carr and Wu (2009)). The information ratio of RV-minus-VIX behaves highly similarly to the other series, also converging to zero over the sample.²⁹

The results here *do not* mean that the variance risk premium is zero by the end of the sample. There is still a premium for variance risk, but the results imply that the premium is no larger than what would be expected from the CAPM beta. Variance rises when the market falls, so it has a negative beta and carries a negative premium. In the past, the premium from trading variance via delta-hedged options was even larger than is implied by the CAPM beta, but by the end of the sample that is no longer true. These results are consistent with those in Heston, Jacobs, and Kim (2022), who also find that there is a negative variance risk premium, but that it cannot be distinguished from simple market (beta) risk.

5.2 Model

This section studies a simple extension of the model of Garleanu, Pedersen, and Poteshman (GPP; 2009) to help clarify how various frictions can affect option prices when markets are segmented. The only addition to their framework is to allow for transaction costs and index-futures basis risk. The main text describes just the key parts of the setup and predictions of the model. The details are in appendix D.

5.2.1 Setup

In GPP’s model, the price of the underlying – which we take here to be the market index – is determined exogenously (presumably by a much larger mass of traders, for example retail investors optimizing between index funds and cash), as is the demand or supply for a set of derivative claims, such as options. GPP show empirically that in general retail investors appear to be long index options, so that dealers must be net short.³⁰

²⁹Specifically, define a (pseudo-) return $R_t^{RV} = (RV_t - VIX_{t-1}^2) / VIX_{t-1}^2$, where RV_t is (annualized) realized variance in month t and VIX_{t-1} is the level of the VIX at the end of month $t - 1$. The information ratio is then the CAPM alpha of R_t^{RV} divided by the residual standard deviation.

³⁰A simple model for the source of asymmetry in retail demand is that it is easier in practice to buy than sell options – buying options does not require posting margin, it has limited downside risk, and it can be

The dealers are assumed to have time-additive CARA preferences over consumption with risk aversion γ . The key equation in the model is the dynamic budget constraint,

$$W_{t+1} = (W_t - C_t) R_f + D_t R_{t+1}^O + F_t R_{t+1}^F - \frac{\kappa}{2} F_t^2 \quad (17)$$

W_t is wealth and C_t consumption. The risk-free rate, R_f is constant for simplicity, R_{t+1}^F is the gross return on index futures, and R_{t+1}^O is the return on the derivatives. The dealers endogenously choose consumption and the allocations to derivatives and futures D_t and F_t , respectively.

We add two frictions: a quadratic trading cost, $\frac{\kappa}{2} F_t^2$, and a wedge between the futures return, R_{t+1}^F , and the underlying index return, R_{t+1}^I ,

$$R_{t+1}^F = R_{t+1}^I + z_{t+1} \quad (18)$$

z_{t+1} represents *basis risk*. Ideally the dealers would like to hedge the options they trade with the underlying, like the S&P 500. But the S&P 500 is not itself directly tradable (except at significant cost by buying 500 stocks). Instead, dealers must buy futures (or ETFs or other instruments) whose price is not guaranteed to perfectly track the index. z_{t+1} captures the risk associated with imperfect tracking.³¹

While the dealers choose D_t , markets must clear, meaning that in equilibrium their choice of D_t must perfectly offset the (exogenous) demand from retail investors. The core idea in GPP is to understand how derivative prices, denoted by P_t , vary with quantities, D_t .

5.2.2 Predictions

In the model, intermediaries hedge their options each period with a position in the underlying – it can be thought of as a delta hedge that is adjusted each period. The optimal position, in the absence of any frictions, is denoted by β_t^I (which is simply the local sensitivity of option returns to the underlying index). The unhedgeable risk is defined as

$$\sigma_{\varepsilon,t}^2 \equiv \text{var}_t^d (R_{t+1}^O - \beta_t^I R_{t+1}^I) \quad (19)$$

done in both regular and tax-protected accounts. So if retail investors are roughly split between those who would to buy and sell, the net demand can still be positive due to asymmetry in frictions. And as those frictions decline, so should the asymmetry.

³¹The S&P 500 index is the underlying for CBOE options, but not CME futures options. For the futures options, an interpretation of basis risk would be that intermediaries price options based on a model for the underlying, meaning that deviations of the futures price from the index create risk.

where var_t^d is a variance taken under the intermediaries' pricing measure d based on date- t information.

The model's key prediction is for the sensitivity of option prices, P_t^O , to demand:

Proposition 4 *Up to first order in the transaction cost κ and the index-futures basis risk $\text{var}_t^d(z_{t+1})$,*

$$\frac{\partial P_t^O}{\partial D_t} = -\gamma(R_f - 1) \left(\underbrace{\sigma_{\varepsilon,t}^2}_{\text{Unhedgeable risk}} + \underbrace{(\beta_t^I)^2 \text{var}_t^d(z_{t+1})}_{\text{Basis risk}} \right) + \underbrace{\kappa R_f^2 (\beta_t^I)^2}_{\text{Imperfect hedging}} \quad (20)$$

The sensitivity is proportional to risk aversion, γ , and has three terms.

The first component, $\sigma_{\varepsilon,t}^2$, is the unhedgeable risk from (19). Dealers hedge by taking positions exposed to the underlying, but since options have nonlinear exposure, that hedge is inevitably imperfect, due to discrete hedging, jumps, and unspanned volatility. The synthetic options studied in our empirical analysis exactly map into the hedge that the dealers use here – they are updated discretely and inherit risk from deviations between the discrete replication and the traded option return.

The second term represents basis risk. When there are larger random gaps between the hedging instrument and the true underlying index, dealers face greater risk and thus demand larger premia. Finally, the third term arises due to the quadratic trading cost, κ , which causes dealers to hedge incompletely, further raising their risk from holding derivatives.

In the context of the general theoretical analysis in section 2, this is a model in which traded options are not priced by the marginal utility of retail equity investors, but instead by that of dealers. And the exogenous option demand, since it must be borne only by dealers, drives option prices up, creating negative CAPM alphas.

5.3 Empirics

The results in section 5.1 show that the premium for traded options has shrunk toward zero over time. Proposition 4 predicts that the options premium declines if any of the three factors in equation (20) have shrunk (without offsetting rises in the others). This section examines those factors and how they have changed over time.

5.3.1 Trading frictions

The top-left panel of figure 10 plots measures of bid-ask spreads for equities as a measure of trading costs. The gray line is the average spread for the Dow 30 (from Jones (2002)), blue

is for the DIA Dow 30 ETF, and red is for the SPY S&P 500 ETF. Spreads were around 100 basis points until the late 1990's, falling with both the rise in electronic trading and decimalization.

The top-right panel of figure 10 plots effective spreads based on the Roll (1984) estimator for S&P 500 futures. Effective spreads also have consistently fallen over time. While the decline looks close to linear, note that the y-axis is on a \log_{10} scale, so that in absolute terms the declines were much larger in the early part of the sample. Overall, effective spreads fell by about a factor of 100 over the 1982–2022 period.

5.3.2 Basis risk

We measure basis risk empirically from the gap between the level of the S&P 500 index and the futures price. The middle-left panel of figure 10 plots the three-month rolling standard deviation of that gap. The y-axis is again on a \log_{10} scale. Over time, basis risk has fallen by about an order of magnitude. While there is a large decline early in the sample, similar to trading frictions, basis risk seems to settle at its current level around the early 2000's.

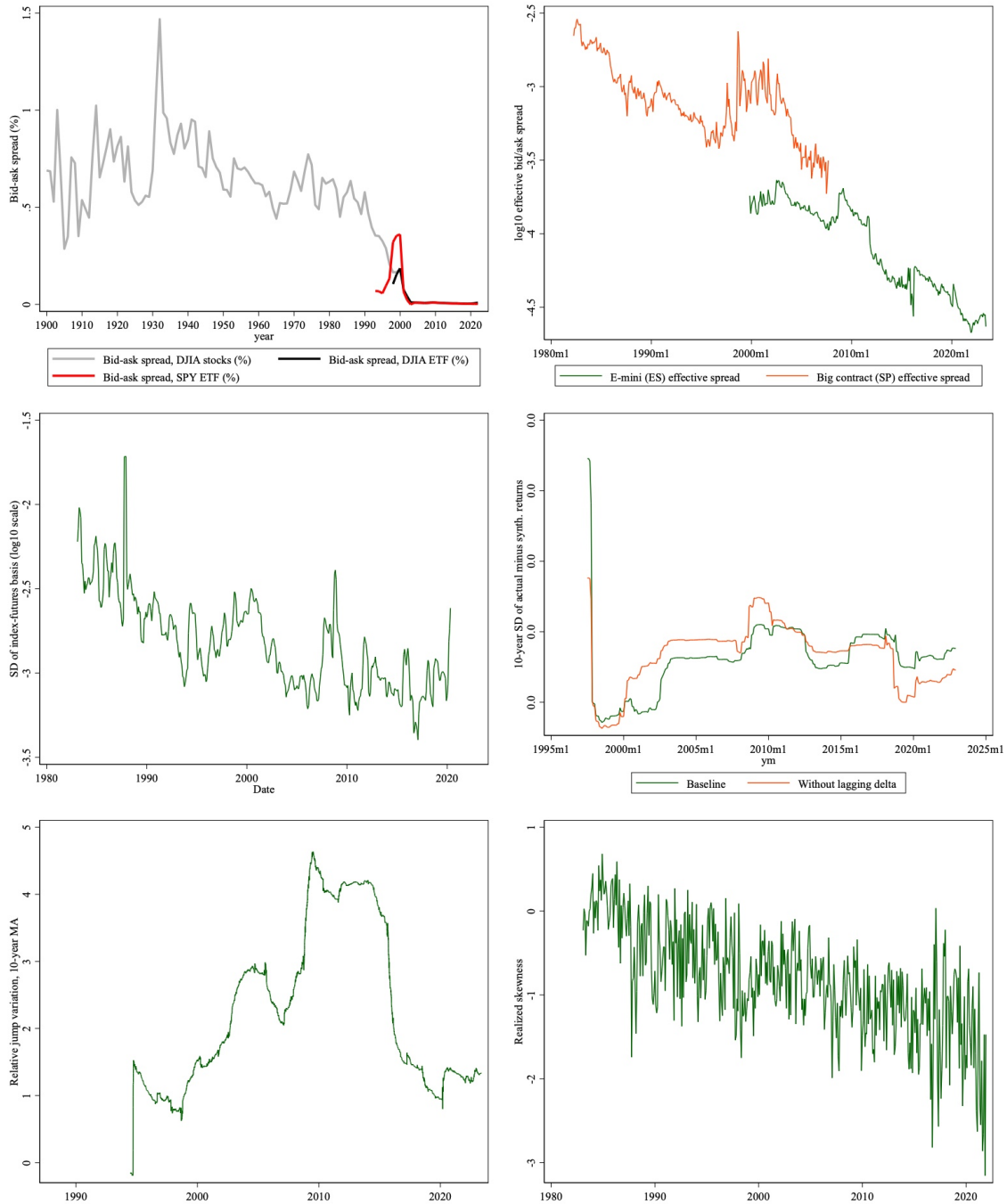
Also similar to trading frictions, basis risk is significantly higher during market crashes – that is clearly apparent for 1987, 2008, and 2020. However, note that in the latter two episodes basis risk was far, far smaller. During the 1987 crash, SP futures were underpriced by as much as 20% for a period of three days. During the 2008 crash, on the other hand, the mispricing was never larger than 5%. In the 2008 and 2020 crashes, the volatility of the basis is lower by an order of magnitude than in 1987. To the extent that it is basis risk in crashes that dealers are worried about – futures prices separating from the underlying index at exactly the moment when the hedge is most important – that is something that can really only be learned about in a crash. In other words, 2008 represented a stress test that the futures market appears to have passed in this sense, after which the model predicts a smaller options premium.

5.3.3 Unhedgeable risk

Figure 10 plots three measures of unhedgeable risk. The first is the standard deviation of the daily delta-hedged options return, which measures the gap between the return on the option and the daily rebalanced hedge (equation (19)) and maps directly into the model. It shows no clear trend. While the 10-year rolling standard deviation is high when the 1987 crash is included, outside that event it has been flat, with no downward trend matching the change in traded option returns.

The second measure of unhedgeable risk is based on the idea that jumps are a potential

Figure 10: Various measures of hedging risk **figure_frictions.jpg**



Note: The to-left panel plots bid-ask spreads: for the Dow-30 from Jones (2002) and the DIA and SPY ETFs from CRSP. The top-right panel plots effective spreads based on the Roll estimator for S&P 500 futures. The middle-left panel plots the three-month rolling average of the (\log_{10}) standard deviation of the gap between the S&P 500 futures price and the level of the index based on 15-minute averages for each. The middle-right panel plots the 10-year rolling standard deviation of the gap between synthetic and traded option returns (which is simply the volatility of the delta-hedged return). The bottom-left panel plots jump variation for the S&P 500 measured as total quadratic variation minus bipower variation from 15-minute returns. The bottom-right panel plots realized skewness of S&P 500 returns from Dew-Becker (2022).

major driver. It is the measure of jumps as the gap between quadratic and bipower variation in the S&P 500 return (see Bollerslev et al. (2009)). Relative jump variation rose during the 2008 financial crisis, and has been lower subsequently, but again does not have a clear trend. The period when traded option returns were most negative does not match the period when jump variation was highest.

Finally, unhedgeable risk is, more broadly, driven by higher moments in returns, so the bottom-right panel of figure 10 plots the measure of S&P 500 return skewness from Neuberger (2012). Realized skewness has, over time, trended consistently more negative (implied skewness does the same; see the CBOE’s SKEW index and Dew-Becker (2022)).

Overall, figure 10 shows that unhedgeable risk does not appear to have clearly fallen along with options alphas.

5.3.4 Tests for changes in the premia

The evidence shows that two major sources of risk to dealers – trading costs and basis risk – have declined over time, which the model predicts should reduce the premia on traded options. This section first examines simple time-series tests for a break in options premia and then tries connecting those premia more directly to the risks from the previous section.

We examine two tests of whether the gap between traded and synthetic option returns has shrunk over time, one reduced-form and the other based on the GPP model. For the reduced-form test, we test for a structural break in the difference in the information ratios for traded and synthetic options at various dates. This is a standard econometric way to look for a structural shift, but it is not terribly well supported economically – one would not really expect a discrete shift in the gap between the returns to happen on a single date, so the test is probably not well specified, impairing its power.

We look at two specific dates, 12/2003 and 12/2008, based on the decline in bid/ask spreads in the former case and the index-futures basis in the latter (both discussed further below). The date in the bottom row, 1/2013, is chosen as the optimal date from a Wald test perspective. The p-values in that case are calculated using the optimal exponential Wald statistic of Andrews and Ploberger (1994), which corrects for multiple testing.

Difference in information ratios			
Breakpoint	Before break	After	p-value
12/2003	-0.62	-0.42	0.37
12/2008	-0.67	-0.26	0.03
12/2012	-0.67	-0.12	0.01

The changes in the information ratios are economically significant in all cases and statistically significant for the two later dates. The magnitude of the difference is also larger for the later break dates, implying that any shift happened closer to, say, 2010, than the early 2000's.

Our second test is more tightly linked to the model studied in this section. We regress the gap between traded and synthetic option returns on basis risk.³² Periods in which basis risk is high are associated with lower future average returns of traded options compared to synthetic options (i.e., a more negative gap), with a coefficient of -0.035 statistically significant at the 0.01 level).

Overall, then, this section shows that returns on exchange-listed options have converged to those on synthetic options in a way that is economically meaningful and statistically significant. Over time the positive excess return from shorting traded options has shrunk until by the end of the sample there is no significant difference between the true and synthetic options, and their alphas are both approximately equal to zero. The results are consistent with the idea that in the earlier part of the sample, up to the mid-2000's, perhaps, traded options were segmented from the overall equity market and hence priced differently. As that segmentation has shrunk, due to declines in the costs and risks of hedging for dealers, the results from the two methods have converged and now both agree on the proposition that there is no particular premium for derivatives that pay convex functions of the market return.

All of that said, these are certainly not the only factors driving options premia, they are just ones that are relatively easy to measure. Many other factors may have also contributed to the decline. For example, Jurek and Stafford (2015) show that hedge fund returns are highly similar to those from writing S&P 500 puts. As the hedge fund sector has grown over the past three decades, it may have contributed to the supply of options, increasing risk-bearing capacity in the context of the GPP model and reducing premia. Increased liquidity in options markets, development of other derivatives markets (e.g. VIX futures and the related ETFs), and changes in risk management practices may have also played important roles.

³²We estimate this regression model via maximum likelihood, accounting for the fat-tailed nature of the hedging errors by modeling them with a student-t distribution with degrees of freedom chosen to match the empirical kurtosis of the hedging residuals. We measure basis risk as the log of the exponentially-weighted moving average of the squared daily basis realizations, with smoothing parameter estimated to minimize the in-sample sum-of-squared forecast errors, lagged by one month.

6 Conclusion

The fact that options can reveal state prices is a foundational result in asset pricing, and it is well known that in the data equity index option prices imply that state prices are especially high (relative to the associated physical probabilities) for states in which the market has significant declines. This paper takes a novel approach to measuring the characteristics of state prices, showing that they can be recovered from the dynamics of stock market returns. The results of that method contrast starkly with those from options, with index returns implying that there is nothing particularly special about the left tail of the return distribution.

Standard models of intermediary constraints and market segmentation imply that as liquidity increases in the options market and it becomes better integrated and accessible, the returns in the options and equity markets should converge. We provide evidence that the convergence seems to be going in the direction of equities – the tail and variance risk premia in options have been shrinking and approaching the values recovered from equity returns.

There are two additional implications of the results that we would like to highlight. First, they show that it is important to take nonstationarity in options returns into account in empirical analyses. The results that one obtains for options premia are very sensitive to the sample used, and research using the full sample currently available through 2022 will get answers that are both economically and statistically significantly different from earlier work using data only through the 1990's.

Second, if synthetic options premia are the superior measure of the preferences of the typical investor – with derivatives markets being segmented – that suggests in the future calibrating and estimating structural models not based on the returns on traded options, but on those for synthetic options. The synthetic options have the added benefits of being available over the same period that equity returns are available and being able to be constructed not only for the total stock market, but also for bonds, anomaly portfolios, or any other asset class.

References

- Ait-Sahalia, Yacine and Andrew W Lo**, “Nonparametric risk management and implied risk aversion,” *Journal of econometrics*, 2000, *94* (1-2), 9–51.
- Ait-Sahalia, Yacine, Mustafa Karaman, and Lorian Mancini**, “The term structure of equity and variance risk premia,” *Journal of Econometrics*, 2020, *219* (2), 204–230.

- Alvarez, Fernando and Urban J. Jermann**, “Using Asset Prices to Measure the Cost of Business Cycles,” *Journal of Political Economy*, 2004, *112*(6), 1223–1256.
- Andersen, Torben G, Nicola Fusari, and Viktor Todorov**, “The risk premia embedded in index options,” *Journal of Financial Economics*, 2015, *117* (3), 558–584.
- Andrews, Donald WK and Werner Ploberger**, “Optimal tests when a nuisance parameter is present only under the alternative,” *Econometrica: Journal of the Econometric Society*, 1994, pp. 1383–1414.
- Backus, David, Mikhail Chernov, and Ian Martin**, “Disasters implied by equity index options,” *The Journal of Finance*, 2011, *66* (6), 1969–2012.
- Bakshi, Gurdip and Nikunj Kapadia**, “Delta-Hedge Gains and the Negative Market Volatility Risk Premium,” *The Review of Financial Studies*, 2003, *16*(2), 527–566.
- Bates, David S**, “The market for crash risk,” *Journal of Economic Dynamics and Control*, 2008, *32* (7), 2291–2321.
- , “US stock market crash risk, 1926–2010,” *Journal of Financial Economics*, 2012, *105* (2), 229–259.
- , “Empirical Option Pricing Models,” *Annual Review of Financial Economics*, 2022, *14*, 369–389.
- Beason, Tyler and David Schreindorfer**, “Dissecting the equity premium,” 2022.
- Black, Fischer and Myron Scholes**, “The Pricing of Options and Corporate Liabilities,” *The Journal of Political Economy*, 1973, *81* (3), 637–654.
- Bollen, Nicolas PB and Robert E Whaley**, “Does net buying pressure affect the shape of implied volatility functions?,” *The Journal of Finance*, 2004, *59* (2), 711–753.
- Bollerslev, Tim and Viktor Todorov**, “Tails, fears, and risk premia,” *Journal of Finance*, 2011, *66*(6), 2165–2211.
- , **Uta Kretschmer, Christian Pigorsch, and George Tauchen**, “A discrete-time model for daily S & P500 returns and realized variations: Jumps and leverage effects,” *Journal of Econometrics*, 2009, *150* (2), 151–166.

- Broadie, Mark, Mikhail Chernov, and Michael Johannes**, “Model Specification and Risk Premia: Evidence from Futures Options,” *The Journal of Finance*, 2007, *62*(3), 1453–1490.
- , — , and — , “Understanding Index Option Returns,” *The Review of Financial Studies*, 2009, *22*(11), 4493–4529.
- Büchner, Matthias and Bryan Kelly**, “A factor model for option returns,” *Journal of Financial Economics*, 2022, *143* (3), 1140–1161.
- Carr, Peter and Dilips B. Madan**, *Towards a Theory of Volatility Trading*, London: Risk Books,
- and **Liuren Wu**, “Variance Risk Premiums,” *Review of Financial Studies*, 2009, *22*(3), 1311–1341.
- Chambers, Donald R, Matthew Foy, Jeffrey Liebner, and Qin Lu**, “Index option returns: Still puzzling,” *The Review of Financial Studies*, 2014, *27* (6), 1915–1928.
- Cochrane, John H and Jesus Saa-Requejo**, “Beyond arbitrage: Good-deal asset price bounds in incomplete markets,” *Journal of political economy*, 2000, *108* (1), 79–119.
- Constantinides, George M, Jens Carsten Jackwerth, and Alexi Savov**, “The puzzle of index option returns,” *Review of Asset Pricing Studies*, 2013, *3* (2), 229–257.
- Corsi, Fulvio**, “A simple approximate long-memory model of realized volatility,” *Journal of Financial Econometrics*, 2009, *7* (2), 174–196.
- Coval, Joshua D. and Tyler Shumway**, “Expected Option Returns,” *The Journal of Finance*, 2001, *56*(3), 983–1009.
- Cuesdeanu, Horatio and Jens Carsten Jackwerth**, “The pricing kernel puzzle: Survey and outlook,” *Annals of Finance*, 2018, *14*, 289–329.
- Dew-Becker, Ian**, “Real-time forward-looking skewness over the business cycle,” 2022. Working paper.
- and **Stefano Giglio**, “Cross-sectional uncertainty and the business cycle: evidence from 40 years of options data,” 2020. Working paper.
- , — , **Anh Le, and Marius Rodriguez**, “The price of variance risk,” *Journal of Financial Economics*, 2017, *123* (2), 225 – 250.

- Drechsler, Itamar**, “Uncertainty, Time-Varying Fear, and Asset Prices,” *The Journal of Finance*, 2013, *68* (5), 1843–1889.
- and **Amir Yaron**, “What’s Vol Got to Do with it?,” *The Review of Financial Studies*, 2011, *24*(1), 1–45.
- Du, Du**, “General equilibrium pricing of options with habit formation and event risks,” *Journal of Financial Economics*, 2011, *99*, 400–426.
- Frazzini, Andrea and Lasse Heje Pedersen**, “Embedded leverage,” *The Review of Asset Pricing Studies*, 2022, *12* (1), 1–52.
- Gabaix, Xavier**, “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance,” *Quarterly Journal of Economics*, 2012, *127*(2), 645–700.
- Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M Poteshman**, “Demand-based option pricing,” *The Review of Financial Studies*, 2008, *22* (10), 4259–4299.
- Haddad, Valentin and Tyler Muir**, “Do intermediaries matter for aggregate asset prices?,” *The Journal of Finance*, 2021, *76* (6), 2719–2761.
- Hall, Robert E**, “High discounts and high unemployment,” *American Economic Review*, 2017, *107* (2), 305–30.
- Han, Bing**, “Investor sentiment and option prices,” *The Review of Financial Studies*, 2008, *21* (1), 387–414.
- Hansen, Lars Peter and Ravi Jagannathan**, “Implications of Security Market Data for Models of Dynamic Economies,” *Journal of Political Economy*, 1991, *99*(2), 225–262.
- and **Robert J Hodrick**, “Forward exchange rates as optimal predictors of future spot rates: An econometric analysis,” *Journal of Political Economy*, 1980, *88* (5), 829–853.
- He, Zhiguo and Arvind Krishnamurthy**, “Intermediary asset pricing,” *American Economic Review*, 2013, *103* (2), 732–770.
- Heston, Steven, Kris Jacobs, and Hyung Joo Kim**, “Exploring Risk Premia, Pricing Kernels, and No-Arbitrage Restrictions in Option Pricing Models.” Working paper.
- Hull, John and Alan White**, “Optimal delta hedging for options,” *Journal of Banking & Finance*, 2017, *82*, 180–190.

- Jackwerth, Jens Carsten**, “Recovering risk aversion from option prices and realized returns,” *The Review of Financial Studies*, 2000, 13 (2), 433–451.
- Jegadeesh, Narasimhan and Sheridan Titman**, “Short-horizon return reversals and the bid-ask spread,” *Journal of Financial Intermediation*, 1995, 4 (2), 116–132.
- Jones, Charles M.**, “A century of stock market liquidity and trading costs.” Working paper.
- Jurek, Jakub W and Erik Stafford**, “The cost of capital for alternative investments,” *The Journal of Finance*, 2015, 70 (5), 2185–2226.
- Kozhan, Roman, Anthony Neuberger, and Paul Schneider**, “The skew risk premium in the equity index market,” *The Review of Financial Studies*, 2013, 26 (9), 2174–2203.
- Lamoureux, Christopher G and William D Lastrapes**, “Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities,” *The Review of Financial Studies*, 1993, 6 (2), 293–326.
- Lewellen, Jonathan and Stefan Nagel**, “The Conditional CAPM does not Explain Asset-Pricing Anomalies,” *Journal of Financial Economics*, 2006, 82 (2), 289–314.
- Martin, Ian**, “What is the Expected Return on the Market?,” *The Quarterly Journal of Economics*, 2017, 132 (1), 367–433.
- Muir, Tyler**, “Financial crises and risk premia,” *The Quarterly Journal of Economics*, 2017, 132 (2), 765–809.
- Muravyev, Dmitriy and Xuechuan Charles Ni**, “Why do option returns change sign from day to night?,” *Journal of Financial Economics*, 2020, 136 (1), 219–238.
- Neuberger, Anthony**, “Realized skewness,” *The Review of Financial Studies*, 2012, 25 (11), 3423–3455.
- Pan, Jun**, “The jump-risk premia implicit in options: Evidence from an integrated time-series study,” *Journal of Financial Economics*, 2002, 63 (1), 3–50.
- Paoli, Bianca De and Pawel Zabczyk**, “Cyclical risk aversion, precautionary saving, and monetary policy,” *Journal of Money, Credit and Banking*, 2013, 45 (1), 1–36.
- Roll, Richard**, “A simple implicit measure of the effective bid-ask spread in an efficient market,” *The Journal of finance*, 1984, 39 (4), 1127–1139.

Rosenberg, Joshua V and Robert F Engle, “Empirical pricing kernels,” *Journal of Financial Economics*, 2002, *64* (3), 341–372.

Schreindorfer, David, “Macroeconomic tail risks and asset prices,” *The Review of Financial Studies*, 2020, *33* (8), 3541–3582.

— and **Tobias Sichert**, “Volatility and the Pricing Kernel.” Working paper.

Seo, Sang Byung and Jessica A Wachter, “Option prices in a model with stochastic disaster risk,” *Management Science*, 2019, *65* (8), 3449–3469.

A Proofs from section 2

A.1 Proposition 2

Consider a regression of $M_{t,t+j}$ on $R_{t,t+j}^m$ and $R_{t,t+j}^{O\perp}$. Since $R_{t,t+j}^m$ and $R_{t,t+j}^{O\perp}$ are, by construction, conditionally uncorrelated with each other, we have

$$M_{t,t+j} = \text{const.} + \frac{\text{cov}_t(R_{t,t+j}^m, M_{t,t+j})}{\text{var}_t[R_{t,t+j}^m]} R_{t,t+j}^m + \frac{\text{cov}_t(R_{t,t+j}^{O\perp}, M_{t,t+j})}{\text{var}_t(R_{t,t+j}^{O\perp})} R_{t,t+j}^{O\perp} + \text{resid.} \quad (21)$$

Additionally, $R_{t,t+j}^m$ is conditionally uncorrelated with $\hat{M}_{t,t+j}$ by construction, so that

$$\text{cov}(R_{t,t+j}^m, M_{t,t+j}) = \text{cov}(R_{t,t+j}^m, \bar{M}_{t,t+j}) \quad (22)$$

The same fact works for $R_{t,t+j}^{O\perp}$ since, conditional on date- t information, $R_{t,t+j}^{O\perp}$ is a (nonlinear) function of $R_{t,t+j}^m$. We then can replace $M_{t,t+j}$ on the left-hand side above with $\bar{M}_{t,t+j}$.

Under the assumption that $R_{t,t+j}^m$ and $R_{t,t+j}^{O\perp}$ are priced,

$$\text{cov}(R_{t,t+j}^m, M_{t,t+j}) = -E_t[R_{t,t+j}^m - 1] \quad (23)$$

$$\text{cov}(R_{t,t+j}^{O\perp}, M_{t,t+j}) = -E_t[R_{t,t+j}^{O\perp} - 1] \quad (24)$$

But since $R_{t,t+j}^{O\perp}$ is uncorrelated with $R_{t,t+j}^m$, its (conditional) CAPM beta is zero by construction, and hence

$$E_t[R_{t,t+j}^{O\perp} - 1] = \alpha_{t,t+j}^O \quad (25)$$

The result then follows.

A.2 Proposition 3

The proof follows similar lines to that for proposition 2. The only required adjustment is to note that

$$\text{cov}_t(R_{t,t+j}^S, M_{t,t+j}) = -E_t[R_{t,t+j}^S - 1] = -\alpha_{t,t+j}^S \quad (26)$$

and

$$\text{cov}_t(R_{t,t+j}^S, \bar{M}_{t,t+j}) = \text{cov}_t(R_{t,t+j}^S, M_{t,t+j}) - \text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S) \quad (27)$$

$$= -\left(\alpha_{t,t+j}^S + \text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)\right) \quad (28)$$

B Synthetic puts and replicating portfolio

B.1 Standard delta-hedging

Consider an option with price P_t , and the underlying with price S_t . The payoff from holding the option from time 1 to T is:

$$\Pi_{1,T}^{option} = P_T - P_1 = \sum_{t=1}^{T-1} (P_{t+1} - P_t)$$

The Black-Scholes-Merton replication portfolio is a dynamic strategy that buys a time-varying number of shares of the underlying, Δ_t , and invests a time-varying amount B_t in the risk free rate. These numbers are chosen so that, in the original setup of Black and Scholes, they guarantee an exact replication of the option value as it evolves over time: equivalently, they guarantee that at maturity the payoff of the option and the replicating portfolio are equal, while cash flows are zero in every period except the first and the last.

To achieve this, Δ_t is chosen as the Black-Scholes delta, and B_t is chosen as the difference between the BS option value P_t and the cost of buying the underlying $\Delta_t S_t$:

$$B_t = P_t - \Delta_t S_t$$

This portfolio costs $B_t + \Delta_t S_t$ to buy at time t , and has a value of $B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}$ at time $t + 1$, where r_t is the annualized interest rate at time t . At time $t + 1$, the strategy requires buying the new portfolio $B_{t+1} + \Delta_{t+1} S_{t+1}$, and so on. In every period between the first and the last one, this strategy generates an *intermediate cash flow* of:

$$ICF_{t+1} = (B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}) - (B_{t+1} + \Delta_{t+1} S_{t+1})$$

and at the last period T , it generates a final cash flow of $B_{T-1}(1 + \frac{r_{T-1}}{365}) + \Delta_{T-1} S_T$ which is equal to the option payoff. In the BS model, the portfolio is self-financing, so $ICF_{t+1} = 0$.

Therefore, the total P&L that this replication portfolio generates is:

$$\Pi_{1,T}^{hedge} = \underbrace{\sum_{t=1}^{T-2} \left[(B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}) - (B_{t+1} + \Delta_{t+1} S_{t+1}) \right]}_{=0 \text{ under BS}} + \underbrace{\left[B_{0T-1}(1 + \frac{r_{T-1}}{365}) + \Delta_{T-1} S_T \right]}_{\text{final cash flow}} - \underbrace{[B_1 + \Delta_1 S_1]}_{\text{initial cost}} \quad (29)$$

This can be also rewritten in the following way:

$$\Pi_{1,T}^{hedge} = \sum_{t=1}^{T-1} \left[(B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}) - (B_t + \Delta_t S_t) \right] = \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t) + \sum_{t=1}^{T-1} B_t \frac{r_t}{365} \quad (30)$$

which corresponds to the formula of Bakshi and Kapadia (2003) and Buchner and Kelly (2022) when setting $B_t = P_t - \Delta_t S_t$. When the assumptions of Black-Scholes do not hold, the rebalancing of the portfolio generates intermediate cash flows (i.e., CF_t is not zero), which is accounted for by the formula above.

Finally, we examine *excess* returns of the delta-hedged portfolio. Both $\Pi_{1,T}^{option}$ and $\Pi_{1,T}^{hedge}$ are dollar payoffs that correspond to initial investments of P_1 and $B_1 + \Delta_1 S_1$, respectively. If the objective is to compute delta-hedged returns, then one can compute:

$$\Pi_{1,T}^{option} - \Pi_{1,T}^{hedge} = \sum_{t=1}^{T-1} (P_{t+1} - P_t) - \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t) - \sum_{t=1}^{T-1} B_t \frac{r_t}{365} \quad (31)$$

and this is an excess return: the cost of buying the option is P_1 , the income from shorting the hedge portfolio is $B_1 + \Delta_1 S_1 = (P_1 - \Delta_1 S_1) + \Delta_1 S_1 = P_1$. This is because the hedge portfolio is designed to borrow exactly an amount B_1 that fully matches the price of the option P_1 (which, in this case, is observed). The hedge strategy updates B_t over time by always setting $B_t = P_t - \Delta_t S_t$, using the observed price of the option P_t at each point in time. Therefore, the delta-hedging strategy described above (eq. 31) is an excess return whether P_t conforms or not to the BS prices. The only difference is, if the BS model is correct, then the ICF_t and the last period cash flow will all be zero.

B.2 Synthetic options: P&L of zero-cost strategy

Next, we consider the case in which we do not observe the option price P_t . In that case, we cannot use it as an input for computing B_t and Δ_t . We also cannot obviously compute $\Pi_{1,T}^{option}$. However, we note that equation (29) still describes the total P&L of *any* dynamic trading strategy that at each point in time buys Δ_t units of the underlying and invests B_t in the risk free rate, whether or not those are chosen as per the BS model. Therefore, we can still compute $\Pi_{1,T}^{hedge}$ for a choice of P_t and Δ_t ³³ (and hence B_t). In particular, we determine the Black-Scholes price of an option, P_t , using as input the current underlying S_t and an estimated value for σ_t^2 , and then build the hedging portfolio for that idealized option. The

³³ Δ_t corresponds to $\delta_{t,t+j}^S$ in the main text. To evaluate Δ_t , we follow Hull and White (2017); specifically, we use equation (5) of the referenced paper and select $a = -0.25, b = -0.4, c = -0.5$.

fact that that option is not directly tradable is irrelevant, in the sense that the P&L we build as described above is an actual return of a portfolio that is just a dynamic portfolio of the market.

While the delta-hedged P&L, $\Pi_{1,T}^{option} - \Pi_{1,T}^{hedge}$, is, as described above, the P&L of a zero-cost portfolio, $\Pi_{1,T}^{hedge}$ is not. So we next describe the P&L of hedge portfolios that yield $\Pi_{1,T}^{option}$ and $\Pi_{1,T}^{hedge}$ separately but are funded at the risk-free rate at inception. For the option (i.e. when we do observe P_t), we have:

$$\Pi_{1,T}^{option,exc} = P_T - P_1 \left(1 + \frac{r_1}{365}\right)^T$$

Funding the hedge portfolio requires borrowing $B_1 + \Delta_1 S_1 = P_1$, where P_t is the theoretical BS price of an option. So the total P&L can be written as:

$$\Pi_{1,T}^{hedge,exc} = \underbrace{\sum_{t=1}^{T-2} \left[(B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}) - (B_{t+1} + \Delta_{t+1} S_{t+1}) \right]}_{=0 \text{ under BS}} + \left[B_{T-1} \left(1 + \frac{r_{T-1}}{365}\right) + \Delta_{T-1} S_T \right] - [B_1 + \Delta_1 S_1] \left(1 + \frac{r_1}{365}\right)^T$$

Note that in these formulas the intermediate cash flows are assumed not to be reinvested. One can also reinvest them to obtain:

$$\begin{aligned} \Pi_{1,T}^{hedge,exc} &= \sum_{t=1}^{T-2} (1 + r_{t+1})^{T-t-1} \left[(B_t(1 + \frac{r_t}{365}) + \Delta_t S_{t+1}) - (B_{t+1} + \Delta_{t+1} S_{t+1}) \right] \\ &\quad + \left[B_{T-1} \left(1 + \frac{r_{T-1}}{365}\right) + \Delta_{T-1} S_T \right] - [B_1 + \Delta_1 S_1] \left(1 + \frac{r_1}{365}\right)^T \end{aligned}$$

An alternative procedure is to get the same exposure Δ_t every period, but entirely fund it at the risk-free rate every period. The P&L of this zero-cost portfolio is:

$$\tilde{\Pi}_{1,T}^{hedge,exc} = \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t (1 + \frac{r_t}{365})) = \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t) - \sum_{t=1}^{T-1} \Delta_t S_t \frac{r_t}{365}$$

Note that this relates closely to $\Pi_{1,T}^{hedge,exc}$, since

$$\Pi_{1,T}^{hedge,exc} = \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t) + \sum_{t=1}^{T-1} (P_t - \Delta_t S_t) \frac{r_t}{365} - P_1 \left[\left(1 + \frac{r_1}{365}\right)^T - 1 \right]$$

where the latter term is the total interest paid on the original loan of P_1 . So we can write:

$$\Pi_{1,T}^{hedge,exc} - \tilde{\Pi}_{1,T}^{hedge,exc} = \sum_{t=1}^{T-1} P_t \frac{r_t}{365} - P_1 \left[\left(1 + \frac{r_1}{365}\right)^T - 1 \right]$$

The difference is effectively only coming from the different timing of the borrowing (every period vs. at the beginning of the month), and is unlikely to make any substantial difference empirically.

In fact, we can also consider funding the original hedging strategy, $\Pi_{1,T}^{hedge}$ day by day instead of once at the very beginning. Modifying eq. (30):

$$\hat{\Pi}_{1,T}^{hedge} = \sum_{t=1}^{T-1} \left[\left(B_t \left(1 + \frac{r_t}{365} \right) + \Delta_t S_{t+1} \right) - \left(B_t + \Delta_t S_t \right) \left(1 + \frac{r_t}{365} \right) \right] = \sum_{t=1}^{T-1} \Delta_t \left(S_{t+1} - S_t \left(1 + \frac{r_t}{365} \right) \right)$$

So that:

$$\hat{\Pi}_{1,T}^{hedge,exc} = \tilde{\Pi}_{1,T}^{hedge,exc} \simeq \Pi_{1,T}^{hedge,exc}$$

B.3 P&L and returns

The P&Ls described above ($\tilde{\Pi}_{1,T}^{hedge,exc}$ and $\Pi_{1,T}^{hedge,exc}$) correspond to strategies that have zero cost. Therefore, they also represent excess returns. Scaling that excess return by any time-1 quantity is also an excess return. Just like Buchner and Kelly (2022), we scale P&Ls by the underlying at time 1:

$$R_{1,T}^{hedge,exc} = \frac{\Pi_{1,T}^{hedge,exc}}{S_1}$$

and

$$\tilde{R}_{1,T}^{hedge,exc} = \frac{\tilde{\Pi}_{1,T}^{hedge,exc}}{S_1}$$

Finally, we consider another related zero-cost trading strategy. Instead of scaling by S_1 , we scale the position of the strategy that funds every day ($\tilde{\Pi}_{1,T}^{hedge}$) by S_t every day:

$$\tilde{R}_{1,T}^{hedge,scaled} = \sum_{t=1}^{T-1} \Delta_t \frac{S_{t+1} - S_t \left(1 + \frac{r_t}{365} \right)}{S_t}$$

Defining $R_{t+1}^M = \frac{S_{t+1} - S_t}{S_t}$ we obtain:

$$\frac{S_{t+1} - S_t \left(1 + \frac{r_t}{365} \right)}{S_t} = \frac{S_{t+1} - S_t}{S_t} - \left(1 + \frac{r_t}{365} \right) = R_{t+1}^M - R_{t+1}^f$$

and therefore

$$\tilde{R}_{1,T}^{hedge,scaled} = \sum_{t=1}^{T-1} \Delta_t (R_{t+1}^M - R_{t+1}^f)$$

A final point concerns dividends. While dividends make a small difference over short time

horizons, we can incorporate them easily in our analysis since we are not trying to hedge a traded option. In other words, we consider a synthetic option that aims to hedge a value S_t that tracks the value of an investment in the underlying that reinvests all the dividends. In that case, R^M is the one-day gross return (including dividends) of the market.

B.4 Comparison of the different approaches

In this section, we compare our baseline excess returns ($R_{1,T}^{hedge,exc}$, with reinvested intermediate cash flow) to the one obtained by funding the position each day, $\tilde{R}_{1,T}^{hedge,scaled}$. The table below reports, for different combinations of maturity M and strike K , the correlation between $R_{1,T}^{hedge,exc}$ and $\tilde{R}_{1,T}^{hedge,scaled}$ and the information ratio of each of them.

C Robust confidence bands for alpha

To combine statistical uncertainty with uncertainty from $\text{cov}_t(\hat{R}_{t,t+j}^S, \hat{M}_{t,t+j})$, we treat them as two independent sources of error. Specifically, suppose one starts with a diffuse prior for

$$\alpha_{t,t+j}^{S,adjusted} \equiv \alpha_{t,t+j}^S + \text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S) \quad (32)$$

Denote by $\alpha_{t,t+j}^{S,estimated}$ the empirical estimate. Asymptotically, $\alpha_{t,t+j}^{S,estimated} \sim N(\alpha_{t,t+j}^S, SE^2)$, where SE is the standard error for the estimate. We also treat the second term as though it is drawn from the distribution,

$$\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S) \sim N\left(0, \left(\frac{1}{2} \times 0.5 \times \text{std}_t(\hat{R}_{t,t+j}^S) \frac{E[R_{t,t+j}^m - 1]}{\text{std}(R_{t,t+j}^m)}\right)^2\right) \quad (33)$$

Recall from the main text that we take $0.5 \times \text{std}_t(\hat{R}_{t,t+j}^S) \frac{E[R_{t,t+j}^m - 1]}{\text{std}(R_{t,t+j}^m)}$ as an upper bound for $\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)$. To incorporate that with the estimation uncertainty, we treat $\text{cov}_t(\hat{M}_{t,t+j}, \hat{R}_{t,t+j}^S)$ as though it has a standard deviation of $0.5 \times \text{std}_t(\hat{R}_{t,t+j}^S) \frac{E[R_{t,t+j}^m - 1]}{\text{std}(R_{t,t+j}^m)}$, so that the upper bound is two standard deviations from the prior mean – i.e. at the edge of a ± 2 standard deviation interval.

Given those two assumptions along with the diffuse prior, we then have

$$\alpha_{t,t+j}^{S,adjusted} \sim N\left(\alpha_{t,t+j}^{S,estimated}, SE^2 + \left(\frac{1}{2} \times 0.5 \times \text{std}_t(\hat{R}_{t,t+j}^S) \frac{E[R_{t,t+j}^m - 1]}{\text{std}(R_{t,t+j}^m)}\right)^2\right) \quad (34)$$

D Theoretical results for intermediary model

This section presents our version of the GPP model in more detail. The vast majority of the content is due to them; the only change is the addition of the trading friction and index-futures basis.

D.1 Basic setup

There is a constant gross risk-free rate R_f . The underlying index has an exogenous excess return R_{t+1}^I . We consider a simplified version of the model where there is a single option traded that has some price P_t . Its excess return is then $R_{t+1}^O = P_{t+1} - R_f P_t$.

Dealers/intermediaries maximize discounted utility over consumption C_t ,

$$E_t \sum_{j=0}^{\infty} \rho^j (-\gamma^{-1}) \exp(-\gamma C_t) \quad (35)$$

subject to a transversality condition and budget constraint, which is

$$W_{t+1} = (W_t - C_t) R_f + D_t R_{t+1}^O + F_t R_{t+1}^F - \frac{\kappa}{2} F_t^2 \quad (36)$$

where W_t is wealth. The intermediaries optimize over D_t , F_t , and C_t subject to the budget constraint and taking the returns as given.

As described in the text, the term $\frac{\kappa}{2} F_t^2$ is a deviation from GPP, as is the distinction between R^F and R^I .

It is assumed that the futures contract on the underlying that the dealers trade is available in infinite supply. For the options, there is some exogenous demand from outside investors, d_t , and the market clearing condition is $D_t + d_t = 0$.

Lemma 5 *In this model, assets are priced under a probability measure d which is equal to the measure P multiplied by the factor $\frac{\exp(-k(W_{t+1} + G(d_{t+1}, X_{t+1})))}{E_t[\exp(-k(W_{t+1} + G(d_{t+1}, X_{t+1})))]}$. In addition,*

$$\kappa F_t = E_t^d [R_{t+1}^F] \quad (37)$$

$$P_t = R_f^{-1} E_t^d P_{t+1} \quad (38)$$

where P_t is the price of the option (equivalently, $0 = E_t^d R_{t+1}^O$).

Proof. The value function and budget constraint satisfy

$$V_t = \max_{C_t, D_t, F_t} -\gamma^{-1} \exp(-\gamma C_t) + \rho E_t V_{t+1} \quad (39)$$

$$W_{t+1} = (W_t - C_t) R_f + D_t (P_{t+1} - R_f P_t) + F_t R_{t+1}^F - \frac{\kappa}{2} F_t^2 \quad (40)$$

Now guess that

$$V_t = -k^{-1} \exp(-k(W_t + G_t))$$

for some variable G_t that is exogenous to the dealers, and where

$$k = \gamma \frac{R_f - 1}{R_f} \quad (41)$$

We have

$$\frac{\partial}{\partial W_t} V_t = -k V_t \quad (42)$$

$$\text{and } \frac{dW_{t+1}}{dC_t} = -R_f \quad (43)$$

So then the FOC for consumption under this guess is

$$0 = \exp(-\gamma C_t) + k R_f \rho E_t V_{t+1} \quad (44)$$

Noting that

$$V_t = -\gamma^{-1} \exp(-\gamma C_t) + \rho E_t V_{t+1} \quad (45)$$

$$\rho E_t V_{t+1} = V_t + \gamma^{-1} \exp(-\gamma C_t) \quad (46)$$

We have

$$\exp(-\gamma C_t) = \exp(-k(W_t + G_t)) \quad (47)$$

Now consider the FOC with respect to F_t . First,

$$\frac{dW_{t+1}}{dF_t} = R_{t+1}^F - \kappa F_t \quad (48)$$

And hence the FOC is

$$0 = \rho E_t \left[\exp(-k(W_{t+1} + G_{t+1})) (R_{t+1}^F - \kappa F_t) \right] \quad (49)$$

$$\kappa F_t = E_t^d [R_{t+1}^F] \quad (50)$$

where E^d is the expectation under the risk-neutral measure, which is the physical measure distorted by the factor

$$\frac{\exp(-k(W_{t+1} + G_{t+1}))}{E_t[\exp(-k(W_{t+1} + G_{t+1}))]} \quad (51)$$

Next, for D_t ,

$$\frac{dW_{t+1}}{dD_t} = R_{t+1}^O \quad (52)$$

So then

$$0 = \rho E_t \left[\exp(-k(W_{t+1} + G_{t+1})) R_{t+1}^O \right] \quad (53)$$

$$0 = R_f^{-1} E_t^d R_{t+1}^O \quad (54)$$

It is straightforward to get a recursion for G_t by following the derivation in GPP. ■

Proposition 6 *The effect of options demand on prices is*

$$\frac{\partial P_t}{\partial D_t} = -\gamma (R_f - 1) E_t^d \left[\left(R_{t+1}^O - \hat{\beta}_t R_{t+1}^F \right) P_{t+1} \right] \quad (55)$$

where

$$\hat{\beta}_t \equiv \beta_t^F \frac{E_t^d \left[(R_{t+1}^F)^2 \right]}{E_t^d \left[(R_{t+1}^F)^2 \right] + k^{-1} R_f \kappa} \quad (56)$$

$$\beta_t^F \equiv \frac{\text{cov}_t^d (R_{t+1}^F, R_{t+1}^O)}{\text{var}_t^d (R_{t+1}^F)} \quad (57)$$

Proof. Based on the analysis from the previous proof, the pricing kernel can be written as

$$m_{t+1}^d \equiv \frac{\exp(-k(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1}))}{R_f E_t \exp(-k(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1}))} \quad (58)$$

Differentiate m_{t+1}^d with respect to D_t to get

$$\frac{\partial m_{t+1}^d}{\partial D_t} = \frac{-k \left(R_{t+1}^O + R_{t+1}^F \frac{\partial F_t}{\partial D_t} \right) \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)}{R_f E_t \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)} \quad (59)$$

$$- \frac{\exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)}{\left(R_f E_t \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right) \right)^2} \quad (60)$$

$$\times E_t \left[-k R_f P_{t+1} \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right) \right] \quad (61)$$

$$= -k \left(R_{t+1}^O + R_{t+1}^F \frac{\partial F_t}{\partial D_t} \right) m_{t+1}^d - \frac{\exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)}{R_f E_t \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)} \quad (62)$$

$$\times E_t \left[-k R_f R_{t+1}^O \frac{\exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)}{R_f E_t \exp \left(-k \left(F_t R_{t+1}^F + D_t R_{t+1}^O + G_{t+1} \right) \right)} \right] \quad (63)$$

$$= -k \left(R_{t+1}^O + R_{t+1}^F \frac{\partial F_t}{\partial D_t} \right) m_{t+1}^d \quad (64)$$

Next, we differentiate the first-order condition for F_t with respect to D_t ,

$$\kappa \frac{\partial F_t}{\partial D_t} = E_t \left[\frac{\partial}{\partial D_t} m_{t+1}^d R_{t+1}^F \right] \quad (65)$$

$$= E_t \left[-k \left(R_{t+1}^O + R_{t+1}^F \frac{\partial F_t}{\partial D_t} \right) R_{t+1}^F m_{t+1}^d \right] \quad (66)$$

Solving for the derivative yields

$$k^{-1} \kappa \frac{\partial F_t}{\partial D_t} = E_t \left[-R_{t+1}^O R_{t+1}^F m_{t+1}^d - R_{t+1}^F \frac{\partial F_t}{\partial D_t} R_{t+1}^F m_{t+1}^d \right] \quad (67)$$

$$k^{-1} \kappa \frac{\partial F_t}{\partial D_t} + \frac{\partial F_t}{\partial D_t} R_f^{-1} E_t^d \left[\left(R_{t+1}^F \right)^2 \right] = R_f^{-1} E_t^d \left[-R_{t+1}^O R_{t+1}^F \right] \quad (68)$$

$$\frac{\partial F_t}{\partial D_t} = - \frac{E_t^d \left[R_{t+1}^O R_{t+1}^F \right]}{E_t^d \left[\left(R_{t+1}^F \right)^2 \right] + k^{-1} R_f \kappa} \equiv -\beta_t^F \frac{E_t^d \left[\left(R_{t+1}^F \right)^2 \right]}{E_t^d \left[\left(R_{t+1}^F \right)^2 \right] + k^{-1} R_f \kappa} \quad (69)$$

where

$$\beta_t^F \equiv \frac{E_t^d \left[R_{t+1}^O R_{t+1}^F \right]}{E_t^d \left[\left(R_{t+1}^F \right)^2 \right]} = \frac{\text{cov}_t^d \left(R_{t+1}^F, R_{t+1}^O \right)}{\text{var}_t^d \left(R_{t+1}^F \right)} \quad (70)$$

The price sensitivity comes from differentiating the pricing equation for the option

$$\begin{aligned}
\frac{\partial P_t}{\partial D_t} &= E_t \left[\frac{\partial m_{t+1}^d}{\partial D_t} P_{t+1} \right] \\
&= -k E_t \left[\left(R_{t+1}^O + R_{t+1}^F \frac{\partial F_t}{\partial D_t} \right) m_{t+1}^d P_{t+1} \right] \\
&= -k E_t \left[\left(R_{t+1}^O - \hat{\beta}_t R_{t+1}^F \right) m_{t+1}^d P_{t+1} \right] \\
&= -\gamma (R_f - 1) E_t^d \left[\left(R_{t+1}^O - \hat{\beta}_t R_{t+1}^F \right) R_{t+1}^O \right]
\end{aligned}$$

where the last line uses the fact that $E_t^d [R_{t+1}^O] = E_t^d [R_{t+1}^F] = 0$ since they are excess returns that are fairly priced under the pricing measure d . ■

D.2 Proof of proposition 4

The proof involves simply analyzing the expectation in 6 above. We have

$$E_t^d \left[\left(R_{t+1}^O - \hat{\beta}_t R_{t+1}^F \right) R_{t+1}^O \right] = E_t^d \left[\left(R_{t+1}^O - \beta_t^F R_{t+1}^F - \left(\hat{\beta}_t - \beta_t^F \right) R_{t+1}^F \right) R_{t+1}^O \right] \quad (71)$$

$$= E_t^d \left[\left(\varepsilon_{t+1}^F - \left(\hat{\beta}_t - \beta_t^F \right) R_{t+1}^F \right) \left(\beta_t^F R_{t+1}^F + \varepsilon_{t+1}^F \right) \right] \quad (72)$$

$$= \text{var}_t^d \left[\varepsilon_{t+1}^F \right] - \left(\hat{\beta}_t - \beta_t^F \right) \beta_t^F E_t^d \left[\left(R_{t+1}^F \right)^2 \right] \quad (73)$$

$$= \text{var}_t^d \left[\varepsilon_{t+1}^F \right] - \left(\frac{-k^{-1} R_f \kappa}{E_t^d \left[\left(R_{t+1}^F \right)^2 \right] + k^{-1} R_f \kappa} \right) \left(\beta_t^F \right)^2 E_t^d \left[\left(R_{t+1}^F \right)^2 \right]$$

Next, we want to further decompose $\text{var}_t^d \left[\varepsilon_{t+1}^F \right]$. We have

$$R_{t+1}^F = R_{t+1}^I + z_{t+1} \quad (75)$$

$$\beta_t^F = \beta_t^I \frac{\sigma_{I,t}^2}{\sigma_{I,t}^2 + \sigma_{z,t}^2} \quad (76)$$

where $\sigma_{I,t}^2 = \text{var}_t^d \left(R_{t+1}^I \right)$. We can write

$$R_{t+1}^O = \beta_t^I R_{t+1}^I + \varepsilon_{t+1}^I \quad (77)$$

where β_t^I is the (d -measure) regression coefficient. Then

$$\varepsilon_{t+1}^F = R_{t+1}^O - \beta_t^F R_{t+1}^F \quad (78)$$

$$= \beta_t^I R_{t+1}^I + \varepsilon_{t+1}^I - \beta_t^I \frac{\sigma_{I,t}^2}{\sigma_{I,t}^2 + \sigma_{z,t}^2} (R_{t+1}^I + z_{t+1}) \quad (79)$$

$$= \beta_t^I \frac{\sigma_{z,t}^2}{\sigma_{I,t}^2 + \sigma_{z,t}^2} R_{t+1}^I + \varepsilon_{t+1}^I - \beta_t^I \frac{\sigma_{I,t}^2}{\sigma_{I,t}^2 + \sigma_{z,t}^2} z_{t+1} \quad (80)$$

$$\text{var}_t^d [\varepsilon_{t+1}^F] = (\beta_t^I)^2 \frac{\sigma_{z,t}^2 \sigma_{I,t}^2}{\sigma_{I,t}^2 + \sigma_{z,t}^2} + \sigma_{\varepsilon,t}^2 \quad (81)$$

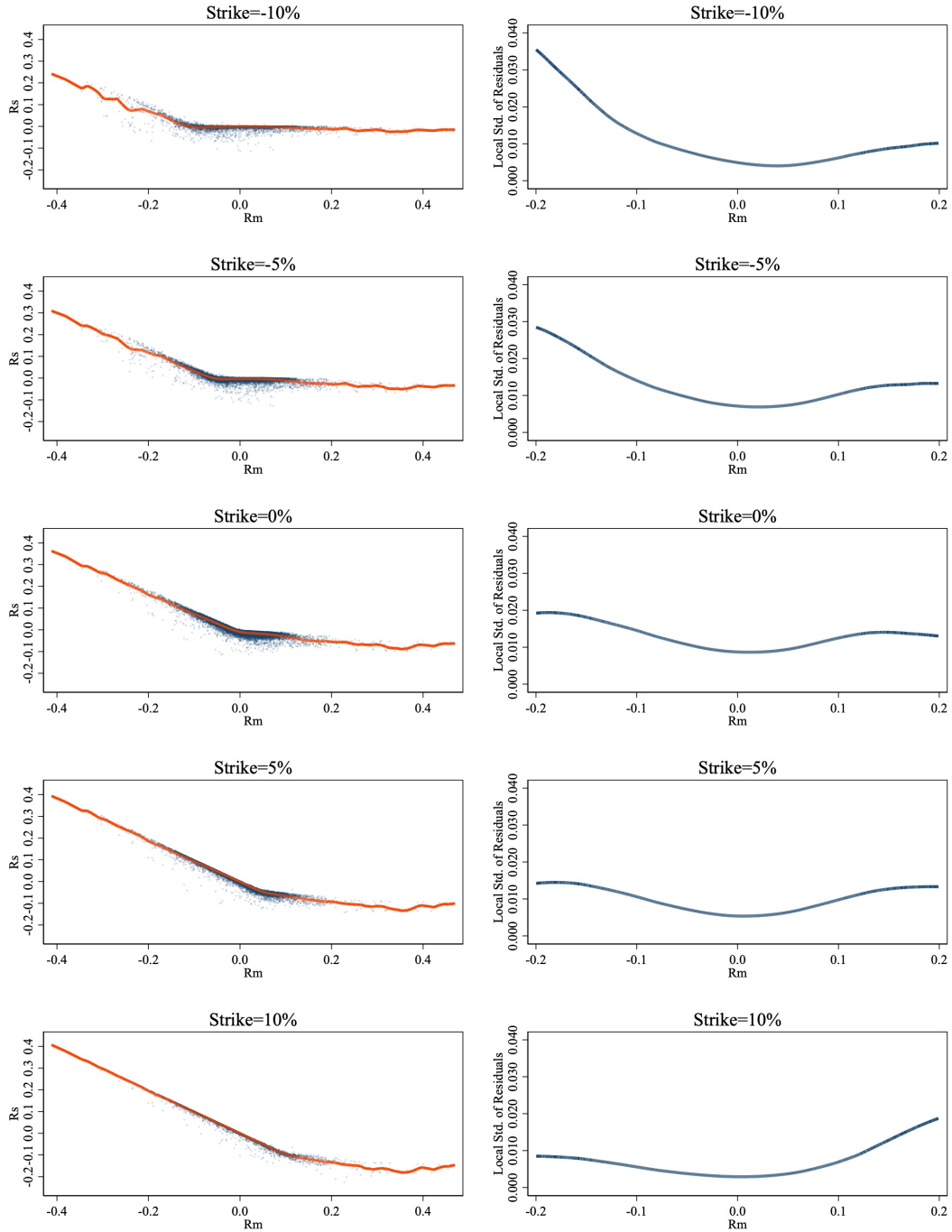
where $\sigma_{\varepsilon,t}^2 \equiv \text{var}_t^d [\varepsilon_{t+1}^I]$.

Up to first order in κ and σ_z^2 ,

$$\frac{\partial P_t}{\partial D_t} = -\gamma (R_f - 1) \left(\underbrace{(\beta_t^I)^2 \sigma_{z,t}^2}_{\text{Basis risk}} + \underbrace{\sigma_{\varepsilon,t}^2}_{\text{Unhedgeable risk}} \right) + \underbrace{\kappa R_f^2 (\beta_t^I)^2}_{\text{Imperfect hedging}} \quad (82)$$

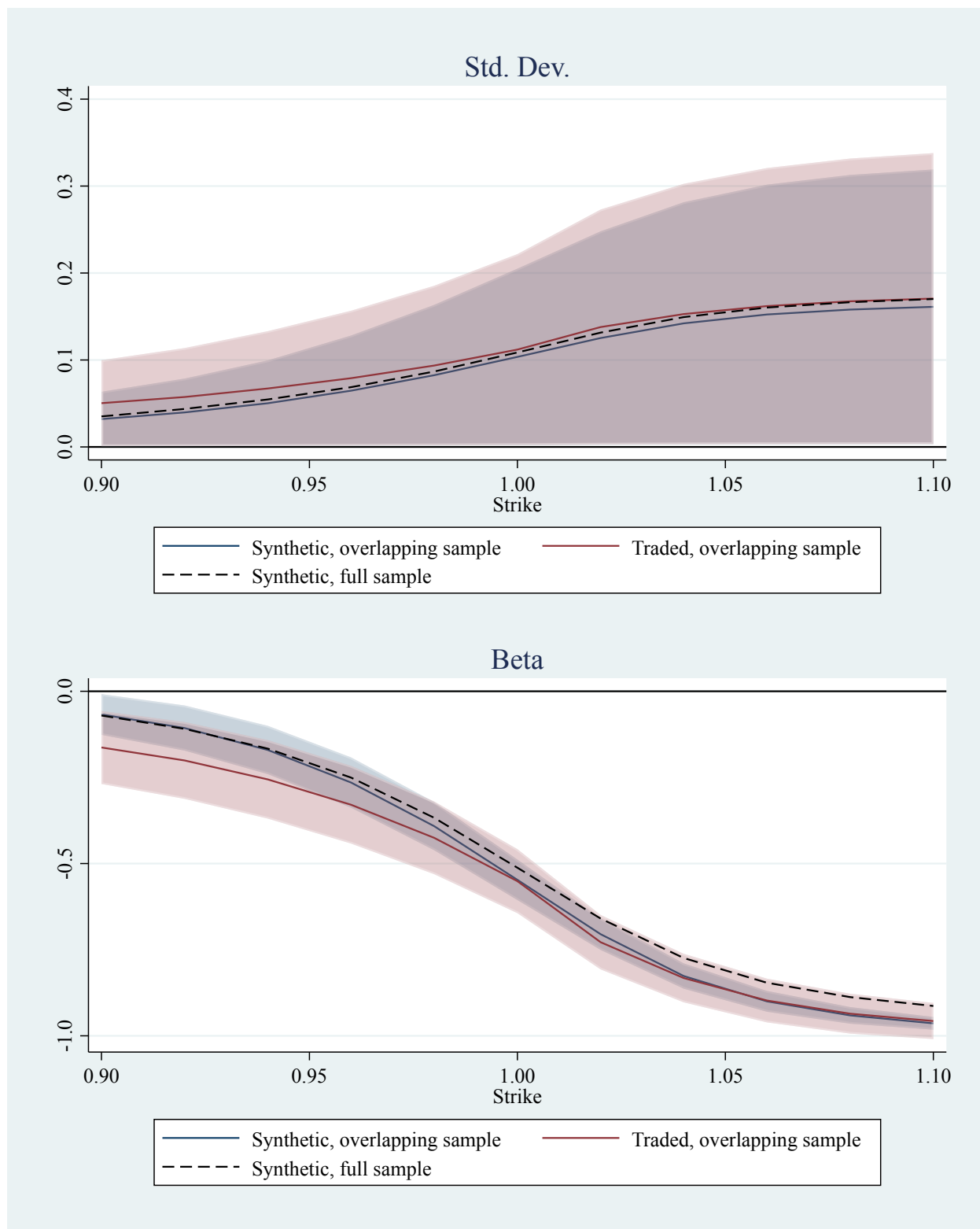
E Additional figures

Figure A.1: Synthetic put returns for various strikes



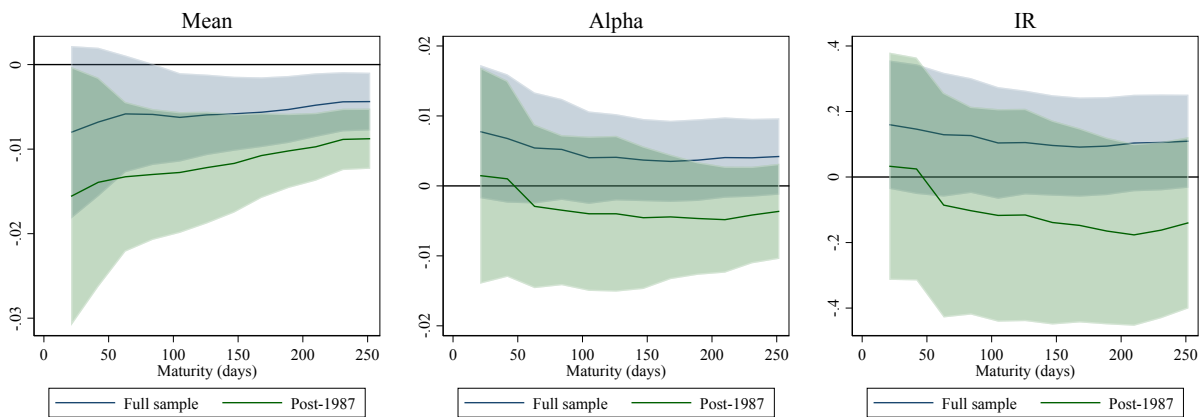
Note: these figures replicate figure 3 varying the strike used for the synthetic option.

Figure A.2: Risk measures for synthetic and traded options.



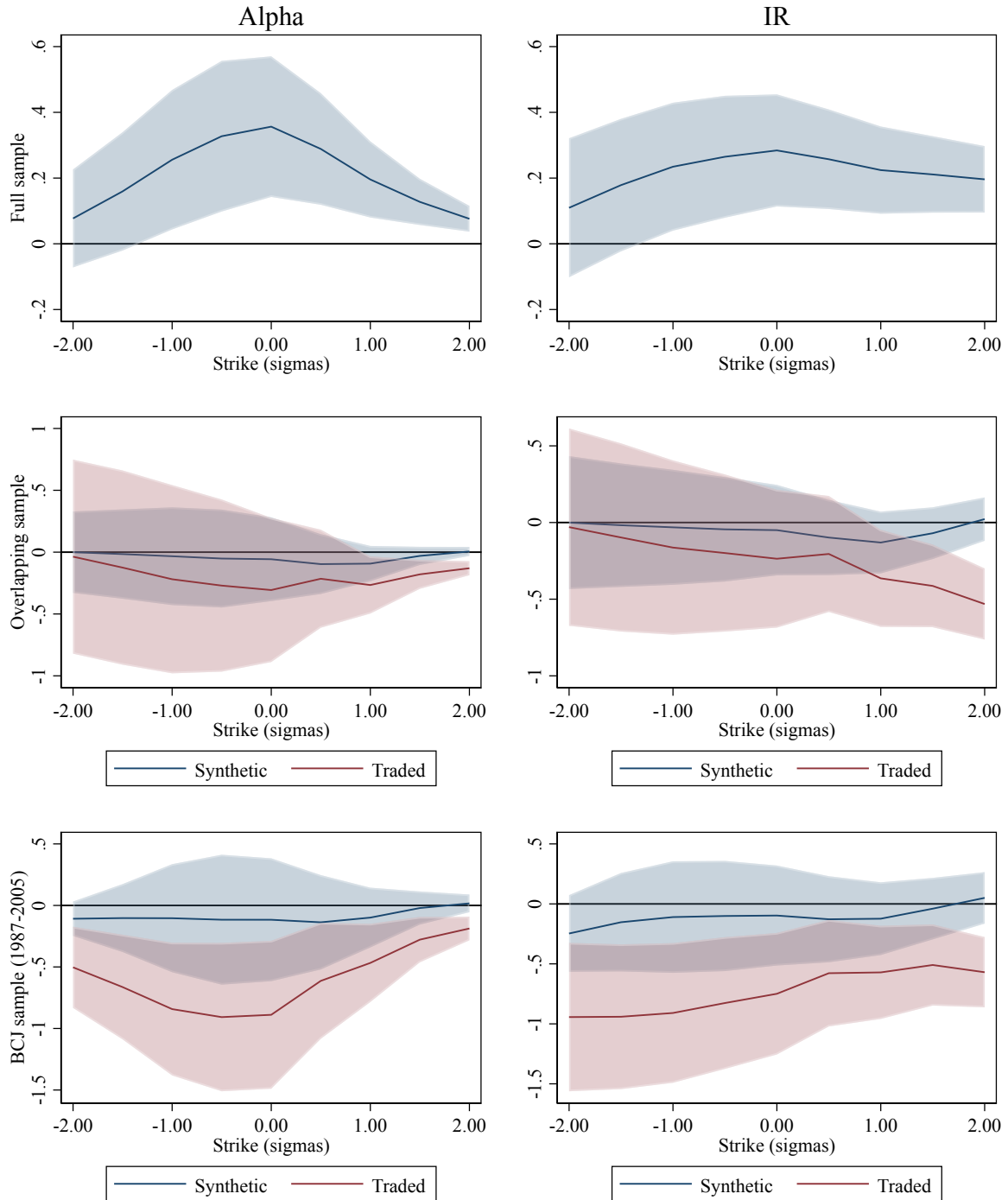
Note: Standard deviations and CAPM betas for synthetic and traded option returns. The shaded regions are 95% confidence intervals (no confidence interval is shown for the dotted line)

Figure A.3: Synthetic option returns across maturities



Note: Average return, CAPM alpha, and information ratio for synthetic options across maturities. In all cases, the strike is set to be equivalent to -5% at the monthly maturity (scaling with the square root of the maturity). Note that the two series plotted are not traded versus synthetic options but rather synthetic options in two different samples (the longer maturities are much less liquid for the traded options, especially early in the sample, and present a number of issues in implementation).

Figure A.4: Option returns for moneyness in volatility units



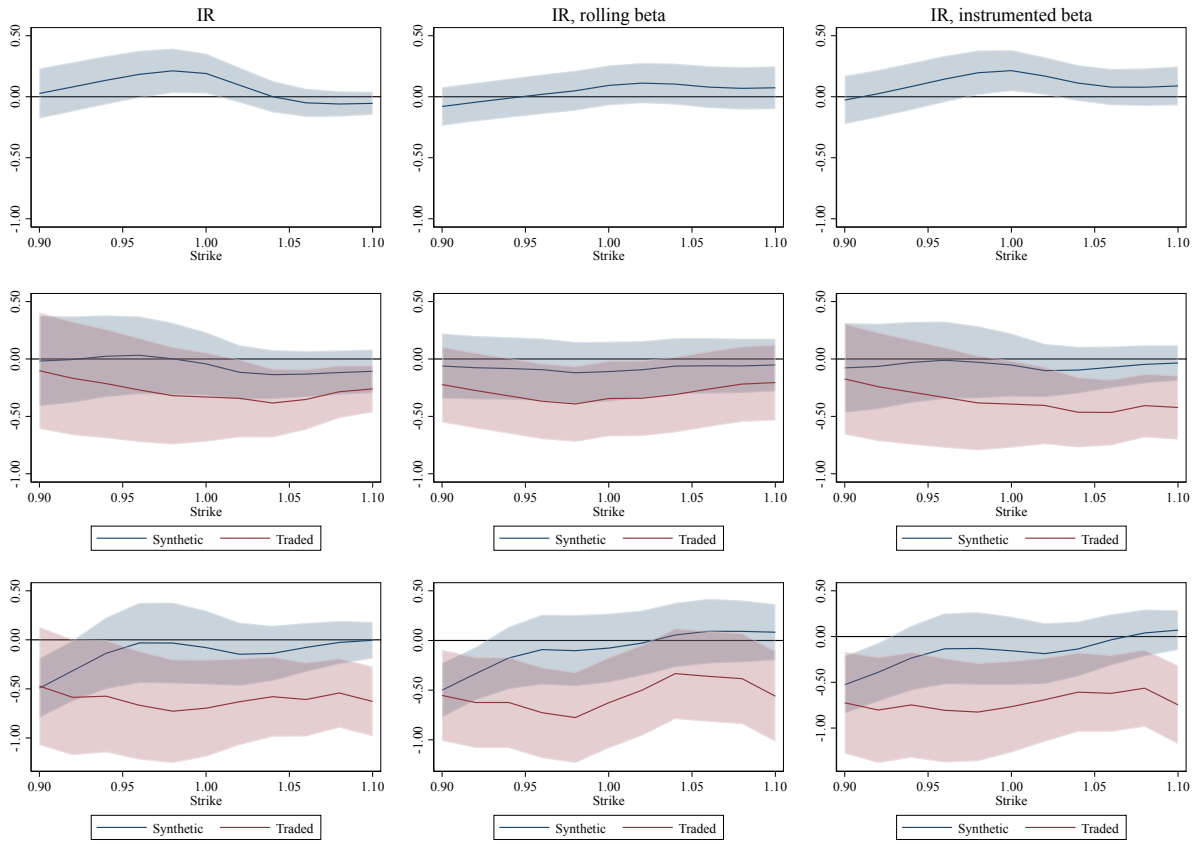
Note: Average return, CAPM alpha, and information ratio for synthetic and traded options. Strikes here are selected in units of volatility instead of as a fixed percentage of the price of the underlying.

Figure A.5: Option returns scaling by option price



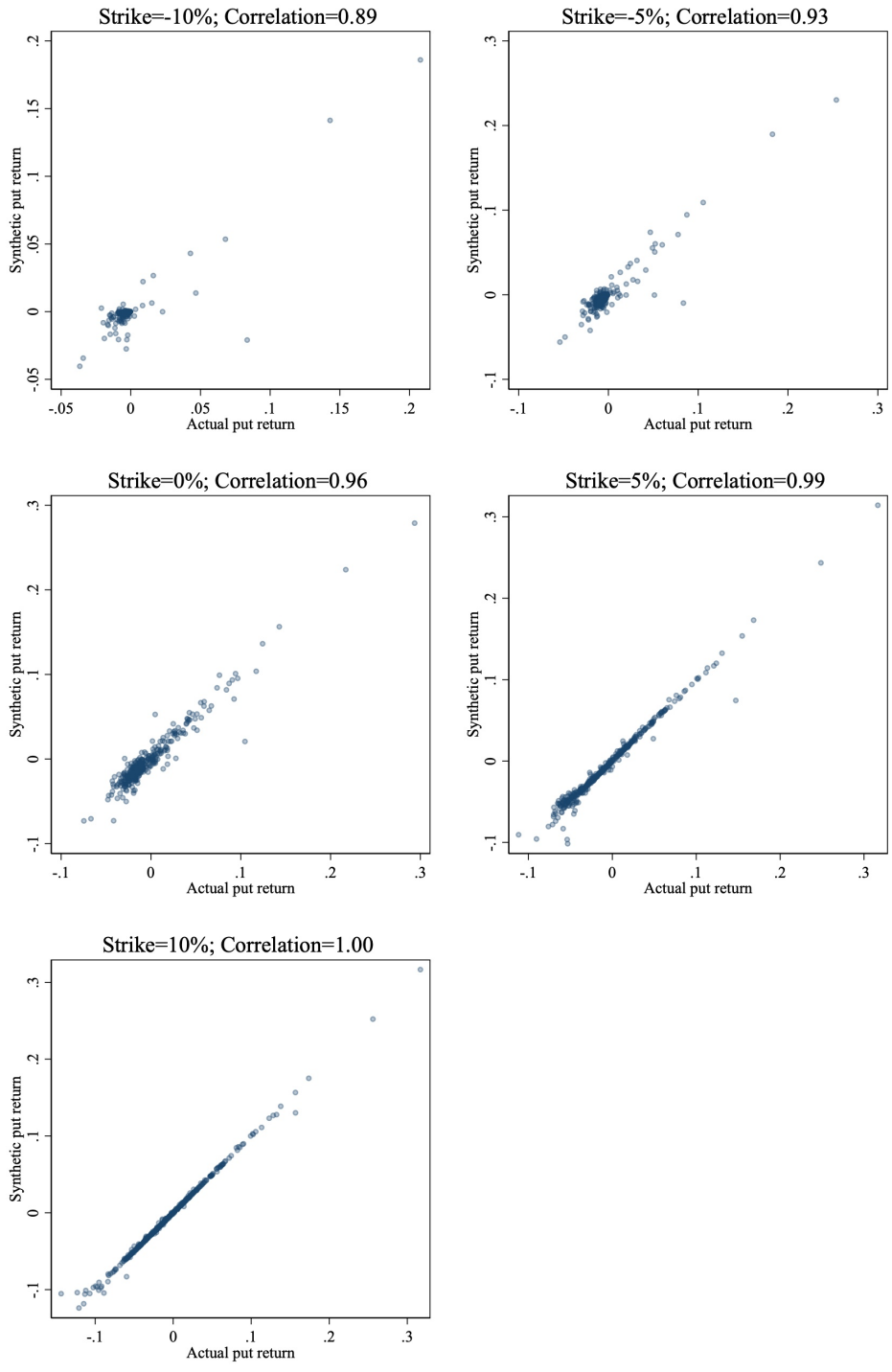
Note: Reports results where the denominator of the return is the price of the option instead of the underlying – so purchasing a fixed dollar amount of insurance, instead of insurance on a fixed number of units of the underlying.

Figure A.6: Information ratios under various specifications for beta



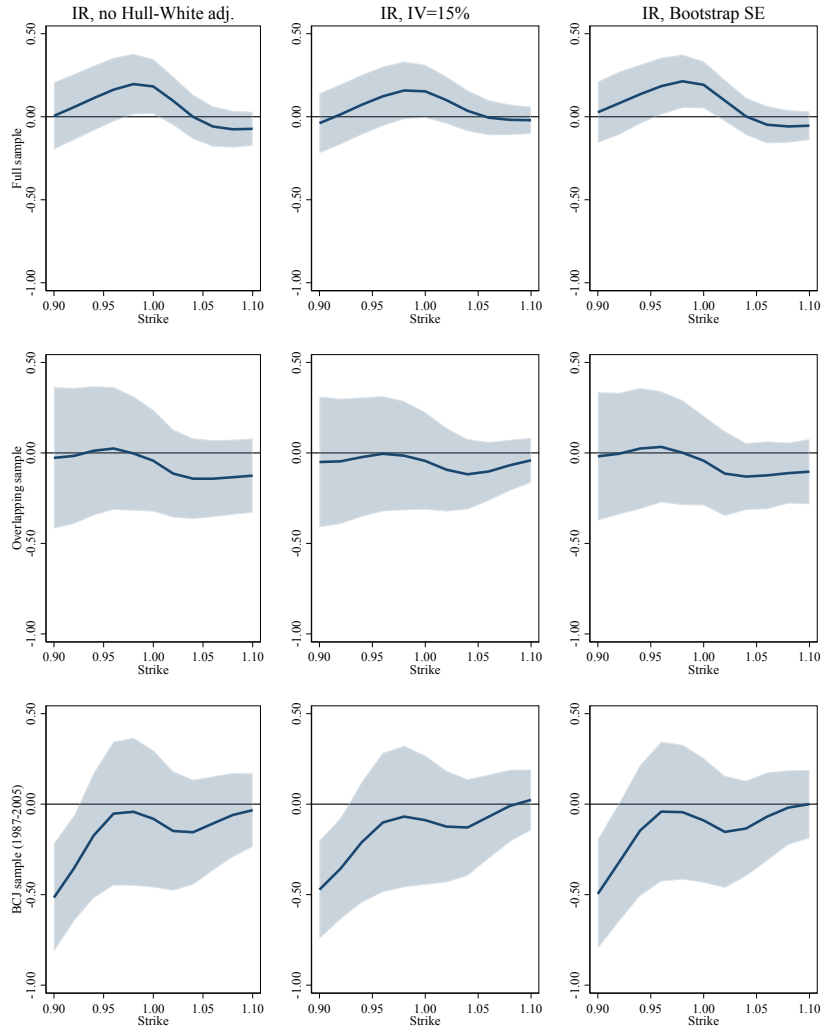
Note: Information ratios under various specifications for the betas. The left-hand column uses the full-sample beta, which is the benchmark specification. The middle column uses betas estimated from a rolling three-month window. Finally, the right-hand column instruments for the conditional beta, as described in the text.

Figure A.7: Synthetic versus traded option returns



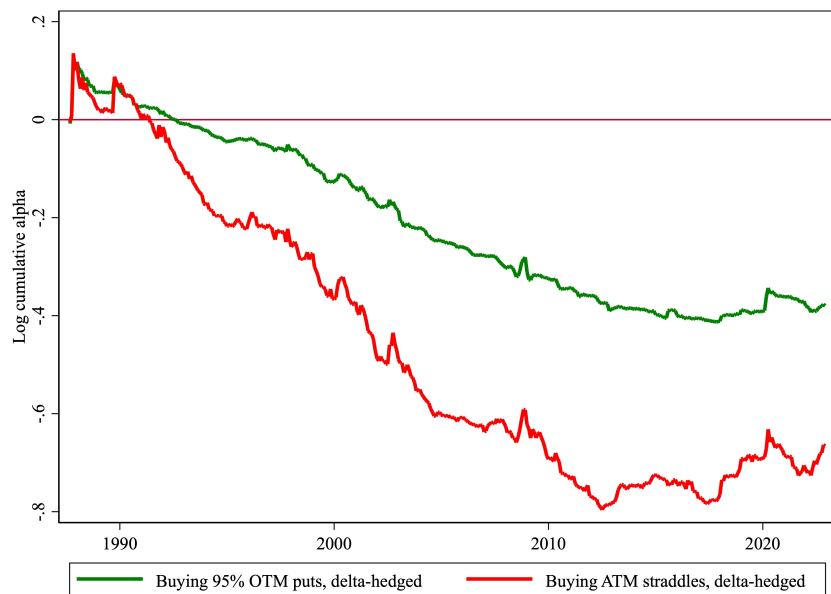
Note: The scatter plots are for traded versus synthetic option returns over the period 1987–2022. The returns are monthly, rolling on the third Friday.

Figure A.8: Robustness for information ratios



Note: The figure reports information ratios for synthetic options of different strikes, for alternative choices in the construction of the option returns. The first column uses standard Black-Scholes to compute delta (instead of Hull-White). The second column fixes implied volatility at 15% instead of estimating it using historical data. The last column computes standard errors via block bootstrap. The three rows correspond to different samples: the full 1926-2022 sample, the sample in which both traded and synthetic options are available (1987-2022) and the BCJ sample (1987-2005).

Figure A.9: Cumulative alphas for delta-hedged options



Note: This graph reports results analogous to those in figure 5, but giving cumulative alphas for delta-hedged option returns.